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Testing the Validity of Multiparticipant Distributed Simulation for Understanding and Modeling Road User Interaction

Amir Hossein Kalantari , Yi-Shin Lin , Ali Mohammadi, Natasha Merat , and Gustav Markkula 

Abstract—Understanding driver–pedestrian interactions at unsignalized locations has gained additional importance due to recent advancements in vehicle automation. Naturalistic observations can only provide correlational data of limited value for understanding and modeling the mechanisms underlying road user interaction. Therefore, controlled studies in virtual reality (VR) are an important complement, but conventional methods can only accommodate a single human participant. Recently, there has been some interest in studying interactions in VR, by means of distributed simulation, involving multiple human participants. However, there is a lack of validation of this method. Here, we provide a validation study, focusing on a distributed vehicle–pedestrian interaction setup, where pairs of one driver and one pedestrian interacted under various kinematic conditions in a connected virtual environment. To test the validity of the distributed simulation, we used a naturalistic dataset collected in the same U.K. city, at similar locations, and compared the observed behavior between the two settings. Our results indicate a good relative validity of the simulator study, where road users showed similar nonverbal communication behavior in both datasets. As an additional means of validation, we also leveraged a set of game theoretic models that were developed based on the simulator studies, and found that when applied to the naturalistic dataset, we obtained similar (although not identical) model selection results. The findings suggest that distributed simulation can also be useful for development of computational models of interaction. Overall, the findings suggest that distributed simulation can be a highly valuable tool for studying and modeling road user interactions.

Index Terms—Behavioral sciences, decision making, human–computer interaction, mathematical models, vehicle driving.

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I. INTRODUCTION

NEGOTIATING right-of-way at unsignalized locations and shared spaces has become a topic of interest over recent years. This happened partly due to the recent advances in vehicle automation in which highly automated vehicles (HAVs) are expected to take over most (if not all) of the human driver tasks [1]. This requires an in-depth understanding of human road user interaction and its challenges in the first place so that HAVs can be fully prepared for deployment on the roads. If not, they may face the challenge of having nontransparent driving behavior due to associated complexities with their architecture and different decision-making mechanisms compared to humans [2]; this nontransparent driving behavior can eventually lead to mistrust among human road users. The problem is exacerbated when interacting with vulnerable road users, especially pedestrians, as they are omnipresent [3] and unpredictable in their crossing behaviors [4], which targets safety and efficiency in future urban traffic scenarios. This situation has led researchers to investigate [5] communication strategies that exist between pedestrians and vehicles, examining them through both qualitative and quantitative approaches [5]. Among these approaches, quantitative models of traffic interaction have proven particularly valuable for predicting and simulating such strategies [6], [7].

To understand each other’s intentions, road users employ a wide range of communication strategies to convey information about their position, trajectory, and intention. A growing body of research suggests that this heavily relies on implicit communication [8], [9], which consists of time-based [e.g., time-to-arrival (TTA)] [10], [11]; and movement-based (e.g., vehicle speed) [12], [13] factors. Other situational factors such as crossing location type (e.g., presence of zebra) [14], [15]; and vehicle distance at interaction onset [16] have also been found to play an important mediating role [17]. For instance, pedestrians have been found to cross less often at an unmarked crossing compared to a zebra crossing for the same TTA [14]. In addition, age and gender have been found to be associated with interaction outcomes and pedestrian crossing behaviors [18], [19].

Previous research has relied on either controlled or naturalistic studies to investigate and model vehicle–pedestrian interactions. Naturalistic data constitute a greater proportion of past studies where they were used as validation tools for the mathematical models of road user behavior. Naturalistic studies are usually conducted either using instrumented vehicles [20] or traffic data

collection devices, including drones [21] and installed cameras on fixed objects such as light poles [22]. Using instrumented vehicles provides the advantage of studying road user behavior over a longer period of time and with high quality and the capability of tracking a large number of road user parameters [23]. However, the downside is that this method is expensive and drivers are usually aware that they are being watched [20] and this might make them alter their behavior [24]. On the other hand, video and sensor-based studies offer this opportunity to record road user behavior in an unnoticed manner, gaining more knowledge of road user behavior and its features while avoiding behavioral adaptation [24].

While naturalistic data are important, they primarily offer correlational information and do not establish causal relationships between different factors. To comprehend and model the causal mechanisms underlying behavior, controlled studies are more helpful. Controlled studies are divided into two categories: test track studies [25] and studies in virtual reality (VR) [12]. In both categories, one can study traffic scenarios in a way not possible in reality, not least with respect to safety. Among these types of studies, distributed simulation (also known as co/coupled simulation) is one of the few methodologies capable of precisely, repeatably, and controllably investigating interactive communications among road users [14], [26], [27]. In this type of study, two or more simulators (e.g., a driving and a pedestrian simulator) are connected over a network where two or more human participants can interact in a safe and controlled environment, and the experimenter(s) can manipulate the conditions of interest to study the impact of traffic conditions regarding interactive behaviors and outcomes. The technique also allows participants to be observed multiple times, offering a deeper understanding of interindividual differences [28]. These features are undoubtedly beneficial for developing human behavior models for vehicle automation such as those in the game theory category [28]. Game-theoretic models offer a well-established framework for understanding road user interactions by considering interdependencies, typically suggesting optimal decisions for each party [29].

However, distributed simulation is expensive and requires advanced hardware and software installations, plus a comprehensive experiment design. Hence, it is imperative to ensure that the results coming out of the simulators are in line with reality to justify all the costs and complexities to leverage such methodology. Nevertheless, while numerous validation studies exist for driving simulators [30], those focused on pedestrian simulators are rare [31], and to the best of our knowledge, no study has validated road user behavior within a connected virtual environment (i.e., distributed simulation). Thus, there is a gap in the literature regarding the validity of the data that comes out of the simulator with respect to pedestrian simulators and distributed simulation [32]. Validity in the context of simulators pertains to how faithfully they replicate real-world driving or walking behavior. Researchers have identified different types of validity [33], [34], with the most common ones being absolute validity and relative validity, which are often evaluated in studies [35]. Absolute validity is achieved when measured metrics in the simulator match those in real traffic, whereas relative

validity is established when the patterns and/or effects of the studied variables in the simulator resemble those observed in a naturalistic setting [30]. Overall, the practice of connecting simulators has the potential to enhance the validity of simulator data by incorporating an interaction channel that enables nonverbal communication, thereby promoting more realistic behavior [32].

