

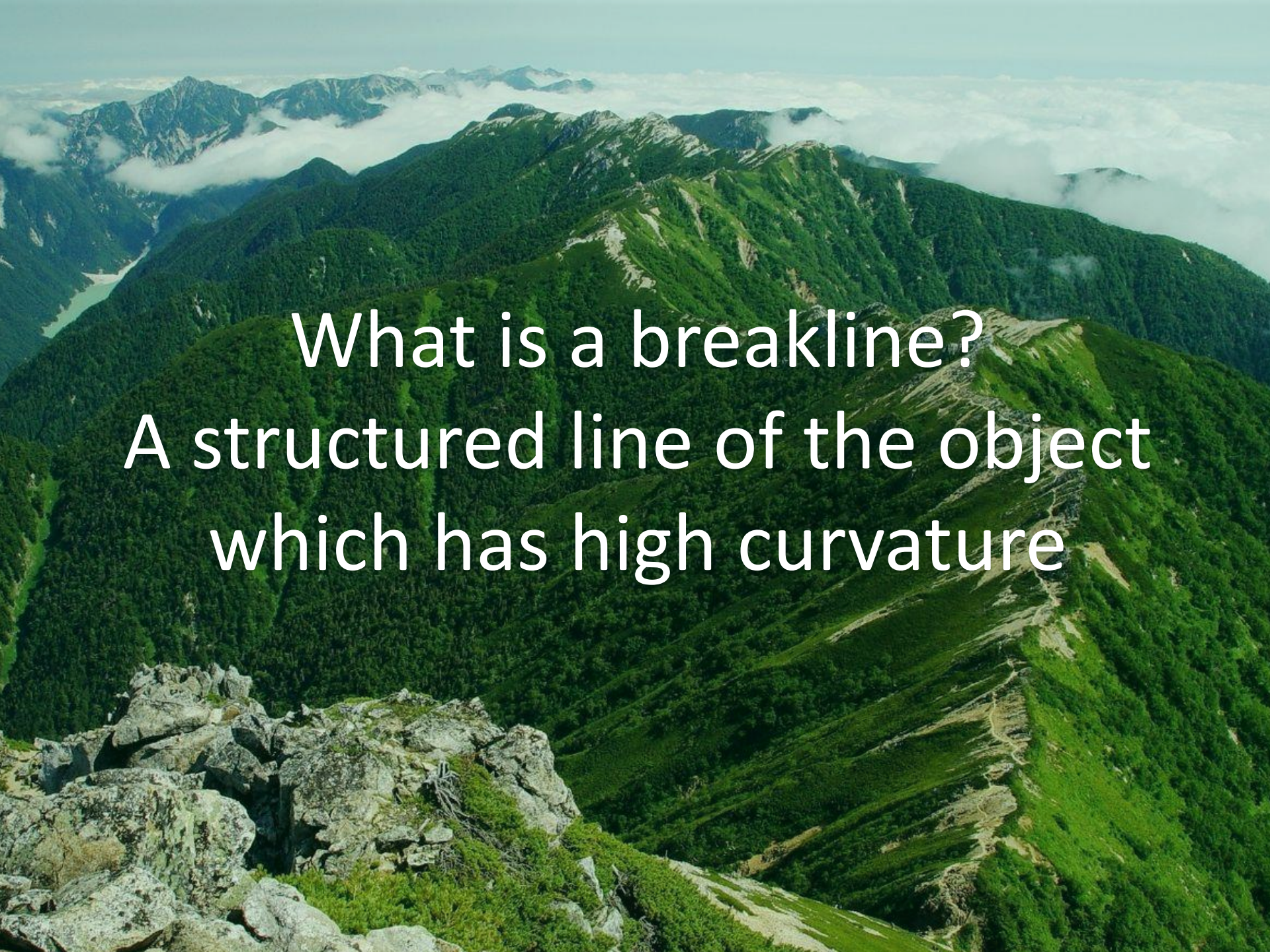
# 3D breakline extraction from point clouds with the Medial Axis Transform



An aerial photograph of a mountain range. The foreground shows a rocky, light-colored ridge. The middle ground is dominated by a large, dark green forested valley. In the background, more mountain peaks are visible, some partially obscured by white clouds. The sky is a pale blue. The text "What is a breakline?" is overlaid in white, sans-serif font across the center of the image.

What is a breakline?



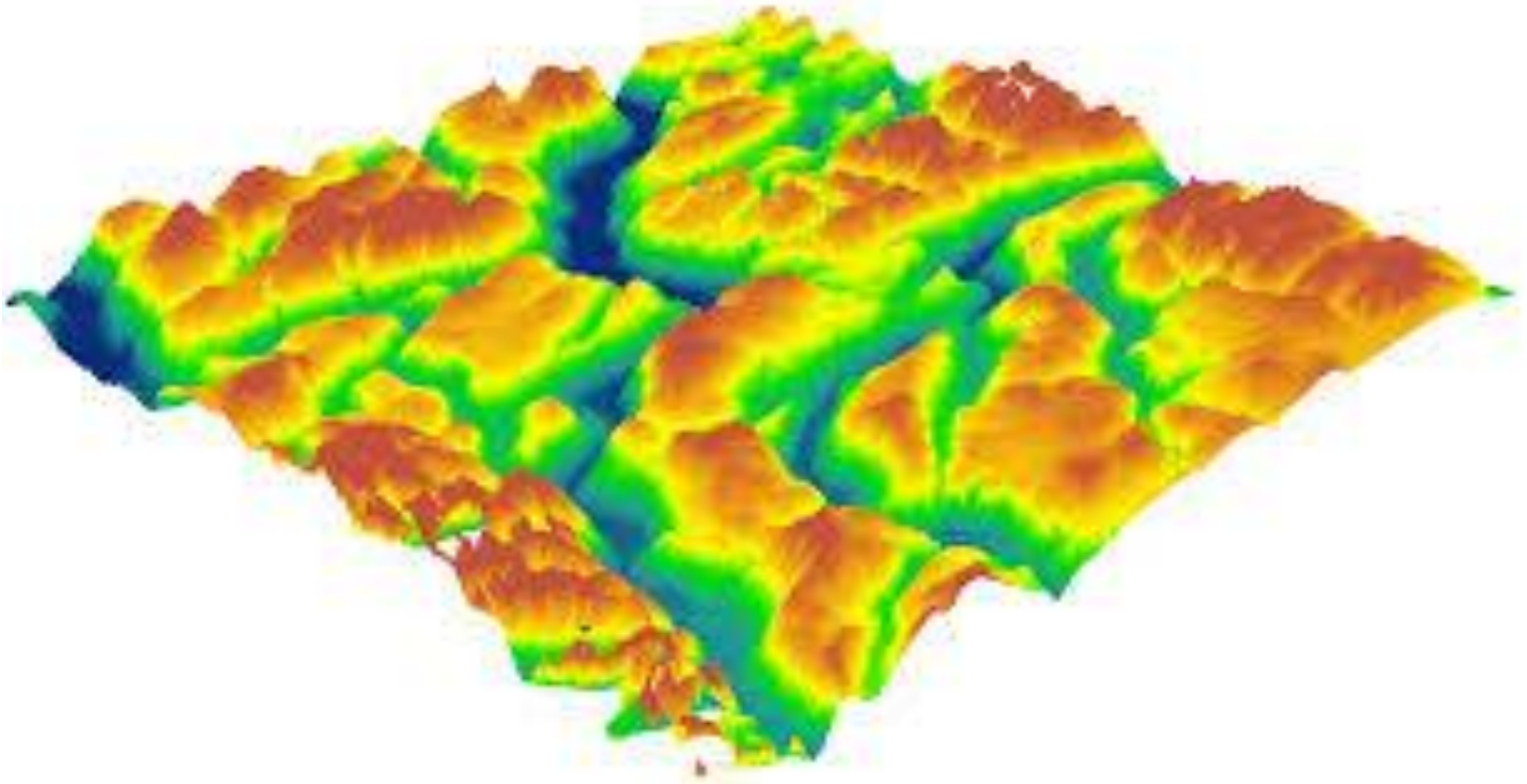
An aerial photograph of a mountain range. The foreground shows rocky terrain with sparse vegetation. The middle ground features a prominent ridge line, which is highlighted with a white dashed line, representing a breakline. The background shows more mountain peaks, some partially obscured by low-hanging clouds or mist. The overall scene is a lush, green mountain landscape.

What is a breakline?  
A structured line of the object  
which has high curvature



# Motivation

- Application
  - Topographic (e.g. generate DEM)



# Motivation

- Application
  - Hydrological (e.g. flood simulation)



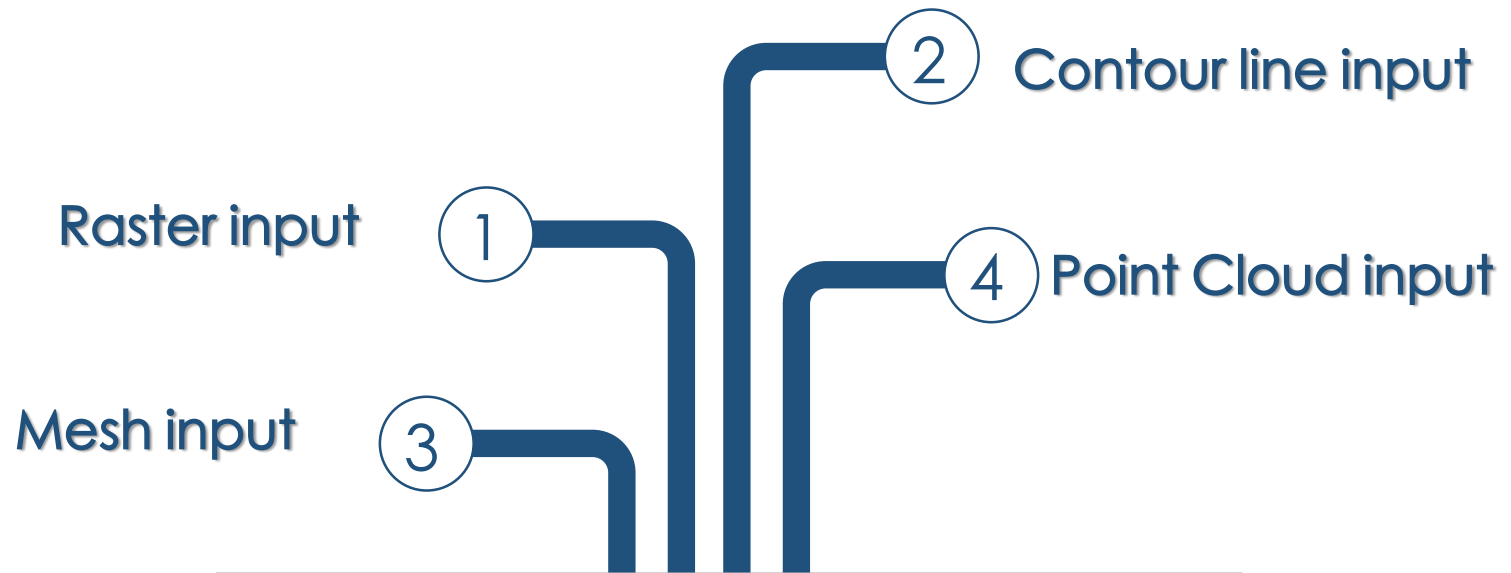


# Motivation

- Application
  - Monitoring

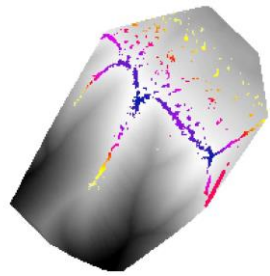


# Existing method



Idea: detect points in breaklines;  
make connection

# Existing method



Raster input

Mesh input

1

3

2

Contour line input

4

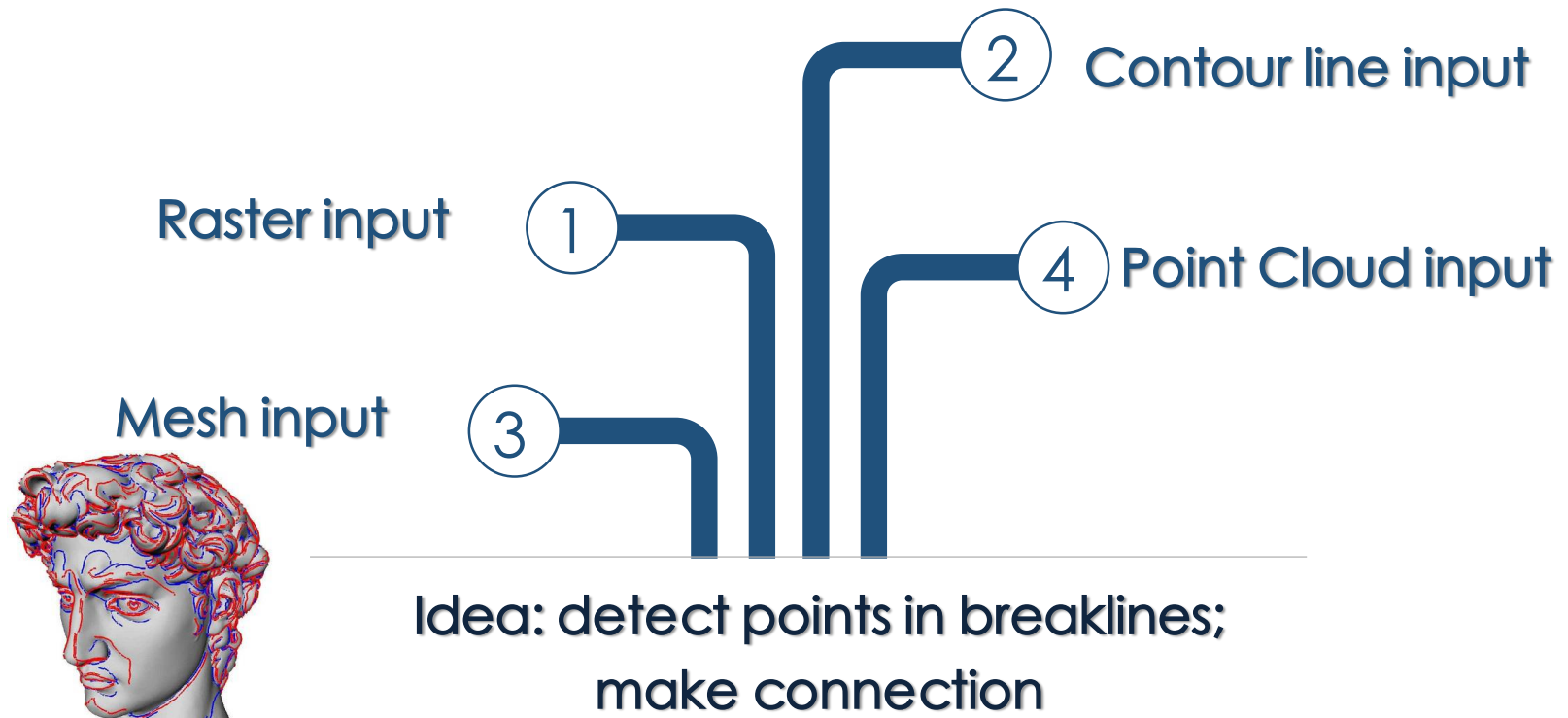
Point Cloud input

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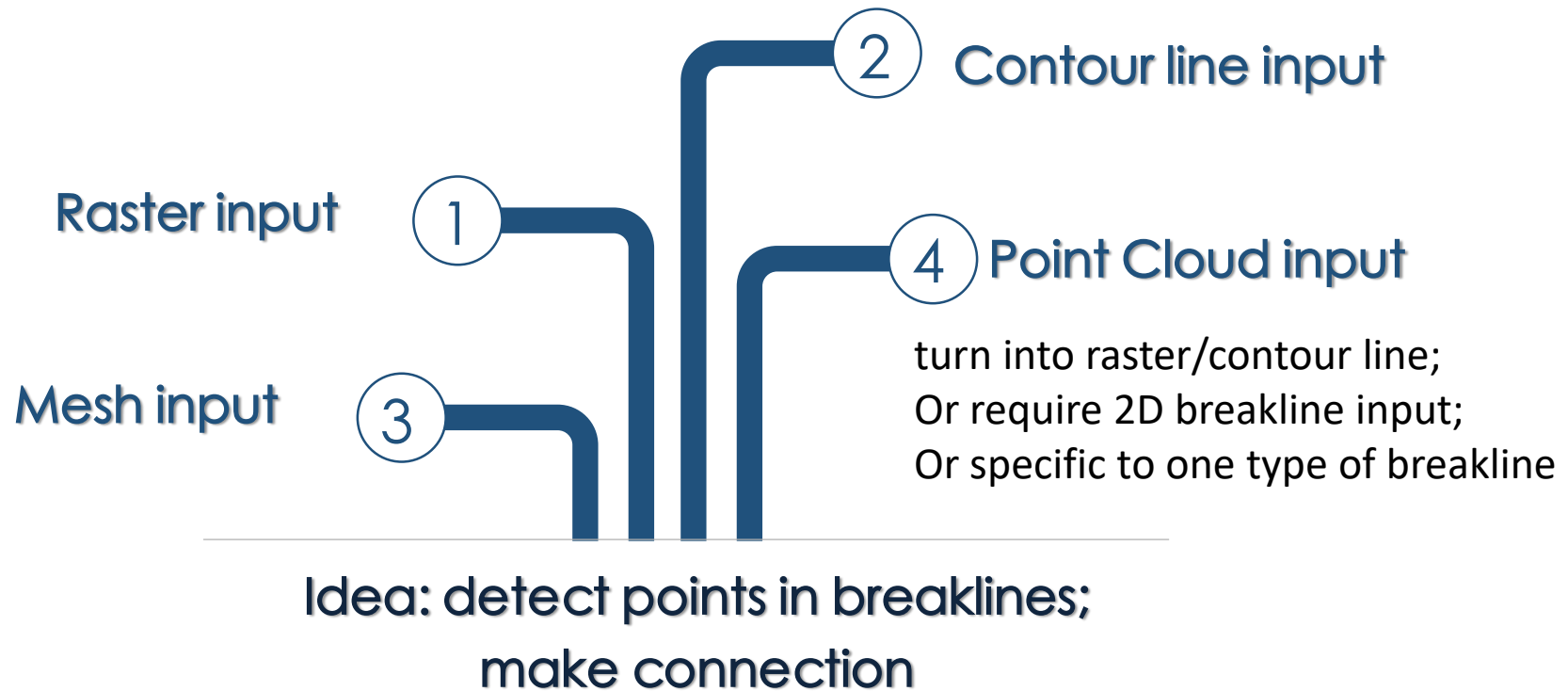
Idea: detect points in breaklines;  
make connection



# Existing method



# Existing method



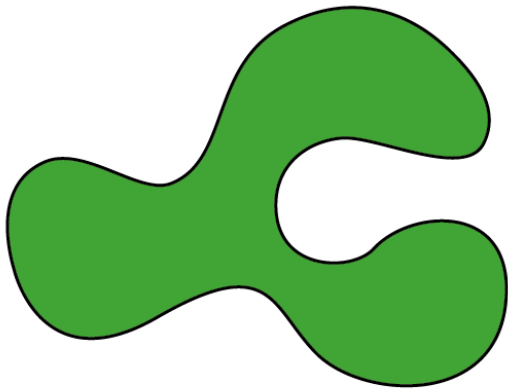


# Motivation

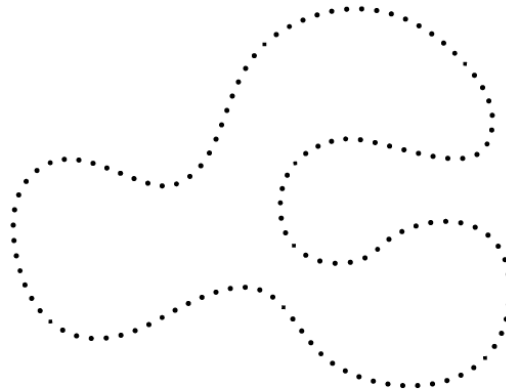
- No efficient method to generate breaklines from point clouds directly
- Converting point cloud into other formats will lose information

# Related work: MAT

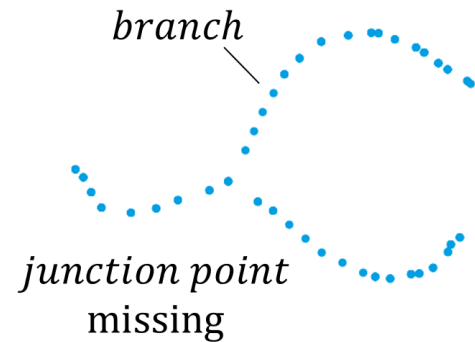
- Medial Axis Transform (MAT)
  - Represent the skeleton



2D object



Boundary points



Interior MAT



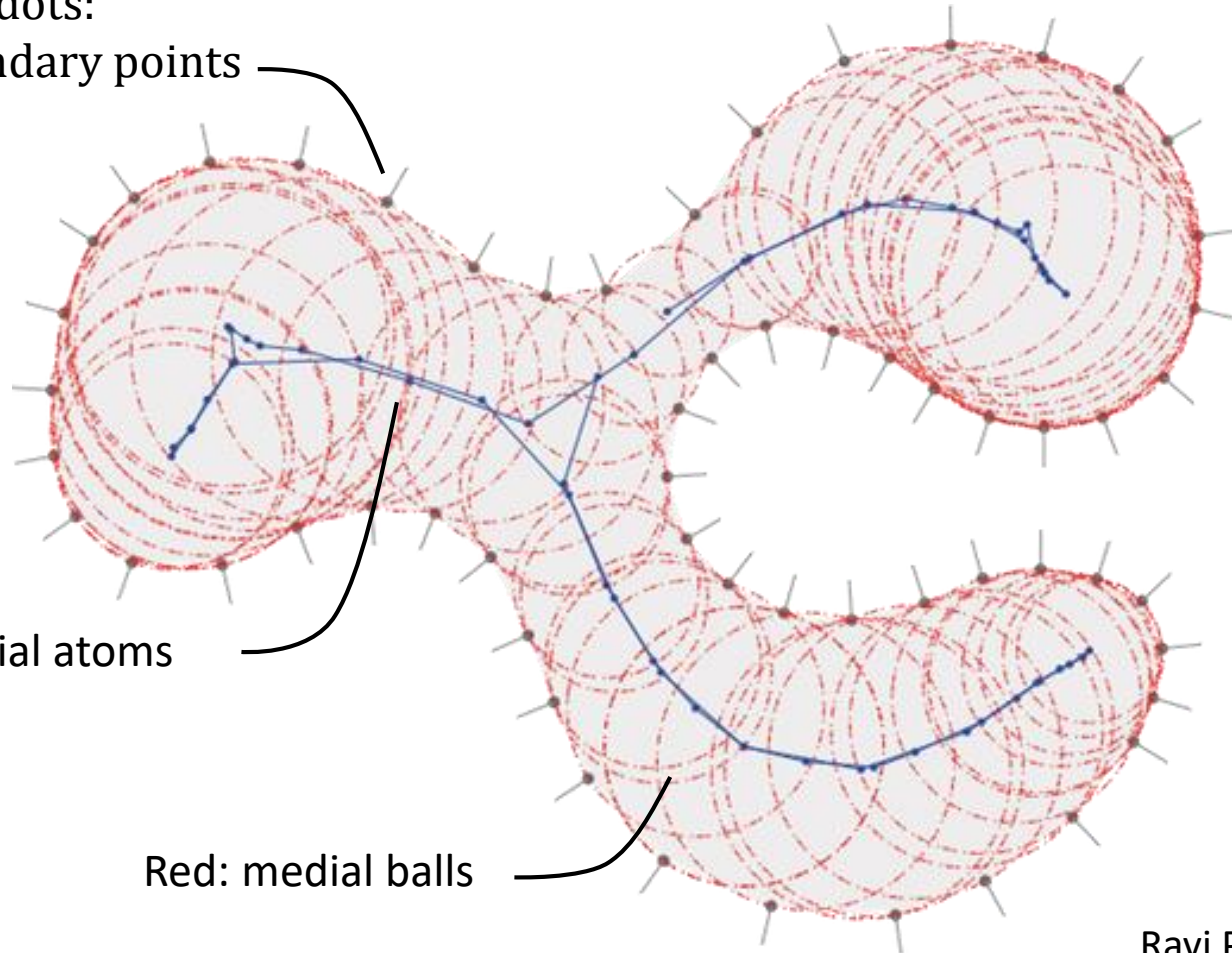
# Related work: MAT

- Definition: medial ball

Black dots:  
object's boundary points

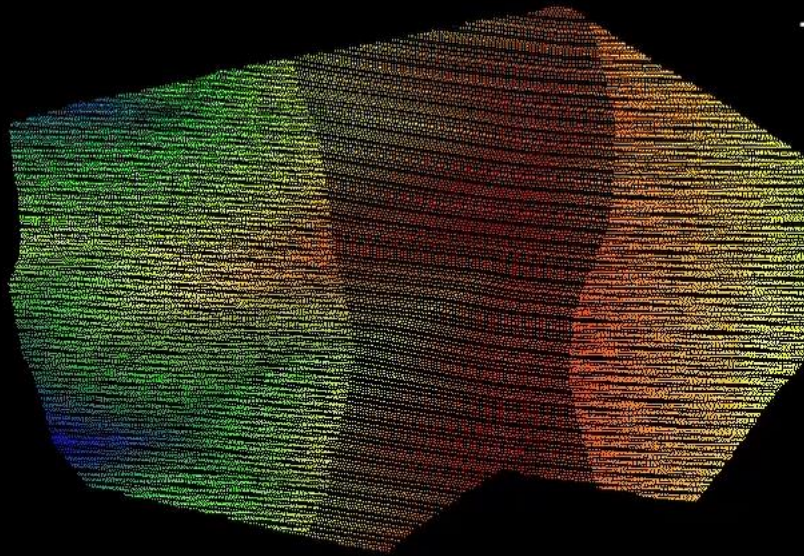
Blue dots: medial atoms

Red: medial balls



# Related work: MAT

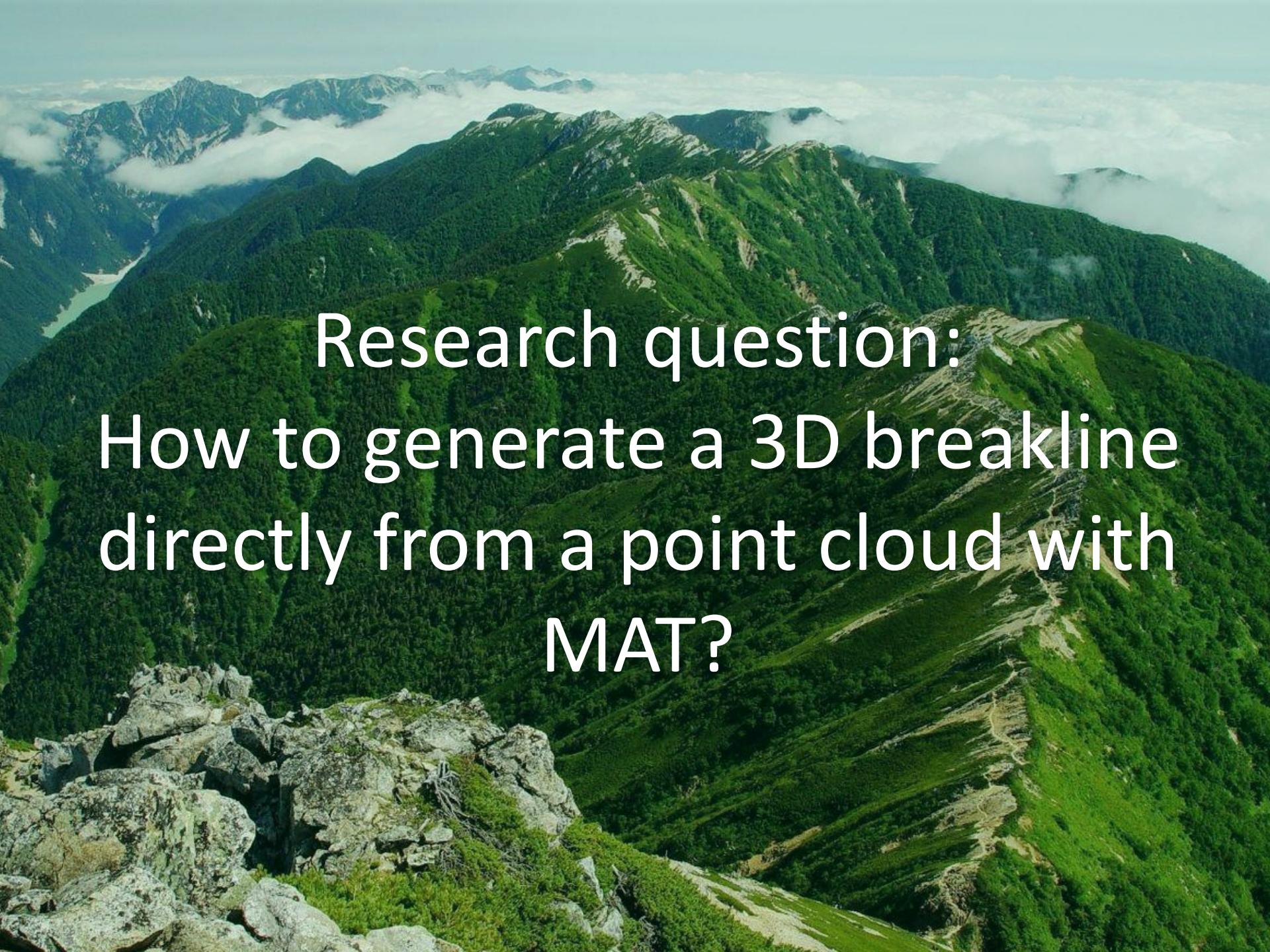
point cloud



100







Research question:  
How to generate a 3D breakline  
directly from a point cloud with  
MAT?



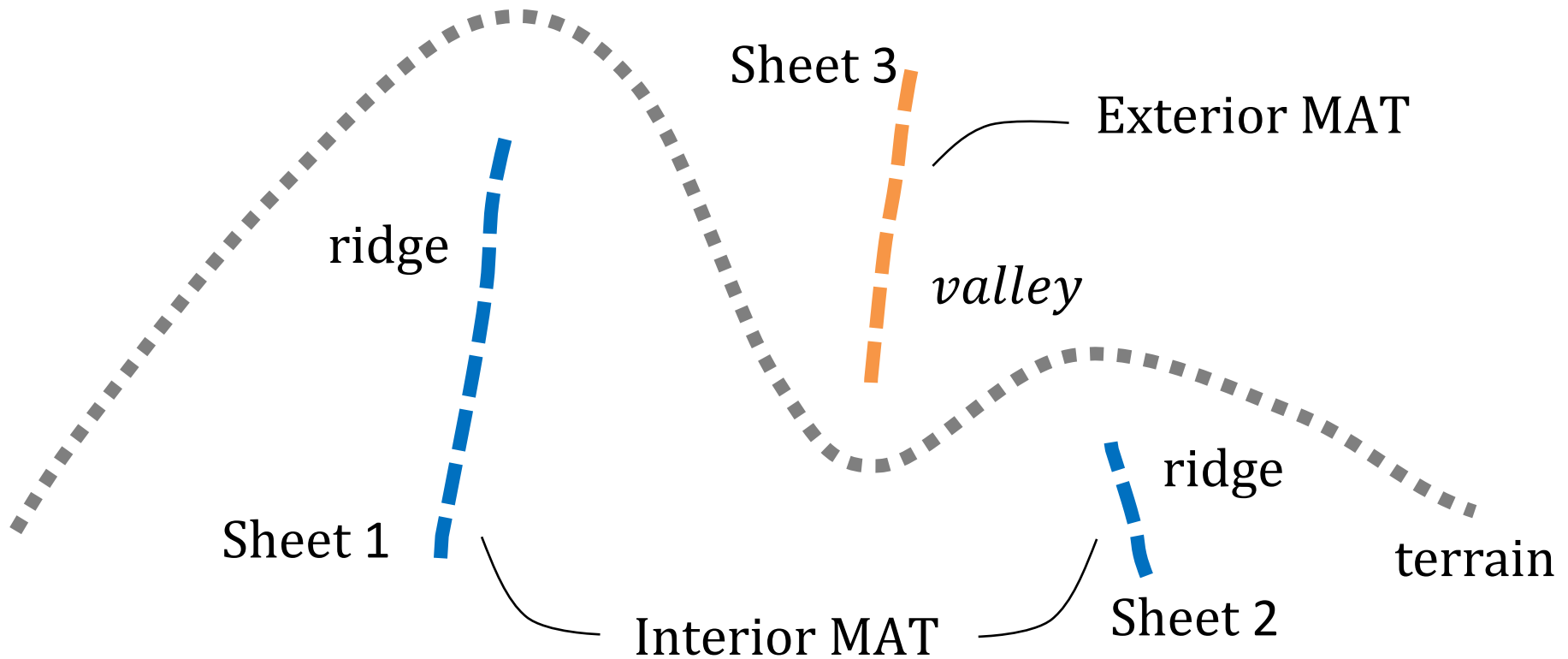
# Scope

- Focusing on natural ground surfaces, such as ridges and valleys in mountains;
- Point cloud contains trees will not be considered;
- This project can not deal with holes caused by river or lake in the point cloud.



