Delft University of Technology

Document Version
Final published version

Citation (APA)
Dash, T., Krasnov, O., Driessen, H., & Yarovoy, A. (2025). Parametric Estimation of Elevation-Doppler Profiles with Phased Array Radar for Precipitation. In *Proceedings of the 2025 22nd European Radar Conference (EuRAD)* (pp. 169-172). (2025 22nd European Radar Conference, EuRAD 2025). IEEE. https://doi.org/10.23919/EuRAD65285.2025.11234103

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright
In case the licence states "Dutch Copyright Act (Article 25fa)", this publication was made available Green Open Access via the TU Delft Institutional Repository pursuant to Dutch Copyright Act (Article 25fa, the Taverne amendment). This provision does not affect copyright ownership.
Unless copyright is transferred by contract or statute, it remains with the copyright holder.

Sharing and reuse
Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy
Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.

Green Open Access added to TU Delft Institutional Repository
as part of the Taverne amendment.

More information about this copyright law amendment
can be found at https://www.openaccess.nl.

Otherwise as indicated in the copyright section:
the publisher is the copyright holder of this work and the
author uses the Dutch legislation to make this work public.

Parametric Estimation of Elevation-Doppler Profiles with Phased Array Radar for Precipitation

Tworit Dash, Oleg Krasnov, Hans Driessen, Alexander Yarovoy

Microelectronics, TU Delft, The Netherlands

{t.k.dash, o.a.krasnov, j.n.driessen, a.yarovoy}@tudelft.nl

***Abstract*— A novel approach for estimating the elevation-Doppler profiles for precipitation using phased array radars is presented. The proposed technique is parametric, where a semi-analytical model of the *Power Spectral Density* (PSD) as a function of the normalized Doppler velocities and the elevation profiles of the Doppler moments is used as a reference. The inverse problem of jointly estimating the Doppler moments at each elevation angle is addressed using the maximum likelihood estimation (MLE). The proposed technique is compared with the traditional non-parametric techniques using synthetic radar echoes and shown to be superior to these traditional techniques.**

***Keywords*— Phased Array, Weather Radars, Precipitation, Elevation-Doppler, Beamforming, Maximum Likelihood Estimation.**

I. INTRODUCTION

Timely accurate updates of the parameters related to precipitation are essential for a number of applications, such as weather prediction, hydrology, and aviation safety. Traditional mechanically scanning (in azimuth) weather radars provide information regarding the precipitation intensity (reflectivity), the mean Doppler velocity, and the Doppler spectral width (these parameters are related to the statistical Doppler moments) at a fixed elevation angle. The radar manually performs sequential azimuthal scans at different elevation angles to obtain a full Doppler moment profile across elevation. These consecutive elevation scans are not coherent and bring issues when it comes to a non-stationary atmosphere.

With the advent of phased array technology (phased array in elevation and mechanical scan in azimuth) [1], it is possible to make electronic scans across the elevation instantaneously. Instantaneous elevation scans can help in applications related to constructing three-dimensional wind fields [2]. However, for such applications, first making instantaneous profiles of the Doppler moments in space is necessary. Although phased array radars bring the advantage of measuring the elevation instantaneously, there remain challenges to processing the data. The signal received at each antenna element is a weighted superposition of the signals received from all the elevation directions of interest. The number of antenna elements is also usually limited, posing challenges in the post-processing. Due to the limited antenna elements (for elevation processing) and limited time of observation (for Doppler processing at a fixed azimuthal sector), the traditional non-parametric techniques for Doppler moment estimation suffer from low resolution and can be very biased [3], [4].

