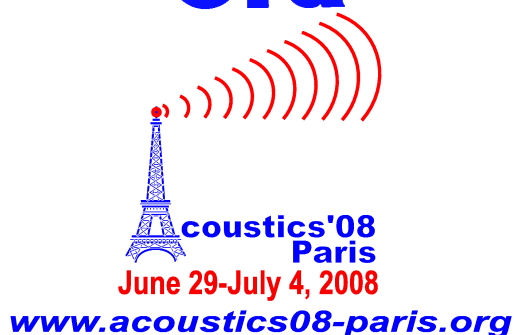




Correcting bathymetry measurements for water sound speed effects using inversion theory

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We present a method to accurately estimate the bathymetry and water sound speed in shallow waters using overlapping swaths obtained from a Multi-Beam Echo Sounder (MBES). The method is designed to correct the errors in the bathymetry manifested mainly in the outer beams of the MBES. The errors are caused by deviations in sound speed, which occur, for example, in estuaries where fresh river water mixes with seawater. Simulations show that we are able to simultaneously estimate the mean sound speed and the bathymetry. This is accomplished by minimising the differences between the MBES measurements in the overlap region using a Levenberg-Marquardt optimisation routine. Preliminary tests on real data revealed that the quality of the results strongly depends on the track-to-track distance. Therefore, we discuss the optimal distance between sailed tracks. The method appears to be a promising tool for the accurate mapping of sea floors in shallow-water areas with a complex sound-speed profile.

1 Introduction

Measuring the bathymetry using Multi-Beam Echo Sounders (MBES) has proven to be a cost-efficient technique to map a seafloor within a reasonable amount of time. In order to cover a large area of the seafloor at once, an MBES sends out an acoustic pulse with a large angular width in the across-track direction. From the time delay between the emission of the pulse and the reception of the echos, the depths can be estimated for nearly the entire ensonified area, provided that the local sound speed in the water column is known. This way, a ship sailing at moderate speed can cover a relatively large area in a limited amount of time. Therefore, MBES instruments are one of the most widely used tools to map seafloors.

However, the sound speed in the water column, which is essential for an accurate determination of the bathymetry, is often not well known. The sound speed at the transducer of the MBES is usually measured continuously using a sensor under the keel of the ship. In addition, probes are used to measure the sound speeds at larger depths. But due to the constantly changing seawater conditions, the obtained bathymetry is sometimes not acceptable. If the true sound speed in the water column differs from the measured sound speeds, then the bathymetry shows 'smiley'- or 'droopy'-like effects [3]. In these cases, the depths measured in the outer parts of the area that are probed per ping are under- or over-estimated. For a flat seafloor, the measured swaths resemble smiley- or droopy-like curves. This is most obvious when parts of the probed areas overlap, because then the measured depths in the overlap regions are not consistent with each other.

A real-life example of the 'smiley' and 'droopy' effects can be found in the fairway toward the entrance to the harbour of Rotterdam, The Netherlands. There, fresh water from the largest Dutch rivers mixes with the salty seawater of the North Sea, which leads to large variations of the sound speed in the water column. MBES measurements near the river-mouth therefore show 'droopy' effects, which is shown in Figure 1. Accurate estimations of the depth of this fairway are of great economic importance, therefore a fast and efficient method is needed to correct the bathymetry for sound-speed effects.

A substantial number of publications have been written on how to correct for these sound speed effects, but with varying success. Many solutions (e.g. [1, 2, 5]) are based on reprocessing the echo-sounder data using improved corrections to the sound speed profile and the ships attitude. More recent papers have introduced methods to estimate the sound speed or sound speed profile by using inversion techniques on overlapping swaths. They obtain the sound speed by minimising