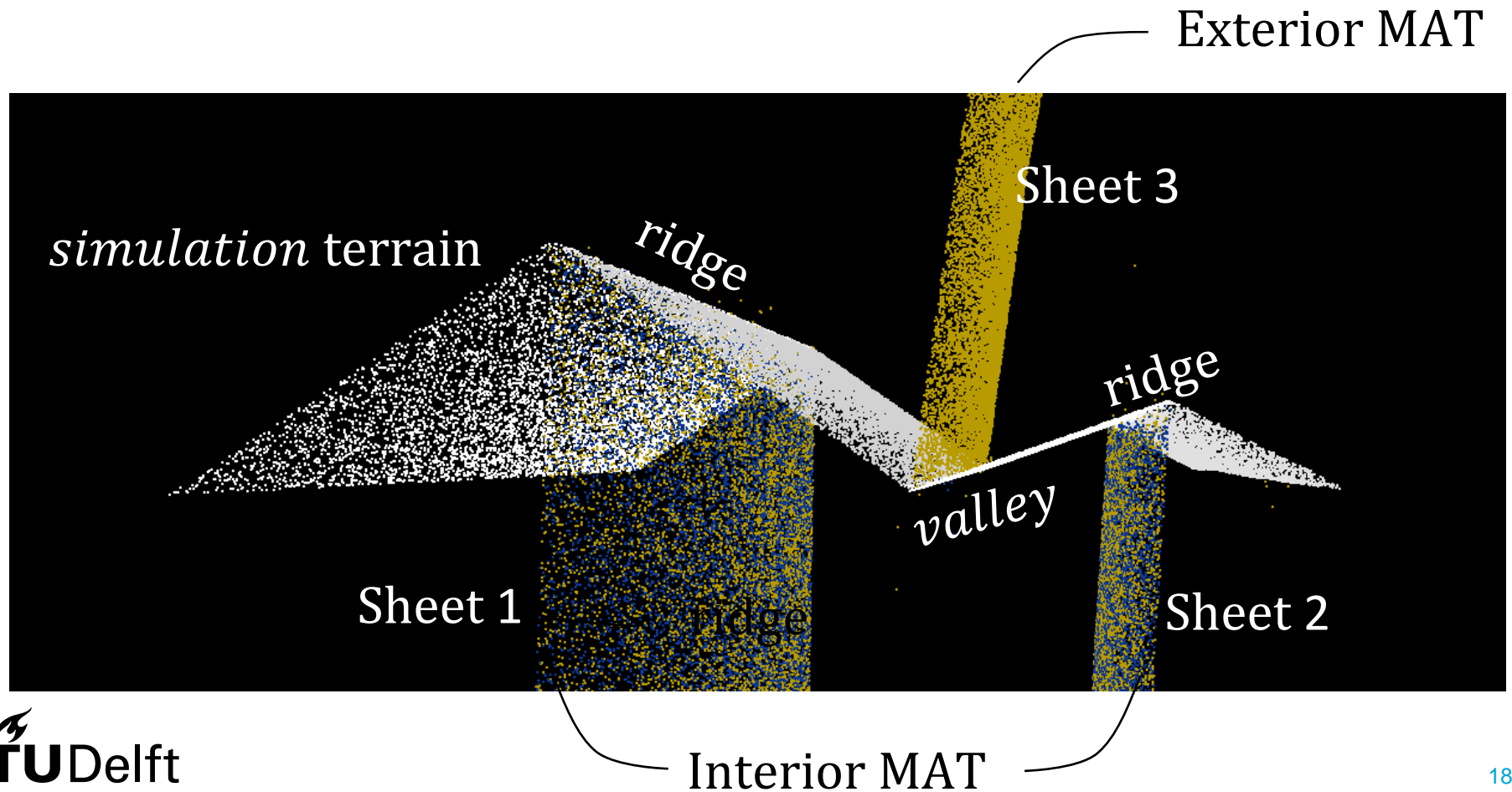
# Methodology: MAT & breakline

- Link between breaklines and MAT



# Methodology: MAT & breakline

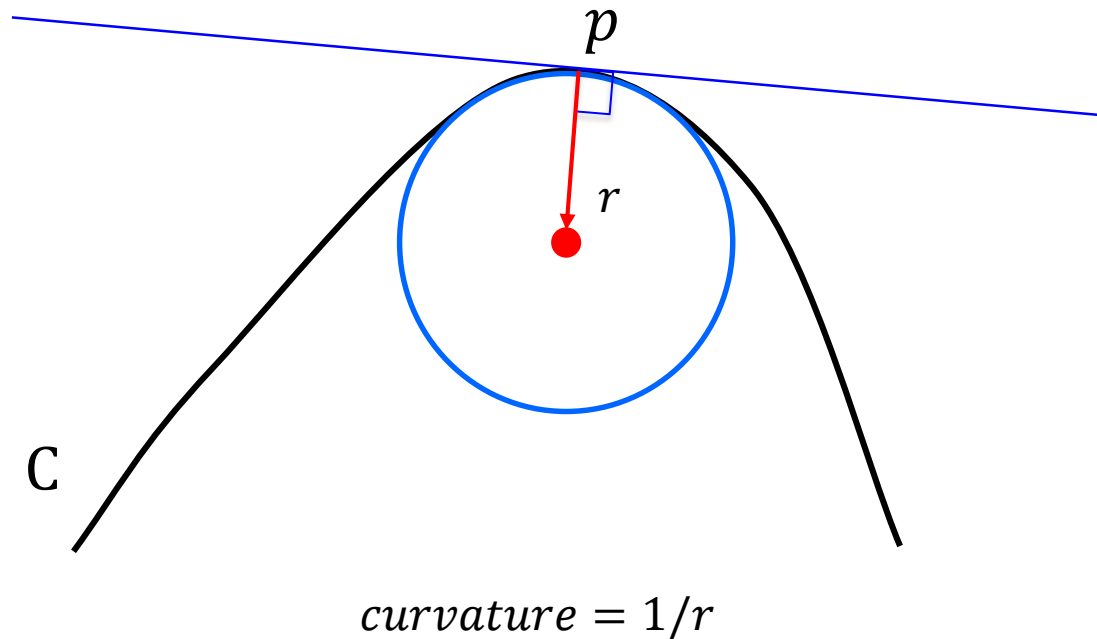
- Link between breaklines and MAT





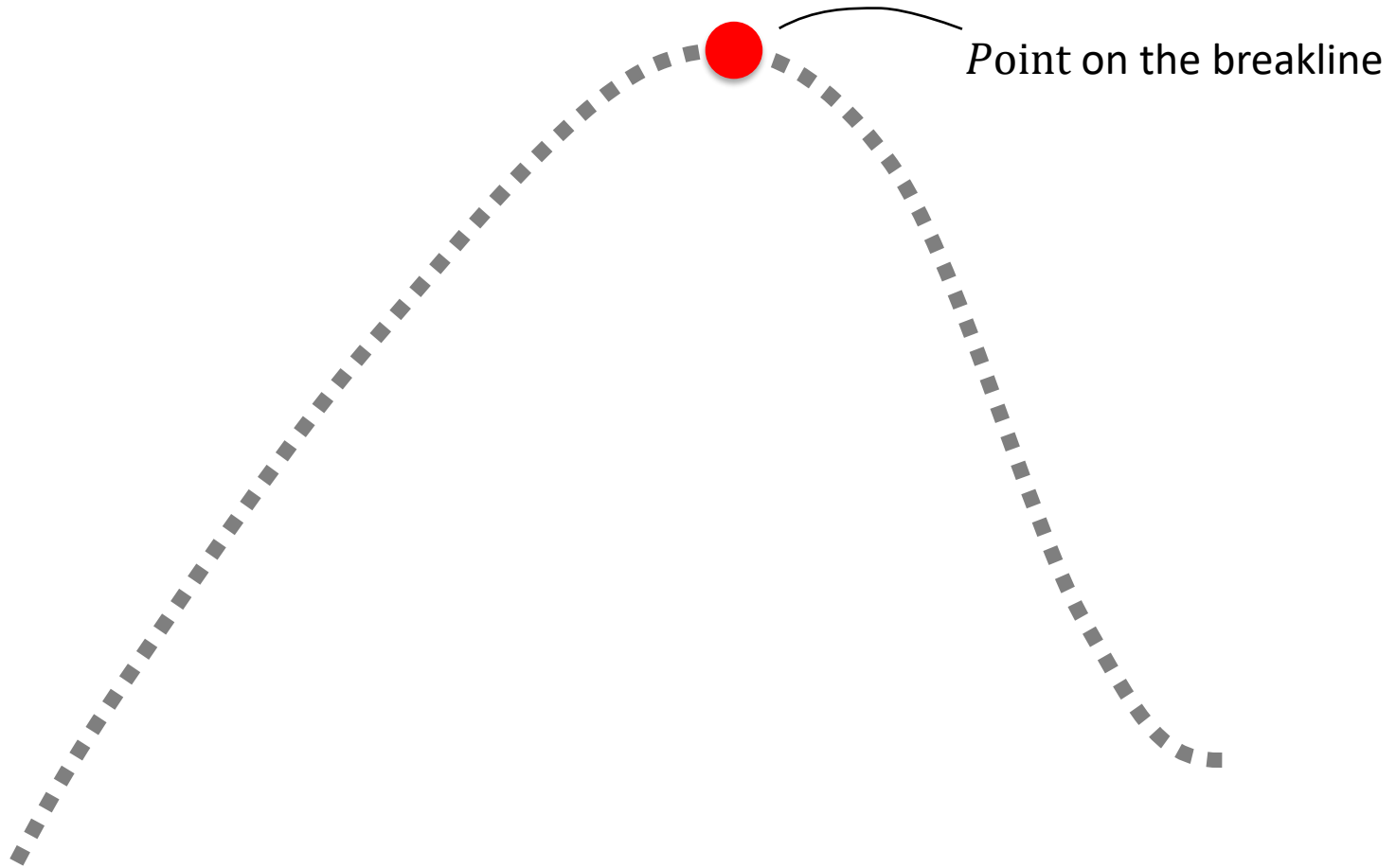
# Methodology: MAT & breakline

- Link between breaklines and MAT
  - Curvature and medial ball



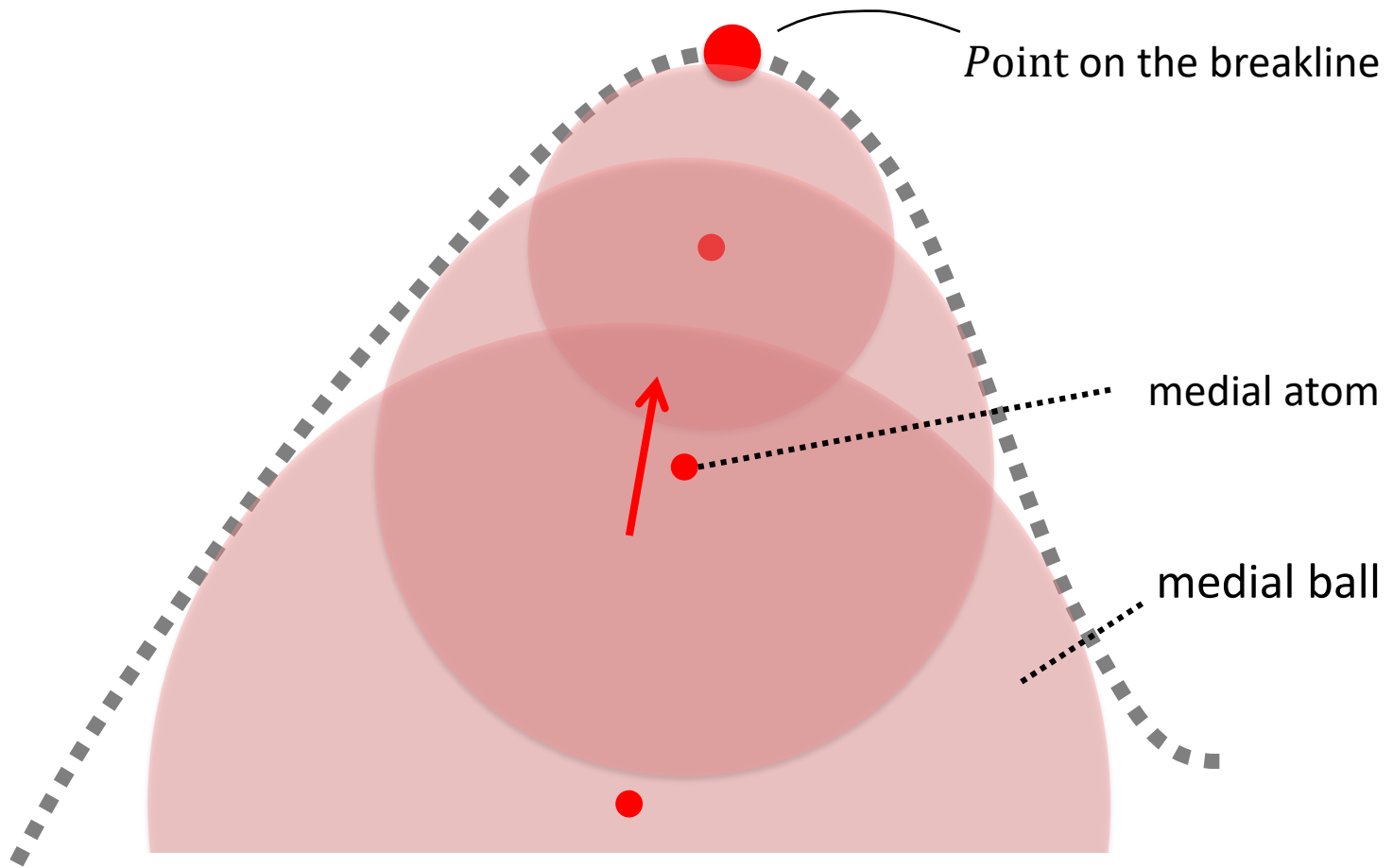
# Methodology: MAT & breakline

- Link between breaklines and MAT



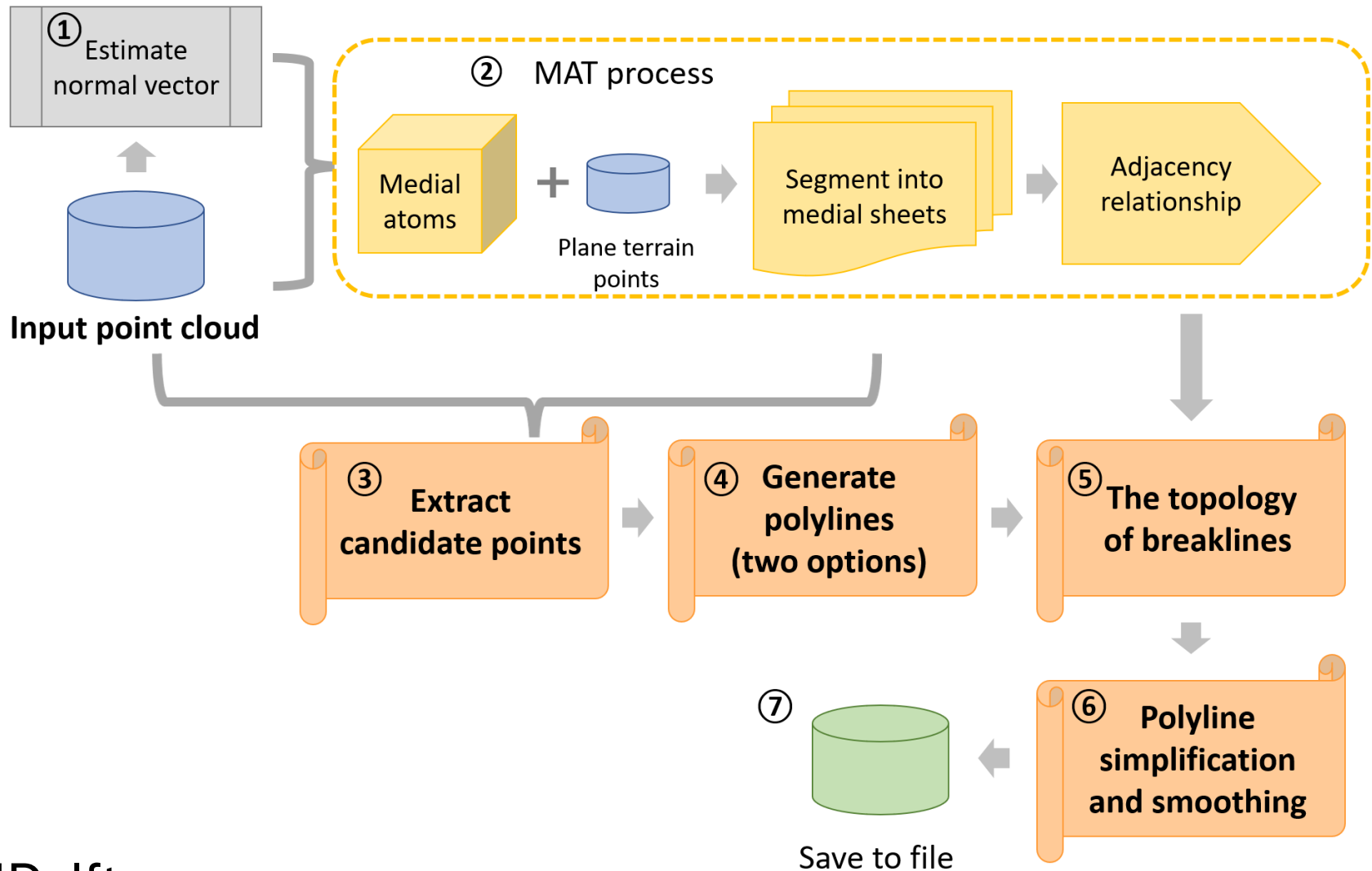
# Methodology: MAT & breakline

- Link between breaklines and MAT

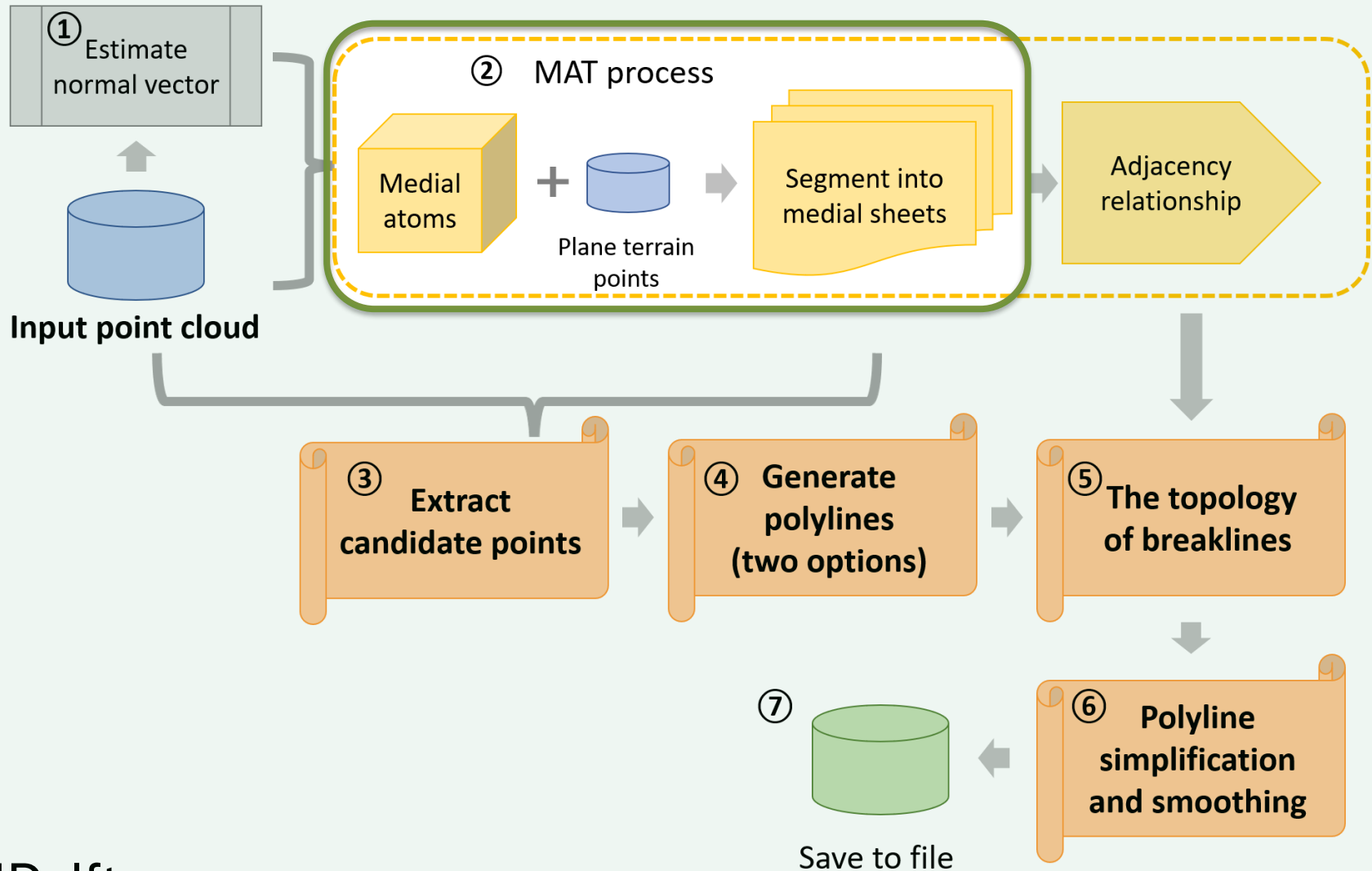




# Methodology overview

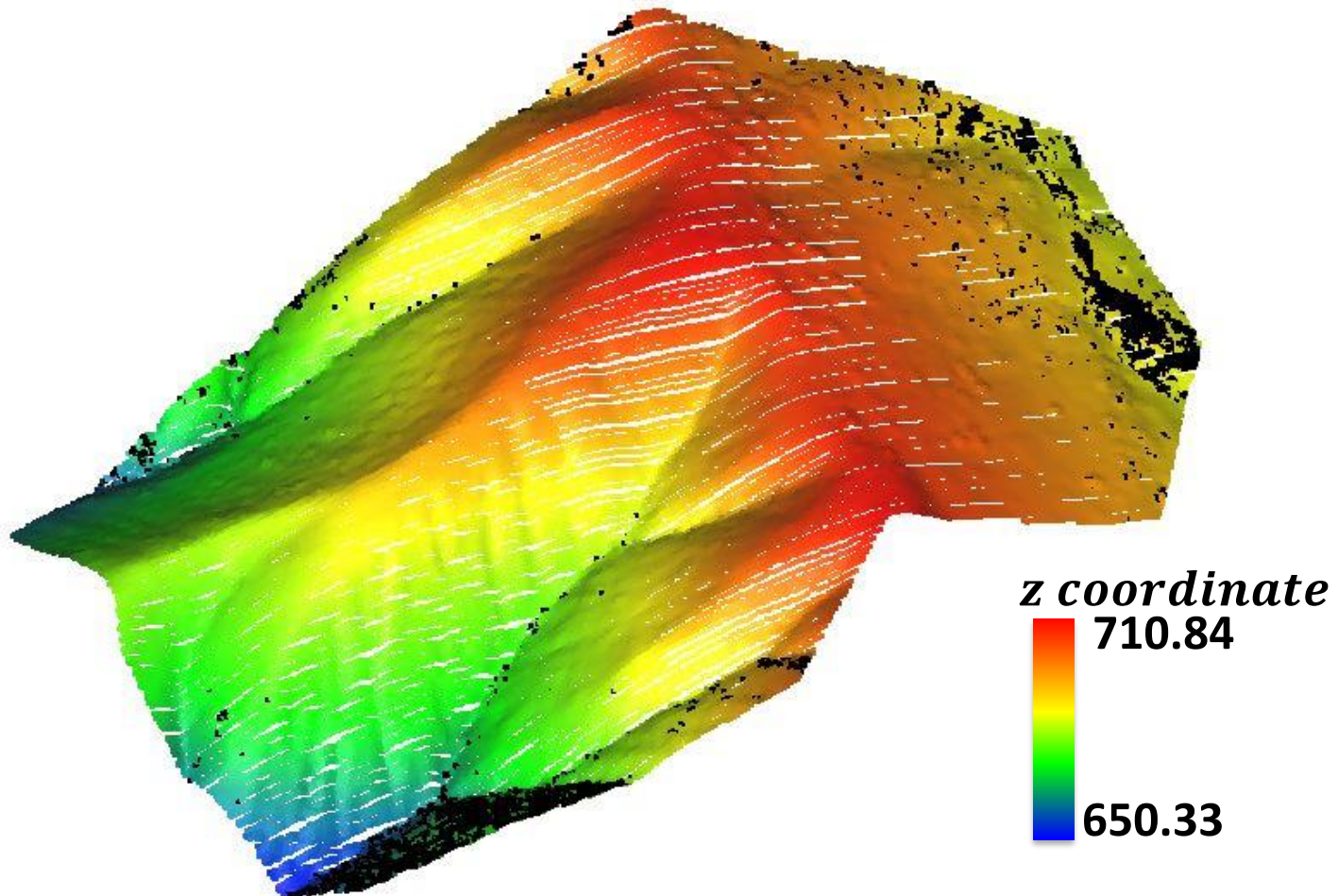


# Methodology overview



# Methodology: MAT process

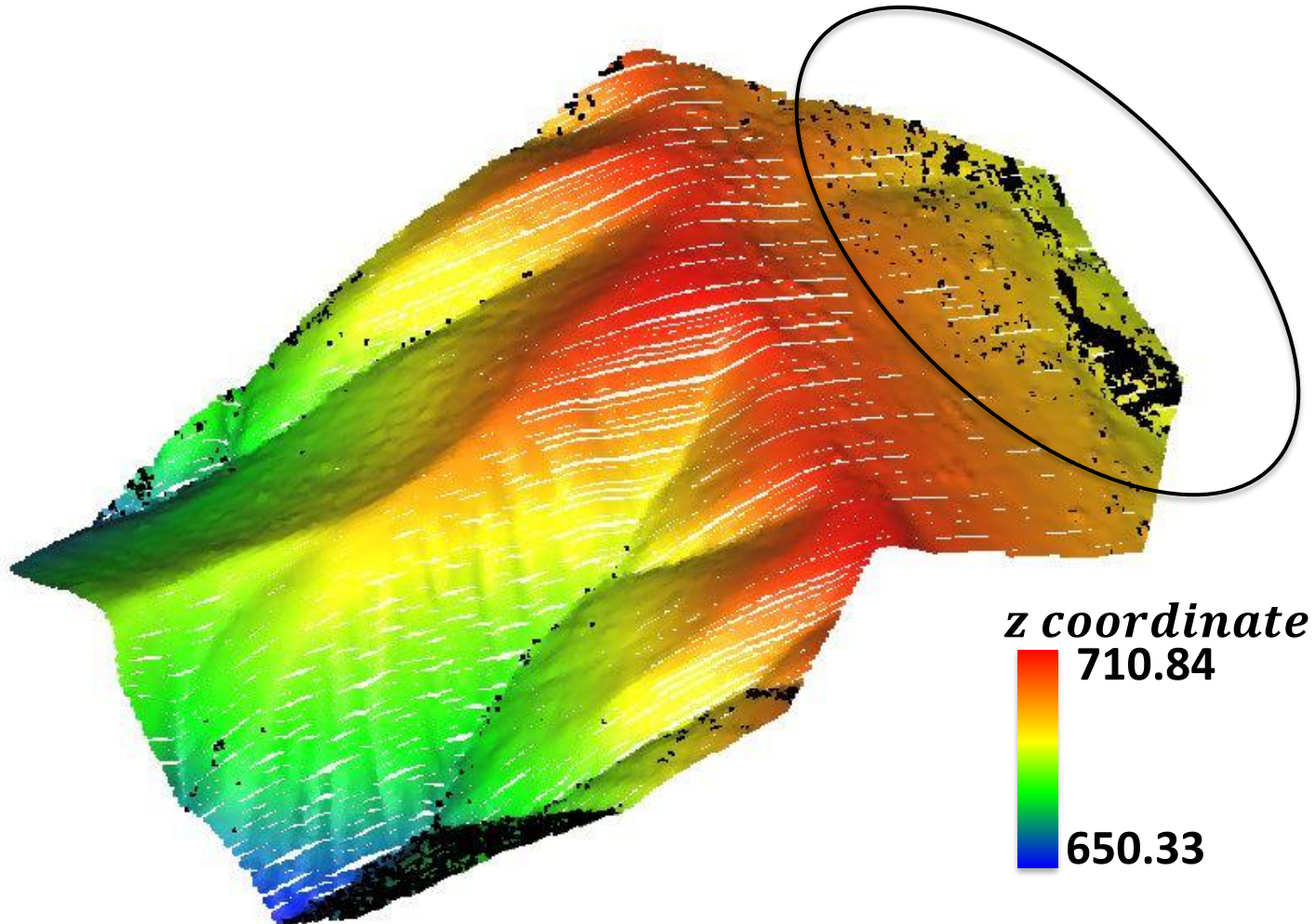
- Unshrinken points (black) → planar area





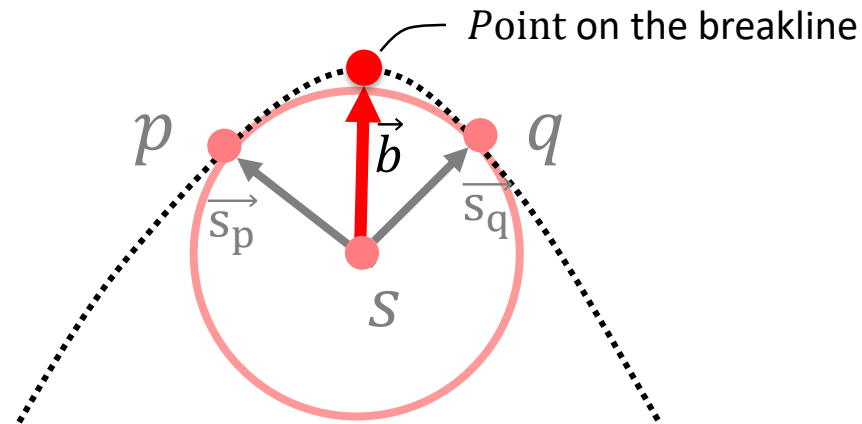
# Methodology: MAT process

- Unshrinken points (black) → planar area



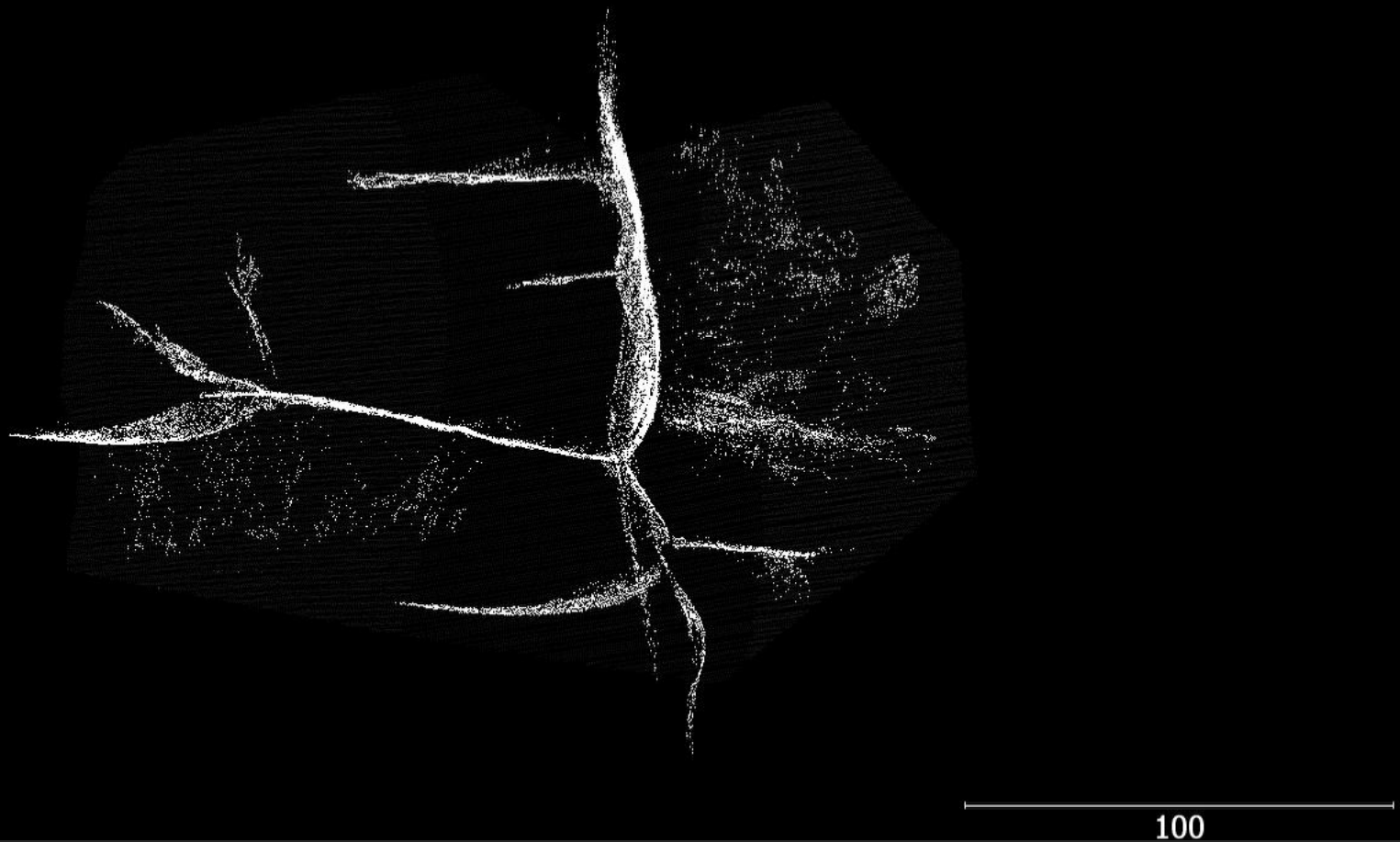
# Methodology: MAT process

- Medial segmentation  $\rightarrow$  medial sheet
- Medial geometry: bisector  $\vec{b}$



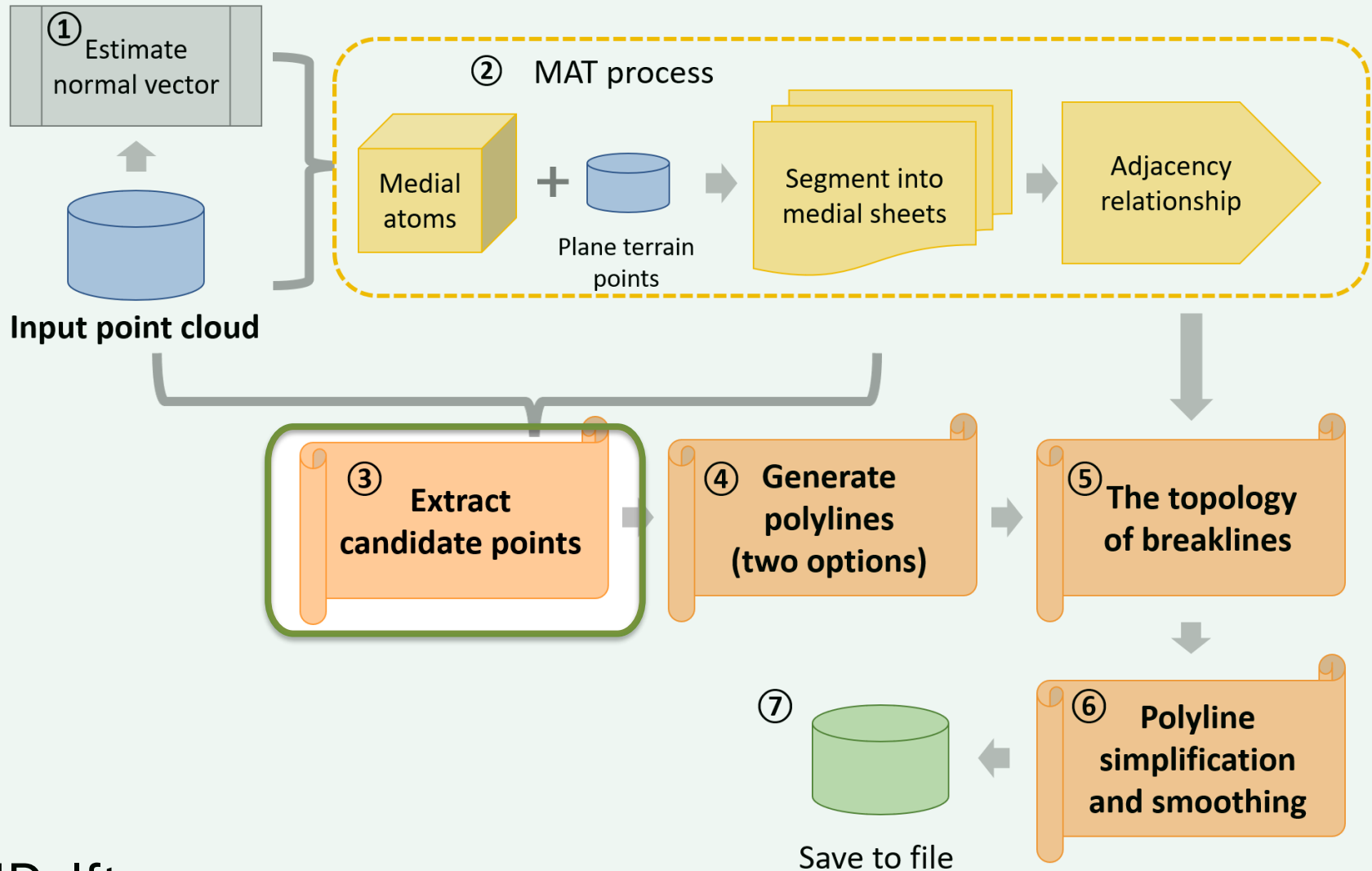
# MAT segmentation

- Medial segmentation → medial sheets



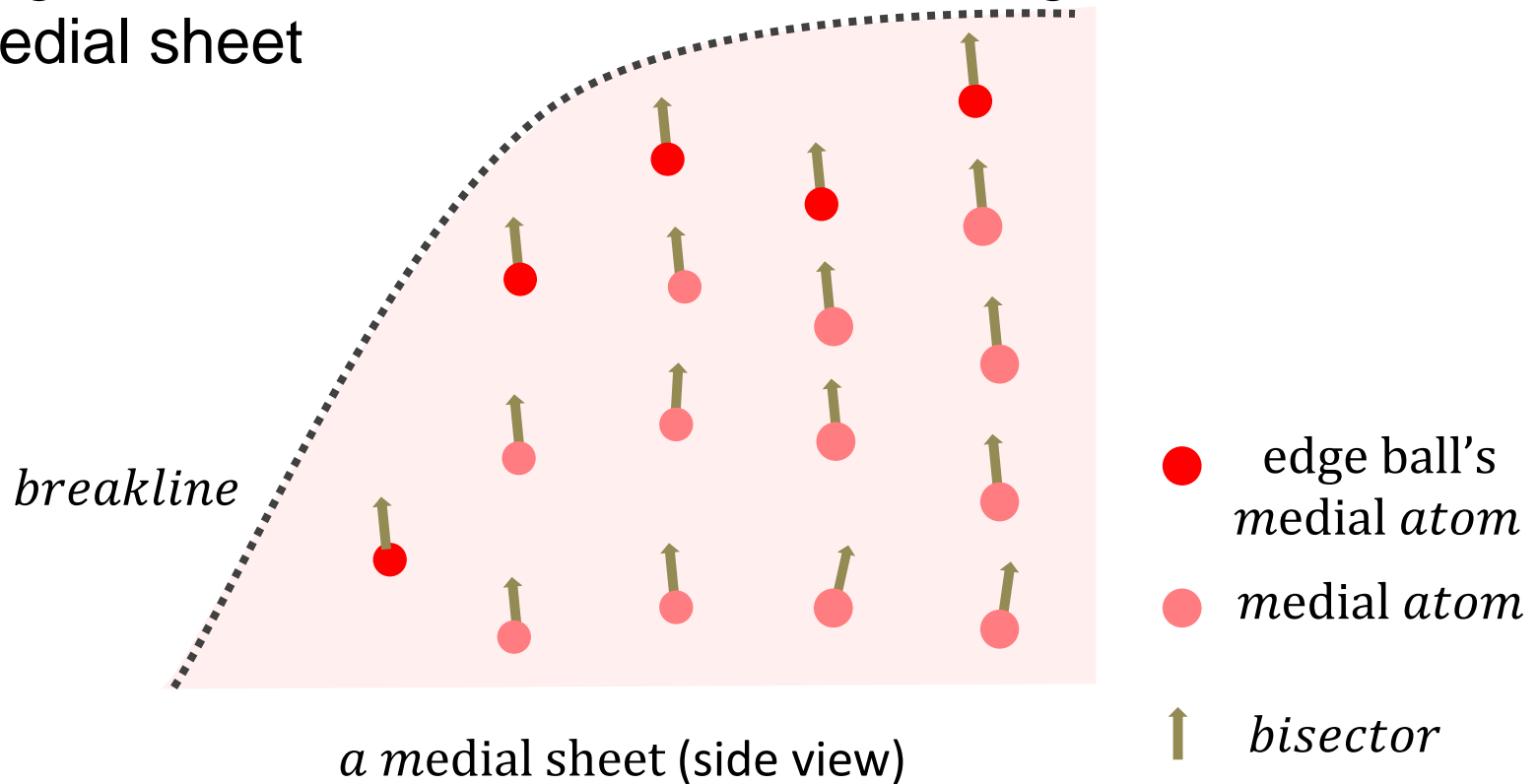


# Methodology overview



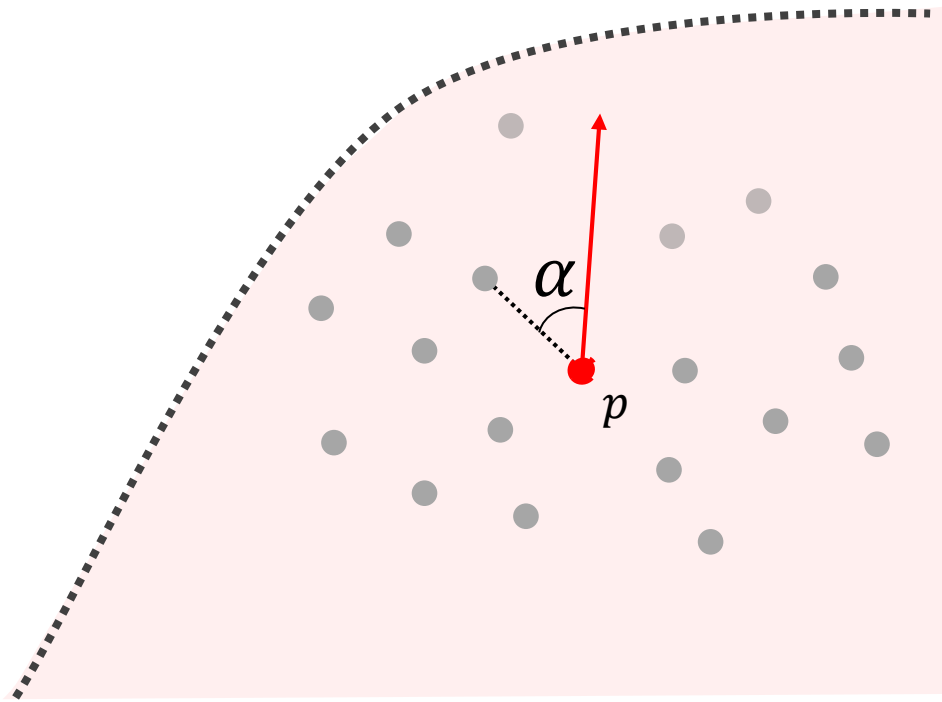
# Methodology: extracting candidate points