This paper proposes a model-based retrieval of the profiles of the Doppler moments in elevation. The Doppler moments at several elevation angles are estimated jointly with *Maximum Likelihood Estimation* (MLE). The model for the MLE is developed by extending the semi-analytical Doppler *Power Spectral Density* (PSD) presented in [5, Eq. 10] towards the phased array architecture. This two-dimensional (elevation-Doppler velocity) PSD model is a function of the Doppler moments in elevation, the limited observation interval, and the limited number of antenna elements. Therefore, the model accounts for the limited angular and Doppler velocity resolutions. The proposed technique can also accommodate several frames (snapshots). A frame here refers to a radar signal matrix (at a fixed range-azimuth resolution volume) having several echo samples in time (N_T) received at several antennas in the phased array (L). If several frames (i.e., Q) exist, the signal dimension becomes $Q \times L \times N_T$. To accommodate several such frames (which are not necessarily coherent), it is assumed that the Doppler moments do not change drastically during the whole observation interval (stationary atmosphere assumption). In a realistic scenario, such frames can be obtained from several fast azimuthal scans [3], or several closely spaced range resolution volumes in each azimuthal scan [6].

The main body of the paper is organized as follows. Section II explains the signal and covariance model for precipitation with a phased array. Section III presents the inverse problem (parameter estimation) by defining the model of the PSD as a function of the elevation and the Doppler moments. It also explains how the synthetic radar echoes are simulated in time and at each antenna location, along with the optimization procedure to estimate the Doppler moments jointly for all elevation angles of interest. Section IV presents the results and discussions. Finally, section V concludes the paper.

II. SIGNAL MODEL

The antenna-slow time signal model for the phased array radar can be represented as (after range processing):

$$\mathbf{Z} = \mathbf{A}\mathbf{S} + \mathbb{N}, \tag{1}$$

where $\mathbf{Z}$ (size $L \times N_T$) is the measurement matrix, $\mathbf{A}$ steering matrix (size $L \times K$), and $\mathbf{S}$ is the source matrix (size $K \times N_T$), $\mathbb{N}$ is the complex noise matrix (size $L \times N_T$) (zero mean white Gaussian noise with noise variance σ_n^2). Here, L is the number of antennas (with half-wavelength spacing), N_T is the number

Authorized licensed use limited to: TU Delft Library. Downloaded on December 11,2025 at 07:35:42 UTC from IEEE Xplore. Restrictions apply.

of echo samples in time for Doppler processing, and K is the number of elevation directions of interest. One entry of $\mathbf{Z}$ at antenna location l and time t:

$$Z(l,t)=\sum_{k=0}^{K-1}\exp\left(-j2\pi l\frac{d}{\lambda}\sin(\psi_k)\right)s(\theta_k,t)+\mathbb{N}(l,t), \quad (2)$$

where ψ_k is the kth elevation, d is the distance between antenna elements, λ is the radar central wavelength, θ_k is the parameters of the Doppler spectrum from the rain at elevation ψ_k, such as the mean Doppler velocity (μ_k) and Doppler spectrum width (σ_k).

The signal model $s(\theta_k,t)$ at each elevation ψ_k is given by:

$$s(\theta_k,t)=\sum_{m=1}^{M_k}a_{(m,k)}\exp\left(-j\beta_{(m,k)}\right)\exp\left(-j\frac{4\pi T}{\lambda}v_{r,(m,k)}t\right), \quad (3)$$

where $v_{r,(m,k)}$ is the radial velocity of the mth scatterer, $\beta_{(m,k)}$ is the initial phase by the mth scatterer, T is the *Pulse Repetition Time* (PRT), $a_{(m,k)}$ is the amplitude of the echo return of the mth scatterer, M_k is the number of scatterers at elevation ψ_k.

The $v_{r,(m,k)}$ is a random variable:

$$v_{r,(m,k)}\overset{i.i.d.}{\sim}\mathcal{N}(\mu_k,\sigma_k^2), \quad (4)$$

and $\beta_{(m,k)}$ is also a random variable:

$$\beta_{(m,k)}\overset{i.i.d.}{\sim}\mathcal{U}[-\pi,+\pi] \quad (5)$$

Let's assume the sizes of all the scatterers are the same (i.e., $a_{(m,k)}=a_k$). Let's derive the covariance of the signal in space (antenna location) and time (echo sample time):

$$\mathbb{E}[Z(l_p,t_a)^*Z(l_q,t_b)]=\mathbb{E}\left[\left(\sum_{k=0}^{K-1}\exp\left(j2\pi l_p\frac{d}{\lambda}\sin(\psi_k)\right)\right.\right. \quad (6)$$
$$\left.\left.s^*(\theta_k,t_a)\times\sum_{k=0}^{K-1}\exp\left(-j2\pi l_q\frac{d}{\lambda}\sin(\psi_k)\right)s(\theta_k,t_b)\right)\right] + \sigma_n^2\delta(t_a-t_b)\delta(l_p-l_q).$$

