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Beyond Gaussian beams



Optical and Quantum
satellite communications



Mario Badás Aldecocea

Beyond Gaussian beams

Optical and Quantum satellite communications

Beyond Gaussian beams
Optical and Quantum satellite communications

Dissertation

for the purpose of obtaining the degree of doctor
at Delft University of Technology
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chair of the Board for Doctorates
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A izeko Cristi, a todos y a Paula,

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Summary

Ever since data has been created and stored, communicating it has been of primary importance for the progress and growth of the modern society we live in today. As the amount of generated and stored data has increased, so has the need to transmit it at higher data rates. Furthermore, as the information shared through these communication channels has become more sensitive, the risk of security breaches has also become a major concern. Driven by these needs, optical and quantum communications present promising solutions to address these challenges. The higher bandwidth available in the optical electromagnetic spectrum, compared to the radiofrequency spectrum, can be exploited to achieve higher data rates. Although fiber-optic communication is widely used and reliable, losses in the fiber, along with the need for extensive infrastructure to deploy and operate it, make free-space optical and quantum communications an attractive alternative for bridging large distances. In particular, satellite-based optical communication links could provide a global high-throughput network capable of meeting the connectivity demands of the modern era. Regarding the security requirements mentioned above, the narrow beams produced by lasers in this regime are difficult to intercept, thereby providing an initial layer of physical security. On top of this security layer, by exploiting the principles of quantum mechanics, quantum communication channels can be developed to generate quantum keys which, unlike current encryption techniques, are secure against attacks from future quantum computers. Furthermore, these quantum communication channels could be used for additional purposes in the future, such as connecting remote quantum computers to enhance their individual capabilities.

However, in order to build such a global optical and quantum network based on satellite terminals orbiting the Earth or navigating through deep space, it is necessary to overcome several challenges arising from the harsh space environment. Satellites are subjected to extreme and periodic thermo-mechanical loads, which cause the performance of onboard optical systems to deviate from their ideal operation. Specifically, the optical components in satellite-based optical and quantum communication terminals are affected by optical aberrations, misalignment, and stress-induced birefringence. These effects, together with onboard microvibrations that cannot be fully mitigated by the attitude control system and the pointing optical mechanisms, influ-

ence the pointing of transmitted and received optical and quantum beams in a complex manner. The stochastic nature of the pointing jitter induced by these microvibrations, combined with changes in laser beam propagation caused by optical aberrations, modulates the power received at the detector and, consequently, affects the overall performance of the optical and quantum communication system. At present, it is not well understood or quantitatively characterized how these phenomena jointly impact system performance, nor what strategies can be employed to mitigate their effects. To address this gap, and to enable the reliable deployment of future optical and quantum communication terminals, this thesis aims to identify, model, and mitigate the effects of opto-thermo-mechanical phenomena in satellite-based optical and quantum communication systems.

This thesis first provides an overview of the opto-thermo-mechanical phenomena occurring in satellite-based optical and quantum communication systems (§3 and §4 for optical and quantum systems, respectively). To understand how pointing jitter and optical aberrations interact and affect channel performance, a mathematical model is developed. This model combines a stochastic description of pointing jitter with an optical Fourier propagation framework to quantify their joint effects in an intersatellite optical communication link (see §6). The results show that coma aberrations have the most significant impact on system performance compared to other types of aberrations. Furthermore, the developed model enables the evaluation of several communication performance metrics while accounting for the combined effects of both Seidel and higher-order optical aberrations.

To evaluate the entanglement rates that quantum communication channels can achieve under realistic conditions, §7 and §8 model the propagation of single-photon beams under finite aperture sizes and realistic pointing jitter conditions. Furthermore, these chapters incorporate additional effects arising from imperfect quantum hardware, such as multi-photon events and dark counts. In §7, a model of a satellite constellation equipped with onboard quantum memories for entanglement distribution across intercontinental distances is proposed. This model defines the system requirements and evaluates the corresponding performance of the quantum communication link. It is shown that, to achieve a rate of 100 entangled pairs per second between ground stations over intercontinental distances, approximately ~ 500 memory modes are required. Building on this framework, §8 presents an analysis of satellite-assisted entanglement-based quantum key distribution at a national scale. This chapter proposes an optimization of the photon transmission channel by considering the coupled effects of pointing jitter and the finite size of the transmitter aperture onboard the satellite. By comparing the achievable key rates, this model provides an evaluation of the requirements for spaceborne entangled photon sources to meet the demands of various use cases.

Although understanding and modeling the effects of stochastic pointing jitter is essential for the development of these technologies, it is not sufficient. Part III proposes a novel method to mitigate the impact of residual pointing jitter in satellite terminals. By shaping the transmitted beam, *beyond the fundamental Gaussian mode*, it is shown that the performance of intersatellite optical communication links can be significantly improved. First, §9 proposes the superposition of orthogonally polarized Laguerre-

Gaussian beams to generate beam shapes that outperform the conventional Gaussian beam. Numerical results demonstrate that this approach can achieve power savings of up to 40% while maintaining the same communication performance. In §10, the optical system proposed in §9 is experimentally realized to shape the far-field irradiance of the transmitted beam. By using a spiral phase plate, higher-order annular beams are generated and superposed with the fundamental Gaussian mode. Although both experimental results and analytical derivations show that pure Laguerre-Gaussian beams cannot be achieved with the proposed setup, the resulting beam profiles still outperform the fundamental Gaussian mode. Furthermore, the gains obtained through beam shaping are shown to outweigh the additional losses introduced by the extra optical components, compared to a conventional Gaussian beam. Extending the concept of mitigating pointing jitter through beam shaping, §11 introduces a variational method to determine the optimal far-field irradiance profile. This method shows that the optimal distribution resembles a flat-top beam, capable of providing power savings exceeding 60%. To assess how closely this ideal profile can be approximated, both incoherent superpositions of Laguerre-Gaussian and Hermite-Gaussian modes are investigated. Additionally, a coherent beam-shaping technique is numerically evaluated, where a fundamental Gaussian beam is transformed into a higher-order super-Gaussian profile using an iteratively designed phase mask. These incoherent and coherent approaches are shown to achieve power savings of up to 47% and 50%, respectively.

In this way, this thesis aims to provide a deeper understanding of how various phenomena affecting satellite-based optical and quantum communication systems influence their performance, as well as how these effects can be mitigated. By building upon previous research, this thesis seeks to serve as an additional step for future developments in optical and quantum communications, contributing to a more connected and secure world.

Samenvatting

Sinds data wordt gecreëerd en opgeslagen, is het communiceren ervan van primair belang geweest voor de vooruitgang en groei van de moderne samenleving waarin wij vandaag leven. Naarmate de hoeveelheid gegenereerde en opgeslagen data is toegenomen, is ook de behoefte gegroeid om deze met hogere datasnelheden te verzenden. Bovendien, aangezien de informatie die via deze communicatiekanalen wordt gedeeld steeds gevoeliger is geworden, is het risico op beveiligingslekken eveneens een belangrijk aandachtspunt geworden. Gedreven door deze behoeften vormen optische en kwantumcommunicatie veelbelovende oplossingen om deze uitdagingen aan te pakken. De hogere bandbreedte die beschikbaar is in het optische elektromagnetische spectrum, in vergelijking met het radiofrequente spectrum, kan worden benut om hogere datasnelheden te bereiken. Hoewel glasvezelcommunicatie wijdverbreid en betrouwbaar is, maken verliezen in de vezel, samen met de noodzaak van uitgebreide infrastructuur om deze te implementeren en te exploiteren, vrije-ruimte optische en kwantumcommunicatie een aantrekkelijk alternatief voor het overbruggen van grote afstanden. In het bijzonder zouden satellietgebaseerde optische communicatielinks een wereldwijd netwerk met hoge doorvoer kunnen bieden dat voldoet aan de connectiviteitsbehoeften van het moderne tijdperk. Wat betreft de eerder genoemde beveiligingseisen zijn de smalle bundels die door lasers in dit regime worden geproduceerd moeilijk te onderscheppen, en bieden zij daarmee een eerste fysieke beveiligingslaag. Bovenop deze beveiligingslaag kunnen, door gebruik te maken van de principes van de kwantummechanica, kwantumcommunicatiekanalen worden ontwikkeld die kwantumsleutels genereren die, in tegenstelling tot huidige encryptietechnieken, bestand zijn tegen aanvallen van toekomstige kwantumcomputers. Bovendien kunnen deze kwantumcommunicatiekanalen in de toekomst ook voor aanvullende toepassingen worden gebruikt, zoals het verbinden van op afstand gelegen kwantumcomputers om hun individuele capaciteiten te vergroten.

Om echter een dergelijk wereldwijd optisch en kwantumnetwerk te bouwen op basis van satellietterminals die om de aarde draaien of zich door de diepe ruimte bewegen, is het noodzakelijk om verschillende uitdagingen te overwinnen die voortkomen uit de zware ruimteomgeving. Satellieten worden blootgesteld aan extreme en periodieke thermo-mechanische belastingen, die ervoor zorgen dat de prestaties van optische sys-

temen aan boord afwijken van hun ideale werking. In het bijzonder worden de optische componenten in satellietgebaseerde optische en kwantumcommunicatieterminals beïnvloed door optische aberraties, uitlijningfouten en spanningsgeïnduceerde dubbelbreking. Deze effecten, samen met microtrillingen aan boord die niet volledig kunnen worden gecompenseerd door het standregelsysteem en de richtmechanismen, beïnvloeden de uitlijning van uitgezonden en ontvangen optische en kwantumbundels op een complexe manier. Het stochastische karakter van de richtingsfluctuaties veroorzaakt door deze microtrillingen, gecombineerd met veranderingen in de voortplanting van de laserbundel als gevolg van optische aberraties, moduleert het vermogen dat door de detector wordt ontvangen en beïnvloedt daarmee de algehele prestaties van het optische en kwantumcommunicatiesysteem. Momenteel is het niet goed begrepen of kwantitatief gekarakteriseerd hoe deze fenomenen gezamenlijk de systeemprestaties beïnvloeden, noch welke strategieën kunnen worden toegepast om hun effecten te beperken. Om deze kenniskloof te dichten en de betrouwbare inzet van toekomstige optische en kwantumcommunicatieterminals mogelijk te maken, heeft deze proefschrift tot doel de effecten van opto-thermo-mechanische fenomenen in satellietgebaseerde optische en kwantumcommunicatiesystemen te identificeren, modelleren en mitigeren.

Deze proefschrift geeft eerst een overzicht van de opto-thermo-mechanische fenomenen die optreden in satellietgebaseerde optische en kwantumcommunicatiesystemen (§3 en §4 voor respectievelijk optische en kwantumsystemen). Om te begrijpen hoe richtingsfluctuaties en optische aberraties met elkaar interageren en de kanaalprestaties beïnvloeden, wordt een wiskundig model ontwikkeld. Dit model combineert een stochastische beschrijving van richtingsfluctuaties met een optisch Fourier-voortplantingskader om hun gezamenlijke effecten in een intersatelliet optische communicatielink te kwantificeren (zie §6). De resultaten tonen aan dat coma-aberraties de meest significante impact hebben op de systeemprestaties in vergelijking met andere soorten aberraties. Bovendien maakt het ontwikkelde model het mogelijk om verschillende communicatieprestatieparameters te evalueren, waarbij rekening wordt gehouden met de gecombineerde effecten van zowel Seidel- als hogere-orde optische aberraties.

Om de verstrengelingssnelheden te evalueren die kwantumcommunicatiekanalen onder realistische omstandigheden kunnen bereiken, introduceren §7 en §8 het modelleren van de voortplanting van enkel-fotonbundels onder eindige apertuurgroottes en realistische richtingsfluctuaties. Daarnaast nemen deze hoofdstukken aanvullende effecten mee die voortkomen uit imperfecte kwantumhardware, zoals multi-fotongebeurtenissen en donkertellingen. In §7 wordt een model voorgesteld van een satellietconstellatie uitgerust met kwantumgeheugens aan boord voor de distributie van verstrengeling over intercontinentale afstanden. Dit model definieert de systeemeisen en evalueert de bijbehorende prestaties van de kwantumcommunicatielink. Er wordt aangetoond dat, om een snelheid van 100 verstrengelde paren per seconde tussen grondstations over intercontinentale afstanden te bereiken, ongeveer ~ 500 geheugensmodi vereist zijn. Voortbouwend op dit kader presenteert §8 een analyse van satellietgeassisteerde, op verstrengeling gebaseerde kwantumsleuteldistributie op nationaal niveau. Dit hoofdstuk stelt een optimalisatie van het fotontransmissiekanaal voor door rekening te houden met de gekoppelde effecten van richtingsfluctuaties en de eindige grootte van de

zendapertuur aan boord van de satelliet. Door de haalbare sleutelsnelheden te vergelijken, biedt dit model een evaluatie van de vereisten voor ruimtegebaseerde bronnen van verstrengelde fotonen om te voldoen aan de eisen van verschillende toepassingen.

Hoewel het begrijpen en modelleren van de effecten van stochastische richtingsfluctuaties essentieel is voor de ontwikkeling van deze technologieën, is dit niet voldoende. Deel III stelt een nieuwe methode voor om de impact van resterende richtingsfluctuaties in satellietterminals te mitigeren. Door de uitgezonden bundel te vormen, *voorbij de fundamentele Gaussische modus*, wordt aangetoond dat de prestaties van intersatelliet optische communicatielinks aanzienlijk kunnen worden verbeterd. Ten eerste stelt §9 de superpositie van orthogonaal gepolariseerde Laguerre-Gauss-bundels voor om bundelvormen te genereren die beter presteren dan de conventionele Gaussische bundel. Numerieke resultaten tonen aan dat deze aanpak vermogensbesparingen tot 40% kan opleveren bij behoud van dezelfde communicatieprestaties. In §10 wordt het optische systeem voorgesteld in §9 experimenteel gerealiseerd om de verre-veld intensiteitsverdeling van de uitgezonden bundel te vormen. Door gebruik te maken van een spiraalfaseplaat worden hogere-orde annulaire bundels gegenereerd en gesuperponeerd met de fundamentele Gaussische modus. Hoewel zowel experimentele resultaten als analytische afleidingen aantonen dat zuivere Laguerre-Gauss-bundels niet kunnen worden verkregen met de voorgestelde opstelling, presteren de verkregen bundelprofielen nog steeds beter dan de fundamentele Gaussische modus. Bovendien blijkt dat de winst die wordt verkregen door bundelvorming groter is dan de extra verliezen die worden geïntroduceerd door de extra optische componenten, vergeleken met een conventionele Gaussische bundel. Voortbouwend op het idee om richtingsfluctuaties te mitigeren door middel van bundelvorming, introduceert §11 een variatiemethode om het optimale verre-veld intensiteitsprofiel te bepalen. Deze methode toont aan dat de optimale verdeling lijkt op een flat-top bundel, die vermogensbesparingen van meer dan 60% kan opleveren. Om te beoordelen hoe dicht dit ideale profiel kan worden benaderd, worden zowel incoherente superposities van Laguerre-Gauss- en Hermite-Gauss-modi onderzocht. Daarnaast wordt een coherente bundelvormtechniek numeriek geëvalueerd, waarbij een fundamentele Gaussische bundel wordt omgevormd tot een hogere-orde super-Gaussisch profiel met behulp van een iteratief ontworpen fase-masker. Deze incoherente en coherente benaderingen blijken vermogensbesparingen tot respectievelijk 47% en 50% te kunnen realiseren.

Dit werk heeft tot doel een dieper inzicht te verschaffen in hoe verschillende fenomenen die optreden in satellietgebaseerde optische en kwantumcommunicatiesystemen hun prestaties beïnvloeden, evenals hoe deze effecten kunnen worden gemitigeerd. Door voort te bouwen op eerder onderzoek, beoogt deze proefschrift een aanvullende stap te zijn voor toekomstige ontwikkelingen in optische en kwantumcommunicatie, en zo bij te dragen aan een meer verbonden en veilige wereld.

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Mario Badas Aldecocea
Cambridge, July 2026

Part I



Introduction and background

Chapter 1

Introduction

There must be no barriers to freedom of inquiry.

Robert Oppenheimer

THE act of transferring information lies at the heart of all communication systems, whether natural or engineered. At its most fundamental, information can be conveyed through variations in physical quantities that can be controlled by a sender and detected by a receiver. Over the course of history, various physical carriers – such as mechanical vibrations, electrical signals, and electromagnetic waves – have been exploited to serve this purpose. Among these carriers, light plays a particularly intriguing and increasingly important role due to its unique properties in both classical and quantum regimes.

In the natural world, one of the earliest and most evolutionarily successful methods for conveying information between animals has been sound. Many animals, including humans, use acoustic waves to encode and decode information. Sound propagates through a medium, usually air or water, by mechanical vibrations, and has long served as a reliable and biologically feasible way to share information over short distances [1]. With the development of human societies and technology, however, the demand for faster, longer-range, and more reliable means of communication led to innovations that extended far beyond acoustic signaling. The advent of electrical communication in the 19th century marked a significant technological leap. The invention of the telegraph, followed by Alexander Graham Bell's telephone, enabled information to be transmitted over long distances using conductive wires. These systems modulated electrical currents to represent bits of information – typically binary on-off patterns – paving the way for modern digital communication. Parallel to these developments, wireless communication emerged as a new paradigm. Initially relying on the radiofrequency spectrum of electromagnetic waves, wireless systems enabled information to be transmitted without the need for a physical medium. The first successful demonstration of wireless telegraphy by Popov, Marconi, and others in the late 19th century exploited the properties of electromagnetic waves to carry Morse code signals through the air [2]. This breakthrough introduced humanity to a new era of long-distance communication, laying the foundation for modern wireless technologies such as radio, television, and eventually mobile networks.

Among the various spectral ranges of electromagnetic radiation, light – encompassing visible, infrared, and ultraviolet wavelengths – occupies a special position. Its high frequency makes it well-suited for carrying information at high rates. Moreover, as both a wave and a particle, light offers unique capabilities for communication that span both classical and quantum domains. From fiber optics that underpins the modern internet to the emerging fields of free-space optical and quantum communication, light continues to redefine the limits of what is possible in information transfer.

The higher carrier frequency of optical electromagnetic waves, compared to those in the radiofrequency spectrum, enables significantly higher modulation bandwidths for encoding information. This property directly supports the increasing demand for high data rates in modern communication systems. While optical fiber networks exploit this advantage and form the backbone of today's internet infrastructure, free-space optical communication offers complementary benefits. In particular, it enables connectivity over large distances without the need for extensive physical infrastructure, making it especially attractive for remote or hard-to-access regions. Additionally, free-space optical systems typically experience lower channel loss and provide an inherent level of

physical security, as the interception of the narrow laser beams becomes considerably more challenging than in radiofrequency-based systems.

Beyond classical optical communication, further advantages can be realized by harnessing the principles of quantum mechanics. Quantum communication leverages uniquely quantum phenomena –such as superposition and entanglement– to enable fundamentally new modes of information transfer. One of the most prominent applications is quantum cryptography, where protocols such as quantum key distribution offer provable security based on the laws of quantum mechanics. This represents a significant advancement over classical cryptographic schemes, which may become vulnerable with the advent of powerful quantum computers. In addition to secure communication, quantum information networks hold promise for a broader range of applications, including distributed quantum computing and enhanced sensing capabilities, thereby extending the role of communication systems into new technological domains.

To fully realize the potential of both optical and quantum communication technologies, it is necessary to establish networks on a global scale. Bridging such vast distances poses significant challenges. Fiber-based infrastructure, while useful over short terrestrial links, becomes increasingly costly and lossy over intercontinental spans. These limitations motivate the exploration of alternative approaches. In this context, satellite-based free-space optical links emerge as a compelling solution, offering the capability to connect distant nodes across the globe with reduced loss and infrastructure requirements. This paradigm forms the central focus of this thesis: the development and analysis of satellite free-space optical and quantum communication systems.

1.1. Optical and quantum communications in space

Spaceborne platforms enable a range of free-space optical and quantum communication links, including space-to-ground, intersatellite, and deep-space connections. Each of these link configurations imposes distinct requirements on the communication system, driven by factors such as propagation distance, relative motion, and environmental disturbances. The realization of such space-based optical and quantum links requires systems capable of operating reliably under the unique conditions of the space environment. This motivates a detailed analysis of the physical limitations affecting link performance, as well as the development of techniques to mitigate their impact.

1.1.1. The problem: Opto-thermo-mechanical phenomena

The space environment poses significant challenges to the operation of optical communication systems. Satellites are exposed to a range of perturbations, including radiation from energetic particles, impacts from micrometeoroids, and large temperature fluctuations induced by orbital dynamics. These factors impose stringent requirements on the design and reliability of onboard systems. For free-space optical and quantum links, microvibrations and thermo-mechanical deformations are among the most critical disturbances. Microvibrations, originating from onboard subsystems such as reaction wheels or mechanical actuators, induce stochastic deviations in the pointing direction of the transmitted beam. This so-called pointing jitter leads to fluc-

tuations in the received optical power –or, in the quantum regime, in the probability of photon detection– thereby degrading link performance.

In addition, thermo-mechanical loads can deform optical components, giving rise to a range of opto-thermo-mechanical effects. These include distortions of the optical wavefront and alterations of the polarization state of the transmitted beam. In contrast to imaging systems, where performance is typically quantified in terms of angular resolution, the power-transfer efficiency and stability of the optical field are the primary figures of merit in communication systems. Consequently, accurate characterization of pointing jitter and wavefront errors is essential for defining performance limits and system requirements for future space missions.

1.1.2. The solution: Beam shaping

In parallel with the characterization of these perturbations, it is crucial to develop strategies to mitigate their impact on system performance. Conventional approaches rely on a combination of coarse and fine pointing mechanisms to compensate for beam misalignment. While these systems can significantly reduce pointing errors, practical limitations in sensing, control, and actuation result in residual jitter that cannot be fully eliminated. On the transmitter side, a common mitigation strategy consists of adjusting the divergence of a Gaussian beam to reduce sensitivity to pointing errors. By considering alternative, non-Gaussian beam shapes, it is possible to study whether improved robustness against pointing jitter can be achieved in long-distance classical communication links.

1.2. Scope of the thesis and research questions

To overcome the challenges imposed by the space environment in optical and quantum communications described above, this thesis aims to answer the following main research question:

How can opto-thermo-mechanical perturbations in spaceborne optical and quantum communication terminals be modelled, and how can their impact on communication performance be mitigated?

To address the main research question, a *divide and conquer* approach is employed, whereby three subquestions are considered independently:

1. Which opto-thermo-mechanical phenomena perturb spaceborne optical and quantum communication terminals?
2. How can these phenomena be modelled within a unified mathematical framework that quantifies their impact on the performance of optical and quantum communication?
3. How can beam shaping mitigate the impact of the studied phenomena on optical and quantum communication performance?

The first and second research questions aim to deepen the understanding of the phenomena that affect spaceborne optical and quantum communication terminals. The first question focuses on (1) *identifying and classifying* the relevant opto-thermo-mechanical phenomena occurring in space, while the second aims to (2) *model and quantify* their effects. Finally, the third research question addresses (3) *mitigating* these effects by proposing a novel technique based on beam shaping, going beyond the conventional Gaussian beams most commonly used in optical communication systems.

1.3. Structure of the thesis

To address the research questions posed, this thesis is organized into three main parts. Part I introduces the fundamental concepts necessary for understanding optical and quantum communication systems. Chapter 2 revisits the physics of light as an information carrier and the theory of information, covering both classical and quantum perspectives. Chapters 3 and 4 review environmental opto-thermo-mechanical effects in space-based optical and quantum communication terminals, identifying and characterizing the key phenomena that impact system performance and inform the first research question.

Part II develops models and analytical tools to quantify the impact of the phenomena identified in Part I. Chapter 5 introduces Fourier-based optical propagation methods, which are widely used throughout the thesis to analyze truncation effects, optical aberrations, and beam shaping. In Chapter 6, this optical propagation model is combined with a stochastic model for pointing-jitter-induced power fluctuations to evaluate the coupled effect of Seidel aberrations and pointing jitter in satellite optical communications. Chapters 7 and 8 extend these models to the quantum domain, enabling the quantification of stochastic photon transmission probabilities in space-based quantum communication systems. These chapters also account for hardware imperfections, such as multiphoton emission from entangled-photon sources and limitations in quantum memories, to assess the performance and capabilities of satellite-assisted quantum entanglement distribution. Overall, Part II provides accurate modeling frameworks to address the second research question.

Part III focuses on mitigating, through a novel beam shaping technique, one of the main contributors to power loss in satellite optical communication systems: transmitter pointing jitter. This part proposes the use of non-Gaussian beam shapes to redistribute the power on the far-field irradiance and reduce the effects of jitter. Chap-

ter 9 introduces a superposition of orthogonally polarized Gaussian and higher-order Laguerre–Gaussian modes for far-field beam shaping. Chapter 10 presents an experimental demonstration of this approach, evaluating system feasibility while accounting for real-world losses on the optical components. Chapter 11 identifies the optimum far-field irradiance profile for mitigating transmitter pointing jitter using a variational approach and analyzes both coherent and incoherent optical techniques for approximating this optimal beam shape.

As customary in any research endeavour, in the process of answering the questions posed above, this thesis has probably raised more new questions than it has managed to answer. To guide future researchers, Part IV presents recommendations for further work, suggesting ways to extend the ideas developed here toward a more secure and interconnected future in optical and quantum communications.

Chapter 2

Light as an information carrier

Waves and bits, photons and qubits

*Science is what you know,
philosophy is what you don't know.*

Bertrand Russell

The objective of this chapter is to introduce some fundamental concepts of light beams and information theory, which will set the ground for satellite free-space optical and quantum communications. One of the concepts that is most recurrent throughout this thesis is that of a light beam. First, the classical interpretation of what a light beam is, and how it naturally arises from the classical electromagnetic field theory, is presented. Extending this concept towards the quantum behaviour of light, the quantum field theoretical interpretation of what a light beam is follows. Then, a short introduction to the classical and quantum information theory is given –focusing on some specific applications that will be covered in later chapters of the thesis. Finally, the potential of light as both a classical and quantum information carrier is briefly discussed.

2.1. Light beams: waves and photons

The propagation of light through free-space plays a foundational role in both classical and quantum free-space communication systems. In both cases, the concept of a *light beam* is central to system design, modeling, and performance analysis. A rigorous understanding of what a light beam is and how it behaves requires a proper interpretation of the physical nature of light. As we will see in this and later chapters, the dual interpretation of what a light beam is, will yield a different interpretation of the measurables in optical and quantum communication systems.

This section explores the dual nature of light by examining its classical and quantum descriptions. In particular, it addresses how each framework defines and treats the notion of a light beam, and what implications this has for practical communication systems. In the classical theory of light, electromagnetic waves are described by Maxwell's equations. This framework is essential for understanding the macroscopic propagation of optical fields, including the influence of diffraction, optical aberrations, and beam alignment errors, which will be one of the issues addressed in this dissertation. However, classical electromagnetism falls short in explaining phenomena central to quantum communication –such as entanglement, and the discrete detection events observed in single-photon detectors. To fully capture these effects, we must turn to the quantum theory of light, where the electromagnetic field is quantized, and light is described in terms of quantum states and operators. This quantum mechanical viewpoint redefines the concept of a light beam at the level of individual photons and their associated probabilistic behavior.

2.1.1. Classical electromagnetic fields: waves

The nature of light has been one of the most controversial topics throughout the history of physics [3]. By compiling several laws, James Clerk Maxwell came up with a unified theory for electric and magnetic phenomena in the second half of the 19th century. The so-called electromagnetic theory describes light as a coupled phenomenon that propagates at a finite speed. Maxwell's equations in the vacuum can be written in the following differential form

$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) = \frac{1}{\varepsilon_0} \rho(\mathbf{r}, t) \quad (\text{Gauss's Law}), \quad (2.1)$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}}{\partial t}(\mathbf{r}, t) \quad (\text{Faraday's Law}), \quad (2.2)$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0 \quad (\text{Gauss's Law for magnetism}), \quad (2.3)$$

$$\nabla \times \mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{J}(\mathbf{r}, t) + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}(\mathbf{r}, t) \quad (\text{Ampère-Maxwell Law}), \quad (2.4)$$

where $\mathbf{E}(\mathbf{r}, t)$ is the electric field, $\mathbf{B}(\mathbf{r}, t)$ is the density of the magnetic flux, ρ is the electric charge density, $\mathbf{J}(\mathbf{r}, t)$ is the current density, ε_0 is the permittivity of vacuum, μ_0 is the permeability of vacuum and $\mathbf{r}$ and t are the space-time coordinates. To use the powerful tools of classical field theory, Maxwell's electromagnetic theory can be rewritten more adequately in terms of the scalar and vector potentials $\Phi(\mathbf{r}, t)$ and

$\mathbf{A}(\mathbf{r}, t)$.

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\Phi(\mathbf{r}, t) - \frac{\partial\mathbf{A}(\mathbf{r}, t)}{\partial t}, \quad \mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t). \quad (2.5)$$

After constructing a Lagrangian density for the free electromagnetic field [4, 5], we can now describe light using the classical field theory [6, 7]. By considering only the vacuum solutions of the electromagnetic fields and choosing the Coulomb gauge of the scalar $\Phi(\mathbf{r}, t)$ and vector $\mathbf{A}(\mathbf{r}, t)$ potentials, the classical solutions to Maxwell's equations can be found to satisfy the following homogeneous wave equation for the vector potential $\mathbf{A}(\mathbf{r}, t)$ [5]

$$\nabla^2\mathbf{A}(\mathbf{r}, t) - \mu_0\varepsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2}(\mathbf{r}, t) = 0. \quad (2.6)$$

The classical solution to the previous homogeneous wave equation is given by

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\varsigma} \int \frac{d^3\mathbf{k}}{\sqrt{\varepsilon_0}} \frac{A_{\varsigma}(\mathbf{k}) \boldsymbol{\epsilon}_{\varsigma}(\mathbf{k})}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}} e^{j(\mathbf{k}\cdot\mathbf{r} - \omega_{\mathbf{k}}t)} + \text{c.c.}, \quad (2.7)$$

where $A_{\varsigma}(\mathbf{k})$ is the amplitude of the mode with wave vector $\mathbf{k}$ and polarization ς , the unit vector $\boldsymbol{\epsilon}_{\varsigma}$ gives the polarization state, and c.c. denotes the complex conjugate of the previous expression. The complete set of known polarization states (i.e. linear, elliptical, circular) can be derived by assigning complex values to the components of the polarization unit vector $\boldsymbol{\epsilon}_{\varsigma}$. Furthermore, the dispersion relation for the free field needs to be satisfied

$$|\mathbf{k}|^2 - \varepsilon_0\mu_0\omega_{\mathbf{k}}^2 \equiv k^2 - \frac{\omega_{\mathbf{k}}^2}{c^2} = 0. \quad (2.8)$$

Looking at Eq. (2.7) above, we see that any free-space electromagnetic field can be expressed as the superposition of the plane waves in free-space, i.e., its Fourier spatial components.

To further simplify Eq. (2.7) towards finding a description of light beams in the classical electromagnetic field theory, we will consider the case of monochromatic waves, i.e. $\mathbf{A}(\mathbf{r}, t) = \mathbf{A}(\mathbf{r}) \exp(-j\omega_{\mathbf{k}}t)$. Under this condition, the homogeneous wave equation (Eq. (2.6)) is converted into Helmholtz's equation

$$\nabla^2\mathbf{A}(\mathbf{r}) - k^2\mathbf{A}(\mathbf{r}) = 0. \quad (2.9)$$

When we talk about a light beam, we think about something propagating in a certain spatial direction. However, the equation above treats all three dimensions of space on equal grounds. To obtain beam-like solutions to this equation, we need to introduce the *paraxial approximation*. This approximation will be implied on all the optical propagation techniques used throughout this book, unless specified otherwise. Under the paraxial approximation, the light is assumed to propagate primarily along a certain direction. Choosing this direction to be z , we can assume the following form for the vector potential

$$\mathbf{A}(\mathbf{r}) = \boldsymbol{\varphi}(x, y, z) e^{jkz}, \quad (2.10)$$

where $\varphi(x, y, z)$ is a slowly varying vectorial function of z . Substituting this form into Helmholtz's equation and assuming $\left| \frac{\partial^2 \varphi}{\partial z^2} \right| \ll \left| k \frac{\partial \varphi}{\partial z} \right|$, we obtain the paraxial wave equation [8]

$$\nabla_{\perp}^2 \varphi + 2jk \frac{\partial \varphi}{\partial z} = 0. \quad (2.11)$$

where $\perp$ refers to the bidimensional plane perpendicular to the z coordinate. This equation can be solved by solving the partial differential equation for each of the transversal components φ_x and φ_y of the vector field $\varphi(x, y, z)$ —the Coulomb gauge chosen implies that the polarization is transversal to the direction of propagation z [9]. In this way, with the monochromatic and paraxial approximation, Maxwell's equations for the vectorial electromagnetic field in vacuum have been converted into a scalar field equation for each of the transversal components in x and y . With this scalar field theory, which will be further developed in §5, the solution to the scalar components φ_j of the vector field φ can be found. In Cartesian and cylindrical coordinates, these solutions are given by the Hermite-Gauss and Laguerre-Gauss beams, respectively (see also Ince-Gaussian beams in elliptical coordinates [10]). By including the temporal dependence assumed for the monochromatic case before Eq. (2.9) and the factor e^{jkz} from Eq. (2.10), the families of Hermite-Gaussian and Laguerre-Gaussian beams are obtained for each of the transversal components A_x, A_y (where $\mathbf{A}(\mathbf{r}, t) = A_x(\mathbf{r}, t)\vec{\mathbf{x}} + A_y(\mathbf{r}, t)\vec{\mathbf{y}}$)¹

$$\begin{aligned} u_{nm,\mathbf{k}}^{\text{HG}}(\mathbf{r}, t) = & \frac{C_{nm}}{w(z)} H_n \left(\frac{\sqrt{2}x}{w(z)} \right) H_m \left(\frac{\sqrt{2}y}{w(z)} \right) \exp \left(-\frac{x^2 + y^2}{w^2(z)} \right) \\ & \times \exp \left(-jk \frac{x^2 + y^2}{2R(z)} \right) \exp [j\psi(z)] \exp [j(kz - \omega_{\mathbf{k}}t)], \end{aligned} \quad (2.12)$$

$$\begin{aligned} u_{\ell p,\mathbf{k}}^{\text{LG}}(\mathbf{r}, t) = & \frac{C_{\ell p}}{w(z)} \left(\frac{r\sqrt{2}}{w(z)} \right)^{|\ell|} L_p^{|\ell|} \left(\frac{2r^2}{w^2(z)} \right) \exp \left(-\frac{r^2}{w^2(z)} \right) \\ & \times \exp \left(-jk \frac{r^2}{2R(z)} \right) \exp(-j\ell\phi) \exp [j\psi(z)] \exp [j(kz - \omega_{\mathbf{k}}t)], \end{aligned} \quad (2.13)$$

where the Hermite-Gaussian beams are expressed in Cartesian coordinates and the Laguerre-Gaussian beams are expressed in cylindrical coordinates. In the previous equations, C_{nm} and $C_{\ell p}$ are normalization constants, w_0 is the beamwaist radius, (m, n) and (p, ℓ) are the mode orders of the Hermite-Gaussian and Laguerre-Gaussian beams, respectively. $H_m(x)$ and $L_p^{|\ell|}(x)$, are the Hermite and generalized-Laguerre polynomials, respectively. The rest of the terms are

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2} \quad (\text{Beamwidth at } z), \quad (2.14)$$

¹It can be shown that as a consequence of Eq. (2.5) and Eq. (2.6) the electric and magnetic vector fields ($\mathbf{E}$ and $\mathbf{B}$) fulfill also the Helmholtz equation. Therefore, the paraxial beam solutions obtained are also applicable to $\mathbf{E}$ and $\mathbf{B}$, as well as to the vector potential $\mathbf{A}$.

$$R(z) = z \left[1 + \left(\frac{z}{z_R} \right)^2 \right] \quad (\text{Wavefront radius of curvature at } z), \quad (2.15)$$

$$\psi(z) = (N + 1) \arctan \left(\frac{z}{z_R} \right) \quad (\text{Gouy phase at } z), \quad (2.16)$$

where $z_R = \pi w_0^2 n / \lambda$ is the Rayleigh range, and $N = n + m$ and $N = |\ell| + 2p$ for Hermite-Gauss and Laguerre-Gauss beams, respectively. As the basis created by Hermite-Gauss and Laguerre-Gauss modes (see Fig. 2.1) is complete, orthogonal, and a solution to the original homogeneous wave equation (Eq. (2.6)), the transversal components A_j (with $j = x, y$) of any monochromatic paraxial beam can be expressed as a superposition of either of the mode families as

$$A_j(\mathbf{r}, t) = \sum_{n,m=0}^{\infty} a_{nm} u_{nm,\mathbf{k}}^{\text{HG}}(\mathbf{r}, t) + a_{nm}^* u_{nm,\mathbf{k}}^{\text{HG},*}(\mathbf{r}, t), \quad (2.17)$$

where $*$ represents the complex conjugate. An analogous expansion can be performed for the Laguerre-Gaussian beam family. In the beam families above derived, we can notice the different properties of light that will be fundamental for the latter application of optical communications: frequency (represented by ω), polarization (represented by the vectorial nature of $\mathbf{A}$), amplitude (the magnitude of the complex field) and phase (the argument of the complex field). The phase determines the spatial structure of the wavefront and its curvature, and thus plays a central role in how the beam propagates and diffracts through space. Particularly, the orbital angular momentum (OAM) of Laguerre-Gaussian beams, represented by the helical phase factor $\exp(-j\ell\phi)$, is worth emphasizing as a characteristic property of the spatial phase distribution of this beam family. The connection between phase and beam propagation is naturally described within the framework of Fourier optics, which will be explained in §5.

There is a beam of primary relevance in the field of optics, i.e., the *Gaussian beam*. The Gaussian beam is the zeroth order of the Hermite-Gauss and Laguerre-Gauss modes presented before. We will use this beam as an example to illustrate how a beam can transport energy. Using Poynting's theorem for electromagnetic waves in vacuum, the directional energy flow of a free electromagnetic field is given by the Poynting vector $\mathbf{S} = (\mathbf{E} \times \mathbf{B}) / \mu_0$. The time-average of the Poynting vector is defined as the irradiance of the field. For a Gaussian beam, the irradiance is given by

$$I(\mathbf{r}) = \langle \mathbf{S}(\mathbf{r}, t) \rangle = \frac{|\mathbf{E}(\mathbf{r}, z)|^2}{2\eta_0} = I_0 \left(\frac{w_0}{w(z)} \right)^2 \exp \left(\frac{-2r^2}{w(z)^2} \right) \quad (2.18)$$

$\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$ being the impedance of vacuum, and I_0 is the peak irradiance of the Gaussian beam. The irradiance at each point in space gives the time-averaged energy received by a surface per unit area at that point. Under this classical interpretation, a beam of light is an electromagnetic wave that carries energy within and is propagated predominantly in a certain direction (z in this case).

Under the classical electromagnetic field theory approach presented, it can be seen that light beams appear as monochromatic paraxial solutions of Maxwell's equations.

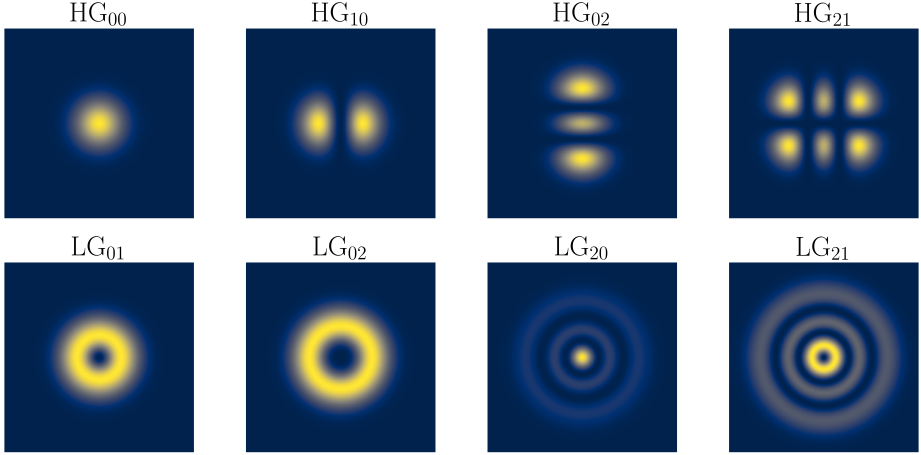


Figure 2.1: Irradiance fields of some example Hermite-Gaussian and Laguerre-Gaussian modes.

Hence, classically, a light beam is an electromagnetic field (a complex vector field) propagating predominantly in a certain direction of space. This light beam carries energy with it, and this can be used to communicate information in a classical sense, as will be seen in §2.3.

2.1.2. Quantum electromagnetic fields: photons

Shortly after Maxwell and others laid the ground for what seemed to be the ultimate wave nature of light, early 20th-century physicists started developing the theory of quantum mechanics. Later on, building upon the framework of classical field theory and the principle of relativity, quantum field theory was developed. Quantum field theory is, so far, the most accurate model of light (and matter) that we have. Under this interpretation of light as a quantum field, the concept of a light beam also arises in the quantum mechanical context. In this section, we present the concept of a light beam in the context of quantum field theory. This will be key to understanding what a light beam is in the single-photon regime in which quantum communication systems can operate.

In the framework provided by quantum field theory the classical free electromagnetic field (discussed in the previous section) is quantized by promoting the vector potential $\mathbf{A}(\mathbf{r}, t)$ to an operator $\hat{\mathbf{A}}(\mathbf{r}, t)$ acting on the Hilbert space of the electromagnetic field, spanned by the photon-number (Fock) states associated with each field mode. Under the selection of the Coulomb gauge, defined by the transversality condition $\nabla \cdot \hat{\mathbf{A}} = 0$, which eliminates the longitudinal component of the vector potential in the absence of sources, and by imposing the equal-time canonical commutation relations, the quantized vector potential (now an operator) is expanded in plane waves

as [5]

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \sum_{\varsigma=1}^2 \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}} \sqrt{\frac{\hbar}{\varepsilon_0}} \left[\boldsymbol{\epsilon}_{\varsigma}(\mathbf{k}) \hat{a}_{\varsigma}(\mathbf{k}) e^{j(\mathbf{k} \cdot \mathbf{r} - \omega_{\mathbf{k}} t)} + \boldsymbol{\epsilon}_{\varsigma}^*(\mathbf{k}) \hat{a}_{\varsigma}^{\dagger}(\mathbf{k}) e^{-j(\mathbf{k} \cdot \mathbf{r} - \omega_{\mathbf{k}} t)} \right], \quad (2.19)$$

where $\hbar$ is the reduced Planck's constant, the polarization of the electromagnetic quantized field is contained within the unit vector $\boldsymbol{\epsilon}_{\varsigma}(\mathbf{k})$, the sum over ς represents the sum over orthogonal polarization states of light, and the creation and annihilation operators, $\hat{a}_{\varsigma}^{\dagger}(\mathbf{k})$ and $\hat{a}_{\varsigma}(\mathbf{k})$, satisfy the canonical commutation relations

$$\left[\hat{a}_{\varsigma}(\mathbf{k}), \hat{a}_{\varsigma'}^{\dagger}(\mathbf{k}') \right] = \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\varsigma\varsigma'} \quad \left[\hat{a}_{\varsigma}(\mathbf{k}), \hat{a}_{\mathbf{k}',\varsigma'} \right] = \left[\hat{a}_{\varsigma}^{\dagger}(\mathbf{k}), \hat{a}_{\varsigma'}^{\dagger}(\mathbf{k}') \right] = 0. \quad (2.20)$$

In this way, Eq. (2.19) shows the plane wave expansion, analogous to the one derived for the classical field formalism (Eq. (2.7)). In a similar way to how the vector potential operator was constructed from the classical vector potential, the Hamiltonian operator of the free electromagnetic field can be derived [11, 12]. By defining a ground state $|0\rangle$ of the eigenstates of this Hamiltonian operator so that $\hat{a}_{\varsigma}(\mathbf{k})|0\rangle = 0$, we can construct a set of basis states that are also eigenstates of the Hamiltonian. The ground state $|0\rangle$ contains no particles and is therefore also called the vacuum state. To construct a set of basis states, the operator $\hat{a}_{\varsigma}^{\dagger}(\mathbf{k})$ can be applied repeatedly to the ground state $|0\rangle$. Although the eigenvalue (or energy) associated with these eigenstates is infinite, the relative energy with respect to the ground state $|0\rangle$ is finite. The relative energy, related to these energy eigenstates or Fock states $|n_{\mathbf{k},\varsigma}\rangle$, is

$$E_n(\mathbf{k}) = n \hbar \omega_{\mathbf{k}} \quad \text{with } n = 0, 1, 2, \dots \quad (2.21)$$

The explanation above involves creating states of the kind $|1_{\mathbf{k},\varsigma}\rangle$ from the ground state $|0\rangle$ by applying the creation operator $\hat{a}_{\varsigma}^{\dagger}(\mathbf{k})$. This physically means populating a specific field mode of the electromagnetic field. Nevertheless, the states $|1_{\mathbf{k},\varsigma}\rangle$ created are not physically realizable because they are non-normalizable. This fact can also be understood considering that the states created are associated with plane waves (which are infinitely spread waves that are nonphysical) [5].

To construct physical states, we can define a new set of mode functions $v_{\mathbf{k}}(\mathbf{r}, t)$, that will have an associated set of operators $\hat{b}_{\varsigma}(\mathbf{k})$, to express the vector potential operator above as

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \sum_{\varsigma=1}^2 \int d^3\mathbf{k} \sqrt{\frac{\hbar}{\varepsilon_0}} \left[\hat{b}_{\varsigma}(\mathbf{k}) v_{\mathbf{k}}(\mathbf{r}, t) \boldsymbol{\epsilon}_{\varsigma}(\mathbf{k}) + \hat{b}_{\varsigma}^{\dagger}(\mathbf{k}) v_{\mathbf{k}}^*(\mathbf{r}, t) \boldsymbol{\epsilon}_{\varsigma}^*(\mathbf{k}) \right]. \quad (2.22)$$

By superposing plane waves as

$$f(\mathbf{r}, t) = \int d^3\mathbf{k} \left[\alpha(\mathbf{k}) \frac{1}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}} \sqrt{\frac{\hbar}{\varepsilon_0}} e^{j(\mathbf{k} \cdot \mathbf{r} - \omega_{\mathbf{k}} t)} + \beta^*(\mathbf{k}) \frac{1}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}} \sqrt{\frac{\hbar}{\varepsilon_0}} e^{-j(\mathbf{k} \cdot \mathbf{r} - \omega_{\mathbf{k}} t)} \right], \quad (2.23)$$

the new mode operators to use in Eq. (2.22) can be defined as

$$\hat{b}_{\zeta} = \sqrt{\frac{\varepsilon}{\hbar}} (\epsilon_{\zeta} f, \hat{\mathbf{A}}) = \int d^3\mathbf{k} [\alpha(\mathbf{k}) \hat{a}_{\zeta}(\mathbf{k}) + \beta(\mathbf{k}) \hat{a}_{\zeta}^{\dagger}(\mathbf{k})]. \quad (2.24)$$

Consider now that f is part of a set of orthonormal functions $f_j(\mathbf{r}, t)$, with $\beta(\mathbf{k}) = 0$ and the functions $\alpha(\mathbf{k})$ to be well-behaved and sharply peaked around a central value $\mathbf{k}_0$. Then, a single excitation of the mode f_j (which are no longer nonphysical plane waves) can be created as

$$|1_{j\zeta}\rangle = \hat{b}_{j\zeta}^{\dagger} |0\rangle = \int d^3\mathbf{k} \alpha_j^*(\mathbf{k}) \hat{a}_{\zeta}^{\dagger}(\mathbf{k}) |0\rangle. \quad (2.25)$$

This excitation, that is basically a wavepacket of infinite duration, follows the rules

$$|n_{j\zeta}\rangle = \frac{(\hat{b}_{j\zeta}^{\dagger})^n}{\sqrt{n!}} |0\rangle, \quad (2.26)$$

$$\hat{b}_{j\zeta} |n_{j\zeta}\rangle = \sqrt{n} |n-1_{j\zeta}\rangle, \quad (2.27)$$

$$\hat{b}_{j\zeta}^{\dagger} |n_{j\zeta}\rangle = \sqrt{n+1} |n+1_{j\zeta}\rangle. \quad (2.28)$$

This set of rules strongly suggests considering the excitations of mode f_j of the electromagnetic field as particle states, i.e. *photons*. These particles are created by the operator $\hat{b}_{j\zeta}^{\dagger}$ and annihilated by the operator $\hat{b}_{j\zeta}$. These wavepacket-like excitations, called photons, have well-defined energy and momentum due to the sharply peaked nature of the related modes f_j . Hence, with the formalism provided by the quantum field theory, a fundamental result has been achieved: the interpretation of light as being composed by photons (which are excitations with well-defined energy and momentum of the quantum electromagnetic field).

The objective of this section is to find the interpretation of a single-photon beam of light within quantum field theory. To do so, the paraxial approximation needs to be introduced (similarly to how it was done for the classical field in the previous section). In the paraxial regime, a field is typically represented as an envelope function modulating a carrier plane wave with wave vector $\mathbf{k}_0$ and angular frequency $\omega_0 = c|\mathbf{k}_0|$. Consider the excitation of the field to have a predominant direction of propagation along z and momentum $k_0 > 0$, i.e. $\mathbf{k}_0 = k_0 \hat{\mathbf{z}}$. Following the paraxial wave equation formalism in Ref. [13], the corresponding single-photon wave function can be derived. Under these assumptions, the vector potential operator is written as

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \sum_{\zeta=1}^2 \int d\omega \frac{e^{-j\omega(t-z/c)}}{\sqrt{4\pi\varepsilon_0 c \omega / \hbar}} \int d^2\mathbf{r}'_{\perp} \mathcal{F}_{\zeta}(\mathbf{r}_{\perp}, z, \mathbf{r}'_{\perp}, \omega, t) \hat{a}_{\zeta}(\mathbf{r}'_{\perp}, \omega) + \text{H.c.}, \quad (2.29)$$

with

$$\mathcal{F}_\varsigma(\mathbf{r}_\perp, z, \mathbf{r}'_\perp, \omega, t) = \frac{1}{(2\pi)^2} \int d^2\mathbf{q} \, \epsilon_\varsigma(\mathbf{k}_\perp, \vartheta) \left(\frac{1 + \vartheta^2}{\sqrt{1 + \vartheta^4}} \right)^{1/2} e^{-j\omega t(\sqrt{1+\vartheta^4}-1)} e^{j\mathbf{k}_\perp \cdot (\mathbf{r}_\perp - \mathbf{r}'_\perp) - iq^2 cz/(2\omega)}, \quad (2.30)$$

where $\mathbf{r}_\perp$ and $\mathbf{k}_\perp$ are the position and momentum coordinates perpendicular to the paraxial direction z , $\vartheta = |\mathbf{k}_\perp|/\sqrt{2k_0^2}$, and H.c represents the Hermitian conjugate. Although in general, no position operator can be defined for a relativistic massless particle with helicity, e.g., the quantum electromagnetic field, a transverse position operator can be defined in the paraxial regime [12]. The eigenstates of this transverse position operator are then given by $|\mathbf{r}_\perp, \omega, \varsigma\rangle \equiv \hat{a}_\varsigma^\dagger(\mathbf{r}_\perp, \omega)|0\rangle$ where the annihilation operator $\hat{a}_\varsigma(\mathbf{r}_\perp, \omega)$ is used in Eq. (2.29). Using these position states we can generate, in general, any single-photon state $|n, m, \omega, \varsigma\rangle = \int d^2\mathbf{r}_\perp \psi_{nm}(\mathbf{r}_\perp, \omega) |\mathbf{r}_\perp, \omega, \varsigma\rangle$ given a basis set of orthogonal transverse functions ψ_{nm} (e.g. Hermite-Gaussian basis). The transverse position operator in the paraxial approximation allows the representation of the exact quantum mechanical state of each of the photons in the classical paraxial beam². This leads to the important observation that the normalized irradiance (see Eq. (2.18)) of a single-photon beam in the paraxial regime can be interpreted as the probability density for the transverse localization of the single-photon.

Under this interpretation enabled by the paraxial wave equation and the well-behaved transverse position operator, analogous to the way it was done for the classical electromagnetic field, the vector potential operator can be written in terms of the Hermite-Gaussian mode basis (by properly choosing the spatial-temporal modes $v_{\mathbf{k}}(\mathbf{r}, t)$ in Eq. (2.22))

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \sqrt{\frac{\hbar}{\epsilon_0}} \sum_{\varsigma=1}^2 \sum_{n,m=0}^{\infty} \int d^3\mathbf{k} \left[\epsilon_\varsigma(\mathbf{k}) \hat{b}_{nm}(\mathbf{k}) u_{nm,\mathbf{k}}^{\text{HG}}(\mathbf{r}, t) + \epsilon_\varsigma^*(\mathbf{k}) \hat{b}_{nm}^\dagger(\mathbf{k}) u_{nm,\mathbf{k}}^{\text{HG},*}(\mathbf{r}, t) \right], \quad (2.31)$$

where the annihilation and creation operators $\hat{b}_{nm}(\mathbf{k})$ and $\hat{b}_{nm}^\dagger(\mathbf{k})$ are associated with the Hermite-Gaussian mode basis $u_{nm,\mathbf{k}}^{\text{HG}}(\mathbf{r}, t)$ presented in Eq. (2.12). Considering the previous discussion for a well-behaved mode basis f_j , the annihilation and creation operators for the Hermite-Gaussian mode basis annihilate and create photons with the respective spatial-temporal mode $u_{nm}^{\text{HG}}(\mathbf{r}, t)$. An analogous expression to Eq. (2.31) can be found for the operator when expanded in the Laguerre-Gaussian mode basis, in which it can be seen that these beams carry a quantized orbital angular momentum [14].

As it is seen in the expansions of the vector potential operator in Eq. (2.31), the polarization of the electromagnetic quantized field is contained within the unit vector $\epsilon_\varsigma(\mathbf{k})$. As a consequence of the Coulomb gauge chosen, the polarization vector points in a perpendicular direction to the direction of propagation $\mathbf{k}$, i.e. $\epsilon_\varsigma(\mathbf{k}) \cdot \mathbf{k} = 0$ for

²The reader is referred to Ref. [12] for a detailed derivation of the above.

each ς [9]. By allowing the components of the polarization unit vector $\epsilon_{\varsigma}(\mathbf{k})$ to be complex, all the polarization states known can be obtained. The amplitude of the field is now related to the number of photons on the quantum state (in this thesis, we will be interested mainly in single-photon states). Finally, the phase of single-photon states, mathematically represented in the argument of the complex spatial mode, will also be necessary to understand the diffraction of these states (see §5). This shows that the different properties of light that were discussed in the classical field theory (frequency, phase, and polarization) are also present in the quantized electromagnetic field.

In summary, the formalism provided by the quantum field theory allows us to generate single-photon states associated with classical paraxial beams in which the transverse position operator is well behaved. With the latter, the interpretation of the normalized classical irradiance of a single-photon state as a localization probability density is obtained, which will be central to the discussions presented in §7 in the context of quantum communication.

2.2. Information theory: bits and qubits

In the previous section, the fundamental nature of light has been presented. Starting from Maxwell's equations, the solutions for paraxial beams in both the classical and quantum theories have been obtained. The directionality provided by these beams can be used, among other applications, to carry information between two spatially distant locations. This is what free-space optical and quantum communications are all about. By modulating and analyzing different properties of the electromagnetic light fields, information can be transmitted and received effectively. In this section, the fundamental notions of classical and quantum information theory relevant to the analysis of space-based optical and quantum communication systems investigated in this thesis are introduced. The reader is referred to the literature for a more in-depth discussion of these topics [5, 15–18].

2.2.1. Classical information theory

We can start with a simple question: *what is information?* A rigorous mathematical answer to this question was given by the foundational work of Claude Shannon when he established the concept of *information entropy* [19]. The information entropy measures the average uncertainty or information content associated with a random variable. For a discrete source X emitting symbols x_i with probabilities $p_X(x_i)$, the Shannon entropy is defined as [16]

$$H(X) \equiv - \sum_i p_X(x_i) \log_2 p_X(x_i). \quad (2.32)$$

A detailed discussion on how this definition comes from four axioms that are expected for an uncertainty measurement, such as $H(X)$, can be found in Ref. [15]. If the logarithm is base 2 as above, the units of the entropy are *bits*. Shannon's entropy sets the fundamental limit on the minimum average number of bits required to encode the source output. In other words, Shannon's information entropy tells how much information we gain on average when we learn the value of the random variable X (or

equivalently, how much uncertainty we have on the variable before we learn its value). In this sense, a bit can be understood as the amount of information required to reduce the uncertainty by a factor of 2. The information-theoretic bit introduced so far is not to be confused with the *digital bit*, which represents a *binary digit* in a physical or computational system and can take only the discrete values 0 or 1, whereas the information-theoretic bit is a unit of information that measures the average reduction in uncertainty and can take any real-valued quantity depending on the underlying probabilities [16]. The relation between them can be understood as follows: when we say that we give 1 bit of information in the information-theoretic sense, this can be represented by a single digital bit that takes the value 0 or 1 to distinguish between the two equally likely alternatives.

In optical communication, the abstract random variable X introduced above is physically encoded in specific degrees of freedom of the electromagnetic field, such as the amplitude, phase, polarization, or spatial mode of the light beam, which are modulated to represent the transmitted symbols. In optical communication systems, the channel is modeled as a physical process that transforms the transmitted signal (the modulated light beam) into a received signal, possibly corrupted by noise. In free-space optical channels, noise sources include background light, detector noise, and atmospheric turbulence, which can distort different properties of the light beam carrying the information (e.g., amplitude, phase and polarization). The different noise sources and perturbations to the signal that will be of interest to the present thesis are presented in detail in §3. A diagram of the basic operation of such a communication channel is shown in Figure 2.2. Physically, a communication channel refers to the physical media through which the signal is propagated and subject to physical disturbances. The figure illustrates a noisy communication system, showing the interaction between the encoder, the channel, and the decoder. The encoder maps, through the operator f_n , the input information into codewords designed to introduce redundancy, enabling error detection and correction. These codewords are transmitted through a noisy channel characterized by the conditional probability $p(y|x)$, which models the uncertainty introduced by noise. At the receiver, the decoder estimates the transmitted message from the received sequence through the operator g_n , aiming to minimize the bit error probability.

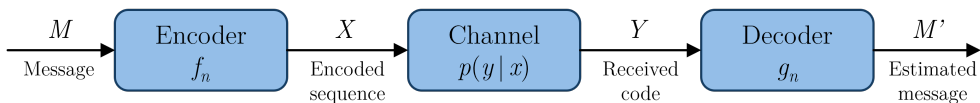


Figure 2.2: Block diagram of a classical communication channel

The upper bound of the amount of information that can be effectively transmitted through such a noisy communication channel is given by *Shannon's noisy channel theorem*. This theorem establishes that for a noisy communication channel with capacity $\mathcal{C}$, reliable transmission is theoretically possible if the transmission rate $\mathcal{R}$ is below $\mathcal{C}$, as coding techniques exist that make the probability of error arbitrarily small. Conversely, when $\mathcal{R} > \mathcal{C}$, no coding scheme can achieve an arbitrarily low error rate, and the probability of bit errors increases as $\mathcal{R}$ exceeds $\mathcal{C}$. Thus, $\mathcal{C}$ represents the

fundamental limit for error-free communication. For a memoryless channel (a channel acting the same way every time it is used), with input X and output Y , the so-called mutual information $H(X : Y)$ quantifies the amount of information shared between X and Y [16, §4]

$$H(X : Y) \equiv H(X) + H(Y) - H(X, Y) = H(X) - H(X|Y), \quad (2.33)$$

where the joint entropy $H(X, Y)$ and conditional entropy $H(X|Y)$ are given by

$$H(X, Y) \equiv \sum_{i,j} p_{XY}(x_i, y_j) \log_2 \frac{1}{p_{XY}(x_i, y_j)}, \quad (2.34)$$

$$H(X|Y) \equiv - \sum_{i,j} p_{XY}(x_i, y_j) \log_2 \frac{p_{XY}(x_i, y_j)}{p_Y(y_j)}. \quad (2.35)$$

The latter is a measure of how uncertain we are on average about the value of X ,

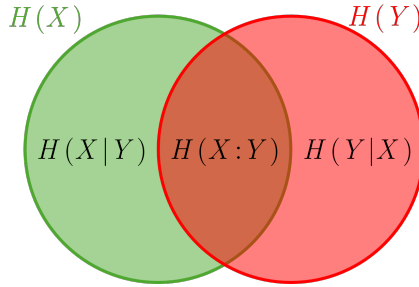


Figure 2.3: Graphical representation of the entropies, conditional entropies, and mutual information of two random variables X and Y .

after knowing the value of Y . In Fig 2.3, a graphical representation of the different entropies and mutual information is shown. Then, the *Shannon channel capacity* for a discrete memoryless channel can be expressed as

$$\mathcal{C} = \max_{p_X(x)} H(X : Y), \quad (2.36)$$

where the maximization is over all possible input distributions. The previous equations are valid for discrete variables X and Y . However, analogous definitions for differential entropies in continuous random variables can be easily obtained by substituting the summatories for integrals [16]. For example, for continuous random variables with band-limited channels under Gaussian noise, the capacity $\mathcal{C}$ can be determined using the Shannon–Hartley theorem, linking the channel’s physical properties to its maximum achievable data rate as [16]

$$\mathcal{C} = B \log_2 \left(1 + \frac{S}{N} \right), \quad (2.37)$$

where B is the bandwidth, S/N is the signal-to-noise power ratio.

There are plenty of systems in which classical information theory can be applied. The system of interest for this thesis is optical communication. In this way, the concepts presented in the theory of information are physically implemented on free-space light beams, and the information is encoded by manipulating different properties of light (such as amplitude, phase, frequency, or polarization modulation). The relation between the classical information theory and the physical implementation of this through light beams is discussed in detail in §2.3.

2.2.2. Quantum information theory

As seen in the previous section, applications of classical information theory can be physically implemented by exploiting the properties of light predicted by the classical theory of light (§2.1.1). Analogously, quantum information theory studies the information processing on systems that exploit the properties predicted by the quantum theory of light (§2.1.2). For almost all classical information processing systems (e.g., computer or communication system), there is nowadays a quantum analog (e.g., quantum computer or quantum communication system). In this section, a brief introduction to quantum information theory, which sets the foundations for quantum communication systems, is presented. For a more in-depth discussion of quantum information theory, the reader is referred to the literature [5, 18].

The fundamental unit of quantum information is the quantum bit, or *qubit*. Unlike a classical bit, which is either 0 or 1, a qubit can exist in a superposition. In general, the state of a qubit in the two-dimensional complex Hilbert space $\mathcal{H}$ can be expressed as

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle, \quad (2.38)$$

where α and β are complex amplitudes satisfying the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. Among the different possible physical implementations of a qubit [18], they can be realized using the polarization, spatial mode, or time-bin degree of freedom of single-photon states (§2.1.2). For most applications of quantum information theory, things start to be interesting when at least a pair of qubits is used, and combined operations on them are performed. The Hilbert space for two qubits $\mathcal{H}_2 = \mathcal{H} \otimes \mathcal{H}$ can be spanned by the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. Another interesting basis that explicitly showcases a foremost important property of quantum systems is the *Bell state basis*

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle), \quad |\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle). \quad (2.39)$$

This basis highlights a property that has no equivalent in classical systems, i.e. *entanglement*. An entangled state of two qubits cannot be described as a product of the individual qubits' state vectors, i.e., it cannot be written in the separable form $|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$ (where $|\psi_1\rangle$ and $|\psi_2\rangle$ are the states of the single qubits). This property of quantum systems is key to enabling the outstanding capabilities of quantum computation and communication systems, as it will be explained later in this section.

Exploiting the idea of entanglement, a key operation in entangled photon pairs that can distribute entanglement over large distances is *entanglement swapping* or *quantum teleportation*. Two entangled pairs of qubits, say (A, B) and (C, D) , where

B and C are sent to a central node. Performing a Bell state measurement (BSM) on B and C projects the remaining qubits A and D into an entangled state, even though they have never interacted directly. Mathematically, if the initial state of the two pairs is $|\Psi\rangle_{AB} \otimes |\Psi\rangle_{CD}$, a BSM on qubits B and C can be represented by the projection operators

$$P_{\Phi^\pm} = |\Phi^\pm\rangle\langle\Phi^\pm|, \quad P_{\Psi^\pm} = |\Psi^\pm\rangle\langle\Psi^\pm|, \quad (2.40)$$

and the outcome of the measurement determines which Bell state the distant qubits A and D collapse into. As presented in §7, entanglement swapping and quantum memories –physical systems capable of storing quantum states for a finite duration– can be used to overcome distance limitations in quantum communication systems. However, due to imperfections in physical implementations, the generated entangled states are often not maximally entangled. In such cases, entanglement distillation can be employed to extract a smaller number of highly entangled states from a larger set of degraded ones. Thus, entanglement distillation enables a trade-off in which multiple low-quality entangled pairs are converted into fewer, higher-quality pairs.

So far, the quantum states have been described by vectors in Hilbert space. However, another formalism based on the density operator is much used in quantum mechanics. By using the density operator formalism, fundamental concepts of quantum information theory are made more approachable. The density operator can describe not only pure quantum states such as the ones presented with the vector formalism in Eqs. (2.38)-(2.39), but also quantum systems whose state is not completely known. For example, consider a quantum system X is in a state $|\psi_i\rangle$ with a probability p_i . Then, such an ensemble of pure states can be described by a density operator

$$\rho_X \equiv \sum_i p_i |\psi_i\rangle\langle\psi_i|. \quad (2.41)$$

With this formalism, the description of unitary operations and measurements in the quantum system is easily described mathematically [18]. In the same way that in classical information theory a random variable is characterized by its probability distribution, a quantum random variable is characterized by its density operator. Therefore, there is an analogue to the Shannon information entropy for a quantum random variable X with a density operator ρ_X . The entropy is now called the *von Neumann entropy* and is defined as

$$S(X) \equiv -\text{Tr}(\rho_X \log_2 \rho_X) = -\sum_x \lambda_x \log_2 \lambda_x, \quad (2.42)$$

where a Tr refers to the trace operation, and λ_x are the eigenvalues of the density operator ρ_X . Analogous to how it was presented in the framework of classical information theory, the mutual information can be defined in quantum information theory for the quantum random variables X and Y with states given by the density operator ρ_{XY}

$$S(X : Y) \equiv S(X) + S(Y) - S(X, Y) \quad (2.43)$$

where $S(X, Y) \equiv -\text{Tr}(\rho_{XY} \log_2 \rho_{XY})$ is the joint entropy and the density operator of the subsystems X and Y can be obtained through the partial trace operation $\rho_X = \text{Tr}_Y(\rho_{XY})$.

So far, we have briefly described some of the concepts underlying the theory of qubits and quantum information. However, how can the properties of qubits be exploited to obtain a useful benefit with respect to their classical counterparts? A bit and a qubit differ in that while the bit can be in one of two possible values, the qubit has a whole continuum of possible states between these two values (see Eq. (2.38)). Not only that, but by combining several of these qubits, entanglement between them can be created (see Eq. (2.39)). The properties of superposition and entanglement give rise to certain phenomena that can be exploited for several applications, such as quantum computing and quantum cryptography.

To understand how the superposition and entanglement properties give rise to these applications, we first need to introduce the concept of *accessible information*. Consider two non-orthogonal quantum states $\langle\psi|\phi\rangle \neq 0$. It can be proven that there exists no quantum mechanical operation that can reliably distinguish between them [18]. The accessible information quantifies the maximum amount of classical information that can be extracted from a quantum system, and the indistinguishability of non-orthogonal states fundamentally limits this quantity—an upper bound for the accessible information in a quantum system is given by the Holevo bound [18, §12]. In contrast, in classical information theory, distinct states are always perfectly distinguishable in principle, meaning that the accessible information equals the total information encoded. Another consequence related to the concept of accessible information is the *no-cloning theorem*. This theorem, which lies in the heart of quantum cryptography protocols presented later, states that it is impossible to perfectly clone an unknown quantum state using unitary evolution [18, §12].

When quantum states are used and sent, the question can be asked of how much classical information can be transferred between two parties over a noisy channel. In other words, we want to find an analogue to Shannon’s noisy channel theorem, but when transmitting quantum states. This topic continues to be an open problem, although some advances and bounds have been found for specific conditions (see the Holevo–Schumacher–Westmoreland theorem [20, 21]). An even greater challenge in quantum information theory is determining the amount of quantum information that can be transmitted over a noisy channel (see Ref. [18] for a detailed discussion).

Quantum cryptography

Among the different applications of quantum information theory [18], we will be focusing mainly on two: entanglement distribution and quantum cryptography. Entanglement distribution consists of distributing quantum entangled states between several different parties that are separated in space. As will be seen in later chapters, entanglement distribution between different parties can be used for several applications, such as distributed quantum computing [22] and cryptography [23]. Specifically, quantum entanglement can also be used for quantum key distribution (QKD).

Quantum key distribution is a set of provably secure protocols that create private key bits between two parties over a public channel (see Fig. 2.4). At the time of this writing, most of the key generation through public channels is done through classical public-key encryption schemes, such as RSA [24]. However, the development of large-scale quantum computers threatens these classical public-key encryption schemes, as

algorithms such as Shor's algorithm can efficiently break their security assumptions [18, 25–27]. There are two main solutions to this threat: post-quantum cryptography and quantum key distribution. On the one side, post-quantum cryptography seeks to design classical encryption methods that remain secure even against eavesdroppers with quantum computers. On the other side, QKD provides a solution by enabling information-theoretically secure key exchange exploiting the no-cloning theorem mentioned above. We now proceed to explain the two most popular protocols for quantum key distribution, i.e., BB84 and BBM92.

The protocol BB84 is able to create a secure key without requiring entanglement distribution between the parties involved [17, 28]. The diagram in Fig. 2.5 shows the basic steps of the protocol. The protocol is started by Alice preparing a state of random qubits in one of the states $\{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$ according to a random bit string s . The states $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ create a basis diagonal to $\{|0\rangle, |1\rangle\}$. The string of random qubits created by Alice is then transmitted through the quantum channel to Bob, who measures each in a randomly chosen basis. Then Alice and Bob share information through a classical channel to compare which measurements of Bob is done in the same basis as prepared by Alice. Then, a subset of the resulting bits is chosen by Alice to check the presence of any eavesdropper. If the number of coincidences is acceptable, considering other errors that occur due to imperfections on the channel, information reconciliation and privacy amplification procedures are applied, creating a secure key between the parties. The security of the protocol relies on the fact that an eavesdropper cannot measure the qubits without disturbing them: since the states are prepared in non-orthogonal bases, any attempt to measure without knowing the correct basis introduces detectable errors. Furthermore, the no-cloning theorem prevents an eavesdropper from perfectly copying unknown quantum states, ensuring that any interception attempt inevitably alters the transmitted qubits, and can thus be detected.

The BBM92 protocol proposed in Ref. [29] is a QKD scheme that, unlike BB84, is based on the use of quantum entanglement. The protocol is illustrated at the bottom of Fig. 2.5. Firstly, pairs of maximally entangled qubits are prepared in the entanglement source –this source can be located away from both Alice and Bob or with one of them. The elements of the Bell state basis are maximally entangled states. Each of the qubits of the maximally entangled states, e.g. the singlet state $|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2} = (|+-\rangle - |-+\rangle)/\sqrt{2}$, are sent through the quantum channels to Alice and Bob, respectively. Then, Alice and Bob independently choose random measurement bases from predefined sets (e.g., $\{|0\rangle, |1\rangle\}$ and $\{|+\rangle, |-\rangle\}$), and they measure the received qubits in these bases. As the qubits received by Alice and Bob are entangled, their results should be correlated (or anticorrelated). After all the measurements have been made, both parties announce publicly the randomly chosen bases, and keep only those measurements in which their bases coincide. In these remaining qubits, under perfect channel conditions and no eavesdropper attacks, the results of Alice and Bob should be perfectly correlated (or anti-correlated). This correlation is checked by choosing a subset of all the remaining qubits, and the protocol continues as in BB84. The security of the BBM92 protocol arises from the fact that any eavesdropping attempt necessarily disturbs the entanglement and reduces the strength of the observed

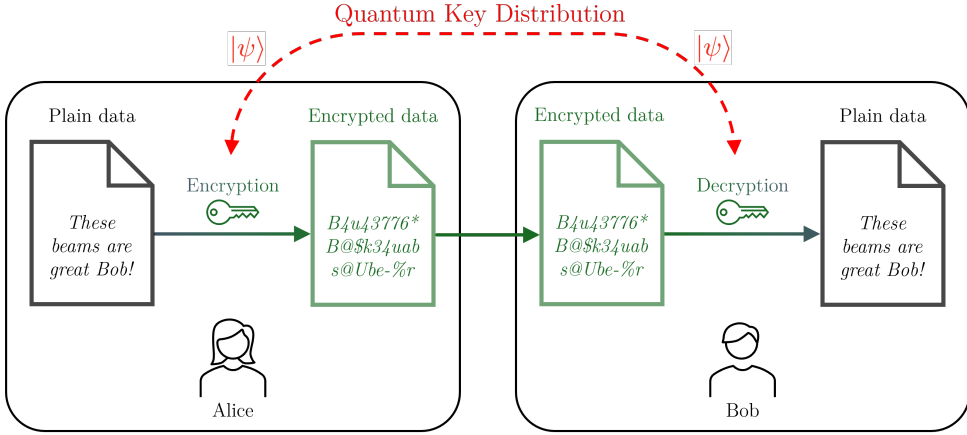


Figure 2.4: Diagram of public key encryption based on quantum key distribution.

correlations (or anti-correlations). By testing these correlations, Alice and Bob can verify whether the shared states are genuinely entangled; if an eavesdropper intercepts or manipulates the qubits, the correlation will not be perfect, and the intrusion will be detected. In the protocol explained, the security check is done through simplified correlation checks of the remaining subset. For a more secure check, Bell's inequality violation proof with the selected measurements is required [18], yielding to the E91 protocol proposed in Ref. [30].

The physical implementation of the applications discussed above can be done by different means: quantum computers use photons, ions, and other quantum systems to store and process quantum information [31]. However, for quantum communication purposes, photons can be transmitted through long distances carrying with them the quantum states of interest [5]. Therefore, in our case, the logical qubits presented previously are implemented in different properties of single-photon states (§2.1.2). In the following section, more details on the physical implementation of photonic bits and qubits are given.

2.3. Light as an information carrier

In this chapter, we have explored the physical properties of light beams in both the classical and quantum regimes, alongside the foundational principles of classical and quantum information theory. Finally, we bring these concepts together by addressing a central question: *how can light serve as a medium for encoding both bits and qubits?*

Building on the physical description of light beams as carriers of energy §2.1.1, we can now proceed to exploit them as information carriers §2.2.1. Light beams propagating through free-space can carry information by modulating their physical properties, such as amplitude, phase, polarization, frequency, and spatial mode. This forms the basis of free-space optical communications (FSOC), where data is encoded onto a light beam, transmitted through a channel, and then decoded at the receiver. The ability to encode information onto a light beam arises from its various degrees of freedom.

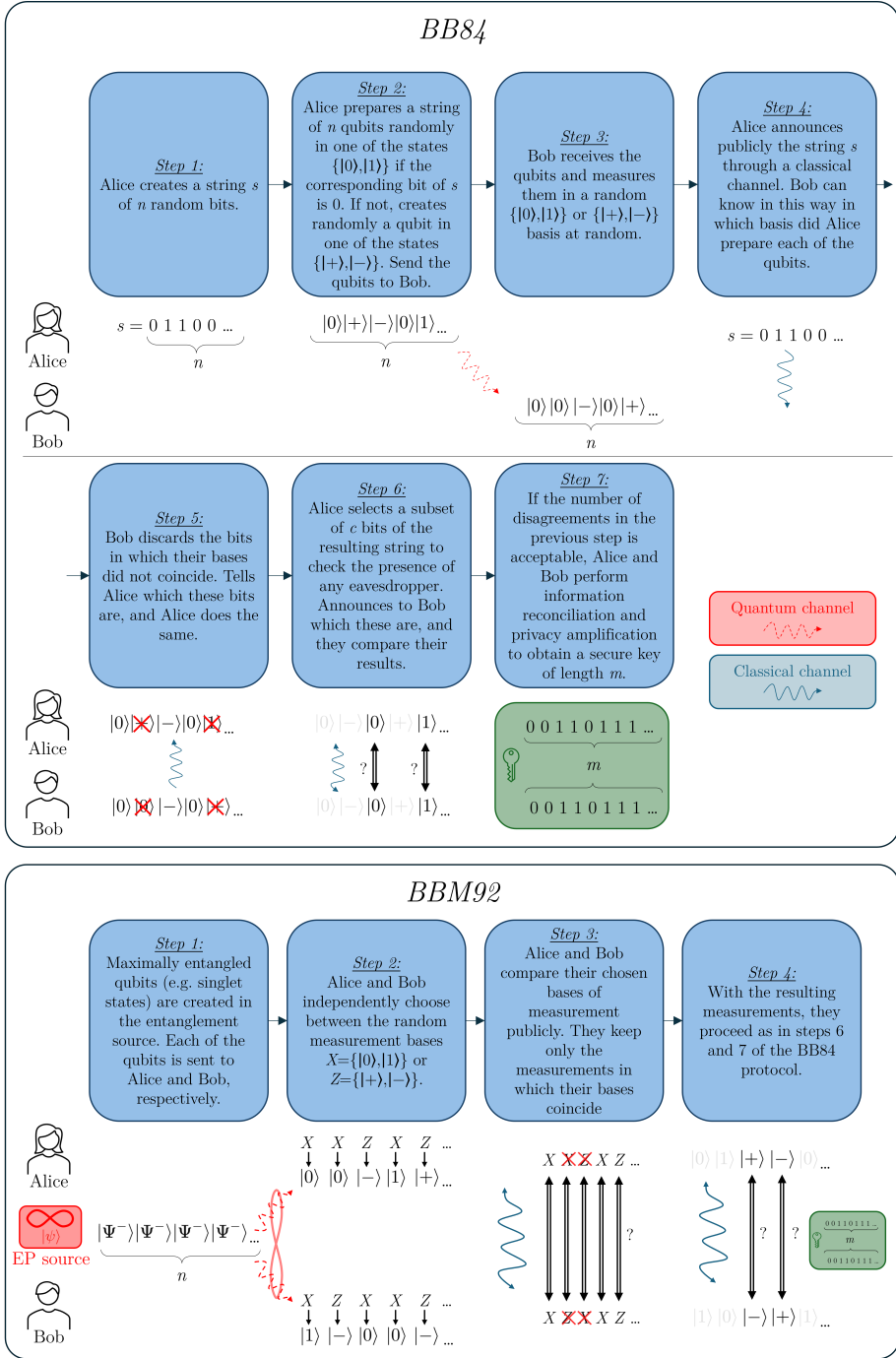


Figure 2.5: Diagram of the BB84 and BBM92 QKD protocols.

Modulation techniques exploit these properties to represent digital data: for example, amplitude modulation varies the intensity of the beam to encode bits, while polarization modulation alters the optical field's polarization state [32]. Similarly, frequency and spatial mode modulation can be used to encode logical bits of information. Beyond modulation, multiplexing techniques allow several data streams to be transmitted simultaneously. For instance, wavelength-division multiplexing assigns different wavelengths to separate channels [33], while spatial-division multiplexing uses orthogonal spatial modes –such as Hermite-Gaussian or Laguerre-Gaussian beams discussed in §2.1– to increase capacity [34]. By controlling these properties of light, multiple parallel streams of information can be reliably transmitted between two parties.

However, imperfections in the physical channel and optical components introduce noise and distortions that can corrupt the transmitted information. Classical information theory, introduced in §2.2.1, provides the mathematical framework to quantify and mitigate such corruption through different techniques (e.g., error-correcting codes [35]). For example, information-theoretical concepts such as capacity can explain why optical communication channels have a higher capacity than the radiofrequency ones: according to the Shannon–Hartley theorem (Eq. (2.37)), the capacity of a communication channel increases with both the available bandwidth and the signal-to-noise ratio. Since optical frequencies provide orders of magnitude more available bandwidth than radio frequencies (due to the higher carrier frequencies involved), free-space optical communication systems inherently offer much higher potential data rates.

When information needs to be transmitted over global distances, or when data must be collected directly in space, the limited range of optical fibers makes satellite-based FSO links essential. However, the optical terminals used in space are far from perfect and are subject to various environmental challenges, including thermal fluctuations, mechanical vibrations, and radiation effects. These factors, along with their impact on system performance and design, will be reviewed in detail in §3.

In the same manner as classical bits can be encoded into the different properties of classical light beams, qubits can also be physically implemented using the single-photon states presented in §2.1.2 as carriers of quantum information §2.2.2. Although quantum information is not restricted to single-photon states alone [36, 37], we will only treat single-photon states in this thesis.