- Detecting edge balls
  - Edge balls: medial balls close to the edge of the medial sheet



# Methodology: extracting candidate points

- Detecting edge balls

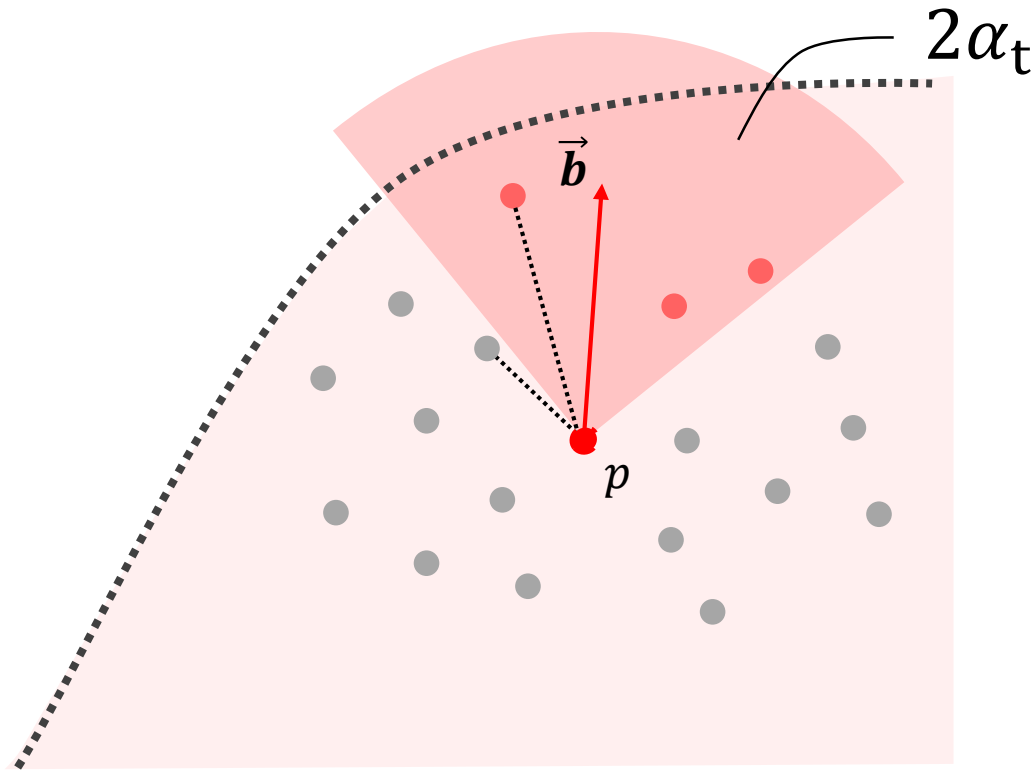


Calculate  $\alpha$  for each neighbor



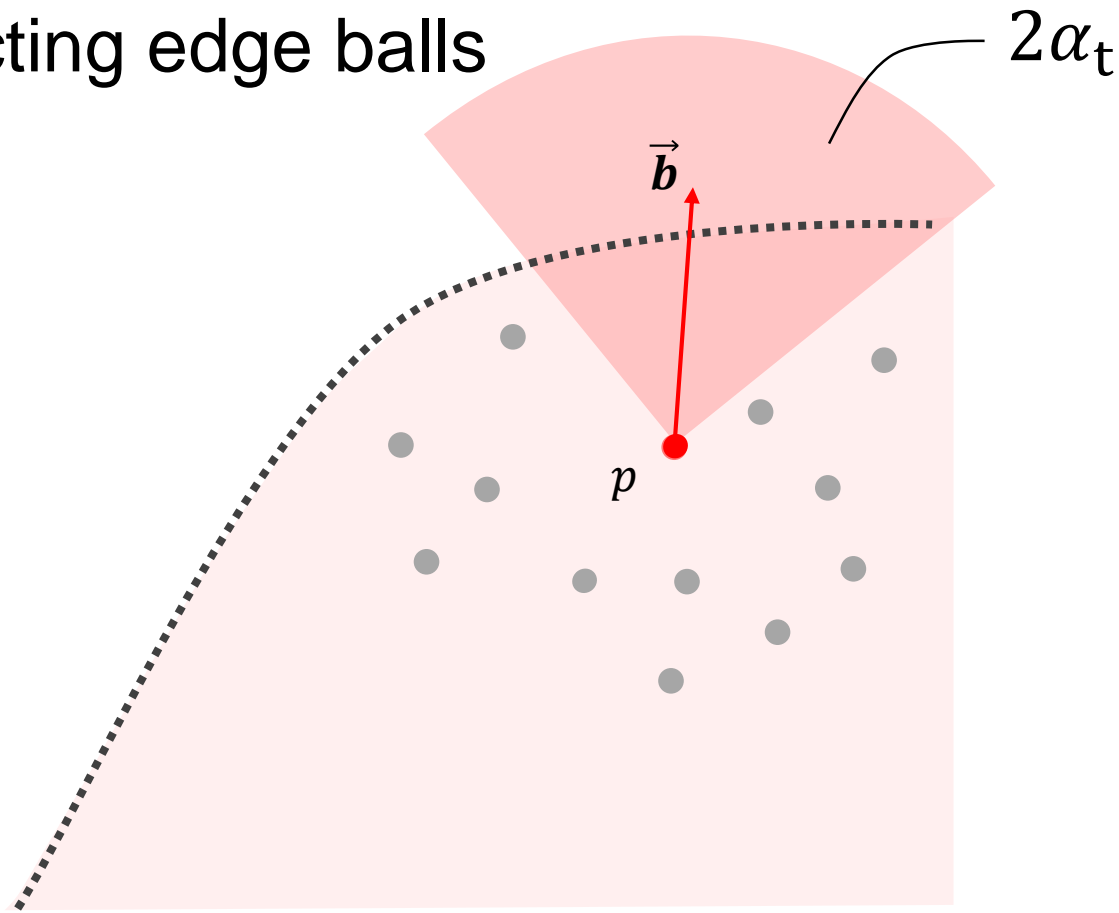
# Methodology: extracting candidate points

- Detecting edge balls



# Methodology: extracting candidate points

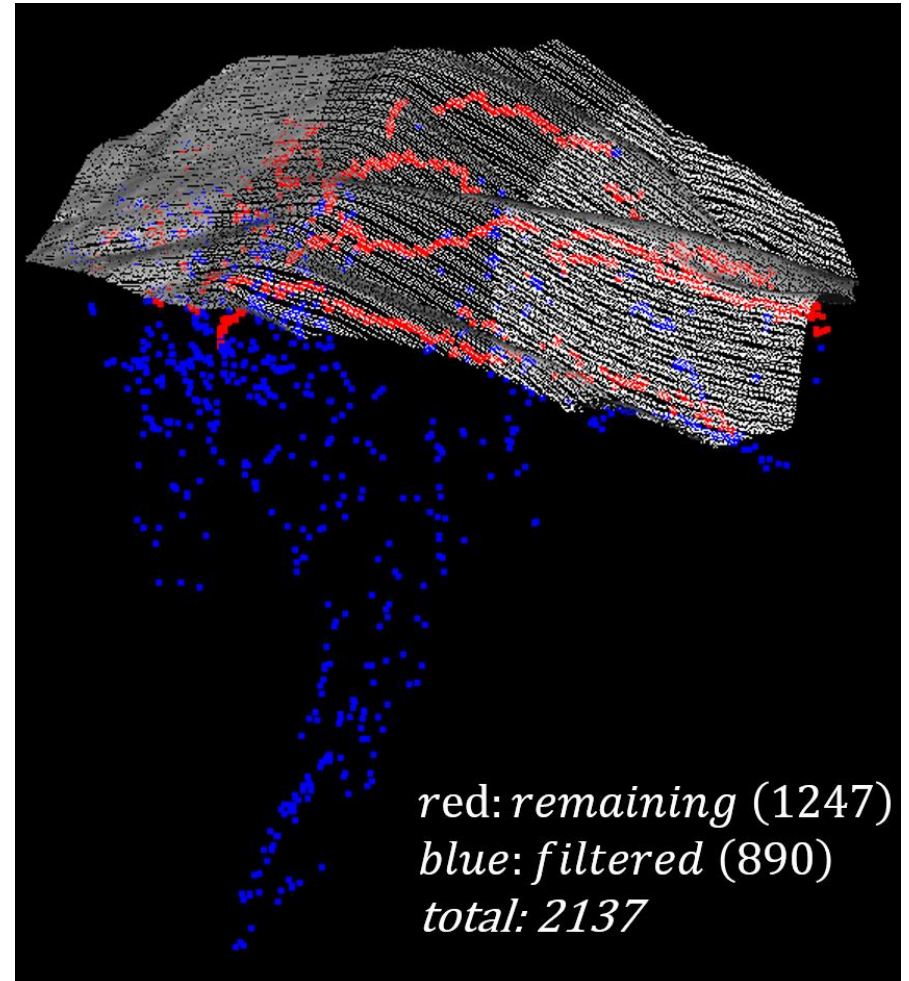
- Detecting edge balls



$p$  is an edge ball

# Methodology: extracting candidate points

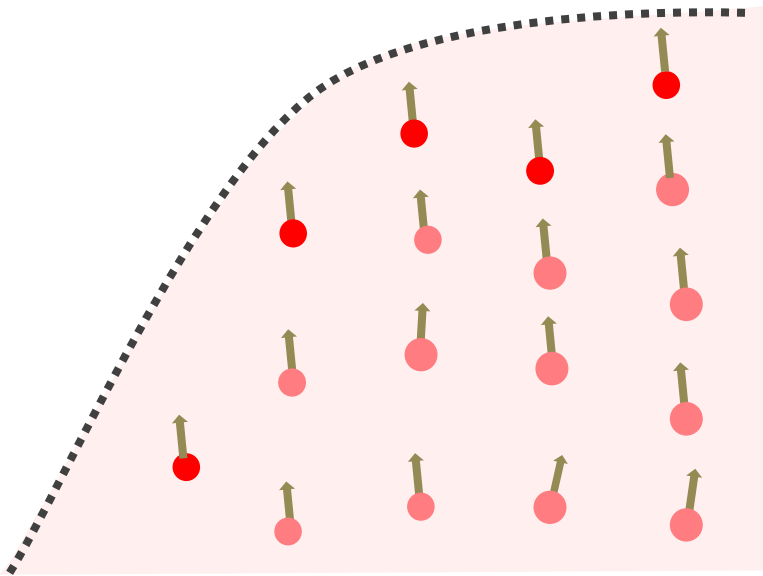
- Filtering edge balls
  - Radius ( $r_{min}, r_{max}$ )  
→ curvature  
( $1/r_{max}, 1/r_{min}$ )
  - Distance to the point cloud (3D)
  - Distance to the planar area (2D)





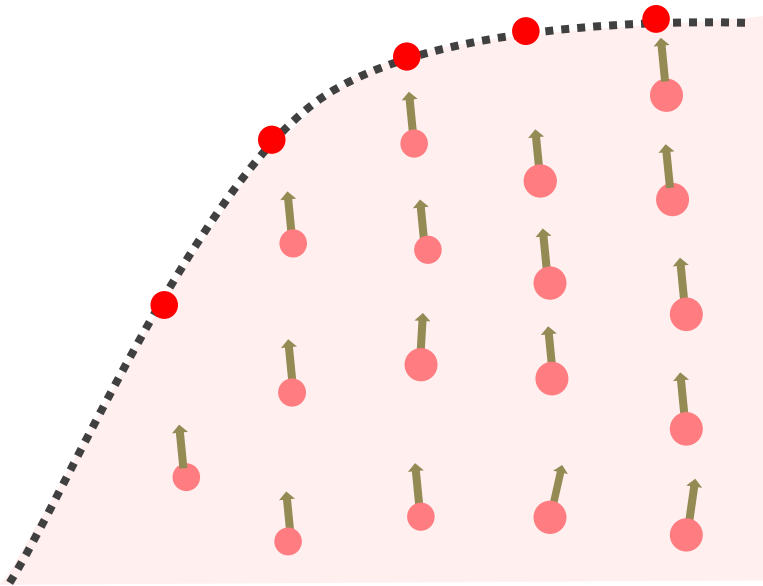
# Methodology: extracting candidate points

- Remaining edge ball  $\rightarrow$  candidate point



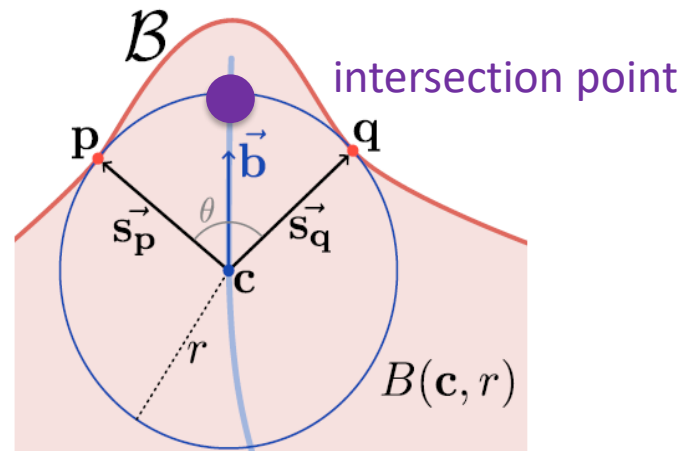
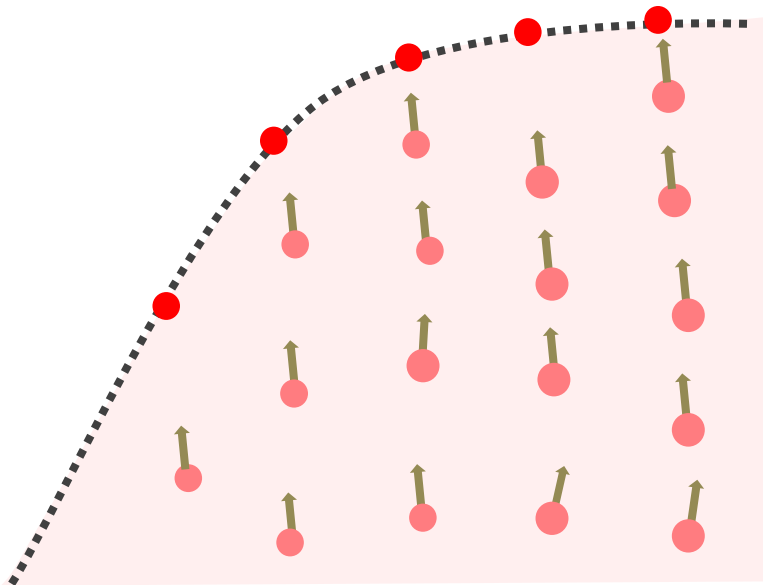
# Methodology: extracting candidate points

- Remaining edge ball  $\rightarrow$  candidate point



# Methodology: extracting candidate points

- Remaining edge ball  $\rightarrow$  candidate point

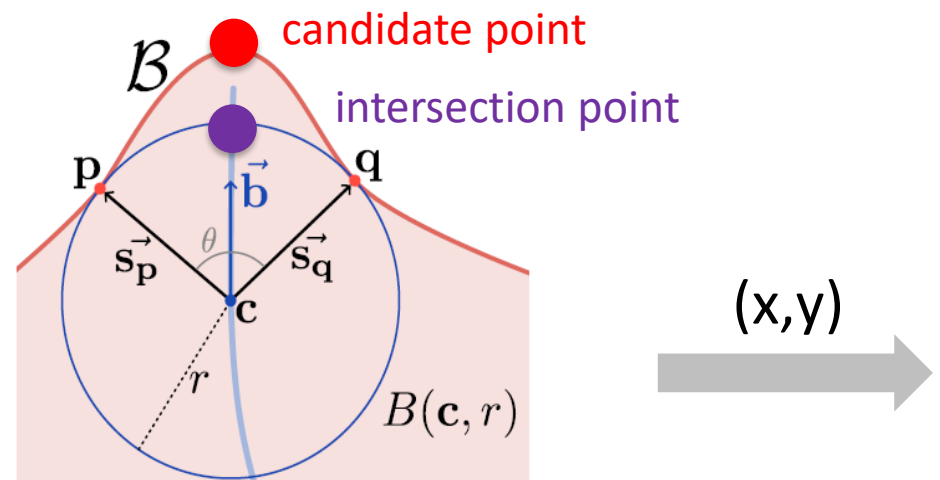


Intersection of bisector  
and the medial ball

$(x, y, z)$

# Methodology: extracting candidate points

- Remaining edge ball  $\rightarrow$  candidate point



Intersection of bisector  
and the medial ball

$(x, y, z, y, z)$

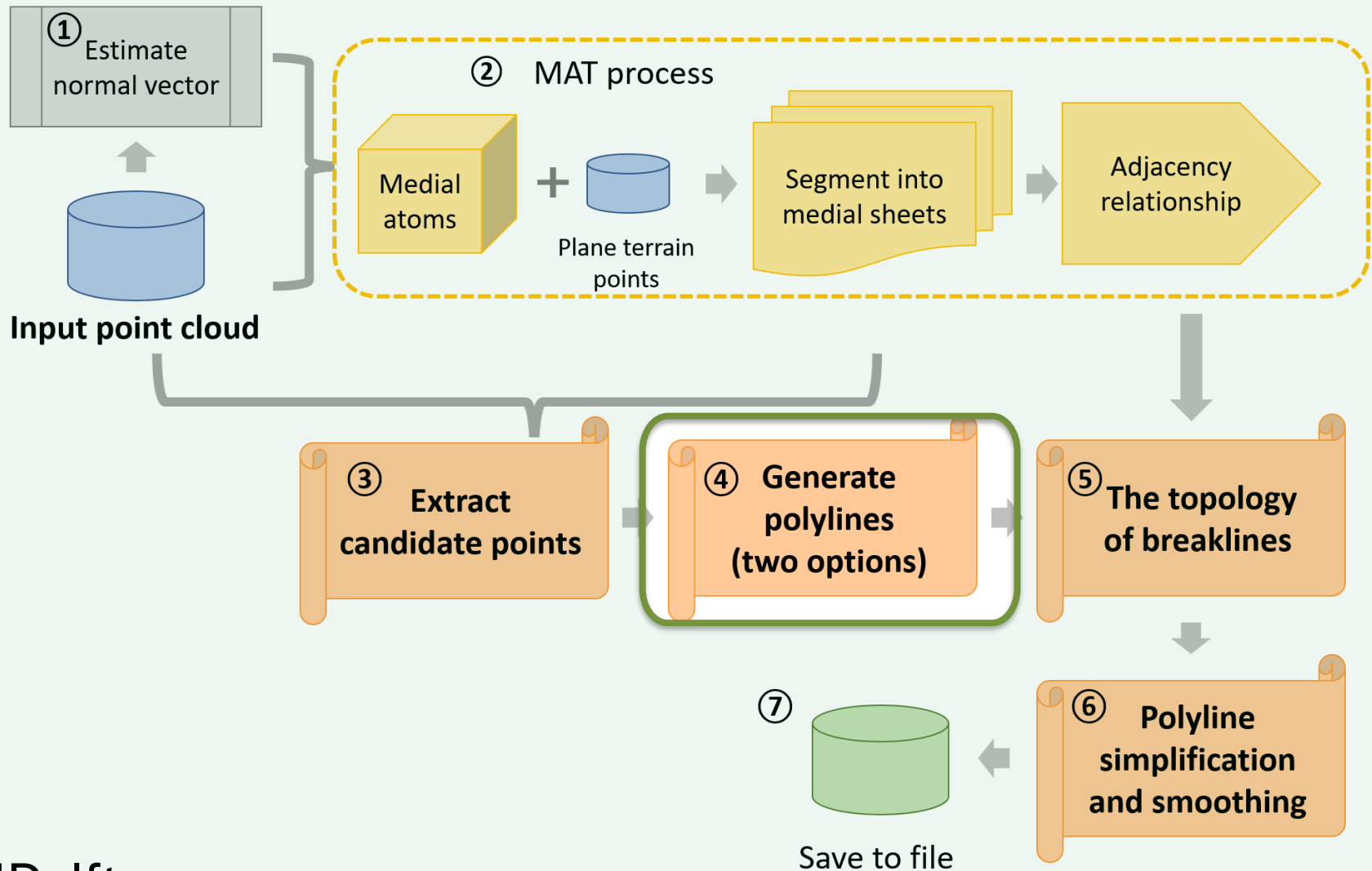


# Methodology: extracting candidate points



(red: ridge points;  
Green: valley points)

# Methodology overview



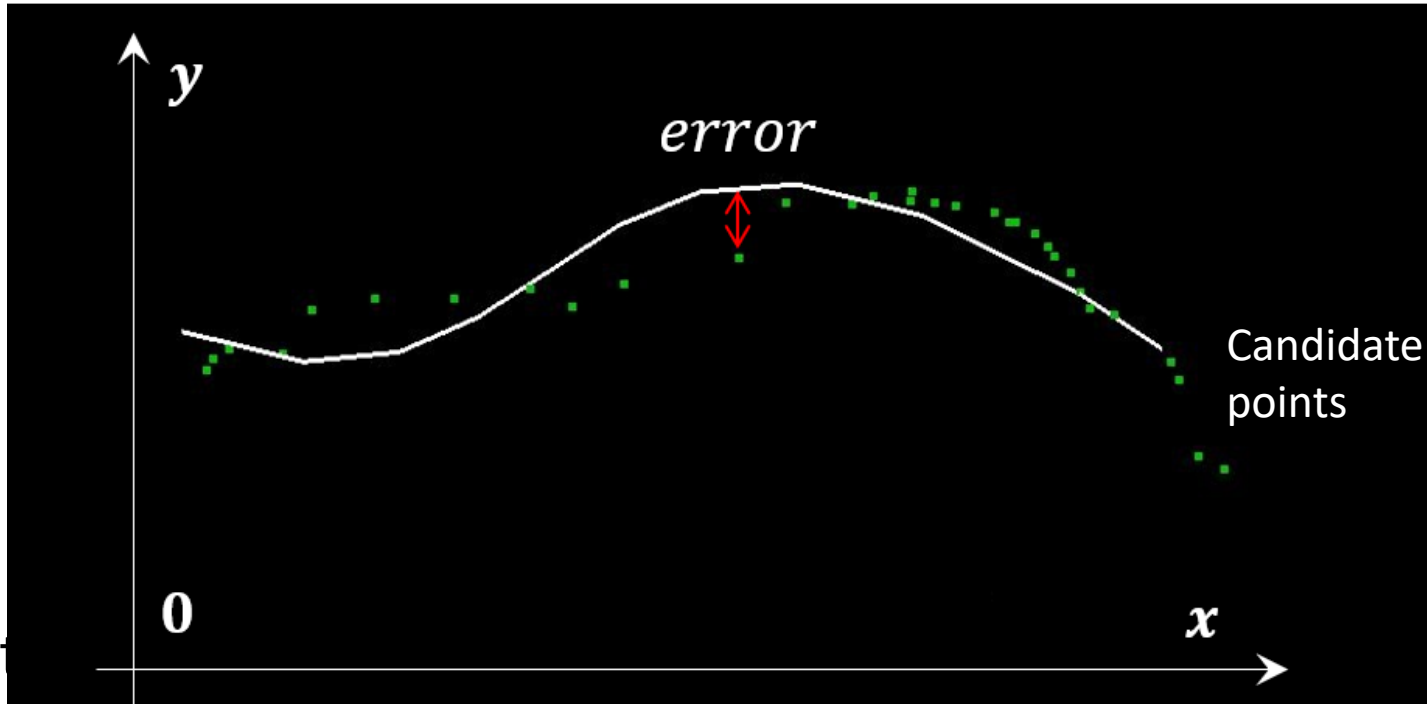
Option 1 : generating polylines  
using the polynomial fitting

Option 2 : generating polylines  
using the graph theory

# Methodology: generating polylines using the polynomial fitting

- For each medial sheet:
  - Fitting a cubic polynomial function with the candidate points

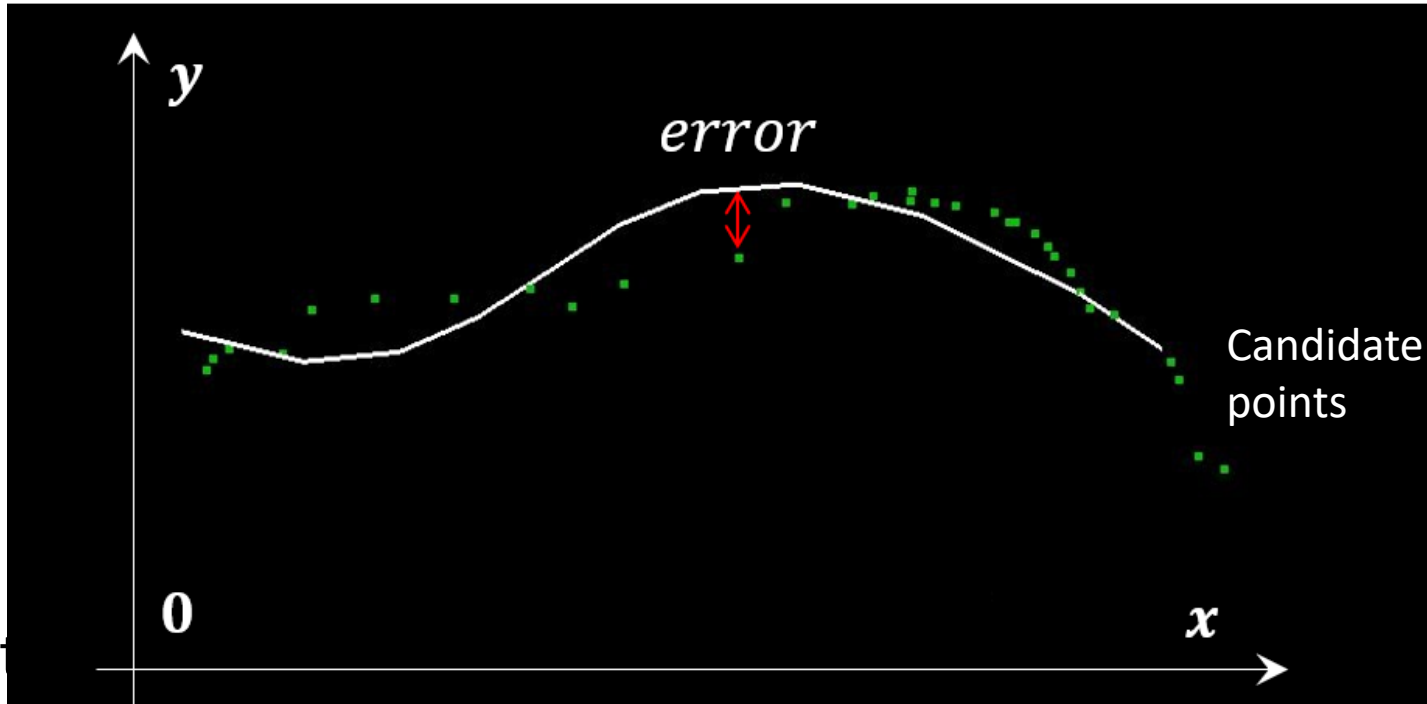
$$y = f(x) = t_3 \cdot x^3 + t_2 \cdot x^2 + t_1 \cdot x + t_0$$



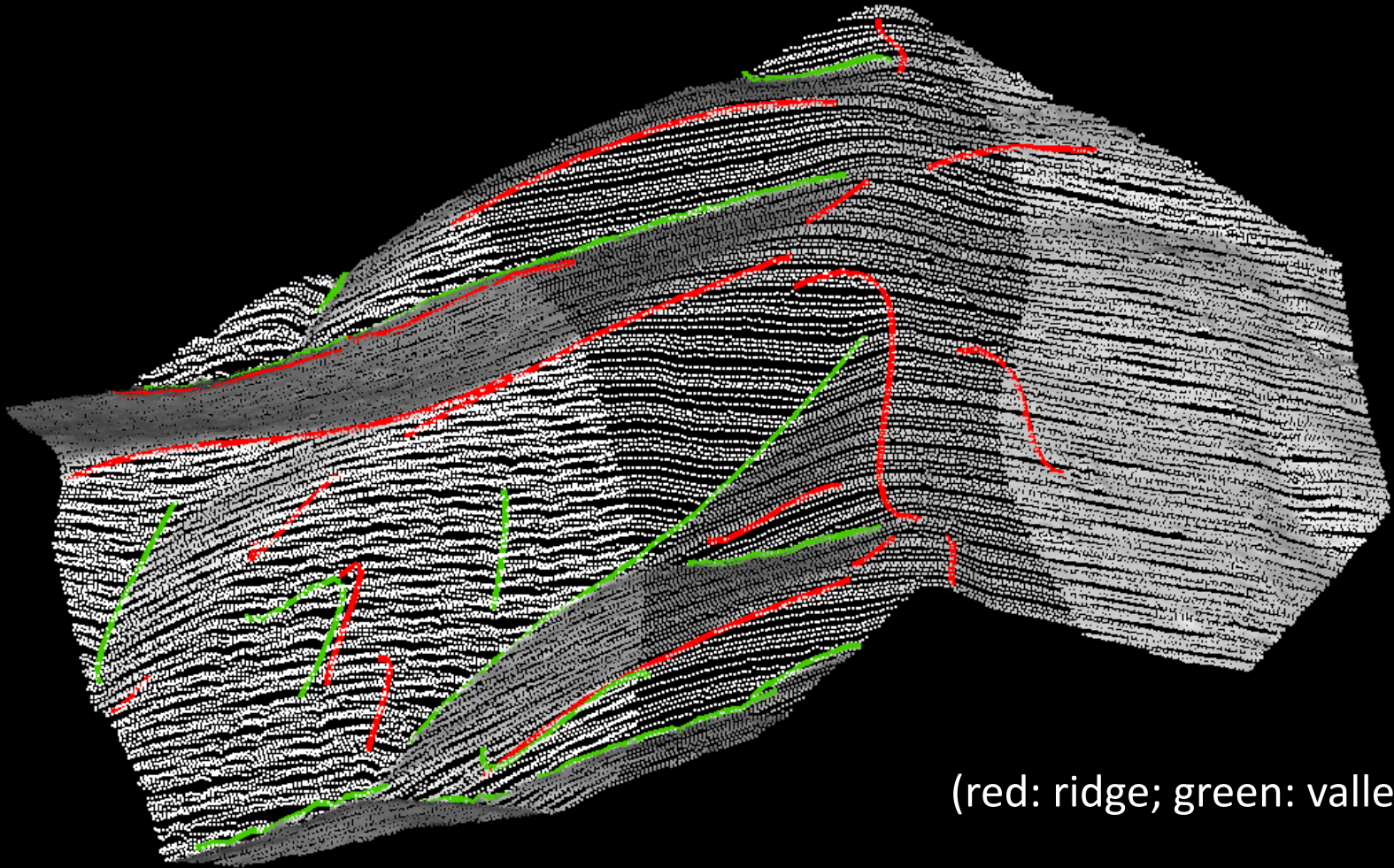


# Methodology: generating polylines using the polynomial fitting

- For each medial sheet:
  - Fitting a cubic polynomial function with the candidate points
  - Eliminating unexpected breaklines by RMSR



# Methodology: generating polylines using the polynomial fitting



(red: ridge; green: valley)

