

Department of Precision and Microsystems Engineering

Noise robustness improvement of reset control systems using measurement line filtering

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Report no : 2023.024
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Specialisation : Mechatronic System Design
Type of report : Master Thesis
Date : 25 April 2023

Noise robustness improvement of reset control systems using measurement line filtering

April 14, 2023

by

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in partial fulfillment of the requirements for the degree of

Master of Science

in Mechanical Engineering

at the Delft University of Technology,

to be defended publicly on Tuesday April 25th, 2023 at 09:30 AM.

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An electronic version of this thesis is available at <http://repository.tudelft.nl/>.

Abstract

Inherent limitations of linear control, including PID, are Bode's phase-gain relationship and the waterbed-effect. Literature showed that reset control can overcome these limitations. The 'Constant in Gain, Lead in Phase' (CgLp) reset element was developed, which can be used in combination with a conventional PID approach. The benefit is that CgLp increases the broadband phase without altering the gain.

Recently, a continuous reset (CR) CgLp was proposed to eliminate the discontinuous output of the CgLp, which for example, excites resonances in mechanical structures. Unfortunately, CR CgLp is more sensitive to measurement noise than CgLp due to CR CgLp taking the derivative of its input, amplifying noise. This causes resets to occur more frequently. Literature has shown that these excessive resets can deteriorate the system performance.

The main contribution of this work is to improve the noise robustness of CR CgLp by implementing a filter on the measurement line to attenuate measurement noise. The attenuation of noise causes a reduction of excessive resets. Two filters are designed and critically compared. First, a low-pass filter succeeded by a lead element (LL). Second, an accelerometer-aided Kalman filter (aaKF). These were validated in practice on a precision-positioning stage.

Simulations have shown that the proposed designs improve performance of CR CgLp. The overshoot of CR CgLp could be reduced by 32% and IAE could be reduced by 53%. Besides this, CR CgLp with aaKF showed to outperform a PID controller with similar bandwidth and phase margin. The overshoot could be reduced by 46% and the IAE by 34%.

Test results on a physical setup have shown that the LL filter could improve the IAE of a step response by 13% compared to a CR CgLp without filter. Compared to PID control, the CR CgLp with LL filter has a superior transient response.

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Nomenclature

Abbreviations

Abbreviation	Definition
aaKF	Accelerometer-aided Kalman filter
ADC	analogue-to-digital converter
BLS	Base-linear system
CgLp	Constant in Gain, Lead in Phase
CI	Clegg Integrator
CPSD	Cumulative power spectral density
CR	Continuous Reset
DF	Describing function
DoF	Degree of freedom
FFT	Fast Fourier transform
FIR	Finite impulse response
FORE	First order reset element
FRF	Frequency response function
GM	Gain margin
IAE	Integral absolute error
IIR	Infinite Impulse Response
ISR	Integral square error
KF	Kalman filter
LL	Low-pass filter + lead filter
LO	Luenberger observer
LPF	Low-pass filter
LTI	Linear time-invariant
MM	Modulus margin
MPZ	Matched Pole-Zero
PM	Phase margin
PSD	Power spectral density
RMS	Root-mean-square
SF	Shaping filter
ZOH	Zero-Order Hold

Preface

First of all, I would like to thank my supervisors Ruben Buitenhuis, Nima Karbasizadeh and Hassan HosseinNia. They have challenged, motivated and helped me in many different ways.

Ruben helped me with the both the practical and the theoretical side. At DEMCON he supported me during the component selection and system integration of the experimental setup. Besides this, he shared a lot of knowledge on reset control and control theory in general.

I would like to thank Nima and Hassan for allowing me to do my graduation project at DEMCON and to give me the freedom to include practical internship into my graduation project. During this internship I could acquire a lot of practical knowledge on the shift from simulation environment like Simulink to a practical setup and system integration.

Besides this, I would like to thank Nima for sharing his knowledge and experience on reset control. You were able to explain some of the more abstract and complex parts of reset control. Hassan, thank you for your feedback and ideas on how to tackle some of the problems I encountered.

Finally, I would like to thank my friends and family. They have supported me to stay motivated, showed interest in my work and provided me with distraction. A great way of gaining insight in the theory was to try explaining reset control to people who are not educated in control or mechanical engineering at all. This is quite a challenge. Thanks for your attention and patience. Without you I would not have been able to do what I did.

*Dennis van de Ketterij
Delft, April 14, 2023*

1

Introduction

PID controllers form the backbone of the precision industry. They cover approximately 90% of industrial controllers [1]. An ever-increasing demand for higher precision and speed pushes the requirements of controllers further and further. PID controllers, and linear time-invariant (LTI) controllers in general, are subjected to inherent limitations including the waterbed-effect and Bode's gain-phase relationship [2, 3]. These limitations put restrictions on the maximal achievable performance of linear controllers. To keep meeting those increasing requirements in the future, industry needs ways to overcome these limitations. Fortunately, nonlinear control can be utilized to overcome these restrictions [4, 5].

One of those nonlinear control strategies is reset control. A reset controller is an element that resets its state or states, introducing nonlinearity. The first reset element was introduced in the fifties [6]. This element, called the Clegg Integrator (CI), has its frequency response analogous to an integrator, but with 52° less phase lag, thus breaking Bode's gain-phase relationship. Hence, reset control could overcome the aforementioned limitations inherent to linear control [38].

However, it took reset control a couple of decades to gain momentum and to become a serious alternative for PID control. In 2019, [5] proposed the 'Constant in Gain, Lead in Phase' (CgLp) reset element. A key benefit of this element is that it can be used in combination with a conventional PID approach. CgLp increases the broadband phase without altering the gain. The controller gains can be tuned using loop-shaping, a technique that is well-known within the precision industry. The downside of this element is that the resetting states cause discontinuities in the output. This could lead to excitation of resonances in mechanical structures and amplifier saturation. To eliminate these issues, [4] proposed a continuous reset (CR) structure as a modification of CgLp, resulting in CR CgLp.

The CR structure consists of a CgLp preceded by a lead element preceding and succeeded by a lag element. Unfortunately, CR CgLp is more sensitive to measurement noise than CgLp. This is due to the lead element. The lead element contains a differentiator which amplifies measurement noise. This increased sensitivity to measurement noise causes resets to occur more frequently. Literature has shown that these excessive resets deteriorate both transient and steady-state performance [11, 12].

The main contribution of this work is to improve the noise robustness of CR CgLp by implementing a filter to attenuate measurement noise. This decreases the noise on the reset line leading to a reduction of excessive resets. To this end, two filter designs are proposed. The first design is a low-pass filter (LL) succeeded by a lead element. The second design is a more advanced accelerometer-aided Kalman filter (aaKF). Firstly, simulations are used to study the feasibility of the designs. Secondly, the simulation results of the proposed designs are validated in practice on a 1 degree of freedom (DoF) precision-positioning stage.

In the remainder of this thesis, chapter 2 provides a report of the literature study. This report contains the theoretical framework and a feasibility study of the proposal. Chapter 3 contains the problem definition based on the literature study and it presents the proposed control structure. Details on the design of the LL filter is given in chapter 4. Chapter 5 presents the design of the aaKF. In chapter 6 the

two proposed designs are compared to each-other. This is followed by an analysis of the test results in chapter 7. The conclusions and recommendations are given in chapter 8. This chapter also contains a discussion of the differences between the simulation and test results. The appendices contain a characterization of additional preliminaries, a study on the cause and impact of excessive resets, a study on the impact of accelerometer noise, details on the experimental setup, the used controllers and the Simulink model.

2

Literature Review

This chapter contains a literature review on reset control systems and the impact of noise on the performance of the CR CgLp. Section 2.2 provides the definitions that will be used, an overview of reset elements and theory on low-pass and Kalman filters. Next, section 2.3 addresses the motivation for the use of a CR CgLp element on a physical system. It also discusses the main drawback of this reset element. The problem statement is given in section 2.4. Section 2.5 introduces techniques to use when implementing a reset controller on a physical system with a discrete time controller. Next, the proposal to counter the performance deterioration due to noise is presented in section 2.6. Section 2.7 contains the validation of the proposals using a Simulink model. To conclude, section 2.8 presents the conclusion and discusses the next steps.

Literature Review and Project Proposal on Noise Attenuation when Implementing a CR CgLp on a Physical Setup

J.D. van de Ketterij¹

Abstract—Reset control has shown that it enables controller designs that overcome the Bode gain-phase relationship. This relationship limits the performance of linear controllers, including PID. When implementing reset control in a PID framework a substantial phase advantage can be obtained. This could lead to an improvement in both overshoot and settling time. In contrast to other reset elements the Continuous Reset Constant Gain and Lead in Phase reset element (CR CgLp) has a continuous output. Because of this continuous output, issues such as saturation of actuators and introduction of vibrations are prevented. To that end this reset element is used in this paper.

The CR CgLp is more sensitive to noise than other reset elements. This because the reset instants depend on both the error signal and its derivative. The differentiation of the error signal leads to a amplification of noise, which can lead to an increase in the number of resets. This can lead to performance deterioration due to excessive resets.

This paper proposes two solutions to overcome this problem. By implementing either a low-pass filter (LPF) or an accelerometer aided Kalman filter (KF), the impact of noise can be reduced. These proposals are validated using simulations. Compared to a CR CgLp without a filter, the CR CgLp+KF can reduce overshoot by a factor 17 and the settling time by a factor 2.9. When using a CR CgLp+LPF the overshoot can be reduced by a factor of 4.5 and settling time can be reduced by a factor of 2.5.

Index Terms—Mechatronics, reset control, continuous reset, CR CgLp, noise attenuation, Kalman filter, accelerometer, Simulink

I. INTRODUCTION

Nowadays PID control forms the backbone of control applications in industry, approximately 90% of the controllers are PIDs [1], [2]. This is mainly due to its ease of implementation by means of loop-shaping. The PID controller has some fundamental drawbacks because it is a linear controller. From (1) follows that the sensitivity function $S(j\omega)$ has to distribute the disturbance rejection over different frequencies. Therefore, if the attenuation is increased for one frequency, the attenuation decreases for other frequencies. This is also known as the waterbed-effect [3], [4], [5], [6]. In practice this means that an increase of low-frequency disturbance rejection results in a decrease of high-frequency disturbance rejection, and vice-versa. In Equation 1 p_k is the number of right half-plane poles.

$$\int_0^\infty \log|S(j\omega)|d\omega = \pi \sum p_k \quad (1)$$

The introduction of reset control (RC) breaks this relationship [7]. The first reset element, called the Clegg Integrator

(CI), was introduced in 1958 by J.C. Clegg. He showed that by resetting an integrator when the error signal is equal to zero an improvement in phase margin of 51.9° could be realized [7]. His work was continued by Horowitz and Rosenbaum who introduced the First Order Reset Element (FORE). This element allows for the tuning of the pole of the integrator [8]. The Generalized FORE (GFORE), introduced in [9], is a FORE with the addition of a non-zero reset matrix. This gives the engineer an extra tuning parameter. Continuing this work, the ‘Constant in Gain, Lead in Phase’ (CgLp) element was designed. The CgLp is a GFORE element with subsequently a lead filter. This reset element is designed to work within a PID framework. Therefore, it offers compatibility with the industry [10]. A problem with these reset elements is that they have discontinuities in their control signal. In [11] a lead filter is added before the CgLp. A lag element is added after the CgLp. This results in a smooth control signal. Therefore, it is called a Continuous Reset (CR) CgLp.

This paper focusses on the implementation of CR CgLp in an industrial real-time system. Up until now the controllers were predominantly tested in simulation software and the lab environment. This study will focus on the implementation on an industrial machine. Solving problems imposed by the noise present in such a physical system will be the aim of this study. Using a KF, fused position and accelerometer data will be used to attenuate the measurement noise. The presence of noise causes reset controllers to reset excessively. This then could cause performance deterioration. Both a LPF and KF are used to attenuate the noise. This should then limit the amount to resets to the point the system performance is improved compared to linear control.

The remainder of this paper is structured as follows: In Section II, along with the theoretical framework, some definitions are given. Section III the choice for the CR CgLp will be motivated. This section also addresses the issues with this reset element. Next, Section IV presents the problem statement. Then some previously encountered issues with implementation and some solutions are discussed in Section V. Section VI introduces the proposal to reduce excessive resets. Section VII presents the model, the controllers used for the validation of the proposal and discusses the simulation results. Lastly, the conclusion and some next steps are presented in Section VIII.

This literature review is written before any experiments took place. This review serves the purpose of constructing a theoretical framework for the proposed solution. The validation of the proposed solution is part of the next phase. Details of the physical system can be found in Section B.

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II. DEFINITIONS AND THEORY

A. Definition of reset controller

The general definition of the reset controller is derived by Zaccarian and shown in Equation 2. In this equation A_r, B_r, C_r and D_r are state matrices of the base linear system (BLS). Reset matrix A_ρ resets the appropriate states. For most reset elements covered in this literature review the resets depend only on the error signal, $x_l(t) = e(t)$. In (2) $e(t)$ is the error fed into the controller, $u(t)$ is the output of the controller [12].

$$R := \begin{cases} \dot{x}(t) = A_r x(t) + B_r e(t) & \forall x_l(t) \neq 0 \\ x(t^+) = A_\rho x(t) & \forall x_l(t) = 0 \\ u(t) = C_r x(t) + D_r e(t) \end{cases} \quad (2)$$

B. Reset elements

Several reset elements have been developed. They can all be written in the form of (2). The resetting behaviour of a reset element can be tuned using either γ for first order reset element or γ_1 and γ_2 for second order reset element. If γ is set to zero, the reset element is called fully resetting. When set to one the system does not reset at all. In the case of a second order reset element the system is called partially resetting if either γ_1 or γ_2 is not equal to zero. Elements which contain γ defined as $\gamma \in [-1, 1]$ are denoted as *Generalized* [10], [11].

1) *Clegg Integrator*: The first reset element, a Clegg Integrator (CI), was introduced in [7]. Resetting the integrator to zero when the input is crossing zero results in a reduction of phase lag from 90° to 38.1° . When used in control systems a reduction of overshoot due to the increase in phase margin was observed. The matrices describing this element are defined in (3).

$$A_r = 0, B_r = 1, C_r = 1, D_r = 0, A_\rho = \gamma \quad (3)$$

2) *FORE*: In [8] Clegg's work was continued. Compared to the CI, the FORE adds the freedom to tune the filter frequency. The matrices of the GFORE are defined in (4).

$$A_r = -\omega_r, B_r = \omega_r, C_r = 1, D_r = 0, A_\rho = \gamma \quad (4)$$

3) *SORE*: To provide more flexibility of design compared to FORE, [13] introduced the Second Order Reset Element (SORE). The element resets both states when the error $e(t)$ is zero. As can be seen in (5) this elements provides the engineer with more parameters to tune. In this equation ω_r is the corners frequency and β_r is the damping coefficient. This offers more flexibility, but it also introduces more complexity.

$$A_r = \begin{bmatrix} 0 & 1 \\ -\omega_r^2 & 2\beta_r\omega_r \end{bmatrix}, B_r = \begin{bmatrix} 0 \\ \omega_r^2 \end{bmatrix}, \quad (5) \\ C_r = [1 \quad 0], D_r = [0], A_\rho = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}$$

4) *CgLp*: Based on the work of [14] and [15] on phase lag reduction by reset elements, [10] introduced the CgLp. The advantage of this element is that it has been shown to work within a PID framework. The CgLp can be used with either a GFORE or a GSORE element. The GSORE offers more flexibility of design, however it also introduces more complexity into the analyses and tuning. The tuning is usually done by engineers and not by algorithms. Having more tuning parameters could make optimal tuning more complicated [10].

Therefore, the GSORE will not be used in this study. The matrices of the CgLp using a GFORE are given in (6).

$$A_r = \begin{bmatrix} -\omega_{r\alpha} & 0 \\ \omega_f & \omega_f \end{bmatrix}, B_r = \begin{bmatrix} \omega_{r\alpha} \\ 0 \end{bmatrix}, \quad (6) \\ C_r = \begin{bmatrix} \omega_f & \left(1 - \frac{\omega_f}{\omega_r}\right) \end{bmatrix}, D_r = [0], A_\rho = \begin{bmatrix} \gamma & 0 \\ 0 & 1 \end{bmatrix}$$

5) *CR CgLp*: As a further development of the CgLp, the CR CgLp was introduced in [11]. This is a CgLp with a lead element $L(s)$ in front and a lag element $R(s)$ after it. The matrices of a CgLp are defined in (6). Figure 1 shows the implementation of this element within a PID framework.

In (7) the elements surrounding the reset element are shown.

$$L(s) = \frac{s/\omega_l + 1}{s/\omega_h + 1}, R(s) = \frac{1}{s/\omega_l + 1} \quad (7)$$

Figure 1 gives a schematic of the CR CgLp structure within a PID framework. Due to the lead element in front of the

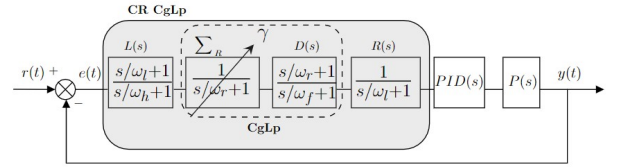


Fig. 1: Control loop using CR CgLp. Courtesy of N. Karbasizadeh [11]

reset element the reset law differs to that of the CgLp. When choosing $\omega_h \gg \omega_l$ the denominator of $L(s)$ goes to 1. Consequently, $L(s)$ can be written as $\frac{s}{\omega_l} + 1$. Hence, the reset law can be approximated by (8).

$$x_l(t) = \frac{\dot{e}(t)}{\omega_l} + e(t) = 0 \quad (8)$$

C. Sinusoidal Input Describing Function (DF)

A commonly used design tool for controllers is loop shaping. This technique makes use of Bode and Nyquist plots. This does not work for most reset elements because their non-linear behaviour. To be able to design such controllers sinusoidal input describing functions are used to approximate the response of reset elements. A prominent limitation of this technique is that it only accounts for the first harmonic of the Fourier series expansion of the steady-state response [10]. Higher order dynamics are not considered meaning that the DF does not take into account non-linear behaviour. Therefore, describing reset elements using the DF can be inaccurate. A general description of a describing function for the system in (2) is given by (9) with $n = 1$ [9].

D. Higher Order Sinusoidal Input Describing Function (HOSIDF)

Saikumar et al. proposed in [16] to use so called HOSIDF. This technique is based on the use of a virtual harmonic generator which was introduced in [17]. The usage of HOSIDF allows for open-loop frequency domain analysis using analytical descriptions of the non-linear system. The HOSIDF is defined in (9). For $n = 1$, (9) gives the DF [16], [17]. Using the HOSIDF's regular loop shaping techniques can be used.

$$H_n(\omega) = \begin{cases} C_r(j\omega I - A_r)^{-1}(I + j\Theta(\omega))B_r + D_r & n = 1 \\ C_r(j\omega n I - A_r)^{-1}j\Theta(\omega)B_r & \text{odd } n > 2 \\ 0 & \text{even } n \geq 2 \end{cases} \quad (9)$$

With

$$\begin{aligned} \Theta(\omega) &= -\frac{2\omega^2}{\pi} \Delta(\omega) [\Gamma(\omega) - \Lambda^{-1}(\omega)] \\ \Lambda(\omega) &= \omega^2 \bar{I} + A_r^2 \\ \Delta(\omega) &= I + e^{\frac{\pi}{\omega} A_r} \\ \Delta_\rho(\omega) &= I + A_\rho e^{\frac{\pi}{\omega} A_r} \\ \Gamma(\omega) &= \Delta_\rho^{-1}(\omega) A_\rho \Delta(\omega) \Lambda^{-1}(\omega) \end{aligned}$$

E. Pseudo-sensitivity

Pseudo-sensitivity is first introduced in [16]. For the calculation of the sensitivities the maximum value at steady state is used, better known as the L^∞ norm. This norm is denoted by $\|\cdot\|_\infty$. The L^∞ norm is preferred above the commonly used L^2 norm. L^∞ is critical for precision motion control because the performance is determined by the peak error [16]. The physical setup used for validation is a precision motion control platform. Therefore, it makes sense to use the L^∞ norm. Besides this, experiments have shown that the prediction at low frequencies is significantly better when using $\|e\|_\infty$ instead of $\|e\|_2$.

In [18] the pseudo-sensitivity is calculated as is defined in (10). The complementary and control pseudo-sensitivity is shown in (11) and (12) respectively. In these equations t_{ss} denotes the time when the system reaches steady-state.

$$\hat{S}(\omega) = \frac{\|e(t)\|_\infty}{\|r(t)\|_\infty} \text{ for } t \geq t_{ss} \quad (10)$$

$$\hat{T}(\omega) = \frac{\|y(t)\|_\infty}{\|r(t)\|_\infty} \text{ for } t \geq t_{ss} \quad (11)$$

$$\hat{CS}(\omega) = \frac{\|u(t)\|_\infty}{\|r(t)\|_\infty} \text{ for } t \geq t_{ss} \quad (12)$$

In [18] sufficient conditions are given that show that a closed-loop reset control system converges to a steady-state when driven by a periodic input. These conditions are based on the H_β condition in combination with the assumptions that the external input is a Bohl function, that there are infinitely many resets, and that time goes to infinity.

F. Stability

This subsection describes two methods for the analysis of the stability of a reset controller. First the H_β condition and secondly the Nyquist Stability Vector.

1) *H_β -stability*: Using H_β condition the quadratic stability of the closed-loop reset system can be studied [3], [19], [20]. Stability is studied using a Lyapunov theorem that requires the Lyapunov function to be decreasing in both flow and jump mode. Note that there are no explicit assumptions on the reset times [21].

Let $V : \mathbb{R}^n \rightarrow \mathbb{R}$ be a continuous differentiable, positive-definite, radially unbounded function such that

$$\dot{V}(x) := \left(\frac{\partial V}{\partial V} \right)^T A x < 0 \text{ for } x \neq 0 \quad (13)$$

and

$$\Delta V(x) := V(A_R x) - V(x) \leq 0, \text{ for } x \in \mathcal{M} \quad (14)$$

with the reset surface $\mathcal{M}$ defined as

$$\mathcal{M} = \{x \in \mathbb{R}^n : Cx = 0, (I - A_R)x \neq 0\} \quad (15)$$

To prove quadratic stability the basic result derived above can be specialized to the quadratic Lyapunov case. The reset system defined in (2) is quadratically stable if there exists a constant $\beta \in \mathbb{R}^{n_\rho \times 1}$ and a positive definite $P_\rho \in \mathbb{R}^{n_\rho \times n_\rho}$ such that the restricted Lyapunov equation has a solution for P [19], [20].

$$P > 0, A_{cl}^T P + P A_{cl} < 0 \quad (16)$$

$$B_0^T P = C_0 \quad (17)$$

with

$$C_0 = [\beta C_p \quad 0_{n_\rho \times n_{\bar{\rho}}} \quad P_\rho], B_0 = \begin{bmatrix} 0_{n_p \times n_\rho} \\ 0_{n_{\bar{p}} \times n_\rho} \\ I_{n_\rho \times n_\rho} \end{bmatrix}$$

In these matrices n_ρ and $n_{\bar{\rho}}$ denote the number of resetting and partial resetting states respectively. n_p is the number of plant states. The matrices with a p as a subscript are part of the plant. The closed loop A -matrix is defined below.

$$A_{cl} = \begin{bmatrix} A_\rho & B_p C_r \\ -B_r C_p & A_r \end{bmatrix}$$

For the H_β condition to hold the Base Linear System (BLS) has to be stable as well [19], [20].

Unfortunately, the H_β -condition has some limitations. The first one is that the BLS must be asymptotically stable. This follows from the notion that $V(x)$ is a continuous derivative over $\mathbb{R}^n$. Therefore, (13) is equivalent to $x^T (A^T P + P A) < 0$ for all $x \in \mathbb{R}^n$ and $x \neq 0$ [19]. Secondly, there are stable reset systems that do not have a strictly positive real H_β . For example, if reset actions cause small positive increments $\Delta V(x) > 0$ that are compensated by large negative increments, because of $\dot{V}(x) < 0$, this could cause a net negative increment of $V(x)$, yielding the system stable [19].

Besides this, assessing H_β for high dimensional plants in the frequency domain becomes a complex task [22]. In [22] an easy-to-use frequency domain method to study the stability of reset systems. This is further elaborated in the next subsection.

Concluding, the H_β -condition has two basic limitations. First, the BLS must not be unstable. Secondly, it is a sufficient condition. Besides these two limitations, applying this technique becomes increasingly complex for high dimensional plants.

2) *Nyquist Stability Vector*: An alternative way to study the stability of a reset system is to use the Nyquist Stability Vector (NSV). In [11] the analysis of the stability of the CR CgLP uses theories presented in [22]. This paper introduces a frequency-domain based stability analysis using a NSV, see (18) for its definition. With $L(s) = \mathcal{L}(s) C_R(s)$. $\mathcal{L}(s)$ is the combination of all linear controllers and the plant model. The reset compensator is defined in C_R .

$$\begin{aligned} \vec{\mathcal{N}}(\omega) &= [\mathcal{N}_x \mathcal{N}_y]^T = \\ &[|L(j\omega) + \frac{1}{2}|^2 - \frac{1}{4}, \Re(L(j\omega) \cdot C_R(j\omega)) \Re(C_R(j\omega))]^T \end{aligned} \quad (18)$$

In addition to this the H_β circle is defined in the complex plane with its centre at $[-\frac{1}{2}, 0]$ and its radius being $\frac{1}{2}$. In [22] stability conditions are derived for two types of systems. The Type I and Type II systems are defined in *Definition 1* and *Definition 2* respectively, see Section A. Using the definitions of the NSV, Type I and Type II systems with the following stability conditions were derived.

For Type I systems:

$$\left(-\frac{\pi}{2} < \theta_1 < \pi\right) \wedge \left(-\frac{\pi}{2} < \theta_2 < \pi\right) \wedge (\theta_2 - \theta_1 < \pi) \quad (19)$$

For Type II systems:

- 1) $\mathcal{L}(s)$ does not have any pole at the origin.
- 2)

$$\left(0 < \theta_1 < \frac{3\pi}{2}\right) \wedge \left(0 < \theta_2 < \frac{3\pi}{2}\right) \wedge (\theta_2 - \theta_1 < \pi) \quad (20)$$

In (19) and (20) θ_1 and θ_2 are defined as:

$$\theta_1 = \min_{\omega \in \mathbb{R}^+} \angle \vec{N}(\omega) = \angle \vec{N}_1 \quad (21)$$

and

$$\theta_2 = \max_{\omega \in \mathbb{R}^+} \angle \vec{N}(\omega) = \angle \vec{N}_2 \quad (22)$$

G. Kalman filter theory

The algorithm of a Kalman filter (KF) consists of two steps, the time update or prediction step and the measurement update or correction step. The first step predicts the new states based on a state space model of the plant. This model consists of the state transition matrix F_k and the control vector B_k . Subscript k denotes the discrete time instant. A mathematical description of the time update is given in (23). In (23a) $\hat{x}_{k+1}^-$ is the *a priori* state estimate at $k+1$, $\hat{x}_k$ is the *a posteriori* state estimate from the previous time step. u_k is the control signal. (23b) gives the covariance matrix of the estimate error. In this equation Q_k is the covariance of the process noise. This is a measure for the accuracy of the plant model [23], [24].

$$\hat{x}_{k+1}^- = F_k \hat{x}_k + B_k u_k \quad (23a)$$

$$P_{k+1}^- = F_k P_k F_k^T + Q_k \quad (23b)$$

Secondly, the state will be corrected based on the measurement update. This step is defined in (24). In (24a) the Kalman gain K_k is updated based on the covariance of the error and the covariance of the measurement noise, R_k . This gain is used in (24b) to compute the *a posteriori* state estimate $\hat{x}_k$ using z_k , the measurement at time k . The measurement matrix H_k maps the *a priori* state estimate $\hat{x}_k^-$ to z_k . The last step is to update the error covariance P_k using the updated Kalman gain. This is done in (24c) [23], [24].

