PROBABILISTIC SENSITIVITY ANALYSIS OF DUNE EROSION CALCULATIONS

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ABSTRACT: Coastal dunes protect low lying coastal areas against the sea. Extreme waves and water levels during severe storms may cause breaching of the dunes. Consequently, serious damage due to flooding and direct wave attack could occur, resulting in loss of life and property. Proper coastal management implies that reinforcement measures will be taken if the actual safety level does not meet the agreed standard. It is therefore essential to be able to assess the safety of a dune coast against breaching. This study concerns a probabilistic sensitivity analysis of various variables that are included in the current Dutch safety assessment method. The aim is to get more insight in the influence of the stochastic characteristics of the various variables which are taken into account in the current method. Although for the actual assessment a semi-deterministic method is used, the design values of the variables are based on a probabilistic investigation. Using the underlying probabilistic investigation as a reference, the various distribution functions have been varied in order to get more insight in the influence of each of these stochastic characteristics on the rate of dune erosion.

Key Words: Dune erosion, Extreme hydraulic conditions, Storm surges, Uncertainty, Probabilistic approach

1. INTRODUCTION

In The Netherlands, law prescribes a maximum probability of failure of the dune coast of 10^{-5} per year (for the most important parts of the coast, for other parts, probabilities are slightly larger). For the actual assessment a semi-deterministic method is used, for which the design values are derived based on probabilistic investigations (WL | Delft Hydraulics, 2007). This study aims at more insight in the sensitivity of the rate of erosion, during extreme situations corresponding to a probability of exceedance of 10^{-5} per year, for the stochastic characteristics of the various variables which are currently taken into account. Based on such sensitivity study it can be concluded which of the variables need most attention in further research and e.g. field measurements. Prioritization in the list of variables for modeling dune erosion is one of the main outcomes.

2. MODEL DESCRIPTION

The probabilistic sensitivity analysis of dune erosion calculations, presented in this paper, is strongly related to the investigation of WL | Delft Hydraulics (2007). This report describes the probabilistic background for the semi-deterministic safety assessment method.

2.1 Probabilistic model

The generic probabilistic toolbox 'Prob2B' (former 'Probox'; Courage & Steenbergen, 2007), developed by TNO Built Environment and Geosciences, has been used for this study. Within Prob2B seven reliability calculation methods are available. The First Order Reliability Method (FORM) has been applied in the current study. Since Prob2B can easily be coupled to other software, for this case the dune erosion model, it is very suitable to apply to dune safety assessment.