The main objective of this study is to validate both behavioral observations and computational modeling coming from a distributed simulator study (DSS) [14], [28]. The validation is achieved by comparing the findings of both studies with the real traffic data presented in this article.

In the DSS, 32 driver–pedestrian pairs interacted with each other in different crossing scenarios. To maximize experimental control, the simulator study only had participants cross a single lane. In practice, this was achieved by making the crossing location staggered, letting the participants cross only a single lane, to a refuge in the middle of the simulated road. Many crossings in the real world are not of this nature, but to the best of our knowledge, there are no existing comparisons of pedestrian crossing behavior between staggered and normal (nonstaggered) two-lane zebra crossings. Thus, it remains uncertain whether road user behavior would have been the same if the pedestrians had the opportunity to cross both parts of the crossing, something that this study can help us resolve. Therefore, a secondary objective of this article is to compare naturalistic pedestrian crossing between these two types of crossing locations to provide additional insight about the generalizability of the findings from our DSS.

II. METHODS

This section describes all the methods used in the study, beginning with a description of the DSS, naturalistic data collection locations and data extraction algorithms, followed by data preparation and modeling details.

A. Simulator Study

In the DSS, described in full detail in [14], 32 pairs of one driver and one pedestrian (32 drivers; Age: $M = 31.53$, $R = 21\text{--}50$, $SD = 1.72$; paired with 32 pedestrians; Age: $M = 25.09$, $R = 19\text{--}34$, $SD = 0.87$) interacted with each other in a VR environment (see Fig. 1). The study included eight pairs for each possible gender combination: male–male, male–female, female–male, and female–female, assigned to the driver and pedestrian roles, respectively. The study was approved by the University of Leeds Ethics Committee (Reference No AREA 21-022). In the DSS, the VR environment was built in Unity and by connecting a motion-based driving simulator known as the University of Leeds Driving Simulator (UoLDS) to a CAVE-based pedestrian lab named Highly Immersive Kinematic Experimental Research (HIKER) lab. The entire scene in the HIKER responded to the pedestrians' head movements using the HIKER glasses to show a perspective-correct VR. Both participants were instructed to approach the study with the mindset of being late for an important meeting, highlighting the importance of avoiding unnecessary delays while prioritizing safety. Therefore, the concept of time pressure was



Fig. 1. Experimental scene in the DSS. Top left shows the driver's view of the pedestrian, with the pedestrian represented by pink spheres to the driver. Bottom left displays the driver behind the wheel of the UOLDS. On the right, the pedestrian is seen crossing the zebra in the CAVE-based pedestrian lab [14].

contextualized as an “instruction of being in a hurry” to avoid the potential of passiveness after a number of trials while ensuring that participants remain concerned about their safety when interacting with each other. In this setup, participants (both drivers and pedestrians) had the choice to decide whether they would wait for the other to pass first or proceed themselves. As for the procedure, the pedestrians were asked to stay behind a vision obstruction until an auditory cue prompted them to approach the curb, observe for oncoming traffic, and cross the road if they deemed it safe. The auditory cue was activated based on the temporal distance of the subject vehicle to the center of the crossing (2–7 s). Drivers were informed that they would navigate a two-way road with traffic flowing in both directions, engaging with pedestrians at various points. They were instructed to drive in their usual manner and adhere to the prescribed speed limit of 30 mi/h. Due to the straight nature of the virtual road, drivers and pedestrians encountered each other at perpendicular angles [14]. Each participant pair experienced 40 trials, which were designed as a combination of the approaching vehicle's TTA (2–7 s) and the presence of zebra. A number of interaction-related metrics, including interaction outcomes, pedestrians' crossing duration, and vehicles' delay as a result of yielding to pedestrians, were recorded and analyzed. Also, participants' demographics and personality traits were collected. The DSS also included trials where the pedestrian crossed at unmarked locations, i.e., jay-walking. This type of pedestrian crossing location is left outside of the scope in the article [14].

B. Naturalistic Data Collection

Two marked crossings in the city of Leeds, England, were surveyed to collect real-time traffic data. Following several roadside observations by two independent observers and consultations with Leeds City Council regarding each location's crash history, the crossings were chosen based on both safety concerns (the number of crashes in the past) and the prevalence of one-to-one interactions between vehicles and pedestrians criteria. This implies that certain locations with a higher incidence of crashes in the past five years were excluded, as they did not involve direct (one-to-one) interactions between individual road users. A staggered crossing on Belle Isle



Fig. 2. Bird's eye view of Queensway (top) and Belle Isle roads (bottom); note that the photos represent the coverage of one sensor per location. A second sensor complemented the road scenery to provide a more comprehensive picture of interactions.

Road ($53^{\circ}46'06.6''\text{N } 1^{\circ}31'48.1''\text{W}$) and a zebra crossing on Queensway Road ($53^{\circ}44'46.2''\text{N } 1^{\circ}36'13.1''\text{W}$) were chosen. Two Viscando camera sensors known as OTUS3D [36] were used to collect data over 14 days (7 days for each location). The sensors detect the types of road users (light vehicles, heavy vehicles, cyclists, and pedestrians) and track their trajectories and speeds over discrete time stamps. Camera sensors were installed on two light poles, one at Belle Isle Road and the other at the Queensway, with heights of 6.46 m and 8.3 m, respectively. Traffic data were recorded from the 9th of May to the 15th of May, 2022, at the first site. For the second site, road user data were recorded from the 17th of May to the 23rd of May, 2022. Data collection occurred from 00:00 to 20:00 each day, except for the first day, which spanned from 20:00 to 00:00 at each location. Fig. 2 shows the bird's-eye view of the two locations.

