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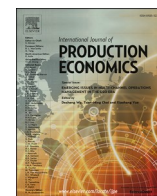
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Food supply chain coordination for growing items: A trade-off between market coverage and cost-efficiency

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ABSTRACT

The accurate operation of a Food Supply Chain (FSC) is a critical issue as it directly interfaces with health and safety matters. This study addresses coordination and conflict management in a three-level FSC that embraces a new inventory type known as growing items like poultry and livestock. The chain involves a rearing farm as the supplier, a processed food producer as the manufacturer, and multiple processed food retailers. Vendor Managed Inventory (VMI) is applied by the manufacturer to handle the retailers' systems and prevent replenishment mismatches and thereby food waste. To increase its market coverage, the manufacturer needs to provide the retailers with enough incentives to enter this setting. So, a cost-sharing contract is designed under which the manufacturer undertakes a fraction of the retailers' holding costs. Accordingly, the manufacturer faces two contradictory targets, increasing its market coverage by convincing the retailers to enter the system on the one hand and managing its costs efficiently on the other hand. An analytic solution approach with a game-theoretic perspective is developed to solve the model. Extensive numerical experiments and a case study are provided, presenting fruitful managerial insights that can be utilized by the policymakers and chain members under different settings. The results highlight the efficiency of our VMI and cost-sharing collaboration scheme in enhancing the performance of the chain.

Credit author statement

Nadia Pourmohammad-Zia: Conceptualization, Methodology, Investigation, Validation, Software, Writing- Original draft, Behrooz Karimi: Conceptualization, Supervision, Jafar Rezaei: Conceptualization, Supervision, Writing - Review & Editing.

1. Introduction

About one-third of the food produced for human consumption is wasted yearly. An estimated 60 percent of these food losses occur through the processes of the Food Supply Chain (FSC) and before the products reach the end consumers (Göbel et al., 2015). Coordination deficiency among chain members and concentrating on individual echelons of FSC play a key role in this regard (Fritz and Schiefer, 2008). Accordingly, applying efficient inventory management policies for individuals is not enough, and coordination mechanisms should be designed under which the entities interact while making decisions. Vendor Managed Inventory (VMI) is one of these coordination

mechanisms.

VMI is a collaborative supply chain initiative under which the manufacturer manages its retailers' inventory and specifies their replenishment policies (Cai et al., 2016). Accordingly, it can dramatically diminish production stops, excess inventory, mismatching, and unsatisfied demand. When it comes to the food industry, these benefits are of higher value due to the quality losses and confined lifetime of the items. Food Storage and Distribution Federation (2010) claimed that VMI is a potent implement to achieve competitive advantage through improving the interactions of FSC's members. To transform VMI into a practice where both parties are eager to apply, the manufacturer needs to design incentive mechanisms for the retailers. Otherwise, they will not enter this setting, and the manufacturer will lose its retailers, which is referred to as decreased market coverage. The cost-sharing contract is among these incentive schemes based on which the manufacturer is responsible for undertaking a fraction of the costs imposed on its retailers. This, in turn, increases the costs of the manufacturer. Accordingly, the manufacturer faces two contradictory targets: increasing its market coverage by convincing the retailers to enter the system on the one hand and managing the costs, on the other hand, which is referred to

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Notations

The following notations are used in the mathematical formulation

C_S	Supplier's unit purchasing cost
P_S	Supplier's unit selling price
C_p	Manufacturer's unit production cost
P_p	Manufacturer's unit selling price
P_R^i	Retailer i 's unit selling price
C_B	Supplier's breeding (feeding, nourishment, and holding) cost per unit item
C_{HW}	Manufacturer's unit holding cost per unit time for raw material
C_{HP}	Manufacturer's unit holding cost per unit time for finished products
C_{HR}	Retailer's unit holding cost per unit time
C_{OS}	Supplier's fixed ordering cost per cycle
C_{OW}	Manufacturer's fixed ordering cost per cycle
C_{OP}	Manufacturer's fixed production set-up cost per cycle
C_{OR}^i	Retailer i 's fixed ordering cost per cycle
D_i	Demand rate at the retailer i
w_t	Weight of a unit item at time t
$I_S(t)$	Supplier's inventory level at time t

Q_0	Supplier's order quantity (units)
Q_W	Manufacturer's order quantity for raw material (units)
Q_R^i	Retailer i 's order quantity (units)
y	Number of growing items purchased at the beginning of a cycle (unit items)
T_S	Breeding period at the supplier
T_{Pi}	Production cycle of order i at the manufacturer
T_R	Inventory cycle at the retailer (common replenishment cycle)
ρ	The production rate
σ	The depletion rate of raw material
μ_{T_S}	Fraction of discarded inventory during quality control
γ	Raw material consumption rate to produce a unit of the finished product
η_i	Fraction of retailer i 's holding cost undertaken by the manufacturer
m	Shelf-life of the items at the retailer
M	The number of retailers
TP_S	Supplier's total profit per unit time
TP_M	Manufacturer's total profit per unit time
TP_R^i	Retailer i 's total profit per unit time

as cost-efficiency. Fig. 1 presents this conflict in a generic form. That is to say, focusing on cost-efficiency could come at the cost of reduced market coverage and vice versa.

A broad class of inventories experiences an alteration through FSC, which is almost neglected by the researchers. The phenomenon is known as *growth* and is common in the fishery, poultry, and livestock industry. It is defined as natural development leading to physical changes such as size and weight increase. It describes a process where the newborn animals enter the system of a rearing farm at the beginning of a period called the breeding phase. They are fed and nourished, resulting in their physical weight increase and thereby inventory level increase. The items are raised until they reach a slaughtering point where the breeding period terminates. The slaughtered items such as beef and chicken meat are used as raw materials or finished products in various FSCs.

There exists a well-grounded concept in inventory management known as amelioration. Although it has some similarities with the growth, they should not be mistaken for each other. Both concepts illustrate the inventory level increase during storage. Amelioration mostly considers the opposite procedure of the deterioration, which projects the inventory utility increase and is common in the wine industry. This is while growth depicts a biological procedure through which the items undergo weight and size increase. It relies on the nature of the growing animals and the breeding period.

Inspired by the significance of efficient FSC management, our study investigates a three-level FSC, including a rearing farm as the supplier, a processed food producer as the manufacturer, and multiple processed food retailers. Newborn animals, such as broiler chickens, enter the

supplier's system and undergo growth during their breeding period. They are then slaughtered and sent to the manufacturer. The slaughtered items (e.g., chicken meat) are used as the raw materials to produce processed food (e.g., chicken sausage and nugget) at the manufacturer. The manufacturer uses VMI with common epochs to handle the replenishments of different retailers (Viswanathan and Piplani, 2001). The manufacturer undertakes a fraction of the retailers' holding costs (illustrated as a cost-sharing contract) to provide them with the incentives to comply with this setting. This, in turn, imposes additional costs to the manufacturer's system, and here the battle is commenced: *market coverage* versus *cost-efficiency*. Handling such conflicting paradigms is a fundamental factor in the process management of successful firms (Benner and Tushman, 2003). Thereafter, the design of the cost-sharing contract is a key point, and we have incorporated reference profit of the retailers in this sense.

The retailers face a free market for which price competition is a dominant factor. So the selling price of each retailer influences its own demand as well as the other retailers' pricing strategies. As the prices are optimized, each retailer's order size is specified, which affects the manufacturer's system. Accordingly, there exist two interactive decision-making procedures; one between the manufacturer and the retailers and the other among the retailers. Then, two games are formed: A Stackelberg competition between the manufacturer and the retailers and a Nash competition among the retailers. Fig. 2 provides a schematic view of this chain.

Revenue and inventory management in FSCs are usually focused on two-echelon structures, and more complex configurations are almost overlooked. Although coordinated decision-making in FSCs is highly

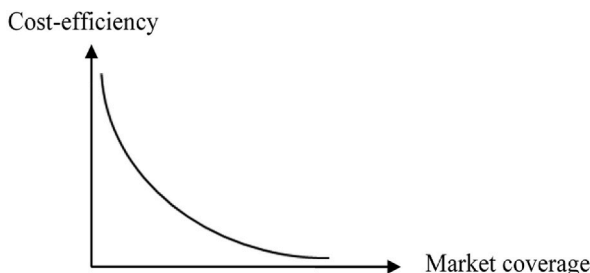


Fig. 1. Paradoxical tension between cost efficiency and market coverage.

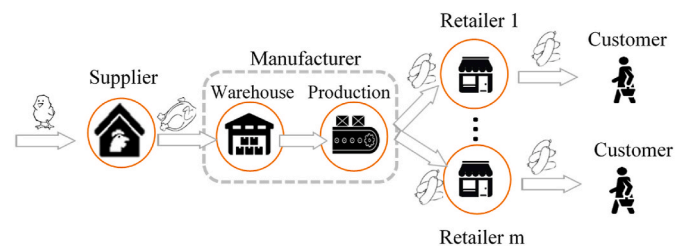


Fig. 2. Schematic view of the prescribed supply chain.

essential due to food safety, collaboration schemes have been rarely taken into account. On the other hand, growing items are the main components of various FSCs. However, no academic heed has been paid to address them within the structure of FSCs. Surprisingly, this inventory class has not been dealt with in-depth, even in the company-level frameworks. This research contributes to the existing literature by filling several existing gaps in the area, which are discussed comprehensively in Section 2. Explicitly, the contributions of this study are as follows:

- Outlining a three-level FSC, including the growing inventory and handling a main source of contradiction in complex three-level FSCs by designing a VMI system complied with a cost-sharing contract.
- Modeling growth as a biological weight increase function and simultaneously considering the impact of the breeding period and the number of young-born animals entering the system.
- Taking the negative impact of overbreeding into account to preserve the quality of the growing items and incorporating an age-dependent feeding-holding cost.
- Investigating integrated inventory management and pricing decisions of growing-perishable items in the context of FSCs.
- Modeling the interactions of the chain members through game theory.

The characterized FSC highly complies with various real-world instances in the processed food industry, such as McDonald's and Jimmy Dean Foods. As an instance, McDonald's supply chain for chicken patties in the USA involves Tyson Foods as the supplier of poultry, the chicken patties production lines as the processed food manufacturer, and McDonald's restaurants as the retailers (Goldberg and Yagan, 2007).

The rest of the paper is structured as follows. In Section 2, the relevant literature is reviewed. Section 3 provides the mathematical model, while the solution approach is outlined in Section 4. Numerical results, sensitivity analysis, and managerial insights are derived in Section 5. Section 6 concludes the paper and presents some possible directions for future research.

2. Literature review

This research mainly fits in with two streams of the literature: *Growth in inventory management* and *Revenue and inventory management in FSCs*.

2.1. Growth in inventory management

Studying growth in the context of inventory management is still in its infancy. The problem is not investigated through the supply chain, and there exist very few studies even for the single-level structures.

Rezaei (2014) for the first time incorporated growth concept in inventory management by introducing an EOQ system for growing items. Growth is measured by a mathematical weight function, and feeding costs are linked to the age of the items by using a time-dependent production function. Several studies have extended the original 'EOQ model for growing items' (Rezaei, 2014) which are discussed here. Zhang et al. (2016) extended EOQ model with growing items by taking into account environmental considerations. Nobil et al. (2018) studied the problem with admissible shortages. They treated the growth period as a known parameter by specifying the initial and final weights transforming the model into a simple EOQ framework.

Sebatjane and Adetunji (2019) developed an EOQ model for growing items with quality considerations. They also treated the growth period as a known parameter by defining a targeted final weight for each unit item. In addition to logistic growth, they applied linear and split linear estimations of the growth function. They showed that the split linear function outlines the growth better than linear. Sebatjane and Adetunji (2021) extended their previous study by studying the problem in the context of the supply chain.

Khalilpourazari and Pasandideh (2019) studied a multi-product and multi-constraint EOQ model for growing items. On-hand budget, warehouse capacity, and total allowable holding cost are considered to be limited. They used sequential quadratic programming to obtain near-optimal solutions in small size and also developed two meta-heuristics for medium and large sizes.

Malekitabar et al. (2019) proposed an inventory model for Rainbow trout. They considered the initial inventory level of the growing items to be known and applied a deterioration rate to depict mortality and an increasing rate to project growth. The nature of the items is identical during the cycle, and the breeding period has not been taken into account. These transform the model into an EOQ model in the presence of deterioration and amelioration. Gharaei and Almedhawe (2019) developed an economic quantity model for growing inventory where a fraction of the items die during their growth. They considered the growth and mortality rates to be linear functions of time, which, as shown by Sebatjane and Adetunji (2019) is unable to depict the growth of these items accurately. Pourmohammad-Zia and Karimi (2020) proposed one of the few models for the growing items for which the joint impact of the number of newborn animals and breeding period on the final inventory level is studied. For the first time, they took the negative effect of overbreeding on the quality of the slaughtered items into account.

Ameliorating inventory is a rich area of academic research, and there exist few papers in existing literature with some features of the growth. Law and Wee (2006) investigated a two-echelon supply chain of ameliorating items, including a manufacturer and a retailer. The manufacturer buys young animals and raises them. After a while, his production period starts, and the animals gradually enter the production system. After being slaughtered, the items are used as raw material. So, the ages of slaughtered animals are not the same, and thereby there is no specific breeding period. They have considered that the final products are sent to the retailer in fixed batches. However, as a simplification, the stock level of the manufacturer declines continuously due to demand. Wee et al. (2008) studied a similar problem with the same features in a retailer-level context.

2.2. Revenue and inventory management in FSCs

Food supply chain management is a relatively active research area. There exist several papers that outline the standard supplier-retailer scenario in the food industry, which is mostly referred to as two-level FSC. Wang and Li (2012) proposed one of the key studies in this context. They developed pricing policies based on the remaining shelf life of the items to decrease food waste and increase profit through a perishable food supply chain. Their results highlighted the importance of tracing food quality in the chain. Chen et al. (2014) also analyzed the impact of different pricing schemes and food quality in a two-level FSC under centralized and decentralized structures.

Sustainability has been heeded by several studies in FSCs. Govindan et al. (2014) proposed an integrated framework to specify the optimal location, routing, and production decisions in a sustainable FSC, taking both economic and environmental objectives into account. Azadnia et al. (2015) studied optimal supplier selection and ordering decisions in a multi-period multi-product setting with an application in FSC. In addition to the costs, they took sustainability into account and illustrated that this approach could yield better performance measures. Soysal et al. (2015) studied the inventory routing problem in a two-level FSC where the vendor applies VMI, and the demand is uncertain. They took environmental considerations into account by minimizing food waste and emissions. Later, Soysal et al. (2018) extended their work by analyzing the benefits of collaboration in terms of costs, emissions, driving time, and food waste in a multi-period setting. Govindan (2018) developed a conceptual framework based on different theories that identifies indicators, drivers, and barrier to achieve sustainable production and consumption in FSCs.

Three-level FSCs are hardly heeded by the researchers due to their

complexities and conflicting issues. Yu (2016) investigated inventory policies in a multi-echelon food cold chain. They optimized capacity and ordering decisions in cold storage at the manufacturer, central distribution place, and marketing point. Tabrizi et al. (2018) analyzed coordinated pricing and ordering policies for a three-level fish supply chain. They developed a bi-level Nash-Stackelberg equilibrium model, which is claimed to be NP-hard. Their results indicated that the proposed coordinated scheme leads to higher profit for the chain members.