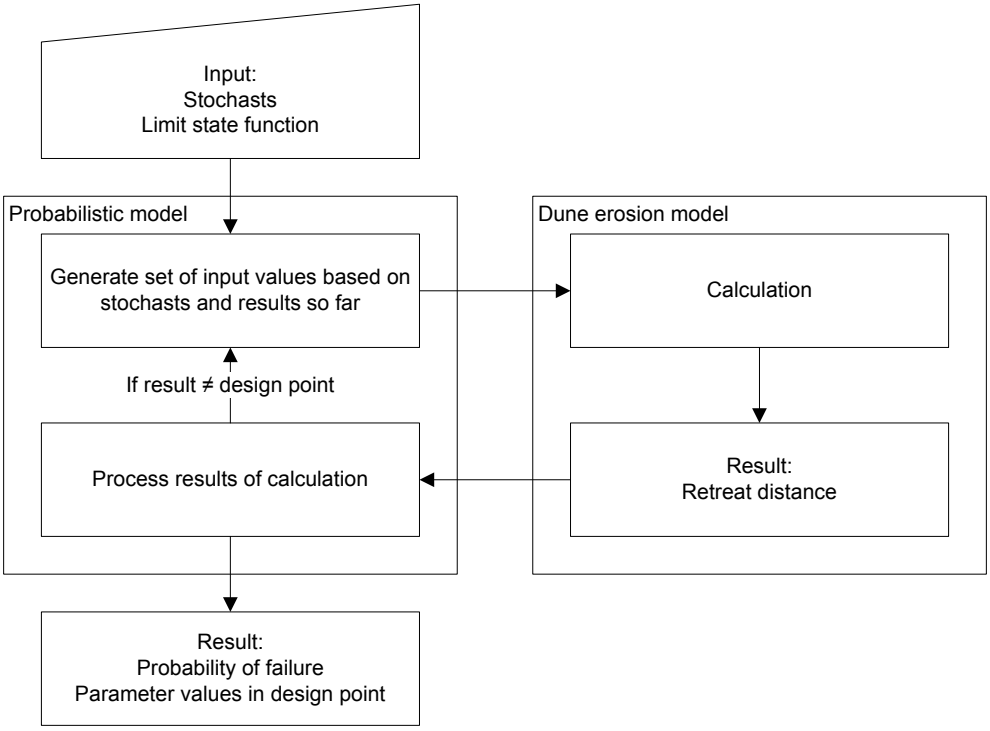

Figure 1: Flowchart of probabilistic model coupled with dune erosion model.

2.2 Dune erosion model

The empirical dune erosion model DUROS-plus (WL | Delft Hydraulics, 2006) has been used for this investigation. This model is similar to the DUROS model (CUR/TAW, 1989/1984; Vellinga, 1986), except that an extra contribution for the influence of the wave period has been added. The cross-shore profile just before the storm surge, the grain size of the sediment, the maximum storm surge level and the wave characteristics at the MSL – 20 m depth contour (wave height and peak wave period) are governing the erosion rate in the model.

The DUROS-plus algorithm available in the Marine and Coastal Toolbox (McTools; Van Koningsveld, Stive & Mulder, 2005), has been applied for this study.

2.2.1 Shape of the erosion profile

The cross-shore erosion profile is described by three parts: the dune face, the parabolic part, and the toe slope.

The dune face is described by a 1:1 slope, from the maximum storm surge level and upward.

At the maximum storm surge level, the dune face is connected to the parabolic part. This connection is 'the after storm' dune foot (x = 0; y = 0). From the dune foot seaward, the parabolic part is described by the formula:

[1] $$\left(\frac{7.6}{H_{0s}}\right) y = 0.4714 \sqrt{\left[\left(\frac{7.6}{H_{0s}}\right)^{1.28} \left(\frac{12}{T_p}\right)^{0.45} \left(\frac{w}{0.0268}\right)^{0.56} x + 18\right]} - 2.0$$

Equation [1] is valid until the cross-shore location where:

[2] $$x = x_{\max} = 250 \left(\frac{H_{0s}}{7.6}\right)^{1.28} \left(\frac{0.0268}{w}\right)^{0.56}$$

For this value of x, the y coordinate is given by:

[3] $$y = y_{\max} = \left[0.4714 \sqrt{\left(250 \left(\frac{12}{T_p}\right)^{0.45} + 18\right)} - 2.0\right] \left(\frac{H_{0s}}{7.6}\right)$$

From the point x_{max}, y_{max} seaward, a constant slope of 1:12.5 is connected to the parabolic part until it intersects with the initial profile.

The fall velocity w is calculated with the formula (Waterloopkundig Laboratorium, 1983):

[4] $$^{10}\log\left(\frac{1}{w}\right) = 0.476 \left(^{10}\log D_{50}\right)^2 + 2.180\, ^{10}\log D_{50} + 3.226$$

In formulae [1], [2], [3] and [4] and the rest of the paper, the following notation is adopted:

H_{0s}	significant wave height at the MSL – 20 m depth contour (≈deep water) [m]
T_p	(spectral) peak wave period [s]
w	fall velocity of the sediment in seawater of 5° Celsius [m/s]
x	horizontal coordinate (positive seaward) w.r.t. dune foot after storm [m]
y	vertical coordinate (positive downward) w.r.t. maximum storm surge level [m]
D_{50}	measure for the grain size [m] (50% in weight of the sample is finer)

Formulae [1] and [3] are valid for peak wave periods in the range 12 s < T_p < 20 s. In case T_p < 12 s, T_p = 12 s is used, which in fact means that the DUROS model (TAW, 1984) is applied. In case T_p > 20 s, T_p = 20 s is used.