# Methodology: generating polylines using the graph theory

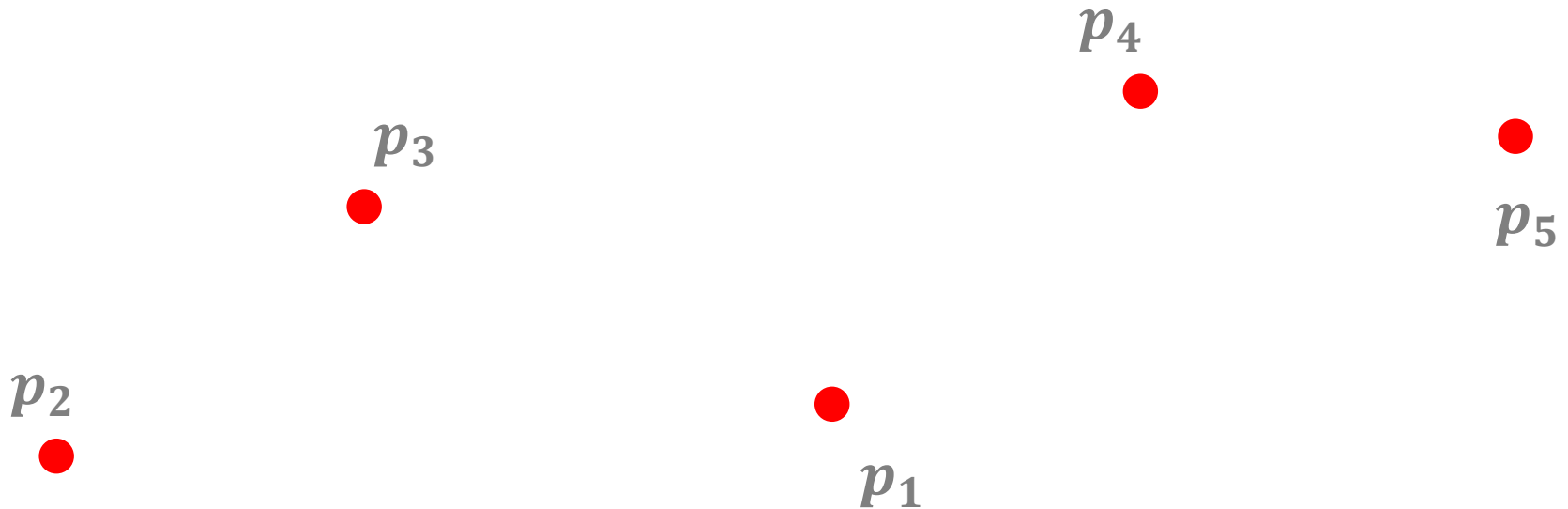
# Methodology: generating polylines using the graph theory

- For each medial sheet:
  - Connecting candidate point to its closest point by Minimum Spanning tree
  - Simplify to one polyline



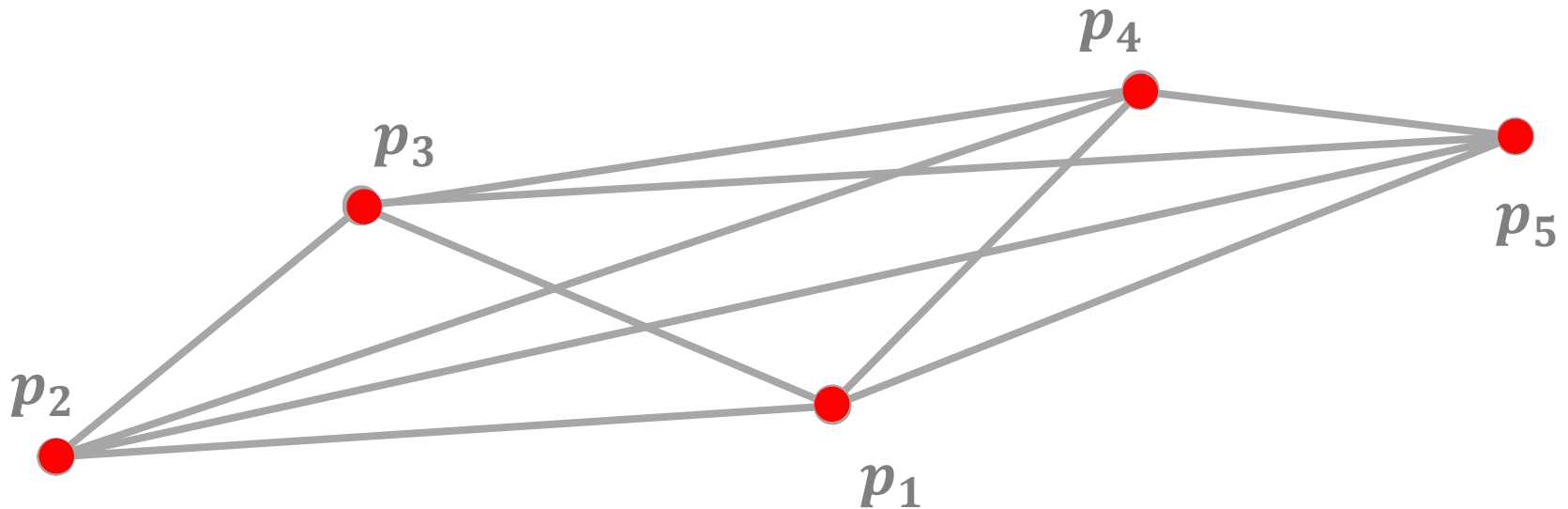
# Methodology: generating polylines using the graph theory

- Connecting candidate point to its closest point by Minimum Spanning tree



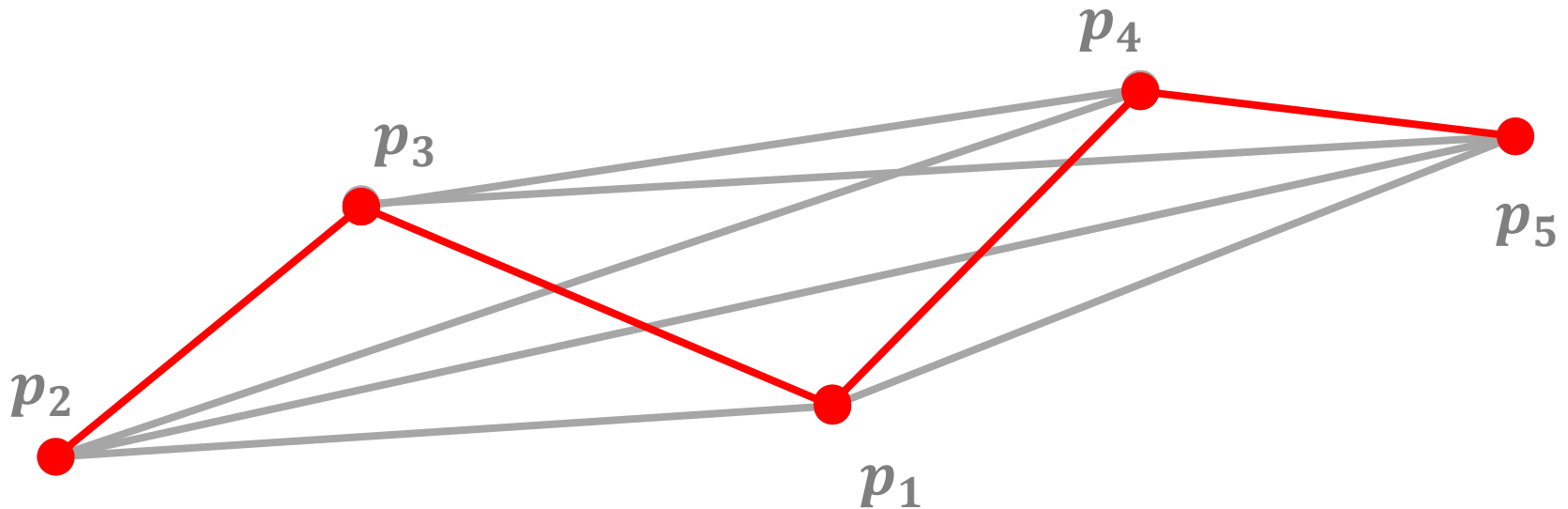
# Methodology: generating polylines using the graph theory

- Connecting candidate point to its closest point by Minimum Spanning tree



# Methodology: generating polylines using the graph theory

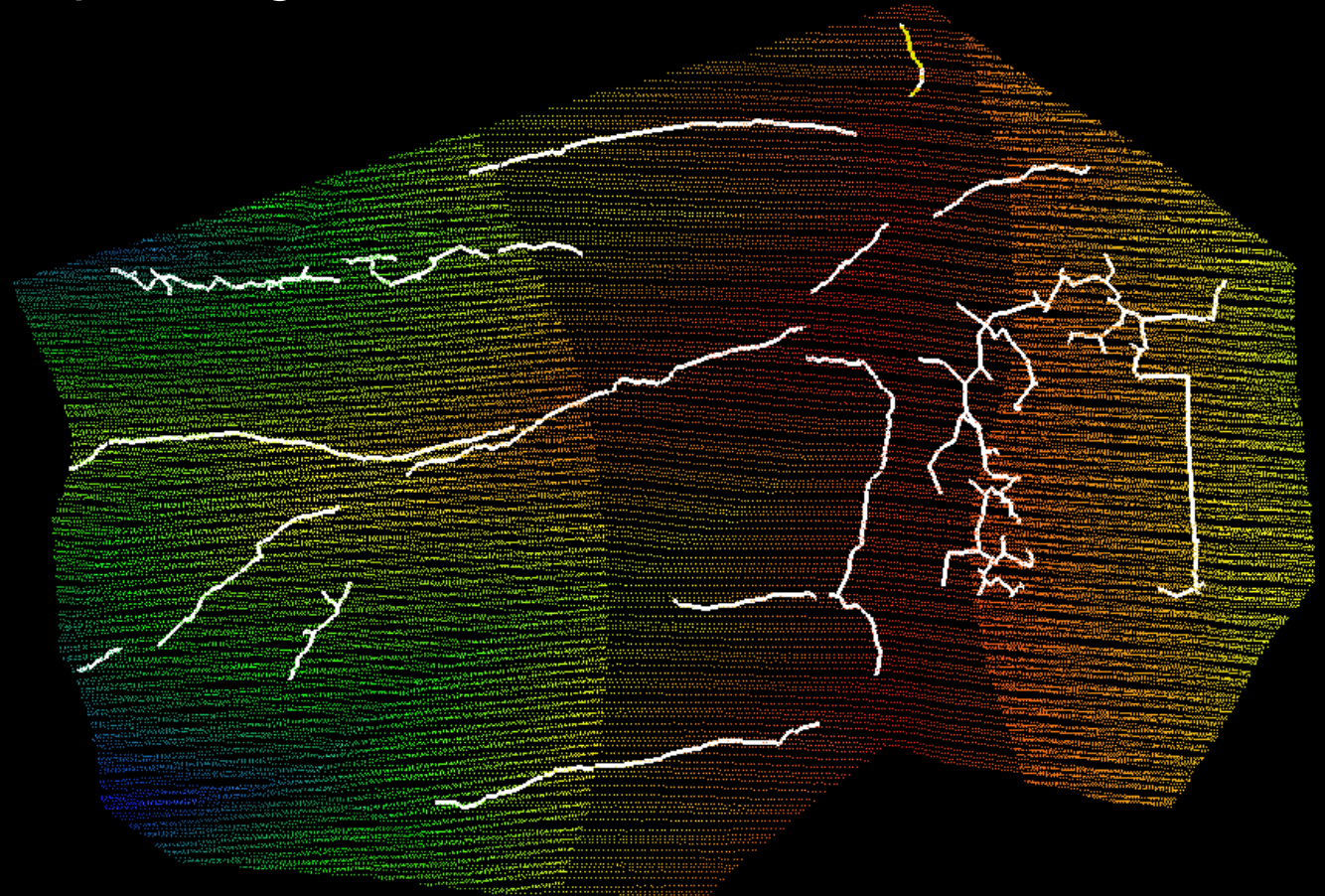
- Connecting candidate point to its closest point by Minimum Spanning tree



# Methodology: generating polylines using the graph theory

- Connecting candidate point to its closest point by Minimum Spanning tree

Example: ridge

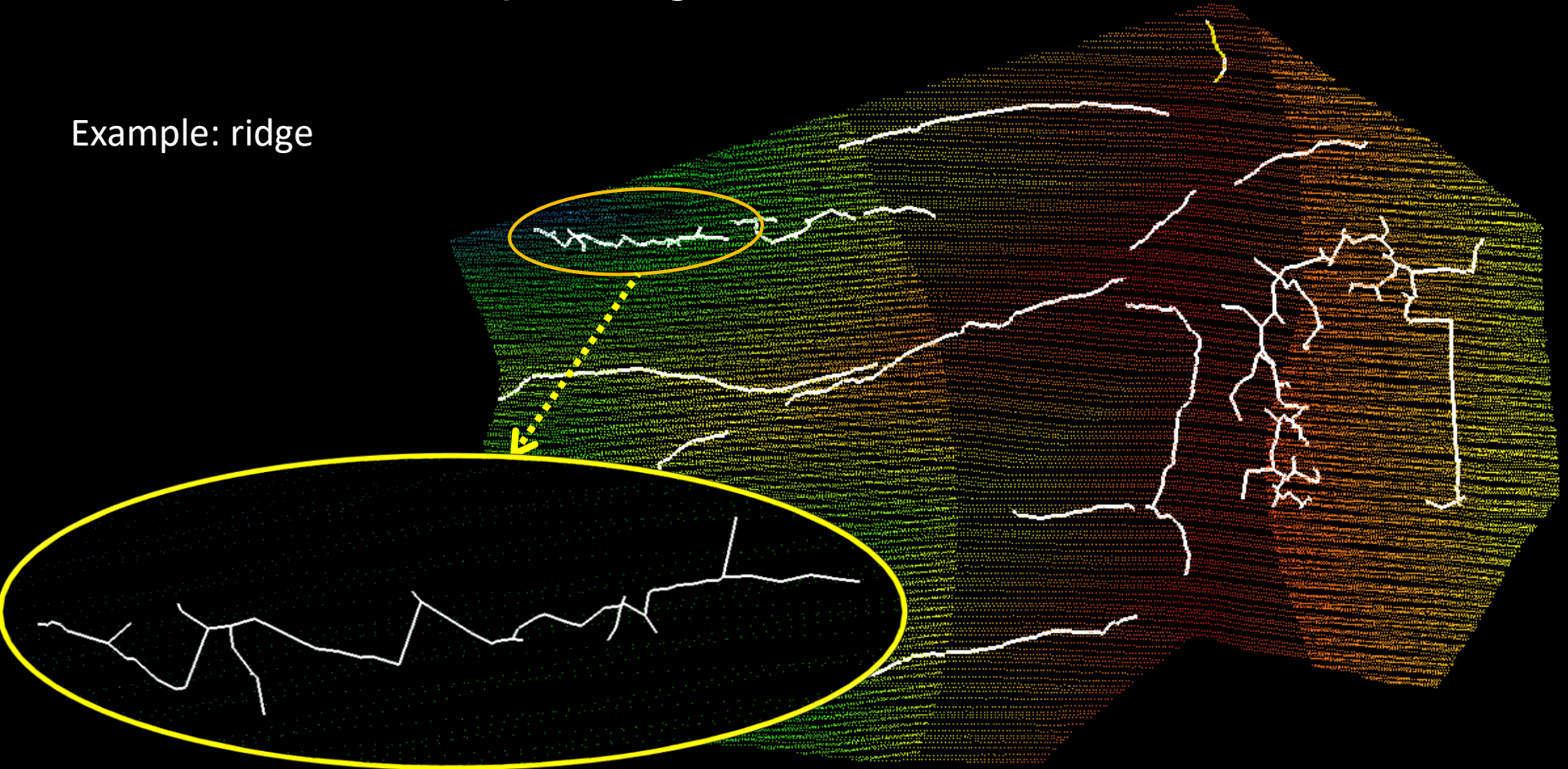




# Methodology: generating polylines using the graph theory

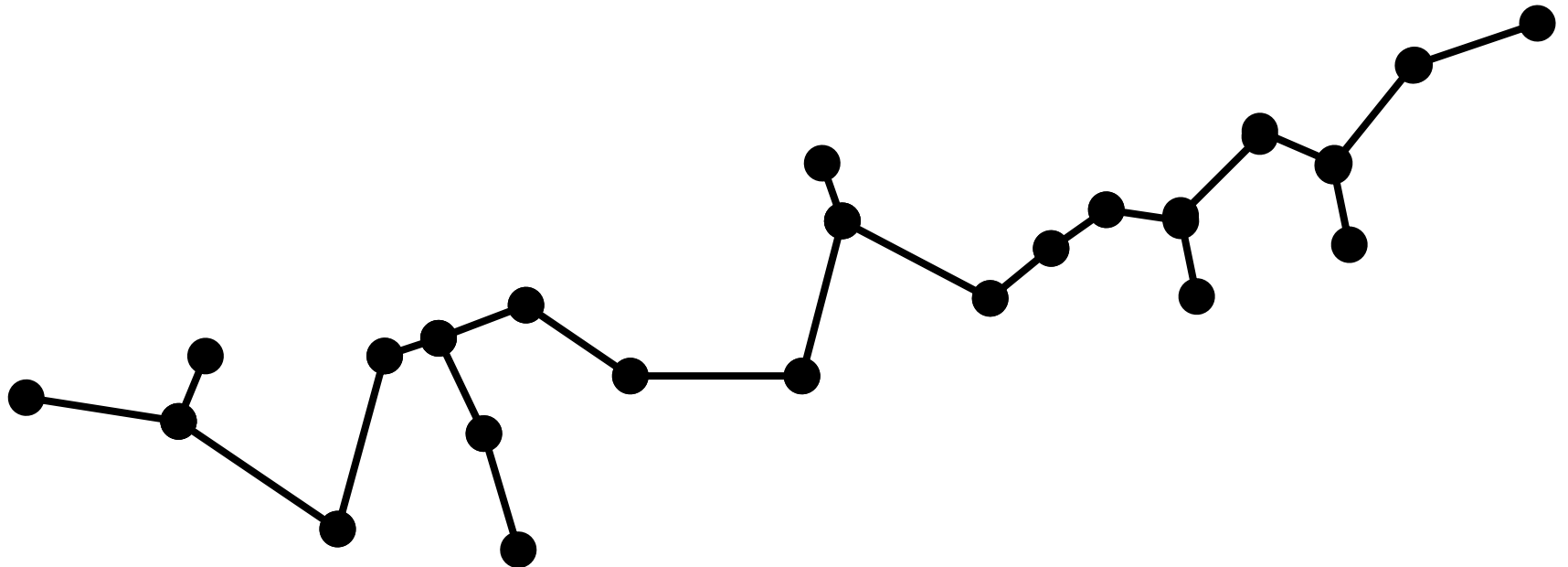
- Connecting candidate point to its closest point by Minimum Spanning tree

Example: ridge



# Methodology: generating polylines using the graph theory

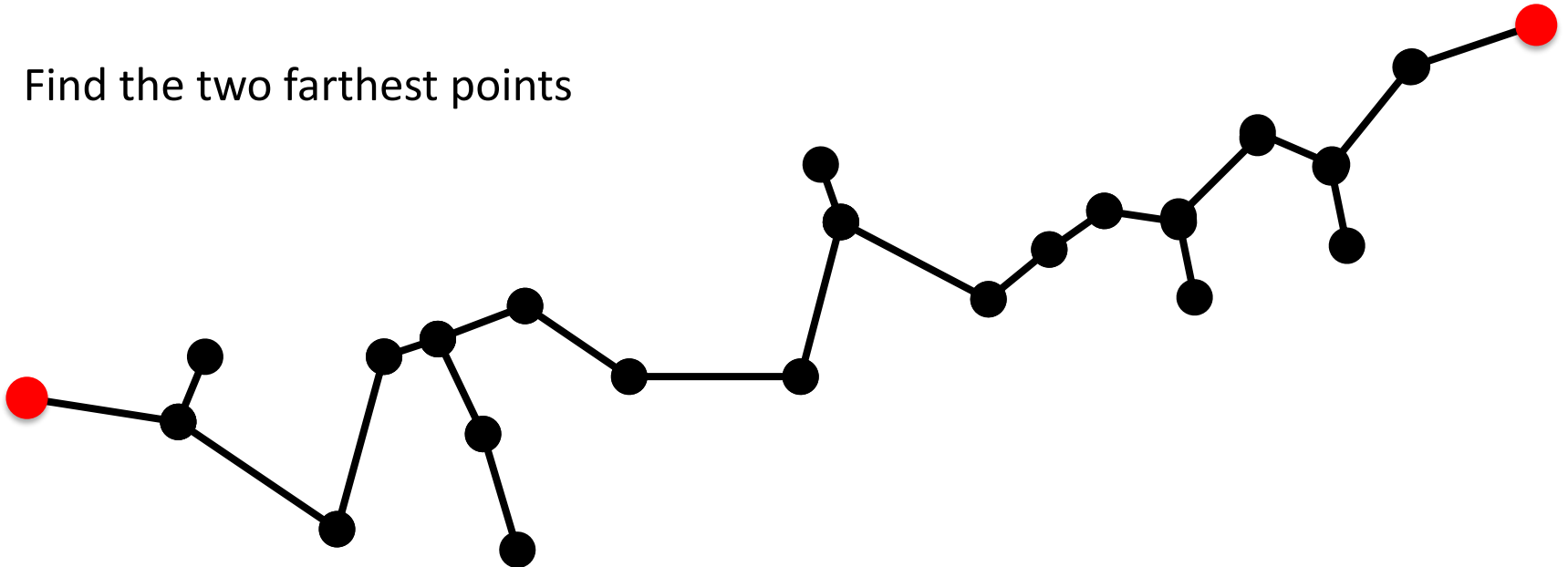
- Simplify to one polyline



# Methodology: generating polylines using the graph theory

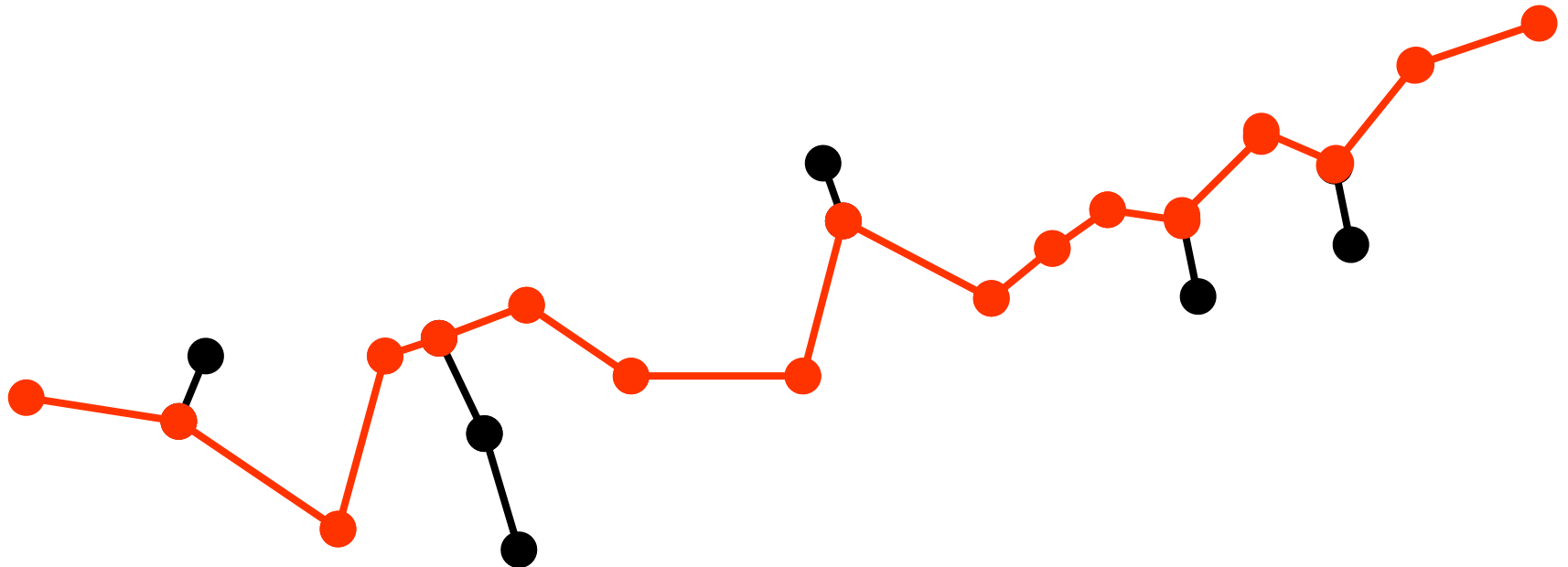
- Simplify to one polyline

Find the two farthest points



# Methodology: generating polylines using the graph theory

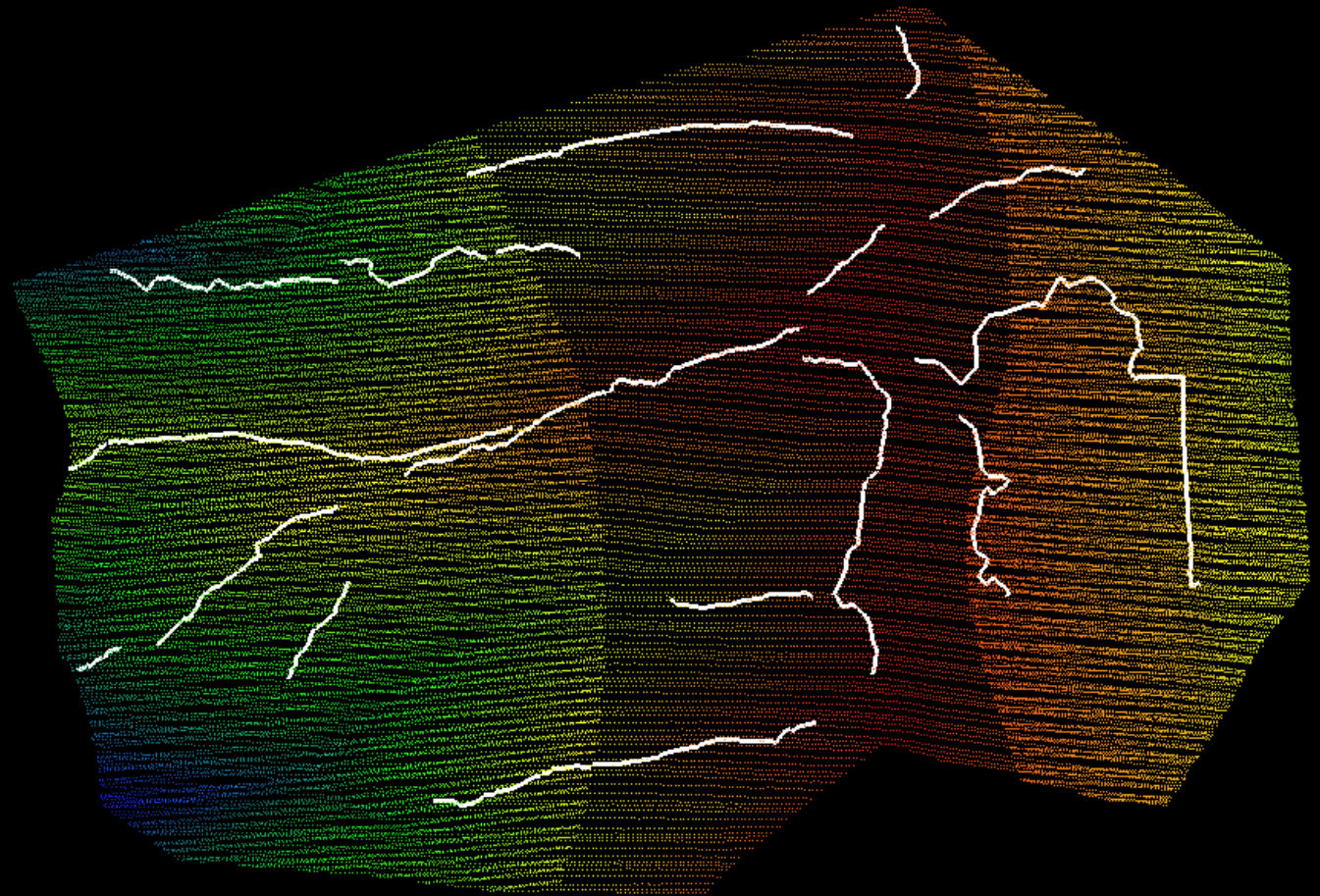
- Simplify to one polyline



# Methodology: generating polylines using the graph theory

- Simplify to one polyline by the shortest path algorithm

Example: ridge

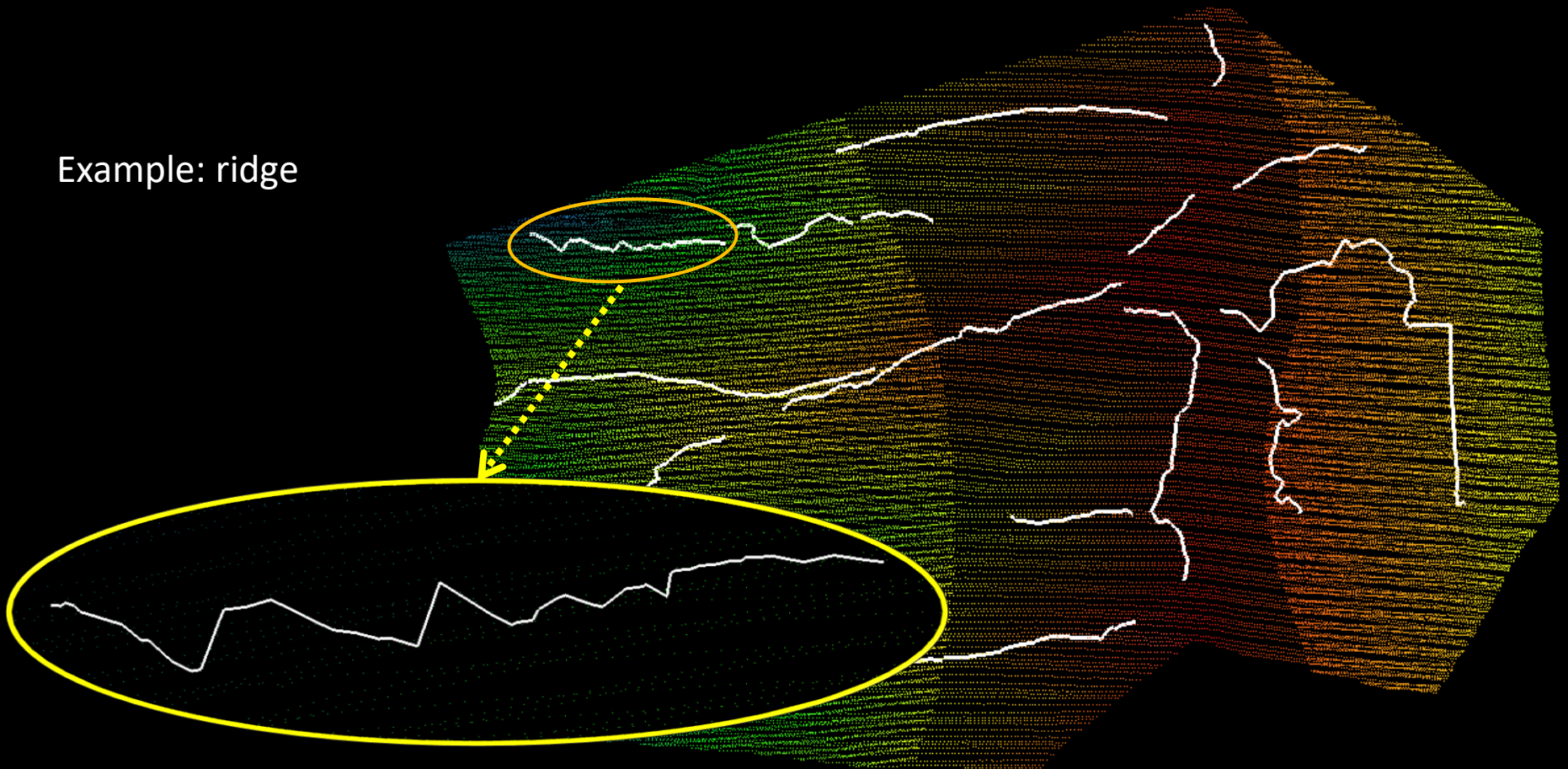




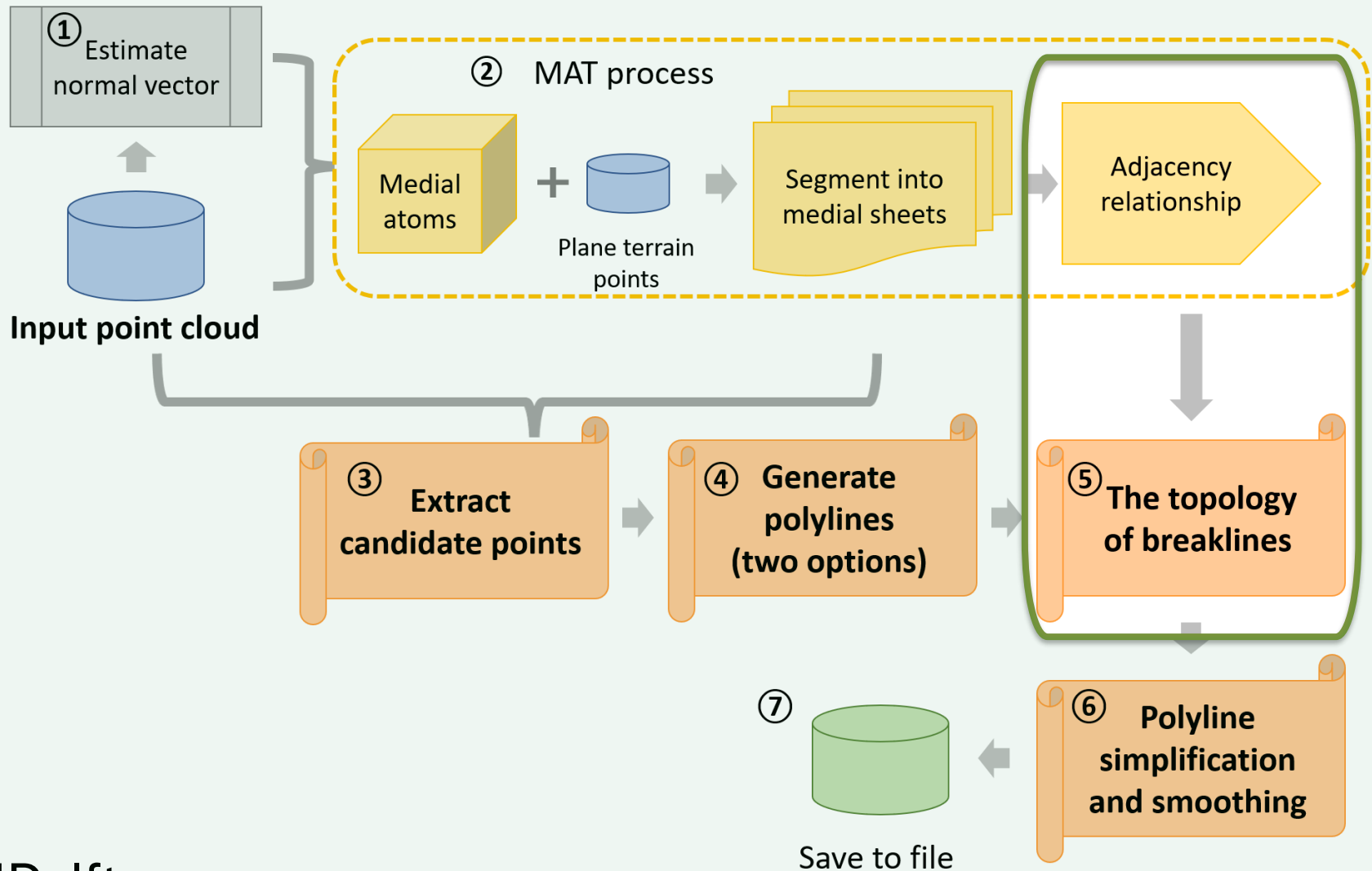
# Methodology: generating polylines using the graph theory