C. Naturalistic Data Extraction

The data were provided in terms of both trajectories and videos, following the necessary ethical and data protection requirements by the U.K. law. To detect and investigate road user interactions, a potential conflict zone was defined from 1 m before the curb on one side to 1 m after the curb to the other side of each crossing location (see Online Supplementary, Fig. S1). Potential interaction was defined as the time that both pedestrian and car entered the potential conflict zone within a specific temporal distance of ± 7 s and a car was approaching the pedestrian on the near lane. If there were multiple vehicles interacting with a pedestrian, only the nearest car that interacted with the pedestrian was considered. This was done by selecting

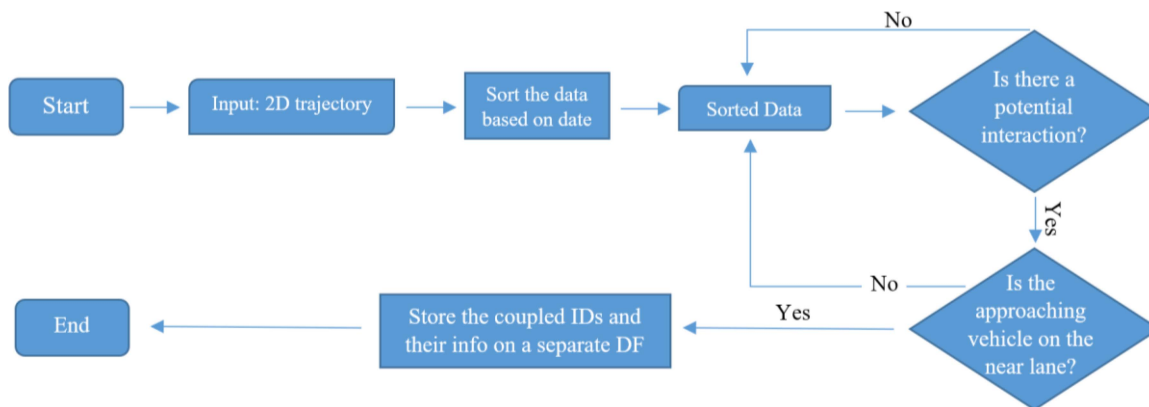


Fig. 3. Flowchart for extracting the potential interactions (ID = identification; DF = data frame).

the car that had a minimum time difference of entering the potential conflict zone after which the pedestrian entered the zone. An algorithm was developed to detect the potential interactions and extract and store the interaction-related metrics according to Fig. 3. A total of 813 potential interactions were detected using the algorithm. Afterward, to verify and extract the actual one-to-one interactions, the related video clips were watched according to the timestamps obtained by the algorithm. Several exclusion criteria were applied to the data. The exclusion criteria were the following.

- 1) There were some cases where interaction took place between a bus/truck and a pedestrian; in these instances, trucks were detected as cars (e.g., two cars) by the sensors, erroneously. A total of 26 events were removed.
- 2) In some cases where the vehicle passed first, the pedestrians did not change their direction toward the crossing nor did they turn their head towards the crossing before the vehicle reached the crossing, suggesting a lack of interaction. Consequently, 131 additional events were excluded.
- 3) Every potential interaction that had a group of three or more pedestrians interacting with a car was removed. As the time of entering and exiting the potential conflict zone for the couples was roughly the same, they were considered in the analysis. The sensors almost always considered these cases as a single pedestrian. Another 62 events were removed.
- 4) There were a few instances where there was no direct car–pedestrian interaction: although there was a car on the near lane approaching satisfying the potential interaction definition, the car was behind a bus/truck and the pedestrian interacted with the bus/truck, not the car. A total of 20 additional events were excluded from the analysis.
- 5) There were some instances where sensors detected pedestrian presence, but in the corresponding video clip, no pedestrian was present in the scene. Another 12 events were removed.

After applying the aforementioned criteria, 562 interactions were identified—243 for the normal zebra crossing and 319 for the staggered zebra crossing—and used for statistical analysis and modeling. Similar to the experiment layout, the interactions of drivers and pedestrians occurred perpendicularly.

D. Naturalistic Data Preparation

Table I displays the variables and parameters utilized in the study for analysis, including their type, description, symbol, and the source from which each variable is defined. The classification of variables as dependent or independent was based on the related literature and the DSS [14].

E. Inferential Testing

Similar to Kalantari et al. [14], a logistic regression and a linear regression model were used to predict interaction outcomes and vehicle delay, respectively. Also, to provide a more precise comparison between the two studies, regression analyses were conducted on pedestrian walking speed instead of crossing duration for both studies. This is due to the difference in the length of the staggered crossings in the two studies (2.5 m in this study versus 4.55 m in the DSS). Finally, only the trials in which the pedestrian crossed first were considered in the models for walking speed and vehicle delay similar to the DSS.

F. Computational Models

The five computational models introduced in [28] were tested and fitted to the naturalistic dataset to predict the interaction outcomes. These are original formulation (solved by) conventional game theory (OCGT), alternative formulation (solved by) conventional game theory (ACGT), original formulation (solved by) behavioral game theory (OBGT), alternative formulation (solved by) behavioral game theory (ABGT), and a Logit model. To achieve this objective, we considered both an original payoff formulation from the game theory literature and an alternative formulation proposed by us, based on road users' risk perception and efficiency in interactions. The payoff formulations were then solved by two algorithms from conventional game theory (i.e., mixed-strategy Nash equilibrium) and behavioral game theory (i.e., dual accumulation; [37]) literature resulting in OCGT, ACGT, OBGT, and ABGT models, respectively. Slight modifications were made to the Logit model and the payoff formulation of the ABGT/ACGT model. For the modeling details and formulation, refer Online Supplementary Material (see Section I-B).