Huang et al. (2018) studied a three-level FSC under production disruptions. The problem is investigated under isolated and coordinated scenarios to obtain optimal pricing, ordering, and preservation policies. The results showed that vertical cooperation enhances the profit of the chain members. Ma et al. (2019) developed a three-level FSC, including a supplier, a 3 PL, and a retailer where the demand is price and freshness-dependent. They considered both quality and quantity degradations, which can be controlled by preservation technologies. The problem is analyzed under decentralized and centralized structures. Rezaei et al. (2020) developed a collaboration scheme for procurement in a supply chain involving multiple suppliers, intermediate storage facilities, and retailers with an application in the food industry. They studied lot-sizing and transportation decisions for a multi-period setting where the retailers can collaborate with each other. Their results suggest that collaboration can significantly enhance the performance of the system. They found that the centralized chain with revenue-sharing provides higher profits for all chain members. Table 1 provides a general overview of the existing studies in these areas.

Coming up to a general overview, a notable academic deficiency exists in analyzing complex and three-level FSC configurations. Furthermore, modeling growth in inventory management problems is still in its preliminary stage. There is limited research at a company level, and only one effort has been made to investigate the growing items in the context of supply chains which confronts several simplifications. Even among the existing ones, the majority fails to consider the simultaneous effect of the breeding period and the initial number of newborn animals that enter the system. The negative impact of overbreeding that largely influences the quality of the items is almost overlooked. Despite their efficiency, integrated pricing and inventory control decisions are neglected in existing growth models.

Table 1

Brief overview of closely related models in the literature.

Ref	Growth	Simultaneous impact of BP and IIL	Overbreeding effect	Pricing	Structure				Coordination Mechanism	Solution Approach
					CL	2LSC	3 L SC	MR		
Rezaei (2014)	✓	✓			✓					An
Zhang et al. (2016)	✓				✓					An
Nobil et al. (2019)	✓				✓					An
Sebatjane and Adetunji (2019)	✓				✓					An
Khalilpourazari and Pasandideh (2019)	✓	✓			✓					He
Malekitabar et al. (2019)	✓				✓					An
Gharraei and Almeshdawe (2019)	✓	✓			✓					An
Pourmohammad-Zia and Karimi (2020)	✓	✓	✓		✓					An
Sebatjane and Adetunji (2021)	✓						✓			An
Wang and Li (2012)				✓		✓				An
Chen et al. (2014)				✓		✓			✓	An
Govindan et al. (2014)						✓				He
Azadnia et al. (2015)						✓				An
Soysal et al. (2015)						✓				He
Soysal et al. (2018)						✓				An
Yu (2016)							✓		✓	An
Tabrizi et al. (2018)				✓			✓		✓	An & GT
Huang et al. (2018)				✓			✓		✓	An & GT
Ma et al. (2019)				✓			✓			An
Rezaei et al. (2020)							✓	✓	✓	An
This paper	✓	✓	✓	✓			✓	✓	✓	An & GT

BP: Breeding Period, IIL: Initial Inventory Level, CL: Company Level, 2 L SC: Two-level Supply Chain, 3 L SC: Two-level Supply Chain, MR: Multi-retailer, An: Analytical Approach, He: Heuristic, GT: Game Theory.

3. Model development

3.1. Assumptions

The model is developed based on the following basic assumptions:

1. The planning horizon is infinite, and shortages are not allowed through the chain.
2. The rearing farm uses “delayed equal-size shipments” policy and delivers the slaughtered items to the processed food manufacturer.
3. The manufacturer applies VMI with common epochs to handle the retailers’ orders.
4. Unit production cost is a U-shaped convex function of the production rate and is formulated as $C_p = \varepsilon \left(\frac{a_1}{\rho} + a_2 \rho \right)$ where a_1 , a_2 and ε are non-negative scale parameters. This is in line with Khouja and Mehrez (1994).
5. The items at the retailers have shelf-lives and need to be sold out no later than this time or will be discarded.
6. Demand of retailer i is a function of its selling price and the other retailers’ prices. It is formulated as $D_i(P_R^1, \dots, P_R^i, \dots, P_R^M) = MB_i - \omega P_R^i + \sum_{j \neq i} \chi (P_R^j - P_R^i)$, where MB is the potential demand for price zero and $\omega > 0, \chi > 0$ are price sensitivity factors (Bernstein and Federgruen, 2003).
7. To ensure that the manufacturer would be able to handle all the orders per cycle, it is assumed that

$\sqrt{\frac{a_1}{a_2}} \geq \sum_i MB_i - \omega P_p$, where $\sqrt{\frac{a_1}{a_2}}$ is the lower bound for optimal production rate obtained by minimizing unit production cost and $MB_i - \omega P_p$ is the upper bound for demand.

3.2. Mathematical formulation

Consider a three-level FSC, including a rearing farm as the supplier, a processed food producer as the manufacturer, and multiple processed food retailers. The supplier, which is the rearing farm, buys newborn

animals at the beginning of each cycle, raises, and then slaughters them. The growth is outlined by measuring the weight of the items through time. A mathematical function is applied where the weight of a unit item at time t is formulated as $w_t = A(1 + be^{-jt})^{-1}$ (Richards, 1959). A is the ultimate limiting value ($A > 0$) representing the maximum possible weight of the growing item. b is the integration constant, which reflects the choice of time zero ($b > 0$). j is a constant rate, which determines the spread of the growth curve during the time axis ($0 < j < 1$). In this formulation, time is expressed in days. As the time basis of our model is year, $k=365j$ is substituted to change the time basis. (i.e. $w_t = A(1 + be^{-kt})^{-1}$, t in years).

The supplier bears feeding costs for each unit item of the growing inventory. As the unit items grow, they impose higher feeding costs on the system. To reflect this cost increase, an age-dependent function is used to calculate feeding, nourishment, and holding costs of the inventory system at the supplier. There are a number of functions in the existing literature, where polynomial and exponential functions are the most vastly applied ones (Golomytis et al., 2003). In this paper, the exponential function ($B_t = e^{\beta t}$, $\beta > 0$) is applied.

At the end of the breeding period, the items are slaughtered. For each unit item, a fraction of the weight is not utilizable (such as fat). As the items grow, they might experience illness and quality losses. Consequently, after slaughtering the items, quality control is performed, and a fraction of the inventory units, including bad quality items and non-useable parts, are discarded. This process is assumed to be instantaneous. Since overbreeding increases the fat deposition and risk of diseases, the fraction of discarded inventory should be an increasing function of T_S . This indicates that a longer breeding period brings more food waste to the system of the supplier. This fraction should also hold two more features: First, in time zero, this fraction is negligible (i.e. $\mu_0 = 0$). Second, as the breeding period takes very large values, it approaches one (i.e. $\lim_{T_S \rightarrow \infty} \mu_{T_S} = 1$). Accordingly, $\mu_{T_S} = 1 - e^{-\alpha T_S}$, $\alpha > 0$ holds these features. Hence, the negative impact of overbreeding is taken into account.

The slaughtered and quality controlled items are sent to the manufacturer. These are used continuously as raw materials to produce finished goods at the manufacturer. The manufacturer applies VMI with common epochs to handle its retailers' inventories. It fills each order as

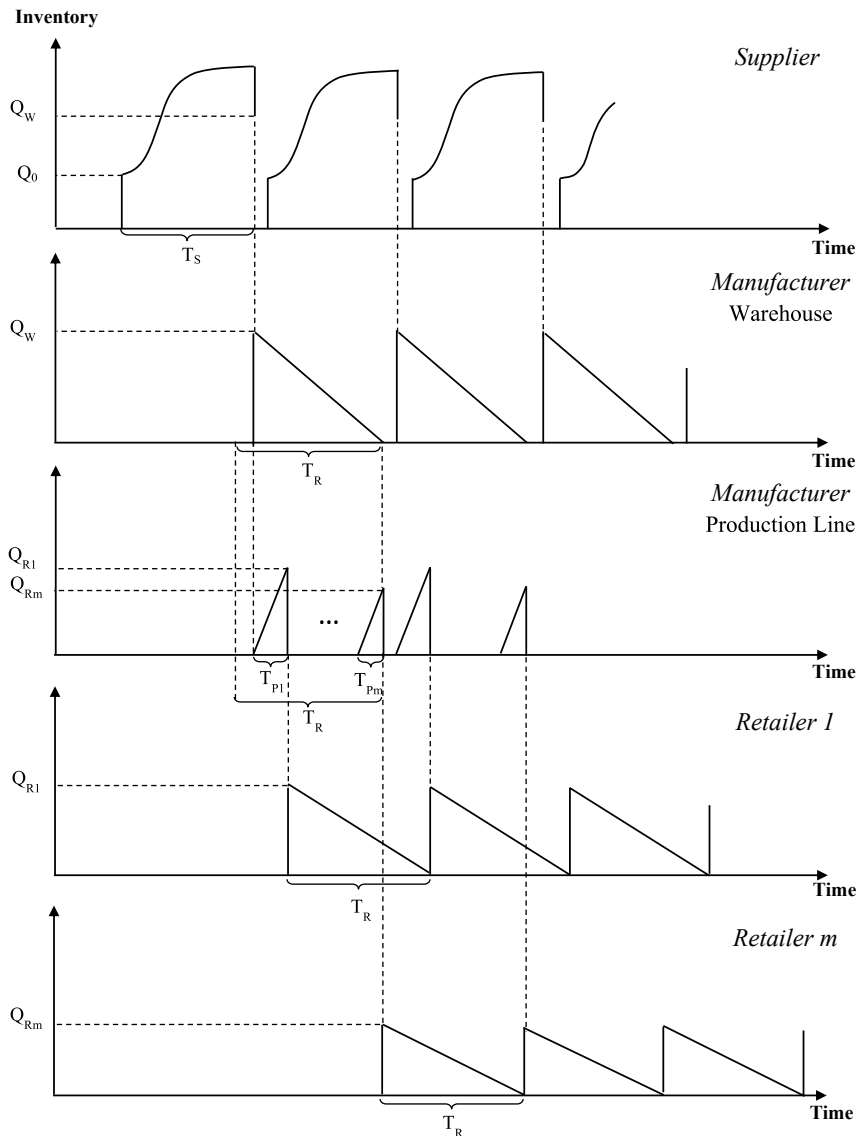


Fig. 3. The prescribed inventory system.

the items are prepared. So the dispatching procedure for different retailers is considered to be independent, which is rational when the retailers activate in a competitive environment. In such a setting, the fixed ordering cost can partially illustrate the transportation costs as well. A global set of holding-cost-sharing fractions $\eta_i = \{0, 0.1, 0.2, \dots, 1\}$ is considered among which the manufacturer chooses one to bear a part of the retailers' holding costs and provide them with the incentives to comply with this setting. The appropriate η_i for the manufacturer is the lowest possible value, which still makes the retailers eager to enter the VMI setting. This can be distinguished with respect to a term known as *reference profit* that characterizes the minimum expected profit of the retailers. Precisely, the manufacturer can reduce η_i as long as the reference profits of the retailers are met.

The manufacturer specifies the retailers' replenishment cycle, and then they optimize their prices. That is, the manufacturer moves first, and the retailers move afterward. As each party's decisions impact the other one's system, this scheme forms a two-stage game, which is commonly known as Stackelberg competition, where the manufacturer is the leader, and the retailers are the followers. The demand at each retailer is a function of its selling price as well as the other retailers' prices. Therefore, the retailers enter a Nash competition among themselves to specify their selling prices. Fig. 3 illustrates the prescribed inventory system.

FSCs embrace some features that differentiate them from other SCs. Among those are food quality and safety concerns, inventory alterations, limited lifetime, coordination complexity, and intense competition (Zhong et al., 2017). These features are addressed in our modeling process by:

- Introducing growing items
- Implementing quality control and discarding low-quality slaughtered items
- Considering limited shelf-life for the final products
- Applying VMI together with a cost-sharing contract to facilitate collaboration
- Linking the customers' demand to the retailer's selling price as well as its competitors' prices

3.3. The inventory system at the retailers

At the beginning of each inventory cycle, retailer i receives Q_{Ri} units of finished goods. During the time

Interval $[0, T_R]$ the inventory depletes to zero due to the customer's demand (recall that T_R is determined by the manufacturer). Since shortages are not admissible, the ordering quantity should satisfy the demand during the inventory cycle. That is:

$$Q_{Ri} = D_i(P_R^i, P_R^j)T_R = \left(MB_i - \omega P_R^i + \sum_{j \neq i} \chi (P_R^j - P_R^i) \right) T_R \quad (1)$$

The total profit of the inventory system at the retailer i comprises the following components:

RE_{Ri}: The sales revenue

$$RE_{Ri} = P_{Ri}Q_{Ri} = P_{Ri}D_i(P_R^i, P_R^j)T_R \quad (2)$$

PC_{Ri}: The purchasing cost

$$PC_{Ri} = P_P Q_{Ri} = P_P D_i(P_R^i, P_R^j)T_R \quad (3)$$

HC_{Ri}: The inventory holding cost

$$HC_{Ri} = (1 - \eta_i) \frac{C_{HR}}{2} Q_{Ri} T_R = (1 - \eta_i) \frac{C_{HR}}{2} D_i(P_R^i, P_R^j) T_R^2 \quad (4)$$

Afterward, the retailer i 's total profit per unit time is formulated as:

$$TP_{Ri} = \frac{RE_{Ri} - PC_{Ri} - HC_{Ri}}{T_R} \quad (5)$$

3.4. The inventory system at the manufacturer

As Fig. 3 depicts, the manufacturer dispatches each order independently. So each production size i , is determined by ρT_{Pi} , which should be equal to the retailer i 's order size. That is:

$$\rho T_{Pi} = D_i(P_R^i, P_R^j)T_R \quad (6)$$

Therefore:

$$T_{Pi} = \frac{D_i(P_R^i, P_R^j)}{\rho} T_R \quad (7)$$

The manufacturer receives Q_W units of raw material in each inventory cycle. The items are depleted to zero during $\sum_i T_{Pi}$ time units. The depletion rate of the raw material is characterized by the consumption rate to produce a unit item and the production rate $\sigma = \gamma\rho$. Then the raw material order size is obtained by the following equation:

$$Q_W = \sigma \sum_i T_{Pi} = \gamma\rho \sum_i \frac{D_i(P_R^i, P_R^j)}{\rho} T_R = \gamma \sum_i D_i(P_R^i, P_R^j) T_R \quad (8)$$

The total profit of the inventory system at the manufacturer involves the pursuant components:

RE_M: The sales revenue

$$RE_M = P_P \sum_i Q_{Ri} = P_P \sum_i D_i(P_R^i, P_R^j) T_R \quad (9)$$

PC_W: The raw material purchasing cost

$$PC_W = P_S Q_W = P_S \gamma \sum_i D_i(P_R^i, P_R^j) T_R \quad (10)$$

PC_P: The production cost

$$PC_P = C_\rho \sum_i Q_{Ri} = \epsilon \left(\frac{a_1}{\rho} + a_2 \rho \right) \sum_i D_i(P_R^i, P_R^j) T_R \quad (11)$$

HC_W: The inventory holding cost of raw material

$$HC_W = \frac{C_{HW}}{2} Q_W \sum_i T_{Pi} = \frac{C_{HW}}{2\rho} \gamma \left(\sum_i D_i(P_R^i, P_R^j) \right)^2 T_R^2 \quad (12)$$