- Simplify to one polyline by the shortest path algorithm

Example: ridge

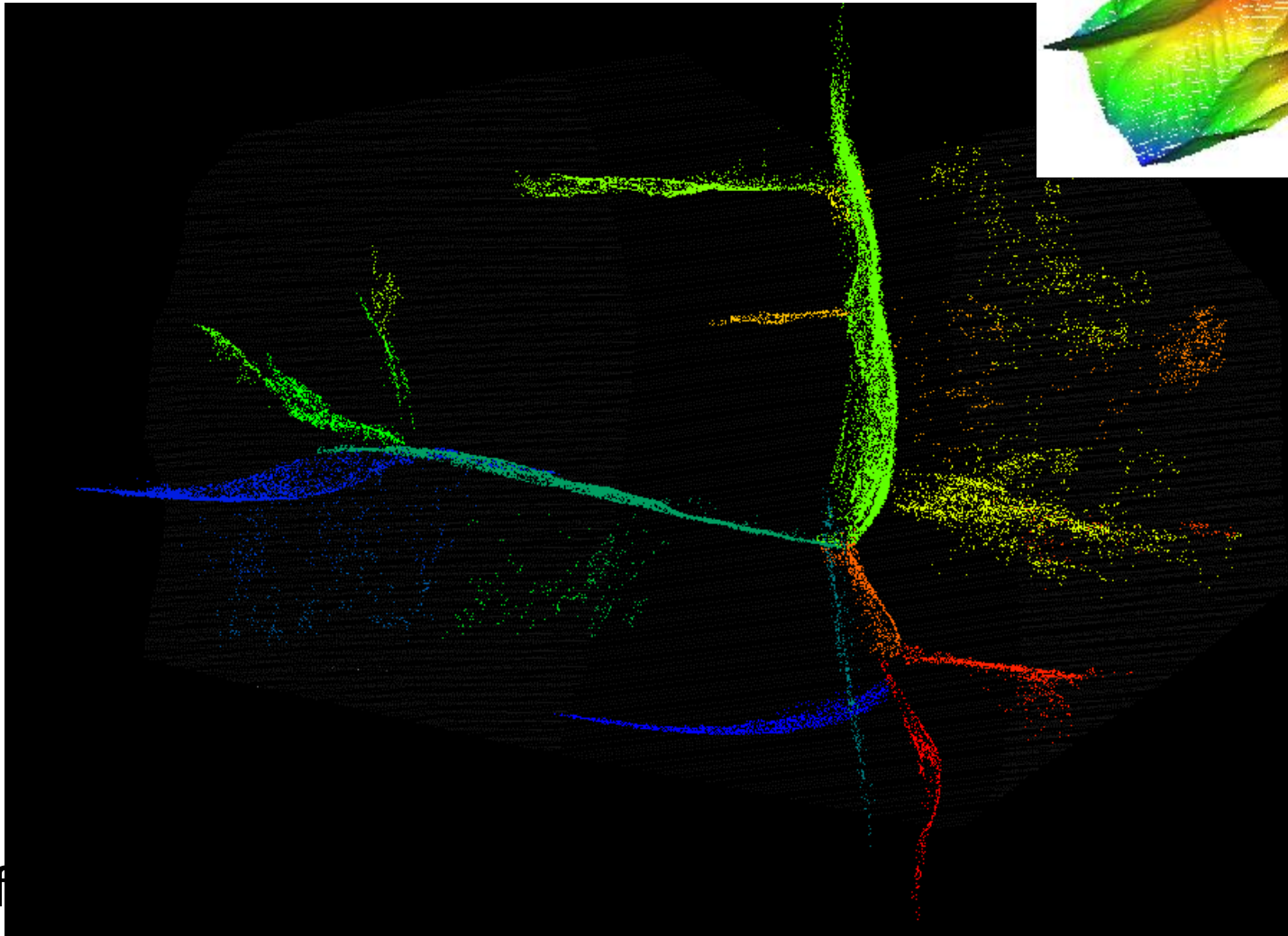


# Methodology overview



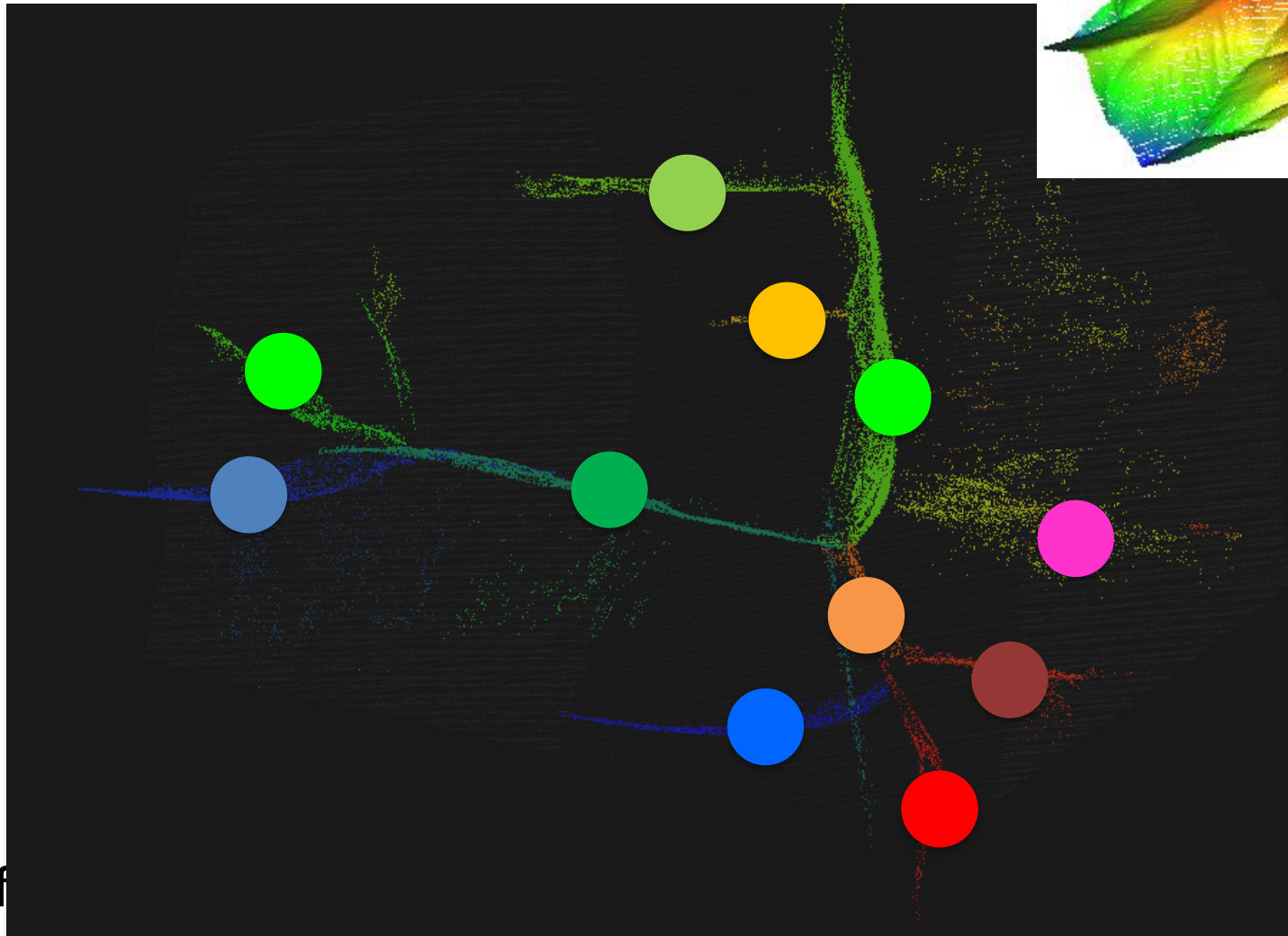
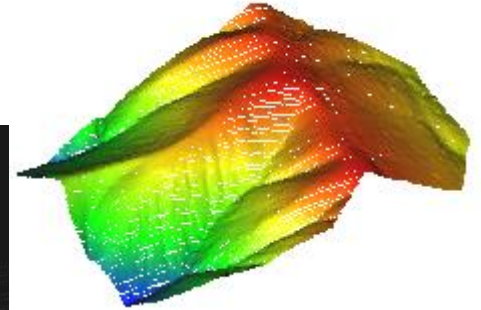
# Methodology: the topology of breaklines

- Using the adjacency graph from MAT



# Methodology: the topology of breaklines

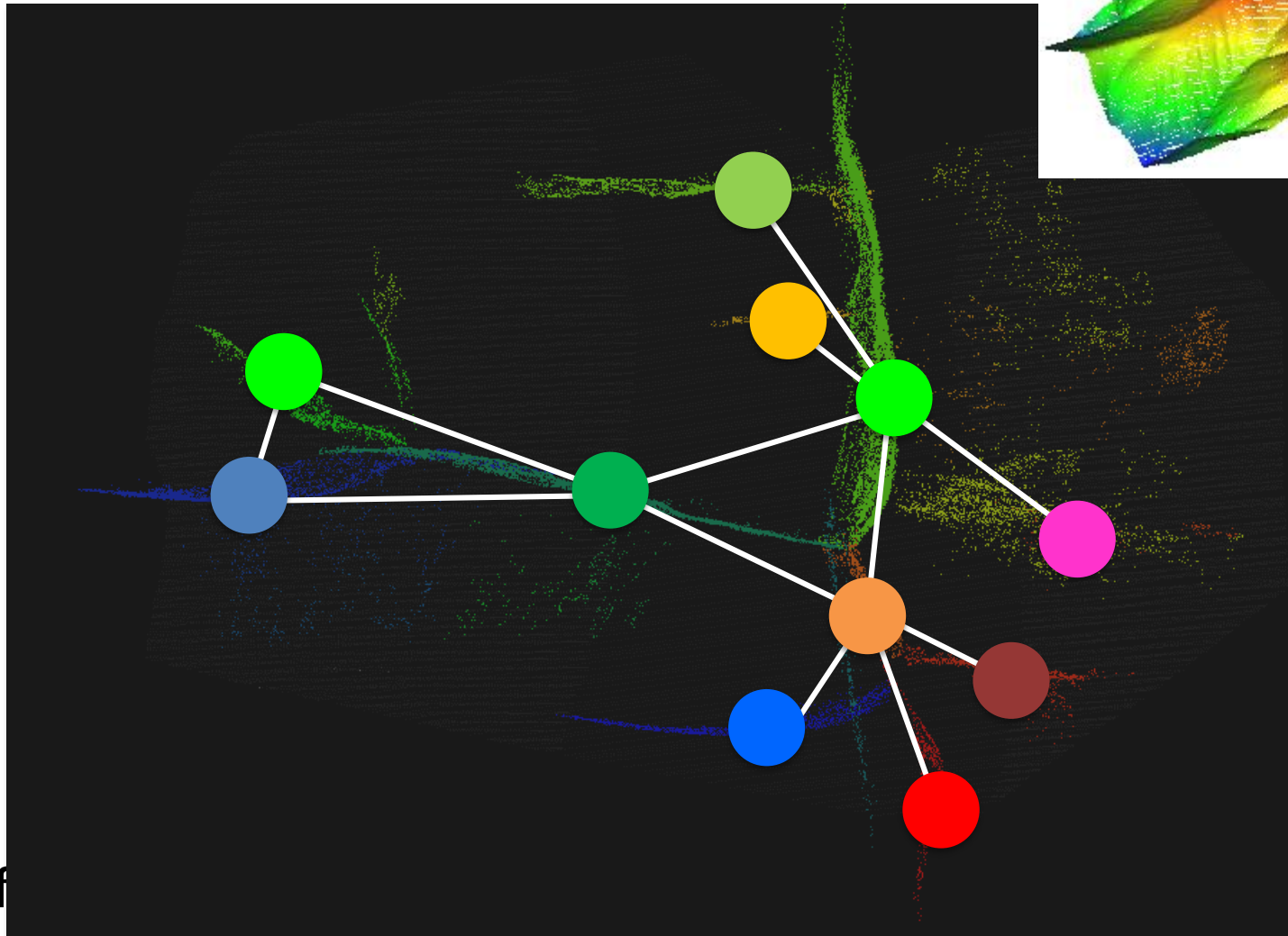
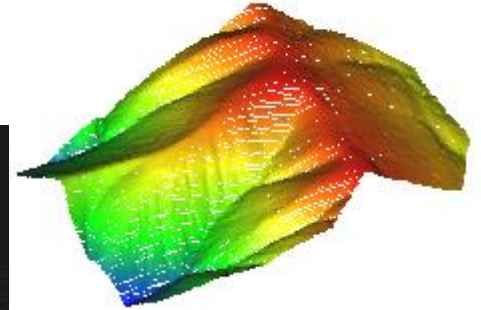
- Using the adjacency graph from MAT





# Methodology: the topology of breaklines

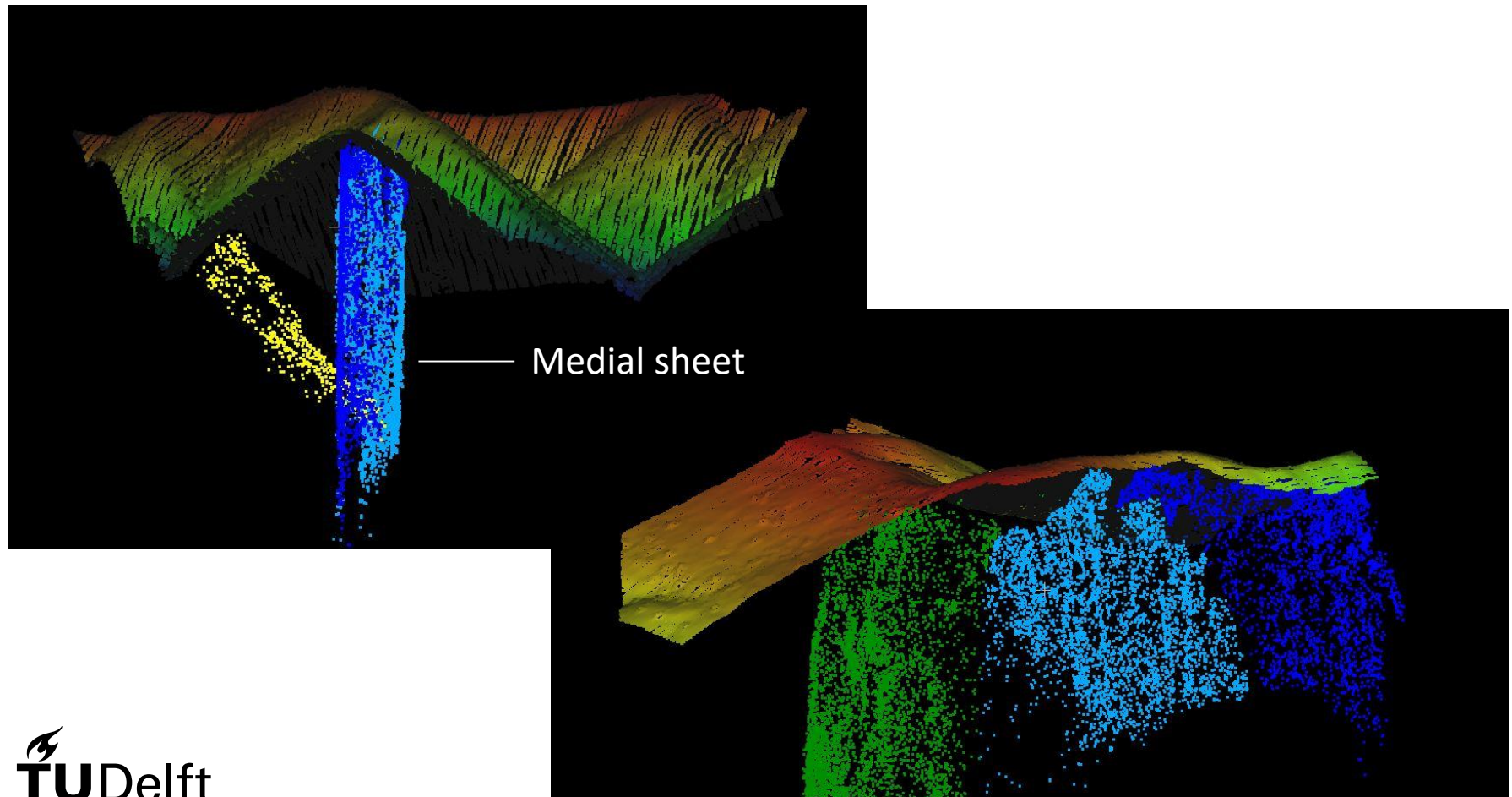
- Using the adjacency graph from MAT





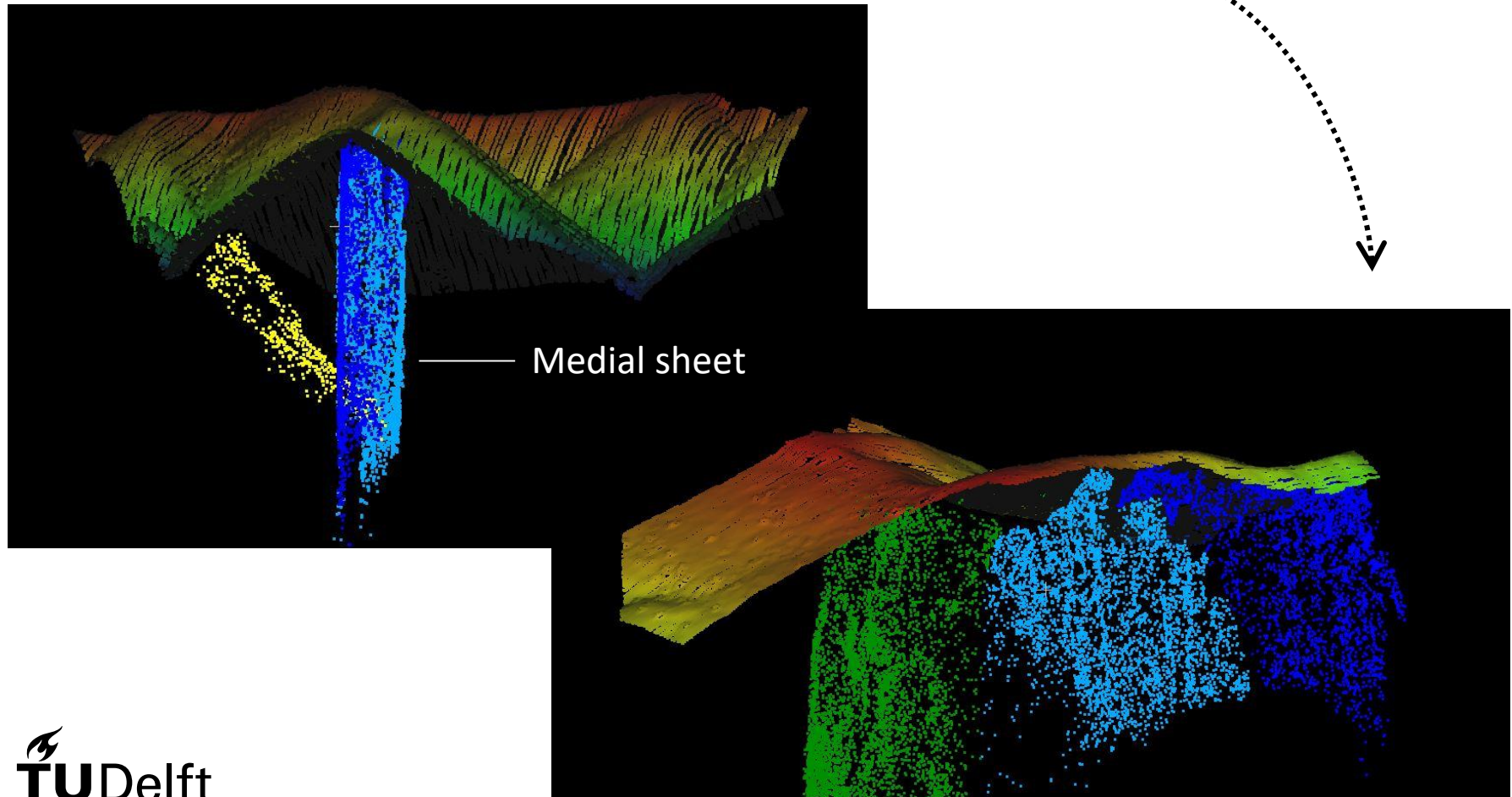
# Methodology: the topology of breaklines

- Using the adjacency graph from MAT



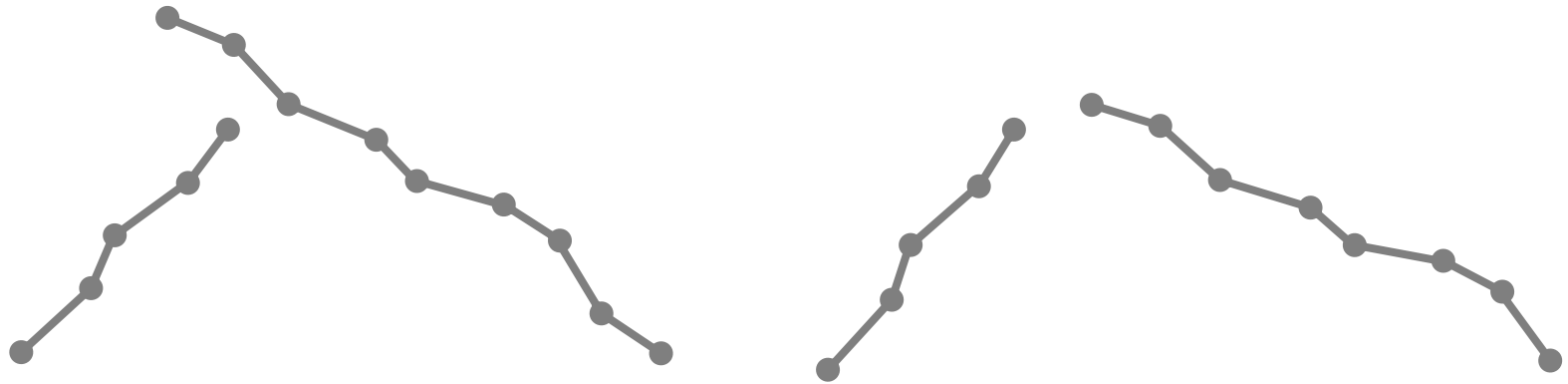
# Methodology: the topology of breaklines

- Using the adjacency graph from MAT
- Only the surface adjacency is required



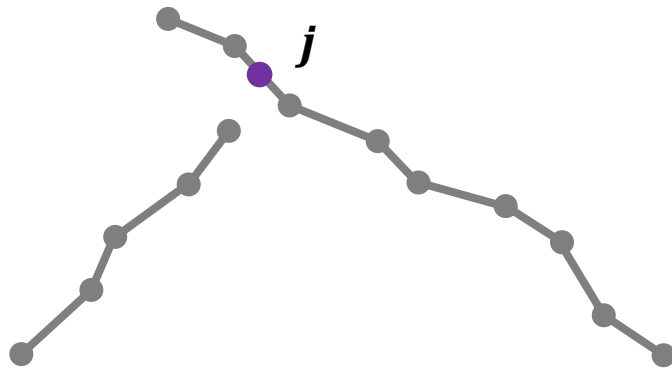
# Methodology: the topology of breaklines

- connecting two breaklines

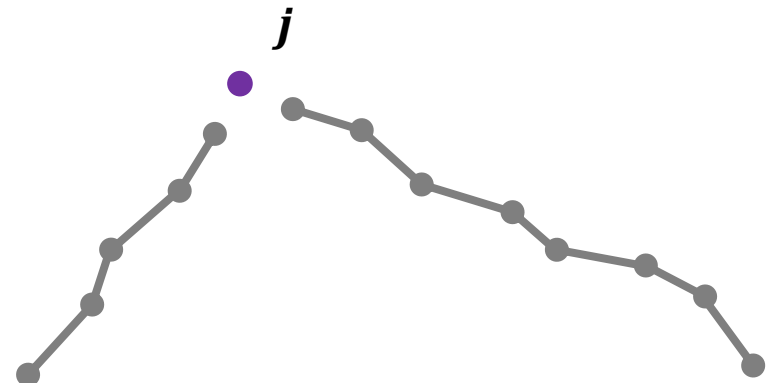


# Methodology: the topology of breaklines

- connecting two breaklines



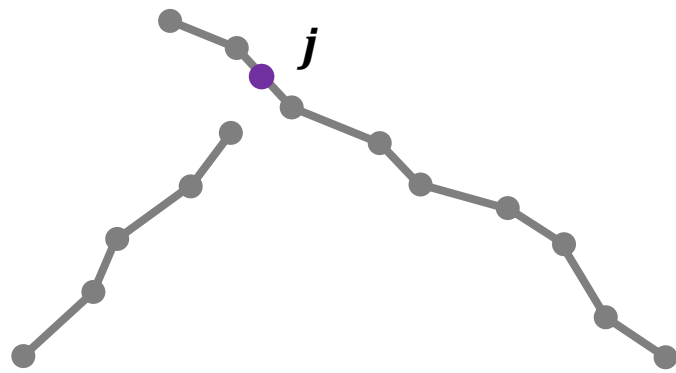
Junction point  $j$  in the polyline



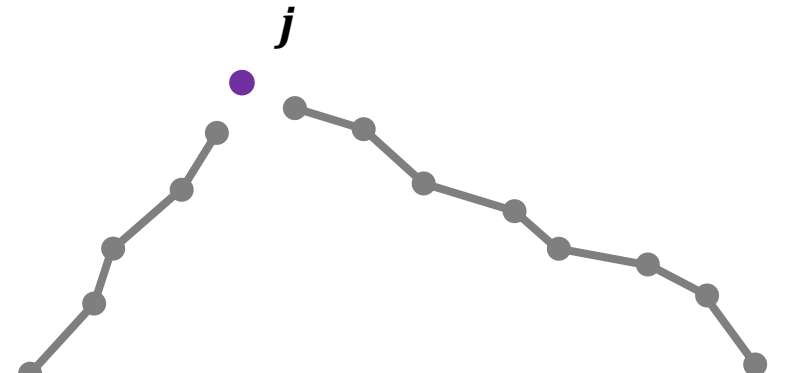
Junction point  $j$  out side the polyline

# Methodology: the topology of breaklines

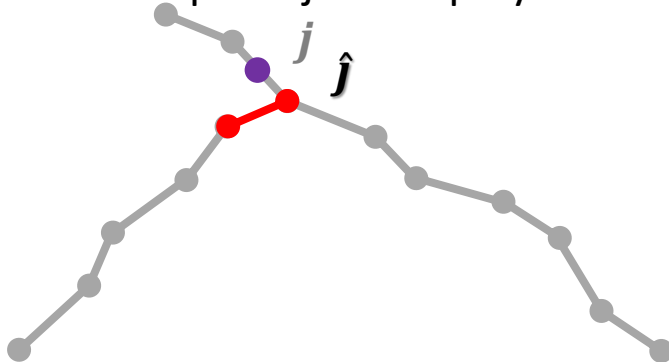
- connecting two breaklines



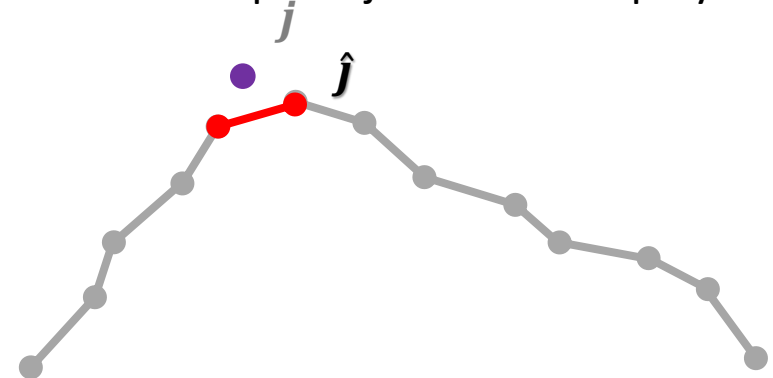
Junction point  $j$  in the polyline



Junction point  $j$  out side the polyline



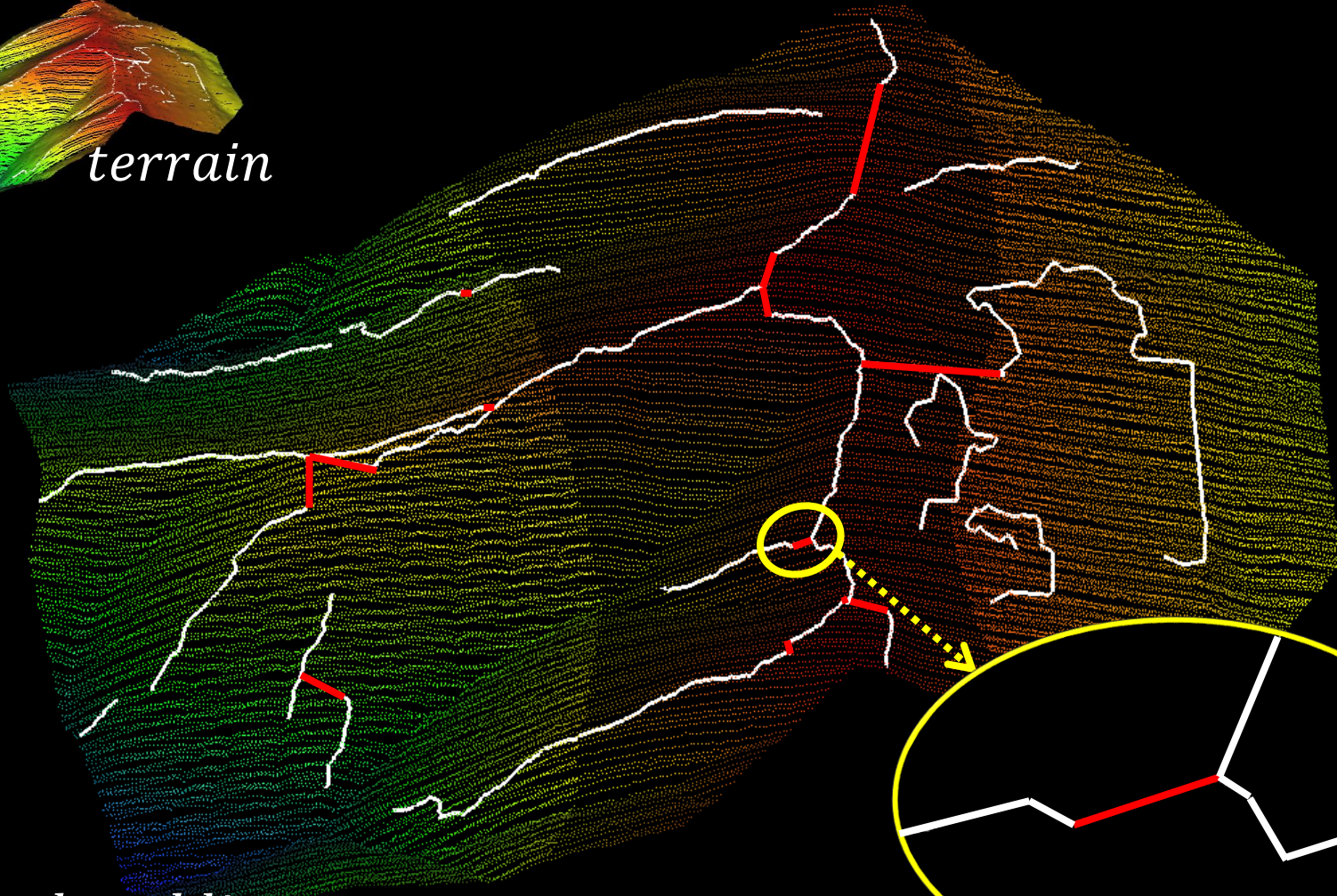
connection &  
estimated junction  $\hat{j}$



connection &  
estimated junction  $\hat{j}$



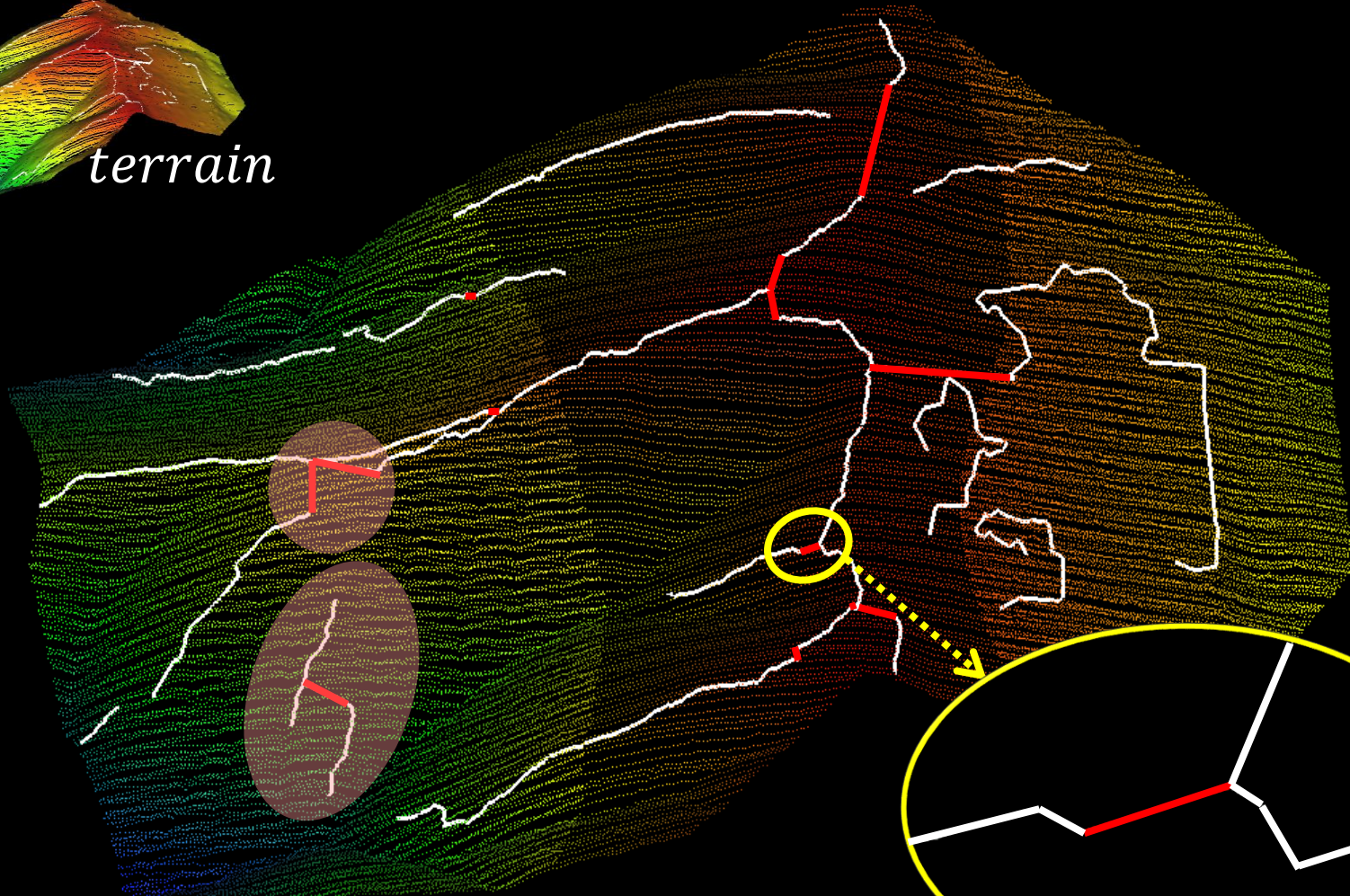
# Methodology: the topology of breaklines