TABLE I
PARAMETERS OF THE STUDY

Variable	Type	Description	Symbol	Unit	Source
Interaction onset time	Independent	Defined as 2 s before the pedestrian entered the potential conflict zone.	T_o	s	Trajectory data
Vehicle arrival time	Independent	The time at which the vehicle reached the edge of the crossing.	T_a	s	Trajectory data
Vehicle distance	Independent	The distance of the vehicle to the edge of the crossing at interaction onset time.	L	m	Trajectory data
Vehicle speed (onset)	Independent	The speed of the vehicle at interaction onset time.	v_{vo}	m/s	Trajectory data
Time gap	Independent	$TTA = L/v_{vo}$	t_v	s	Trajectory data
Pedestrian approach speed	Independent	The speed of the pedestrian when they entered the potential conflict zone.	v_p	m/s	Trajectory data
Age group	Independent	The age group of pedestrians: (1: <20, 2: 20–60, 3: >60).	Age	-	Video
Gender	Independent	The detected gender of the pedestrians.	Gender	-	Video
Location	Independent	The type of crossing (1 = normal zebra, 2 = staggered zebra).	Location	-	Video
Crossing duration	Independent	The time it took the pedestrians to cross their nearest driving lane (i.e., half of the normal, two-lane zebra crossing, and the first of the two single-lane crossing in the staggered zebra).	t_p	s	Trajectory data
Walking speed	Dependent	The average walking speed of pedestrians during the crossing behavior. This was obtained by dividing the width of each driving lane by crossing duration.	WS	m/s	Trajectory data
Interaction outcome	Dependent	The pedestrian was considered to have crossed first when they crossed before the car had reached the crossing, and then continued walking until reaching the other end of the crossing location, i.e., the pedestrian did not abort the crossing (1 = pedestrian crossed first, 0 = waited).	IO	-	Video
Vehicle delay	Dependent	The time the driver lost as a result of yielding to the pedestrian is defined as the difference between the time that it actually took the vehicle to reach the edge of the crossing (i.e., $T_a - T_o$) and the time it would have taken if the driver had kept their initial speed (i.e., t_p).	VD	s	Trajectory data

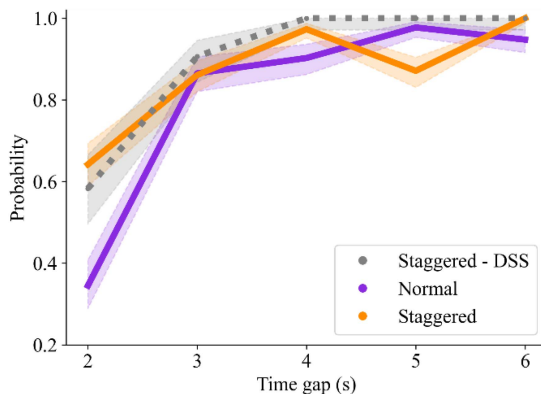


Fig. 4. Percentage of pedestrians crossing first among three crossing locations and two datasets with 95% confidence intervals.

All the models were fitted to the dataset using maximum likelihood estimation and Powell's method implemented in Scipy similar to [28].

III. RESULTS

A. Interaction Outcomes

Fig. 4 shows the probability of pedestrians crossing first over time gaps with 95% CI at both marked crossings of the current

study and the staggered crossing in the DSS. The crossing pattern is generally similar between the two staggered crossings, as indicated by the high Pearson correlation between the staggered DSS and naturalistic datasets (Pearson's $r = 0.938$, $p = 0.018$). This suggests that the time gap effect on pedestrian crossing behavior is consistent across both datasets, especially for the staggered crossings. Notably, the crossing pattern remains closely aligned across all time gaps, with a slight dip observed in the naturalistic data at 5 s, which shows a significant difference according to Fisher's Exact Test ($p = 0.000$). However, the effect size analysis further supports the overall similarity, with Cohen's h values being small across all time gaps (Cohen's $h < 0.35$), indicating small effects in both datasets. For the normal zebra crossing, the pattern closely aligns with what has been observed in the DSS (Pearson's $r = 0.900$, $p = 0.037$), although some variations remain. Overall, the positive effect of time gap on pedestrian crossing first is evident in all crossings, with the staggered crossings showing higher proportions of accepting the gap at higher time gaps.

Table II lists the results for logistic regression of interaction outcomes. The distance of the vehicle at the interaction onset had a significant positive relationship with the pedestrian's choice to pass first, meaning that at greater distances, pedestrians crossed more often. Vehicle speed, on the other hand, showed a negative association with interaction outcomes, indicating that at higher speeds, pedestrians were less likely to cross first. This finding

TABLE II
LOGISTIC REGRESSION RESULTS OF INTERACTION OUTCOMES (1 =
PEDESTRIAN CROSSED FIRST, 0 = WAITED)

	Estimate	Std. Error	Pr(> z)	95% CI					
				L	U				
(Intercept)	2.349	0.767	0.002	0.84	3.85				
Gender	-0.164	0.221	0.457	-0.59	0.26				
Age	-0.294	0.186	0.113	-0.65	0.07				
Pedestrian approach speed	-0.046	0.056	0.407	-0.15	0.06				
Distance	0.036	0.007	0.000	0.02	0.05				
Speed	-0.269	0.037	0.000	-0.34	-0.19				
Location	1.032	0.311	0.001	0.42	1.64				
AIC	538.29	BIC	568.61	logLik	-262.15	R-squ.	0.144	Observations	562

TABLE III
LINEAR REGRESSION RESULTS OF WALKING SPEED

	Estimate	Std. Error	Pr(> z)	95% CI					
				L	U				
(Intercept)	0.501	0.177	0.005	0.153	0.850				
Gender	0.005	0.042	0.924	-0.076	0.087				
Age	-0.114	0.036	0.001	-0.184	-0.044				
Pedestrian approach speed	0.828	0.065	0.000	0.700	0.957				
Distance	0.000	0.001	0.746	-0.003	0.002				
AIC	538.29	BIC	568.61	logLik	-262.15	R-squ.	0.144	Observations	562

aligns with previous naturalistic studies [13]. The combined results of both distance and speed of the vehicle at interaction onset correspond well with our previous simulator study (DSS) finding regarding TTA [14]. In addition, pedestrian behavior differed between the two locations, with a higher probability of crossing observed at the staggered crossing, as shown in Fig. 4. Finally, similar to the DSS, both age and gender did not show any association with interaction outcomes.