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad (24a)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-) \quad (24b)$$

$$P_k = (I - K_k H_k) P_k^- \quad (24c)$$

The Kalman gain is the weighting factor that determines the ratio between the time update and measurement update. If $K_k = 0$ the algorithm disregards the measurement and completely trusts the plant model given in (23a). For $K_k = 1$ vice-versa. For time-invariant systems the covariance of the error converges. Consequently, the Kalman gain also converges. The process of converging to a final value takes some time. To eliminate this transient behaviour Riccati equations can be used to calculate the converged error covariance and Kalman gain. The Riccati equations are given in (25). When using the converged Kalman gain the algorithm simplifies to only (23a) and (24b) [25]. Please note that in (25) the process and measurement covariances Q and R are also time-invariant.

$$P_\infty = F_k \left(P_\infty - P_\infty H_k^T (H_k P_\infty H_k^T + R)^{-1} H_k P_\infty \right) F_k^T + Q \quad (25a)$$

$$K_\infty = P_\infty H_k^T (R + H_k P_\infty H_k^T)^{-1} \quad (25b)$$

1) *Augmented-state KF*: According to [26] a kinematic state estimator does not require any knowledge of the plant if a direct measurement of the load acceleration is available. This makes the estimation process robust against plant uncertainties and variations. However, this theory assumes that the accelerometer is accurate and bias-free [27]. In [27] and [28] an augmented-state KF is introduced that is not based on these assumptions. This model proved to be more robust to accelerometer output drift. Besides this, [28] shows that discretization induced robustness deterioration is suppressed when using an augmented-state KF instead of a conventional KF.

2) *Accelerometer-aided KF*: The two states of a kinematic KF are the position and the velocity. To add the accelerometer data a third dummy state is added, v_0 . This state is a measure for the drift of the accelerometer. The summation of this variable with the integrated accelerometer data yields the contribution of the accelerometer to the velocity state, see (26). In this equation $\dot{z}_k$ is the second entry of the measurement vector at time k . $\ddot{z}_k$ is the accelerometer data.

$$\dot{z}_k = \int_0^t \ddot{z}_k + v_0 \quad (26)$$

With the addition of the dummy variable the state vector of the KF is defined as:

$$\hat{x}_k = \begin{bmatrix} x \\ \Delta x \\ v_0 \end{bmatrix} \quad (27)$$

The KF uses two types of sensors, the capacitive sensors and the accelerometer. The specifications of those sensors are used to define the R matrix. In (28) σ_{cap} is the standard deviation of the capacitive sensor and σ_{acc} is the standard deviation of the accelerometer.

$$R = \begin{bmatrix} \sigma_{cap}^2 & 0 \\ 0 & \sigma_{acc}^2 \end{bmatrix} \quad (28)$$

III. CR CgLP

A. Motivation for CR CgLP

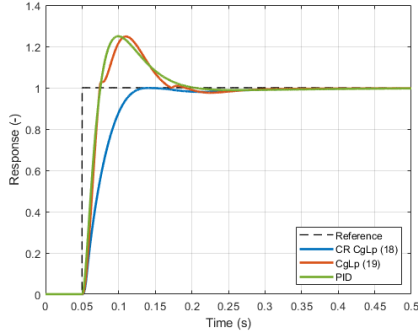
An issue most other reset elements have in common is the discontinuity in the control signal. Two possible consequences of this discontinuity are saturation of actuators and the introduction of high frequency vibrations. When using the CR CgLP these two problems might be prevented, because of the continuous nature of the control signal. Another benefit of this structure is that higher-order harmonics are reduced. This increases the accuracy of the DF approximation, which makes the tuning of the controller more straightforward [11].

B. Issues with CR CgLP

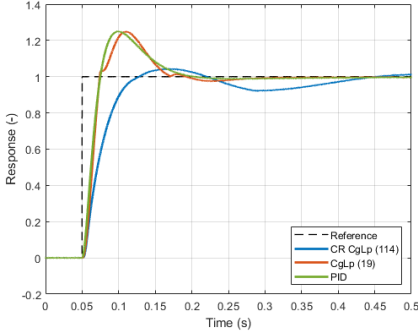
The downside of this element is its sensitivity to noise. Measurements in real world systems always contain noise. Examples of noise sources are vibrations from the environment, electrical noise and sensor precision. There are solutions for the damping of noise, for example an active air-damped vibration isolation table. Regardless of the effort some noise will

remain. The source of this sensitivity to noise can be explained by the reset law. Because of the lead filter $L(s)$ before the reset element the reset law does not only consist of the error $e(t)$, but also its time derivative, see (8). Differentiating a noisy signal amplifies the noise.

The influence of noise on the performance of a CR CgLP is illustrated in Figure 2. This figure shows the step response of a CR CgLP, a CgLP and a PID controller. Note that the controllers are tuned to have approximately equal bandwidth and phase margin. Figure 2a shows that the CR CgLP outperforms both the PID and CgLP. It has less overshoot and a faster settling time. In Figure 2b it is shown that both overshoot and settling time increase for all controllers when noise is added to the simulation. The signal-to-noise ratio (SNR) is 30dB. However, the performance of the CR CgLP deteriorates the most. The settling time is almost three times longer than that of the PID. Please note that during these simulations time regularization, as discussed in Section V-A, is applied.



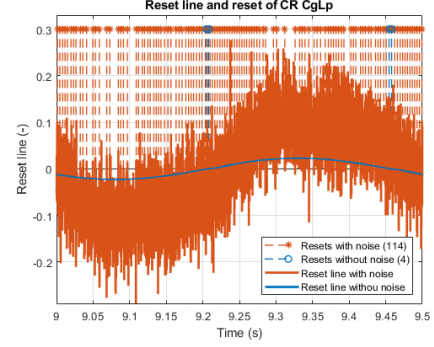
(a) Step response without noise



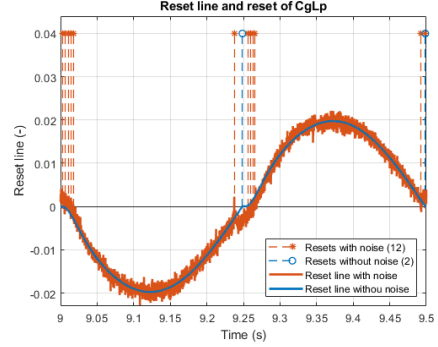
(b) Step response with added noise

Fig. 2: Comparison of impact of noise on performance controllers between PID, CgLP and CR CgLP

The deterioration of the performance of the CR CgLP is mainly due to excessive resets. This can be seen in Figure 3. Here the CR CgLP and CgLP are excited by a sinusoidal reference signal. The number of resets of the CR CgLP increases with more than a factor 25, while the number of reset of the CgLP increases to 6 times the amount of reset without added noise. The excessive resets of the CR CgLP cause both the overshoot and the settling time to increase. Resets are needed to obtain the phase advantage compared to linear control, but too many resets can result in a reset controller that performs worse than a linear controller.



(a) Plot reset line of CR CgLP



(b) Plot reset line of CgLP

Fig. 3: Illustration of impact of noise on the number of resets of both CR CgLP and CgLP. Number of resets is shown in legend.

IV. PROBLEM STATEMENT

Due to the continuousness of the control signal of the CR CgLP the implementation of this element is promising. However, noise in the system is identified as a cause for the performance of the CR CgLP to deteriorate. The performance deterioration is caused by excessive resets. Therefore, a method needs to be developed that reduces the impact of noise on the number of resets, and consequently the performance of the CR CgLP.

V. CONSIDERATIONS ON PRACTICAL IMPLEMENTATION

This section describes considerations about the practical implementation of reset controllers. Identified causes of performance deterioration are noise and quantization. Quantization occurs when a continuous, analogue signal is covered to a discrete value [29]. This is needed because the controller is implemented on digital systems. The quantization level Q is defined in (29). In this equation $Range$ is the measurement range of the sensor and n is the number of bits of the analogue-to-digital converter (ADC). Q determines the maximal resolution of the system.

$$Q = \frac{Range}{2^n} \quad (29)$$

Quantization-induced performance degradation comes to light when measuring the sensitivity. In the ideal case the sensitivity levels out at $\frac{E}{Q}$ at low frequencies, $E = 1$ for a normalized reference. [30], [31]. For an 8-bit system this would yield a minimum sensitivity of $\frac{1}{2^8} = -48\text{dB}$. In

Figure 4 we observe that the blue line goes to -48dB for low-frequencies indeed. At mid-range frequencies the sensitivity of the discrete controller is much higher than that of the continuous controller, approximately 10dB at 1Hz . Examination of the resets yields that at 1Hz the discrete time CR CgLP resets almost 964.9 times and the continuous controller 2.4 times per period. When the error signal is within $-\frac{1}{2}Q \leq e(t) \leq \frac{1}{2}Q$ it meets the resetting condition as defined in (2), and therefore it resets [30], [31]. Below two remedies for this problem are discussed.

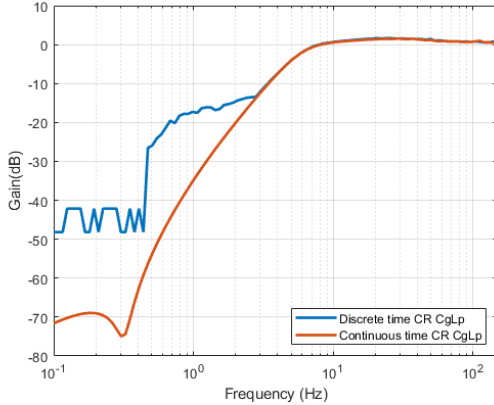


Fig. 4: Pseudo-sensitivity $\hat{S}(\omega)$ of continuous and discrete system

A. Time regularization

An excessive number of resets causes the performance to deteriorate. Based on the phenomenon of Zenoness [32], [30] introduced time regularization to limit the number of resets. The idea is to define a holding time ρ that prevents reset to happen. ρ is defined in (30). In this equation f_c is the cross-over frequency in Hz, and k_{tr} is a safety factor. The tuning of k_{tr} introduces a trade-off. Choosing k_{tr} small reduces the effect of the time regularization. Choosing k_{tr} too high might cause the system to miss reset. This can lead to a higher peak-of-sensitivity. A safety factor $k_{tr} = \frac{5}{2}$ results in the longest time without increasing the peak-of-sensitivity [30].

$$\rho = \frac{1}{2k_{tr} \cdot 2\pi f_c} \quad (30)$$

B. Reset band

In [31] a reset band is introduced to solve the performance deterioration caused by excessive resets. This leads to a new definition of a reset controller, see (31)[19]. With this structure the controller will reset then the error enters the area of $|\delta|$ around zero. δ should be designed such that it is bigger than Q [31].

$$R := \begin{cases} \dot{x}(t) = A_r x(t) + B_r e(t) & \text{if } (e(t), \dot{e}(t)) \notin B_\delta \\ x(t^+) = A_\rho x(t) & \text{if } (e(t), \dot{e}(t)) \in B_\delta \\ u(t) = C_r x(t) + D_r x e(t) & \end{cases} \quad (31)$$

Where $B_\delta = (x, y) \in \mathbb{R}^2 \mid (x = -\delta \wedge y > 0) \vee (x = \delta \wedge y < 0)$. In the presence of noise δ should not be tuned too tightly. This can cause excessive reset at lower frequencies. Since at lower frequencies $e(t)$ is close to Q . Wrong tuning of δ can

cause performance deterioration. δ should be tuned for the correct frequency and amplitude of the reference signal. The deterioration caused by noise can be solved by introducing a safety factor k_{rb} , see (32). If the maximum noise could be measured, this value should be included in k_{rb} . The same hold for disturbances and plant uncertainties [31].

$$\delta_{robust} = k_{rb} \delta \quad (32)$$

VI. PROPOSAL

To make the CR CgLP more robust against noise two strategies will be applied and compared to each other. The first one is to use a low-pass filter to attenuate the high-frequency noise. The second strategy is to apply a KF. The location of the filters within the control loop might have an impact on the overall system performance. Therefore, the filters will be tested on both the reset line (RL) and the measurement line (ML).

A. Low-pass filter

Using a LPF the high-frequency content on the reset signal can be attenuated. Dampening these frequencies could then lead to the reduction of excessive resets. Which should lead to less performance deterioration caused by noise. The downside of a LPF is that it introduces significant phase lag into the system. This can have a negative effect on the control performance at mid- and high-frequency range [33]. Therefore, a second solution is proposed.

B. Kalman filter

A common way to attenuate noise is to implement a Kalman filter (KF) [34], [35], [27]. This is often done by combining data from several sensors [36], [37], [28]. One example is a hand motion tracking. Data from a motion capture glove and camera is combined using a KF to increase the overall precision and accuracy of the measurement [38]. Another example is presented in [37] where a GPS sensor and an inertial measurement unit are combined for positioning of an autonomous land vehicle. In particular, accelerometer aided KF's could reduce the impact of noise, by smoothing a signal. It is shown that even with low-cost accelerometers the performance of controllers can be improved [27]. Therefore, this technique will be utilized to prevent excessive resets by attenuating noise on the reset line.

C. Measurement line vs. reset line

The proposed methods will be implemented in two different configurations. The first configuration is to place the filters on the RL. Placing the filters on the RL does not directly impact the states of the system. Because the RL only determines when the integrator resets. Therefore, the higher order harmonics remain less affected. The second configuration is to place the filters on the measurement line, they will have a direct impact on the states of the system.

VII. VALIDATION

The proposed methods are simulated in Simulink. Let $P(s)$ be a transfer function of the plant as defined in (33). $P(s)$ represents a mass-spring-damper system that is used to study the impact of the proposed methods on the reduction of excessive resets. The parameters of this plant are $m = 250\text{kg}$, $c = 100\text{Ns/m}$ and $k = 1\text{kN/m}$.

$$P(s) = \frac{1}{ms^2 + cs + k} \quad (33)$$

To study the behaviour of reset controllers in relation to conventional control, a PID controller is used. The PID is defined in (34).

$$C_{PID}(s) = K_p \cdot \left(\frac{\omega_i}{s} + 1 \right) \cdot \left(\frac{\frac{s}{\omega_d} + 1}{\frac{s}{\omega_t} + 1} \right) \quad (34)$$

A. Controller tuning

The parameters of the controllers are listed in Table I. The CgLp is tuned for tracking and precision improvement. The tuning guidelines from [10] are used for the design of the CgLp. The CR CgLp makes use of the same CgLp-block, but ω_l and ω_h are used to tune the extra lead and lag elements such that there is no overshoot. The reset controllers are tuned to have a bandwidth of around 10 Hz and a phase margin of approximately 56°. The table below includes the tuned parameters used in the simulations. ω_c is the desired bandwidth and G_{BW} is the open-loop gain of the plant at the frequency of the desired bandwidth.

$$k_p = \frac{0.3}{G_{BW}} \quad (35)$$

TABLE I: Parameters of the controllers

Parameter	CR CgLp	CgLp	PID
ω_i	$\omega_c/10$	$\omega_c/10$	$\omega_c/10$
ω_d	$\omega_c/2.5$	$\omega_c/2.5$	$\omega_c/3.95$
ω_t	$\omega_c \cdot 2.5$	$\omega_c \cdot 2.5$	$\omega_c \cdot 3.95$
ω_f	$\omega_c \cdot 20$	$\omega_c \cdot 20$	-
ω_l	$\omega_c \cdot 0.35$	-	-
ω_h	$\omega_c \cdot 20$	-	-
γ	0	0	-
K_p	$1.15k_p$	$1.15k_p$	$0.85k_p$

Figure 5 shows the open-loop frequency response of the different controllers. In this figure the blue line represents the first order harmonic of the CR CgLp. Please note that for low frequencies the BLS and CR CgLp overlap.

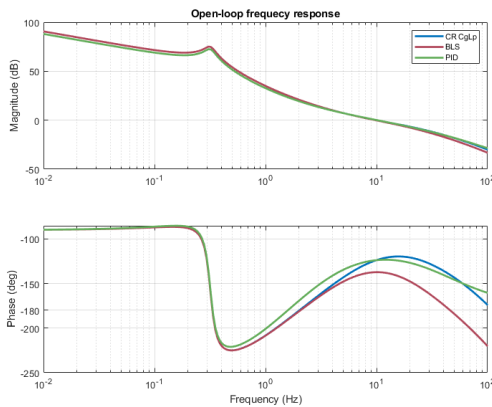


Fig. 5: Open-loop frequency response of CR CgLp and PID

B. LPF tuning

Figure 6 shows the frequency response of a first and second order LPF. Although a first order LPF dampens the high-frequency content in a signal it also adds 90° phase lag after

its cutoff frequency. In the case of a second order filter the added phase lag becomes 180°. When choosing the cutoff frequency too low, this could counteract the phase lead provided by the reset element. It could even render the controller unstable if the phase drops below 180° at bandwidth.

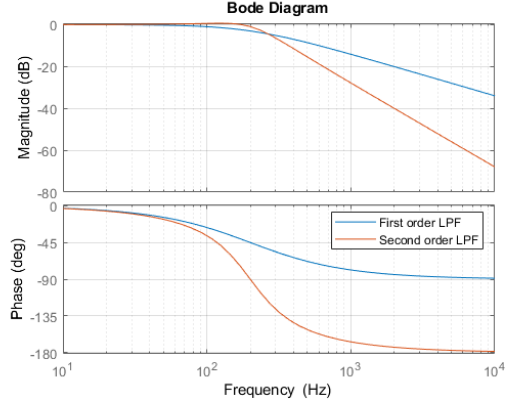


Fig. 6: Bode plot of first and second order LPF

For the remainder of this paper, we will only consider a first order LPF. The second order LPF falls outside the scope of this research due to the addition of an extra tuning parameter, which makes tuning in general more complex.

The idea of adding a LPF to the reset line is not new. In [39], a shaping filter (SF) consisting of a LPF and a lead filter (LL) is presented. In this paper the SF is used to reduce the noise of on the reset line of a CgLp. The definition of this SF is given in (36). The lead filter is added to compensate for the phase lag caused by the LPF. In this equation f_f is the cutoff frequency of the LPF, f_c is the bandwidth and a is used to tune the lead filter.

$$C_{LPF+L} = \left(\frac{1}{1 + \frac{s}{2\pi f_f}} \right) \left(\frac{1 + \frac{s}{2\pi f_c/a}}{1 + \frac{s}{2\pi f_c \cdot a}} \right) \quad (36)$$

To determine the value of a the phase of the LPF at bandwidth must be calculated, see (37).

$$\phi_{LPF} = -\tan^{-1} \left(\frac{f_c}{f_f} \right) \quad (37)$$

Subsequently, the tuning variable a is defined using the relation given in (38).

$$\tan^{-1}(a) - \tan^{-1} \left(\frac{1}{a} \right) = -\phi_{LPF} \quad (38)$$

Choosing a small f_f increases the noise attenuation. However, it also requires a large a . This can create a magnitude peak, see Figure 7. According to [39] a LL filter with $f_f = 2f_c$ is a good trade-off. This would result in an a of 1.62. Besides, Figure 7 shows that the LPF with a higher f_f , in yellow, does not add an amplitude peak. The phase lag at bandwidth is 2.9°. This is worse than the LL, but much less than the 26.8° of the LPF without lead filter. The remainder of this literature review will only consider the LPF with high cutoff frequency. A study on the difference in performance of the LL compared to the high cut-off frequency LPF is part of future work.

As mentioned earlier the cutoff frequency has to be high enough not to counteract the phase lead provided by the

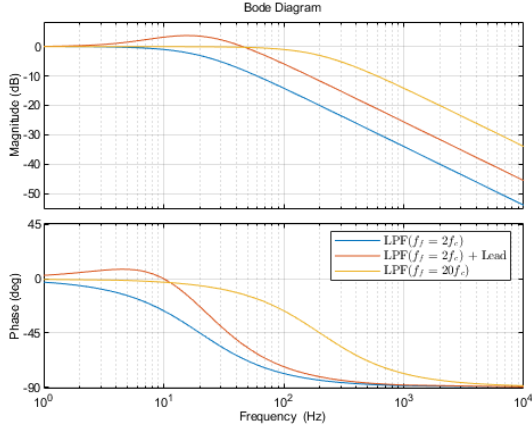


Fig. 7: Bode plot of low-pass filters with different cutoff frequencies

reset element. Figure 8a shows the DF of a CR CgLP for five different LPFs. The LPFs are located on the reset line. The LPFs have cutoff frequencies of $5\omega_c$, $10\omega_c$, $20\omega_c$, $30\omega_c$ and $50\omega_c$. ω_c is the bandwidth of the controllers, set to 10Hz.

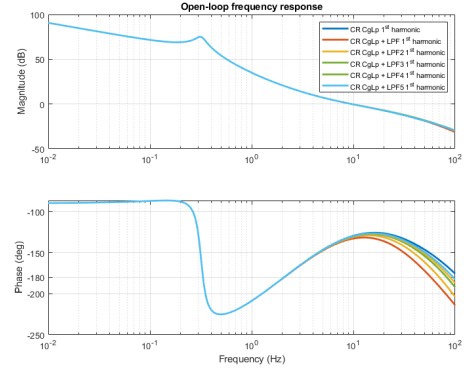
Comparing Figure 8a with Figure 8b it can be concluded that placing the LPF on the reset line results in less loss of phase margin than placing the filter on the measurement line. However, looking at the magnitude plot it can be seen that the LPFs of the measurement line have lower gains at higher frequencies. In other words, it has better noise attenuation. In Figure 9 the same LPFs are used. In this figure it can be seen that for a cutoff frequency of approximately $20\omega_c$ the overshoot reduces and the response without added noise, showed in Figure 2a, is approximated. Hence, following from the step response the cutoff frequency should be set to $20\omega_c$. Figure 9b shows that indeed for LPFs with a higher cutoff frequency the response becomes slower in both rise time and settling time.

C. KF tuning

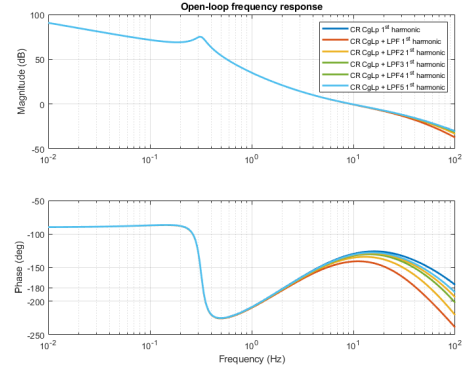
The KF can be tuned by adjusting the process and the measurement uncertainties, Q and R respectively [24]. The entries of R are given by the precision of the used sensors, see (28). The physical setup that will be used has two types of sensors, capacitive distance sensors and an accelerometer.

The process covariance matrix is given in (39). Using the state vector as defined in (27) this yields that the KF completely trusts the state transition matrix F_k for the first state. The other two values are not deterministic. They are found experimentally. Setting the third entry to a lower value implicates that the filter should trust the accelerometer data more than the position measurement. A closer study of the impact of the precision of the accelerometer on the performance of the KF falls outside the scope of this review but it will be part of future work.

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1e-3 \end{bmatrix}, \quad (39)$$



(a) LPF's on reset line



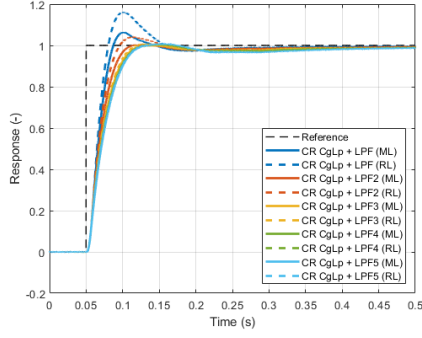
(b) LPF's on measurement line

Fig. 8: Step responses for LPFs with different cutoff frequencies

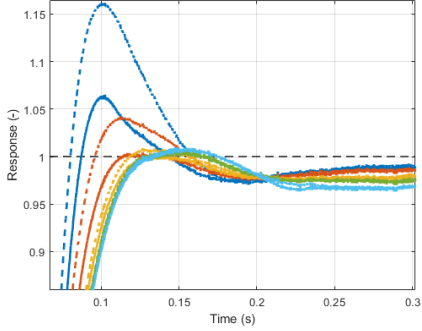
D. Measurement line vs. reset line

To determine which one of the two alternatives performs better both the transient and steady-state responses must be studied. The transient response will be studied by applying a step input to the system, see Figure 10. From this simulation one learns that the response with the filter on the reset line the response converges quicker towards the reference. This holds for both the LPF and the KF. For the KF the difference in overshoot is negligible. In the case of the LPF the difference is noticeable, but within reasonable bounds ($< 1\%$).

The steady-state behaviour is studied using the pseudo-sensitivity. These results can be found in Figure 11 and Figure 12. The former contains simulations with added noise and the later contains simulations without added noise. This figure shows that at low frequencies the filter on the ML perform best in noise attenuation. At low-frequencies it can be seen that all sensitivities in Figure 11a settle at a higher gain than the sensitivities in Figure 12a. This ≈ 20 dB difference is caused by the addition of noise. Mid-frequency range there is no significant difference between the filter configurations. Looking at Figure 11b one sees that the peak-of-sensitivity is lowest for the KF. The difference between the KF and LPF are 0.25dB and 0.5dB for reset and measurement line respectively. Therefore, based on the pseudo-sensitivity placing the filters on the measurement line is the preferred option.



(a) Step response



(b) Step response zoomed in

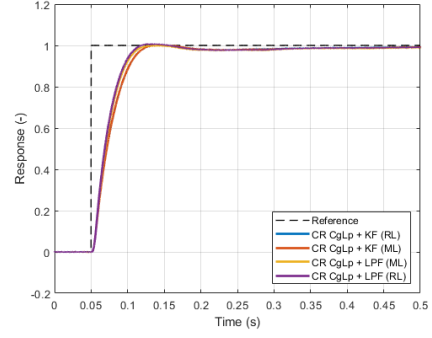
Fig. 9: Comparison between different LPFs on either measurement line (ML) or reset line (RL)

E. Results

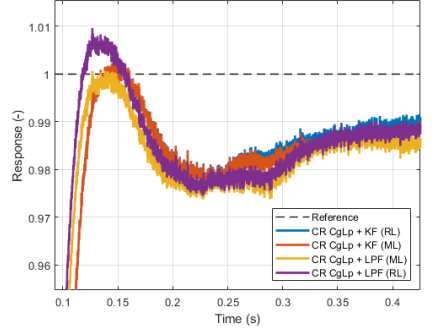
Now that the preferred configuration has been determined, the performance of the filters will be compared to that of a conventional PID. Both the transient and steady-state response are studied.