HC_P: The inventory holding cost of finished products

$$HC_P = \frac{C_{HP}}{2} \sum_i Q_{Ri} T_{Pi} = \frac{C_{HP}}{2\rho} \sum_i D_i(P_R^i, P_R^j)^2 T_R^2 \quad (13)$$

HC_{RP}: The proportional inventory holding cost at the retailers

$$HC_{RP} = \frac{C_{HR}}{2} \sum_i \eta_i D_i(P_R^i, P_R^j) T_R^2 \quad (14)$$

OC_P: The production set up cost

$$OC_P = C_{OP} \quad (15)$$

OC_W: The ordering cost of raw material

$$OC_W = C_{OW} \quad (16)$$

OC_R: The ordering cost at the retailers

$$OC_R = \sum_i C_{OR}^i \quad (17)$$

Accordingly, the manufacturer's total profit per unit time is outlined as:

$$TP_M = \frac{RE_M - (PC_W + PC_P + HC_W + HC_P + HC_{RP} + OC_P + OC_W + OC_R)}{T_R} \quad (18)$$

3.5. The inventory system at the supplier

Suppose y unit items enter the supplier's inventory system at time zero. The weight of each unit item at time t is $w_t = A(1 + be^{-kt})^{-1}$. Therefore, the inventory level during $t \in [0, T_S]$ is illustrated by:

$$I_S(t) = yw_t = yA(1 + be^{-kt})^{-1}, 0 \leq t < T_S \quad (19)$$

So the supplier's ordering size is equal to:

$$Q_0 = I_S(0) = yA(1 + b)^{-1} \quad (20)$$

Eq. (20) gives $y = \frac{Q_0(1+b)}{A}$. Then Eq. (19) can be reformulated as:

$$I_S(t) = \frac{Q_0(1+b)}{(1 + be^{-kt})}, 0 \leq t < T_S \quad (21)$$

The final inventory before quality control is depicted by $I'_S(T_S) = \frac{Q_0(1+b)}{(1 + be^{-kT_S})}$. So the quantity of the items which should be discarded is expressed as:

$$\mu_{T_S} I'_S(T_S) = (1 - e^{-\alpha T_S}) \frac{Q_0(1+b)}{(1 + be^{-kT_S})} \quad (22)$$

Accordingly, the final inventory level after inspection yields:

$$I_S(T_S) = (1 - \mu_{T_S}) I'_S(T_S) = (1 - \mu_{T_S}) \frac{Q_0(1+b)}{(1 + be^{-kT_S})} = Q_0 \frac{e^{-\alpha T_S}(1+b)}{(1 + be^{-kT_S})} \quad (23)$$

This is the quantity that enters the manufacturer's system. So it should match Q_w :

$$Q_0 \frac{e^{-\alpha T_S}(1+b)}{(1 + be^{-kT_S})} = \gamma \sum_i D_i(P_R^i, P_R^i) T_R \quad (24)$$

Then Q_0 can be expressed as a function of T_S and T_R :

$$Q_0 = \frac{\gamma \sum_i D_i(P_R^i, P_R^i)}{(1+b)} (1 + be^{-kT_S}) e^{\alpha T_S} T_R \quad (25)$$

The total profit of the inventory system at the supplier embodies the pursuant components:

RE_S: The sales revenue

$$RE_S = P_S Q_W = P_S \gamma \sum_i D_i(P_R^i, P_R^i) T_R \quad (26)$$

PC_S: The purchasing cost

$$PC_S = C_S Q_0 = C_S \frac{\gamma \sum_i D_i(P_R^i, P_R^i)}{(1+b)} (1 + be^{-kT_S}) e^{\alpha T_S} T_R \quad (27)$$

BC_S: The breeding cost

$$\begin{aligned} BC_S &= C_B y \int_0^{T_S} B_t dt = \frac{C_B Q_0(1+b)}{A} \int_0^{T_S} e^{\beta t} dt \\ &= C_B \frac{\gamma \sum_i D_i(P_R^i, P_R^i)}{A\beta} (1 + be^{-kT_S}) e^{\alpha T_S} (e^{\beta T_S} - 1) T_R \end{aligned} \quad (28)$$

OC_S: The ordering cost

$$OC_S = C_{OS} \quad (29)$$

The supplier's inventory cycle is repeated every T_R units of time. Consequently, the supplier's total profit per unit time is:

$$TP_S = \frac{RE_S - PC_S - BC_S - OC_S}{T_R} \quad (30)$$

The lead-time has been considered to be negligible through the chain. However, non-zero lead-time (say LT) does not impact the inventory level and total cost of the system. In this case, each order should be placed LT time-units before the time that the inventory level reaches zero. So, the reorder points of the manufacturer and retailers can be easily calculated by applying equations (1) and (8), respectively.

3.6. The constraints

Recall that the retailers sell items with shelf-life, and shortages are not admissible. Since the upstream entity specifies the retailers' inventory cycle, this assumption should be heeded by that entity. So, the inventory cycle at the manufacturer, which is equal to the retailers' inventory cycles, cannot exceed the shelf-life. Otherwise, the retailers face shortages when the time oversteps the shelf-life during the cycle. This is regarded by the pursuant constraint:

$$T_R \leq m \quad (31)$$

Then the manufacturer's problem is outlined as:

$$\begin{aligned} \max TP_M \\ \text{s.t.} \\ T_R \leq m \end{aligned} \quad (32)$$

4. Solution approach

Game theory is a rigorous tool for illustrating the interrelationships among different parties in a system by identifying how their decisions influence each other. In our proposed setting, the common replenishment cycle, which is determined by the manufacturer, directly affects the systems of the retailers. On the other hand, the selling prices of the retailers influence the manufacturer's system. Since the manufacturer has applied a VMI policy and owns the power, these interactions are modeled as a non-cooperative Stackelberg game. The manufacturer is the leader who specifies the common replenishment cycle (T_R) and the production rate (ρ). Then the retailers, as the followers of the game, tend to optimize their selling prices (P_R^i) based on their observations. The retailers act in a competing setting with equal power. Accordingly, they enter a Nash competition to specify their retailing price.

Multi-stage games are commonly solved through backward induction. First, the problem of the follower is solved to obtain the follower's best response for different actions of the leader (Osborne, 2004). Taking this as the input, the solution procedure steps backward to determine the best response function for the leader.

4.1. The inventory system at the retailers

The first-order optimality condition for TP_R^i gives:

$$\frac{dTP_R^i}{dP_R^i} = MB_i - 2\delta P_R^i + \chi \sum_{j \neq i} P_R^j + (1 - \eta_i) \delta \frac{CH_R}{2} T_R + \delta P_P = 0 \quad (33)$$

Eq. (33) rules the best response function of each retailer, where $\delta = \omega + (M - 1)\chi$. In order to obtain the Nash equilibrium, the equations should be solved simultaneously for M retailers. So, we face a system of M linear equations with M variables. For notational simplicity, from this point, we step forward with two retailers. The solution procedure for M retailers is identical to the two-retailer case except that the system of linear equations involves M variables instead of two. Solving Eq. (33) yields:

$$P_R^i = \left(\frac{2\delta}{4\delta^2 - \chi^2} \right) \left[MB_i + \frac{\chi}{2\delta} MB_i + \left(1 + \frac{\chi}{2\delta} \right) P_P + \left((1 - \eta_i) + \frac{\chi}{2\delta} (1 - \eta_j) \right) \frac{CH_R \delta}{2} T_R \right] \forall i, j = 1, 2, i \neq j \quad (34)$$

The Nash equilibrium is outlined as (P_R^1, P_R^2) . In order to illustrate that these are unique optimal solutions, it is enough to show that TP_R^i is concave:

$$\frac{d^2 TP_R^i}{dP_R^i{}^2} = -2\delta \leq 0 \quad (35)$$

Accordingly, Eq. (34) provides unique optimal solutions.

4.2. The inventory system at the manufacturer

The shelf-life constraint is linear. Then if the concavity of the objective function with respect to the variables can be demonstrated, we face a convex programming optimization problem for which the Karush-Kuhn-Tucker (K.K.T.) conditions are necessary and sufficient optimization requisites. Thereafter, as the problem faces just one constraint, the K.K.T conditions provide a simple optimization scheme:

Optimize T_R without considering the constraint. If the shelf-life constraint is satisfied then $T_R^* = T_R$, otherwise $T_R^* = m$.

The necessary conditions for TP_M to be optimal, are $\frac{\partial TP_M}{\partial T_R} = 0$ and $\frac{\partial TP_M}{\partial \rho} = 0$. It should be demonstrated that these equations give unique optimal solutions. Regarding the complexity of the formulations, the concavity of the profit function cannot be demonstrated by the Hessian matrix. Then we use a similar approach like the one proposed by [Pentico and Drake \(2009\)](#) which has been later applied by many researchers ([Rabbani et al., 2014, 2017](#); [Soleymanfar et al., 2015](#); [Taleizadeh et al., 2019](#)).

Lemma 1. For a fixed ρ , there exists a unique value T_R^* which maximizes TP_M where $\frac{\partial TP_M}{\partial T_R} \big|_{T_R=T_R^*} = 0$

Proof: Regarding the backward induction, the best response function of each retailer is substituted in TP_M . The first-order optimality conditions for T_R gives:

$$\begin{aligned} \frac{\partial TP_M}{\partial T_R} = & \left[P_P - \epsilon \left(\frac{a_1}{\rho} + a_2 \rho \right) - P_S \gamma \right] (\varphi_1 + \varphi_2) - \frac{CH_P}{2\rho} [2(D_1 \varphi_1 + D_2 \varphi_2) T_R + D_1^2 + D_2^2] \\ & - \frac{CH_W}{2\rho} \gamma [2(\varphi_1 + \varphi_2)(D_1 + D_2) T_R + (D_1 + D_2)^2] \\ & - \frac{CH_R}{2} [(\eta_1 \varphi_1 + \eta_2 \varphi_2) T_R + \eta_1 D_1 + \eta_2 D_2] + \frac{C_{OP} + C_{OW} + C_{OR}^1 + C_{OR}^2}{T_R^2} = 0 \end{aligned} \quad (36)$$

where $D_i = D_i(P_R^i, P_R^j)$, $(i, j = 1, 2, i \neq j)$ for which P_R^i and P_R^j are the best responses obtained from Eq. (34) and $\varphi_i = \frac{CH_R}{2} \frac{2(\omega + \chi)^2}{4(\omega + \chi)^2 - \chi^2} \left((1 - \eta_i) \left(\omega + \chi - \frac{\chi^2}{2(\omega + \chi)} \right) + (1 - \eta_j) \frac{\chi}{2} \right)$. In order to guarantee that Eq. (36) provides a unique optimal solution for a fixed ρ , it is enough to show TP_M is concave with respect to T_R :

$$\begin{aligned} \frac{\partial^2 TP_M}{\partial T_R^2} = & -\frac{CH_P}{2\rho} [2(\varphi_1^2 + \varphi_2^2) T_R + 4D_1 \varphi_1 + 4D_2 \varphi_2] - 2 \frac{C_{OP} + C_{OW} + C_{OR}^1 + C_{OR}^2}{T_R^3} \\ & - \frac{CH_W}{2\rho} \gamma [2(\varphi_1 + \varphi_2)^2 T_R + 4(D_1 + D_2)(\varphi_1 + \varphi_2)] - \frac{CH_R}{2} [\eta_1 \varphi_1 + \eta_2 \varphi_2] \end{aligned} \quad (37)$$

If $\varphi_1, \varphi_2 \geq 0$, Eq. (37) would be negative.

$$-\omega - \frac{3\chi}{2} + \frac{4(\omega + \chi)^2 - \chi^2}{2(\omega + \chi)} = \omega + \frac{\chi}{2} - \frac{\chi^2}{2(\omega + \chi)} \geq \omega + \frac{\chi}{2} - \frac{\chi^2}{2\chi} = \omega \geq 0 \quad (38)$$

Therefore $\varphi_1, \varphi_2 \geq 0$ and Lemma 1 is proven. \square

Lemma 2. For a fixed T_R , there exists a unique value ρ^* which maximizes TP_M where $\frac{\partial TP_M}{\partial \rho} \big|_{\rho=\rho^*} = 0$

Proof: The first-order optimality conditions for ρ gives:

$$\begin{aligned} \frac{\partial TP_M}{\partial \rho} = & -\epsilon \left(-\frac{a_1}{\rho^2} + a_2 \right) (D_1 + D_2) + \frac{CH_P}{2\rho^2} (D_1^2 + D_2^2) T_R + \frac{CH_W}{2\rho^2} \gamma (D_1 + D_2)^2 T_R = 0 \end{aligned} \quad (39)$$

Then we have:

$$\rho = \sqrt{\frac{CH_P (D_1^2 + D_2^2) T_R + CH_W \gamma (D_1 + D_2)^2 T_R + 2\epsilon a_1 (D_1 + D_2)}{2\epsilon a_2 (D_1 + D_2)}} \quad (40)$$

In order to show that Eq. (40) provides a unique optimal solution for fixed T_R , it is enough to show TP_M is concave with respect to ρ :

$$\frac{\partial^2 TP_M}{\partial T_R^2} = -\epsilon \frac{2a_1}{\rho^3} (D_1 + D_2) - \frac{CH_P}{\rho^3} (D_1^2 + D_2^2) T_R - \frac{CH_W}{\rho^3} \gamma (D_1 + D_2)^2 T_R \leq 0 \quad (41)$$

Accordingly, Lemma 2 is proven.

\square In order to solve the problem at the manufacturer and the retailers, start

with the lowest value of η_i for each retailer and apply the following procedure:

- Calculate the inventory cycle (T_R) by substituting Eq. (40) into Eq. (36) and using a numerical root-finding method such as Newton-Raphson.
- If the obtained inventory cycle satisfies shelf-life constraint, then: $T_R^* = T_R$; otherwise: $T_R^* = m$.
- Obtain the optimal production rate (ρ^*) applying Eq. (40) for $T_R = T_R^*$.
- Obtain the optimal selling prices (P_R^i) applying Eq. (34) for $T_R = T_R^*$.

If the optimal total profit per unit time of each retailer meets its reference profit, $\eta_i^* = \eta_i$ and the obtained values are valid. Otherwise, use the next minimum η_i for the retailers whose reference profits are not met and again follow the instructions. Repeat the procedure until the optimal profits of both

retailers are at least as large as the reference profits.