2.2.2 Procedure

To determine the position of the erosion profile and subsequently the rate of erosion, cross-shore conservation of volume is assumed. The erosion profile, as defined in Section 2.2.1 is shifted in horizontal

direction in such a way that erosion and accretion volumes are equal. The erosion volume above storm surge level (the water level as applied in that particular calculation) is designated as 'volume A' (see Figure 2).

It is obvious that a cross-shore profile is not constant in time. Therefore, also a contribution for the profile fluctuation is taken into account. This is done by creating a virtual bar or trough in the profile, somewhere in the accretion zone. This is implemented by striving after a volume balance in which Accretion – Erosion = Profile Fluctuation. When the profile fluctuation is set to zero, a closed sediment balance will be found, as described above. However, when the profile fluctuation is set negative (trough), more erosion will be found (for a positive profile fluctuation it works the other way around).

2.2.3 Additional erosion

Because the water level, wave height, peak wave period and grain size are the only governing variables in the DUROS-plus model, the so-called additional erosion 'volume ΔA' (see Figure 2) is used to include contributions from surge duration and model accuracy. These extra contributions are expressed in an additional erosion volume as a portion of volume A (see Section 2.2.2). The 1:1 dune face (above water level) will be shifted landward until the additional erosion fits.

2.2.4 Retreat distance

In this study, the retreat distance has been used as a measure for the erosion rate. The retreat distance is defined as the horizontal distance between the MSL + 5 m contour and the dune edge (top of the 1:1 dune face of the additional erosion profile).

2.3 Procedure for probabilistic model coupled with dune erosion model

This study focuses on the rate of dune erosion for a probability of exceedance of 10^{-5} per year. However, the probabilistic model does not have the possibility to predefine the probability of failure to find the corresponding erosion rate. Therefore it has been chosen to calculate for each situation the probability of exceedance for a series of retreat distances (with step sizes of 5 m). Consequently, the 10^{-5} per year retreat distance has been estimated by inverse analysis. Backup calculations for the estimated retreat distance can confirm the 10^{-5} per year probability, as well as the results of the design values of the involved variables.

3. APPROACH

Starting from a reference situation which is strongly linked to the investigation of WL | Delft Hydraulics (2007), the stochastic characteristics have been varied in order to study the sensitivities in the rate of dune erosion. The results described below are based on a simplified cross-shore profile, and statistics near the location of Hoek van Holland along the Dutch coastline.

3.1 Reference situation

The cross-shore profile as shown in Figure 2 has been used for this investigation. This profile is considered to be more or less representative for most of the Dutch dune coast.

For the water level distribution, a so-called conditional Weibull distribution function is applied (see among others RIKZ (2000) for more details).

[5] $$F_e\left(H > h\right) = \rho \exp\left[-\left(h/\sigma\right)^{\alpha} + \left(\omega/\sigma\right)^{\alpha}\right]$$

Where:
F_e frequency of exceedance of the highest level h during a storm surge [year^{-1}]
h highest water level during a storm surge [m]
α shape parameter that depends on the location along the coast
ω threshold above which the function is valid [m above MSL]
σ scale parameter that depends on the location along the coast
ρ frequency of exceedance of the threshold level ω