As explained in §2.1.2, the quantum theory of light allows the generation of single-photon states associated with classical paraxial beams. The different properties of these single-photon states, such as polarization, frequency, spatial mode, and time-bin degree of freedom, can be exploited to encode qubits (see Fig. 2.6). For example, polarization states such as $|H\rangle$ and $|V\rangle$ can represent the logical states $|0\rangle$ and $|1\rangle$ of a qubit, while superpositions of these states can be used to create arbitrary qubit states as described in Eq. (2.38) –this type of encoding is the preferred option in free-space optical quantum systems due to the high stability in free-space propagation and high-quality polarization commercial elements [38]. Time-bin encoding, on the other hand, uses early and late arrival times of photons to represent logical states, typically separated by a few microseconds [39, 40]. Similarly, spatial modes (or paths) and photon frequency can be used to represent different logical states [41, 42]. Finally, transversal modes like Hermite-Gaussian or Laguerre-Gaussian beams can also be used to define

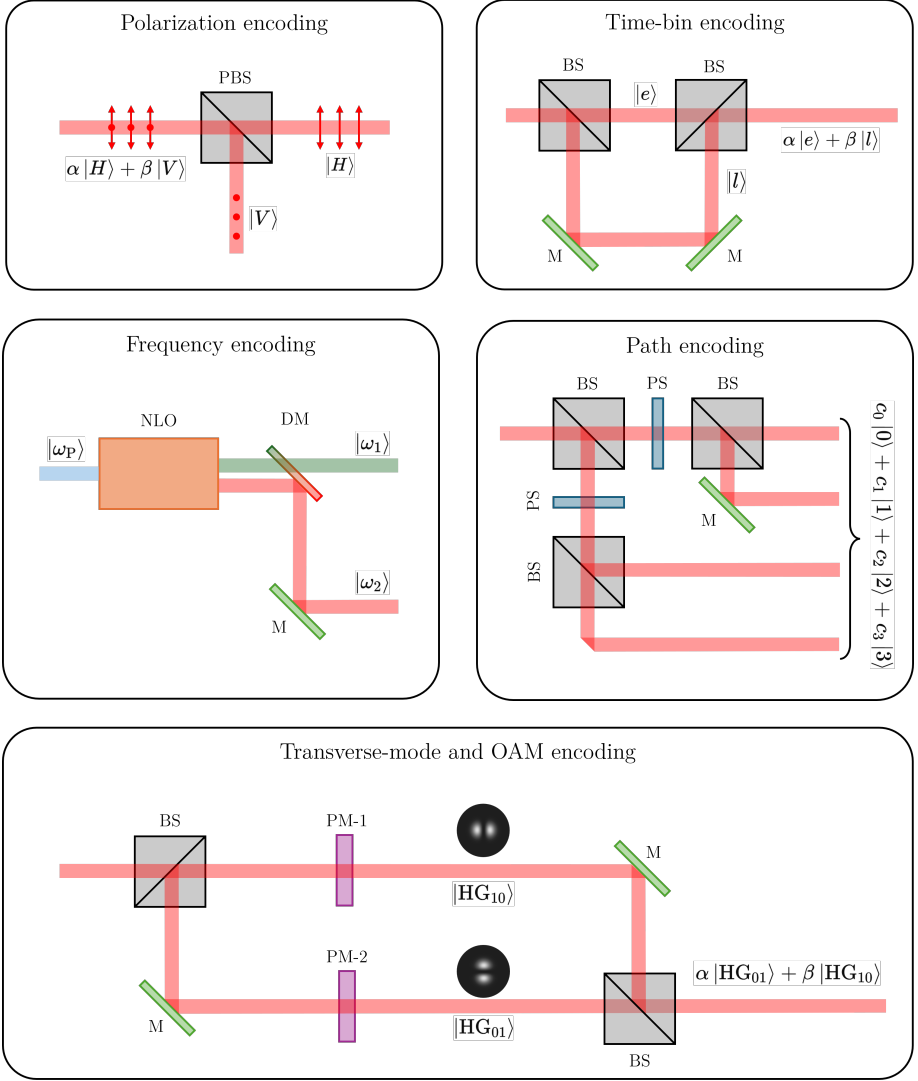


Figure 2.6: Photonic qubits. (a) Polarization qubit: polarization states, e.g. $|H\rangle$ and $|V\rangle$ states, represent logical qubits (b) Time-bin qubit: early $|e\rangle$ and late $|l\rangle$ states represent logical qubits or qudits (c) Frequency-bin encoding: different frequencies encode the logical qubits (d) Path encoding: the presence of a photon in one of the spatial modes (paths) encodes the logical states (e) Transverse-mode and OAM qubit: different transverse spatial modes, e.g. Hermite-Gaussian modes HG_{10} and HG_{01} , represent logical states. PBS: Polarization beam splitter, BS: Beam splitter, M: Mirror, NLO: Non-linear optical system, DM: Dichroic mirror, PS: Phase shifter, and PM: Phase mask.

qubit states based on their transverse mode structure [43]. It is worth noting, however, that time-bin, path, frequency and transverse-mode encodings naturally extend beyond two-level systems, allowing the encoding of *qudits*, i.e., d -dimensional quantum states [44–46]. These qudits can, in principle, enhance the information capacity per photon. By using optical systems such as the ones depicted in Fig. 2.6, logical operations can be performed in qubits. For example, polarization qubits can be converted into spatial qubits through a polarization beam splitter. Furthermore, nonlinear optical sources can generate entangled pairs of optical qubits, enabling the distribution of entanglement across remote parties while preserving their quantum correlations over long distances [47].

Although photonic classical bits and photonic qubits can be implemented using the same physical properties of light –such as polarization, spatial modes, or time bins– they behave fundamentally differently. In classical optics, bits are encoded in orthogonal states (e.g., $|H\rangle$ and $|V\rangle$), which can be measured without disturbance and perfectly copied or amplified for long-distance communication. In contrast, a photonic qubit can exist in an arbitrary superposition, such as $\alpha|H\rangle + \beta|V\rangle$, and if the state is unknown, the no-cloning theorem forbids making an exact duplicate. Any measurement collapses the superposition, preventing perfect copying and making amplification nontrivial. This fundamental difference underpins the physical security provided by quantum communication.

When trying to bridge long distances between the parties involved in quantum communication links, satellite-based free-space optical quantum channels become extremely useful, just as their classical counterparts do. Furthermore, combining the idea of satellite-based quantum communications with entanglement swapping using quantum memories, even larger distances can be bridged. However, as for classical optical links, quantum communication systems are also affected by the imperfections of the physical channel. Moreover, as discussed in the context of QKD protocols in §2.2.2, quantum channels alone are not sufficient for secure key distribution. A classical communication channel is also required to enable the public discussion between the parties, such as basis reconciliation and error correction. A short review of the challenges and limitations of satellite-based quantum communication systems will be discussed in detail in §4.

Chapter 3

Satellite Free-Space Optical Communications

*My greatest concern was what to call it. [...]
Von Neumann told me, "You should call it entropy, [...]
no one really knows what entropy really is,
so in a debate you will always have the advantage."*

Claude Shannon

In the previous chapter, we established how light can serve as both a classical and a quantum information carrier. This fundamental property enables the development of global optical and quantum communication networks. In particular, satellite-based optical communications offer the potential to bridge the large distances involved in such networks. However, spaceborne optical and quantum communication terminals are subject to various opto-thermo-mechanical phenomena induced by the harsh space environment. In this chapter, we review these phenomena as they occur in classical optical communication terminals, highlighting their relevance and impact. Methods for quantifying their effects on communication performance are collected, establishing the connection between optical behavior and communication performance parameters. Finally, we classify available mitigation techniques and identify the key research challenges that remain. A similar analysis for space-based quantum communication will be presented in §4.

The content of this chapter is based on the Ref. [48] Badás, M. et al. "Opto-thermo-mechanical phenomena in satellite free-space optical communications: survey and challenges". Optical Engineering 63, 041206 (2023).

3.1. Introduction

Due to an increasing demand for higher data rates and more secure communications, free-space optical communications (FSOC) has already been shown to be a technology that can become a crucial part of future communication networks [49]. The higher bandwidth, directivity, and security, as well as lower power and mass requirements, make FSOC an interesting alternative to radiofrequency communication. The applications of this technology range from solving the last mile problem [50] to deep space optical communications [51, 52].

Satellite FSOC terminals are paramount for the further development of applications such as deep-space communications and data downlinks of Earth observation missions. These terminals have to be resilient to the severe space environment. Radiation, thermal, and mechanical loads in space will affect the operational performance of the components of the terminal. Furthermore, these terminals must be designed considering the limitations of spacecraft volume and mass. Hence, a profound understanding of the coupled opto-thermo-mechanical phenomena and their impact on the communication performance of the system is needed to identify requirements and optimize the design. This chapter aims to explain the relevant opto-thermo-mechanical phenomena occurring in satellite terminals, identifying which of these phenomena are more important by providing a framework to evaluate the impact of these phenomena in FSOC. Furthermore, the mitigation techniques to counteract these effects are classified, and future research directions for the field are proposed.

In §3.2, a general satellite FSOC terminal is presented, describing its components and their function in the system. In §3.3, the mechanical and thermal loads to which satellite terminals are subjected in different orbits and scenarios are presented. The opto-thermo-mechanical phenomena occurring at the component level of the terminal are presented in §3.4, along with the effect that these have on the optical performance of the system. Furthermore, the modeling workflow to simulate these coupled multiphysics problems is presented, i.e. structural, thermal, and optical performance (STOP) analysis. Section §3.5 builds a bridge to translate the optical effects to communication performance parameters, to evaluate the impact on the communication due to the phenomena previously explained. This section will focus on the impact of opto-thermo-mechanical phenomena on intersatellite links, although the explanation could be generalized to other scenarios (i.e. satellite-to-ground) by considering also atmospheric effects. The mitigation techniques to decrease the detrimental impact of opto-thermo-mechanical phenomena in FSOC are presented in §3.6. Finally, future research topics in this area are proposed in §3.7. In Fig. 3.1, the structure of the chapter is shown and the relation between the phenomena, effects, and impact can be tracked.

3.2. System overview

A satellite FSOC terminal will send or receive information modulated in a laser beam that travels through free-space without the need of optical fibers to guide the light. To achieve this objective, a satellite FSOC terminal is formed by several optical components that steer and shape the beams going through it. Fig. 3.2 shows a possi-

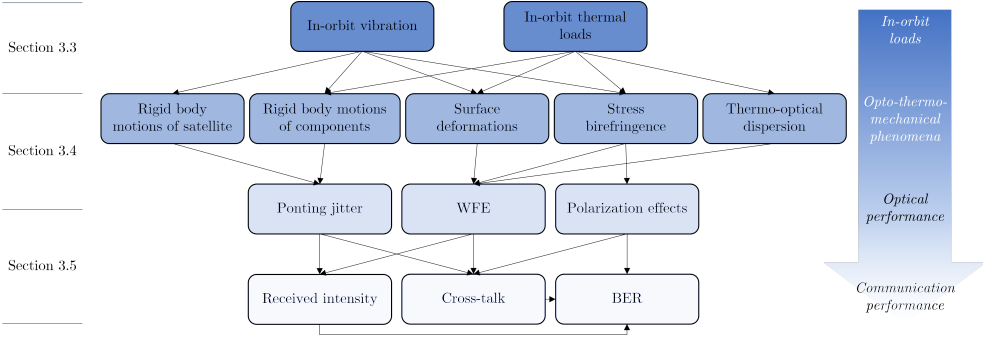


Figure 3.1: Structure of the chapter emphasizing the main topics covered and the sections in which these are presented. The arrows represent the link between the different phenomena and parameters, relating the different causes to their respective effects. (wavefront error (WFE), bit error rate (BER))

ble architecture in which the components that are relevant to the purpose of this chapter are shown. The optical terminal will be in most cases capable of operating both as a receiver and transmitter (i.e. transceiver) as shown in the figure. The architecture presented, has the objective of including the components to which opto-thermo-mechanical phenomena can be relevant and does not pretend to be an optimum architecture of an FSO transceiver.

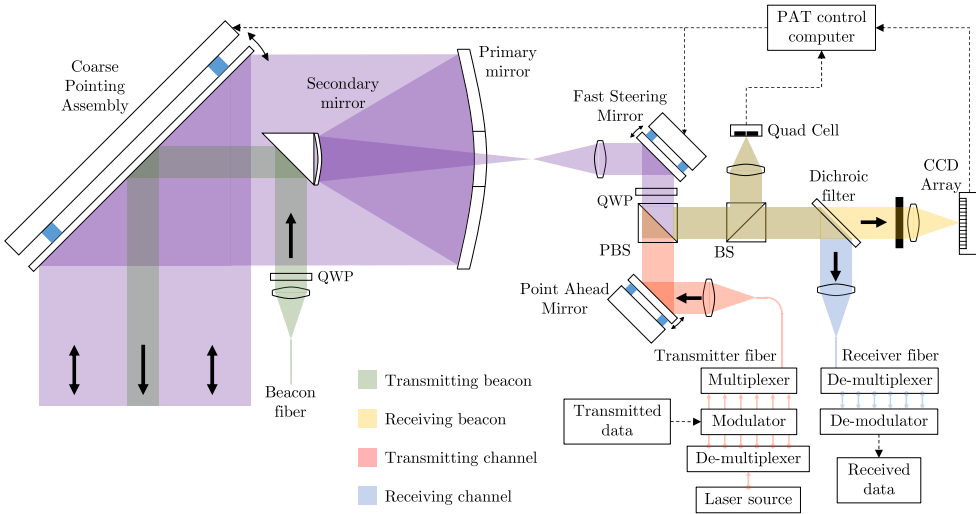


Figure 3.2: General system diagram architecture for satellite FSO transceiver terminal. The lenses shown are a simplified representation of lens assemblies. Colors not stated in the legend, are combinations of others (beam splitter (BS), charge-coupled device (CCD), pointing, acquisition and tracking (PAT), polarization beam splitter (PBS), quarter wave plate (QWP)).

When transmitting a signal, the output data is modulated into the beam coming from the laser source. This transmitted beam is collimated and the point ahead angle compensates for the relative velocity with respect to the other terminal. By sending the laser beam to the location where the receiving satellite will be after the time it takes for light to travel from the transmitter to the receiver, the point ahead angle corrects for the relative velocities between the terminals. This angle is applied to the beam with the point-ahead mirror in the architecture shown in Fig. 3.2. Finally, the beam is transmitted through the telescope after being steered by the pointing and tracking mechanisms (i.e. fast steering mirror (FSM) and coarse pointing assembly (CPA)). These mechanisms are controlled by the spot location of the light received in the CCD array and the Quad Cell, coming from the other terminal's beacon. When receiving the signal, a beacon is emitted from the receiver to enable tracking from the other transmitter. The received light is coupled into a fiber and the data is extracted through demultiplexing and demodulation of it. Alternative architectures include large free-space detectors instead of a fiber coupler. The decision between these architectures depends usually in practical considerations as, fiber based detectors and amplifiers are often used in free-space optical communication terminals.

There are different criteria that have to be considered when choosing a laser source for a satellite communication terminal, from the modulation and multiplexing techniques used to compatibility with optical fiber amplifiers [53]. Compatibility with optical fiber technologies in the design of FSOC terminals will enable the use of current fiber optic technologies to improve the performance of the system. Such is the case of fiber amplifiers to amplify both transmitting [54] and receiving [55, 56] signals.

The presented architecture in Fig. 3.2 shows a polarization beam splitter (PBS) to differentiate the transmitting and receiving waves, representing one of the many applications of the use of the polarization property of light in FSOC. Therefore, this chapter will also study the effects on the polarization state of light due to opto-thermo-mechanical phenomena. Polarization can also be used in optical communications for multiplexing [57] and modulating the signal (Polarization Shift Keying) [32, 58–61]. Polarization of light is also especially relevant for a technology that is closely related to FSOC, i.e. quantum communication [62]. Furthermore, several technologies used in optical communications terminals, such as wavelength division multiplexing devices, use the polarization of light as a baseline for their operation [63]. Finally, the polarization state of the beam has to be maintained in order to efficiently detect the beam in coherent detection receivers [64]. Considering all this, it is paramount to consider the changes in the polarization state induced by opto-thermo-mechanical phenomena in the terminal. The polarization state of light can be linear, elliptical, or circular. For circularly polarized light, each photon carries a spin angular momentum (SAM). The left- and right-handed circular polarization states correspond to an SAM of $+\hbar$ and $-\hbar$, respectively.

Another property of light that can be exploited for communication purposes is the orbital angular momentum (OAM). The OAM is the momentum carried by a beam that is dependent on the spatial distribution of the electromagnetic field (polarization being the other component of angular momentum of light) [65]. Using a set of orthogonal OAM states, this property can be used in FSOC for both modulation [66] or

demultiplexing [67] (see Fig. 3.3). This property of light can be combined with other modulation and demultiplexing techniques that exploit other properties of light, e.g. wavelength, phase, and polarization [60]. Fig. 3.4 shows the different modulation techniques using the polarisation and OAM properties of light. The reader can therefore verify the difference between these two angular momenta; while the helicoid exploiting SAM to modulate the encoded information represents the polarization status of the whole beam, the one using OAM represents the spatial distribution of the wavefront. The generation of SAM is attained by using polarization components, e.g., impinging a linearly polarized beam through a quarter wave plate, while the generation of OAM is achieved by using diffractive optical elements such as spiral phase plates (as will be seen in §9-10).

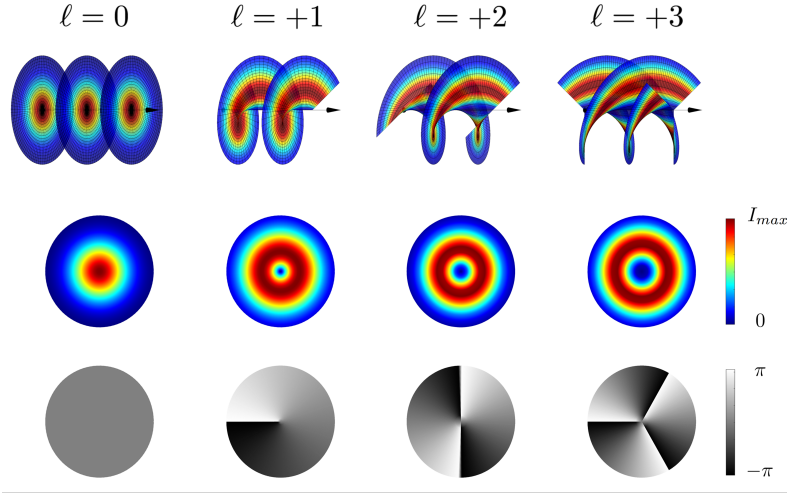


Figure 3.3: OAM of Laguerre-Gaussian beams. The radial zeroth order of the modes is shown for different ℓ azimuthal orders. The wavefront, the intensity profile, and the phase for each OAM mode given by the order ℓ are shown in the first, second, and third rows respectively. The negative values of the ℓ order will yield a different handedness of the helical wavefront conserving the intensity profile

Furthermore, different wavelengths are used in the proposed architecture for the received beacon and channel to differentiate them with a dichroic filter. The selection of the wavelength to be used in a communication link is dependent on several factors such as the atmospheric effects, availability of technology, safety regulations, etc. When the FSOC laser beam travels partially or totally through the atmosphere, the wavelength selection is driven by the reduction of detrimental atmospheric effects (absorption, rain, fog, turbulence, etc.). Several of these reasons support using the near-infrared spectra as the main range of interest for FSOC [68], primarily due to attenuation and atmospheric scattering being lower at these wavelengths. Wavelengths on the mid-wavelength and long-wavelength infrared ($3 - 15 \mu\text{m}$) have also been proposed to lower the detrimental atmospheric effects in the link [69–71]. However, due to

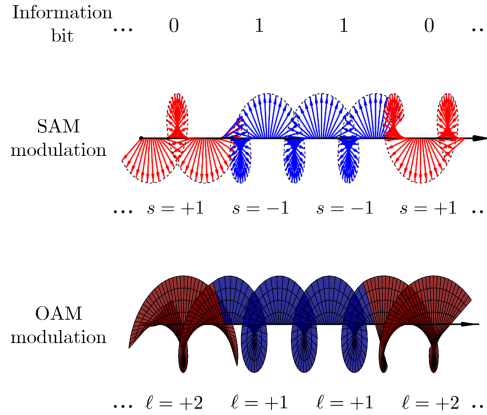


Figure 3.4: Schematic diagram showing the use of SAM and OAM modulation respectively. The SAM states $s = \pm 1$ and the OAM states $l = +1, +2$ are used as an example

the maturity of technologies developed for fiber-based communications, near-infrared wavelengths are still the preferred choice for most FSOC systems. Furthermore, the role that the wavelength of the light plays in FSOC becomes paramount when using wavelength division multiplexing to increase the capacity of the channel.

In general, all properties of light (amplitude, phase, wavelength, and polarization) can be exploited with a different purpose in an FSOC satellite terminal (see Tab. 3.1). This will provide a large number of design possibilities to optimize the performance of the system. In order to assure the correct operation of the system, the effects of the environmental conditions encountered in space on the performance of the terminal must be thoroughly assessed.

3.3. Space environment

Satellite FSOC terminals, will have to survive the harsh space environment consisting of radiation, plasma, thermal, and mechanical loads. Furthermore, these terminals and their constituting components will have to operate correctly in this environment. In this section, only the mechanical and thermal environments encountered in space are treated. An overview of other space environmental conditions was presented by Lu et al. [72] These other environmental conditions will also affect the performance of the FSOC system and have to be considered during the design of the terminal. For example, radiation effects can be especially harmful to the optical components [73] and laser sources [74] required to build such a terminal.

The mechanical and thermal loads impinged on a satellite will affect the performance of the communication link through different opto-thermo-mechanical phenomena (§3.4). The sources and magnitudes of the mechanical and thermal loads on a satellite have to be studied in order to quantify their effects on FSOC.

3.3.1. Microvibrations

Due to the narrow beam divergence (usually smaller or equal to milliradians), FSO systems require a high pointing accuracy (in the order of microradians) to minimize the average bit error rate. The main source of pointing error in an FSO satellite terminal, is the vibration of the satellite due to different in-orbit sources. These in-orbit vibration sources can be classified into internal and external vibration sources [75]:

- *External vibration sources*: are due to loads coming from outside the satellite such as solar radiation pressure, micrometeorite/space debris impacts [72], elastic forces of tension/bending, and gravitational dynamic effects (solar and lunar gravity, Earth oblateness effects, ellipticity of orbit or Earth's central gravitational field).
- *Internal vibration sources*: are due to loads originating inside the satellite such as solar array mechanism, thruster operation, pointing, acquisition and tracking (PAT) mechanisms, gyroscope noise, and attitude actuators based on angular momentum (e.g. reaction wheels) [76]. Other internal sources of vibrations that could be relevant for future FSO satellites are the vibrations induced by a cryocooler. The cryocooler could be needed to operate superconducting nanowire single-photon detectors in future FSO satellites [77]. The microvibrations induced by this turbomachinery can contribute significantly as it was shown on the Hubble Space Telescope [78].

To quantify the contribution of each vibration source to the overall satellite vibration, in-orbit measurements of the vibration have been done on FSO satellites. Given the multiple sources of microvibrations and their statistical nature, experimental in-orbit measurements of these are especially relevant [79]. A review of the in-orbit microvibration detection methods is presented by Tang et al. [80]. In Fig. 3.5, the in-orbit measurements of the power spectral density (PSD) of the angular microvibrations for the most relevant missions are shown. This figure shows similar trends for all the missions and specifications collected. In particular, it can be seen that the energy due to angular microvibrations is mainly concentrated at frequencies lower than 30 Hz for Micus and Olympus, 100 Hz for OICETS, and 102 Hz for ETS-VI [81].

The in-orbit measurements done through a three-axis accelerometer package onboard the Olympus satellite [82] set the ground for the characterization of the vibration environment of an optical communication payload onboard a satellite. This experimental campaign quantified the relative weight of each of the internal vibration sources. The typical duration of internal sources that are relevant to the FSO satellite terminals can be found in the literature [83, 84]. This analysis shows that the longest lasting source of vibrations onboard the Olympus satellite is the antenna pointing mechanism (in the order of seconds).

More recent in-orbit measurements of microvibrations for the OICETS FSO satellite were presented by Toyoshima et al. [85]. The authors of this work reported that the worst-case angular microvibration is created during the tracking and slewing operations of the gimballed optical antenna. The angular microvibrations onboard the

Micius quantum communication satellite and the suppression of these with the PAT mechanism are presented by Wang et al. [81].

The analysis of the in-orbit measurements of other internal vibration contributors such as the control momentum gyroscope, the solar array mechanism, and the reaction wheel was reported by Yu et al. [79]. This work concludes that in the absence of an optical gimbal antenna mechanism, the main contributors to satellite vibrations are the control moment gyros and reaction wheels.

Another source of microvibrations in space that has been studied in depth is a coupled thermo-mechanical phenomenon known as the thermal snap. This phenomenon occurs when the solar arrays (which are usually the most flexible structure in the satellite) are subjected to rapid changes in temperature when entering or leaving an eclipse. This phenomena is mainly due to a varying temperature gradient across the thickness of the solar panels [86]. In-orbit measurements of the thermal snap phenomena were presented by Oda et al. [87]. As it is presented in §3.6, several techniques can be implemented to reduce some of the microvibrations sources presented in the previous paragraphs.

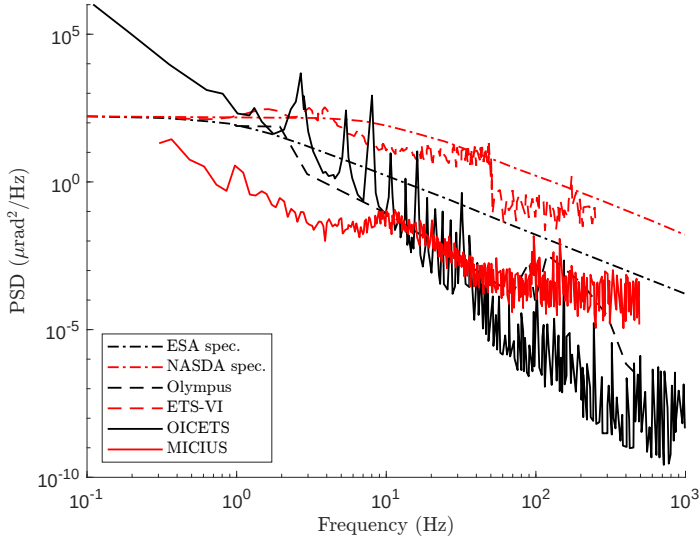


Figure 3.5: In-orbit measurements of vibration PSD of the angular displacement for different space missions. *ESA spec.* and *NASDA spec.* refer to the specifications set by the respective space agencies, while the rest are in-orbit measurements of different satellites. Data extracted from [81, 82, 85, 88].

3.3.2. Thermal loads

Due to the orbit around the Earth, a satellite will be subjected to large temperature gradients during an orbital period. Inside an FSO terminal there are optical elements that are sensitive to these temperature variations. As it will be explained in the next section, the thermal loads can cause optical elements, such as lenses and mirrors, to

deviate from their nominal performance yielding a detrimental effect on the FSOC link. Therefore, it is paramount to know the thermal loads to which the satellites in different orbits will be subjected.

Due to the high to ultra-high vacuum environment encountered in space, radiation is the dominant heat transfer mechanism of the satellite with the external environment. Fig. 3.6 shows the external radiation heat loads that appear in the space thermal environment.

The solar radiation from the sun will impinge the satellite with an approximately $\sim 1361 \text{ W/m}^2$ at one astronomical unit in a plane normal to the vector direction to the sun [89]. For a satellite in orbit around the Earth, this value is not constant due to various factors such as the solar forcing [90] or the ellipticity of the Earth's orbit around the sun (that results in a change in the distance between these two). On the other hand, when the satellite enters the shadow of the Earth, this heat load disappears and the cooling down of the whole satellite occurs, until it is heated again at the exit of the shadow. Some of the sunlight is reflected on the Earth yielding to the short wavelength albedo radiation, which will be dependent on the location and time of each of the Earth's locations. Measurements of the albedo coefficients necessary to accurately quantify this heat load can be found in the literature [91]. The Earth-emitted radiation creates long wavelength radiation also known as earth orbital long-wavelength radiation (OLR), that will also be part of the space thermal environment. The time and location-dependent measurements of this radiation can be found in [92]. Finally, the satellite dissipates heat by radiating into the deep space background, which acts as a thermal sink at approximately $\sim 2.7 \text{ K}$.

Another external source of heat in space is the charged-particle heating [93], which is due to collisions of the charged particles trapped in the Van Allen belts with the spacecraft. This mode of heating is usually confined to the first $\sim 0.1 \text{ mm}$ from the material surface and is much weaker than the radiation heat sources stated above. These heat loads can have relevant contributions in spaceborne cryogenic systems [94], such as the future superconducting nanowire single-photon detectors for FSOC [77, 95]. For example, a radiator designed for 70 K steady-state operation in a circular equatorial Earth orbit at 3200 km altitude, will increase its temperature by 2.9 K [93].

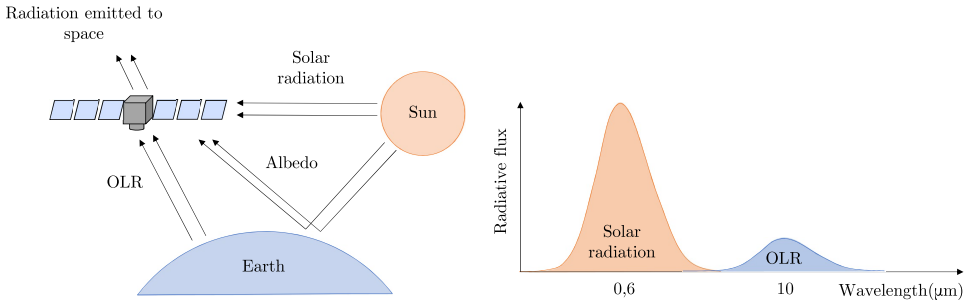


Figure 3.6: External thermal loads of a satellite in orbit around the Earth. On the right side, the qualitative spectrum of the direct solar radiation and the earth orbital long-wavelength radiation (OLR)

The phenomena explained above will altogether set a periodic transient temperature for a satellite in-orbit. low Earth orbit (LEO) is where most of the satellites are placed nowadays. In LEO, defined as 160-2000 km altitude [72], the satellite has an orbital period equal or less than 128 minutes. The thermal loads for a satellite in LEO will change in each orbit due to the changing thermal environmental parameters in time (i.e. albedo coefficient, direct solar irradiance, and OLR). The analysis done by Gonzalez et al. [89] presents a methodology to obtain the worst-case thermal scenarios for a satellite in LEO taking into account the variation of these parameters in time. This analysis derives temperatures in the 150-210K range, for a satellite in a sun-synchronous orbit. Furthermore, an analysis for different solar beta angles¹ to select the worst-case thermal environment was developed by Hengeveld et al. [96]. The guidelines and methodologies developed by NASA to obtain the worst-case thermal scenario in LEO can be found in the literature [97, 98]. The approach followed by Yenchesky et al. [99] for the thermal analysis of a FSOC cubesat, is characterizing statistically the environmental thermal parameters and computing a Monte Carlo simulation.

The Geosynchronous Orbit is defined as an orbit with an orbital period that matches Earth's rotation, the most famous of which is the geostationary orbit (GEO). The thermal environment for a satellite in GEO can be calculated with the same methodologies that are presented by Gonzalez-Barcena et al. [89] for LEO. Another approach followed by Zhang et al. [100], is to neglect both the albedo and OLR heat flows considering only the prevalent direct solar radiation at these altitudes.

There is also an increasing interest in using FSOC for the communication systems to transfer scientific data from deep space probes back to Earth [51, 101]. The thermal environment in such a scenario can be quite diverse due to the broad spectra of missions and trajectories found in deep space. The external radiation heat loads in deep space probes will be due to the albedo and self-emitted radiation of other planets and asteroids (such are the cases of the LLCD [102], MLCD [103] and DSOC onboard Psyche mission [104]). In addition, the direct solar radiation will also change as a function of the distance to the sun and therefore this value has to be accurately reevaluated in each of the scenarios. An overview of the specific characteristics of the thermal environment around other solar system objects is presented by Gilmore [93].

On top of that, internal heat loads will also appear due to the operation of different systems onboard the satellite, such as Joule dissipation in circuits, the propulsion system [105], the laser source [106], or the cryocooler [107]. Notably, the onboard optical amplifiers used to boost the source beam constitute a significant portion of the satellite's thermal load [108]. Optical amplifiers, e.g. erbium-doped fiber amplifier (EDFA), with output optical powers on the order of watts dissipating tens of watts of heat, can be severely self-heated in satellite terminals [109]. The self-heating of these devices not only limits their performance [110] but also usually limits their duty cycle [111]. These sources of internal heat load can be enough to generate optical misalignment issues onboard the terminal [112]. Finally, the laser beam itself can be a significant thermal load in small components without enough thermal sinking. Such

¹The beta angle is the angle between the orbital plane of the satellite and the geocentric position of the Sun

is the case of microelectromechanical FSMs [113]. For satellite FSOC terminals, the thermal lensing effect [114, 115] is usually negligible due to the low powers (maximum 10 W average power) used in satellite transmitter lasers [64].

3.4. Opto-thermo-mechanical phenomena

The mechanical and thermal loads in an optical terminal in space will induce displacements, deformations in the surfaces of optical elements, and changes in the index of refraction. These effects will in turn deviate the system from the nominal optical performance and therefore have to be well understood and analyzed during the design process of the FSOC space terminal. In this section, the relevant opto-thermo-mechanical phenomena are presented and their effects on the optical performance of the system are discussed. The classification of these phenomena has been done by attending to the physical description and modeling techniques that are used to quantify them (see Fig. 3.1).

The effects on the optical performance of the system are quantified by three different optical performance parameters that are relevant to FSOC: pointing jitter, wavefront error (WFE), and polarization state (see Fig. 3.1). The pointing jitter refers to the dynamic angular error between the communication line of sight and the optical axis of the terminal (transmitter or receiver). In this chapter, the WFE is used to quantify the optical aberrations due to the opto-thermo-mechanical phenomena explained below. Other parameters, such as the Strehl ratio [116], can also be used to evaluate the optical performance in FSOC. Finally, changes in the polarization state created by opto-thermo-mechanical phenomena will also be relevant to the performance of the system and will be quantified by means of the polarization extinction ratio (PER).

3.4.1. Satellite rigid body angular displacements

At the largest scale of the system, the vibrations of the satellite will induce rigid body angular displacements of the whole satellite. These rigid body displacements will in turn create a pointing jitter, and therefore deteriorate the performance of the link.

The satellite's rigid body angular displacements will be mainly due to the in-orbit induced microvibrations. The highly stochastic nature of these microvibrations and their high dependency on the interfaces and the details of the satellite [117], make experimental measurements the most valuable to understand the dynamics of the satellite as a whole (such as the in-orbit results shown in Fig. 3.5). The angular deviation of the satellite with respect to the optical axis is statistically characterized by a probability density function (PDF) for the satellite's angular pointing error $\theta^2 = \theta_E^2 + \theta_A^2$. Where θ_E and θ_A are the elevation and azimuth pointing errors, respectively. For a zero-centered normal PDF in both azimuth and elevation pointing error, the PDF of the angular pointing error is a Rayleigh distribution given by (see App. D)

$$p(\theta) = \frac{\theta}{2\pi\sigma^2} \exp\left(-\frac{\theta^2}{2\sigma^2}\right) \quad (3.1)$$

where σ^2 is the variance of the radial pointing error. For a normal PDF non-centered

at zero in both elevation and azimuth angles, a positive angular static error² has to be included. In this case, the PDF containing pointing jitter and static error will be given by a Rician distribution [118, 119]

$$p(\theta, \phi) = \frac{\theta}{\sigma^2} \exp\left(-\frac{\theta^2 + \phi^2}{2\sigma^2}\right) \mathcal{J}_0\left(\frac{\theta\phi}{\sigma^2}\right) \quad (3.2)$$

where ϕ is the angular static error and $\mathcal{J}_0$ is the zeroth order Bessel function. For the remaining analysis in this chapter, the boresight error is taken into account through the PDF of the pointing jitter. Hence, from the experimentally obtained time series of microvibrations in-orbit, the variance σ^2 can be derived. The PDF constructed with this value will characterize the performance of the communication as explained in §3.5.

3.4.2. Rigid body motions of optical components

The satellite microvibrations and the thermal loads can also induce rigid body motions of the optical components inside the satellite [120]. These rigid body motions will, in turn, induce pointing jitter, defocus, and other high-order optical aberrations (see Fig. 3.7a).

In order to model the effects on the optical performance of the system due to rigid body displacements induced by in-orbit random vibrations in the satellite, finite element analysis (FEA) can be computed to obtain the natural frequencies and mode shapes of the optical system. The pointing sensitivity coefficients (also referred to as line of sight sensitivity coefficients in the literature) can be calculated by obtaining the pointing error due to unitary displacements on each of the degrees of freedom that are being analyzed. Then, the frequency response function (FRF) is obtained using modal analysis, and the PSD response is obtained through the FRF and the input force PSD functions. Finally, the pointing jitter can be obtained via the pointing sensitivity coefficients [121, 122]. An analysis of the pointing jitter for the FSOC terminal of the Mars Laser Communications Demonstration Project is presented by Doyle et al. [123]

3.4.3. Surface deformations of optical components

In case the vibrations of the satellite are strong in magnitude, significant elastic deformations will be induced in the optical surfaces. Thermal loads will also create thermo-elastic deformations of the surfaces of the optical components. These, in turn, will create optical aberrations (see Fig. 3.7b) that will also affect the performance of the link. Surface deformations will occur together with changes in volume on the optical components that affect the optical path length. The thermo-elastic phenomena are partially governed by the properties of the materials in the optical assembly of the system. The relevant thermo-elastic properties for the materials used in the infrared are discussed in detail by Klocek [124].

Surface deformations induced by thermal static or transient loads are usually computed through finite element analysis (FEA) and/or finite difference analysis (FDA)

²Also referred to as foresight pointing error or static pointing error in the literature

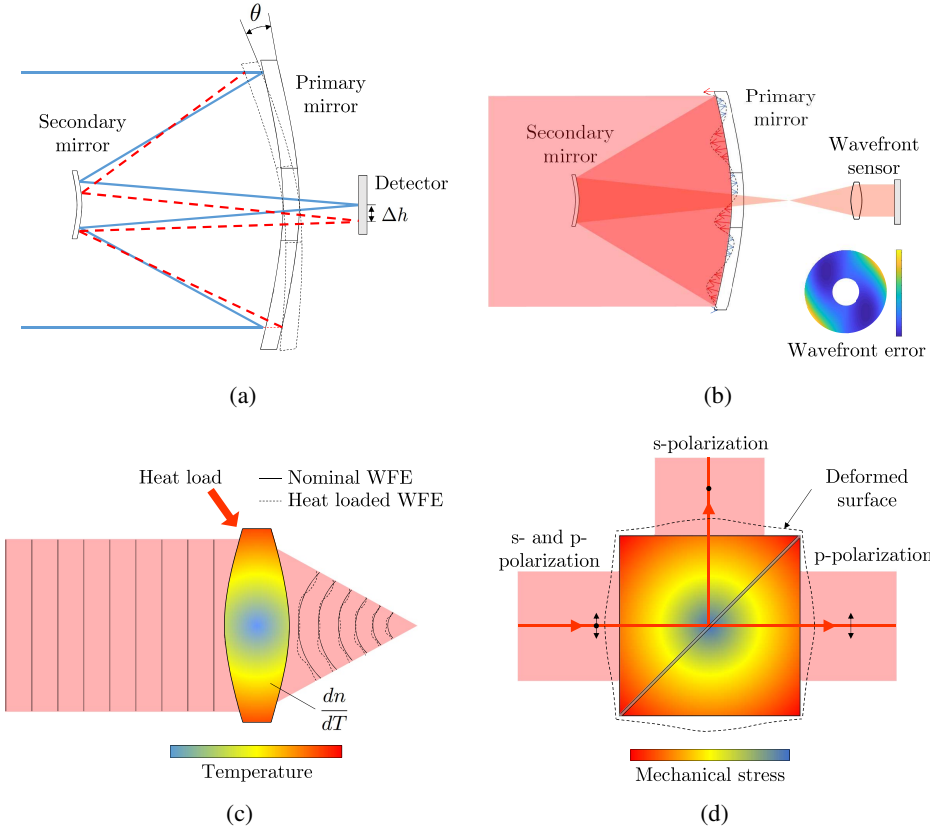


Figure 3.7: Opto-thermo-mechanical phenomena (a) Rigid body motions of optical components inducing pointing jitter (b) Surface deformations of optical components inducing WFE (c) Thermal dispersion on refractive elements (d) Stress birefringence in a polarization beam splitter

[125]. The heat transfer problem can be computed through any of the methods, while the thermo-elastic problem is usually solved by FEA [100, 126–128].

To compute surface deformations induced by in-orbit vibration loads, the surface deformations are computed with FEA and statistical energy analysis (SEA). The first is used to obtain the response to low excitation frequencies, which enables a more deterministic analysis of the system. SEA [129] is used to compute the behavior of the system for high excitation frequencies because the deterministic methods become too sensitive to structural details for these frequency bands [117] (see Fig. 3.8). Other approaches consist of using stochastic FEA or hybrid FEA-SEA models [130].

3.4.4. Thermal dispersion on refractive optical components

Refractive optical materials used in FSOC components exhibit a temperature-dependent index of refraction. The dependence on the index of refraction due to the variation of

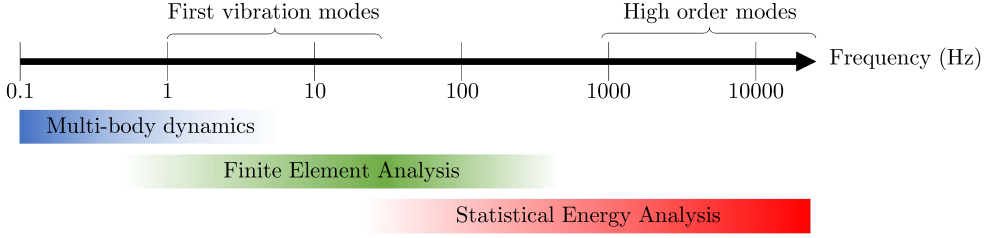


Figure 3.8: Modeling techniques for vibration transmission for different frequency bands. Adapted from Calvi et al. [117]

temperature can be expressed as [131]

$$\frac{dn}{dT} = \left(\frac{\partial n}{\partial T} \right)_{\rho} + \left(\frac{\partial n}{\partial \rho} \right)_T \left(\frac{\partial \rho}{\partial T} \right) \quad (3.3)$$

the first term is the change in the index of refraction solely due to the change in temperature, while the second represents the change in the index of refraction due to the temperature-induced change in the densities of the material. A detailed theoretical expression of each of the terms presented above can be obtained through the theory of thermo-optic dispersion for solids presented in [132, 133]. It is important to remark that although the coefficient of thermal expansion is also captured in Eq. (3.3) through the factor $\partial \rho / \partial T$, the optical path difference created in a ray due to the thermal expansion of the material has to be added to that created by the change in the index of refraction. Hence, the optical path length (OPL) and therefore the WFE for a thermally loaded material must be computed as

$$\text{OPL} = \int_0^{L'} n(s, T) ds \quad (3.4)$$

where s is the path followed by the ray, $n(s)$ will be obtained through dn/dT and L' is the thermally expanded length of the path that the ray travels. The thermally induced optical path difference is obtained directly by combining the two factors explained above: the change in the index of refraction and the thermal expansion of the material [134, 135]. The thermo-optic coefficients for the typical materials used in infrared refractive optical components are presented by Klocek [124]. Rogers et al. [136] analyze the importance of the accuracy in the value of dn/dT for large operational temperature range scenarios (as could be the one for satellite FSO terminals). The importance of making custom measurements of this parameter for wide operational temperature range applications is emphasized in the study.

The thermal load will induce a gradient index profile in the optical component that results in a non-nominal OPL for each ray [137], and therefore a WFE as illustrated in Fig. 3.7c. Two ways of computing the thermal dispersion in refractive components with ray tracing algorithms are discussed by Michels et al. [137].

3.4.5. Stress birefringence of the optical refractive components

Finally, stress states due to the external loads on the refractive optical components will create a stress state that will induce birefringence. Optical birefringence is an anisotropic phenomenon in which the index of refraction of the material depends on the polarization and the propagation direction of light. Some materials are birefringent in nature, while others will become birefringent when subjected to a stress field. In the latter, the stress field will vary the dielectric impermeability tensor β_{ij} , therefore, varying the index of refraction of the material in an anisotropic manner [63]. Mathematically, this phenomenon is usually represented via the stress-optical coefficient tensor q_{ijkl}

$$\Delta\beta_{ij} = q_{ijkl}\sigma_{kl} \quad (3.5)$$

where σ_{kl} are the components of the stress state tensor in each point of the domain, and the subindices represent the spatial coordinates. For isotropic materials, the variation in the dielectric impermeability will create a difference between the two principal optical indices in the electromagnetic waves perpendicular to the ray direction (resulting in an anisotropic behaviour). Hence, the principal optical axes will coincide with the principal stress axes, and retardation between the orthogonally polarized waves will be produced as depicted in Fig. 3.9. This retardation is equivalent to a change in the polarization state of the beam. Furthermore, the phase retardation in each of the optical principal axis components of the electromagnetic wave will induce a WFE in the light that can be approximated as the average of the OPL differences of each of these components [128]. Moreover, the beam can split into two beams due to the birefringence, but this effect is so small in FSOC that it can usually be neglected [138]. It is important to mention that stress birefringence will also occur in nominally birefringent materials often used in optical systems for polarization manipulation.

For an isotropic material subjected to a hydrostatic stress state, Eq. (3.5) will yield an isotropic index variation. Therefore, in the latter, no change in the polarization state will be induced by the hydrostatically stressed element. Hence, only stress dispersion of the index of refraction will occur in this element, i.e., an isotropic variation of n .

The stress birefringence phenomena can be modeled with Jones calculus [139] by transforming the Jones vector through rotation and retardation matrices as presented in Yiu et al. [138]. Jones calculus represents the polarized light as a vector, and the optical components are represented as matrix operations changing these vectors. To compute the effects of stress birefringence, the Jones matrix of the stressed element has to be computed. In order to do so, the stress state tensor in the domain points encountered by the ray path has to be obtained in a FEA structural mechanics software [138, 140, 141]. When the optical modeling of the system under study is done through wave optics instead of ray tracing, Eq. (3.5) is used to compute the variation on the index of refraction in every direction in each element of the domain, and the wave equations are solved, assigning the calculated values to each element in the domain [142]. The effect of stress-induced birefringence in a cube polarization beam splitter used in an optical circulator (widely used in FSOC systems), was analyzed by Doyle et al. [143], where the effect of the temperature and stress in the WFE and

the polarization state are analyzed through a combination of FEA and ray-tracing software.

In an FSOC terminal onboard a satellite, the opto-thermo-mechanical phenomena explained above act altogether, creating a strongly coupled opto-thermo-mechanical problem. Therefore, to simulate the optical performance of the system, all the phenomena above have to be included simultaneously in the model. The approach for modeling opto-thermo-mechanical phenomena combining mechanical, thermal, and optical modeling techniques is presented in Fig. 3.10. These multiphysics modeling techniques that analyze the coupled opto-thermo-mechanical phenomena are generally referred to as structural, thermal, and optical performance (STOP) analysis in the literature [128, 144–146]. As shown in Fig. 3.10, there are cases in which the opto-thermo-mechanical analysis has to be done in an iterative process, such as when the laser beam itself is the source of heat [113]. This is usually not the case for FSOC, and the vast majority of opto-thermo-mechanical phenomena can be characterized by following the diagram in Fig. 3.10 non-iteratively from top to bottom. The thermal and mechanical effects on the optical performance can be analyzed with several analytical and/or numerical methods depending on the complexity of the problem that is being modeled. Analytical methods can be used when the boundary conditions and the geometry of the system are simple. The increasing complexity of any of them increases the necessity of numerical methods to model the problem. A summary of the different commercial software available to model opto-thermo-mechanical problems is presented by Heijmans et al.[147]. An example of a complete STOP analysis done by combining the methods presented in this section is given by Driscoll et al. [148] for the beam expander telescope onboard the NASA Psyche Deep Space Optical Communications mission. The STOP analysis enables the calculation of the permitted WFE to maintain the beam divergence required below $14.5 \mu\text{rad}$ over a temperature range of 80°C . Furthermore, some of the key challenges, such as maintaining the alignment between the primary and secondary off-axis mirrors, are discussed.

The optical performance of the system has been characterized via pointing jitter, WFE, and polarization state in this section. The following section explains how these optical parameters affect communication performance on an FSOC link.

3.5. Impact of opto-thermo-mechanical phenomena on FSOC

The pointing jitter, WFEs, and polarization state changes induced by the opto-thermo-mechanical phenomena on the optical terminal onboard the satellite will impact the performance of the FSOC link. This section presents the methods to quantify the impact on the communication link due to these phenomena. A classification has been created according to where the phenomena occur (transmitter or receiver) and the type of optical effect created by these (jitter, WFE or polarization change). In the subsections below, the impact on communication of each of these effects acting separately is explained. It is worth mentioning that while some FSOC terminals act as transceivers, the following classification categorizes the opto-thermo-mechanical effects affecting terminals, whether they operate as transmitters or receivers. In the remainder of this section, the use of transmitter and receiver terminals refers to terminals (that also include transceivers) operating as such.

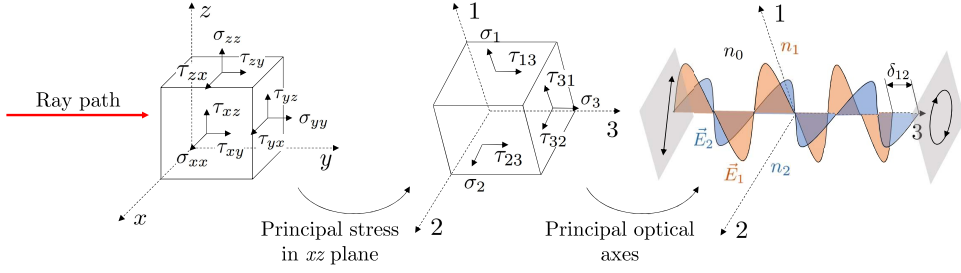


Figure 3.9: WFE induced by stress birefringence in optical components. In the figure, the linear polarization of the incoming ray is changed to an elliptically polarized one. σ_{ij} and τ_{ij} are the normal and shear stress components of the element. δ_{12} is the retardation induced by the stress state between the light polarized parallel to the optical axes 1 and 2. Being 1 and 2 the principal stress axes perpendicular to the ray path

Property	Use in FSO	Effect of opto-thermo-mechanical phenomena
Amplitude	Modulation	A minimum amplitude of the light field is required on the receiver side in order to detect the signal. The amplitude of the light field entering the receiver is affected by opto-thermo-mechanical phenomena (see §3.5).
Phase	Modulation, Multiplexing	Apart from phase modulation, the spatial distribution of the field's phase can generate orthogonal beam modes that can be used for multiplexing. Unexpected changes in the phase due to opto-thermo-mechanical phenomena are detrimental for the system.
Wavelength	Modulation, Multiplexing, Tx/Rx	The wavelength of the light field is a key link parameter mainly dependent on the channel characteristics (atmosphere, vacuum...) and available hardware. Deformations of optical components due to opto-thermo-mechanical phenomena can be more restrictive in shorter wavelengths (see §3.5.2).
Polarization	Modulation, Multiplexing, Tx/Rx	The polarization state of light can be perturbed by opto-thermo-mechanical phenomena, yielding to crosstalk and BER (see §3.5.5)

Table 3.1: Properties of light used in FSO and a brief description of how they are affected by opto-thermo-mechanical phenomena. Tx/Rx refers to the separation between transmitting and receiving light.

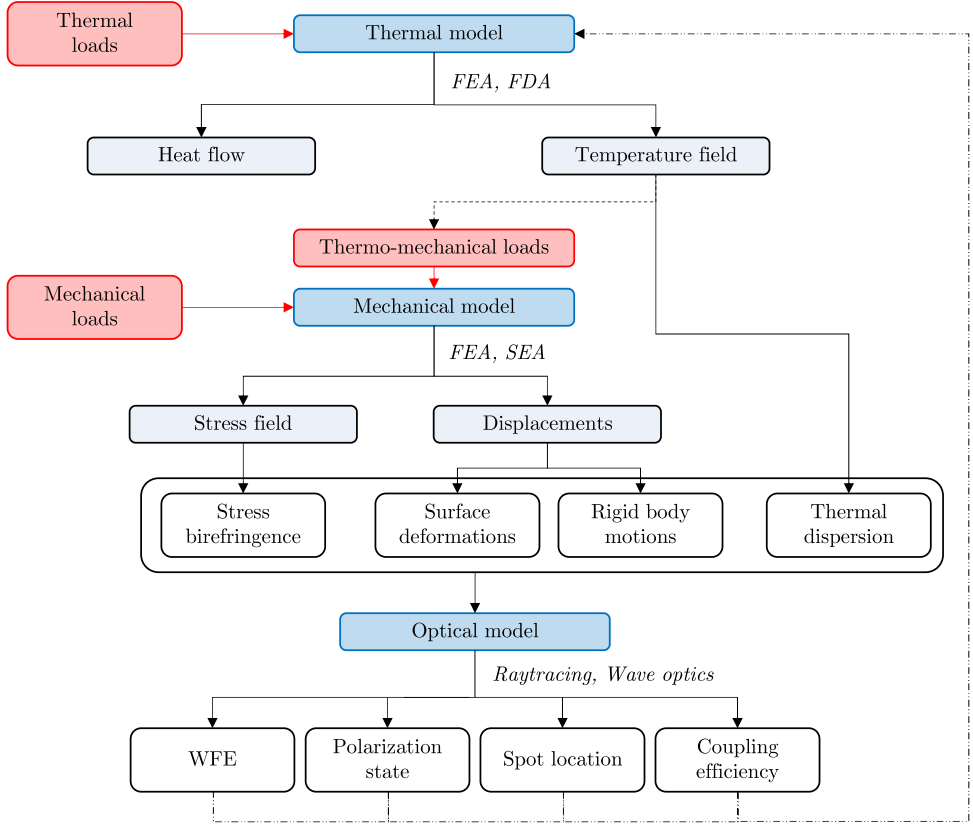


Figure 3.10: Road-map for modelling opto-thermo-mechanical phenomena, STOP analysis. The loads (boundary conditions necessary to solve the problem) are shown in red boxes. The models and their results are shown in blue and light blue boxes, respectively. Finally, the parameters to characterize opto-thermo-mechanical properties are shown in white boxes. The dashed-dotted line shows the iterative process for scenarios in which the optical performance determines the thermal loads.

To evaluate the impact of the phenomena presented above in the communication performance, the effect of these phenomena in the transmission equation is evaluated first. The transmission equation is used in FSOC to evaluate the value of the signal intensity at the receiver's detector. The Friis transmission equation can be written as [149]

$$P_R = P_T G_T \mu_T L_S L_{PJT} L_A G_R \mu_R \mu_C \quad (3.6)$$

where P_R is the instantaneous received power, P_T is the average transmit power, G_T is the transmit antenna gain, μ_T is the transmitter feeder loss, L_S is the space loss, L_A is the term grouping all atmospheric effects, L_{PJT} is the transmitter pointing jitter loss, G_R is the receiver antenna gain, μ_R is the receiver transmission loss, and μ_C is

the fiber coupling loss³.

Furthermore, the different noise sources appearing in the communication link can be computed to obtain the total noise [64]. Combining the transmission equation and the noise, the signal to noise ratio (SNR) of the communication link can be obtained. The probability that a transmitted information bit is read incorrectly at the receiver is called the bit error rate (BER). The BER is evaluated through the SNR of the communication link according to the modulation technique used.

The impact on the communication performance of the opto-thermo-mechanical phenomena stated above is evaluated by assessing how these phenomena alter the values of the received intensity, the cross-talk between multiplexed signals, and finally the BER (see Fig. 3.1). In what follows, satellite-to-satellite links are mainly discussed to quantify the impact of opto-thermo-mechanical phenomena. Satellite-to-satellite scenarios are discussed because of the absence of atmospheric effects in the link, hence isolating the impact of opto-thermo-mechanical phenomena solely. However, the phenomena discussed in this section can be translated to other scenarios (e.g. satellite-to-ground) by including the relevant atmospheric effects.

3.5.1. Transmitter pointing jitter

In order to assess the impact of the pointing jitter and the aberrations in the communication link, a distinction must be made according to where these phenomena occur, i.e., in the transmitter or in the receiver. For the sake of explanation, an inter-satellite communication scenario is considered. The pointing jitter in the transmitter (Fig. 3.11), created primarily due to the rigid body motions of the satellite and the optical components in the terminal due to microvibrations, will degrade the communication by diminishing the signal intensity received in the receiver's aperture (see Fig. 3.12a). This is usually quantified in the link budget by introducing a pointing jitter loss L_{PJ} in the transmission equation Eq. (3.6). In general, the contribution of the microvibrations to the pointing jitter will be added to many other pointing errors (see Fig. 2.2 by Hemmati [64]). The instantaneous loss due to the transmitter pointing jitter can be expressed for a Gaussian beam at the aperture of the transmitting telescope as [150]

$$L_{PJT} = \exp(-G_T \theta_T^2) \quad (3.7)$$

where G_T is the antenna gain and θ_T is the angle radial pointing error. For a Gaussian, non-aberrated beam, the wavefront at sufficiently large distances (when the distance between the terminals is much bigger than the Rayleigh range) will be a spherical surface centered on the transmitter terminal. Hence, the electromagnetic field entering the aperture of the receiver can be considered as a plane wavefront. Under random vibrations, such as those experienced by the onboard optical terminal, θ_T will be characterized by a PDF (Eq. (3.1) and (3.2)). For a link using a Gaussian beam and under transmitter pointing jitter given by a Rayleigh PDF (Eq. (3.1)), the PDF of the intensity at the receiver aperture is described by a beta distribution [151] (see also

³ Atmospheric effects are not considered in this chapter. Opto-thermo-mechanical phenomena onboard satellites will not impact the value of L_A

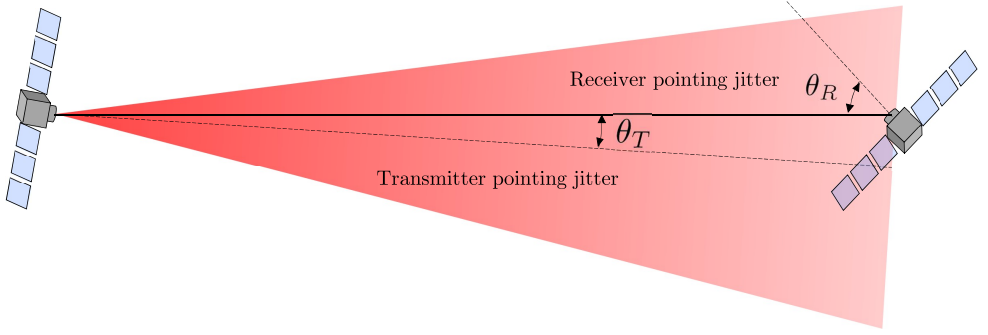


Figure 3.11: Definition of the transmitter and receiver pointing jitter. The continuous line represents the direct line of sight between the transmitter and the receiver, connecting the central point of both apertures.

App. D)

$$p(\mathcal{I}) = \beta \mathcal{I}^{(\beta-1)} \quad \text{where} \quad \beta = \frac{w_0^2}{4\sigma^2} \quad \text{for} \quad 0 \leq \mathcal{I} \leq 1 \quad (3.8)$$

where w_0 is the beamwaist of the Gaussian beam, $\mathcal{I}$ is the normalized intensity, and the average intensity is given by $I_{\text{avg}} = \beta/(\beta+1)$.⁴ Therefore, the average intensity in the receiver's aperture can be obtained by combining the far-field Fraunhofer diffraction intensity profile in the receiver's aperture plane and the PDF of the θ_T . Through this reduction of the average intensity of the signal, the SNR will decrease, giving an increase in the BER. For different modulation techniques [152, 153] and receiver architectures (direct or coherent) [149, 154], different BER models will be used to compute the effect of transmitter pointing jitter in communication.

Furthermore, the pointing jitter will also affect the communication performance of multiplexed links. The transmitter's pointing jitter could create cross-talk between the signals multiplexed in the same beam. This effect is significant in OAM multiplexed links because of the spatial dependence of this multiplexing technique. The result of the transmitter pointing error in OAM multiplexed FSO is discussed Xie et al. [34], Li et al. [155] and Dabiri et al. [156]. In mode-division multiplexed links that make use of other orthogonal beam modes (e.g., Hermite-Gaussian beams) to multiplex the signal, the transmitter pointing error can also be a potential source of crosstalk between the signals [157].

3.5.2. Transmitter wavefront error

The WFE induced by the thermo-mechanical phenomena occurs in both transmitter and receiver terminals. The WFE induced in the transmitter can result in a decrease of the intensity at the aperture plane of the receiver and an increase in the beam

⁴This equation considers that the beamwidth of the beam in the aperture plane of the receiver is much larger than the receiver aperture size. In this way, the variation of the intensity across the aperture is negligible (see App. C).

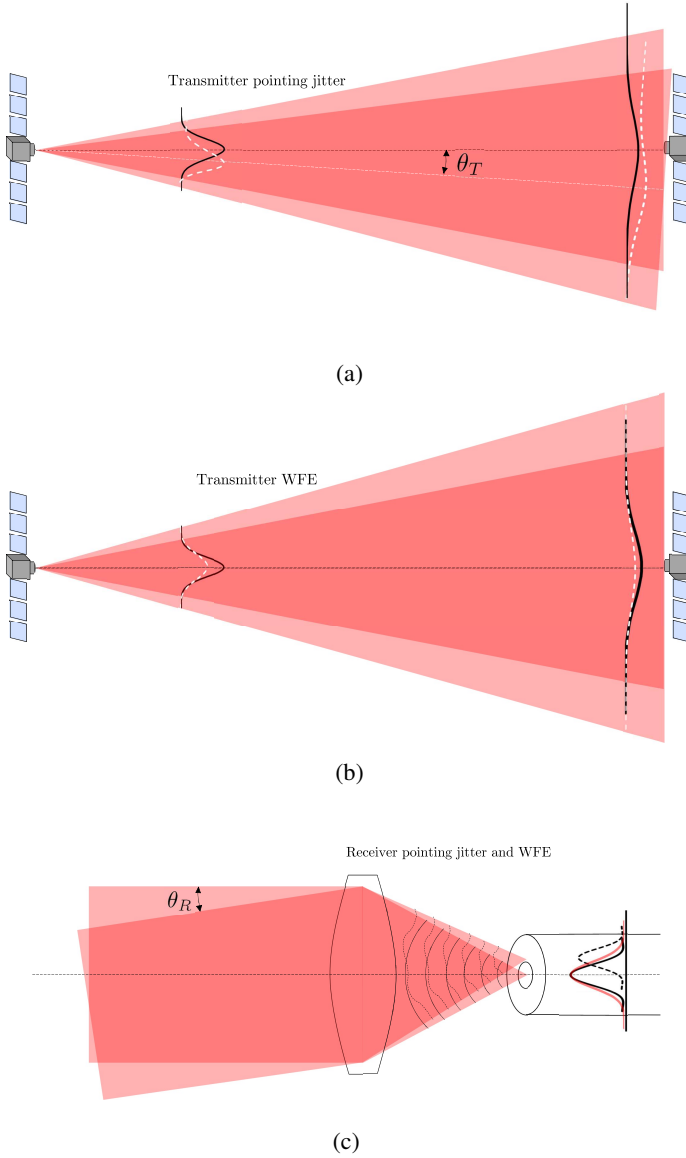


Figure 3.12: Impact of opto-thermo-mechanical phenomena in FSOC (a) Transmitter pointing jitter, the white dashed-line and the black line are the intensity profile of the beam with and without pointing error respectively (b) Transmitter WFE, the white dashed-line and the black line are the intensity profile of the beam with and without WFE respectively (c) Receiver pointing jitter and WFE, the red line is the intensity profile of the mode of the fiber, the black dashed and continuous lines are the intensity profile of the focused light with and without WFE and pointing jitter, respectively

divergence angle (Fig. 3.12b). It is important to mention that although the figure shows an increase in the divergence of the beam due to the WFE this is not necessarily always the case, as it depends on the specific wavefront error under analysis (as it will be analyzed in §6). This effect can be quantified by propagating the field at the aperture of the transmitter to the aperture plane of the receiver through Fraunhofer's diffraction. Yang et al. [158] analyzed the impact of the transmitter WFE in an FSO communication link using on-off keying modulation. This analysis shows that for the same root mean square value of the primary aberrations, coma aberration will have the largest impact while spherical aberration the least (see also §6).

Requirements on the maximum WFE for FSO are dependent on the specific link parameters, but usually $\text{WFE}_{\text{RMS}} < \lambda/30$ for the on-axis field of view and $\text{WFE}_{\text{RMS}} < \lambda/15$ for the whole field of view are a good rule of thumb for a preliminary terminal design in the wavelengths used in FSO [148, 159, 160].

As explained in §3.3.2, the laser beam itself can induce thermo-elastic deformations in optical microcomponents without enough thermal sinking, such as the microelectromechanical mirrors [113]. This type of load will only occur in the transmitter terminal because the intensity of the received signal in the receiver will usually be orders of magnitude below that of the transmitter.

3.5.3. Receiver's pointing jitter

The receiver's pointing jitter (see Fig. 3.11) will create a random beam walk-off at the detector plane. The impact on the communication performance of this random beam walk-off will be dependent on the architecture of the receiving terminal. If the photodetector is sufficiently large that the focal spot radius is contained within it, then the degradation due to the random walk-off is negligible. When the detector is not big enough, then the effect on the intensity received can be computed through the degradation factor proposed by Zaman et al. [149]. In fact, the pointing jitter in the detector will have the largest effect when the incoming link is coupled into an optic fiber onboard the receiver's terminal (see Fig. 3.12c).

When the communication system couples the receiving light to a fiber through a coupling lens, the pointing jitter will decrease the fiber-coupling efficiency. The instantaneous fiber coupling efficiency can be calculated in the presence of receiver pointing jitter, θ_R , for both single-mode [161] and multi-mode [162] fibers as:

$$\eta_{C,mn} = \frac{P_C}{P_A} = \frac{\left| \int \int \int_A E_s(\theta_R, \rho, \phi) E_{mn}^*(\theta_T, \rho, \phi) p(\theta_R) \rho d\rho d\phi d\theta_R \right|^2}{\int \int_A |E_s(\rho, \phi)|^2 \rho d\rho d\phi} \quad (3.9)$$

where $\eta_{C,mn}$ is the coupling efficiency of the LP_{mn} mode ⁵, P_C is the power coupled in the fiber and P_A is the total available power at the aperture of the coupling lens (considering negligible the transmission losses through the terminal, this is equal to the power available at the aperture of the receiver). E_s is the electric field in the focal plane, E_{mn} is the fiber mode corresponding to mode LP_{mn} and $\{\rho, \phi\}$ are the polar coordinates at the fiber coupling plane. $p(\theta_R)$ is the PDF of the receiver's pointing jitter. This coupling efficiency can then be introduced to the Gaussian-Q function

⁵LP mode refers to the characteristic linearly polarized modes of the fiber optic considered

using the theory shown in [149] to obtain the BER (although this is not the kind of coupling considered in the chapter, the rationale behind the approach is equivalent).

When the link is OAM multiplexed, the receiver's pointing jitter will also induce crosstalk between the multiplexed signals, similarly to the case of the transmitter jitter [34]. The crosstalk in the case of the receiver's pointing jitter is due to a tilted phase added to the phase of the OAM multiplexed beam at the receiver plane.

3.5.4. Receiver's wavefront error

As for the WFE induced on the optical terminal of the receiver, the spot at the detector plane will increase in size and move its position. Usually, these two effects created by thermo-mechanical loads on the satellite are small enough that the spot remains contained within the detector [116, 158].

However, when the incoming beam is coupled into a fiber (see Fig. 3.12c), the WFE induced by the thermo-mechanical phenomena in the receiver terminal will decrease the coupling efficiency [163, 164]. This effect can be computed by obtaining the field E_s in the focal plane of the fiber coupler in the presence of WFE in the receiver, and using Eq. (3.9).

The overall effects in the communication performance due to all the phenomena presented above can be quantified for an on-off keying modulation scenario through the BER [149, 150]:

$$\text{BER} = \int_0^\infty \int_0^\infty \frac{1}{2} \text{erfc} \left(\frac{Q(\theta_T, \text{WFE}_T, \theta_R, \text{WFE}_R, \mathbb{S})}{\sqrt{2}} \right) p(\theta_T) p(\theta_R) d\theta_T d\theta_R \quad (3.10)$$

where Q is the Gaussian-Q function, and $\mathbb{S}$ represent the design parameters of both terminals. Therefore, all the effects presented above can be quantified through the Gaussian-Q factor defined as:

$$Q(\theta_T, \text{WFE}_T, \theta_R, \text{WFE}_R, \mathbb{S}) = \frac{i_{D-1} - i_{D-0}}{\sigma_{N-1} + \sigma_{N-0}} \quad (3.11)$$

where $i_{D-1,0}$ is the instantaneous signal current at the detector and $\sigma_{N-1,0}$ is the noise variance for the 1 and 0 bits, respectively. The instantaneous signal current values are affected by the pointing jitters and WFE in both the transmitter and the receiver (therefore are dependent on the parameters $\{\theta_T, \text{WFE}_T, \theta_R, \text{WFE}_R, \mathbb{S}\}$, not explicitly shown in the equation above for simplification). The pointing jitter and the WFE of the transmitter will reduce the value of instantaneous signal current through the reduction of the power received at the aperture plane of the receiver. The pointing jitter of the receiver will reduce the signal current by affecting the degradation factor [149]. Finally, the WFE in the receiver will reduce the instantaneous current signal by a fiber coupling efficiency reduction.

3.5.5. Polarization effects

As stated in §3.1, the polarization property of the laser beam can be exploited for communication purposes in FSOC. Therefore, changes in the polarization state due to stress birefringence can affect the performance of the link [54, 165]. There are several

devices (see §3.2) in FSOC terminals that can be affected by stress birefringence and therefore affect the polarization of the beam. The most susceptible component on a terminal to polarization effects is the polarization beam splitter (PBS), which is used to split the incoming beam into two orthogonally polarized beams. In the presence of stress birefringence, crosstalk between the two output beams could occur due to a change in polarization induced by the stress field [143]. This could, in turn, yield several effects depending on the role polarization is playing in the communication link:

- If the beam is linearly polarized and to be used in a coherent detection system, then the changes in polarization due to stress birefringence (both in transmitter and receiver) will yield a reduction in the intensity received at the detector and therefore an increase in BER. In this case, an effort has to be made to assure that the polarization is maintained within boundaries in the adverse space environment [54]. The maintenance of polarization required for this kind of link is quantified through the polarization extinction ratio (PER).
- If polarization modulation is used in the link, then the change in polarization will directly increase the BER due to the higher probability of receiving an erroneous bit value.
- If polarization division multiplexing is used in the link, then crosstalk between the signals multiplexed in the same beam could occur.

In this section, several optical performance parameters that are affected by opto-thermo-mechanical phenomena have been investigated. An overview of these parameters, the metrics used for their characterization, and a summary of their effects in FSOC are presented in Tab. 3.2.

3.6. Mitigation techniques

The opto-thermo-mechanical phenomena in an FSOC satellite can greatly impact the communication performance as stated before. In order to diminish the impact of these phenomena, several mitigation techniques can be applied. In this section, a review of these techniques is presented.

The mitigation techniques are classified according to the source of the opto-thermo-mechanical phenomena (i.e., in-orbit microvibrations or thermal loads). It has been seen before that these sources could create similar opto-thermo-mechanical phenomena and therefore have an effect on the same optical performance parameters (see Fig. 3.1). On the one hand, microvibrations will mainly create rigid body motions (of both the satellite and components) and therefore pointing jitter, the mitigation techniques associated with these items are covered first. On the other hand, the thermal sources will mainly create surface deformations, stress birefringence, and thermo-optical dispersion. These will, in turn, give polarization effects and WFE of the system. Hence, the mitigation techniques related to these items are covered second.

3.6.1. Microvibrations

In order to reduce satellite microvibrations and diminish their detrimental effects on FSOC, mitigation techniques can be applied on several layers. The first layer includes

Phenomenon	Performance metrics	Effect on FSOC
Pointing errors	Angular pointing jitter PDF and σ^2 (see Eq. (3.1)-(3.2))	Transmitter pointing error decreases the signal on the receiver. Receiver pointing error changes the angle of incidence of the incoming beam, reducing fiber coupling efficiency. In spatially multiplexed beams, pointing errors cause crosstalk between channels.
Wavefront errors	Zernike coefficients of the WFE and WFE_{RMS}	Transmitter WFE reduces far-field intensity along the beam axis. Receiver WFE lowers fiber coupling efficiency. In spatially multiplexed beams, WFEs also introduce crosstalk.
Polarization effects	PER	Depending on how polarization is used, this may reduce received intensity, introduce bit errors, or cause crosstalk.

Table 3.2: Optical performance parameters affected by opto-thermo-mechanical phenomena, their performance metrics, and their effects on FSOC.

reducing the microvibrations onboard by smart design of the attitude determination and control system (ADCS) and structural system of the satellite. These mitigation techniques are common to any satellite that require good pointing stability on their specific application (e.g., Earth observation, astronomy). Three different measures can be taken at this layer, classified according to where in the satellite the measure is applied [166]:

- *Microvibration source suppression.* The intensity of some of the internal microvibration sources listed in §3.3.1 can be suppressed or reduced by a proper design of the systems involved, as presented by Li et al. [166]. Special attention must be dedicated to the reduction of the reaction wheel microvibrations as this is one of the main sources of microvibrations [76, 167]. Furthermore, operational improvements can be made to suppress or reduce the intensity of these sources [168]. This could mean turning off some of the equipment or limiting the operation ranges of these whenever possible. The latter could be applied by not operating the satellite communication under the thermal snap induced microvibrations when entering or going out of the eclipse.
- *Terminal isolation.* As shown in Fig. 3.13 the terminal can be isolated through both passive and active isolators. A review of the techniques to isolate the optical terminal from the spacecraft microvibrations is presented by Liu et al. [169] The isolation techniques developed for other space-borne instruments can be transferred to FSOC systems. Hyde et al. [170] present a specific example of the use of a passive hexapod isolation system for FSOC.
- *Structural transmission path suppression.* The microvibrations arriving at the payload will be transferred to the optical relay of the terminal as represented

in Fig. 3.13. Through a detailed structural dynamics analysis [130] and the optimization of the optical mounts for the different components, the vibrations can be isolated on a component level through the transfer path suppression technique. The analysis of such a technique to design the supporting structure of a satellite FSOC telescope is presented by Nevin et al. [159] This mitigation technique can also be applied at a more general satellite level, for the design of the bus structure and the optimization of the location of the internal microvibration sources (e.g., reaction wheels and solar array drive mechanism).

Each of the microvibration suppression and isolation techniques described will have an effect on different frequency ranges of the PSD of the microvibration (see Fig. 3.5). Several examples of the different methods of suppression and isolation are presented in Tab. 3.3, stating the frequency ranges in which they will attenuate the PSD of the microvibration. High-frequency microvibrations, above 500 Hz, have a much lower influence on the pointing jitter and are usually suppressed by the structure itself (transmission path suppression) [171].

The second layer would be to use active correcting components on the optical terminal to compensate for microvibrations (i.e. CPA and FSM). An extensive survey of the available fine and coarse steering mechanisms that can be used in mobile FSOC terminals, such as a satellite terminal, is presented by Kaymak et al. [172]. Different fine steering architectures are investigated by Miller et al. [173], and the impact of including a fine steering assembly in a satellite communication terminal is quantified in the satellite-to-ground experiment onboard Micius [81]. The terminal microvibration requirements can also be relaxed through the use of wide-field angle receivers [174].

Finally, the mitigation of the impact of microvibrations on FSOC systems can be attained through the shaping and modulation of the laser beam itself. Beam size optimization can be used to reduce the impact of transmitter pointing jitter [118, 152, 175, 176], receiver pointing jitter [161] or a combination of both [177] –this will be analyzed in detail in Part III. The impact of the pointing jitter in different modulation techniques [152, 154, 178] and different terminal architectures [149] will determine the optimal combination of these to minimize the effects of pointing jitter in FSOC.

	Low-frequency (< 5 Hz)	Mid-frequency (5 Hz – 500 Hz)
Source	Drag free technology, multifunctional attitude control	Fluid viscous damper, viscoelastic materials, magnetorheological damper, particle damper, tuned mass damper, metal rubber damper, voice coil damper, piezoelectric hybrid damper, electromagnetic hybrid damper
Payload isolation	Drag free payload	Magnetic flux pinning
Transmission path	Metamaterial damper	Metamaterial damper

Table 3.3: Spacecraft microvibration suppression methods and frequency ranges attenuated by them [171]

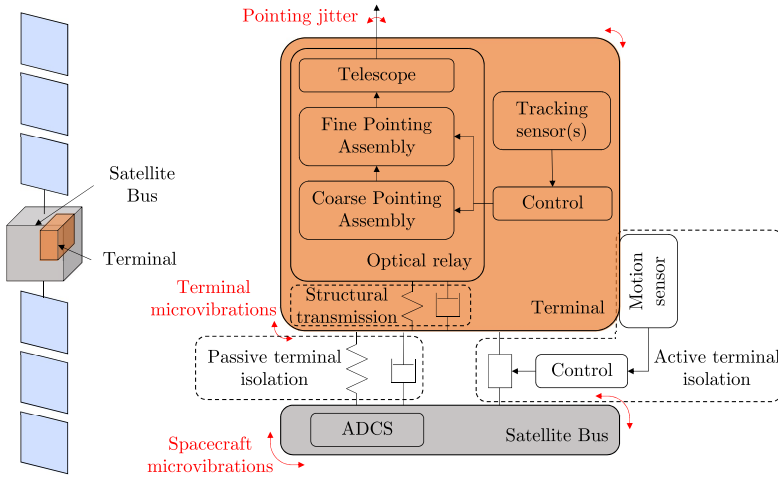


Figure 3.13: FSO satellite microvibration mitigation techniques. Microvibrations at different system levels of the satellite are represented through red arrows. ADCS system in the bus is depicted to represent one of the many possible onboard vibration sources

Other mitigation techniques that are not specific but can also attenuate the effects of microvibrations can be found in the literature [49, 179]. The right combination of the previously presented mitigation techniques will enable the optimal design of the communication system.

3.6.2. Thermal phenomena

The effects induced by thermal loads and their impact on an FSO satellite can be mitigated in several ways. Given that the thermal loads will mainly create surface deformations, stress birefringence, and thermo-optic dispersion, the mitigation techniques related to reducing the WFE and polarization effects are considered in this section (considering the structure presented in Fig. 3.1). Depending on where the mitigation technique is implemented, these can be classified as:

- *Satellite thermal control.* Proper control of the external (see Fig. 3.6) and internal thermal loads on a satellite level can be achieved through different techniques [93, 180, 181]. Both active and passive thermal control mechanisms will help to obtain a more stable operating temperature environment for the terminal. Furthermore, the thermal management of internal heat loads is paramount when these loads are high (e.g. EDFA). An example of a passive thermal management technique onboard the TBIRD satellite terminal is done by placing the components that generate most of the heat close to the external faces of the enclosure, increasing their radiation to space [111].
- *Component design and athermalization.* It is not possible to achieve a completely constant and uniform temperature in the satellite with the satellite-level

thermal control techniques. Therefore, the optical mounts and components of the terminal must be carefully designed to reduce the mechanical stresses and deformations under the given thermal environment. In this way, the WFE and stress birefringence phenomena can be reduced. The stability of the optical performance of the terminal under the cyclic thermal space environment can be achieved by athermalizing optical components [182]. To do so different design parameters such as geometry (flexures [183]), materials, and coatings (attending to their thermomechanical and thermo-optical properties) have to be combined to achieve the optimum solution [159]. A combination of these techniques is presented for a catadioptric telescope design for an FSOC satellite by Rahman et al. [184] and in the design of the FSOC TBIRD terminal [112]. A hermetic sealing of the mirror inside a container filled with inert gas has been proposed to create convection thermal transfer and mitigate the effects of thermal loads in a microelectromechanical FSM [113].

- *WFE compensation.* The relative effect on the propagation of the transmitter WFE on different beam profiles will be different. In links where non-Gaussian beams are used for purposes such as multiplexing [67] or atmospheric turbulence mitigation [185], the link could also be more resilient to the effects of the transmitter WFE. In this way, the requirements for the WFE in the transmitter terminal could be relaxed. On-ground receiver terminals using adaptive optics systems to compensate for the atmospheric turbulence [163, 186], can also correct for the WFE induced by opto-thermo-mechanical phenomena. In this way, the coupling efficiency can be increased when an optical fiber is used for the detection of the signal. Adaptive optics systems onboard satellites have also been proposed to correct for the aberrations produced in satellite FSOC terminals [187].
- *Polarization compensation.* An active compensation in the receiver side to correct for polarization mode dispersion occurring in optical fibers is presented by Nabavi et al. [188]. This method could be applied on optical coherent receivers to compensate for polarization mode dispersion created by opto-thermo-mechanical phenomena in FSOC.

The WFE and polarization compensation techniques should not be applied as the main solution to reduce the impact of these induced by opto-thermo-mechanical phenomena in the communication link. These solutions require complex active systems that are difficult to qualify and operate onboard satellites. Instead, thermal control techniques and smart component design should be applied to reduce the WFE and the change in polarization state created by opto-thermo-mechanical phenomena.

3.7. Future work

To improve the communication performance of FSOC, several challenges related to the opto-thermo-mechanical phenomena occurring in the optical terminals have to be studied.