As β are uniformly distributed from $-\pi$ to $+\pi$, the covariance is given by:

$$\begin{aligned}C(l_p,t_a;l_q,t_b)&=\mathbb{E}[Z(l_p,t_a)^*Z(l_q,t_b)] \quad (7)\\&=\sum_{k=0}^{K-1}R_k\exp\left(-j2\pi(l_p-l_q)\frac{d}{\lambda}\sin(\psi_k)\right)\\&\times\exp\left(-\frac{1}{2}\left(\frac{4\pi T}{\lambda}\right)^2\sigma_{k,v}^2(t_a-t_b)^2\right)\\&\times\exp\left(-j\left(\frac{4\pi T}{\lambda}\right)\mu_{k,v}(t_a-t_b)\right)+\sigma_n^2\delta(t_a-t_b)\delta(l_p-l_q),\end{aligned}$$

where R_k is the total power ($R_k=a^2M_k$). It is a block matrix of size ($LN_T\times LN_T$). If we use normalized Doppler moments for simplicity, the covariance becomes:

$$\begin{aligned}C&=\sum_{k=0}^{K-1}R_k\exp\left(-j2\pi(l_p-l_q)\frac{d}{\lambda}\sin(\psi_k)\right) \quad (8)\\&\times\exp\left(-2\pi^2\sigma_{k,fn}^2(t_a-t_b)^2\right)\\&\times\exp\left(-j2\pi\mu_{k,fn}(t_a-t_b)\right)+\sigma_n^2\delta(t_a-t_b)\delta(l_p-l_q),\end{aligned}$$

where $\sigma_{k,fn}=\sigma_{k,v}/(2V_a)$ and $\mu_{k,fn}=\mu_{k,v}/(2V_a)$. The V_a is the Nyquist unambiguous velocity $V_a=\lambda/(4T)$.

III. Inverse problem:

A. *Maximum Likelihood Estimation*

Let's define $\mathbf{Z}$ as a Complex Gaussian process in 2D with $\mathbf{0}$ mean and the block covariance $\mathbf{C}$.

$$\mathrm{vec}(\mathbf{Z})\sim\mathcal{CGP}(\mathbf{0},\mathbf{C}(\theta)), \quad (9)$$

where θ is a vector containing all the θ_ks ($\theta=[\theta_1,\theta_2,\ldots,\theta_k]^{\mathrm{T}}$), and $\mathrm{vec}(\mathbf{Z})$ is the vectorized form of the matrix $\mathbf{Z}$. If we want to estimate all the parameters θ for all the elevations, the following marginal log-likelihood of the Gaussian process can be maximized [7]:

$$\begin{aligned}\log(p(\mathbf{Z}|\theta))&=-N_T\times L\log(2\pi)-\log(|\mathbf{C}|) \quad (10)\\&-\mathrm{vec}(\mathbf{Z})^{\mathrm{H}}\mathbf{C}^{-1}\mathrm{vec}(\mathbf{Z}),\end{aligned}$$

where H on the superscript is the Hermitian operator. The estimation problem can, therefore, be expressed as:

$$\hat{\theta}=[\hat{\theta}_k]_{k=1}^{K}=\arg\max_{\theta}\log(p(\mathrm{vec}(\mathbf{Z}|\theta)). \quad (11)$$