1) *Transient response*: To study the transient response the system is subjected to a step input. The performance is then quantified using the overshoot and settling time. During the simulations the signal-to-noise ratio (SNR) of the position sensors was set to approximately 30dB. Please note that in Figure 13a the response of the *CR CgLp + KF* and the *CR CgLp no filter* are on top of each other. Since there is no added noise in the system, the KF does not filter out any. In Table II the number of resets for a step response is shown. Both filters cause a reduction in number of resets compared to the situation without SF. Namely, a reduction of 48% and 41% for the KF and LPF respectively. This reduction in reset instances reduces the overshoot, this is shown in Figure 13b. Column two of Table II contains the settling time to the $\pm 2\%$ domain around the reference. It shows that the addition of shaping filters decreases the settling time by more than a factor 2. However, it could not match the settling time of the PID controller with similar bandwidth and phase margin. It can be concluded that a reduction in overshoot comes with the cost of a slight increase in settling time.

2) *Steady-state response*: For analysis of the steady-state response the pseudo-sensitivity data is studied. The simulation results can be seen in Figure 14. At low frequencies (0.1 – 0.4Hz) the *CR CgLp* with KF has the lowest values.



(a) Step response of CR CgLp



(b) Step response of CR CgLp zoomed in

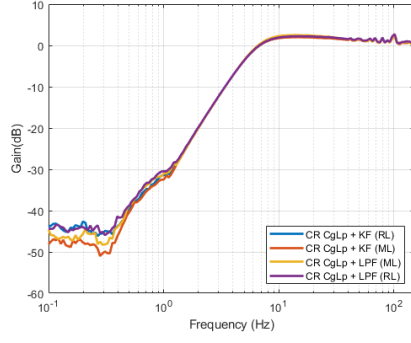
Fig. 10: Comparison filter on measurement line (ML) and reset line (RL)

TABLE II: Simulation results of step response. Number of resets in last column come from Figure 13. The number of resets of the simulation without added noise is shown in parentheses.

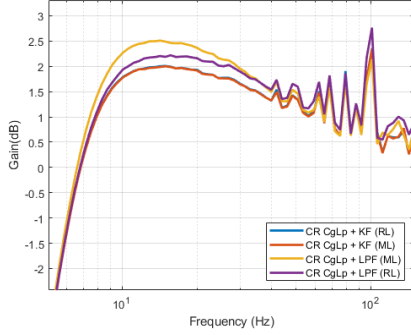
Controller	Overshoot	2%-settling time (s)	Resets
PID	25% (25%)	0.18 (0.18)	-
CR CgLp	4.5% (0%)	0.7 (0.22)	113 (4)
CR CgLp + LPF	1% (0.6%)	0.3 (0.22)	45 (2)
CR CgLp + KF	0.26% (0%)	0.27 (0.22)	39 (4)

This means that it is less sensitive to low frequency disturbances. At the lower side of mid-range frequencies (0.4 – 1.0Hz) the PID controller performs somewhat better than the reset controller. This could be caused by precision of the accelerometer. Studying this is part of the next phase. Between 1Hz and 4Hz the filtered reset controllers perform better than the linear controller. The peak-of-sensitivity of the PID is 0.1dB lower than the peak of the *CR CgLp* with KF, which is 2.0dB. The peak of the *CR CgLp* with the LPF reaches 2.5dB, the highest. This renders the *CR CgLp* with the LPF the most sensitive to noise around the bandwidth. At even higher frequencies, > 40Hz, the pseudo-sensitivity of the reset controllers shows some peaks. The study on the source of those peaks is part of future work.

Figure 15 shows the reset line and the reset instances when the model is driven by a sinusoidal reference. Based on the



(a) Pseudo-sensitivity



(b) Pseudo-sensitivity zoomed in on peak-of-sensitivity

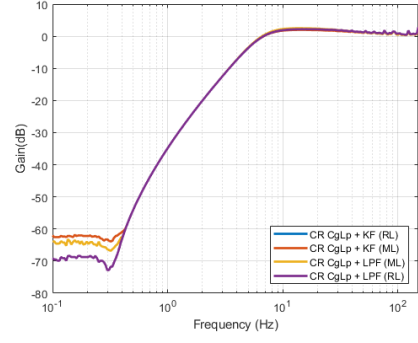
Fig. 11: Closed-loop steady-state comparison between KF and LPF on either measurement line (ML) or reset line (RL). Simulation with added noise.

simulation without added noise (see Figure 15a) it can be seen that the numbers of reset when using a KF is matching the numbers of resets in Figure 3a. In the case of the LPF the number of resets is reduced to two. This can be because the KF is not compensating anything, but the LPF is still attenuating some high frequency content of the measurement signal.

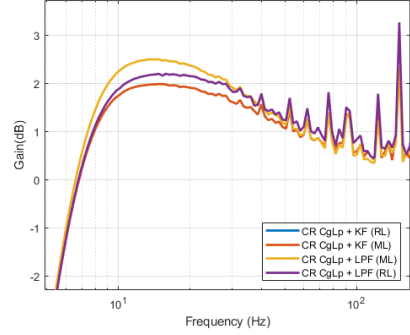
As expected the response of the CR CgLp without SF in Figure 15b is similar to the response of the CR CgLp in Figure 3a. Compared to the controller without SF both KF and LPF decrease the number of resets. The KF reduces the number of resets by a factor of 2.9 and the LPF by a factor of 2.5. Noise on a signal cannot be attenuated completely, and the filtering comes at a price of losing phase when using a LPF. For a linear element like a LPF there is always a trade-off between noise attenuation and reference tracking. A further reduction of the number of reset by reducing the cutoff frequency of the LPF could lead to an overall performance degradation. Therefore, some excessive resets will remain. But their impact on the system performance is limited. Further study of these excessive resets is part of another chapter.

VIII. CONCLUSION & FUTURE WORK

Based on the simulation data we can conclude that the addition of either a KF or a LPF leads to a reduction of resets of 65% and 60% respectively. Consequently, this leads to less performance deterioration caused by excessive resets.



(a) Pseudo-sensitivity



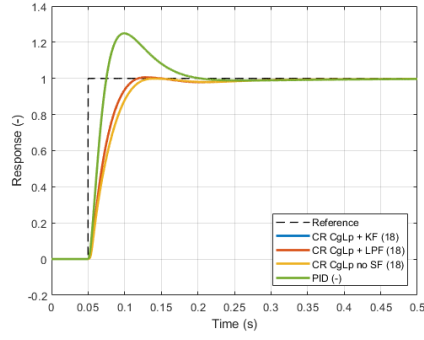
(b) Pseudo-sensitivity zoomed in on peak-of-sensitivity

Fig. 12: Closed-loop steady-state comparison between KF and LPF on either measurement line (ML) or reset line (RL). Simulation without added noise. Note that both lines of the CR CgLp + KF are coinciding.

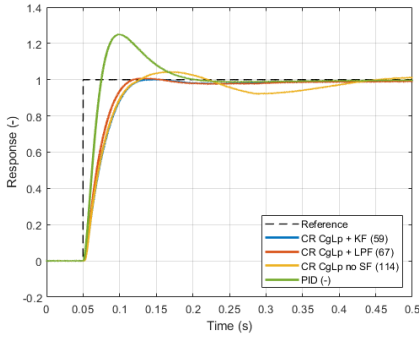
For this model this leads to reduction of the settling time by a factor of 2.6 and a decrease in overshoot of factor 17 when using a KF on the measurement line. The addition of a LPF on the measurement line can lead to a 2.3 times reduction of the settling time and a 4.5 reduction of the overshoot. The most improvement can be obtained when using a KF on the measurement line. Compared to a conventional PID controller with similar bandwidth and phase margin the overshoot is reduced with a factor 96. The settling time is approximately the same.

Further research is needed to gain insight in the relation between the precision of the accelerometer and the performance of the KF. To know the limits of the system, the impact of an increasing noise on the performance of the system should be studied. Besides this, the improvement gained by implementing an extended KF could be studied.

Future work will contain a study on the relation between the performance of the KF and the precision of the accelerometer. Secondly, a study on the impact of the magnitude of the noise on the system performance will provide an understanding of the limitation of the proposed solution. Thirdly, the performance difference between a LPF+L and a LPF with high cutoff frequency will be studied. The aim of this project is to study the performance of a reset element when it is implemented on a physical system. To that end, the proposed solution will be implemented and validated on the setup that is introduced in the remainder of this section. The setup that

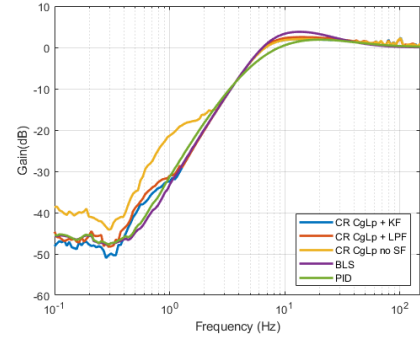


(a) Step response without added noise

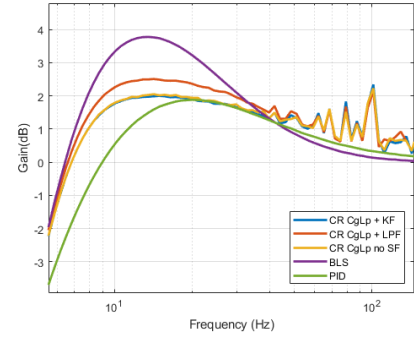


(b) Step response with added noise

Fig. 13: Step response of CR CgLP with either a KF, a LPF or no filter



(a) Pseudo-Sensitivity function



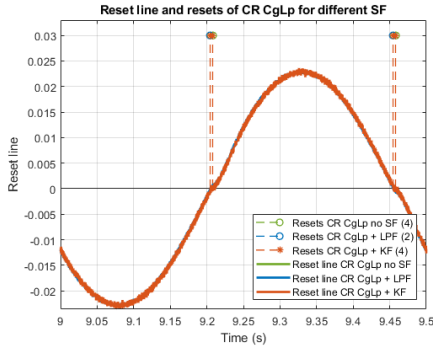
(b) Pseudo-Sensitivity function zoomed in on peak-of-sensitivity

Fig. 14: Closed-loop steady state comparison between PID and CR CgLP with either KF or LPF

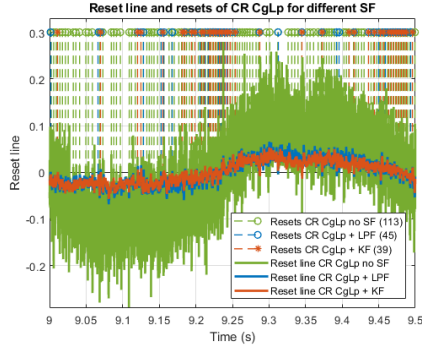
will be used is a 6 DoF motion stage.

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(a) Plot reset line and resets without added noise



(b) Plot reset line and resets with added noise

Fig. 15: Illustration of impact of noise on the number of resets of CR CgLP with either a KF, a LPF or no filter. Number of resets is shown in legend. The SNR used in this simulation is 30dB.

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APPENDIX

A. Definitions used for NSV

Below the definitions of two types of systems are given. These systems are used to assess the stability of reset systems.

Definition 1: For the reset system to be of Type I the following conditions should hold [22]:

- 1) For all $\omega \in \mathcal{M} = \{\omega \in \mathbb{R}^+ : \mathcal{N}_\chi(\omega) = 0\}$ one has $\mathcal{N}_\gamma(\omega) = 0$.
- 2) For all $\omega \in \mathcal{Q} = \{\omega \in \mathbb{R}^+ : \mathcal{N}_\gamma(\omega) = 0\}$ one has $\mathcal{N}_\chi(\omega) = 0$.
- 3) At least one of the following is true:
 - a) $\forall \omega \in \mathbb{R}^+ : \mathcal{N}_\gamma(\omega) \geq 0$,
 - b) $\forall \omega \in \mathbb{R}^+ : \mathcal{N}_\chi(\omega) \geq 0$,

3

Problem Statement

Literature has shown that reset control is able to overcome limitation inherent to linear control [4, 5]. However, quantization and measurement noise can lead to performance deterioration [11, 12]. Both quantization and measurement noise can cause additional resets. These additional resets increase the reset action when they follow each other fast compared to the oscillation period [15]. This is mainly due to the excessive resets caused by the amplified noise on the reset line. Further analysis on the causes and effects of excessive resets is covered in appendix B.

This thesis presents two proposals that should improve the system performance by attenuating the noise on the reset line. The first proposal is a low-pass filter (LPF) with lead element (LL). This design is based on the work of [7]. The second proposition is to use an accelerometer-aided KF (aaKF). This design is predominantly inspired by [8]. Both methods have previously been used to attenuate noise. The LPF by attenuating high-frequency content. Measurement contains predominantly high-frequency content, so the amplitude of the measurement noise gets reduced. The configuration of the aaKF used in this thesis uses two types of sensors, position sensors and an accelerometer, and an internal plant model. Each of those sensors have their own noise covariance. The aaKF combines those sources to estimate the position state with an improved combined noise covariance. This improved covariance should limit the impact the noise has on the performance of the CR CgLP.

Figure 3.1 shows the proposed control structure. The filter, which is either a LL or aaKF, is located on the measurement line. Not on the reset line as is done in [7]. The filter is placed on the measurement line because this allows for noise attenuation on the position measurement and consequently the error signal. As shown in Figure 11 in section 2.7, this can result in an reduced low-frequency pseudo-sensitivity floor. Hence, improved trajectory tracking.

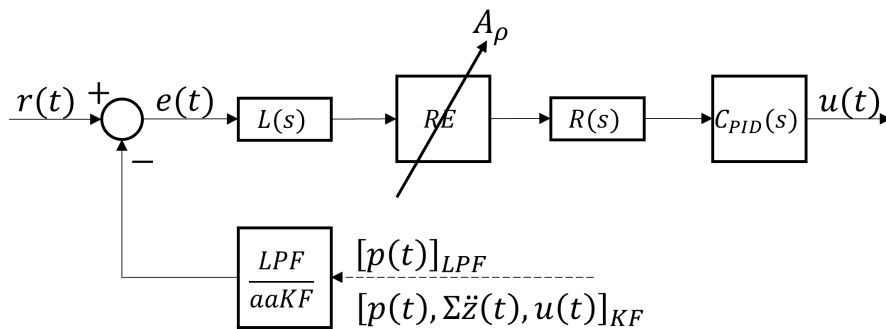


Figure 3.1: Control structure with PID, CR CgLP and filter on measurement line. For both filters the required input signals are shown. $\Sigma \ddot{z}(k)$ is the integrated accelerometer data.

4

Design of Low-Pass Filter

This chapter discusses the design of a LPF for the CR CgLP. The aim of this filter is to attenuate measurement noise, so the LPF is placed on the measurement line. The aim is to come up with a design that improves the noise robustness of the CR CgLP. The design of the filter on the measurement line is based on a shaping filter introduced by [7]. In this paper this filter was designed to attenuate the noise on the reset line, called a shaping filter (SF). It was concluded that the SF can reduce the effect of noise improves trajectory tracking. However, some performance deterioration around bandwidth was observed. How these observations effect the system performance of the system discussed in this thesis will be addressed in this chapter.

4.1. Design of Low-Pass Filter + Lead filter

In [7] a SF that attenuates noise on the reset line of a CgLP is introduced. This SF consist of a first order low-pass filter and a tamed lead filter. This structure will be referred to as LL. The lead filter is implemented to compensate for the phase lag introduced by the LPF. Below the transfer function of this filter is given. In (4.1) f_f is the corner frequency of the LL, f_c is the bandwidth of the system and a is a tuning parameter.

$$C_{LL} = \left(\frac{1}{1 + \frac{s}{2\pi f_f}} \right) \left(\frac{1 + \frac{s}{2\pi f_c/a}}{1 + \frac{s}{2\pi f_c \cdot a}} \right) \quad (4.1)$$

As is shown in [7], the design of the LL consists of two steps. Firstly, the phase of the LPF at bandwidth f_c is calculated using (4.2). Secondly, based on the phase at bandwidth, the tuning variable a is defined using the relation given in (4.3). Choosing a small f_f increases the noise attenuation. However, it also requires a large a to counteract the phase lag. This can create a magnitude peak, see Figure 4.2. According to [7] a LL with $\omega_f = 2\omega_c$ is a good trade-off. This would result in an a of 1.62, see Figure 4.1.

$$\phi_{LPF} = -\tan^{-1} \left(\frac{f_c}{f_f} \right) \quad (4.2)$$

$$\tan^{-1}(a) - \tan^{-1} \left(\frac{1}{a} \right) = -\phi_{LPF} \quad (4.3)$$

Figure 4.2 shows the Bode plot of a LPF and a LL. The phase lag introduced by the LPF is compensated by the lead element which provides the LL with a phase lead at bandwidth. This improves the stability but comes with the cost of a reduction in noise attenuation. The magnitude peak of the LL filter has an direct impact on the gain margin of the system. For the current configuration, the peak has a

magnitude of 3.8dB at 1000Hz. This means that for the worst case scenario the gain margin might be reduced by 3.8dB. Moreover, this peak might lead to an increase of the modulus margin (MM). This would then reduce stability.

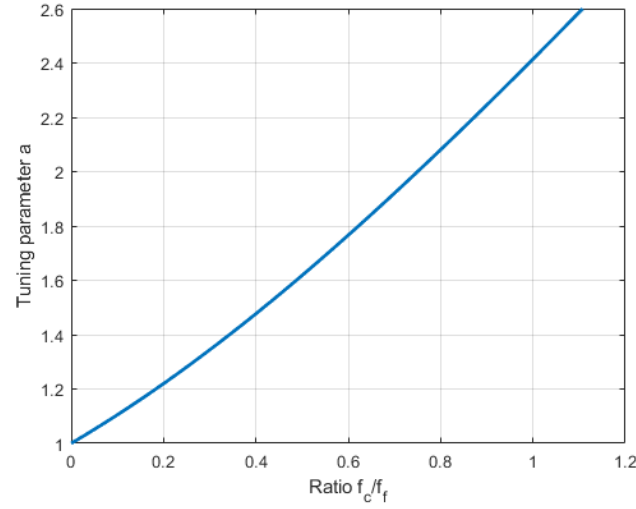


Figure 4.1: Plot of tuning parameter α as function of ratio between system bandwidth f_c and cutoff frequency f_f .

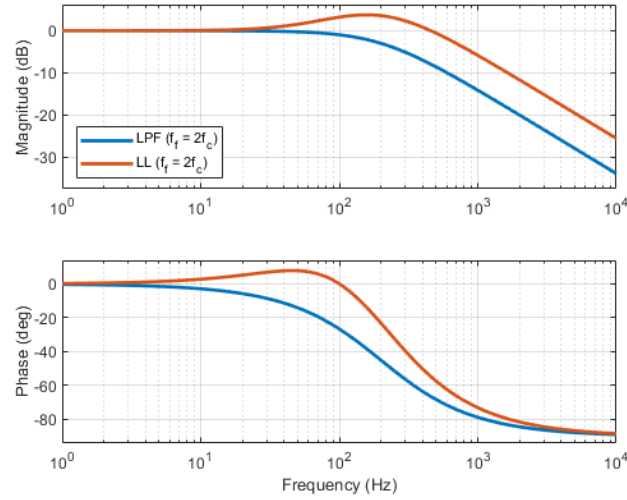


Figure 4.2: Bode plot of LL and LPF.

4.2. Performance of Low-Pass Filter + Lead Filter

In order to assess the performance of the LL, the design is tested in Simulink. The plant is a mass system with its mass equal to that of the physical test setup. Details on the plant and controller can be found in appendix E.3.1.

4.2.1. Transient response

Figure 4.3 shows the step response of a CR CgLP with LL. Compared to the CR CgLP without filter, the CR CgLP with LL has a reduced overshoot and reduced settling time. Based on the step response it can be concluded that the transient response is improved when a LL filter is added to the measurement line. The oscillatory behavior of the CR CgLP with LL could indicate a reduction in stability. This can be contributed to the reduction of the MM.

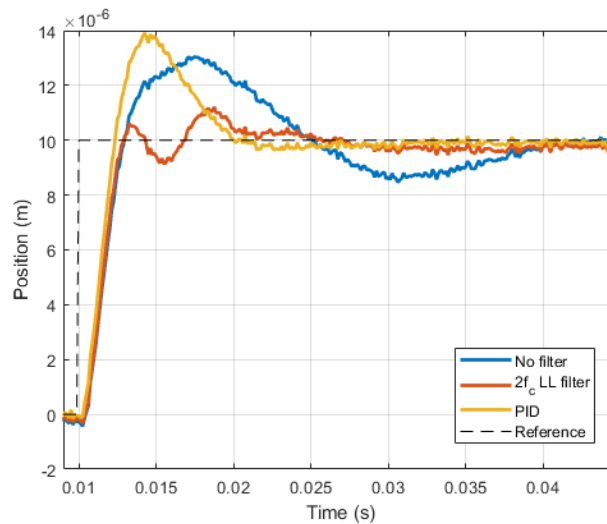


Figure 4.3: Step response of PID and CR CgP with and without LL filter.

4.2.2. Steady-state response

Next, the steady-state behavior of the LL filter is discussed. Please note that the measured pseudo-sensitivity depends on the magnitude of the reference signal. For the simulations the magnitude of the reference is scaled to the reference used in the physical setup. For low frequencies the LL filter reduces the pseudo-sensitivity compared to the CR CgP without filter, see Figure 4.4. The low-frequency sensitivity floor drops below the sensitivity floor of the PID. Hence, trajectory tracking is improved compared to both the PID and the CR CgP without a filter. The peak of sensitivity of the CR CgP with LL is approximately 8dB. The peak of sensitivity is directly related to $1/MM$, see (A.4) & (A.5), this value should remain below 8dB to ensure stability.

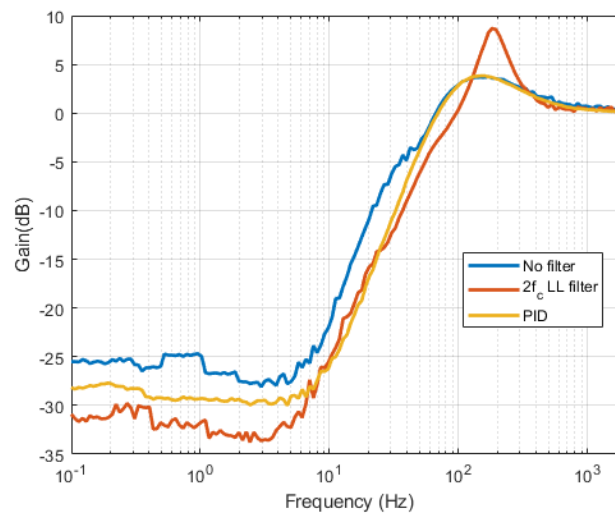


Figure 4.4: Pseudo-sensitivity of PID and CR CgP with and without LL filter.

For this plant and controller combination the mid-frequency range can be divided in frequencies below and above 30Hz. For frequencies below 30Hz the pseudo-sensitivity rises above the PID-line, above 30Hz the pseudo-sensitivity drops below the PID-line. To get more insight in the behavior of the pseudo-sensitivity the error signals at 20Hz and 70Hz are given, see Figure 4.5. The main difference between the two situations is the ratio of the noise compared to the measurement. At 70Hz the dynamics of the system become more prominent, causing the error to increase. The measurement noise is modelled as additive noise and thus the magnitude of this noise remains constant. This translates behavior directly

to the reset line of the CR CgLP which is discussed next.

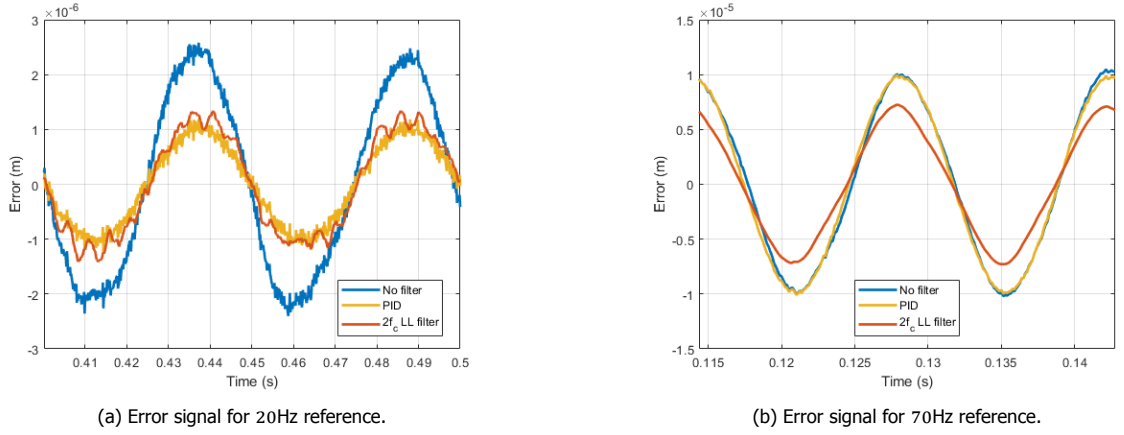


Figure 4.5: Error signals of of PID and CR CgLP with and without LL filter.

Figure 4.6 gives the reset lines of the CR CgLP with and without LL for a reference of 20Hz and 70Hz. From Figure 4.6a it can be seen that zero crossing occur for almost the entire oscillation. Hence, resets are triggered during the entire oscillation causing the number of resets to diverge from the theoretical number of two resets per oscillation. At 70Hz this problem is less pronounced. Figure 4.6b shows that the filtered reset line has two zero crossings per oscillation, resulting in two resets. The number of zero crossings of the unfiltered reset line is still greater than two. This might cause to deteriorate the performance of the CR CgLP.