*white: breaklines*

*red: connection of adjacent breaklines*

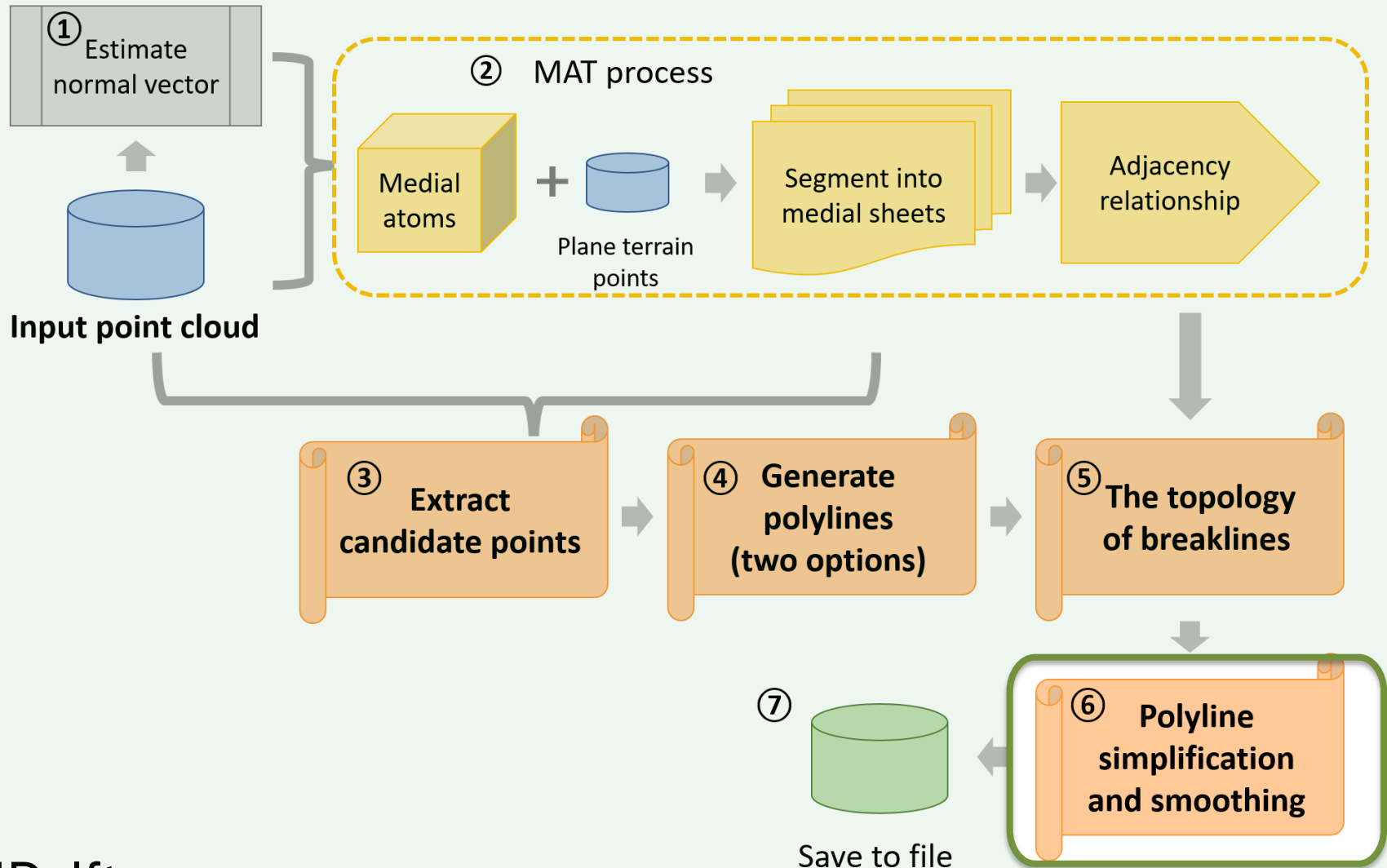
# Methodology: the topology of breaklines



*white: breaklines*

*red: connection of adjacent breaklines*

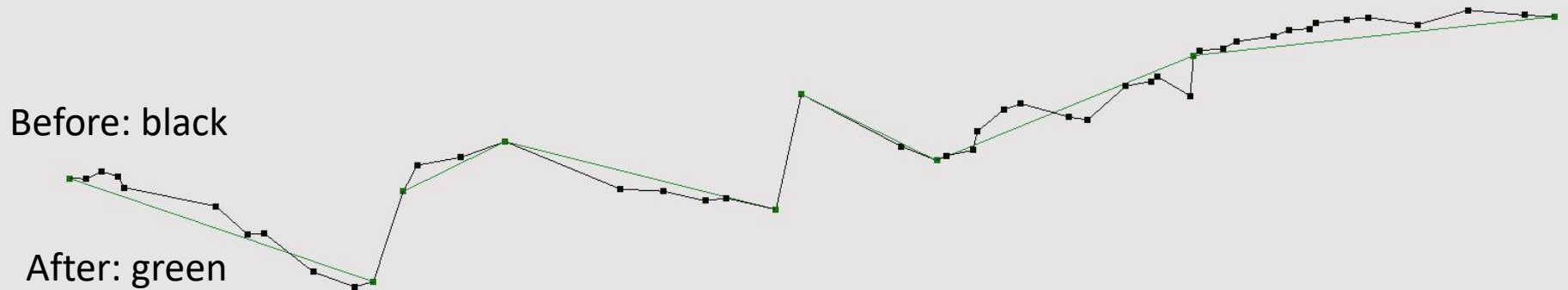
# Methodology overview





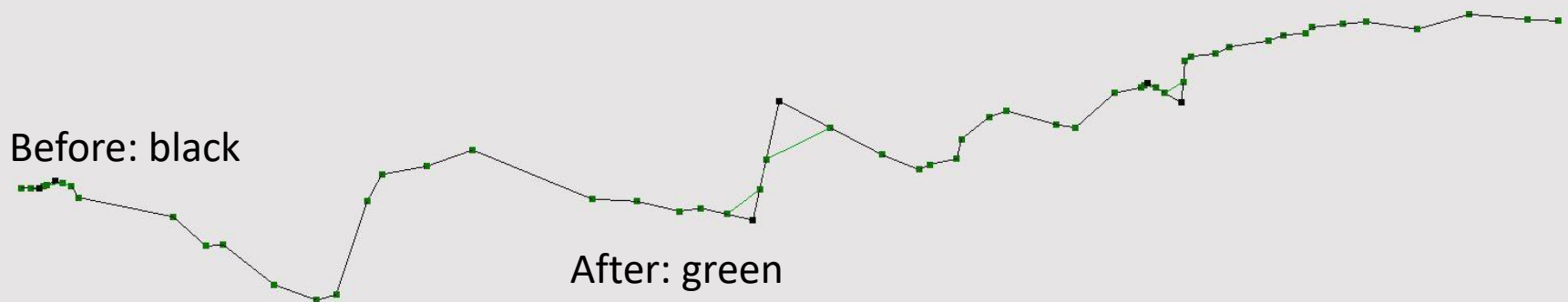
# Methodology: polyline simplification and smoothing

- Simplification: remove less important points\*



# Methodology: polyline simplification and smoothing

- Smoothing: adding points to remove sharp angle





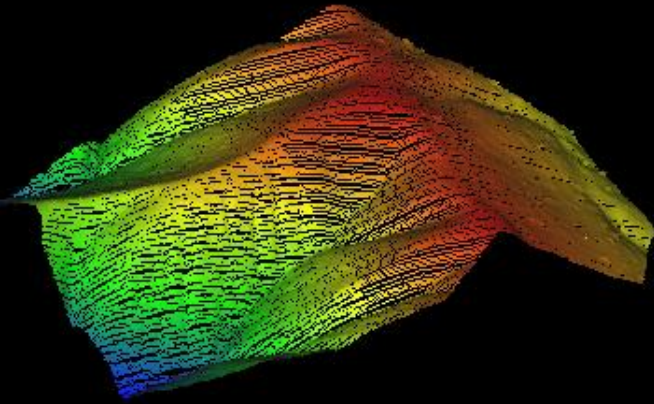
An aerial photograph of a mountain range. The foreground shows rocky terrain with sparse green vegetation. The middle ground features steep, green mountain slopes. In the background, a valley contains a lake, and the mountains are partially obscured by a layer of white clouds. The sky is clear and blue.

# Result & analysis

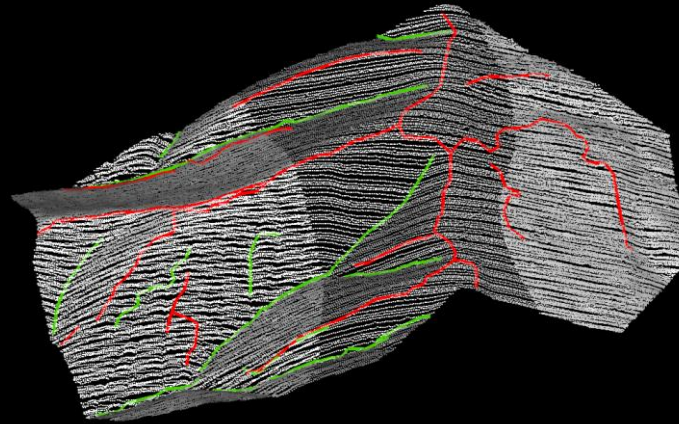


# Result1 (red ridge; green valley)

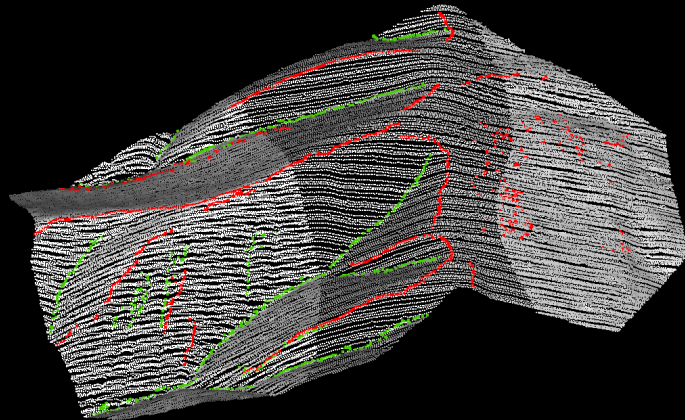
Point cloud



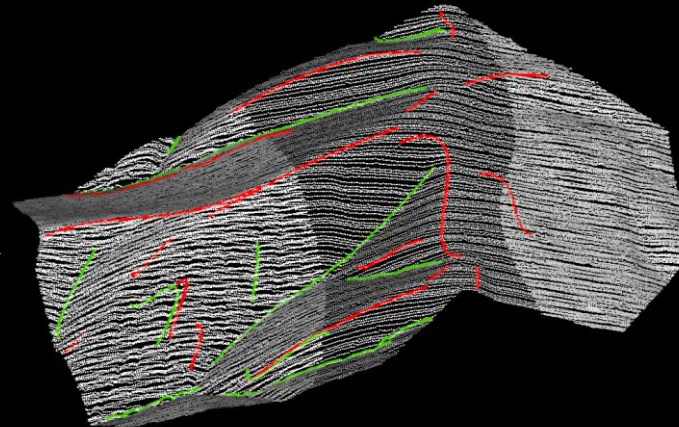
Breaklines with graph theory



Candidate points

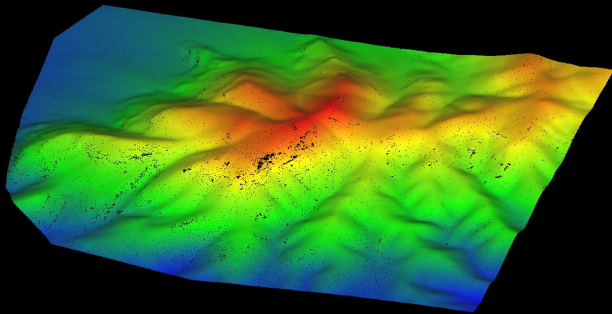


Breaklines with polynomial fit

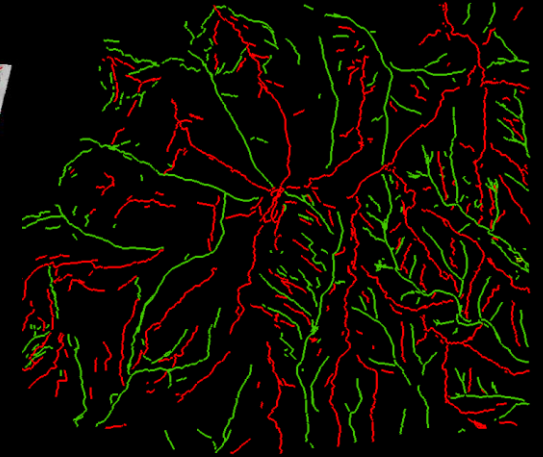
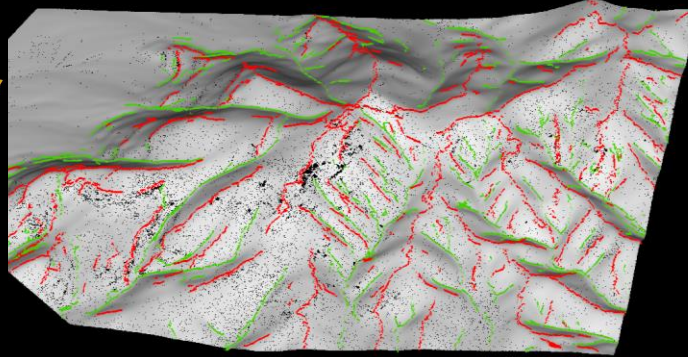


# Result2 (red ridge; green valley)

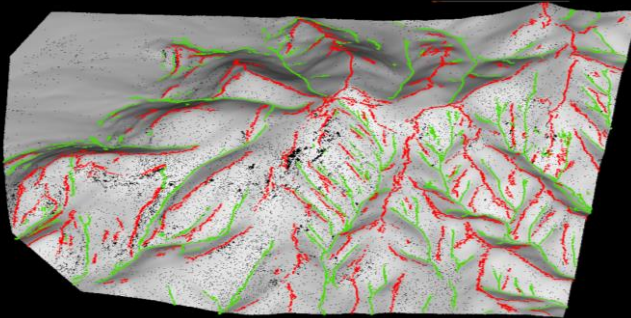
Point cloud



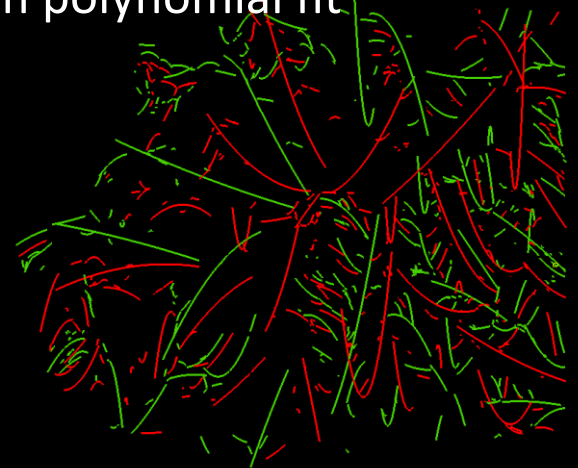
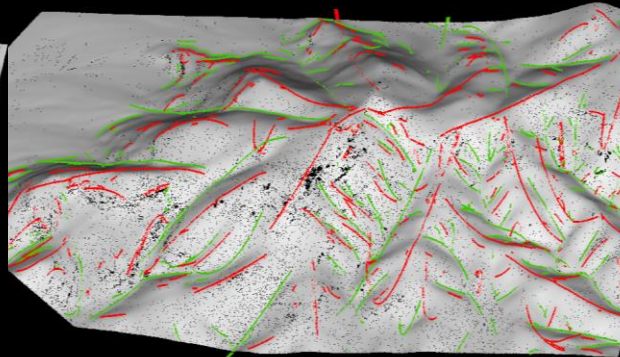
Breaklines with graph theory



Candidate points

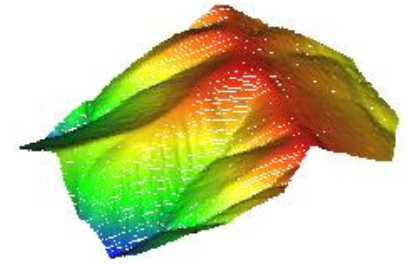


Breaklines with polynomial fit





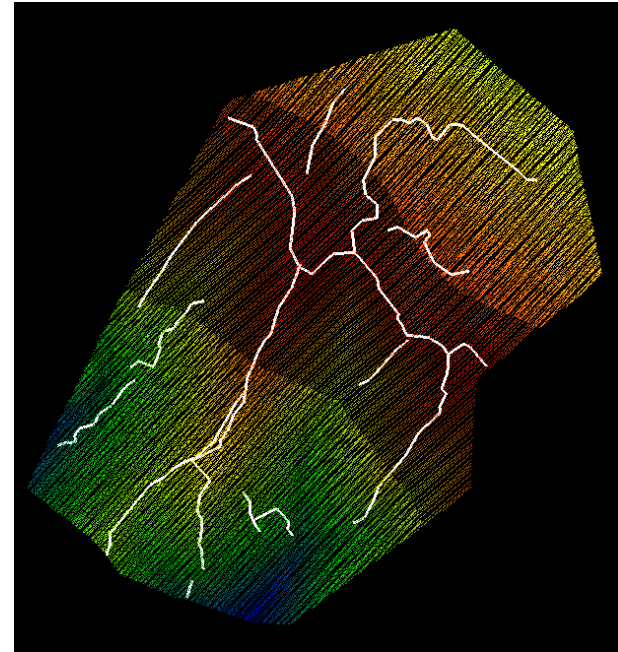
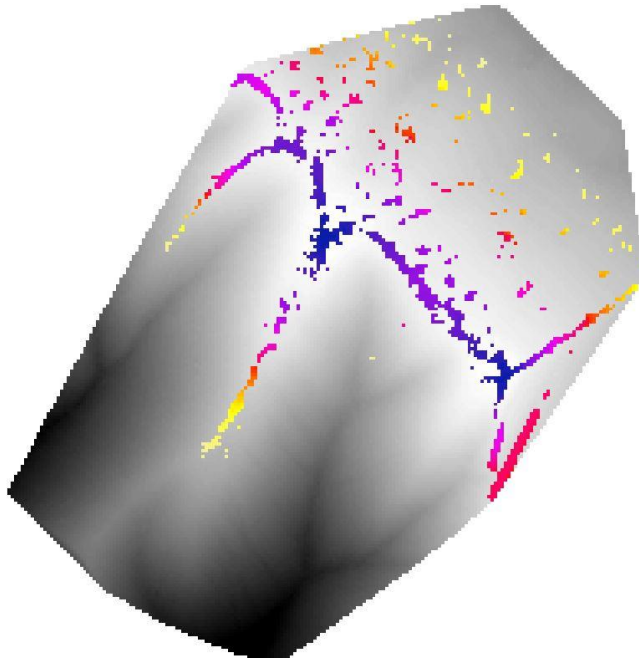
# Comparing with ArcMap: ridge



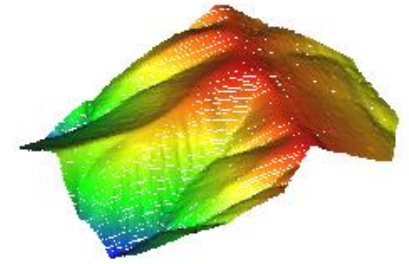
Elevation



Ridge cell



# Comparing with ArcMap: ridge



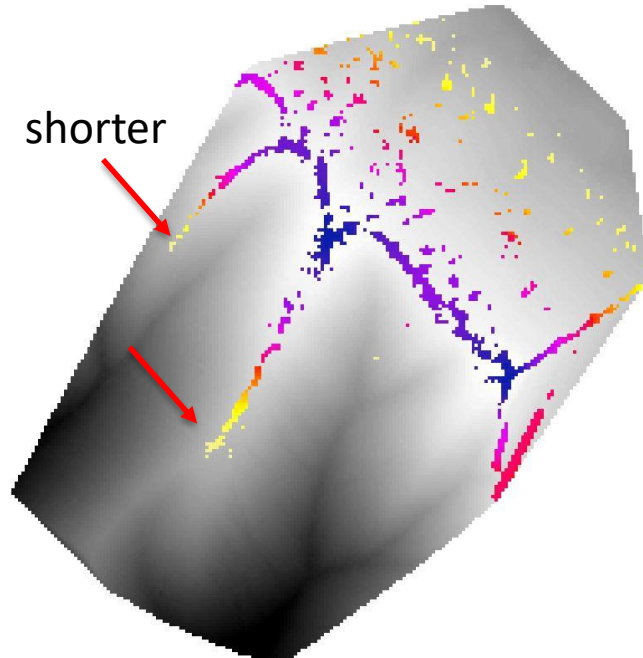
Elevation



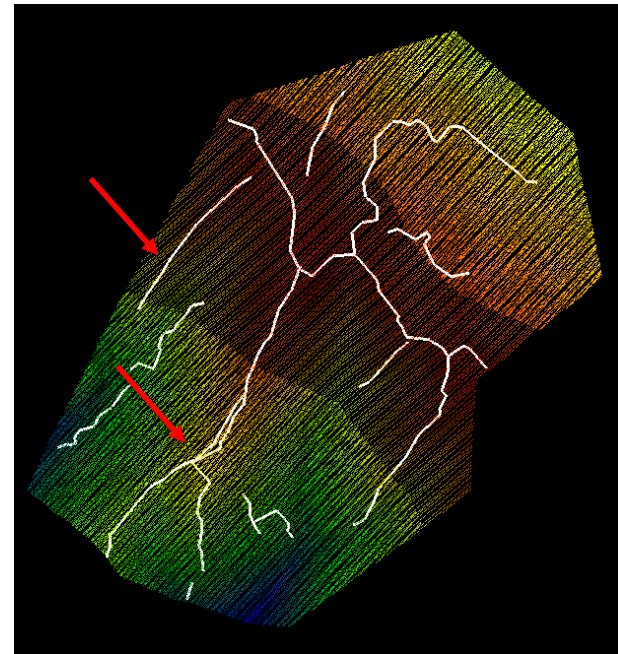
shorter



Ridge cell



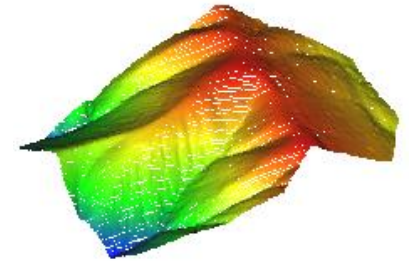
Cells  $\rightarrow$  points  
Incomplete



3D polylines + topology  $\checkmark$   
Complete  $\checkmark$



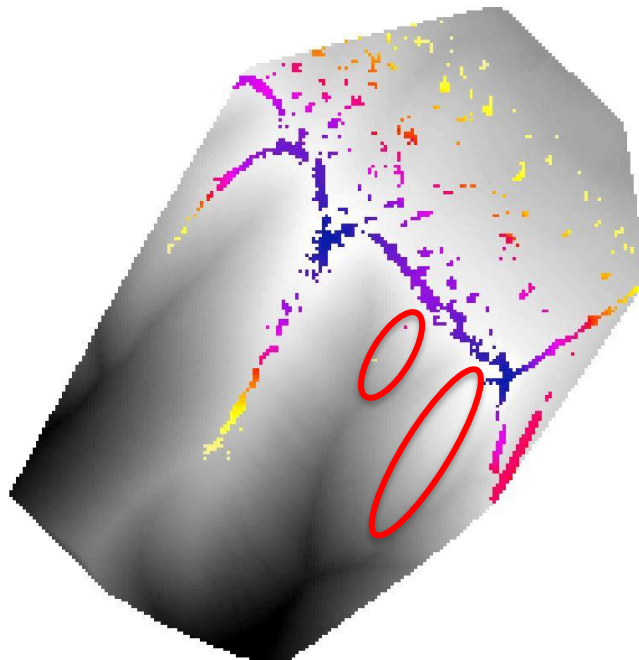
# Comparing with ArcMap: ridge



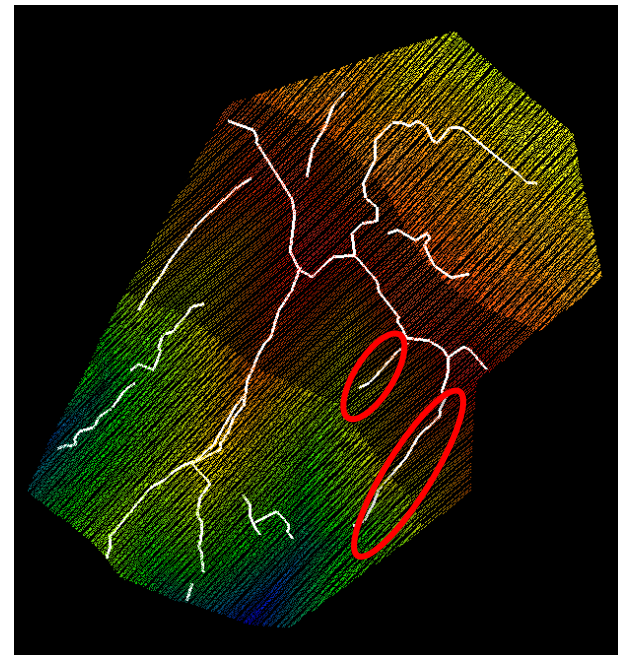
Elevation



Ridge cell

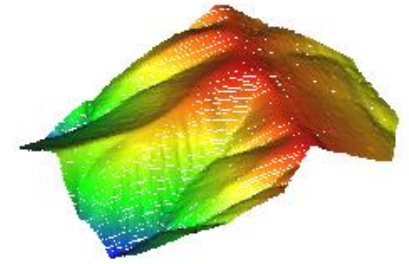


Cells  $\rightarrow$  points  
Incomplete  
Missing ridges



3D polylines + topology ✓  
Complete ✓  
Missing small ridges

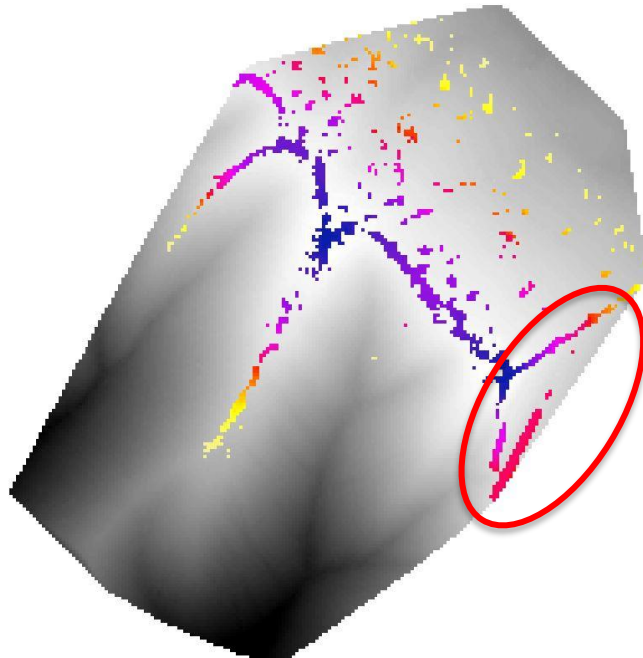
# Comparing with ArcMap: ridge



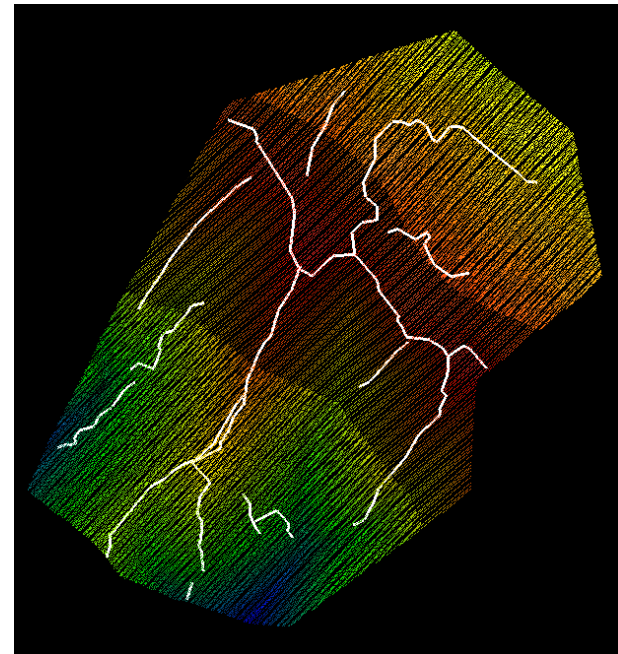
Elevation



Ridge cell



Cells  $\rightarrow$  points  
Incomplete  
Missing ridges  
Effected by spatial interpolation

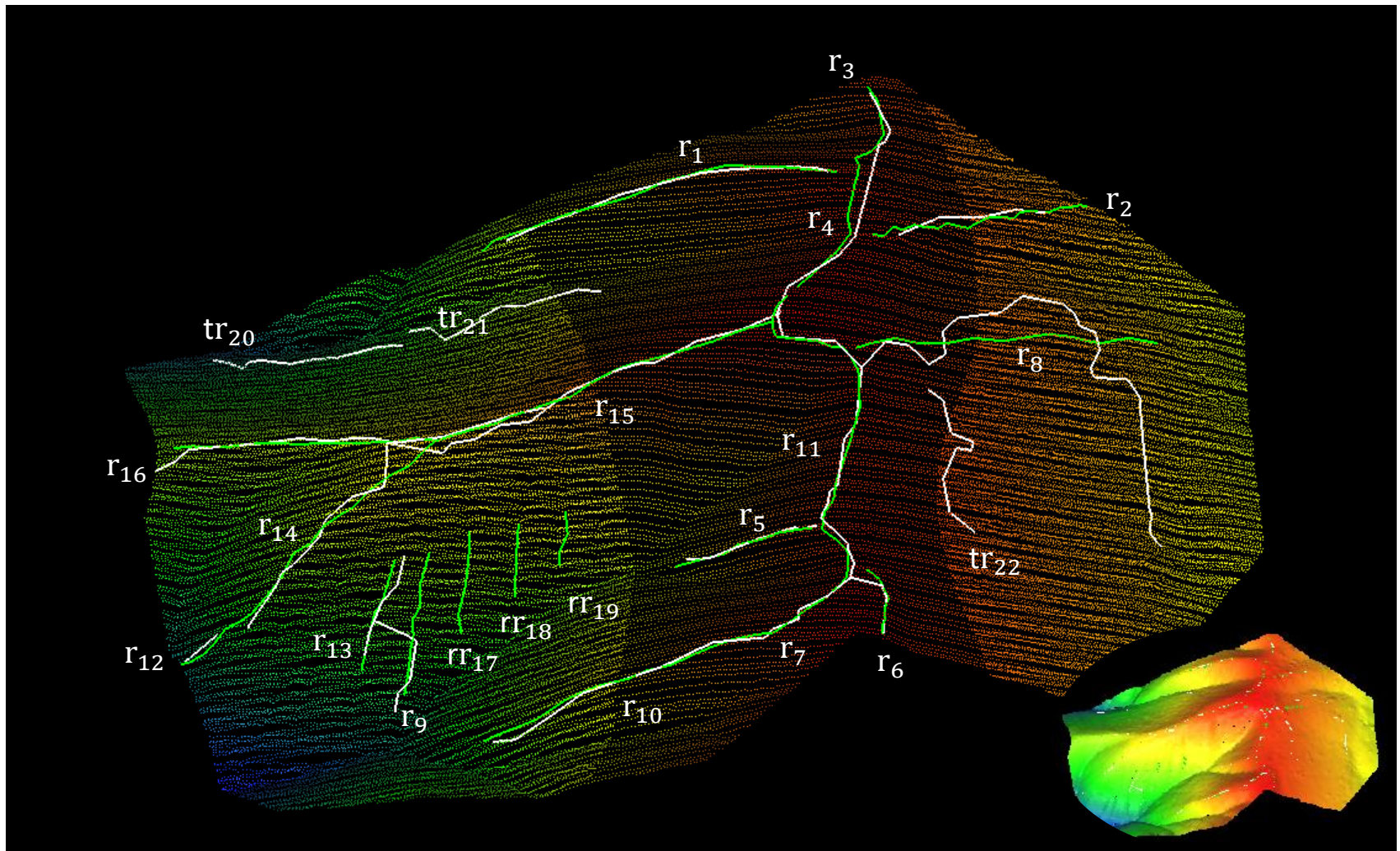


3D polylines + topology ✓  
Complete ✓  
Missing small ridges

# Validation:

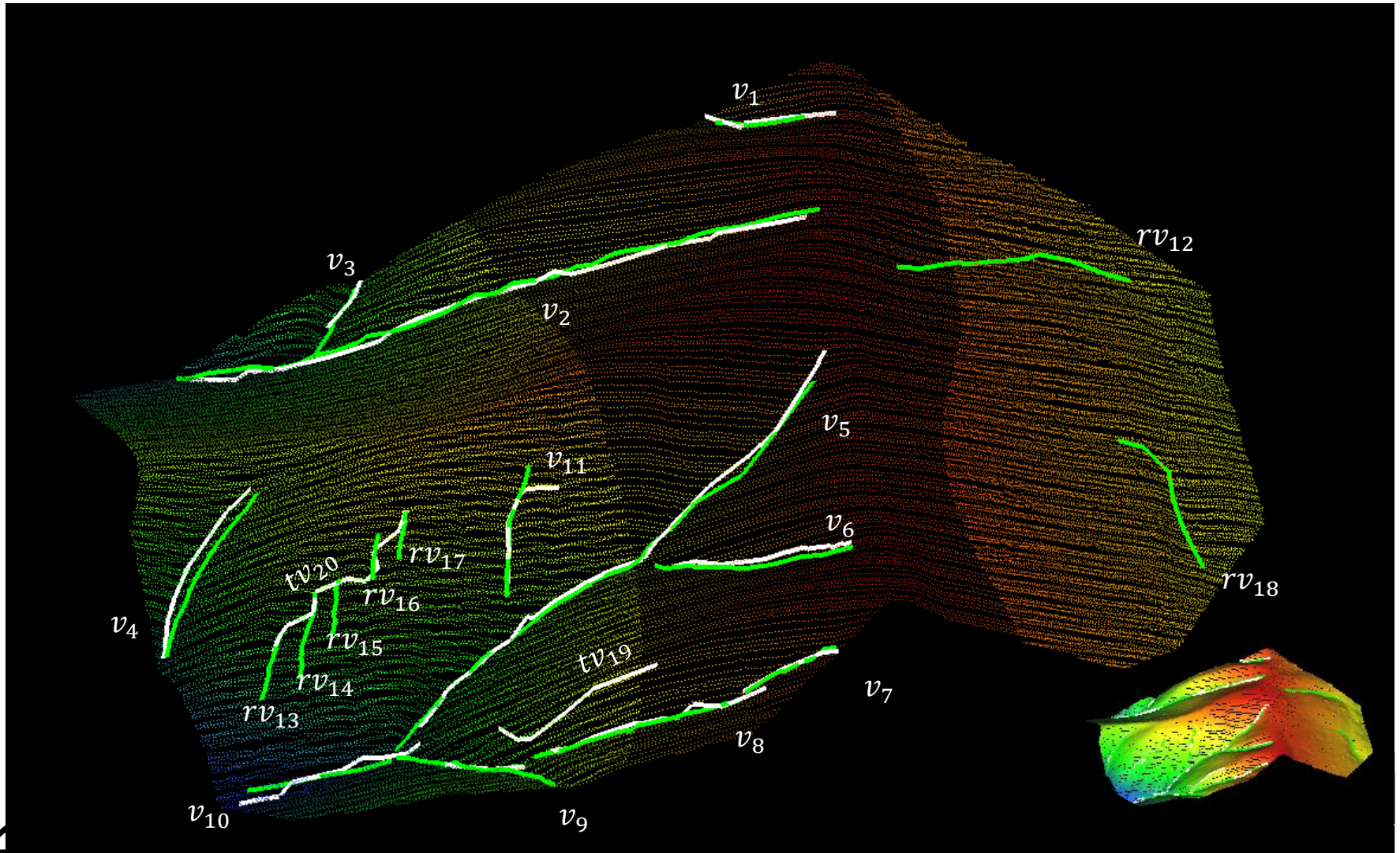
- If the breaklines are correctly identified?
- How is the accuracy and precision of the correctly identified breaklines?

