COMPLEX WAVE ACTION ON SUBMERGED BODIES

J. SOMMET and PH. VIGNAT

Société Grenobloise d'Etudes et d'Applications Hydrauliques S.A.
SOGREAH, Grenoble, France

SUMMARY

This paper describes a method of calculating exciting forces on free or fixed bodies in waves and its application to some examples. The method lays upon an accurate computation of the transitory pressures applied by a potential flow on a submerged body.

The calculation may be used for the case of a free body in complex waves.

First, we give the calculation hypothesis on flow conditions and the formulae which proceed from assumptions.

Second, we give the computation results on well known examples.

- On set parallel flow around a sphere,
- Fixed triaxial ellipsoïd in waves.

Third, we apply this method to a free caisson, steadied by schematic mooring device in stationnal waves computed with second orders term.

1. INTRODUCTION

Sogreah has investigated a wide variety of hydraulic problems during its lifetime, among which especially the effects of waves on marine structures and both immersed and non-immersed floating bodies.

With the advent of computers, digital computation proved itself a valuable adjunct to scale model research, and mathematical models are now being used instead of physical ones for certain applications.

Sogreah has developed a method for the calculation of flow around an immersed body and has used it to determine wave forces acting on a floating platform caisson.

The immersed body flow computation method described here is a conventional one assuming potential flow which can be represented by a single-layer potential. The original feature of the Sogreah investigation, however, is that it more specifically considered transient-state pressures and forces with a view to determining the behaviour of an immersed body under complex wave conditions. This method gives the response of a body immersed at a given depth to waves that are chromatic as regards height and phase, and it can be confirmed in this case by comparison with scale model tests. It is particularly useful as a means of studying the behaviour of an immersed body in complex wave conditions, and it scores over the physical model in that it enables any complex waves given by their spectrum to be investigated for either finite or infinite depth assumptions.

This note gives the confirmation of the method for simple bodies (sphere, ellipsoid) and describes its application to the motion of a single caisson suitably anchored for stability and exposed to Atlantic swell conditions.

2. DESCRIPTION OF THE METHOD - THEORETICAL STUDY

2.1 Remark

As the calculation method used in this study is quite conventional, the mathematical formulation of the problem will be discussed very briefly and only the basic formulae required to understand the method will be mentioned. We have applied this method to the special case of the determination of wave forces acting on an immersed body, but it is also suitable for other two - or three - dimensional flow problems.

2.2 Physical assumptions

We have supposed that the viscosity forces are low and do not perturb the flow around the body, and that the speeds and pressures on the body surface are identical both in ideal fluid and in viscous fluid.

This assumptions involves that the body motions are slow and of the same order of magnitude as the water motions. We will not take into account the wake effects which can occur in certain places of the body.

On the other hand, it is possible to take in account the drag and lift effects either as a whole, or with the aid of a shear term in each point of the body, this term being a function of the relative water body speed.

Then again, we have not taken in account the influence - on the flow characteristics - of the free surface distortion owing to the presence of ** the body.

This limitation leads us to the following point. This method is merely valid when the immersed body stands at a depht more than about twice the body height.

2.3 Type of flow

The water is considered as an incompressible fluid in irrotational motion, so that the flow is derived from a potential Φ which is the solution of the Laplace equation :

$$\Delta \ \Phi = 0 \qquad (2.1)$$

The fluid velocity at any point is :

$$\overline{V}_F = - \overline{\text{grad } \Phi} \qquad (2.2)$$

Without a body, the flow is simply the motion of the water, and the above assumptions require that we consider a wave scheme of potential Φ_H

Determination of the flow is then a matter of solving an exterior Neumann problem, i.e. the determination of a harmonic function Φ which is regular at infinity, knowing the normal derivative $d\Phi/dn$ on the body surface Σ .

2.4 Determination of potential Φ

The given condition $d\Phi/dn$ at point M on surface Σ is met when the normal velocity components for point M associated with the body and the fluid velocity at that point are equal, i.e. :

$$\frac{d\Phi}{dn} = - (\overline{V}_c + \overline{\Omega}_{\Lambda} \overline{CM}) \ \overline{n} \ (M) \qquad (2.3)$$

As the potential satisfies the Laplace equation we can apply the principle of superimposed flows and break down the overall potential Φ into the three following elementary potentials :

- Φ_H giving the flow of water without the body,
- Φ_{PH} giving the flow of water around the body, which is assumed to be stationary under the influence of Φ_H ,
- Φ_{PC} giving the flow of water due to the motion of the body in calm water.

hence :

$$\Phi = \Phi_H + \Phi_{PH} + \Phi_{PC} \qquad (2.4)$$

Potentials Φ_{PH} and Φ_{PC} are expressed conventionally by a single-layer potential of respective densities σ_{PH} and σ_{PC} , i.e. :

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$$\Phi_{PH}(P) = \iint_{\Sigma} \frac{\sigma_{PH}(M)}{|MP|} \, ds\,(M) \tag{2.5}$$

**

$$\Phi_{PC}(P) = \iint \frac{\sigma_{PC}(M)}{|MP|} \, ds\,(M) \tag{2.6}$$

The source densities σ_{PH} and σ_{PC} are solutions of the Fredholm equation with the given condition $d\Phi/dn$:

$$2\pi\,\sigma_{PC}(P) + \iint_{\Sigma} \sigma_{PC}(M) \frac{\overline{MP}.\overline{n}(P)}{|MP|^3} \, ds\,(M) = [\overline{V}_c + \overline{\Omega}_\Lambda \overline{CM}]\,\overline{n}\,(P) \tag{2.7}$$

$$2\pi\,\sigma_{PH}(P) + \iint_{\Sigma} \sigma_{PH}(M) \frac{\overline{MP}.\overline{n}(P)}{|MP|^3} \, ds\,(M) = -\,\overline{V}_H(P)\,\overline{n}\,(P) \tag{2.8}$$

Potential Φ_{PC} only depends on the velocity of the body and can be expressed as a function of unit potentials φ_1 , φ_2 , φ_3 , χ_1 , χ_2 , χ_3 (ref 1)

$$\Phi_{PC} = u\varphi_1 + v\varphi_2 + w\varphi_3 + p\chi_1 + q\chi_2 + r\chi_3 \tag{2.9}$$

Potentials φ_1 , φ_2 , φ_3 , χ_1 , χ_2 , χ_3 are calculated once and for all for the body surface area Σ . Potential Φ_{PH} is calculated for any moment of time in terms of the position of the body and wave conditions.

2.5 Determination of wave forces acting on the body

Knowing the overall flow potential the water pressure point - especially on the body surface - can be calculated by the following formula :

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$$P = Po + \rho \left(gz + \frac{\partial\Phi}{\partial t} - \frac{V_F^{\,2}}{2} \right) \tag{2.10}$$

Integrating pressure over the surface area Σ gives the wave force and moment resultants on the body at any instant of time, i.e. :

$$\left.\begin{aligned} \overline{F}_H &= -\iint_\Sigma P(M)\,\overline{n}\,(M)\,ds\,(M) \\ \overline{M}_H &= -\iint_\Sigma P(M)\,[\overline{CM}_\Lambda\,\overline{n}\,(M)]\,ds\,(M) \end{aligned}\right\} \qquad (2.11)$$

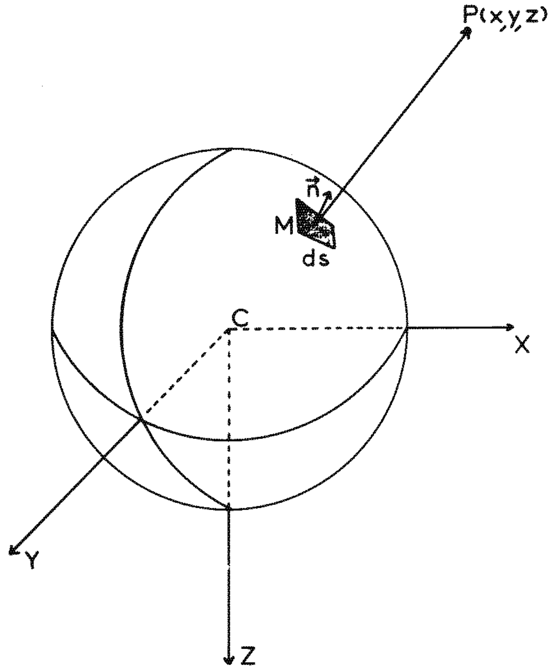


Figure 1 - The body surface. Notation used in describing the potential due to a surface source density distribution.