The problem with (11) is that the inverse of the block covariance matrix can take a substantial amount of computational resources. To avoid this, the likelihood, i.e., $\left(\left[\mathrm{vec}(\mathbf{Z})^{\mathrm{H}}\mathbf{C}^{-1}\mathrm{vec}(\mathbf{Z})+\log|\mathbf{C}|\right]\right)$ can be carried out in the frequency domain with the help of the expectation of the Schuster's periodogram or the PSD model of the signal ($\mathbf{F}=\mathbb{E}\left[\hat{\mathbf{Z}}^{(\mathrm{PSD})}\right]$). The measurements, in this case, are the PSD of $\mathbf{Z}$, i.e., $\hat{\mathbf{Z}}^{(\mathrm{PSD})}$. Therefore, instead of $\log(\mathbf{Z}|\theta)$, we use, $\log\left(\hat{\mathbf{Z}}^{(\mathrm{PSD})}|\theta\right)$. As the PSD of a Gaussian process is exponentially distributed, the log-likelihood can be rewritten as:

$$\log\left(p(\hat{\mathbf{Z}}^{(\mathrm{PSD})}|\theta)\right)=-\left[\sum_{l=1}^{L}\sum_{n=1}^{N}\left(\log\left(\pi\left(\mathbf{F}_{l,n}(f_l,f_n;\theta)\right)\right)\right.\right. \quad (12)$$
$$\left.\left.+\frac{\hat{\mathbf{Z}}_{l,n}^{(\mathrm{PSD})}(f_l,f_n)}{\mathbf{F}_{l,n}(f_l,f_n;\theta)}\right)\right],$$

where f_l and f_n are the spatial and temporal frequencies, respectively. To avoid the broadening caused by limited resolution (due to limited antenna elements and observation interval, i.e., N_T and L), we chose to use a model of

Authorized licensed use limited to: TU Delft Library. Downloaded on December 11,2025 at 07:35:42 UTC from IEEE Xplore. Restrictions apply.

$\mathbf{F} = \mathbb{E}\left[\hat{\mathbf{Z}}^{(\mathrm{PSD})}\right]$, that also contains N_T and L to get better performance in terms of the bias [7].

$$F(f_l, f_n; \theta) = \sum_{k=0}^{K-1} R_k \sum_{l=1}^{L-1} \left(1 + \left(1 - \frac{l}{L}\right) \cos\left(2\pi l d (f_l - f_k)\right)\right) \tag{13}$$
$$\times \sum_{n=1}^{N_T-1} \left(1 + \left(1 - \frac{n}{N_T}\right) \exp\left(-2\pi^2 \sigma_{k,fn}^2 n^2\right) \times \cos(2\pi n (f_n - \mu_{k,fn}))\right) + \sigma_n^2.$$

If several frames (Q) are obtained, then the dimension for $\hat{\mathbf{Z}}^{(\mathrm{PSD})}$ becomes $Q \times L \times N_T$. In that case, the log-likelihood becomes:

$$\log\left(p(\hat{\mathbf{Z}}^{(\mathrm{PSD})}|\theta)\right) = -Q\left[\sum_{l=1}^{L}\sum_{n=1}^{N_T}\left(\log\left(\pi\left(\mathbf{F}_{l,n}\left(f_l, f_n; \theta\right)\right)\right) + \frac{\sum_{q=1}^{Q} \hat{\mathbf{Z}}_{q,l,n}^{(\mathrm{PSD})}(f_l, f_n)}{Q\mathbf{F}_{l,n}\left(f_l, f_n; \theta\right)}\right)\right], \tag{14}$$

The spatial frequencies are of the form $f_l = \sin(\psi_l)/\lambda$ with elevation direction ψ_l. We choose to represent the MLE solution as:

$$\hat{\theta}^{(\mathrm{MLE})} = [\hat{\theta}_k]_{k=1}^{K} = \arg\max_{\theta} \log\left(p\left(\hat{\mathbf{Z}}^{(\mathrm{PSD})}|\theta\right)\right). \tag{15}$$

1) Simulation

The phased array signals are generated using (1). The parameters θ as a function of the elevation are generated using Gaussian processes with RBF (Radial Basis Function) kernels. More rigorous profiles of these parameters can be developed using relevant atmospheric parameters such as the *Drop Size Distributions* (DSD) parameters [8] or the empirical profiles of Doppler moments as a function of height. However, in this paper, we stick to a generic approach to generate continuous arbitrary profiles of these parameters using Gaussian processes to have more focus on PSD modeling and retrieval.

$$\{\theta_k\}_{k=1}^{K} \sim \mathcal{GP}\left(\mathbf{m}_\theta, \mathbf{C}_\psi(\xi_\theta)\right)\right), \tag{16}$$

where $\mathbf{m}_\theta$ is a vector containing the mean of the parameter θ and $\mathbf{C}_\psi(\xi_\theta)$ is the RBF kernel with parameters ξ. The $_\theta$ subscript refers to the fact that the parameters ξ are different for different θs. The PSD measurements $\hat{\mathbf{Z}}^{(\mathrm{PSD})}$ are computed using the Schuster's periodogram [9] in 2D (across antennas and time).