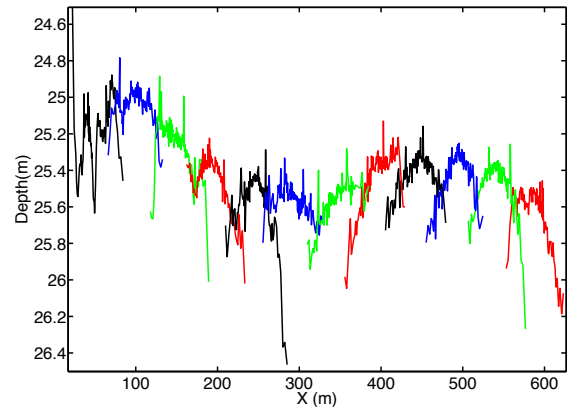


Figure 1: Uncorrected MBES measurements near the entrance to the harbour of Rotterdam. We show twelve overlapping swaths obtained from a dataset provided by the Dutch Directorate-General for Public Works and Water Management (RWS).

a 'cost' function defined by either differences in depth (e.g. [10]) or differences in two-way travel times [9, 8] in the overlap region.

The Dutch Directorate-General for Public Works and Water Management is very interested in a method that corrects their depth measurements of river estuaries for sound-speed effects. In this paper, we aim to design an efficient method that directly estimates the sound-speed and the bathymetry using two or more overlapping swaths in shallow waters. If we assume that the seafloor does not change during the mission, then there may be enough redundancy in the overlapping regions to derive the sound speed in the water column, and thus the bathymetry, from the echo-sounder data by inversion. The method that we present is an improved version of the method presented in [9, 8].

2 MBES Theory

A typical Multi-Beam Echo Sounder (MBES) in operation emits acoustic pulses with an opening angle of about 120–150 degrees in the across-track direction. The two-way travel times along the swath can be derived using a process called beam forming. An MBES is equipped with a receiver array that allows the signal to be split into beams. For every beam at a certain angle, a depth can be derived for the reflection point of the beam on seafloor. However, the process highly depends on the sound speed in the water (see [3] for an overview of beam steering and sound speed profile errors

for several MBES systems).

For flat transducer arrays, errors in the sound speed influence the measurements in two ways [3]:

- Errors in the sound speed at the transducer result in different steering angles.
- Errors in the sound speed profile result in differences in the ray tracing, because of sound refraction and different travel times.

Steering angles in the beam forming process are influenced by the sound speed, because it is used to determine the phase delay that has to be applied to the individual array elements. In order to form a beam (i) with incidence angle θ_i , the phase-delay τ_i that the MBES has to apply to data of neighbouring transducer elements is defined as:

$$\tau_i = \frac{d \sin(\theta_i)}{c_m}, \quad (1)$$

where d is the distance between two elements, and c_m the measured sound speed at the transducer. Because the measured sound speed can be different from the true sound speed c_t , the true beam angle θ_t has to be derived using Snell's Law:

$$\sin(\theta_t) = \frac{c_t}{c_m} \sin(\theta). \quad (2)$$

Sound-speed differences in the water column also affect the ray tracing. For a constant sound speed with depth, the distance to the point on the seafloor where the sound ray reflected is given by $\frac{1}{2}cT$. If the profile is more complicated, the ray path is refracted. The expression for the two-way travel time T then becomes $\frac{1}{2} \int_0^T c(z)dt$, where $c(z)$ represents the sound speed as a function of depth z . The changes in angle due to refraction (θ) follow also from Snell's law:

$$\frac{\sin(\theta(z))}{c(z)} = \frac{\sin(\theta_0)}{c_0} = \text{constant} \quad (3)$$

The curvature of the sound ray leads to a position change of the reflection point on the seafloor with respect to a straight ray, which is able to compromise the accuracy of the obtained bathymetry [3].

3 Methodology

In order to estimate the true bathymetry, we try to exploit the redundancy of measurements in the overlap region between two neighbouring swathes. If we assume that the seafloor does not change during a survey, then the depth measurements in the overlap regions should be the same. With our method, we try to construct the most likely model of the seafloor that fits the measured two-way travel times best.