The forces are calculated along the body axes. Integration of the pressure term ρgz gives the buoyancy force and will not be carried out.

The pressure term $\partial\Phi/\partial t$ gives the forces at wave period and takes the motion of the immersed body into account. These forces are much greater than those due to the pressure term $V_F^2/2$ representing the surface attraction effect. In certain cases where only body motion at the wave period is considered the velocity term can be considered negligible compared to the pressure term $\partial\Phi/\partial t$.

<u>Note</u> : The pressure term $\partial\Phi/\partial t$ is the sum of derivatives :

$$\frac{\partial\Phi_H}{\partial t}\ , \quad \frac{\partial\Phi_{PH}}{\partial t} \quad \text{and} \quad \frac{\partial\Phi_{PC}}{\partial t}\ ,$$

the last of which is as follows :

$$\frac{\partial\Phi_{PC}}{\partial t} = u'\varphi_1 + v'\varphi_2 + w'\varphi_3 + p'\chi_1 + q'\chi_2 + r'\chi_3 \tag{2.12}$$

By integrating this term over surface area Σ the twenty-one added mass coefficients (ref 2) can be calculated, which are of the following form :

$$A = \iint_\Sigma \varphi_1 \quad \alpha \quad ds \tag{2.13}$$

If the added mass coefficients of the body are known, the calculation method used can be checked.

3. CALCULATION METHOD

The basic problem involved in determining the flow around an immersed body is to solve the Fredholm (2.7) and (2.8) equation, which generally defies analytical solution. Its digital solution method is conventional and consists in replacing the Fredholm integral by a linear system of n equations with n unknowns.

This system is obtained by replacing the continuous functions defined on the body surface Σ by their values at n points on the surface and by calculating the integrals by summation over the n considered points.

3.1 Discretisation - Approximate body definition

In order to solve the Fredholm equation (2.7) and(2.8) we divide the body surface area into n surface elements ("facets") (fig. 1) defined by the following :

(i) Facet area δs_ι

(ii) The vector normal to the facet $\overline{n}_\iota$.

The facet centre M_ι .

The ι-th facet is determined by its circumference Γ_ι ; the components of vector $\overline{n}_\iota$ and the facet area are then given by the following integral :

$$n_\iota \, \delta s_\iota = \iint_{\delta s_\iota} \overline{n}\,(M)\, ds\,(M) = \int_{\Gamma_\iota} \overline{CM}_\Lambda \, \overline{d\ell} \tag{3.1}$$

The centre of the i-th facet (M_ι) has been assumed to coincide with the centre of gravity of the projection of the facet on a plane perpendicular to the mean normal given by formula (3.1), i.e. :

$$\overline{CM}_{\iota\Lambda} \, \overline{n}_\iota \, \delta s_\iota = \iint_{\delta s_\iota} \overline{CM}_\Lambda \, \overline{n} \, ds = -\frac{1}{2}\int_{\Gamma_\iota} |\overline{CM}|^2 \, \overline{d\ell} \tag{3.2}$$

The integrals we have to use are of the following type :

$$\iint_\Sigma f\,(M)\, ds\,(M) \quad \text{or} \quad \iint_\Sigma f\,(M,P)\, ds\,(M) \tag{3.3}$$

We shall calculate these integrals by summing over all n body facets, i.e. :

$$\left.\begin{aligned} \iint_\Sigma f\,(M)\, ds\,(M) &= \sum_{\iota=1}^{n} f\,(M_\iota)\, ds_\iota \\ \iint_\Sigma f\,(M,P)\, ds\,(M) &= \sum_{\iota=1}^{n} f\,(M_\iota,M_j)\, ds_\iota \end{aligned}\right\} \tag{3.4}$$

3.2 Determination of the flow around the body

The characteristic flow quantities (source density, potential, velocity and pressure) are calculated at the centre of the n facets defining the body.

The Fredholm integral is calculated by summation and the source density at the centre of the facets is determined by solving the following linear system :

$$\sigma_\iota \, \delta s_\iota + \sum_{j=1}^{n} K_{\iota j} \, \sigma_j \, \delta s_j = \frac{1}{2\pi} \left[\overline{V}_c + \overline{\Omega}_\Lambda \, \overline{CM}_\iota - \overline{V}_{H\iota} \right] \overline{n}_\iota \, \delta s_\iota \qquad (3.5)$$

where $K_{\iota j}$ is the general term of a square matrix of rank n. This term solely depends on the body characteristics and is calculated in terms of the quantities defining each facet, i.e. :

$$K_{\iota j} = \frac{1}{2\pi} \frac{\overline{M_\iota M_j}}{|M_\iota M_j|^3} \, \overline{n}_\iota \, \delta s_\iota \qquad (3.6)$$

The linear equation system was solved by inverting the matrix $A = I + K$.

The quantity :

$$Q_\iota = \frac{1}{2\pi} \left[\overline{V}_c + \overline{\Omega}_\Lambda \, \overline{CM}_\iota - \overline{V}_{H\iota} \right] \overline{n}_\iota \, \delta s_\iota$$

can be accurately calculated for any instant of time and any point on the body.

Knowing the inverted matrix A^{-1} it is easy to find $\sigma_\iota \, \delta s_\iota$, and the values for σ_ι provide a practical means of determining the flow around the body, for the potential and velocities can be calculated from the source densities by simple summation over the body surface Σ, from which the pressure at each point on the body are then found.

3.3 Representation of waves

In order to determine the effect of waves on the body, we must introduce a wave scheme ensuring adequate representation of the motion of the water at any instant of time and at any point. A wave scheme of potential Φ_H was adopted for the purpose, in which complex waves are represented by a certain number of elementary waves whose heights and pulsations were selected to ensure adequate representation of the complex wave spectrum. By this method, given recorded waves can be reproduced.

The wave potential is given by the following formula :

$$\Phi_H = \sum_{\iota=1}^{I} \frac{A_\iota \omega_\iota}{K_\iota} e^{-K_\iota z} \sin(\omega_\iota t - K_\iota x + \psi_\iota)$$

$$- \sum_{\iota=1}^{I} \sum_{j=1}^{\iota-1} A_\iota A_j \omega_\iota e^{-(K_\iota - K_j)z} \sin[(\omega_\iota - \omega_j)t - (K_\iota - K_j)x + \psi_\iota - \psi_j] \qquad (3.7)$$

where I is the wave component number.

The (3.7) formula alows the explicit computation of the datas required for the Φ_{PH} potential computation.

The wave spectrum is parted in ten equal energy band. This sharing gives a satisfactory repruduction of the statistical properties of waves.

4. REMARKS ON THE COMPUTATION PROGRAMME

4.1 General considerations

• With the computation programme used to determine the effects of waves on an immersed caisson all the intermediate quantities required to calculated the forces can also be determined, i.e. source density, potential, water velocity and pressure. Our purpose in using this programme was to follow the various computation phases and to establish the degree of accuracy of the method by comparisons considering cases known by analytical calculation.

On the other hand, we intended to show how a method of this type can be used for very varied applications both for the investigation of transient wave effects as considered here and for the determination of water velocities at a given point of a fixed body immersed in a known flow.

We would like to draw attention to the following remarks regarding the application of the computation method in this paper to the case of an immersed body under wave condition :

Forces $\overline{F}_\varphi$ due to the $d\Phi/dt$ pressure term are linear functions of the Q_ι values.

The intermediate summations can be done once and for all and calculation of these forces boils down to the following summation :

$$\overline{F}_{\varphi} = \Sigma_{\iota=1}^{n} \; \overline{C}_{\iota} \, Q_{\iota} \qquad (4.1)$$

Forces F_V due to $\frac{V_{\overline{F}}^2}{2}$ pressure terms are not linear functions of the Q_{ι} values, however, which makes it necessary to also calculate the velocities in between.

$$\overline{V}_{Fj} = \sum_{\iota=1}^{n} \overline{B_{\iota j}} \; Q_{\iota} \qquad (4.2)$$

The calculation of $\overline{F_V}$, therefore, will take about n times longer than for $\overline{F}_{\varphi}$.