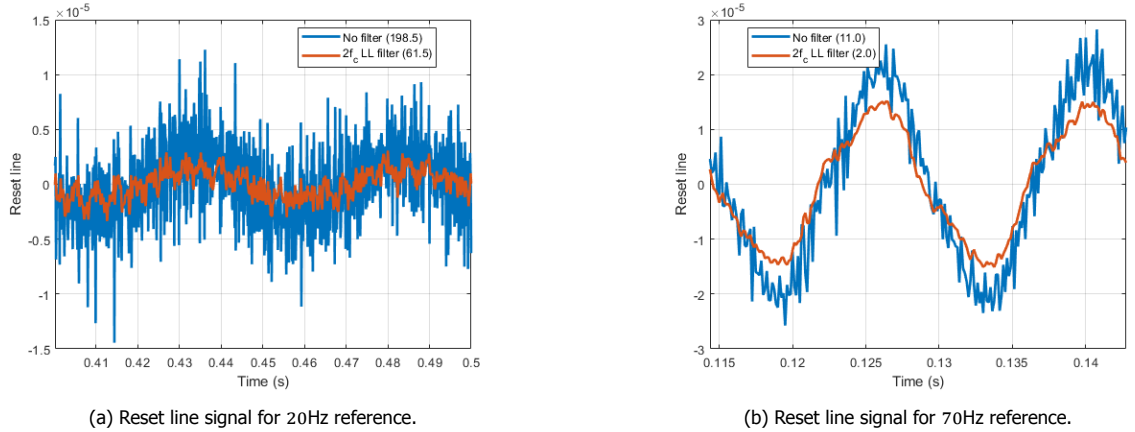


Figure 4.6: Reset line signals of of PID and CR CgLP with and without LL filter. The number of resets per oscillation is given in brackets.

4.2.3. Sub-conclusion

The designed LL filter improves the transient response compared to both a PID controller and a CR CgLP without filter. However, with a peak of sensitivity at 10dB, this comes at the cost of stability. The proposed corner frequency, $f_f = 2f_c$ is too low. Increasing the corner frequency might provide better results. This option is discussed in the next section.

4.3. Redesign of Low-Pass Filter

The design parameters suggested in [7] do not seem to fulfil the requirements in this particular application. One way to reduce the effect of the magnitude peak of the LL around bandwidth is to increase the cutoff frequency. A higher cutoff frequency requires a lower value of a . This results in a lower magnitude peak. A second approach could be to use a second order LPF. By reducing the damping of such a filter, the phase lag around bandwidth can be reduced. A second advantage of a second order

filter is the steep roll-off. This improves the high-frequency noise attenuation. A downside is that a resonance peak will be present. Both methods will be discussed next.

The cutoff frequency of the LL filter is set to $10 \times f_c$, with f_c the bandwidth of the system. Consequently, the tuning variable α is set to 1.1. The cutoff frequency f_f of the second order filter, see (4.4), is also set to $10 \times f_c$ and the damping d is set to 0.1.

$$C_{LPF}(s) = \frac{1}{\tau^2 s^2 + 2d\tau s + 1}, \text{ with } \tau = \frac{1}{2\pi f_f} \quad (4.4)$$

The Bode plots of the redesigned filters are shown in Figure 4.7. The magnitude peak of the LL filter is reduced to 1.2dB and is shifted to 1500Hz. This reduces the amplification of high-frequency noise compared to the previous LL filter. Therefore, it has less effect on the gain margin (GM) and MM, thus on the system stability.

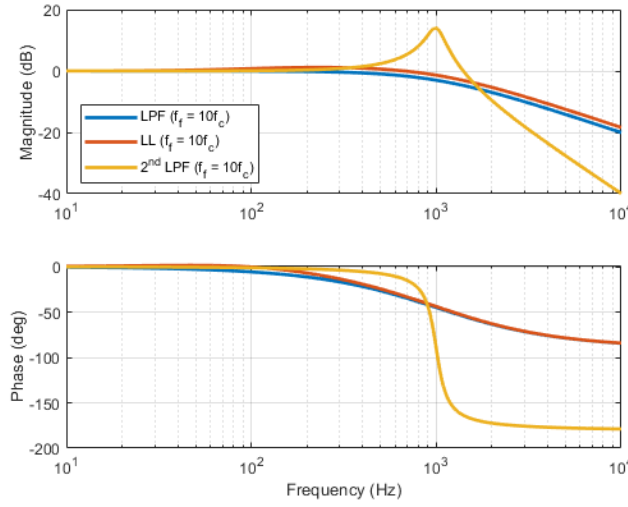


Figure 4.7: Bode plot of redesigned LL and LPF.

4.3.1. Transient response

The step responses of the redesigned filters are shown in Figure 4.8 and numerical data can be found in Table 4.1. Compared to the CR CgP without filter most filters improve the settling time of the reset controller. The settling time is not changes by the LPF. The PID and LPF do not improve the overshoot, PID even increasing it. The PID remains having the shortest settling time, but both the 2nd order LPF and the LL improve it as well. All filter improve the IAE and the RMS. Nonetheless, based on the transient response all filters are viable.

Table 4.1: Transient response characteristics of PID and CR CgP with different filters. For all filters the corner frequency is set to $10 \times$ bandwidth. *Res.* contains simulation results and *Impr.* contains the performance improvement compared to CR CgP without filter. The settling time is determined based on the 5%-reference band. The IAE is calculated for the interval $0 \leq t \leq 1s$, including the step. Th last column contains the noise RMS for $t > 0.2$.

Controller	Overshoot		Settling time (s)		IAE ($\times 10^{-7}$)		RMS ($\times 10^{-9}m$)	
	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>
PID	38.2%	-25.5%	0.0096	66%	1.05	29.1%	91.5	24.4%
No Filter	30.4%	-	0.0286	-	1.48	-	121.1	-
LPF	29.7%	2.41%	0.0286	0%	0.91	39.0%	96.3	20.5%
2 nd order LPF	23.0%	24.4%	0.0239	16%	1.34	10.0%	95.0	21.5%
LL	21.4%	30.0%	0.0205	28%	0.96	35.4%	96.2	20.5%

In Figure 4.9 the CPSD of the different filter designs. All filters reduce the total error of the CR CgP. Despite the improvement of the performance of the CR CgP, the PID outperforms the reset controller.

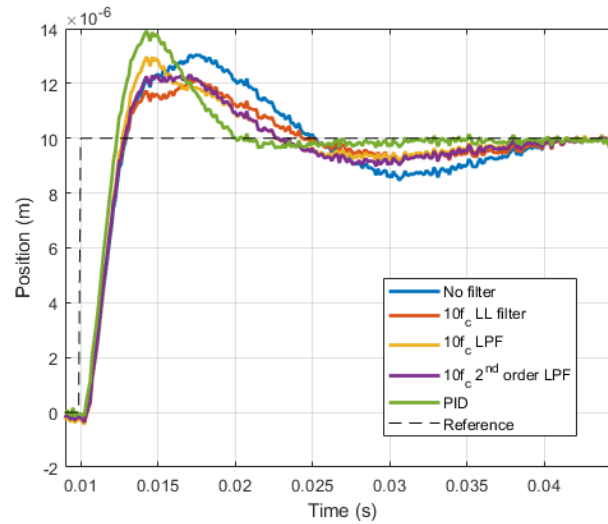
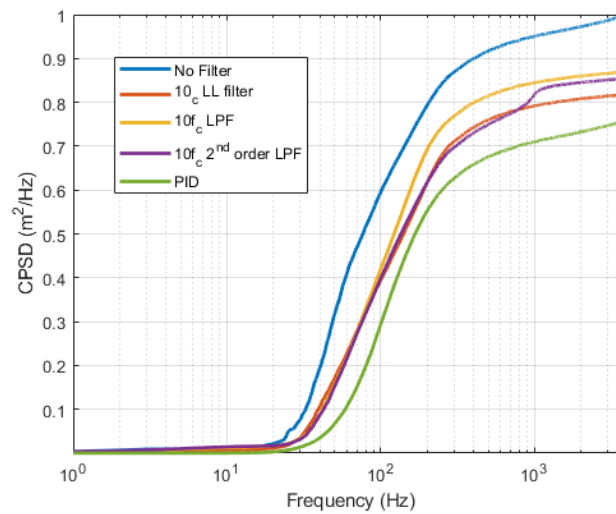


Figure 4.8: Step response of redesigned filters.

The total error remains the lowest. Up until 700Hz total error of the 2nd order LPF remains below the LL filter. The jump around 900Hz can be caused by the magnitude peak of the filter. This magnitude peak is caused by the damping used to increase the phase at bandwidth.

Figure 4.9: Cumulative power spectral density of the error of the redesigned filters. The CPSD is measured over a time of 1s and includes the step response as shown in [Figure 4.8](#)

4.3.2. Steady-state response

[Figure 4.10](#) shows the pseudo-sensitivity of the redesigned elements. It is clear that the peak created by the LL filter, see [Figure 4.4](#), is no longer present. Numerical data of the peaks can be found in [Table 4.2](#). Unfortunately, the peak of sensitivity is increased by all the filters, and the height of the peak of sensitivity is directly related to $1/MM$, see [\(A.5\)](#). However, the effect of these peaks cannot be seen in the step response. This might indicate a nonlinear effect. The second order LPF provides no improvement compared to the reset element without filter. Compared to the CR CgLp without filter, both the first order LPF and the LL filter improve the sensitivity up to $\approx 50\text{Hz}$. Moreover, at frequencies up to 6Hz those two filters cause the sensitivity to drop below the sensitivity floor of the PID controller. At mid-range frequencies the sensitivity of the LL with higher cutoff frequency is above that of the PID. However, it remains below the sensitivity of the CR CgLp without filter.

Table 4.2: Pseudo-sensitivity characteristics of PID and CR CgLp with different filters. *Res.* contains simulation results and *Impr.* contains the performance improvement compared to CR CgLp without filter.

Controller	Peak (dB)	
	<i>Res.</i>	<i>Impr.</i>
PID	3.83	-2.6%
No Filter	3.73	—
LPF	4.82	-29.3%
2 nd order LPF	4.10	-10.0%
LL	4.59	-23.0%

To analyse this the reset line signals at 15Hz and 35Hz are given in Figure 4.11.

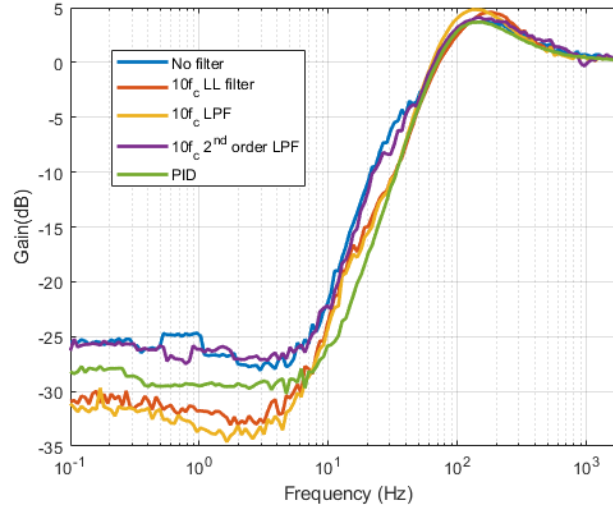


Figure 4.10: Pseudo-sensitivity of redesigned filters

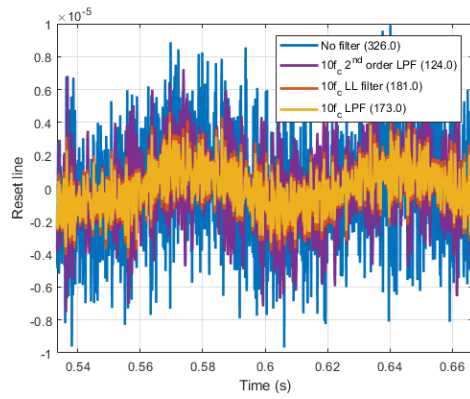
Figure 4.11a shows that the reset line signal is dominated by noise. For all filter configurations the reset line is crossing zero during the entire oscillation. This means that resets can occur at any time. The number of resets per oscillation is much higher than the ideal two resets per oscillation. At 35Hz the system dynamics start dominating the shape of the reset line, causing both the error and reset line signals to gain in magnitude. Figure 4.11b shows that the zero crossings are more concentrated towards the true zero crossings. Consequently, the resets instances are more concentrated around these zero crossings. This might improve the overall system performance. Looking at pseudo-sensitivity around 35Hz this is confirmed for the first order LPF and LL filter. However, the pseudo-sensitivity of the second order LPF remains high, even though the number of resets per oscillation drops. This indicates that the performance is limited by another cause. Finding the exact reason for this is beyond the scope of this thesis. Moreover, trajectory tracking using the second order LPF results in worse performance than the PID. Therefore, this is not a desired design.

4.3.3. Sub-conclusion

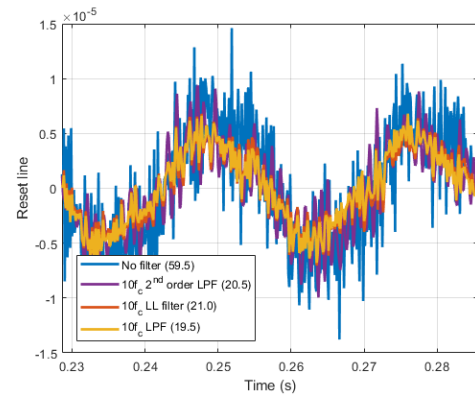
Based on the transient response all designed filters show improvements compared to the CR CgLp without filter. The pseudo-sensitivity shows that for trajectory tracking the first order LPF and LL filter are preferred, due to their lower low-frequency sensitivity floor. Of those two the LL has a lower peak of sensitivity, which indicates a higher MM. This makes the LL filter the favoured design.

4.4. Conclusion

The design of a LPF-based filter on the measurement line is a trade-off between noise attenuation and controller stability margins. With the cutoff frequency of the LPF with lead element (LL) set to $10 \times f_c$ the filter improves the response compared to the CR CgLp without filter. This without the negative



(a) Reset line signal for 15Hz reference.



(b) Reset line signal for 35Hz reference.

Figure 4.11: Reset line signals of of PID and CR CgLp with and without LL filter.
The number of resets per oscillation is given in brackets.

consequences of the magnitude peak caused by the lead filter. With the use of LPFs and lead filters the performance of a CR CgLp can be improved. The 2^{nd} order LPF and LL improve both the overshoot, settling time, IAE and RMS of the CR CgLp. For the steady-state response the proposed filters results in superior trajectory tracking performance to that of the PID up to 6Hz. However, this came at the cost of an increase of the peak of sensitivity, reducing the modulus margin. Despite this, the addition of a LL filter on the measurement line does improve noise attenuation of CR CgLp, reducing measurement noise induced performance deterioration.

5

Design of Kalman Filter

In this chapter the design and tuning of the Kalman filter (KF) are discussed. The KF is used to attenuate the noise on the position signal. As shown before the performance of the CR CgLP deteriorates when noise is present. Using a KF to fuse position data with accelerometer data can improve the position estimation [8, 16]. This could lead to an overall performance improvement of the CR CgLP. Ideally, leading to a CR CgLP that outperforms the a PID controller.

5.1. Motivation for Kalman filter

Already from its introduction in [17] the KF has proven its added value in noise attenuation [18, 19]. A common way to achieve this is to fuse data from multiple sensors. This data fusion results in a state estimate with a lower covariance compared to the covariance of the sensors themselves [8, 20, 21].

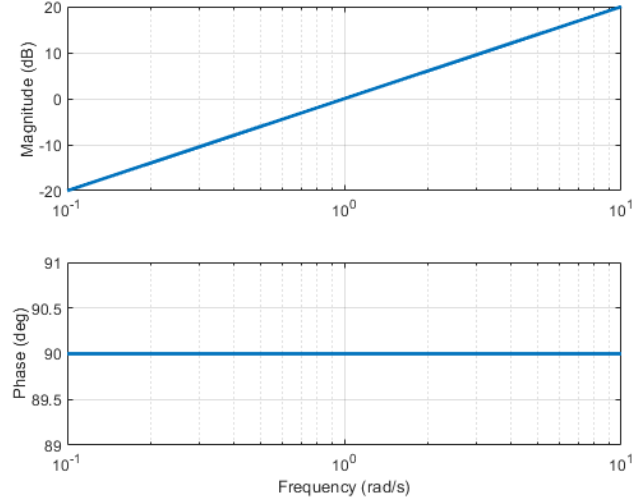
The performance of a KF depends among others on the accuracy of the modelled plant and the precision of the sensors [22]. The freedom of choosing sensors for a setup might be limited by the budget, available volume or required measurement techniques. A KF offers a solution when sensors specification and performance requirements do not meet. An example is the fusion of GPS data and accelerometer measurement. In this case precision of the position estimate is increased by the accelerometer and the GPS data compensated for the drift inherent to accelerometers [8, 16]. A seconds example of the application of a KF is the Apollo Onboard Trajectory Estimation, that combined data from the internal accelerometer with an optical space sextant that tracked the angles between stars. The sextant was used to filter out the drift of the accelerometer, while the accelerometer provided the system with more accurate measurements to counter the noise of the sextant angle measurements [23]. Both examples show a KF being applied to attenuate noise.

As in appendix B, performance deterioration of the CR CgLP is caused by the measurement noise. $L(s)$ is the lead element of the CR CgLP located before the reset element. This element differentiates the error signal $e(t)$, therefore it amplifies the noise. This can cause performance deterioration by provoking excessive resets.

In (B.2) the output signal of $L(s)$ is given. This signal contains a differentiator. Figure 5.1 shows the bode plot of a pure differentiator. From this figure it can be seen that high frequencies are amplified. Measurement noise contains mostly high-frequency signals. Therefore, it can be stated that a differentiator amplifies noise.

Differentiation could be omitted by constructing this signal from position data and integrated accelerometer data directly, see (5.1). In this equation $\ddot{z}$ is the accelerometer measurement. By directly constructing the signal x'_l way a KF could be omitted. Unfortunately, accelerometers are subjected to drift. After a some time (5.1) is no longer valid and thus this straightforward solution is not a feasible option.

$$x'_l(t) = e(t) + k \cdot \int_0^t \ddot{z} dt \quad (5.1)$$

Figure 5.1: Bode plot of differentiator $H(s) = s$

On the one hand, a KF is able to counter this drift by combining the accelerometer measurement with position measurement and an internal model. On the other hand, the precision of the position measurement can be improved using accelerometer data to calculate a position estimate [16, 18]. In [24] an accelerometer-aided KF (aaKF) is introduced which uses a MEMS-accelerometer to calculate a velocity estimate that is insensitive to quantization and modelling uncertainties. According to [20, 24, 25] this method provides a cost-effective solution that works well with low-resolution encoders. In [24] a kinematic KF was used to estimate the velocity based on accelerometer data. In this thesis no such KF is used, but the idea of velocity estimation by accelerometer data is used to design a KF.

5.2. Kalman Filter Theory

In both simulation and experimental testing the KF is implemented in discrete time. A discussion of the continuous time formulation of the KF is irrelevant to this thesis. This section gives a high-level description of the KF. The KF estimates the state using a linear stochastic difference equation (5.2a) and the measurement in (5.2b). The variables w_k and v_k are the process and measurement noise. The noises are assumed have a normal probability distribution, zero mean and independent of each other [17, 22, 26].

$$x_{k+1} = F_k x_k + G_k u_k + w_k \quad \text{with} \quad p(w) \sim N(0, Q_k) \quad (5.2a)$$

$$z_k = H_k x_k + v_k \quad \text{with} \quad p(v) \sim N(0, R_k) \quad (5.2b)$$

The algorithm consists of two steps, a prediction and a correction step. The prediction step is also denoted by the time update. The time update, given in (5.3), provides a projection of the the current state and updates the error covariance of this estimate. In this equation the $\hat{x}_{k+1}^-$ is the *a priori* state estimate given by a linear difference equation. Hence, F_k is the state transition matrix and G_k the input matrix. The *a priori* error covariance P_{k+1}^- depends on the process covariance matrix Q_k . This matrix contains the model uncertainties, like unmodelled dynamics and plant variations [17, 22, 27]

$$\hat{x}_{k+1}^- = F_k \hat{x}_k + G_k u_k \quad (5.3a)$$

$$P_{k+1}^- = F_k P_k F_k^T + Q_k \quad (5.3b)$$

Secondly, the *a priori* state estimate $\hat{x}_{k+1}^-$ then corrected in the measurement update. The first step of this update is to determine the Kalman gain K_k . K_k is calculated based on the coefficients of the linear difference equation, (5.2), and the measurement covariance R_k containing the covariances of the sensors. Next, the *a posteriori* state estimation $\hat{x}_k$ is calculated. In (5.4b) it can be seen that the

Kalman gain K_k determines how much the algorithm *trusts* the measurement compared to the internal model of the KF. A high Kalman gain results in an $\hat{x}_k$ particularly estimated upon measurement data. For a low Kalman gain the state estimated is predominantly determined by the internal model. Lastly, the estimation error covariance P_k is updated [22].

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad (5.4a)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-) \quad (5.4b)$$

$$P_k = (I - K_k H_k) P_k^- \quad (5.4c)$$

This concludes the general description of the KF. The next section discusses the specifics of the KF implemented in this thesis.

5.3. Design and Implementation

This section discusses aspects of the KF specific to this design. Examples of these specifics are the dummy variable v_0 and the definition of the internal model. The aaKF is designed to estimate a mass system defined in (B.6) using a linear stochastic difference equation (5.2). Rewriting (B.6) to state-space notation yields (5.5). In this equation $x(t)$ is the state vector representing the position, $u(t)$ is the applied force and $z(t)$ is the position measurement.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\dot{x}}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u(t) \quad (5.5a)$$

$$z(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \quad (5.5b)$$

The model in (5.5) is defined for continuous time. However, the KF is implemented in discrete time. Therefore, the model needs to be discretized. The discretization is performed using a zero-order hold (ZOH). All discretization methods offer an approximation of the continuous model, none of them give an exact match. In order to assure a close approximation, the sampling time should be at least 20×the bandwidth of the controller [10, 28].

5.3.1. Dummy variable

The aaKF attenuates the position measurement noise based on an internal model of the plant and accelerometer data. The accelerometer data is fed to the aaKF using discrete integration and a dummy variable, v_0 . This variable is the integrated accelerometer noise, also known as drift, see (5.6). In this equation $\hat{z}$ is the *a posteriori* state measurement, $\ddot{z}$ is accelerometer data, t is end time of one simulation step and v_0 is the accelerometer drift. The dummy variable is added as the third state to the linear stochastic difference equation. This allows the aaKF to update for the measured drift, since the drift can change over time.

$$\hat{z}(t) = \int_0^t \ddot{z}(t) dt + v_0 \quad (5.6)$$

When tuning the aaKF for optimal performance the process covariance matrix Q_k can be used to model the reliability of the accelerometer drift. Since the aaKF is implemented in discrete time, (5.6) should be rewritten to its discrete equivalent, see (5.7).

$$\hat{z}_k = T_s \sum_{m=0}^{k-1} \ddot{z}_m + v_0 \quad (5.7)$$

With the addition of state v_0 the state-space notation given in Equation 5.5 becomes:

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \\ \dot{v}_0(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ v_0(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \\ 0 \end{bmatrix} u(t) \quad (5.8a)$$

$$z(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ v_0(t) \end{bmatrix} \quad (5.8b)$$

5.3.2. Steady-State Kalman Filter

In case the R_k and Q_k are constant the Kalman gain K_∞ and process estimation error covariance P_∞ will converge. These matrices can then be pre-computed [22]. The downside of this is that the Kalman gain cannot change during a test, rendering the aaKF suboptimal if changes in either plant or sensors occur.

For this setup it is assumed that R is constant. This means that the RMS of the sensor noise does not change. Typically, this is not true. The precision of both capacitive distance sensors and accelerometer depends on the measurement frequency content. Environmental conditions such as temperature could also lead to a change in precision.

However, measurements on the physical setup have shown that noise levels are orders of magnitude higher than then sensor dynamics given in the specifications sheets. Fast Fourier transform (FFT) analysis of this noise measurement showed that the noise can be approximated by white noise. Ultimately, the measurement noise floor is higher than the noise levels specified in the data-sheets. Sensor dynamics and variations in sensors noise can be neglected. Therefore, it can be stated that during operation the noise RMS values do not change because of sensor dynamics. Besides this, it is assumed that plant dynamics do not change during testing, hence Q remains constant as well [19, 22].

The pre-computed Kalman gain K_∞ and estimation error covariance P_∞ are obtained using the Riccati Equation, see (5.9). This equation can be solved using by the `dlqe` command in Matlab [19].

$$K_\infty = F^T P_\infty H^T (H P_\infty H^T + R) \quad (5.9a)$$

With

$$P_\infty = F P_\infty F^T - F P_\infty H^T (R + H P_\infty H^T)^{-1} H P_\infty F^T + Q \quad (5.9b)$$

If changes of R and Q cannot be neglected a time-variant KF should be implemented. This makes the state estimation worse initially but after a short time K_k and P_k should converge. Downside of this method is the required computational power. For implementation on the test setup the required computational power should be kept to a minimum.

With the addition of the dummy variable v_0 , representing the drift, the discretized time-invariant, linear differential equation (5.8) becomes (5.10a) for the prediction step and (5.10b) for the correction step.

$$\begin{bmatrix} \widehat{x} \\ \widehat{\frac{dx}{dt}} \\ \widehat{v_0} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0.0001 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \widehat{x} \\ \widehat{\frac{dx}{dt}} \\ \widehat{v_0} \end{bmatrix}_k + \begin{bmatrix} 7.81 \times 10^{-9} \\ 1.25 \times 10^{-4} \\ 0 \end{bmatrix} [u]_k \quad (5.10a)$$

$$\begin{bmatrix} \widehat{x} \\ \widehat{\frac{dx}{dt}} \\ \widehat{v_0} \end{bmatrix}_k = \begin{bmatrix} \widehat{x} \\ \widehat{\frac{dx}{dt}} \\ \widehat{v_0} \end{bmatrix}_k + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \end{bmatrix} \left(\begin{bmatrix} z \\ \frac{dz}{dt} \end{bmatrix}_k - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \widehat{x} \\ \widehat{\frac{dx}{dt}} \\ \widehat{v_0} \end{bmatrix}_k \right) \quad (5.10b)$$

5.4. Accelerometer noise

The magnitude of the accelerometer noise influences the performance of the aaKF. In Equation 5.9a it can be seen that Kalman gain K is related to the measurement covariance matrix R . Matrix R contains the RMS of the accelerometer noise, see Equation C.1. The accelerometer noise is modelled based on measurement on the physical setup. This to obtain a noise profile that is representable to the

real setup. Details on this procedure can be found in appendix D. To prove the added value of the accelerometer to the aaKF, simulations using various accelerometer noise levels were performed. The noise level P_{noise} is given in g^2/Hz . With P_{noise} the input parameter of the Band-Limited White Noise block in Simulink. The simulations also include a KF without an accelerometer. In order to be of added values, the aaKF with accelerometer should outperform the KF without accelerometer, denoted by KF. This should then yield an upper bound on the measurement noise of the accelerometer. Besides this, a aaKF with ideal accelerometer is added. The P_{noise} is set to $0g^2/Hz$, this to find a bound on the performance of the aaKF.

The optimal value of one of the parameters of the process covariance matrix Q depends on the precision of the accelerometer. Therefore, this matrix is tuned for each noise power. Detail on the tuning process can be found in appendix C.