A common way to compare modelled and measured two-way travel times is to use the least-squares principle. We aim to minimise the function:

$$\chi^2 = \sum_{k=0}^S \sum_{i=0}^N \frac{(t_{k,i} - T_{k,i})^2}{\sigma_m^2}, \quad (4)$$

where N and S are the total numbers of beams and swathes, respectively. The modelled two-way travel times are denoted by $t_{k,i}$ and σ_m is the 1- σ error on the measured two-way travel times ($T_{k,i}$).

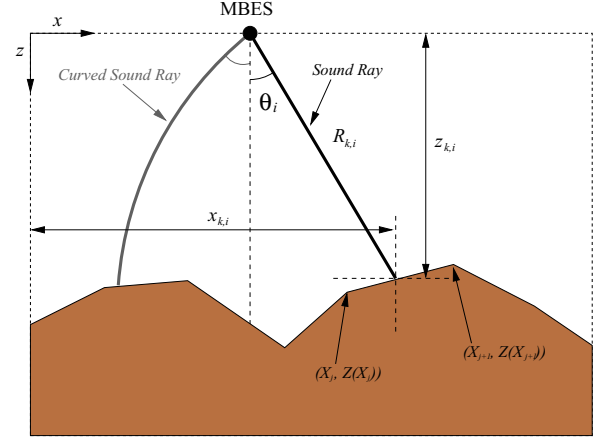


Figure 2: Schematic overview of our model to describe the MBES Measurements. On the right side of the MBES in the figure we show the path of a sound ray through water with a constant sound speed. The ray reflects in between the grid points X_j and X_{j+1} at coordinates $(x_{k,i}, z_{k,i})$. On the left side of the MBES a curved sound ray is shown, which occurs when the sound speed increases linearly with depth.

The minimisation routine needs a model to calculate $t_{k,i}$ for a number of free parameters. We choose to calculate them by modelling the seafloor with an interpolated grid function. For simplicity and computational speed, we approximate the sound speed profile by a constant sound speed (c_k). This may seem inappropriate, but we will show that this approximation is valid for shallow waters with not too strong sound-speed gradients in Section 4.

Figure 2 shows a schematic overview of our model for $t_{k,i}$. For the seafloor, we use a fixed grid of horizontal positions (X_j). At every position X_j there is a variable parameter $Z(X_j)$ denoting the depth. In between the grid points X_j , the depth is interpolated linearly. For every beam i at angle θ_i , we can derive the point where the acoustic beam reflects on the model seafloor $(x_{k,i}, z_{k,i})$. Using the variable parameter for the sound speed c_k , we can subsequently derive $t_{k,i}$.

Mathematically, the function for $t_{k,i}$ can be derived by calculating the intersection between two lines: the sound ray and the line in between the grid points in Figure 2. We need the coordinates of the MBES ($X_{k,MBES}$, $Z_{k,MBES}$), and the grid points ($X_j, Z(X_j)$) and ($X_{j+1}, Z(X_{j+1})$). For every beam i with angle θ_i of swath k , we can write

$$x_{k,i} = \frac{\frac{X_{k,M}}{\tan \theta_i} + Z(X_j) - Z_{k,MBES} - X_j \frac{Z(X_{j+1}) - Z(X_j)}{X_{j+1} - X_j}}{\frac{1}{\tan \theta_i} - \frac{Z(X_{j+1}) - Z(X_j)}{X_{j+1} - X_j}}, \quad (5)$$

$$z_{k,i} = \frac{x_{k,i} - X_{k,M}}{\tan \theta_i} + Z_{k,MBES}. \quad (6)$$

The corresponding two-way travel time $t_{k,i}$ can be written as:

$$t_{k,i} = \frac{2R_{k,i}}{c_k} = \frac{2(x_{k,i} - X_{k,M})}{c_k \sin \theta_i}, \quad (7)$$

where c_k is the sound speed.

In the optimisation routine, the sound speed and the depths $Z(X_j)$ are varied to minimise Eq. (4). We use the Levenberg-Marquardt (LM) algorithm [6, 7] of MATLAB® to find the optimal solution for c_k and $Z(X_j)$. This algorithm is designed to solve non-linear least-square problems.