Where $\overline{F_V}$ is negligible compared to $\overline{F}_{\varphi}$ and especially where second-order wave effects are not to be considered, the computation time cane be reduced considerably by neglecting $\overline{F_V}$ and considering a first-order wave formulation.

4.2 Features of the programme

The immersed body wave force computation programme was written in Fortran IV and is being used with IBM 360-65 equipment. All the computations are done with central storage and we have limited the number of facets (surface elements) defining the body to 190. A body with a place of symmetry can be divided up into 270 facets.

The mathematical model comprises three main programmes in the following sequence :

(i) The body characteristics computation programme, which calculates the matrix of the Fredholm equation K and gives the inverted form A^{-1} of the corresponding matrix $A = I + K$.

(ii) The programme for computing tables B and C from A^{-1} , which enables the velocities and forces to be calculated by formulas (4.1) and (4.2).

(iii) The programme to compute the wave forces and immersed body motion for various wave characteristics from tables B and C .

The computation times given below for these three phases are only a rough indication. For a body divided into 60 facets, these times are as follows :

- Computation and inversion of matrix A ...... 1 minute.
- Computation of tables B and C .... 3 minutes.
- Time to compute forces $\overline{F}_{\varphi}$ and to determine body motion, for a first-order approximation of complex waves represented by eight rays .... 0.02 second per time ste[p]
- Time to compute $\overline{F}_V$ and $\overline{F}_{\varphi}$ for a second-order approximation of complex waves represented by eight rays .. 0.5 second per time step, (i.e. very much longer than above).

5. COMPARISON BETWEEN COMPUTED AND ANALYTICAL DATA

In order to establish the accuracy of the method described in the previous section, we applied it to simple bodies for which some of the calculations can be done analytically.

In the comparison with analytical solutions, the flow itself (i.e. source density, potential, velocities, added mass coefficients) and wave effects on the body (heaving, rolling, pitching and yawing force coefficients were considered.

As a general rule we chose a number of facets giving and accuracy of one to two per cent for the calculated values, which we considered to be adequate for the wave calculations.

As the wave characteristics are approximate, it did not seem necessary to require more accurate computations. This enables us to achieve very short computation times, and so to represent the history of the studied phenomena during a time sufficiently long to reproduce their random aspects.

5.1 Study of a sphere

As flow around a sphere is a very well-known subject, this seemed a reasonable choice for the initial comparisons.

The sphere was divided up as shown in Fig. 2 , and though this is not the best method of subdivision, it is the one generally used for long bodies. In the considered example, the sphere was divided into 162 facets bounded by meridians and parallels of latitude every 20 degrees.

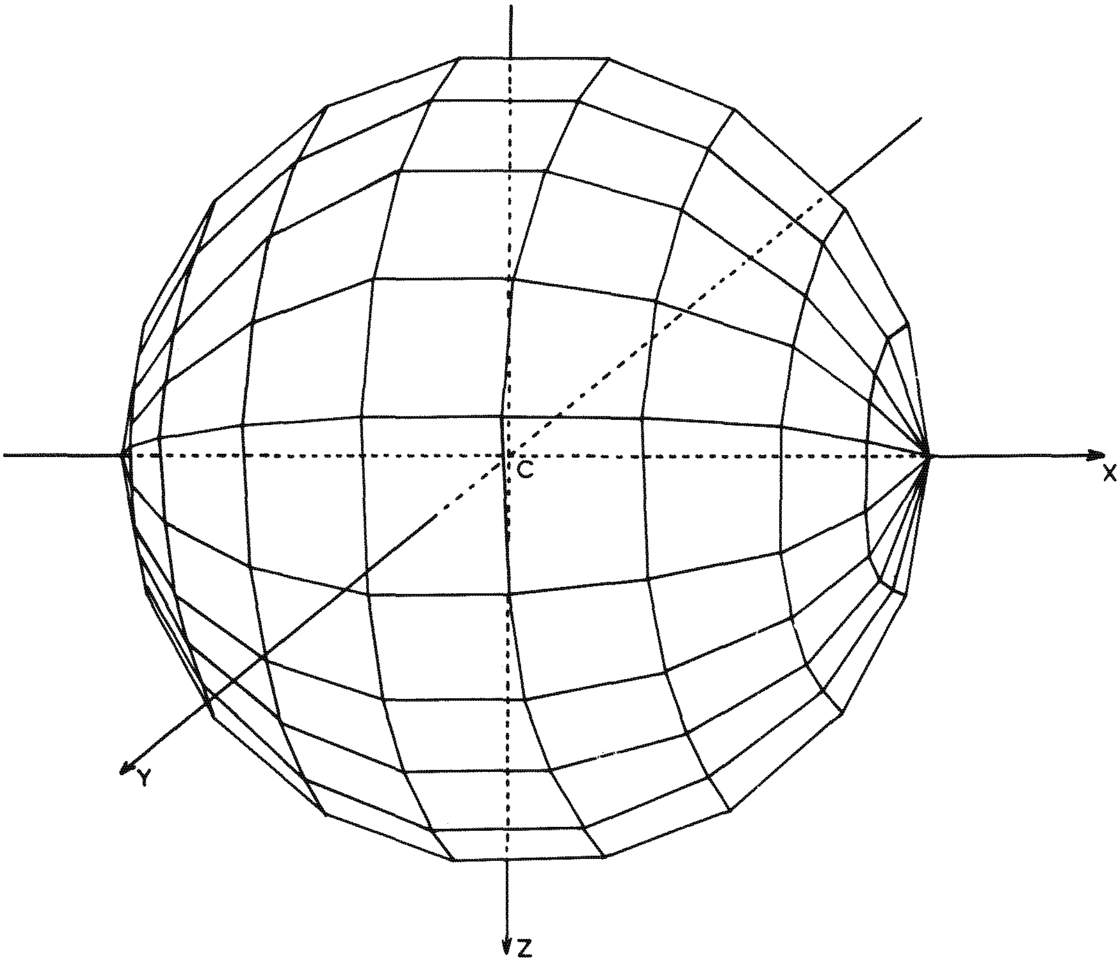


Figure 2 - The approximate representation of the sphere.

5.1.1 Sphere in uniform steady flow

Two uniform flows of unit velocity are considered, one along Cy and the other along Cz . We know the theoretical flow around the sphere in this case, and comparing this with the analytical solution in Fig. 3 we observe the following :

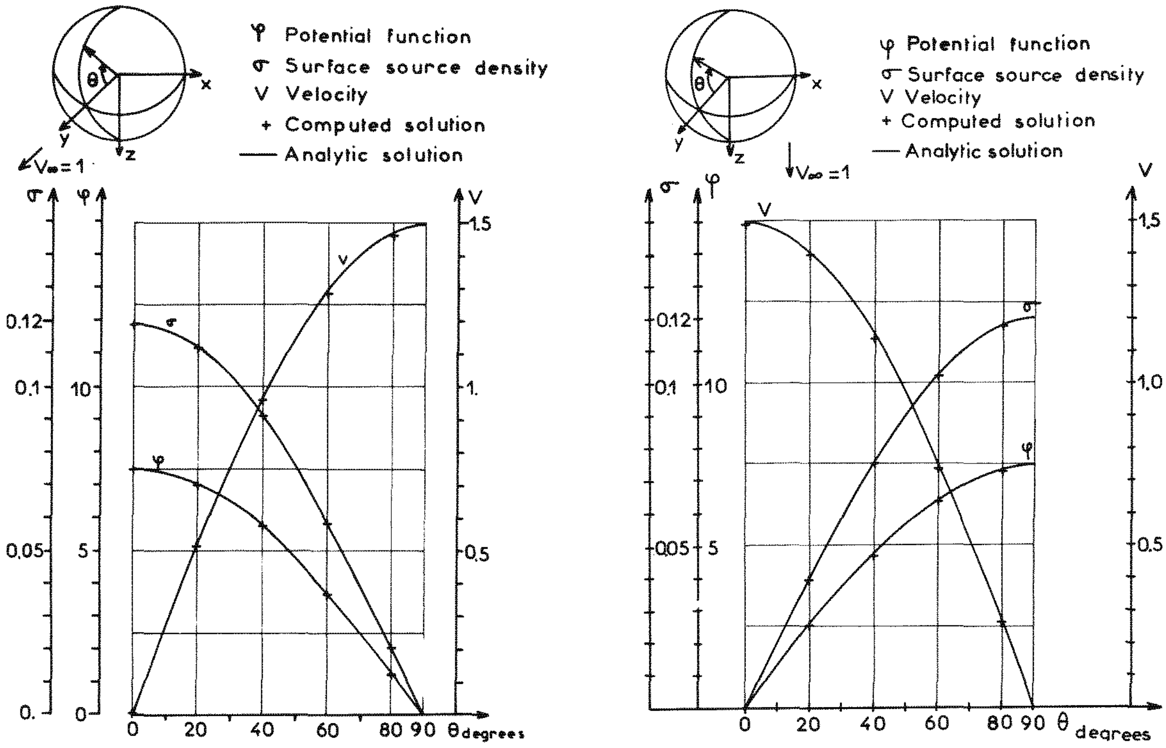


Figure 3 - Comparison of analytic and calculated values on a sphere for an onset uniform flow.