B. Walking Speed

As mentioned previously, we considered the average walking speed of pedestrians instead of their crossing duration, which was reported in the DSS paper [14] to provide a more direct comparison between the two studies. Table III lists the results of linear regression for the walking speed of pedestrians. From both the table and Fig. 5(top), it can be seen that those pedestrians who approached the crossing with a higher speed tended to keep their high speed during the crossing. They also walked faster at staggered compared to the normal zebra crossing. From the table and Fig. 5(bottom), an effect of age group can be seen where the older pedestrians walked significantly slower compared to younger pedestrians, which is in correspondence with the literature [38], [39]. Table IV lists the results of linear mixed-effects modeling of the pedestrians' average walking speed in the DSS. Similar to the naturalistic study, only the type of crossing was important, suggesting that pedestrians walked faster at unmarked crossings compared to marked crossings. Also, unlike the naturalistic study, the age of participants was not a predictor for their walking behavior in the simulator. From Fig. 5, it can be observed that the pedestrians in the simulator

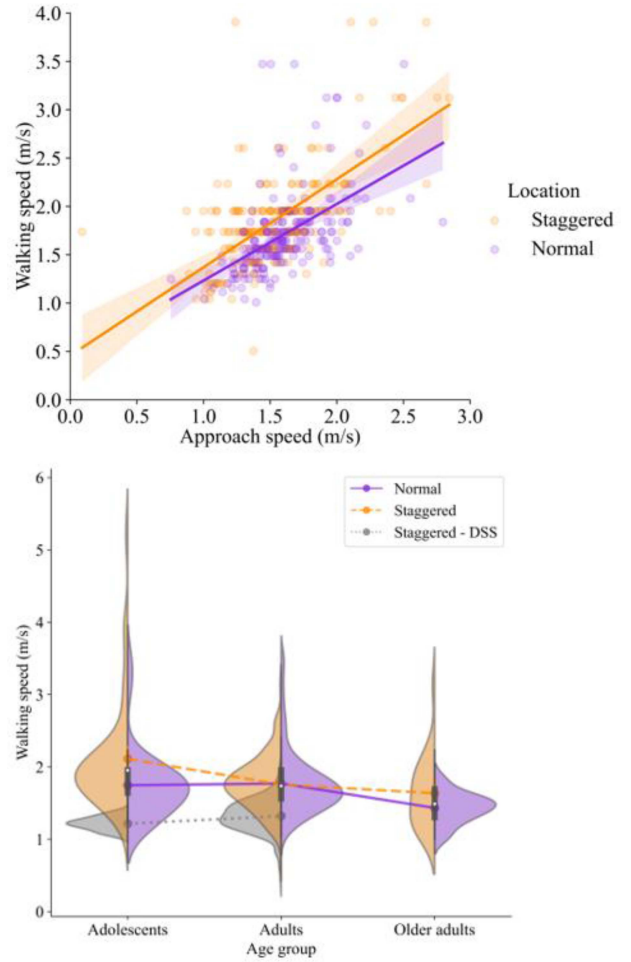


Fig. 5. Relationship between pedestrians' approach speed and walking speed (top) and violin plots of walking speed as a function of age group and crossing type (bottom); connected dots show the means for each category.

TABLE IV
LINEAR MIXED-EFFECTS MODELING FOR WALKING SPEED IN THE DSS

	Estimate	Std. Error	Pr(> z)	95% CI							
				L	U						
(Intercept)	1.372	0.208	0.000	0.964	1.780						
Gender	-0.113	0.072	0.119	-0.254	0.029						
Age	-0.005	0.007	0.493	-0.020	0.009						
TTA	-0.004	0.004	0.377	-0.011	0.004						
Location	0.132	0.012	0.000	0.109	0.155						
AIC	-712.9	BIC	-679.8	logLik	363.4	Marginal	0.1	df.resid	831	Observations	836

walked more slowly compared to those in the naturalistic study. This slower walking speed may be attributed to a lower perceived risk in the lab setting compared to the real world, suggesting that the reduced sense of risk outweighed the sense of urgency among participants when determining their walking speed. It should also be noted that the pedestrians in the DSS had a narrower age range (19–34) than those in the naturalistic study, and therefore, no older adult category can be seen for the DSS.

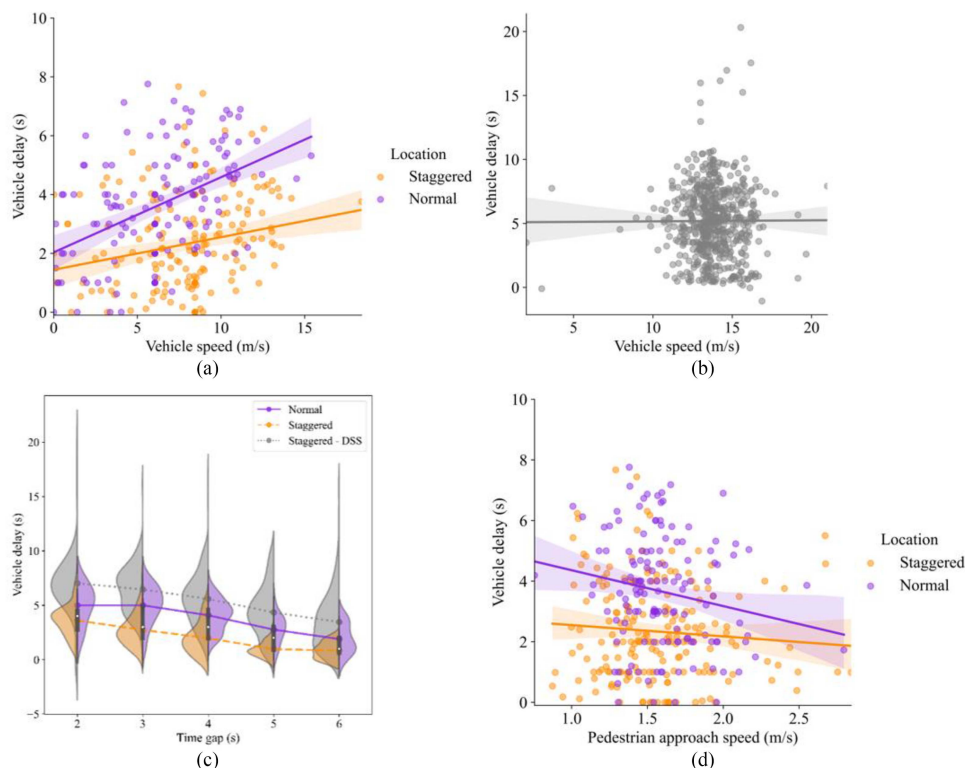


Fig. 6. Relationship between vehicle delay and speed at (a) interaction onset in the naturalistic study and (b) DSS, (c) violin plots of vehicle delay as a function of time gap and crossing type, and (d) relationship between vehicle delay and pedestrian approach speed as a function of crossing type.