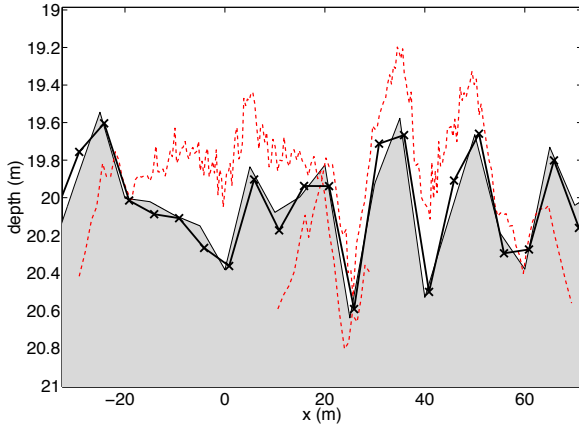


Figure 3: Fit of a simulated seafloor using a constant sound speed. The 'true' seafloor, shown in gray, is well approximated by the thick black line. The black crosses on it show the location of the grid points. The two dashed lines show the initial estimate of the seafloor using the wrong sound speed.

4 Application to simulated data

We test our method by applying the inversion procedure that was described in the previous section to simulated data. For the simulations, we assume a constant $\sigma_{k,i}$ on the two-way travel times of 10^{-4} s, which is about three times the inverse of the typical MBES sampling frequency of 30kHz. We generate an artificial dataset from a seafloor using a random number generator. For an evenly spaced grid of x -coordinates, we randomly generate depth values using a normal distribution with a mean of 20 meters and with a σ_N of 0.3 meters. In between the grid points, the depth is interpolated linearly to form a continuous seafloor.

We calculate the two-way travel times using Eq. (7), our generated seafloor, and a chosen true sound speed of 1480 m s^{-1} for both swaths. Then we deliberately choose a wrong sound speed of 1450 m s^{-1} as initial parameter in our optimisation routine in order to show the droopy effect. In Figure 3, the initial measurements of the seafloor using the wrong sound speed are shown with a dashed line for both swaths. The distance between the tracks is set to 40 meters. The initially measured depths are clearly not consistent with each other and indeed show a 'droopy' effect.

Then, we start the optimisation routine with reasonable initial guesses for $Z(X_j)$. Both sound speeds c_k are set to the 'measured' sound speed. The fit using LM converges to the correct solution within a few iterations. The result is shown in Figure 3. The estimated bathymetry reproduces the true seafloor quite well. Sharp features are not always estimated correctly, depending on the resolution of the grid X_j . Additional simulations show that when the resolution of the grid is increased, the sharp features on the seafloor are also better fitted. However, more resolution also results in longer computation times. Moreover, if the resolution of the grid becomes smaller than the typical distance between two neighbouring beams, then some of the $Z(X_j)$ will not be constrained. This can lead to unwanted artifacts in the bathymetry, like spikes. In order to test whether the constant sound speed approximation is correct, we simulate two-way travel time measurements for a linear sound speed profile ($c(z) = c_0 - gz$). The sound speed c_0 is chosen to be 1480 m s^{-1} and the gradient

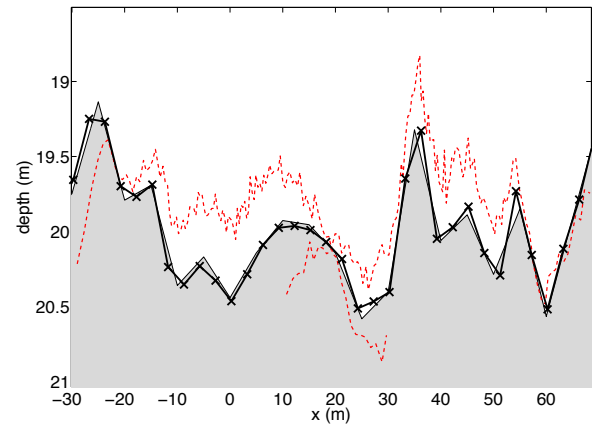


Figure 4: Fit of a simulated seafloor using a constant sound speed, but to a simulated data set including a gradient in the sound speed. The 'true' seafloor, shown in gray, is again well approximated by the thick black line.