The simulations results can be found in Figure 5.2. An extensive analysis of these results can be found in appendix D. Figure 5.2b shows that for $P_{noise} \geq 4.35 \times 10^{-12} g^2/Hz$ the accelerometer does not have an added value. In that case a KF without accelerometer would be sufficient. However, for $P_{noise} \leq 4.35 \times 10^{-14} g^2/Hz$ the sensitivity floor can be reduced, improving trajectory tracking performance. The found results apply only to the system described in this report. For other systems a similar analysis should be performed.

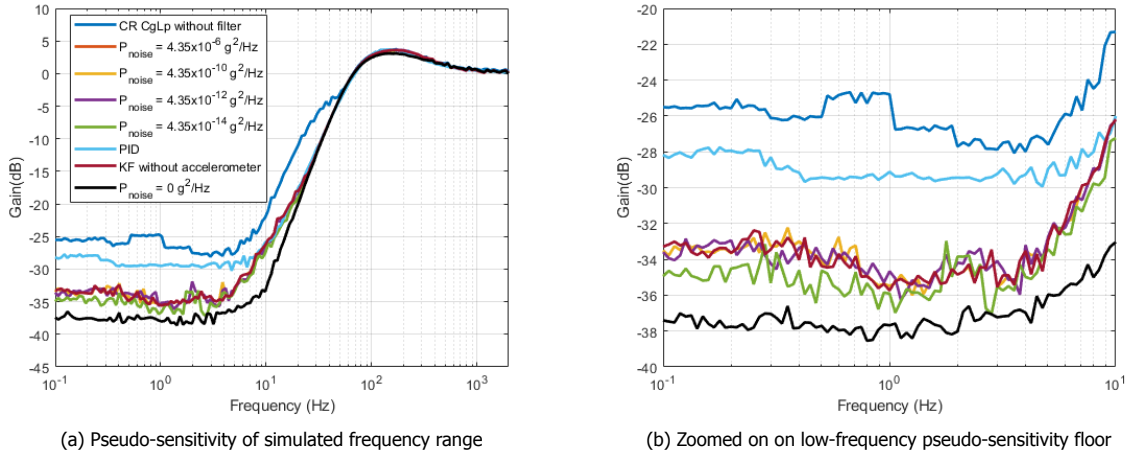


Figure 5.2: Pseudo-sensitivity of PID and CR CgLP with and without KF for different P_{noise} . P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.

5.5. Sensor configuration

This section focusses on the performance of a KF with and without an accelerometer. An extensive research on the use of an observer to improve the robustness to noise is presented in [9]. [9] showed that both the steady-state and the transient response of a CR CgLP are improved by the addition of a filter to the feedback line. The observer used in [9] is a Luenberger observer (LO) [29]. The LO in [9] uses an internal state model to reduce the impact of noise on the system performance. This in contrast to the KF used in this thesis that makes use of an additional accelerometer. To study the added value of the accelerometer a CR CgLP + KF with and without an added accelerometer are compared.

5.5.1. Transient performance

Figure 5.3 shows the step response of the CR CgLP in several KF configurations. Numerical data of the transient response can be found in Table 5.1. In Figure 5.3 the ideal accelerometer stands for an accelerometer with a P_{noise} of approximately zero. All filters of the CR CgLP reduce the overshoot and the settling time. The PID controllers have a superior settling time, but the overshoot is larger than that of the CR CgLP. The integral absolute error (IAE) and the RMS are both improved by the designed filters. Based on the transient performance the addition of an accelerometer to the KF has no benefit.

Table 5.1: Transient response characteristics of PID and CR CgLP with different filters. For all filters the corner frequency is set to $10\times$ bandwidth. *Res.* contains simulation results and *Impr.* contains the performance improvement compared to CR CgLP without filter. The settling time is determined based on the 5%-reference band. The IAE is calculated for the interval $0 \leq t \leq 1s$, including the step. The last column contains the noise RMS for $t > 0.2$.

Controller	Overshoot		Settling time (s)		IAE ($\times 10^{-7}$)		RMS ($\times 10^{-9}m$)	
	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>
PID	38.2%	-25.5%	0.0096	66%	1.05	29.1%	91.5	24.4%
PID + aaKF	37.5%	-23.2%	0.0096	66%	0.62	58.0%	91.5	23.9%
No Filter	30.4%	-	0.0286	-	1.48	-	121.1	-
KF	20.5%	32.5%	0.0239	16%	0.75	49.7%	95.4	21.5%
aaKF	20.7%	32.0%	0.0231	19%	0.69	53.4%	96.1	20.6%
aaKF (ideal)	20.3%	33.3%	0.0108	62%	0.79	47.1%	107.2	11.5%

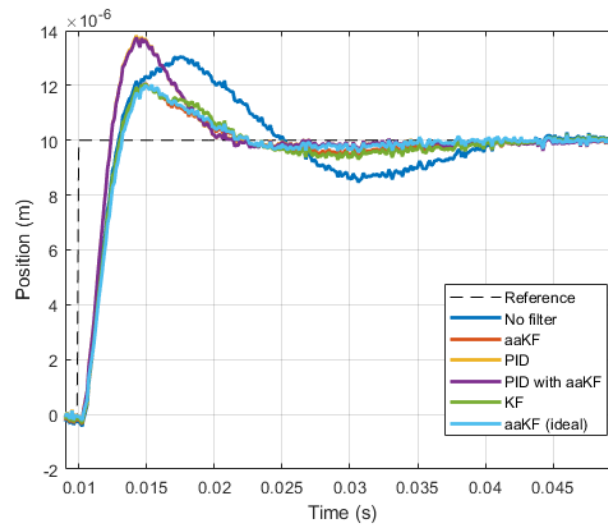


Figure 5.3: Step response of PID and CR CgLP with several KF configurations. P_{noise} of the accelerometer is based on measurements and P_{noise} of the ideal accelerometer is approximately zero. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$. Note that the results of both PIDs overlap.

In Figure 5.4 the cumulative power spectral density of the error of both the PID and CR CgLP in several configurations is shown. From this figure it can be seen that at all filters reduce the CPSD of the CR CgLP. The CPSD of the aaKF remains below that of the KF, but the difference is small. Moreover, the KF without accelerometer has already a lower total error than the PID.

5.5.2. Steady-state performance

Figure 5.5 shows the pseudo-sensitivities of the same controller discussed above. The aaKF and KF show superior trajectory tracking behavior compared to both the PID and CR CgLP without filter. This is due to the low-frequency sensitivity floor of the reset elements with KF being lower than that of the PID. This holds for frequencies up to 8Hz. The sensitivities of the KF with and without accelerometer are quit similar. This indicates that the addition of the used accelerometer yields no performance improvement. The sensitivity of the ideal accelerometer shows that with a more precise accelerometer the performance can be improved. There is no significant difference between the peaks. This means that the MM of the controllers is not changed with the addition of the KF.

5.6. Conclusion

For the used mass-plant model the KF and aaKF contribute to the noise robustness of the CR CgLP. The 5%-settling time can be reduced by 20%, compared to a CR CgLP without filter. The IAE and RMS can be improved with 50% and 20% respectively. The addition of the KF or aaKF improves the trajectory tracking performance. For low frequencies ($< 8Hz$) the sensitivity was reduced by $\approx 8dB$ compared to

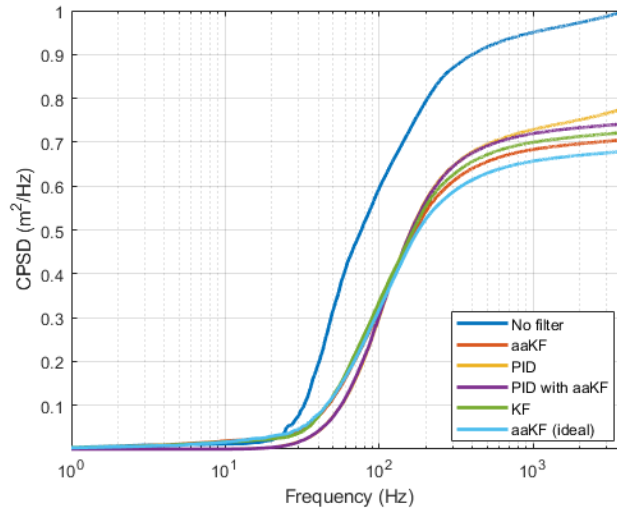


Figure 5.4: CPSD of PID and CR CgLp with several KF configurations. P_{noise} of the accelerometer is based on measurements and P_{noise} of the ideal accelerometer is approximately zero. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$. The CPSD is measured over a time of 1s and includes the step response as shown in Figure 5.3.

Table 5.2: Pseudo-sensitivity characteristics of PID and CR CgLp with different filters. *Res.* contains simulation results and *Impr.* contains the performance improvement compared to CR CgLp without filter.

Controller	Peak (dB)	
	<i>Res.</i>	<i>Impr.</i>
PID	3.69	1.1%
PID + aaKF	3.52	5.8
No Filter	3.73	—
KF	3.60	3.6%
aaKF	3.48	6.9%
aaKF (ideal)	3.08	17.3%

the sensitivity of the CR CgLp without filter. Both the transient and steady-state behavior showed that the currently used accelerometer does not improve the system performance. But the results of the simulations with the ideal accelerometer show that accelerometer with higher precision might improve the performance. Compared to a PID controller with similar bandwidth and phase margin the CR CgLp with aaKF improves the trajectory tracking up to $\approx 8Hz$. For frequencies higher than 8Hz the sensitivities of the PID and CR CgLp with aaKF or KF perform similar.

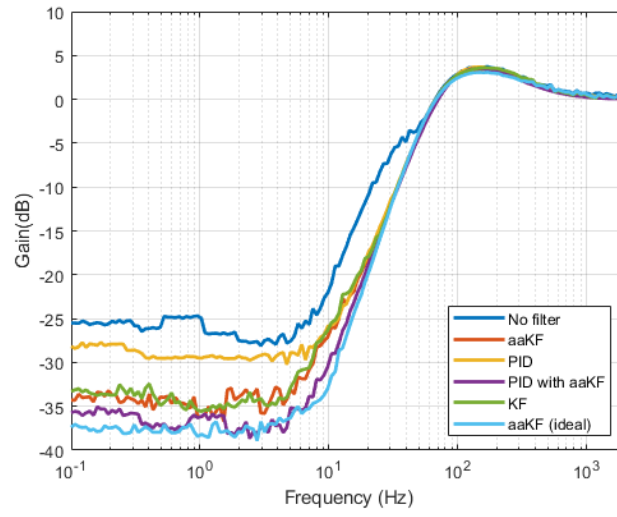


Figure 5.5: Pseudo-sensitivity of the CR CgLP + KF. The accelerometer P_{noise} is set to $4.35 \times 10^{-14} g^2/Hz$. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.

6

Kalman Filter vs Low-Pass Filter

This chapter compares the implementation effort and the performance of the two proposed designs, a accelerometer-aided Kalman filter (aaKF) and a low-pass + lead (LL) filter. Both the transient and steady-state performance are compared. The aim of this chapter is to make a comprehensive trade-off between the more complex aaKF and the more straight-forward LL filter. The filter designs are compared using simulation data, using the mass-model and controllers given in appendix E.

6.1. Implementation

The LL requires no extra sensors. This design can be implemented using only software and industry-preferred loop shaping. In contrast, the KF requires an internal model and careful tuning of its matrices. The tuning can be done by iteratively minimizing the mean squared error, see appendix C for further details. Besides this, if the system requirements demand an aaKF, this design requires both an accelerometer and its power supply to be installed on the system. This increases the costs and mounting an accelerometer on a setup can be challenging. If a KF without accelerometer is sufficient the complexity of implementation is reduced. Between the two designs, the implementation of the LL filter requires the least effort.

6.2. Transient performance

To study the transient response a step input is applied to the system. The results can be seen in Figure 6.1 and numeric data is given in Table 6.1. This table contains both the measurement data and the performance improvement compared to the CR CgLP without filter. Besides the PID controller all designs reduce the overshoot. The settling time is reduced for most design, the settling time of the LL filter is equal to that of the CR CgLP without filter. The IAE is reduced by all filters. The RMS of the position measurement noise is measured for 0.8s starting when the system is settled after the step. All of the designed filters attenuate the measurement noise similarly. Still, PID control results in a lower RMS level.

Table 6.1: Transient response characteristics of PID and CR CgLP with different filters. *Res.* contains simulation results and *Impr.* contains the performance improvement compared to CR CgLP without filter. The settling time is determined based on the 5%-reference band. The IAE is calculated for the interval $0 \leq t \leq 1s$, including the step. The last column contains the noise RMS for $t > 0.2s$.

Controller	Overshoot		Settling time (s)		IAE ($\times 10^{-7}$)		RMS ($\times 10^{-9}m$)	
	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>
PID	38.2%	-25.5%	0.0096	66%	1.05	29.1%	91.5	24.4%
No Filter	30.2%	-	0.0286	-	1.48	-	121.1	-
LL	20.0%	34.3%	0.0286	0%	0.97	34.8%	96.3	20.5%
KF	20.5%	32.5%	0.0239	16%	0.75	49.7%	95.4	21.2%
aaKF	20.7%	32.0%	0.0231	19%	0.69	53.4%	96.1	20.6%

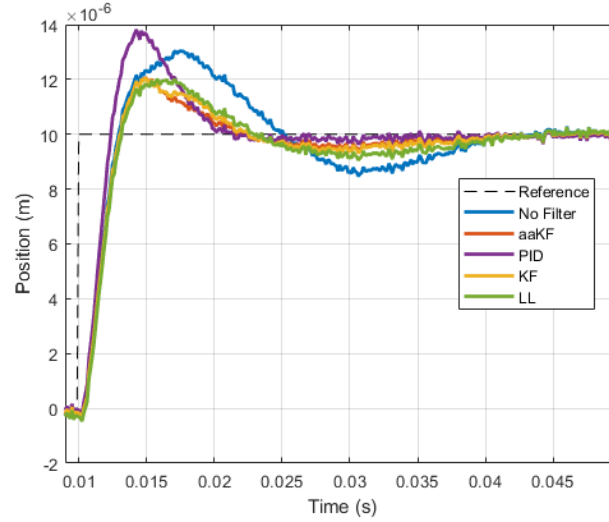


Figure 6.1: Step response CR CgLP with several filter configurations. P_{noise} of the accelerometer is set to $4.35 \times 10^{-14} g^2/Hz$. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$. Note that the results of both PIDs overlap.

A second measure for the transient performance is the CPSD. Figure 6.2 shows that the CPSD of the CR CgLP with KF and aaKF outperform both the CR CgLP without filter and with LL. Compared to the case without filter, the total error is reduced by 30% for the KF and 15% for the LL. Concluding, based on the step response and the CPSD, the aaKF and KF outperform the LL as far as transient behavior is concerned.

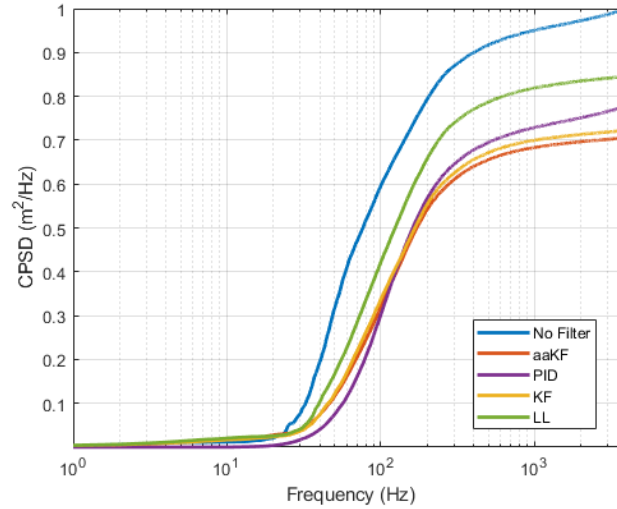


Figure 6.2: CPSD of CR CgLP with several filter configurations. P_{noise} of the accelerometer is set to $4.35 \times 10^{-14} g^2/Hz$. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$. The CPSD measurement starts x_s prior to the step as shown in Figure 6.1 and ends 1s later.

6.3. Steady-state performance

The pseudo-sensitivity is used to compare the steady-state performance between the proposed filter designs. The results of the simulation can be found in Figure 6.3. The peak of sensitivity is directly related to the stability margins of a system. A higher peak results in a reduction of the modulus margin (MM), see (A.5) in appendix A.2. Table 6.2 shows that both KF designs do not deteriorate the peak of sensitivity. The LL filter increases the peak of sensitivity, resulting in a reduction of the MM.

For trajectory tracking the pseudo-sensitivity should be as low as possible. In practice most reference

signals consists of low-frequency content. This means that for low frequencies, the pseudo-sensitivity should be as low as possible. Figure 6.3 shows that all filter designs reduce the sensitivity floor compared to the reset controller without filter. Up to 6Hz all filter perform better than the PID controller. Beyond this frequency the LL filter has a higher pseudo-sensitivity and is thus more sensitive to noise. The pseudo-sensitivities of the aaKF and KF remain below that of the PID.

Table 6.2: Pseudo-sensitivity characteristics of PID and CR CgLp with different filters. *Res.* contains simulation results and *Impr.* contains the performance improvement compared to CR CgLp without filter.

Controller	Peak (dB)	
	<i>Res.</i>	<i>Impr.</i>
PID	3.85	−2.7%
No Filter	3.75	—
LL	4.23	−13.0%
KF	3.66	2.3%
aaKF	3.58	4.5%

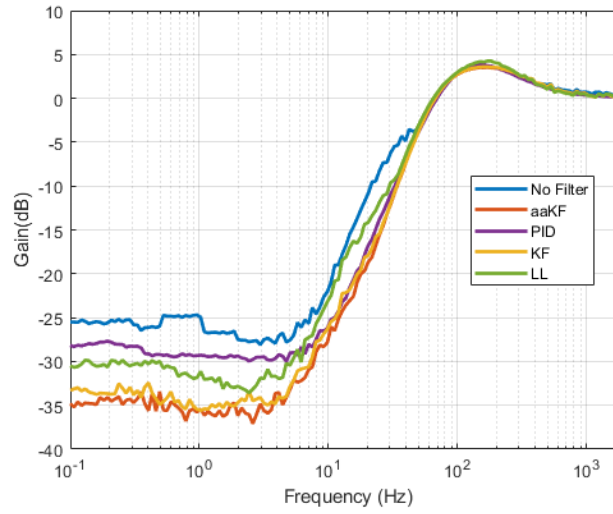


Figure 6.3: Pseudo-sensitivity of the CR CgLp with several filters. The accelerometer P_{noise} is set to $4.35 \times 10^{-14} g^2/Hz$. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.

6.4. Conclusion

Based on the transient response, all designed filters improve the transient performance compared to the CR CgLp without filter. Moreover, the aaKF and KF outperform the LL filter. The overshoot is reduced by 30% for all three filter designs. The settling time of the LL is equal to that of the CR CgLp without filter. The aaKF and KF improve the settling time of the reset element without filter by 19% and 16% respectively. The IAE of the CR CgLp could be improved by 50% when using an aaKF or KF. All three filter designs reduced the RMS by 20%. Compared to the CR CgLp without filter, PID settles 66% faster, but at the cost of a 25% increase of the overshoot. An accelerometer with lower noise power could further improve the performance of the aaKF, but for the accelerometer used in the simulation the addition of the accelerometer does not justify the implementation complexity of the aaKF compared to the KF. In choosing to utilize a KF over a PID, a trade-off is made between a reduction of overshoot and an increase in settling time.

The pseudo-sensitivity of the LL is inferior to that of the aaKF and the KF. Up until 30Hz the pseudo-sensitivity of the LL filter remains at least a couple of dB above the aaKF and KF. For frequencies below 5Hz all filter designs perform better than the PID. Hence, for trajectory tracking the CR CgLp designs are superior to PID. At mid-range the performance improvement of LL compared to the CR CgLp without filter almost disappears. The difference between the pseudo-sensitivities of the aaKF and the KF is thus small, that the complexity of the addition of the accelerometer is not justified. Hence,

considering both the transient and steady-state performance it is concluded that the CR CgP with KF is the preferred design.

7

Results

This chapter discusses the results of the tests performed on the physical setup. Appendix E provides a detailed description of this setup and the used controllers.

7.1. Transient response

To study the transient behavior of the system the results contain measurements of the step response. Figure 7.1 shows the step response of the physical setup for several controllers. Numerical data can be found in Table 7.1. Compared to PID, CR CgLP without filter reduces the overshoot with 90%. However, when the designed filters are applied the overshoot is increased with 12% for the LL filter and with 28% for the aaKF. The settling time of the PID is approximately equal to that of the CR CgLP without filter, moreover the LL filter reduces the settling time. Unfortunately, the CR CgLP using the aaKF does not settle to the 5%-reference band. In relation to the PID, the integral absolute error (IAE) is improved by the CR CgLP and an even better IAE is obtained when adding the LL filter. The LL filter does not deteriorate the measurement noise RMS of the CR CgLP, this in contrast to the aaKF which causes the RMS to increase. In conclusion, it can be stated that the LL filter improves the transient behavior of the CR CgLP and that this design is an improvement on PID.

Table 7.1: Transient response characteristics of PID and CR CgLP with different filters. *Res.* contains simulation results and *Impr.* contains the performance improvement compared to CR CgLP without filter. The settling time is determined based on the 5%-reference band. The IAE is calculated for the interval $44.99 < t < 45.99$ s, including the step. The last column contains the noise RMS for $53 < t < 55$.

Controller	Overshoot		Settling time (s)		IAE ($\times 10^{-7}$)		RMS ($\times 10^{-9}$ m)	
	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>	<i>Res.</i>	<i>Impr.</i>
PID	46.7%	-90.1%	0.347	-0.9%	5.43	-23.7%	72.7	44.5%
No Filter	24.6%	-	0.344	-	4.39	-	131.0	-
LL	27.7%	-12.5%	0.231	32.8%	3.81	13.1%	128.6	1.8%
aaKF	31.6%	-28.4%	-	-	5.51	-25.5%	194.9	-48.8%

In Figure 7.2 the cumulative power spectral densities (CPSD) of several controllers are shown. This measurement lasted for one second including a step input and the settling time afterwards. The CPSD shows that the aaKF does not improve the performance of the CR CgLP. The increase of the aaKF just below 0.9Hz causes the response not to converge the 5%-reference band after the step and to increase the total CPSD. This oscillation can be attributed to the resonance of the vibration isolation table. The LL filter reduces the total error of the CR CgLP by approximately 17%. For frequencies below 9Hz the error of the PID remains below the errors of the CR CgLP controllers.

Figure 7.3 shows the positions, reset lines and reset instances of the CR CgLP with the filter designs. Both filters reduce the number of resets, the LL filter with 28% and the aaKF with 75%. Hence, applying the filters reduces the noise on the reset line resulting in less resets.

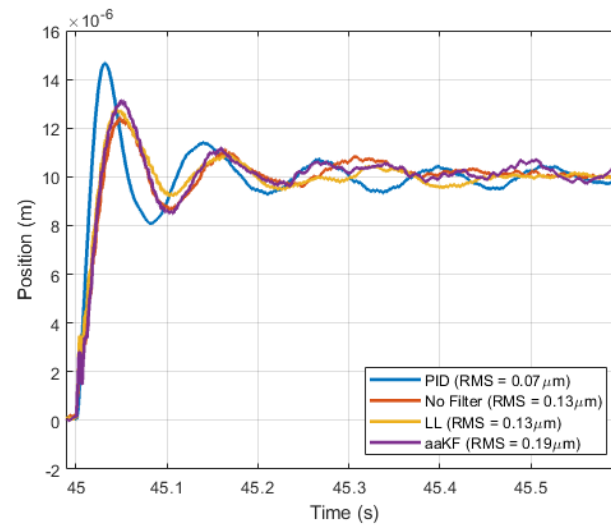


Figure 7.1: Step response of physical setup with 1 Hz bandwidth

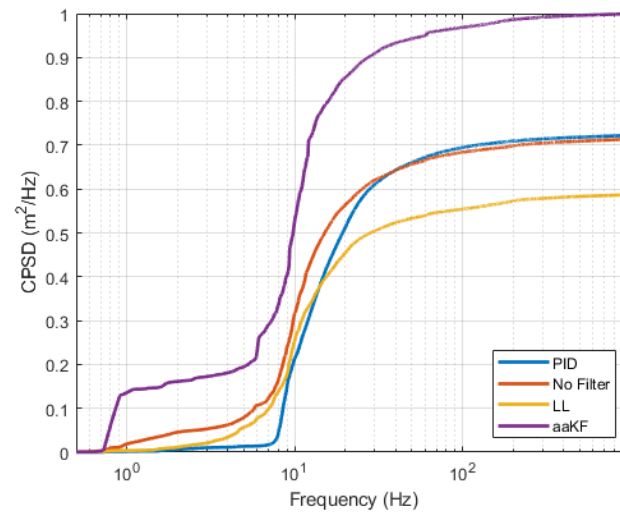


Figure 7.2: Cumulative power spectral density of physical setup with 1 Hz bandwidth. The duration of the measurement is 11s starting 0.01s before the step as shown in Figure 7.1.

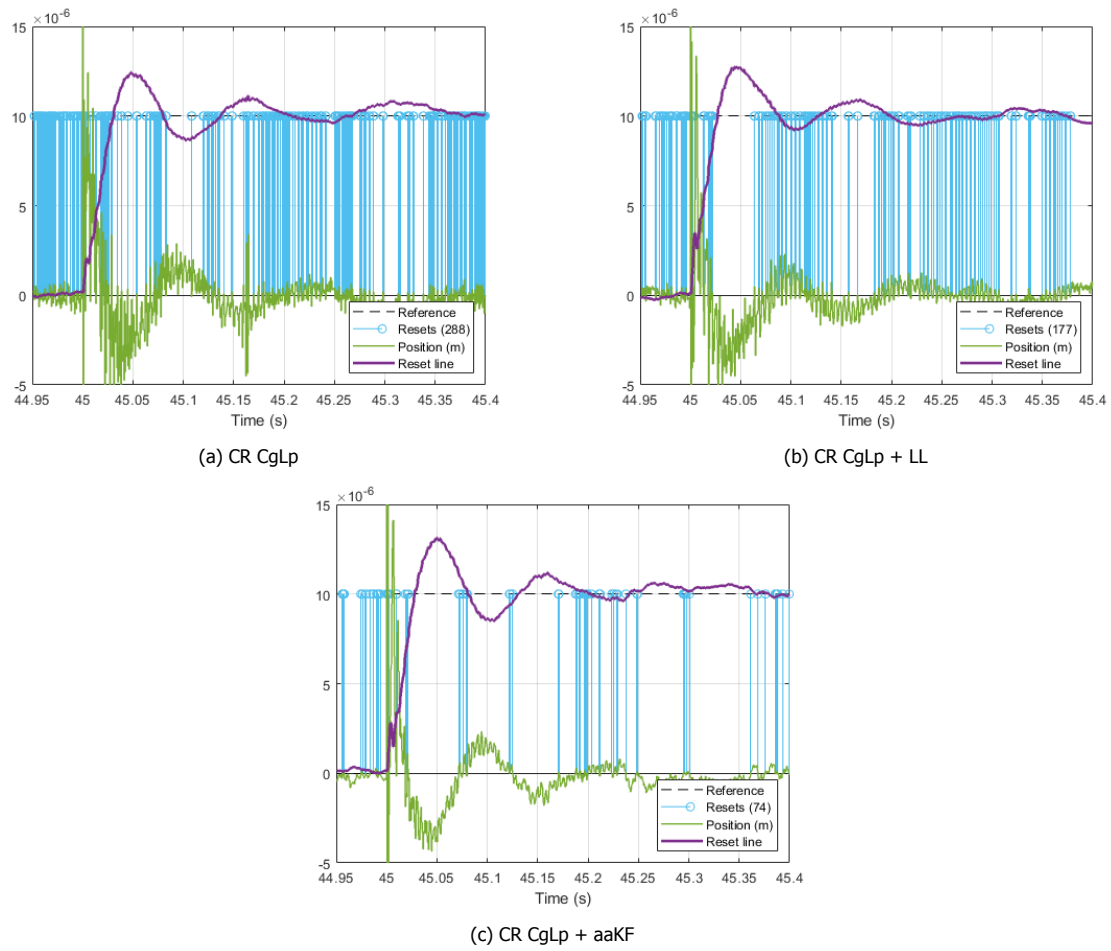


Figure 7.3: Step responses of the reset controller with reset line and reset instances.