Source density and potential computation accuracy is satisfactory, there being less than 1 per cent error between computed and theoretical data throughout. The computed velocities are less satisfactory, however, as they differ from the theoretical values by as much as 5 per cent at certain points.

The difficulty of obtaining accurate velocity data is due to the $[M_i M_j]^3$ term in the denominator of the velocity computation formula.

We have not attempted to improve the velocity computation method yet as the corresponding force term is nearly always small enough to be neglected with respect to the forces at wave period.

Velocity computation accuracy can be improved either by increasing the number of facets or by improving the velocity integration formula by extrapolating the source densities. We intend to try out this second method for future problems as it does not result in an excessive increase in computation time.

5.1.2 Sphere in unsteady flow Added mass coefficients

The inertia tensor for the water set in motion by the body is symmetrical and defined by twenty-one coefficients (ref. 2).

For a body with three planes of symmetry this tensor becomes the main diagonal, and in the case of a sphere it is as follows :

$$\left.\begin{array}{l} A = B = C = \frac{\rho V}{2} \\ P = Q = R = 0 \end{array}\right\} \quad (5.1)$$

The theoretical and computed date compare well for a sphere with a radius of 5 metres, with differences alvays less than 2 per cent, as follows :

Quantity	Theoretical value	Computed value
$A = K_x \rho V$	$261.8 = 0,5\ \rho V$	$257.4 = 0,4916\ \rho V$
$B = K_y \rho V$	$261.8 = 0,5\ \rho V$	$256.8 = 0,4904\ \rho V$
$C = K_z \rho V$	$261.8 = 0,5\ \rho V$	$256.6 = 0,4901\ \rho V$

5.2 Tri-axial ellipsoid study

5.2.1 Some ellipsoid characteristics can be obtained by analytical methods.

Lamb (ref. 3) gives the values of Green's integrals which allow the computation of A, B, C, P, Q, R .

On the other hand, in the case of a tri-axial ellipsoid, Newman (Ref. 4) gives calculation formulae for the pressure term $\partial\Phi/\partial t$ forces, produced by monochromatic waves.

The comparison between our mathematical model and the analytical results is done for an ellipsoid determined by 120 facets.

5.2.2 Added mass coefficients

All the terms but those of the main diagonal of the inertia tensor are equal to zero. We have put the theoretical data computed from Lamb's formulae and the mathematical model data in a table (Fig. 4).

	ANALYTIC DATA	COMPUTED DATA
A	90 T = 0.0192 $V \times \rho$	88 T = 0.0188 $V \times \rho$
B	4880 T = 1.0443 $V \times \rho$	4970 T = 1.052 $V \times \rho$
C	4150 T = 0.8881 $V \times \rho$	4 270 T = 0.914 $V \times \rho$
P	275 T $\times$ m^2	264 T $\times$ m^2
Q	1 935 000 T $\times$ m^2	1991 000 T $\times$ m^2
R	2 281 000 T $\times$ m^2	2 344 000 T $\times$ m^2

Ellipsoid: $a_1 = 50.00$ m, $a_2 = 4.50$ m, $a_3 = 5.00$ m; $\rho = 1$, $V = 4710\ m^3$

Figure 4 - Added mass coefficients for tri-axial ellipsoid.

5.2.3 Waves forces on an ellipsoid - Exciting force coefficients

Newman gives theoretical formulae for the heaving, rolling, pitching, yawing coefficients (Cz, Cxx, Cyy, Czz) in the case of a triaxial ellipsoid. These formulae are more general than those of Havelock (Ref. 5) in which only a spheroid is taken in account.

In the latter case, Newman assumes that the body is in a fixed position. We have taken the same assumptions, i.e. for the ellipsoid computation :

- The major axis parallel to the wave direction with various wave period (6, 7, 8, 9, 10, 12 sec.) ;
- The major axis at 30, 60 and 90 degrees to the wave direction and with two wave periods (8 and 10 sec.) ;

The mathematical model gives the exciting forces on the ellipsoid in waves, from which we find the coefficients Cz, Cxx, Cyy and Czz using the following formulae :

$$\left.\begin{aligned} Fz &= \rho g\ VAK\ e^{-Kz}\ Cz \cos \omega t \\ Mx &= \rho g\ L\ VAK\ e^{-Kz}\ Cxx \sin \omega t \\ My &= -\rho g\ L\ VAK\ e^{-Kz}\ Cyy \sin \omega t \\ Mz &= -\rho g\ L\ VAK\ e^{-Kz}\ Czz \cos \omega t \end{aligned}\right\} \quad (5.3)$$

It is known that for λ / L tending to infinity the limit of Cz is :

$$1 + K_z = (\rho V + C) / \rho V$$

and the pitching coefficient Cyy decreases and tends to zero. Fig. 5 shows this very clearly.

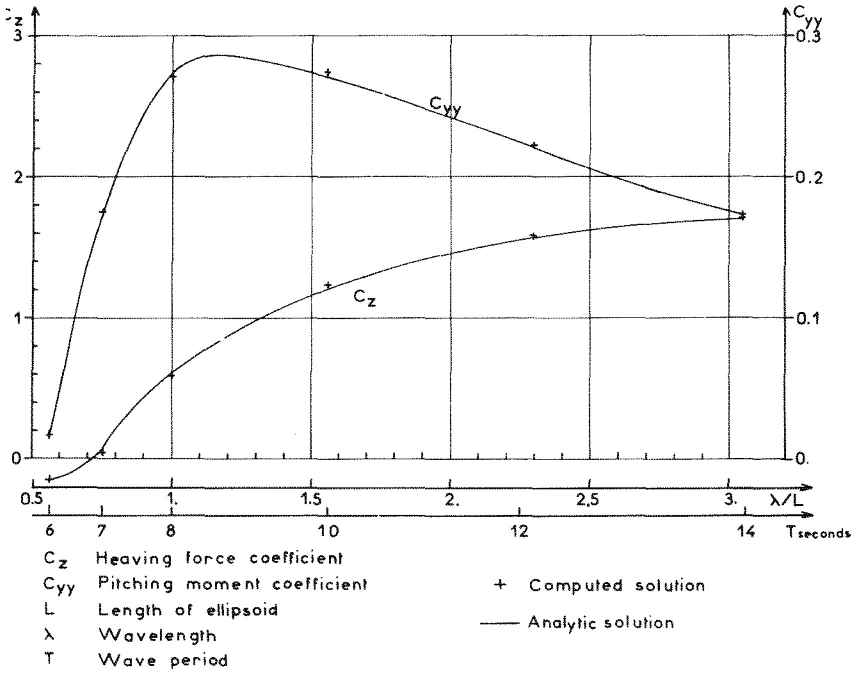
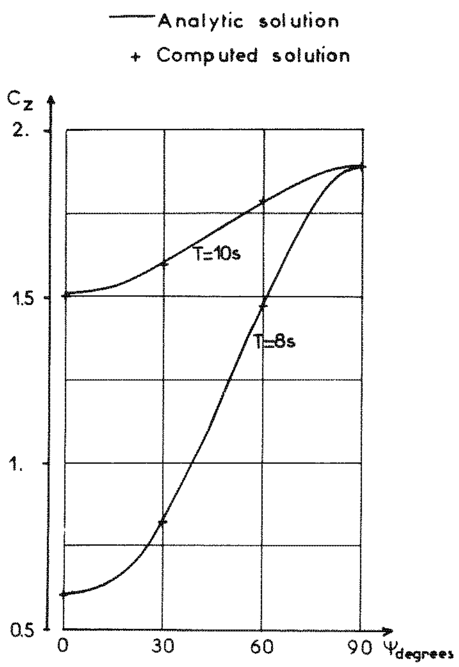


Figure 5 - Heaving force and pitching moment coefficientsfor varying λ/L

For the same reasons, the heaving force acting on an elongated ellipsoid broadside on to the waves is independent of wave period, so that the value of coefficient Cz is :

$$1 + K_z = 1 + \frac{C}{\rho V}$$

This property shows up well in the computations.



