# Seismic interferometry by midpoint integration

Elmer Ruigrok*, Carlos Almagro Vidal†, Kees Wapenaar†

*Department of Geoscience and Engineering, Delft University of Technology, Delft, Netherlands, Email: e.n.ruigrok@tudelft.nl

†Department of Geoscience and Engineering, Delft University of Technology, Delft, Netherlands

*Abstract*—**With seismic interferometry reflections can be retrieved between station positions. In the classical form, the reflections are retrieved by an integration over sources. For a specific dataset, however, the actual source distribution might not be sufficient to approximate the source integral. Yet, there might be a dense distribution of receivers allowing integration over the receiver domain. We rewrite the source integral to an integration over midpoints. With this formulation, a reflection can be retrieved even in the limiting case of only a single source. However, with respect to the classical formulation, an additional stationary-phase analysis is required.**

## I. INTRODUCTION

With seismic interferometry reflections can be retrieved between station positions. In the classical form (e.g., *Wapenaar and Fokkema* (2006)) the reflections are retrieved by an integration over sources or by using a highly-scattered wavefield (e.g., *Derode et al.* (2003)). For specific configurations, the actual source distribution might not be sufficient to approximate the source integral, nor might there be enough scattering. When the velocity model is known, still only a few sources could suffice to build a structural image using seismic interferometry (e.g., *Schuster et al.* (2004)). However, to find more quantitative information about the medium it is desirable to obtain reflections at a few different offsets. In this abstract we show that, for specific settings, it is possible to retrieve high-quality reflections, using only one, or a few, sources. We take advantage of the availability of a well-sampled receiver array and write an interferometric relation that employs an integration over midpoints instead of over sources. The midpoints are defined between virtual-source and actual receiver locations. As an example, we apply the method to core-reflected phases.

## II. SI BY MIDPOINT INTEGRATION

In this abstract we only consider a 2D configuration (Fig. 1). Assume we have one large array of receivers. E.g., a line of receivers from the USArray (*Levander*, 2003). We use the location of the southernmost receiver in the array $\phi_1$ as the reference point. Thus, $\phi_1$ is at $0°$. The northernmost station is at location $\phi_n$. An arbitrary station within the array is denoted with $\phi_i$. We consider a source at $\phi_s$ that is either relatively close to the array, or a source that is at the other side of the globe. Both sources lead to reflection travel paths within the array. The precise location of the sources is not relevant. We define midpoint $m$ as a location within the array and the offset $h$ as the distance from $m$ along the array, where a northwards direction is taken as positive. If the medium of consideration
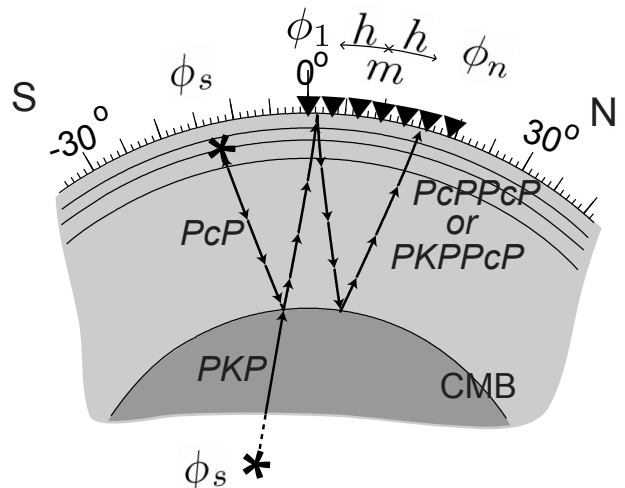


Fig. 1. A global-scale configuration for seismic interferometry by midpoint integration (MSI). The location of sources (stars) and receivers (triangles) is given in degrees with respect to the southernmost station in the array of receivers. The source leading to a $PKP$ and $PKPPcP$ arrival at the array is at the other side of the globe.

is approximately radially symmetric, $m - h \geq \phi_1$ and $m + h \leq \phi_n$ for a range of $m$, we can evaluate the following interferometric relation

$$\int_{\partial\mathbb{S}_1} X((m-h), \phi_s, -t) * XY((m+h), \phi_s, t) dm \propto$$
$$Y((\breve{m}+h), (\breve{m}-h), t), \quad (1)$$

where the asterisk $*$ denotes a temporal convolution and a proportionality sign is used since we have left out all the amplitude terms. $X((m-h), \phi_s, t)$ denotes a phase observed at location $(m - h)$ (one of the receivers in the array) due to a source at $\phi_s$ and $XY$ is a free-surface multiple of $X$. $\partial\mathbb{S}_1$ is the line segment of midpoints over which is integrated.

By evaluating the left-hand side of equation 1 we retrieve $Y((\breve{m}+h), (\breve{m}-h), t)$, which is the response of phase $Y$ for a source at $(\breve{m}-h)$ and a receiver at $(\breve{m}+h)$. If the retrieved phase is a primary reflection or a turning wave (refraction), it has its reflection point or turning point below the stationary midpoint $\breve{m}$. When we denote the integrand of equation 1 with $I$, $\breve{m}$ is the location where $dI/dm = 0$. Hence, $\breve{m}$ can directly be estimated from the crosscorrelated data. If $X$ in equation 1 is substituted by phase $P$ and $XY$ by $PP$ we would retrieve $P$ within the array. We would retrieve $PcP$ within the array

by substituting $X$ with $PcP$ or $PKP$ and $XY$ with $PcPPcP$ or $PKPPcP$ (Fig. 1).