8

Conclusion and recommendations

8.1. Conclusion

PIDs form the backbone of the precision-industry as far as controllers are concerned. The implementation of such a controller is relatively straightforward using loop-shaping techniques. Besides this, its stability properties are well understood. However, this controller is subject to limitations like Bode's gain-phase relationship and the waterbed-effect. These phenomena limit the performance of a PID controller. Reset control has proven to overcome these limitations. The introduction of a reset element that can be used in combination with a conventional PID approach paved the way for the application of reset control in industrial machines. This element is called the 'Constant in Gain, Lead in Phase' (CgLp). The Continuous Reset (CR) CgLp removes discontinuities from the control signal, but this element is more sensitive to measurement noise than other reset elements. This is due to CR CgLp taking the derivative of its input, amplifying noise. To improve measurement noise robustness two designs are proposed: a low-pass filter with a lead element and an accelerometer-aided Kalman filter.

8.1.1. Simulations

Simulation results have shown that (LL), Kalman filter (KF) and accelerometer-aided Kalman filter (aaKF) designs attenuate the measurement noise and improve the performance of the CR CgLp. Both transient response and steady-state response can be improved. The LL filter can reduce the low-frequency noise floor by approximately 5dB, improving the trajectory tracking behavior of the CR CgLp without filter. This however, at the cost of an increase of the peak of sensitivity by approximately 2dB, reducing the stability margins. Compared to the CR CgLp without filter the LL could improve the overshoot by about 34% and the IAE by 35%.

The performance of the KF depends on the precision of the accelerometer. For this setup the accelerometer is of added value only if $Noise_{RMS} \leq 0.2\mu g$. If this requirement is not met, the KF without accelerometer has superior performance. The aaKF has shown to improve the trajectory tracking behavior of the CR CgLp without filter because the low-frequency sensitivity floor was lowered by approximately 8dB. The overshoot could be reduced by 32% and the IAE by 50% when applying an aaKF.

Compared to PID the LL, KF and aaKF have better overshoot and IAE. The settling time and RMS of the PID remains superior to that of the reset element with either of the designed filters. At frequencies $< 6\text{Hz}$ the pseudo-sensitivity of the CR CgLp with either of its filters remains below that of the PID. Hence, trajectory tracking performance of the PID is worst. For frequencies $> 6\text{dB}$ the pseudo-sensitivity of the LL is inferior to that of the PID. The pseudo-sensitivity of the CR CgLp with aaKF or KF is similar to that of PID.

The performance of the aaKF compared to the KF is similar for all used metrics. Considering the implementation complexity of the aaKF compared to the KF and the lack of performance gain of the aaKF, it is concluded that the CR CgLp with KF is the preferred design.

8.1.2. Tests

Based on the tests it can be stated that the CR CgP with LL filter improves the transient response of the CR CgP without filter. This filter reduces overshoot, settling time and IAE without deteriorating the RMS. The aaKF causes performance deterioration of the CR CgP. The test showed that overshoot, settling time, IAE and RMS deteriorate when this filter is applied. Compared to PID, CR CgP without filter improves the overshoot and IAE. Between PID and CR CgP without filter the settling time is similar, but the RMS of PID is smaller than CR CgP. The CPSD measured over a step response of the CR CgP with LL remains below the CR CgP without filter. Concluding, besides the RMS the CR CgP with LL filter has improved transient response compared to CR CgP without filter and to PID control.

Repeatability of the step response measurements was poor. Most test results were corrupted by oscillations or even instabilities. The oscillations and instabilities are caused by resonances inherent to the mechanical design. These results could not be used to study the transient response because the measurements were dominated by oscillations. Most of the measurements shown in chapter 7 belong to test runs that did not trigger resonances. In order to formulate firm conclusions based on the test results, the results should be more reliable.

Unfortunately, the pseudo-sensitivity could not be measured due to inherent mechanical properties of the used setup. This means that the steady-state performance could not be tested and used to validate simulation results.¹

8.1.3. Simulations vs. tests

First, difference between the simulation and test results could be due to the modelling errors in system dynamics. Simulations used a mass-plant model not containing resonances at 10 Hz and 100 Hz. The 10 Hz resonance is caused by the vibration oscillation table and the 100 Hz resonance is a known resonance in the system. They could be attenuated by either increasing the total gain of the controller. Increasing the broadband gain also increases the high-frequency content, reduced stability margins. At high frequencies the system contains some resonances. Most of them could be attenuated using a notch filter, see appendix E.3.1 for the details. Unfortunately, the resonance at 100 Hz could not be attenuated by a notch filter without reducing the phase below the phase margin below 30°. Over time a rubber part has dried out and should be replaced to damp this resonance. As explained¹ this limits the bandwidth of the controller.

Second, different controllers were used for the simulations and the tests. The simulations were done using controllers with a 100 Hz bandwidth and the test with 10 Hz bandwidth controller. The stability margins also differed between tests and simulations, see Table E.3 and Table E.5. This alone causes differences in overshoot, rise time and other parameters, making comparing the two cases difficult.

8.2. Recommendations

This section contains recommendations for further research.

Internal KF model

The internal KF model assumes a pure mass-plant. Frequency response data has shown that this only holds for frequencies between 10 Hz and 100 Hz. The performance for frequencies above 100 Hz is not relevant because it is way above bandwidth. For frequencies lower than 10 Hz, a more advanced model could improve the performance of the KF. A more accurate KF could reduce the error leading to a lower pseudo-sensitivity at low frequencies.

White noise assumption

The Simulink model assumes white noise sources, meaning that the contribution to the total noise is equal for each frequency. This is typically not the case. For example, accelerometer noise tends to

¹When tuning the controllers, mechanical resonances limited the optimization of the controllers. For low bandwidth controllers, up to 10 Hz, low-frequency vibrations caused oscillatory behavior. One source is the vibration isolation table on which the system is mounted. Typically, this issue is resolved by increasing the bandwidth. More gain at low frequencies should allow the controller to dampen those resonances. However, when increasing the controller bandwidth high frequency resonances dominated the response. Notch filters could be used to dampen these, but the measured resonances seem to lack any damping. In other words, their peak goes to infinity. Notch filters damped the resonances slightly but not to a satisfactory level. Time constraints withheld an improvement of the mechanical design, so the only option was to settle for a lower bandwidth.

increase for low frequencies. Modelling the noise sources more realistically could impact the simulation results.

Time-regularization

In [11] time-regularization was introduced to reduce the performance deterioration caused by quantization. The designs proposed in this thesis focus on performance deterioration caused by measurement noise. The two methods address different error sources, so it could be useful to study if combining time-regularization with the designed filter could lead to further performance improvements.

Multiple DoFs

Both physical tests and simulations were performed in a single DoF. Measurements performed by DEMCON showed that the crosstalk between the DoFs was below 1%. Based on this result the assumption was made that the system could be controlled using 6 SISO controllers. Implementation of the reset elements in MIMO-structures is not studied, but could be valuable since not all systems can be decoupled into multiple SISO systems.

Feed forward control

In industry, most precision-positioning systems contain a feed back and a feed forward controller. The feed forward controller is supposed to provide the majority of the control action. The purpose of the feed back controller is to compensate for modelling inaccuracies and disturbances. Up until now, reset control has not been studied in combination with a feed forward controller. It could be useful to industry to understand and be able to predict the behavior of a reset controller when used in a control structure containing a feed forward controller.

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Preliminaries

This appendix is dedicated to preliminaries that were not included into the literature review. The first part covers theory on the sensitivity and complementary sensitivity. The second part describes limitations of linear control. Next, discretization of controllers and filters in general are discussed. This is followed by a description of various noise sources in a high-tech system.

A.1. Reset line filtering

A downside of this configuration is that the filter changes the open-loop of the system. [9] proposes a similar filter but located on the reset line. In this configuration the feedback-loop is not affected by the delay of the filter, only the reset line itself is filtered. Besides this the system performance is less sensitive to model inaccuracies and incorrect tuning of the Kalman filter. The downside is that the error signal contains the unfiltered measurement noise. This limits the low-frequency pseudo-sensitivity floor of the reset system. When the aim of the system is to improve trajectory tracking, the design should be optimized towards this goal. Therefore, it is acceptable to improve trajectory tracking at the cost of reducing robustness to model inaccuracies.

A.2. Sensitivity and complementary sensitivity

The sensitivity function gives the relation between measurement y and output disturbance n , see (A.1). With n containing sensor disturbances and noise. $L(s)$ is the open-loop transfer function. For ideal systems, systems without noise, the sensitivity drops to minus infinity for ω going to zero. However, when there is noise present this can be visually interpreted as a floor in the sensitivity that cannot be crossed. When dealing with white noise the gain of this noise is constant for the entire frequency range. At low frequencies this noise dictates the minimal sensitivity of a system. Consequently, the low frequency error is determined by the noise in the system. For high frequencies the sensitivity goes to 0dB. This means that the system does not respond to these frequencies. The peak of the sensitivity denotes the maximum disturbance amplification. Hence, when optimizing a controller, the peak of sensitivity should be lowered as much as possible [13].

$$S(s) = \frac{y}{n} = \frac{1}{1 + L(s)} \quad (\text{A.1})$$

A second tool to study the closed-loop behavior of a system is the complementary sensitivity function, see (A.2). The complementary sensitivity function describes the system's ability to follow reference r . For low frequencies the function approaches 0dB, meaning that low frequency references can be followed with gain of 0dB. A second property of $T(s)$ is that it has a high frequency roll-off. In other words, high frequency content is attenuated. The steeper the roll-off the stronger the effect of high frequency noise on the system is reduced [13].

$$T(s) = \frac{y}{r} = \frac{L(s)}{1 + L(s)} \quad (\text{A.2})$$

Looking at equations (A.1) and (A.2) it can be seen that they relate to each other. There relation is given in (A.3).

$$T + S = 1 \quad (\text{A.3})$$

At low frequencies the sensitivity should be as low as possible. In practice most reference signals for trajectory tracking are low frequency. The lower sensitivity at low frequencies the higher the precision at the frequencies. If a controller design is optimized for reference tracking the low-frequency sensitivity should be as low as possible yielding the complementary sensitivity as close to 0dB as possible.

The peak of sensitivity (A.4) is related to the stability of the system through the MM, see (A.5). As a rule of thumb, the inverse of the MM should be smaller than 8dB. This to ensure robust control [19, 36].

$$M_s = \max_{\omega} |S(\omega)| \quad (\text{A.4})$$

$$MM = \frac{1}{M_s} \quad (\text{A.5})$$

Both the GM (A.6) and phase margin (PM) (A.7) can also be written as a function of the maximum sensitivity [36]. These inequalities give an upper bound on the stability margins.

$$GM \geq \frac{M_s}{M_s - 1} \quad (\text{A.6})$$

$$PM \geq 2 \sin^{-1} \left(\frac{1}{2M_s} \right) \quad (\text{A.7})$$

A.3. Limitations of linear control

Two limitations of linear control are the waterbed-effect and Bode's gain-phase relationship. The waterbed-effect can be explained by Bode's sensitivity integral, see (A.8) [3, 37, 38]. In Equation A.8 p_k is the number of right half plane poles of the open-loop transfer function. This integral shows that that the sensitivity function cannot be made small for all frequencies. Decreasing the sensitivity at low frequencies, increases the sensitivity at high frequencies.

$$\int_0^{\infty} \log |S(j\omega)| d\omega = \pi \sum p_k \quad (\text{A.8})$$

If the system in minimum-phase Bode's gain-phase relationship can be approximated by (A.9) [38]. This relation states that for a slope of -20dB/decade and -40dB/decade , the pahse is -90° and -180° respectively. For more robustly stable control both the phase should be as large, and slope should be as low as possible around bandwidth. This forms a conflict which results in a trade-off between the two. For example, if the phase margin is increased to improve stability, the gain is increased as well. This results in an increased amplification in high-frequency content, reducing the achievable precision.

$$\angle L(j\omega_0) \approx 90^\circ \times \frac{d \log |L(j\omega)|}{d \log \omega} \quad (\text{A.9})$$

A.4. Controller discretization

As stated before, the controllers are designed in continuous time domain, but they are implemented in discrete time domain. Hence, they need to be discretized. Several discretization methods exist, examples of the most common methods are the ZOH, Matched Pole-Zero (MPZ) and Tustin's Method [10]. Each of these controllers have their pro's and con's. As a rule of thumb, the ZOH is used for discretizing the plant when designing the controllers in discrete time domain. Tustin's method is preferred to be used when converting the continuous time controller to the discrete time domain. This because this method maps the imaginary axis on the unit circle of the z -plane. Meaning that the magnitude and phase can be matched closely. The transformation between the s -plane and z -plane is given in (A.10). In this equation T_s is the sample time.

$$s = \frac{2}{T_s} \frac{z - 1}{z + 1} \quad (\text{A.10})$$

It is possible that the magnitude and phase of the continuous time and discrete time model do not match. This can be remedied using pre-warping. To do this (A.10) should be replaced by (A.11) [13]. In this equation ω_0 is the frequency for which a closely matched magnitude and phase are desired.

$$s = \frac{\omega_0}{\tan(\omega_0 T_s / 2)} \frac{z - 1}{z + 1} \quad (\text{A.11})$$

The design method used during this project is called the discrete equivalent. With this method the controller is first designed in the continuous time domain. This allows for the use of frequency-response design methods, like loop shaping. When the controller meets the requirements in continuous time, the filter can be discretized using either (A.10) or (A.11).

The controllers are implemented using Infinite Impulse Response (IIR) filters. IIR filters are preferred over Finite Impulse Response (FIR) filters because they have a better approximation of the frequency behaviour of an analogue filter [13]. Besides this, FIR filters are computationally expensive and can induce group-delays. The used structure is the Direct-Form 2. Compared to the Direct-Form 1 this saves half of the registers, which reduces the required memory [13]. When using numeric floating-point numbers are used the Transposed Direct Form is beneficial. The Transposed form minimized the effect of numeric swamping [40].

A.5. Noise sources

The essence of a measurement system is to convert a physical quantity into a measurable value. Such a system consists of a few steps. First, a sensing device converts the physical quantity into an analog electrical signal. Second, the processing device converts the analog signal into another analog signal or a digital signal, which can be used for active control. Each of the above-mentioned steps introduce some errors and noise into the system [13]. The remainder of this section describes a couple of noise sources and possible remedies. It is not a complete list, but it gives a brief overview.

A.5.1. Electrical noise

The precision of the digital components in the system should not determine the final resolution of the sensors. Their precision should be chosen appropriately. This precision limit is often determined in the analog domain [41]. This noise is generally broadband, it is usually a combination of a whole range of frequencies. To improve the precision, a low-pass filter could reduce the higher frequency noise in the signal. However, this filter could reduce the system phase margin.

A.5.2. Saturation

Saturation can take place on several place in the system. Every ADC operates on a specific voltage range. When a signal exceeds this range, the digital output does not change with the subsequent analogue input, it saturates. Therefore, the voltage output range of the sensor should not exceed the voltage input range of the ADC. This error could be prevented by either choosing an appropriate ADC or apply signal conditioning steps or amplifying the measurement signal with a value smaller than 1. The sensors used in this setup, have a maximum output of $\pm 10\text{V}$. Fortunately, ADCs matching this

range are widely available [42]. A second possibility is that the actuators saturate. They cannot deliver an infinite force, and their range is also limited. When the controller tries to exceed those limits, the system experiences sudden discontinuities.

A.5.3. Grounding

The ground potential may differ from location to location. If one system has multiple connections to the ground, there may exist a potential difference between those connections. This can induce currents, which will cause noise in the measured signals. Therefore, double grounding should be avoided [13]. Grounding loops can result in a 50 – 60Hz oscillation in the system. This could potentially impact the precision and therefore the performance of the system. When the loop cannot be found and removed a notch filter can dampen this oscillation.

B

Excessive resets

When stepping from a controlled MATLAB/Simulink environment to the real world, the system gets to deal with phenomena like external disturbances and measurement noise. The CR CgLp has shown to be very sensitive to noise. This causes the performance to deteriorate rendering the benefits it has to offer over the PID to vanish. Or even causing it to perform worse than the PID. The aim of this chapter is to identify some causes of excessive resets and to prove that the excessive resets cause the performance of the CR CgLp to deteriorate.

B.1. Ideal situation

For the ideal situation the reset elements are modelled in continuous time without any noise present. In [Figure B.1](#) the response of a CI to a sinusoidal input. From this figure it can be seen that there are two resets per input period.

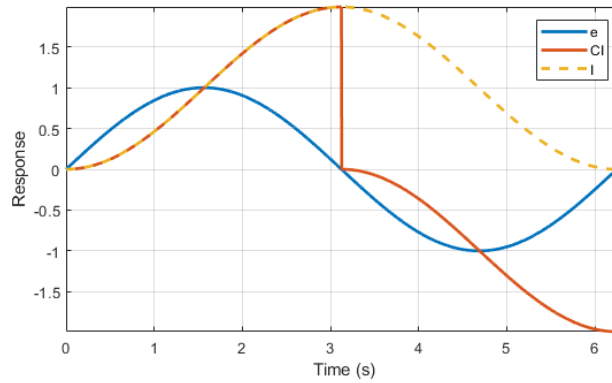
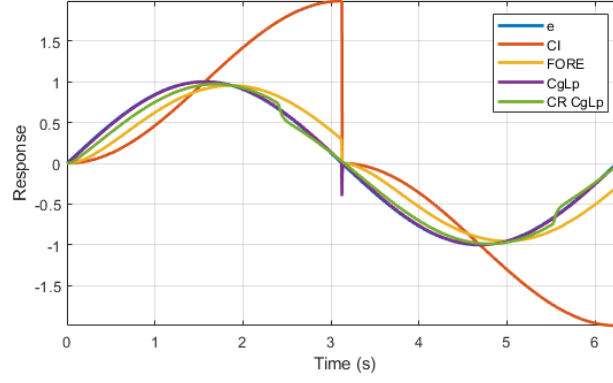


Figure B.1: CI and integrator (I) response $u(t)$ for sinusoidal $e(t)$

Below, in [Figure B.2](#), the responses of some other reset controllers are given. From this figure it can be observed that all reset controllers perform two resets per oscillation. It can be seen that the timing of the reset of the CR CgLp differs from the others. This is caused by the lead filter of the CR CgLp. Due to this filter the reset law should be approximated as in (B.2) [4]. The reset law of the other reset elements in [Figure B.2](#) is given by (B.1).

$$e(t) = 0 \quad (\text{B.1})$$

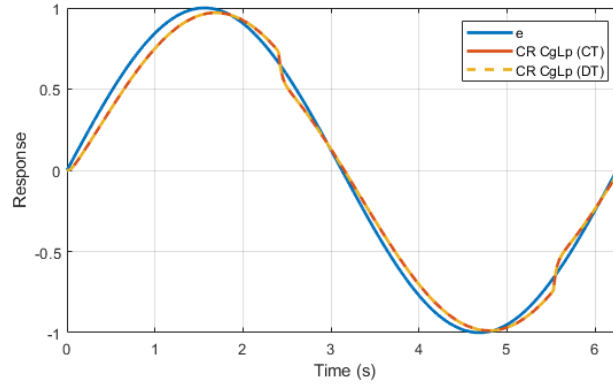
$$x_l(t) = \frac{\dot{e}(t)}{\omega_l} + e(t) = 0 \quad (\text{B.2})$$

Figure B.2: CI, FORE, CgLP and CR CgLP response $u(t)$ for sinusoidal $e(t)$

The remainder of this chapter addresses two mechanisms that increase the number of resets: discretization, quantization and measurement noise.

B.2. Discretization

Most industrially used controllers are implemented digitally. While the controllers are designed in continuous time they have to be converted to discrete time. This is usually done using Tustin's Method. When using a sampling frequency that is sufficiently higher than the bandwidth of the system this process should not have an impact on the system performance. A common rule of thumb is $f_s > 10f_{BW}$, with f_s sampling frequency [10]. From Figure B.3 it can be seen that the two responses are similar. The only difference is the time delay caused by the sampling period, $T_s = 1/f_s$, which was $1.25E - 4s$ during this simulation.

Figure B.3: Continuous time (CT) and discrete time (DT) CR CgLP response $u(t)$ for sinusoidal $e(t)$

B.3. Quantization

Digital computers cannot handle continuous signals. Measurement signals have to be quantised. This process is performed by an analogue-to-digital converter (ADC). In [11] and [12] it is shown that quantization of the measurement signal causes pseudo-resets. When the reset signal $x_l(t)$ is within $\frac{1}{2}Q \geq x_l(t) \geq -\frac{1}{2}Q$ crossings of the zero-line may occur. Q is the digital quantization level as defined in (B.3). The impact of quantization on the performance of a reset system can be studied using the sensitivity. The physical test setup uses a 16-bit ADC. In order to be able to compare test and simulation results, this number of bits is used in both instances.

$$Q = \frac{Range}{2^{\#bits}} \quad (B.3)$$

For linear system the sensitivity is given by (B.4) [13]. $L(s)$ is defined as the open-loop transfer function. However, this relation does not hold for reset systems. Reset systems contain higher-order harmonics. When using (B.4) to describe the sensitivity of reset systems only the describing function (DF) is considered. This will only give a rough approximation of the true sensitivity [14]. Alternatively, pseudo-sensitivity is used as defined in (B.5) with $r(t) = A_n \sin(2\pi\omega t)$ [7].

$$S(s) = \frac{1}{1 + L(s)} \quad (\text{B.4})$$

$$\hat{S}(\omega) = \frac{\|e(t, \omega)\|_{\infty}}{\|r(t, \omega)\|_{\infty}} \text{ for } t \geq t_{ss} \quad (\text{B.5})$$

For the experimental analysis a CR CgLP is used in combination of a mass-system (B.6). In (B.6) m is 22.6kg. A mass-system is used because it has a predictable sensitivity function and no resonances. The used mass is equal to that of the physical setup. The parameters of the controller are given in Table E.2.

$$P(s) = \frac{1}{ms^2} \quad (\text{B.6})$$

Before studying the impact of quantization on the CR CgLP the CgLP is discussed. This to observe the difference in behavior between the reset laws as defined in (B.1) and (B.2). Figure B.4a shows the simulated pseudo-sensitivity of the CgLP for both the ideal and quantized case. It can be seen in this figure that for low frequencies the quantized pseudo-sensitivity approaches the -96dB line. This shows that for these frequencies the sensitivity is dictated by the quantization level. The measurement signal is quantized with a 16-bit ADC. This yields a minimum precision of $20\log_{10}\left(\frac{1}{2^{16}}\right) = -96\text{dB}$. For frequencies of 2Hz and higher the pseudo-sensitivities are similar. This is explained by the system dynamics being dominant at higher frequencies.

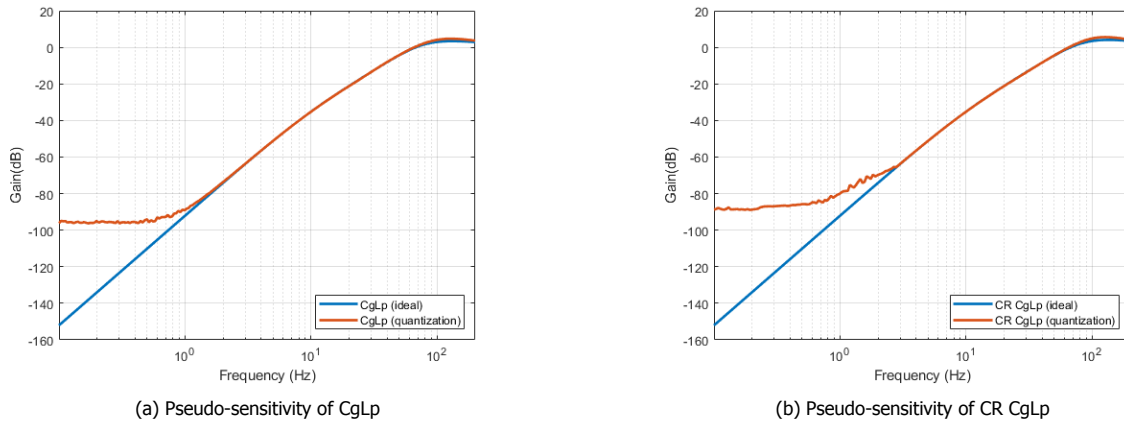
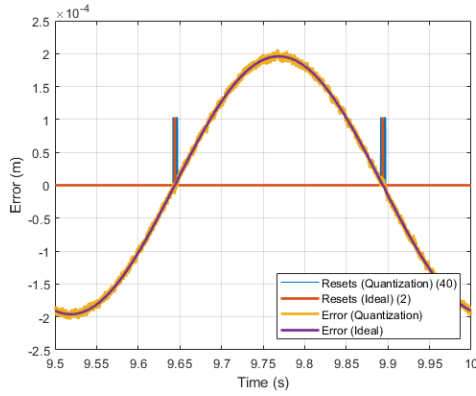
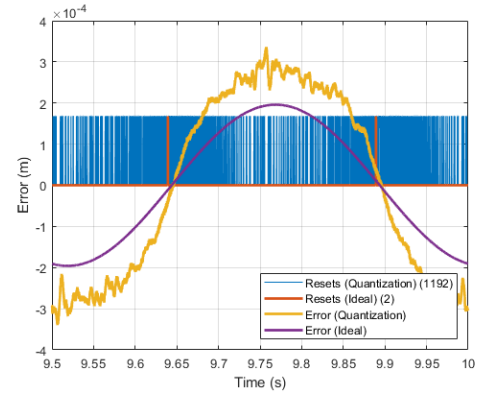


Figure B.4: Pseudo-sensitivity with and without quantization of measurement data. Measurement is conducted using a reference signal with amplitude of 1m and position sensor sensitivity of 1m/V.