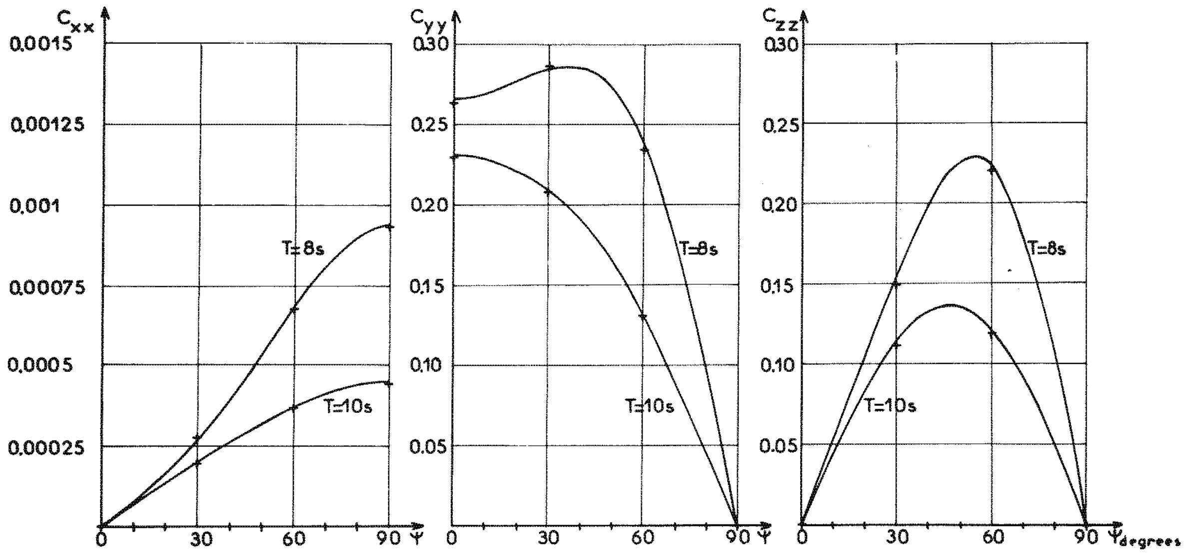


Figure 6 - Heaving force and rolling, pitching and yawing moment coefficients for varying directions.

The results are shown in figures 5 and 6. They show quite close agreement, the computation error being less than 3 per cent.

From the satisfactory agreement between of the various results and the theoretical data it can be concluded that our computation method is adequate for the $\partial\Phi/\partial t$ pressure term.

6. THE EFFECT OF WAVES ON AN IMMERSED CAISSON

6.1 The effect of waves on an immersed caisson can be considered from two aspects, as follows :

(i) An aspect associated with forces of the first order, which are periodic, have the same period as the waves and are proportional to wave height. These forces are of considerable magnitude and give rise to movements which cannot be eliminated by any form of anchoring or other stabilisation method. The corresponding movements are usually periodic, with the body oscillating about a mean position. The sole purpose of anchorings is to correct deviations from this mean position, but considerable deviations may nevertheless occur, even with a taut hawser, to the point of causing it to break. It is important to know whether such situations are likely to arise and to have a very sound statistical knowledge of these movements.

(ii) An aspect associated with second-order forces, by which we mean any forces that are non-periodic or with a period in excess of 30 seconds. The force of attraction on the surface, effects due to second-order terms in wave representation and various force and motion coupling cases are considered to come under this heading.

We have now seen the various aspects of wave action on a submerged body of any shape : the method we have described is equally suitable for the determination of forces of the first and second orders and provides a very thorough means of investigating wave action on an immersed body.

For the caisson discussed in this paper we have considered first-order forces and more specifically the motion of a free caisson under complex wave action.

The computation method can also be used to calculate forces on a caisson in forced motion, which is the case if the caisson submerged, is part of a complex structure such as a drilling platform.

6.2 Caisson characteristics

The outlines of the considered caisson are shown in Fig. 7.

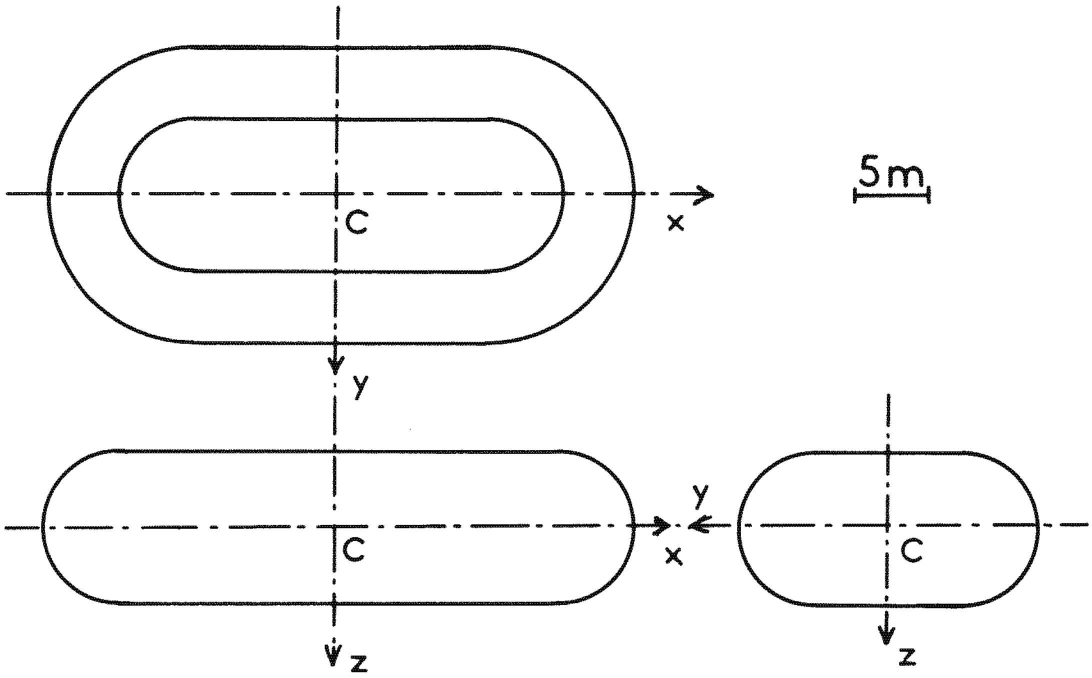


Figure 7 - Caisson outline

Its characteristic dimensions are as follows :

- Length : 40 metres
- Breadth : 20 metres
- Height : 10 metres
- Volume : 6110 cubic metres.

6.3 Added mass coefficients

As the considered caisson has three planes of symmetry only the coefficients of the main diagonal are not zero. From the results obtained for the sphere and ellipsoid it can be estimated that the values are accurate to within 3 per cent (Fig. 8).

CAISSON

$V = 6110\ m^3$

$\rho = 1$

	COMPUTED DATA
A	$931\ T = 0.152\ V \times \rho$
B	$2\ 400\ T = 0.394\ V \times \rho$
C	$7\ 480\ T = 1.225\ V \times \rho$
P	$47\ 100\ T \times m^2$
Q	$414\ 900\ T \times m^2$
R	$102\ 600\ T \times m^2$

Figure 8 - Added mass coefficients for caisson.

6.4 Computation of Cz , Cxx , Cyy and Czz - Computation of forces

Fig. 9 shows the force of attraction toward the surface and Fig. 10 and 11 the amplitudes of the first-order forces on the caisson due to 2 m waves (crest to through height). It will be noted that the attraction force is invariably less than 50 sthenes, which is negligible compared to the first-order forces.