The pointing accuracy of the transmitted beam is one of the key performance parameters on FSOC links. The classical solutions for steering the beam have been mainly based on mechanical PAT mechanisms, such as gimbals and FSMs. However, during the last years, non-mechanical beam steering systems have raised a lot of interest in the field due to their lower weight and bulkiness, and their extended operating lifetime [189]. Several non-mechanical beam steering mechanisms have been proposed for their application in FSOC [190]. There is a vast diversity of non-mechanical beam steering techniques such as based liquid crystal devices [191–193], optical phased arrays [194], and focal plane switch arrays [195]. Furthermore, several techniques have been proposed to simultaneously steer and shape the beam, combining the previous with variable focal lenses [189, 196]. Further research is required to enable the use of these devices in FSOC terminals in space, especially regarding their performance and lifetime under the periodic temperature changes to which they would be exposed.

Some of the non-mechanical beam steering techniques presented above are based on integrated photonic circuits [194]. In fact, integrated photonic devices are currently being investigated for their application in FSOC. Especially in the space segment, FSOC terminals would hugely benefit from the multifunctionality and small size of these devices [197, 198]. Many applications have been proposed for the use of integrated photonic devices in FSOC [199–201] and more specifically in spacecraft terminals [202]. The effects of radiation and thermal loads in space can be critical for the operation and lifetime of these components [203, 204]. Furthermore, the optical powers onboard the satellites can sometimes be too high for integrated waveguides due to non-linear effects [202]. Further research on the radiation hardness and the thermal management of these devices will be key for their implementation in FSOC satellite terminals.

As it has been mentioned in §3.6, adaptive optics systems could be used to compensate WFE due to opto-thermo-mechanical phenomena onboard satellites. Firstly, adaptive optics systems in space to correct perturbations due to opto-thermo-mechanical phenomena [205] have been studied in depth for their application in astronomical telescopes [206–209]. Furthermore, adaptive optics have been widely used in the ground segment of FSOC systems during the last decades for correcting atmospheric distortions [210]. However, little research has been done on the use of adaptive optics systems onboard FSOC satellite terminals [187], to compensate for WFE generated by both atmospheric and opto-thermo-mechanical phenomena.

The benefits of mechanical metamaterials for vibration isolation can be exploited in space applications in the future [211]. Periodic macroscopic lattice cell structures can be customized with the aid of additive manufacturing to obtain the desired vibration mitigation characteristics. Combining the design flexibility with the low mass, the use of these metamaterials seems to be promising for satellites using FSOC. Studies on the applicability of these to FSOC satellites and their qualification for the space environment should be investigated in the coming years [212].

Due to their versatility and low mass, diffractive optical elements have been proposed for FSOC for signal multiplexing [213, 214], multi-beam transmitter design [215], system size reduction [216, 217] and beam spot size reduction [218]. Although athermalization of diffractive elements has been investigated previously [219, 220], a better

understanding of the opto-thermo-mechanical phenomena in this type of component is required to be used in FSOC satellite terminals.

Similar to diffractive optics, metasurfaces have shown to have several benefits compared to classical refractive and reflective optics used generally in FSOC [221]. The metasurface technology is maturing rapidly and will enable a lot of new possibilities for FSOC, e.g., compact wide-field-angle receivers [222] and multiplexing [223, 224]. The opto-thermo-mechanical behavior of these components and their impact on communication have to be further understood to extend their applicability to satellite terminals in the future. Some opto-thermo-mechanical phenomena occurring in metasurfaces have already been investigated in mechanically [225, 226] and thermally [227] tunable metasurfaces.

Considering the impact of the pointing jitter, WFE and polarization on the communication link, several research gaps relevant to this work have been identified:

- The impact of pointing jitter and WFE on links with different modulation techniques has to be methodologically assessed –as it will be presented in §5. As a result, the optimum combination of the requirements on satellite microvibrations and WFE for different modulation techniques can be derived.
- The combined effect of pointing jitter and WFE on the propagation of different beam shapes (e.g. Hermite-Gaussian, Laguerre-Gaussian, Top Hat) has to be assessed.

As has been discussed in the previous section, microvibration suppression on satellite FSOC terminals is key for increasing the performance of the link. The application of novel microvibration mitigation techniques has to be applied to satellite FSOC terminals; these include drag-free devices, voice coil dampers, and magnetic flux pinning. Some of the problems that should be investigated in the future regarding microvibration suppression are presented by Jiao et al. [171].

In a similar manner, novel spacecraft thermal management methods and their use in satellite FSOC have to be further investigated. Recent advances in spacecraft thermal control have been significant in both passive and active devices [228, 229]. Research on passive components includes phase change materials for heat storage to reduce radiator size [230], novel materials such as graphene and aerosols [231], and multifunctional nanostructured surfaces that modulate the emissivity and absorptivity to optimize the thermal performance [232]. Miniaturization of active thermal devices, such as cryocoolers [228] and pumped fluid loops for small satellites [233] will be also relevant for future FSOC systems.

3.8. Summary

The role of opto-thermo-mechanical phenomena in FSOC and their impact on communication on satellite terminals have been described in this chapter. The characterization of the thermal and mechanical environment to which the terminals are subjected has been explained. The opto-thermo-mechanical phenomena created by these loads on FSOC terminals and how to model them have also been described. These phenomena include satellite rigid body displacements and rigid body motion, surface deformation,

thermal dispersion, and stress birefringence of the optical components. The impact of these phenomena on the communication link has been described by categorizing them according to where they occur (i.e., transmitter or receiver). Furthermore, the changes in the state of polarization due to the opto-thermo-mechanical phenomena have also been assessed.

Several mitigation techniques to reduce the effects on the optical performance of the terminal and to diminish the impact on the link have also been presented. Finally, research challenges that exist for the development of future FSOC terminals have been identified. Further investigation on the topics covered in §3.7 will be essential to the development of future satellite FSOC.

Chapter 4

Satellite Free-Space Quantum Communications

*We have learned that it is possible
to fight entanglement with entanglement*

John Preskill

As introduced in §2, light can serve not only as a classical information carrier but also as a quantum information carrier. Leveraging the quantum properties of light, it is possible to envision a global quantum information network enabled by spaceborne quantum communication terminals. However, just as the space environment impacts classical communication systems (as discussed in §3), it similarly affects their quantum counterparts. In this chapter, we present different space-based quantum key distribution (QKD) architectures and assess the influence of opto-thermo-mechanical phenomena on satellite terminals. To this end, the relationship between the physical effects and the resulting quantum communication performance parameters will be clarified. Finally, we analyze available mitigation techniques designed to suppress these detrimental effects.

4.1. Introduction

Quantum communication is an emerging technology that leverages the principles of quantum mechanics to enable fundamentally secure information exchange. Unlike classical communication systems, whose security relies on computational complexity assumptions (e.g., RSA encryption), quantum key distribution (QKD) offers security at the physical layer [18]. This is particularly relevant given the anticipated development of large-scale quantum computers, which threaten to render classical cryptographic schemes vulnerable [26, 27]. Beyond secure communication, quantum communication constitutes a building block of broader quantum information networks, enabling distributed quantum computing, entanglement distribution, and advanced metrology applications [234]. While, in principle, quantum communication can be implemented using different physical carriers, long-distance quantum communication is most efficiently realized in the optical domain, where photons act as qubits. Photons are well suited for communication because they are easily transmitted across long distances and, unlike matter-based carriers, they interact only weakly with the environment [39, 235].

A central challenge in quantum communication is overcoming photon transmission loss. In optical fibers, losses scale exponentially with distance, imposing fundamental limits on long-distance QKD and entanglement distribution. While other ground-based concepts have been proposed, such as the vacuum-assisted transmission channels described in [236], these concepts remain technologically demanding. An alternative approach is to exploit free-space optical channels, where propagation losses follow the same physical principles as in classical optical communications (see §3) but scale much more favorably over large distances. In particular, satellite-based quantum communication has emerged as the most promising method to bridge intracontinental distances, as satellites can overcome the severe attenuation present in terrestrial fibers [237, 238]. To bridge over even larger distances, space-based quantum repeater-assisted architectures have been proposed [239].

Over the past decade, numerous reviews have assessed the state of the art of practical and technological challenges in quantum communications [38, 240, 241], some of them focusing more on space-based schemes [234, 237, 242–244]. Experimental demonstrations have been particularly advanced by the Micius satellite mission, which achieved a series of groundbreaking results, including intercontinental QKD, satellite-to-ground entanglement distribution, and quantum teleportation [245]. Following that mission, the microsatellite Jinan demonstrated similar capabilities despite being only one-sixth the weight of its predecessor [246]. Current efforts are also exploring nanosatellite-based QKD [247], quantum memories for space applications [239], and diverse architectures for quantum information networks [234, 248].

Despite these advances, the realization of a global satellite-based quantum communication network faces a wide range of challenges. These include technical issues such as atmospheric turbulence, scintillation, daylight operation, weather-dependent transmission windows, and background noise, as well as broader challenges such as system standardization and network integration [234, 249]. Turbulence-induced scintillation is believed to have a negligible impact on the performance of some QKD protocols [250], and it has been experimentally proven that turbulence does not significantly change

the polarization state of light in polarization-encoded QKD systems [251]. However, atmospheric turbulence reduces the photon detection probability [252, 253], and adaptive optics can be used to counteract these effects and improve system performance [254, 255]. Daylight operation introduces strong background noise due to the sunlight [256, 257], which requires the implementation of advanced filtering techniques across spatial, temporal, and spectral domains [249]. Weather effects, such as clouds, further complicate scheduling and availability [258, 259].

However, in the context of satellite-based QKD, it is also of primary importance to evaluate the effects that the harsh space environment will have on the performance of these systems. This chapter focuses specifically on the opto-thermo-mechanical phenomena induced by space-environmental loads. The aim is not to provide a comprehensive review of all challenges in satellite-based QKD, but rather to identify, analyze, and quantify the impact of opto-thermo-mechanical effects. These include some of the phenomena already presented in §3, as well as other opto-thermo-mechanical effects specific to quantum communication links, such as polarization perturbations and thermal instabilities in single-photon sources and detectors. In this way, this chapter extends the treatment of technical realizations given in Ref. [237] and complements the subsystem-oriented review in Ref. [234]. By systematically examining opto-thermo-mechanical effects in the satellite segment of quantum information networks, we aim to clarify their relevance for system design, identify possible mitigation strategies, and highlight their implications for the performance and scalability of global quantum communication. The topics presented here will also be directly relevant to other space-based quantum experiments and missions, including distributed quantum computing and sensing [248, 260], enhancing the precision of astronomical interferometry [261], and testing fundamental physics in space [262–268].

This chapter begins by summarizing the different satellite-based QKD architectures and the past, present, and future missions that implement them (§4.2). In §4.3, we present the optical components that constitute the transmitter and receiver satellite-based QKD terminals. The space-environmental loads and the opto-thermo-mechanical phenomena induced by these are discussed in §4.4. Their impact on the performance of the secret key generation process is then analyzed in §4.5 by evaluating photon transmission probability, crosstalk, and background noise in relation to the quantum bit error rate (QBER) and the secret key rate (SKR). Finally, methods to suppress these effects and mitigate their impact are examined in §4.6.

4.2. Satellite-based QKD architectures and missions

To understand how the different space environmental effects affect satellite-based quantum communication systems, we first need to understand how these systems operate. Specifically, it is important to understand what are the different types of satellite-based quantum key distribution architectures and associated protocols. In this section, the fundamentals of the types of satellite-based quantum key distribution protocols are revisited, and the physical components needed in satellite terminals to operate these different protocols are presented.

4.2.1. Light as an information carrier: photons and qubits

Satellite-based quantum key distribution protocols consist on preparing, transmitting, receiving, and measuring photonic quantum states. These photonic quantum states work as quantum information carriers. In order to encode this information on light, one or several of its properties has to be manipulated (see §2.3). Among all the properties of light, the polarization state of light has been the one that has been exploited the most so far for satellite-based QKD. In this way, the logical qubit states $\{|0\rangle, |1\rangle\}$ are encoded in orthogonal polarization bases of our selection, e.g., horizontal and vertical polarization states $\{|H\rangle, |V\rangle\}$. Another option is to encode the logical qubits in time bins [269, 270], by encoding the logical qubits in different temporal states of the transmitted signal, e.g., early and late photons $\{|e\rangle, |l\rangle\}$ –typically separated by a few microseconds [41, 42]. By encoding the qubits using the frequency property of light [42, 271], we can represent the logical basis through photons with different frequencies $\{|\omega_1\rangle, |\omega_2\rangle\}$. Another way to encode the logical qubits is by utilizing the different spatial modes or paths that the photons travel through. Finally, transversal spatial orthonormal modes of light in the paraxial regimes can be used to encode logical qubits. For example, Laguerre-Gaussian beams with orbital angular momentum can be used to create the bases $\{|\text{LG}_{00}\rangle, |\text{LG}_{01}\rangle\}$. In general, the bases presented to encode logical qubits can be extended from bidimensional qubits to n -dimensional qudits, e.g. $\{|\omega_1\rangle, |\omega_2\rangle, \dots, |\omega_n\rangle\}$ or $\{|\text{LG}_{00}\rangle, |\text{LG}_{01}\rangle, \dots, |\text{LG}_{0n}\rangle\}$. This is possible in all but polarization-encoded systems, as the polarization state of light has only two degrees of freedom.

Nevertheless, in satellite-based QKD, polarization has been the most often chosen [245, 272, 273], due to the high stability in free-space propagation and high-quality polarization commercial elements [38]. Using time-bin and frequency-energy encoding in satellite-based QKD has also been investigated, although not demonstrated in the space environment [274, 275]. Space mode or path-based encoding can be used in short distances through light guiding devices, but exploiting them in free-space channels would require extremely collimated beams and huge transmit and receive apertures. Finally, although transverse mode and orbital angular momentum encoding have received some attention for their application in satellite-based communications [276–278], we will comment on some of the challenges associated with this in the following sections.

Furthermore, satellite-based QKD can be implemented using either single-photon (discrete-variable) protocols or continuous-variable protocols. In discrete-variable QKD, information is encoded into distinct quantum states, such as the polarization or time-bin degree of freedom of single-photons, providing well-defined symbols for key generation [5]. By contrast, continuous-variable QKD relies on encoding information into quadratures of the electromagnetic field, where the key is extracted from continuous measurements of amplitude and phase, offering potential advantages in terms of detection efficiency and compatibility with existing telecommunication technologies [37, 279]. Both approaches have been explored for satellite-based implementations, but in this work, we focus on single-photon (discrete-variable) QKD. This choice is motivated by the higher technology maturity of DV systems –most on-orbit demonstrations to date have employed discrete-variable protocols– as well as their lower

sensitivity to atmospheric scintillation effects [279] and their ability to tolerate significantly higher channel losses compared to continuous-variable schemes [280], which is a critical requirement for long-distance satellite links.

4.2.2. Satellite-based QKD architectures

The encoding chosen for satellite-based QKD can be implemented with different architectures (see Fig 4.1), which are related to the QKD protocol to be used. In general, most of the satellite-based QKD systems aim to generate a secret key between two ground stations on Earth by using the satellite as a relay. The basic operation is similar across different QKD protocols and involves the transmission, measurement, and post-processing of quantum states. A source on the satellite (or on the ground) generates quantum states of light –typically single-photons, weak coherent pulses, or entangled photon pairs– that are transmitted through free-space to the receiver. The receiver measures the quantum states in randomly chosen bases, producing raw key data correlated with the sender’s. The raw keys shared between sender and receiver are inevitably affected by losses, noise, and channel imperfections, requiring several post-processing steps to ensure security. Error correction (information reconciliation) aligns the mismatched keys of Alice and Bob, leaking some information to the eavesdropper (Eve). After reconciliation, privacy amplification is applied using hash functions to compress the key into a shorter version that removes any residual knowledge Eve might possess [242]. In entanglement-based protocols, entanglement distillation can further enhance security by extracting a smaller set of highly entangled pairs from a noisy ensemble, thereby improving the fidelity of the final shared key [281]. We can therefore say that the key generated is secure in the sense of composable security. Specifically, a QKD protocol between Alice and Bob is considered secure if, for some positive parameters $a, b > 0$, any eavesdropping strategy allowed by the laws of quantum physics either causes the protocol to abort or, if the protocol completes successfully, it does so with probability $1 - O(2^{-a})$. Moreover, in this case, the mutual information between the generated key and the environment is smaller than 2^{-b} [5, §5.4].

According to where the sources, detectors, and other quantum hardware is located, the satellite-based QKD schemes can be classified by (1) uplinks in which the source is located in the ground and (2) downlinks in which the source is located in space. Furthermore, there are schemes that involve flying trusted-nodes (see Fig. 4.1(a) and(b)) and entanglement-based schemes that do not (see Fig. 4.1(c) and(d)). According to the different architectures presented, different QKD protocols can be operated in them. There is a huge amount of discrete variable protocols proposed in the literature [235], being decoy-state BB84 ¹ the only one so far demonstrated in space [283].

Starting with satellite-based QKD architectures that require a flying trusted-node, we can distinguish downlink and uplink schemes illustrated in Fig. 4.1(a)-(b). Under these architectures, denominated prepare-and-measure architectures, a secret key is created between the satellite and the first ground station (we can call this key $\mathcal{K}_A$).

¹In prepare-and-measure QKD systems based on weak coherent pulses, there is a risk of photon-number-splitting attacks. To counteract this, decoy-state protocols randomly vary the intensity of pulses. By monitoring detection statistics, Alice and Bob can detect and prevent these attacks, ensuring the security of the key [282].

Later, when the satellite passes above the second ground station, another key is generated (we can call this key $\mathcal{K}_B$). Finally, the satellite having both keys can announce publicly the bit-wise parity $\mathcal{K}_A \oplus \mathcal{K}_B$, which enables both ground stations to obtain the other ground station's key securely, e.g., $\mathcal{K}_A = \mathcal{K}_B \oplus (\mathcal{K}_A \oplus \mathcal{K}_B)$. In this way, both ground stations now share the same secret key. In case of performing a downlink (see Fig. 4.1(a)), a single-photon source needs to be located in the satellite and single-photon detectors on the ground. When performing an uplink, however (see Fig. 4.1(b)), the single-photon states are transmitted from the ground and detected in space. The effects of the atmospheric turbulence are more detrimental in the latter configuration. While adaptive optics systems, particularly those relying on laser guide stars, can mitigate this [239], uplink schemes are generally less favorable. Furthermore, the installation of single-photon detectors requiring cryogenic temperatures is more challenging in the satellite than on the ground [234], yielding to downlink configurations having more heritage in space than their uplink counterparts (see §4.2.3). A detailed comparison of uplink and downlink performances for satellite-based QKD can be found in Ref. [284].

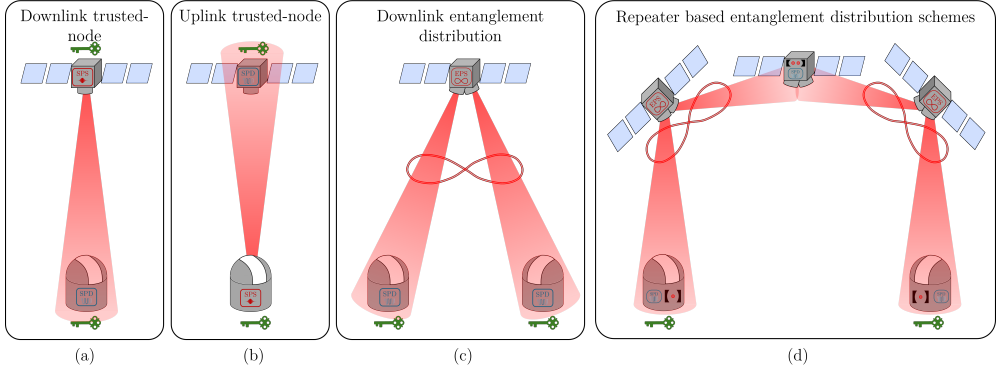


Figure 4.1: Examples of satellite-based quantum key distribution architectures. SPS: single-photon source, SPD: single-photon detector, EPS: entangled-photon source, and the cavity with circles in illustrates a quantum memory.

However, in order to operate the previously explained protocols, it is necessary that both ground stations trust the satellite. Although there have been proposals for an operation similar to the one explained in the previous paragraph with an untrusted-node satellite, they have been proven wrong [285]. In this context, entanglement-distribution-based protocols with untrusted nodes arise (see Fig. 4.1(c)-(d)). Distributing entanglement through a satellite-assisted scheme can generate secret keys between two ground stations. In the simplest scheme (see Fig. 4.1(c)), an entangled-photon source is located in space, and protocols like E91 or BBM92 are used[29, 30], a secret key is distributed between two ground stations. However, in this scheme, both ground stations need to be in the line of sight of the satellite, limiting the maximum distance in between the ground stations through which a key can be generated (this distance is ~ 1000 km for a LEO satellite [285]). For bridging larger distances with-

out trusted nodes, schemes consisting of quantum repeaters have been proposed [275, 286–290]. In these schemes, quantum states are stored in quantum memories, and through entanglement swapping protocols, the secret key can be established between the ground stations [291]. Quantum memories allow for storing quantum states for a certain storage time and releasing them on demand. Some schemes proposed ground-based quantum memories [292] while others proposed to install the memories onboard the satellite [284, 289]. However, quantum memories are still in early development stages, and many challenges need to be overcome before qualifying them for their operation in space [293–295]. An overview of the different architectures and challenges associated with quantum-memory-based schemes can be found in [239]. Finally, other interesting schemes have been proposed for bridging large distances without trusted nodes or quantum memories by guiding the light through focusing stages in a satellite relay [238].

Regarding the orbit selection for satellite-based QKD, most of the architectures proposed in the literature use LEO orbits due to the huge losses in the link for higher altitude orbits [237] (there is a planned mission for a GEO downlink [296]). In Refs. [234, 237] a comparison of the losses between different orbits can be found. Furthermore, some network aspects of satellite-based QKD systems are covered in Ref. [297].

However, a fundamental limitation of QKD is that it does not eliminate the need for classical authentication. While QKD ensures information-theoretic security for the key exchange, Alice and Bob must still verify that they are communicating with the intended party rather than an adversary posing as one of them. This authentication step currently relies on classical cryptographic primitives, typically based on pre-shared symmetric keys or public-key infrastructure [298]. In practice, initial authentication can be bootstrapped from a small pre-shared key, but maintaining long-term trust in large-scale quantum networks may require integration with post-quantum cryptography. Unlike QKD, post-quantum cryptography algorithms do not provide unconditional security but are designed to resist attacks from quantum computers [299]. Therefore, QKD and post-quantum cryptography should be seen as complementary: QKD can deliver information-theoretically secret keys, while post-quantum cryptography can secure authentication.

4.2.3. Overview of demonstrators and missions

Several pioneering missions have been carried out to investigate the feasibility of satellite-based QKD. The purpose of this section is not to provide an exhaustive review –comprehensive overviews can be found in [237, 239]– but rather to highlight some representative milestones and demonstrations that illustrate the technological progress to date. Several early proof-of-principle experiments focused on characterizing the feasibility of free-space quantum links. Being polarization-encoding the most popular of all encodings for satellite-based QKD, measurements of polarization transmission from LEO to ground were reported in Ref. [251, 300], while polarization measurements from GEO were demonstrated in Ref. [301]. In addition, experiments employing retroreflectors in LEO and MEO enabled the reception of weak coherent polarization states on the ground, providing valuable insight into channel losses and background noise [302,

303]. These initial tests laid the foundation for the deployment of quantum sources and detectors in space. Notably, entangled photon sources suitable for space have been demonstrated [304, 305], and more recently, an analysis of single-photon detectors in space has been carried out [306].

The Micius satellite marked the first dedicated quantum science satellite and achieved several landmark results. A direct satellite-to-ground QKD demonstration was reported in Ref. [307], followed by intercontinental QKD enabled through satellite-based trusted-node operation [308]. Long-distance key distribution integrated with terrestrial quantum networks has also been demonstrated [309]. Beyond prepare-and-measure QKD, Micius enabled the first satellite-based entanglement distribution, accompanied by a Bell-inequality violation over large distances [310]. Furthermore, satellite-to-ground entanglement-based QKD has been demonstrated, with one photon detected on board the satellite and its entangled partner transmitted to Earth [283]. Following up on the success with Micius, the recent smaller satellite Jinan has also demonstrated space-to-ground QKD [246].

A number of missions are now planned to extend these capabilities. The Eagle-1 mission aims to demonstrate a LEO downlink based on a double-pulse phase-encoding scheme [311]. At higher altitudes, the GARBO mission will test downlinks from GEO [296]. In parallel, Canada's QEYSSat project will explore for the first time an up-link configuration to a LEO satellite, advancing both experimental capabilities and architectural diversity [249, 312, 313].

4.3. System diagram and physical components

To assess the space environment's effect on satellite-based QKD, it is necessary to understand the typical components and technologies that constitute satellite-based QKD terminals. In this section, some of these optical components and technologies will be reviewed. The reader is referred to Refs. [234, 237, 244] for a detailed review of all the critical technologies needed in satellite-based QKD systems.

Fig. 4.2 shows example diagrams of a typical satellite-based QKD transmitter and receiver. In practice, there are many possible variants of satellite-based QKD terminals, since the exact design depends on factors such as system architecture, encoding methods, and mission requirements. The setups illustrated here are therefore not meant to represent unique designs, but rather examples inspired by currently flying or planned satellite-based QKD implementations [246, 249, 283, 305, 314]. The system depicted in Fig. 4.2 employs polarization-encoding. For other types of encoding, such as time-bin encoding, the optical setups would require different generation and measurement modules corresponding to those degrees of freedom, and the associated optical components would also change. To enable simultaneous transmission and reception of multiple optical signals, wavelength-division multiplexing is employed for separating the beacon and other beams from the quantum channel. The choice of operating wavelengths is motivated by several considerations: minimizing atmospheric effects, enabling daylight operation, and ensuring compatibility and high efficiency of the hardware components involved (including sources, detectors, and, in future systems, quantum memories) [234].

To explain the components of the satellite-based QKD terminals, we follow the

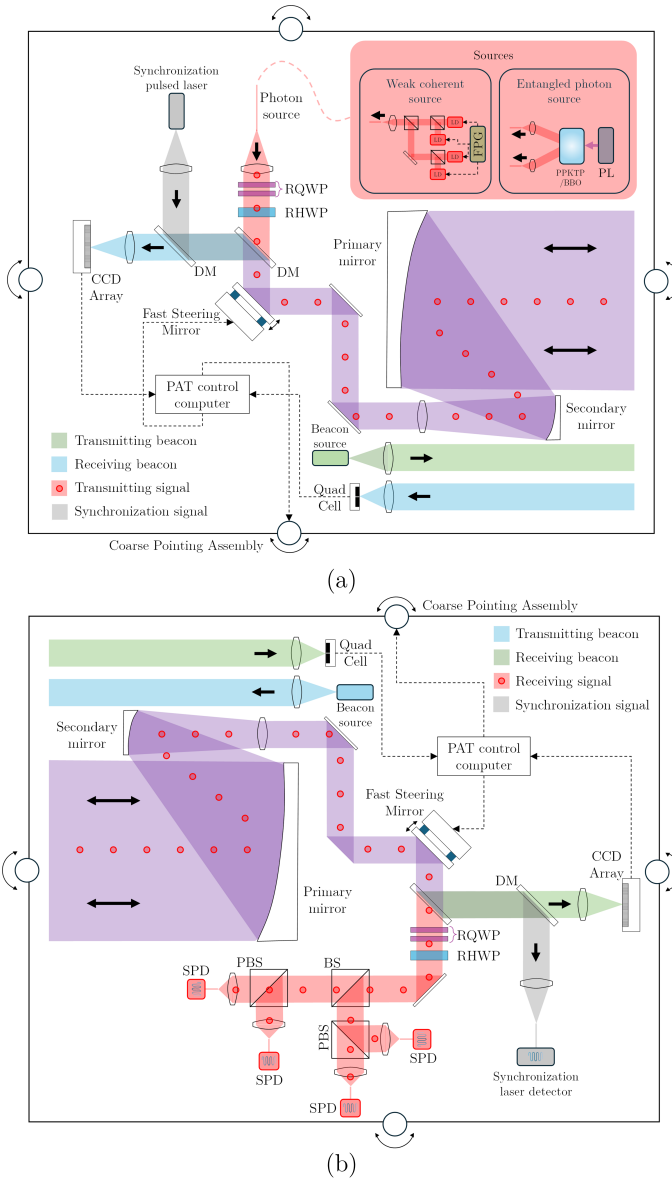


Figure 4.2: General system diagram for satellite-based QKD (a) transmitter and (b) receiver. The lenses shown are a simplified representation of lens assemblies. Colors not stated in the legend are combinations of others (dichroic mirror (DM), rotatable half-wave plate (RHWP), rotatable quarter-wave plate (RQWP), pump laser (PL), fast pulse generator (FPG), single-photon detector (SPD)).

sequential path of the photon through the system, beginning with the photon sources.

Two main types of sources are typically considered in satellite-based QKD: weak coherent pulses (WCP) sources and entangled-photon source (EPS), both of which have already been demonstrated in space [246, 283, 315].

Weak coherent pulse (WCP) sources are used in decoy-state QKD architectures. They are obtained by attenuating the laser pulse down to a mean photon number that results in a single-photon level intensity, that maintains the phase coherence of the source laser. In WCP sources, the probability of the attenuated laser pulse containing a single-photon is small, but the probability of it containing two or more photons is negligible. They can be implemented using multiple laser diodes, as illustrated in Fig. 4.2, or by employing Sagnac-interferometer-based designs, as demonstrated in Ref. [246]. Entangled photon sources (EPS), on the other hand, are required for entanglement-based QKD architectures. The most common approach is spontaneous parametric down-conversion (SPDC), where nonlinear crystals generate pairs of entangled photons. Different implementations have been proposed or demonstrated in space missions: periodically poled potassium titanyl phosphate (PPKTP) crystals in the Micius satellite [310], periodically poled lithium niobate (PPLN) crystals in the source in Ref. [314], and beta barium borate (BBO) crystals in the SpooQy nanosatellite [305]. Intrinsic timing correlation between photon pairs in entangled photon SPDC sources is in the order of femtoseconds [237]. Although not depicted in Fig. 4.2(a), in entanglement-based protocols each photon of the entangled pair is directed towards a different receiver—one of which may also be located on board the satellite itself [283]. Comprehensive reviews of WCP and SPDC sources for satellite-based QKD are provided in Refs. [5, 234, 237, 247]. Most current sources are based on bulk optical implementations [234], but there is increasing interest in developing photonic integrated circuits (PIC)-based sources, which offer the promise of improved compactness, scalability, and stability for future satellite missions [38, 316].

Once the photons are generated, they can be transported within the satellite system using optical fibers [246, 283], although this is not strictly necessary. A drawback of fiber transmission is that it can alter the polarization state of the photons. To compensate for this effect, polarization control modules are introduced, typically consisting of a combination of two rotatable quarter-wave plates and one half-wave plate. This configuration allows the implementation of any unitary transformation on the polarization state of the light, thereby enabling full compensation of the polarization distortions introduced by the fibers. As noted in [246], this setup ensures that the transmitted photons maintain the intended polarization states for proper operation of the QKD protocol.

After polarization compensation, the photon beam is directed to a fast steering mirror (FSM), which provides fine pointing of the beam. Coarse pointing is typically achieved using the satellite's attitude control system [249], although dedicated coarse pointing assemblies with steering mirrors can also be implemented (see §3.6). Both fine and coarse tracking subsystems are guided by feedback from quadrant detectors (QDs) and CCD cameras in the beacon beam received from the other terminal, processed through the pointing, acquisition, and tracking (PAT) control computer. There are certain advantages of using CCDs over QDs for tracking, as pointed out in Ref. [249]. In particular, the pixelated readout of CCDs enables sub-windowing to reduce the

impact of background light and radiation events across the full sensor area. Moreover, accurate centroiding can be performed anywhere on the detector, relaxing boresight alignment tolerances during assembly and thermal operation.

The single-photon beam exits the system through the telescope of the terminal. In the design of satellite-to-ground (or ground-to-satellite) QKD links, two main approaches are considered regarding telescope sizes: either small-aperture telescopes in space combined with large ground-based telescopes, or vice versa [234]. On the space segment, both on-axis and off-axis telescope designs have been employed. For example, the Micius satellite used an off-axis telescope [283], while the Jinan mission used an on-axis one [246]. In the case of QEYSSat [249], an on-axis catadioptric design was preferred over an all-reflective three-mirror anastigmat despite the requirement for a wide field of view. This choice was motivated by the fact that the on-axis design offers lower mass and inherently better performance in terms of polarization extinction ratio—an important factor in minimizing the quantum bit error rate (QBER). Such considerations highlight the broader trade-offs in QKD telescope design, which are discussed in detail in [317].

Along with the quantum signal and beacon beams, a time synchronization beam—typically implemented as a pulsed laser—is transmitted to enable precise time alignment between the clocks of the transmitter and receiver. This synchronization channel is essential, since a primary challenge in all QKD systems is the reliable identification of single transmitted photons against a very high background light level. Narrow temporal gating, enabled by accurate clock synchronization, has been demonstrated as an effective technique to improve the signal-to-noise ratio by reducing the probability of detecting spurious background photons. Moreover, the synchronization beam provides robustness against clock dynamics between the sender and receiver, which otherwise degrade the temporal alignment and the overall performance of the quantum link [318].

The transmitted beams propagate through the free-space channel from the satellite to the ground (or vice versa)—in §5 the diffraction and propagation of single-photon beams will be treated. In satellite-to-ground links, this path includes both the vacuum of space and the Earth’s atmosphere. At the receiver, the incoming beams are collected by a telescope and then directed toward their respective detectors, as illustrated in Fig. 4.2. In polarization-encoded QKD systems, relative rotational motion between the transmitter and receiver introduces misalignment of the polarization bases. To address this, a rotatable half-wave plate can be employed to dynamically compensate for the relative rotation in real time [246]. In addition, to compensate for other polarization effects introduced by the receiver’s terminal optical relay, a pair of rotatable quarter-wave plates is included in the system diagram of Fig. 4.2(b).

In the receiver, the single-photon beam is directed to a polarization analyzer, typically implemented as a combination of beam splitters and polarization beam splitters that separate the incoming photons into the measurement bases. Alternatively, the choice of polarization basis can be achieved using an electro-optic device such as a Pockels cell, as demonstrated in [310], rather than the static optical arrangement shown in Fig. 4.2. After the polarization analysis stage, the photons are detected by single-photon detectors. Two main types of detectors are considered for satellite-based QKD: avalanche photodetector (APD) and superconducting-nanowire single-photon

detectors (SNSPS). APDs, which can operate with passive or active cooling (as in QEYSSat [249]), are compact and relatively simple to integrate but have limited efficiency and higher dark count rates compared to SNSPSs. SNSPSs, on the other hand, provide superior detection efficiency, lower dark counts, and lower timing jitter, but require cryogenic operation and thus the use of a cryocooler [95]. Reviews of detector technologies and their trade-offs for satellite-based QKD can be found in Ref. [234]. In the diagram in Fig. 4.2(b), fiber coupling is employed to connect detectors with the optical system. The reasons to use fiber coupling in satellite-based QKD are several. First, when detectors are placed on the satellite, fiber coupling (as in QEYSSat using a multimode fiber [249]) simplifies the integration with the optical relay. Second, entangled photon sources often employ fiber-coupled outputs, as in Ref. [310]. Third, in the ground segment, fiber coupling enables compatibility with terrestrial fiber networks, which is a key requirement for hybrid ground-space quantum networks [258, 319].

Although not depicted in Fig. 4.2, some satellite-based QKD architectures require the integration of quantum memories on board the satellite terminal (see Fig. 4.1(b)). In such schemes, entanglement swapping is typically performed through Bell-state measurements (BSM) on photons retrieved from the quantum memories, using an arrangement similar to the polarization analyzer shown in Fig. 4.2, but with two input photons impinging on the first beam splitter. Optical quantum memories function by exploiting light-matter interactions to coherently map -without quantum information loss- an incoming optical field onto a collective excitation distributed across an ensemble of atoms, a single atom or another material medium. A variety of physical systems have been proposed for this purpose, ranging from laser-cooled atomic ensembles to single-atom memory systems [320]. The performance of quantum memories is determined by several key parameters: storage time, efficiency, noise, and multiplexing capacity. Storage times ranging from milliseconds to seconds are required, since they must at least match the light travel time in satellite-to-ground links [239]. High efficiency and low noise are equally critical, as they directly impact the achievable entanglement distribution rates. The number of memory modes supported is also an important parameter to enable longer secure-key lengths and higher rates [321]. While individual milestones such as long storage times, high efficiency, or multimode operation have been achieved separately, they have not yet been demonstrated in combination [239, 295, 322–324]. Although proving all these requirements for quantum memories in the space environment will be challenging, microgravity environments may even offer potential advantages for extending coherence times in quantum memories, as suggested in Ref. [325]. Comprehensive reviews of the state of the art on quantum memories can be found in Refs. [291, 293, 294, 316, 326, 327].

Some simplifications for the sake of clarity have been made in the system diagram of Fig. 4.2. First, the transmitter and receiver have been completely separated (unlike in Fig. 3.2 for the classical communication optical terminal). For simplicity, the classical communication channels –whether optical or radio– necessary for the operation of QKD protocols are not explicitly shown in Fig. 4.2. Consequently, a realistic QKD terminal is best understood as a combination of the elements shown in Fig. 3.2 and Fig. 4.2, as shown in the systems diagrams of several missions [246, 249, 283]. In addition to the main optical chain, QKD terminals often incorporate several filter-

ing mechanisms to suppress background noise. Spatial filtering can be implemented with pinholes to reduce stray light [249]. Spectral filtering is also applied, typically by inserting narrowband filters that match the emission wavelength of the quantum source. Together, these techniques improve the signal-to-noise ratio and are essential for reliable operation in the presence of background light, especially during daylight conditions. While the system shown in Fig. 4.2 is based on polarization-encoding, other encoding schemes, such as time-bin encoding, require other optical setups. In this case, the generation and measurement modules must be adapted to manipulate time-bin states rather than polarization, involving interferometers and phase modulators [328]. Alternatively, it is possible to convert between different encoding types using linear optical elements [5, §5.1].

While the previous discussion focused on the internal design of QKD terminals, their performance cannot be fully understood without considering the external environment in which they operate. For satellite-based systems, the harsh conditions of space –radiation, vacuum, and extreme thermal and mechanical loads– can significantly affect both optical components and the quantum states they manipulate. The following section examines these space-induced opto-thermo-mechanical phenomena and their impact on QKD terminals.

4.4. Space sources and opto-thermo-mechanical phenomena

Satellite-based QKD systems, consisting of complex optical setups as described in §4.3, must be able to survive and operate reliably in the harsh space environment. In orbit around the Earth, as well as in more remote deep-space locations, satellites are exposed to radiation, vacuum, and significant thermal and mechanical loads. In this section, the main environmental loads encountered in space will be summarized. Furthermore, the opto-thermo-mechanical phenomena affecting the different components of a QKD terminal –and their effect on the quantum states of light carrying quantum information– will be assessed. The following section (§4.5) will then evaluate how these phenomena influence the overall performance of satellite-based QKD systems. These sections are therefore built following the same logic as in §3 for satellite free-space optical communications (see Fig. 3.1) –as we will see, there will be many commonalities.

Although crucial for the successful operation of satellite-based QKD, certain effects such as atmospheric turbulence, radiation phenomena, and background noise [329–331] will not be covered here for the sake of conciseness. For a detailed treatment of atmospheric effects in satellite-based QKD, the reader is referred to [253, 284, 332]. While further research is needed to fully characterize the behavior of satellite-based QKD components in the radiative space environment, initial studies have already been conducted on both photon sources and detectors [306, 333–337].

4.4.1. Space environment

In the space environment, several disturbance sources affect the performance of the satellite-based QKD terminal’s components. This section focuses on microvibrations and thermal loads. These environmental loads induce opto-thermo-mechanical effects

that can misalign optical paths, degrade pointing accuracy, and impair the fidelity of quantum states used for secure communication- this will be seen in the following sections. In the following, a summary of the sources presented for satellite optical communications in §3.3 is presented, adapting the explanation wherever the consideration for quantum communications makes a difference.

- *Microvibration sources.* Satellite-based QKD systems rely on narrow beam divergences, which require pointing accuracies at the microradian level to assure a high enough photon transmission probability. The microvibrations sources on these systems are the same as the ones discussed in §3.3. Cryogenic systems required for single-photon detectors such as SNSPDs [95], which may be necessary in quantum communication terminals, can introduce significant mechanical vibrations, as observed in space platforms like the Hubble Space Telescope [78]. Measurements onboard the Micius quantum satellite provide the power spectral density of the microvibrations that can be expected onboard a quantum communication terminal (most of the vibration energy lies below 100 Hz, as can be seen in Fig. 3.5).
- *Thermal loads.* Temperature fluctuations in orbit, as seen in §3.3, present another major challenge for satellite-based QKD terminals. The internal temperature inside a satellite can fluctuate significantly, as seen in the LEO quantum in-orbit experiment in Ref. [338], where the internal temperature varied in the range 0-30°C.

4.4.2. Opto-thermo-mechanical phenomena

As in satellite free-space optical communication systems, the microvibrations and thermal loads present in the space environment give rise to several opto-thermo-mechanical phenomena. These effects directly impact the performance of satellite-based QKD terminals, where extreme optical stability and polarization fidelity are required. In this section, we review the most relevant opto-thermo-mechanical phenomena with a special focus on polarization effects, quantum sources, and detectors, which are critical for the operation of QKD systems. For an in-depth discussion of the other effects (rigid body motions, surface deformations, therm-optic dispersion, and stress-birefringence), the reader is referred to §3.4.

- *Rigid body motions of the satellite.* Due to microvibration sources, the satellite's orientation continuously fluctuates around its nominal pointing direction. The power spectral density of this angular motion, measured in orbit for several missions, is shown in Fig. 3.5. Rigid body motion is of particular importance in polarization-entangled QKD systems, which are currently the most widely used. A natural consequence of the relative motion between satellite and ground station is a time-dependent polarization rotation, arising from the continuously changing reference frames [339]. This effect requires active polarization compensation during the communication window between the transmitter and receiver terminals.

- *Rigid body motions of optical components.* Beyond global satellite jitter, individual optical elements inside the terminal also undergo rigid body displacements due to mechanical loads. Both static and dynamic misalignments can occur. The effects of these misalignments are similar to the ones explained in the classical beam case in §3.4 and can be critical. Polarization optics often used in satellite QKD, such as polarizers and birefringent elements, are especially sensitive to angular rotations. Furthermore, entanglement sources also need to be strictly aligned with the pump laser, where angular alignment tolerances on the order of $100\ \mu\text{rad}$ are required [247, 338]. Another important effect is the changes induced in the polarization state of light due to the reflections on optical components [283]. An example of the latter can be found in the study of the influence of steering mirrors in free-space quantum links presented in Ref. [340].
- *Surface deformations.* Thermal and mechanical stresses can deform optical surfaces. These deformations induce wavefront errors on the beams that, in turn, change their shape as they propagate through the system. The changes in the shape of the beam will in turn, change the transmission probability of the photons as they are coupled into the aperture of the receiver (transmitter's wavefront errors) or into the fiber (receiver's wavefront errors).
- *Thermo-optical dispersion.* Changes in the refractive index of optical materials with temperature lead to additional wavefront errors. This thermo-optic effect compounds surface deformations and can degrade beam quality. If this phenomenon is wavelength-dependent, then different wavelengths will be affected differently. In the case of dichroic mirrors used mainly for multiplexing and de-multiplexing different beams (beacon and quantum beams), this could create crosstalk between the optical paths followed by them.
- *Stress birefringence.* Thermo-mechanical stress fields within optical components can induce unintended birefringence (see Fig. 3.9). This birefringence will create unwanted changes in the polarization state of the quantum and beacon beams travelling through the terminal. This effect can be very significant when using optical fibers in the quantum communication terminals [246]. For polarization-encoded QKD, this is particularly harmful, as it can create spurious correlations between Alice's and Bob's measurement outcomes, degrading the security of the key –similar to the effect that polarization rotation due to the relative rotation motion of the satellite would have.
- *Other polarization effects.* Apart from the birefringence and reference system rotation effects on polarization-encoded quantum states, relativistic effects on the polarization state of the photon have also been investigated [341, 342]. Due to the relative motion between the transmitter and the receiver, there is a special relativistic effect that rotates the polarization state of the photon, i.e. Wigner rotation. Furthermore, the Faraday rotation will be a similar effect induced by the gravitational field. The analysis performed in Ref. [341] shows that both of the effects, in near-Earth quantum communications, are orders of magnitude

smaller than the effect of the nonrelativistic misalignment between the transmitter and receiver reference frames due to their relative rotation motion. Hence, these polarization-related relativistic effects can usually be neglected.

- *Effects on quantum sources.* In quantum communication sources (WCP and EPS), the effect of thermal loads is very complex due to several multiphysical phenomena occurring simultaneously. In WCP sources, based on laser diodes, the temperature variations on the semiconductor will affect their performance through different mechanisms (bandgap energy shifting and consecutive wavelength shift, refractive index change due to thermo-optic dispersion, etc.). In the case of entangled-photon sources, the performance of the usual nonlinear-crystal-based sources is highly temperature-dependent. For example, periodically-poled crystals require stabilization to within 0.1°C to maintain phase matching [247]. Among the different SPDC entangled-photon sources, although the BBO crystals have lower nonlinearities ($\chi^{(2)}$) than PPKTP, they are less sensitive to temperature fluctuations and allow larger apertures [237]. Temperature variations in SpooQy's BBO-based source were experimentally measured in Ref. [305], demonstrating that the source is very stable within the range $\sim 16 - 22^\circ\text{C}$. As several unwanted effects will occur due to temperature variations on quantum communication photon sources, proper thermal isolation and stabilization of them needs to be performed through thermo-mechanical design [338]. For example, to ensure thermal and mechanical stability, the entangled-photon source onboard the Micius satellite employed a thick titanium alloy baseplate (due to its good balance between rigidity and thermal expansion) and an invar plate mounting for the onboard Sagnac interferometer [310].
- *Effects on single-photon detectors.* Single-photon detectors used in satellite-based QKD systems are very sensitive to temperature fluctuations. For APDs, dark count rates increase with temperature, directly impacting the performance of the system. In Ref. [237], an analysis for a typical APD shows an order of magnitude increase of the dark counts due to an increase of $\sim 30^\circ\text{C}$ on the detector. Although the intrinsic time-jitter of the detector will not be much affected by the temperature, the increased number of dark counts will increase the effective time-jitter of the detector. SNSPDs operate at cryogenic temperatures, therefore requiring active thermal control for their operation. If the temperature control is not adequate, the superconducting state of the nanowires could be lost, resulting in their complete inoperability. In general, active thermal control through cryocoolers is needed to efficiently operate single-photon detectors on-board satellite-based QKD terminals.
- *Effects on quantum memories.* Quantum memories are still in very early stages of development, and active research is ongoing to develop reliable physical systems to be operated in space. Depending on the physical system of the space-based quantum memory, the effects of temperature fluctuations will differ. However, temperature fluctuations will most securely impact storage time, efficiency, and fidelity [343], thereby affecting overall entanglement distribution rates.

- *Other phenomena.* Another important effect due to the temperature variations and the microvibrations is the stability in optical cavities and clocks that are relevant for performing time filtering in satellite-based QKD [344].

4.5. Impact on satellite-based QKD

Opto-thermo-mechanical phenomena above, due to the thermal and mechanical loads on board a satellite-based QKD terminal, will impact the performance of the system in many different ways. In this section, a detailed explanation of how the phenomena explained in the section above affect a space quantum communication system is discussed. The main effects due to the phenomenon above explained are: impact on photon transmission probability, impact on polarization state, impact on wavelength, and other effects.

4.5.1. Photon transmission probability and transverse-mode crosstalk

Due to the free-space channel in satellite-based QKD, a photon exiting the transmitter will have a lower-than-unity probability of being received by the single-photon detector (or stored in the quantum memory) on the receiver's side. This loss will be referred to as the photon transmission probability over the free-space channel. This photon transmission probability will depend on the divergence and the pointing jitter of the single-photon beam in the free-space channel. Furthermore, once the photon is captured by the receiver aperture, its spatial mode has to be coupled into a usually small-sized detector. In some receiver configurations, the incoming single-photon beam in the receiver is first coupled into a fiber to ease the interface with a single-photon detector or photonic quantum memories. When coupling into the detector or the optical fiber, the photon transmission probability is decreased. Altogether, these effects are the most important contributors to photon loss in satellite-based QKD systems [239].

When satellite-based QKD terminals are affected by the opto-thermo-mechanical phenomena presented above, the photon transmission probability of the free-space channel is decreased. There are several opto-thermo-mechanical phenomena that affect the photon-transmission probability: (1) transmitter pointing jitter, (2) receiver pointing jitter, (3) transmitter wavefront error, and (4) receiver wavefront error (these are the same that affect classical optical communication systems as seen in §3.5). In Fig.4.3, an illustration of how these effects impact the photon transmission probability is depicted.

1. *Transmitter pointing jitter.* As a justification for the need for satellite quantum communications, it is often mentioned that the fiber losses scale exponentially with distance, while the free-space losses increase quadratically. While this statement is strictly right for a perfectly pointing system, in satellite quantum communications, beam pointing losses are unavoidable. Although these are not usually big enough to make them comparable to losses over current fiber optics [307].

Single-photon quantum communication sources (WCP or EPS) typically emit photons in the fundamental Gaussian transverse mode [38, 283]. As the beam

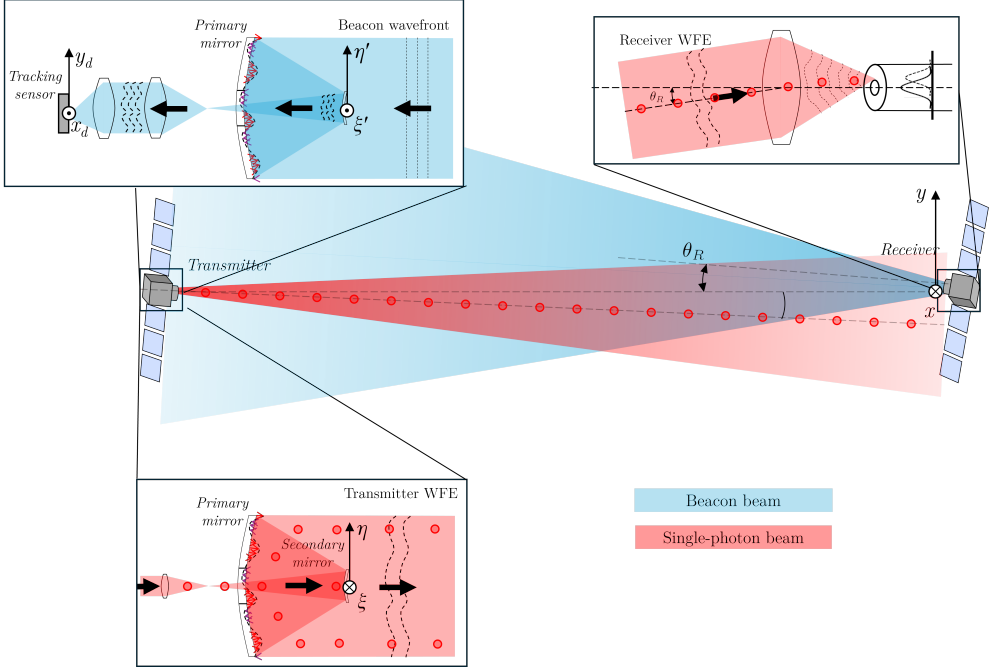


Figure 4.3: Effects of transmitter and receiver pointing jitter and wavefront errors on the photon transmission probability. Due to the transmitter's pointing jitter, the angle of arrival of the received beacon in the top-left figure should be non-zero, but to emphasize the effects of the transmitter wavefront error, a zero angle of arrival is illustrated.

propagates through free-space, it diverges, so only a fraction falls within the receiver aperture. Additionally, transmitter pointing jitter causes misalignment between the beam and the receiver, making photon capture probabilistic. These effects are often approximated via link budget averaging [329], with more accurate statistical models available [238, 275, 284, 345, 346]. While analogous to classical links, in the quantum regime, they affect photon transmission probability rather than optical power seen in §3.5.

The deviations of the center of the beam with respect to the receiver's optical axis will induce a stochastic photon transmission probability on the quantum communication system. In polarization, time and frequency-encoded systems, this will be the only effect. However, transverse-mode/OAM encoding systems have been proposed for their use in reference basis resilient satellite-based QKD systems [276–278]. In this case, not only is the photon transmission probability affected. As the quantum state of the transmitted single-photon is encoded in the transversal spatial distribution of the phase of the beam, this state needs to be manipulated by non-uniform spatial phase components that are aligned with the optical axis of the beam. Due to the misalignment induced by the transmitter

pointing jitter between the optical axis of the transmitted single-photon beam and the receiver's optical axis, the decomposition of the single-photon beam's spatial phase would be faulty [34]. Moreover, while Gaussian beams exhibit maximum irradiance at their center, beams carrying orbital angular momentum (OAM) feature a phase singularity at the center, resulting in a null intensity. As a consequence, especially when such a single-photon beam is perfectly aligned with the receiver's optical axis, the probability of capturing the photon remains very low. Further research is required to enable the applications of OAM beams in satellite quantum and classical communications [268, 347].

2. *Receiver pointing jitter.* In-orbit microvibrations also affect the receiver, inducing angle-of-arrival fluctuations that lead to stochastic coupling efficiency at the detector. When detection involves fiber coupling (see §4.3), this can significantly impact the photon transmission probability, analogous to the discussion detailed in §3.5 for the optical power. The effect of receiver pointing jitter-induced angle of arrival can be very similar to the angle of arrival fluctuations due to atmospheric turbulence in terrestrial or satellite-to-ground links [348]. As it happened for the transverse mode encoding quantum communications for the transmitter pointing jitter, the receiver pointing jitter will induce a spurious tip-tilt in the received beam. This, in turn, will create cross-talk between different transversal modes and will be detrimental for the performance of the quantum communication system (an analysis of angle of arrival impaired OAM multiplexed channels can be found in Ref. [34]).
3. *Transmitter wavefront error.* The thermo-mechanically induced aberrations in the transmitter's terminal will change the shape of the transmitted single-photon beam. In this way, the change of the shape of the single-photon beam will induce a change in the photon transmission probability. To compute the far-field irradiance in the receiver aperture plane of the single-photon beam affected by the transmitter's aberrations, Fourier wave optics propagation methods can be used (see §5). By combining the transmitter's pointing jitter and the shape of the single-photon beam in the receiver aperture plane, the probability of the receiver's aperture capturing the single-photon can be computed. Analysis of the effect of certain aberrations on an intersatellite reflector-relayed quantum communication system can be found in Ref. [238].

Furthermore, although not an aberration, it is worth mentioning that a similar effect to the one induced by the transmitter aberrations will be given by the clipping at the transmitter's aperture (including a central obscuration due to on-axis secondary mirrors). Due to the finiteness of the transmitter's aperture, the single-photon beam will be clipped, i.e., abruptly cut on the external (and internal) side of the beam. This will induce two main effects that will affect the photon transmission probability. On the one hand, the clipping will reduce the transmission probability of the photon going out from the transmitter in the expected direction. On the other side, the clipping will induce further diffraction of the transmitter beam due to the abrupt cut of the single-photon beam in the aperture. These effects, and their impact on satellite-based QKD [238, 349], will

be analyzed in detail in §7.

For transverse-mode/OAM encoding, if they can be used in practice in long-distance satellite-based QKD links, the effect of the transmitter aberrations will create cross-talk between the different transverse modes. This cross-talk will, in turn, induce spurious correlations between Alice and Bob's measurements, increasing the QBER. An analysis of the impact of low-order aberrations in the quantum states of OAM encoded quantum communications can be found in Ref. [350]. Furthermore, although not analyzed in the literature, the clipping on the transmitter will also change the single-photon state beam (both in amplitude and phase), inducing further crosstalk between different modes.

4. *Receiver wavefront error.* At the receiver, the incoming single-photon beam must be coupled into the detector (or quantum memory). For long distances, the captured field is often approximated as a uniform plane wave limited by the receiver aperture (see Fig. 4.2), leading –after diffraction– to a point spread function at the detector. Wavefront errors can distort this pattern and reduce coupling efficiency, thus affecting photon transmission probability. When a larger portion of the beam is captured, the uniform plane wave assumption breaks down, and the coupling must be evaluated for the actual clipped beam profile.

In transverse-mode/OAM encoded quantum communications, the effects of the receiver wavefront error will be that of increasing the crosstalk between the transmitted orthonormal modes (in a similar way to what happens for the transmitter's wavefront error). Furthermore, the diffraction due to the receiver's finite aperture will also induce further crosstalk between the different modes.

Although the explanation of photon transmission probability effects has been discussed referring to the quantum information-carrying single-photon beam, the other beams travelling through the system will also be affected by the effects explained. As these signals (beacon and synchronization beams) do not carry along quantum states, they are not subject to the no-cloning theorem and can therefore be amplified. In this sense, the beacon and synchronization beams are less sensitive to the free-space channel losses discussed above, although these need to be computed and properly accounted for. Specifically, the wavefront errors on the transmitter's impinging on the received beacon can be very detrimental as they induce an apparent angle of arrival of the beacon beam. This apparent angle of arrival creates a static bias on the transmitter's pointing error (this effect is analyzed for classical free-space optical communication links in §6). Furthermore, as a classical communication channel is required for performing QKD protocols, the impact of opto-thermo-mechanical phenomena explained in §3.5 is also important in satellite-based QKD.

4.5.2. Polarization effects

In polarization-encoding, being one of the most popular methods in satellite-based QKD, the perturbations in the polarization state encoding the logical qubits are critical. The polarization rotations and, more generally, the polarization state changes created by rigid body motions, stress birefringence in bulk optics and optical fibers,

and other phenomena can be characterized through the polarization extinction ratio (PER). The polarization state of light can be, in general, described by the vector [62]

$$\begin{pmatrix} a_x \\ a_y \exp(i\delta) \end{pmatrix} \quad (4.1)$$

where the phase delay between the x and y components is given by δ . The amplitude ratio angle can be defined as $\alpha = \arctan(a_y/a_x)$ and the basis deviation angle as $\tan 2\theta = \tan 2\alpha \cos \delta$. The ellipticity angle β can be defined according to δ , α and θ as

$$\beta = \pm \frac{\sqrt{a_x^2 + a_y^2}}{2} \arcsin \left(\sqrt{\frac{\sin^2 \delta}{\tan^2 2\theta + \cos^2 \delta}} \right), \quad -\frac{\pi}{4} \leq \beta \leq \frac{\pi}{4}. \quad (4.2)$$

Using this ellipticity angle, the polarization extinction ratio can be defined as

$$\text{PER} = \frac{1}{\tan^2 \beta}. \quad (4.3)$$

By combining the polarization extinction ratios of both the transmitter and receiver terminals, the QBER induced by perturbations on the polarization state of the single-photon states can be easily computed [62]. As stated in Ref. [62], for a QBER value below 0.1% the bias deviation angle needs to be less than 35 mrad. Apart from the PER, the degree of polarization can also be used to evaluate the effect of the polarization in satellite-based QKD [273].

The previous explanation only covers the polarization effects on the single-photon beam carrying the quantum information. However, the effects of polarization perturbations on the other beams are also critical. Specifically, as polarization compensation systems are used to match the orientation of the transmitter and receiver bases, the effect on the polarized beam used as a reference for compensation is also critical. This can be seen in Ref. [62]. If, instead of encoding the qubits, polarization is used to discern between the different beams going through the system, the polarization state changes will deviate the light, creating photon transmission losses and increasing the background noise. In satellite-to-ground QKD systems, the perturbations of the polarization state of the single-photon beam will have their primary origin in the phenomena covered above, as the effects on the polarization due to atmospheric turbulence are negligible [251]. Finally, when interferometric setups are used in satellite-based QKD systems (e.g., as frequency analyzers in frequency-bin encoding systems), the effects of stress birefringence also need to be considered, as it will induce unwanted effects [274, §4.3].

4.5.3. Wavelength effects

In most of the satellite-based QKD implementations, different wavelengths are used to separate the different beams (e.g., single-photon, beacon, and synchronization beams) travelling through the system. However, surface deformations and thermal dispersion on the filters and dichroic mirrors that are used to guide the different wavelengths through the system can induce crosstalk between different wavelengths by changing

the bandpass behaviour of the optical element. In this case, the quantum channel would again be subjected to both a decrease in the photon transmission probability and an increase in the background noise.

Furthermore, the wavelength of light can be used for encoding the logical qubits on satellite-based QKD systems. In this scenario, it has been found that frequency-bin encoding is sensitive to the random phase noise induced by thermal fluctuations on fiber optical components [271] –these effects should be small for the fiber lengths involved in satellite-based QKD as long as the temperature variations are minimized.

4.5.4. Figures of merit in satellite-based QKD

In the previous sections, the effects of opto-thermo-mechanical phenomena on photon transmission loss and crosstalk were discussed. However, to fully evaluate their impact on satellite-based QKD systems, it is necessary to consider more appropriate figures of merit. Three metrics are particularly important in this context: the secret key rate (SKR), the quantum bit error rate (QBER), and the fidelity of quantum states ($\mathcal{F}$).

In general, for QKD applications, the primary performance indicator is the SKR, which quantifies the amount of secret key that can be established through the link. Different models exist to compute the SKR while accounting for photon transmission probability, crosstalk, and background noise. Based on the explanation given in Ref. [235, §IV] to define the SKR, we can define:

1. Total detection rate $\mathcal{R}$. The total number of detections we have on the receiver side.
2. Photon-number resolver detection rate $\mathcal{R}_n$. The detection rate for the events in which n photons are sent by Alice ($\sum_n \mathcal{R}_n = \mathcal{R}$). This gives the photon-number resolved yield $Y_n = \mathcal{R}_n / \mathcal{R}$.
3. Error rates ε_n . Among $\mathcal{R}_n$, a subset corresponds to wrong counts $\mathcal{R}_n^w$. This gives the photon-number-resolved error rate $\varepsilon_n = \mathcal{R}_n^w / \mathcal{R}_n$.

Now we can define the quantum bit error rate as $\text{QBER} = \sum_n Y_n \varepsilon_n$. The QBER is the fraction of mismatched bits in the raw key before error correction and privacy amplification. Its value is crucial: bounds on the QBER determine whether a finite secret key can be distilled. For example, in BB84 with one-way post-processing, the QBER must remain below 11% to allow secret key generation [235].

Then, we can obtain the most relevant parameter for QKD, i.e., the secret key rate (SKR). For a discrete variable protocol under imperfect error correction codes, the secret key rate can be written as

$$\text{SKR} = \mathcal{R} [1 - \text{leak}_{\text{EC}}(\text{QBER}) - I_{\text{E}}], \quad (4.4)$$

where

$$I_{\text{E}} = \min(I_{\text{AE}}, I_{\text{BE}}) = \max_{\text{Eve}} \sum_n Y_n I_{\text{E},n}, \quad (4.5)$$

Here, $I_{\text{AE}} = \max_{\text{Eve}}(I(A : E))$ is the maximum of the mutual information for individual attacks (for collective attacks, see Ref. [235]) and the maximization is performed

over Eve's attack strategies. The term $\text{leak}_{\text{EC}}(\text{QBER}) \geq H(\text{QBER})$ accounts for the imperfections of error-correction codes, with $H(\text{QBER})$ the binary entropy.

For specific protocols, these expressions can be refined further [235, 349]. This formalism depicts how photon transmission probability, crosstalk, background noise, and detector dark counts affect satellite-based QKD performance. Transmission probability first scales the detection rate $\mathcal{R}$, directly impacting the SKR. In addition, transmission losses influence the QBER, as explicitly discussed in Ref. [253, §VI] and illustrated in [237, Fig. 6], further degrading the SKR.

Crosstalk between different beams or quantum states also plays a significant role. It reduces the transmission probability to the intended detector (e.g., single-photon light entering a beacon detector) and increases background noise (e.g., beacon light leaking into a single-photon detector). Increased background noise raises the number of wrong counts $\mathcal{R}_n^w$, thereby increasing the QBER and lowering the SKR. Similarly, when crosstalk between different quantum states arises due to opto-thermo-mechanical effects (such as reduced polarization extinction ratio in polarization-encoded QKD), the QBER rises because of higher wrong count rates. Finally, dark counts increase due to thermal loads in space-based detectors and contribute in the same way as background noise. A concrete example of these effects, incorporating stochastic photon transmission probabilities in a satellite-based BB84 decoy-state protocol, is given in Ref. [253].

Beyond QKD, other satellite-based quantum communication applications employ additional figures of merit. In entanglement distribution, for instance, the fidelity of the shared quantum state relative to an ideal reference is key. Fidelity is defined as [5, §3.2]

$$\mathcal{F}(\rho, \sigma) = \left(\text{Tr} \sqrt{\rho^{1/2} \sigma \rho^{1/2}} \right)^2, \quad (4.6)$$

where ρ is the density matrix of the distributed state and σ the one of the reference. In entanglement-based QKD protocols, the fidelity of the distributed state after propagation through the free-space channel is critical for evaluating the overall system performance [275].

4.6. Mitigation techniques

We can now track the different effects on the different properties of the single-photon beam (and the other beams), due to opto-thermo-mechanical phenomena and the impact that these effects have on the performance of satellite-based QKD systems. As it has been shown, the photon transmission losses, crosstalk, and noise effects induced by these phenomena can be very detrimental to the final performance of the system in terms of SKR. In order to counteract this, we present in this section a summary of several mitigation techniques that can be pursued to minimize these detrimental effects. Most of the mitigation effects explained in §3 can be implemented in satellite-based QKD systems. In this section, we will focus on the mitigation techniques applied specifically for QKD protocols, with a focus on the most popular polarization-encoded terminals.

One of the main sources of photon transmission loss is the one related to the pointing jitter errors. In the mechanical domain, to minimize these errors, we can first

act on reducing the microvibration sources, isolating the terminal and suppressing the structural transmission path (see §3.6). In the optical domain, optical components acting directly on the light beam need to be used to reduce the pointing jitter of the incoming and outgoing beams. This is attained with the pointing assemblies and fast steering mirrors onboard the satellite. Using these systems, the Micius satellite could achieve a sub-microrad pointing jitter level [81]. Finally, in the quantum domain, photon loss can be compensated with higher brightness sources [239] (unlike optical classical communications, the non-cloning theorem, see §2, forbids the copying of unknown arbitrary qubits, and therefore amplifying the signal).

To minimize the effects of photon loss due to thermo-mechanically induced optical aberrations, adequate opto-themo-mechanical analysis and design is required [128]. Furthermore, athermalization of sources, optical components, and detectors will ensure the correct performance of the system under the harsh space environment [338].

Single-photon detectors operating for quantum experiments and applications usually require the use of both passive [351] and active thermal control through cryocoolers or other systems [77, 95]. For instance, the entangled-photon source onboard the Micius satellite achieved passive thermal and mechanical stability by using a thick titanium alloy baseplate together with invar mounts [310]. It is important that the microvibrations induced by these cryocoolers are properly damped so as not to increase the pointing jitter of the terminals significantly. Other thermal effects, such as the random phase noise induced by thermal fluctuations of the environment when using frequency-bin encoding, can be corrected via real-time adaptive phase rotation of the measurement basis [271].

A special focus must be given to polarization perturbation mitigation techniques in satellite-based QKD, since most current implementations rely on polarization-encoding. Several strategies have been proposed and demonstrated to counteract polarization degradation arising from opto-thermo-mechanical effects and relative motion. One approach is the use of polarization-maintaining designs, as demonstrated in Ref. [62]. By employing polarization-maintaining coatings, carefully designed waveplates, and compensation across different optical components, the Micius satellite achieved a polarization extinction ratio (PER) well below 500:1, ensuring high-fidelity transmission of polarization states. Another critical source of polarization errors is attitude drift between the satellite and ground stations. Mitigation techniques include active polarization tracking systems [273, 339, 352, 353], which dynamically compensate for relative rotations of the polarization basis. Alternatively, some approaches avoid active tracking altogether by performing quantum state tomography to reconstruct and correct the polarization states at the receiver [354]. A practical implementation of polarization stabilization is described in Ref. [246], where a two-step polarization compensation scheme with rotatable waveplates was employed. In this design, compensation first corrects the birefringence introduced by the onboard single-mode fiber, followed by adjustment for the polarization rotations due to satellite attitude drift. With proper temperature control, the onboard waveplates can remain calibrated for months, while compensation of relative rotations must still be performed in real time. Beyond active compensation, more fundamental solutions involve encoding strategies that are inherently robust to reference-frame misalignment.

For example, decoherence-free subspaces for polarization communication [5, §5.3] with alignment-free encoding techniques [277] could provide alternative ways to mitigate polarization rotation without requiring real-time feedback systems.

4.7. Summary and future work

We have reviewed the different architectures proposed for distributing quantum keys between widely separated stations using space-assisted solutions. An overview of the optical and quantum hardware required in both transmitter and receiver satellite-based QKD terminals has been presented, considering different architectures and encoding schemes. To ensure reliable satellite-based QKD systems, it is essential to assess the impact of the harsh thermo-mechanical environment in space. Accordingly, the various opto-thermo-mechanical phenomena induced by these loads –such as rigid body motions, surface deformations, and stress birefringence–have been systematically described, along with their effects on the optical and quantum components of the terminal. To enable an end-to-end characterization of these effects, their influence on photon transmission probability, crosstalk, and background noise has been translated into final performance parameters of satellite-based QKD schemes: secret key rate, quantum bit error rate, and fidelity. Finally, mitigation techniques to suppress or compensate for these opto-thermo-mechanical effects have been presented.

It has been observed that significant effort has been devoted to the development of polarization-encoded satellite-based QKD systems. This has provided a deep understanding of polarization perturbations in physical systems and led to the advancement of various polarization compensation technologies. However, other encoding techniques –such as frequency, time-bin (see §7) or OAM–still require further investigation to assess their suitability for space deployment and to fully characterize their fundamental limitations (e.g. challenges of orbital angular momentum encoding over large distances and finite apertures). Moreover, as highlighted in Ref. [239] and in the spirit of extending communication distances, the development and eventual space qualification of quantum memories will be crucial for enabling a truly global quantum information network (see §7). Beyond quantum cryptography applications, future work on satellite-based quantum network architectures should expand on applications such as blind quantum computing [355, 356], quantum-enhanced sensing networks [267, 357–359], global time-keeping [357], and experimental tests probing fundamental physics [360].

Part II



Optical aberrations and pointing jitter

Among the various opto-thermo-mechanical phenomena discussed in the previous chapters that influence optical and quantum free-space optical communications, this section focuses on three key effects: beam truncation, optical aberrations, and pointing jitter. On the transmitter side, beam truncation due to a finite aperture leads to diffraction, thereby reducing the transmitted power –or photon transmission probability. Optical aberrations further distort the far-field irradiance, or photon spatial distribution, at the receiver aperture plane. These two effects alter the ideal Gaussian beam typically emitted by the source, requiring the consideration of more complex field distributions. We therefore go beyond the Gaussian beam approximation to accurately model the propagation of realistic optical fields. Furthermore, in combination with these wavefront imperfections, the transmitter’s pointing jitter introduces stochastic fluctuations in the received power– or photon transmission probability.

This part presents models to quantitatively assess these effects in satellite-based optical and quantum free-space communication systems. To evaluate the impact of truncation and optical aberrations on electromagnetic fields, §5 provides a concise overview of wave-optical propagation using Fourier optics. The influence of finite transmitter apertures –i.e., truncation– on free-space optical communication performance is examined in §6, which also analyzes how optical aberrations and pointing jitter degrade inter-satellite communication links. Subsequently, §7 explores a quantum communication application, modeling the effects of imperfections in satellite-based quantum-repeater entanglement distribution, including those introduced in §4 (e.g., dark counts, pointing jitter, and entanglement-swapping errors). Finally, §8 takes a further practical step by modeling satellite-assisted entanglement distribution for quantum key distribution. Building upon the framework established in §7, this chapter incorporates optical truncation effects and proposes beam optimization strategies to enhance system performance under realistic conditions.

Chapter 5

Physical optics

Wave propagation with Fourier optics

A body actually enlightened may become obscure by adding new light to that which it has already received.

Francesco Maria Grimaldi

All models are wrong, but some are useful.

George E.P. Box

As discussed in previous chapters, the effects of finite apertures and optical aberrations must be carefully evaluated in both free-space optical and quantum communication systems. To accurately model the influence of Gaussian beam truncation and optical aberrations, this chapter introduces the Fourier optics formalism. The underlying assumptions, approximations, and hypotheses of the employed methods –namely, the angular spectrum, Fresnel diffraction, and Fraunhofer diffraction approaches– are examined in detail to assess their applicability across different propagation regimes. In addition, a brief discussion is presented on the applicability of this formalism to quantum fields in the single-photon regime, which is essential for the analysis of quantum communication systems in the subsequent chapters.

5.1. Introduction

In this thesis, the simulation and analysis of light propagation play a central role in understanding and predicting the behavior of optical fields in various configurations. Accurately modeling the propagation of optical waves is essential, as many of the phenomena investigated throughout this work depend on the spatial evolution of both the amplitude and phase of the electromagnetic field. In particular, optical wave propagation will be required in several parts of this thesis. The effects of clipping and optical aberrations, which alter the phase and amplitude distribution of a propagating beam, will be analyzed in detail in §6 and §8. Furthermore, the results presented in Part III rely heavily on accurate propagation models. For instance, the generation of annular beams through a spiral phase plate is simulated using Fourier optical propagation methods, as described in §9-10. Additionally, in §11, the phase screen required for beam shaping is obtained via phase retrieval algorithms based on Fourier propagation principles.

The use of Fourier-based methods, rather than ray-tracing or geometrical optics approaches, is justified by the need to account for wave phenomena such as diffraction and interference. When an optical beam is clipped, truncated, or encounters apertures, diffraction effects become significant and cannot be accurately modeled within the ray-optics formalism. Fourier optics, being rooted in the scalar wave equation, provides a rigorous framework to capture these effects, enabling the analysis of both near-field and far-field propagation regimes.

In this chapter, the main formulations of physical optics propagation methods will be derived and discussed. Specifically, we will present the angular spectrum method, the Fresnel diffraction integral, and the Fraunhofer diffraction approximation. Emphasis will be placed on the assumptions underlying each approach, the approximations introduced during their derivation, and the limitations that result from these simplifications. Understanding these aspects is crucial to ensure that the appropriate propagation model is applied in subsequent chapters, depending on the optical setup and the propagation distance involved. For a more comprehensive treatment of physical optics and propagation theory using the Fourier optics framework, the reader is referred to [8, 361].

We begin our derivation of the propagation equations by considering the vectorial wave equation for the electric field. As introduced in §2, the electromagnetic field can be described in the framework of classical field theory by the vectorial electric and magnetic fields, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$, which satisfy Maxwell's equations. Particularly, in Eq. (2.9), the vectorial plane wave equation for a *monochromatic wave* (Assumption 1), in a *linear, homogeneous, isotropic, and non-dispersive medium* (Assumption 2), was obtained as

$$(\nabla^2 + k^2) \mathbf{A}(\mathbf{r}) = 0. \quad (5.1)$$

Under these conditions, the components of the electric field are decoupled, and each component satisfies an independent vectorial wave equation. When these assumptions do not hold—such as in nonlinear, anisotropic, or inhomogeneous media—coupling between the field components arises, and the scalar approximation breaks down. Where it is nonlinear, anisotropic, or inhomogeneous media would directly enter the vectorial wave equation through n no longer being a scalar but a tensor $n_i^j(\mathbf{E}, \mathbf{r}, t, \lambda)$. In

some practical cases, such as propagation through atmospheric turbulence where $n(\mathbf{r})$ varies spatially, these variations are extremely small, and the coupling between field components can be safely neglected.

Although it is not strictly necessary to invoke the paraxial approximation to reduce a vectorial field to a scalar one, the *small-angle approximation* (Approximation 1) is often convenient. When the aperture size is on the order of the wavelength, diffraction can occur at relatively large angles with respect to the optical axis (e.g., the z direction). In contrast, the small-angle approximation effectively corresponds to considering apertures that are large compared to the wavelength of interest, such that diffraction occurs primarily near the forward direction [8]. As shown by Lax *et al.* [362], under the small-angle approximation, a first-order expansion of Maxwell's equations yields the standard scalar wave equation for each of the transverse field components, A_x and A_y . The longitudinal component, A_z , can then be obtained to first order from the transverse components by enforcing the divergence-free condition, $\nabla \cdot \mathbf{E}(\mathbf{r}, t) = 0$. In this regime, A_z is typically much smaller - often negligible - compared to the transverse components. For larger diffraction angles, higher-order corrections become significant (see Ref. [362] for second-order terms). Although partial decoupling of the field components remains possible, the resulting scalar differential equations lose their simple form. Furthermore, under conditions of large-angle diffraction - or equivalently, for apertures small compared to the wavelength - the complete vectorial treatment of the field becomes necessary [363].

Considering all the above approximations (Ass. 1-2 and App. 1), the transversal components of the field can be considered independently and the longitudinal component can be neglected. In this case, Maxwell's equations are reduced to the scalar Helmholtz equation (see (2.9)) for the transversal components as

$$(\nabla^2 + k^2) U(\mathbf{r}) = 0. \quad (5.2)$$

The goal of the following sections is to determine the scalar field $U(x, y, z)$ at any propagation distance z , given the field distribution $U(x, y, 0)$ at the input plane. This framework forms the foundation for the Fourier optical propagation methods, i.e., the angular spectrum, Fresnel, and Fraunhofer formulations, that will be derived in the next subsections.

5.2. Angular spectrum of light

Having established in the previous section that the optical field can be described by the scalar Helmholtz equation, we now seek an explicit solution that allows the field to be propagated from one plane to another. One powerful and general approach to this problem is the angular spectrum method, which expresses any optical field as a superposition of plane waves, each being an exact solution of the Helmholtz equation. This representation naturally connects the spatial and frequency-domain descriptions of optical propagation and forms the basis of many Fourier optics analyses. If the complex optical field $U(x, y, 0)$ is known on the input plane $z = 0$, it can be expressed as a weighted superposition of plane waves using the two-dimensional Fourier

transform:

$$U(x, y, 0) = \iint_{-\infty}^{\infty} A(f_x, f_y, 0) \exp[j2\pi(f_x x + f_y y)] df_x df_y, \quad (5.3)$$

where $A(f_x, f_y, 0)$ represents the angular spectrum of plane-wave components at the plane $z = 0$. Each of these plane waves is itself a solution to the scalar wave equation. Therefore, the propagation of each plane-wave component to a plane at distance z is obtained simply by the accumulation of a phase factor corresponding to its longitudinal wavenumber k_z

$$A(f_x, f_y, z) = A(f_x, f_y, 0) \exp(j k_z z), \quad (5.4)$$

where $k_z = \sqrt{k^2 - k_x^2 - k_y^2} = \sqrt{k^2 - (2\pi f_x)^2 - (2\pi f_y)^2}$. A necessary condition for k_z to be real is that the *transverse wavenumber components satisfy the non-evanescent condition* $k_x^2 + k_y^2 < k^2$ (Hypothesis 1). When this condition is not met, i.e., $k_x^2 + k_y^2 > k^2$, the corresponding components represent evanescent waves, for which k_z becomes imaginary. In this case, Eq. (5.4) results in an exponential decay $A(f_x, f_y, z) = A(f_x, f_y, 0) \exp(-\mu z)$, where $\mu = \sqrt{k_x^2 + k_y^2 - k^2}$ is real and positive. These evanescent components decay over a distance on the order of one wavelength and therefore do not contribute significantly to the field at larger propagation distances relevant to this thesis. Physically, in regions dominated by evanescent waves with a single interface, as in our case, the time-averaged Poynting vector is zero, indicating that no net energy is transported.

Since the scalar wave equation is linear, the total field at any propagation distance z can again be expressed as a superposition of plane waves. The propagated field $U(x, y, z)$ is therefore given by the inverse Fourier transform of the propagated angular spectrum

$$U(x, y, z) = \iint_{-\infty}^{\infty} A\left(\frac{\alpha}{\lambda}, \frac{\beta}{\lambda}, 0\right) \exp\left[j \frac{2\pi}{\lambda} \sqrt{1 - \alpha^2 - \beta^2} z\right] \times \text{circ}\left(\sqrt{\alpha^2 + \beta^2}\right) \exp\left[j \frac{2\pi}{\lambda} (\alpha x + \beta y)\right] d\alpha d\beta, \quad (5.5)$$

where the direction cosines $\alpha = \lambda f_x$ and $\beta = \lambda f_y$ are used for convenience, and the circular aperture function is used to exclude the negligible contribution of the evanescent components. This cutoff imposed by the circ function provides a physical interpretation: it represents the fundamental resolution limit of conventional optical systems. Only spatial frequencies satisfying $f_x^2 + f_y^2 < 1/\lambda^2$ are transmitted, meaning that no imaging system relying solely on propagating waves can resolve features with spatial periods smaller than the optical wavelength. Finally, the diffraction process described by the angular spectrum method in Eq. (5.5) can be interpreted as a linear spatial filtering operation, where the transfer function is determined by the propagation phase term and the circular cutoff of the evanescent spectrum.

5.3. Fresnel propagation

In the previous subsection, the angular spectrum method provided a frequency-domain solution to the Helmholtz equation, describing wave propagation as a superposition

of plane waves. An equivalent approach can be formulated directly in the spatial domain by invoking Green's theorem, leading to the Rayleigh–Sommerfeld diffraction integrals. From these, the Fresnel and Fraunhofer diffraction approximations naturally arise, providing practical tools to model near- and far-field propagation from apertures.