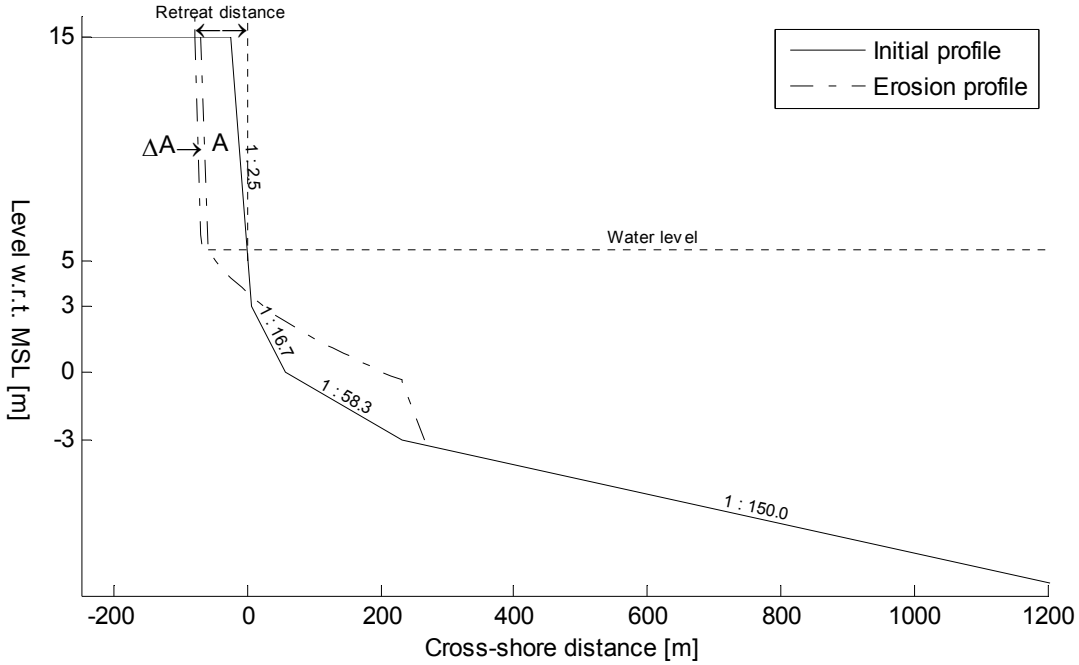

Figure 2 Simplified cross-shore profile.

The mean significant wave height is related to the water level by the following expression (see WL | Delft Hydraulics, 2007):

[6] $$H_s = 4.35 + 0.6h - 0.0008(7-h)^{4.67}$$

Where:
H_s significant wave height
h highest water level during a storm surge [m]

Table 1 Summary of distribution functions for the reference situation

Variable	Mean value	Uncertainty/variance	Distribution type
Water level	Based on Equation [5]	-	Conditional Weibull
Wave height	Equation [6]	0.6 m	Normal
Wave period	Table[1] (see HKV, 2005)	1 s	Normal
Grain size	225 μm	10% of mean (22.5 μm)	Normal
Profile fluctuation	0	60 m^3/m^1	Normal
Surge duration	0	10 % * A	Normal
Model accuracy	0	15 % * A	Normal

[1] Numerical relation between wave height and peak wave period

3.2 Sensitivity analysis

To investigate the sensitivity of the dune erosion rate by the stochastic characteristics of the involved variables, all of the characteristics have been varied one by one (keeping the others to their reference values). Table 2 summarizes the values which have been used (see Figure 3 for results of underlined characteristics).

Table 2 Summary of stochastic characteristics which have been varied (reference values in **bold**)

Variable	Mean value	Uncertainty/variance
Water level	Based on equation [5] + [-0.5, -0.25, **0**, 0.25, 0.5] m	-
Wave height	Equation [6] + [-0.5, -0.25, **0**, 0.25, 0.5] m	[0, 0.3, **0.6**, 0.9 and 1.2] m
Wave period	Table[1] (see HKV, 2005) + [0, 1 and 2] s	[0, 0.5, **1.0**, 1.5 and 2.0] s
Grain size	[200, **225**, 250, 275 and 300] μm	[0, 5, **10** and 15] % of 225 μm
Profile fluctuation	[-20, **0** and 20] m^3/m^1	[0, 30, **60**, 90 and 120] m^3/m^1
Surge duration	[**0**, 5 and 10] % * A	[0, 5, **10**, 15 and 20] % * A
Model accuracy	[**0**, 5 and 10] % * A	[0, 5, 10, **15** and 20] % * A

[1] Numerical relation between wave height and peak wave period

For most characteristics, five different values are given in the table (for others only three). For each of these values, a full FORM calculation has been carried out in which all other characteristics where set to the reference value (the bold ones).