2) Optimization

The initial values of the parameters for the optimization are taken as the Pulse Pair estimates (PP) [10], [11]. For this, first a Discrete Fourier Transform (DFT) is computed across the antenna elements (with the same number of points as the number of antennas $K_{(\mathrm{DFT})} = L$). Then, the PP approach is applied in slow time for each elevation ψ (ψ is discretized from $-90°$ to $+90°$ with the same number of points $K_{(\mathrm{DFT})}$). The gradients of the log-likelihood is computed analytically.

$$\frac{\partial \log\left(p\left(\hat{\mathbf{Z}}^{(\mathrm{PSD})}|\theta\right)\right)}{\partial\theta} = -\sum_{k=1}^{K}\sum_{n=1}^{N_T}\left[\frac{1}{\mathbf{F}(f_k, f_n; \theta) + \sigma_n^2} - \frac{\hat{\mathbf{Z}}^{(\mathrm{PSD})}(f_k, f_n)}{\left(\mathbf{F}(f_k, f_n; \theta) + \sigma_n^2\right)^2}\right] \times \frac{\partial \mathbf{F}(f_k, f_n; \theta)}{\partial\theta}, \tag{17}$$

The noise variance σ_n^2 is assumed to be known in this optimization. The optimization is performed using the "fmincon" function in MATLAB, which performs a Newton-based constrained optimization [12].

IV. Results and Discussion

The 2D profile (Elevation x Doppler) using the model (13) with the true value of parameters θ, the measurement $\hat{\mathbf{Z}}^{(\mathrm{PSD})}$, and the retrieved profile using MLE is shown in Fig. 1. The number of points in the elevation is $K = 64$, the number of antennas is $L = 64$, and the number of echoes is $N_T = 32$. Only one frame of radar data is used for this estimation $Q = 1$. It can be observed that the MLE reconstruction is adequate enough and agrees with the truth.

The parameter estimate $\hat{\theta}$ results are shown in Fig. 2 with respect to the elevation angle ψ. It can be seen that the non-parametric techniques overestimate the normalized spectral widths σ_{fn} because of limited antenna elements and the number of echo samples. The normalized mean Doppler velocity μ_{fn} is recovered adequately with the non-parametric techniques. The proposed MLE approach outperforms the non-parametric techniques for the estimation of the normalized spectral width σ_{fn}.

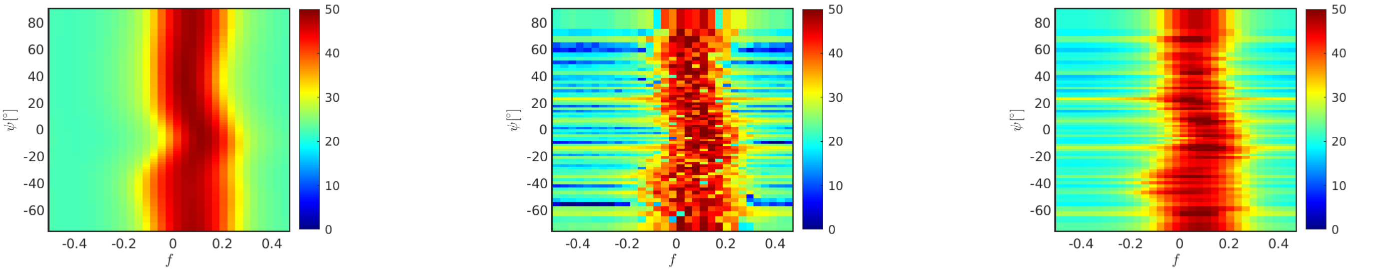
In Fig. 3, the estimation results are shown with an increase in the number of radar frames Q. With increasing Q, the estimation improves. It can be seen that the estimated normalized spectral width $\hat{\sigma}_{fn}$ becomes smoother with increasing Q.