Starting from Green's theorem and defining a suitable Green's function $G(\mathbf{r})$ that satisfies the homogeneous scalar wave equation in (5.2), the field at a point $\mathbf{r}_0$ can be expressed as

$$U(\mathbf{r}_0) = \frac{1}{4\pi} \iint_S \left(\frac{\partial U}{\partial n} G - U \frac{\partial G}{\partial n} \right) ds, \quad (5.6)$$

where S is an arbitrary closed surface enclosing $\mathbf{r}_0$, and n denotes the local outward direction normal to the surface (see Fig. 5.1(a)). The quantity $r_{01} = |\mathbf{r}_1 - \mathbf{r}_0|$ represents the distance between the observation point $\mathbf{r}_1$ and the differential surface element located at $\mathbf{r}_0$. This relation expresses the field at any point as a function of the field and its normal derivative on a surrounding surface.

To obtain a diffraction formula for an aperture, we specify the integration surface S as the union of the aperture plane S_1 and a spherical dome of radius R centered on the observation point $\mathbf{r}_0$, as shown in Fig. 5.1(a). The spherical dome lies in the same half-space as the observation point relative to the aperture plane. At this point, another hypothesis about the asymptotic behaviour of the field at large distances needs to be made, i.e., the *Sommerfeld radiation condition* (Hypothesis 2). This condition, is imposed in the field $U(\mathbf{r})$ at the spherical surface shown in Fig. 5.1(a) as

$$\lim_{R \rightarrow \infty} R \left(\frac{\partial U}{\partial n} - jkU \right) = 0. \quad (5.7)$$

This ensures that the field U decays at least as rapidly as an outgoing spherical wave, thereby excluding unphysical incoming waves. Consequently, the contribution from the spherical cap in Fig. 5.1(a) vanishes as $R \rightarrow \infty$.

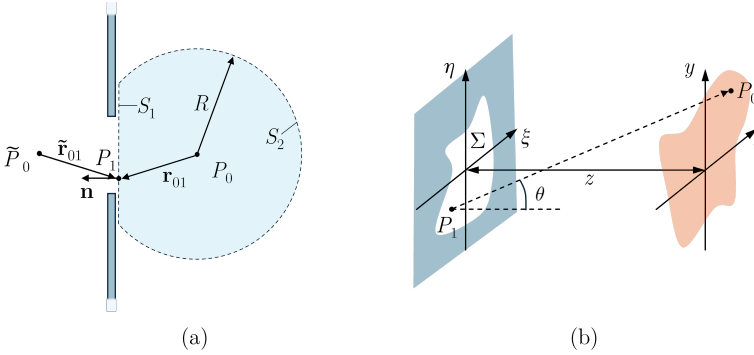


Figure 5.1: Diffraction geometries. (a) Integration surface used in scalar diffraction theory. (b) Geometry of diffraction from a planar aperture.

By choosing the Green's function as

$$G_{\pm}(\mathbf{r}_1) = \frac{\exp(jkr_{01})}{r_{01}} \pm \frac{\exp(jk\tilde{r}_{01})}{\tilde{r}_{01}}, \quad (5.8)$$

where $\tilde{r}_{01} = |\mathbf{r}_1 - \tilde{\mathbf{r}}_0|$ and $\tilde{\mathbf{r}}_0$ is the mirror image of the integration point with respect to the aperture plane S_1 . Letting the propagation distance be much larger than the wavelength, i.e., $r_{01} \gg \lambda$ (Approximation 2), and $R \rightarrow \infty$, we obtain the Rayleigh–Sommerfeld diffraction formulas

$$U_-(\mathbf{r}_0) = \frac{1}{j\lambda} \iint_{\Sigma} U(\mathbf{r}_1) \frac{\exp(jkr_{01})}{r_{01}} \cos(\mathbf{n}, \mathbf{r}_{01}) ds, \quad (5.9)$$

$$U_+(\mathbf{r}_0) = \frac{1}{j\lambda} \iint_{\Sigma} \frac{\partial U(\mathbf{r}_1)}{\partial n} \frac{\exp(jkr_{01})}{r_{01}} ds, \quad (5.10)$$

where the subscript signs correspond to the choice of $G_{\pm}$ in Eq. (5.8) and the integration is done over the aperture area Σ . The first of these expressions, $U_-(\mathbf{r}_0)$, is consistent with the Huygens-Fresnel principle, representing the field at a point as the superposition of secondary wavelets emitted by the aperture. Although the derivation assumes monochromatic illumination (Ass. 1), it can be generalized to broadband fields by superposing the different temporal frequency components of the considered broadband field. For sufficiently narrowband light, the monochromatic assumption introduces negligible error, which applies in this work.

From the first Rayleigh-Sommerfeld diffraction formula in Eq. (5.9), and using the planar geometry in Fig. 5.1(b), the field at an observation plane z can be written as

$$U(x, y) = \frac{z}{j\lambda} \iint_{-\infty}^{\infty} U(\xi, \eta) \frac{\exp(jkr_{01})}{r_{01}^2} d\xi d\eta, \quad (5.11)$$

The distance r_{01} can be approximated using a binomial expansion as

$$r_{01} = \sqrt{z^2 + (x - \xi)^2 + (y - \eta)^2} \approx z \left[1 + \frac{1}{2} \left(\frac{x - \xi}{z} \right)^2 + \frac{1}{2} \left(\frac{y - \eta}{z} \right)^2 \right]. \quad (5.12)$$

when the *paraxial approximation* $(x - \xi)^2 + (y - \eta)^2 \ll z^2$ (Approximation 3) is fulfilled. Substituting this binomial approximation into Eq. (5.11), and retaining only the first term $r_{01} \approx z$ in the denominator¹, we obtain the Fresnel diffraction integral

$$\begin{aligned} U(x, y) = \exp \left[\frac{jk}{2z} (x^2 + y^2) \right] & \frac{\exp(jkz)}{j\lambda z} \\ & \times \iint_{-\infty}^{\infty} \left\{ U(\xi, \eta) \exp \left[\frac{jk}{2z} (\xi^2 + \eta^2) \right] \right\} \\ & \times \exp \left[-\frac{j2\pi}{\lambda z} (x\xi + y\eta) \right] d\xi d\eta. \end{aligned} \quad (5.13)$$

This equation then represents a Fourier transform of the product of the input field and a quadratic phase factor. The binomial approximation (Eq. (5.12)) remains valid as long as the additional phase error due to neglected higher-order terms is much smaller than one radian. A sufficient condition for this to hold is [8, §4.2]:

$$z^3 \gg \frac{\pi}{4\lambda} [(x - \xi)^2 + (y - \eta)^2]_{\max}^2. \quad (5.14)$$

¹In the exponential factor $\exp(jkr_{01})$ the second terms of the expansion in Eq. (5.12) also need to be considered, as the error in this factor can be significant otherwise.

When this condition is fulfilled, Eq. (5.13) provides an accurate and computationally efficient description of optical wave propagation in the near-field (Fresnel) regime, bridging the gap between the exact Rayleigh–Sommerfeld formulation and the far-field Fraunhofer approximation.

5.4. Fraunhofer propagation

When considering propagation to planes located at distances z even larger than those for which the Fresnel approximation holds, the quadratic phase term $\exp [jk(\xi^2 + \eta^2) / (2z)]$ can be approximated as unity, provided that

$$z \gg \frac{k(\xi^2 + \eta^2)_{\max}}{2}. \quad (5.15)$$

Under this condition (Approximation 4), Eq. (5.13) simplifies to

$$\begin{aligned} U(x, y) = & \exp \left[\frac{jk}{2z}(x^2 + y^2) \right] \frac{\exp(jkz)}{j\lambda z} \\ & \times \iint_{-\infty}^{\infty} U(\xi, \eta) \exp \left[-\frac{j2\pi}{\lambda z}(x\xi + y\eta) \right] d\xi d\eta, \end{aligned} \quad (5.16)$$

which defines the Fraunhofer propagation approximation. Another way of satisfying the App. 4 is usually written as

$$z > \frac{2\mathcal{D}^2}{\lambda}, \quad (5.17)$$

where $\mathcal{D}$ denotes the characteristic size of the aperture.

The Fraunhofer propagation formula is relevant not only when the propagation distance z is large, but also when light is focused by a thin lens. For a thin lens under the paraxial approximation, the phase introduced by the lens is given by

$$t_{\text{thin-lens}}(x, y) = \exp \left[-\frac{jk}{2f}(x^2 + y^2) \right], \quad (5.18)$$

where f is the focal length of the lens. Using the Fresnel propagation equation (Eq. (5.13)), and assuming that the field $U(x, y)$ is known immediately in front of the lens (i.e., the object field is located against the lens), the combined effect of the lens phase and its finite aperture –described by the pupil function $\mathcal{A}(\xi, \eta)$ – leads to the following expression for the field at the focal plane (see [8, §5.2]):

$$\begin{aligned} U_f(x, y) = & \frac{1}{j\lambda f} \exp \left[\frac{jk}{2f}(x^2 + y^2) \right] \\ & \times \iint_{-\infty}^{\infty} \mathcal{A}(\xi, \eta) \exp \left[-\frac{j2\pi}{\lambda f}(x\xi + y\eta) \right] d\xi d\eta, \end{aligned} \quad (5.19)$$

This expression is identical in form to the Fraunhofer diffraction integral in Eq. (5.16). Therefore, the Fraunhofer approximation not only describes far-field diffraction from an aperture but also the field distribution at the focal plane of a thin lens when an object is placed directly against it.

In summary, the scalar diffraction theory (that is satisfied under the Ass./Hyp. 1-1) is most exactly predicted by the angular spectrum formalism in Eq. (5.5), which does not consider the evanescent field. For increasing propagation distances, the Fresnel propagation approximation in Eq. (5.13) becomes applicable (that is satisfied under the additional App./Hyp. 2-3). Finally, for larger propagation distances involving the condition App. 4, the Fraunhofer propagation approximation in Eq. (5.16) becomes valid.

In general, the field at the source plane, $U(x, y, 0)$, cannot be propagated analytically using the formulations presented in this section. To address this limitation, numerical Fourier propagation techniques are employed. In these methods, the bidimensional Fourier transforms appearing in Eqs. (5.5), (5.13), and (5.16) are replaced by discrete Fourier transforms (DFT), which can be efficiently implemented computationally [364]. However, such numerical implementations can introduce artefacts—for instance, aliasing effects inherent to discrete Fourier transforms. To minimize these errors, the numerical considerations and sampling conditions discussed in Ref. [364, §7] must be satisfied. In summary, the conditions that need to be fulfilled in all the formulations above are

$$\delta_2 \leq -\frac{D_2}{D_1}\delta_1 + \frac{\lambda z}{D_1}, \quad N \geq \frac{D_1}{2\delta_1} + \frac{D_2}{2\delta_2} + \frac{\lambda z}{2\delta_1\delta_2}, \quad (5.20)$$

where D_1 and D_2 are the source and observation planes extents, and δ_1 and δ_2 are the spacing in these planes, respectively. The number of elements in the grid is $N \times N$, and is the same for both source and observation planes. For Fresnel propagation (which will be the most used in the results presented in this thesis), as the spacing relation $\delta_2 = \lambda z / N \delta_1$ needs to be fulfilled, the previous conditions and another extra condition that avoids aliasing (considering the source field to have a parabolic wavefront with radius R) are

$$N \geq \frac{D_1 \lambda z}{\delta_1 (\lambda z - D_2 \delta_1)}, \quad z \geq \frac{D_1 \delta_1 R}{\lambda R - D_1 \lambda_1}, \quad (5.21)$$

the latter condition converting to $z \geq D_1 \delta_1 / \lambda$ for $R \rightarrow \infty$.

5.5. Propagation of single-photon states

So far, the propagation of classical electromagnetic waves ruled by Maxwell's equations under certain hypotheses has been presented. This will be valuable for the propagation of light beams in classical free-space optical communications (see §6, §9 and §10). However, as presented in §2 and §4, quantum communications can operate in the single-photon regime. Then the question arises: *how do we propagate single-photon quantum states over the free-space channel and how do diffraction phenomena affect these states?*

To answer this question, we remember that the irradiance associated with a single-photon state in the paraxial regime can be interpreted as the probability density function of the localization of the single-photon (see §2.1.2). Although phenomenologically this can be easily understood by considering the double slit experiment with single-photons—that results in the same interference pattern as a classical electromagnetic

wave– the mathematical justification of this in the context of quantum field theory poses some challenges.

For free-space quantum communications, what is of primary importance is to evaluate the probability of capturing a single-photon state that was emitted by a distant transmitter. Although this question is simple, its answer is not straightforward. This is due to the fact that the position operator for the photon is ill-defined in the context of quantum field theory [365]. Unlike other particles (e.g., nonrelativistic particles following the Schrödinger equation), this poses a challenge when trying to answer *where* the photon is, and therefore what is the probability of capturing a photon by a finite aperture. To solve this issue a first approach that requires no approximations to any radiation regime was developed by Bialynicki-Birula [365]. Under the interpretation presented by the author, the photon wave function $\Psi(\mathbf{r}, t)$ is defined as the normalized positive energy (positive frequency) solutions to the photon wave equations derived from Maxwell's equations, in which the defined photon wave function in coordinate representation no longer follows the usual rule $\Psi(\mathbf{r}) = \langle \mathbf{r} | \Psi \rangle$. Furthermore, the normalization is required to give the probabilistic interpretation sought for. This convenient picture of the photon, although controversial, is useful as it allows us to interpret the quantum excitations of the electromagnetic vector field (see §2.1.2) as the probability of finding the energy associated with the excitation (i.e., the photon) in each point of space. By using the well-defined momentum representation of the single-photon states and their related scalar product, the scalar product in the coordinate representations is obtained. Equipped with this scalar product in coordinate representation, several operators of interest, such as the energy operator, can be obtained. Finally, this energy operator allows for the interpretation of the photon wave function in coordinate representation to give probabilistic information of *where* the energy of the excitation of the field (i.e., the energy of the photon) is located. This last factor is why several authors have called the photon wave function the energy wave function [366]. Under this interpretation, and bounded by the limitations of the so-called photon wave function in coordinate representation (arising from the ill-defined photon position operator), the photon wave function can be used to evaluate the probability of the energy of the single-photon state being in a certain volume or crossing a certain area (which is useful to evaluate the performance of free-space quantum communication channels).

Another approach that is useful for our context is the one followed by Aiello et al. [11–13]. Not only does it allow for the probabilistic interpretation of the photon as Bialynicki-Birula [365], but it also provides the formalism for the propagation of single-photon states. Aiello et al. solve the problem of not having a well-defined position operator by using the paraxial approximation in scalar wave optics (App. 3). Under this condition, the well-behaved position operator in the transversal plane – perpendicular to the propagation paraxial direction– can be defined, and it can be shown that eigenstates of this position operator satisfy a Schrödinger equation [12, Eq. (109)]. Hence, the wave function of the scalar field associated with these photon states can now be interpreted in a similar manner to Bialynicki-Birula, but without the inconvenience of an ill-defined position operator.

Thanks to this interpretation by Aiello et al., we can finally answer our original question on how to propagate single-photon states in free-space. In Ref. [13, Eq. (22)],

the authors derive the quantum mechanical analogue to the first Rayleigh-Sommerfeld diffraction formula in Eq. (5.9) (or the equivalent Huygens-Fresnel principle). Therefore, as the single-photon states are also affected by the diffraction on the apertures in the same way as classical waves are, the propagation phenomena of the single-photon states can be treated on the same grounds as classical wave optics by using the Fourier optics techniques presented above. Combining the propagation of single-photon states and their probabilistic interpretation, the probability of a single-photon state that is diffracted by a transmitted aperture, being captured by a finite receiver aperture, can be computed in the context of free-space quantum communications (see §7-§8).

Using the Fourier propagation formalism presented in this chapter, the effects of truncation and optical aberrations in the far-field irradiances can be analyzed. In the following chapters, these diffraction effects and the pointing jitter will be analyzed in a coupled manner to assess their impact on satellite free-space optical and quantum communications.

Chapter 6

Optimal truncation and Seidel optical aberrations

*Finite aperture and aberration effects on intersatellite
free-space optical communications*

*The idea of distinguishing probability
distributions is slippery business.*

Christopher Fuchs

This chapter analyzes the combined effects of transmitter pointing jitter and optical aberrations on intersatellite free-space optical (classical) communication links. Building upon the Fourier propagation formalism introduced in §5, a mathematical framework is developed that integrates this approach with a statistical captured power model to evaluate key performance metrics, including the average bit error probability, outage probability, and link reliability under realistic perturbations. To establish a reference baseline, optimal non-aberrated truncated Gaussian beams under pointing jitter are first computed for several telescope architectures, revealing a strong dependence of system performance on the far-field irradiance distribution. The subsequent inclusion of Seidel aberrations shows that free-space optical terminals are particularly sensitive to coma, primarily due to the induced apparent angle-of-arrival deviation on the beacon beam. Finally, Monte Carlo simulations incorporating multiple aberrations confirm that the sensitivity of communication performance is strongly influenced by the telescope architecture. Together, these results offer valuable insights for the design of robust and high-reliability intersatellite optical communication systems.

The content of this chapter is based on the Ref. [367] Badás, M. et al. "Seidel optical aberrations and optimum truncated Gaussian beams on intersatellite free space optical communications". Optics Express 33, Issue 16, pp. 33686-33703 (2025).

6.1. Introduction

The growing demand for secure, high-throughput global communications positions free-space optical communication as a strong alternative to radio-frequency systems. Its highly directional laser beams enable enhanced security and power efficiency. In this context, intersatellite optical links are essential for long-distance connectivity. However, as discussed in §3, spaceborne optical terminals operate under harsh and dynamic conditions. Thermo-mechanical loads induce microvibrations and structural deformations, leading to pointing jitter that stochastically misaligns the transmitted beam from the receiver. Although partially mitigated by pointing control systems, a residual jitter remains and degrades system performance. Combined with the finite transmitter aperture, this limits the achievable beam directionality. Additionally, thermo-mechanical effects introduce optical aberrations that distort both transmitted and received beams, altering their irradiance profiles and further reducing performance (see Fig. 6.1). While similar effects impact reception –causing fluctuations in angle of arrival and reduced coupling efficiency– these are beyond the scope of this work [116, 368–370]. Overall, pointing jitter and optical aberrations degrade intersatellite link performance, for example, by increasing data loss in LEO-GEO links.

Analyzing the effect of the coupled phenomena given by the transmitter pointing jitter and optical aberrations is necessary for establishing requirements for the optical design of space terminals. On the one side, there is a fair amount of literature analyzing the effect of pointing jitter on intersatellite optical links, such as the early works by Kiasaleh and Toyoshima to mention a few [118, 151]. Analyses combining the effect of the pointing jitter and other in-orbit perturbations can be found in Refs. [315, 371]. Furthermore, the effect of transmitter optical aberrations on the mutual alignment error has also been studied in depth [372–376]. The effect of optical aberrations in the propagation of a Gaussian beam can also be found in the literature [377–381]. Finally, a simplified analysis of the combined effect of transmitter pointing jitter and optical aberrations has been developed by Yang et al. [158, 382]. However, the combined effect of transmitter pointing jitter and optical aberrations, considering the mutual alignment error (or the related apparent angle of arrival that is defined later in the present chapter) and the finite aperture effects, on different communication parameters, is lacking in the literature. In this chapter, we present a model that encompasses all this coupled phenomena, enabling a more accurate evaluation of the perturbations mentioned. This chapter will provide insight in the interplay between these perturbations and how the communication performance of the system is affected (§6.3.2). Furthermore, it will help determine the requirements for admissible pointing jitter and optical aberrations in FSOC satellite terminals. In summary, this chapter aims to provide a step forward in developing the next generation of reliable intersatellite optical communications.

We first explain the mathematical model developed to evaluate the impact of pointing jitter and transmitter optical aberrations in §6.2. The effects of optical aberrations on both the transmitted communication beam and the received beacon beam are accounted for through wave optics propagation as presented in §6.2.1 and §6.2.2. The mathematical models to compute the received power statistics due to the stochastic transmitter pointing jitter are presented in §6.2.3 and §6.2.4. From these statistics,

different communication performance parameters can be obtained (§6.2.5). Using the proposed model, the optimum far-field irradiance distributions for different communication performance parameters, when considering a truncated Gaussian beam are obtained for different transmitter architectures (§6.3.1). This is done considering only the transmitter pointing jitter, without optical aberrations. These optimized solutions are then perturbed with transmitter optical aberrations to evaluate the impact of these on the communication performance. An analysis is presented showing the effect of each of the Seidel aberrations for different telescope architectures. Finally, the combined effect of these Seidel aberrations acting altogether is assessed statistically through Monte Carlo simulations.

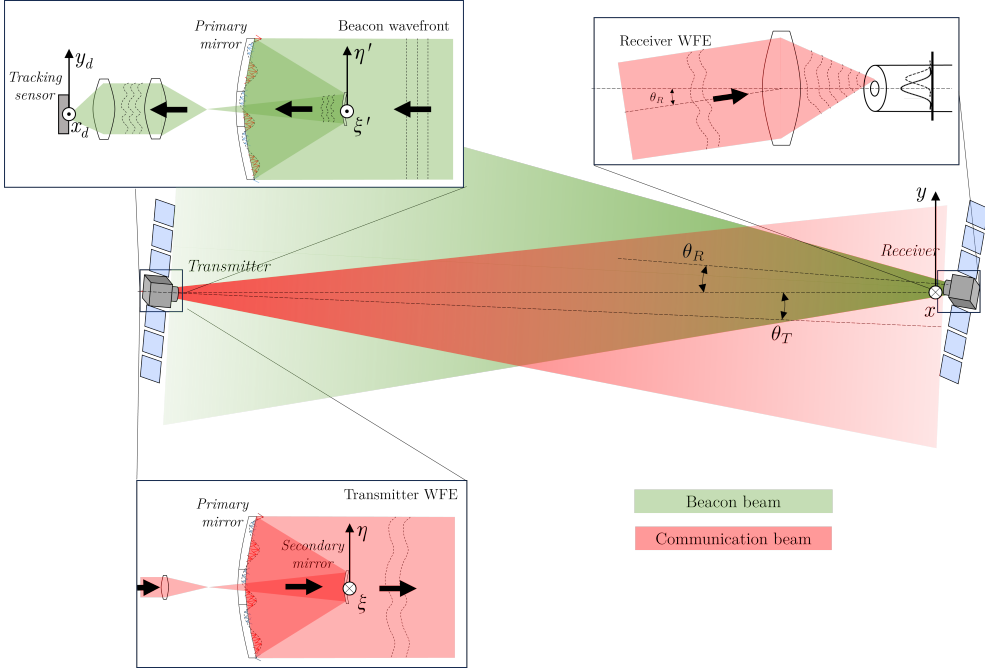


Figure 6.1: Illustration of the effect of the transmitter and receiver wavefront errors and pointing jitters. θ_T and θ_R represent the transmitter pointing error and receiver angle of arrival at a certain instant of time, respectively. In the top left figure, only the sensor of the tracking system is shown for simplicity, while the complete tracking system would also have actuators (i.e. coarse and fine-pointing assemblies).

6.2. Wave optics propagation and communication performance

In this section, the mathematical model to compute the performance of an intersatellite link under transmitter pointing jitter and wavefront errors is presented. Firstly, the computational optical model to obtain the far-field diffraction pattern is presented. Secondly, the effect of the optical aberrations on the received beacon, which materialize as mutual alignment errors, are presented. Then, the far-field irradiance and the

pointing jitter statistics are combined to obtain the statistics of the received power. Finally, the received power statistics are used to compute different communication performance parameters of the intersatellite link. The block diagram in Fig. 6.2 shows the overview of the steps followed in the model.

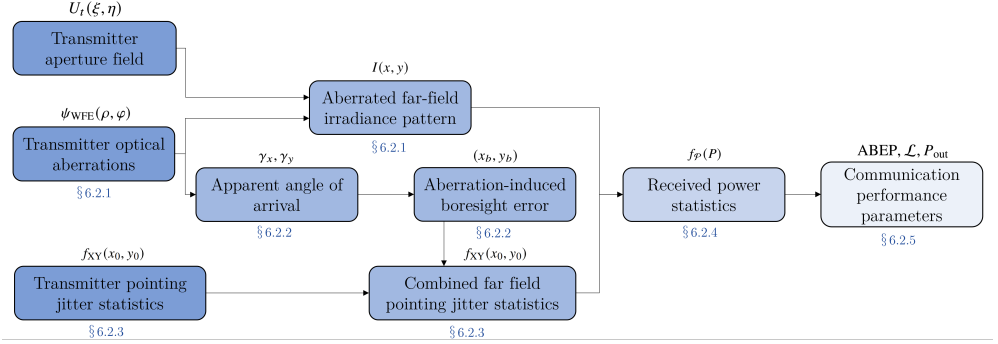


Figure 6.2: Diagram of the steps of the mathematical model to compute the effect of optical aberrations on intersatellite FSO. The mathematical expressions and the relations between them are presented in the respective sections of the chapter.

6.2.1. Wave optics propagation: far-field diffraction patterns

For a complex electromagnetic scalar field in the transmitter aperture plane (including the optical aberrations under study), the far-field irradiance pattern at the receiver aperture plane can be obtained through Fourier optics (see §5). A Gaussian beam is considered as the output of the laser source in the transmitter [118]. However, due to the diffraction effects of the transmitter aperture and the optical aberrations, the outgoing beam is no longer purely Gaussian. Given that different transmitter telescope architectures will yield different far-field diffraction patterns, three different architectures are considered in this chapter. In Fig. 6.3 these architectures are illustrated. Fig. 6.3(a) shows the case of a telescope-centered Gaussian beam with a central *obscuration* due to the secondary mirror. Fig. 6.3(b) shows the case for a *clear* aperture telescope. This case is also representative of a clear refractive telescope, not necessarily an off-axis reflective one as shown in the figure. Fig. 6.3(c) shows the case in which the central obscuration of the telescope is avoided by using an off-axis *subaperture*. In §6.3.2 the differences between the clear and subaperture architectures are emphasized. In the nominal case (without optical aberrations), the system should be designed so that the outgoing beam contains as much of the power available, and the far-field irradiance of the beam is optimum for the given pointing jitter [118]. The electrical field for a Gaussian beam of wavelength λ in a homogeneous medium with index of refraction n is

$$U_G(x, y, z) = U_0 \frac{w_0}{w(z)} \exp\left(-\frac{x^2 + y^2}{w(z)^2}\right) \exp\left[-i\left(kz + k\frac{x^2 + y^2}{2R(z)} - \nu(z)\right)\right], \quad (6.1)$$

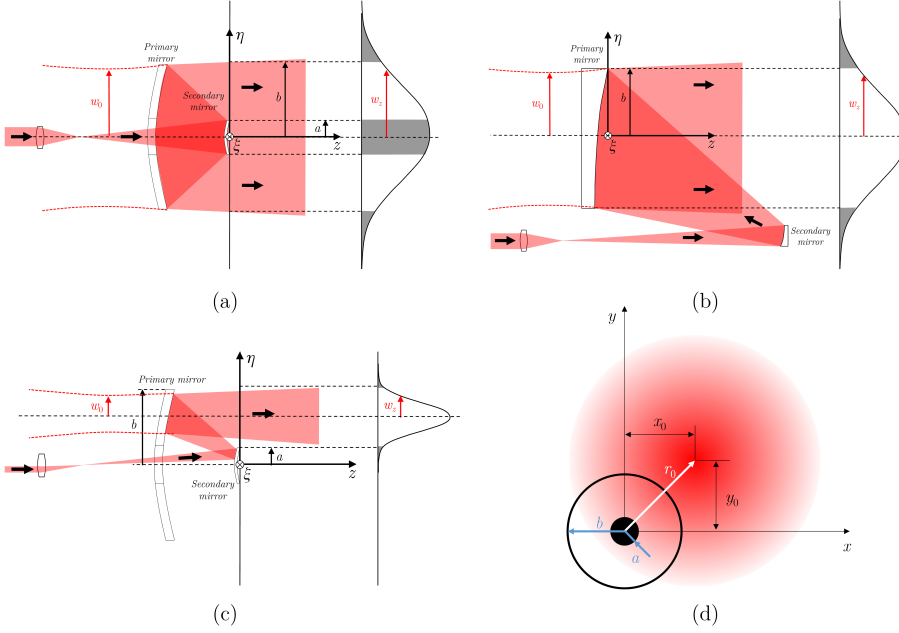


Figure 6.3: (a)-(c) Transmitted beam truncation and obscuration for three different transmitter architectures analyzed: (a) obscured, (b) clear, and (c) subaperture architectures. (d) The receiver's aperture plane shows the far-field irradiance pattern at a certain instant with pointing error (x_0, y_0) as seen from the inside of the receiver.

where

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \quad R(z) = z \left[1 + \left(\frac{z_R}{z}\right)^2\right], \quad \nu(z) = \arctan\left(\frac{z}{z_R}\right),$$

$w(z)$ is the beamwidth of the beam at a distance z from the beamwaist, $R(z)$ is the radius of curvature of the Gaussian beam wavefront, $\nu(z)$ is the Gouy phase and $z_R = \pi w_0^2 n / \lambda$ is the Rayleigh distance. When transmitting a Gaussian beam, the finiteness of the telescope aperture will truncate and obscure the Gaussian beam to a higher or lower extent. The need for each of these different architectures is given by the telescope design trade-off [317]. For each of these architectures, the divergence of the outgoing beam can be adjusted by adjusting the beamwidth $w(z)$ and the curvature of the wavefront $R(z)$ in the transmitter's aperture plane. Instead, in this chapter, the outgoing Gaussian beam will be characterized by the beamwidth $w(z) = w_z$ at the transmitter aperture plane, and the underlying beamwaist w_0 (see Fig. 6.3(c)). In the supplemental document of Ref. [367], a simplified explanation of how these parameters

can be adjusted through optical design of the system is presented by using the ABCD matrix method.

Depending on the transmitter telescope architecture considered, the field in the transmitter aperture is given by

$$U_t(\xi, \eta) = U_G(\xi, \eta, z_t) \mathcal{A}(\xi, \eta) \exp[j \psi_{\text{WFE}}(\xi, \eta)] \quad \text{Fig. 6.3a and 6.3b, (6.2)}$$

$$U_t(\xi, \eta) = U_G(\xi, \eta - \eta_0, z_t) \mathcal{A}(\xi, \eta) \exp[j \psi_{\text{WFE}}(\xi, \eta)] \quad \text{Fig. 6.3c, (6.3)}$$

where η_0 corresponds to the decentering of the beam axis with respect to the telescope optical axis and $z_t = z_R \sqrt{(w_z/w_0)^2 - 1}$ is given by the beamwidth at the transmitter aperture w_z and the underlying beamwaist w_0 . The optical aberrations have been included via the additional phase $\psi_{\text{WFE}}(\xi, \eta)$, and $\mathcal{A}(\xi, \eta)$ is the transmitter aperture function is given by

$$\mathcal{A}(\xi, \eta) = \begin{cases} 1 & \text{if } a \leq \sqrt{\xi^2 + \eta^2} \leq b \\ 0 & \text{else,} \end{cases} \quad (6.4)$$

where b and a are the primary and secondary mirror radii, respectively. The clear aperture architecture in Fig. 6.3(b), has no secondary mirror, i.e., $a = 0$. In this model, the effects of the transmitter's optical aberrations are considered as if they were being applied in the aperture plane of the transmitter. This is because most of the optical aberrations due to in-orbit thermo-mechanical loads, will happen in the primary mirror of the transmitter's telescope. This is usually the optical component that is the largest and most exposed to the space environmental loads [372]. The additional phase $\psi_{\text{WFE}}(\xi, \eta)$ representing the optical aberrations can be expressed in polar coordinates using the Zernike polynomials $Z_n^m(\rho, \varphi)$ [383]

$$\psi_{\text{WFE}}(\rho, \varphi) = \sum_{n=0}^{\infty} \left(\sum_{m=-n}^n a_{nm} Z_n^m(\rho, \varphi) \right), \quad (6.5)$$

where a_{nm} are the coefficients corresponding to the radial index n and the azimuthal index m . Fig. 6.4 shows the first Zernike polynomials along with their classical denominations. To compute the far-field irradiance distribution, the Fresnel diffraction equation (see Eq. (5.13)) for wave propagation can be numerically evaluated with

$$U_r(x, y) = \frac{e^{jk\Delta z}}{j\lambda\Delta z} e^{j\frac{k}{2\Delta z}(x^2+y^2)} \iint_{\mathbb{R}^2} \left\{ U_t(\xi, \eta) e^{j\frac{k}{2\Delta z}(\xi^2+\eta^2)} \right\} e^{-j\frac{2\pi}{\lambda\Delta z}(x\xi+y\eta)} d\xi d\eta, \quad (6.6)$$

where $U_r(x, y)$ is the field in the receiver aperture plane at a given distance Δz from the transmitter, and $U_t(\xi, \eta)$ is the field in the transmitter aperture plane. In reality, the receiver plane is tilted a small angle with respect to the perpendicular plane to the transmitter optical axis (see Fig. 6.1). This tip-tilt is the combination of both the transmitter and receiver angular pointing jitters. This deviation would create not only a displacement as seen by the receiver considered in §6.2.3, but also a tip-tilt of the received field [384–387]. However, for the small angular pointing jitters involved in satellite optical communications, this effect is negligible. Combining the

equations (6.2)-(6.5) above, Eq. (6.6) can be numerically evaluated using the fast Fourier transform (FFT) to obtain the far-field $U_r(x, y)$ on the receiver aperture plane. Finally, the far-field irradiance distribution in the receiver's aperture plane is obtained by simply evaluating $I_r(x, y) = |U_r(x, y)|^2 / 2\zeta_0$, where the wave impedance is $\zeta_0 = 377 \Omega$ for free-space. The source of the optical aberrations will be in orbit variations of temperature over time [372], hence the variation of the far-field irradiance shape will occur on the order of minutes at least (see §3). While large optical elements may exhibit high-frequency temporal stochastic deformations due to microvibrations, this work considers elements smaller than one meter, for which such effects are neglected. Therefore, the effect of the optical aberrations is considered quasi-static compared to the pointing jitter dynamics, the latter having a characteristic time on the order of milliseconds [81, 112].

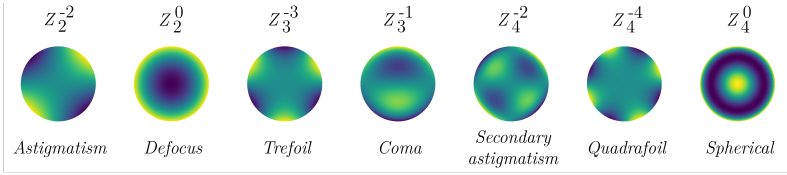


Figure 6.4: Zernike polynomials $Z_n^m(\rho, \varphi)$ for different radial and azimuthal indices, n and m respectively

6.2.2. Apparent angle of arrival and mutual alignment error

In the previous section, the model to compute the effect of the transmitter optical aberrations in the far-field irradiance pattern has been presented. Furthermore, these aberrations will also affect the diffraction pattern on the tracking detector plane of the beacon coming from the receiver (see Fig. 6.1). As a result of the aberrated beacon image on the tracking sensor, an *apparent* angle of arrival of the beacon signal will be detected. This artifact, created by the optical aberrations, will command the tracking system to point the outgoing beam towards the direction of this apparent angle of arrival. In this chapter, we consider that the tracking sensor uses a center of gravity algorithm to detect the direction of the incoming beacon. The type of algorithm that can be used depends on the type of tracking sensor considered. When using a quadrant photodiode with an axially symmetric irradiance shape, the usual quadrant algorithm can yield similar results to that of the center of gravity algorithm [388]. When using a higher spatial resolution tracking sensor (e.g., a CCD), the center of gravity and maximum irradiance algorithms are mostly used. For a comparison of the apparent angle of arrival obtained when using a center of gravity or maximum irradiance algorithms, see App. A.

At the large distances involved in intersatellite optical communications and the big divergence used for the beacon beam, the beacon's electric field is seen as a uniform plane wave in the transmitter aperture plane. When the transmitter is pointing directly in the line of sight of the incoming beacon, the beacon's complex field in the

transmitter aperture plane (considering unit amplitude of the field) is given by

$$U_b(\xi', \eta') = \mathcal{A}(-\xi', \eta') \exp[-j \psi_{\text{WFE}}(-\xi', \eta')], \quad (6.7)$$

where again the optical aberrations and aperture diffraction effects have been included. The negative sign in the phase field of the optical aberrations represents the fact that the received beacon sees the inverse aberration of what the transmitted beam sees (see Fig. 6.1). Furthermore, the (ξ', η') reference system used is consistent for propagating the received beacon towards the transmitter terminal instead of outwards (see App. A for more details on the reference systems used). The complex field in the tracking sensor plane, U_d , is obtained via Fraunhofer diffraction equation [8], with an effective focal length f that focuses the incoming beacon to the tracking sensor plane (see Eq. (5.19)),

$$U_d(x_d, y_d) = \frac{1}{j\lambda f} e^{j\frac{k}{2f}(x_d^2 + y_d^2)} \iint_{\mathbb{R}^2} U_b(\xi', \eta') e^{-j\frac{2\pi}{\lambda f}(x_d \xi' + y_d \eta')} d\xi' d\eta', \quad (6.8)$$

The irradiance can then be computed as $I_d = |U_d(x_d, y_d)|^2 / 2\zeta_0$. Finally, by computing the center of gravity (x_G, y_G) of this irradiance field as

$$x_G = \frac{\iint I_d(x_d, y_d) \cdot x_d dx_d dy_d}{\iint I_d(x_d, y_d) dx_d dy_d}, \quad y_G = \frac{\iint I_d(x_d, y_d) \cdot y_d dx_d dy_d}{\iint I_d(x_d, y_d) dx_d dy_d}, \quad (6.9)$$

the apparent angle of arrival directions, under small-angle approximation, on the tracking detector reference system can be obtained as

$$\gamma_{d,x} = \frac{x_G}{f}, \quad \gamma_{d,y} = \frac{y_G}{f}, \quad (6.10)$$

where f is the effective focal length. As the tracking algorithm corrects for this apparent angle of arrival, the transmitted beam will be mispointed from the real beacon direction (real direction of the receiver terminal) by a static boresight error. Hence, the transmitted beam will be pointing in the direction given by the directional angles $(\gamma_x, \gamma_y) = (\gamma_{d,x}, -\gamma_{d,y})$ as seen by the transmitter's outgoing reference system (see App. A for more details on the reference systems used). Toyoshima et al. [372] define the mutual alignment error as the angle between the direction of the apparent angle of arrival and the direction of the maximum irradiance of the aberrated transmitted beam. While the apparent angle of arrival tells us the direction error between the real receiver direction and the optical axis of the transmitted beam, the mutual alignment error accounts for the direction error between the real receiver direction and the peak irradiance direction of the transmitted beam. As commented in the previous section, the variation of the optical aberrations is considered to be quasi-static compared to the dynamics of the pointing jitter (which occur at $\sim$ kHz, see §3). Therefore, the apparent angle of arrival will be treated statically, including an additional boresight error in the transmitter pointing jitter statistics.

6.2.3. Transmitter pointing jitter

Along with the effects of the transmitter optical aberrations presented in the previous sections, in-orbit microvibrations will generate a pointing jitter of the transmitted

beam's optical axis. Although this is usually mitigated by the pointing system, there will always remain a residual pointing jitter affecting the transmitted beam (see §3). Residual pointing jitter must be estimated by accounting for onboard microvibrations and the mitigation from the optical pointing assembly. This jitter will create a stochastic fading of the received power that will affect the performance of the system as presented in the next section. The transmitter angular pointing jitter is usually characterized by a centered symmetrical bivariate Gaussian distribution [118, 149, 154]. Theoretically, this assumption is justified by the central limit theorem, given that the transmitted pointing jitter statistics are determined by a large number of variables [389]. Furthermore, experimental verification of the previous can be found in the literature [81, 85, 112, 390]. For the small-angle approximation, it can be shown that this translates into a decenter error between the transmitted beam center and the receiver's axis (see Fig. 6.3(d)), characterized by a centered bivariate Gaussian distribution. Finally, considering that the standard deviation in x and y axes are the same (or in azimuth and elevation angles in the transmitter pointing), the probability density function (PDF) of the pointing jitter is given by

$$f_{XY}(x_0, y_0) = \frac{1}{2\pi\sigma^2} \exp \left[-\frac{(x_0 - x_b)^2 + (y_0 - y_b)^2}{2\sigma^2} \right], \quad (6.11)$$

where (x_0, y_0) are the coordinates of the instantaneous error, (x_b, y_b) are the coordinates of the boresight error and σ is the standard deviation of the bivariate Gaussian distribution. Under the small angle approximation, the angular pointing jitter is defined as $\sigma_\theta = \sigma/\Delta z$, where Δz is the link distance. In our model, the only source of boresight error is the apparent angle of arrival, as explained in §6.2.2. For zero boresight error, the previous PDF is converted to a Rayleigh distribution, $f_{\mathcal{R}}(r)$, in radial coordinates. When the problem at hand is fully axially symmetric, this distribution can be useful [391].

6.2.4. Received power statistics

As seen by the receiver aperture plane, the effect of the thermo-mechanically induced optical aberrations presented above is to have an aberrated far-field irradiance pattern (§6.2.1), that is quasi-statically displaced by the mutual alignment error (§6.2.2). Furthermore, the transmitter's angular pointing jitter stochastically further deviates the center of the aberrated far-field from the receiver aperture plane (§6.2.3). All these effects together create a fluctuation of the power collected by the receiver aperture. In this section, the expressions to obtain the received power statistics from the far-field irradiance pattern and the pointing jitter statistics are presented.

The location of the far-field irradiance distribution in the receiver's aperture plane is affected by the transmitter pointing jitter. This stochastic process will produce received power dynamics that will degrade the performance of the system. The received power as a function of the location of the center of the transmitted beam, given by the coordinates (x_0, y_0) , can be computed through a bidimensional convolution of the receiver's aperture pupil and the far-field irradiance at the receiver's aperture plane

(see Fig. 6.3d)

$$P = g(x_0, y_0) = \iint_{\mathbb{R}^2} I(x, y) \mathcal{A}(x - x_0, y - y_0) dx dy = \mathcal{F}^{-1}\{\mathcal{F}(I) \cdot \mathcal{F}(\mathcal{A})\}, \quad (6.12)$$

where $\mathcal{A}(x, y)$ is the pupil function of the receiver, and the convolution theorem has been applied to obtain the last equality in terms of Fourier transforms. This chapter considers symmetric intersatellite links, in which the pupil function of the receiver is the same as the transmitter's (see Eq. (6.4)). When the change on the far-field irradiance across a length equal to the diameter of the receiver telescope is negligible, the previous expression can be simplified as $g(x_0, y_0) \approx A \cdot I(x_0, y_0)$, where A is the total area of the receiver's aperture.

To evaluate the performance of a communication system, the received power statistics are needed. Considering a slow fading channel, in which the modulation frequency of the communication signal is orders of magnitude higher than the dynamics of the power fading, the received power can be characterized by a PDF to later obtain communication performance parameters [175]. As the dynamics of the pointing jitter are on the order of kHz (see §3), compared to the usual MHz modulation frequencies, the slow fading channel is a good approximation for our analysis. To obtain the PDF of the received power, the power as a function of the displacement of the beam (Eq. (6.12)) can be combined with the PDF of the position of the center of the beam in the receiver aperture plane (Eq. (6.11)). Using these functions and the expression to obtain PDFs from statistically dependent variables [392], the PDF of the received power can be obtained as

$$f_{\mathcal{P}}(P) = \iint_{-\infty}^{\infty} f_{XY}(x_0, y_0) \delta[P - g(x_0, y_0)] dx_0 dy_0, \quad (6.13)$$

where $\delta(x)$ is the Dirac delta function. When both the irradiance field and the transmitter pointing jitter PDF are axially symmetric, the simplified equations presented in Ref. [391] can be used. However, in general, the truncation geometry, the optical aberrations and the mutual alignment error will break this axial symmetry and therefore, the expressions above are required to obtain the received power statistics. By normalizing the received power P by the average transmitted power P_t , the PDF of the channel loss $f_{\mathcal{P}}(h)$ can be obtained, where $h = P/P_t$.

6.2.5. Communication performance parameters

In this section, we present the parameters that characterize the performance of an intersatellite optical communication system. We will be considering three main performance parameters in this chapter: average bit error probability (ABEP), probability of outage (P_{out}), and the reliability ($\mathcal{L}$). It is relevant to mention that some papers presented in the literature have mainly focused on the peak irradiance or Strehl ratios of the far-field beam as an estimate of the performance parameter [116, 372, 393]. This, however, as it will be proven in the next section, is not enough to fully characterize the performance of an intersatellite communication system when considering the effects of pointing jitter. Furthermore, the widespread link budget approach only gives partial information, and it is not able to capture the stochastic behaviour inherent to the system. That is why in this chapter we develop an exact model to capture the

stochastic nature of the problem at hand and obtain the probabilistic communication performance parameters presented below. The stochastic behavior of the system is captured by the received power probability density function.

Average bit error probability

From the normalized received power density function $f_{\mathcal{P}}(h)$, the statistics of the bit error probability can be obtained. Using the scalar-to-scalar conversion of variables related by monotonic functions [392], the PDF of the bit error probability can be obtained as

$$f_{\mathcal{B}}(P_e) = \left| \frac{d}{dP_e} P_e^{-1} \right| \cdot f_{\mathcal{P}}(P_e^{-1}), \quad (6.14)$$

where the $P_e^{-1}(P_e)$ is the inverse function of the instantaneous conditioned bit error probability. The latter can be computed for on-off keying intensity modulation direct detection (OOK IM/DD) as [394]

$$P_e(h) = \frac{1}{2} \operatorname{erfc} \left(\frac{h P_t R}{\sqrt{2} \sigma_n} \right), \quad (6.15)$$

where P_t is the transmitted power, R is the responsivity of the detector, and $\sigma_n^2 = N_0 R_0 / R_{\text{load}}$ is the signal-independent additive white-Gaussian thermal noise (dependent on the bit rate R_0). From the PDF of the bit error probability, the ABEP can be computed as

$$\text{ABEP} = \langle P_e \rangle = \int_0^\infty P_e(h) f_{\mathcal{P}}(h) dh = \int_0^{1/2} P_e f_{\mathcal{B}}(P_e) dP_e, \quad (6.16)$$

where the first equality uses the channel loss PDF and the second uses the bit error probability PDF for the calculation.

Reliability

The reliability is defined as the probability that the instantaneous bit error probability is smaller than a certain threshold that is considered reasonable by the designer [395]. Mathematically, this parameter is computed through the PDF of the bit error probability in Eq. (6.14) as

$$\mathcal{L}(P_e^{\text{th}}) = \text{Prob} \{ P_e < P_e^{\text{th}} \} = \int_0^{P_e^{\text{th}}} f_{\mathcal{B}}(P_e) dP_e, \quad (6.17)$$

where P_e^{th} is the bit error probability threshold defined by the designer. This threshold is established depending on how the error correction code of interest performs under different levels of BEP. Usually, values on the range between $P_e^{\text{th}} \in (10^{-6}, 10^{-9})$ are considered [395].

When comparing the ABEP and reliability, ABEP represents the long-term expected value of the BEP under the time dynamics of pointing jitter. However, since the probability distribution of the BEP is often highly skewed, ABEP can be misleading. In such cases, reliability offers a more practical and representative measure of system performance.

Probability of outage

An outage is an event in which the data rate is above the instantaneous capacity of the channel [175]. This is equivalent to saying that there exists no error correction code that can correct the instantaneous bit error probability of the channel. In this case, the channel can not handle such a data rate and the communication between the terminals fails. The outage event differs from the fade [396], in that during the outage, there might be enough power to be detected by the detector, but the data can not be successfully decoded (i.e., the bit error rate is too high to be handled by the error correction code). In the same manner, the reliability can also be interpreted as the probability of being above a certain power threshold that is provided by the relation between the signal-to-noise ratio and the BEP of the modulation technique of interest. In summary, both the reliability, the fade, and the outage are failures of the same physical nature. However, while the first refers to real practical limitations of the system, the third one is an information-theoretical limit that would be the ultimate mode of failure of the system.

As shown by Farid et al. [175], the probability of outage can be computed for a nonergodic slow-fading channel, OOK IM/DD modulation, and complete channel state information on the receiver as

$$P_{\text{out}}(D_0) = \text{Prob}[h < h_0] \quad \text{with} \quad h_0 = \sqrt{\frac{\mathcal{C}^{-1}(D_0)\sigma_n^2}{2P_t^2 R^2}}, \quad (6.18)$$

where h is the normalized received power. D_0 is the achievable rate in bits/channel use, and represents the fraction of bits of information over all the bits sent through the channel (including the ones needed for bit error correction). The expression for the instantaneous capacity of the channel $\mathcal{C}$ is shown in the supplemental document of Ref. [367].

6.3. Numerical simulation of an intersatellite optical link

In this section, an intersatellite free-space optical link is simulated with the model presented above. First, the optimal transmitted beam is computed for minimizing/maximizing the average bit error probability, probability of outage, and reliability (for each of the transmitter architectures presented in Fig. 6.3). Then, the performance variation under optical aberrations is studied.

6.3.1. Transmitted beam optimization

To have a more realistic evaluation of how the optical aberrations affect the system, the *nominal* non-aberrated transmitted beam is first computed. In this way, when computing the effect of the optical aberrations on the performance of the system, the wave physics propagation will compute the far-field irradiance due to both the truncation of the aperture and the aberrations. Previous studies have presented a Gaussian beam divergence optimization to mitigate the effects of the transmitter pointing jitter in the communication performance [118, 175]. Furthermore, not only the divergence, but the shape of the far-field irradiance has also been found to be key to mitigating this

effect, as will be shown in Part III. However, these works do not consider the limitations posed by the finite-sized apertures of the transmitter telescope, which are critical to determining the far-field irradiance of the beam, especially under aberrations.

The nominal non-aberrated transmitted beam is found by optimizing each performance parameter –varying the Gaussian beam shape in the transmitter aperture plane to reach the extremal value of the performance parameter. Considering the rest of the parameters to be constant, the Gaussian beam at the aperture of the transmitted telescope can be characterized by two parameters, e.g., the beamwidth w_z and the underlying beamwaist w_0 (see Fig. 6.3(c)). In the supplemental document of Ref. [367], the expression to obtain the Gaussian beam’s radius of curvature R_z in the transmitter aperture plane from these two parameters is presented. Qualitatively, the beamwidth w_z is going to determine the amount of truncation that occurs to the transmitted beam, while the beamwaist w_0 will determine the divergence of the untruncated pure Gaussian beam. Both together will determine the far-field irradiance field and therefore the performance of the system. In the supplemental document of Ref. [367], the way to adjust for these parameters, (w_0, w_z) , in an equivalent optical system through ABCD matrices is presented.

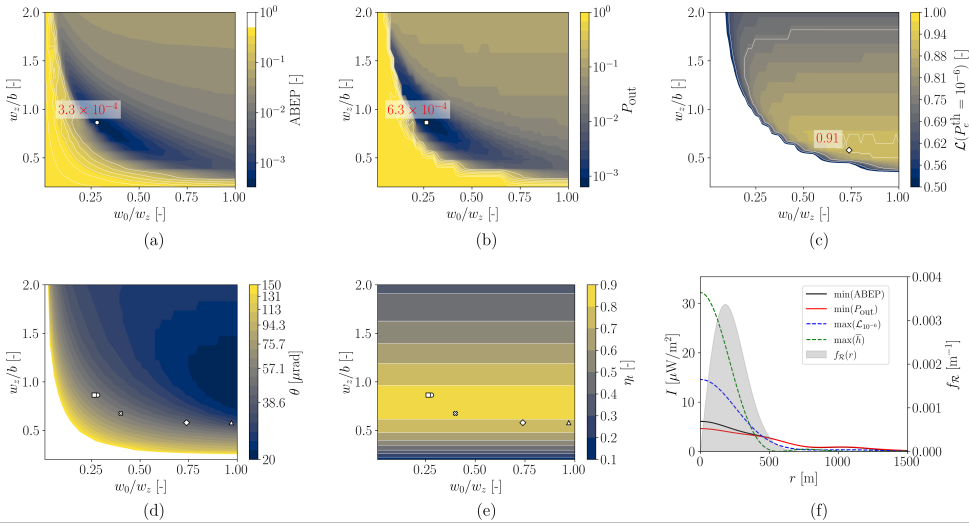


Figure 6.5: Optimization of the transmitted beam for an intersatellite link given by the parameters in Tab. 6.1 and the telescope architecture shown in Fig. 6.3(a). (a), (b), and (c) are the optimizations for each of the performance parameters considered. (d) and (e) show the transmitted beam divergence and the telescope transmission efficiency due to the truncation, respectively. The cross and the triangle give the optimum for $\mathcal{L}(P_e^{th} = 10^{-3})$ and $\mathcal{L}(P_e^{th} = 10^{-9})$ respectively. (f) are the far-field irradiances as a function of the radial coordinate in the receiver aperture plane for the optimum points found in (a)-(c), along with the irradiance field that maximizes the received power average $\bar{h}$, and the PDF $f_{\mathcal{R}}(r)$ of the pointing jitter.

Table 6.1: Parameters used for an intersatellite LEO-GEO free-space optical link.

Parameter	Value	Ref.	Parameter	Value	Ref.
λ	1550 nm	-	b	6.75 cm	[397]
P_t	10 W	-	a	$\{0, 1.35\}$ cm	-
σ_n	4.7×10^{-9} V	[389]	Modulation	OOK-NRZ	[389]
σ_θ	5 μ rad	[398]	Data rate	10 Gbps	[389]
R	0.8153 A/W	[154]	Δz	36000 km	-

For an intersatellite LEO-GEO link given by the system parameters shown in Tab. 6.1, nine optimizations are performed. Each of these optimizations corresponds to one of the three telescope architectures presented in Fig. 6.3 above and for each of the three performance parameters (ABEP, P_{out} and $\mathcal{L}(P_e^{\text{th}} = 10^{-6})$). Figs. 6.5(a)-(c) show the optimization of the transmitted beam for these three performance parameters in the case of an obscured architecture (Fig. 6.3(a)). Furthermore, the divergence of the beam is shown in Fig. 6.5(d) for each point. The divergence is defined as the full-width angle described by a circular contour containing the 86% of the total power of the beam as it propagates (see App. A for the formal mathematical definition). This definition is an extension of that of the pure Gaussian beam. Finally, the transmission efficiency due to the truncation of the beam on the transmitted telescope is shown in Fig. 6.5(e). It can be seen that the optimum for each of the parameters is given by different far-field irradiance distributions, given by different beam parameters w_z and w_0 of the transmitted beam. This is due to the fact that different far-field irradiance distributions will result in different received power statistics when combined with the pointing jitter. Each of the parameters is maximized for different received power statistics in the domain of achievable PDFs $f_{\mathcal{P}}(h)$ given by the limitations of the aperture size of the telescope and its architecture. In Fig. 6.5(f) the optimum far-field irradiances for each of the performance parameters are presented along with the pointing jitter PDF (due to the axially symmetrical nature of all the elements considered, the pointing jitter is also axially symmetric). In App. A, the optimization for the two remaining architectures can be found, yielding similar qualitative results. Compared to the results presented here for the obscured architecture, the clear aperture case yields better values of the performance parameters, primarily due to the higher power available after the truncation and obscuration of the beam. In the subaperture case, the interplay between the achievable divergence and the power loss due to the truncation also limits the performance of the system compared to the clear aperture case.

From the analysis above, it can be seen that the optimum characteristics of the Gaussian beam in the transmitter aperture plane are extremely dependent on the performance parameter that is being optimized. For the minimization of the ABEP and the probability of outage, it can be seen that more uniform distributions of the far-field are interesting (see Fig. 6.5(f)). This is due to the underlying received power statistics given by these types of far-field irradiances. For a spatially more uniform

(compared to Gaussian) type of far-field irradiance distributions, the resulting received power statistics are more condensed around a central received power. In the limit of a perfectly uniform far-field irradiance, the received power statistics would be given by a discrete probability distribution (see §11). In the case of outage probability, this tendency is justified by the fact that the Eq. (6.18) sets a minimum received power threshold above which any extra power is not beneficial for this performance parameter. In the case of the ABEP, the results show that there is a trade-off between the maximum achievable received power and the uniformity of the beam. This is in agreement with the findings in Part III. Furthermore, the divergence value of $52.8 \mu\text{rad}$ obtained for the minimum ABEP (circled marker in Fig. 6.5(d)), is in close agreement with the $51.6 \mu\text{rad}$ that would be obtained using Eq. (12) in [118]. The discrepancy between these two values is due to the non-Gaussian beam shape resulting from the truncation of the beam considered in the present chapter.

For maximizing the reliability, the figure shows that a more Gaussian shape is beneficial. This means that the underlying received power statistics for maximizing this parameter are much more skewed than in the cases before. This effect is more pronounced as the bit error probability threshold of the considered reliability decreases (see App. A). However, attending to the Eqs. (6.18) and (6.17) it can be seen that the reliability and the probability of outage are mathematically similar, in that they set a power threshold above which the performance of the system is considered acceptable. Then, the probability of being above (or below) this power threshold is maximized (or minimized). Hence, given the similar mathematical expression for the computation of these two parameters, we would expect a similar type of far-field irradiances in contrast to what is shown in Fig. 6.5(f). The discrepancy between the far-field irradiances for outage probability and reliability is due to the different power thresholds set by each of these parameters. While reliability is a much more practical parameter, it establishes a received power threshold way above the one set by the probability of outage. The latter being a theoretical limit in which failure would occur due to information-theoretical restrictions. Due to the power threshold of the reliability being higher than that of the probability of outage, the physical limits imposed on the Gaussian beam source are not able to uniformize the beam for the reliability power threshold as it did for the probability of outage. Hence, the physical limitations imposed (truncated Gaussian beam in the transmitter aperture plane) yield a more Gaussian beam shape far-field irradiance for the reliability threshold compared to the optimum beamshape for the probability of outage. This is in agreement with the tendency shown for the $P_e^{\text{th}} = 10^{-3}$ and $P_e^{\text{th}} = 10^{-9}$ shown in App. A, showing that the reliabilities and the probability of outage belong to the same family of parameters (the one that set a power threshold and computes the cumulative density function of it).

To emphasize the improvement provided by the optimization proposed, a comparison with a baseline truncated Gaussian beam is now discussed. The baseline beam considered is the one that maximizes the average received power $\bar{h}$. The maximization of $\bar{h}$ is in line with the maximization of the peak of the irradiance field at the receiver aperture plane, which is the parameter considered in previous works [372, 393]. The far-field irradiance of the baseline result can be seen in Fig. 6.5(f), for the obscured

architecture. In Fig. A.4, the far-field irradiances and received power PDF of all the architectures can be found. Table 6.2 summarizes the improvements in each parameter achieved by the proposed optimization across different architectures, compared to the baseline field obtained by maximizing the average received power $\bar{h}$. The results show that the optimized fields achieve performance gains of several orders of magnitude in terms of ABEP and P_{out} . Furthermore, it is also seen that a bigger relative improvement is provided by the optimization in the clear and subaperture architectures, compared to the obscured architecture. In summary, these results highlight the relevance of optimizing the optical system attending to communication performance parameters, instead of attending widely used optical imaging performance parameters (e.g., peak irradiance, Strehl ratio).

	Obscured	Clear	Subaperture
$\text{ABEP}_{\max(\bar{h})} / \min(\text{ABEP})$	46.5	5.7×10^2	4.3×10^2
$P_{\text{out},\max(\bar{h})} / \min(P_{\text{out}})$	77.2	9.8×10^4	1.7×10^4
$\max(\mathcal{L}_{10^{-6}}) / \mathcal{L}_{10^{-6},\max(\bar{h})}$	1.03	1.06	1.11

Table 6.2: Improvement provided by the optimization compared to the baseline field performance, given by the field that maximizes the average received power $\bar{h}$.

6.3.2. Seidel optical aberrations

To assess the effect of optical aberrations on intersatellite links, which will enable setting requirements for admissible optical aberrations on intersatellite terminals, the optimum outgoing beams computed in the previous section are perturbed with Seidel aberrations. Using the full model presented in Fig. 6.2, the effect of these aberrations is computed independently of each other. The results are shown in Fig. 6.6. Firstly, several qualitative differences can be discerned between the subaperture architecture (third row) and the rest. In the case of the former, as the symmetry of the transmitted beam is broken with respect to the telescope's optical axis, the coma aberrations that are defined over the whole aperture of the telescope do not have the same effect. That symmetry breaking, due to the non-axially symmetric transmit field, creates unequal behaviors of the horizontal and vertical comas as seen in Fig. 6.6(g)-(i).

In general, for all the architectures, coma aberration seems to be the one that has the highest negative impact on the performance of the system. This effect is expected as coma is the only Seidel aberration creating an apparent angle of arrival. Hence, coma aberration is the only one creating a systematic boresight pointing error of the transmitted beam. The fields in the transmitter aperture plane, far-field receiver plane, and the tracking sensor plane for the case of vertical coma aberration are shown in Fig. 6.7. In this figure, we can see more clearly the differences between the clear and subaperture architectures (Figs. 6.7(b) and (c), respectively). The Gaussian beam in the clear aperture architecture is truncated symmetrically around its axis, while in the subaperture case the truncation occurs in a non-symmetric way. We have considered the same diameter for the aperture in both architectures, and hence, the effective diameter achievable without clipping is smaller for the subaperture architecture. This

is why in the subaperture architecture, the limitation of a smaller effective aperture is mitigated by choosing a more collimated beam in the transmitter's aperture for the nominal case (compare the optimum ratios w_0/w_z in Fig. A.6 and A.7 in App. A). Furthermore, while the apparent angle of arrival is the same in both cases for the same optical aberration, the transmitted beam on the subaperture architecture is only affected by a local section of the full optical aberration.

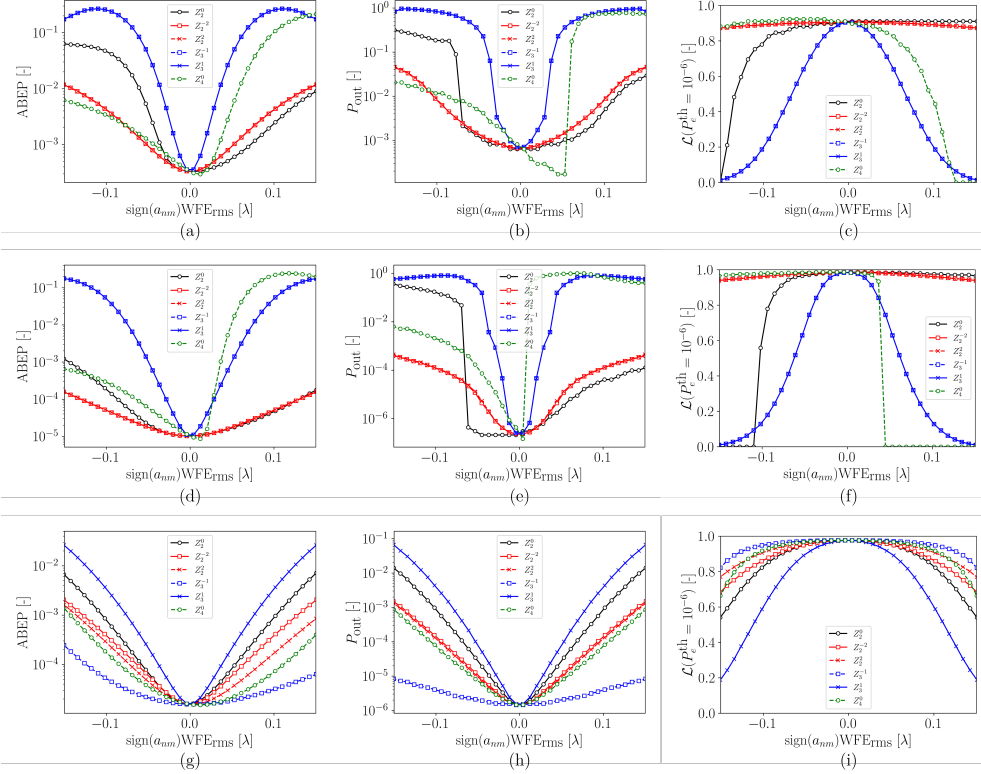


Figure 6.6: Effect of optical aberrations on the performance of a LEO-GEO intersatellite link. First, second, and third rows correspond to the obscured, clear, and subaperture architectures, respectively.

Furthermore, the effect of defocus aberration is analogous to changing the radius of curvature of the transmitted beam, and this in turn is equivalent to changing the beamwaist w_0 of the transmitted beam. As these parameters have been optimized as shown in the previous section, defocus has the same effect as traveling in w_z constant lines in Fig. 6.5(a)-(c). As the outgoing beam is optimized before any aberrations are applied, the effect of further defocus is to degrade the respective performance parameter as shown in Fig. 6.6. Finally, it is relevant to mention that the effect of a slight spherical aberration can be beneficial in some cases, as is shown for all the parameters in the obscured and clear aperture cases (rows one and two in Fig. 6.6).

This is in agreement with the findings that will be shown in Part III, in which a redistribution of the far-field irradiance, reducing the peak irradiance and transferring it towards the outside of the beam, can be beneficial for mitigating the pointing jitter (as will be seen in Part III). This is indeed the effect of spherical aberration [391].

Although the results presented for each of the aberrations applied independently provide insight into the physics behind this perturbation, in reality, the optical aberrations that will appear on telescopes due to thermo-mechanical loads are a combination of all these aberrations [372]. In order to evaluate the admissible values for the root mean square of the optical aberrations statistically, Monte Carlo simulations for combinations of all the Seidel aberrations have been considered in a similar manner as done by Kenny et al. for the LISA gravitational interferometry mission [399]. The results of these Monte Carlo simulations are shown in Fig. 6.8. These simulations are performed by randomly allocating to each aberration a wavefront error root mean square magnitude coming from a uniform distribution, in accordance with the experimental results presented in [372]. It can be seen that the least sensitive architecture of all is the subaperture architecture, in which both the percentiles and the mean remain the most unaffected by the increasing magnitude of the optical aberrations. This is due to the fact that the Zernike polynomials are defined over the whole aperture of the transmitter telescope, while the transmitted beam in the subaperture architecture only uses a small section of this aperture. Hence, the Seidel aberrations on the whole aperture basis are seen as similar to a combination of tip-tilt and astigmatism in the transmitted Gaussian beam basis. The results presented enable us to discern the effect of optical aberrations in the apparent angle of arrival and the far-field irradiance. Using the reliability $\mathcal{L}$ for the sake of the explanation, the apparent angle of arrival is the same in both the obscured and subaperture architectures. However, the effect of the optical aberrations is much more pronounced in the obscured case as seen in Fig. 6.8(c). This means that the alterations of the far-field irradiance generated by the optical aberrations have a determining role in the performance of the system. This effect can also be seen by comparing the clear and subaperture architectures in the same figure. We can see in Fig. A.3 of the App. A, that the difference in the apparent angle of arrival for the clear and subaperture cases is very similar. Therefore, the much higher sensitivity of the clear aperture architecture to the performance parameters shown in Fig. 6.8 can only be due to the effect of the Seidel aberrations on the transmitted beam. As for the obscured case, the transmitted beam in the clear aperture case is also subjected to full aberrations (see Fig. 6.6(a)-(b)). In contrast, the transmitted beam on the subaperture architecture is only affected by a localized section of the full aberrations (see Fig. 6.6(c)). By using simulations as the ones shown in Fig 6.8, the requirements for acceptable wavefront errors can be determined for optical terminals working as transmitters. However, when the terminal is also working as a receiver, the effects of optical aberrations in the received beam when coupling it to the detector also need to be considered.

For the sake of better understanding how the results presented in Fig. 6.8 can be used, an evaluation of the performance of the system is done under Maréchal's criterion. Maréchal's criterion is determined by assuring that the peak irradiance of the diffraction pattern of a uniform circular aperture under aberrations is reduced

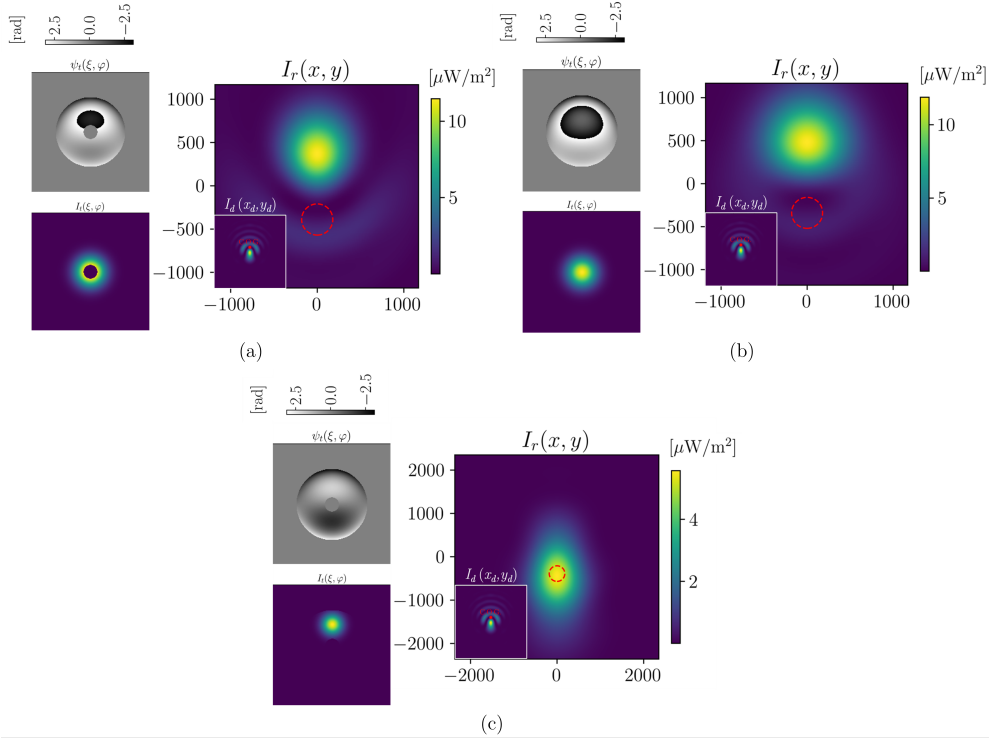


Figure 6.7: Fields in the case of vertical coma aberration with $WFE_{rms} = 0.15 \lambda$ for (a) obscured, (b) clear, and (c) subaperture architectures, respectively. The small figures on the left show the transmitter aperture phase and irradiance. The big figure shows the far-field irradiance in the receiver aperture planes along with a red circle enclosing the area of $< 1\sigma$ of the pointing jitter (the center of which is the computed static boresight error due to the apparent angle of arrival). The receiver is located at the origin in the receiver aperture plane. The snippet shows the tracking sensor irradiance field.

by less than 20%. This in turn yields a maximum allowable aberration given by $WFE_{rms} \leq \lambda/14$ [400]. This criterion is in principle, only useful for imaging optical systems, as it does not account for truncated Gaussian beams, stochastic pointing jitter, and captured power statistics. However, we can now evaluate the adequacy of Maréchal's criteria for an intersatellite optical communication system. By considering the effect that optical aberrations have on the reliability shown in Fig. 6.8, we can see that the average of the reliability in the clear architecture goes from 98.4% in the non-aberrated case to 72.4% in the limiting Maréchal condition. The latter architecture is the one that is most affected as the obscured and subaperture architectures go from 90.4% and 97.3% to 77.8% and 95.3%, respectively. By extending this analysis to the performance parameter of interest, we can assess the implications of imposing the classical diffraction-limited condition (Maréchal's criterion), commonly used in imag-

ing optics, on the design of optical communication terminals. All things considered, in order to evaluate the effect of optical aberrations on intersatellite optical links, it is paramount to use a complete end-to-end model that considers altogether these coupled phenomena.

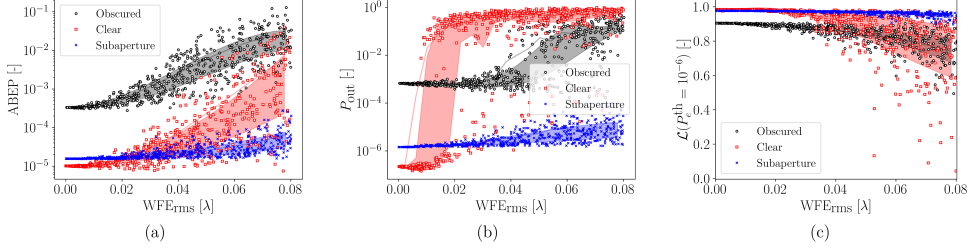


Figure 6.8: Effect of optical aberrations in the performance of a LEO-GEO intersatellite link given by Monte Carlo simulations of the Seidel aberrations. The circle markers show the Monte Carlo simulations, while the mean is given by a solid fitted curve and the 25% and 75% percentiles are bounding the filled areas.

6.4. Conclusions and further work

We have presented in this chapter a complete framework that models the coupled effects of the transmitter pointing jitter and optical aberrations on intersatellite free-space optical communications. The presented model enables the evaluation of the effect of these perturbations on the communication performance of the system, i.e., average bit error probability, probability of outage, and reliability. Three different architectures have been analyzed, namely the obscured, clear, and subaperture architectures. Firstly, an optimization of the transmitted Gaussian beam considering the transmitter pointing jitter and the finite aperture effects (without optical aberrations) has been performed. This optimization shows that the optimum far-field irradiances depend on the communication parameter chosen as the target. Indeed, non-Gaussian beams generated by using the aperture truncation effects of the transmitter telescope are beneficial in some of these optimizations. Finally, the full framework developed has been used to evaluate the effect of the optical aberrations by perturbing the previously obtained optimal beams with different Seidel aberrations. It has been found that the effect of coma is extremely detrimental as a result of the apparent angle of arrival created. Finally, given that in practical scenarios these aberrations will appear altogether, Monte Carlo simulations of combinations of Seidel aberrations are presented to statistically determine the impact of optical aberrations on system performance. The latter will help determine the requirements for admissible optical aberrations in intersatellite free-space optical communication terminals.

Further investigation into the coupled effects of optical aberrations and receiver angle-of-arrival at the receiving terminal is recommended as a future research direction. In addition, the results presented here could be extended to other applications, such as space-to-Earth downlinks –accounting for atmospheric effects and alternative

modulation schemes— as well as quantum communications. In this context, the impact of Gaussian beam truncation on quantum communication links will be specifically examined in §8.

Chapter 7

Satellite-assisted entanglement distribution

Modelling single-photon sources, atomic memories, pointing jitter, and free-space losses

One must always do what one really cannot.

Niels Bohr

Similarly to satellite classical optical communication channels, satellite quantum communication channels are also affected by imperfections of the system. Building upon the phenomena discussed in §4, this chapter quantifies the impact of several key imperfections within a quantum communication system. We propose a satellite-based quantum repeater architecture for distributing entanglement through large distances. In this scenario, we analyze the effects of imperfections in trapped atomic qubits –which serve both as single-photon sources and as quantum memories– and the influence of pointing jitter in the optical links. The model developed allows us to estimate that high-rate and high-fidelity entanglement distribution can be achieved over inter-continental distances. Specifically, our results indicate that entanglement distribution at rates on the order of 100 Hz and over thousands of kilometres is feasible, while requiring orders of magnitude fewer memory modes than conventional architectures relying on probabilistic optical Bell-state measurements.

The content of this chapter is based on the Ref. [401] Dominguez, V., Badás M. et al. "Satellite-assisted quantum communication with single-photon sources and atomic memories". Physical Review Research 8, 013099 (2026). The author of the thesis contributed to developing the mathematical model for evaluating the performance of the system.

7.1. Introduction

The implementation of a quantum internet opens a range of new opportunities for secure communication [28, 30, 402], enhanced sensing networks [357–359], and distributed quantum computing [403]. To exploit these opportunities in applications such as protecting and optimizing large-scale power distribution networks [404] or probing fundamental constants and geodesy [405], it is necessary to extend the range of quantum networks to distances of thousands of kilometres.

To carry out quantum communication over large distances, photons seem like the best choice due to their low interaction with the environment compared to other particles §4. However, transmission loss of an optical quantum signal cannot be compensated with standard classical amplification techniques due to the quantum no-cloning theorem [406]. Instead, quantum repeaters have been proposed, where the total distance is divided into smaller segments over which direct transmission is feasible [239, 286–288, 290]. The segments are then combined either through quantum teleportation [291, 407] or quantum error correction [408, 409] at the repeater nodes to enable faithful transmission over the total distance.

For optical fiber-based quantum repeaters, the photon transmission probability between the repeater nodes decreases exponentially with distance, and hundreds of repeater nodes are required to cover distances at the continental scale. Alternatively, fiber-based connections can be replaced with satellite-assisted free-space optical links where transmission decreases only polynomially with the distance [234, 258, 286–289, 297]. For continental scales, this can reduce the required number of repeater nodes significantly, compensating for the arguably higher cost of space-based quantum repeater nodes.

Among the different applications of quantum communication, quantum key distribution (QKD) has already been demonstrated with the Micius and Jinan satellites performing as a trusted node for distances ranging from 500 to 7000 km [246, 308, 310]. However, assuming that the satellite is a trusted node poses a security concern (see §4.2.2). In a more advanced approach, entanglement-based QKD was achieved without relying on a trusted node, spanning distances of up to 1,200 km [410]. In this case, the satellite served as a source of entangled photon pairs, paving the way for future entanglement-based quantum networks. Reaching larger distances by direct transmission, however, requires a high-orbit satellite due to the limitation of the line of sight of the ground receivers. This substantially increases the transmission loss, making high-rate quantum communication extremely challenging. Alternatively, multiple low-orbit satellites can operate in a quantum repeater architecture to efficiently compensate for the transmission loss and ensure line of sight between distant locations [239, 287–289].

Previous theoretical work on satellite-based quantum repeaters assumes the use of probabilistic optical Bell state measurements (BSM) [287], which requires multiplexing of thousands of memory modes in order to reach high-rate communication over global distances. Additionally, the availability of on-demand entangled photon pair sources is often assumed [286, 287, 289, 297], which remains an outstanding technological challenge [314]. Promising candidate systems are solid-state semiconductor quantum dots [411], which require cryogenic temperatures to function, or multiplexing

of spontaneous parametric down-conversion (SPDC) sources [412] and single-photon sources [413].

This chapter proposes a satellite-assisted quantum repeater protocol based on trapped individual atomic qubits. Individually trapped alkali atoms can both function as efficient single-photon sources [414] and atomic memories due to their long coherence times [415, 416]. Furthermore, they enable *nearly deterministic* Bell state measurements through Rydberg-mediated two-atom gates [417–419] and the possibility of scaling to hundreds of qubits per repeater node [420, 421]. Additionally, laser-cooling of the atoms is sufficient to ensure long coherence times, which avoids the need for costly cryogenic technology in space.

A detailed analytical model of the repeater protocol is developed, which considers the main imperfections of both the quantum hardware and the optical link budget. In contrast to Monte-Carlo-based simulations [289], the analytical model allows us to efficiently simulate long chains of quantum repeater nodes. We show that, for realistic satellite parameters and quantum hardware errors, we can get high-fidelity (≥ 0.9) entanglement distribution rates of 100 Hz over continental distances of up to 1500 km using 5 satellites with less than 200 quantum memory modes per satellite.

7.2. Architecture

We consider the simulation of a down-link scenario where photons are sent from the end-satellites to the ground stations, which is more robust to atmospheric turbulence than the reverse up-link scenario [288, 422]. A high-level sketch of a 3-node repeater is shown in Fig. 7.2(a). The quantum repeater consists of different types of satellites. Some satellites act as transmitters, sending photons to their neighbouring satellites or to the ground station. Others are receivers that collect the photons sent by the transmitter satellites. Both types of satellites are equipped with atomic quantum memories and act as quantum repeater nodes, as shown in Fig. 7.2(b). When adding more satellites, an odd number of satellites is always ensured to maintain the structure of the receiver and transmitter satellites.

To distribute entanglement between the ground stations, atom-photon entanglement is generated at the repeater stations through pulsed excitation of the atoms (see Fig. 7.1). The generated photons are sent from the transmitter nodes to the receiver ones, where the atom-photon entanglement is swapped to atom-atom entanglement by means of a probabilistic photonic BSM as in Fig. 7.2(b). Once entanglement between two neighbouring elementary links has been successfully heralded, a nearly deterministic entanglement swap operation is performed through Rydberg-mediated atom-atom interactions. If the entanglement generation is successful in all elementary links, the two ground states will share an entangled pair.