# Validation: ridge





# Validation: valley

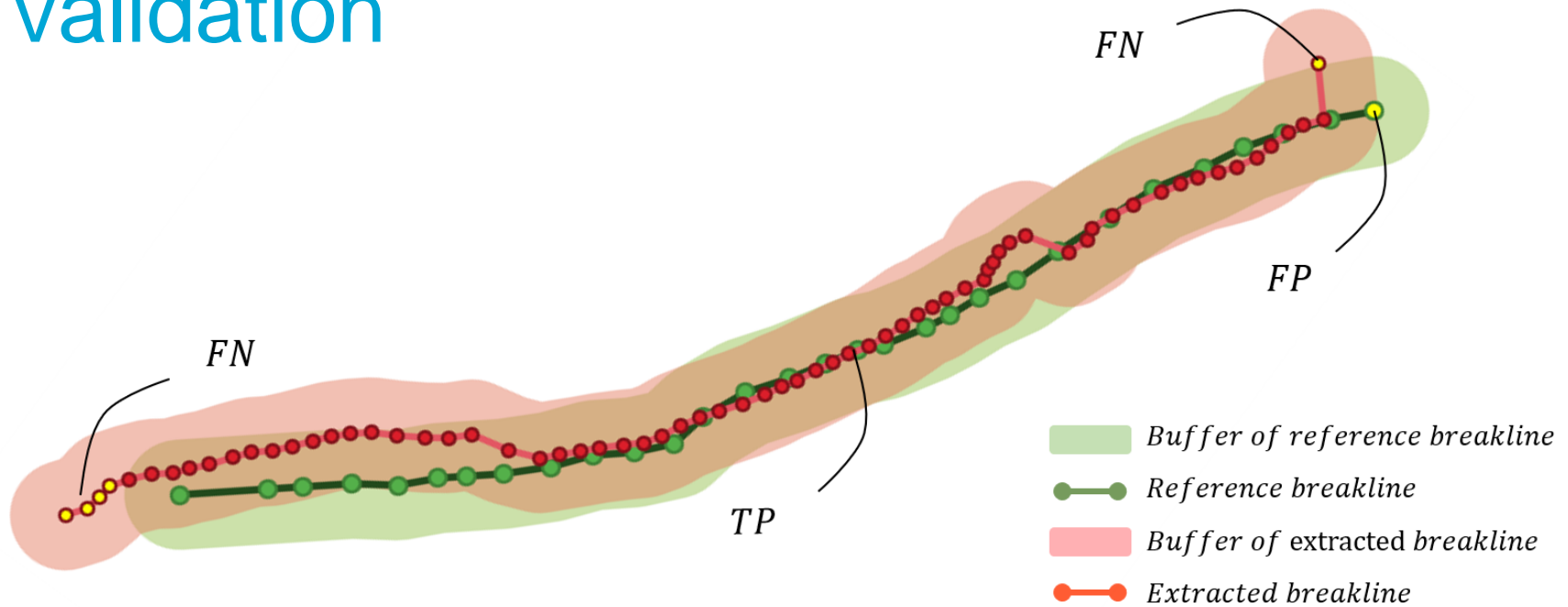




# Validation

- If the breaklines are correctly identified?
  - 16/19 ridges and 11/13 valleys (27/32 breaklines) are correctly identified

# Validation



Accuracy:

$$correctness = \frac{TP}{TP + FP \times scalar}$$

$$completeness = \frac{TP}{TP + FN}$$

$$quality = \frac{TP}{TP + FN + FP \times scalar}$$

( $scalar = \frac{TP + FN}{NR}$ )

Precision:

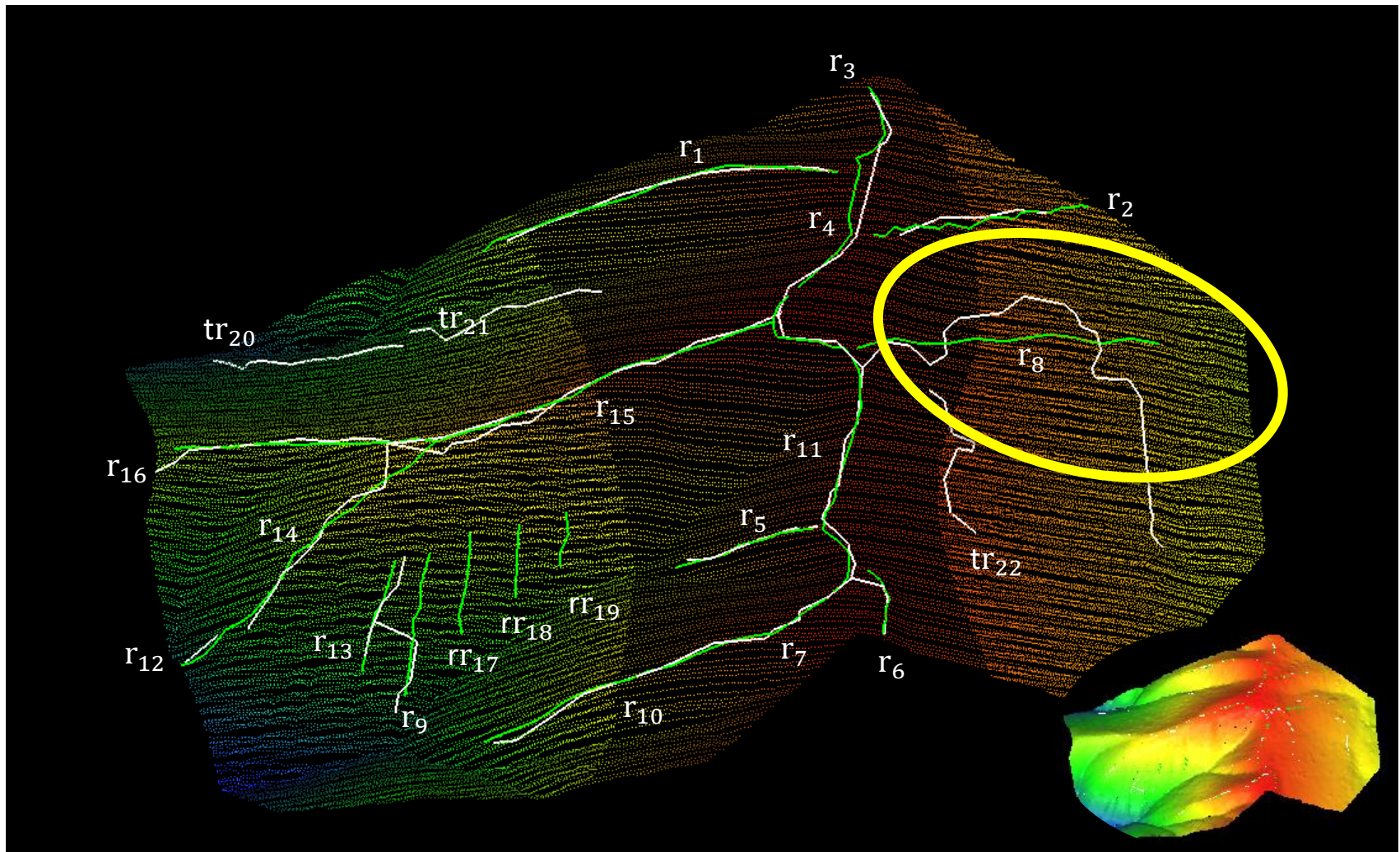
$$p_l = \frac{\sum d(v, r_l)}{\text{the length of } l}$$

$$p_v = \frac{\sum d(v, r_l)}{TP}$$

# Validation: ridge

TP pair	scalar	correctness	quality	completeness	polyline precision	vertex precision
r1	100%	100.00%	100.00%	100.00%	0.06	0.46
r2	21.74%	88.46%	88.46%	100.00%	0.09	0.94
r3	85.71%	100.00%	100.00%	100.00%	0.26	0.70
r4	28.57%	100.00%	100.00%	100.00%	0.15	1.44
r5	54.55%	100.00%	100.00%	100.00%	0.10	0.49
r6	44.44%	100.00%	100.00%	100.00%	0.27	1.09
r7	33.33%	100.00%	100.00%	100.00%	0.20	0.78
r8	191.67%	89.87%	68.23%	73.91%	1.23	4.62
r9	66.67%	71.43%	62.50%	83.33%	0.26	1.48
r10	52.38%	100.00%	100.00%	100.00%	0.13	0.85
r11	64.00%	100.00%	100.00%	100.00%	0.18	0.86
r12	40.00%	100.00%	100.00%	100.00%	0.07	0.42
r13	83.33%	100.00%	100.00%	100.00%	0.19	0.84
r14	80.00%	93.22%	85.94%	91.67%	0.20	0.99
r15	100%	100.00%	61.54%	61.54%	0.18	1.10
r16	76.47%	100.00%	92.31%	92.31%	0.16	0.93
average		96%	91.19%	93.92%	0.23	1.12

# Validation: ridge



# Validation: valley

TP pair	scalar	correctness	quality	completeness		polyline precision	vertex precision
v1	83.33%	100%	80.00%	80.00%		0.32	1.37
v2	75.76%	100%	100%	100%		0.25	1.28
v3	30.00%	90.91%	90.91%	100%		0.17	1.16
v4	72.73%	100%	100%	100%		0.36	1.58
v5	73.33%	96.63%	92.38%	95.45%		0.18	0.94
v6	146.15%	100%	100%	100%		1.03	2.17
v7	44.44%	100%	100%	100%		0.13	0.62
v8	76.47%	100%	84.62%	84.62%		0.21	0.78
v9	17.65%	77.27%	77.27%	100%		0.06	0.69
v10	100.00%	100%	88.89%	88.89%		0.49	1.68
v11	66.67%	100%	83.33%	83.33%		0.05	0.25
average		97%	90.67%	93.84%		0.30	1.14



# Validation

- If the breaklines correctly identified?
  - 16/19 ridges and 11/13 valleys (27/32 breaklines) are correctly identified
- How is the accuracy and precision of the correctly identified breaklines?
  - accurate and precise

	correctness	quality	completeness	polyline precision	vertex precision
ridge	96%	91.19%	93.92%	0.23	1.12
valley	97%	90.67%	93.84%	0.30	1.14

Point density: 2pts/m<sup>2</sup>

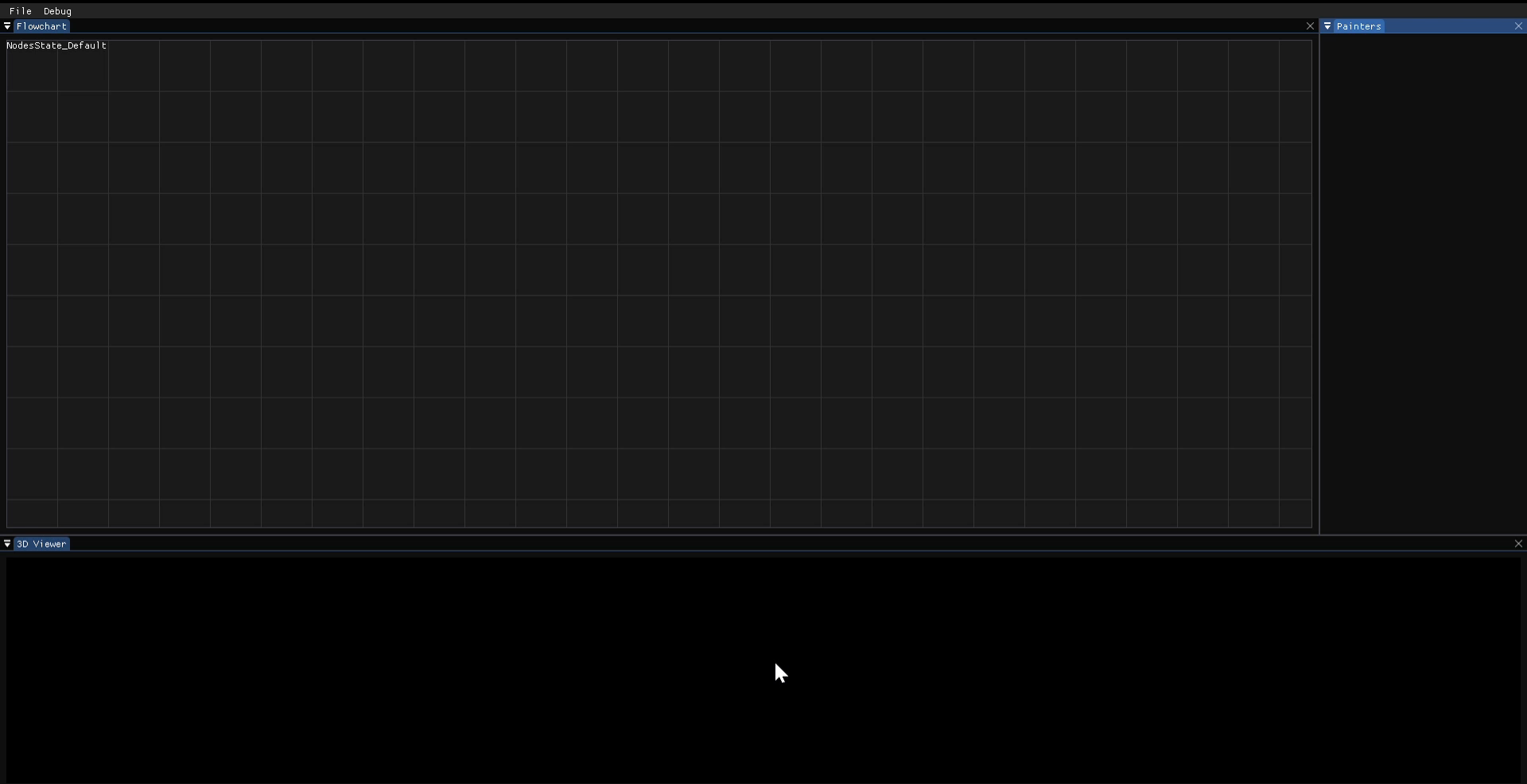


An aerial photograph of a mountain range. The foreground shows rocky terrain with sparse green vegetation. The middle ground features steep, forested slopes in various shades of green. In the background, a valley contains a lake, and further mountains are partially obscured by a layer of white clouds. The sky is a pale, clear blue.

Demo



# Demo







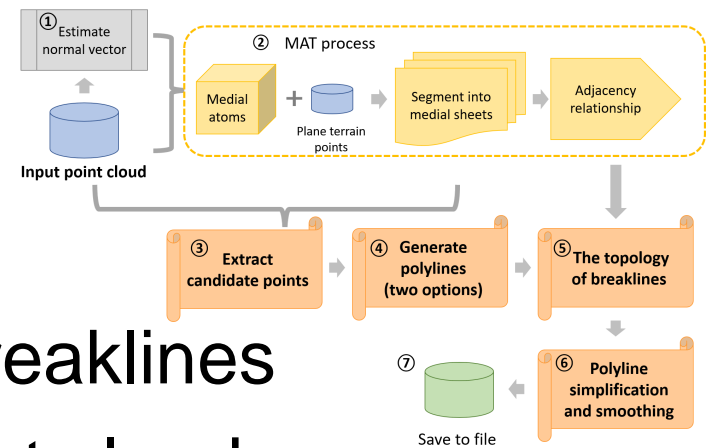
An aerial photograph of a vast mountain range. The foreground shows a rocky, grey peak with sparse green vegetation. Below, the mountain slopes are covered in dense, vibrant green forests. In the distance, a valley contains a turquoise lake, and the mountains recede into the background, partially shrouded in white mist or low clouds under a clear sky.

# Conclusion



# Conclusion

- A new method to generate breaklines
- Directly processing on point cloud
- long breaklines:
  - Correctly identified;
  - Accurate and precise
- small breaklines & planar area:
  - missing identified or incorrectly identified
  - topology error



# Future work

- parameters:
  - User defined → estimate parameters  
→ better result & automatically
  - Global → adapt to different sheet  
→ eliminate error in small breaklines
- Topology
  - The junction points & connection  
→ require better estimation
- Validation on more datasets

# Tools and datasets used

- Language: C++
- Libraries: Geoflow, boost, CGAL, eigen, spline;
- Input data
  - Mountain: *OpenTopography* LiDAR point cloud data
- Source code: <https://github.com/qq2012/geoflow-nodes>

MAT : <https://github.com/tudelft3d/geoflow-nodes>

Boost: <https://www.boost.org>

CGAL: <https://www.cgal.org>

Eigen: <http://eigen.tuxfamily.org>

Spline: <https://github.com/chen0040/cpp-spline>

OpenTopography: <https://opentopography.org/start>





Thank you ^\_^



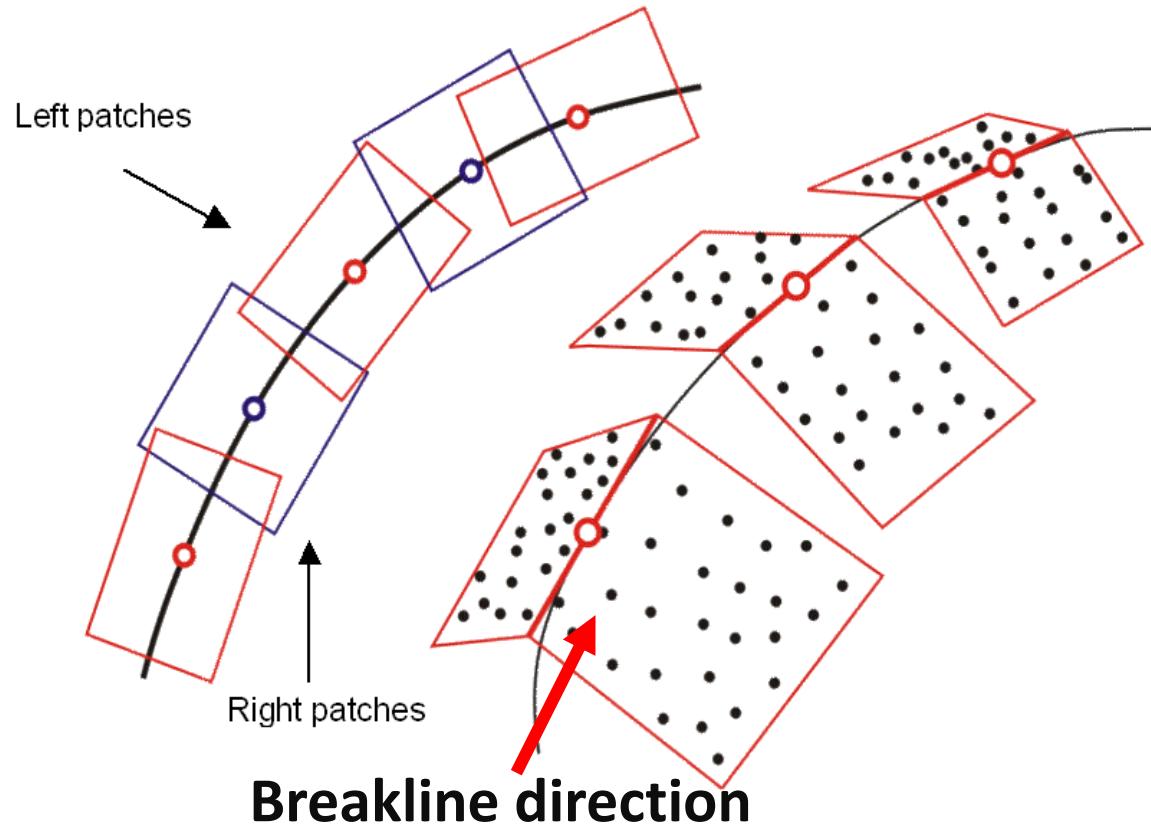
An aerial photograph of a vast mountain range. The foreground shows a rocky, light-colored ridge with sparse green vegetation. Below the ridge, the slopes are covered in dense, vibrant green forests. In the distance, a large lake is visible in a valley, surrounded by more mountain peaks. The sky is a pale blue, and there are wispy white clouds scattered across the landscape, particularly around the higher mountain peaks.

Questions?



- Why Medial Axis Transform (MAT)?
  - Fully 3D, directly processing on point cloud
  - Provide useful geometry features
    - the radius corresponds to the curvature,
  - Medial sheet indicates the breaklines
    - interior & exterior → ridge & valley
    - One sheet indicates one breakline
  - Provide adjacency graph to estimate the topology of the breaklines

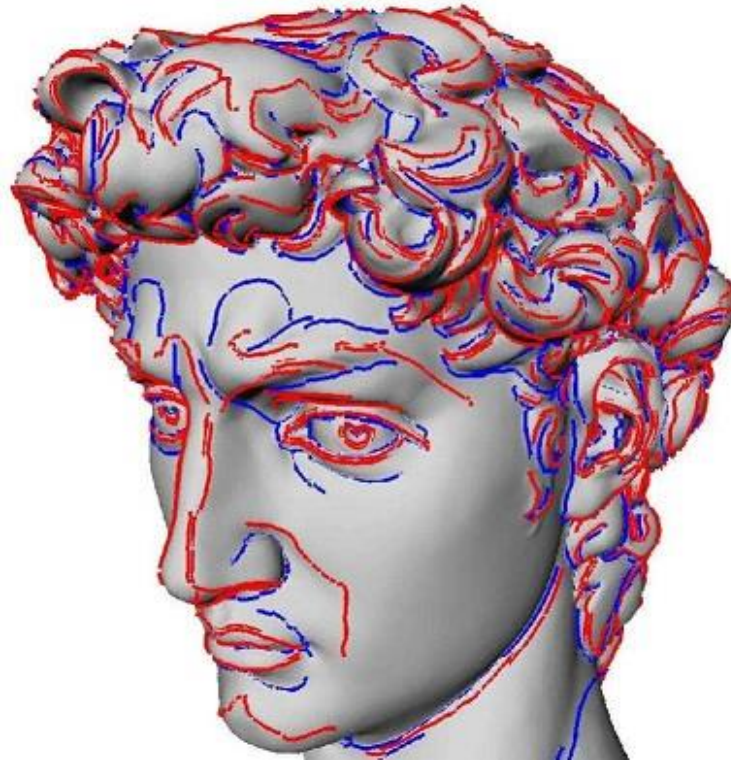
# Existing method: Point cloud



- Need manual intervention for every breakline
- can not handle branching

# Existing method: Mesh

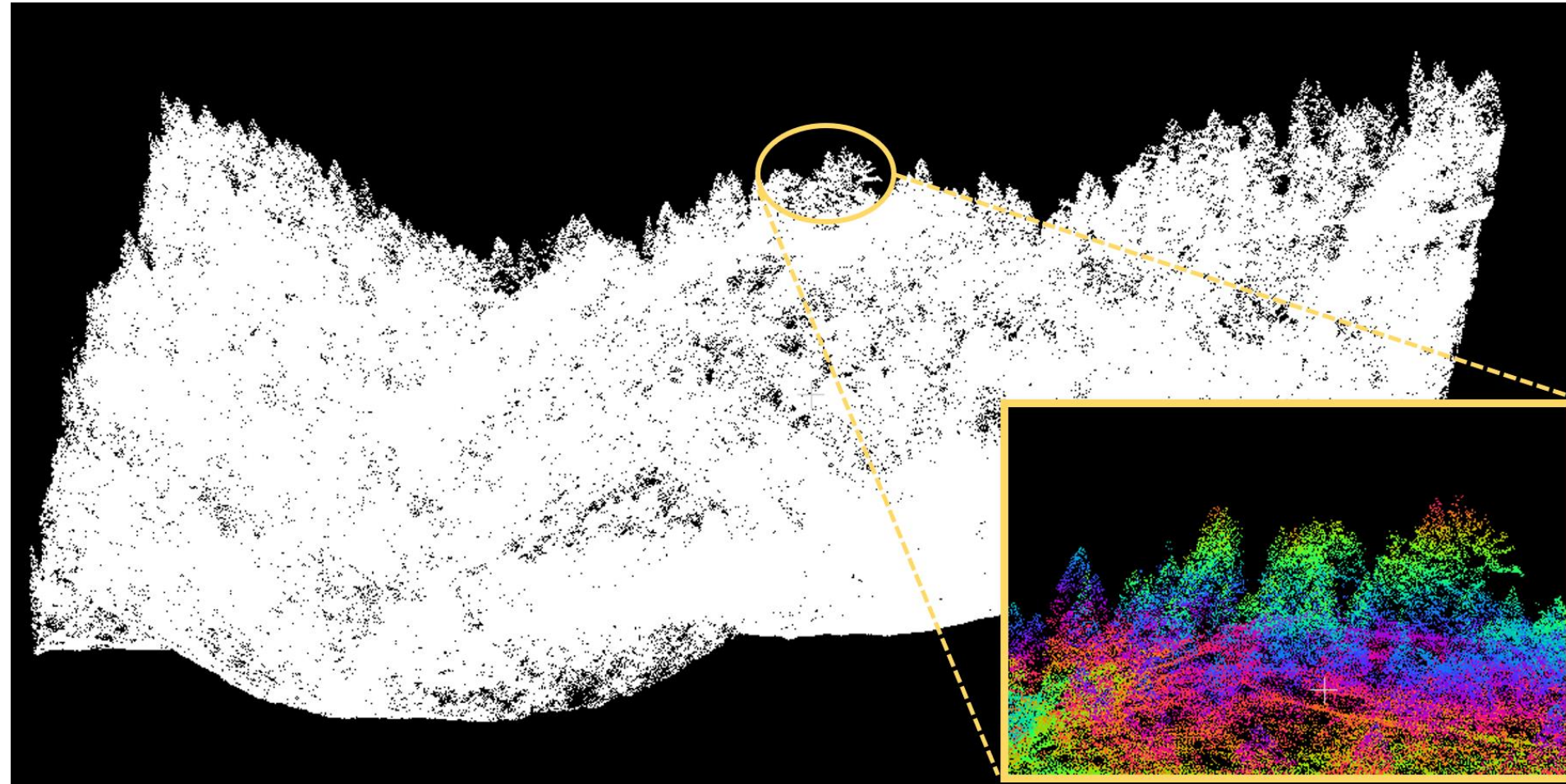
- Input: TIN mesh
- Estimate curvature for each vertices





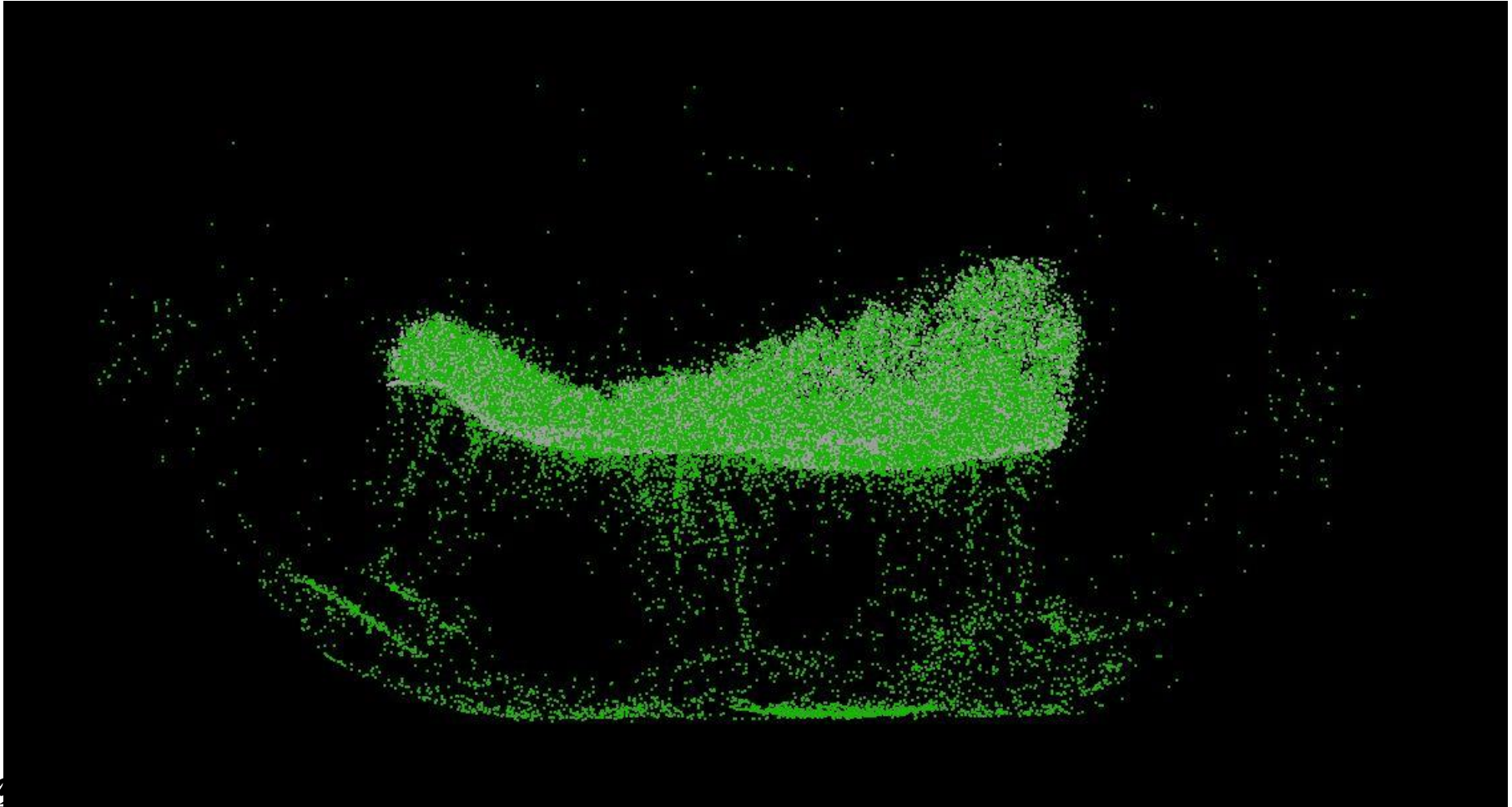
# Scope

- Point cloud contains trees will not be considered



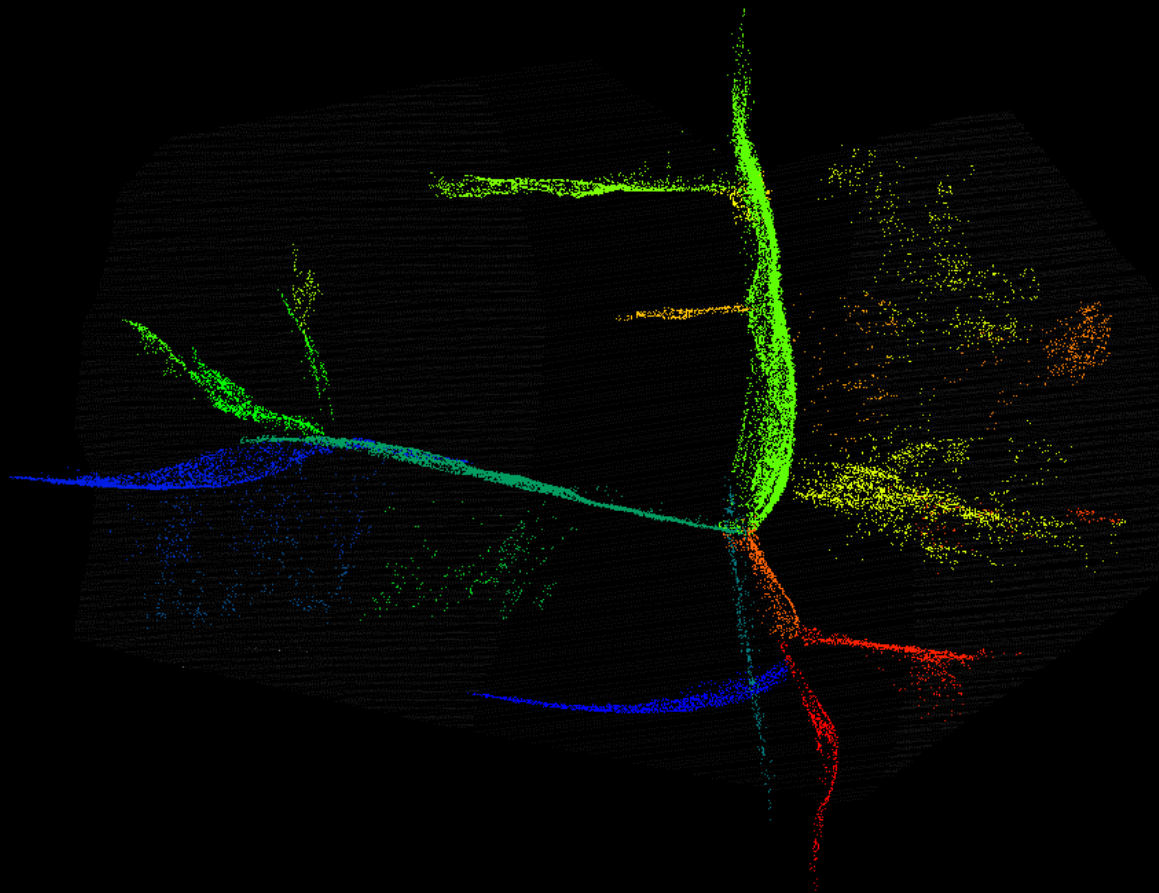
# Scope

- Point cloud contains trees will not be considered

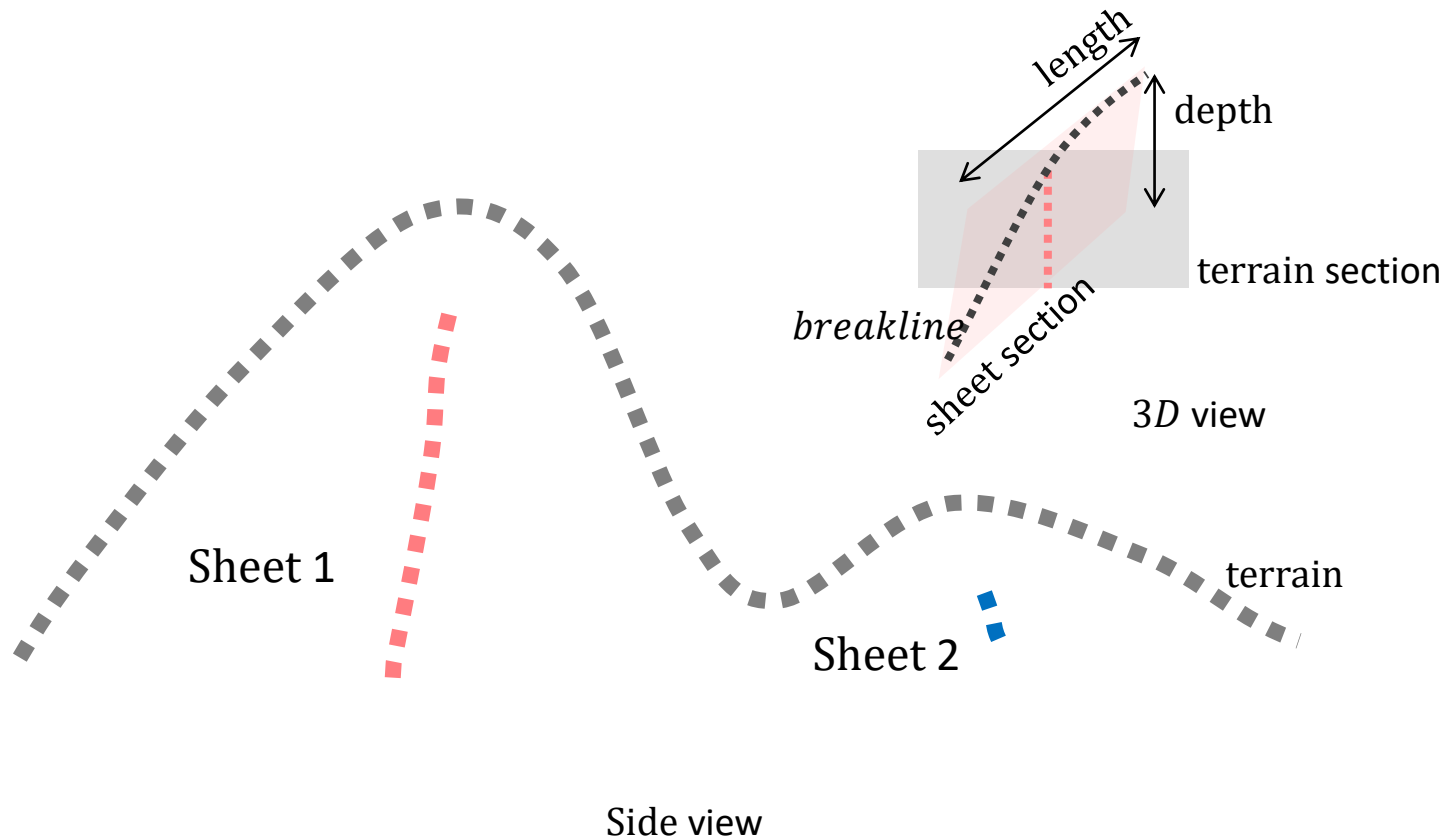


# MAT segmentation

- Medial segmentation → medial sheet

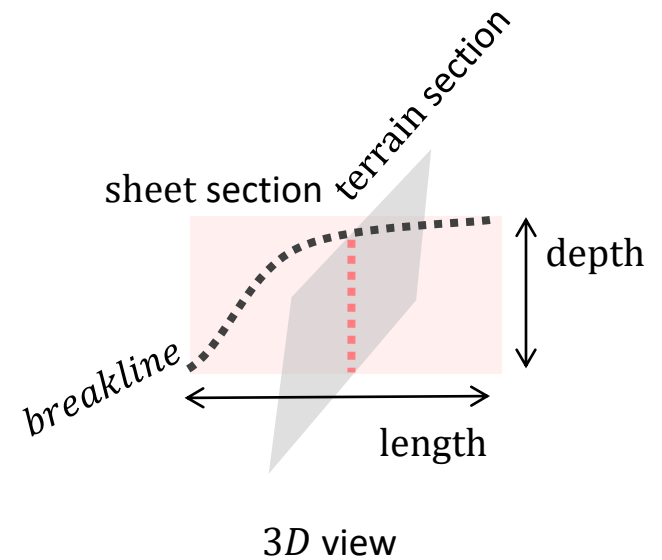
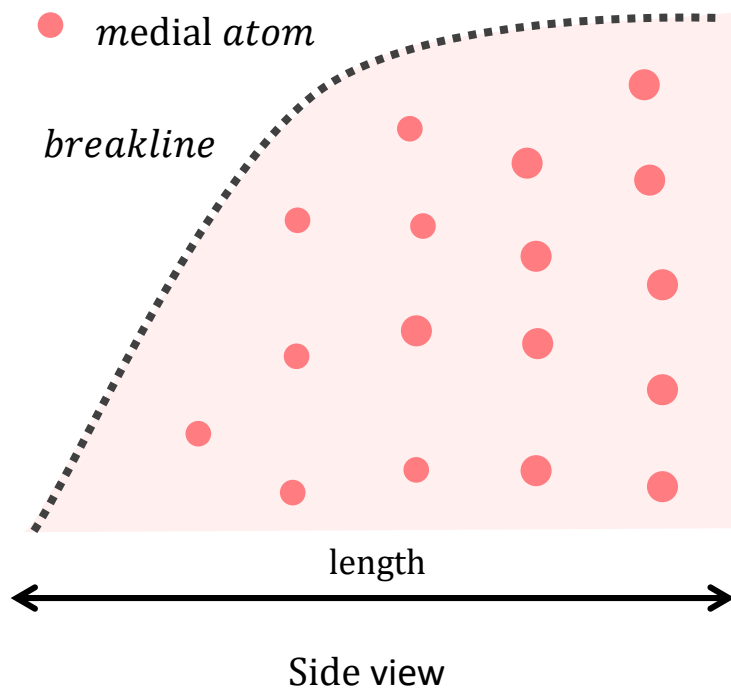