4.3. The inventory system at the supplier side

The first-order optimality condition for TP_S is:

$$\begin{aligned} \frac{dTP_S}{dT_S} = & -C_S \gamma \frac{D_1 + D_2}{(1+b)} [\alpha e^{\alpha T_S} (1 + b e^{-k T_S}) - b k e^{\alpha T_S} e^{-k T_S}] \\ & - C_B \gamma \frac{D_1 + D_2}{A \beta} \left[\frac{\alpha e^{\alpha T_S} (1 + b e^{-k T_S}) (e^{\beta T_S} - 1) - b k e^{\alpha T_S} e^{-k T_S} (e^{\beta T_S} - 1)}{+\beta e^{\alpha T_S} (1 + b e^{-k T_S}) e^{\beta T_S}} \right] = 0 \end{aligned} \quad (42)$$

After some simplifications, we have:

$$\begin{aligned} & \frac{C_S}{1+b} [\alpha + b(\alpha - k) e^{-k T_S}] \\ & + \frac{C_B}{\beta A} [(\alpha + \beta) e^{\beta T_S} + (b k - \alpha b) e^{-k T_S} + (\alpha b - b k + \beta b) e^{-k T_S} e^{\beta T_S} - \alpha] = 0 \end{aligned} \quad (43)$$

To illustrate that Eq. (43) provides a unique optimal solution, it is enough to show that TP_S is concave:

$$\frac{d^2 TP_S}{dT_S^2} = -\gamma (D_1 + D_2) \left[\frac{C_S}{1+b} e^{\alpha T_S} \underbrace{\left[\begin{aligned} & \alpha^2 + b \alpha (\alpha - k) e^{-k T_S} \\ & - b k (\alpha - k) e^{-k T_S} \end{aligned} \right]}_I + \frac{C_B}{A \beta} e^{\alpha T_S} \underbrace{\left[\begin{aligned} & (\alpha + \beta)^2 e^{\beta T_S} + b(k - \alpha)(\alpha - k) e^{-k T_S} \\ & + b(\alpha - k + \beta)^2 e^{-k T_S} e^{\beta T_S} - \alpha^2 \end{aligned} \right]}_{II} \right] \quad (44)$$

It can be shown that if $2\alpha(1+b) + \beta(1+b) \geq 2bk$ or $\frac{C_S}{1+b} - \frac{C_B}{A\beta} \geq 0$ Eq. (44) is always non-positive (Pourmohammad-Zia and Karimi, 2020). Accordingly TP_S is concave and Eq. (43) provides unique optimal value for T_S .

5. Numerical experiments

Our proposed framework is illustrated through numerical experiments for a specific type of newborn animal: *Broiler chicken*. MATLAB R2019b is applied to solve the closed-form, and non-closed-form equations of the problem and the experiments are carried out on a computer with Intel® Core i7-8650U CPU 1.9 GHz, 2.11 GHz, and 7.88 GB memory available. We have used both experimental and derived input data as follows:

Function approximation is used to estimate the parameters of the weight function. More precisely, a data set including the weights of the broiler chickens during their lifetime in an industrial rearing farm located in the southeast of the Netherlands is used as the training input

Table 2

The value of the parameters of the problem.

Parameters	Parameters
$C_S = 0.005 \text{€}/\text{gr}^1$	$C_{OS} = 5000 \text{€}/\text{cycle}^1$
$P_S = 0.006 \text{€}/\text{gr}$	$C_{OW} = 300 \text{€}/\text{cycle}$
$\varepsilon = 6 \times 10^{-5} \text{€}/\text{gr}^2$	$C_{OP} = 200 \text{€}/\text{cycle}$
$\alpha_1 = 10^{9.2}$	$C_{OR}^b = 100 \text{€}/\text{cycle}$
$\alpha_2 = 10^{-6.2}$	$C_{OR}^c = 100 \text{€}/\text{cycle}$
$P_P = 0.012 \text{€}/\text{gr}$	$MB_1 = 5.2 \times 10^7 \text{gr}/\text{year}^1$
$C_B = 0.02 \text{€}/\text{unititem}^1$	$MB_2 = 4.8 \times 10^7 \text{gr}/\text{year}^1$
$C_{HW} = 0.0012 \text{€}/\text{gr}/\text{year}^1$	$\omega = 2 \times 10^9 \text{€}/\text{gr}$
$C_{HP} = 0.002 \text{€}/\text{gr}/\text{year}$	$\chi = 10^9 \text{€}/\text{gr}$
$C_{HR} = 0.0025 \text{€}/\text{gr}/\text{year}$	$\gamma = 0.7$

¹ From Rezaei (2014).

² From Majumder et al. (2018).

data in a neural network-based function approximation approach that provides the estimated weight function as follows:

$A = 3200, b = 69.4$ and $g = 0.12, k = 0.12 \times 365 = 43.8$. Then the growth function is formulated as $w_t = 3200(1 + 69.4e^{-43.8t})^{-1}$. The exponential breeding function $B(t)$ and the disposal rate $\mu(T_S)$ are ruled by $B(t) = e^{76t}$ and $\mu(T_S) = 1 - e^{-T_S}$, respectively.

The values of the remaining parameters are provided in Table 2. The identical parameters of the problem are taken from the related literature and adapted to our model.

The reference profit is usually equal to the profit obtained by the classical RMI system. In our problem, optimizing the classical RMI system for retailers 1 and 2 gives reference profits equal to 89,192.61 and 71,515.26, respectively. Solving the outlined problem provides the following solutions: $\eta_1 = \eta_2 = 0.5$

Supplier	$T_S = 0.08175 \text{year}$	$Q_0 = 113600.53 \text{gr}$	$TP_S = 79352.46 \text{€}$
Manufacturer	$T_R = 0.1157 \text{year}$	$\rho = 32482813.51 \text{gr}/\text{year}$	$TP_M = 112773.35 \text{€}$
Retailer 1	$Q_R^1 = 1894012.34 \text{gr}$	$P_R^1 = 0.0175291 \text{€}/\text{gr}$	$TP_R^1 = 89329.66 \text{€}$
Retailer 2	$Q_R^2 = 1695673.61 \text{gr}$	$P_R^2 = 0.0169576 \text{€}/\text{gr}$	$TP_R^2 = 71600.26 \text{€}$

The results indicate that the processed food manufacturer replenishes the inventory system of the retailers every 42 days and undertakes 50 percent of each retailer's holding cost. Afterward, the retailers specify their selling prices, which characterize their annual demand rate. So, the manufacturer should produce 3590 kg of processed food in each cycle to fill the system of both retailers. Regarding the manufacturer's optimal production rate, the production time yields 0.1105 time units. That is to say, in each inventory cycle, the manufacturer spends 95 percent of its time to produce the items. In order to meet the chicken meat ordering quantity, the rearing farm buys 114 kg (2508 chicks) of newborn broiler chickens at the beginning of the breeding period. The chicks are grown during $T_S = 31$ days. So the final weight of each broiler chicken reaches 1.091 kg. The slaughtered items are quality controlled, and 7.85 percent of the inventory which does not pass the quality standards is discarded. Fig. 4a and b project the profit values of Retailers 1 and 2 regarding different holding-cost-sharing fractions (η), respectively.

As depicted in Fig. 4a and b, for any cost-sharing fraction belonging to $\{0, 0.1, 0.2, 0.3, 0.4\}$, the reference profits of the retailers are not met. Under the remaining $\eta_i \in \{0.5, 0.6, 0.7, 0.8, 0.9, 1\}$ the retailers will attend the VMI. As the manufacturer tends to lower its costs, the contract would be designed under the lowest cost-sharing fraction, which meets the retailers' requirements ($\eta_1^* = \eta_2^* = 0.5$).

If the manufacturer focuses on reducing its costs by choosing any $\eta_i \in \{0, 0.1, 0.2, 0.3, 0.4\}$, it will lose the market coverage for both of the retailers. In such a case, suppose the manufacturer serves the retailer with higher demand (retailer 1 here) according to the traditional RMI system. Then its total profit is 59497.58 which is lower than its previous profit. Although its costs are decreased by eliminating the cost-sharing scheme, market coverage of the second retailer is lost, and so is its revenue. Expressly, concentrating on cost efficiency comes at the price of losing the market coverage (and revenue), which is in line with the idea discussed in Fig. 1. In a system with several retailers and non-identical η_i , as cost-sharing rates decrease, the retailers leave the system one by one, and the gradual decrease in market coverage is perceivable.

5.1. Comparison with the usual practice

In the EU (and some other regions), the common slaughter age of broiler chickens is 42 days (Mebratie et al., 2018), while our research suggests the breeding period should be reduced to 31 days. Solving the

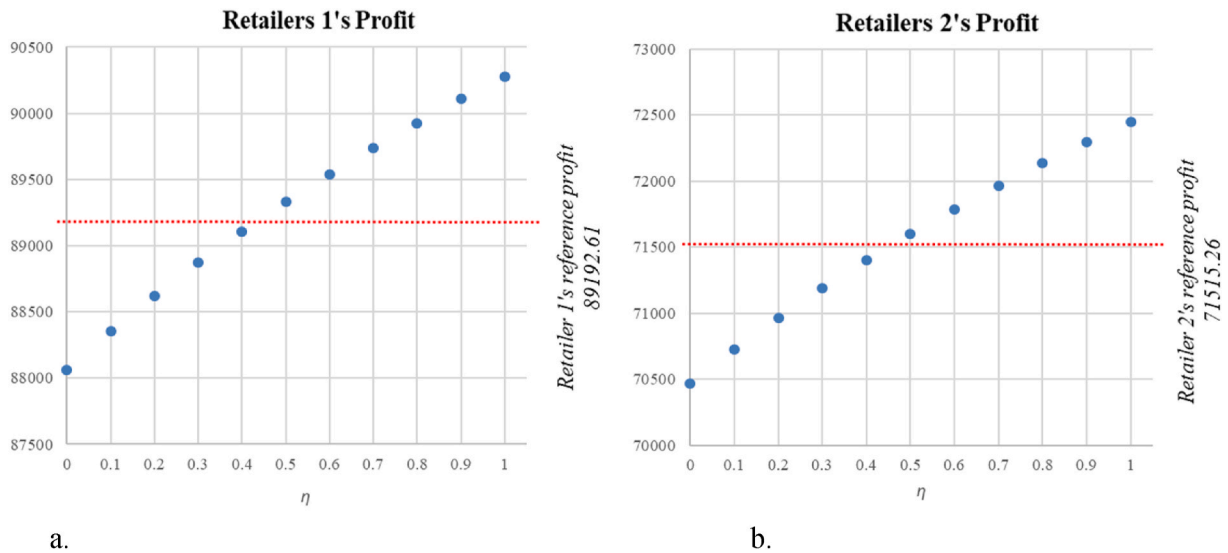


Fig. 4. a. Retailer 1's profit for different values of η . b. Retailer 2's profit for different values of η .

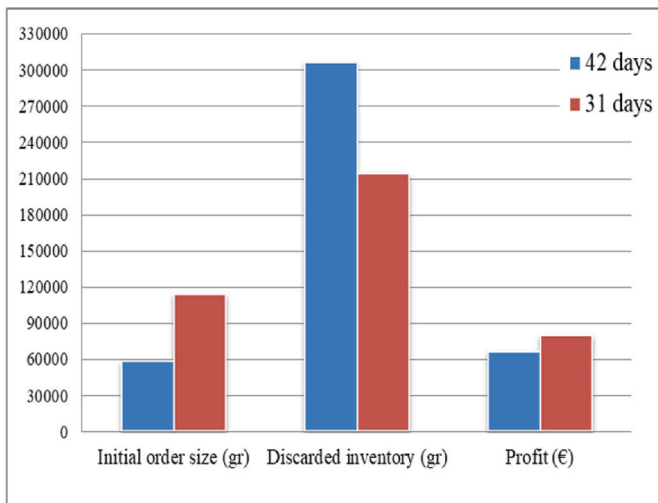


Fig. 5. Comparison of results between our approach and the usual practice.

supplier's problem under known $T_s = 42$ days yields the following results:

Supplier	$T_s = 0.1151\text{year}$	$Q_0 = 58014.32\text{gr}$	$TP_s = 66314.61\text{€}$
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This indicates that the supplier buys 1289 newborn broiler chickens at the beginning of each cycle and raises them for 42 days when the final weight of each unit item reaches 2.21 kg. Fig. 5 outlines the results for two breeding periods.

By breeding the items for 31 days, the initial order size increases in comparison to the usual practice (42 days), which is due to the decrease in the breeding period. As Fig. 5 projects, an improvement in the system is observed in terms of enhanced profit and lower food wastes. More precisely in comparison to the usual practice, 19.66% increase in profit and 30.18% decrease in food wastes (discarded inventory after quality control) are obtained.

It should be noted that this 31-day breeding period is not a one-size-fits-all policy. The optimal breeding period largely depends on the growth pattern of the broiler chickens (outlined as the weight function), which can vary for different rearing farms and growth conditions. In particular, the value is optimal for our data-set and the estimated weight function. This highlights the significance of applying the exact

optimization method instead of empirical practice.

5.2. Lifetime impact

In the preceding problem, the optimal replenishment cycle of the retailers satisfied the shelf-life constraint. What if the constraint is violated?

Consider $m = 0.1$ year. Then $T_R = 0.1157$ is not an authentic solution, and the optimal inventory cycle should be equal to $m = 0.1$:

Supplier	$T_s = 0.08175\text{year}$	$Q_0 = 98261.98\text{gr}$	$TP_s = 72661.59\text{€}$
Manufacturer	$T_R = 0.1\text{year}$	$\rho = 32368022.37\text{gr/year}$	$TP_M = 112646.12\text{€}$
Retailer 1	$Q_R^1 = 1638214.28\text{gr}$	$P_R^1 = 0.0175232\text{€/gr}$	$TP_R^1 = 89458.2\text{€}$
Retailer 2	$Q_R^2 = 1466785.71\text{gr}$	$P_R^2 = 0.0169518\text{€/gr}$	$TP_R^2 = 71715.34\text{€}$

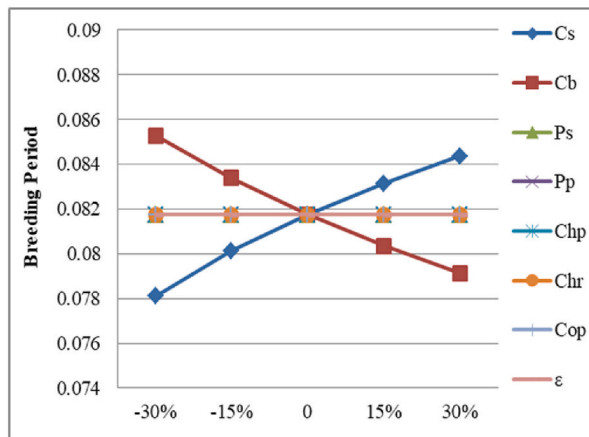
As the results depict, the manufacturer faces a decline in its profit. If the two parties do not act coherently and the manufacturer replenishes the retailers' systems without heeding this lifetime constraint, we will observe a food waste of 257.1 kgs at retailer 1 and 230.09 kgs at retailer 2. This highlights the significance of consonant decision-making in this system.