Overall, the proposed technique can reconstruct the Doppler profiles in elevation till a scanning angle of around $\psi = \pm 60°$. Beyond this scanning angle, due to aliasing effects in the angular domain, the estimates become increasingly biased. In other words, for scanning angles larger than $\pm 60°$, the antenna beams are wider, and the spectral energy starts leaking from either direction circularly.

V. Conclusion

A novel parametric maximum likelihood estimation for the profiles of the Doppler moments for precipitation events in elevation (with a phased array radar) is presented in this paper. The estimated profiles are compared against the non-parametric techniques such as the Discrete Fourier Transform (DFT) and the pulse pair statistics (PP 0-lag). The proposed approach is shown to be superior to these traditional techniques. The estimation results are shown by using synthetic radar data. The proposed technique also can accommodate

Authorized licensed use limited to: TU Delft Library. Downloaded on December 11,2025 at 07:35:42 UTC from IEEE Xplore. Restrictions apply.



(a) (b) (c)

Fig. 1. 2D profiles (Elevation x Doppler): (a) True, (b) PSD measurements, (c) MLE reconstruction for one frame of radar data (Q = 1), number of echo samples $N_T = 32$. The number of antennas $L = 64$ and the number of elevation directions $K = 64$.

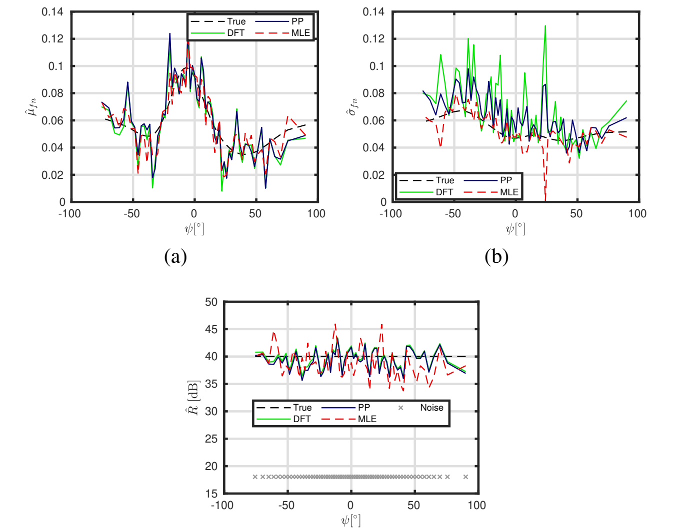


Fig. 2. $\hat{\theta}$ vs ψ: (a) $\hat{\mu}_{fn}$, (b) $\hat{\sigma}_{fn}$, (c) $\hat{R}[dB]$ for one frame of radar data (Q = 1), number of echo samples $N_T = 32$. The number of antennas $L = 64$ and the number of elevation directions $K = 64$.

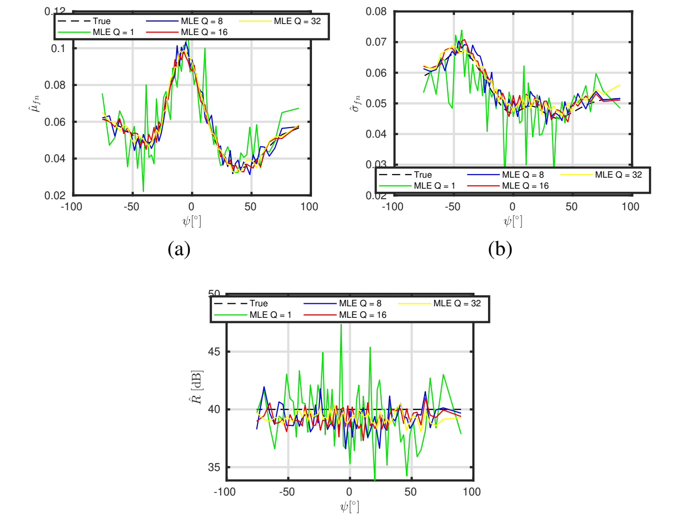


Fig. 3. $\hat{\theta}$ vs ψ: (a) $\hat{\mu}_{fn}$, (b) $\hat{\sigma}_{fn}$, (c) $\hat{R}[dB]$. with increasing number of radar frames Q. The number of echo samples $N_T = 32$, the number of antennas $L = 64$ and the number of elevation directions $K = 64$.