TABLE V
LINEAR REGRESSION RESULTS OF VEHICLE DELAY

	Estimate	Std. Error	Pr(> z)	95% CI L	U
(Intercept)	4.8207	0.674	0.000	3.496	6.145
Gender	0.008	0.158	0.960	-0.303	0.319
Age	-0.092	0.136	0.496	-0.359	0.174
Pedestrian approach speed	-0.710	0.248	0.004	-1.197	-0.223
Distance	-0.008	0.005	0.107	-0.018	0.002
Speed	0.220	0.026	0.000	0.170	0.272
Location	-1.535	0.225	0.000	-1.977	-1.094
AIC	BIC	logLik	R-squ.	Observations	
1642	1671	-814.15	0.196	429	

C. Vehicle Delay

Table V lists the results of linear regression for vehicle delay. The table illustrates that when the vehicle speed was higher at interaction onset, the driver waited longer for the pedestrian to pass first. Fig. 6(a) shows this relationship and that different vehicles had different speeds at interaction onset (mostly in the range of 0–12 m/s). In the DSS, however, this was different. As shown in Fig. 6(b), most drivers had a speed of 12–15 m/s at interaction onset, and the relationship between vehicle delay and speed was almost nonexistent.

Moreover, drivers experienced a significantly longer delay at the normal zebra crossing compared to the staggered crossing, which can be confirmed by looking at Fig. 6(c). Vehicle delay was greater at shorter time gaps and at the normal zebra crossing.

The figure also shows the violin plots of vehicle delay for the DSS at the staggered zebra. Similar to the naturalistic data, a trend of decreasing vehicle delay by increasing time gap can be seen.

However, compared to the naturalistic setting, drivers experienced longer delays in the simulator ($M = 2.56$ s versus $M = 4.49$ s). Finally, the speed at which the pedestrians approached the crossings was negatively associated with the delays the drivers experienced [see Fig. 6(d)].

D. Computational Models

Fig. 7 shows the pedestrians' probability of crossing first over time gap at the (a) normal zebra, the (b) staggered zebra, and the (c) total data for all the computational models. Table VI lists information loss criteria (AIC, BIC) and error indices (MAE, RMSE) for all models and datasets (S for staggered, N for Normal zebra, and T for total data). Both from Fig. 7 and Table VI, it is evident that all the models except for ACGT performed close to each other and well. However, it can be seen that both behavioral game-theoretic models performed best for the normal zebra crossing with the OBGIT in lead. However, the situation for the staggered crossing is more complex: in terms of model parsimony, the Logit and OCGT models performed better, but when it comes to prediction accuracy, again behavioral game-theoretic models did a better job. Finally, for the total data, the ABGT model performed best, replicating our previous findings for the DSS.

TABLE VI
MODEL COMPARISON

Model	ABGT			ACGT			Logit			OBGT			OCGT			
	N	S	T	N	S	T	N	S	T	N	S	T	N	S	T	
MAE ¹	Case by case	0.130	0.227	0.188	0.197	0.296	0.279	0.137	0.247	0.219	0.115	0.231	0.151	0.179	0.238	0.212
	Average	0.036	0.028	0.019	0.132	0.103	0.125	0.041	0.062	0.048	0.019	0.040	0.025	0.077	0.042	0.040
RMSE ²	Case by case	0.252	0.339	0.305	0.320	0.379	0.356	0.250	0.335	0.314	0.252	0.337	0.300	0.270	0.335	0.310
	Average	0.056	0.039	0.025	0.235	0.163	0.178	0.053	0.086	0.082	0.023	0.044	0.040	0.104	0.048	0.049
AIC ³	116.178	267.544	366.74	159.7	289.088	459.13	119.822	242.92	376.934	111.622	265.838	371.3	129.576	247.34	375.442	
BIC ⁴	126.657	278.839	379.963	170.179	300.383	472.124	133.794	257.980	394.260	118.608	273.368	379.734	136.562	254.870	384.105	
NLL ⁵	55.089	130.772	180.370	76.850	141.544	226.565	55.911	117.460	184.467	53.811	130.919	183.65	62.788	121.670	185.721	
NO params ⁶	3			3			4			2			2			

¹ Mean absolute error ² Root mean squared error ³ Akaike information criterion ⁴ Bayesian information criterion ⁵ Negative log-likelihood ⁶ Number of free parameters
The bolded cell that the corresponding value is the highest (or lowest, depending on the metric) among all models' evaluation results.

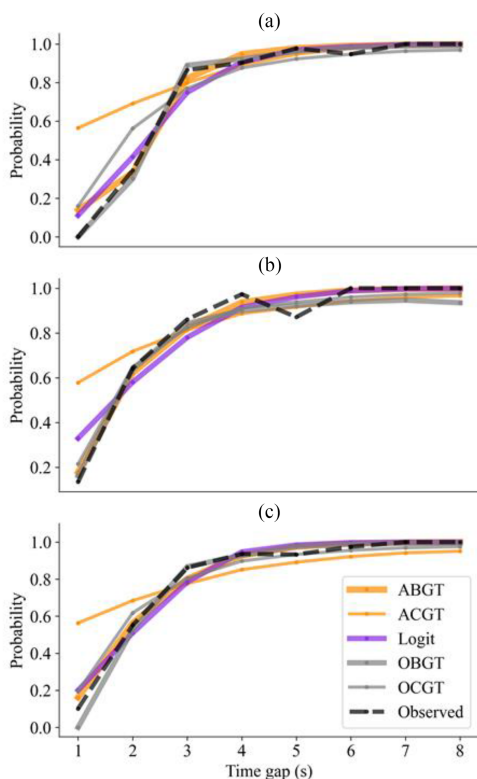


Fig. 7. Pedestrians' probability of passing first over time gap for all models and for (a) normal zebra, (b) staggered zebra, and (c) total data.