The attraction force is due to the difference between flow velocities over the top and bottom caisson surfaces and is proportional to the difference between the squares of these velocities. It remains constant during a wave period ; its magnitude is proportional to wave height and varies with depth of submersion according to a e^{-2K_z} law. In calculating the forces the caisson is assumed to be held stationary at a depth of 15 metres below the surface.

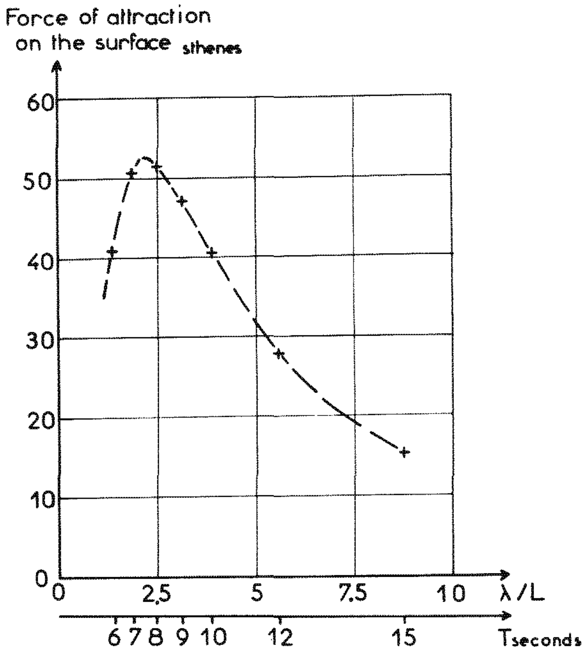


Figure 9 - Force of attraction toward the surface for varying λ/L

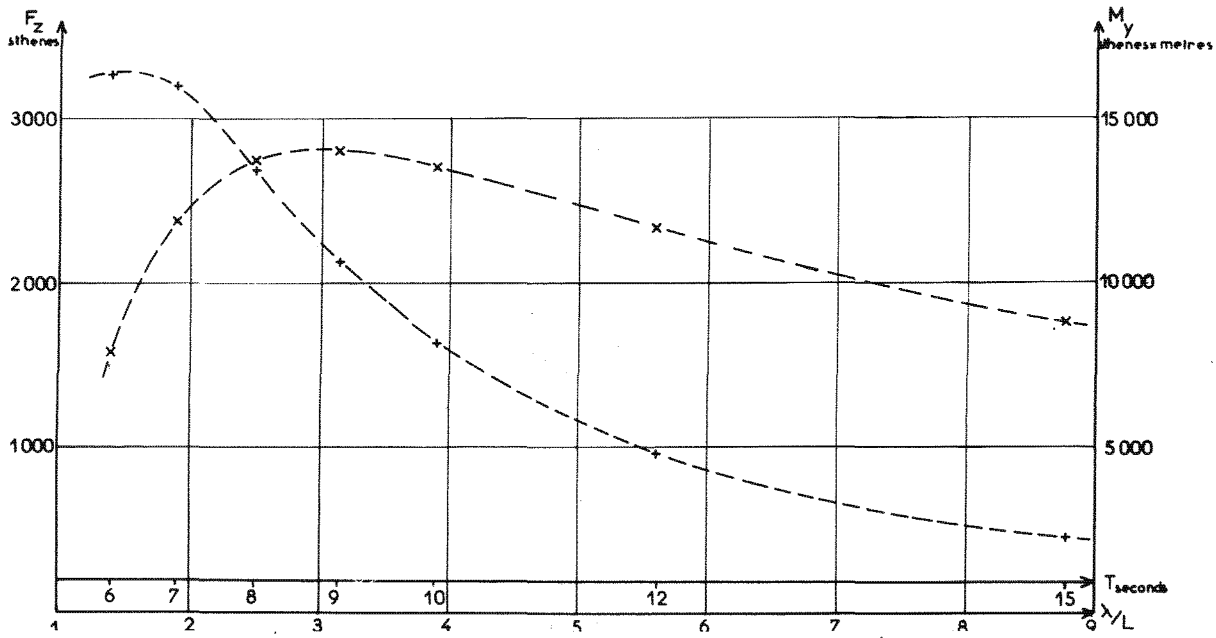


Figure 10 - Heaving force and pitching moment for varying λ/L

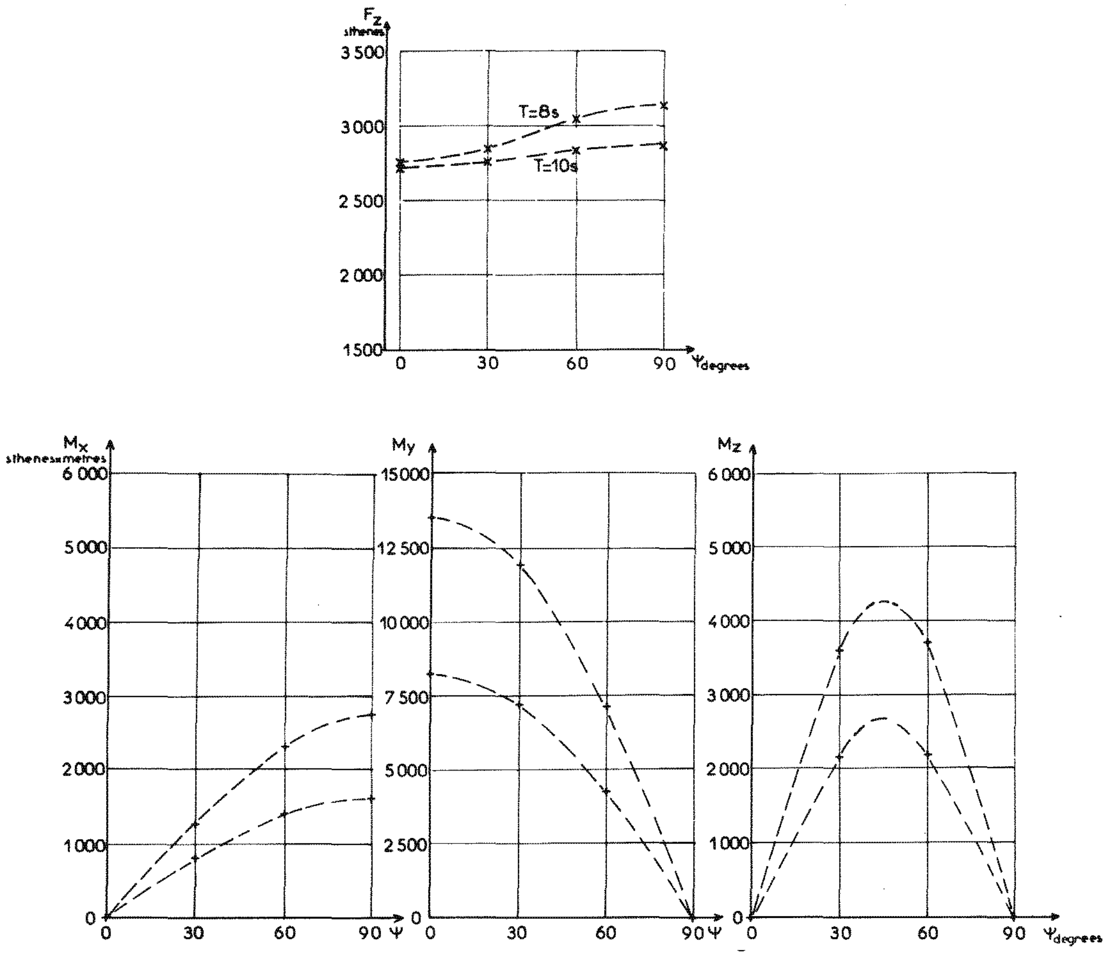


Figure 11 - Heaving force and rolling, pitching and yawing moments for various directions.