g is 0.16 s^{-1} . Then, we fit the simulated data with a constant sound-speed model and a 3-meter grid. The result of the fit is shown in Figure 4. Although the estimated sound speed should now be regarded as a kind of average over the water column instead of the sound speed at the transducer, the estimated bathymetry approximates the true seafloor very well. The constant sound speed model strongly appears to be an adequate approximation for the estimation of the bathymetry of shallow waters that show variation in sound speed in the water column.

4.1 Overlap

For the fairway to the harbour of Rotterdam, the Dutch Directorate-General for Public Works and Water Management uses a common rough-and-ready rule to determine the track-to-track distance. This rule is to sail tracks with a separation of two times the local water depth. For example, if the depth is about 25 meters, then the track separation is usually set to about 50 meters. The inset in Figure 5 shows the situation for two neighbouring swaths. The rays for -45° and 45° are shown, because they approximately hit the sea floor at the same spot.

A preliminary analysis of MBES data obtained at the Rotterdam fairway shows that our method has difficulties estimating the bathymetry correctly. This is due to an effect which is shown in the main plot of Figure 5. The arrow points to a special point. At this spot, which was also noticed in other data by [4, 3], the measured depth is always equal to the true depth. In our case, it acts like a pivot point. Due to the choice for the track separation distance, the points for both swaths fall on top of each other. The angle associated with these special points can be easily calculated. The (true) angle θ_t at that point is:

$$\theta_t = \arcsin \left(\sqrt{\frac{c_t^2}{c_m^2 + c_t^2}} \right) \quad (8)$$

where c_t and c_m are the true and measured sound speeds respectively. Eq. (8) shows that for reasonable values for c_m and c_t , $\theta_t \approx 45^\circ$.

The fact that these points overlap in the dataset is a problem for our method, because we try to minimise the differences between smilies and droopies in the overlap region. It can

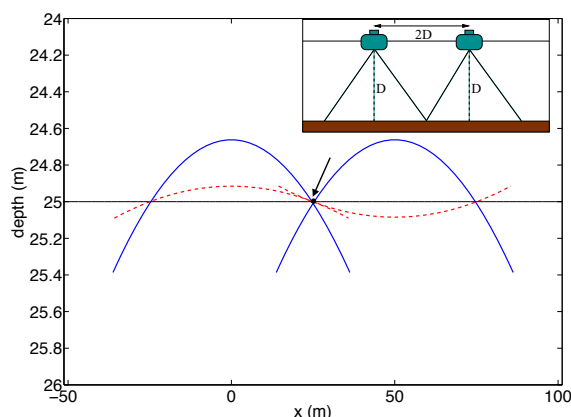


Figure 5: Example of a 'wrong' model solution (red dashed lines) after correcting two droopies. The true seafloor is flat and lies at a depth (D) of 25 meters. The track separation was chosen to be $2D$. The arrow indicates the point where the measured sea floor (the blue droopies) intersects with the ground truth. If these points of two swaths overlap, then they act like a pivot point allowing multiple model solutions of a smiley next to a droopy. The inset shows the sound rays at an angle of 45° for a situation where the track separation is 2 times the depth.

be deduced from Figure 5 that multiple models with varying sound speeds (c_1 and c_2) exist for which χ^2 is very small. If we decrease the sound speed c_1 for the red-dashed droopy on the left, then the sound speed c_2 of the smiley on the right can be increased so that χ^2 remains minimal. Hence, c_1 and c_2 show a negative correlation. The differences in the overlap region between the smiley and the droopy in Figure 5 are small, while it is clearly not the right solution. Only the outermost beams show large enough differences for the model to constrain the true minimum. In practice, these outermost beams suffer from higher noise due to the reduced signal strength. If the signal is detected at all. In datasets of the fairway toward the harbour of Rotterdam, the outermost beam is registered at at most 55° , which leaves little useful points to constrain the sound speeds. Due to the correlation effect and the lack of constraining points in this particular case, the fit often finds the wrong sound speeds.