5.3. The performance of the VMI setting

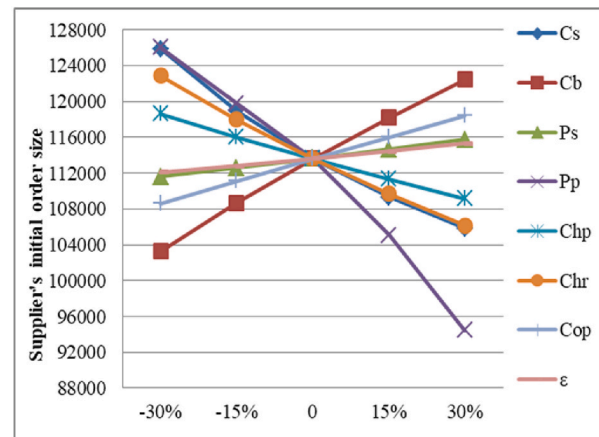
A critical question is whether the proposed VMI improves the performance of our FSC. To answer this question, the problem is studied under a different setting where the two retailers specify a joint cycle length that optimizes their overall profit. Solving the problem under this setting yields the following results:

Supplier	$T_s = 0.08175\text{year}$	$Q_0 = 50799.41\text{gr}$	$TP_s = 74621.53\text{€}$
Manufacturer	$T_R = 0.05171\text{year}$	$\rho = 32010219.8\text{gr/year}$	$TP_M = 110229.68\text{€}$
Retailer 1	$Q_R^1 = 846930.22\text{gr}$	$P_R^1 = 0.0175245\text{€/gr}$	$TP_R^1 = 86437.599\text{€}$
Retailer 2	$Q_R^2 = 758290.58\text{gr}$	$P_R^2 = 0.0169531\text{€/gr}$	$TP_R^2 = 68808.46\text{€}$

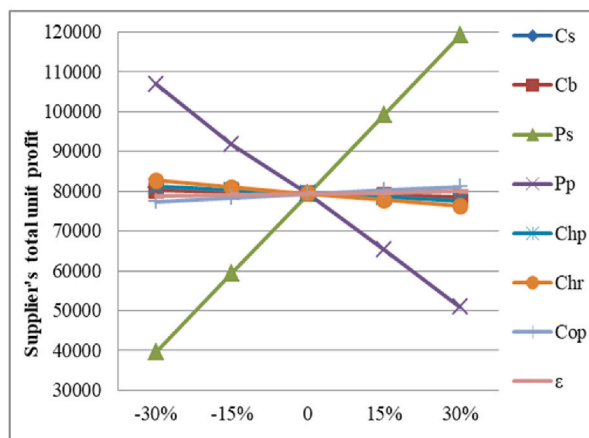
As the results suggest, the profits of the four parties decrease in the non-VMI case. By applying this collaborative scheme, the manufacturer



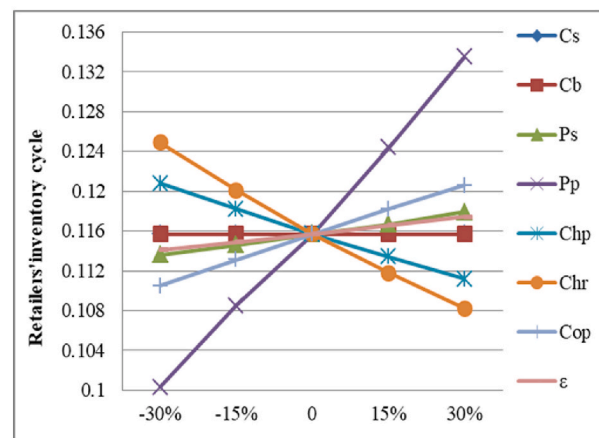
a.



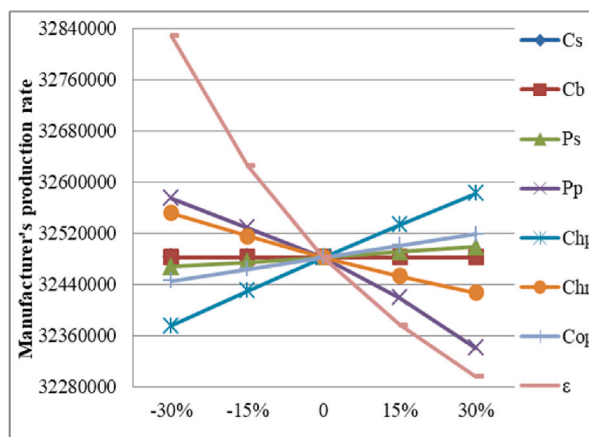
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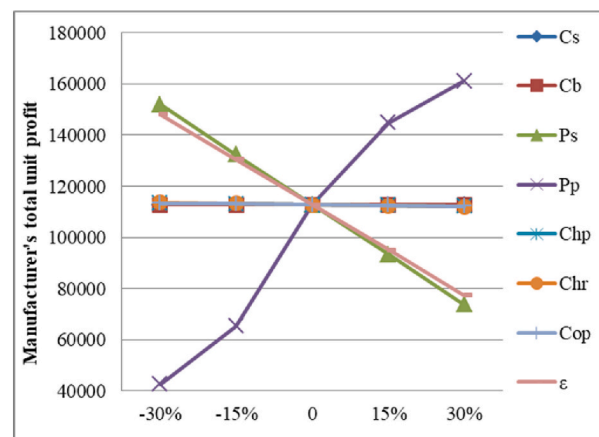
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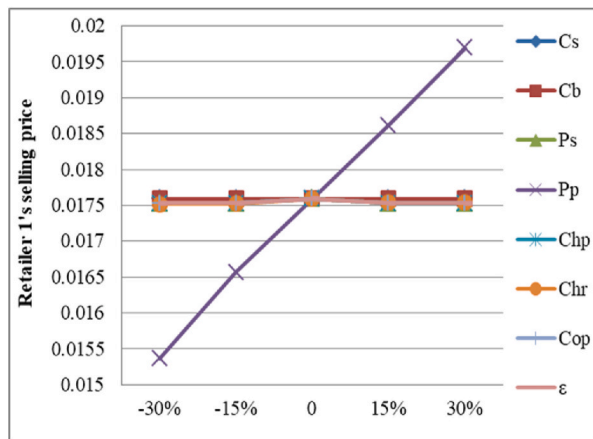


e.

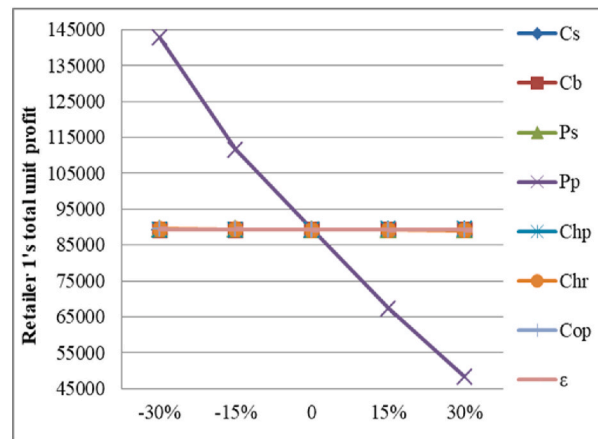


f.

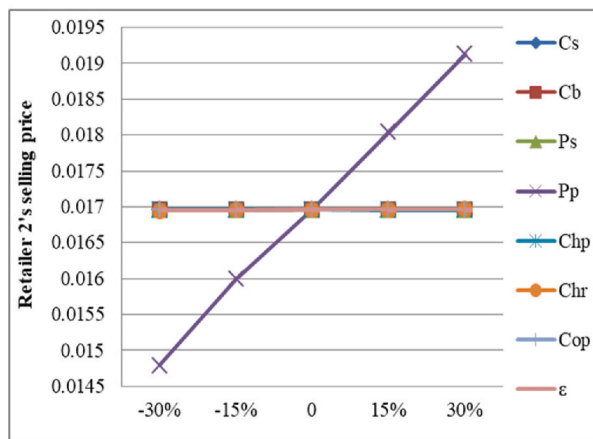
Fig. 6. a. Changes in the optimal T_S with variations in input parameters. b. Changes in the optimal Q_0 with variations in input parameters. Changes in the optimal TP_S with variations in input parameters. d. Changes in the optimal T_R with variations in input parameters. e. Changes in the optimal ρ with variations in input parameters. f. Changes in the optimal TP_M with variations in input parameters. g. Changes in the optimal P_R^1 with variations in input parameters. h. Changes in the optimal TP_R^1 with variations in input parameters. i. Changes in the optimal P_R^2 with variations in input parameters. j. Changes in the optimal TP_R^2 with variations in input parameters. k. Changes in the optimal η_i with variations in input parameters.



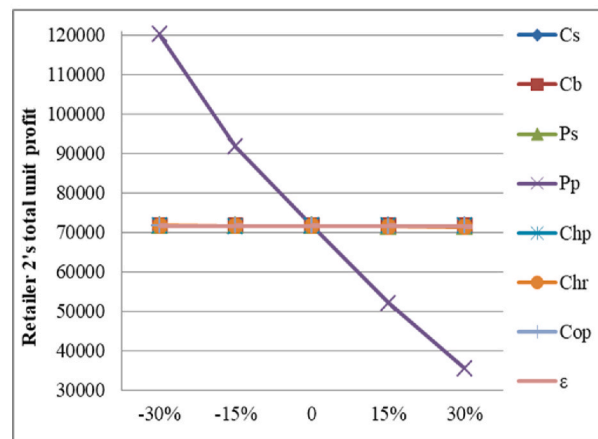
g.



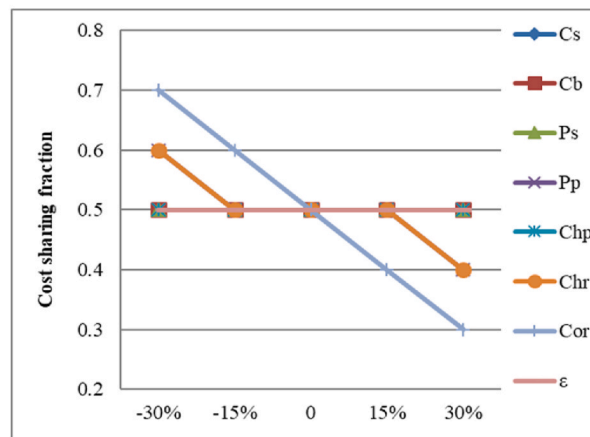
h.



i.



j.



k.

Fig. 6. (continued).

is able to handle the orders from different retailers without facing any shortages, production interruptions, and excess inventory. Since the manufacturer specifies the replenishment cycles of the retailers, it can take its production capacity constraint and supply capacity constraint of the supplier into account while determining the replenishment cycle. It is mainly important due to the bullwhip effect of the shortages through the chain. For that reason, the VMI can decrease the risk of shortages.

The retailers can take advantage of this coalition by lowering their costs and still satisfying the customer demand. The supplier can also gain benefits due to the regular raw material replenishment at the manufacturer.

5.4. The problem with more than two retailers

As the behavior of the model for different sizes of the retailers cannot be analytically derived, it is beneficial to investigate the problem under a setting with more than two retailers, to fully understand the features of the optimal solutions. To do so, we have considered two instances with three and four retailers and obtained the following results:

The instance with three retailers:

Supplier	$T_s = 0.08175\text{year}$	$Q_0 = 143802.26\text{gr}$	$TP_s = 147580.46\text{€}$
Manufacturer	$T_R = 0.08775\text{year}$	$\rho = 32513036.4\text{gr/year}$	$TP_M = 189907.12\text{€}$
Retailer 1	$Q_R^1 = 1592683.72\text{gr}$	$P_R^1 = 0.0165923\text{€/gr}$	$TP_R^1 = 82347.39\text{€}$
Retailer 2	$Q_R^2 = 1436673.84\text{gr}$	$P_R^2 = 0.0161482\text{€/gr}$	$TP_R^2 = 67004.98\text{€}$
Retailer 3	$Q_R^3 = 1514678.78\text{gr}$	$P_R^3 = 0.016379\text{€/gr}$	$TP_R^3 = 74478.65\text{€}$

The instance with four retailers:

Supplier	$T_s = 0.08175\text{year}$	$Q_0 = 168831.11\text{gr}$	$TP_s = 220210.76\text{€}$
Manufacturer	$T_R = 0.0731\text{year}$	$\rho = 32712135.6\text{gr/year}$	$TP_M = 268898.29\text{€}$
Retailer 1	$Q_R^1 = 1405585.87\text{gr}$	$P_R^1 = 0.0160514\text{€/gr}$	$TP_R^1 = 77127.66\text{€}$
Retailer 2	$Q_R^2 = 1271966.64\text{gr}$	$P_R^2 = 0.0156931\text{€/gr}$	$TP_R^2 = 63495.09\text{€}$
Retailer 3	$Q_R^3 = 1330191.43\text{gr}$	$P_R^3 = 0.0158944\text{€/gr}$	$TP_R^3 = 70053.25\text{€}$
Retailer 4	$Q_R^4 = 1327183.81\text{gr}$	$P_R^4 = 0.0159317\text{€/gr}$	$TP_R^4 = 69894.86\text{€}$

By increasing the number of retailers from two to three, a slight increase in the production rate takes place to meet the orders of the retailers. However, this increase cannot fully compensate for the third retailer's order. This is justifiable as the manufacturer cannot increase its production rate dramatically. Therefore, the manufacturer reduces the cycle length of the retailers so that their ordering quantities get smaller. With the entrance of the third retailer, the price competition gets intensified. Accordingly, the prices of the two retailers and thereby their profits decrease. On the other hand, we observe an increase in the profits of the manufacturer and supplier, which is due to having further orders from another retailer. The same pattern of the changes takes place by increasing the number of retailers from three to four. This suggests that we can expect a similar behavior by having larger numbers of retailers.

5.5. Sensitivity analysis

It is fruitful to analyze the model's behavior under different settings to deep dive into the characteristics of the decision variables and validate the proposed framework. In this regard, sensitivity analysis is carried out by varying the parameters which affect the profits, including revenue and cost components. The related results are provided in the Appendix. Fig. 6a to k illustrate the results graphically.

The numerical results provide fruitful insights, which are stated as follows:

- Variation in the supplier's unit purchasing cost does not affect the systems of the manufacturer and the retailers. As the unit purchasing cost decreases, the supplier can afford to buy more newborn chickens. Since the manufacturer's raw-material order size is fixed, there exists a reverse link between the newborn chickens' order size

and their breeding period. Thereby the breeding period gets shorter. This is in line with the pattern observed in Rezaei (2014). As the reference profits of the retailers remain constant, no changes in the cost-sharing fraction are observed.

- By decreasing the unit item breeding cost, the supplier undertakes lower feeding and holding expenses. So, the breeding period can be extended, which brings smaller sizes of the newborn chickens. Thereafter, the supplier expends lower total purchasing costs in each period. This is in line with the results in Rezaei (2014).
- Surprisingly, varying the supplier's ordering cost parameter has no impact on its optimal decision variables. This might seem odd, but it is completely rational: The supplier's inventory system recurs based on the retailers' inventory cycle, which is specified by the manufacturer.
- The supplier's unit selling price is the manufacturer's unit purchasing cost. Accordingly, its variations lead to considerable changes in both systems. As the supplier's unit selling price decreases, the raw-material ordering size can take larger values implying that the retailers' inventory cycle increases. We also observe an increase in the order size of the newborn chickens to meet the raw-material ordering quantity. The retailer's selling prices remain almost constant, and so do the demand rates. Subsequently, the retailers' ordering sizes rise as their inventory cycle increases. Then, the production rate increases to meet these orders. Following the analysis, it is worth highlighting that varying the supplier's unit selling price causes a chain of changes in the system, and as Fig. 6c suggests, it is the most influential factor on the supplier's total profit per unit time.
- The manufacturer's unit selling price plays the role of the retailers' unit purchasing cost. By decreasing the value of this parameter, the retailers' reference profits increase, and the manufacturer needs to increase the cost-sharing fraction to meet these minimums. So, reducing the manufacturer's unit selling price results in a considerable decrease in its total profit and an increase in the retailers' profits. As the manufacturer's unit selling price decreases, the retailers get the chance to reduce their selling prices and still earn higher profits through stimulating sales. The manufacturer shortens the retailers' inventory cycle in response to the reduction in its selling price. The impact of the increase in demand is higher than the inventory cycle decrease. So, the retailers' ordering sizes increase. This, in turn, raises the raw material order size and leads to an increase in the order size of the newborn chickens.
- Changes in the manufacturer's holding cost factors for raw-material and final products have the same impact on the optimal solutions of the problem. As in classical inventory models, by decreasing the holding cost, the inventory cycle increases, which leads to a rise in the retailer's order sizes. The breeding period is insensitive to changes in these parameters, and subsequently, an increase in the order size of the newborn chickens is observed. As Fig. 6f projects, these cost factors have light impacts on the manufacturer's unit profit compared to other cost parameters. This suggests that the manufacturer's profit is more resilient to changes in its holding cost parameters, and controlling these parameters does not need to be the manufacturer's first priority.
- The impact of changes in the retailer's holding cost parameter is similar to the manufacturer's holding cost parameters, and just some distinctions are observed. The scale of changes in the order sizes and profits of the manufacturer and the retailers are considerably larger than in the previous case. This stems from the point that the manufacturer's holding cost parameters are charged only during the production cycle, which is a part of the retailers' inventory cycle.
- The raw material ordering cost, the production set-up cost, and the retailers' ordering costs are all expended by the manufacturer. So,

changes in these parameters lead to similar outcomes except for the cost-sharing fraction. While changes in the raw material ordering cost and the production set-up cost have no impact on the cost-sharing fraction, by decreasing the retailers' ordering costs, their reference profits increase, which in turn raises the cost-sharing fraction. Compared to the other parameters, reducing the retailers' ordering costs results in greater changes in the cost-sharing fraction. This is because, in VMI, the retailer bears no ordering cost, and changes in its ordering cost factor do not influence its profit. Accordingly, by reducing the retailer's ordering cost factor, a larger gap between obtained and reference profits is made, which forces the manufacturer to increase the cost-sharing fraction by larger amounts. As any of the ordering cost parameters decreases, the number of replenishments increases, which brings a reduction in the inventory cycle. This reduction leads to drops in the retailers' ordering sizes.