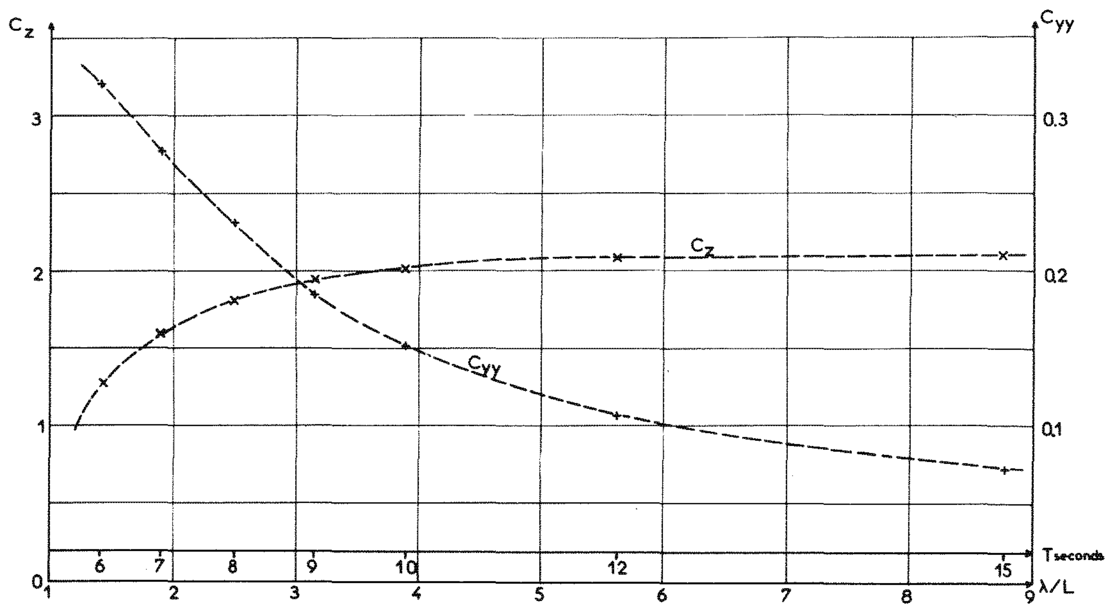


Figure 12 - Heaving force and pitching moment coefficients for various λ/L

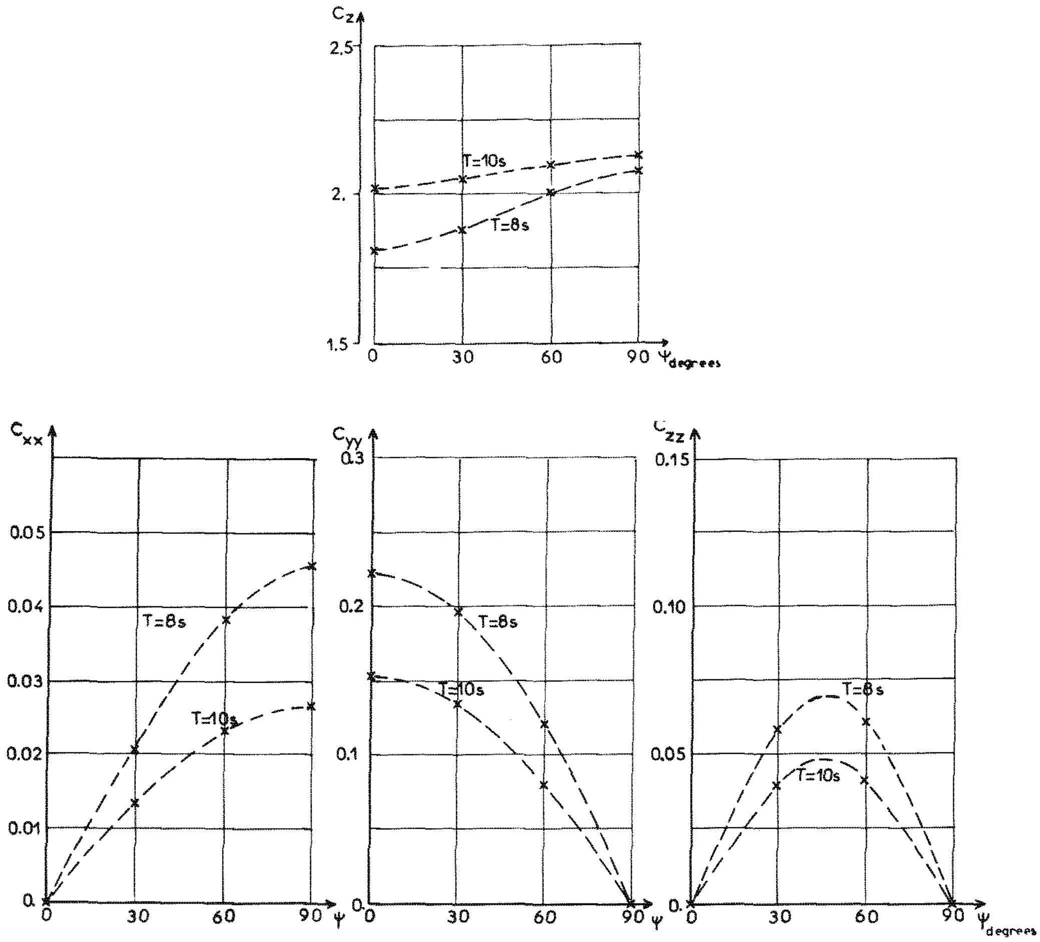


Figure 13 - Heaving force and rolling, pitching and yawing moment coefficients for various directions.

Coefficients Cz , Cxx , Cyy and Czz were calculated from the forces by formulae (5.3) and taking the biggest length of the caisson for L .

Figs. 12 and 13 show how these coefficients vary with the waves and caisson position.

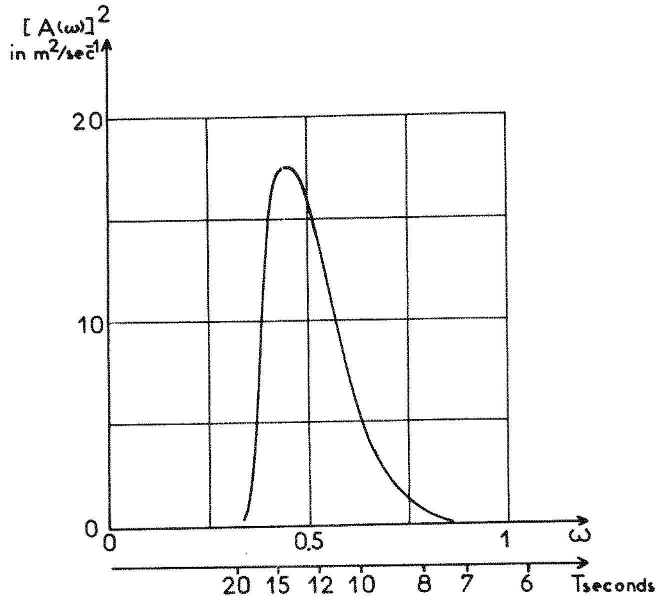


Figure 14 - Energy wave spectrum representing an Atlantic type swell.

6.5 Caisson motion when immersed under complex waves

The computation method described in this paper was u ed to determine the motion of a free caisson maintained at a depth of 15 metres by a schematic anchoring at its centre of thrust. The tension displacement relation ship for this anchoring is linear.

The considered complex waves are given by their energy spectrum (Fig 14) which is divided into ten constant-energy bands. This spectrum represents an Atlantic-type swell with an average period of 14 seconds.

Caisson heaving and pitching motion, corresponding wave forces and the difference in the free surface level vertically above the centre of thrust are all plotted in Fig. 15. It will be noted that as the caisson dimensions are small compared to the wave length, its motion is in phase with the wave motion. A low-frequency motion is superimposed upon the motion in phase with the coaves at a period close to the natural period of the system comprising the caisson and anchoring.

For this test, the caisson wa placed with its major axis in the wave direction and only one wave direction was considered. Use of the computation programme is not limited to this one case, however, and we have successfully applied it to the motion of a free caisson facing in any direction subjected to multi-directional waves.