7.2.1. Entanglement generation

For the generation of atom-atom entanglement over each elementary link, we consider a ‘two-click’ protocol [423], where each atom is entangled with a time-bin encoded single-photon through pulsed excitation. This is then swapped to atom-atom entanglement by a probabilistic photonic BSM as shown in Fig. 7.2(b). We focus on this scheme

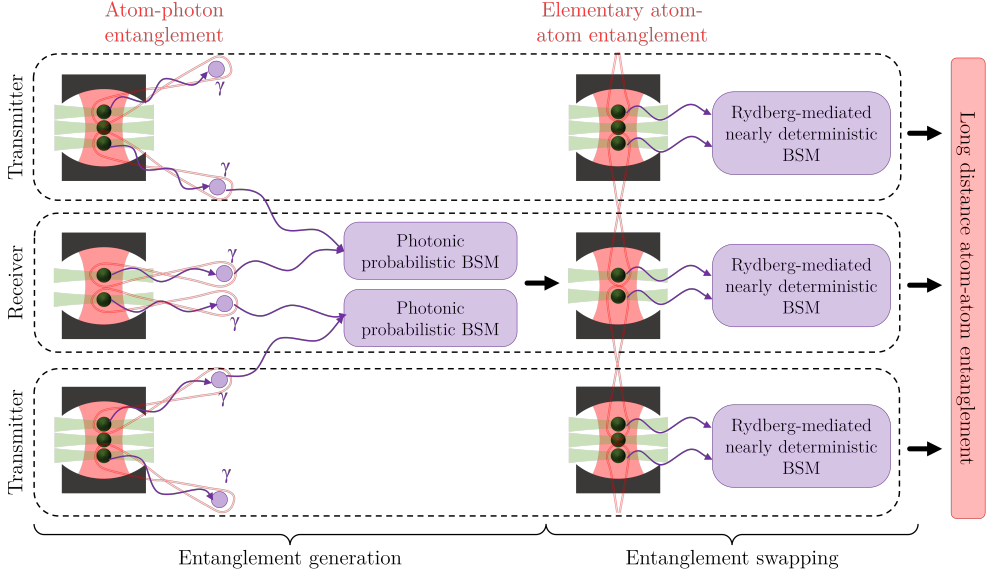


Figure 7.1: Overview of the proposed architecture and steps. Only three satellites. *i.e.*, two elementary links, are illustrated. γ refers to the photons, and the infinite loops represent entanglement.

since it only requires phase stability on the time-scale of the time-bin separation and does not have the fundamental trade-off between rate and fidelity of ‘single-click’ protocols [424]. Furthermore, we consider a scenario where the entanglement swapping (from atom-photon to atom-atom entanglement) happens at a heralding station placed on the receiving satellite. The loss will thus only affect one of the photons, the other being generated in the heralding station.

We now describe the steps of the entanglement generation in more detail. Fig. 7.3(a) shows how individual atoms are trapped with optical tweezers inside macroscopic, near-concentric optical cavities [420, 421, 425]. Throughout this chapter, we will focus on an implementation with Rubidium (Rb) atoms, though other Alkali atoms such as Cesium could also be used. The closed optical transition between the $5S_{1/2}$, $|F = 2, m_F = 2\rangle$ hyperfine ground state and the $5P_{3/2}$, $|F' = 3, m_{F'} = 3\rangle$ excited state in ^{87}Rb is used for atom-photon entanglement generation. Here, F denotes the total atomic angular momentum, while m_F is the magnetic quantum number. This specific transition enables the generation of atom-photon entanglement, whereby the internal spin state of the atom becomes entangled with the temporal mode of the emitted photon following pulsed optical excitation. For simplicity, we first describe this process for a single atom and then discuss how to perform the operation for a collection of atoms. First, the atom is prepared in a superposition of the spin states,

$$|\phi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle).$$

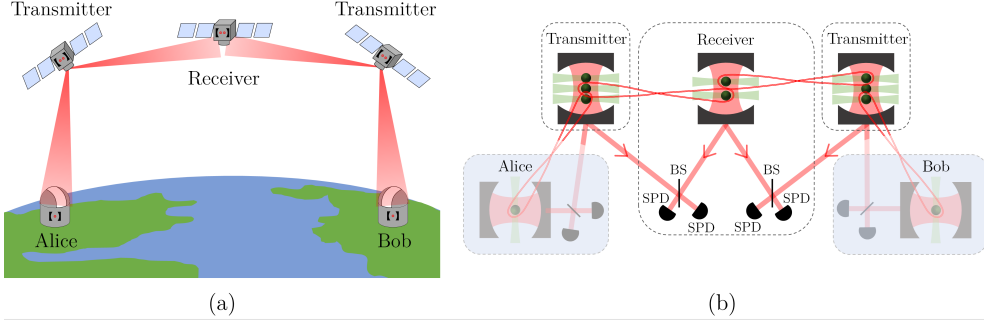


Figure 7.2: Schematic diagram of the proposed scheme. (a) We consider a downlink scenario where the photons are transmitted from the end-satellites to the ground stations. Each of the satellites contains quantum memories, and entanglement swaps are performed on the satellites to link the segments of the repeater (satellite-satellite or satellite-ground stations) once entanglement has been established. (b) Probabilistic photonic BSM after entanglement generation. The atom-photon entanglement generated through the emission of a photon from an atom is swapped to atom-atom entanglement within a receiver satellite by means of a linear optics Bell state measurement. This is done using a 50/50 beam splitter (BS) and single-photon detectors (SPD). As shown in the figure, transmitter telescopes typically need more memory modes than the receiver telescope because we are operating in a low transmission probability regime.

by means of standard optical pumping and two-photon Raman driving [415, 426]. We imagine that $|0\rangle = |F = 1, m_F = 1\rangle$ and $|1\rangle = |F = 2, m_F = 2\rangle$ in the $5^2S_{1/2}$ ground state manifold. Next, a short optical π -pulse¹ is applied to induce the transition, $|1\rangle \rightarrow |ex\rangle$, where $|ex\rangle = |F' = 3, m_F' = 3\rangle$ in the excited $5^2P_{3/2}$ manifold. This π -pulse is a precisely timed laser pulse that fully transfers the atomic population from the ground state $|1\rangle$ to the excited state $|ex\rangle$. The excited state will subsequently decay back to $|1\rangle$ by emission of an early cavity photon, $|e\rangle$. Next, two-photon Raman driving is used to coherently swap the populations of the ground states, $|0\rangle \leftrightarrow |1\rangle$; this process uses two lasers detuned from the excited state to induce a direct, controllable transition between the ground states without populating the excited state. After this, a second optical π -pulse is applied, resulting in the emission of a late photon, $|l\rangle$, if the atom is in the $|1\rangle$ state. Ideally, this procedure results in the spin-photon entangled state

$$|\psi_{\text{sp}}\rangle = \frac{1}{\sqrt{2}} (|0\rangle|e\rangle + |1\rangle|l\rangle). \quad (7.1)$$

When performing this operation on a string of atoms, it is important to prevent the emission from one atom from interfering with another. To this end, we propose to operate in a sequential manner where all atoms are initially prepared in the state $(|0\rangle +$

¹A π -pulse is a laser pulse whose time-integrated coupling strength (pulse area) equals π , meaning it drives a complete coherent transition between two quantum states (well-defined phase relationship between the quantum states). Physically, this is achieved by carefully controlling the pulse's intensity and duration.

$|1\rangle\rangle/\sqrt{2}$ and then addressed sequentially with the optical π -pulse for the generation of atom-photon entanglement. An additional laser at 1530 nm addressing the transition between the $5^2P_{3/2}$ and $4^2D_{3/2}$ excited manifolds is applied to all atoms except the one subject to the optical π -pulse. Specifically, this laser will couple the excited $|ex\rangle$ to the $|ex'\rangle = |F'' = 3, m_{F''} = 3\rangle$ hyperfine level of the $4^2D_{3/2}$ manifold. This is done to effectively shift the $|1\rangle \leftrightarrow |ex\rangle$ transition out of resonance for the other atoms since the excited dressed states will be detuned from the cavity resonance by $\pm\hbar\Omega$, where Ω is the Rabi frequency² of the laser-driven $|ex\rangle \leftrightarrow |ex'\rangle$ transition [427]. Having $\Omega \gg g\sqrt{N}$, where N is the number of atoms in the cavity, g is the single-photon Rabi frequency of the cavity-coupled $|1\rangle \leftrightarrow |ex\rangle$ transition, ensures that there is effectively no coupling of the atoms to the cavity field. In this way, atom-photon entanglement can be attempted with each atom sequentially as described above.

The photons collected from the cavity are transmitted to either one of the ground stations or another satellite, depending on the specific location of the satellite in the repeater chain. In both cases, a probabilistic photonic BSM is performed at the destination (receiver) to herald atom-atom entanglement, as shown in Fig. 7.2(b). The latter is carried out with a 50/50 beam splitter and single-photon detectors following the scheme of Ref. [423]. If the BSM is successful, meaning we have measured an early and a late photon, we have accomplished entanglement in an elementary link, i.e., satellite-satellite or satellite-ground entanglement.

$$|\psi_{\text{sp}}\rangle \otimes |\psi_{\text{sp}}\rangle \xrightarrow{\text{BSM}} |\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle). \quad (7.2)$$

The phase of the superposition is determined by which detectors on the BSM record the photons. Since this information is known, it is possible to change the phase with local qubit operations, if necessary.

7.2.2. Entanglement swapping

Following the successful atom-atom entanglement generation of neighbouring links in the setup, we carry out an entanglement swapping between the entangled atoms of the different links. Previous satellite-based quantum repeater schemes have considered probabilistic photonic BSM similar to the entanglement generation step [286–288] to achieve this. However, the downside of performing a probabilistic photonic BSM is that it has an intrinsic failure probability of 50%, which has a detrimental effect on the overall rate of the repeater. To circumvent this, we consider nearly-deterministic BSM between pair of atoms through the Rydberg interaction, which relies on the strong dipole-dipole forces between atoms excited to Rydberg states (high-energy level states), allowing for fast and high-fidelity entangling operations. To do so, the atoms are spatially rearranged. This can be performed on a timescale of milliseconds [415, 420], which is on the order of the communication time between segments for the parameters considered below. This is different from the situation in the entanglement generation step, where the typical time scale of the local operations is on the

²The Rabi frequency Ω , quantifies the rate of coherent oscillations between two quantum states under a resonant driving field.

microsecond scale for typical experimental parameters [420, 421] and thus negligible compared to the communication time.

The nearly deterministic BSM is implemented through a controlled-Z (CZ) gate by means of the dipole-dipole interactions of Rydberg states in the atomic memories [417, 428, 429]. CZ is a two-qubit gate that applies a phase flip to the $|11\rangle$ state while leaving the other basis states unchanged. Since the originally proposed Rydberg-mediated schemes in Ref. [428], there have been a number of further developments to increase the performance of Rydberg-mediated two-atom gates [417, 429], leading to experimentally reported gate fidelities exceeding 99% [419]. To illustrate the basic idea of how the Rydberg interaction allows for a two-atom controlled phase gate, we focus, however, on one of the original schemes of Ref. [428] illustrated in Fig. 7.3(b) for simplicity. First, a π pulse is applied to the control atom, which makes the transition $|1\rangle \rightarrow |r\rangle$, where $|r\rangle$ is a Rydberg state with a high principal quantum number ($n \approx 60$). Then, a 2π -pulse is applied to the target atom. If the control atom is in state $|0\rangle$, this pulse will make the transition $|1\rangle \rightarrow |r\rangle \rightarrow -|1\rangle$ on the target atom. However, if the control atom is in the Rydberg state, the Rydberg interaction will shift the transition out of resonance such that the target atoms remain essentially unperturbed by the pulse. A final π -pulse brings the population of the control atom back to $|1\rangle$ and concludes the gate. This operation amounts ideally to a controlled phase gate between the atoms, which is sufficient to perform a Bell state measurement.

To realise the nearly deterministic Bell measurement, it is necessary to apply a Hadamard gate on the target atom before the CZ gate and a Hadamard gate on the control atom after the gate, which can be done with two-photon Raman driving [415, 426]. The Hadamard gate is a single-qubit operation that maps $|0\rangle \rightarrow (|0\rangle + |1\rangle)/\sqrt{2}$ and $|1\rangle \rightarrow (|0\rangle - |1\rangle)/\sqrt{2}$, effectively creating or undoing superpositions necessary for Bell-state measurements. The qubit states are then measured by first driving the closed $|1\rangle \leftrightarrow |ex\rangle$ transition and collecting the transmitted light, which detects if the atom is in the $|1\rangle$ state. If no light is detected, the atom is either in the $|0\rangle$ state or it could have been lost from the trap. The latter can result from the rearrangement step, the two-atom gate, or simply from imperfect vacuum. To herald whether the atom was lost, a Raman drive is applied to bring the population from $|0\rangle$ to $|1\rangle$ followed by a second detection. If light is collected, the atom is in the $|0\rangle$ state, while if no light is collected, the atom is assumed lost. We note that this means that the Bell measurement is, in fact not deterministic since there is a non-zero probability that the atom was lost, in which case the operation fails.

7.3. Model

To evaluate the performance of the repeater protocol, we model a number of imperfections at both the quantum hardware level as well as in the optical transmission budget. Below, we describe the imperfections in the spin-photon entanglement generation, the losses due to the free-space transmission, the losses in the probabilistic photonic BSM, quantum memories, and entanglement swapping operation. A summary of the different imperfections considered is shown in Fig. 7.4.

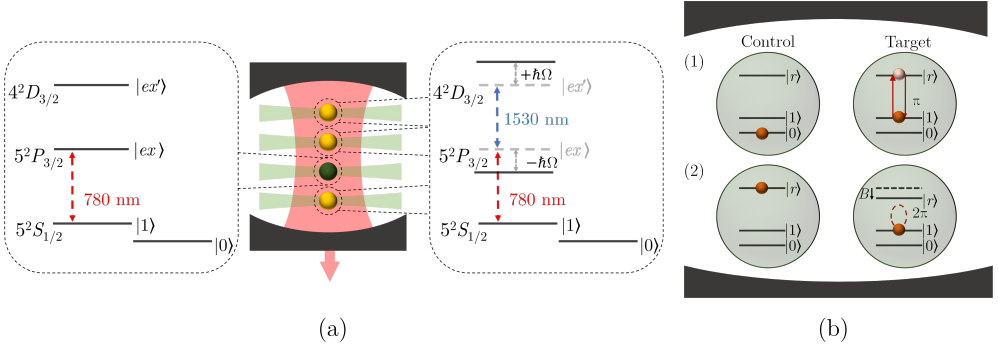


Figure 7.3: Individually trapped atoms in a cavity as the main hardware. (a) The quantum memories chosen are Rubidium (Rb) atoms in a cavity, which also work as transmitters. With a laser resonant with the $|1\rangle \rightarrow |ex\rangle$ transition, the atom is excited to the $|ex\rangle$ state from which it will decay back to the $|1\rangle$ state with the emission of a photon. To selectively address only one atom in an entangling attempt, the other atoms are shifted out of resonance through strong driving of the $|ex\rangle \rightarrow |ex'\rangle$ transition. (b) Nearly deterministic BSM. After the elementary links are entangled, a nearly deterministic Bell state measurement is carried out to distribute entanglement between Alice and Bob. The latter is performed by applying a CNOT gate between the atoms and a Hadamard gate in the control atom. To perform the 2-qubit gate, we exploit the well-known Rydberg blockade: The $|1\rangle \rightarrow |r\rangle$ transition of the target atom is resonant with our driving field if (1) the control atom is in state $|0\rangle$ but (2) shifted out of resonance by the Rydberg interaction if the control atom is in state $|r\rangle$.

7.3.1. Spin-photon entanglement

In the generation of spin-photon entanglement, we consider undesired two-photon emissions and imperfect coupling to the cavity mode. Two-photon emissions and imperfect coupling to the cavity mode are captured by considering the following state

$$\rho_{\text{emit}} = \alpha_e |\psi_{\text{sp}}\rangle \langle \psi_{\text{sp}}| + \beta_e |\psi_{\text{multi}}\rangle \langle \psi_{\text{multi}}| + \eta_e \rho_{\text{deph1}} + \gamma_e \rho_{\text{deph2}}, \quad (7.3)$$

where $|\psi_{\text{sp}}\rangle$ is the desired state given in Eq. 7.1, obtained with probability α_e . The state $|\psi_{\text{multi}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|2e\rangle + |1\rangle|2l\rangle)$ represents undesirable two-photon emissions where both photons are collected, which we obtain with probability β_e . We neglect any coherence between the state $|\psi_{\text{sp}}\rangle$ and $|\psi_{\text{multi}}\rangle$ for simplicity since it will have a negligible effect in the later parts of the protocol, as detailed below. The dephased states $\rho_{\text{deph,1}}$ and $\rho_{\text{deph,2}}$, represent the loss of one (or multiple) photon(s) to the environment (e.g., through emission into non-collected modes), which leaks information about the spin state. Specifically, we define $\rho_{\text{deph,1}} = (|0\rangle\langle 0| \otimes |e\rangle\langle e| + |1\rangle\langle 1| \otimes |l\rangle\langle l|)/2$ and $\rho_{\text{deph,2}} = [(|0\rangle\langle 0| + |1\rangle\langle 1|) \otimes |0_{\text{ph}}\rangle\langle 0_{\text{ph}}|]/2$ —where $|0_{\text{ph}}\rangle$ is the vacuum photon state. The probability of getting a dephased state with a photon in the collected mode is thus η_e , while the probability of having a dephased state with zero photons collected is γ_e . The estimation of the probabilities as a function of the cavity collection efficiency and the probabilities of emitting different photon states can be found in App. B.1.

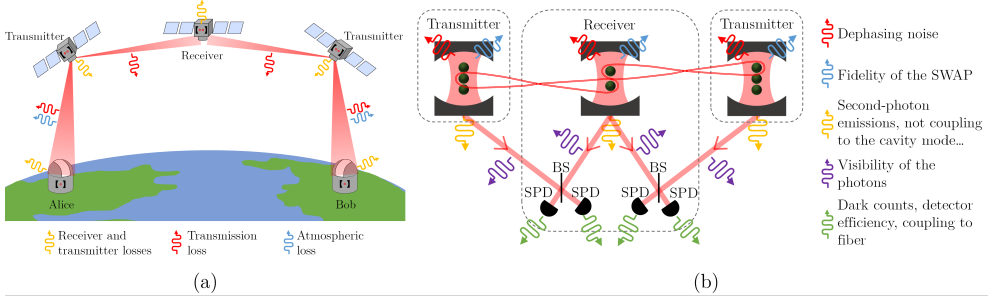


Figure 7.4: Errors considered in the protocol. (a) Schematic representation of the transmission losses. In the satellite-satellite link, we take into account transmission losses due to the propagation of the Gaussian beam and the pointing jitter of the emitter. In the satellite-ground station link, we also consider the effect of the atmosphere, i.e., the Rayleigh scattering due to molecules. Additionally, losses originating in the terminals are also added to the model. (b) Schematic representation of the quantum setup used and possible errors that can happen. Two-photon emissions and imperfect coupling to the cavity mode are taken into account when modelling the atom-photon entanglement generation. Dark counts, detector efficiency, and coupling to the fiber are included as imperfections in the Bell state measurements carried out in the receivers. Non-perfect visibility of the photons involved in the Bell state measurement is also included. Finally, we include dephasing of the atoms and imperfections of the entanglement swapping operation.

7.3.2. Free-space transmission

The free-space and atmospheric loss is modelled as a fictitious beam splitter with transmission p_T determined from the optical link budget. Using the beam splitter model, the effect of the free-space transmission on a single-photon state can be described as

$$\hat{a}_{\text{ph}}^\dagger |0_{\text{ph}}\rangle |0_e\rangle \rightarrow \sqrt{p_T} \hat{a}_{\text{ph}}^\dagger |0_{\text{ph}}\rangle |0_e\rangle + \sqrt{1-p_T} \hat{a}_e^\dagger |0_{\text{ph}}\rangle |0_e\rangle, \quad (7.4)$$

where subscript 'e' denotes the loss mode. So, for example, for the state $|\psi_{\text{sp}}\rangle$ we will have that either the state is transmitted with probability p_T , or that the photon is lost with probability $1-p_T$ and the atom is left in a dephased state $\rho_{\text{deph},2}$. Building upon the state given in Eq. (7.3), the state reaching the receiver after free-space transmission can be written as

$$\rho_{\text{trans}} = \alpha_T |\psi_{\text{sp}}\rangle \langle \psi_{\text{sp}}| + \beta_T |\psi_{\text{multi}}\rangle \langle \psi_{\text{multi}}| + \eta_T \rho_{\text{deph},1} + \gamma_T \rho_{\text{deph},2}, \quad (7.5)$$

where the probabilities α_T , β_T , γ_T and η_T are given by a combination of the probabilities considered for the spin-photon entanglement and the free-space transmission losses (see App. B). To compute the photon transmission probability p_T , we consider the photon states to be given by the fundamental Gaussian spatial mode. This mode is the fundamental solution to the free-space paraxial wave equation (see §2.1.2). Furthermore, this is the mode transmitted through single-mode fibers, which are sometimes used to guide light from the source to the optical terminal. Experimental campaigns

strongly agree with this Gaussian shape for describing single-photon states, even after atmospheric propagation in a downlink [307]. To model the free-space transmission losses, the effect of the transmitter pointing jitter is also considered. The latter is assumed to be described by a radially varying Rayleigh probability distribution function. Furthermore, in satellite-to-ground links, we include the atmospheric effects of Rayleigh scattering and beam widening due to turbulence (although the latter are sometimes negligible, see App. B). Finally, internal losses in the terminals, due to non-ideal operation of the optical elements (i.e., absorption in lenses and mirrors) are included.

In a link between two terminals, the beam describing the photon states diverges, creating a far-field intensity pattern in the aperture plane of the receiver. In the single-photon regime, the probability density function of finding the photon on the xy -plane of the receiver (see §2.1.2 and §5.5 for a detailed explanation of this), considering a given link distance z and a fundamental Gaussian beam profile, is

$$p_g(x, y) = \frac{2}{\pi w(z)^2} \exp\left(\frac{-2(x^2 + y^2)}{w(z)^2}\right), \quad (7.6)$$

where $w(z)$ is the beam width given by Eq. (6.1). The transmitter pointing jitter will displace the center of the Gaussian beam from the center of the receiver optical axis. For a given pointing error (x_0, y_0) , the probability of capturing the photon is given by the convolution

$$p(x_0, y_0) = \iint_{-\infty}^{\infty} p_g(x, y) \mathcal{A}(x - x_0, y - y_0) dx dy, \quad (7.7)$$

where $\mathcal{A}(x, y)$ is the pupil aperture function of the receiver given by

$$\mathcal{A}(x, y) = \begin{cases} 1 & x^2 + y^2 \leq r_R^2 \\ 0 & \text{else} \end{cases}, \quad (7.8)$$

with r_R being the radius of the receiver and (x_0, y_0) being the displacement of the optical axis of the receiver with respect to the optical axis of the beam. As discussed in §6, the transmitter pointing jitter is usually characterized by a Gaussian probability density function for x_0 and y_0 of the form [151]

$$f_{PJ}(x_0, y_0) = \frac{1}{2\pi\sigma_{xy}^2(z)} \exp\left(-\frac{x_0^2 + y_0^2}{2\sigma_{xy}^2(z)}\right), \quad (7.9)$$

where $\sigma_{xy}(z)$ is the standard deviation of the displacement. The displacement of the beam in the xy plane is determined by the transmitter angular pointing error, θ . Under the small angle approximation, and considering that both the azimuthal and elevation angular errors are statistically equal, the standard deviation is $\sigma_{xy}(z) \approx \sigma_\theta z$, where σ_θ is the standard deviation of the angular transmitter pointing error.

For the rate analysis, it is assumed that the probability of receiving k out of N_{mem} photons in an elementary link follows a binomial distribution (see §7.3.6), using

the averaged probability $\bar{p} = \iint p(x_0, y_0) f_{XY}(x_0, y_0) dx_0 dy_0$. This approximation is valid in the regime where we are dominated by beam divergence rather than pointing jitter, i.e., when $w(z) \gg \sigma_{xy}$. For long distances, $z \gg 1$, hence $w(z) \approx w_0 z / z_R$, where $z_R = \pi w_0^2 / \lambda$ is the Rayleigh distance. Since $\sigma_{xy}(z) \approx \sigma_\theta z$, it follows that by choosing a small enough beamwaist, w_0 , the condition $w(z) \gg \sigma_{xy}$ is ensured. In our simulations, we have assumed a pointing jitter of $\sigma_\theta = 0.47 \mu\text{rad}$ in the transmitter, consistent with the Micius satellite [410], and light emitted from the Rb atoms has a wavelength of $\lambda = 780 \text{ nm}$. A beamwaist of $w_0 = \lambda / (10\pi\sigma_\theta) = 0.054 \text{ m}$ is chosen, which ensures that $w(z)/\sigma_{xy} = 10$ in the simulations. We note that since the value of w_0 is much smaller than the smallest transmitter's radius $r_T = 0.15 \text{ m}$ considered for the Micius-like satellite, the effect of clipping in the transmitter's telescope is negligible.

To check the validity of the binomial approximation for $w(z)/\sigma_{xy} = 10$, we compare the true average probability distribution of a link

$$\overline{\text{Pr}}(k; N_{\text{mem}}) = \int \text{Pr}(k; N_{\text{mem}}, p(x_0, y_0)) f_{\text{PJ}}(x_0, y_0) dx_0 dy_0, \quad (7.10)$$

with a binomial distribution $\text{Pr}(k; N_{\text{mem}}, \bar{p})$. The two distributions in the limiting cases (shortest and largest elementary link distance considered) are plotted for specific values in Figs. 7.5. The difference in mean and standard deviation is shown in Figs. 7.6. The error in the mean is negligible ($< 10^{-10}$) and the standard deviation error is between 2% and 14% of the mean. The skewness for both distributions was found to be $\lesssim 0.4$ and thus non-significant.

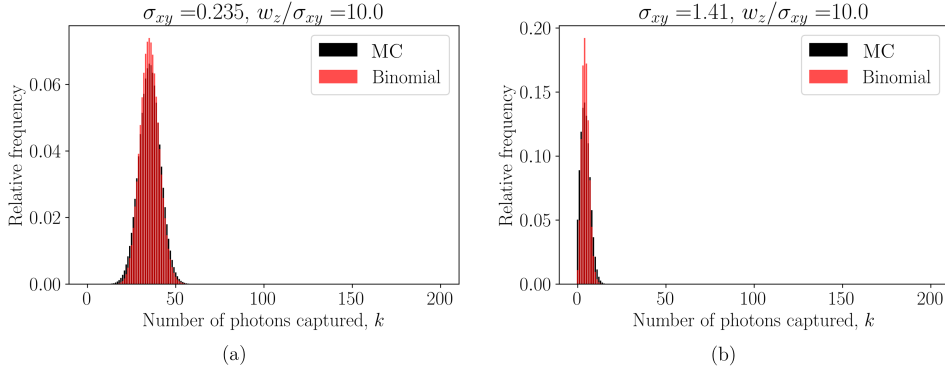


Figure 7.5: Histograms corresponding to the number of photons captured ($\sigma_\theta = 0.47 \mu\text{rad}$, $r_T = 0.5 \text{ m}$, $r_R = 0.5 \text{ m}$ and $N_{\text{mem}} = 200$). In black is the histogram obtained through Monte Carlo simulations, and in red is the approximate binomial distribution (with the same average $\bar{k}$), for (a) $z = 500 \text{ km}$ and (b) $z = 3000 \text{ km}$.

7.3.3. Probabilistic optical Bell state measurement

After the entangled photon is sent from the transmitter to the receiver, a probabilistic photonic BSM is performed to obtain atom-atom entanglement across the elementary

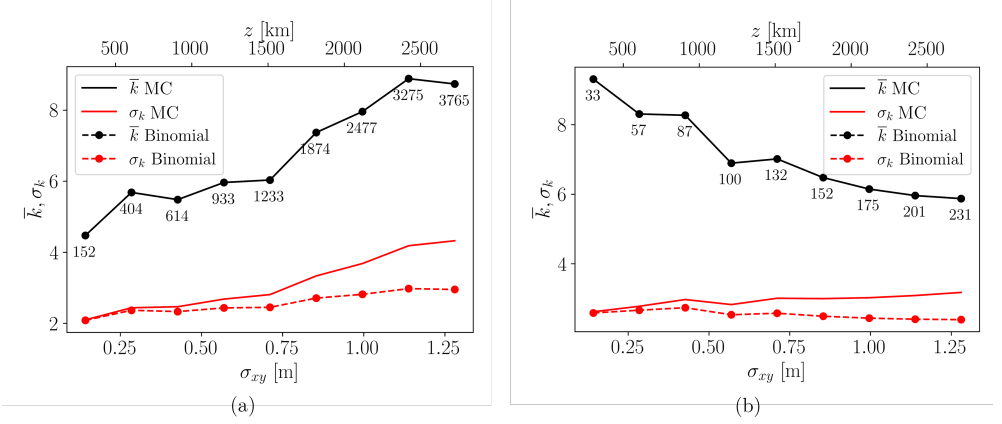


Figure 7.6: Values for mean $\bar{k}$ and standard deviation σ_k of the distributions of the number of photons captured for different distances between intersatellite elementary links for the (a) Micius satellite and (b) upgraded satellite. The numbers below the mean value are the number of memory modes considered for each point. Values are for elementary link distances and the number of memory modes obtained from the $R = 10$ Hz in Fig. 7.7(a). Note that the error in the mean is negligible, resulting in overlapping curves for the binomial and exact distribution.

links. The BSM is performed with a 50/50 beam splitter, and we assume that photonic interference only occurs for the case where the two input states are of the form $|\psi\rangle$, i.e., that there is no photonic interference for the error terms in Eq. (7.5). The photons interfering may not be perfectly indistinguishable, which is accounted for by including a non-unity visibility. Physically, the visibility quantifies the degree of indistinguishability between the photons, with reduced visibility indicating partial distinguishability—and thus reduced interference—arising from effects such as timing jitter, spectral mismatch, polarization misalignment, and imperfect spatial mode overlap. The efficiency of the detector and the coupling losses from free-space to fiber are also included in the link budget. Moreover, dark counts can generate a "click" in the detectors, despite no photon being transmitted, which we include with a non-zero dark count probability.

In the case of input state $|\psi\rangle \otimes |\psi\rangle$, the output state of the beam splitter is given by the superposition of states resulting from the different combinations of the input states. Post-selecting on detecting an early and a late photon, we will project the atoms into an entangled state of the form $|\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle)$. In App. B, the explicit expression of this state is shown, in which the probability of successful projection is one-half in the ideal case. We assume that the rest of the possible combinations between input states do not result in photonic interference as explained above. As shown in App. B, input states $|\psi_{\text{multi}}\rangle \otimes |\psi_{\text{multi}}\rangle$ in the beam splitter will result in a projection into states $|01\rangle$ or $|10\rangle$ by heralding on an early and late detection, with probability one half.

So far, we have neglected the effect of dark counts in the detectors. The probability of this happening is p_{dark} , i.e., a detector will click when receiving vacuum input with probability p_{dark} . In principle, dark counts could affect the outcome of the Bell

measurements when the incident fields contain the desired number of photons, as considered above. However, since $p_{\text{dark}} \ll 1$ for realistic detectors and the probability of receiving two photons is also very small, we can neglect higher-order terms such as the cases where we receive two photons and have a dark count. However, we do need to include the effect of dark counts for terms with only one or no photons arriving at the receiver station—the combinations $\{|\psi\rangle\langle\psi|, \rho_{\text{deph},2}\}$, $\{\rho_{\text{deph},1}, \rho_{\text{deph},2}\}$, and $\{\rho_{\text{deph},2}, \rho_{\text{deph},2}\}$. For the first two combinations, the probability of a false success due to dark counts will be p_{dark} to lowest order, and for the third combination, it will be p_{dark}^2 . For all combinations, the resulting state of the atoms will be an equal mixture of the states $|01\rangle, |10\rangle, |00\rangle$, and $|11\rangle$.

Collecting all possible combinations of input states, the state after the beam splitter is the following:

$$\alpha_{\text{BS}} \mathcal{V} |\psi\rangle\langle\psi| + (\beta_{\text{BS}} + \alpha_{\text{BS}}(1 - \mathcal{V})) \rho_{\text{deph}} + \gamma_{\text{BS}} \rho_{\text{garb}}, \quad (7.11)$$

where we have defined ρ_{garb} as a garbage state which contains all terms of the form $|00\rangle, |11\rangle$ and $\rho_{\text{deph}} = |01\rangle\langle 10| + |10\rangle\langle 01|$. We have also introduced the visibility, $\mathcal{V}$ of the photons, which accounts for the photons not being perfectly indistinguishable even for the input state $|\psi_{\text{sp}}\rangle \otimes |\psi_{\text{sp}}\rangle$. The coefficients in Eq. (7.11) are given in App. B.2 in terms of the detector efficiency, dark count probabilities, and the factors defined in the previous sections.

7.3.4. Quantum memories

The atom-atom entanglement will decohere over time due to dephasing of the spin state of the atoms. We model this as single-qubit dephasing channels acting on each atomic qubit, leading to an exponential decrease of coherence with time. Furthermore, the atoms can get lost from the traps due to imperfect vacuum, which we model as an erasure channel with an exponential decay of the qubit population with time. As the quantum states need to be stored for different amounts of time depending on where the quantum memory is located (transmitter satellite, end-satellite, or ground station), the previous effect will depend on the location of the quantum memory. The mathematical model for both of these imperfections is considered in App. B.

Including the effect of the decoherence of the atomic memories on the stored Bell pairs in the elementary links updates Eq. (7.11) to

$$\alpha |\psi\rangle\langle\psi| + \beta \rho_{\text{deph}} + \gamma \rho_{\text{garb}}, \quad (7.12)$$

where the coefficients as a function of the coefficients of the previous sections and the decoherence for a Pauli noise channel can be found in B. As we will discuss later, atomic loss can be treated as an overall probability of losing an end-to-end entangled pair.

7.3.5. Entanglement swapping

After entanglement between neighboring links is successfully achieved, we carry out the atomic Bell measurements in all the satellites of the chain to enable entanglement swapping to the ground stations. To account for imperfections in the CZ-gate, we

assume that the gate succeeds perfectly with some probability p_{swap} while with probability $1 - p_{\text{swap}}$ the swapping results in a 'garbage' state with zero fidelity with the desired Bell state. In addition, we also include a finite probability that the atoms participating in the Bell measurement are lost, which also destroys the entanglement but is a heralded error. The input state of the first entanglement swapping is:

$$(\alpha_{12}|\psi\rangle\langle\psi| + \beta_{12}\rho_{\text{deph}} + \gamma_{12}\rho_{\text{garb}})_{12} \otimes (\alpha_{34}|\psi\rangle\langle\psi| + \beta_{34}\rho_{\text{deph}} + \gamma_{34}\rho_{\text{garb}})_{34}, \quad (7.13)$$

where the coefficients $\{\alpha_{12}, \beta_{12}, \gamma_{12}\}$ and $\{\alpha_{34}, \beta_{34}, \gamma_{34}\}$ are the coefficients of Eq. (7.12) corresponding to each of the elementary links involved in the entanglement swapping, respectively. This state is followed by a controlled-NOT (CNOT) gate on qubits 2-3, a Hadamard gate on qubit 2, and the measurement of both qubits. As a final note, we can lose an atom as a result of a gate operation. This loss is heralded in the measurement of the entanglement swap and, therefore, not considered in the computation of the fidelity. However, it does affect the rate of entanglement and we assign a probability $p_{\text{loss,swap}}$ of losing an atom in the entanglement swapping operation.

After a successful (i.e. no atom loss detected) entanglement swapping operation, the swapped state will be of the form

$$A[1]|\psi\rangle\langle\psi| + B[1]\rho_{\text{deph}} + C[1]\rho_{\text{garb}}, \quad (7.14)$$

where

$$A[1] = \alpha_0\alpha_1, \quad (7.15)$$

$$B[1] = \beta_0(\alpha_1 + \beta_1) + \alpha_0\beta_1, \quad (7.16)$$

$$C[1] = \gamma_0(\alpha_1 + \beta_1) + \gamma_1(\alpha_0 + \beta_0) + p_2\gamma_0\gamma_1. \quad (7.17)$$

The state after performing n entanglement swaps is described by the recursive expressions:

$$A[n] = A[n-1]\alpha_n, \quad (7.18)$$

$$B[n] = A[n-1]\beta_n + B[n-1](\alpha_n + \beta_n), \quad (7.19)$$

$$C[n] = \gamma_n(A[n-1] + B[n-1]) + C[n-1](\alpha_n + \beta_n + p_2\gamma_n). \quad (7.20)$$

Finally, we model the imperfect entanglement swapping at the repeater nodes as succeeding with probability p_{swap} , resulting in an error-free swap, while with probability $1 - p_{\text{swap}}$, the entanglement swapping results in a garbage state. This allows for obtaining a compact expression of the final entangled state between the end-nodes of the repeater

$$\begin{aligned} \rho_{AB} = & p_{\text{swap}}^{n_{\text{sat}}} (A[n_{\text{sat}}]|\psi\rangle\langle\psi| + B[n_{\text{sat}}]\rho_{\text{deph}}) + C[n_{\text{sat}}]\rho_{\text{garb}} \\ & + (1 - p_{\text{swap}}^{n_{\text{sat}}}) (A[n_{\text{sat}}] + B[n_{\text{sat}}])\rho_{\text{garb}}, \end{aligned}$$

where $p_{\text{swap}}^{n_{\text{sat}}}$ is the probability of no faulty entanglement swapping operations across the chain of n_{sat} satellites. The coefficients $A[n_{\text{sat}}]$, $B[n_{\text{sat}}]$ and $C[n_{\text{sat}}]$ can straightforwardly be found from combining $(n_{\text{sat}} + 1)$ states of the form ρ_{link} . Note that, in

general, the coefficients α, β , and γ will be different for each of the elementary links. From Eq. (7.20), it follows that the fidelity of the final state with the target Bell state is

$$\mathcal{F}_{AB} = p_{\text{swap}}^{n_{\text{sat}}} \frac{A[n_{\text{sat}}] + B[n_{\text{sat}}]/2}{A[n_{\text{sat}}] + B[n_{\text{sat}}] + C[n_{\text{sat}}]}. \quad (7.21)$$

7.3.6. Rate

For the computation of the rate, we assume that N_{mem} photons are transmitted sequentially in each elementary-link entangling attempt, using $2N_{\text{mem}}$ atoms per transmitter satellite since each satellite covers two elementary links. In each attempt, we assume that the N_{mem} photons are emitted within a time assumed to be negligible compared to the communication time between the links, which is on the order of milliseconds. We therefore set the repetition time of the entangling attempt to match the longest communication time between links in the repeater chain.

After each attempt, the entanglement in a link is either swapped if the neighboring link is also successful or discarded before a new attempt is made. We choose this mode of operation because for quantum memories with second-long coherence times, as considered in this work, the communication time between elementary links is too long to maintain high-fidelity entanglement by storing successful links for multiple entanglement attempts. It is therefore desirable to have enough multiplexing to ensure near-deterministic entanglement generation in the elementary links [430].

To estimate the rate of the repeater, we first consider the probability of generating n Bell pairs in a single elementary link, which is simply

$$p_{(g,s)}(n) = \binom{N_{\text{mem}}}{n} p_{\text{link},(g,s)}^n (1 - p_{\text{link},(g,s)})^{N_{\text{mem}}-n}, \quad (7.22)$$

where $p_{\text{link},(g,s)}$ is the success probability per photon for ground-satellite (g) or satellite-satellite links (s). This expression is valid in the regime where the optical transmission is limited by beam divergence rather than pointing errors, since the latter can induce correlated errors, whereas the binomial distribution assumes uncorrelated loss. We verify that this is the case for realistic pointing errors through Monte-Carlo simulations of the distribution of entangled pairs in an elementary link (see App. B for details). Since the number of entangled pairs between the ground stations will be limited by the link with the smallest number of successful pairs, we can express the probability of generating n entangled pairs between the ground stations, assuming deterministic swapping, as

$$\begin{aligned} p(n) = & \left(\sum_{i=0}^{n_{\text{sat}}-1} \binom{n_{\text{sat}}-1}{i} p_s(n)^i p_s(>n)^{n_{\text{sat}}-(i+1)} \right) \\ & \times \left(\sum_{j=0}^2 \binom{2}{j} p_g(n)^j p_g(>n)^{2-j} \right) \\ & - p_s(>n)^{n_{\text{sat}}-1} p_g(>n)^2, \end{aligned} \quad (7.23)$$

where $p_{(g,s)}(> n) = \sum_{i=n+1}^{N_{\text{mem}}} p_{(g,s)}(i)$. Since the entanglement swapping is only near-deterministic due to the possibility of atom loss, we need to include the probability of losing some of these pairs due to failed entanglement swappings. This modifies the probability for establishing n entangled pairs between the ground stations to

$$p_f(n) = \sum_{i=n}^{N_{\text{mem}}} \binom{i}{n} p(i) (1 - p_{\text{loss}})^n p_{\text{loss}}^{i-n}, \quad (7.24)$$

where p_{loss} is the probability of unsuccessfully combining the entangled pairs in the elementary links to achieve an entangled pair between the ground stations due to loss of at least one atom in any of the elementary link pairs. The final average rate of the repeater is then estimated as

$$R = \frac{\sum_{i=1}^{N_{\text{mem}}} i p_f(i)}{T_{\text{com}}}, \quad (7.25)$$

where we have assumed that the repetition rate is set by T_{com} , which is the longest round-trip communication time between elementary links. Additional details about the calculation of the rate and fidelity can be found in the App. B, where we provide detailed expressions for $p_{\text{link},(g,s)}$, p_{loss} , and T_{com} .

7.4. Performance: numerical simulations

Using the model explained above, the computation of the rate and fidelity of the entanglement distribution protocol can be carried out for different parameters of the system. In our simulations, we consider two different satellites. One has the characteristics of the Micius satellite [410], namely a telescope with 15 cm radius and $\sigma_\theta = 0.47 \mu\text{rad}$ pointing error. In addition, we consider an improved second satellite, with a 50-centimeter radius telescope. In both cases, the satellite orbit is at a height of 500 km from the surface of the Earth, the same as the Micius satellite, and both ground stations have a 60-centimeter radius telescope. We also assume the satellites to be in a ‘string of pearls’ configuration following an equatorial orbit. The ground stations are assumed to be at a height of 2000 meters to avoid Mie scattering. Mie scattering is produced by particles of size comparable to the wavelength of light, mainly due to atmospheric aerosols. These particles are more abundant in the lower atmosphere and can be neglected for higher elevations [431].

Rather than using the rate analysis described above to estimate the achievable rate and fidelity for a fixed number of atoms, N_{mem} , we instead estimate the number of atoms needed to achieve a certain target rate and fidelity. We do this to get a sense of the multiplexing capabilities needed to reach fidelities above 90% compatible with secret key distillation [242, 432], and rates above 10 Hz (100 Hz) such that at least 10 (100) entangled pairs could be distributed within the second long memory time that we assume for the ground state memories. Fidelities above 50% indicate the presence of entanglement. Since the satellites are moving in orbit, the rate is estimated as the average rate during the time window where ground-satellite communication is possible. This communication window is about 5 min per satellite pass above the ground stations for the satellite height of 500km considered here.

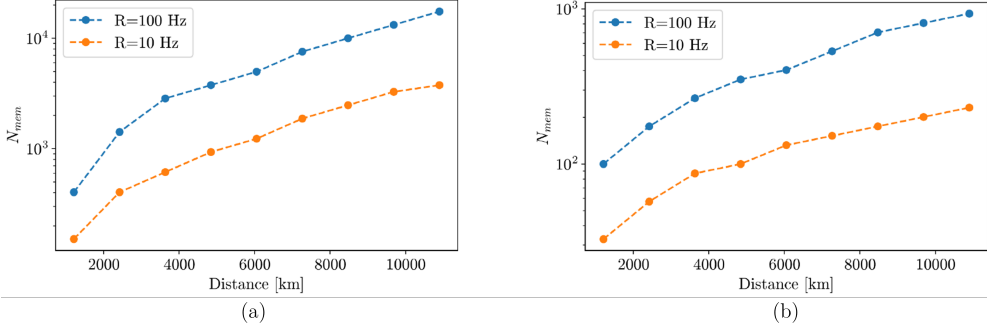


Figure 7.7: Multiplexing capabilities required to reach a rate of 10Hz (orange) or 100Hz (blue) and a fidelity of 0.9 in the entanglement distribution protocol in a satellite chain of 5 satellites. (a) Number of memory modes, N_{mem} , needed in a satellite with the characteristics of Micius, i.e. the same size of the telescope ($r = 0.15$ m), pointing error ($\sigma = 0.47$ μ rad) and height ($h = 500$ km), as a function of the distance between ground stations. (b) Number of memory modes, N_{mem} in the text, needed in a satellite with a bigger telescope radius than Micius ($r = 0.5$ m) as a function of the distance between ground stations.

In Fig. 7.7 we plot the necessary number of memory modes per repeater node to reach a certain rate of entanglement between the ground stations, in this case 10 Hz or 100 Hz, and a fidelity ≥ 0.9 of the final Bell pair as a function of distance, assuming a chain of five satellites. The parameters considered in the simulation are shown in Table 7.1.

From the comparison of Figs. 7.7 a) and b), it is seen that increasing the radius of the satellite mirrors relaxes the multiplexing requirements up to two orders of magnitude. While the Micius-like satellite requires more than 1000 memory modes to get a rate of 100 Hz for a distance of 3500km between ground stations, the upgraded satellite only requires ~ 200 memory modes. Additionally, the upgraded satellite only requires around 1000 memory modes to reach a rate of 100 Hz over distances ≥ 8000 km. This is an order of magnitude less than found in photonic BSM architectures for similar satellite characteristics [287].

We chose to target a final fidelity of ≥ 0.9 since this allows for direct extraction of secure encryption keys through quantum key distribution [242, 432]. If higher fidelity entanglement is required, this can, in principle, be achieved through entanglement purification [433]. Increasing the rate from the targeted 10Hz or 100Hz can be achieved by increasing the amount of multiplexing. Increasing the number of memory modes in Fig 7.7 by a certain factor will result in roughly the same increase in rate.

Table 7.1 shows the assumed quality of the hardware and quantum operations, e.g., the entanglement swapping quality and photon detection efficiency, in order to reach a final fidelity ≥ 0.9 . Lower error budgets in the quantum hardware will naturally increase the performance of the repeater. While errors at the 0.1% level are within reach of current neutral atom-based hardware, entanglement purification techniques

Parameters considered for the simulations		
Prob. of the transmitter to emit a single-photon	p_1	0.99
Prob. of the transmitter to emit two photons	p_2	0.002
Collection efficiency	η_{coll}	0.49
Visibility of the photons	$\mathcal{V}$	0.999
Prob. of losing an atom in the entanglement swap	$p_{\text{loss,swap}}$	0.1
Effective loss time in satellite	$T_{\text{loss,sat}}$	0.01 s
Effective loss time in ground station	$T_{\text{loss,gstat}}$	1.5 s
Fidelity of the entanglement swap	p_{swap}	0.995
Dark detection probability	p_{dark}	10^{-6}
Decoherence time in satellite	$\tau_{\text{c,sat}}$	1.5 s
Decoherence time in ground station	$\tau_{\text{c,gstat}}$	10 s
Photon detection efficiency	η	0.98
Transmission probabilities in the elementary links		
Trans. prob. sat.-ground. ($r = 0.15\text{m}$)	$p_{\text{T,sat-ground}}$	0.14
Trans. prob. sat.-ground. ($r = 0.5\text{m}$)	$p_{\text{T,sat-ground}}$	0.33
Trans. prob. sat.-sat. ($r = 0.15\text{m}$)	$p_{\text{T,sat-sat}}$	0.052 - 0.0007
Trans. prob. sat.-sat. ($r = 0.5\text{m}$)	$p_{\text{T,sat-sat}}$	0.55-0.03

Table 7.1: Assumed values of the parameters in the model. The assumed values are compatible with current or near-term technology. The parameters that are different from the ones shown in this table are indicated in the figures. The collection efficiency is defined as $\eta_{\text{coll}} = p\eta_{\text{cav,coll}}$, where $p = 0.99$ is the probability of emitting in the cavity mode, and $\eta_{\text{cav,coll}} = 0.5$ is the collection efficiency from the cavity to free-space. The effective loss coherence times depends on the quality of the vacuum in the atomic memories and is assumed to be higher for ground stations than on the satellites. The values of the transmission probability between satellites shown are the ones for the shortest distance between ground stations (~ 1300 km) and for the longest one ($\sim 12000\text{km}$) to provide a sense of the relevant regime.

could also be employed to leverage the effect of non-perfect operations at the expense of a slower distribution rate.

7.5. Conclusion

In summary, we have proposed a satellite-assisted quantum repeater architecture based on individually trapped alkali atoms, which has several desirable features for functioning as quantum payloads. In particular, the ability to perform nearly deterministic BSMs with Rydberg-mediated two-qubit gates significantly lowers the amount of multiplexing needed to establish entanglement at continental distances compared to protocols based on probabilistic photonic BSMs [287]. Additionally, the use of atoms as single-photon sources circumvents the need for entangled photon sources and absorptive quantum memories.

We developed a simple but accurate analytical model of the repeater architecture that allowed us to compute how expensive, in the sense of the number of memory modes per repeater station, it is to get high-fidelity entanglement at continental distances

using the setup we propose. From this model, we estimated that a chain of 5 satellites with a 50-centimeter radius telescope enables high-fidelity ($F \geq 0.9$) entanglement at a rate of 100 Hz with less than 200 atoms per repeater station at a range of 1500 km.

We believe that our model can be readily adapted to provide first-order estimates of the performance for other satellite-assisted quantum repeater architectures based on heralded entanglement generation and entanglement swapping, also with different quantum hardware. This is due to the characterization of the effect of the physical errors into simple lower-bound estimates of the fidelity of the final state. Notably, our model allows for easy and fast simulation of long quantum repeater chains in contrast to Monte Carlo-based simulations [289] where the computational overhead increases rapidly with the length of the repeater chain.

In addition to the promise of long-distance entanglement distribution, satellite-assisted quantum repeaters also open up new tools for more fundamental tests of nature. In particular, the atomic hardware considered here opens up new opportunities for quantum-enhanced sensor networks relevant for searches for topological dark matter [267], global time-keeping [357], and tests of the interplay between gravity and quantum mechanics [360].

Chapter 8

Entanglement-based satellite-assisted quantum key distribution

Practical considerations and truncated beam optimization

Nature is not embarrassed by difficulties of analysis.

Augustin-Jean Fresnel

Taking a more practical and near-future approach, in this chapter, the practical implementation of entanglement-based quantum key distribution is studied. In this chapter, a satellite-based quantum key distribution (QKD) setup covering the Iberian Peninsula is proposed and evaluated, linking Madrid with Barcelona, Bilbao, and Lisbon. Our proposed setup uses a Low-Earth-Orbit state-of-the-art satellite equipped with a spontaneous parametric down-conversion source to distribute entangled photon pairs to ground stations. Considering vibrations in the satellite, we optimize the truncated beam waist, in a similar manner to §6, to enhance the transmission probability and improve the secret key rate. Our results show that key rates sufficient for real-world applications, such as secure communication between hospitals, using hybrid classical-quantum protocols, are feasible with existing protocols. Our results highlight the viability of near-term satellite-based QKD networks for national-scale secure communications.

The content of this chapter is based on the Ref. [434] Dominguez, V., Badás M. et al. "Quantum key distribution in the Iberian Peninsula". Optica Quantum 4, Issue 1, pp. 22-30 (2026)

8.1. Introduction

The realization of a quantum internet holds the potential to revolutionize multiple fields, including secure communication [28, 30, 402], high-precision sensing networks [357–359], and distributed quantum computing [403]. In particular, quantum key distribution (QKD) enables communication with information-theoretic security, a level of protection unattainable by classical methods.

As discussed in §4 and §7, photon loss fundamentally limits long-distance fiber-based quantum communication, as quantum signals cannot be amplified due to the no-cloning theorem. Quantum repeaters between the fiber links address this by segmenting the total distance and reconnecting the links via teleportation or error correction. However, in fiber-based implementations, exponential attenuation necessitates a large number of repeater nodes for continental or national-scale distances.

Satellite-based free-space QKD offers a promising alternative for long-distance communication without the need for complex repeater infrastructures. Notably, the Micius satellite has demonstrated QKD over distances from 500 to 7,000 km using a trusted-node architecture [283, 308]. More recently, entanglement-based QKD was successfully achieved without relying on a trusted node, decreasing the risk of eavesdroppers, reaching up to 1,200 km [310, 410]—distances well beyond the capability of terrestrial fiber networks.

Fiber-based QKD has also made significant progress, enabling high-speed, secure communication across metropolitan areas. Recent demonstrations include the protection of multiple virtual private networks (VPN) [435] and secure data exchange between hospitals in the Madrid area via a trusted-node QKD network [436]. The next logical step is to extend these capabilities from city-scale to national-scale coverage.

To this end, we propose a satellite-based QKD architecture to interconnect major cities across the Iberian Peninsula—Madrid with Barcelona, Bilbao, and Lisbon—as shown in Fig. 8.1(a). Our system employs a satellite like Micius [310], in a low Earth orbit (LEO) equipped with a spontaneous parametric down-conversion (SPDC) source—see §4.3 for more details on this type of source—to distribute entangled photon pairs to multiple ground stations, implementing the BBM92 protocol presented in §2.2.2. In our setup, we consider the vibrations in the satellite and propose a novel optimization of the beam waist that generalizes the static Gaussian beam clipping problem [437], to the dynamic regime. Unlike classical optical communication links that prioritize signal stability to minimize fading outages [175], our method maximizes the average photon yield even at the cost of higher signal variance. This optimization allows us to mitigate the detrimental effects of pointing jitter and consequently increase the photon transmission probability to the ground stations.

We evaluate the system’s performance with and without optimization, demonstrating that the latter significantly enhances the secret key rate (SKR)—by a factor of approximately 10. The non-optimized beam refers to a Gaussian with a beam waist of radius $w_{0,\text{non-opt}} = 2.7$ cm, which is compatible with the $10\ \mu\text{rad}$ divergence at a wavelength of 850 nm, as reported in Ref. [245] for the Micius satellite. In our model, we also consider atmospheric conditions in real time, following the work done in [258]. In other words, the weather data is collected for the desired experiment

duration at the target locations using the Free Weather API software [438] and fed into an empirical mathematical model [258] that extracts the attenuation experienced by an optical beam operating at the wavelength of 1550 nm. A single satellite link proves sufficient to support secure communication across the Iberian Peninsula. Our results show that the SKR is adequate to support existing QKD use cases such as hospital-to-hospital encryption demonstrated in the MAD-QCI project by Telefónica, using hybrid protocols where the quantum layer handles key generation and classical layers manage key expansion via IPsec [436]. Furthermore, we check that for more demanding use cases such as supporting virtual private network (VPN) services using the ETSI-QKD-014 protocol for JPMorgan [435], we would need to improve the entangled photon pair source rate by 3 orders of magnitude, i.e., 1 GHz. While such a high rate has not currently been demonstrated on a satellite, it is compatible with performances demonstrated in laboratory environments simulating LEO satellite conditions [439].

8.2. Model

We focus on a downlink configuration, where photons are transmitted from a satellite to ground stations. This setup is preferred because atmospheric disturbances are generally less severe than in the uplink case, where photons travel from the ground to the satellite [288, 422]. In the uplink, atmospheric disturbances are more pronounced because turbulence affects the beam early in its propagation path, significantly increasing its divergence compared to the downlink case. The satellite carries an onboard entangled photon source that probabilistically generates polarization-entangled photon pairs. Specifically, we consider a single satellite like Micius, with telescopes of radius of 15 cm, a rate of entangled photon pairs per second of $5.9 \cdot 10^6$ pairs/s and pointing jitters of $\sim 10^{-6}$ rad [81, 310], in a low Earth orbit (LEO) at an altitude of approximately 400 km, following a medium-inclination trajectory at a latitude of 42° , passing over the Iberian Peninsula, as illustrated in Fig. 8.1(b).

We consider the BBM92 protocol explained in §2.2.2, where the photons are measured randomly in either the Z or X basis at the ground stations. Only photon pairs detected in the same basis are retained in classical memories through postselection. We employ the BBM92 protocol, following the approach demonstrated by the Micius satellite [310]. The figure of merit that we base our analysis on is the extracted key material; in general, the value is presented as a certain amount of *bits per second* (bps), but eventually it may also be convenient to also express it in terms of *bits per satellite pass*. The calculation involves three main steps, which are discussed in the Sections to follow: (1) the model of the photonic qubit states generated by the entangled photon-pair source on board the satellite; (2) the effect of the optical channels between the satellite and the ground stations; (3) the effect of the measurement performed by the receivers in the presence of noise.

8.2.1. Entangled photon pair source

A pump laser is used to drive a nonlinear optical crystal (e.g., PPKTP or others presented in §4.2.2) in both the clockwise and anticlockwise directions of the Sagnac interferometer simultaneously, generating down-converted polarization-entangled pho-

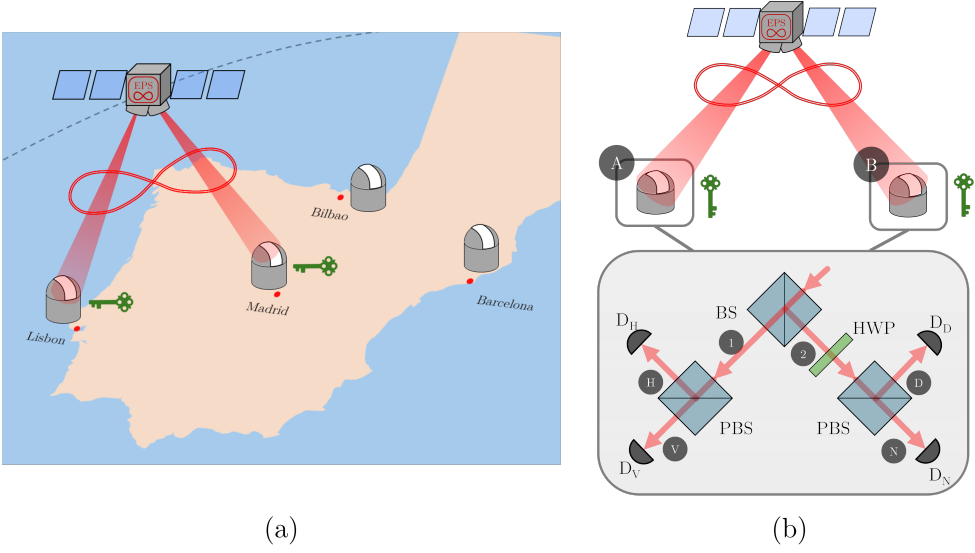


Figure 8.1: *Experimental setup. (a) Ground stations of the regional quantum key distribution (QKD) network that we want to implement. The central node is Madrid, and the key distribution is always between Madrid and any of the other cities (Barcelona, Bilbao, and Lisbon). (b) We consider a downlink scenario, with an entangled photon source firing photons from the satellite to the ground stations. At the ground stations, the photons are randomly measured in the X or Z basis. To perform such a measurement, the photons go through a 50/50 beam splitter (BS), followed by a half-wave plate (HWP) with the fast axis at 22.5° in one of the outputs, which changes the basis of the input photons. Finally, before measuring, the photons go through a polarization beam splitter (PBS) such that we measure different polarizations- horizontal (D_H), vertical (D_V), diagonal (D_D) and antidiagonal (D_N)- at the output ports of the B.S. The measurement results are fed into a classical memory from which we can then extract secret keys for the two applications we consider.*

ton pairs with orthogonal polarizations [410, 440],

$$\begin{aligned}
 |\psi_{\text{source}}\rangle \approx & (1 - \lambda) [|0, 0\rangle_{A,B} \\
 & + \frac{\sqrt{2}\lambda^{1/2}}{(1 - \lambda)^{1/2}} \overbrace{\frac{1}{\sqrt{2}} (|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B)}^{|\Psi^+\rangle} \\
 & + \frac{\sqrt{3}\lambda}{1 - \lambda} \frac{1}{\sqrt{3}} (|2H\rangle_A |2V\rangle_B + |HV\rangle_A |HV\rangle_B + |2V\rangle_A |2H\rangle_B) + \dots] ,
 \end{aligned} \tag{8.1}$$

where the subscripts A and B correspond to two different spatial modes directed toward the ground stations of Alice and Bob respectively, $|H\rangle$ and $|V\rangle$ denote the horizontal and vertical polarization states, respectively. The state $|\Psi^+\rangle = 1/\sqrt{2}$

$(|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B)$ denotes the desired state, which corresponds to a maximally entangled state with anticorrelated polarization. Equation (8.1) neglects higher-order multi-pair contributions and is therefore truncated at second order, as shown.

The term $\mu = 2\lambda$ is the average number of photon pairs generated per pulse, characterised by the brightness of the SPDC source [440]. The source operates in a weak-pump regime, where $\lambda \ll 1$, implying that the probability of emitting a photon pair is low. However, operating in this regime is necessary since it suppresses multi-photon events ($\propto |2H\rangle_A|2V\rangle_B + |HV\rangle_A|HV\rangle_B + |2V\rangle_A|2H\rangle_B$), which causes errors and lowers the secret key rate. In other words, there is a trade-off between the rate and the fidelity of entanglement. In our simulation, we optimize λ (proportional to the average number of photon pairs generated) to obtain the highest SKR.

8.2.2. Photon transmission

Photon losses during transmission to the two ground stations will change the received state from the one emitted by the SPDC source. We model this by passing the state in Eq.(8.1) through fictitious beam splitters in each spatial mode, where one output of the beam splitters corresponds to the loss, while the other is the transmitted mode. In general, the two spatial modes experience different transmission probabilities due to the asymmetric distances from the satellite to each of the ground stations. This is also relevant during satellite orbit, since the photons from the same pair can only be captured by both optical ground stations when the satellite is within their mutual range. In the calculations that follow, the maximum zenith angle of each ground station that determines whether it is in range of the satellite or not during the orbital movement is seventy-five degrees.

The free-space transmission probability is estimated from the model in §7, which includes divergence, atmospheric absorption, and pointing jitter. For a more accurate estimation of the number of entangled photon pairs captured by the ground station per second, an optical propagation model based on §6, which accounts for the truncation-induced diffraction of the beam in the transmitter's telescope, is used. As discussed in §5.5, in the paraxial regime, the diffraction effects of single-photon states –arising here from truncation of the transmitted beam– can be described using the Rayleigh-Sommerfeld, Fresnel, and Fraunhofer integrals. These formulations carry over from classical field theory to quantum field theory due to their formal equivalence. In this chapter, Fresnel propagation is employed to model the evolution of single-photon states and obtain the spatial mode at the receiver aperture plane.

Previous works (see §7) often assume a Gaussian beam where the truncation of the beam in the transmitter can be neglected. Instead, we explicitly take this effect into account using the model from §6. This model enables the accurate evaluation of the average photon capture probability of each downlink channel, accounting for both the diffraction effects and the pointing jitter.

The beamwaist w_0 of the spatial Gaussian mode is located at the satellite's telescope aperture. The telescope on board the satellite has a radius a . The field at the ground station's aperture plane, located at a distance z , can be computed numerically through Fresnel propagation, as explained in §5.5. This propagation gives the probability density function of the spatial location of the single-photon, i.e. $I(x, y)$, in the

ground station's aperture plane. Then, a convolution can be performed to compute the probability of capturing the single-photon $g(x_0, y_0)$ when the center of the beam is displaced to (x_0, y_0) in the ground station's reference system. This convolution can be computed using the convolution theorem by

$$g(x_0, y_0) = \mathcal{F}^{-1}\{\mathcal{F}(I) \cdot \mathcal{F}(\mathcal{A})\} \quad (8.2)$$

where $\mathcal{A}(x, y)$ is the aperture function of the ground station's telescope and $\mathcal{F}$ is the bidimensional Fourier transform. The satellite pointing jitter is modeled as a bidimensional Gaussian probability density function of the location of the center of the beam at the ground station frame of reference (x_0, y_0) ,

$$f_{\text{PJ}}(x_0, y_0) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x_0^2 + y_0^2}{2\sigma^2}\right) \quad (8.3)$$

where $\sigma = z\sigma_\theta$, and σ_θ is the angular pointing jitter. By combining the previous equations, the probability density function of the photon capture probability can be computed as [441]

$$f(\eta) = \iint_{\mathbb{R}^2} f_{\text{PJ}}(x_0, y_0) \delta[\eta - g(x_0, y_0)] dx_0 dy_0 \quad (8.4)$$

where η is the photon capture probability and $\delta(x)$ is the Dirac delta function. Finally, the average of this probability density function gives the average photon capture probability. As shown in App. C.3.2, the latter is to be maximized in the context of quantum entanglement-based QKD for optimal performance of the system. This maximization is done by varying the beamwaist w_0 of the spatial Gaussian mode transmitted.

By changing the beamwaist w_0 of the transmitted spatial Gaussian mode, the average photon capture probability $\bar{\eta}$ can be maximized. Fig. 8.2 shows the optimization of the beamwaist for the apertures used in the present work. The results show that an optimum beamwaist can be found that maximizes the average photon capture probability (this maximum barely changes with the distance between the terminals). This maximum is the result of two counteracting effects when increasing the beamwaist. On the one hand, increasing the beamwaist increases the collimation of the beam and therefore results in a smaller spread of the spatial mode on the receiver aperture plane. On the other hand, as the beamwaist increases, the truncation effects due to the finite aperture of the transmitter start to be relevant. These truncation effects account for the telescope's transmission probability and the diffraction induced by the edges of the aperture.

As a result of this optimization, the spatial spread of the single-photon mode in the ground station's aperture plane can be significantly reduced, compared to a case in which $w_0 \ll a$ (where a is the transmitter's telescope radius). This optimum beam, when combined with the pointing jitter statistics, results in a higher average photon capture probability $\bar{\eta}$. This formulation generalizes the classic optimization of static on-axis irradiance [437] to include the dynamic and stochastic effects of the pointing jitter. While the static approach maximizes the peak far-field intensity, our optimization targets the average collected power by explicitly integrating over the probability

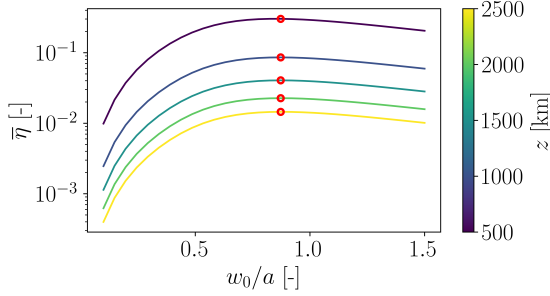


Figure 8.2: Optimization of the average photon capture probability $\bar{\eta}$ as a function of the ratio w_0/a , where w_0 is the beam waist transmitted from the satellite and a is the satellite aperture radius. The simulation assumes a pointing jitter $\sigma_{PJ} = 0.47 \mu\text{rad}$ [81, 310], a satellite aperture radius $a = 15 \text{ cm}$, and a ground station aperture radius of 60 cm . Curves correspond to different link distances z and red dots to the maxima found.

density function of the pointing jitter. Consequently, the optimal beam-to-aperture ratio is not fixed, but strictly depends on the magnitude of the pointing jitter as shown in App. C. However, the maximization of the average photon capture probability comes along with an increase on the fluctuation of the photon capture probability η around the average $\bar{\eta}$. Although these fluctuations can be very detrimental in classical communication links –as was analyzed in §6– this is not the case for quantum entanglement distribution. As shown in App. C, the average photon capture probability is what matters on quantum entanglement distribution protocols affected by the pointing jitter stochastic process. These differences arise because classical systems decode information from large correlated bit strings, which require a stable power to get as many of these correlated bits as possible. Then, classical error correction codes are applied to correct the bits flipped in the transmission. If the probability of being below a certain power threshold is high, the classical error correction codes can not compensate for the flipped bits, and information contained in the full bit string is lost. For quantum key distribution, however, we detect individual independent events on each receiver. Hence, in this case, the maximization of the number of photons captured in each receiver can be pursued.

The effect of the atmospheric channel in the performance of the system has also been modeled. The atmospheric attenuation and scattering model presented in Ref. [258] has been used. Regarding the atmospheric turbulence, under weak turbulence conditions, the effect of beam spread and wander has been considered negligible for the downlinks at limited zenith angles considered in this work [422]. Moreover, the turbulence-induced scintillation and the resulting coupling efficiency on the detector are accounted for in the coupling efficiency –this efficiency is obtained assuming an adaptive optics system on the ground station [442]. Enforcing wavefront correction can be achieved by spectrally multiplexing a beacon channel alongside the photonic qubit beam on the downlink path, as shown in Ref. [443]. We assume that a spec-

tral channel separation of a few tens of nanometers is enough to guarantee both an optimal fiber coupling performance of the photonic qubits via the adaptive optics and efficient spectral filtering with free-space and fiber filters. Furthermore, based on satellite-to-ground experimental observations reported in Ref. [273], the depolarizing effect induced by atmospheric turbulence can be regarded as negligible.

Other dynamic atmospheric effects that significantly impact the quality of the QKD link, depending on time of day and weather, are also taken into account. This is possible due to the framework developed in Ref. [258], where the sun irradiance –used to calculate the stray light impinging on the single-photon detectors–, and atmospheric parameters –such as cloud coverage factor, weather visibility, and the relative angle between the satellite and the ground station telescope– are extracted at the ground station locations and used to determine noise levels and photon losses dynamically. To limit the analysis to reliable links, we exclude observations where the satellite’s zenith angle exceeds 75° with respect to the ground stations. We also model false detections assuming that there is a probability p_{dark} that a detector clicks despite no photon being received. We assume that this event translates into a detected state with zero fidelity with respect to the desired entangled state, leading to a lower bound in the overall fidelity measured at the ground.

8.2.3. Photon measurement

At the ground stations, the incoming photons are randomly measured in either the Z basis –defined by the states $|H\rangle$ (horizontal) and $|V\rangle$ (vertical)– or the X basis, defined by $|D\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$ (diagonal) and $|N\rangle = (|H\rangle - |V\rangle)/\sqrt{2}$ (antidiagonal). To implement this random-basis measurement, the photons first pass through a 50/50 beam splitter (BS). At one of the BS output ports, a half-wave plate (HWP) is used to rotate the measurement basis from Z to X. Both outputs are then directed into a polarization beam splitter (PBS), which spatially separates the polarization components, as illustrated in Fig. 8.1(b).

Assuming the maximally-entangled state $|\Psi^+\rangle$ is received at the ground stations, it evolves as follows:

$$\begin{aligned} \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B) &\xrightarrow{\text{B.S.}} \\ \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}}(|H\rangle_{A1} + i|H\rangle_{A2}) \frac{1}{\sqrt{2}}(|V\rangle_{B1} + i|V\rangle_{B2}) \right. \\ &\quad \left. + \frac{1}{\sqrt{2}}(|V\rangle_{A1} + i|V\rangle_{A2}) \frac{1}{\sqrt{2}}(|H\rangle_{B1} + i|H\rangle_{B2}) \right]. \end{aligned} \quad (8.5)$$

where a phase i corresponding to the reflected state (corresponding to path 2 in Fig. 8.1(b)) in each beam splitter appears. Here, the subscripts A1, A2, B1, and B2 refer to the different spatial modes after the beam splitter for the photon at the ground station A and B, respectively. The different spatial modes are shown in Fig. 8.1(b). The output ports labelled A2 and B2 undergo a basis transformation via a HWP: $|H\rangle \rightarrow |D\rangle$ and $|V\rangle \rightarrow |N\rangle$. Subsequently, each output passes through a PBS, which

separates the polarization components before detection. The resulting state just before measurement is:

$$\begin{aligned}
 |\psi_{\text{final}}\rangle = \frac{1}{2\sqrt{2}} [& |H\rangle_{\text{AH}}|0\rangle_{\text{AV}}|0\rangle_{\text{BH}}|V\rangle_{\text{BV}} + i|H\rangle_{\text{AH}}|0\rangle_{\text{AV}}|0\rangle_{\text{BD}}|N\rangle_{\text{BN}} \\
 & + i|D\rangle_{\text{AD}}|0\rangle_{\text{AN}}|0\rangle_{\text{BH}}|V\rangle_{\text{BV}} - |D\rangle_{\text{AD}}|0\rangle_{\text{AN}}|0\rangle_{\text{BD}}|N\rangle_{\text{BN}} \\
 & + |0\rangle_{\text{AH}}|V\rangle_{\text{AV}}|H\rangle_{\text{BH}}|0\rangle_{\text{BV}} + i|0\rangle_{\text{AH}}|V\rangle_{\text{AV}}|D\rangle_{\text{BD}}|0\rangle_{\text{BN}} \\
 & + i|0\rangle_{\text{AD}}|N\rangle_{\text{AN}}|H\rangle_{\text{BH}}|0\rangle_{\text{BV}} - |0\rangle_{\text{AD}}|N\rangle_{\text{AN}}|D\rangle_{\text{BD}}|0\rangle_{\text{BN}}].
 \end{aligned} \tag{8.6}$$

where the first subscript corresponds to the ground stations (A or B) and the second subscript to the detector path (H, V, D, or N), as depicted in Fig. 8.1(b). From Eq. (8.6), we observe that in half of the cases, the photon pairs are measured in the same basis (either Z or X). Consequently, during post-selection, approximately 50% of the events are discarded to ensure only measurements in matching bases are kept. The fidelity, F , of the final state, relevant to the calculations of secret key rate in the next section, is taken with respect to a maximally-entangled Bell state. The final state is of the form

$$\rho_{\text{final}} = A|\psi\rangle\langle\psi| + B\rho_{\text{deph}} + C\rho_{\text{garb}}, \tag{8.7}$$

where $|\psi\rangle$ is the desired target state which gives fidelity 1, ρ_{deph} are the multi-photon states, where one photon for each elementary link has been lost, resulting in a dephased states which gives fidelity smaller than one, and ρ_{garb} is the non-specified 'garbage' density matrix with zero overlap with the desired target state. The (normalized) density matrix of the dephased state is the following

$$\begin{aligned}
 \rho_{\text{deph}} = & \left(5|H\rangle_{\text{A}}|V\rangle_{\text{B}}\langle H|_{\text{A}}\langle V|_{\text{B}} + 5|V\rangle_{\text{A}}|H\rangle_{\text{B}}\langle V|_{\text{A}}\langle H|_{\text{B}} \right. \\
 & \left. + |H\rangle_{\text{A}}|H\rangle_{\text{B}}\langle H|_{\text{A}}\langle H|_{\text{B}} + |V\rangle_{\text{A}}|V\rangle_{\text{B}}\langle V|_{\text{A}}\langle V|_{\text{B}} \right) / 12,
 \end{aligned} \tag{8.8}$$

being the fidelity of the previous state $\mathcal{F}_{\text{deph}} = \langle\psi|\rho_{\text{deph}}|\psi\rangle = 5/12$. Therefore, the overall fidelity is

$$\mathcal{F} = \frac{\langle\psi|\rho_{\text{final}}|\psi\rangle}{\text{Tr}(\rho_{\text{final}})} = \frac{A + \frac{5}{12}B}{A + B + C}, \tag{8.9}$$

where the coefficients A , B and C depend on the transmission probabilities, the source brightness, and the dark counts (see App. C.2 for details).

8.2.4. Secret key rate calculation

The computation of the SKR follows the model shown in [258], where both the assumption of a depolarizing channel and Werner states produced by the entangled photon-pair source have been used. This analysis provides a lower bound on the achievable secret key rate and takes as input the calculated fidelity of the state measured on the ground. Furthermore, the model enables one to split the effect of losses and decoherence (loss of fidelity) in the channel, calculating them separately only to combine them when calculating the final secret key rate expression:

$$\text{SKR} = R_{\text{final}} \cdot (\max(0, 1 - (1 + \xi) \mathcal{H}(\text{QBER}))). \tag{8.10}$$

In the expression above, $\mathcal{H}$ is Shannon's entropy function, ξ is a factor representing the inefficiency of the classical protocol required for key extraction [440], and the QBER is calculated based on the total state fidelity.

$$\text{QBER} = \frac{1}{2} \left(1 - \frac{3\mathcal{F} + 1}{4} \right). \quad (8.11)$$

$R_{\text{final}} = R p_{T,A} p_{T,B}$ depends on the rate R of the source in the satellite and the total channel efficiency, i.e., source-to-ground-stations transmission probabilities. This term can be decomposed into three main ones: the free-space path loss, governed by the divergence of the beam and the pointing jitter; the atmospheric attenuation factor, governed by the weather condition; and the detector efficiency, which varies significantly in case one chooses cryogenically-cooled detectors (higher efficiency, but fiber coupling is required), or free-space-coupled detectors (lower efficiency, no fiber coupling). For the purpose of this analysis, we choose to operate under the assumption that fiber coupling can be efficiently implemented via adaptive optics correction strategies, leading to an overall net improvement in the channel quality with the more efficient superconducting nanowire single-photon detectors (SNPDs). This assumption is also tied to a lower system jitter and lower detector dead times, which lead to a higher signal-to-noise ratio (due to narrower available detection time windows and detection saturation rates). For the purposes of this study, the fiber-coupling efficiency is set to 40%; inclusion of the turbulence and wavefront correction on the QKD link performance is left as future work.

In summary, the transmission from the satellite to the ground stations follows the expression

$$p_{T,A} = \bar{\eta}_{S-GA} \gamma_{S-GA} \alpha_{S-GA} \quad (8.12)$$

$$p_{T,B} = \bar{\eta}_{S-GB} \gamma_{S-GB} \alpha_{S-GB}. \quad (8.13)$$

$\bar{\eta}$ accounts for the diffraction and pointing jitter losses (see Section 8.2.2), γ for atmospheric attenuation and scattering loss, α for coupling loss, and GA/GB for the two ground stations of interest. A coupling loss of $\alpha = 0.5$ is considered in agreement with simulations and experimental data in optical downlink channels with adaptive optics and single-mode fiber coupling [254, 255, 442]. The detailed description of how each of these terms is evaluated can be found in App. C. The reason for evaluating the two paths individually is the channel asymmetry during satellite transit, not only in terms of the dynamic conditions (such as weather conditions) but also the distance between the spacecraft and either ground station.

Equipped with the overall loss term, the probabilities –per attempt– associated with the detection of a valid pair can be translated into raw detection rates via the source pair emission rate R . These are further translated into the expected secret key rate using a key extraction algorithm that implements: basis reconciliation, error correction, and privacy amplification. As mentioned before, depending on the channel quality, the raw detection material that must be spent in order to produce a final key will change according to the classical protocols being used. Since this procedure can be quite computationally intensive and would have to be executed for each time step

at each satellite pass, a look-up table was generated by feeding the software with a grid of parameters (QBER and block length) –refer to App. C.4 for discussions and results.

8.3. Results

For the course of one month, specifically June 2025 (when the calculations presented in this chapter were computed), we compute the SKR per satellite pass that reached the ground stations, while accounting for real-time weather conditions. Our analysis focuses on three links: Madrid–Barcelona (504 km), Madrid–Bilbao (322 km), and Madrid–Lisbon (503 km). We begin by computing the SKR per pass and, based on this, derive the usable secret key material per pass. By averaging the usable key material per pass and taking into account the percentage of successful passes, i.e., the passes where communication between the satellite and the ground stations is established, we obtain the average secret key material rate, which we then compare against the requirements of Telefónica and JPMorgan use cases.

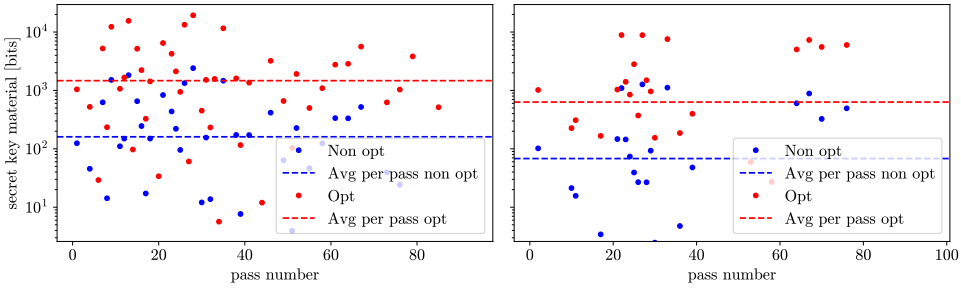


Figure 8.3: Secret key material between Madrid and Barcelona (left), and Madrid and Bilbao (right). The blue data shows the usable key without carrying out the beam optimization, and the red data shows the results when applying the optimization of the beam. The discontinuous red line shows the average key per pass. In the link Madrid-Barcelona, the successful passes, where the key is shared between the satellite and the ground stations, are around 39% for the non-optimized case and 48% for the optimized case. While in the Madrid-Bilbao link, the successful passes for the non-optimized case are around 22% and for the optimized case 24%, leading to a lower average of secret key material.

In Fig. 8.3, we present the secret key material per satellite pass and its average for both the non-optimized and optimized cases on the Madrid Barcelona and Madrid–Bilbao links. The non-optimized beam refers to a Gaussian with a beam waist of radius $w_{0,\text{non-opt}} = 2.7$ cm, which is compatible with the $10\,\mu\text{rad}$ divergence at a wavelength of 850 nm, as reported in Ref. [245] for the Micius satellite. The figure shows that beam optimization results in an increase of the average secret key material by an order of magnitude, which increases the SKR to a point where enough secret bits are produced to run the applications.

Link	Non-Optimized Scenario			Optimized Scenario		
	Rate [keys/s]	Telefónica Req. met?	JPMorgan Req. met?	Rate [keys/s]	Telefónica Req. met?	JPMorgan Req. met?
Mad-Bilb	0.003	No	No	0.028	Yes	No
Mad-Bcn	0.012	Yes	No	0.132	Yes	No
Mad-Lis	0.004	No	No	0.055	Yes	No

Table 8.1: Summary of results for an entangled photon source (Micius-like, $5.9 \cdot 10^6$ Hz [310]). The table details whether the achievable key rates meet the distinct service requirements for Telefónica (0.006 keys/s) and JPMorgan (2.13 keys/s).

Table 8.1 summarizes the computed results for the different links and compares them against two use cases: Telefónica [436], which requires 256-bit keys every 12 hours (equivalent to 0.006 bits/s), and JPMorgan [435], which requires 256-bit keys every 2 minutes (2.13 bits/s). For the Telefónica case, the 256-bit size ensures compliance with the standard RFC 8784, while the 12-hour rotation is a conservative reduction of the NIST 24-hour recommendation to protect sensitive medical data [444]. We observe that the 12-hour key renewal, used for communication between hospitals, is achieved across all links when the beam optimization is carried out. We do also see that for the link Madrid-Barcelona, this last use case can also be implemented without the optimization. This is due to the high percentage of successful passes, i.e., the key is shared between the satellite and the ground stations, we observe in that link during the month we are evaluating: around 39% for the non-optimized case and 48% for the optimized. This contrasts with the other two links, where the percentage of successful passes is around 20–25%, and up to 34% for the optimized case between Madrid and Lisbon. The passes where the communication is established between the satellite and the ground station are higher in the optimized case due to the fact that in those cases the overall channel efficiency improves; therefore, there will be cases where the SKR is zero in the non-optimized cases and non-zero in the optimized case.