- The production rate is optimized through a trade-off between the inventory holding cost (which tends to exceed the lower bound to shorten the production period and decrease the holding costs) and the production cost (which tends to minimize the gap between lower bound and optimal production rate to decrease the production costs). Then, as the scale parameter of the production cost (ϵ) decreases, the same overages from the lower-bound production rate lead to smaller rises in the production cost, which is also indicated by Majumder et al. (2018). On this account, decreasing this parameter induces increases in the production rate and the manufacturer's profit. As the production rate increases, the orders are produced at a faster pace, and thereby the inventory cycle gets shorter. A shorter cycle brings smaller retailers' order sizes and subsequently smaller order sizes of newborn chickens. As Fig. 6e depicts, ϵ is the most prominent factor for variations in the production rate.

As Fig. 6a illustrates, the breeding period is only sensitive to changes in the supplier's unit purchasing cost and the unit item breeding cost. This indicates that the breeding period is highly influenced by the growth pattern of the items rather than external cost factors. As depicted by Fig. 6f, h, and j, the manufacturer's unit selling price is the most dominant factor influencing the profits of the manufacturer and the retailers. It also has significant impacts on the supplier's profit. Then, it can be considered as most significant input parameter of the problem. Moreover, Fig. 6g and i shows that the retailing prices are not notably sensitive to most of the parameters of the problem. This suggests that the retailers face a highly competitive, price-oriented market in which any price increase can lead to high demand losses.

5.6. Case study

In previous subsections, we provided extensive numerical analysis under different experimental settings. In order to investigate the application of the proposed framework under a practical setting, a case study is outlined in this subsection. We consider a processed FSC in the Netherlands, involving a poultry rearing farm as the supplier, a processed food manufacturer, and three retailers. In order to preserve the privacy of the firms, their customers and business partners, the company names remain unfolded. The input parameters at the rearing farm are estimated based on the available data provided by the report on "Economics of broiler production systems in the Netherlands" (van Horne, 2020). We were not able to take all the production parameters directly from the manufacturer and the unavailable data were fabricated based on Barbut (2016), together with online facts and figures provided by Keystone Foods, GEA group, and other sources. Finally, a description of the sales reports and the inventory policies were available for the retailers.

The manufacturer produces allergy-free processed food products, including bread, stew, Croquettes, Nuggets, and several other snacks, and has three production facilities in the Netherlands. We investigate the facility located in the vicinity of Rotterdam that covers the selling points in the provinces Utrecht, Gelderland, and North and South Holland. Different products are produced in distinct production lines, and we focus on chicken nuggets as the final product of our studied FSC, which contain 71% chicken meat as their main ingredient. The chicken meat is provided by a rearing farm that is the supplier of quality broiler chicken meat. The supplier breeds several flocks separately for different purposes, and we will only study the breeding of the broiler chicken flocks related to our considered production facility. Three supermarket chains sell the final products, and each includes several selling points. Since the ordering and pricing process of each retailer's selling points is handled centrally, they are treated as one retailing entity by accumulating their demands in our problem. The first, second, and third retailers involve eight, six, and five selling points, respectively. The entities of our studied FSC are illustrated in Fig. (7).

Since the data set, including weights of the broiler chickens during their lifetime, were previously obtained from an industrial rearing farm located in the southeast of the Netherlands, the parameters of the weight function remain identical: $A = 3200, b = 69.4$ and $g = 0.12, k = 0.12 * 365 = 43.8$. Then, the growth function is formulated as $w_t = 3200(1 + 69.4e^{-43.8t})^{-1}$.

As the broiler chickens are slaughtered at age 42 days, there was no concise basis for estimating the disposal fraction function $\mu(T_s)$. Accordingly, we take 0.5, 1, 1.5, and 2 as the value of α and represent the results of the problem for each value. The value of β is estimated as 65

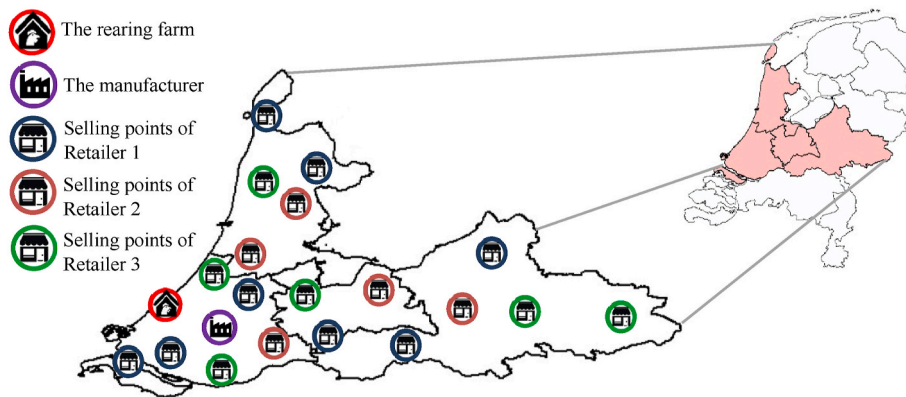


Fig. 7. The graphic representation of our case study.

Table 3

The results of the case study.

α	T_S	Q_0	TP_S	T_R	ρ	TP_M	Q_R^1	P_R^1
0.5	0.092188	10585.86	25011.17	0.04924	16220795.19	41966.52	174068	0.021259
1	0.091843	11174.31	24901.66	0.04924	16220795.19	41966.52	174068	0.021259
1.5	0.091496	11792.51	24787.43	0.04924	16220795.19	41966.52	174068	0.021259
2	0.091147	12441.8	24668.31	0.04924	16220795.19	41966.52	174068	0.021259
α	TP_R^1	Q_R^2	P_R^2	TP_R^2	Q_R^3	P_R^3	TP_R^3	
0.5	21229.52	149897.3	0.02044	15791.09	126750.1	0.019657	11335.84	
1	21229.52	149897.3	0.02044	15791.09	126750.1	0.019657	11335.84	
1.5	21229.52	149897.3	0.02044	15791.09	126750.1	0.019657	11335.84	
2	21229.52	149897.3	0.02044	15791.09	126750.1	0.019657	11335.84	

based on the input data, projecting the average of accumulated breeding costs at ages 7, 14, 21, 28, 35, and 42 days for industrial rearing farms in the Netherlands. The breeding cost involves feeding, health, heating, catching, and litter costs and is estimated as $C_B = 0.029\text{€}/\text{unititem}$. The purchasing cost of newborn broiler chickens and the selling price of slaughtered items are $C_S = 0.0069\text{€}/\text{gr}$ and $P_S = 0.0103\text{€}/\text{gr}$, respectively. The supplier's ordering cost involves costs of preparing purchase requisition and ordering, fixed terms of transportation, labor cost for inspecting received newborn chicks, and handling received orders. The value of this parameter is estimated as $C_{OS} = 1950\text{€}/\text{cycle}$ for our studied rearing farm.

The production cost function parameters are estimated based on the available data on the production cost for various production rates. We have $\varepsilon = 5.73 \times 10^{-5}\text{€}/\text{gr}$, $a_1 = 2.5 \times 10^8$ and $a_2 = 10^{-6}$. These values satisfy the required relationship between the maximum demand and the lower bound on the production rate. The wholesale price of final products is $P_P = 0.0152\text{€}/\text{gr}$. The holding costs of the manufacturer include warehouse storage fees, insurance, and opportunity charges. Apparently, this cost is higher for the final chicken nuggets compared to the frozen meat. The values of the holding costs for the raw material and final products at the manufacturer are taken as $C_{HW} = 0.0028\text{€}/\text{gr}/\text{year}$ and $C_{HP} = 0.0039\text{€}/\text{gr}/\text{year}$. The raw material ordering cost is estimated as $C_{OW} = 220\text{€}/\text{cycle}$. The production set up cost includes arranging the production line, preparing and moving frozen meat and other input material, and testing the final product, and is estimated as $C_{OP} = 170\text{€}/\text{cycle}$.

The holding cost of the retailers and ordering costs of the first, second, and third retailer are estimated as $C_{HR} = 0.0054\text{€}/\text{gr}/\text{year}$, $C_{OR}^1 = 137\text{€}/\text{cycle}$, $C_{OR}^2 = 125\text{€}/\text{cycle}$, and $C_{OR}^3 = 115\text{€}/\text{cycle}$, respectively. The inventory system of these retailers is currently handled by themselves,

and by switching to VMI, the ordering cost parameters should decrease. This is because, under the new setting, the orders are handled by the manufacturer, and he is responsible for the ordering cost of the retailers. Regarding SC experts' opinions, the approximate decrease in these values is taken as 35%. The parameters of the demand functions are generated based on an empirical distribution following the retailers' annual demand reports and the market's price sensitivity. We have $MB_1 = 1.05162 \times 10^7 \text{gr}/\text{year}$, $MB_2 = 9.41664 \times 10^6 \text{gr}/\text{year}$, $MB_3 = 7.88364 \times 10^6 \text{gr}/\text{year}$, $\omega = 3.12 \times 10^8 \text{€}/\text{gr}$, and $\chi = 1.44 \times 10^8 \text{€}/\text{gr}$.

Solving the outlined problem yields the following results (see Table 3).

Changing the value of α does not impact the inventory systems of the manufacturer or the retailers. The manufacturer orders chicken meat every 18 days and fills the retailers' inventory system with the same frequency. The nuggets are produced in packs of 250 gr. Then, the manufacturer produces 64883 packs each year to meet the customers' demand at the retailers, bringing 41966 € profit to its system. Considering the average annual production of different products in three manufacturing facilities and the firm's total annual profit, this value is roughly estimated as 37500 € in practice. Then, our obtained profit is 11.9% higher than the current profit, highlighting the positive impact of VMI in these FSCs.

The selling price of the first, second, and third retailers are 5.31, 5.11, and 4.91 per pack, respectively. These values are similar to the current average price of the product at the market, which is 5.17 €. Currently, the products are refilled every 12–15 days at the retailers. Then, taking this value and the selling price to estimate the retailers' current profit, we can conclude that they will experience a profit boost of at least 7.5% (up to 13.6%) by switching to VMI.

In order to meet the required chicken meat for the studied production facility, the rearing farm buys a flock of 235–276 newborn chicks every 18 days and raises them for 33–34 days. It slaughters the chickens at age 42 days currently, which, as previously discussed, imposes additional costs to the system. Figure (8) projects the optimal annual profit of the rearing farm for different values of the breeding period raising from 25 to 45 days.

By varying the value of α from 0.5 to 1, the breeding period and initial size of newborn chickens undergo 1.12% and −17.53% changes, respectively. This implies that, although the breeding period is not very sensitive to changes in this value, the initial order size is highly affected. Any inaccurate estimation of this value can lead to unsatisfied orders or excess inventory after slaughtering the chickens, which are both potentially undesirable. As an instance, if the accurate value of α is 1, estimating it as 0.5 and 2 will lead to 4.7% unsatisfied orders and 4.4% excess inventory, respectively. Then, estimating this value is a crucial step in incorporating our proposed approach, which can be achieved by empirical studies on the discarded fraction of the slaughtered broiler chickens at different ages. This is not only important from the cost perspective but also is critical in view of sustainability, as it has a direct impact on food waste.

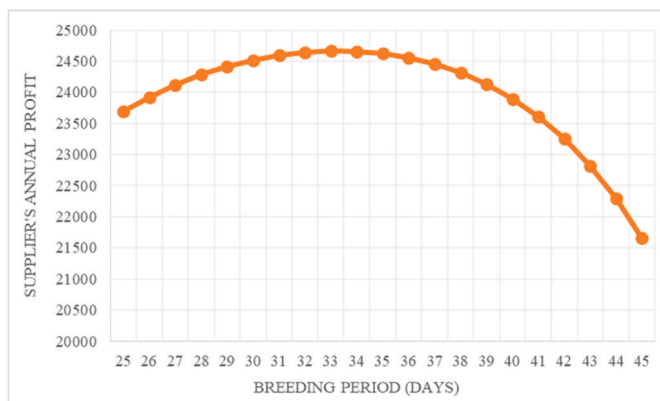


Fig. 8. The annual profit of the rearing farm for different values of the breeding period.

5.7. Managerial implications

As stated by Food Storage and Distribution Federation (2010), VMI is a rigorous mechanism to handle conflicts in FSCs by improving the interactions between the involved parties. Several papers also indicated the efficiency of VMI in complex chain structures, such as Laganà et al. (2016), Stellingwerf et al. (2018), and Amiri et al. (2020). Our results support this claim as the proposed framework provides a coordination mechanism to reduce the mismatches through the FSC that leads to the chain members' enhanced profits. It can also decrease the food waste that introduces VMI as a more sustainable mechanism. Our findings suggest that VMI's success highly relies on the designed cost-sharing contract, which complies with the results of Zhao et al. (2019) and Zhang et al. (2020). This indicates that the manufacturer should pay particular attention to the specification of the cost-sharing fraction. However, this requires a smooth flow of information among the vendor and retailers, which suggests that VMI is an appropriate coordination mechanism in long-run business cooperation, where the trust is already built.

The growth function imposes limitations on the speed of the weight increase and the ultimate weight of the broiler chickens. Furthermore, since quality standards and the negative impact of overbreeding are considered in the model, the changes in the supplier's purchasing cost and even breeding cost do not change the breeding period on a severe scale in comparison to other variables. This is while the breeding period is highly affected by the pattern of the growth, which is modeled as the growth function. This indicates that the first step in applying our proposed framework is estimating the growth parameters accurately. Inaccurate estimation might lead to financial losses.