Overall, fewer differences in performance can be observed among the models using the naturalistic data compared to the DSS dataset [28].

IV. DISCUSSION

In this study, we compared our analyses and modeling findings from the DSS to naturalistic data. Overall, the findings of this article suggest a good match between the two methodologies, especially regarding behavioral observations. We found a well-established relative validity of what was observed in the simulator study, which is a promising achievement. This is because simulating pedestrian and driver behavior simultaneously in a VR

environment has the potential to radically change how we study and understand traffic safety, including with the development of HAVs. However, it is worth noting that driving in a simulator, for example, will never be exactly the same as driving in reality [40], both from the exact sensory stimuli perspective [35] and the sense of awareness of driving in a virtual environment.

We found that road infrastructure plays an important role in driver-pedestrian interaction metrics. This is because crossing location type not only conveys the regulatory messages (e.g., the right of way) but also acts as a mediating factor influencing road users' kinematics, which is the pivotal point of interactions. The pedestrians crossed first more often at staggered zebra versus normal zebra and normal zebra versus unmarked crossing (in the DSS [14]), suggesting a difference in risk perception even between the two types of marked crossings. We did not find a study regarding a comparison of unsignalized crossings including staggered crossings, but very few papers studied pedestrians' preferences and behavior at different types of signalized crossings; among them, Anciaes and Jones [41] found that pedestrians rated staggered crossings higher in terms of safety compared to straight crossings, primarily because they perceived the former as more convenient and offering greater safety. The preference could be strengthened in the case of unsignalized staggered crossings as road users usually have a higher risk perception at these locations.

In addition, there was a good match between the data of the two studies for staggered zebra crossing: a similar pattern was observed in two studies except for a time gap of 5 s in which a dip in the naturalistic dataset was seen (see Fig. 4). We tend to believe that this is due to the limitations associated with this type of data including some unknown confounding variables or the errors associated with the sensors regarding the road users' trajectories rather than a fundamental behavioral phenomenon.

Overall, differences in road user behavior at marked crossings are influenced by cultural, geographic, and legal factors. For instance, in the U.K., drivers are legally obligated to yield to pedestrians waiting to cross as well as those already on a zebra crossing, as outlined in Rule H2 of The Official Highway Code (2023) [42]. This practice is similarly observed in many western European countries. Despite these legal requirements and perceived safety advantages, challenges persist in ensuring

adherence to pedestrian right of way. Instances have been documented where drivers fail to yield to pedestrians even when aware of their legal obligation to do so [43], [44]. Factors contributing to this noncompliance include time constraints, urgency to reach destinations, and instances where drivers fail to perceive pedestrians in a timely manner, leading to aborted crossings and pedestrian retreat [43]. This suggests that even when pedestrians have the right of way, they should remain cautious while crossing the road and continuously assess the situation, particularly observing whether vehicles show signs of yielding. This behavior was especially evident in both our naturalistic and simulator studies, where drivers were less likely to yield to pedestrians at lower time gaps, making pedestrians more hesitant to cross. Therefore, efforts to promote pedestrian safety and ensure adherence to right-of-way regulations should account for these multifaceted influences.

In addition, some other factors such as personality traits, driving or walking style preferences, and different states of mind (e.g., stress or tiredness) can influence road users' behavior in real-life interactions at crossings. From a methodological standpoint, incorporating these factors into a controlled study is not straightforward. Consequently, it is essential to acknowledge the inherent challenges of replicating real-world dynamics in a controlled environment. Controlled studies typically aim to hold most of these factors constant, occasionally varying one at a time to investigate its specific impact on behavior. These caveats underscore the complex interplay between legal mandates, cultural norms, and individual behaviors in shaping road user interactions at marked crossings, as well as the challenges associated with studying these dynamics using different methodologies.

The crossing location also played a role in the pedestrians' walking speed and the vehicles' delay. A novel finding here was that there is a correlation between pedestrian approach and crossing speed and that this correlation is stronger for the normal zebra crossing ($r(243) = 0.633$, $p = 0.000$, [0.529, 0.714]) compared to the staggered crossing ($r(319) = 0.557$, $p = 0.000$, [0.439, 0.653]). This is an aspect that could not be investigated in the DSS due to the physical limitations of the apparatus (HIKER lab) [14]. Previous research has shown pedestrian approach phase [45] plays an inevitable role in interaction outcomes [5] and a higher pedestrian velocity when approaching the crossing suggests a higher likelihood of pedestrians passing first [46]. In addition, the pedestrian approach speed has been found to strongly correlate with their aggressiveness [46] suggesting more assertive pedestrians are more likely to pass first, regardless of the other agent's status. This is an important finding for the virtual testing of HAVs as they need to continuously track and react to pedestrians' behavior as soon as they are detectable and do not wait, for instance, until they reach the curb which might be too late to react in some cases.

Vehicle delay was longer for the normal crossing compared to the staggered crossing. This is in line with the primary applications of staggered crossings where they are being used to minimize clearance time in wider roads such as dual carriageways. Also, the speed of the pedestrians approaching the crossing could predict the vehicle delay. This novel finding can be explained by looking at Figs. 5 and 6(d). By comparing the

results of this study and the DSS, it becomes evident that, first, drivers waited less for pedestrians in real traffic compared to the DSS [see Fig. 6(c)]. This could be because in the DSS, both participants were told that they should assume they are in a rush, and this could make interactions more competitive where both agents were more assertive to pass first resulting in longer delays for the vehicle [14]. Another reason could be the simulated driving, which might have resulted in a different deceleration or acceleration behavior by the drivers compared to a real vehicle. Second, unlike this naturalistic study, vehicle speed had a dense distribution at interaction onset and was not a predictor of vehicle delay in the DSS. This finding is not surprising as the definition for interaction onset was different between the two studies: in this naturalistic study, interaction onset was defined about 3 s before the pedestrian reached the curb (accounting for the pedestrian approach phase), whereas in the DSS, road users could only see each other after the pedestrian had reached the curb which was set in this way to control for TTAs [14]. Hence, less speed variation was observed in the DSS.