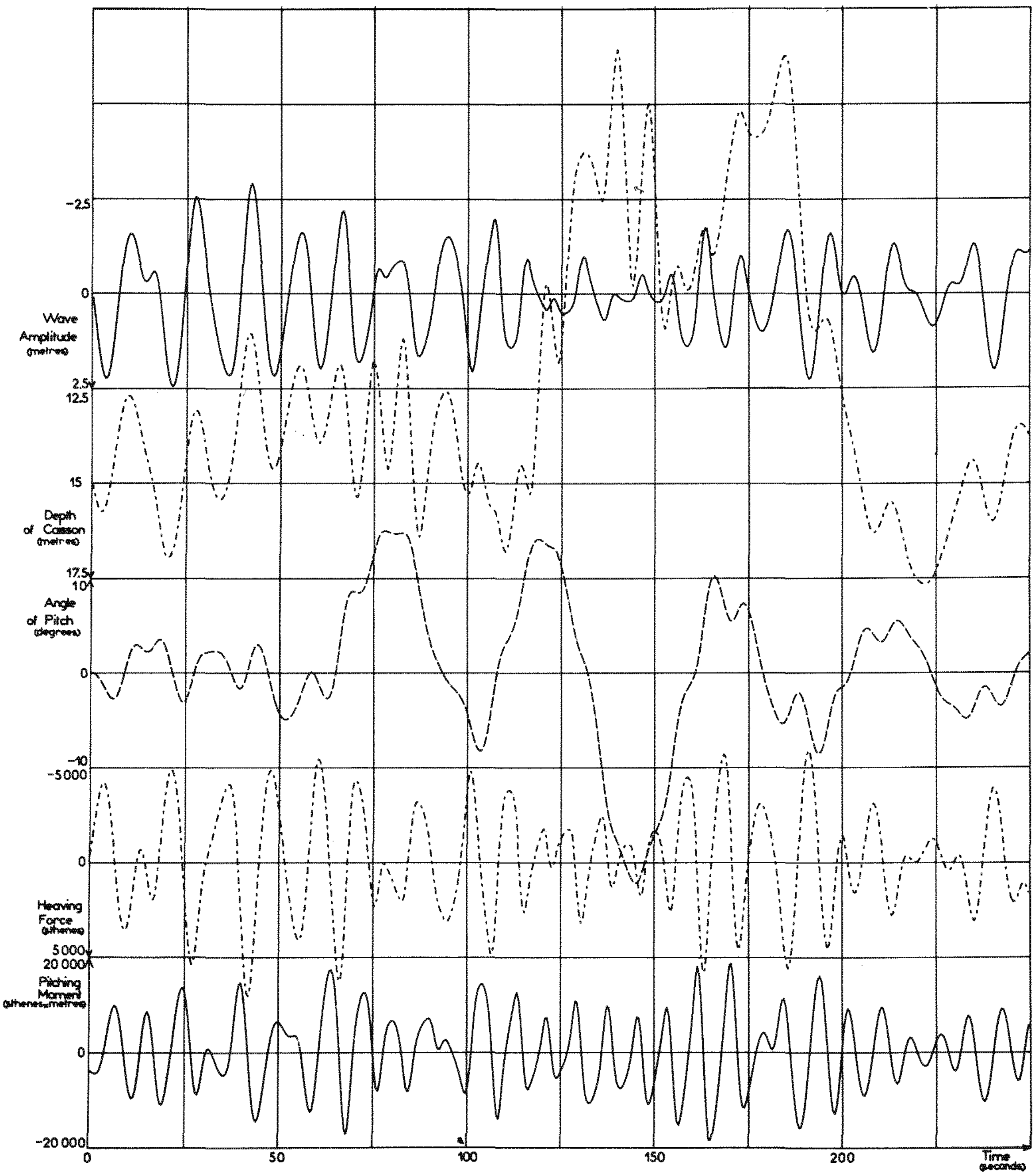


Figure 15 - Caisson heaving and pitching motion.

CONCLUSION

This study resulted in the design of a mathematical model for the computation of wave forces on body of any shape with and without sharp
** edges submerged at an adequate depth.

It is proposed to develop this model for calculations at any depth and allowing for free surface effects.

The model can already cope with viscosity forces which are computed from local friction coefficients and vary as the square of velocity.

Complete mathematical models of complex structures (e.g. semi-submersible drilling platforms) can thus be constructed for use in calculating real life wave forces and motion. Simplifying flow assumptions have to be made, however, especially as regards the mutual action of the structural members, and this leads to certain approximations which can then be narrowed down by carrying out a few tests on a model under monochromatic wave conditions.

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LIST OF SYMBOLS

$2A_\iota$	Wave height
A, B, C, P, Q, R.	Added mass coefficients
Cz	Heaving force coefficient
Cxx	Rolling moment coefficient
Cyy	Pitching moment coefficient
Czz	Yawing moment coefficient
$\overline{F}z$	Heaving force
$\overline{F}_\varphi$	Force due to the pressure term $\partial\Phi/\partial\Gamma t$
$\overline{F}_V$	Force due to the pressure term $V_F{}^2/2$
g	Acceleration of gravity
K_ι	Wave number
L	Length of body
$\overline{n}$	Unit vector normal to an element
$\overline{n}_\iota$	Unit vector normal to the i-th element
P(M)	Total pressure
Mx, My, Mz.	Rolling, pitching and yawing moments
t	Time
T_ι	Wave period
k_x , k_y , k_z	Virtual inertia coefficients

V	Volume of immersed body
$\overline{V}_F$	Fluid velocity vector
$\overline{V}_C$	Buoyancy centre velocity vector
$\overline{V}_H$	Wave induced fluid velocity
u, v, w	Velocity components
p, q, r	Angular velocity components
z	Depth of submergence
x, β, γ	Components of the unit normal vector
δs_i	Area of the i-th surface element
Γ_i	Circumference of a surface element
θ	Angle of pitch
λ	Wavelength
ρ	Fluid density
Σ	Area of body surface
σ	Surface source density
Φ	Overall potential
Φ_H	Potential due to incident wave
Φ_{PH}	Potential due to the presence of a fixed body in waves
Φ_{PC}	Potential due to the body motion
ω_i	Wave angular frequency
$\overline{\Omega}$	Angular velocity
$\varphi_1 , \varphi_2 , \varphi_3$ χ_1 , χ_2 , χ_3	Potentials associated to unit velocity components
ψ_i	Wave phase angle
ψ	Heading of immersed body

USED UNITS

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Time	:	Second
Lenght	:	Metre
Mass	:	Metric ton
* Force	:	Sthene = 10^3 Newton

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DISCUSSION ON PAPER 12

J. B. MILLER
University of Manchester, England

The authors neglect the influence of the body on the free surface motion and hence they have to assume that the depth of the body is more than twice its vertical height. This restriction can be removed if the source potential function is modified to satisfy the free surface boundary conditions.

We take horizontal co-ordinates x and y in the mean free surface and as vertical co-ordinate, z, measured positive downwards. A fluctuating source with strength varying as $\sigma \cos \omega t$ at $x = a$, $y = b$, $z = f$ produces diverging waves at infinity. The potential which satisfies the boundary conditions for infinitesimal height waves is,

$$\phi(x, y, z, t) = \sigma \left[\frac{1}{r} - PV \int_0^\infty \frac{\upsilon + k}{\upsilon - k} e^{-k(z+f)} J_0(kR)\, dk \right] \cos \omega t + $$
$$+ \sigma \alpha \pi \upsilon e^{-\upsilon(z+f)} J_0(\upsilon R) \sin \omega t,$$

where $\upsilon = \sigma^2/g$,

J_0 is the Bessel function the first kind and order zero,

$R = \sqrt{(x-a)^2 + (y-b)^2}$,

$r = \sqrt{(x-a)^2 + (y-b)^2 + (z-f)^2}$,

and PV indicates that the Cauchy principal value of the integral is to be taken.

A similar expression can be obtained for the case of finite depth, see Thorne (1953).

This expression can be used in place of $1/|MP|$ in equation (2.5) and (2.6) of the paper. The solution for the source densities σ_{PH} and σ_{PC} would then be obtained from suitably modified versions of equations (2.7) and (2.8).

The Cauchy principal value integral can be evaluated by contour integration and this will increase the time taken to set up matrix A, but the time penalty incurred should not be too great.

The conditions under which the authors' solution is valid may be determined from this expression. If the depth of the source, f , is greater than half a wave length then the extra terms will be negligible. But at this depth we do not expect any appreciable wave motion, so it would appear that the extra terms should always be considered. However, the strength of the sources on a body must sum to zero, and if the body is sufficiently deep the potentials due to the sources will cancel at the surface, and it will not be necessary to consider the extra wave terms. This, of course, is the condition the authors impose.

Ref.: Thorne, R.C., Multipole expansions in the theory of surface waves, Proc. Camb. Phil. Soc., Vol. 49, 1953, pp. 707 - 716.