The results of our case study suggest that any inaccurate estimation of disposal fraction function can lead to unsatisfied orders or excess inventory after slaughtering the chickens, which are both potentially undesirable. Then, estimating the respective value is a crucial step in incorporating our proposed approach, which can be achieved by empirical studies on the discarded fraction of the slaughtered broiler chickens at different ages.

The results propose fruitful insights to the decision-makers in different echelons of FSCs under various operational settings. The most influential cost factors of involved parties are distinguished, providing them with a guideline to prioritize their focus in improving their systems and handling costs. Since the studies on growing items are still in a very initial step, the findings can be particularly beneficial for the rearing farms as the suppliers of FSCs. The results imply that if the rearing farm has a chance to choose among different hatcheries, selecting the one with the lowest purchasing cost can not only reduce the costs but also shorten the breeding period. This is specifically advantageous under the conditions of a newly emerging disease among the broiler chickens or

the customers' desire to buy younger items. Moreover, if the rearing farm faces limitations on its periodic purchasing budget, the firm can manage its costs by the application of better holding technologies and feeding processes, which decreases the unit-item breeding cost. The latter can be achieved by manipulating feed ingredients, fermenting the feed, and incorporating grit and probiotics.

It is shown that a larger retailer size brings more intense price competition that reduces the retailers' profits. This suggests that firms that want to enter the retailing food industry have to be cautious in selecting their customer market and applying sophisticated pricing schemes to survive.

6. Conclusion

In this paper, coordinated replenishment, production, and pricing policies are studied in a three-level FSC comprising one supplier, one manufacturer, and multiple retailers. The chain involves a new type of inventory known as growing items. The manufacturer applies VMI as a coordination mechanism to manage the orders of different retailers. It undertakes a part of the retailers' holding costs to provide them with enough incentives to enter this collaborative setting. Accordingly, there exists a battle between increasing market coverage and optimizing the manufacturer's costs. To handle these two conflicting targets, the cost-sharing contract is designed with respect to reference profits of the retailers. An analytic solution approach with a game-theoretic perspective is used to solve the outlined problem. The numerical experiments provide the chain members with helpful insights that can be incorporated in different operational situations. The results support the claim that VMI is a strong mechanism in the FSCs to achieve a competitive advantage by improving the chain members' interactions. It is also shown that the setting of the designed cost-sharing contract plays an important role in VMI's success.

There exist some interesting research directions to extend this study: Various coordination incentives such as quantity discounts or overstocking penalties can be applied to enhance the cooperation through the FSC. Taking uncertainty into account is a promising future research direction that can address different growth speeds of the flocks of newborn animals. More precisely, incorporating the uncertainty of growth parameters can lead to a distributional value for the breeding period instead of a constant one. Considering transportation as a major factor in food logistics is another future research avenue. It can influence assumptions of the model, such as the dispatching scheme and the structure of the formulations, specifically in non-competing environments. Finally, considering admissible shortages in the problem under some modifications can lead to new understandings of the FSCs with growing items.

Appendix. -Numerical Results

Table 1 provides the results of sensitivity analysis on the key parameters of our model.

Table 1
Sensitivity analysis on parameters of profit components

Parameter	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
C_S	-30%	0.07817 (-4.38%)	125890.22 (+10.82%)	80897.92 (+1.94%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	-15%	0.08013 (-1.98%)	118936.5 (+4.69%)	80105.47 (+0.95%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	+15%	0.08314 (+1.7%)	109335.07 (-3.75%)	78630.36 (-0.91%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	+30%	0.08435 (+3.18%)	105821.45 (-6.85%)	77933.36 (-1.79%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%							

(continued on next page)

Table 1 (continued)

Parameter	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
Parameter C_B		0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	-15%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	+15%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	+30%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08528 (+4.31%)	103254.83 (-9.11%)	80275.35 (+1.16%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	-15%	0.08337 (+1.98%)	108667.23 (-4.34%)	79794.07 (+0.56%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	+15%	0.08035 (-1.71%)	118163.74 (+4.02%)	78941.77 (-0.52%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	+30%	0.07912 (-3.2%)	122430.22 (+7.77%)	78556.04 (-1.01%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	-15%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	+15%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	+30%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	113600.53 (0%)	92317.29 (+16.34%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	-15%	0.08175 (0%)	113600.53 (0%)	85834.88 (+8.17%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	+15%	0.08175 (0%)	113600.53 (0%)	72870.05 (-8.17%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
	+30%	0.08175 (0%)	113600.53 (0%)	66387.63 (-16.34%)	0.1157 (0%)	32482813.51 (0%)	112773.35 (0%)	0.5 (0%)
Parameter C_{OS}	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	-15%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	+15%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	+30%	0.5 (0%)	1894012.34 (0%)	0.0175291 (0%)	89329.66 (0%)	1695673.61 (0%)	0.0169576 (0%)	71600.26 (0%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	115731.72 (+1.88%)	39482.32 (-50.24%)	0.1179 (+1.9%)	32498730.91 (+0.05%)	151868.65 (+34.67%)	0.5 (0%)
	-15%	0.08175 (0%)	114651.02 (+0.92%)	59416.58 (-25.42%)	0.1167 (+0.86%)	32490660.38 (+0.02%)	132320.51 (+17.33%)	0.5 (0%)
	+15%	0.08175 (0%)	112578.79 (-0.89%)	99289.93 (+25.13%)	0.1146 (-0.95%)	32475179.57 (-0.02%)	93227.19 (-17.33%)	0.5 (0%)
	+30%	0.08175 (0%)	111584.63 (-1.77%)	119229.03 (+50.25%)	0.1136 (-1.82%)	32467749.97 (-0.05%)	73682.06 (-34.61%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1929555.34 (+1.88%)	0.0175299 (+0.005%)	89311.8 (-0.02%)	172474.37 (+1.88%)	0.0169585 (+0.005%)	71584.27 (-0.02%)
	-15%	0.5 (0%)	191531.89 (+0.92%)	0.0175295 (+0.002%)	89320.86 (-0.009%)	1711348.67 (+0.92%)	0.0169581 (+0.003%)	71592.38 (-0.01%)
	+15%	0.5 (0%)	1876972.33 (-0.89%)	0.0175287 (-0.002%)	89338.23 (+0.009%)	1680427.43 (-0.89%)	0.0169572 (-0.002%)	71607.93 (+0.01%)
	+30%	0.5 (0%)	1860392.36 (-1.77%)	0.0175283 (-0.005%)	89346.56 (+0.019%)	1665592.7 (-1.77%)	0.0169569 (-0.011%)	71615.39 (+0.02%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	121787.23 (+7.2%)	105315.81 (+32.71%)	0.09687 (-16.27%)	32543328.11 (+0.18%)	1598.75 (-98.58%)	0.6 (+20%)
	-15%	0.08175 (0%)	119787.77 (+5.45%)	91710.27 (+15.57%)	0.1085 (-6.22%)	32528693.55 (+0.14%)	65228.59 (-42.15%)	0.5 (0%)
	+15%	0.08175 (0%)	105060.33 (-7.52%)	65248.11 (-17.77%)	0.1244 (+7.52%)	32419408.47 (-0.19%)	144707.47 (+28.31%)	0.5 (0%)
	+30%	0.08175 (0%)	95797.75 (-15.67%)	51855.7 (-34.65%)	0.1341 (+15.98%)	32350606.01 (-0.4%)	160606.96 (+42.41%)	0.4 (-20%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.6 (+20%)	2007228.27 (+5.97%)	0.0153558 (-12.39%)	143092.53 (+60.18%)	1841150.75 (+8.58%)	0.0147833 (-12.82%)	120393.25 (+68.14%)
	-15%							

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Table 1 (continued)

Parameter	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
Parameter C_{HW}		0.5 (0%)	1985623.93 (+4.48%)	0.0165664 (-5.49%)	111578.11 (+24.91%)	1799573.75 (+6.12%)	0.015995 (-5.68%)	91648.27 (+27.99%)
	+15%	0.5 (0%)	1766512.76 (-6.73%)	0.0186124 (+6.18%)	67249.94 (-24.72%)	1553309.71 (-8.39%)	0.0180409 (+6.39%)	51996.54 (-27.37%)
	+30%	0.4 (-20%)	1628586.67 (-14.03%)	0.0196461 (+12.07%)	49097.17 (-45.04%)	1398545.69 (-17.52%)	0.0190747 (+12.48%)	36206.62 (-49.43%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	117767.29 (+3.67%)	80864.81 (+1.91%)	0.1199 (+3.63%)	32395278.21 (-0.27%)	113213.7 (+0.39%)	0.5 (0%)
	-15%	0.08175 (0%)	115395.13 (+1.58%)	80017.44 (+0.84%)	0.1175 (+1.56%)	32443810.69 (-0.12%)	112991.4 (+0.19%)	0.5 (0%)
	+15%	0.08175 (0%)	111681.5 (-1.69%)	78617.35 (-0.93%)	0.1137 (-1.73%)	32525486.94 (+0.13%)	112559.31 (-0.19%)	0.5 (0%)
	+30%	0.08175 (0%)	109859.73 (-3.29%)	77895.35 (-1.84%)	0.1119 (-3.28%)	32566973.23 (+0.26%)	112349.07 (-0.37%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1963504.09 (+3.67%)	0.0175307 (+0.009%)	89294.73 (-0.039%)	1757848.05 (+3.67%)	0.016959 (+0.008%)	71568.99 (-0.04%)
	-15%	0.5 (0%)	1923941.85 (+1.58%)	0.0175298 (+0.004%)	89314.62 (-0.017%)	1722451.97 (+1.58%)	0.0169584 (+0.005%)	71586.79 (-0.02%)
	+15%	0.5 (0%)	1862007.89 (-1.69%)	0.0175284 (-0.004%)	89345.75 (+0.018%)	1667038.19 (-1.69%)	0.0169569 (-0.004%)	71614.67 (+0.02%)
	+30%	0.5 (0%)	1831625.87 (-3.29%)	0.0175277 (-0.008%)	89361.02 (+0.035%)	1639853.84 (-3.29%)	0.0169562 (-0.008%)	71628.34 (+0.03%)
Parameter C_{HP}	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	118632.73 (+4.43%)	81165.38 (+2.28%)	0.1208 (+4.41%)	32375532.4 (-0.33%)	113301.2 (+0.46%)	0.5 (0%)
	-15%	0.08175 (0%)	116032.05 (+2.14%)	80248.42 (+1.13%)	0.1182 (+2.16%)	32430173.1 (-0.16%)	113034.21 (+0.23%)	0.5 (0%)
	+15%	0.08175 (0%)	111320.46 (-2.01%)	78476.17 (-1.1%)	0.1134 (-1.98%)	32533629.46 (+0.16%)	112518.21 (-0.22%)	0.5 (0%)
	+30%	0.08175 (0%)	109176.81 (-3.89%)	77618.38 (-2.18%)	0.1112 (-3.89%)	32582775.41 (+0.31%)	112268.44 (-0.45%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1977937.77 (+4.43%)	0.0175314 (+0.009%)	89287.48 (-0.047%)	1770761.54 (+4.43%)	0.0169596 (+0.012%)	71562.49 (-0.05%)
	-15%	0.5 (0%)	1934564.17 (+2.14%)	0.0175304 (+0.004%)	89309.28 (-0.023%)	1731955.79 (+2.14%)	0.0169591 (+0.008%)	71582.01 (-0.02%)
	+15%	0.5 (0%)	1855986.74 (-2.01%)	0.0175282 (-0.004%)	89348.78 (+0.021%)	1661650.8 (-2.01%)	0.0169568 (-0.005%)	71617.38 (+0.02%)
	+30%	0.5 (0%)	1820236.47 (-3.89%)	0.0175274 (-0.008%)	89366.75 (+0.041%)	1629663.01 (-3.89%)	0.016956 (-0.009%)	71633.46 (+0.05%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	120296.96 (+5.9%)	81944.86 (+3.27%)	0.1223 (+5.65%)	32532793.89 (+0.15%)	113434.98 (+0.59%)	0.6 (+20%)
	-15%	0.08175 (0%)	117956.56 (+3.83%)	81001.82 (+2.07%)	0.1201 (+3.81%)	32515336.95 (+0.1%)	113215.07 (+0.39%)	0.5 (0%)
	+15%	0.08175 (0%)	109692.88 (-3.44%)	77764.58 (-2.01%)	0.1118 (-3.37%)	32453609.98 (-0.089%)	112347.33 (-0.51%)	0.5 (0%)
	+30%	0.08175 (0%)	108780.86 (-4.2%)	77206.91 (-2.7%)	0.1112 (-3.98%)	32446796.27 (-0.11%)	112254.66 (-0.46%)	0.4 (-20%)
Parameter C_{HR}	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.6 (+20%)	2005419.72 (+5.88%)	0.0175185 (-0.06%)	89716.78 (+0.43%)	1795868.11 (+5.9%)	0.0169471 (-0.06%)	71946.89 (+0.48%)
	-15%	0.5 (0%)	1966570.55 (+3.83%)	0.017524 (-0.029%)	89441.44 (+0.12%)	1760762.33 (+3.84%)	0.0169526 (-0.03%)	71705.33 (+0.14%)
	+15%	0.5 (0%)	1828921.26 (-3.44%)	0.0175339 (+0.027%)	89224.45 (-0.12%)	1637285.9 (-3.44%)	0.0169625 (+0.03%)	71506.07 (-0.13%)
	+30%	0.4 (-20%)	1813939.86 (-4.23%)	0.0175385 (+0.054%)	89124.91 (-0.56%)	1623448.19 (-4.26%)	0.0169671 (+0.06%)	71149.2 (-0.63%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	105981.71 (-6.71%)	76274.49 (-3.88%)	0.1079 (-6.74%)	32425846.25 (-0.18%)	113578.38 (+0.71%)	0.5 (0%)
	-15%	0.08175 (0%)	109855.44 (-3.29%)	77893.62 (-1.84%)	0.1118 (-3.37%)	32454823.26 (-0.086%)	113168.85 (+0.35%)	0.5 (0%)
	+15%	0.08175 (0%)	117229.13 (+3.19%)	80675.63 (+1.67%)	0.1194 (+3.19%)	32509910.13 (+0.083%)	112390.55 (-0.34%)	0.5 (0%)
	+30%	0.08175 (0%)	120751.67 (+6.29%)	81882.79 (+3.19%)	0.1230 (+6.31%)	32536193.2 (+0.16%)	112019.32 (-0.69%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1766952.15 (-6.71%)	0.0175262 (-0.016%)	89293.52 (+0.07%)	1581985.11 (-6.7%)	0.0169547 (-0.02%)	71657.44 (+0.08%)
	-15%	0.5 (0%)	1831554.23 (-3.29%)	0.0175277 (-0.008%)	89361.06 (+0.03%)	1639789.73 (-3.29%)	0.0169562 (-0.008%)	71628.37 (+0.04%)
	+15%							

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Table 1 (continued)