multiple radar frames into the estimation provided that the stationary atmosphere condition is satisfied.

References

[1] *Robin Radar Systems full 3D Advanced Bird Detection Radar - MAX3D*. [Online]. Available: https://www.robinradar.com/max-avian-radar-system.

[2] X. Qiu, Q. Xu, C. Qiu, K. Nai, and P. Zhang, "Retrieving 3D Wind Field from Phased Array Radar Rapid Scans," *Advances in Meteorology*, vol. 2013, 2013, ISSN: 1687-9309. DOI: 10.1155/2013/792631.

[3] T. Dash, H. Driessen, O. Krasnov, and A. Yarovoy, "Doppler Spectrum Parameter Estimation for Weather Radar Echoes Using a Parametric Semi-analytical Model," pp. 1–14, 2023.

[4] T. Dash, A. Gîrdianu, O. A. Krasnov, and A. G. Yarovoy, "Beamforming for a Fast Scanning Phased Array Weather Radar," in *Proceedings of the 20th European Radar Conference*, 2023, pp. 290–293, ISBN: 9782874870743. DOI: 10.23919/eurad58043.2023.10289351.

[5] T. Dash, H. Driessen, O. Krasnov, and A. Yarovoy, "Doppler Spectrum Parameter Estimation for Weather Radar Echoes Using a Parametric Semi-analytical Model," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 62, pp. 1–18, 2024. DOI: 10.1109/TGRS.2023.3338233.

[6] S. M. Torres and D. S. Zrnić, "Whitening in Range to Improve Weather Radar Spectral Moment Estimates. Part I: Formulation and Simulation," *Journal of Atmospheric and Oceanic Technology*, vol. 20, no. 11, pp. 1433–1448, Nov. 2003, ISSN: 0739-0572. DOI: 10.1175/1520-0426(2003)020<1433:WIRTIW>2.0.CO;2.

[7] T. Dash, H. Driessen, O. A. Krasnov, and A. Yarovoy, "Counter-Aliasing Is Better Than De-Aliasing: Application to Doppler Weather Radar With Aperiodic Pulse Train," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 62, pp. 1–17, 2024, ISSN: 0196-2892. DOI: 10.1109/TGRS.2024.3438567.

[8] C. W. Ulbrich, "Natural Variations in the Analytical Form of the Raindrop Size Distribution," *Journal of Applied Meteorology and Climatology*, vol. 22, no. 10, 1764–1775, 1983.

[9] A. Schuster, "On the investigation of hidden periodicities with application to a supposed 26 day period of meteorological phenomena," *Terrestrial Magnetism*, vol. 3, pp. 13–41, 1898. DOI: https://doi.org/10.1029/TM003i001p00013.

[10] D. Sirmans and B. Bumgarner, "Numerical Comparison of Five Mean Frequency Estimators," *Journal of Applied Meteorology*, vol. 14, no. 6, pp. 991–1003, Sep. 1975, ISSN: 0021-8952. DOI: 10.1175/1520-0450(1975)014<0991:NCOFMF>2.0.CO;2.

[11] E. G. Baxa and J. Lee, "The Pulse-Pair Algorithm as a Robust Estimator of Turbulent Weather Spectral Parameters Using Airborne Pulse Doppler Radar," Tech. Rep., 1991.

[12] Mathworks, *Optimization Toolbox™ User's Guide R2024a*, 2024. [Online]. Available: www.mathworks.com.

Authorized licensed use limited to: TU Delft Library. Downloaded on December 11,2025 at 07:35:42 UTC from IEEE Xplore. Restrictions apply.