We can determine the optimal track-to-track distance for our method by simulating measurements for multiple track-to-track distances. In Figure 6, we show the mean difference between estimated and true seafloor as a function of track-to-track distance. In order to make this figure, we ran 4000 simulations in total on fifty randomly generated seafloors. Per seafloor, eighty simulations of an EM 3000 echo sounder with track-to-track distances between 10 and 50 meters were performed. Because the distance between the MBES and the seafloor is in reality about 20 metres, we set the depth to this value in the simulation. The plot shows a clear increase of the error around 40 metres. For convenience, we average the data points within a 2.5-metre wide bin and represent this average with a bar. The individual data points are much more noisy, due to random effects.

Using Figure 6, we can estimate the most efficient track-to-track distance. The plot shows that the largest increase in error occurs around 25-30 metres. In general, the errors are largest in the 30 to 50 metre interval. Below 25 metres, the

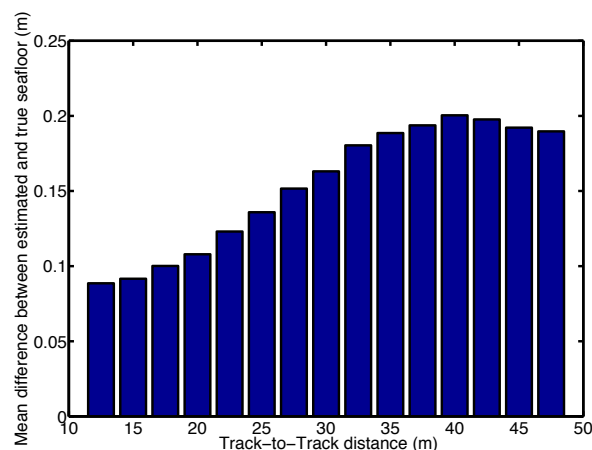


Figure 6: Mean differences between estimated and true seafloor for various track-to-track distances for the Simrad EM 3000 echo sounder.

error slowly levels off due to the instrumental noise that we set to a σ_m of 10^{-4} seconds, which corresponds to about 7.5 centimetres. Considering the shape of the plot, the optimal track-to-track distance would be around 25 metres. At that point, the results are nearly a factor of two better than the results at 40 metres. Unfortunately, the time needed to measure a certain area also increases with about the same factor.

5 Discussion

We have designed a method to correct MBES bathymetry measurements for errors in the sound speed profile. It uses the overlap region between two neighbouring swaths to estimate the 'average' sound speed in the water column and the bathymetry that fits best to the observed two-way travel times. Simulations show that the method performs well for shallow waters, even for rough terrain and linear sound speed profiles. However, the distance between sailed tracks has a large influence on the accuracy of the fit. The uncertainty peaks at a track-to-track distance of two times the water depth. Therefore, it may be greatly beneficial to consider sailing tracks closer to each other.

It should be noted that the assumed uncertainty for the two-way travel time of 10^{-4} seconds (7.5 cm) is very conservative. In reality, the uncertainties may turn out to be centimetres smaller. However, the high mean depth error for track-to-track distances that are about twice the local water depth does not change for different values of σ_m , according to our simulations.

In our method, we do not take into account possible biases in the ship's attitude (roll angle) and tide data. Because biases in these parameters can have a substantial effect on the measurement (see e.g. [3, 10]), we need to make sure that they are removed from a real dataset before we apply our method. At the moment of writing, we are planning a field test in the Maasgeul. We will investigate the accuracy of the method for several track-to-track distances. We aim to find the most optimal sailing parameters to accurately measure the bathymetry of the Maasgeul at the lowest costs.

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