Parameter	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
Parameter C_{OP}		0.5 (0%)	1954528.76 (+3.19%)	0.0175305 (+0.008%)	89299.24 (-0.03%)	1749817.97 (+3.19%)	0.0169591 (+0.009%)	71573.03 (-0.04%)
	+30%	0.5 (0%)	2013277.5 (+6.29%)	0.0175318 (+0.015%)	89269.71 (-0.07%)	1802378.7 (+6.29%)	0.0169604 (+0.02%)	71546.58 (-0.07%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	108579.16 (-4.42%)	77373.11 (-2.49%)	0.1105 (-4.49%)	32445279.06 (-0.11%)	113303.73 (+0.47%)	0.5 (0%)
	-15%	0.08175 (0%)	111117.43 (-2.19%)	78396.36 (-1.21%)	0.1131 (-2.25%)	32464257.9 (-0.057%)	113035.53 (+0.23%)	0.5 (0%)
	+15%	0.08175 (0%)	116031.86 (+2.14%)	80248.35 (+1.13%)	0.1182 (+2.16%)	32500972.01 (+0.056%)	112516.81 (-0.23%)	0.5 (0%)
	+30%	0.08175 (0%)	118414.63 (+4.24%)	81090.05 (+2.19%)	0.1206 (+4.23%)	32518757.95 (+0.11%)	112265.57 (-0.46%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1810269.64 (-4.42%)	0.0175272 (-0.011%)	89371.76 (+0.057%)	1620744.99 (-4.42%)	0.0169557 (-0.01%)	71637.95 (+0.05%)
	-15%	0.5 (0%)	1852600.77 (-2.18%)	0.0175281 (-0.006%)	89350.48 (+0.02%)	1658621.21 (-2.18%)	0.0169567 (-0.005%)	71618.9 (+0.03%)
Parameter C_{OR}	+15%	0.5 (0%)	1934561.04 (+2.14%)	0.01753 (+0.005%)	89309.28 (-0.02%)	1731952.99 (+2.14%)	0.0169586 (+0.006%)	71582.02 (-0.02%)
	+30%	0.5 (0%)	1934561.04 (+4.24%)	0.017531 (+0.011%)	89289.3 (-0.04%)	1767507.16 (+4.24%)	0.0169595 (+0.01%)	71564.13 (-0.05%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	103327.84 (-9.04%)	75247.27 (-5.17%)	0.1049 (-9.27%)	32405974.08 (-0.24%)	112949.43 (+0.15%)	0.7 (+40%)
	-15%	0.08175 (0%)	108328.87 (-4.64%)	77352.31 (-2.52%)	0.1102 (-4.77%)	32443404.18 (-0.12%)	112847.99 (+0.06%)	0.6 (+20%)
	+15%	0.08175 (0%)	119184.72 (+4.91%)	81258.55 (+2.11%)	0.1216 (+5.06%)	32524507.71 (+0.13%)	112727.16 (-0.04%)	0.4 (-20%)
	+30%	0.08175 (0%)	125131.11 (+10.15%)	83079.84 (+4.69%)	0.1278 (+10.48%)	32568848.11 (+0.26%)	112708.47 (-0.057%)	0.3 (-40%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.7 (+40%)	1722510.84 (-9.05%)	0.0175093 (-0.113%)	89761.68 (+0.48%)	1542566.15 (-9.03%)	0.0169379 (-0.12%)	71987.09 (+0.54%)
	-15%	0.6 (+20%)	1805994.46 (-4.64%)	0.0175188 (-0.058%)	89555.32 (+0.25%)	1617111.29 (-4.63%)	0.016947 (-0.06%)	71802.31 (+0.28%)
Parameter ε	+15%	0.4 (-20%)	1987267.16 (+4.92%)	0.0175404 (+0.064%)	89082.83 (-0.27%)	1778874.79 (+4.91%)	0.016969 (+0.067%)	71379.29 (-0.31%)
	+30%	0.3 (-40%)	2086591.74 (+10.17%)	0.0175528 (+0.13%)	88812.56 (-0.57%)	1867451.13 (+10.13%)	0.0169814 (+0.14%)	71137.39 (-0.65%)
	Changes	T_S	Q_0	TP_S	T_R	ρ	TP_M	η_1
	-30%	0.08175 (0%)	112083.18 (-1.34%)	78773.33 (-0.73%)	0.1141 (-1.38%)	32828488.27 (+1.06%)	148113.55 (+31.34%)	0.5 (0%)
	-15%	0.08175 (0%)	112808.19 (-0.69%)	79052.02 (-0.38%)	0.1149 (-0.69%)	32625299.19 (+0.44%)	130441.66 (+15.67%)	0.5 (0%)
	+15%	0.08175 (0%)	114443.53 (+0.74%)	79667.47 (+0.39%)	0.1166 (+0.78%)	32377425.14 (-0.32%)	95107.56 (-15.66%)	0.5 (0%)
	+30%	0.08175 (0%)	115328.53 (+1.52%)	79993.13 (+0.81%)	0.1174 (+1.47%)	32296365.95 (-0.58%)	77443.73 (-31.33%)	0.5 (0%)
	Changes	η_2	Q_R^1	P_R^1	TP_R^1	Q_R^2	P_R^2	TP_R^2
	-30%	0.5 (0%)	1868706.78 (-1.33%)	0.0175285 (-0.003%)	89342.39 (+0.01%)	1673031.95 (-1.33%)	0.0169571 (-0.003%)	71611.65 (+0.02%)
	-15%	0.5 (0%)	1880798.17 (-0.69%)	0.0175288 (-0.002%)	89336.31 (+0.007%)	1683850.53 (-0.69%)	0.0169574 (-0.001%)	71606.21 (+0.008%)
Parameter ε	+15%	0.5 (0%)	1908071.41 (+0.74%)	0.0175294 (+0.002%)	89322.6 (-0.008%)	1708252.5 (+0.74%)	0.016958 (+0.002%)	71593.94 (-0.008%)
	+30%	0.5 (0%)	1922831.09 (+1.52%)	0.0175298 (+0.004%)	89315.18 (-0.02%)	1721458.17 (+1.52%)	0.0169583 (+0.004%)	71587.29 (-0.02%)

References

- Amiri, S.A.H.S., Zahedi, A., Kazemi, M., Soroosh, J., Hajiaghaei-Keshteli, M., 2020. Determination of the optimal sales level of perishable goods in a two-echelon supply chain network. *Comput. Ind. Eng.* 139, 106156.
- Azadnia, A.H., Saman, M.Z.M., Wong, K.Y., 2015. Sustainable supplier selection and order lot-sizing: an integrated multi-objective decision-making process. *Int. J. Prod. Res.* 53 (2), 383–408.
- Barbut, S., 2016. *Poultry Products Processing: an Industry Guide*. CRC press.
- Benner, M.J., Tushman, M.L., 2003. Exploitation, exploration, and process management: the productivity dilemma revisited. *Acad. Manag. Rev.* 28 (2), 238–256.
- Bernstein, F., Federgruen, A., 2003. Pricing and replenishment strategies in a distribution system with competing retailers. *Oper. Res.* 51 (3), 409–426.
- Cai, J., Hu, X., Han, Y., Cheng, H., Huang, W., 2016. Supply chain coordination with an option contract under vendor-managed inventory. *Int. Trans. Oper. Res.* 23 (6), 1163–1183.
- Chen, C., Zhang, J., Delaurentis, T., 2014. Quality control in food supply chain management: an analytical model and case study of the adulterated milk incident in China. *Int. J. Prod. Econ.* 152, 188–199.
- Food Storage and Distribution Federation, 2010. Food industry slow to embrace vendor-managed inventory. Retrieved from: <https://www.foodmanufacture.co.uk/Article/2010/04/22/Food-industry-slow-to-embrace-vendor-managed-inventory>.
- Fritz, M., Schiefer, G., 2008. Food chain management for sustainable food system development: a European research agenda. *Agribusiness: Int. J.* 24 (4), 440–452.
- Gharai, A., Almelhdawe, E., 2019. Economic growing quantity. *Int. J. Prod. Econ.* 107517.
- Göbel, C., Langen, N., Blumenthal, A., Teitscheid, P., Ritter, G., 2015. Cutting food waste through cooperation along the food supply chain. *Sustainability* 7 (2), 1429–1445.

- Goldberg, R.A., Yagan, J.D., 2007. McDonald's Corporation: Managing a Sustainable Supply Chain, vol. 16. Harvard Business School.
- Goliomytis, M., Panopoulou, E., Rogdakis, E., 2003. Growth curves for body weight and major component parts, feed consumption, and mortality of male broiler chickens raised to maturity. *Poultry Sci.* 82 (7), 1061–1068.
- Govindan, K., 2018. Sustainable consumption and production in the food supply chain: a conceptual framework. *Int. J. Prod. Econ.* 195, 419–431.
- Govindan, K., Jafarian, A., Khodaverdi, R., Devika, K., 2014. Two-echelon multiple-vehicle location–routing problem with time windows for optimization of sustainable supply chain network of perishable food. *Int. J. Prod. Econ.* 152, 9–28.
- Huang, H., He, Y., Li, D., 2018. Pricing and inventory decisions in the food supply chain with production disruption and controllable deterioration. *J. Clean. Prod.* 180, 280–296.
- Khalilpourazari, S., Pasandideh, S.H.R., 2019. Modeling and optimization of multi-item multi-constrained EOQ model for growing items. *Knowl. Base Syst.* 164, 150–162.
- Khouja, M., Mehrez, A., 1994. Economic production lot size model with variable production rate and imperfect quality. *J. Oper. Res. Soc.* 45 (12), 1405–1417.
- Laganà, D., Longo, F., Vocaturo, F., 2016. Vendor-managed inventory practice in the supermarket supply chain. *Int. J. Food Eng.* 12 (9), 827–834.
- Law, S.-T., Wee, H.-M., 2006. An integrated production-inventory model for ameliorating and deteriorating items taking account of time discounting. *Math. Comput. Model.* 43 (5–6), 673–685.
- Ma, X., Wang, S., Islam, S.M., Liu, X., 2019. Coordinating a three-echelon fresh agricultural products supply chain considering freshness-keeping effort with asymmetric information. *Appl. Math. Model.* 67, 337–356.
- Majumder, A., Jaggi, C.K., Sarkar, B., 2018. A multi-retailer supply chain model with backorder and variable production cost. *Oper. Res.* 52 (3), 943–954.
- Malekitabar, M., Yaghoubi, S., Gholamian, M.R., 2019. A novel mathematical inventory model for growing-mortal items (case study: Rainbow trout). *Appl. Math. Model.* 71, 96–117.
- Mebratie, W., Bovenhuis, H., Jensen, J., 2018. Estimation of genetic parameters for body weight and feed efficiency traits in a broiler chicken population using genomic information. In: Paper Presented at the Proceedings of the World Congress on Genetics Applied to Livestock Production.
- Nobil, A.H., Sedigh, A.H.A., Cárdenas-Barrón, L.E., 2018. A generalized economic order quantity inventory model with shortage: case study of a poultry farmer. *Arabian J. Sci. Eng.* 1–11.
- Osborne, M.J., 2004. An Introduction to Game Theory, vol. 3. Oxford university press, New York.
- Pentico, D.W., Drake, M.J., 2009. The deterministic EOQ with partial backordering: a new approach. *Eur. J. Oper. Res.* 194 (1), 102–113.
- Pourmohammad-Zia, N., Karimi, B., 2020. Optimal replenishment and breeding policies for growing items. *Arabian J. Sci. Eng.* 1–11.
- Rabbani, M., Zia, N., Rafiei, H., 2014. Optimal dynamic pricing and replenishment policies for deteriorating items. *Int. J. Ind. Eng. Comput.* 5 (4), 621–630.
- Rabbani, M., Zia, N.P., Rafiei, H., 2017. Joint optimal inventory, dynamic pricing and advertisement policies for non-instantaneous deteriorating items. *Oper. Res.* 51 (4), 1251–1267.
- Rezaei, J., 2014. Economic order quantity for growing items. *Int. J. Prod. Econ.* 155, 109–113.
- Rezaei, J., Pourmohammadzia, N., Dimitropoulos, C., Tavasszy, L., Duinkerken, M., 2020. Co-procurement: making the most of collaborative procurement. *Int. J. Prod. Res.* 1–12.
- Richards, F., 1959. A flexible growth function for empirical use. *J. Exp. Bot.* 10 (2), 290–301.
- Sebatjane, M., Adetunji, O., 2019. Economic Order Quantity Model for Growing Items with Imperfect Quality. *Operations Research Perspectives*.
- Sebatjane, M., Adetunji, O., 2021. Optimal lot-sizing and shipment decisions in a three-echelon supply chain for growing items with inventory level-and expiration date-dependent demand. *Appl. Math. Model.* 90, 1204–1225.
- Soleymannfar, V.R., Taleizadeh, A.A., Zia, N.P., 2015. A sustainable lot-sizing model with partial backordering. *Int. J. Adv. Oper. Manag.* 7 (2), 157–172.
- Soysal, M., Bloemhof-Ruwaard, J.M., Haijema, R., van der Vorst, J.G., 2015. Modeling an inventory routing problem for perishable products with environmental considerations and demand uncertainty. *Int. J. Prod. Econ.* 164, 118–133.
- Soysal, M., Bloemhof-Ruwaard, J.M., Haijema, R., van der Vorst, J.G., 2018. Modeling a green inventory routing problem for perishable products with horizontal collaboration. *Comput. Oper. Res.* 89, 168–182.
- Stellingwerf, H.M., Laporte, G., Cruijsen, F.C., Kanellopoulos, A., Bloemhof, J.M., 2018. Quantifying the environmental and economic benefits of cooperation: a case study in temperature-controlled food logistics. *Transport. Res. Transport Environ.* 65, 178–193.
- Tabrizi, S., Ghodspour, S.H., Ahmadi, A., 2018. Modelling three-echelon warm-water fish supply chain: a bi-level optimization approach under Nash–Cournot equilibrium. *Appl. Soft Comput.* 71, 1035–1053.
- Taleizadeh, A.A., Pourmohammad-Zia, N., Konstantaras, I., 2019. Partial linked-to-order delayed payment and life time effects on decaying items ordering. *Operational Research* 1–23.
- van Horne, P.L.M., 2020. Economics of Broiler Production Systems in the Netherlands: Economic Aspects within the Greenwell Sustainability Assessment Model. Wageningen Economic Research.
- Viswanathan, S., Piplani, R., 2001. Coordinating supply chain inventories through common replenishment epochs. *Eur. J. Oper. Res.* 129 (2), 277–286.
- Wang, X., Li, D., 2012. A dynamic product quality evaluation based pricing model for perishable food supply chains. *Omega* 40 (6), 906–917.
- Wee, H.-M., Lo, S.-T., Yu, J., Chen, H.C., 2008. An inventory model for ameliorating and deteriorating items taking account of time value of money and finite planning horizon. *Int. J. Syst. Sci.* 39 (8), 801–807.
- Yu, S., 2016. Study ON ordering model OF food cold chain logistics with multi-echelon inventory. *Carpathian Journal of Food Science & Technology* 8 (4).
- Zhang, Y., Li, L.-y., Tian, X.-q., Feng, C., 2016. Inventory Management Research for Growing Items with Carbon-Constrained. Paper Presented at the Control Conference (CCC), 2016 35th Chinese.
- Zhang, P., Xu, X., Shi, V., Zhu, J., 2020. Simultaneous inventory competition and transshipment between retailers. *Int. J. Prod. Econ.* 107781.
- Zhao, X., Si, D., Zhu, W., Xie, J., Shen, Z.J., 2019. Behaviors and performance improvement in a vendor-managed inventory program: an experimental study. *Prod. Oper. Manag.* 28 (7), 1818–1836.
- Zhong, R., Xu, X., Wang, L., 2017. Food Supply Chain Management: Systems, Implementations, and Future Research. *Industrial Management & Data Systems*.