Relation 1 also holds for finding phases with midpoints in between two (similarly oriented) arrays of receivers. In this case, a range of $m$ is chosen in between the arrays, for which $m-h$ coincides with locations in array 1 and $m+h$ coincides with locations in array 2.

In the following, application of equation 1 and a subsequent stationary-phase analysis to find $\breve{m}$, we will call midpoint seismic interferometry (MSI).

### III. NUMERICAL ILLUSTRATION

We illustrate equation 1 numerically using the configuration as depicted in Fig. 1. We do not take exactly the same source positions as indicated on Fig. 1, however. We consider a well-sampled array of stations from $\phi$=0 to $\phi$=20°. A source at $\phi$=−11°, at a depth of 200 km, occurs at $t$=0. The source-time function (STF) is a delta pulse. Figs. 2(a) and (b) are the traveltimes of the forward modeled responses at the array, time-windowed around $PcP$ and $PcPPcP$, respectively. We aim to retrieve $PcP$ within the array with $h$=7°. For this offset, the integral (equation 1) can be evaluated between ∼7 and ∼ 13°. Note that the midpoint limits are not exactly 7 and 13° due to the (unknown) depth of the source. Computing the integrand of equation 1 results in Fig. 2(c). From this integrand we determine $\breve{m}$, which equals 9.84°. Evaluating the integral would give a pulse at $t(\breve{m})$=519.62 s, and two minor pulses at later times, which are due to the edges of the integration line. The latter pulses could be suppressed by tapering the integrand. The main pulse retrieved is the response that would be found if there was a delta-pulse source at $\phi$=2.84° and the $PcP$ phase was measured at $\phi$=16.84°. Indeed, raytracing $PcP$ for an epicentral distance of 14° from a source at the Earth's surface gives an arrival time of 519.62 s. Using the same source, we could repeat the upper procedure to find $PcP$ at different offsets and midpoints. However, by using only one source we cannot retrieve $PcP$ at more than one offset per midpoint.

To find one more offset ($h$=5°), midpoint combination we use the response from a distant source ($\phi$=−160°) as illustrated in Fig. 2(c)-(e).

### IV. DISCUSSION

The correlation integral (equation 1) can be evaluated for varying $h$. The possible range depends on the configuration. Consequently, for a range of midpoints, the response of phase $Y$ would be obtained, for a different offset at each midpoint. This dataset in $h$ and $m$ can directly be imaged using a prestack migration when the velocity model is known.

A reflector can be characterized by repeating an MSI procedure as illustrated in Section III for a few sources. After application to a few sources, per midpoint reflection information can be obtained for varying offset. The traveltime-vs-offset and amplitude-vs-offset curves can be used to quantify the impedance contrast (e.g., *Castagna* (1993)).
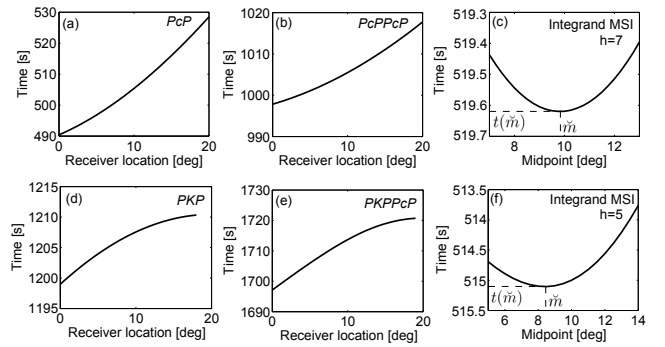


Fig. 2. A numerical illustration of seismic interferometry by midpoint integration (MSI). The left and middle panels depict arrival times of different phases, measured at an array of receivers (Fig. 1), induced by (a)&(b) a source near the array and (d)&(e) a source at the other side of the globe. (c)&(f) depict the correlation panels for the application of MSI to the responses on the left-hand side, for an offset of 7 and 5°, respectively. From these correlation panels, a response for midpoint location ($\breve{m}$) is retrieved by integration over midpoint

We illustrated MSI for a global-scale setting, with the aim of retrieving reflections. MSI can also be used for retrieving multiple reflections and turning waves. Furthermore, MSI can be applied in an exploration-scale setting, for updating a clear reflection, when there is a good receiver network, but there is only a limited amount of (natural) sources. E.g., for monitoring it would be advantageous to use only limited time windows of noise. A noise source might be stationary within this limited time window, allowing the implementation of MSI. Similarly, MSI can be applied to short time windows of microbaroms (*Fricke et al.*, 2011) to retrieve stratospheric or thermospheric refractions within a infrasound receiver network. The refractions would be used to monitor conditions in the stratosphere and thermosphere.

### REFERENCES

Castagna, J. (1993), *Offset-dependent reflectivity: theory and practice of AVO analysis*, SEG.

Derode, A., E. Larose, M. Campillo, and M. Fink (2003), How to estimate the Green's function of a heterogeneous medium between two passive sensors? Application to acoustic waves, *Applied Physics Letters*, *83*, 3054–3056.

Fricke, J., E. Ruigrok, L. Evers, D. Simons, and K. Wapenaar (2011), A large aperture infrasound array for interferometric studies, in *Geophysical Research Abstracts*, vol. 13, EGU2011-4471.

Levander, A. (2003), USArray design implications for wavefield imaging in the lithosphere and upper mantle, *The Leading Edge*, *22*, 250–255.

Schuster, G., J. Yu, J. Sheng, and J. Rickett (2004), Interferometric/daylight seismic imaging, *Geophysical Journal International*, *157*, 838–852.

Wapenaar, K., and J. Fokkema (2006), Green's functions representations for seismic interferometry, *Geophysics*, *71*(4), SI33–SI46.