However, for the more demanding use case of QKD-based VPNs, where the key must be refreshed every 2 minutes, the required key rate is not met. To get to such key requirements, we would need to improve the entangled photon source rate to the order of 1 GHz [439] (see Tab. 8.2). Additionally, we could also boost the SKR by not only increasing the photon source rate, but also combining it with the decrease in coupling loss, γ , which we consider 0.5 due to the scintillation effect of the lowest parts of the atmosphere [254, 255, 442]. This coupling efficiency could be increased by considering free-space coupling into the single-photon detectors [445], or multimode fiber coupling [446, 447]. For distances beyond the Iberian Peninsula, constellations of satellites could be used as proposed in §7.

8.4. Conclusion

In summary, we have demonstrated how single-satellite architectures enable the extension of quantum key distribution (QKD) from metropolitan-scale deployments [436] to national-scale distances. Our approach employs an entanglement-based QKD protocol, which eliminates the need for trusted nodes in the key distribution process, thereby

Link	Non-Optimized Scenario			Optimized Scenario		
	Rate [keys/s]	Telefónica Req. met?	JPMorgan Req. met?	Rate [keys/s]	Telefónica Req. met?	JPMorgan Req. met?
Mad-Bilb	0.565	Yes	No	5.005	Yes	Yes
Mad-Bcn	2.367	Yes	Yes	22.978	Yes	Yes
Mad-Lis	1.031	Yes	No	10.378	Yes	Yes

Table 8.2: Summary of results for an entangled photon source of 10^9 Hz. The table details whether the achievable key rates meet the distinct service requirements for Telefónica (0.006 keys/s) and JPMorgan (2.13 keys/s).

reducing the risk of eavesdropping. Furthermore, we introduced a beam optimization method that accounts for satellite pointing jitter and the truncation effects due to the finite size of the transmitter’s aperture, maximizing the transmission probability. This optimization allows us to meet the key rate requirement for the current use cases of secure communication between two hospitals [436], using state-of-the-art satellite technology [310]. Furthermore, by upgrading the rate of the entangled photon source to 1GHz, which can be reached with current technology [439], we could also meet the requirement for the most demanding use case [435].

In those use cases, hybrid quantum algorithms are employed, where only the first layer involves quantum processing. As a natural progression, future work should investigate fully quantum algorithms and their corresponding key rate requirements. Furthermore, limitations arising from photon arrival-time correlations have not been considered in this thesis (where sub-nanosecond time synchronization has been considered, see App. C.4), particularly those influenced by clock synchronization imperfections. Their impact on the number of usable detected photons –especially in the high photon capture rate regime enabled by the optimization proposed in this chapter– should be addressed in future work.

Additionally, in our current setup, we assume trusted ground stations for classical information storage. Incorporating quantum memories at the ground level would allow for the use of untrusted ground stations and enable the distribution of entanglement across nodes. This would open the door to a broader range of quantum technologies beyond QKD, including blind quantum computing [355, 356] and quantum-enhanced sensing networks [357–359].

Part III



Beam shaping

In the previous part, we quantified the effects of pointing jitter and optical aberrations in satellite optical and quantum communications and showed how the residual pointing jitter can be one of the dominant performance-limiting factors in intersatellite optical links. In this part, we propose a beam-shaping strategy specifically aimed at mitigating the impact of such residual stochastic pointing errors. By moving beyond the fundamental Gaussian beam and engineering the transverse field distribution, we demonstrate that a redistribution of optical power within the receiver aperture can substantially enhance link robustness to pointing jitter, relaxing the requirements of pointing control.

In §9, we introduce a scheme that combines Gaussian and higher-order Laguerre–Gaussian beams to generate optical field shapes that are intrinsically resilient to pointing jitter. The approach relies on the superposition of orthogonally polarized beams to tailor irradiance profiles in the receiver aperture plane. Chapter §10 presents the first experimental demonstration of this orthogonal superposition technique. Although ideal Laguerre–Gaussian modes cannot be reproduced exactly with the proposed setup, the experimentally generated annular beams already exhibit the intended resilience and clearly demonstrate the potential of beam shaping to mitigate pointing-induced losses. Building upon these ideas, §11 develops a rigorous mathematical framework to derive optimal far-field beam shapes using variational methods and proposes several optical systems capable of approximating these optimal solutions.

Chapter 9

Orthogonal superposition of Laguerre-Gaussian beams

*Towards pointing jitter resilient beam shapes
for intersatellite optical communications*

*The whole is equal to the sum of its parts
when the parts are unaware of one another.*

Almudena Grandes

This chapter proposes a novel approach to improve the performance of intersatellite optical links by combining fundamental Gaussian and higher-order Laguerre-Gaussian beams. To generate the desired beam profiles, we propose an optical system capable of creating a superposition of orthogonally polarized Laguerre-Gaussian beams that yield the far-field irradiance distributions that optimize the communication performance. Our theoretical analysis demonstrates that the combination of fundamental Gaussian and higher-order modes can significantly enhance system performance compared to conventional fundamental Gaussian beams. In some scenarios, the proposed approach offers savings on the order of 20% to 40% of the power required in the transmitter.

The content of this chapter is based on the Ref. [389] Badás M. et al. "On the optimum far-field irradiance distribution using Laguerre-Gaussian beams for intersatellite free-space optical communications," Optics Express 32, Issue 18, 31597-31620 (2024)

9.1. Introduction

As we have seen in the previous chapters, on an intersatellite link, the modulated laser is directed toward the receiver as accurately as possible. However, as seen in the previous chapters, several perturbations occurring in the space environment will induce microvibrations of the satellite, e.g., reaction wheels, thermomechanical and gravitational effects (see §3.3.1). All these sources combine to give rise to a stochastic pointing jitter that will deviate the optical axis of the transmitter from the center of the receiver aperture. The effect of this stochastic process is a fluctuation in the power collected by the receiver aperture that is detrimental to the communication performance. The high directivity of narrow laser beams can only be exploited if the transmitter pointing jitter is maintained within some operational range. In fact, as it is shown in §6.3.1 the divergence of the laser beams needs to be adjusted for the amount of pointing jitter that is expected in the transmitter terminal.

The effect of this power reduction on the communication performance of the link has been thoroughly studied in the past. Most of the work available in the literature combines the atmospheric and the pointing jitter effects [119, 175, 448], although some of them consider the intersatellite link scenario in which only the pointing jitter plays a role [118, 149, 150, 154]. Some of these papers not only present models to compute the effect of the pointing jitter but also optimize the link performance considering the detrimental effect of the pointing jitter (see §6). There are several approaches to optimize the link performance by varying different parameters and maximizing (or minimizing) different objective functions. Most of them consist of varying the beam divergence of the transmitted beam to minimize the average bit error probability (ABEP) [118, 154, 176] or the outage probability [175]. Furthermore, research has also been done on the receiver architectures and the modulation techniques used to communicate [149, 449]. On the other hand, a huge effort is being devoted to reducing the microvibrations on board satellites to reduce the pointing jitter of free-space optical communications (FSOC) terminals in space [171].

However, on intersatellite links, mostly Gaussian beam profiles have been investigated. Nevertheless, under the effect of the transmitter pointing jitter, the Gaussian irradiance distribution might not be the optimum for the communication performance on an intersatellite link. In this chapter, we propose to use a superposition of linearly orthogonally polarized higher-order Laguerre-Gaussian beams to improve the communication performance on intersatellite FSOC links. It is investigated whether these beams can improve the communication performance on intersatellite links and how they can be generated onboard an FSOC terminal. In §11, the concept of far-field beam shaping is further developed to determine the optimal far-field irradiance for mitigating transmitter pointing jitter.

Firstly, the model to compute the statistics of the received power and the resulting communication performance is presented for a given irradiance distribution on the receiver aperture (see §9.2). Furthermore, an optical system to generate these beams is also proposed, explaining the limitations of such a system and the design decisions made (see §9.3). Finally, the numerical results showing the optimum far-field distributions are presented, demonstrating an improvement in performance due to the beams proposed by the authors.

9.2. Received power statistics and communication performance

The high directionality of the laser beam in an FSOC link enables lower free-space power losses, meaning that compared to a less directional beam, more power will reach the receiver's aperture. However, this directionality can only be exploited up to a certain point due to the transmitter pointing jitter. For the satellite-based links studied in this chapter, this pointing jitter is due to micro-vibrations of the platform that originate in the orbit environment. The magnitude of the pointing jitter has been measured in several optical and quantum communication satellites and ranges from sub-microradians to milliradians (see §3). Although the deviation of the beam due to the microvibrations can be reduced with coarse and fine pointing assembly mechanisms (e.g., gimbals and fast steering mirrors), the pointing jitter in the transmitter terminal onboard a satellite will always occur to a greater or lesser extent. The transmitter pointing error deviates the center of the beam from the optical axis of the receiver. For a Gaussian beam, as the maximum power is concentrated in the center of the beam, this mispointing diminishes the power collected by the aperture of the receiver. Finally, these power fluctuations deteriorate the performance of the optical communication link. Hence, it is very important to correctly model and understand the magnitude of these power fluctuations and how they affect the communication performance of the system. In this section, a brief summary of the model presented in §6.2 for analyzing the received power statistics and the communication performance of the system is presented.

9.2.1. Received power statistics

As discussed in §6.2, the power received will vary over time as the transmitter pointing error changes due to jitter (see Fig. 9.1). However, as the received power variations due to the pointing jitter are orders of magnitude slower than the modulation period of the communication signal (the power fluctuations due to the pointing jitter will be below kHz while the modulation will be at least in the MHz domain), the slow-fading channel model can be considered for the intersatellite link [175].

The power captured by the receiver aperture for a given pointing error can be computed by integrating the irradiance field across the aperture (see (6.12)). Then, the expression of the power as a function of the displacement of the center of the beam w.r.t the optical axis of the receiver, $g(x_0, y_0)$, can be combined with the probability density function (PDF) of the displacement of the beam center to obtain the PDF of the received power (see (6.13)). In general, the PDF of the pointing jitter, and therefore of the displacement of the beam center from the receiver's optical axis, will be axially symmetric with respect to the optical axis of the receiver. Given that the PDF of the pointing jitter is axially symmetric and that the aperture of the receiver is also considered axially symmetric with respect to the same axis, the irradiance distributions that will be investigated throughout this chapter will keep this symmetry. Therefore, the irradiance distributions in the far-field will also be considered axially symmetric. Under these considerations, the power collected as a function of the displacement of the center of the beam will maintain this symmetry and can be expressed as (see

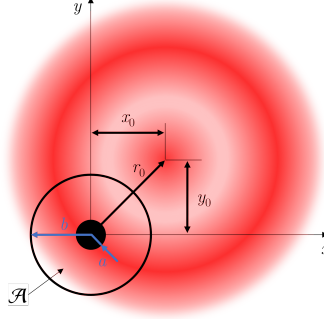


Figure 9.1: Generic far-field irradiance distribution in the receiver's aperture plane for a decentered beam. a and b are the inner and outer radii of the aperture, and r_0 is the instantaneous pointing error in meters.

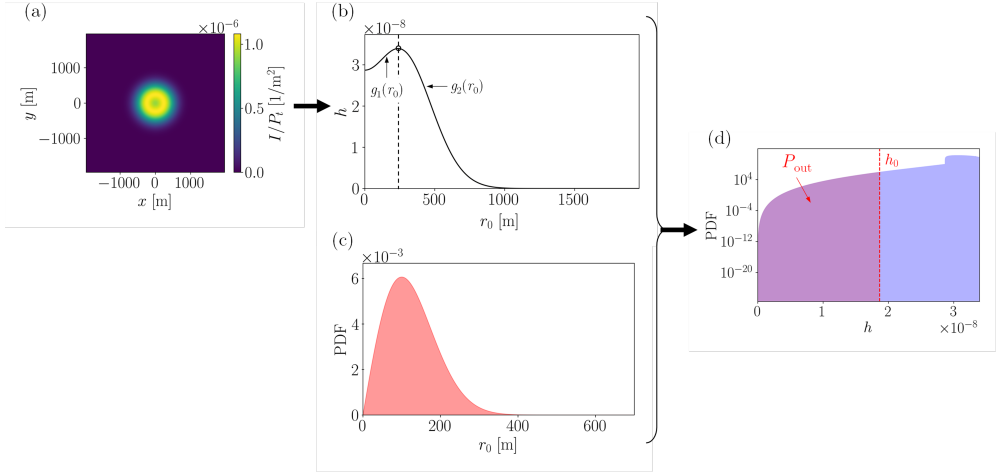


Figure 9.2: Process followed to obtain the received power statistics and the communication performance parameters for $LG_{00} + LG_{10}$ with $a_0 = 0.345$, $a_1 = 0.655$, $w_z/\sigma = 4.903$, $\sigma = 100$ m, $P_t = 25$ dBm, and the rest of the parameters given in Table 9.1. a_0 and a_1 are the contributions of the fundamental and the higher-order Laguerre-Gaussian modes to the total power, see Eq. (9.12). (a) Far-field irradiance distribution at the receiver, (b) normalized received power as a function of the pointing error, (c) probability density function (PDF) of the pointing error, and (d) PDF of the normalized received power.

Fig. 9.2b)

$$P = g(x_0, y_0) = g\left(\sqrt{x_0^2 + y_0^2}\right) = g(r_0), \quad (9.1)$$

where r_0 is the radial displacement of the center of the beam due to the transmitter pointing jitter. Finally, to obtain the PDF of the received power, given a PDF of the radial pointing jitter $f_R(r_0)$ (see Fig. 9.2c), in general, the expression for a non-monotonic function $g(r_0)$ will be considered. In the Gaussian beam case, the function $g(r_0)$ will be monotonic, and this conversion will be simpler. However, for the far-field irradiance patterns given by the orthogonally polarized superposition of Laguerre-Gaussian beams proposed in this chapter, the function $g(r_0)$ can be non-monotonic. To obtain the PDF of a given scalar variable (the received power P) from the PDF of another variable (the pointing error r_0), given that these variables are interdependent, the general expression for a non-monotonic $g(r_0)$ function is (see Ref. [392], Eq. 7.2-23)

$$f_P(P) = \sum_{k=1}^{n(P)} \left| \frac{d}{dP} g_k^{-1}(P) \right| \cdot f_R(g_k^{-1}(P)), \quad (9.2)$$

where $f_P(P)$ is the PDF of the received power, and $n(P)$ is the number of intervals between successive maxima and minima values of the function $g(r_0)$. $g_k^{-1}(P)$ is the k th inverse function of $g(r_0)$ in each of these intervals (see example in Figs. 9.2b and 9.2c, where the index k takes values 1, 2). In the following, different assumptions and considerations are made, and some analytical approximations are obtained for each of these.

Gaussian beam

First of all, a Gaussian beam will be considered. Hence, the pattern in the far-field irradiance will be Gaussian and can be written as

$$I_G(r, z) = I_0 \left(\frac{w_0}{w(z)} \right)^2 \exp \left(-\frac{2r^2}{w(z)^2} \right), \quad w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2}, \quad (9.3)$$

where I_0 is the irradiance at the center of the beam, w_0 is the beamwaist, r is the radial coordinate, z is the distance from the beamwaist location along the propagation axis of the beam, and $z_R = \pi w_0^2 n / \lambda$ is the Rayleigh range. The main body of research done in FSO has focused on these types of beams for several reasons. First of all, Gaussian beams can be generated by most of the commercially available laser sources, as they are the fundamental mode of a laser cavity. Furthermore, the Gaussian beam is the fundamental beam mode of the single-mode optical fibers that are frequently used in free-space optical communication terminals, i.e., optical amplification on the transmitter side [108]. Finally, the irradiance profile of the main lobe of the diffraction pattern due to the clipping of the transmitter aperture can be fairly approximated to a Gaussian irradiance profile [393].

Regarding the PDF of the angular pointing jitter, this is usually considered to be given by a Rayleigh distribution. The Rayleigh distribution is the result of the combination of independent centered normal PDFs in both elevation and azimuth angles with the same standard deviations [118]. For the small angle approximation (up to ~ 100 mrad), it can be proven that the angular Rayleigh distribution is converted into a radial Rayleigh distribution of the displacement of the center of the beam r_0

in the receiver's aperture plane (see App. D). Due to the stochastic nature of the pointing jitter and its multi-variable origin, the normal PDF in both elevation and azimuth angles is justified by the central limit theorem. Furthermore, the in-orbit experimental measurements of the micro-vibrations of different optical and quantum communication satellites have shown this type of angular pointing jitter PDFs [81, 85, 390]. The Rayleigh PDF of the displacement of the center of the beam, r_0 , is written as

$$f_R(r_0) = \frac{r_0}{2\pi\sigma^2} \exp\left(-\frac{r_0^2}{2\sigma^2}\right), \quad (9.4)$$

where σ is the scale parameter and is the same as the standard deviation of the underlying normal distributions in the x and y axes. The scale parameter is related to the scale parameter of the angular pointing jitter σ_θ by $\sigma = \sigma_\theta z$, where z is the distance from the center of the transmitter to the center of the receiver (small angle approximation).

Considering a Gaussian beam under a pointing jitter given by a Rayleigh distribution, Eqs. (9.1)-(9.2) can be combined with Eqs. (9.3) and (9.4) to find the following approximation of the received power [175]

$$f_{\mathcal{P}}(h) = \frac{\gamma_1^2}{B_0^{\gamma_1^2}} h^{\gamma_1^2-1}, \quad 0 \leq h \leq B_0, \quad (9.5)$$

$$B_0 = [\text{erf}(v_b)]^2, \quad v_b = \frac{\sqrt{\pi}b}{\sqrt{2}w_z}, \quad \gamma_1 = \frac{w_{\text{eq},1}}{2\sigma}, \quad w_{\text{eq},1}^2 = w_z^2 \frac{\sqrt{\pi}\text{erf}(v_b)}{2v_b \exp(-v_b^2)},$$

where $h = P/P_t$ is the received power normalized by the transmitted power, b is the radius of the receiver aperture, and $w_z = w(z)$ has been written to simplify the notation. This approximation is obtained by approximating the circular aperture (without the secondary mirror obscuration) to a square aperture of the same area and expanding the integral of the received power in a Taylor series. This approximation gives a normalized mean squared error smaller than 10^{-3} for $w(z)/b$ bigger than six [175]. However, when bigger apertures are needed in the receiver terminal, on-axis secondary mirrors are usually used for reflective telescopes in satellite communication terminals [102, 450]. Although off-axis telescope configurations can also be considered, they have several drawbacks such as more complex alignment or maximum field of view. Furthermore, due to the higher angles of incidence of the light impinging on the mirrors in off-axis configurations, the effects on the polarization have to be taken care of [451]. Therefore, an expression for the PDF of the received power has to be considered when there is an obscuration due to a secondary mirror. By approximating the secondary mirror obscuration area by an equivalent squared section with the same area, a more general approximation can be obtained (see App. D)

$$f_{\mathcal{P}}(h) = \frac{\gamma_2^2}{C^{\gamma_2^2}} h^{\gamma_2^2-1}, \quad 0 \leq h \leq C, \quad (9.6)$$

$$C = B_0 - A_0, \quad A_0 = [\text{erf}(v_a)]^2, \quad v_a = \frac{\sqrt{\pi}a}{\sqrt{2}w_z},$$

$$\gamma_2 = \frac{w_{\text{eq},2}}{2\sigma}, \quad w_{\text{eq},2}^2 = \left| \frac{A_0 - B_0}{B_2 - A_2} \right| w_z^2,$$

$$A_2 = -\frac{2}{\sqrt{\pi}} \text{erf}(v_a) [v_a \text{erf}(-v_a^2)], \quad B_2 = -\frac{2}{\sqrt{\pi}} \text{erf}(v_b) [v_b \text{erf}(-v_b^2)],$$

where B_0 and v_b are defined in Eq. (9.5). The Rayleigh distribution considered for characterizing the pointing jitter is a special case of the more general Rice distribution. The Rice distribution has also been considered to model the pointing jitter when considering non-zero mean normal distributions in both elevation and azimuth angles [119]. This non-zero mean is referred to as boresight error in optical communications and can be due to several factors. For an intersatellite link, it is usually considered that the pointing jitter has a negligible boresight error because optical axis calibration of the transmitted beam is carried out periodically [118]. However, wavefront aberrations could induce misalignment between the transmitted beam path and the received beacon paths, which could cause a systematic bias in pointing jitter [372]. Therefore, for the sake of completeness, the non-zero boresighted PDF of the pointing jitter, i.e., the Rice probability distribution [452], is also considered in this chapter. The Rice distribution is given by

$$f_R(r_0) = \frac{r_0}{\sigma^2} \exp \left[\frac{-(r_0^2 + s^2)}{2\sigma^2} \right] \mathcal{J}_0 \left(\frac{r_0 s}{\sigma^2} \right), \quad (9.7)$$

where σ is the scale parameter, $\mathcal{J}_0$ is the zeroth-order modified Bessel function of the first kind, and s is the boresight error. For a Gaussian beam under a transmitter pointing jitter characterized by a Rice PDF, the PDF of the power for a circular aperture can be approximated as [119]

$$f_{\mathcal{P}}(h) = \frac{\gamma_1^2 \exp \left(\frac{-s^2}{2\sigma^2} \right)}{B_0^{\gamma_1^2}} h^{\gamma_1^2-1} \mathcal{J}_0 \left(\frac{s^2}{\sigma^2} \sqrt{\frac{-w_{\text{eq},1} \ln \frac{h}{B_0}}{2}} \right), \quad 0 \leq h \leq B_0, \quad (9.8)$$

where the terms are defined in Eq. (9.5). Following the same process as for the Rayleigh distribution, this PDF can be generalized to the case in which there is an obscuration due to the secondary mirror to

$$f_{\mathcal{P}}(h) = \frac{\gamma_2^2 \exp \left(\frac{-s^2}{2\sigma^2} \right)}{C^{\gamma_2^2}} h^{\gamma_2^2-1} \mathcal{J}_0 \left(\frac{s^2}{\sigma^2} \sqrt{\frac{-w_{\text{eq},2} \ln \frac{h}{C}}{2}} \right), \quad 0 \leq h \leq C, \quad (9.9)$$

where the terms are defined in Eqs. (9.5) and (9.6). When the aperture of the receiver is much smaller than the beamwidth $w(z) \gg b$, then it can be shown (see App. D for the proof) that under a Rayleigh PDF of the transmitter pointing jitter the PDF of the received power can be modeled as a beta distribution

$$f_{\mathcal{P}}(h') = \left(\frac{\pi w_z^2}{2A^2} \right)^{\beta-1} \beta h'^{\beta-1}, \quad 0 \leq h' \leq \frac{2A^2}{\pi w_z^2 P_t}, \quad (9.10)$$

where $\beta = w_z^2/(4\sigma^2)$, A is the area of the receiver aperture, $h' = P/I_{0,z}A$ and $I_{0,z}$ is the irradiance at the center of the Gaussian beam in the receiver aperture plane. In this case, both expressions for the clear aperture telescope and on-axis secondary mirror configurations will be the same, as only the area of the aperture is involved and the variation of the irradiance across the aperture of the receiver is completely neglected.

From the expressions above, it can be seen that the PDF of the received power can be varied by changing the beamwidth of the Gaussian beam in the receiver aperture plane $w(z)$. This is equivalent to changing the divergence of the Gaussian beam, and the approach has been thoroughly investigated in the literature [175, 176, 453]. The divergence of the Gaussian beam is adjusted by properly designing the optical relay on the transmitter side to adjust its underlying beamwaist w_0 in Eq. (9.3). Furthermore, dynamic beam divergence adjustment methods have also been proposed to adjust the divergence [454]. Indeed, an intuitive explanation can be found for the existence of an optimum beamwidth in the far-field to optimize the power received: when the beamwidth is very small compared to the aperture of the receiver, it will be very sensitive to the pointing error, and the signal will be completely lost when the beam center is slightly displaced. This effect can be compensated by increasing the beamwidth; however, maintaining the transmitted power constant, for large beamwidths, the power collected by the receiver aperture will be very low. Hence, an optimum point will exist somewhere in between these two extremes given a pointing jitter and a receiver aperture. The optimum received power PDF for communication performance will be discussed in the next section.

Orthogonally polarized Laguerre-Gaussian beams

In this chapter, the use of higher-order Laguerre-Gaussian beams is proposed to change the far-field irradiance distribution. This new degree of freedom (added to the far-field beamwidth) will allow us to further change the received power PDF to increase the performance of intersatellite communication links. The far-field irradiance distribution obtained when non-interfering higher-order Laguerre-Gaussian modes are considered is (a way of achieving the non-interference condition is presented in §9.3)

$$I(r, z) = \frac{1}{2\eta_0} \sum_{\ell} (a_{\ell} |U_{\ell p}(r, \phi, z)|^2), \quad (9.11)$$

where $\eta_0 = 377 \Omega$ is the wave impedance of free-space, a_{ℓ} are the weighting coefficients for each azimuthal mode superposed, and $U_{\ell p}$ is the complex electromagnetic field of the Laguerre-Gaussian mode with radial index $p \geq 0$ and azimuthal index ℓ . When the radial index is set to $p = 0$, and only the fundamental Gaussian mode and a higher-order Laguerre-Gaussian mode are considered (as will be done in this chapter), the far-field irradiance expression above simplifies to

$$I(r, z) = \frac{1}{2\eta_0} (a_0 |U_{00}(r, \phi, z)|^2 + a_{\ell} |U_{\ell 0}(r, \phi, z)|^2), \quad (9.12)$$

where ℓ is the azimuthal order of the higher-order Laguerre-Gaussian mode considered. Considering the $a_0 + a_{\ell} = 1$, the final irradiance field is normalized to the power unit.

The complex electromagnetic field for each Laguerre-Gaussian mode is given by

$$U_{\ell p}(r, \phi, z) = C_{\ell p} \frac{1}{w(z)} \left(\frac{r\sqrt{2}}{w(z)} \right)^{|\ell|} \exp\left(-\frac{r^2}{w^2(z)}\right) L_p^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right) \times \exp\left[-i\left(k\frac{r^2}{2R(z)} + \ell\phi - \psi(z)\right)\right], \quad (9.13)$$

where

$$R(z) = z \left[1 + \left(\frac{z_R}{z} \right)^2 \right], \quad (9.14)$$

$$\psi(z) = (|\ell| + 2p + 1) \arctan\left(\frac{z}{z_R}\right), \quad (9.15)$$

$$C_{\ell p} = \sqrt{\frac{2p!}{\pi(p + |\ell|)!}}, \quad (9.16)$$

$\psi(z)$ being the Gouy phase, $C_{\ell p}$ a normalization constant and $L_p^{|\ell|}$ the generalized Laguerre polynomials. The Laguerre-Gaussian modes are the solution to the scalar paraxial Helmholtz equation in polar coordinates [437]. The complex scalar field of these beams generates an orthonormal basis. Furthermore, these beams have an orbital momentum of light associated with the azimuthal order ℓ . The orbital angular momentum of light has been widely investigated to increase the capacity of free-space optical communication links by mode division multiplexing [67, 455–457]. The application of this division multiplexing technique is limited to short-range free-space optical links because the beam has to be fully contained within the aperture of the receiver to demultiplex the signal [458, 459]. However, the scope of this chapter is to use higher-order Laguerre-Gaussian modes along with the fundamental Gaussian beam to change the irradiance distribution on the far-field for intersatellite free-space optical communications. Laguerre-Gaussian beams have been considered to solve the problem because of their symmetry around the optical axis in their irradiance fields. As it has been mentioned before, the pointing jitter in free-space optical communications is symmetric to the optical axis, and hence the solution to it will also keep this property. As will be shown in §11, the far-field irradiance obtained through the orthogonally polarized superposition of Gaussian and Laguerre-Gaussian beams aligns with the rationale of redistributing the irradiance toward a more uniform far-field pattern. Furthermore, Laguerre-Gaussian beams have already been used for other applications in free-space optical communications, such as multiplexing and noise mitigation [460], and their behavior under atmospheric turbulence has also been investigated [461]. Hence, the research presented in this chapter can be expanded to FSOC links through atmospheric turbulence (the irradiance fields on the receiver aperture would need to be computed to account for the speckle patterns generated during the propagation through turbulence). As it will be explained later, the generation of these beams can be accomplished with a simple optical setup and, therefore, implemented on satellite optical communication terminals.

Other types of beams have also been proposed for their application in FSOC links, such as Airy beams [462, 463], Bessel beams [464–466], bottle vortex beams [467]

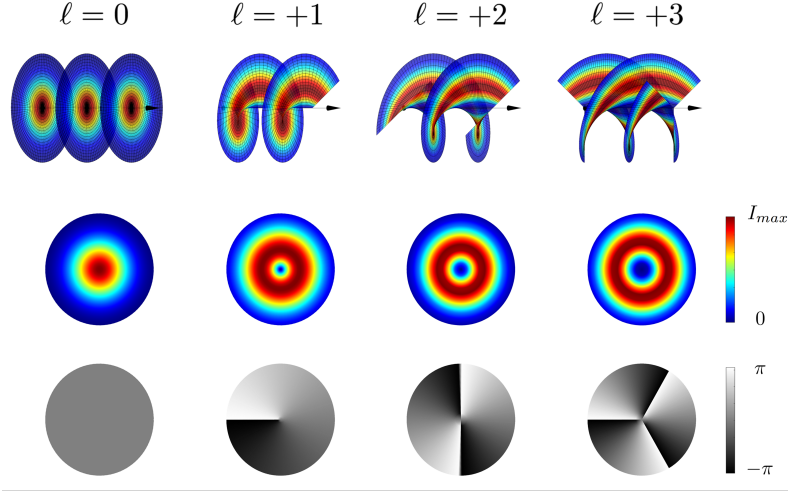


Figure 9.3: Laguerre-Gaussian modes of order $\ell = 0, +1, +2, +3$ and radial order $p = 0$. The first row represents irradiance and phase, the second row represents the irradiance field, and the third row represents the phase field.

and Hermite-Gaussian beams [468, 469]. However, the use of these types of beams has been limited to (1) increasing the capacity of the channel through multiplexing, (2) avoiding obstructions in the link, and (3) mitigating the atmospheric turbulence effects. By contrast, this chapter proposes combining higher-order Laguerre-Gaussian beams to mitigate the detrimental impact of transmitter pointing error by tailoring the far-field irradiance distribution in the receiver's aperture plane.

Some Laguerre-Gaussian modes are presented in Fig. 9.3. In this figure, only the radial order $p = 0$ is presented, as it is the one that will be proposed to obtain the far-field irradiance distributions of interest. Other radial orders could be considered; however, given the nature of the PDF of the pointing jitter, the various null rings in between bright irradiance rings of higher-order modes will not, in principle, be that interesting. Furthermore, the higher the radial mode, the higher the beam divergence, and this could limit their application to the problem at hand [470, 471].

As it has been explained above, under the stochastic behavior of the pointing jitter, the power received in the optical communication terminal will be characterized by the PDF of the power. The analytical approximation obtained for the Gaussian beam for different scenarios can not be in general expanded to the combination of various orthogonally polarized Laguerre-Gaussian modes. Hence, to obtain the PDF of the power received when far-field irradiance fields described by Eq. (9.12) are used, the power as a function of the radial displacement of the center can be obtained with Eq. (9.1). It is important to emphasize that on the receiver side, no polarization or OAM division is performed on the resulting beam, coupling the power captured by the receiver's aperture to the detector. Then, numerically evaluating Eq. (9.2), the PDF of the power can be obtained for Rayleigh and Rice distributions with Eqs. (9.4) and (9.7), respectively (see App. D for more details).

9.2.2. Communication performance

The general stochastic communication model for intersatellite optical links under pointing jitter has already been developed in §6.2.5. In particular, it established how the received power PDF, resulting from the far-field irradiance distribution and the pointing error statistics, enables the computation of relevant performance metrics. In this chapter, we restrict the analysis to two key performance parameters: the average bit error probability (ABEP) and the probability of outage. The slow temporal dynamics of transmitter pointing jitter, whose spectral content lies in the kHz range compared to the MHz–GHz modulation frequencies typical of intersatellite links, justify the use of a slow-fading channel model. Under this assumption, many consecutive bits experience the same pointing-induced attenuation, and both ABEP and outage probability can be directly obtained from the received power PDF.

Specifically, for an on-off keying intensity modulation direct detection (OOK IM/DD) intersatellite link, the ABEP is computed by averaging the instantaneous bit error probability over the normalized received power PDF, while the outage probability is defined as the probability that the instantaneous channel capacity falls below a target transmission rate. The corresponding expressions for both quantities can be found in (6.16) and (6.18). Hence, combining the received power PDF derived in §9.2.1 with the ABEP and outage probability, the communication performance of any far-field irradiance distribution under transmitter pointing jitter can be systematically evaluated.

9.3. Optical system design

In the previous section, the mathematical models to compute the ABEP and outage probability for a given far-field irradiance distribution and a pointing jitter PDF have been presented. Our goal is to find an optimum far-field irradiance distribution that maximizes the communication performance of an intersatellite link by minimizing either of these communication performance parameters. In this section, an optical setup that can generate these types of far-field distributions using linearly orthogonally polarized Laguerre-Gaussian beams is presented.

However, when finding an optimum far-field irradiance distribution to minimize either the ABEP or the outage probability in an intersatellite communication link, there are physical limitations that must be taken into account. The problem of finding the far-field irradiance distribution that gives the minimum ABEP or the minimum P_{out} is highly degenerate (meaning that it has many solutions) if the limits imposed by physics are not considered. This means that there can be many far-field irradiance distributions that yield the same ABEP or outage probability. This can be understood directly in the P_{out} case: two far-field irradiance distributions that yield the same cumulative density function of the power for the value h_0 given by Eq. (6.18) (see Fig. 9.2d), will give the same P_{out} . Therefore, the mathematical problem of finding the optimum far-field without considering any more restrictions is out of the scope of this chapter. However, there are certain limits when we consider the physics of wave propagation and the stable far-field electromagnetic fields allowed by this. Indeed, we are looking for a solution that has the following main characteristics:

1. It has to be a stable far-field solution. Stable here means that the far-field irradiance has to change smoothly with distance from the transmitter. For example, top-hat beams that have been considered in other applications [472], i.e., laser welding, are not stable solutions as they will rapidly decay in all kinds of patterns that are different from the top-hat out of the focal region.
2. It has to be simple to generate. To assure a reasonable practical implementation of such a far-field distribution, there should be a simple optical system that can be put on board a satellite terminal. Indeed, the simplicity will favor lower losses in the transmitter. If the complexity of the system to generate such far-field irradiance distribution is too high, and it involves many optical components, the losses incurred in this system would need to be more than compensated by the resulting far-field distribution gains.

The use of pure or approximate Gaussian beams in free-space optical communications has been thoroughly investigated in previous works [52, 393, 422, 473]. So far, only the adjustment of the divergence of the Gaussian beam has been proposed to mitigate the effects under a given transmitter pointing jitter [118, 154, 175, 176]. In this chapter, the use of superposed orthogonally polarized higher-order Laguerre-Gaussian beams is investigated to generate a better far-field distribution than in the fundamental Gaussian beam case. These beams will comply with the two requirements imposed above as it will be explained in detail in this section.

To generate a far-field irradiance profile that is given by a superposition of the irradiance fields of two Laguerre-Gaussian modes, the two modes involved should not interfere (see Eq. (9.12)). In this chapter, the latter is attained by using orthogonal polarization states for each of the beams. Fig. 9.4 shows an optical setup that combines two orthogonally polarized beams to generate different far-field irradiance distributions. The linearly polarized Gaussian fundamental beam from the laser source is collimated by the collimating lens. The rotatable half-wave plate (RHWP) can be used to determine the direction of the linear polarization for the fundamental Gaussian beam entering the polarization beam splitter. On the one hand, if the polarization state after the RHWP is vertical, then the beam will be reflected in the polarization beam splitter (PBS) and will travel to the mirror M1. On the other hand, when the outgoing beam from the RHWP is horizontally polarized, the beam will go straight through the PBS and reach mirror M2. For intermediate polarization states, the beam will be split in the PBS. Therefore, by rotating the fast axis of the RHWP we can adjust the amount of energy that travels through each of the paths.

The fundamental Gaussian beam traveling to mirror M1 is transmitted through the spiral phase plate (SPP), which imparts a helical phase profile onto the wavefront and thereby transforms the beam into a higher-order beam carrying orbital angular momentum. A SPP is a plate with a helical surface that introduces a corresponding spatially varying phase delay across the beam aperture [474–479] (see upper left side in Fig. 9.4). The SPP is used here to generate the higher-order beam due to its polarization-maintaining property [480]. The helical surface of the SPP is designed

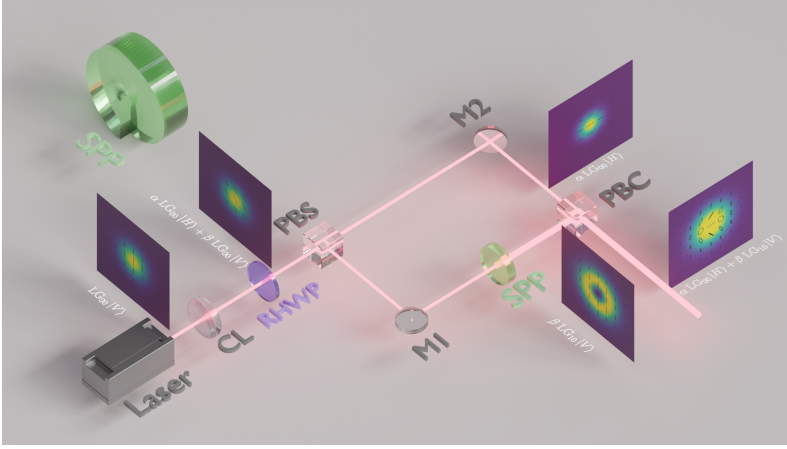


Figure 9.4: Optical setup to generate the far-field irradiance distributions of interest. Collimating lens (CL), rotatable half-wave plate (RHWP), polarization beam splitter (PBS), mirror 1 (M1), mirror 2 (M2), spiral phase plate (SPP), polarization beam combiner (PBC). In the upper left corner, an SPP is illustrated out of scale. $\alpha = a_0$ and $\beta = a_\ell$ in Eq. (9.12).

such that the total height jump s_{SPP} is given by

$$s_{\text{SPP}} = \frac{\lambda \ell}{n_{\text{SPP}} - 1}, \quad (9.17)$$

where λ is the wavelength, ℓ is the azimuthal order associated with the imposed orbital angular momentum, and n_{SPP} is the refractive index of the SPP material.

Strictly speaking, a fundamental Gaussian beam impinging on a spiral phase plate does not generate an exact Laguerre–Gaussian mode. Instead, it produces an annular beam carrying orbital angular momentum whose field distribution closely approximates that of the corresponding Laguerre–Gaussian mode with the same topological order ℓ . An analytical and numerical demonstration of this approximation, and of the resulting modal content of the generated beam, is presented in §10. Nevertheless, for the remainder of this chapter, these generated higher-order beams are modeled as ideal Laguerre–Gaussian modes, since this approximation captures the relevant far-field irradiance features that govern the pointing-jitter sensitivity and link performance.

Later in the optical relay, the horizontally polarized fundamental Gaussian beam is reflected on mirror M2 and combined with the higher-order, vertically polarized beam in the final polarization beam combiner. The resulting composite beam is then directed to the telescope of the transmitter terminal and sent toward the receiver. In this setup, the modulation of the laser beam is not explicitly included. For this chapter, it is assumed that the laser has already been intensity-modulated for communication purposes before entering the proposed optical arrangement.

The proposed optical system and the resulting communication channel have several limitations when it comes to the detection system, multiplexing techniques, and modulation techniques that can be used. Due to the use of two orthogonally polarized

states, the resulting far-field will have a non-uniform linear polarization. Due to these spatial position-dependent polarization states, the use of coherent detection is not trivial for the proposed system. Indeed, coherent detection systems use the interference between the incoming light and a local oscillator onboard the receiver. To create this interference, the polarization state of these two beams has to be the same. As the polarization of the incoming beam to the receiver has both horizontally polarized and vertically polarized components that are dependent on the position, with different pointing errors of the transmitter, we will obtain different incoming polarization states (see ellipses in Figs. 9.4, 9.12 and 9.13). A way to partially compensate for these changes would be a polarization state locker that changes to ensure that the polarization of the local oscillator and the incoming beam remains the same [481]. However, this is out of the scope of this thesis and has to be further investigated. On the other hand, a direct detection system will detect the incoming beam power directly without caring for the polarization state of the beam; hence, the proposed system is fully compatible with this kind of detection system.

Concerning the multiplexing and modulation techniques, polarization multiplexing and polarization modulation can not be used because this property of light is already being exploited to combine without interference the fundamental Gaussian and the higher-order Laguerre-Gaussian beams. Furthermore, spatial division multiplexing or modulation using Laguerre-Gaussian modes can not be used for the same reason. This limitation is not especially relevant for long-range communication links, as spatial division multiplexing or modulation requires that a significant portion of the beamwidth is captured by the receiver aperture [156]. Even for small transmitter pointing jitters, the displacement of the transmitted beam from the receiver optical axis in the far-field will be too high for capturing a significant section of the space division multiplexed beam.

In Fig. 9.5, a simulation using Jones calculus and Fourier propagation with gold-protected mirrors at 1550 nm wavelength is presented for the optical setup proposed in this chapter. In this graph, the variation of the power directed through each of the optical branches in the setup (each with either s-polarization or p-polarization) is shown. The losses due to the imperfections of the other elements have not been considered, e.g., the transmission losses and crosstalk in the PBS. It can be seen that the fraction of power directed through each of the branches, and therefore contributing to either the fundamental Gaussian mode or the higher-order Laguerre Gaussian mode, is dependent on the direction of the RHP's fast axis ψ . The fraction of power P/P_{tot} for $LG_{00}|H\rangle$ and $LG_{\ell 0}|V\rangle$ plotted in Fig. 9.5, corresponds to the weights a_0 and a_ℓ in Eq. (9.12). Therefore, the optical system proposed in Fig. 9.4 allows us to tune the far-field irradiance as aimed.

As explained above, this chapter proposes two orthogonally polarized states to avoid a variation of the irradiance due to interference between the fundamental and the higher-order Gaussian beams. However, other properties of light can be exploited for the purpose of non-interference. Indeed, two sufficiently separated wavelengths coming from different sources could be used. Analogous to what is proposed with two orthogonally polarized states in Fig. 9.4, each of the different beams with respective wavelengths would travel different paths, giving rise to a multispectral far-field. How-

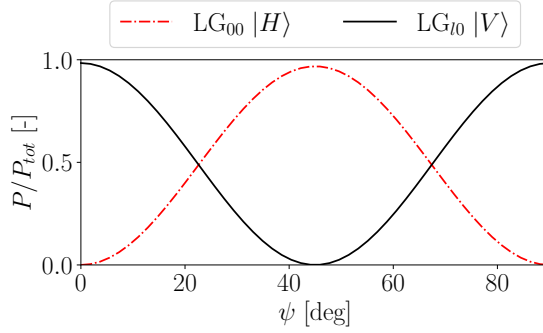


Figure 9.5: Variation of the power going through the two branches of the optical setup shown in Fig. 9.4, as a function of the RHPW fast axis angle w.r.t. the vertical axis, ψ . The light incoming to the RHPW is considered to be vertically polarized (see Fig. 9.4). The complex index of refraction of the gold-protected mirrors is $n = 0.56 + 9.81i$ and the simulated wavelength is $\lambda = 1550$ nm.

ever, there would be several drawbacks related to this kind of setup. Firstly, several laser sources would need to be used and modulated simultaneously. Furthermore, different wavelengths are usually used to separate the different lasers (receiving the beacon, calibration laser, and transmitted beacon) traveling through the terminal or to increase the capacity of the channel through wavelength division multiplexing [55, 455].

Furthermore, other devices aside from the SPP depicted in Fig. 9.4 can be used to generate a higher-order Laguerre-Gaussian beam. Several ways have been proposed to generate these types of beams, including the use of spatial light modulators, micromirror arrays, metalenses, and vortex phase plates [482]. The main reasons to use a SPP have been its polarization-maintaining properties and its simplicity and easier implementation for space terminals. Vortex phase plates generate spatially dependent polarization states of the output higher-order beam, and hence this beam would interfere with the fundamental Gaussian beam traveling through the other path. The authors are currently considering metalenses as a potential device to compactly generate the far-field irradiance patterns of interest.

9.4. Numerical results

The optical design presented in §9.3 can be used to generate the far-field irradiance pattern given by a superposition of the fundamental and a higher-order Laguerre-Gaussian mode. Using different SPPs, the azimuthal order ℓ of the higher-order Laguerre-Gaussian mode can be adjusted. In this chapter the results for the combinations $LG_{00} + LG_{10}$, $LG_{00} + LG_{20}$, $LG_{00} + LG_{30}$ and $LG_{00} + LG_{40}$ will be investigated and compared to the fundamental Gaussian beam LG_{00} . With the mathematical model presented in §9.2, the results obtained for the communication performance parameters are presented in this section. To illustrate the intermediate results obtained through the different steps of the model, the reader is referred back to Fig. 9.2, which

illustrates the power statistics and the communication performance obtained for a certain set of parameters for the case $\text{LG}_{00} + \text{LG}_{10}$. First of all, the far-field irradiance distribution is obtained given the coefficients a_i corresponding to each of the Laguerre-Gaussian modes (see Eq. (9.12)) and the beamwidth in the receiver aperture plane w_z (Fig. 9.2a). Then, the normalized received power, h , as a function of the pointing error can be computed using the convolution theorem in Eq. (6.12). This is done by computing a bidimensional Fast Fourier Transform. Finally, combining the latter with the PDF of the pointing jitter (Fig. 9.2c), the PDF of the normalized received power, h , is obtained (Fig. 9.2d) using Eq. (9.2)). With this PDF, the ABEP and outage probability can be computed to evaluate the communication performance. Depending on the receiver telescope design (with on-axis secondary mirror or clear aperture) and the pointing jitter statistics (Rayleigh distribution for non-boresight pointing error and Rice distribution for boresighted pointing error), the respective analytical approximations presented in §9.2.1 can be used to compute the PDF of the received power for the fundamental Gaussian beam case. For the far-field irradiance pattern given by the superposition of the fundamental and the higher-order Laguerre-Gaussian beam, numerical evaluation is required as illustrated in Fig. 9.2.

Parameter	Symbol	Value	Remarks
Noise standard deviation	$\sigma_n^2 = N_0 R_0 / R_{load}$	4.7×10^{-9} A	[175]
Noise power spectral density	N_0	-174 dBm/Hz	[154]
Receiver load resistance	R_{load}	179700 Ω	[176]
Receiver aperture diameter	$2b$	20 cm	
Pointing jitter standard deviation	σ	10, 100, 1000 m	
Carrier wavelength	λ	1550 nm	
Responsivity	$R = e\eta/h\nu$	0.8153 A/W	[154]
Data rate	R_0	1 Gbps	
Rate	D_0	0.5 bits/channel use	
Average Transmitter power	P_t	$\in (0, 50)$ dBm	

Table 9.1: Parameters used for the numerical simulations.

In an intersatellite link, the distances can vary from the order of hundreds of kilometers for LEO optical links to astronomical distances for deep space optical communications. Furthermore, the transmitter angular pointing jitter, σ_θ , can also vary from sub-microradian [81] to tens of milliradians. In this chapter, several resulting far-field pointing jitter values are analyzed (see Table 9.1). The respective relations between the link distance z and the angular pointing jitter σ_θ can be seen in Fig. 9.6 for several values of the far-field pointing jitter σ considered in this chapter.

The parameters used for the simulations have been obtained from the literature and are presented in Table 9.1. In Fig. 9.7, the values of the ABEP and the outage probability obtained for the optimum combination of LG_{00} and LG_{10} beams can be seen, along with the extreme values obtained using pure LG_{00} ($a_0 = 1$) and LG_{10} ($a_0 = 0$) beams. In these figures, the values for the optimum far-field distribution have been plotted for the respective communication performance parameters. It can be seen that a combination of the fundamental and higher-order Laguerre-Gaussian beams proposed by the authors yields a better performance of the system compared

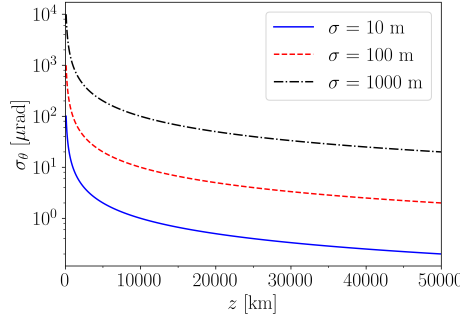


Figure 9.6: Relation between the standard deviation of the angular pointing jitter σ_θ and the link distance z for several far-field pointing jitter values σ .

to the fundamental Gaussian beam, as both the ABEP and the outage probability can be further minimized by using the proposed beams. Furthermore, it can also be seen that the far-field irradiance distribution required to obtain an optimum performance in ABEP is different from the one giving the optimal performance in outage probability. This behavior can also be seen in Fig. 9.8, where the change as a function of both the beamwidth w_z and the contribution of the fundamental LG_{00} Gaussian beam a_0 is presented, along with the locations of the minima. The same results are plotted in Figs. 9.9, 9.10 and 9.11, when using a combination of the $\text{LG}_{00} + \text{LG}_{20}$, $\text{LG}_{00} + \text{LG}_{30}$ and $\text{LG}_{00} + \text{LG}_{40}$ beams, respectively. From these figures, it can be seen that any of the beams proposed present an improvement with respect to the performance obtained for the fundamental Gaussian beam.

This performance improvement provided by the proposed beams is obtained through the following mechanism: as the transmitter pointing changes, the section of the far-field irradiance that is captured by the receiver aperture changes. The stochastic behavior of the value of h due to these dynamics is captured in the PDF of the normalized received optical power h . Hence, the PDF of h is determined by the far-field irradiance shape as well as the PDF of the pointing jitter. Therefore, by changing the far-field irradiance through a superposition of orthogonally polarized Laguerre-Gaussian beams, the PDF of h can be changed. This changes the PDF of the SNR, which finally changes the values of ABEP and outage probability (Eqs. (6.16) and (6.18)).

The optimum far-field irradiance patterns for obtaining the minimum ABEP are shown in Fig. 9.12. Each of these far-field irradiance distributions is the one given by the white circle on the left figures of Figs. 9.8-9.11. The resulting polarization ellipse of the field in each point is plotted in these figures, as it would be created from the optical setup presented in §9.3. These polarization ellipses are obtained by computing the contribution to the total irradiance by each of the modes at each point on the receiver aperture plane (see Eq. (9.12)). Analogously, the optimum far-field irradiance patterns for obtaining the minimum outage probability are shown in Fig. 9.13 (which correspond to the white circles on the right figures of Figs. 9.8-9.11). Several features

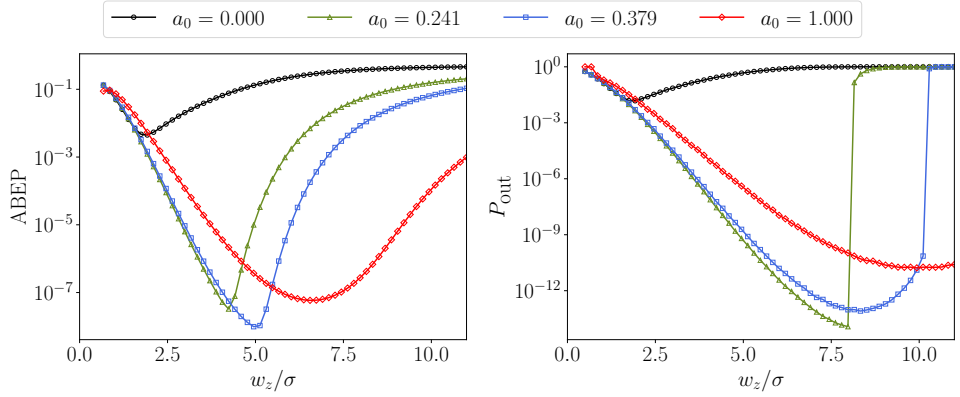


Figure 9.7: ABEP and outage probability values for $\sigma = 100$ m and $P_t = 29$ dBm, for different combinations of LG_{00} and LG_{10} modes as a function of w_z . $a_0 = 0.241$ and $a_0 = 0.379$ give the minimum outage probability and ABEP, respectively.

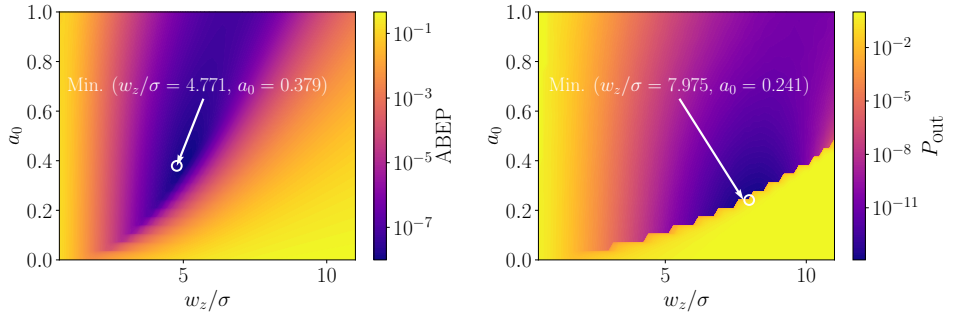


Figure 9.8: ABEP and outage probability values for $\sigma = 100$ m and $P_t = 29$ dBm, for LG_{00} and LG_{10} modes as a function of a_0 and w_z .

can be extracted from these figures

1. The optimum far-field irradiance for minimizing ABEP concentrates the power more towards the center receiver optical axis, than the optimum irradiance minimizing for outage probability. This feature can be seen in Figs. 9.12 and 9.13. This is true even for the fundamental Gaussian beam LG_{00} (see minima of the red curves in Fig. 9.7).
2. The higher the order of the Laguerre-Gaussian beam considered, the lower the beamwidth w_z that results in the optimum far-field irradiance for both ABEP and outage probability (see Figs. 9.8-9.11).
3. In the analyzed figures, it can be seen that the minima given by the superposition of orthogonally polarized Laguerre-Gaussian beams are more pronounced

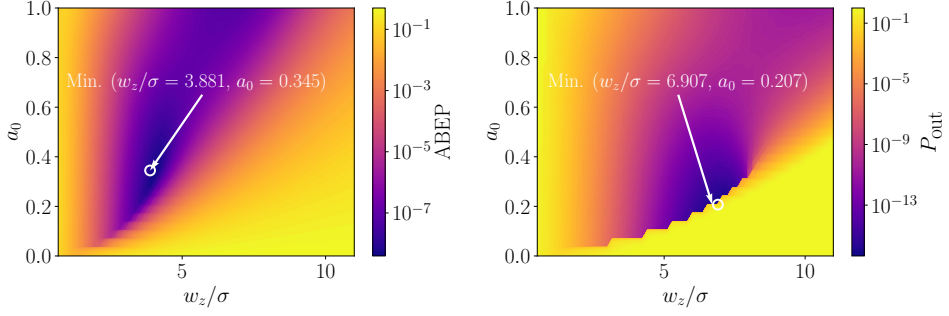


Figure 9.9: ABEP and outage probability values for $\sigma = 100$ m and $P_t = 29$ dBm, for different combinations of LG_{00} and LG_{20} modes as a function of w_z .

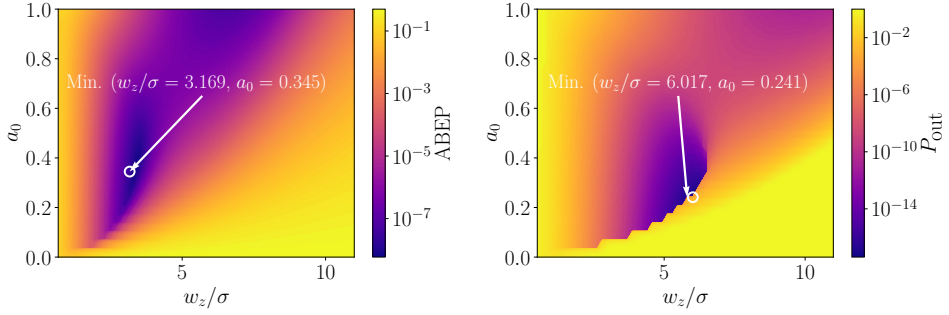


Figure 9.10: ABEP and outage probability values for $\sigma = 100$ m and $P_t = 29$ dBm, for different combinations of LG_{00} and LG_{30} modes as a function of w_z .

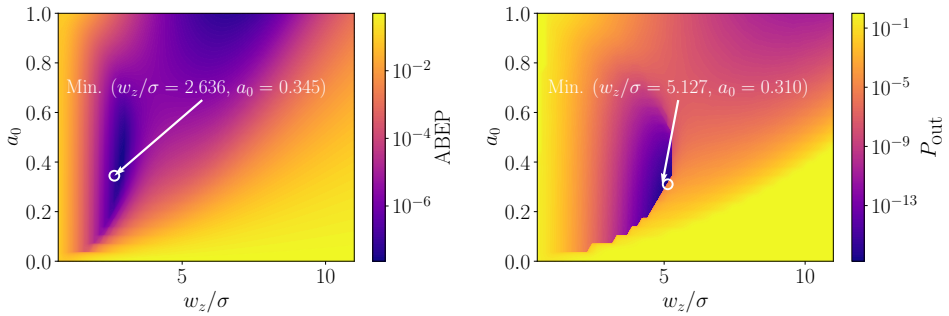


Figure 9.11: ABEP and outage probability values for $\sigma = 100$ m and $P_t = 29$ dBm, for different combinations of LG_{00} and LG_{40} modes as a function of w_z .

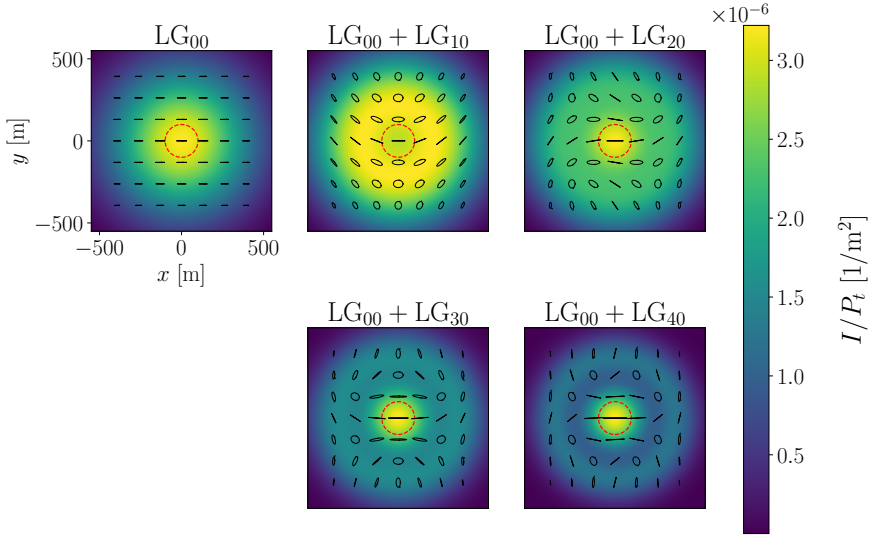


Figure 9.12: Far-field irradiance distributions yielding the optimum ABEP. Each figure shows the far-field distribution for the corresponding combination of Laguerre-Gaussian beams, for $\sigma = 100$ m and $P_t = 29$ dBm. The dashed red line shows the magnitude of the pointing jitter scale factor σ . The lines represent the resulting polarization ellipses of the electric field, where the size of the ellipse is proportional to the irradiance at each point. x and y axis are the same for all figures.

than the ones given by the fundamental Gaussian beam (see Figs. 9.7-9.11). In practice, a system operating in one of these minima would have a more sensitive behavior under changes in the beamwidth w_z , compared to the fundamental Gaussian beam. This sensitivity is even more pronounced for the outage probability, when operating in the optimum far-field outage probability (see for example, the minimum of the green curve in the right figure of Fig. 9.7).

4. Spatially varying polarization states in free-space propagating vortex beams, as shown in the figures, are linked to an underlying topological property, which can be characterized by a Skyrmionic number. Beams exhibiting this feature—referred to as *paraxial Skyrmionic beams*—have been previously proposed [483, 484], where their topological nature is quantified through the spatial distribution of the polarization Stokes vector. For the specific case of the beams $\text{LG}_{00} + \text{LG}_{\ell 0}$ shown in Figs. 9.12-9.13, the Skyrmionic number can be readily shown to be equal to ℓ , as demonstrated in [484]. Although these topological properties are noteworthy, they arise here as a byproduct rather than the fundamental mechanism behind the performance improvements discussed in this chapter. In par-

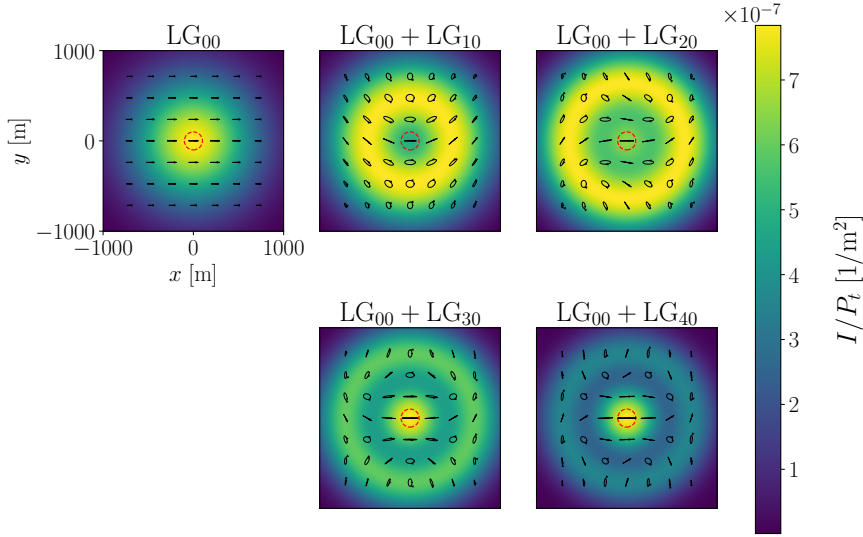


Figure 9.13: Far-field irradiance distributions yielding the optimum P_{out} . Each figure shows the far-field distribution for the corresponding combination of Laguerre-Gaussian beams, for $\sigma = 100$ m and $P_t = 29$ dBm. The dashed red line shows the magnitude of the pointing jitter scale factor σ . The lines represent the resulting polarization ellipses of the electric field, where the size of the ellipse is proportional to the irradiance at each point. x and y axes are the same for all figures.

ticular, the spatially varying polarization state—responsible for the Skyrmionic characteristics—is not desirable in intersatellite optical communication, as it renders the polarization degree of freedom unsuitable for other applications, such as multiplexing. Hence, the topological aspects of the obtained beams (both in this and the next chapters) will not be discussed in this work.

To evaluate how much the proposed beams can drive the design of future intersatellite optical terminals, Figs. 9.14-9.16 show the minimum ABEP and outage probabilities achievable for different transmitter power values using the beams proposed in this chapter. Each of the points plotted in these figures refers to the minimum ABEP (or outage probability) that can be obtained by combining orthogonally polarized higher-order Laguerre-Gaussian beams. It can be seen that the power needed to obtain the same ABEP (or outage probability), is lower for orthogonally polarized Laguerre-Gaussian beams than for fundamental Gaussian beams. Furthermore, this trend is confirmed for different magnitudes of the pointing jitter. For $\sigma = 100$ m (see Fig. 9.15), it can also be seen that up to order $\ell = 2$, the higher the azimuthal order of the Laguerre-Gaussian beam, the better the achievable ABEP. However, for $\ell > 2$, the

minimum achievable ABEP starts to increase. Similar behavior can be seen for the outage probability when higher than $\ell = 3$ Laguerre-Gaussian beams are considered. It is the authors' opinion that this turning point in performance is due to the increased separation between the central irradiance lobe of the Gaussian fundamental beam and the radial lobe of the higher-order Laguerre-Gaussian beam, as the order of the latter is increased. The radial lobe of low irradiance as the order increases (see bottom figures in Fig. 9.12 and 9.13) is frequently captured by the receiver aperture in this case, as the pointing jitter is a bivariate Gaussian distribution in the receiver aperture plane. The trend of higher achievable performance with higher azimuthal order ℓ is also limited in practice by the transmitter aperture size. For a given communication distance z , the achievable far-field beamwidth w_z is limited by the aperture size of the transmitter. Considering that the transmitted beamwaist w_0 of the beam is located in the aperture plane of the transmitter aperture, the size of the beamwidth in the receiver's aperture plane, w_z , is limited by the clipping created in the transmitter's aperture (i.e., the transmitter's aperture size).

In the analysis presented so far, a pointing jitter given by a Rayleigh distribution and a clear receiver aperture has been considered. On one side, many FSOC inter-satellite telescopes have an on-axis secondary mirror [102, 450]. The obscuration of the secondary mirror would change the received power statistics, and therefore, a difference in the results would be expected. However, as the resulting optimum far-field irradiance patterns are in general orders of magnitude above the aperture size (see Figs. 9.12 and 9.13, where the aperture is too small to discern it), the results would be mainly dependent on the total area of the receiver aperture. Hence, the results shown are also representative of a secondary obscured mirror on the receiver side with an equivalent total aperture area. On the other side, in some links, the pointing jitter will have a static boresight error that will yield a pointing jitter given by a Rice PDF (see Eq. (9.7)). In this case, if we were to compute the optimum far-field irradiance distributions, a widening of the distribution would be expected. However, if the boresight error is known, the system will usually be calibrated to account for it and the optimization problem will be again one given by the Rayleigh PDF for the pointing jitter.

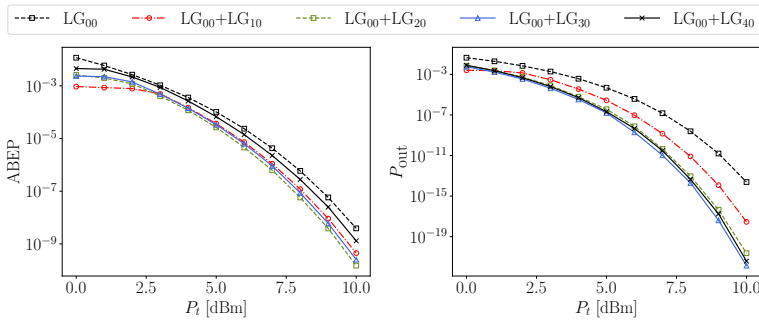


Figure 9.14: Optimum ABEP and outage probability as a function of the transmitter power P_t , for $\sigma = 10$ m.

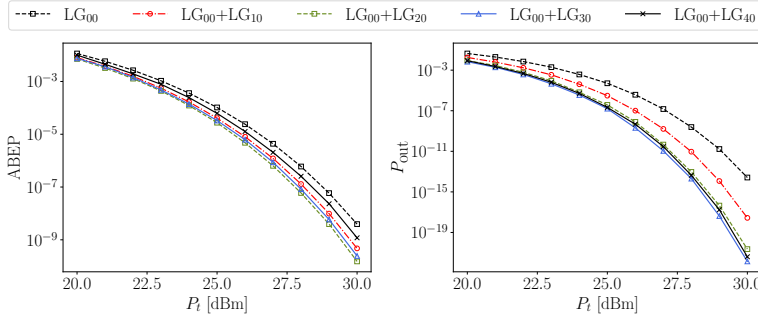


Figure 9.15: Optimum ABEP and outage probability as a function of the transmitter power P_t , for $\sigma = 100$ m.

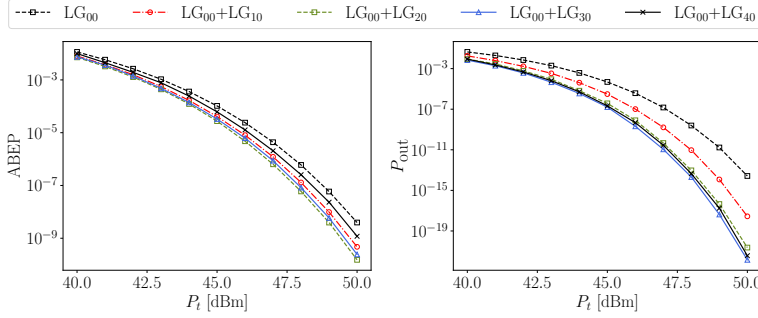


Figure 9.16: Optimum ABEP and outage probability as a function of the transmitter power P_t , for $\sigma = 1000$ m.

To quantify the improvement provided by the proposed beams in an intersatellite link scenario, the reduction in the required transmitted power can be analyzed for a 1000km LEO-LEO link. Considering an angular pointing error of $\sigma_\theta = 100$ μ rad, the resulting far-field pointing jitter is $\sigma = 100$ m. From Fig. 9.15 it can be seen that the same $\text{ABEP} \approx 10^{-6}$ can be obtained by using $\text{LG}_{00} + \text{LG}_{20}$ with $\sim 20\%$ less transmitted power, compared to the conventional fundamental Gaussian beam. Similarly, it can be seen in the same figure that a reduction of $\sim 40\%$ required power can be achieved for $\text{LG}_{00} + \text{LG}_{30}$ when a $P_{\text{out}} \approx 10^{-11}$ is required. For a LEO-GEO link of 45000 km, with the same angular pointing jitter, a far-field pointing jitter of $\sigma = 450$ m would be obtained. As the link is operating somewhere between $\sigma = 100$ m and $\sigma = 1000$ m, and the power savings in both are comparable according to Figs. 9.15 and 9.16, power savings of the same magnitude would be expected. In summary, these results show that a considerable improvement in intersatellite free-space optical communication performance can be obtained by using a superposition of orthogonally polarized fundamental and higher-order Laguerre-Gaussian beams.

9.5. Conclusions and future work

In this chapter, a superposition of orthogonally polarized fundamental Gaussian and higher-order Laguerre Gaussian beams is proposed to improve the performance of intersatellite free-space optical communication links. An optical system to generate the far-field irradiance distributions of interest is presented showing the practical feasibility of generating such far-field irradiance distributions (an experimental demonstration of this setup is presented in §10). Finally, numerical results are presented, showing transmitted power savings of 20–40%, depending on the chosen performance parameter, while achieving the same communication performance as conventional fundamental Gaussian beams.

Several aspects related to the research presented here deserve further investigation. One important direction concerns the influence of atmospheric turbulence on the quality and robustness of the proposed beam shapes, in particular their applicability to satellite-to-ground and terrestrial optical links. Another relevant topic is the practical generation of the desired far-field irradiance distributions in a more compact and integrated form, for instance through the use of metalenses or other flat-optics technologies. Beyond Laguerre–Gaussian modes, it would also be of interest to explore combinations of alternative beam families that may offer improved resilience to pointing jitter, which will be pursued in §11.

Chapter 10

Experimental demonstration of annular beams

*Demonstrating pointing jitter resilient beam shapes
for intersatellite optical communications*

*Only from the tension that arises out
of the interplay between the fullness of facts
and the possibly suitable mathematical forms
can decisive advances come.*

Werner Heisenberg

In the previous chapter, a beam-shaping approach was proposed to mitigate the impact of transmitter pointing jitter in intersatellite free-space optical communication links by redistributing the transmitted power over a broader spatial profile. The concept was shown, under idealized assumptions, to provide significant robustness against pointing-induced power fluctuations compared to a conventional Gaussian beam. In this work, we present the first experimental characterization of this approach using a spiral phase plate (SPP) to generate annular beams as a practical approximation of higher-order Laguerre-Gaussian modes. We demonstrate that the proposed beam superpositions can be reliably generated in a realistic optical setup, quantify the associated shaping errors and optical losses, and evaluate their impact on intersatellite link performance. Despite nonideal beam profiles and implementation losses, the experimental results show that the proposed beam-shaping approach can still yield power savings on the order of 20% compared to a fundamental Gaussian beam.

The content of this chapter is based on the preprint available in Ref. [485] Badás M. et al. "Annular beams for reliable intersatellite optical communications," Optics Letters 51, Issue 14, (2026).

10.1. Introduction

In §9, we introduced a novel approach to mitigate pointing-jitter-induced performance degradation in intersatellite optical communication links. Building on the optimization of beam divergence proposed by Toyoshima *et al.* [118], we extended the concept beyond fundamental Gaussian beams by considering alternative beam shapes obtained through the superposition of Gaussian and higher-order modes. Under idealized assumptions, this approach was shown to significantly improve link robustness against residual pointing jitter, yielding power savings on the order of 20 to 40 %.

As it will be presented in this chapter, the generated annular beams –created by impinging a Gaussian beam onto a spiral phase plate (SPP)– do not exactly correspond to Laguerre-Gaussian (LG) beams. Although both beams are annular in irradiance and carry orbital angular momentum, the SPP generated beam is a superposition of Laguerre-Gaussian modes, being the main contributor the Laguerre-Gaussian mode of the same topological order ℓ as the SPP. However, the performance gain provided by these beams relies on the same principle. The performance gain relies on redistributing the total transmitted power over a broader spatial profile, thereby reducing the sensitivity of the received signal to pointing-induced power fluctuations. In this way, the system deliberately sacrifices peak received power in favor of a more stable and reliable signal, which ultimately improves link-level performance metrics such as outage probability and average bit error probability under pointing jitter.

In this chapter, we present the first experimental demonstration of the proposed concept. The aim of this work is to characterize the practical implementation of the system, with particular emphasis on the beam shaping performance of the SPP used to generate annular beams and on the power losses introduced by a realistic optical setup. This chapter is structured as follows. §10.2 describes the experimental setup and the beam shaping principle. §10.3 presents the experimental results, including beam shape quality and power losses. §10.4 evaluates the impact of the experimentally obtained beams on the performance of an intersatellite FSOC link.

10.2. Experimental setup

To generate a superposition of orthogonally polarized Gaussian and higher-order annular beams, the experimental setup shown in Fig. 10.1 is employed at a wavelength of 532 nm. Minor modifications with respect to the setup proposed in Fig. 9.2 were implemented, the most important being the addition of a wavefront-cleaning stage based on a single-mode fiber. This stage ensures a high-quality Gaussian beam at the input of the beam shaping section. As shown in App. E.1, the SPP used to generate the annular beam is highly sensitive to the quality of the incident Gaussian beam. In a realistic FSOC terminal, such an additional wavefront-cleaning stage would typically not be required, since most FSOC sources are already fiber-coupled, for instance due to the use of fiber-based modulators and erbium-doped fiber amplifiers [108, 486, 487].

After the wavefront-cleaning stage, the Gaussian beam from the fiber collimator passes through a periscopic pair of mirrors. The beam is linearly polarized, and its polarization orientation is adjusted using a rotatable half-wave plate (RHWP). By changing the angle between the incoming polarization and the fast axis of the RHWP,

the polarization state incident on the polarization beam splitter (PBS) is controlled. This, in turn, determines how the optical power is split between the two arms of the setup. The horizontally polarized Gaussian beam –transmitted by the PBS– then passes through the SPP. The fused silica SPP is a diffractive optical element that imposes a helicoidal phase profile on the incoming Gaussian beam. After sufficient propagation distance, this phase modulation leads to the formation of an annular beam shape [474, 488].

Although other techniques exist to generate annular beam shapes, such as spatial light modulators or birefringent vortex plates [65], these approaches often introduce additional complexity, dynamic behavior, or higher losses, which are undesirable for FSOC applications. The SPPs used in this work generate annular beams of topological orders $\ell = 1, 2, 3$, which are the most relevant modes for pointing-jitter mitigation according to §9.

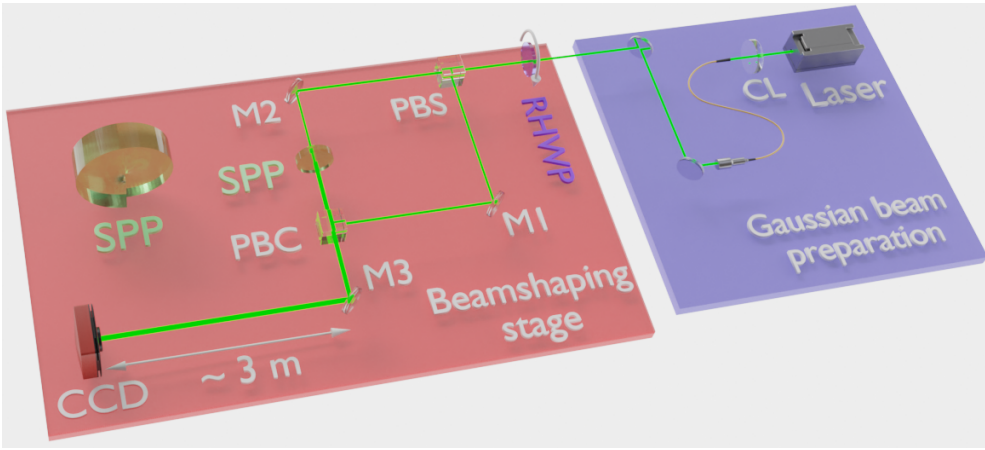


Figure 10.1: Experimental setup proposed for beam shaping. The Gaussian beam cleaning stage and the beam shaping stage are shown. A picture of the experimental setup in the laboratory can be found in Fig. E.18.

The vertical polarization component reflected by the PBS bypasses the SPP and is subsequently recombined with the horizontally polarized annular beam using a polarization beam combiner (PBC). The combined beam is then propagated over a distance of more than ~ 3 m to allow the annular beam shape to fully develop. The resulting irradiance pattern is measured using a camera. Since the Gaussian and annular components are orthogonally polarized, the measured irradiance corresponds to the incoherent sum of their individual irradiance profiles. By rotating the RHWP, the power ratio between the Gaussian and annular components can be continuously adjusted, thereby generating a family of composite beam shapes.

To assess the quality of the experimentally generated beams, the measured irradiance patterns are compared with numerical simulations. These simulations are based on a Fourier-optics propagation model capable of describing both near- and far-field propagation [8, 364], combined with Jones calculus to model polarization effects

throughout the setup [139]. Further details of the developed model are provided in App. E.2.

10.3. Results

In this section, we present and analyze the experimentally obtained beam shapes and quantify the power losses introduced by the proposed optical system. The experimental results are systematically compared with numerical simulations in order to assess the beam shaping performance of the setup.

For a meaningful comparison between simulations and experiments, the Gaussian beam emitted by the fiber collimator was first characterized. This was accomplished by matching simulated results to experimental beam profiles for different assumed Gaussian beam waists and wavefront curvatures at the collimator output. From this procedure, the beam was found to have a waist of $w_0 = 1.19$ mm and a wavefront radius of curvature of $R_{\text{col}} = 12$ m. Details of the retrieval procedure are provided in App. E.3.

10.3.1. Beam shaping

After characterizing the Gaussian beam at the aperture of the fiber collimator, simulated and measured beam shapes can be directly compared. Both the simulated and experimental beams are normalized to unit power, enabling a comparison focused solely on beam shape. Fig. 10.2 shows the results for superpositions of Gaussian and annular beams with topological orders $\ell = 1$ and 2 when the RHPW is set to 22.5° , corresponding to equal power splitting between the two beams (an extended set of figures for other RHPW angles and ℓ can be found in App. E.6, including the cases in which pure Gaussian and pure annular are obtained). As the topological order ℓ increases, the radial position of the annular beam's intensity peak also increases, as shown in the figure. The residuals between the simulated and experimental irradiance patterns are also shown. These residuals reveal a slight asymmetry in the experimental annular beams, with increased power in the top-right region compared to the bottom-left. As will be shown later in §10.4, these small asymmetries do not lead to significant losses in the communication performance of the generated beam shapes.

As discussed in §9, the optimal power distribution between the Gaussian and annular components depends on the chosen performance metric and the operating conditions (e.g., pointing jitter, available power, etc.). Therefore, it is important to characterize the beam shaping performance across the full range of RHPW angles. This is done by computing an R^2 metric that quantifies the agreement between simulated and experimental irradiance patterns. Specifically, for each RHPW angle, the slice-wise coefficient $R^2_{\text{slice}} = 1 - \iint |I_{\text{sim}} - I_{\text{exp}}|^2 dx dy$ is computed over multiple angular slices of the beam. Fig. 10.3 shows the average R^2 across all slices, along with the standard deviation and the extremal values, as a function of RHPW angle for $\ell = 1$ and $\ell = 2$.

Overall, the average R^2 remains above 95%, indicating that the intended beam shapes are reproduced with high fidelity, with higher R^2 values observed as the Gaussian beam contribution to the final shape increases. The observed discrepancies between simulated and experimental beam shapes can be attributed to several factors.