4. RESULTS

4.1 Reference situation

Various calculations with the simplified profile as described in WL | Delft Hydraulics (2007), have been reproduced, resulting in the same results. The results for the 10^{-5} per year dune erosion using the stochastic characteristics as given in the reference situation for this study have been summarized in Table 3.

Table 3 Summary of design point values for the reference situation (10^{-5} per year)

Variable	Value	Relative contribution [%]
Water level	5.48 m	89.53
Model accuracy	58.4 m^3/m^1	3.40
Grain size	208 μm	3.28
Surge duration	26.0 m^3/m^1	1.51
Wave height	7.92 m	1.26
Profile fluctuation	-23.4 m^3/m^1	0.84
Wave period	12.71 s	0.19
Retreat distance	**78.8 m**	

4.2 Sensitivity analysis

Figure 3 presents a small selection of the results of the sensitivity analysis. Each of the panels in the figure show the 10^{-5} per year retreat distance on the vertical axis and a stochastic variable on the horizontal axis. It turns out that the retreat distance is most sensitive to the water level and the grain size distribution characteristics. The results showed in the four panels of Figure 3 are briefly discussed here:

(a) In this set of simulations the water level has been changed for all probabilities of exceedance with the same value. As clearly appears from this figure, water level is a very important variable. In fact this investigation could be seen as some kind of fictitious sea level change. However, in case of real sea level change, the shape of the profile will also be changed.

(b) This panel shows the 10^{-5} per year erosion to be very sensitive to the mean grain size. Although the absolute value of the standard deviation of the grain size is larger for increasing grain sizes (set to 10% of mean), still the mean grain size has much influence. In reality, the shape of the initial profile will also depend on the grain size. This will have a counteracting effect, since coarser sediment will allow a steeper profile which can result in more erosion.

(c) The effect of varying the standard deviation between 0 and 10% (=22.5 μm) of the mean grain size is not extreme, but further increasing the standard deviation appears to have more effect. Interesting detail is that for σ_{D50}=33.75 μm, in the design point the water level is 5.23 m (81.79%) and the grain size 178 μm (10.84%), which is quite different from the situation as represented in Table 3.

(d) The sensitivity of the 10^{-5} per year erosion for the standard deviation of the peak wave period is an example of a characteristic which has hardly any effect.