In Figure B.4b the pseudo-sensitivities of a CR CgLP using quantized and ideal data are shown. For mid-range frequencies the two results do not differ, the system dynamics are dominant for these frequencies. At lower frequencies the CR CgLP using quantized measurement data converges to $\approx -89\text{dB}$. The CR CgLP not being able to converge to -96dB can be contributed to an excessive amount of resets per period. Figure B.5b shows the error signal and the reset instances for both simulations. Comparing this to Figure B.5a it can be seen that the resets per period of the CgLP is 30 times lower than the resets per period of the CR CgLP. Between 1Hz and 3Hz the pseudo-sensitivity deviates from the $+2$ -mass line. According to [12] the increased magnitude is caused by the quantization induced excessive resets.



(a) Steady-state error and reset instances for the CgLp



(b) Steady-state error and reset instances for the CR CgLp

Figure B.5: Steady-state error signal of ideal and quantized measurement data with input reference at 2Hz

When the mean of the error is within $\frac{1}{2}Q \geq e(t) \geq -\frac{1}{2}Q$ resets can occur, which can result in significant increase of the error [11]. Figure B.6 shows the reset line and the error signal of both the CR CgLp and the CgLp. The reset line is equal to the error signal for a CgLp. Hence, only one signal is shown in for the CgLp. During the entire period the reset line of the CR CgLp keeps crossing zero yielding excessive resets.

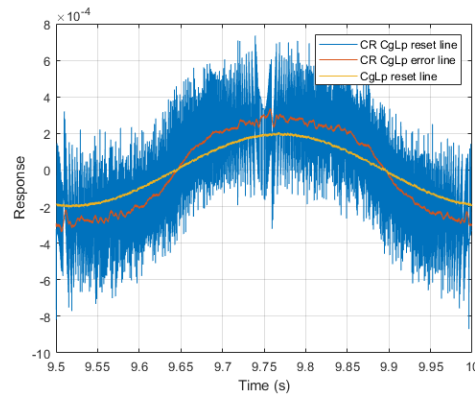


Figure B.6: Steady-state reset line and error signal using quantized measurement data with input reference at 2Hz

B.3.1. Sub-conclusion

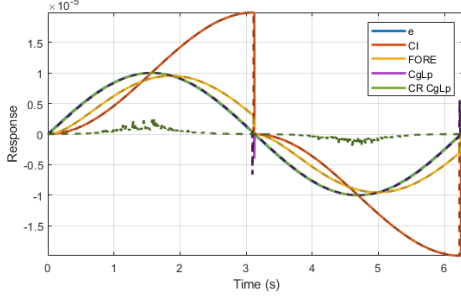
Quantization of measurement data limits the low-frequency pseudo-sensitivity. For low-frequencies the amplitude of the system dynamics is smaller than the least-significant-bit of the ADC. This causes the ADC to determine the low-frequency sensitivity floor. Elements with a lead filter have a sensitivity floor that is a couple of dB higher. This is due to the presence of a differentiator in the reset law. When the true reset signal is within $\frac{1}{2}Q \geq e(t) \geq -\frac{1}{2}Q$ excessive resets can occur. These resets cause the system to move. This movement causes the sensitivity floor to be higher than the sensitivity floor of other reset elements.

B.4. Measurement noise

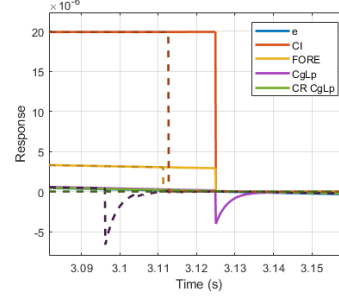
A second cause for an increase of resets is the presence of noise. Measurement signals contain noise, opposed to the ideal case. Examples of causes of noise are: electronic components, high frequency resonances and vibrations from the environment. Because of this noise a reset signal might cross the zero-line wrongfully, leading to a pseudo-crossing of the zero-line. The noise levels used in this section are based on measurements from the physical setup.

B.4.1. Effect of noise on reset elements

The effect of noise on the reset elements is visualized by subjecting open-loop reset elements to a sinusoidal signal containing noise. The magnitude of the reference signal and the noise are matched to those used in simulation in the following chapters. The frequency of the reference signal is set to 0.16Hz. Figure B.7 shows the effect of noise on several reset elements. From Figure B.7b it can be seen that for the CI, first order reset element (FORE) and CgLp the resets shift in time.



(a) Response during one period of reset elements to sinusoid



(b) Zoomed in on the reset instance at 3.12s

Figure B.7: CI, FORE, CgLp and CR CgLp response $u(t)$ for sinusoidal $e(t)$. The dashed lines represent the simulations with added noise. The frequency of reference signal is 0.16Hz.

In Figure B.7a it can be seen that with the addition of noise to the simulation the performance of the CR CgLp deteriorates more than that of the other reset elements. As stated earlier the CR CgLp is more sensitive to noise than the other reset elements due to lead element $L(s)$, see (B.7) [4]. Assuming that ω_h is large the output of this reset law can be defined as (B.2).

$$L(s) = \frac{s/\omega_l + 1}{s/\omega_h + 1} \quad (\text{B.7})$$

In Figure B.8 it is shown that the noise is amplified by the lead element, causing reset during the entire oscillation period. This could cause the performance of the CR CgLp to be more sensitive to noise than the other elements. For low frequencies the trajectory tracking performance is dominated by noise. Because of the CR CgLp being more sensitive to noise than the other reset elements its performance deteriorates for low frequency references, including 0.16Hz.

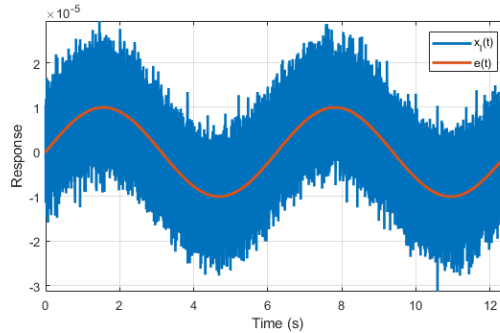


Figure B.8: Input $e(t)$ and output $x_l(t)$ of lead element $L(s)$ with added noise and frequency of reference signal at 0.16Hz.

When increasing the frequency of the reference to 5Hz it can be seen that the trajectory tracking of the CR CgLp is improved, see Figure B.9. Figure B.9b shows that the zero crossings of the reset signal are more concentrated, resulting in more concentrated reset instances. Besides, this reduced the overall amount of resets per oscillation.

Figure B.10 presents the pseudo-sensitivity of a CgLp and CR CgLp with and without noise. The controllers used differ from the controllers previously used in this section. Details of these controllers

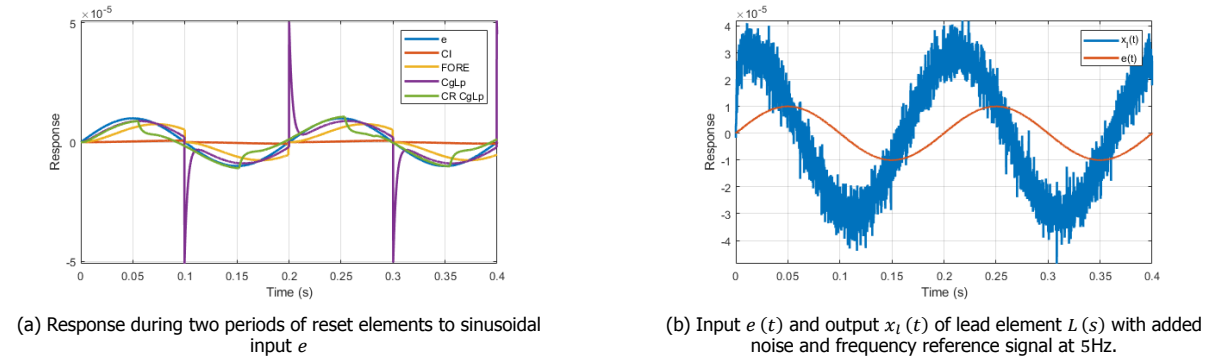


Figure B.9: CI, FORE, CgLp and CR CgLp response for sinusoidal $e(t)$. Frequency of reference signal is 5Hz.

can be found in [subsection E.3.1](#). This figure confirms that the CR CgLp performs worse at low-frequency trajectory tracking then the CgLp. This can be seen at the low-frequency sensitivity floor being $\approx 8\text{dB}$ lower for the CgLp. For mid-range frequencies the pseudo-sensitivity of the CgLp remains $\approx 8\text{dB}$ lower than that of the CR CgLp.

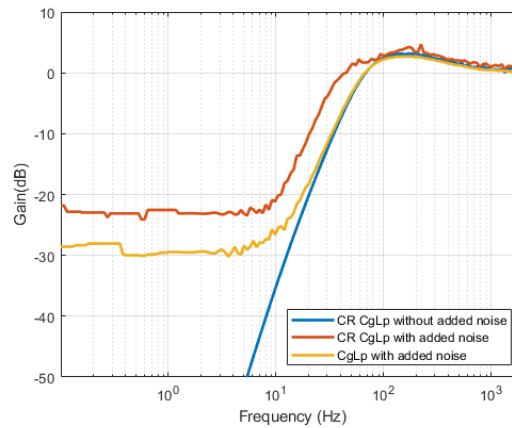


Figure B.10: Pseudo-sensitivity of continuous time CgLp and CR CgLp with and without measurement noise. Measurement is conducted using a reference signal with amplitude of $1 \times 10^{-5}\text{m}$.

Plotting the reset lines of the reset controllers for a 30Hz sinusoidal reference reveals that the noise on the reset line of the CR CgLp causes zero crossings throughout the oscillation, see [Figure B.11](#). This results in an excessive amount of resets. Those reset induce vibration leading to a higher pseudo-sensitivity. Hence, worse trajectory tracking performance. This behavior is exhibited over the entire frequency range. The reason that the pseudo-sensitivities converge is that the high-frequency content beyond the controller bandwidth is attenuated.

B.4.2. Sub-conclusion

The presence of measurement noise introduces a the low-frequency sensitivity floor. This floor limits the trajectory tracking performance of a controller. The performance deterioration depends amongst other on the used reset line. Out of the simulated controllers is the CR CgLp the most sensitive to measurement noise. The lead element of the CR CgLp amplifies the noise resulting in reset throughout the oscillation. This behavior is observed over the entire frequency range.

B.5. Performance deterioration by excessive resets

The aim of this section is to address the relation between excessive resets and performance deterioration. It is already shown that excessive resets cause the performance to deteriorate. This section addresses the will answer the question of why that happens.

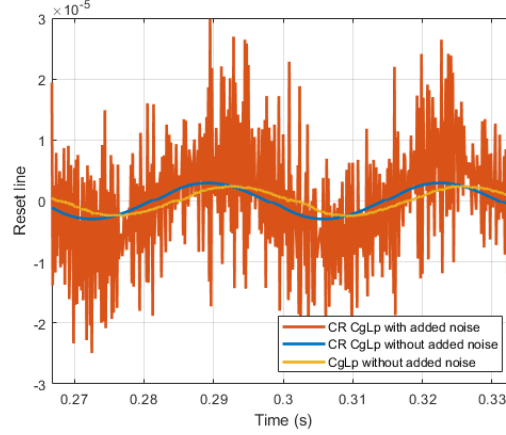


Figure B.11: Reset line of the CgLp and CR CgLp with and without noise with sinusoidal reference at 30Hz.

B.5.1. Impulsive Description

Reset controllers can be modelled as a linear system with impulse train input that is timed based on a state [15]. In [15] it is stated that in closed-loop the reset controller behaves like the base-linear system (BLS) with impulses scaled with parameter A_ρ . In order to rewrite the Laplace domain formulation of the error to closed-loop frequency-domain terms for zero-crossing reset systems periodicity is used in [15]. A reset system is periodic if, amongst other assumptions, resets occur in periodic instants, with period π/ω . Here ω is the frequency of the sinusoidal reference. This assumption does not hold for systems that contain measurement noise. It has been shown in previous sections that measurements noise and quantization cause both additional resets and shifts in reset instances. Those additional resets can occur close together, relative to the period of one oscillation.

Consecutive resets

If reset instances occur together, relative to period π/ω , they are denoted as consecutive resets. [15] showed that in the limit case when two reset instances approach each other the equivalent γ (γ_{eq}) is squared, yielding $\gamma_{eq} = \gamma^2$. With $\gamma \in [0, 1]$, γ_{eq} approaches zero. For example, if $\gamma = 0.6$ then $\gamma_{eq} = 0.6^2 = 0.36$. The decrease in γ results in a stronger reset, or increased impulse. When several consecutive resets occur this effect can cause the system to become fully resetting, i.e. $\gamma_{eq} = 0$. This can cause spikes and vibrations in the position. When resets are interpreted as impulses the system needs time to dampen out the impulse. When a new impulse is applied to the system before the previous one is dampened out this may cause the error to increase [15]. Figure B.8 shows that consecutive crossings occur when a CR CgLp is subjected to noise. In between the resets the system has no time to let the impulse dampen out. This could lead to a $\gamma_{eq} \rightarrow 0$ causing an increase of the error. This could lead to an increase of the pseudo-sensitivity, as is observed in Figure B.10.

Additional resets

Additional resets are resets that occur far, relative to π/ω from the previous reset instance. In [15] it is stated that additional resets can cause an error in the peak of the position when subjected to a sinusoidal reference. This explains part of the increase of the low-frequency sensitivity floor in Figure B.10. For the CR CgLp holds that at low frequencies the magnitude of the reset signal is dominated by the differentiated measurement noise. This leads to reset instances during an entire oscillation, see Figure B.8. Hence, additional resets can occur in a system with a CR CgLp and measurement noise.

B.5.2. Sub-conclusion

Excessive resets cause performance deterioration because each reset gives an impulse to the system. Both successive and additional resets can cause the error to increase. Therefore, causing the performance to deteriorate.

B.6. Conclusion

Discretization, quantization and measurement noise all contribute to the performance deterioration of the CR CgLp. This impact of discretization is limited to a time shift equal to the sample time. Quantization and measurement noise do this by either shifting the reset instances or causing extra resets. Low-frequency trajectory tracking is impacted the most. This can be seen at the height of the low-frequency sensitivity floor. A lower floor means better performance. The next chapters focus on the design of two methods that attenuate noise and this reduce the performance deterioration.

C

Tuning of Kalman filter

The measurement covariance matrix R is given by the specifications of the sensors, see (C.1). It is assumed that the covariances are independent of each other. So, the off-diagonals are zero. These values can be found by measuring the RMS of the noise [27].

$$R = \begin{bmatrix} \sigma_{pos}^2 & 0 \\ 0 & \sigma_{acc}^2 \end{bmatrix} \quad (C.1)$$

According to [27] the design of the process noise matrix Q_k is most difficult aspect of the design of a KF. This is mainly due to the fact that this matrix should compensate for errors made in the plant model F_k . A lot of these errors are unknown or difficult to estimate. Tuning this matrix depends on experiments and experience. A common way to find values for Q_k is to sweep the parameters to find the optimal values [27].

The values of Q have a significant effect on the performance of the KF. If Q is too small the KF trusts the plant model too much leading to diversion of the actual solution. When Q is chosen too large the KF is not able to correct for the noise containing measurements resulting in sub-optimal performance [27]. The off-diagonal terms are zero because the process states are assumed to be independent of the other states, see (C.2).

$$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \quad (C.2)$$

The tuning is done by tracking the mean squared error (MSE), see (C.3), while sweeping the diagonal terms of Q . In (C.3) $\hat{y}$ is the estimated position, y is the position measured before the noise block and n is the length of the measurement.

$$MSE = \frac{\sum (\hat{y} - y)^2}{n} \quad (C.3)$$

After four iterations the tuning appeared to converge. The results can be found in (C.4). This process is repeated for each noise power P_{noise} .

$$Q = \begin{bmatrix} 1.0 \times 10^{-25} & 0 & 0 \\ 0 & 3.0 \times 10^{-10} & 0 \\ 0 & 0 & 7.9 \times 10^{-12} \end{bmatrix} \quad (C.4)$$

D

Accelerometer noise

Accelerometers are available in a wide variety of shapes and forms. Ranging from cheap MEMS-based devices to high-end piezo-based accelerometers. Performance characteristics like measurement range, sensitivity and noise levels differ per type. Higher performance accelerometers come at a higher price. This chapter addresses the requirements for the noise level of the accelerometer is used in the KF as proposed in chapter 5. The aim is to conclude with a minimal requirement for the noise level of the accelerometer. This to prevent unnecessary expenses when choosing expensive accelerometers.

D.1. Noise modelling

Measurement data of an accelerometer is used to establish a valid model of the noise. During this test the system was not actuated, so the sensor remained stationary. The setup is mounted on top of a vibration isolation table. Vibrations from the environment are neglected. The results of the test are shown in Figure D.1. Note that the accelerometer used in this test will also be used for the KF. From Figure D.1b it can be seen that the noise behaves like white noise. A flat power spectral density (PSD) means that wide range of frequencies contributes equally to the total noise. With a Band-Limited White Noise block from Simulink this behavior could be approached.

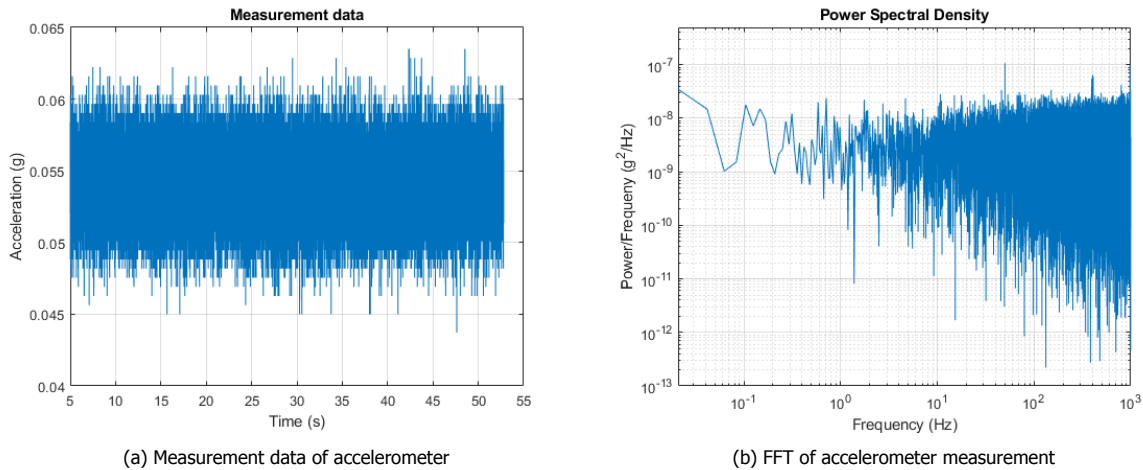


Figure D.1: Results of accelerometer measurement when the chuck stationary and in end-stops

According to [30], the noise power in the Band-Limited White Noise block is given by the PSD. Theoretically, white noise has a flat PSD, meaning that every frequency contributes equally to the total noise. Because of this the Simulink block requires only one value to specify the noise power. The average power of the PSD can be calculated using Parseval's Theorem, see (D.1) [31, 32].

$$P = \int_{-\infty}^{\infty} S_{xx}(f) df \quad (D.1)$$

Calculating the power of a band limited signal using (D.1) is based on two assumptions. The first assumption is that the frequencies within the signal are defined as $f_2 \geq f_1 > 0$. Secondly, it is assumed that both positive and negative frequencies contribute the same amount of power, $S_{xx}(-f) = S_{xx}(f)$. With these assumptions (D.1) can be rewritten to (D.2). When considering vibration, the average power is given in g^2 . The RMS of the measurement is given by the square-root of $P_{bandlimited}$ [31, 32].

$$P_{bandlimited} = 2 \int_{f_1}^{f_2} S_{xx}(f) df, \quad f_2 \geq f_1 > 0 \quad (D.2)$$

The relation between the noise power required in the Band-Limited White Noise block P_{noise} and the covariance σ^2 is given in (D.3) [30]. White noise has a relatively flat PSD profile. Therefore, the relation between P_{noise} and the average PSD is straight-forward. Only half $\overline{PSD}$ is needed because of the translation between two-sided noise power and one-sided PSD. The PSD of the generated white noise is given in Figure D.2. From this figure it can be seen that the simulated white noise matches with the PSD of the measured data. The difference in RMS is in the order of μg , which is 3 orders of magnitude smaller than the RMS of the measured accelerations. Therefore, this difference can be neglected.

$$P_{noise} = \sigma^2 \cdot T_s \cong \frac{\overline{PSD}}{2} \quad (D.3)$$

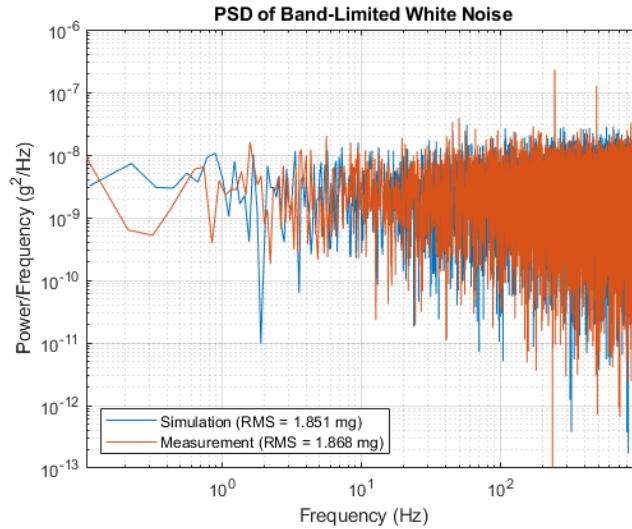


Figure D.2: Power Spectrum Density of measurement and simulation using the Band-Limited White Noise block

The measured RMS of the accelerometer measurement varies when exiting the accelerometer with several single sine reference signals, see Figure D.3. This variation of the RMS is caused by the system dynamics. In other words, the attenuation of the reference signal by the controllers depends on the frequency of the reference signal. Position data shows that, for a sinusoid with an equal amplitude and a higher frequency, the magnitude of the position response increases, which indicates that the increased response of the accelerometer is caused by the controller performance. This measurement was only performed for a limited frequency range.

Figure D.4 shows the relation between the RMS and the frequency of the sinusoidal reference. When using the full frequency range for computing the RMS, there seems to be a relation between the RMS and the input frequency. If only high frequency data is considered, this relation does not hold. Looking

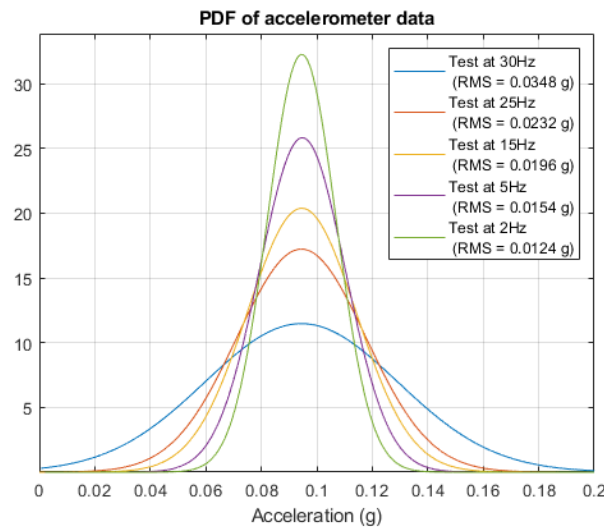


Figure D.3: Probability Density Function of measurement data of accelerometer when excited by a single sine reference with same amplitude and varying frequencies.

only at the noise at frequencies higher than 60Hz shows that the RMS is not correlated with the input frequency. The observation complies with the statement in the paragraph above. Therefore, it can be concluded that the noise is independent from the input frequency and can be modelled as an additive noise source.

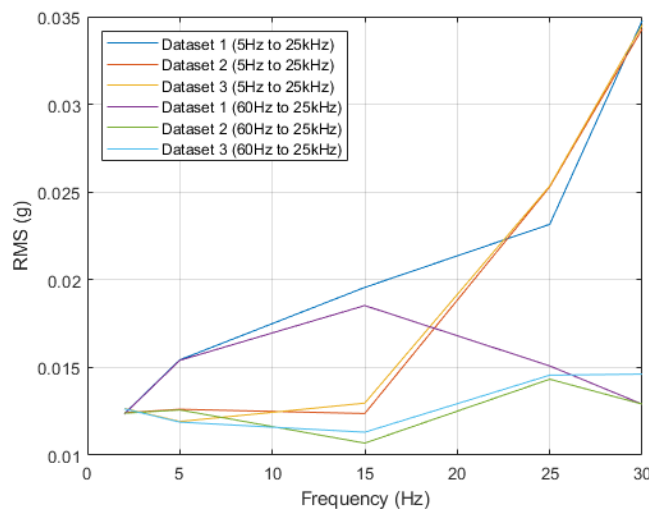


Figure D.4: RMS values of measurement data of accelerometer when excited with single sine reference. The RMS is calculated for both the entire frequency range and for frequencies starting at 60Hz.

When analyzing the PSD itself it can be seen in [Figure D.5](#) that besides the input induced resonance peaks the PSDs do not differ visibly between the different tests. This adheres to the previously made statement on the relation between the noise and the input signal.

D.2. Noise specifications in accelerometer data-sheets

In datasheets of accelerometers the electrical noise is often given in broadband or spectral noise. When not mentioned otherwise broadband noise is the RMS of a measurement without excitation of the system. Spectral noise represents the noise contribution of a specific bandwidth. This is needed because the noise level of an accelerometer might vary over the measured frequency range. Typically, the summation of the spectral noise, as described by ([D.4](#)), yields a lower noise level than is given by

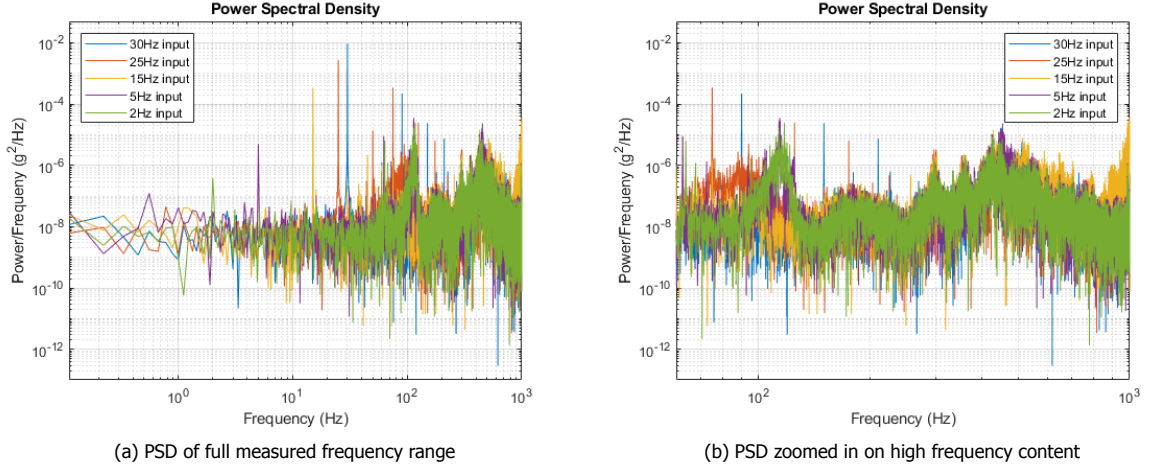


Figure D.5: Power Spectral Density of accelerometer measurement when chuck is in control

the broadband noise [33–35].

$$N_{RMS} = \sqrt{\sum_i N_A^2(i) \cdot (f_{high}(i) - f_{low}(i))} \quad (D.4)$$

This electrical noise gives a noise floor for the accelerometer itself. However, an accelerometer is often-times part of a system where other noise sources are present. In practice the noise level is higher than the noise level given by the accelerometer data-sheet. For example, the noise as shown in Figure D.1 contains noise from other sources. Compare this to noise measurement shown in Figure D.2. These noise levels can be used as a lower bound on the expected measured noise, not as an upper bound. The RMS of the signal in this figure is 1.96mg, while the RMS should be 30μg according to the data-sheet. This shows that noise from other sources is picked up by the accelerometer. In the next section an upper limit of the noise of the accelerometer is determined.