# Medial sheet sideview 1

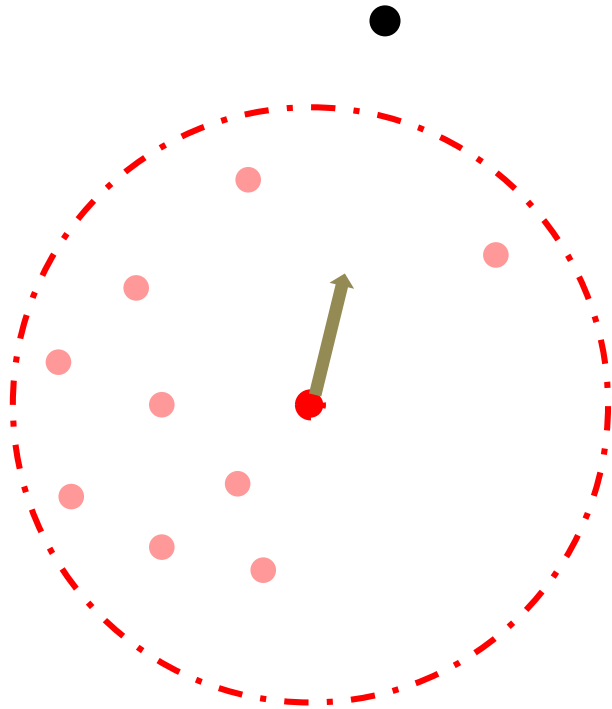




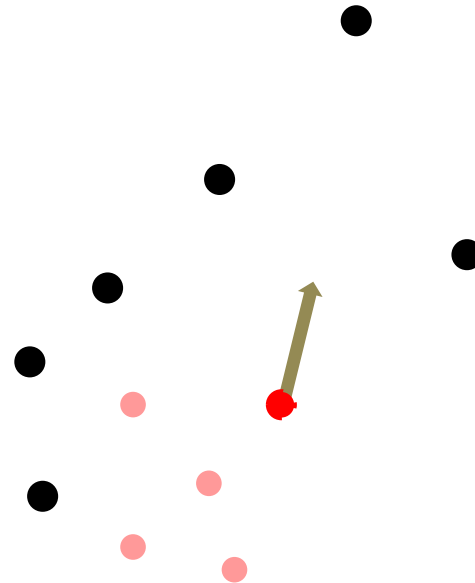
# Medial sheet sideview 2



# Methodology: edge ball detection: neighbor search



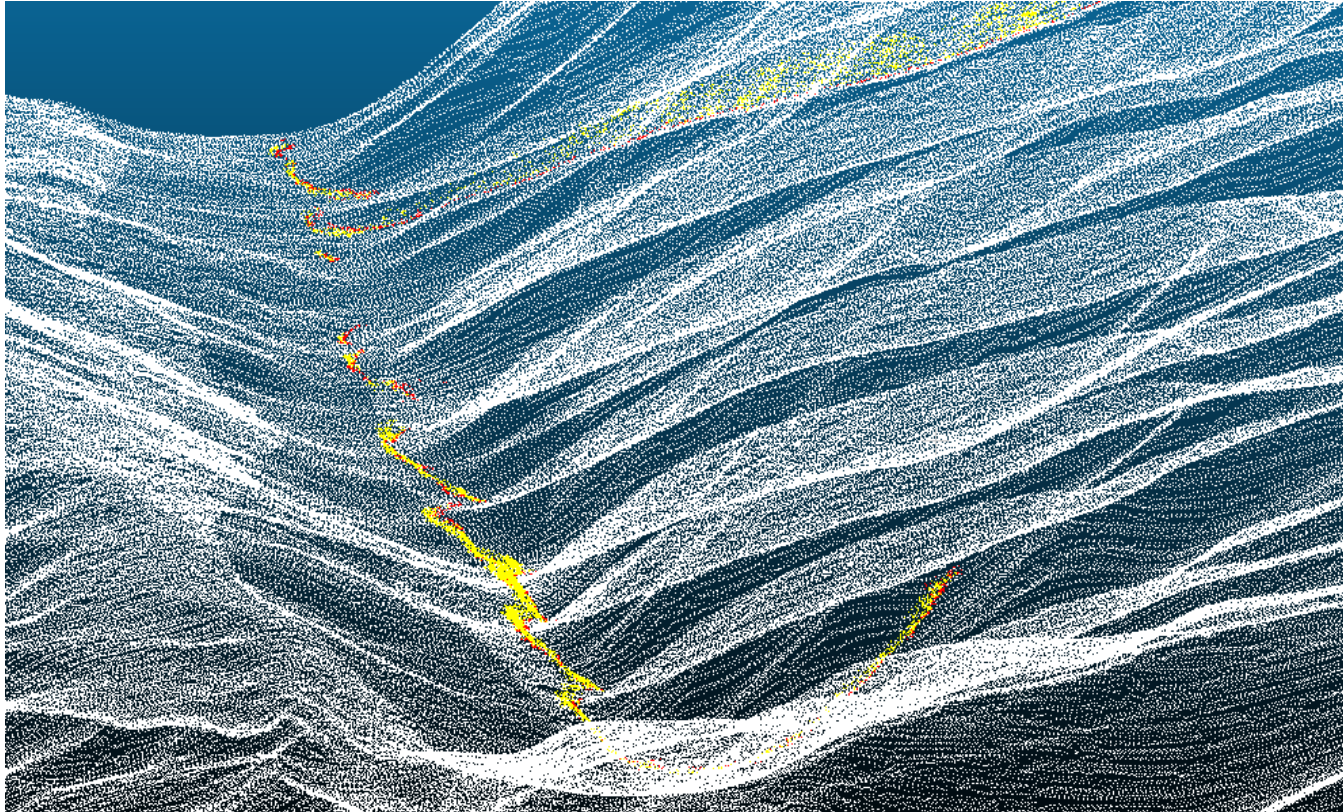
radius neighbour search



5-nearest-neighbour search

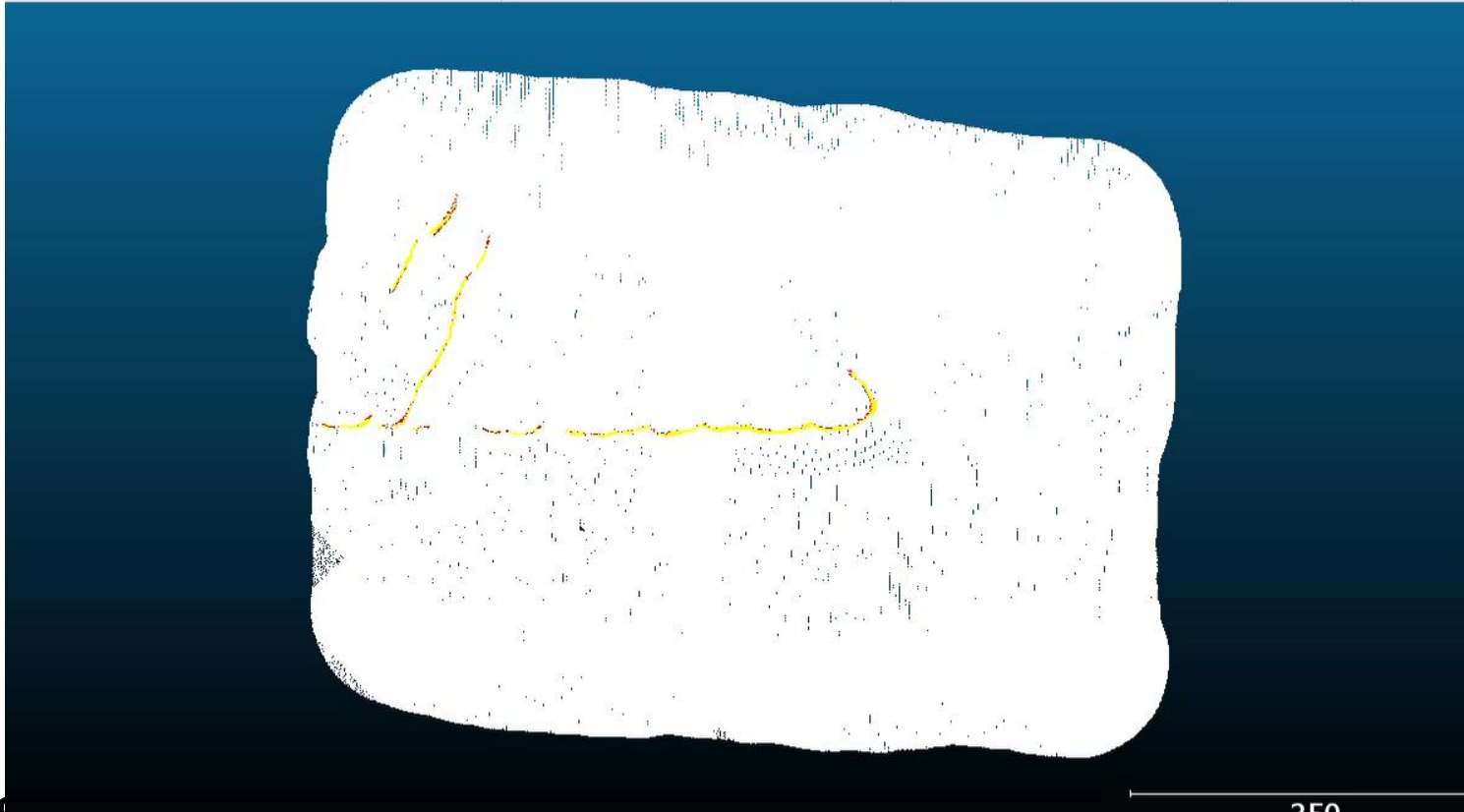
# Methodology: edge ball detection: neighbor search

- Extract candidate points
  - KNN (yellow) vs search radius (red)



# Methodology: edge ball detection: neighbor search

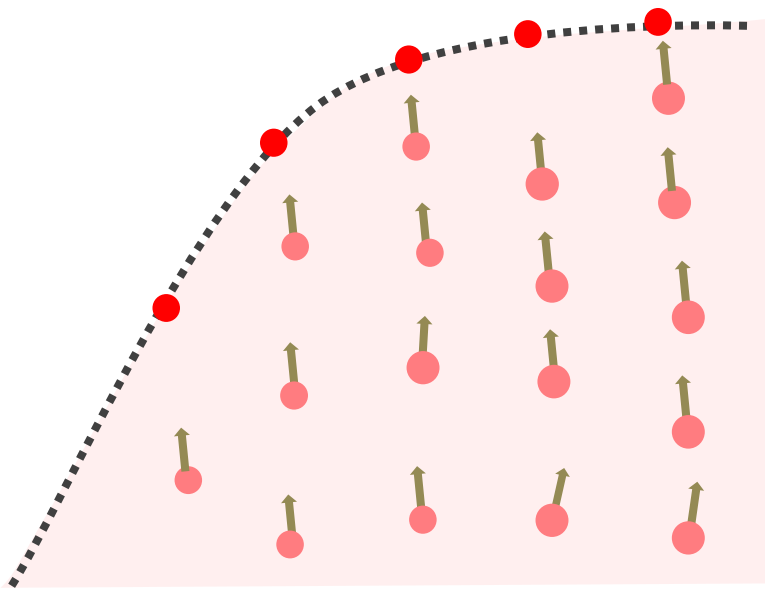
- Extract candidate points
  - KNN (yellow) vs search radius (red)





# Methodology: extracting candidate points

- calculating candidate point coordinates  $(X, Y, Z)$



For each sheet's edge points P:

Find k-nearest-neighbor

$$R = \sum r_i$$

$$\vec{b}_{avg} = \sum (bisector_i \times \frac{r_i}{R})$$

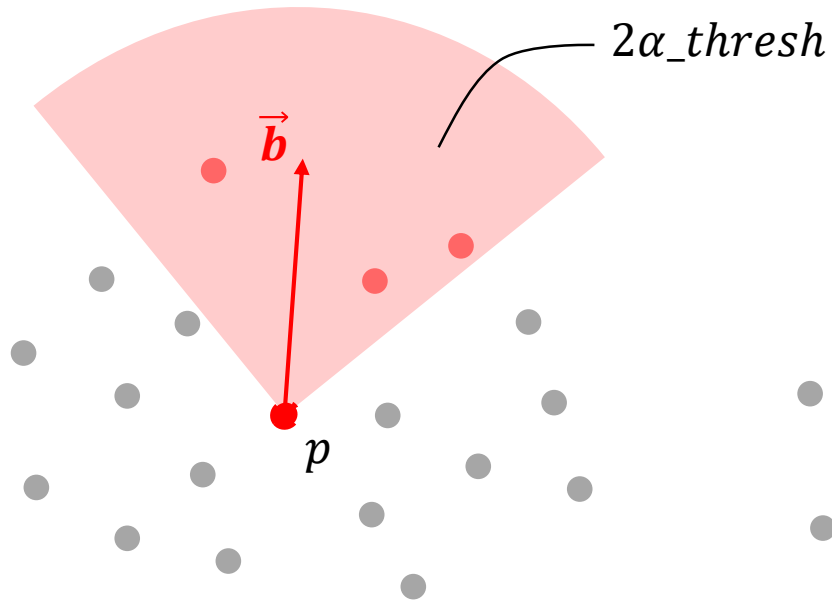
$$X = P_x + r_P \cdot \vec{b}_{avg_x}$$

$$Y = P_y + r_P \cdot \vec{b}_{avg_y}$$

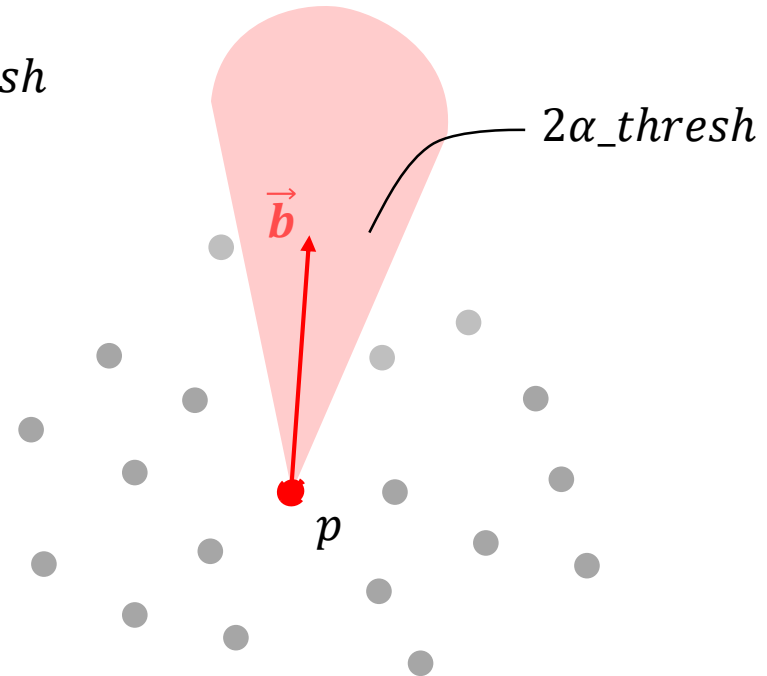
$$Z = P_z + r_P \cdot \vec{b}_{avg_z}$$

# Methodology: extracting candidate points

- Detecting edge balls      Small  $\alpha$  vs large  $\alpha$

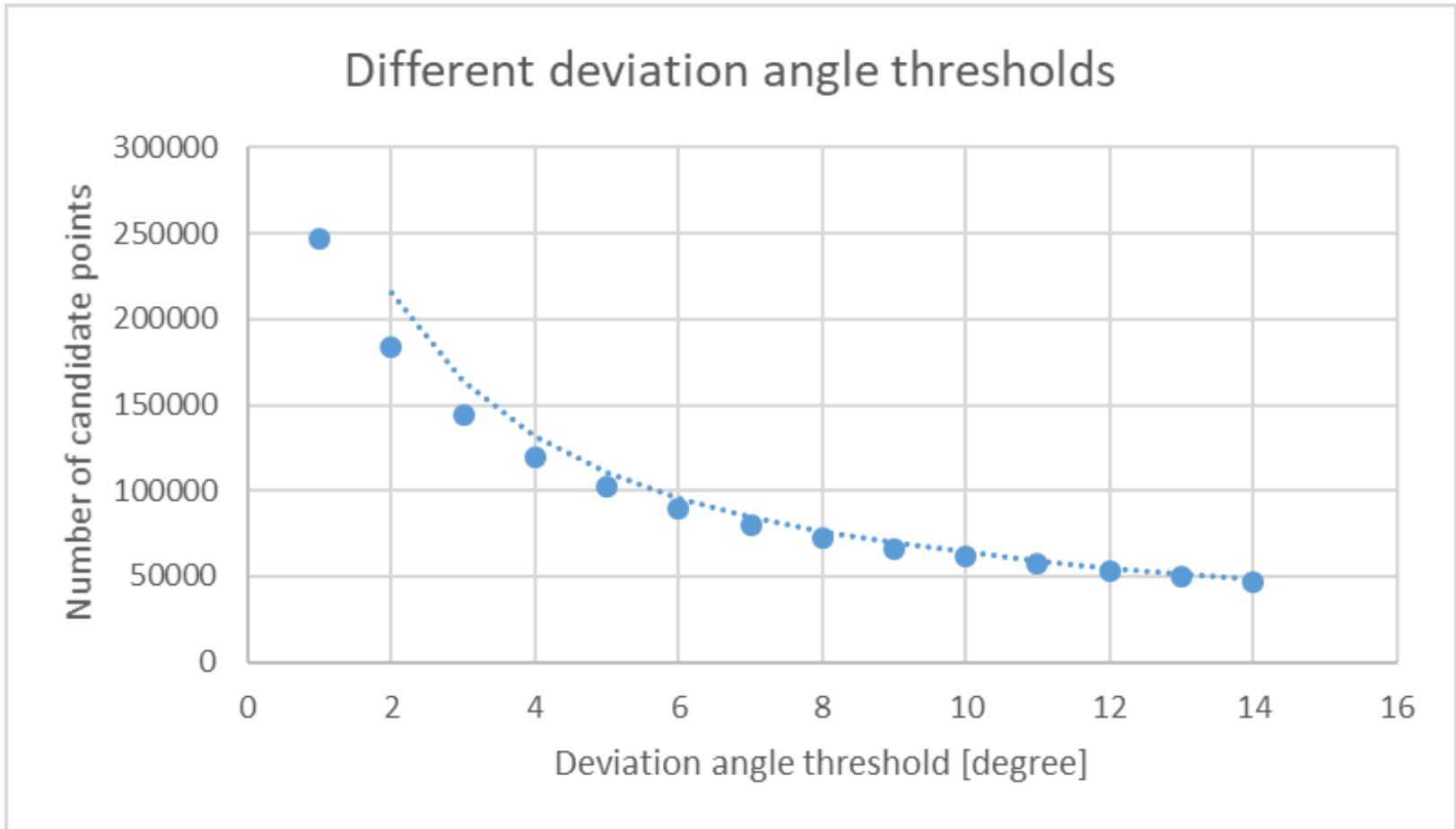


$p$  is not an edge ball



$p$  not an edge ball

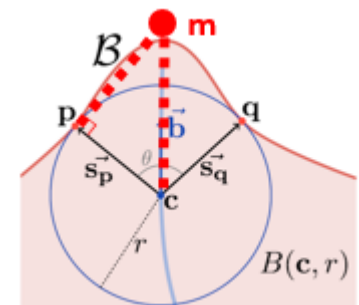
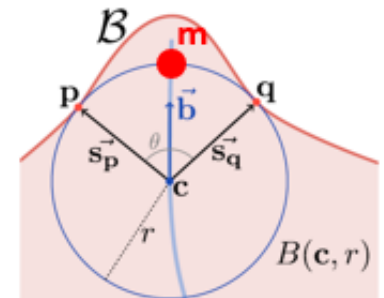
# Methodology: extracting candidate points



total number of medial atom is 336839.

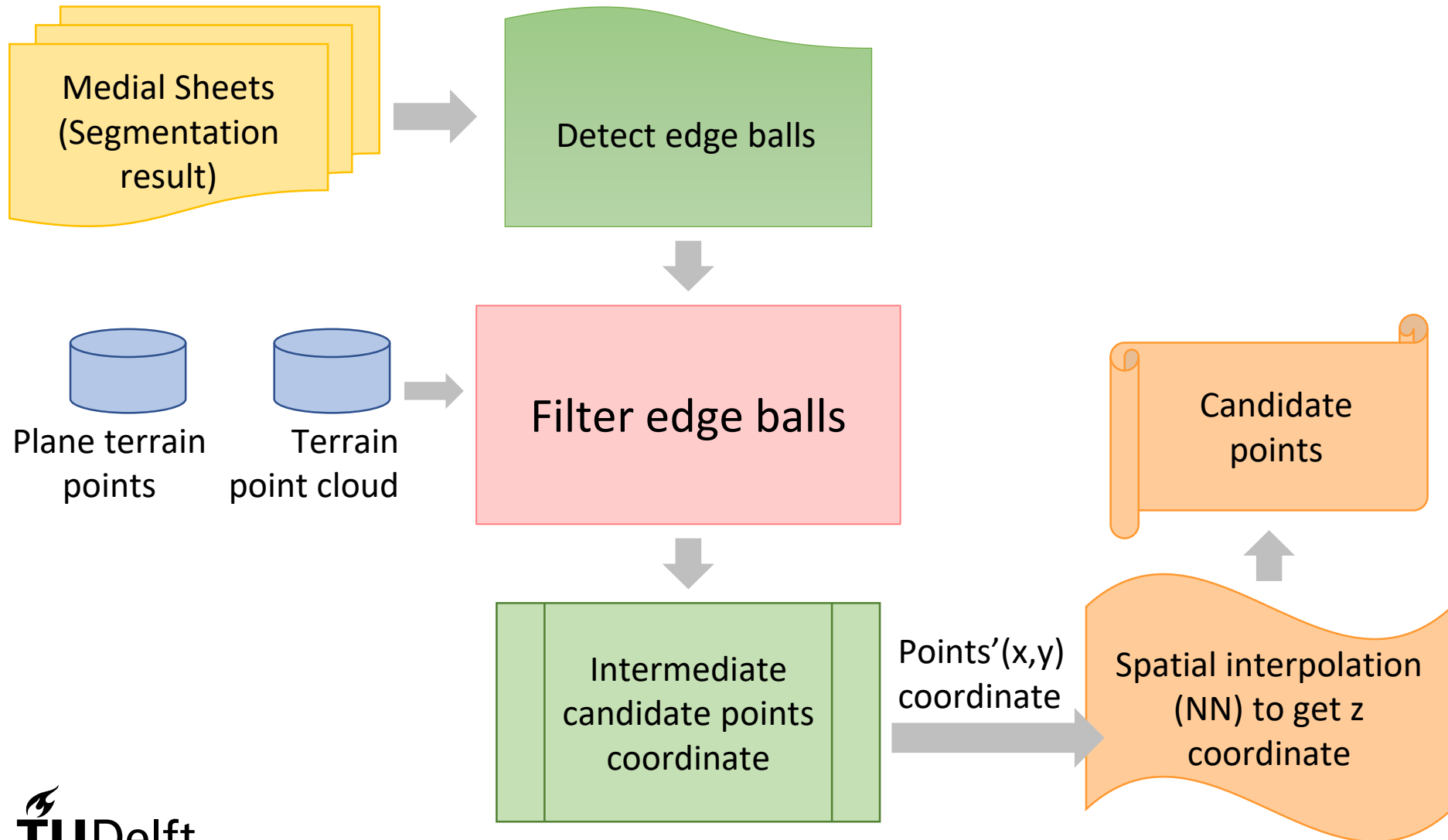
# Methodology: extracting candidate points

- calculating candidate point coordinate  $(X, Y, Z)$ 
  - Intersection of bisector and the medial ball (red)
  - Vs. Intersection of bisector and right- angle side (green)



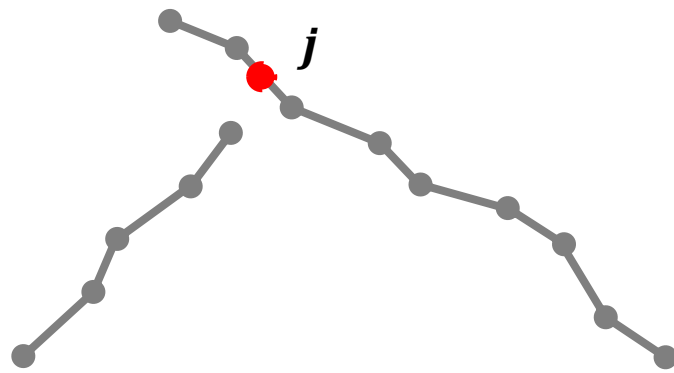


# Methodology: extracting candidate points

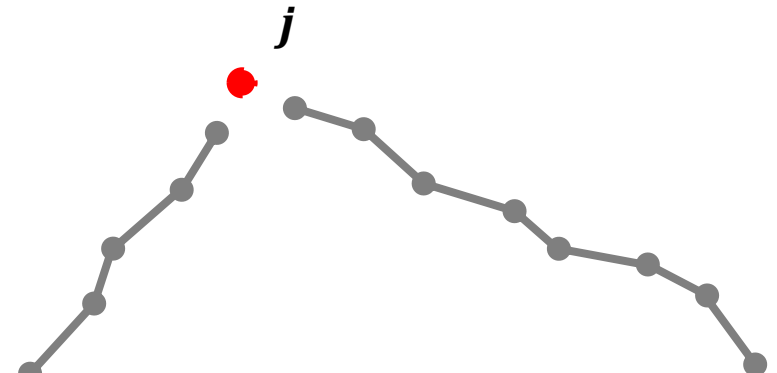


# Methodology: connecting candidate points

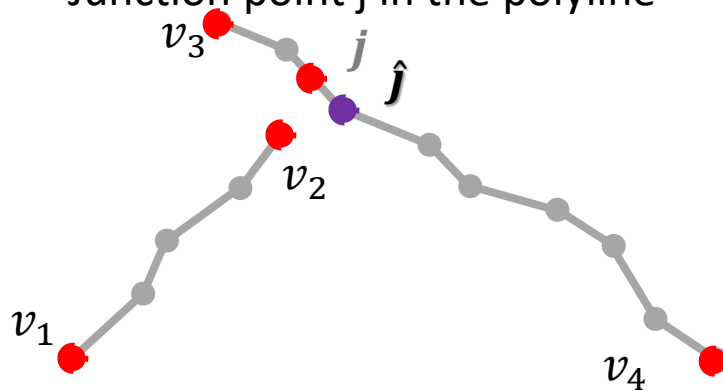
- Finding the topology of breaklines



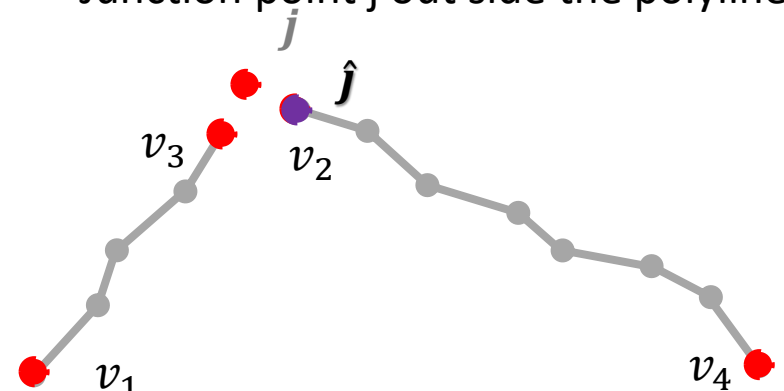
Junction point  $j$  in the polyline



Junction point  $j$  out side the polyline



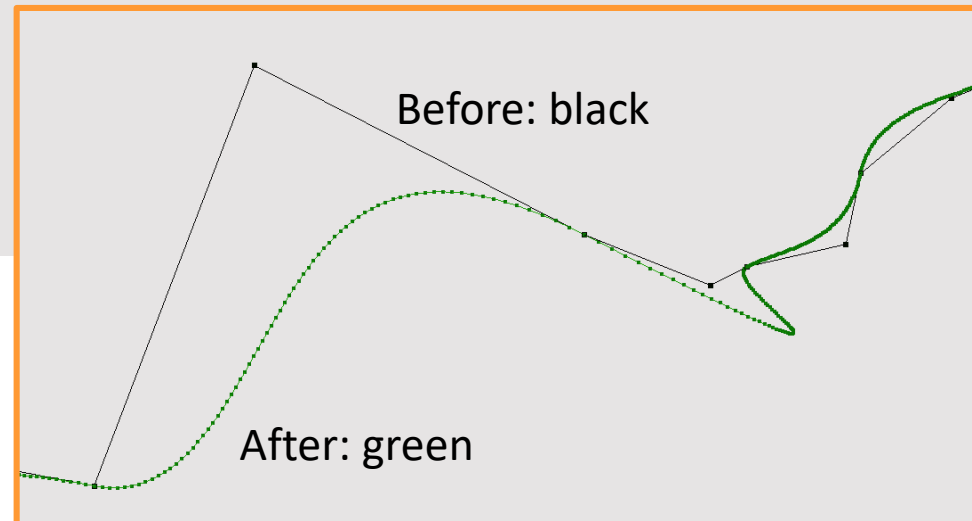
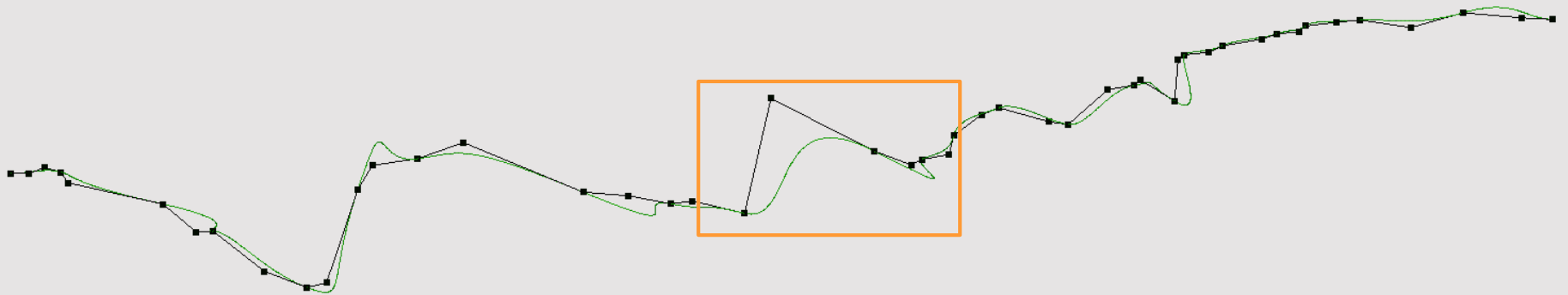
estimated junction  $\hat{j}$



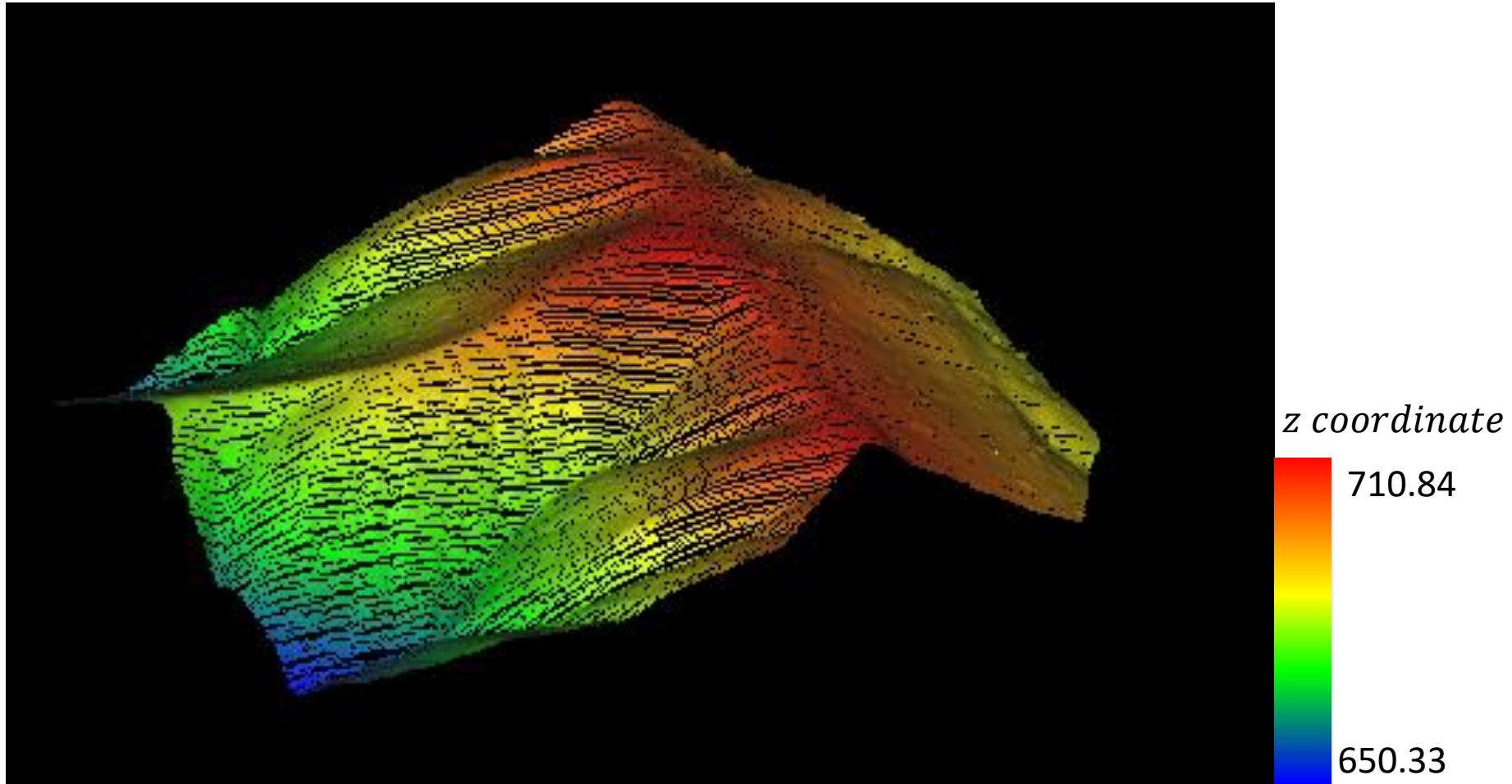
estimated junction  $\hat{j}$

# Methodology: polyline simplification and smoothing

- Smoothing by b-spline



# Data 1



Point count: 72493

Point density: 2pts/m<sup>2</sup> height difference: 60.51 m



# Data 2