At least one of the vehicle kinematics, including TTA, distance, and speed at interaction onset, was a significant predictor of interaction outcomes and vehicle delay. This is in correspondence with both the DSS [14] and previous research [13], [16]. Higher vehicle speeds were associated with longer delays for the drivers probably because they had to brake more harshly and stop completely before having the chance to accelerate again and pass the crossings in these instances. Moreover, lower time gaps imposed longer delays for the drivers in both studies. Finally, age was a predictor of walking speed in this study, confirming previous research that as age increases, pedestrians tend to walk more slowly [39], [47], [48]. However, age did not show an association with either the average walking speed or crossing duration of the pedestrians in the DSS. The most obvious reason was the limited age range (19–34 years) in that study, which was not representative of the whole population [14]. Also, no effect of gender on walking speed was found in either study.

The results for the five previously developed computational models suggest the following: first, all models showed a better fit to the naturalistic data compared to the simulator data, and the differences in the models' performance were less noticeable for the former. However, this does not imply that the simulator data were less useful for the models in any way. One reason for the smaller differences in performance among the models when fitted to the naturalistic data is that, in this study, the models were fitted to the average population, whereas in the previous study, each model was fitted to individual data and then the average of all participant pairs was reported for each model. This finding suggests that the DSS enabled us to account for interindividual differences among participants and to test the models at their highest potential. Hence, the availability of multiple data points per individual in the DSS has an extra advantage for investigating road user behavior and it could support the notion that the models' predictions derived from the DSS dataset possess greater reliability.

Second, the behavioral game-theoretic models performed better than the others in terms of prediction accuracy. This is in line

with our previous findings in which behavioral game-theoretic models came out on top also in the DSS-based model selection. In addition, improved performance was observed moving from mixed-strategy Nash equilibrium to dual accumulation solving algorithms. Specifically, the strong performance of the ABGT model and the weak performance of the ACGT model, both arising from the same payoff formulation, align with our previous findings [28]. This is because the mixed-strategy Nash equilibrium and the designated (alternative) payoff formulation were less compatible compared to the original payoff formulation.

This incompatibility arises from the mixed-strategy Nash equilibrium treating each player's strategy independently, without inherently modeling simultaneous or joint decision-making (beyond influencing equilibrium probabilities). In contrast, the alternative payoff formulation incorporated strategies for both road user types, which exhibit greater adaptability in responding to the strategies chosen by others. These are better addressed by dynamic solutions, such as those found in behavioral game theory. Overall, the results of the computational modeling showed the viability of the DSS as a valuable alternative for developing and testing models of road user interaction, including those involving HAVs. Moreover, it was shown that behavioral game-theory models can be utilized to predict and simulate vehicle-pedestrian interactions, offering a deeper comprehension of road user behavior within the framework of vehicle automation [28]. Accuracy and interpretability are both vital in HAVs' decision-making systems to minimize erroneous predictions. Since machine-learned models serve as the primary method for decision-making and motion planning algorithms in the automotive sector, their dependence on extensive datasets presents challenges such as biases and accidental correlations. Tackling these challenges is intricate due to the opacity and nonlinear characteristics of machine learning algorithms. Conversely, as evidenced in this research, glass box models such as behavioral game theory, can perform effectively with both controlled and naturalistic datasets while furnishing researchers with thorough insights into road user behavior.

Overall, in line with our secondary objective of this study, we found that the findings from the staggered crossing in the DSS can be generalized to nonstaggered (normal straight) crossings in terms of positive time gap-interaction outcomes and negative vehicle delay-time gap relationships. In addition, we found similar model selection results between the staggered crossing in the DSS and the normal crossing in the naturalistic study, with behavioral game theory and logit models outperforming conventional game-theoretic models.

This study is not without limitations. For modeling purposes, only one-on-one interactions were considered and used in the analyses. Hence, the effect of variables such as pedestrian group size [18] on interaction-related metrics was not investigated. Also, road user behavior was investigated for one driving lane and direction, whereas previous research suggests pedestrians behave differently on two-way streets [49], [50]. The road infrastructure, including the crossing locations between the two studies, was not exactly the same, hindering more specific comparisons. For example, we could not compare the two datasets in terms of pedestrians' waiting and crossing initiation time. Also, the instructions of being in a hurry to the DSS participants could

set the simulator study slightly apart from the real-world setting. However, as previously mentioned, this was done to prevent passiveness during the experiment and it was crucial for preserving the naturalness of participants' behaviors [14]. Moreover, representing pedestrians as pink spheres instead of calibrated avatars may have limited the scope of investigations into interactions, such as exploring explicit communication strategies. However, since the primary focus of our study was to examine implicit rather than explicit communication between drivers and pedestrians, we believe this limitation likely had minimal impact on the observed behaviors. Finally, demographic information was extracted by a human observer and, therefore, errors and mistakes are probable.

V. CONCLUSION

Overall, the findings highlight the compatibility of the DSS with naturalistic studies for studying road user behavior, as we observed similar patterns of nonverbal communication between drivers and pedestrians in both datasets. This represents an important step toward simulating critical traffic scenarios, such as near-misses and crashes, which occur infrequently in real-world traffic but are essential for the virtual testing and development of HAVs. However, on a broader level, employing both methodologies (controlled and naturalistic studies) is crucial for a comprehensive understanding of road user behavior as the limitations inherent in each approach can be addressed and complemented by the other.

While there are some potential limitations with conducting studies in a virtual environment, including the fact that participants may not exactly experience traffic scenarios in the simulators as they do in the real world, it is possible to attribute the observed differences more to the study design rather than inherent limitations of the simulator itself. This is promising, as it suggests that optimizing the study design can lead to better results in alignment with one's objectives. Finally, the current study suggests that distributed simulation is an appropriate methodology for studying and modeling road user behavior and can be used as an alternative method to naturalistic studies for some applications.

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