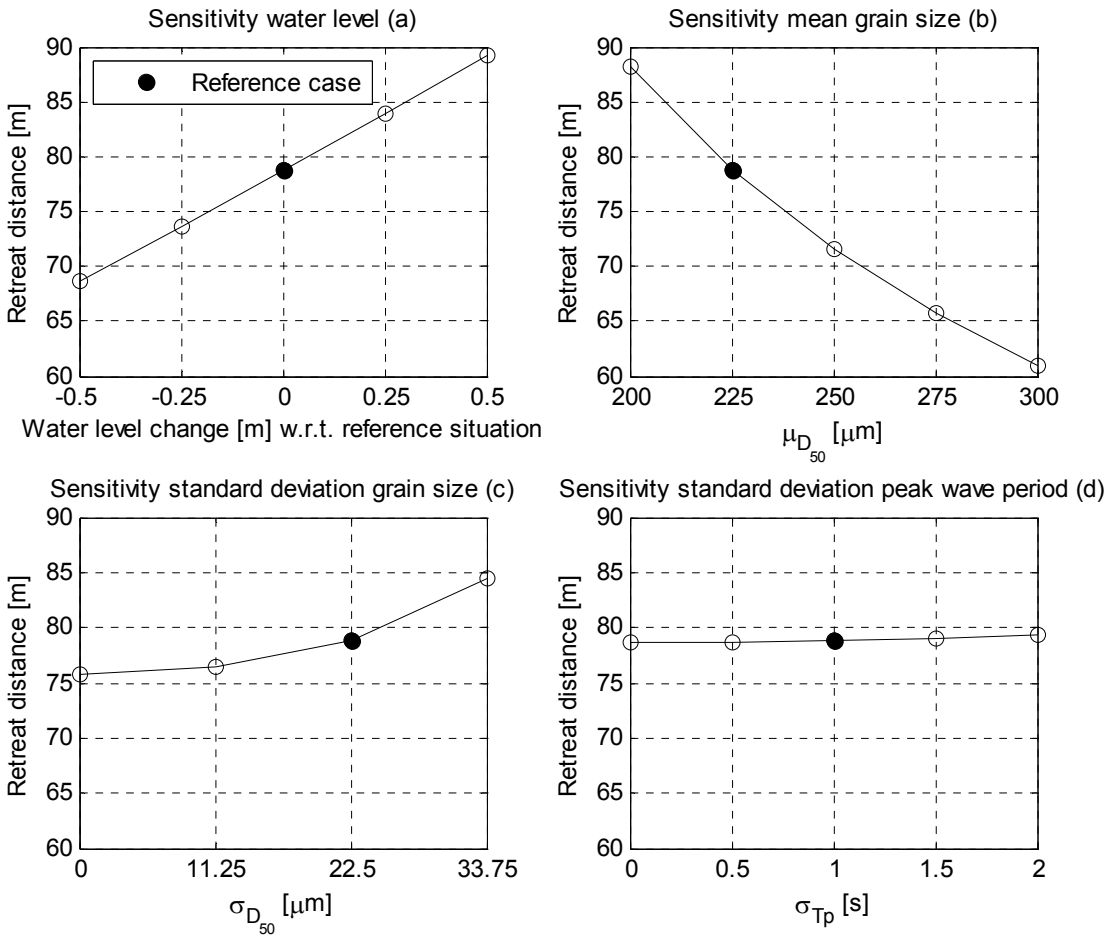

Figure 3 Sensitivity of retreat distance for 10^{-5} per year probability of exceedance.

5. CONCLUSIONS

From this study it can be concluded that the water level and the grain size distribution are the most important variables in the current safety assessment method for the Dutch dune coast. It is therefore crucial to have proper field data of these variables. Most of the other variables do have their influence on the amount of erosion for the normative situation, but a change in their stochastic characteristics does not cause a significant change in the results.

It is recommended to carry out similar investigations with process based models, in order to simulate storms more realistically.

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7. REFERENCES

Courage, W.M.G. & H.M.G.M. Steenbergen, 2007. Prob2B™: variables, expressions and Excel®. Installation and Getting Started. TNO-report 2007-D-R0887/A, TNO Built Environment and Geosciences, Delft. August 2007.

CUR/TAW, 1989/1984. Guide to the assessment of the safety of dunes as a sea defence (English translation). Centre for Civil Engineering Research and Codes, Gouda. Technical Advisory committee for Water defences, The Hague.

HKV, 2005. Deep water boundary conditions (ELD, EUR, YM6, SCW en SON), HKV consultants (prepared for RIKZ). July 2005 (in Dutch).

RIKZ, 2000. Direction dependent extreme values for high water levels, wave height and wave period. Report RIKZ 2000.040 (in Dutch).

Van Koningsveld, M.; M.J.F. Stive & J.P.M. Mulder, 2005. Balancing research efforts and management needs. A challenge to coastal engineering. Proceedings of the 29th Int Conf. of Coast. Eng. Lisbon, Portugal, 2004. pp. 2985 - 2997

Vellinga, P., 1986. Beach and Dune Erosion during Storm Surges. Ph.D. Thesis Delft University of Technology.

Waterloopkundig Laboratorium, 1983. The fall velocity of sand in sea water of 5° C (In Dutch: De valsnelheid van zand in zeewater van 5° C). Report M1263IV[b].

WL | Delft Hydraulics, 2006. Dune erosion; Product 2 – Large-scale model tests and dune erosion prediction, methods. WL | Delft Hydraulics report H4357.

WL | Delft Hydraulics, 2007. Dune erosion; Product 3 – Probabilistic dune erosion method. WL | Delft Hydraulics report H4357.