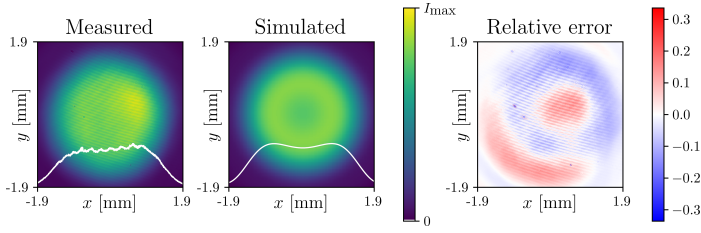
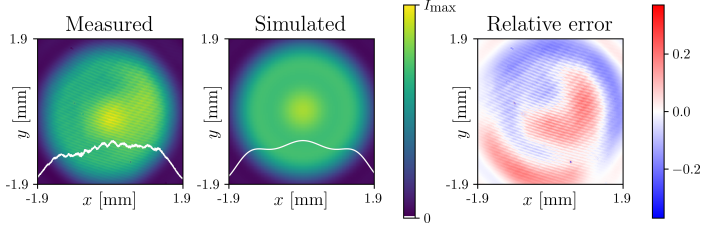
(a) $\ell = 1$ and RHP angle = 22.5° (b) $\ell = 2$ and RHP angle = 22.5°

Figure 10.2: Measured and simulated beam shapes for different topological charges, with equal power contributions from each beam (Gaussian and annular). The white lines indicate the irradiance cross-section at $y = 0$.

Although the wavefront-cleaning stage significantly improves the Gaussian beam quality (see App. E.1), residual wavefront aberrations remain and are transferred to the annular beam generated by the SPP. In addition, the beam quality is sensitive to manufacturing variations between different SPP batches, which, according to the SPP manufacturer, the measured total depth of the SPP has a tolerance of ± 5 nm. Finally, spurious interference fringes caused by internal reflections within the camera system introduce additional distortions in the measured irradiance patterns.

The results presented are valid on the camera plane of the proposed setup (~ 3 m away from the SPP, as shown in Fig. 10.1). The further development of the annular beam after the SPP is discussed in detail in App. E.4. Both simulations and measurements show that the beam continues to evolve with propagation distance, approaching an annular profile that differs from a Laguerre–Gaussian beam. The analysis in §E.4.2 demonstrates that the SPP-generated beam does not exactly correspond to a Laguerre–Gaussian mode, with a larger transverse size and a distinct far-field profile; achieving an exact LG mode would require an additional transmission screen, which would introduce significant power losses. The subsequent alignment sensitivity analysis shows that the beam is highly tolerant to angular misalignments of SPP but extremely sensitive to lateral displacements at the micrometer scale. This indicates that precise lateral positioning of the SPP is critical for maintaining the desired beam shape (see App. E.5 for details).

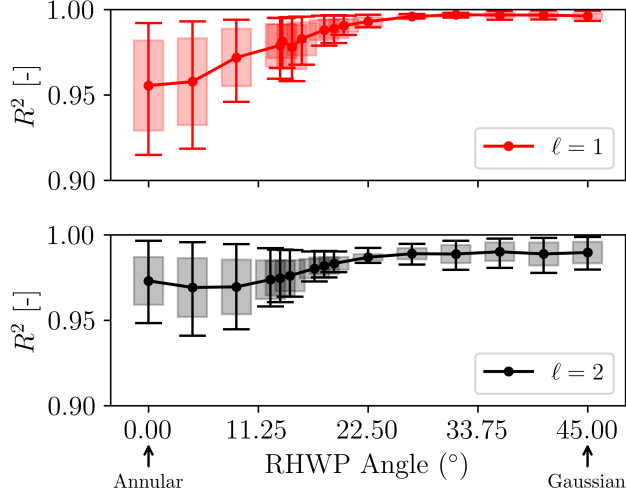


Figure 10.3: R^2 values obtained by comparing simulated and experimental beam shapes for different RHPW angles, for $\ell = 1$ and $\ell = 2$ annular beams. Average (dots), standard deviation (boxes) and extremal values (whiskers) are shown.

10.3.2. Power losses

In addition to beam shaping performance, it is essential to quantify the optical losses introduced by the proposed setup, as these directly impact the achievable communication performance. Here, we focus only on losses in the beam shaping stage shown in Fig. 10.1, since the Gaussian beam cleaning stage is not representative of a realistic FSO terminal, where fiber coupling is typically already present due to the use of fiber-based modulators and amplifiers [108, 486, 487].

The dominant sources of loss arise from the SPP. These include Fresnel reflections, absorption, and scattering due to fabrication imperfections. Other optical components, such as the RHPW and polarization beam splitters, contribute to polarization-dependent insertion losses; however, these are negligible compared to the losses introduced by the SPP. To quantify the SPP-induced losses, power measurements were performed with a power meter before and after the SPP. The measured transmittances are 92.6% for $\ell = 1$ and 87.5% for $\ell = 2$. The annular beam with $\ell = 3$ was generated experimentally by cascading SPPs with $\ell = 1$ and $\ell = 2$, which leads to an accumulated transmission efficiency of 81%. In a practical implementation, the latter could be significantly reduced by using a single SPP with topological charge $\ell = 3$, which would avoid cascading elements and reduce the transmission loss. In the following section, we analyze the experimentally obtained beam shapes both with and without accounting for these measured power losses, evaluating their expected performance in an intersatellite optical communication link.

10.4. Intersatellite optical communication performance

Using the experimentally obtained beam profiles, the performance of an intersatellite FSOC link is evaluated through numerical simulations. Two scenarios are considered. In the first, the power losses discussed in the previous section are neglected, which allows the isolated effect of beam shaping on pointing-jitter mitigation to be assessed. In the second, the measured losses are included, providing a realistic estimate of the current system performance. The model used to evaluate the communication performance is described in §9.2. The intersatellite link parameters employed in the simulations are taken from Tab. 9.1. For each beam configuration, the optimum performance is obtained by simulating different beam sizes and different relative contributions of the Gaussian and annular components, and then identifying the configuration that minimizes the outage probability. As explained in §9, the outage probability is defined as the probability that the instantaneous received power falls below the threshold required for an ideal receiver to sustain communication at a prescribed data rate. The results obtained are summarized in Fig. 10.4, which shows the outage probability for Gaussian beams and for superpositions of Gaussian and annular beams of different topological orders at the camera plane. It is important to note that these beams do not correspond to those that would be encountered at the receiver aperture, since they are affected by the finite clipping of the camera and are not yet fully developed. Nevertheless, this comparison is useful to further validate the simulation framework by allowing the measured fields to be propagated both numerically and analytically to the far-field and to assess the resulting communication performance. In this way, the match found in Fig. 10.4 means that not only are the beam shapes obtained experimentally close to the simulated profiles (see Fig. 10.3), but also the expected communication performance resulting from these is similar. These results already highlight the benefits of using superpositions of Gaussian and annular beams compared with a conventional Gaussian beam. In particular, the outage probability is significantly reduced, even when the power losses induced by the SPP are taken into account. This demonstrates the potential of tailored beam shaping as an effective means of mitigating pointing-jitter impairments in practical intersatellite FSOC links.

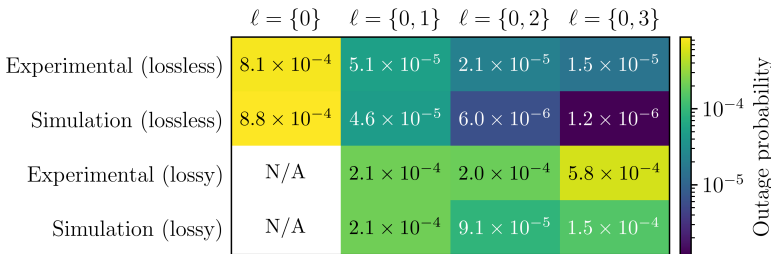


Figure 10.4: Outage probability performance for the measured and simulated beam shapes at the camera plane. The beam shapes are clipped by the camera size and are not fully developed yet.

As can be seen in Fig. 10.4, the good agreement between the measured and sim-

ulated beam profiles shown in Fig. 10.3 is also successfully reflected in the predicted communication performance. This consistency provides confidence in the simulation framework and enables the results to be propagated further to the far-field, where the beam shapes expected at the receiver aperture plane can be used to evaluate the intersatellite link performance under realistic conditions. The resulting outage probabilities for the far-field-simulated beam shapes are presented in Fig. 10.5. In this case, the superpositions of LG far-fields (see §9) are compared with analytical far-field expressions obtained for an SPP-generated annular beam superposed with a Gaussian beam. The analytical far-field irradiance of an SPP-generated annular beam as a function of the radial coordinate ρ is given by (see §E.4.2 for the derivation)

$$I_{\text{FF}}(\rho, z) \propto e^{-\frac{\rho^2}{w^2(z)}} \left| \frac{\rho}{z^2} \left[\mathcal{I}_{\frac{\ell-1}{2}} \left(\frac{\rho^2}{2w^2(z)} \right) - \mathcal{I}_{\frac{\ell+1}{2}} \left(\frac{\rho^2}{2w^2(z)} \right) \right] \right|^2,$$

where $w(z)$ is the Gaussian beam radius at propagation distance z (considering the geometry of the Gaussian beam impinging at the SPP at the beamwaist), and $\mathcal{I}_n(x)$ denotes the modified Bessel function of the first kind of order n . The analytical far-field beam shapes corresponding to the experimentally generated fields are then obtained by a weighted superposition –set by the RHPW angle– of the Gaussian beam irradiance and the annular beam irradiance given by the above expression. These analytically obtained beams match the profiles obtained by wave propagation simulations as can be seen in Fig. E.8. This approach allows to translate the experimental-simulation match found in Fig. 10.4 into the far-field analytical irradiance profiles, and enables a reliable assessment of the communication performance using the beam profiles expected at the receiver aperture. The results in Fig. 10.5 demonstrate the improvement, relative to a conventional Gaussian beam, provided by the beam shaping technique proposed in this chapter for mitigating the effects of transmitter pointing jitter in intersatellite optical communication. Furthermore, the table shows that although the SPP-generated annular beams do not perform as well as LG beams, they still provide a significant enhancement in system performance compared to the conventional Gaussian beam. Notably, even the power losses introduced by absorption in the SPP are compensated by the proposed beam shaping approach, except in the $\ell = 3$ case, where the additional losses arise from the use of two cascaded SPPs. In terms of power efficiency, these results indicate that power savings of up to 20% could be achieved if the SPP losses were further minimized, for example through the use of anti-reflective coatings or other loss-reduction techniques. The reader is referred to App. E.6 for obtaining the performance of these beams in terms of the average bit error probability.

10.5. Conclusions and future work

In this chapter, the experimental performance of orthogonal superpositions of Gaussian and high-order annular beams for the mitigation of pointing jitter in intersatellite communication links has been assessed. To generate annular beams, spiral phase plates of different topological orders have been employed. It has been shown that, by impinging a clean Gaussian beam onto these diffractive optical elements, an average R^2 value above 95% is achieved for the experimental beam shapes compared to simulations, across all superpositions of Gaussian and annular beams.

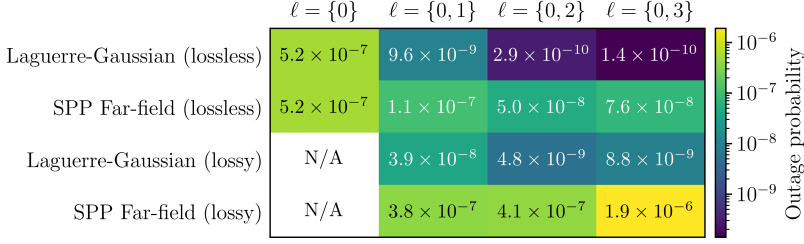


Figure 10.5: Outage probability performance for the analytical and simulated beam shapes at the far-field. The analytical and numerical far-field shapes obtained by the SPP, along with the optimal shapes obtained if LG beams were generated, are shown.

Furthermore, the transmission losses introduced by the inclusion of the SPPs have been analyzed. The transmission efficiencies for single SPPs exceed 87.5%, which remains sufficiently high to provide an overall improvement in communication performance compared to a lossless Gaussian beam. The SPP-generated annular beams have been shown to differ theoretically from the originally proposed Laguerre–Gaussian beams in §9, despite exhibiting an annular irradiance profile and carrying orbital angular momentum.

Finally, the communication performance, evaluated in terms of outage probability, has been computed for both the experimentally obtained beams and those predicted by simulation. The results indicate that, even when accounting for the transmission losses introduced by the SPPs, beam shaping remains a promising approach for pointing jitter mitigation. A comparison between the analytical far-field irradiance of superpositions of SPP-generated annular and Gaussian beams, and those of Laguerre–Gaussian and Gaussian beams (see §9), shows that although Laguerre–Gaussian beams outperform the SPP-generated ones, the annular beams still provide better performance than conventional Gaussian beams in terms of outage probability.

Future work should focus on reducing system losses through improved antireflection coatings on the SPP and on miniaturizing the beam shaping setup for integration into space-qualified FSO terminals. An end-to-end experimental validation using a communication terminal with induced pointing jitter is also envisaged. Beyond the present concept, alternative beam shaping techniques and other structured beam profiles should be investigated for further performance improvements (which will be the focus of §11).

Chapter 11

Variational approach to optimum beam shaping

*Flat-top and super-Gaussian beams for intersatellite optical
communication*

*Simplex sigillum veri
(Simplicity is the sign of truth)*

Building upon the work presented in the previous chapters, we introduce a variational formalism to determine the optimum far-field irradiance for an intersatellite link affected by pointing jitter. Using this formalism, the flat-top beam is found to be the optimum beam shape for minimizing outage probability. In particular, it requires $\sim 37,8\%$ of the power needed by a conventional Gaussian beam to achieve the same outage probability. However, the flat-top profile is discontinuous and physically unrealizable. To address this, we analyze a continuous family of super-Gaussian beam shapes that approximate the flat-top as the order increases. In addition, several coherent and incoherent beam shaping techniques are evaluated to assess their ability to reproduce the optimum irradiance distribution. The results show that these techniques can reduce the required transmitted power by up to $\sim 50\%$ compared with conventional Gaussian beams.

The content of this chapter is based on the Ref. [489] Badás M. et al. "Super-Gaussian approximations for optimum far-field irradiance in intersatellite optical communications: coherent and incoherent beam shaping," submitted for publication in the Journal of Optics.

11.1. Introduction

As discussed in the previous chapters, beam shaping can mitigate the impact of transmitter pointing jitter in intersatellite free-space optical communication links by modifying the far-field irradiance distribution. In this chapter, a variational framework is introduced to determine the optimum far-field irradiance distribution that maximizes communication performance in the presence of pointing jitter. In addition, several optical systems capable of approximating these optimum beam profiles are analyzed. Specifically, the coherent and incoherent beam shaping techniques that provide the calculated optimum beam shapes are studied.

First, the mathematical model of the intersatellite free-space channel is presented in §11.2, where the power statistics and the communication performance of the optical link are derived. Next, §11.3 proposes a variational method to obtain the optimum far-field irradiance. Since the resulting optimum far-field irradiance exhibits a discontinuity, §11.4 analyzes a continuous family of super-Gaussian beam shapes that approximate this optimum field. Finally, in §11.5, incoherent and coherent beam-shaping techniques are studied to approximate the optimum beam shapes identified in the previous sections.

11.2. Mathematical model of the free-space channel

Considering the pointing jitter problem stated above, we aim to determine the optimum far-field irradiance distribution $I(x, y)$ that maximizes the performance of an intersatellite optical link under the influence of transmitter pointing jitter. Formally, the problem can be stated as:

Find the far-field irradiance $I(x, y) \in \mathbb{R}$ with $(x, y) \in \mathbb{R}^2$, given a probability density function of the beam center location $f_{XY}(x, y) \in \mathbb{R}$, that minimizes a chosen communication performance parameter $\mathcal{S}$.

As the pointing jitter probability density function (PDF) will have azimuthal symmetry, the problem can be stated in terms of the radial coordinate ρ instead. Typical communication performance metrics $\mathcal{S}$ to be minimized are: average bit error probability (ABEP) [118], probability of fade, probability of outage and the inverse of the reliability (probability of being below a given bit error probability threshold) as explained in §6.2.5. Using the variational formalism, minimizing any of these parameters is equivalent to finding an extremum of the functional $\mathcal{S}$ as

$$\delta\mathcal{S} = 0. \quad (11.1)$$

For parameters such as outage, fade, and reliability, $\mathcal{S}$ can be expressed as the cumulative density of the received power below a threshold P_{th}

$$\mathcal{S}[I] = \int_0^{P_{\text{th}}} \mathcal{F}_P[I, f_{XY}] dP,$$

where $\mathcal{F}_P$ is the PDF of the received power, which depends on both the far-field irradiance $I(x, y)$ and the PDF of the pointing jitter $f_{XY}(x, y)$. The received power PDF, $\mathcal{F}_P$, can be obtained from the vector to scalar probability mapping of the beam

center location to the collected power as [441]

$$\mathcal{F}_P[I, f_{XY}] = \iint_{-\infty}^{\infty} f_{XY}(x_0, y_0) \delta[P - g(x_0, y_0)] dx_0 dy_0, \quad (11.2)$$

where $f_{XY}(x_0, y_0)$ is the probability density function of the location center of the beam in the far-field and $g(x_0, y_0)$ is the received power in the aperture when the center of the transmitted beam is located at (x_0, y_0) . Mathematically, this power can be written as

$$g(x_0, y_0) = \iint_{\mathcal{A}} I(x, y) dx dy = \iint_{\mathbb{R}^2} I(x, y) \mathcal{A}(x - x_0, y - y_0) dx dy,$$

where $\mathcal{A}(x, y)$ is the receiver aperture pupil function

$$\mathcal{A}(x, y) = \begin{cases} 1 & \text{if } a < x^2 + y^2 < b, \\ 0 & \text{if else,} \end{cases}$$

where a and b are the secondary and primary mirror radii of the receiver telescope. This integral is equivalent to a bidimensional convolution and can be expressed via the Fourier transform $\mathcal{F}$ as

$$g(x_0, y_0) = \{I * \mathcal{A}\}(x_0, y_0) = \mathcal{F}^{-1}\{\mathcal{F}(I) \cdot \mathcal{F}(\mathcal{A})\}. \quad (11.3)$$

Therefore, the problem of finding the optimum far-field irradiance to maximize the performance of an intersatellite optical link impinged by transmitter pointing jitter has been converted into the variational problem given by Eqs. (11.1)-(11.3).

When both the far-field irradiance and the receiver aperture are azimuthally symmetric with respect to the optical axis, the power collected can be written as a function of the radial coordinate $g(x_0, y_0) \rightarrow g(\rho_0)$. Furthermore, for very large distances, the aperture is very small compared to the far-field irradiance footprint due to the divergence of the beam. Hence, the irradiance variation across the aperture is negligible in this regime, and the power as a function of the center of the beam can be written as

$$g(\rho_0) \approx A \times I(\rho_0), \quad (11.4)$$

where A is the area of the receiver aperture. In general, the pointing jitter statistics will be azimuthally symmetric with respect to the optical axis. In this case, the pointing jitter statistics are given by the density function $f_R(\rho)$. This condition, along with the azimuthal symmetry of the receiver aperture, will yield an axially symmetric far-field irradiance distribution $I(\rho)$. Specifically, the pointing jitter is usually considered a Rayleigh distribution in radial coordinates, resulting from a bivariate Gaussian distribution in both elevation and azimuthal angles. The Rayleigh density function for a scale parameter σ (where σ^2 corresponds to the standard deviation of the underlying elevation and azimuth pointing jitter) is given by

$$f_R(\rho) = \frac{\rho}{2\pi\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right). \quad (11.5)$$

Under these assumptions, the received power can be expressed as a function of the radial coordinate $P = g(\rho)$ and the received power statistics can be computed with the scalar-to-scalar $\mathbb{R} \rightarrow \mathbb{R}$ mapping of the density functions [441]

$$\mathcal{F}_P(P) = \sum_{k=1}^{k_{\max}} \left| \frac{d}{dP} \rho_k \right| \cdot f_R(\rho_k), \quad (11.6)$$

where $k_{\max}$ is the number of monotonical piecewise functions between the extreme values of the function $\rho(P)$. Furthermore, the condition of the conservation of energy needs to be included. This means, that if the beam has a total power P_0 in the transmitter aperture plane, the irradiance field at the receiver aperture plane $I(\rho)$ has to fulfill

$$P_0 = \iint_{\mathbb{R}^2} I(x, y) \, dx \, dy = 2\pi \int_{\mathbb{R}^+} I(\rho) \, \rho \, d\rho. \quad (11.7)$$

11.3. Optimum far-field irradiance via variational formalism

Considering the variational statement given in the previous section, no solution $I(x, y)$ has been found when imposing the functional $\mathcal{S}[I]$ to be minimized (see App. F.1). Furthermore, exploiting the azimuthal symmetry to obtain $I(\rho)$ has also not provided any solution through the variational approach. As can be seen in App. F.1, applying both the subsidiary condition through Lagrange multipliers (for the finite power condition) and the Weierstrass-Erdmann corner conditions (for allowing discontinuities in the solution) as in Ref. [490], has not proven successful. This is because the problem is ill-posed. As will be shown later, without enforcing all necessary conditions, the problem becomes degenerate, and no solution can be obtained. Incorporating these conditions within the aforementioned formalisms proved challenging; therefore, the problem was instead addressed by determining the optimal captured power probability density function, and only thereafter finding the irradiance profile that yields the optimum captured power probability density function.

Therefore, the optimization problem can be restated in terms of the probability density function of the received power, $\mathcal{F}_P(P)$. When doing so, if the optimum solution is found for the function $\mathcal{F}_P(P)$, then an irradiance distribution $I(\rho)$ needs to be found resulting in such a PDF. Mathematically, we want to make the functional stationary

$$\mathcal{S}[\mathcal{F}_P] = \int_0^{P_{\text{th}}} \mathcal{F}_P(P) \, dP, \quad (11.8)$$

subject to the two constraints

$$(1) \quad 0 \leq P \leq P_{\max} \quad \text{positivity and upper-bound constraint,} \quad (11.9)$$

$$(2) \quad \int_0^\infty \mathcal{F}_P(P) \, dP = 1 \quad \text{normalization.} \quad (11.10)$$

The first implies that the total power of the beam is finite (through an upper limit of the achievable maximum power), while the second is the normalization condition of

the probability density function. The $P_{\max}$ can be defined as

$$P_{\max} = \max_{(x_0, y_0)} g(x_0, y_0),$$

where $g(x_0, y_0)$ is the power collected when the center of the beam is at (x_0, y_0) (see Eq. (11.3)).

To solve the variational problem, we first introduce a Lagrange multiplier λ to account for the normalization condition in Eq. (11.10). For that, we consider the augmented functional

$$\Phi[\mathcal{F}_P] = \int_0^{P_{\text{th}}} \mathcal{F}_P(P) dP - \lambda \left(\int_0^{P_{\max}} \mathcal{F}_P(P) dP - 1 \right).$$

To include the stationarity condition ($\delta\Phi = 0$), we take a small variation $F_P(P) \mapsto F_P(P) + \varepsilon \eta(P)$ with an arbitrary test function $\eta(P)$. The first variation must vanish as

$$\delta\Phi = \int_0^{P_{\text{th}}} \eta(P) dP - \lambda \int_0^{P_{\max}} \eta(P) dP = 0 \quad \forall \eta(\cdot).$$

By splitting the last integral, the stationary condition given by

$$\int_0^{P_{\text{th}}} (1 - \lambda) \eta(P) dP - \int_{P_{\text{th}}}^{P_{\max}} \lambda \eta(P) dP = 0 \quad \forall \eta(\cdot).$$

must hold. For this condition to be met for any arbitrary test function $\eta(P)$, the coefficients of η on the two intervals must be zero. This gives the pointwise Euler–Lagrange conditions

$$\begin{aligned} \lambda &= 1 & \text{for } 0 \leq P \leq P_{\text{th}}, \\ \lambda &= 0 & \text{for } P_{\text{th}} < P \leq P_{\max}. \end{aligned}$$

These two requirements cannot both hold simultaneously as the multiplier λ is a scalar. Hence, there is no smooth, everywhere-positive density $\mathcal{F}_P$ satisfying the stationarity conditions. What survives are the admissible *extremal* measures which lump all probability at a point. In fact, the only way to satisfy normalization and make $\delta\Phi = 0$ is to place all mass at one of the boundaries of the integration ranges

$$\begin{array}{lll} \text{Mass at } P = 0 \rightarrow & \mathcal{F}_P(P) = \delta(P) \rightarrow & \mathcal{S} = 1, \\ \text{Mass at } P = P_{\text{th}} \rightarrow & \mathcal{F}_P(P) = \delta(P - P_{\text{th}}) \rightarrow & \mathcal{S} = 1/2, \\ \text{Mass at } P = P_{\max} \rightarrow & \mathcal{F}_P(P) = \delta(P - P_{\max}) \rightarrow & \mathcal{S} = 0. \end{array}$$

As we are interested in the minimum of $\mathcal{S}$, the probability density $\mathcal{F}_P(P) = \delta(P - P_{\max})$ is of our primary interest. This probability density function is physically interpreted as the power collected being constant with value $P = P_{\max}$ for any pointing error. Hence, the continuous PDF has been converted to a discrete probability function. Furthermore, the minimality condition of $\mathcal{S} = 0$ is also met by all the PDFs satisfying

$$\mathcal{F}_P(P) = \delta(P - P') \quad \text{for } P' \geq P_{\text{th}}. \quad (11.11)$$

Now the problem remains of finding a far-field irradiance that, in combination with the pointing jitter probability density function (e.g. Rayleigh distribution in Eq. (11.5)), gives the probability function in Eq. (11.11) for the collected power. Finally, it can be seen directly from the definition of $\mathcal{S}$ in Eq. (11.8) that any function

$$\mathcal{F}_P(P) = \begin{cases} 0 & \text{if } 0 \leq P < P_{\text{th}}, \\ n(P) & \text{else,} \end{cases} \quad (11.12)$$

also satisfies the condition $\mathcal{S} = 0$, where $n(P)$ is any positive function satisfying the normalization condition in Eq. (11.10)

$$\int_{P_{\text{th}}}^{P_{\text{max}}} n(P) = 1.$$

Now, for $\mathcal{F}_P(P)$ to be zero for $P \in [0, P_{\text{th}})$ as required by the solution in Eq. (11.12), according to the definition of $\mathcal{F}_P(P)$ in Eq. (11.2)

$$\iint_{-\infty}^{\infty} f_{XY}(x_0, y_0) \delta[P - g(x_0, y_0)] dx_0 dy_0 = 0 \quad \text{for } 0 \leq P < P_{\text{th}},$$

and hence

$$f_{XY}(x_0, y_0) = 0 \quad \forall (x_0, y_0) \in \mathbb{R}^2,$$

or

$$\delta[P - g(x_0, y_0)] = 0 \quad 0 \leq P \leq P_{\text{th}}, \quad \forall (x_0, y_0) \in \mathbb{R}^2.$$

The first condition above is not satisfied for a continuous unbounded pointing jitter (e.g. Rayleigh distribution in Eq. (11.5)). Regarding the second condition, as $g(x_0, y_0) > 0$, this last condition means that the power collected is non-zero, $g(x_0, y_0) \geq P_{\text{th}}$, for any pointing jitter error $(x_0, y_0) \in \mathbb{R}^2$. Under this condition, the total power of the beam would not be bounded, and therefore this solution is not physically possible (see Proof 1 in §F.2).

Considering the above, we have now demonstrated that the $\mathcal{F}_P(P)$ given in Eq. (11.12) can not be fulfilled in general by a finite power far-field irradiance when considering an unbounded pointing jitter PDF. Therefore, now we are looking for a $\mathcal{F}_P(P)$ that gives an $\mathcal{S} > 0$. Although the condition $g(x_0, y_0) \geq P_{\text{th}}$ for $\forall (x_0, y_0) \in \mathbb{R}^2$ can not be met, the best solution would be one in which the condition $g(x_0, y_0) \geq P_{\text{th}}$ is met with the largest probability possible. In other words, we are looking for a function $g(x_0, y_0)$ that maximizes the probability of the collected power being above the given threshold P_{th} . Hence, the probability

$$\Pr\{g(x_0, y_0) \geq P_{\text{th}}\} = \iint_{\{(x_0, y_0): g(x_0, y_0) \geq P_{\text{th}}\}} f_{XY}(x_0, y_0) dx_0 dy_0, \quad (11.13)$$

needs to be maximized. At this point, it is important to prove that the finite power condition in Eq. (11.7) implies the finiteness of the bidimensional integral of $g(x_0, y_0)$

through the relation in Eq. (11.3). The reader is referred to Proof 2 in §F.2 for the proof that given that the power of the beam is finite (see Eq. (11.7)), then

$$\iint_{\mathbb{R}^2} g(x_0, y_0) dx_0 dy_0 \leq G,$$

where G is finite. As the PDF of the pointing jitter $f_{XY}(x_0, y_0)$ is considered radially decreasing (see Eq. (11.5)), we get that the maximization in Eq. (11.13) is achieved when the area described by the condition $\{(x_0, y_0) : g(x_0, y_0) \geq P_{\text{th}}\}$ is a circular region in two-dimensional space with the largest possible radius. In Proof 3 of App. F.2, we show that the maximization of this area is obtained by a circular step function of height P_{th} ,

$$g(x, y) = \begin{cases} P_{\text{th}}, & x^2 + y^2 \leq r_0^2, \\ 0, & \text{otherwise,} \end{cases} \quad (11.14)$$

Having established the condition that $g(x_0, y_0)$ must satisfy in order to minimize $\mathcal{S}$, we can now determine the corresponding optimum far-field irradiance $I(x, y)$ that produces such a power collection profile. To this end, we use the relation between these functions given in Eq. (11.3). By applying the Fourier-transform properties of Eq. (11.3), we obtain, in momentum space,

$$\tilde{I}(k_x, k_y) = \frac{\tilde{g}(k_x, k_y)}{\tilde{\mathcal{A}}(k_x, k_y)}, \quad (11.15)$$

where $\sim$ refers to the Fourier transform. Therefore, for a given aperture $\mathcal{A}$ and considering the optimum power collection function in Eq. (11.14), we can obtain the optimum far-field irradiance operating in the Fourier domain. Considering a circular aperture $\mathcal{A}$, the Fourier components of the irradiance field are (see Proof 4 in App. F.2)

$$\tilde{I}(k) = P_{\text{th}} \frac{r_0}{a} \frac{\mathcal{J}_1(|k| r_0)}{\mathcal{J}_1(|k| a)}. \quad (11.16)$$

where a is the radius of the aperture and $\mathcal{J}_1$ is the Bessel function of the first kind. The inverse Fourier transform (which in polar coordinates results in the inverse Hankel transform of the first kind due to the azimuthal symmetry of the problem) of this ratio is a function in polar coordinates given by

$$I(\rho) = \int_{\mathbb{R}^+} \tilde{I}(k) \mathcal{J}_1(k\rho) \rho d\rho. \quad (11.17)$$

However, the general solution can not be found in a closed form due to the divergences created by the zeros of the Bessel function in the denominator of Eq. (11.16). For the small circular aperture approximation, the inverse Hankel transform involved can be computed (see Proof 4 in App. F.2), yielding the far-field irradiance

$$I(\rho) = \begin{cases} \frac{2P_{\text{th}}r_0}{a^2} & \rho \leq r_0, \\ 0 & \text{otherwise,} \end{cases}$$

which is the flat-top beam (up to a power normalization constant). Imposing the available finite power condition of Eq. (11.7), we get that the flat-top beam is

$$I(\rho) = \begin{cases} \frac{P_{\text{th}}}{\pi a^2} & \rho \leq r_0, \quad \text{where } r_0 = a \sqrt{\frac{P_0}{P_{\text{th}}}}, \\ 0 & \text{otherwise.} \end{cases}$$

For larger apertures, Eq. (11.15) can be numerically computed, although as shown in Fig. 11.1, instabilities arise as the ratio between the aperture radius a and the beam size r_0 increases. It can be seen in the numerical results that the inverse Hankel transform oscillates strongly, giving unphysical negative irradiance values when the ratio a/r_0 is increased. However, it is shown that the shapes obtained numerically for bigger apertures closely resemble the flat-top beam obtained in the small aperture limit. For big a/r_0 ratios, the oscillations obtained mathematically can be understood considering that the variations of the irradiance across the aperture can still compensate each other, yielding the same total power collected by the aperture, i.e. P_{th} .

11.4. Analytical solutions of some beam shapes

Building upon the previous section's finding that a flat-top profile is the optimum beam shape for mitigating pointing jitter, this section derives the analytical performance results, denoted by $\mathcal{S}$, for several key beam shapes. The Gaussian beam is examined first because of its widespread application in free-space optical communications [118, 175, 393]. Subsequently, the study extends to super-Gaussian beam shapes, as they represent a versatile family of functions that converge toward the flat-top profile in the limit of infinite order. Finally, the flat-top beam itself is analyzed in detail, as it represents the ideal solution under the conditions previously identified.

Due to the complexity of the mathematical operations involved, this analysis adopts the small-aperture hypothesis and assumes a Rayleigh density function for the pointing jitter, as expressed in Eq. (11.5). Under the assumption that the receiver telescope aperture is sufficiently small, the irradiance variation across the aperture becomes negligible. Consequently, the power collected as a function of the radial distance ρ_0 between the receiver and the beam center can be expressed as in Eq. (11.4).

11.4.1. Gaussian beam

We begin our derivation by analyzing the Gaussian beam profile, which serves as the benchmark for free-space optical communications systems due to its role as the fundamental mode in laser resonators and single-mode fibers usually employed in optical terminals. At the receiver aperture plane, a Gaussian beam with a total power P_0 is given by

$$I(\rho) = \frac{2P_0}{\pi w^2} \exp\left(\frac{-2\rho^2}{w^2}\right). \quad (11.18)$$

Under the small-aperture approximation established previously in Eq. (11.4), the power collected by a receiver of area A is proportional to the local irradiance $P(\rho) = AI(\rho)$. To determine the statistics of the received power under the influence of pointing jitter, we first find the inverse relationship expressing the radial displacement as a

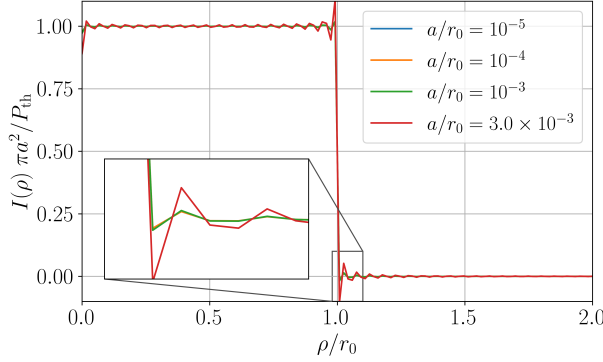


Figure 11.1: Optimum normalized far-field irradiance obtained through the numerical inverse Hankel transform of $\tilde{I}$ obtained through Eqs. (11.16)-(11.17).

function of received power

$$\rho(P) = \sqrt{\frac{w^2}{2} \ln \left(\frac{2AP_0}{P\pi w^2} \right)}.$$

Assuming a Rayleigh distribution for the pointing jitter (as defined in Eq. (11.5)), the probability density function of the received power, $\mathcal{F}_{\text{P,Gaussian}}(P)$, is computed via the transformation of random variables in Eq. (11.6)

$$\begin{aligned} \mathcal{F}_{\text{P,Gaussian}}(P) &= \left| \frac{d}{dP} \rho(P) \right| \cdot f_{\text{R}}(\rho(P)) \\ &\propto \frac{1}{P\sigma^2} w^2 2^{-\beta-3} \pi^{\beta-1} \left(\frac{AP_0}{Pw^2} \right)^{-\beta} \quad \text{with} \quad \beta = \frac{w^2}{4\sigma^2}. \end{aligned}$$

By imposing the normalization condition $\int_0^{P_{\text{max}}} \mathcal{F}_{\text{P,Gaussian}}(P) dP = 1$, where $P_{\text{max}} = 2AP_0/\pi w^2$ corresponds to the peak of the irradiance, the complete PDF is found to be

$$\mathcal{F}_{\text{P,Gaussian}}(P) = \frac{1}{P\sigma^2} w^2 2^{-\beta-2} \pi^{\beta} \left(\frac{AP_0}{Pw^2} \right)^{-\beta} \quad \text{with} \quad \beta = \frac{w^2}{4\sigma^2}.$$

Since the beam profile is constrained to a Gaussian shape, we treat this as a parametric optimization problem where the beamwidth w is the decision variable¹. The objective

¹The average power captured can be obtained from the PDF above, and is given by $\bar{P} = 2AP_0/(\pi w^2 + 4\pi\sigma^2)$ which has a maximum $\max \bar{P} = AP_0/(2\pi\sigma^2)$ in $w \rightarrow 0$. In the regime $w \rightarrow 0$ the approximation of the beam width being bigger than the receiver aperture is no longer valid. In this regime, the average captured power is given by $\bar{P} = P_0 [1 - \exp(-A/2\pi\sigma^2)]$, that under the approximation that the aperture is smaller than the pointing jitter, i.e. $A \ll \sigma^2$, yields the maximum above when expanding $\bar{P}$ in power series.

is to minimize the performance metric $\mathcal{S}$, defined as the probability that the received power falls below a given threshold P_{th}

$$\mathcal{S}(w) = \int_0^{P_{\text{th}}} \mathcal{F}_{\text{P,Gaussian}}(P) dP = \left(\frac{2}{\pi} \frac{AP_0}{w^2 P_{\text{th}}} \right)^{-\beta}.$$

To find the optimum beamwidth, we evaluate $d\mathcal{S}/dw = 0$, that has the only non-trivial root ($w \neq 0$)

$$w_{\text{opt,Gaussian}} = \sqrt{\frac{2AP_0}{e\pi P_{\text{th}}}}. \quad (11.19)$$

Under this condition the minimum $\mathcal{S}$ will be given by

$$\min \mathcal{S}_{\text{Gaussian}} = \exp \left(-\frac{AP_0}{2e\pi P_{\text{th}} \sigma^2} \right). \quad (11.20)$$

The relationships between the optimum beamwidth and the performance metric $\mathcal{S}$ as a function of the system parameters $\{A, P_0, P_{\text{th}}, \sigma\}$ are further illustrated in Fig. 11.2.

11.4.2. Super-Gaussian beams

While the Gaussian beam is the most used in free-space optical communications, it is not necessarily the most robust profile against the detrimental effects of pointing jitter. As demonstrated by the variational analysis in §11.3, flat-top-like distributions offer superior performance by providing a more uniform irradiance over a wider area. Since an ideal flat-top profile is not strictly attainable in practice, due to its discontinuity, it is natural to consider finite-order super-Gaussian beams as realistic approximations. These functions are characterized by a steepness parameter, or order n_{SG} , which allows them to continuously approximate the stepwise flat-top function as n_{SG} increases – a property that has led to their use in several applications, e.g., inertial confinement fusion [472, 491, 492].

The radially varying irradiance profile of a super-Gaussian beam is mathematically defined as

$$I(\rho) = \frac{P_0 4^{1/n_{\text{SG}}}}{\pi w^2 \Gamma[(2 + n_{\text{SG}})/n_{\text{SG}}]} \exp \left[-2 \left(\frac{\rho}{w} \right)^{n_{\text{SG}}} \right],$$

where n_{SG} is the beam order, P_0 is the total power, and $\Gamma(x)$ is the Gamma function. Notably, for $n_{\text{SG}} = 2$, this expression reduces to the standard Gaussian irradiance in Eq. (11.18). Under the small-aperture approximation, the power collected at a radial offset ρ is given by $P(\rho) = A I(\rho)$.

Since the received power $P(\rho)$ is a strictly monotonically decreasing function of the radial displacement ($dP/d\rho \leq 0$) for all $\rho \in [0, \infty)$, there exists a unique, one-to-one mapping between the power and the pointing error. Consequently, the event where the received power falls below a critical threshold P_{th} – defined as the performance parameter $\mathcal{S}$ – is statistically equivalent to the event where the radial jitter exceeds a corresponding spatial displacement ρ_{th} . By solving $P(\rho_{\text{th}}) = P_{\text{th}}$, we define the boundary of the outage probability zone in the spatial domain. Because larger pointing errors result in lower power, the probability of an outage is captured by the tail of

the jitter distribution. Thus, $\mathcal{S}$ can be calculated directly using the complementary cumulative distribution function of the Rayleigh pointing jitter, expressed as

$$\mathcal{S} = \int_{\rho_{\text{th}}}^{\infty} f_{\text{R}}(\rho) d\rho = \exp\left(-\frac{\rho_{\text{th}}^2}{2\sigma^2}\right).$$

where ρ_{th} is the point at which the power collected $P(\rho)$ meets the power threshold $P(\rho_{\text{th}}) = P_{\text{th}}$. To obtain this value,

$$\begin{aligned} P_{\text{th}} &= \frac{AP_0 4^{1/n_{\text{SG}}}}{\pi w^2 \Gamma[(2 + n_{\text{SG}})/n_{\text{SG}}]} \exp\left[-2\left(\frac{\rho_{\text{th}}}{w}\right)^{n_{\text{SG}}}\right] \\ \rightarrow \rho_{\text{th}} &= \frac{w}{2^{1/n_{\text{SG}}}} \left[-\ln\left(\frac{4^{-1/n_{\text{SG}}} \pi P_{\text{th}} w^2 \Gamma[(2 + n_{\text{SG}})/n_{\text{SG}}]}{A P_0}\right) \right]^{1/n_{\text{SG}}}. \end{aligned}$$

This transformation simplifies the analysis by shifting the integration from the complex power domain to the well-defined statistical properties of the pointing error. To achieve the best system performance, we seek to minimize the outage probability $\mathcal{S}$ by maximizing the radial tolerance ρ_{th} . Physically, this represents finding the beamwidth w that allows for the largest possible pointing error before the received power drops below P_{th} . By applying the first-order optimality condition $dw/d\rho_{\text{th}} = 0$, we identify the optimum beamwidth as

$$w_{\text{opt,SG}} = \sqrt{\frac{AP_0}{\pi P_{\text{th}} \Gamma[(2 + n_{\text{SG}})/n_{\text{SG}}]}} \left(\frac{2}{e}\right)^{1/n_{\text{SG}}}. \quad (11.21)$$

In the standard Gaussian case where $n_{\text{SG}} = 2$, this expression reduces to Eq. (11.19). At this optimum beamwidth, the maximum allowable radial displacement is given by

$$\rho_{\text{th}} = \sqrt{\frac{AP_0}{\pi P_{\text{th}} \Gamma[(2 + n_{\text{SG}})/n_{\text{SG}}]}} \left(\frac{2}{n_{\text{SG}} e}\right)^{1/n_{\text{SG}}}.$$

Consequently, the minimized performance parameter for the super-Gaussian beam is determined by substituting this maximized ρ_{th} into the Rayleigh complementary cumulative density function, yielding

$$\min \mathcal{S}_{\text{SG}} = \exp\left(-\frac{\gamma_{\text{SG}}^2}{2}\right), \quad (11.22)$$

with

$$\gamma_{\text{SG}} = \sqrt{\frac{AP_0}{\pi \sigma^2 P_{\text{th}} \Gamma[(2 + n_{\text{SG}})/n_{\text{SG}}]}} \left(\frac{2}{n_{\text{SG}} e}\right)^{1/n_{\text{SG}}}.$$

Consistent with our previous derivations, this result matches the solution for the standard Gaussian beam in Eq. (11.20) when $n_{\text{SG}} = 2$. A comparison between the optimum super-Gaussian beams of several orders and the standard Gaussian beam is illustrated in Fig. 11.2.

11.4.3. Flat-top beam

In the previous section, we introduced super-Gaussian beams and showed that, as the order increases, their transverse intensity profiles approach an ideal flat-top distribution. Moreover, in §11.3 we demonstrated –under suitable approximations– that the flat-top profile constitutes the optimum solution for mitigating the effect of the pointing jitter. Hence, the remaining question is to determine which flat-top solution is optimum. To address this, we consider a flat-top beam characterized by a total power P_0 and a width w . These parameters fully specify the profile and allow us to investigate how the performance depends on the choice of beam width under fixed total power. Mathematically, the flat-top beam is described by the intensity profile

$$I(\rho) = \begin{cases} \frac{P_0}{\pi w^2} & \rho \leq w, \\ 0 & \text{else.} \end{cases}$$

As in the Gaussian-beam case, determining the optimum beam width w constitutes an optimization problem. For a flat-top beam, the received power can take only two possible values depending on whether the receiver aperture lies inside or outside the illuminated region. Consequently, the probability density function of the received power becomes discrete, with values $\{AP_0/(\pi w^2), 0\}$ occurring with probabilities $\{1 - \mathcal{S}, \mathcal{S}\}$, respectively². Here

$$\mathcal{S} = \int_w^\infty f_R(\rho) d\rho = \exp\left(-\frac{w^2}{2\sigma^2}\right),$$

represents the probability that the radial pointing error exceeds the beam radius w . For the Rayleigh-distributed pointing jitter considered, this probability corresponds to the complementary cumulative density function evaluated at w . In principle, increasing the beam width w reduces $\mathcal{S}$, thereby decreasing the probability that the receiver falls outside the beam and improving system performance. However, increasing w simultaneously reduces the received power level $AP_0/(\pi w^2)$. If this power falls below the threshold P_{th} , the received power is always insufficient, and therefore the probability of collecting power below the threshold becomes unity, i.e. $\mathcal{S} = 1$. This situation occurs when

$$\frac{AP_0}{\pi w^2} < P_{\text{th}} \rightarrow w > \sqrt{\frac{AP_0}{\pi P_{\text{th}}}}.$$

Therefore, the optimum performance –corresponding to the minimum cumulative probability of the received power falling below the threshold P_{th} – is achieved when the beam width is chosen such that the received power inside the beam is exactly equal to the threshold value. Under this condition, the optimum intensity profile becomes

$$I(\rho) = \begin{cases} \frac{P_{\text{th}}}{A} & \rho < w_{\text{opt,FT}}, \\ 0 & \text{else,} \end{cases}$$

²The average power captured can be obtained from the discrete probability distribution above, and is given by $\bar{P} = AP_0(1 - e^{-\frac{w^2}{2\sigma^2}})/(\pi w^2)$ which has a maximum $\max \bar{P} = AP_0/(2\pi\sigma^2)$ in $w \rightarrow 0$, as obtained for the Gaussian beam in this limit.

where the optimum beam width is given by

$$w_{\text{opt,FT}} = \sqrt{\frac{AP_0}{\pi P_{\text{th}}}}. \quad (11.23)$$

The corresponding minimum probability that the received power falls below P_{th} , which serves as the performance metric, is

$$\min \mathcal{S}_{\text{FT}} = \exp\left(-\frac{\gamma_{\text{FT}}^2}{2}\right), \quad (11.24)$$

where

$$\gamma_{\text{FT}} = \sqrt{\frac{AP_0}{\pi \sigma^2 P_{\text{th}}}}.$$

This result establishes a direct relation between the maximum achievable performance—in terms of outage probability—and the system parameters: threshold power P_{th} , receiver aperture area A , pointing jitter σ , and total transmitted power P_0 . In Fig. 11.2, the optimum beamwidth and the corresponding performance metric $\mathcal{S}$, as functions of the parameters $\{A, P_0, P_{\text{th}}, \sigma\}$, are shown together with the optimum solutions obtained for Gaussian and super-Gaussian beams. Furthermore, Fig. 11.3 shows the far-field irradiances for the different beams analyzed. It has also been checked that the ring flat-top (a donut-like top hat with a hole in the center) profile results in a worse performance compared to the standard flat-top function (see App. F.3).

In the presence of an *unknown* boresight or a static pointing error, the pointing statistics are described by a Rice distribution (if the static error is known, then the system should be recalibrated, and the effect of the remaining pointing jitter could be mitigated by means of the beams proposed in this chapter). Nevertheless, the resulting optimization problem remains unchanged: the objective is to maximize the central region of the far-field where the collected power exceeds the specified power threshold. Since the area covered by the flat-top beam includes that of a conventional Gaussian beam, the minimum achievable $\mathcal{S}$ will remain smaller for the flat-top, but will differ from the value obtained under the Rayleigh distribution assumption discussed above. By similar reasoning, if the estimation of the pointing jitter σ^2 is inaccurate, the optimization still yields the same result, since the optimum beamwidths depend only on the ratio AP_0/P_{th} and not on the pointing jitter itself. If the actual pointing jitter is smaller than expected, the achieved minimum $\mathcal{S}$ will also be smaller (see Fig. 11.2).

Another practical aspect of interest arises when an overdimensioning coefficient $\mathcal{C} > 1$ is applied to the value of P_{th} in link performance estimations. Equivalently, this can be interpreted as applying the same scaling factor to σ^2 , or, conversely, as introducing an underdimensioning factor $\mathcal{C}^{-1}$ to the parameters $\{A, P_0\}$. In this case, the computed far-field beam widths for Gaussian, super-Gaussian, and flat-top beams are all scaled by a factor $1/\sqrt{\mathcal{C}}$ (see Eqs. (11.19), (11.21), (11.23)). Consequently, the resulting value of $\mathcal{S}$ is no longer the minimum achievable for a given beam shape. Specifically, for a Gaussian beam, the achieved $\mathcal{S}$ becomes $(\min \mathcal{S}_{\text{Gaussian}})^\Sigma$, where

$\Sigma = (1 + \ln \mathcal{C})/\mathcal{C}$. For a super-Gaussian beam, it becomes $(\min \mathcal{S}_{\text{SG}})^\Omega$, where

$$\Omega = \frac{1}{\mathcal{C}} \left[1 + \frac{n_{\text{SG}} \ln \mathcal{C}}{2} \right]^{2/n_{\text{SG}}}.$$

For a flat-top beam, the corresponding expression is $(\min \mathcal{S}_{\text{FT}})^{1/\mathcal{C}}$. In all cases, the system performance degrades relative to the respective optima due to the overestimation of the power threshold. It is therefore of interest to determine the value of the overdimensioning coefficient for which the performance of the Gaussian and flat-top beams becomes identical. This can be obtained by solving for $\mathcal{C}$ such that $(\min \mathcal{S}_{\text{Gaussian}})^\Sigma = (\min \mathcal{S}_{\text{FT}})^{1/\mathcal{C}}$. Numerical evaluation shows that this occurs at $\mathcal{C} \approx 5.57$. Thus, when such an overdimensioning factor is applied, the resulting performance $\mathcal{S}$ of both Gaussian and flat-top beams is the same, and the flat-top beam offers no performance advantage.

To quantify the power savings provided by super-Gaussian and flat-top beams relative to the fundamental Gaussian beam, Fig. 11.4 shows the reduction in required transmitted power for a fixed performance level $\mathcal{S}$. The vertical axis represents the ratio $P_0/P_{0,\text{Gaussian}}$, i.e., the total power required by a super-Gaussian beam to achieve the same performance as an optimally chosen Gaussian beam. Notably, this ratio is independent of the particular value of $\mathcal{S}$ selected and therefore represents an intrinsic measure of the relative efficiency of the different beam profiles. By mathematically equating the $\min \mathcal{S}$ of the super-Gaussian beams in Eq. (11.22) with the Gaussian in Eq. (11.20) the ratio

$$\frac{P_0}{P_{0,\text{Gaussian}}} = \frac{1}{e} \Gamma \left(\frac{2 + n_{\text{SG}}}{n_{\text{SG}}} \right) \left(\frac{n_{\text{SG}} e}{2} \right)^{2/n_{\text{SG}}},$$

is obtained. As the super-Gaussian order n_{SG} increases, the required power decreases monotonically. This behavior reflects the increasing spatial uniformity of the beam profile and its progressive convergence toward the optimum flat-top distribution. In the limit of large n_{SG} , the required power approaches that of the ideal flat-top beam, indicated by the horizontal dashed line, which represents the minimum power required to achieve the specified performance level. The power ratio required for the flat-top can be analytically obtained in a similar way to the super-Gaussian case, yielding a minimum $e^{-1} \approx 36,8\%$ power ratio.

11.5. Incoherent and coherent beam shaping

In the previous sections, the optimum far-field irradiances have been analyzed, and their performance has been evaluated. However, a way of achieving these irradiances has not yet been proposed by optical means. To do so, in this section, several methods for approximating super-Gaussian and flat-top beams are discussed, and their performance is evaluated.

Satellite optical communications are not the only application in which uniform irradiance of the beam is desirable. Other applications include inertial confinement fusion [491], laser material processing [493, 494], molecule manipulation [495] and addressing atomic qubits [492]. Leveraging on the optical systems proposed for achieving uniform

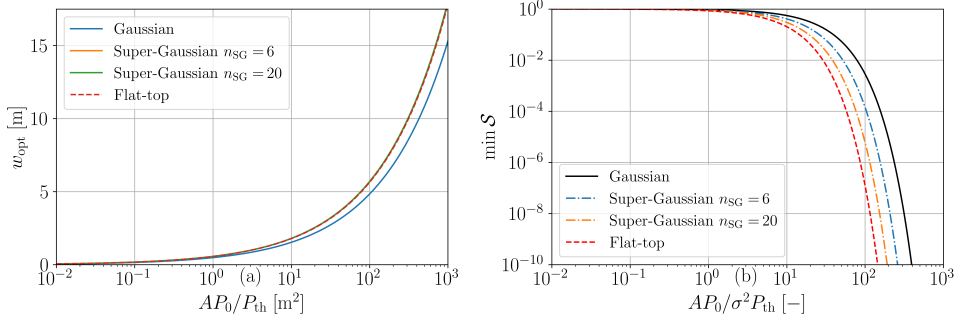


Figure 11.2: Comparison of optimum Gaussian, super-Gaussian, and flat-top beams. The optimum beamwidth expressions and the corresponding performance metric S are shown as functions of the system parameters $\{A, P_0, P_{\text{th}}, \sigma\}$.

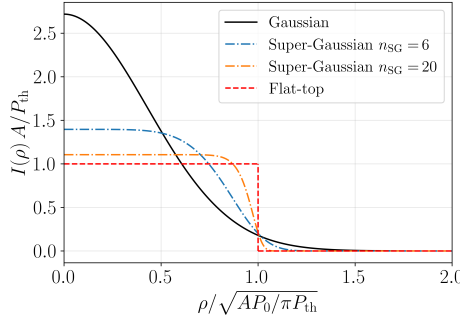


Figure 11.3: Optimum far-field irradiances (normalized) for different Gaussian, super-Gaussian, and flat-top beams.

irradiance fields, in this section, we analyze two main physical means of approximating higher-order super-Gaussian beams. Firstly, incoherent light sources are considered, in which by means of dispersion, different transversal modes are superposed in irradiance to get an approximate flat-top beam. Secondly, a perfectly coherent light beam is assumed. In this case, a phase screen is designed by means of an iterative process to convert the original beam into an approximate flat-top beam. Although other approaches combining phase masks with spectral dispersion have also been proposed for obtaining flat-top beams [496, 497], these will not be investigated in this chapter.

11.5.1. Incoherent light: Spatial mode superposition

Super-Gaussian and flat-top beams can be approximated in the far-field by superposing different free-space optical modes in irradiance. To superpose irradiance fields, however, it is necessary that the different modes are incoherently combined. Physically, this can be done by using different polarizations or wavelengths of light for each

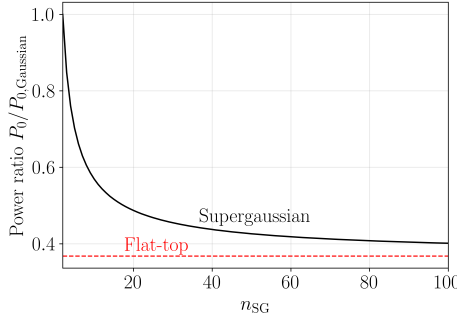


Figure 11.4: Power savings obtained with super-Gaussian and flat-top beams. Ratio $P_0/P_{0,\text{Gaussian}}$ of the total power required to achieve the same performance $\mathcal{S}$ as a function of the super-Gaussian order n_{SG} . The horizontal dashed line indicates the limiting case corresponding to the optimum flat-top beam.

of the modes. Previous works have proposed the use of the superposition of orthogonally polarized modes to shape the far-field irradiance in a convenient way (as in §9-10). This technique can be experimentally implemented by using simple polarization optics and diffractive optical elements as in §10. However, as the polarization of light has only two degrees of freedom, only two modes can be superposed, strongly limiting the achievable variety of beam shapes with this technique. To overcome this limitation, different wavelengths of light can be used for each of the modes [498]. As the frequency of light is a continuous spectrum, there is no theoretical limit on the number of modes that we could combine using this technique. Experimentally, wavelength-based incoherent beam shaping can be achieved by means of dispersive elements and multi-plane light-converters, as has been demonstrated in Ref. [498] for atmospheric turbulence mitigation.

When using orthogonal polarization states to perform incoherent beam shaping through superposition, polarization can no longer be used to modulate, multiplex, or separate transmit and receive optical paths on the optical communication terminal (see §3). Hence, in this case, wavelength would be used to separate these channels, as spatial mode is also being used for shaping the beam incoherently. When mode superposition is done by combining different wavelength beams with different modes, then polarization can be used for other purposes on the optical communication terminal. Not only that, in this case, other wavelength ranges away from the ones used for incoherent beam shaping can be used for modulation, multiplexing, and separation of transmit and receive optical paths.

When incoherently combining several spatial modes, the resulting irradiance is given by the superposition

$$I(x, y) = \sum_{p=0}^{p_{\max}} \sum_{\ell=0}^{\ell_{\max}} c_{p\ell} I_{p\ell}(x, y), \quad (11.25)$$

where $c_{p\ell}$ are the weights of the spatial modes $I_{p\ell}(x, y)$ considered and the sum is done

over the truncated modes, and $\{p, \ell\}$ are the indices of the spatial modes considered. These modes could be Laguerre-Gaussian (LG), Hermite-Gaussian (HG), or another set of spatial modes of our choice. The indices $\{p, \ell\}$ refer to radial and azimuthal indices of the LG modes, and the indices $\{n, m\}$ refer to the horizontal and vertical indices of the HG modes. As the number of modes considered increases, the field should get closer and closer to the desired beam shape. The coefficients $c_{p\ell}$ are calculated via a non-negative least-squares algorithm. More advanced ways of computing the optimum weights of the beams in incoherent superposition have also been proposed [499]. In case polarization-based incoherent beam shaping is used, the summatory in Eq. (11.25) can only be done over a maximum of two spatial modes as in §9-10.

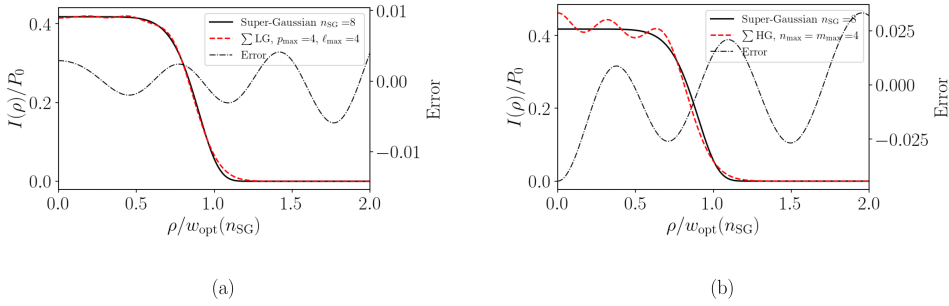


Figure 11.5: Incoherent beam shaping using (a) Laguerre-Gaussian and (b) Hermite-Gaussian modes, targeting the 8th-order super-Gaussian beam.

Using the method above, the approximated super-Gaussian beams of different orders can be obtained using both Laguerre-Gaussian and Hermite-Gaussian modes. In Fig. 11.5, the best fits to a super-Gaussian beam of 8th order are shown for both truncated LG and HG bases. By increasing the number of modes considered, higher-order super-Gaussian beams can be targeted, and the root mean square error (RMSE) of the resulting irradiance can be decreased, as seen in Fig. 11.6. It can be seen in this figure that the LG modes converge faster than HG modes towards super-Gaussian beams. Furthermore, the results also show that a higher number of modes are required as the order of the super-Gaussian beam increases, as expected. To evaluate the communication performance of these beams, the probability $\min \mathcal{S}$ can be computed for different $AP_0/\sigma^2 P_{\text{th}}$ values for the beam shapes obtained through incoherent superposition. The results for different numbers of modes along with the baseline Gaussian solution and the optimum flat-top are shown in Fig. 11.7. It can be seen that the performance of the incoherent superposition is better than the baseline Gaussian beam, and that the performance of LG beams is better than the HG beams, as expected from the better fit to the super-Gaussian beams discussed above.

11.5.2. Coherent light: Phase mask

The previous method can be computationally too expensive, and the truncation order of the basis also limits the obtainable beam shape to a restricted domain. Furthermore,

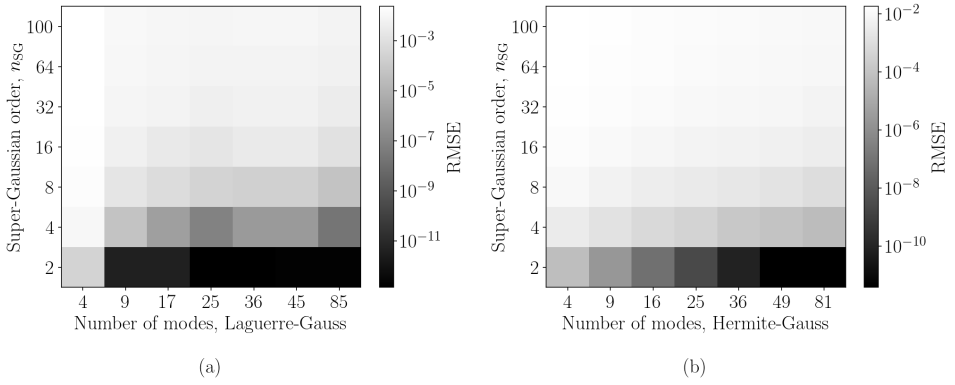


Figure 11.6: Root mean square error of the incoherent superposition of (a) Laguerre-Gaussian and (b) Hermite-Gaussian beams, when targeting several super-Gaussian order beams. The number of modes considered are $(p_{\max} + 1) \times (\ell_{\max} + 1)$ and $(n_{\max} + 1) \times (m_{\max} + 1)$ in the Laguerre-Gaussian and Hermite-Gaussian case, respectively.

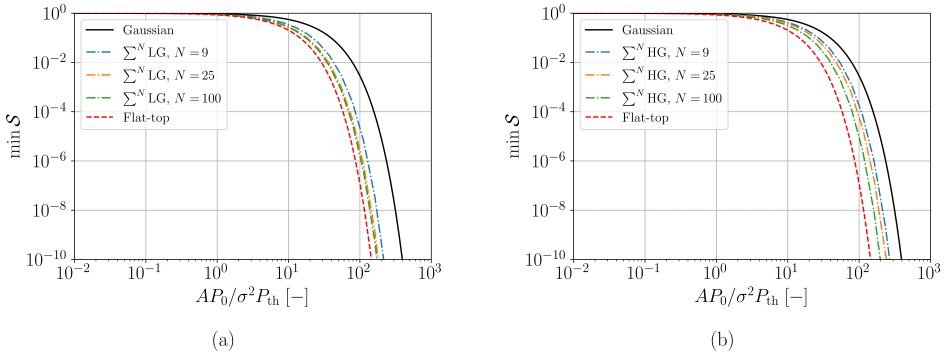


Figure 11.7: Performance parameter $\min \mathcal{S}$ for an incoherent superposition of (a) Laguerre-Gaussian and (b) Hermite-Gaussian beams, for different number of modes N .

using the polarization and wavelength degrees of freedom to incoherently combine the modes limits the applicability of these degrees of freedom for modulation, multiplexing, and receive/transmit optical path splitting. To overcome these limitations, a coherent beam shaping technique can be followed. By considering a fully coherent beam of light in the source, a properly designed phase mask can be used to generate different beam shapes (see Fig. 11.8). For optical communication transmitter terminals, the beam from the source can be considered to be Gaussian, as most of the terminals are built from single-mode fiber components.

There are several optical components that can induce the bidimensional phase

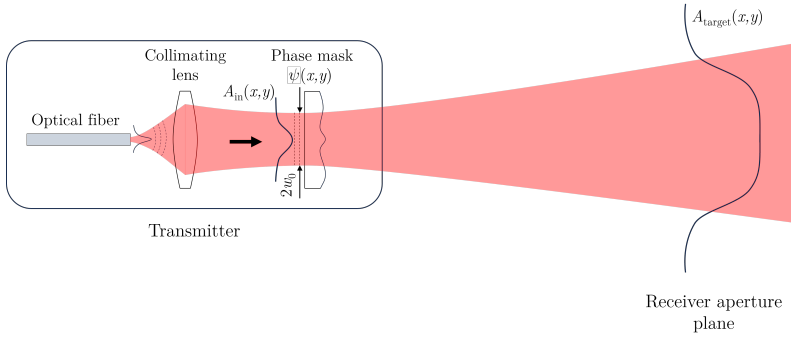


Figure 11.8: Illustration of the coherent beam shaping technique using a phase mask.

masks³ needed for coherent beam shaping. For mitigation of atmospheric turbulence, Billaud et al. proposed the use of multi-plane light converters for coherent beam shaping [500]. Multi-plane light converters consist of both reflective and refractive optical phase masks that convert different spatial modes into one another [501]. Another way of generating a beam shape of interest in the far-field is through the use of metasurfaces, in which subwavelength geometries are used to manipulate the phase induced at each point of the surface (although many other functionalities can also be achieved with these structures) [502]. Dynamical optical elements such as spatial light modulators or deformable mirrors are used for applications in which the phase mask to be applied changes over time [494, 503–505]. For the static beam shaping application targeted in this chapter, diffractive optical elements seem to be one of the most suitable experimental implementations for our phase screens [506]. This is because they can be fabricated with high precision [506, 507], introduce well-defined phase modulations with high optical transmission efficiency, and operate as compact passive components that do not require active control during operation. The performance of such systems has been demonstrated in Refs. [493, 507], where multiple cascaded diffractive optical elements—following a concept similar to multi-plane light converters—are used for laser material processing, enabling the generation of flat-top and other tailored beam shapes. In Ref. [507], the effects of the finite steepness of the 2π phase jumps arising from manufacturing limitations are analyzed. Although binary optical phase elements have been proposed for flat-top beam shaping [508], they are generally less flexible and typically exhibit reduced beam quality compared to multi-level or continuously varying phase profiles.

Considering the source of light to be Gaussian at the transmitter end, a phase mask can be designed that converts this beam into a super-Gaussian-like irradiance distribution in the far-field (see Fig. 11.8). To achieve this, a phase mask can be designed employing phase retrieval algorithms [509–514]. These algorithms—one of the most common being the Gerchberg-Saxton (GS) algorithm or modified versions of it—employ an iterative process that optimizes the targeted far-field irradiance. The basic

³In general, beam shaping can be achieved using a combined phase-transmission mask. However, in the context of this work, the power loss introduced by transmission masks is detrimental to system performance. Therefore, only phase-only masks are considered.

principle is that the algorithm alternates between the spatial domain (phase mask plane) and the Fourier domain (far-field) using forward and inverse Fourier transforms. In each domain, the amplitude constraint is enforced (Gaussian amplitude at the transmitter plane and the desired super-Gaussian amplitude in the far-field), while the phase is allowed to evolve during the iterations. Starting from an initial guess for the phase $\psi_0(x, y)$, the complex field $U_0(x, y) = A_{\text{in}}(x, y)e^{i\psi_0(x, y)}$ is propagated to the far-field through a Fourier transform $\tilde{U}_0(k_x, k_y) = \mathcal{F}\{U_0(x, y)\}$. The amplitude is then replaced by the desired far-field amplitude $A_{\text{target}}(k_x, k_y)$ while keeping the phase, and the field is propagated back with an inverse Fourier transform. Repeating this process iteratively yields a phase distribution that produces approximately the desired far-field irradiance [514]. Moreover, several modifications of the Gerchberg–Saxton algorithm have been proposed to improve convergence speed and generate continuous phase screens [491, 515–518]. For example, a modified iterative scheme presented in Ref. [519] demonstrates the generation of a far-field irradiance that approximates to a super-Gaussian beam of 12th order. Other ways of designing phase masks have also been proposed, such as the Zernike basis decomposition combined with grid-search and gradient-descent optimization introduced in Ref. [502]. This method exploits the azimuthal symmetry of the problem, yielding radially symmetric phase masks. However, despite this advantage, iterative phase retrieval algorithms such as Gerchberg–Saxton generally exhibit faster convergence and remain among the most widely used approaches for practical phase mask design. Furthermore, recent advances in the training of neural networks can also be exploited to design these phase masks iteratively, exploiting the concept of diffractive neural networks [493].

Using the Gerchberg–Saxton algorithm proposed by Liu et al. [519], the phase masks have been designed to convert the Gaussian beam into different super-Gaussian orders. To do so, the algorithm is initialized using a phase mask obtained relying solely on geometrical optics principles [520]. After one hundred iterations, the retrieved phase screen is obtained and the resulting far-field irradiance is analyzed. As an example, the targeted far-field super-Gaussian beam and the achieved approximated beam through this method are shown in Fig. 11.9(a)-(b). The irradiances shown are normalized to total unit power. Furthermore, the retrieved phase mask is also shown. Due to the azimuthally symmetrical nature of the problem and the azimuthally symmetrical initialization of the phase mask provided by the geometrical optical phase mask, the numerically retrieved phase mask also possesses this symmetry. In case random phase masks would have been used [515], this symmetry would be broken, and some more steps would need to be included in the algorithm to force the phase mask to have this symmetry.

In Fig. 11.9(c), the root mean square error of the obtained far-field is compared with the targeted super-Gaussian for different orders. For approximating the Gaussian beam $n_{\text{SG}} = 2$ the GS algorithm is not too appropriate, as it can be seen. This might be due to the initialization of the phase mask. From a certain point on ($n_{\text{SG}} \approx 25$), as the targeted super-Gaussian order increases, the algorithm starts to deviate further from the target as it can not accurately approximate the discontinuity generated in the $n_{\text{SG}} \rightarrow \infty$ flat-top limit.

In a similar manner to what was done for the incoherent case, the performance pa-

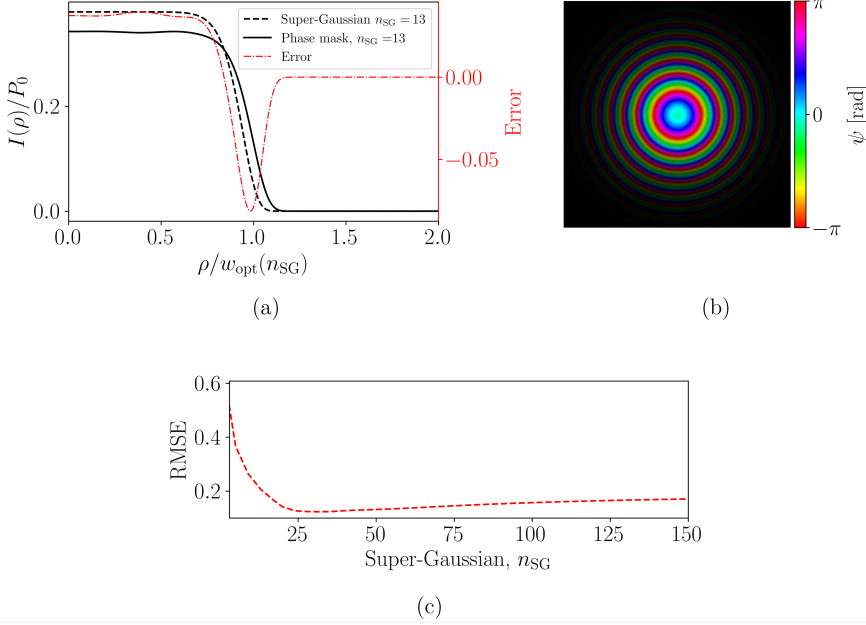


Figure 11.9: Coherent beam shaping using the GS algorithm for obtaining super-Gaussian beams. (a) shows the targeted 13th-order super-Gaussian and the achieved far-field irradiance, and (b) shows the input beam, where the irradiance is Gaussian (brightness of the color) and the retrieved phase mask is given by the colormap. (c) shows the root mean square error of the far-field irradiance compared to the targeted irradiance for increased super-Gaussian orders targeted.

parameter $\min \mathcal{S}$ for far-field irradiances obtained through the designed phase masks has been evaluated. The results for different targeted super-Gaussian beams are shown in Fig. 11.10. The results show a close resemblance to the analytically obtained results in Fig. 11.2(b). However, as the super-Gaussian order increases, the far-field irradiances obtained through the optimized phase mask can no longer closely resemble the respective super-Gaussian beam. This produces the performance saturation of these physically achievable beams to occur faster than the analytical ones (that saturate at the flat-top curve as the super-Gaussian order increases, as shown in Fig. 11.2).

For practical purposes, it might be of interest to the reader to compare the performance of both the theoretical super-Gaussians and the approximated incoherent and coherent beams. To do so, the ratio of the transmitted power P_0 required to obtain the same performance $\min \mathcal{S}$ as that of a conventional Gaussian beam is computed. The results are shown in Fig. 11.11 where the ratio $P_0/P_{0,\text{Gaussian}}$ is plotted as a function of the super-Gaussian order or the number of modes considered. For the super-Gaussian and flat-top beams, the curve is identical to that shown in Fig. 11.4, and it is seen that the super-Gaussian asymptotically approaches the flat-top behaviour as the order n_{SG} increases. For the incoherent superposition of LG and HG families presented in

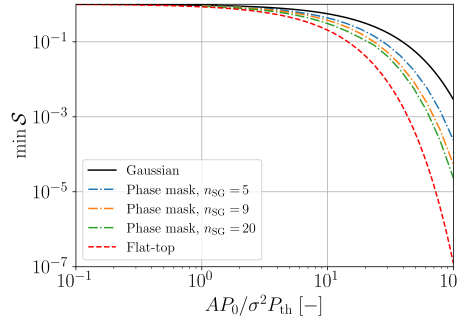


Figure 11.10: Performance parameter $\min \mathcal{S}$ for the irradiance obtained with the phase screens computed through the GS algorithm for different super-Gaussian beam orders, compared to the conventional Gaussian and the optimum flat-top performances.

§11.5.1, the plot shows the power ratio as the number of modes considered in the basis is increased. Following the results shown in Figs. 11.6-11.7, the LG beams converge faster than the HG and saturate at around $\sim 53\%$ (although the ratio continues to decrease slowly for higher N numbers). As discussed in §11.5.1, polarization-based superposition would be limited to $N = 2$ modes. For the LG beam families, the obtained $\sim 60\%$ ratio is in agreement with the power-saving results reported in §9.

For the coherently generated beam shapes, the phase masks obtained through the GS algorithm provide results that are extremely close to the theoretical super-Gaussian ratios up to approximately the 17th super-Gaussian order. However, for higher orders, as shown in Fig. 11.9(c), the phase masks designed with this approach begin to diverge from the corresponding super-Gaussian profiles. For $n_{\text{SG}} > 17$, the phase masks intended to generate higher-order super-Gaussian beam shapes instead produce beams that perform worse than those designed for lower orders. For the phase mask designed for $n_{\text{SG}} = 17$, the resulting far-field irradiance yields a performance ratio of 50%, in agreement with the analytical result for the targeted super-Gaussian profile. This ratio can also be interpreted as the required value of the parameter $AP_0/\sigma^2 P_{\text{th}}$ relative to that of a conventional Gaussian beam. Consequently, the graph may also be interpreted as indicating the reduction in required receiver aperture area for achieving the same performance, for example.

It is important to note that, when evaluating the performance of both the incoherent and coherent beam shapes, the far-field irradiance beam width w_{opt} had to be scaled. In our analysis, we did not consider limitations associated with adjusting the far-field beam width due to the finite aperture of the transmitter, nor the truncation effects on the transmitted beams (analyzed for Gaussian beams in §6). As the targeted far-field beam width decreases—corresponding to the small AP_0/P_{th} regime shown in Fig. 11.2(a)—the divergence of the beams involved may limit the minimum achievable beam width in the far-field. This limitation can be mitigated by increasing the transmitter aperture or by partially sacrificing the minimum achievable performance $\mathcal{S}$.

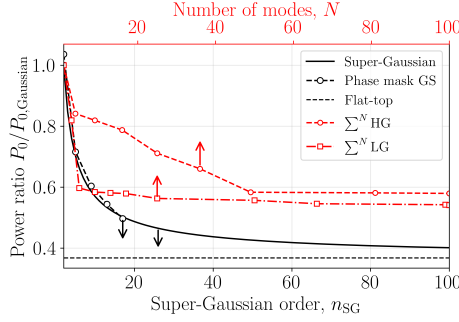


Figure 11.11: Power savings (compared to a conventional Gaussian beam) of the theoretical super-Gaussian, flat-top, incoherent beamshaping with LG and HG beam families, and coherent beam shaping achieved through a phase mask.

11.6. Conclusions and future work

This chapter investigated the optimum far-field irradiance for mitigating the effect of the transmitter pointing jitter in intersatellite optical communication. It has been found through a variational approach that the optimum far-field irradiance corresponds to a flat-top beam in the far-field. Due to the physics of wave propagation limiting the obtention of a flat-top in the far-field, the performance of the super-Gaussian beam family has been investigated (as it approaches the flat-top beam in the infinite order limit). It has been found theoretically that a flat-top requires $e^{-1} \approx 36,8\%$ of the power required by the conventional Gaussian beam to achieve the same performance. Furthermore, incoherent and coherent beam shaping techniques have been proposed to approximate super-Gaussian beams of different orders in the far-field. It has been found that incoherent beam shaping using Laguerre-Gaussian beams converges faster to the super-Gaussian beams than the Hermite-Gaussians due to the azimuthal symmetry of the former. The achievable minimum power ratio through incoherent beam shaping was found to be $\sim 53\%$ when combining one hundred Laguerre-Gaussian modes. Coherent beam shaping, which involved designing a phase mask through the Gerchberg-Saxton algorithm, was able to achieve a very close result to that given by a theoretical super-Gaussian beam of 17th order. In this case, yielding to a power ratio of $\sim 50\%$ compared to a conventional Gaussian beam.

These results extend the previous attempt to optimize the performance of intersatellite optical communication affected by transmitter pointing jitter [118] by providing a fundamental lower limit and approximating it via different optical beam shaping methods. It is important to mention that while the optimization provided is done for minimizing probabilities of being below a reasonable power threshold, other performance parameters (e.g., average capacity, peak capacity) would yield different optimum far-field irradiances. For example, as demonstrated in this chapter, the maximum average captured power is achieved in the limit as the beam width approaches zero ($w \rightarrow 0$), at which point the beam profile converges to a two-dimensional Dirac delta function.

Future work could further expand the beam shaping approaches explored in this chapter in several directions. First, improved algorithms for designing phase masks capable of generating higher-order super-Gaussian beams would allow more optimum far-field irradiances. A realistic performance assessment should also incorporate losses that occur in a real system, similar to the analysis presented in §10, in order to evaluate the actual power savings of the proposed beam shapes. In addition, experimental implementations of the incoherent superposition of higher-order modes –such as through multi-plane light converters or comparable optical architectures– would provide a pathway toward practical realization of the concepts discussed in §11.5.1. Experimental demonstrations of the coherent beam shaping techniques are also required, particularly accounting for manufacturing constraints associated with phase mask elements such as diffractive optical elements. Furthermore, a detailed study of the impact of optical aberrations of the incoming Gaussian beam in coherent beam shaping systems would help quantify performance degradation in realistic setups. Finally, extending the proposed beam shaping to relevant free-space optical communication scenarios would be valuable, including coupling efficiency into detectors or optical fibers in satellite optical communication systems affected by receiver pointing jitter, as well as downlink scenarios with atmospheric turbulence.

Part IV



Closing remarks

In the previous parts, we first identified, then modeled, and finally proposed a novel mitigation technique for some of the phenomena affecting satellite free-space optical and quantum communications. It is now crucial to look forward, address the remaining challenges, and propose a clear path to tackle them. With this goal, this final part summarizes the key findings of the thesis and outlines recommended directions for future research in satellite optical and quantum communications.

Chapter 12

Conclusions and outlook

Prediction is very difficult, especially if it's about the future.

Niels Bohr

12.1. Revisiting the research questions

It is the objective of a thesis to answer the main research question posed in §1. We are now in a position to revisit this question and provide an answer based on the results presented in the previous chapters. The main research question states:

How can opto-thermo-mechanical perturbations in spaceborne optical and quantum communication terminals be modelled, and how can their impact on communication performance be mitigated?

Following a *divide and conquer* approach, we address this main research question by tackling three subquestions independently:

1. *Which opto-thermo-mechanical phenomena perturb spaceborne optical and quantum communication terminals?*
2. *How can these phenomena be modelled within a unified mathematical framework that quantifies their impact on the performance of optical and quantum communication?*
3. *How can beam shaping mitigate the impact of the studied phenomena on optical and quantum communication performance?*

Before addressing these questions, it is important to clarify why answering them is valuable. This rationale can be viewed from two complementary perspectives: societal and scientific.

- *How does society benefit from answering these research questions?*

In the modern era, the flow of information has become one of the most valuable assets of human society. Communication –the exchange of information– has enabled activities that were unimaginable to previous generations, such as real-time communication across continents or collaboration between distant locations. As communication has become essential to daily life, the demand for higher data rates has continuously increased. To meet these demands, optical frequencies within the electromagnetic spectrum offer significantly higher bandwidth than the traditionally used radiofrequency spectrum. Among the technologies that exploit the optical spectrum, both optical fiber networks –already widely deployed– and free-space optical communication systems have been proposed. As communication distances increase and a satellite segment becomes necessary to achieve global coverage, the development of satellite optical communication systems becomes essential. In this context, it is crucial to understand the performance of inter-satellite and satellite-to-ground optical links. By addressing the research questions posed in this thesis, we aim to (1) *identify and weigh* the relevant perturbing effects, (2) *model and quantify* these phenomena, and (3) *mitigate*

their impact on system performance. Through this process, future satellite optical communication terminals can be optimized to achieve more efficient and reliable operation. This, in turn, enables applications from which society can directly benefit, such as the rapid transmission of satellite imagery for emergency response.