D.3. Required noise level

The noise measured by the accelerometer is modelled as additive noise. This means that the RMS of the noise does not scale with the amplitude of the true accelerations. As shown above, the RMS is related to the PSD of the noise. Therefore, the RMS of the accelerometer noise is used to define a bound on the acceptable noise, meaning noise levels for which the KF performs sufficiently. Requirements for the KF will be discussed later. The RMS is related to the P_{noise} through the PSD. The relation between the PSD and P_{noise} is given in (D.3). The RMS can be calculated using the PSD, see (D.5), with $f_2 \geq f \geq f_1$ and $\tilde{q}(f)$ the PSD of the accelerometer measurement [35].

$$g_{RMS} = \sqrt{\int_{f_1}^{f_2} \tilde{q}(f) df} \quad (D.5)$$

The noise power, as defined in (D.3), does not depend on the system dynamics because it is a simulation input. Therefore, P_{noise} can be used to characterize the accelerometer simulation noise. And the RMS is related to the noise specifications given in the data-sheets of accelerometers, see (D.4).

Figure D.6 shows the response of a CR CgLP with a KF on the measurement line. The $P_{noise} = 0g^2/Hz$ -line represents a simulation with added accelerometer noise that is approximately equal to zero, so it has only noise on the position measurement. The CR CgLP without filter illustrates the performance of a CR CgLP without a KF on the measurement line. From this figure it can be seen that the magnitude of the noise impacts the pseudo-sensitivity. At low frequencies the sensitivity floor is lower for lower P_{noise} . This means the trajectory tracking is improved using more accelerometer measuring less noise.

There is no visible difference in the peaks of sensitivity. This means that the KF does not impact the the modulus margin.

Details on the plant and controller used in the simulations can be found in appendix E.3.1. Figure D.7 shows plots of the reset lines of the CR CgLP with different P_{noise} and for different reference signals. For a reference of 20Hz all reset lines keep crossing zero throughout the oscillation period, see Figure D.7a. This causes reset both additional and consecutive reset leading to a high number of resets per oscillation. The number of oscillation can be found in Table D.1. Figure D.7b shows that for a reference of 20Hz most reset line no longer cross zero during the entire oscillation. The zero crossing, and therefore the reset instances, are concentrated towards the 'true' zero crossing. This limits the amount of additional and consecutive resets, improving the performance compared to the reset element without filter. The reset line of the CR CgLP without filter keeps crossing the zero line through an entire oscillation. This leads to a high number of resets per oscillation. Almost $4\times$ higher than the reset element subjected to the higher P_{noise} . For a reference of 130Hz no additional and consecutive reset occur. All reset elements have two resets per oscillation. This is because the system dynamics determine the shape of the reset line increasing the amplitude of the signal.

Table D.1: Resets per oscillation of CR CgLP for different values of P_{noise} at different reference frequencies.

$P_{noise} (g^2/Hz)$	#Resets @ 2Hz	#Resets @ 20Hz	#Resets @ 130Hz
4.35×10^{-6}	1829	54	2
4.35×10^{-10}	1839	54	2
4.35×10^{-14}	1817	38	2
0	1382	16	2
No filter	2592	201	2

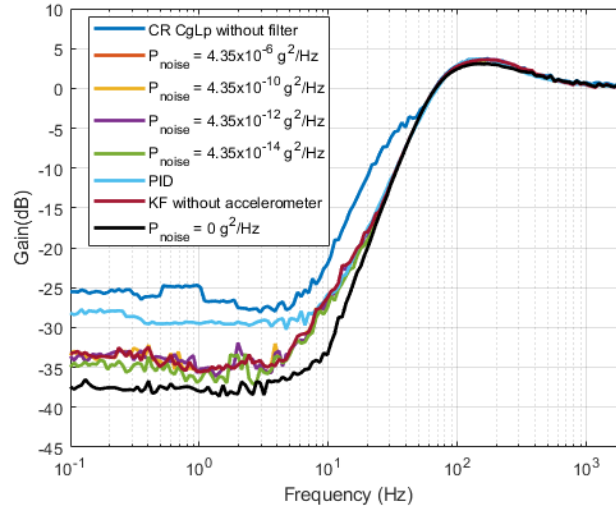


Figure D.6: Pseudo-sensitivity of PID and CR CgLP with and without KF for different P_{noise} . P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.

Figure D.8 illustrates that between 10Hz and 40Hz the pseudo-complementary sensitivity of the CR CgLP without filter is higher than the others. This indicates that these frequencies are amplified more by, which is undesirable. This corresponds to the observation made based on Figure D.6.

Figure D.9 shows the step responses of the PID and CR CgLP + KF with different P_{noise} . The overshoot of the PID is 20% larger than the overshoots of the CR CgLP with filter. The 5% settling time of the PID is approximately 0.001s faster than the settling time of CR CgLP + KF. Based on the step response it can be concluded that the addition of the KF improves the performance of the CR CgLP. Compared to the PID the reduction in overshoot is significantly higher than the reduction in settling time.

Figure D.10 shows the pseudo-sensitivity of a PID and several CR CgLP structures. It can be seen that for the P_{noise} used in this simulation the CR CgLP + KF with and without the accelerometer yield similar

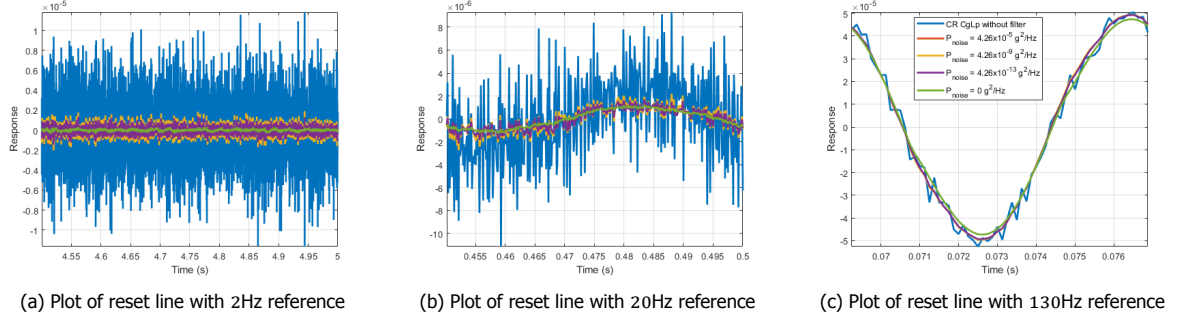


Figure D.7: Error line of PID and CR CgLP with and without KF for different P_{noise} and different reference signals. Each plot shows one oscillation. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.

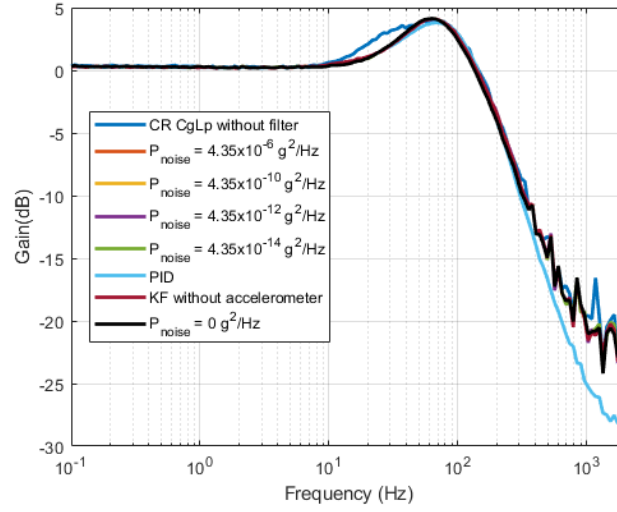


Figure D.8: Pseudo-complementary sensitivity of PID and CR CgLP with and without KF for different P_{noise} . P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.

results. Therefore, this value can be used as an upper limit of the noise level of the accelerometer measurement. Accelerometers with higher noise levels do not improve the system behaviour compared to a KF without an accelerometer. The line of the KF with ideal accelerometer shows the lower bound of the pseudo-sensitivity. From this figure it can be seen that by the addition of an accelerometer to the KF the sensitivity floor can be reduced with $\approx 4dB$.

D.4. Conclusion

The measurement noise of the accelerometer has an impact on the performance of the KF. For $P_{noise} > 4.35 \times 10^{-12} g^2/Hz$ the accelerometer does not have an added value. In that case a KF without accelerometer would be sufficient. However, when the accelerometer noise power is less than $4.35 \times 10^{-12} g^2/Hz$ the sensitivity floor can be reduced, improving the trajectory tracking performance. The mentioned P_{noise} is equal to a measurement noise RMS of $0.186 \mu g$. The found results apply only to the system described in this chapter. For other systems a similar analysis should be performed.

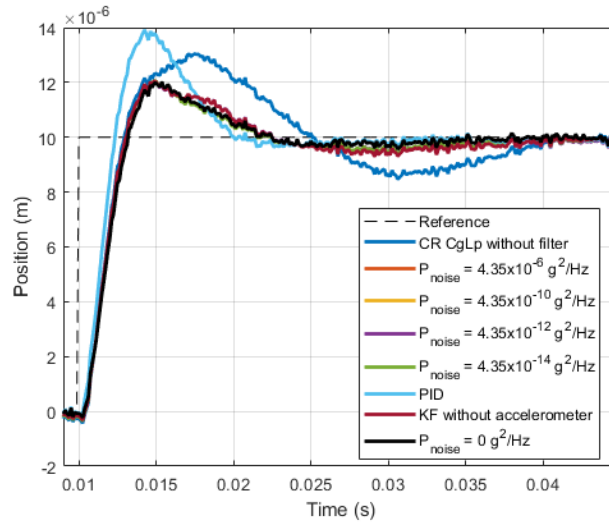


Figure D.9: Step response of the CR CgLP + KF for different values of P_{noise} . P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.

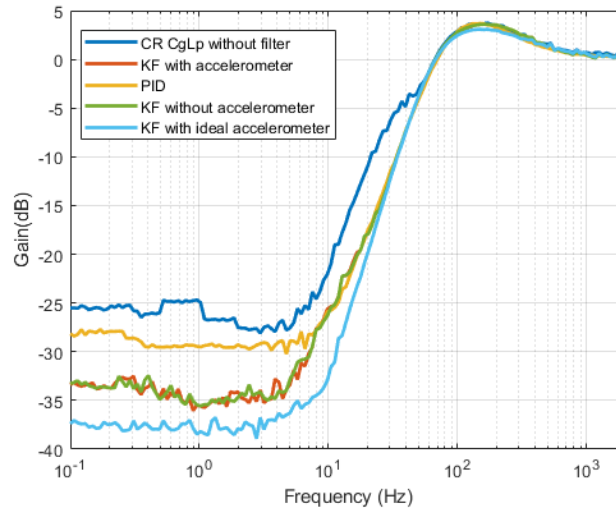
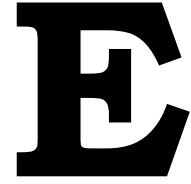


Figure D.10: Pseudo-sensitivity of the CR CgLP + KF. The accelerometer P_{noise} is set to $4.18 \times 10^{-11} g^2/Hz$. P_{noise} of the position sensors was set to $9.77 \times 10^{-19} m^2/Hz$.



Experimental Setup

This appendix discusses the physical setup used for the validation of the designed controllers.

E.1. Physical components

The measurement is taken care of by two groups of sensors. [Figure E.1](#) shows the configuration of the sensors. The *cap*-sensors refer to the high precision sensors. These sensors however have a short range. To position the chuck within the range of the *cap*-sensors, the low precision *diff*-sensors are used.

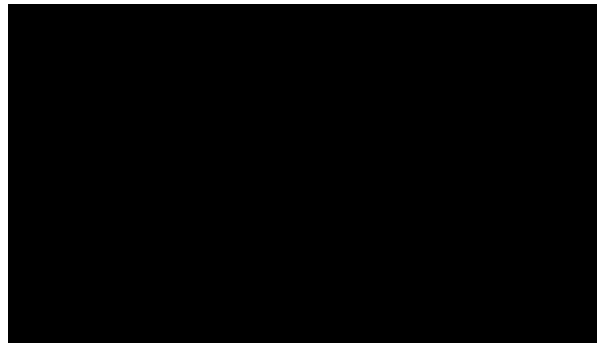


Figure E.1: Configuration of sensors for single DoF with respect to moving chuck

E.2. System Identification

In order to tune a high bandwidth controller a system identification was performed. The results can be seen in [Figure E.2](#). It can be seen that a mass-line can be fitted through the data. The mass is estimated to be 22.6kg.

E.3. Feedback Control Structure

This section elaborates on the feedback control structure. Both the linear controller as the reset controller are addressed. This section contains the continuous time version of the discretization of the controller and the IIR filter implementation.

E.3.1. Continuous time controller

The used controller structure is shown in [Figure E.3](#). The block *Reset controller* contains the reset controller. This can be either the CR CgP or the CgP. The block *Filters* contains the notch filters. When used, *ML Filters* contains the KF or LL filter. A details description of their definitions can be found in [chapter 5](#) and [chapter 4](#).

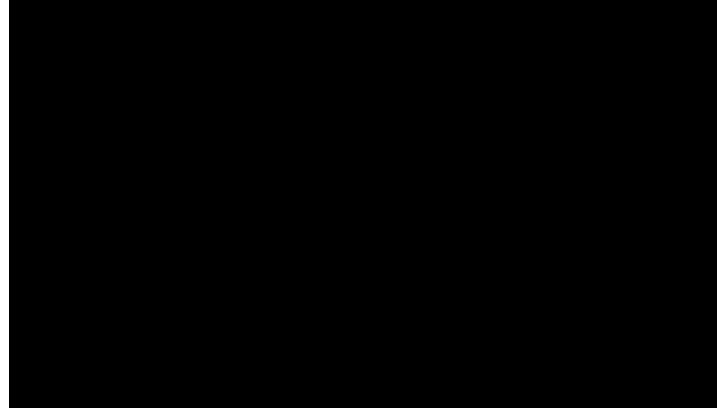


Figure E.2: FRF measurement of Z-axis with fitted mass-model

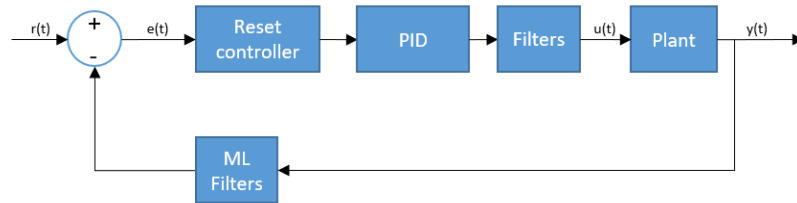


Figure E.3: Controller structure for reset control

Linear controller

The continuous time PID controller is given in (E.1).

$$C_{PID}(s) = K_p \left(1 + \frac{2\pi f_i}{s} \right) \left(\frac{\frac{s}{2\pi f_d} + 1}{\frac{s}{2\pi f_t} + 1} \right) \quad (\text{E.1})$$

In the tables below k_p is defined in (E.2), with $m = 22.6\text{kg}$ the mass of the plant and ω_c^2 the bandwidth of the controller.

$$k_p = \frac{1}{3} m \cdot \omega_c^2 \quad (\text{E.2})$$

Reset controller

Two reset controllers were used during testing, the CgLp and the CR CgLp. Both reset controller are used in a PID framework. The used architecture can be found in Figure E.4. A description of the blocks is given below.

- Lead filter $L(s) = \frac{s/\omega_l + 1}{s/\omega_h + 1}$
- Lag filter $R(s) = \frac{1}{s/\omega_l + 1}$
- Lead filter $D(s) = \frac{s/\omega_r + 1}{s/\omega_f + 1}$
- Reset lag element $C_R(s) = \frac{1}{s/\omega_r + 1}$ with reset parameter γ

For the design of the reset controller only the describing function (DF) is used. This is done under the assumption that the higher-order sinusoidal input describing functions (HOSIDFs) are sufficiently small. This is the case when the HOSIDFs are at least 40dB smaller than the DF. Figure E.5 shows that the difference between HOSIDFs and DF stays greater than 40dB.

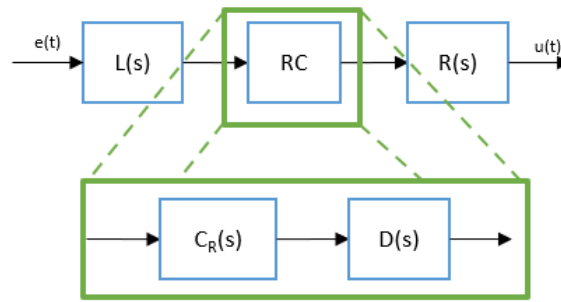


Figure E.4: Controller structure for reset control

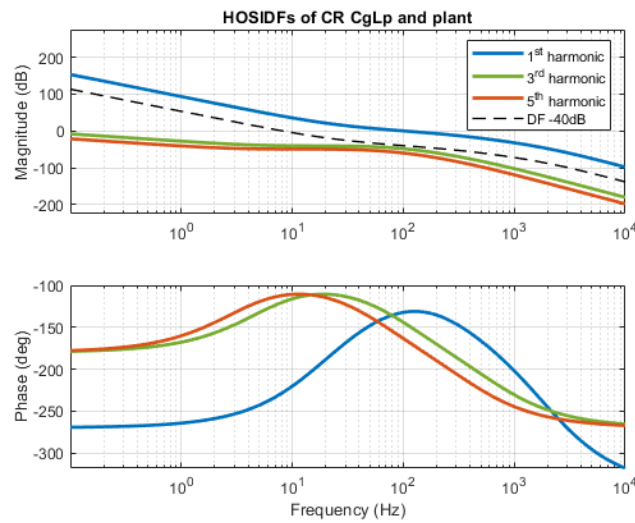


Figure E.5: Bode plot of HOSIDFs

Notch filter

Based on the frequency-response function (FRF) measurement of the plant some notch filter were required to meet the stability requirements. The structure of the notch filter is given in (E.3). The used parameters can be found in Table E.1. These notches were only used during tests of the physical system, not for the simulations.

$$C_{notch}(s) = \frac{s^2 + 2\pi f_n \frac{d_n}{Q_n} s + (2\pi f_n)^2}{s^2 + 2\pi f_n d_n s + (2\pi f_n)^2} \quad (E.3)$$

Table E.1: Parameters of the notch filters used during testing

Parameter	#1	#2	#3	#4	#5
f_n					
d_n					
Q_n					

Parameters

Below three controller designs are presented. The controllers with a 50Hz bandwidth are used on the physical setup and the 100Hz-bandwidth controllers are used for the simulation. Table E.2 gives the parameters of the controllers used on the physical setup. The stability margins of these controllers can be found in Table E.3. The open-loop Bode plots are given in Figure E.6.

Table E.2: Controller parameters for $\omega_c = \blacksquare \cdot 2\pi$ rad/s

Parameter	CR CgLp	CR CgLp+LL	PID
ω_i	$\omega_c/7$	$\omega_c/7$	$\omega_c/10$
ω_d	$\omega_c/3.0$	$\omega_c/3.0$	$\omega_c/3.3$
ω_t	$\omega_c \cdot 3.0$	$\omega_c \cdot 3.0$	$\omega_c \cdot 3.3$
ω_f	$\omega_c \cdot 20$	$\omega_c \cdot 20$	$\omega_c \cdot 20$
γ	0.6	0.6	-
α	1.05	1.05	-
ω_r	ω_c	ω_c	-
ω_l	$\omega_c \cdot 0.4$	$\omega_c \cdot 0.4$	-
ω_h	$\omega_c \cdot 20$	$\omega_c \cdot 20$	-
K_p	$0.94k_p$	$0.86k_p$	$0.9k_p$

Table E.3: Controller parameters for $\omega_c = \blacksquare \cdot 2\pi$ rad/s

Parameter	CR CgLp	CR CgLp + LL	PID
BW	$\blacksquare$	$\blacksquare$	$\blacksquare$
PM	$\blacksquare$	$\blacksquare$	$\blacksquare$
GM	$\blacksquare$	$\blacksquare$	$\blacksquare$
1/MM	$\blacksquare$	$\blacksquare$	$\blacksquare$

Table E.4 lists the controller parameters used for the simulations. The Bode plots of the 100Hz-bandwidth controllers can be found in Figure E.7 and the stability margins of the controllers are shown in Table E.5.

Table E.4: Controller parameters for $\omega_c = 100 \cdot 2\pi$ rad/s

Parameter	CR CgLp	CR CgLp + LL	PID
ω_i	$\omega_c/7$	$\omega_c/7$	$\omega_c/10$
ω_d	$\omega_c/3$	$\omega_c/3$	$\omega_c/3.3$
ω_t	$\omega_c \cdot 3$	$\omega_c \cdot 3$	$\omega_c \cdot 3.3$
ω_f	$\omega_c \cdot 20$	$\omega_c \cdot 20$	$\omega_c \cdot 20$
γ	0.6	0.6	-
α	1.05	1.05	-
ω_r	ω_c	ω_c	-
ω_l	$\omega_c \cdot 0.4$	$\omega_c \cdot 0.4$	-
ω_h	$\omega_c \cdot 20$	$\omega_c \cdot 20$	-
K_p	$0.96k_p$	$0.87k_p$	$0.9k_p$

E.3.2. Controller discretization

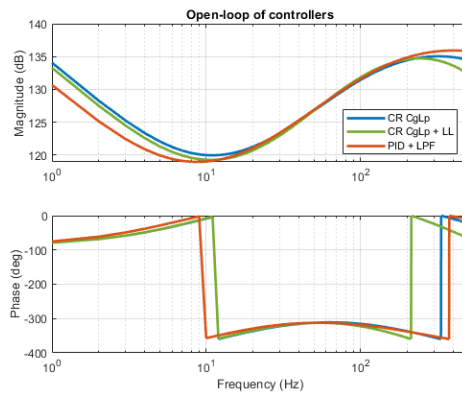
The controllers are discretized using the Tustin's method. The discrete controller are implemented as IIR filters using the bi-linear transform in the transposed direct form 2, see (E.4). This equation is scaled such that $a_0 = 1$.

$$H_d(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}} \quad (\text{E.4})$$

The discretization is done manually using (E.5) with T_s being the sample time.

$$s = K \frac{z-1}{z+1} \quad \text{with} \quad K = \frac{2}{T_s} \quad (\text{E.5})$$

In case the discretized controller does not match the continuous controller for a specific and relevant



(a) Bode plot of controllers



(b) Open-loop Bode plot of controllers and mass-plant

Figure E.6: Open-loop Bode plots of controllers with 50Hz bandwidth. For CR CgLf the DF is used.

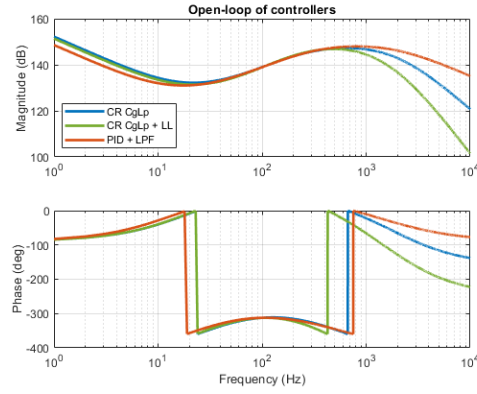
Table E.5: Controller parameters for $\omega_c = 100 \cdot 2\pi$ rad/s

Parameter	CR CgLf	CR CgLf + LL	PID
BW	100Hz	100Hz	100Hz
PM	47.7°	47.4°	47.7°
GM	24.7dB	17.4dB	26.1dB
1/MM	2.5dB	3.1dB	2.8dB

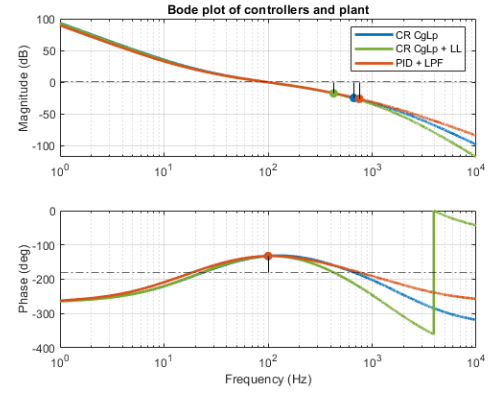
frequency, frequency warping can be used. For the notch filters frequency warping was used. This is done using the definition of K given in (E.6).

$$K = \frac{\omega_0}{\tan\left(\frac{\omega_0 T_s}{2}\right)} \quad (\text{E.6})$$

Figure E.8 shows a schematic of the transposed direct form 2. This is a picture made from the Simulink model. This specific figure shows a second order filter. By varying the number of delays the order can be changes to fit the order of the discrete transfer function.



(a) Bode plot of controllers



(b) Open-loop Bode plot of controllers and mass-plant

Figure E.7: Open-loop Bode plots of controllers with 100Hz bandwidth. For CR CgLp the DF is used.

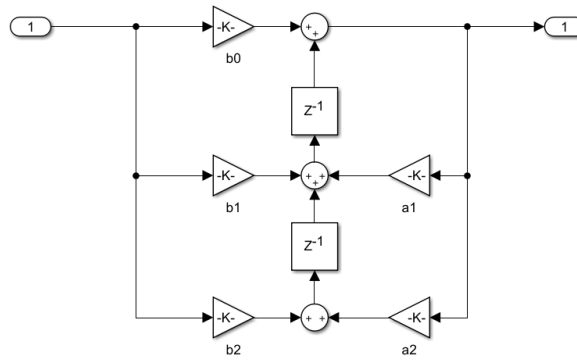


Figure E.8: Schematic of IIR filter implementation in Simulink