In addition to increased bandwidth, optical communication systems provide enhanced security due to the narrow divergence of laser beams, which makes their interception more difficult. While this physical-layer security is valuable, an even more fundamental level of security can be achieved by exploiting the principles of quantum mechanics. This is where society stands to benefit from the development of global quantum communication networks, supported by space-borne quantum communication segments. As quantum computing technologies continue to mature, the possibility of quantum computers capable of breaking current cryptographic schemes –such as RSA– becomes increasingly realistic [26, 27]. In this context, as discussed in §4, quantum key distribution (QKD) offers a promising solution for restoring secure communications. However, QKD is not the only application of global quantum communication networks. In a future quantum network, where satellite links are used to distribute entanglement over large distances as described in §7, many additional applications become possible. One such application is distributed quantum computing. By interconnecting quantum processors located in different regions, the computational capabilities of individual systems can be combined. In this scenario, quantum networks act as interconnects between processors, each with a limited number of physical qubits. By networking these systems, the total computational capacity can scale beyond that of any individual device. This could enable the solution of problems that are currently intractable for classical computers, including the simulation of complex molecules for drug discovery and the design of new materials. Such advances would have a direct and meaningful impact on society.

- *How does the scientific community benefit from answering these research questions?*

In many scientific endeavours, the application of the findings and therefore their impact on the lives of current or future members of society is not very straightforward. In disciplines like pure math or theoretical physics, the researcher sometimes has to find their final motivation of performing that specific research in something more fundamental than its impact on society. Although it is totally true that certain discoveries that at first seem not to have a clear impact on society can find their way to affect our daily lives –history of science is full of these events– this is not always the case. If the research has no direct or predictable societal impact, and hence no economic benefit can be exploited out of it, it can be more challenging to find the *why* of performing that research. In such cases, it is of utmost importance to find a motivation that transcends societal impact and economic growth. This is the pure scientific knowledge motivation. *Knowledge for the sake of knowledge*. There is an interesting debate on whether the accumulation of *knowledge for the sake of knowledge* is an endeavour worth

pursuing. It is the author's opinion that history strongly suggests that it is.

Examples such as the development of quantum mechanics and number theory illustrate how initially abstract research later enabled transformative technologies, including semiconductors, lasers, and modern cryptography. These cases highlight that long-term societal impact can emerge even when it is not initially foreseeable.

Unlike the domains explained above, the research presented in this thesis does have the potential to have a direct societal impact through the application to future optical and quantum satellite communication systems. However, it is the author's belief that it is equally as important to emphasize the contributions that the answers to these research questions could have in the scientific community. By answering the above, the body of knowledge on the environmental effects on optical and quantum systems of any kind is enlarged. Although in our case it applies directly to communication systems, other scientific endeavours could indirectly benefit from the answer to these research questions. Other applications –some known and some yet to be discovered– have the potential to benefit from the answers to these research questions, in the same way as this thesis has been built upon discoveries in other scientific endeavours. A clear example of this is how the flat-top optical beam shaping methods presented in §11 were built upon work that was mainly oriented towards the application of inertial confinement fusion. In a similar manner, the beam shaping approach to mitigate the effect of stochastic power fluctuations proposed in this work could be exploited in long-range optical applications such as space-based gravitational wave interferometry. In this way, enlarging the body of knowledge for the sake of doing so can be beneficial even if, when doing so, it is not clear whether it will have or not a future application that can improve the lives of our contemporary living or those to come.

The reader should, at this point, be convinced of the relevance –and, to some extent, the urgency– of addressing the research questions posed in this thesis. As will be demonstrated in the following sections of this chapter, the results of this thesis provide concrete answers that directly support the development of higher-performance optical and quantum satellite communication systems. In this sense, the work presented here goes beyond the notion of knowledge for its own sake, delivering results that can be directly leveraged in practical implementations. To answer the main research question, we will proceed by addressing each of the subquestions in sequence, structuring the discussion according to the different parts and chapters of this thesis.

12.2. Opto-thermo-mechanical phenomena

In this section, the main findings that answer the first research question are presented. Restating the first research question, we have:

1. Which opto-thermo-mechanical phenomena perturb spaceborne optical and quantum communication terminals?

This subquestion aims to *identify* the different opto-thermo-mechanical phenomena that affect the performance of optical and quantum communication terminals onboard satellites. The answer to this question was methodically given in §3 and §4 for optical and quantum terminals, respectively.

12.2.1. Satellite optical communications

Regarding optical terminals, in §3, firstly, the in-orbit thermal and mechanical loads were identified. These loads, in turn, generate opto-thermo-mechanical phenomena in the optical systems onboard the terminal. In this thesis, a classification of these phenomena was proposed as follows: rigid body motions of the satellite, rigid body motions of the optical components, surface deformations, stress birefringence, and thermo-optical dispersion. Each of these affects the optical performance by inducing pointing jitter, wavefront errors, and polarization effects. Finally, and most relevant for the application of optical communication, these effects degrade communication performance by reducing the received signal, inducing crosstalk between different channels (which could correspond to transmit and receive signals or multiplexed channels), and therefore increasing the bit error rate. All these ideas are summarized in Fig. 3.1.

It is important to emphasize that, depending on how the different properties of light are used for different purposes, the effect of the opto-thermo-mechanical phenomena on both the optical performance and the communication performance of the system will differ. For example, if polarization is used for multiplexing communication channels, stress birefringence occurring in the optical elements will induce crosstalk between the multiplexed signals, ultimately increasing the bit error rate. However, if polarization is used for separating transmit and receive signals, stress birefringence will induce crosstalk between these signals, which may increase the noise in the received signal and, consequently, the bit error rate. For other properties of light (wavelength, phase, and amplitude), analogous phenomena can be tracked and evaluated using the framework proposed in §3. Through this identification and classification of opto-thermo-mechanical effects, researchers in the field can identify and track environmental effects in satellite optical communications. Building upon this work, several of these effects were quantitatively analyzed in different chapters of this thesis. In particular, accurate models were developed to characterize the impact of optical aberrations and pointing jitter, as presented in §6.

Furthermore, existing mitigation techniques for reducing environmental effects onboard satellite terminals have been identified and classified. Regarding microvibrations, source suppression techniques and terminal isolation systems were presented, and the different microvibration frequency ranges suppressed by each were identified. Furthermore, answering this research question enabled the exploration of beam shaping as a potential novel approach to mitigating the effects of pointing jitter, as discussed in Part III. Regarding thermally induced loads, several levels of mitigation were also classified, ranging from active and passive satellite thermal control, through

passive component athermalization, to active compensation of wavefront and polarization errors.

Recommendations for future work

As identified in §3.7, several key research directions are required to further improve the performance of free-space optical communication systems under opto-thermo-mechanical effects. First, the development and space qualification of non-mechanical beam steering technologies represent a major opportunity [189, 190]. Although approaches such as liquid crystal devices, optical phased arrays, and focal plane switch arrays offer clear advantages in terms of mass, volume, and lifetime, their performance and reliability under cyclic thermal loads in space remain insufficiently understood.

Second, integrated photonic devices are a promising avenue for reducing system size and enabling multifunctionality in satellite terminals [202]. However, their susceptibility to radiation, thermal effects, and optical power limitations requires further investigation, particularly regarding radiation hardness and thermal management strategies [203, 204]. In addition, novel structural and optical technologies should be further explored. Mechanical metamaterials offer significant potential for lightweight and customizable microvibration isolation, but their applicability and space qualification need to be demonstrated [211]. Similarly, diffractive optical elements and metasurfaces could enable compact and highly functional optical systems [502], yet their opto-thermo-mechanical behavior and long-term stability in space environments remain open questions. Building upon the work on mechanically and thermally tunable optical metasurfaces [225–227], the opto-thermo-mechanical effects on these systems could be better understood.

Finally, advances in both microvibration mitigation and spacecraft thermal control are essential. Emerging techniques such as drag-free systems, magnetic damping, and voice coil actuators, together with innovative passive and active thermal management solutions, should be further investigated and adapted to the specific needs of satellite optical communication payloads.

12.2.2. Satellite quantum communications

Similar opto-thermo-mechanical phenomena to those identified for optical terminals are also present in satellite-based quantum communication systems, as detailed in §4. The underlying disturbance sources –microvibrations and thermal loads– and the resulting phenomena (rigid body motions, surface deformations, thermo-optic effects, and stress birefringence) remain fundamentally the same. However, their impact must be evaluated through quantum-specific figures of merit, most notably the quantum bit error rate, the secret key rate, and the fidelity of the transmitted quantum states.

A key outcome of §4 is that, in contrast to classical optical communications, the performance of quantum links is not only limited by signal attenuation but also by the preservation of the quantum state. In this context, opto-thermo-mechanical phenomena affect the system through three primary mechanisms: (i) photon transmission probability, (ii) state fidelity degradation (e.g., polarization errors), and (iii) increased background noise and crosstalk.

Pointing jitter induces a stochastic variation in the photon transmission probability by misaligning the transmitted beam with the receiver aperture. At the receiver side, additional jitter leads to fluctuations in the angle of arrival, which significantly degrade fiber-coupling efficiency. These effects directly reduce the detection rate and, consequently, the achievable secret key rate. Wavefront errors, arising from thermo-mechanical deformations and thermo-optic dispersion, further impact system performance by altering the spatial mode of the transmitted photons. This results in reduced photon transmission probability and, in the case of spatial-mode encoding schemes, crosstalk between orthogonal modes. Although most current implementations rely on fundamental Gaussian modes, this effect becomes critical for advanced encoding schemes such as orbital angular momentum. A distinguishing aspect of quantum communication systems is their sensitivity to polarization perturbations. As polarization encoding remains the most mature and widely implemented approach in satellite QKD, significant emphasis was placed on polarization-related effects. Rigid body rotations, stress birefringence in optical components and fibers, and thermally induced anisotropies lead to unintended polarization transformations. These effects reduce the polarization extinction ratio, introduce errors in the measured quantum states, and ultimately increase the quantum bit error rate. Unlike classical systems, where such impairments may only reduce signal quality, in QKD they directly impact security by generating spurious correlations between measurement outcomes.

Thermal effects on quantum sources and detectors were also identified as critical. Temperature fluctuations influence laser diodes in weak coherent pulses (WCP) sources and phase-matching conditions in entangled photon sources, potentially degrading state generation fidelity. Similarly, single-photon detectors exhibit strong temperature dependence, with increased dark count rates leading to higher quantum bit error rates. For superconducting single-photon detectors, inadequate thermal control can result in a complete loss of functionality.

Overall, the analysis in §4 shows that opto-thermo-mechanical phenomena affect quantum communication systems through a complex interplay between losses and errors. While photon losses primarily reduce the detection rate, error mechanisms—such as polarization crosstalk and background noise—have a more severe impact by increasing the QBER and limiting the possibility of secure key generation. This highlights the need to jointly consider optical efficiency and quantum state integrity when designing satellite-based QKD systems. This aspect is explored in §7-8, where accurate models for photon loss and variations in quantum states in satellite-based quantum communication systems are developed.

Recommendations for future work

While many mitigation strategies developed for classical optical communications are directly applicable to quantum systems, several research directions are particularly relevant for advancing satellite-based quantum communications. First, further work is needed to extend beyond polarization encoding. Although polarization-based QKD has reached a high level of maturity, alternative encoding schemes such as time-bin, frequency-bin, and spatial-mode (e.g., OAM) encoding offer potential advantages in

robustness to reference frame misalignment and increased information capacity. However, their sensitivity to opto-thermo-mechanical effects –such as phase noise, modal crosstalk, and finite aperture effects– remains insufficiently characterized, particularly under realistic space conditions.

Second, similar to the classical counterpart, integrated and miniaturized quantum photonic technologies represent a promising avenue for improving stability and scalability. Third, the development and space qualification of quantum memories is a key enabling step toward global-scale quantum networks. Quantum memories would allow buffering of quantum states, enabling advanced protocols such as quantum repeaters proposed in §7. However, their strong sensitivity to environmental perturbations –especially temperature fluctuations and mechanical instability– poses significant challenges that must be addressed.

12.3. Optical aberrations on satellite optical communications

2. How can these phenomena be modelled within a unified mathematical framework that quantifies the impact on the performance of optical and quantum communication?

In §6, an end-to-end modelling framework was developed to quantify the impact of transmitter pointing jitter and optical aberrations on intersatellite optical communication systems. By combining Fourier wave-optics propagation with a statistically captured-power model, it enables the direct evaluation of metrics such as the average bit error probability and outage probability. The framework explicitly links physical optical effects to communication performance. Wavefront errors, described using Zernike polynomials, distort the far-field irradiance, while pointing jitter introduces stochastic misalignment. The inclusion of the apparent angle of arrival further captures systematic boresight errors induced by aberrations. Among Seidel aberrations, coma was identified as the most critical due to its induced static pointing error (see Fig. 6.6). In contrast, other aberrations may have lower impact, or even compensating effects (e.g., spherical aberration). Statistical analyses further indicate that subaperture configurations –in contrast to clear aperture or centrally obscured architectures– offer improved robustness to combined aberrations (see Fig. 6.8). Overall, the framework provides a direct link between wavefront error specifications and communication performance, showing that classical optical imaging criteria can be incomplete for optical communication system design.

Furthermore, §6 also provides a digression on the different communication performance parameters in classical communication. It is shown that one of the most important aspects for the received signal is to minimize the probability that it falls below a power threshold (both the outage probability and the reliability can be computed in this way). This power threshold can be determined in several ways, depending on the objective. In general, for establishing reliable links, it should correspond to the power level at which the instantaneous bit error probability is such that the error correction code can handle this BEP with a determined high probability. Other performance

parameters, such as the average bit error probability (ABEP), have less practical relevance and should be used cautiously. In line with these conclusions, §11 mainly focuses on outage probability-like –or reliability, for that matter– communication performance parameters as the objective of the optimization.

Recommendations for future work

Building upon the modelling framework developed in §6, several research directions are identified to further advance the design of robust optical communication systems. First, the coupled effects of transmitter aberrations and receiver-side impairments should be investigated in greater detail. In particular, the interaction between aberrated wavefronts, angle-of-arrival fluctuations, and receiver coupling efficiency (e.g., fiber coupling) remains an open problem requiring a unified treatment. Second, the extension of the present framework to space-to-Earth links is of high relevance. In such scenarios, atmospheric turbulence introduces additional wavefront distortions and temporal dynamics that interact with the opto-thermo-mechanical effects studied in §6. Incorporating these effects into the current modelling approach would enable a more complete assessment of realistic communication links. Third, the implications of these results for advanced modulation and multiplexing schemes should be explored. In particular, spatial-mode division multiplexing and structured light communication are expected to be highly sensitive to the combined effects of pointing jitter and wavefront errors, especially due to modal crosstalk induced by aberrations and finite receiver aperture effects. Finally, the extension of this framework to quantum communication systems represents a key avenue for future work.

12.4. Quantum communications in space

2. How can these phenomena be modelled within a unified mathematical framework that quantifies the impact on the performance of optical and quantum communication?

This thesis has developed a unified modelling framework that connects opto-thermo-mechanical disturbances to the performance of both classical optical and quantum communication systems. Building upon the optical propagation and stochastic power capture model in §6, the framework was extended to the quantum domain by incorporating the probabilistic nature of photon transmission, quantum state generation, and measurement processes in §7–8. In the context of satellite-assisted quantum communication using quantum repeaters (see §7), the framework integrates physical-layer effects –such as beam diffraction, pointing jitter, and atmospheric losses– into a quantum information model describing entanglement generation, swapping, and storage. Optical losses are consistently modelled as effective beam-splitter channels, enabling a direct mapping between classical link budgets and quantum state degradation.

This approach allows the propagation of physical imperfections through the full protocol, from spin-photon entanglement generation to end-to-end entangled states.

A key outcome is the ability to express the final quantum state in terms of a small set of parameters that capture both optical and hardware imperfections. This enables efficient analytical evaluation of performance metrics such as entanglement distribution rate and fidelity, avoiding the need for computationally intensive Monte Carlo simulations. The framework further supports the use of realistic system parameters, including finite detector efficiency, dark counts, imperfect interference visibility, memory decoherence, and non-ideal entanglement swapping operations. Applied to a satellite-based quantum repeater architecture using trapped atomic qubits in §7, the model demonstrates that high-fidelity entanglement distribution over continental distances is achievable with modest multiplexing resources (e.g., 200 memory modes for 100 entangled photons per second over 3500 kilometers). In particular, the use of nearly deterministic Bell-state measurements enabled by Rydberg-mediated gates significantly reduces the memory requirements compared to photonic-based approaches. These results establish a direct and quantitative link between optical system design, quantum hardware performance, and network-level capabilities.

Building upon this work, §8 further develops the optical propagation model by also considering transmitter aperture clipping effects. The model incorporates the probabilistic generation of entangled photon pairs via a spontaneous parametric down-conversion source on a low-Earth-orbit satellite, as well as the complete propagation dynamics to ground stations, including beam diffraction, finite transmitter aperture, satellite pointing jitter, atmospheric attenuation, and realistic detector characteristics. A central result of this work is the introduction of a beam-waist optimization strategy that maximizes the average photon capture probability by balancing diffraction and aperture truncation under dynamic pointing jitter conditions. Unlike classical optical links, which prioritize power stability, this optimization enhances entanglement distribution rates without requiring constant flux, as QKD detects individual events. Applying this optimization resulted in an order-of-magnitude increase in the average SKR—compared to non-optimized links employing more stable photon-flux beams, following the classical communication approach—for the Madrid–Barcelona, Madrid–Bilbao, and Madrid–Lisbon links, thereby enabling the fulfillment of key distribution requirements for realistic use cases such as secure hospital-to-hospital communication.

Recommendations for future work

Several directions can further extend this unified framework. First, the inclusion of higher-order optical effects, such as wavefront aberrations and receiver coupling efficiency, would enable a more complete description of opto-thermo-mechanical impacts on quantum communication, building upon the results of §6. Second, it is of utmost importance to further develop and space-qualify the quantum communication-specific hardware that is modelled in §7, such as quantum memories, single-photon detectors, and entangled photon sources.

The model in §8 shows that, with current SPDC source rates in the MHz regime [310], near-term QKD applications are feasible. However, high-demand services such as QKD-based VPNs would require source rates in the GHz regime. As these services become increasingly relevant for industry, it is necessary to develop sources operating

in the GHz regime that are suitable for satellite quantum communication. Furthermore, in the model presented in §8, for the specific application of QKD, trusted nodes were assumed on the ground. Including quantum memories at ground stations would enable the use of untrusted nodes, extend entanglement across multiple links, and open the path to fully quantum network protocols beyond QKD, such as blind quantum computing or distributed quantum sensing. For the space-to-ground architecture, it is also recommended to improve the model to account for atmospheric turbulence effects in a more accurate manner, as in Ref. [253]. In this way, an even more realistic performance can be predicted, bringing us closer to a global quantum information network.

12.5. Beam shaping in satellite optical communications

3. How can beam shaping mitigate the impact of the studied phenomena on optical and quantum communication performance?

In Part III, we proposed beam shaping as a method to mitigate the impact of pointing jitter in satellite free-space optical communications. The first approach, theoretically proposed in §9 and experimentally verified in §10, consisted of the incoherent superposition of a Gaussian beam and an orthogonally polarized higher-order Laguerre–Gaussian beam. In §9, it was shown that, by using an optimal combination of these beams, the effect of pointing jitter could be mitigated. This resulted in theoretical power savings on the order of 20–40%, depending on the performance parameter of interest.

To evaluate whether these theoretical gains could be realized in a practical optical system –while accounting for losses introduced by beam-shaping components– the system was implemented in the laboratory. The experiments revealed that a Gaussian beam impinging on a spiral phase plate does not generate an exact Laguerre–Gaussian beam, as assumed in §9. Nevertheless, the resulting annular beams still outperformed a pure Gaussian beam, and the achieved power savings exceeded the additional losses introduced by the beam-shaping system (see Figs. 10.4 and 10.5). This demonstrates that beam shaping remains a promising approach for improving the resilience of optical inter-satellite communication links.

Building upon these results, §11 employed a variational formalism to determine the optimal beam shape for mitigating transmitter pointing jitter. This analysis showed that the optimal far-field irradiance corresponds to a flat-top profile. To assess how closely such physically unrealizable beams can be approximated in practice, two beam-shaping approaches were investigated: incoherent and coherent methods. An ideal flat-top beam can theoretically provide power savings of up to 63.2%. However, incoherent beam shaping falls short of this limit, achieving only about 47% savings even when combining as many as one hundred Laguerre–Gaussian modes. In contrast, coherent beam shaping –based on the design of a phase mask to generate high-order super-Gaussian beams– achieved power savings of around 50% compared to a conventional Gaussian beam (which can be obtained with a single customized phase mask).

Although these more advanced beam-shaping techniques were not directly applied to satellite quantum communication channels for mitigating pointing jitter, in §8, an optimization of the truncation ratio of a Gaussian single-photon beam to maximize the average photon transmission probability, accounting for the coupled effects of truncation, diffraction, and pointing jitter, was presented. This approach is analogous to the beam truncation optimization performed for optical communication in §6.3.1. It is worth noting that the beam-shaping techniques proposed in §11 for optical communication are not, in principle, well suited for quantum communication. As discussed in §8, performance in quantum links is primarily determined by the average photon capture probability. To maximize this quantity, spreading the single-photon beam in the far-field is not advantageous. This is consistent with the results in §6.3.1, where maximizing the average received power –analogous to maximizing the photon capture probability– does not require broadening the beam over the receiver aperture. Instead, a far-field irradiance that minimizes beam spread, approaching a bidimensional Dirac delta (see §11.4.1), maximizes the average photon capture probability. In this sense, the optimization results from §6 and §8 can be regarded as initial beam shaping steps for mitigating pointing jitter in both satellite optical and quantum communication channels.

Recommendations for future work

Future work should explore additional beam shapes, guided by the framework developed in §11. In particular, improved algorithms for designing phase masks capable of generating higher-order super-Gaussian beams ($n_{SG} > 17$) could enable closer approximations to the optimal far-field irradiance. These studies should incorporate realistic system losses, similar to the analysis presented in §10 and Ref. [502].

The development of compact, satellite-compatible beam-shaping systems is also essential. Promising approaches include diffractive optical elements and metasurfaces capable of generating the desired beam profiles. Furthermore, experimental demonstrations of coherent beam-shaping techniques are needed, particularly considering manufacturing constraints associated with phase-mask elements. A detailed investigation of the impact of optical aberrations in the incoming Gaussian beam on coherent beam-shaping performance would also help quantify degradation with respect to the ideal performance in realistic scenarios.

Finally, other application scenarios should be considered, most notably space-to-ground downlinks. Extending these beam-shaping techniques to the receiver side could provide additional performance gains. In particular, shaping the incident field to enable efficient and robust coupling into fibers or detectors could yield improvements comparable to those achieved through far-field beam shaping at the transmitter.

12.6. An exciting future ahead

The research questions posed in this thesis have been addressed; however, the work presented here opens more questions than it answers. These unanswered questions are the guideline without a recipe towards an exciting future in satellite optical and quantum communications.

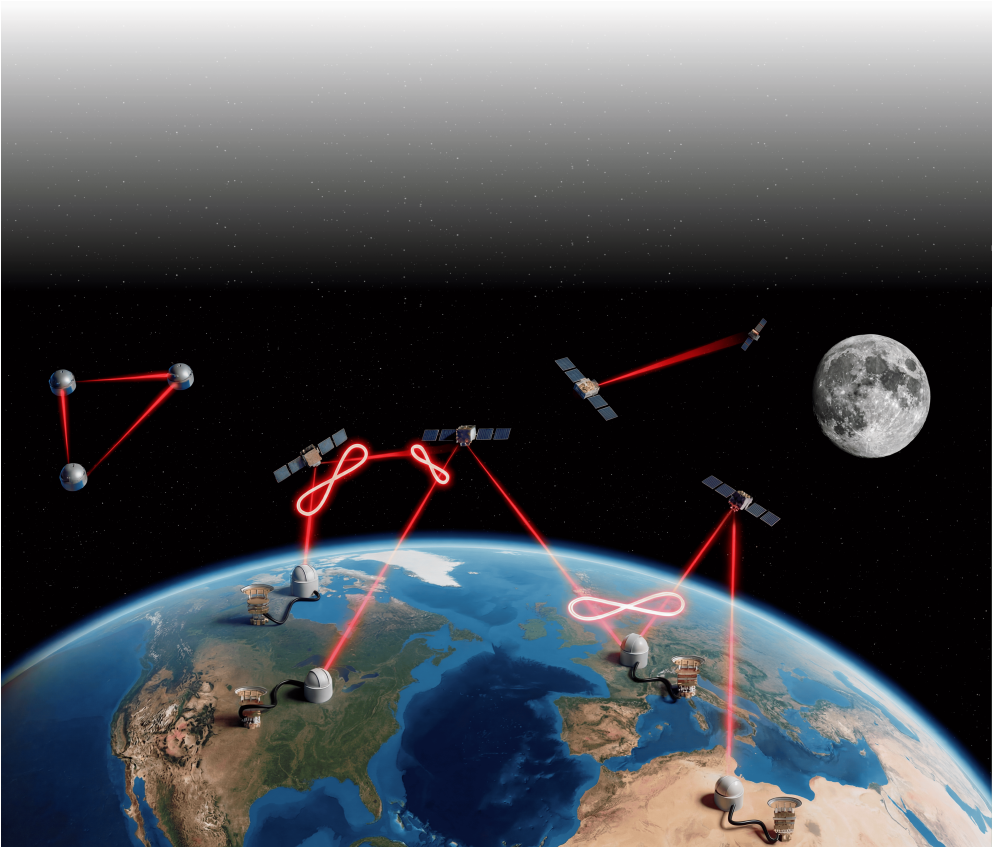
While the discussion in this thesis has primarily focused on these communication systems, the underlying concepts are rooted in broader optical and quantum technologies. The methods and insights developed here are therefore expected to extend beyond the context of communication. For instance, the impact of optical aberrations and the potential of beam shaping could be further explored in other space-based optical systems, such as lidar or gravitational-wave interferometry. As these systems are also affected by pointing jitter, the rationale for employing non-Gaussian beam shapes may prove equally relevant and beneficial.

Imagine a future in which optical and quantum technologies are deeply integrated into everyday life –enhancing global connectivity, strengthening security, and enabling capabilities that are currently beyond reach. A future where laser-based communication allows seamless data transfer between any two points: across Earth, between satellites, and extending to planetary and deep-space missions.

Imagine a future shaped by quantum computing, where problems once considered intractable become solvable. A world in which even the potential misuse of quantum computers to compromise classical encryption methods would no longer pose a threat. A future in which this thread is overcome by a global quantum information network infrastructure.

Imagine future quantum networks that, beyond providing secure keys to society, become large-scale infrastructures that interconnect quantum devices across the globe. These networks could enhance the capabilities of individual quantum computing units, enabling breakthroughs in fields such as materials science and drug discovery. Imagine a world in which quantum information networks are used for quantum sensing and for understanding the interplay between quantum mechanics and gravity, advancing our understanding of fundamental physics.

Imagine.





Appendices

Appendix A

Appendix for Chapter 6

This appendix provides additional details and clarifications on several topics presented in §6. For a comprehensive treatment, the reader is referred to the supplemental document accompanying the paper in Ref. [367]. In that supplemental document, a simplified ABCD-matrix-based calculation is presented to show how the parameters w_z and w_0 can be adjusted to design a system that is optimum for a given performance parameter. Furthermore, the comparison of the apparent angle of arrival with the mutual alignment error is mentioned in [372].

In A.1, some details on the wave optics propagation are given, focusing on the explanation of the several reference systems used in these. In A.2, detailed explanations are given for the sensitivity of the apparent angle of arrival to the tracking algorithm and the telescope architecture. In A.3, complementary results for the truncated Gaussian beam optimization are shown. Finally, A.4 presents the complementary results of the effect of Seidel aberrations.

A.1. Wave propagation

In this section, some more details on the wave optics propagation and the different reference systems used in §6 are given. In Fig. A.1 the convention for the different reference systems used throughout §6 is presented. When, due to the transmitter optical aberrations, the center of gravity of the irradiance field in the tracking sensor plane is located in the coordinates (x_{d0}, y_{d0}) the transmitted beams's optical axis will be pointed in the direction given by the angles (γ_x, γ_y) . Therefore, the optical axis of the transmitted beam will be displaced to the coordinates (x_b, y_b) in the receiver aperture plane. In this plane the receiver's center is located at the origin of coordinates.

There are two wave optics propagation involved in the model when computing the effects of the aberrations. On the one hand, a wave is propagated to compute the far-field irradiance of the transmitted communication beam. This propagates the electromagnetic scalar field from the transmitter aperture plane (ξ, η) to the receiver aperture plane (x, y) . On the other hand, another wave is propagated to compute

the irradiance field of the received beacon in the transmitter. This wave optical propagation considers a uniform plane wave field in the aperture. This assumption is reasonable when the beacon is a Gaussian beam that is much larger than the aperture of the transmitter telescope, which is usually the case. The wave optics propagation involved, gives the field in the tracking sensor plane (x_d, y_d) given by the aberrated incoming uniform plane wave in the transmitter aperture plane (ξ', η') .

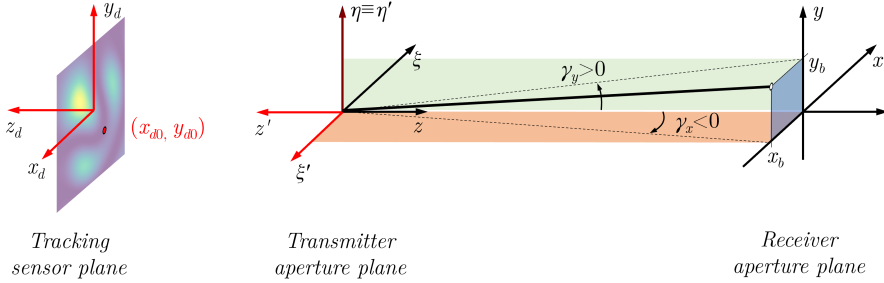


Figure A.1: Convention of the reference systems used in the computational model developed

A.1.1. Beam divergence

In §6 and in Figs. A.5(a), A.6(a), A.7(a) the divergence of the beam is presented. As commented in §6, the divergence of the beam is defined as the full-width angle described by a circular contour containing 86% of the total power of the beam as it propagates. This definition is an extension of that of the pure Gaussian beam. Mathematically, this can be written as

$$\theta = 2 \arctan \left(\frac{r_{86}(\Delta z) - w_z}{\Delta z} \right) \approx 2 \frac{r_{86}(\Delta z) - w_z}{\Delta z}, \quad (\text{A.1})$$

where $r_{86}(\Delta z)$ is defined as the radius of the circle $\mathcal{C}_{86}$ centered at the receiver that contains the 86% of the total power of the received beam as

$$\iint_{\mathcal{C}_{86}} I_r(x, y) dx dy = 0.86 \iint_{\mathbb{R}^2} I_r(x, y) dx dy, \quad (\text{A.2})$$

where $I_r(x, y)$ is the irradiance at the receiver aperture plane computed through wave propagation.

A.2. Apparent angle of arrival

In this section, some aspects of the apparent angle of arrival are discussed in detail. The effect of the effective focal length of the optical system focusing on the tracking sensor has been checked to have no effect on the resulting apparent angle of arrival under aberrations. Furthermore, it has been checked that if the computed apparent

angle of arrival is included as a tip-tilt to the incoming uniform plane wave of the beacon, the resulting diffraction pattern in the tracking sensor plane has its center of gravity in the origin of coordinates. This validates the fact that the aberration will induce a static angular boresight error of the magnitude computed in Eq. (10) of §6.

A.2.1. Sensitivity of different tracking algorithms

Here, the results computed for the mutual alignment error in the case of coma aberrations for two different tracking algorithms are compared: the center of gravity algorithm (COG) and the maximum irradiance algorithm (MI). The first computes the center of gravity of the irradiance field in the tracking sensor plane to evaluate the angle of arrival of the beacon, while the second uses the maximum irradiance point to do so.

In the presence of optical aberrations in the transmitter telescope, the resulting irradiance field in the tracking sensor plane is a distorted point spread function of the transmitter aperture (for an aligned disposition of the transmitter and receiver optical axes). In particular, from the Seidel aberrations considered in §6, only coma will create a displacement of both the center of gravity and the maximum irradiance of the field. Under these conditions, the apparent angle of arrival for a coma aberrated telescope is shown in Fig. A.2. It can be seen that the MI algorithm is much more robust than the COG algorithm to coma aberration. Furthermore, the direction of the apparent angle of arrival in the MI case is the inverse of the COG case due to the peak irradiance being displaced towards the direction contrary to the COG.

A.2.2. Sensitivity to the telescope architecture

In this section, we present an analysis of the sensitivity of the apparent angle of arrival to different telescope architectures studied in this work. Fig. A.3 shows the apparent angle of arrival for horizontal and vertical coma for the obscured and clear architectures presented in Fig. 2 of §6. It can be seen that the obscured telescope is more sensitive to coma aberrations than the clear aperture architecture. This is in agreement with the results found in Ref. [372] for the mutual alignment error.

A.3. Transmit beam optimization for nominal case

In this section, the optimization performed for the three different architectures shown in §6 are presented. In §6, the results for only one of these architectures are given.

The results of the obscured, clear, and subaperture architectures are shown in Figs. A.5, A.6, and A.7, respectively. The figures show the divergence, telescope transmission efficiency, average normalized received power, average bit error probability (ABEP), probability of outage, and reliabilities. It can be seen that the optimum outgoing beam characteristics are different for each of the communication performance parameters (labels (d)-(h) in the figures) shown. Indeed, the comparison of these optimums with the location of the maximum average normalized power $\bar{h}$ proves that maximizing the average received power is not a good approach to maximizing the communication performance. This is due to the fact that for clipped Gaussian beams, the received power statistics underlying the maximum received average power far-field

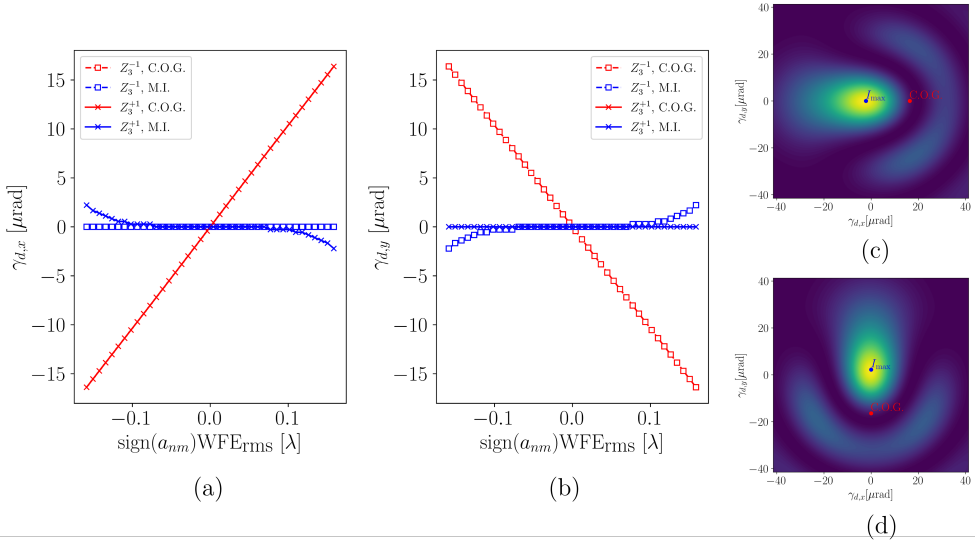


Figure A.2: Comparison of the apparent angle of arrival under coma, for center of gravity (C.O.G) and maximum irradiance (M.I.) tracking algorithms on the beacon tracking sensor in the clear aperture case. (a) and (b) show the apparent angle of arrival in x and y axes. (c) and (d) show the diffraction patterns at the tracking sensor plane under $\text{sign}(a_{nm})\text{WFE}_{\text{rms}} = +0.15 \lambda$.

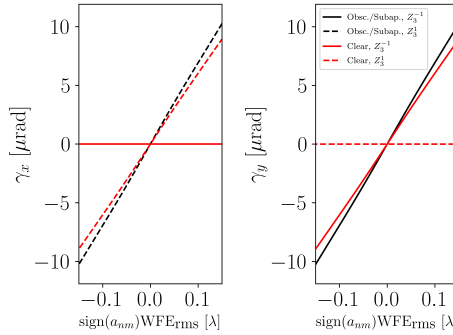


Figure A.3: Apparent angle of arrival for vertical and horizontal coma aberrations, for the telescope architectures studied in this work.

irradiance are such that the fluctuations of the received power are too significant. In this case, the fluctuations degrade the performance of the system so much that having a very high average power can not compensate for this effect. In line with this reasoning, finding the maximum on-axis irradiance in clipped Gaussian beams under pointing jitter as in Ref. [393] is not necessarily beneficial in free-space optical communications. The optimum outgoing beams resulting from this optimization would

be very close to the ones obtained by maximizing the average normalized received power $\bar{h}$. Hence, designing the outgoing beam for maximization of either average received power or on-axis peak irradiance shall not be pursued in free-space optical communications subjected to pointing jitter.

In Fig. A.4(a)-(c) the optimum far-field irradiances are shown for each of the telescope architectures, along with the pointing jitter probability density function in the receiver aperture plane. The reader is referred to §6 for a discussion and analysis of the interpretation of the different shapes leading to different extremal values of each performance parameter. In Fig. A.4(d)-(f), the receiver power statistics for each of these optimum far-field irradiances are given.

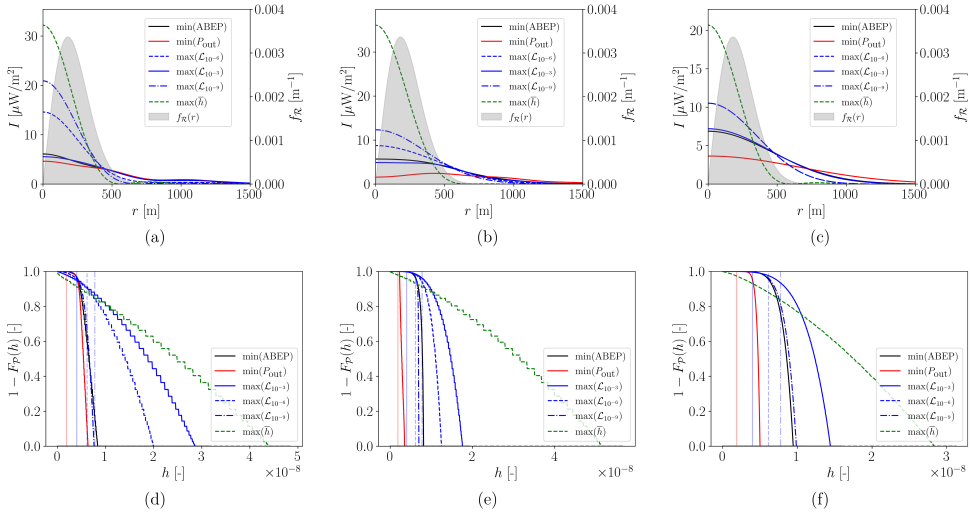


Figure A.4: Radial far-field irradiances for the optimization of different parameters for the (a) obscured, (b) clear, and (c) subaperture architectures. The far-field pointing jitter probability density function (PDF) is shown as $f_R(r)$. These are the far-fields of the optimum beams found in Fig. A.5, A.6 and A.7, respectively. (d)-(f) show the respective $1 - F_P(h)$, where $F_P(h)$ is the cumulative density function of the normalized received power. The vertical lines show the normalized received power thresholds established by each performance parameter (probability of outage and reliability).

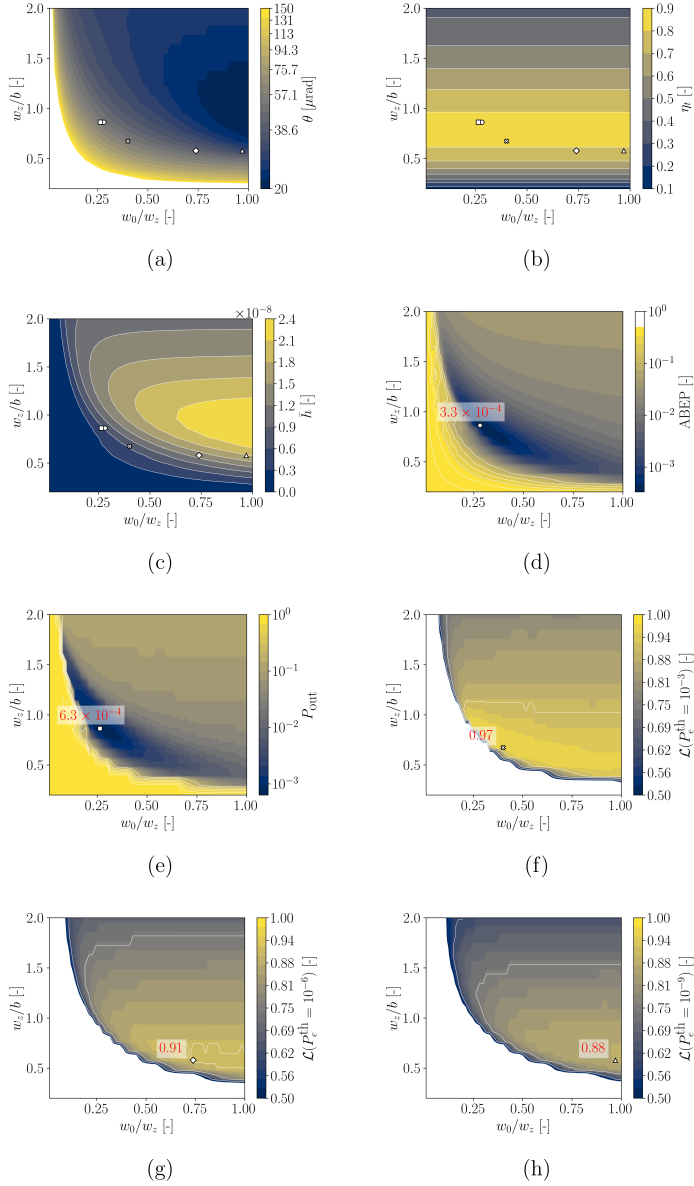


Figure A.5: Optimization of the transmitted beam for the obscured architecture. The $\circ$ symbol is the optimum for ABEP performance, and the $\square$ symbol is the optimum for outage probability. The symbols $\times$, $\diamond$, and $\triangle$ refer to the optimum reliability performances.

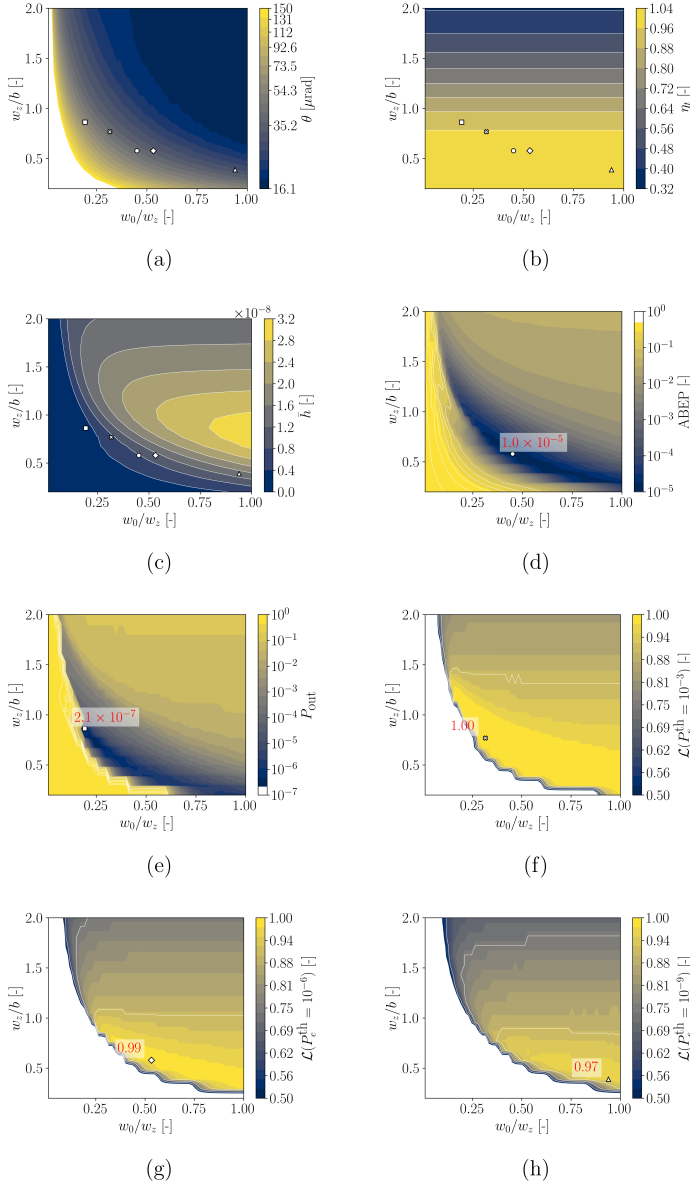


Figure A.6: Optimization of the transmitted beam for the clear architecture. The $\circ$ symbol is the optimum for ABEP performance, and the $\square$ symbol is the optimum for outage probability. The symbols $\times$, $\diamond$, and $\triangle$ refer to the optimum reliability performances.

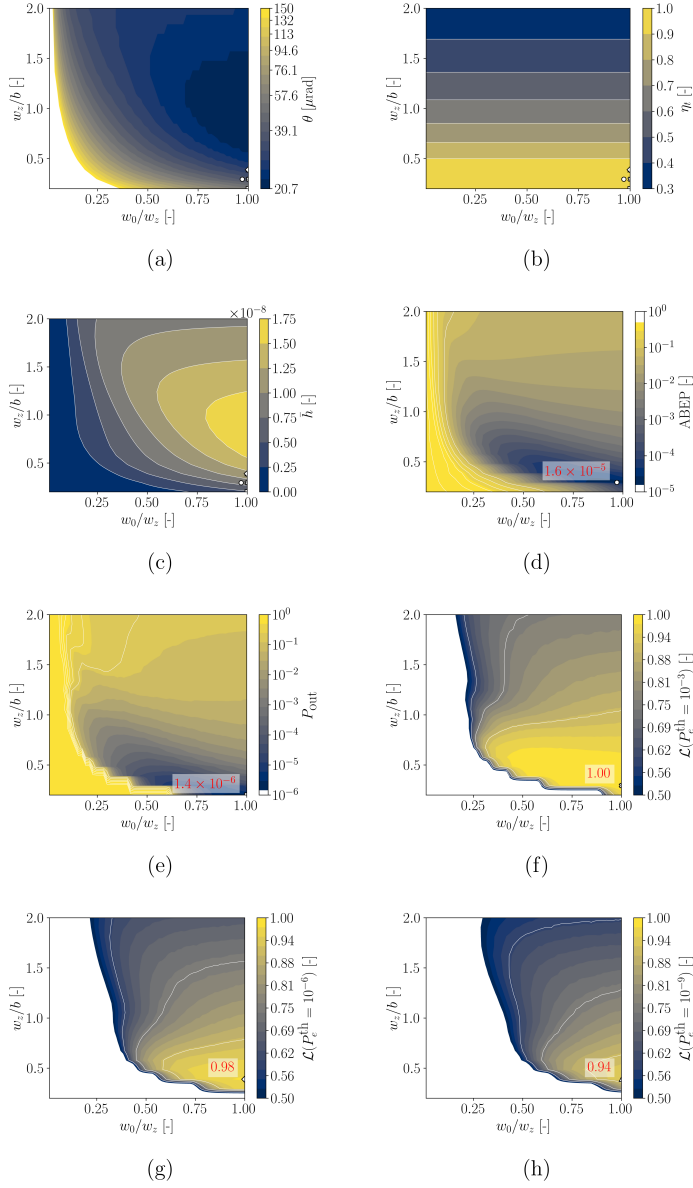


Figure A.7: Optimization of the transmitted beam for the subaperture architecture. The $\circ$ symbol is the optimum for ABEP performance, and the $\square$ symbol is the optimum for outage probability. The symbols $\times$, $\diamond$, and $\triangle$ refer to the optimum reliability performances.

A.4. Optical aberrations

In this section, some complementary results to the ones presented in §6 on the effects of optical aberrations are presented. Fig. A.8 shows the effect on the reliabilities $\mathcal{L}(P_e^{\text{th}} = 10^{-3})$ and $\mathcal{L}(P_e^{\text{th}} = 10^{-9})$ for different architectures. Furthermore, A.9 the Monte Carlo simulations for these parameters are presented for these two parameters along with the results obtained in the Monte Carlo simulations for the apparent angle of arrival. The Monte Carlo simulation is done in the following way. In each iteration, a weight is assigned to each of the Zernike aberrations considered. The coefficient a_{nm} allocated to each Zernike mode is obtained by sampling from a uniform distribution $[-a_{nm}^{\text{max}}, a_{nm}^{\text{max}}]$, therefore also including negative values for the modes. The a_{nm}^{max} for each mode is computed by making sure that the resulting WFE_{RMS} for all the modes at that maximum weighting is the same. This ensures that the WFE_{RMS} (in magnitude) is statistically the same for all the Seidel aberrations. This is supported by the experimental data shown in Fig. 14 in Ref. [372].

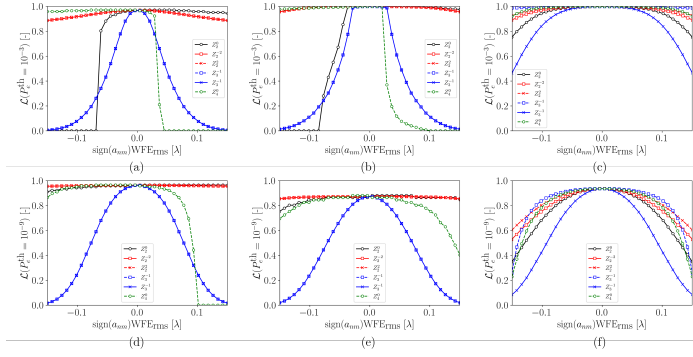


Figure A.8: Effect of optical aberrations for three different telescope architectures and reliabilities for $P_e^{\text{th}} = 10^{-3}$ and $P_e^{\text{th}} = 10^{-9}$. The first, second, and third columns correspond to clear, obscured, and subaperture architectures, respectively.

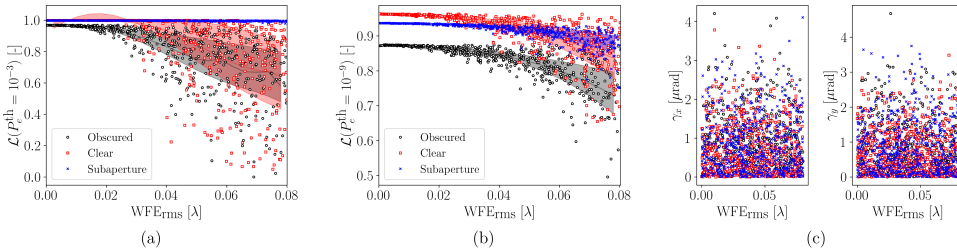


Figure A.9: Effect of optical aberrations on the performance of a LEO-GEO inter-satellite link for different architectures obtained through Monte Carlo simulation.

Appendix B

Appendix for Chapter 7

This appendix provides additional details and clarifications on several topics presented in §7. In Section B.1, the expressions for the coefficients of the spin-photon entangled state are given. In Section B.2, the expressions for the coefficients of the state after free-space transmission are provided. In Section B.3, the details of the probabilistic optical Bell state measurement are explained, along with the expressions for the coefficients of the state after the Bell state measurement. In Section B.3.1, the model for quantum memory decoherence is described, and the expressions for the coefficients of the state after memory decoherence are presented. Finally, in Section B.4, the detailed expression for the rate calculation is provided.

B.1. Spin-photon entanglement

In the spin-photon entanglement, the coefficients of Eq. (7.3) are given by

$$\alpha_e = p_1 \eta_{\text{coll}}, \quad (\text{B.1})$$

$$\beta_e = \eta_{\text{coll}}^2 p_2, \quad (\text{B.2})$$

$$\gamma_e = p_1(1 - \eta_{\text{coll}}) + (1 - \eta_{\text{coll}})^2 p_2 + p_0, \quad (\text{B.3})$$

$$\eta_e = 2p_2 \eta_{\text{coll}}(1 - \eta_{\text{coll}}), \quad (\text{B.4})$$

where $\eta_{\text{coll}} = p\eta_{\text{cav,coll}}$ is the total collection efficiency, where we take into account the collection efficiency of the cavity, $\eta_{\text{cav,coll}}$; and the probability of emitting in the cavity mode, p . The probability of emitting a single-photon is p_1 , and p_2 is the probability of two-photon emissions. As a final note, p_0 is the probability of not emitting any photon at all, which we have assumed results in a dephased state.

The code and data of the results of this paper are openly available on 4TU.ResearchData: "Data underlying the publication *Satellite-assisted quantum communication with single-photon sources and atomic memories*", at <https://doi.org/10.4121/61635423-89d3-46cb-b9bc-bc06f158b9a5>

B.2. Free-space transmission

The coefficients of the state after free-space transmission, Eq. (7.5), are given by

$$\alpha_T = p_T p_1 \eta_{\text{coll}}, \quad (\text{B.5})$$

$$\beta_T = \eta_{\text{coll}}^2 p_T^2 p_2, \quad (\text{B.6})$$

$$\gamma_T = (1 - p_T) p_1 \eta_{\text{coll}} + p_1 (1 - \eta_{\text{coll}}) + (1 - p_T)^2 \eta_{\text{coll}}^2 p_2 \\ + 2p_2 \eta_{\text{coll}} (1 - \eta_{\text{coll}}) (1 - p_T) + (1 - \eta_{\text{coll}})^2 p_2, \quad (\text{B.7})$$

$$\eta_T = 2\eta_{\text{coll}}^2 p_2 p_T (1 - p_T) + 2p_2 \eta_{\text{coll}} (1 - \eta_{\text{coll}}) p_T. \quad (\text{B.8})$$

B.3. Probabilistic optical Bell state measurement

Once the photons from the emitter and from the receiver meet at the receiver station, a Bell state measurement is performed between them to obtain atom-atom entanglement across the elementary links. For the photon generated at the receiver station, we assume negligible transmission loss but imperfect collection from the cavity. The Bell-state measurement (BSM) is performed with a 50/50 beam splitter, and we assume that photonic interference only occurs for the case where the two input states are of the form $|\psi\rangle$ i.e., that there is no photonic interference for the error terms in Eq. (7.5), which has a photonic component. This is justified by the fact that those error terms originate from cases where two photons were initially emitted from an atom. The probability that those two photons are emitted in the exact same mode (e.g., with the same temporal envelope) is limited, and hence we do not expect strong interference between them.

We assume that the beam splitter transformations are of the following form

$$|e_1\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}} (|e_3\rangle + |e_4\rangle) \quad |e_2\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}} (|e_3\rangle - |e_4\rangle) \quad (\text{B.9})$$

$$|l_1\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}} (|l_3\rangle + |l_4\rangle) \quad |l_2\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}} (|l_3\rangle - |l_4\rangle). \quad (\text{B.10})$$

where 1, 2 are the input states to the beam splitter, and 3, 4 are the outputs. In the case of input state $|\psi_{\text{sp},1}\rangle \otimes |\psi_{\text{sp},2}\rangle$, the output state of the beam splitter is

$$|\psi_{\text{sp},1}\rangle \otimes |\psi_{\text{sp},2}\rangle \xrightarrow{\text{BS}} \frac{1}{2} \left[\frac{1}{\sqrt{2}} (|e_3 l_3\rangle |0_4\rangle - |0_3\rangle |e_4 l_4\rangle) \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) |0_e\rangle \right. \\ \left. + \frac{1}{\sqrt{2}} (|l_3\rangle |e_4\rangle - |e_3\rangle |l_4\rangle) \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) |0_e\rangle \right] \\ + \frac{1}{2} \left[\frac{1}{\sqrt{2}} (|e_3 e_3\rangle |0_4\rangle - |0_3\rangle |e_4 e_4\rangle) |00\rangle + \frac{1}{\sqrt{2}} (|l_3 l_3\rangle |0_4\rangle - |0_3\rangle |l_4 l_4\rangle) |11\rangle \right]. \quad (\text{B.11})$$

where the states $|0_e\rangle$, $|0_3\rangle$ and $|0_4\rangle$ represent the vacuum state of the environment, the vacuum state of output port 3 and the vacuum state of output port 4, respectively. Post-selecting on detecting an early and a late photon, we will project the atoms into an entangled state of the form $|\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle)$. The sign of the superposition depends on which detectors recorded the photons and can always be corrected by a

local atomic unitary if desired. The probability of this successful projection is $1/2$ from the expression above.

We assume that the rest of the possible combinations between input states do not result in photonic interference as explained above. Thus, for example, if the input states are $|\psi_{\text{multi}}\rangle \otimes |\psi_{\text{multi}}\rangle$, the beam splitter transformations are assumed to be:

$$|2e_1\rangle \xrightarrow{\text{BS}} \frac{1}{2\sqrt{2}} \left[(b_{e3}^\dagger)^2 + 2(b_{e3}^\dagger b_{e4}^\dagger) + (b_{e4}^\dagger)^2 \right] |0_3\rangle |0_4\rangle, \quad (\text{B.12})$$

$$|2e_2\rangle \xrightarrow{\text{BS}} \frac{1}{2\sqrt{2}} \left[(d_{e3}^\dagger)^2 + 2(d_{e3}^\dagger d_{e4}^\dagger) + (d_{e4}^\dagger)^2 \right] |0_3\rangle |0_4\rangle, \quad (\text{B.13})$$

where $b^\dagger$ and $d^\dagger$ are orthogonal creation operators. The resulting output state is then:

$$\begin{aligned} & |\psi_{\text{multi}}\rangle \otimes |\psi_{\text{multi}}\rangle \xrightarrow{\text{BS}} \\ & \frac{1}{8} (|01\rangle (|2b_e 2d_1\rangle_3 |0_4\rangle + |2b_e\rangle_3 |2d_1\rangle_4 + |2d_1\rangle_3 |2b_e\rangle_4 + |0_3\rangle |2b_e 2d_1\rangle_4) + \\ & |10\rangle (|2b_1 2d_e\rangle_3 |0_4\rangle + |2b_1\rangle_3 |2d_e\rangle_4 + |2d_e\rangle_3 |2b_1\rangle_4 + |0_3\rangle |2b_1 2d_e\rangle_4)) \\ & + \frac{1}{8} (|00\rangle (|2b_e 2d_e\rangle_3 |0_4\rangle + |0_3\rangle |2b_e 2d_3\rangle_4) + |11\rangle (|2b_1 2d_1\rangle_3 |0_4\rangle + |0_3\rangle |2b_1 2d_1\rangle_4)). \end{aligned}$$

Therefore, with probability $1/2$, we will project either into state $|01\rangle$ or $|10\rangle$ by heralding on an early and late detection.

The coefficients of the state after the Bell state measurement, Eq. (7.11), are given by

$$\begin{aligned} \alpha_{\text{BS}} &= \eta^2 \alpha_{\text{T}} \alpha_{\text{e}} / 2, \\ \beta_{\text{BS}} &= 1/2 (p_{\text{dark}} (1 - \eta) (\eta (\alpha_{\text{T}} \gamma_{\text{e}} + \gamma_{\text{T}} \alpha_{\text{e}} + \gamma_{\text{T}} \eta_{\text{e}} + \eta_{\text{T}} \gamma_{\text{e}}) + p_{\text{dark}} (1 - \eta) \gamma_{\text{T}} \gamma_{\text{e}}) + \\ & \quad + \eta^2 (\alpha_{\text{T}} \eta_{\text{e}} + \eta_{\text{T}} \alpha_{\text{e}} + \eta_{\text{T}} \eta_{\text{e}})) \\ & \quad + \eta^2 / 4 (\alpha_{\text{T}} \beta_{\text{e}} + \beta_{\text{T}} \alpha_{\text{e}} + \eta_{\text{T}} \beta_{\text{e}} + \beta_{\text{T}} \eta_{\text{e}}) + \eta^2 \beta_{\text{T}} \beta_{\text{e}} / 8, \\ \gamma_{\text{BS}} &= (1 - \eta) p_{\text{dark}} (1/2 (\eta (\alpha_{\text{T}} \gamma_{\text{e}} + \gamma_{\text{T}} \alpha_{\text{e}} + \gamma_{\text{T}} \eta_{\text{e}} + \eta_{\text{T}} \gamma_{\text{e}}) + (1 - \eta) p_{\text{dark}} \gamma_{\text{T}} \gamma_{\text{e}})), \end{aligned}$$

where η is the single-photon detection efficiency. Note that the state in Eq. (7.11) is not normalized. The normalization constant corresponds to the probability of success, i.e., that an early and late click is recorded.

B.3.1. Quantum memories

We consider the limited coherence time of the quantum memories to be captured by a general Pauli noise channel. Thus, the decoherence of an atomic Bell pair is captured

by the noise channel:

$$\begin{aligned} \Delta(\rho) = & p_{c1}p_{c2}\rho \\ & + \left[(1-p_{c1})p_{c2} \sum_{i=\{x,y,z,0\}} \epsilon_i (\sigma_i \otimes \mathbb{I}) \rho (\sigma_i \otimes \mathbb{I}) \right. \\ & + p_{c1}(1-p_{c2}) \sum_{j=\{x,y,z,0\}} \epsilon_j (\mathbb{I} \otimes \sigma_j) \rho (\mathbb{I} \otimes \sigma_j) \\ & \left. + (1-p_{c1})(1-p_{c2}) \sum_{i,j=\{x,y,z,0\}} (\epsilon_i \epsilon_j (\sigma_i \otimes \sigma_j) \rho (\sigma_i \otimes \sigma_j)) \right], \end{aligned}$$

where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices and σ_0 is the identity matrix; and $p_{c1,2} = e^{-t/\tau_{c,i}}$ where $\tau_{c,i}$ is the coherence time of the atoms. In our simulations, we assumed the dominant noise to be pure dephasing such that $\epsilon_x = \epsilon_y = 0$ and $\epsilon_z = \epsilon_0 = 1/2$. Additionally, we also consider the possibility that atoms are lost due to an imperfect vacuum. This is expressed as an erasure channel with an effective decoherence time, T_{loss} , such that the probability of losing an atom is

$$p_{\text{loss,vac}} = 1 - e^{-t/T_{\text{loss}}}, \quad (\text{B.14})$$

where t is the time the atom stays in the trap. The loss of an atom is, however, ultimately detected once the atom is measured, and we therefore consider it to be a heralded error that does not affect the fidelity of the final pair but only the distribution rate.

Note that quantum states are stored for different amounts of time, depending on whether the memories are in a ground station, an end-satellite, or an emitter/receiver satellite between the two end-satellites. The memories at the end-satellites need to store the quantum states for the time it takes to receive a heralding signal from both the ground station and the neighboring satellite and to subsequently perform an entanglement swap operation. This time is $\max\{2L_{\text{sat-sat}}/c, 2L_{\text{sat-ground}}/c\} + T_{\text{swap}}$, where c is the speed of light, $L_{\text{sat-sat}}$ is the satellite-satellite distance, and $L_{\text{sat-ground}}$ is the distance between the specific end-satellite and its corresponding ground station. The memories at the emitter satellites between the two end-satellites have to store the quantum states for a time $2L_{\text{sat-sat}}/c + T_{\text{swap}}$, while the memories at the receiver satellites only have to store the quantum states for the time of the entanglement swap T_{swap} . Finally, the memories at the ground stations have to store the quantum states for long enough that they can receive signals from all satellites about which pairs were successfully swapped. Therefore, the required storage time will be $\max\{2L_{\text{sat-sat}}/c - L_{\text{sat-ground}}/c, L_{\text{sat-ground}}/c\} + L_{\text{ground-endsat}}/c + T_{\text{swap}}$, where $L_{\text{ground-endsat}}$ is the link distance between the ground station and the end-satellite at the opposite end of the repeater.

The coefficients of the state after memory decoherence, Eq. (7.12), are given by

$$\alpha = p_b \mathcal{V} \alpha_{\text{BS}}, \quad (\text{B.15})$$

$$\beta = p_{\text{deph}} (\beta_{\text{BS}} + \alpha_{\text{BS}}(1 - \mathcal{V})), \quad (\text{B.16})$$

$$\gamma = ((1 - p_b)\mathcal{V}\alpha_{BS} + (1 - p_{\text{deph}})(\beta_{BS} + \alpha_{BS}(1 - \mathcal{V})) + \gamma_{BS}, \quad (\text{B.17})$$

where the probabilities for a general Pauli noise channel are:

$$\begin{aligned} p_b &= p_1 p_2 + (1 - p_1)(1 - p_2) (\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2 + \epsilon_0^2) \\ &\quad + (p_1(1 - p_2) + p_2(1 - p_1)) \epsilon_0, \\ p_{\text{deph}} &= p_1 p_2 + ((p_1(1 - p_2) + p_2(1 - p_1)) (\epsilon_z + \epsilon_0) \\ &\quad + (1 - p_1)(1 - p_2) (\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2 + \epsilon_0^2)). \end{aligned}$$

Since we assume dephasing noise as the dominant decoherence error in our setup, we put $\epsilon_z = \epsilon_0 = 1/2$ and $\epsilon_x = \epsilon_y = 0$. As we will discuss later, atomic loss can be treated as an overall probability of losing an end-to-end entangled pair.

B.4. Rate

As discussed in §7, the average rate of the repeater is estimated as

$$R = \frac{\sum_{i=1}^{N_{\text{mem}}} i p_f(i)}{T_{\text{com}}}, \quad (\text{B.18})$$

where T_{com} which is the longest communication time between elementary links and

$$p_f(n) = \sum_{i=n}^{N_{\text{mem}}} \binom{i}{n} p(i) (1 - p_{\text{loss}})^n p_{\text{loss}}^{i-n}, \quad (\text{B.19})$$

$$p(n) = \left(\sum_{i=0}^{n_{\text{sat}}-1} \binom{n_{\text{sat}}-1}{i} p_s(n)^i p_s(>n)^{n_{\text{sat}}-(i+1)} \right) \quad (\text{B.20})$$

$$\left(\sum_{j=0}^2 \binom{2}{j} p_g(n)^j p_g(>n)^{2-j} \right) - p_s(>n)^{n_{\text{sat}}-1} p_g(>n)^2, \quad (\text{B.21})$$

$$p_{(g,s)}(n) = \binom{N_{\text{mem}}}{n} p_{\text{link},(g,s)}^n (1 - p_{\text{link},(g,s)})^{N_{\text{mem}}-n}, \quad (\text{B.22})$$

and $p_{(g,s)}(>n) = \sum_{i=n+1}^{N_{\text{mem}}} p_{(g,s)}(i)$. The success probabilities of the entanglement generation in the elementary links, $p_{\text{link},(g,s)}$, are

$$p_{\text{link},(g,s)} = \alpha_{BS} + \beta_{BS} + \gamma_{BS}, \quad (\text{B.23})$$

corresponding to the normalization in Eq. (7.11). The overall probability of losing an entangled pair between the ground stations due to atomic loss is p_{loss} . This includes both cases where atoms are lost in the entanglement swap operation and due to imperfect vacuum conditions. We can express this as

$$p_{\text{loss}} = p_{\text{loss,gstat}}^2 (p_{\text{loss,end-sat}})^2 (p_{\text{loss,receiver-sat}})^{(n_{\text{sat}}-1)/2} (p_{\text{loss,emitter-sat}})^{(n_{\text{sat}}-3)/2},$$

where

$$p_{\text{loss,gstat}} = 1 - e^{-t'/T'_{\text{loss}}},$$

$$p_{\text{loss},x\text{-sat}} = e^{(-2t_x/T_{\text{loss}})} p_{\text{loss},\text{swap}} + 2e^{-t_x/T_{\text{loss}}} \left(1 - e^{-t_x/T_{\text{loss}}}\right) + \left(1 - e^{-t_x/T_{\text{loss}}}\right)^2.$$

The different storage times t', t_x ($x = \{\text{end, receiver, or emitter}\}$) are the different storage times discussed in Sec. B.3.1. Note that we allow for different effective loss times of the atoms on ground (T'_{loss}) and on satellites (T_{loss}) since it is conceivable that better vacuum and hence longer effective loss time can be obtained on the ground than on satellites.

B.5. Atmospheric loss

In this section, the photon losses due to atmospheric effects are considered. These losses will be included in the model developed for the space-to-ground links.

B.5.1. Atmospheric turbulence loss model

When an optical beam propagates through the atmosphere, the turbulence will create wavefront errors that will be detrimental to the link. In a downlink scenario, the beam spread induced by the atmospheric turbulence can be neglected (ref. [422], chapter 12). Hence, the beam-spread due to the atmospheric turbulence is not considered in the space-to-ground links in our model. Furthermore, the atmospheric turbulence will also create a beam wander on the receiver's aperture plane. This beam wander can be modeled as a part of the transmitter pointing jitter, as their effects are the same (stochastic displacement of the beam center from the receiver's optical axis). However, for a downlink, the beam wander effect due to the transmitter pointing jitter will be dominant for reasonable turbulence regimes, and the contribution from turbulence can therefore be neglected (ref. [422], chapter 12.6.2).

In our work, we consider that the ground stations are at an altitude higher than 2000 meters, as realised experimentally in Ref. [310]. Furthermore, the telescope in the ground has a diameter of 2 meters, allowing for aperture averaging effects of atmospheric turbulence scintillation [253, 422]. Finally, weak turbulence conditions are considered to impinge on the downlink [422]. Given these three factors, the effects of atmospheric turbulence-induced scintillation will be negligible. Experimental data in Refs. [246, 307, 310], support the Gaussian far-field assumption considered in our model and presented in the previous section.

B.5.2. Atmospheric transmission loss model

The atmospheric transmittance loss is due to four effects: molecular absorption, molecular scattering, aerosol absorption, and aerosol scattering. The beam attenuation due to these can be modeled with Beer's law as [64]

$$\frac{P_\lambda(\mathcal{L})}{P_\lambda(0)} = \exp \left[- \int_0^{\mathcal{L}} \alpha_\lambda(z) dz \right], \quad (\text{B.24})$$

where $P(\mathcal{L})$ is the signal intensity after travelling a distance $\mathcal{L}$ through the atmosphere, z is the distance coordinate, and $\alpha_\lambda(z)$ is the attenuation coefficient (a result of the

sum of the attenuation coefficients of the absorption and scattering of molecules and aerosols). The wavelength dependence is due to the dependence of the absorption and scattering phenomena on the wavelength. Considering that the length $\mathcal{L}$ is given for a beam with zenith angle $\zeta = 0^\circ$, the atmospheric attenuation coefficient can, to a good approximation, be considered as a constant of position [297] such that the atmospheric transmittance loss for a zenith angle ζ can be computed as

$$\frac{P(\mathcal{L}')}{P(0)} = \exp[-\mathcal{L}'\alpha_\lambda] = \eta^{\sec(\zeta)}, \quad (\text{B.25})$$

where $\eta = \exp[-\mathcal{L}\alpha_\lambda]$ is the transmittance for the zenith angle $\zeta = 0^\circ$. For large zenith angles $\zeta > 60^\circ$ the parallel plane approximation of the atmosphere is no longer applicable, and the geometrical correction factor has to be changed to [64]

$$\frac{P(\mathcal{L}')}{P(0)} = \eta^{m(\zeta)}, \quad (\text{B.26})$$

$$m(\zeta) = \begin{cases} \sec \zeta & \zeta \leq 60^\circ, \\ \frac{1}{\cos \zeta + 0.505572(96.07995 - \zeta)^{-1.664}} & 60^\circ < \zeta < 90^\circ, \end{cases} \quad (\text{B.27})$$

Simulated values for the transmittance η , for different zenith angles, can be found in [521]. In space-to-ground links, the main contributor to the atmospheric attenuation of light is the Rayleigh scattering due to molecules [64], assuming that the ground stations are placed at 2 km altitude, where the Mie scattering is negligible [521]. This is because the aerosol density is very low at these altitudes, and therefore the Mie scattering can be neglected. The wavelength considered in this work is 780 nm (see Fig. 7.3), so according to Fig. 15 in Ref. [521], a value of $\eta \approx 0.95$ is used for our simulations.

Appendix C

Appendix for Chapter 8

In this appendix, we present the details of the quantum key distribution model discussed in §8. We start by discussing the different steps at the hardware level, detailing the modelling of the spontaneous parametric down-conversion (SPDC) source, the transmission loss from the satellite to the ground station, and the setup at the ground stations. Finally, we present the details of the transmission loss due to pointing jitter and beam divergence. The average photon capture probability as a performance parameter is justified. Furthermore, the effect of truncation due to the finite transmitter aperture in the transmitted beam and the resulting average photon capture probability are studied. Additionally, the optimum Gaussian beams are presented with and without the effects of truncation.

C.1. Spontaneous parametric down-conversion source

Here, we consider second-photon emissions in the entangled photon source. Following the work shown in [440], the expression for the spontaneous parametric down-conversion (SPDC) source is given in Eq. (8.1). In our model, we always assume that we are in the weak-pump regime, which implies $\lambda \ll 1$. The brightness of the source, B , which is defined as

$$B = \frac{\mu f_{\text{rep}}}{P_{\text{pump}}}, \quad (\text{C.1})$$

where f_{rep} is the repetition rate of the source and P_{pump} is the average power of the pump beam, the results presented in §8 show that the amount of secret key material increases approximately proportionally with the increase in the source's repetition rate. This proportional increase suggests that the quantum bit error rate (QBER) remains roughly constant. Consequently, the fidelity of the quantum link is maintained even as the repetition rate increases. From this, we infer that the parameter λ remains constant and does not increase.

In our model, increasing the repetition rate also entails an increase in the average pump power, yet the system remains within the weak-pump regime, where $\lambda \ll 1$.

This has implications for detector saturation: as the source repetition rate increases, it is assumed that the detectors are also improved –specifically, that their dead time is reduced– allowing them to register single-photons more frequently. Accordingly, the detection window can be narrowed, which in turn requires a shorter detector dead time.

C.2. Free-space, atmosphere propagation and photon measurement

After the SPDC source, the photons are sent from the satellite to the ground stations. The free-space and atmospheric losses are modelled as a fictitious beam splitter with transmission p_T determined by the optical link budget. The transmission of a single-photon is the following

$$\hat{a}_{H,A(B)}^\dagger |0\rangle_{A,(B)} |0\rangle_{E_A,(E_B)} \rightarrow \sqrt{p_{T,A(B)}} \hat{a}_{H,A(B)}^\dagger |0\rangle_{A,(B)} |0\rangle_{E_A,(E_B)} + \sqrt{1-p_{T,A(B)}} \hat{a}_{H,E_A(E_B)}^\dagger |0\rangle_{A(B)} |0\rangle_{E_A,(E_B)}, \quad (\text{C.2})$$

$$\hat{a}_{V,A(B)}^\dagger |0\rangle_{A,(B)} |0\rangle_{E_A,(E_B)} \rightarrow \sqrt{p_{T,A(B)}} \hat{a}_{V,A(B)}^\dagger |0\rangle_{A,(B)} |0\rangle_{E_A,(E_B)} + \sqrt{1-p_{T,A(B)}} \hat{a}_{V,E_A(E_B)}^\dagger |0\rangle_{A(B)} |0\rangle_{E_A,(E_B)}, \quad (\text{C.3})$$

where the subscripts A and B denote the different spatial modes, and the subscript E denotes the loss mode. So, for example, for the state $|\psi\rangle$ we will have that the state is transmitted with probability $p_{T,A}p_{T,B}$, or that we lose a photon in one of the arms with probability $p_{T,A(B)}(1-p_{T,B(A)})$ leaving a dephased state $\rho_{\text{deph0},A(B)} \otimes |0\rangle_{B(A)}\langle 0|$, where $\rho_{\text{deph0}} = \frac{1}{2}(|H\rangle\langle H| + |V\rangle\langle V|)$, or that with probability $(1-p_{T,A})(1-p_{T,B})$ we lose both photons. In other words, the state $|\psi\rangle$

$$\begin{aligned} |\psi\rangle \rightarrow \frac{1}{\sqrt{2}} & \left[\sqrt{p_{T,A}p_{T,B}} (|H\rangle_A |0\rangle_{E_A} |V\rangle_B |0\rangle_{E_B} + |V\rangle_A |0\rangle_{E_A} |H\rangle_B |0\rangle_{E_B}) \right. \\ & + \sqrt{p_{T,A}(1-p_{T,B})} (|H\rangle_A |0\rangle_{E_A} |0\rangle_{E_B} |V\rangle_{E_B} + |V\rangle_A |0\rangle_{E_A} |0\rangle_B |H\rangle_{E_B}) \\ & + \sqrt{p_{T,B}(1-p_{T,A})} (|0\rangle_A |H\rangle_{E_A} |V\rangle_B |0\rangle_{E_B} + |0\rangle_A |V\rangle_{E_A} |H\rangle_B |0\rangle_{E_B}) \\ & \left. + \sqrt{(1-p_{T,A})(1-p_{T,B})} (|0\rangle_A |H\rangle_{E_A} |0\rangle_B |V\rangle_{E_B} + |0\rangle_A |V\rangle_{E_A} |0\rangle_B |H\rangle_{E_B}) \right]. \end{aligned}$$

After the photons pass through the atmosphere, they reach the ground stations. There, photons are randomly measured either in the Z basis- defined by the states $|H\rangle$ and $|V\rangle$ - or in the X basis- defined by $|D\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$ and $|A\rangle = (|H\rangle - |V\rangle)/\sqrt{2}$. To implement this random-basis measurement, the photons first pass through a 50/50 beam splitter (BS). At one of the BS output ports, a half-wave plate (HWP) is used to rotate the measurement basis from Z to X. Both outputs are then directed into a polarizing beam splitter (PBS), which spatially separates the polarization components.

We assume that the beam splitter transformations are of the following form

$$|H\rangle_1 \rightarrow \frac{1}{\sqrt{2}} (|H\rangle_3 + |H\rangle_4) \quad |H\rangle_2 \rightarrow \frac{1}{\sqrt{2}} (|H\rangle_3 - |H\rangle_4) \quad (\text{C.4})$$

$$|V\rangle_1 \rightarrow \frac{1}{\sqrt{2}} (|V\rangle_3 + |V\rangle_4) \quad |V\rangle_2 \rightarrow \frac{1}{\sqrt{2}} (|V\rangle_3 - |V\rangle_4). \quad (\text{C.5})$$

Assuming the ideal entangled state $|\psi\rangle$ is received at the ground stations, it evolves as follows

$$\begin{aligned} \frac{1}{\sqrt{2}} (|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B) &\xrightarrow{\text{B.S.}} \\ \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} (|H\rangle_{A1} + i|H\rangle_{A2}) \frac{1}{\sqrt{2}} (|V\rangle_{B1} + i|V\rangle_{B2}) \right. \\ &\quad \left. + \frac{1}{\sqrt{2}} (|V\rangle_{A1} + i|V\rangle_{A2}) \frac{1}{\sqrt{2}} (|H\rangle_{B1} + i|H\rangle_{B2}) \right]. \end{aligned}$$

Here, the subscripts A1, A2, B1, and B2 refer to the different spatial modes after the beam splitter for photons A and B, respectively. The output ports labelled A2 and B2 undergo a basis transformation via a HWP: $|H\rangle \rightarrow |D\rangle$ and $|V\rangle \rightarrow |N\rangle$.

Subsequently, each output passes through a PBS, which separates the polarization components before detection. The resulting state just before measurement is:

$$\begin{aligned} \frac{1}{2\sqrt{2}} [&|H\rangle_{AH}|0\rangle_{AV}|0\rangle_{BH}|V\rangle_{BV} + i|H\rangle_{AH}|0\rangle_{AV}|0\rangle_{BD}|N\rangle_{BN} \\ &+ i|D\rangle_{AD}|0\rangle_{AN}|0\rangle_{BH}|V\rangle_{BV} - |D\rangle_{AD}|0\rangle_{AN}|0\rangle_{BD}|N\rangle_{BN} \\ &+ |0\rangle_{AH}|V\rangle_{AV}|H\rangle_{BH}|0\rangle_{BV} + i|0\rangle_{AH}|V\rangle_{AV}|D\rangle_{BD}|0\rangle_{BN} \\ &+ i|0\rangle_{AD}|N\rangle_{AN}|H\rangle_{BH}|0\rangle_{BV} - |0\rangle_{AD}|N\rangle_{AN}|D\rangle_{BD}|0\rangle_{BN}]. \end{aligned} \quad (\text{C.6})$$

From Eq. (C.6), we observe that in half of the cases, the photon pairs are measured in the same basis (either Z or X). Consequently, during postselection, approximately 50% of the events are discarded to ensure only measurements in matching bases are kept.

Additionally, we take into account dark counts in the detectors. A dark count is when a detector clicks for a vacuum input. The probability of this happening is p_{dark} , i.e. a detector will click when receiving vacuum input with probability p_{dark} . Since $p_{\text{dark}}, p_T \ll 1$, we discard third-order terms, implying that we will not consider terms with probability $p_T^2 p_{\text{dark}}, p_{\text{dark}}^2 p_T$ and similar. Therefore, from Eq. (C.3), and considering that we postselect states with opposite polarizations in the same basis, we get the desired state, $|\psi\rangle$, with probability $\lambda(1-\lambda)p_{T,A}p_{T,B}$, with probability $\lambda(1-\lambda)p_{T,A(B)}(1-p_{T,B(A)})p_{\text{dark}}$ one of the photons has gotten lost in transmission, and with probability $\lambda(1-\lambda)(1-p_{T,A})(1-p_{T,B})p_{\text{dark}}^2$, both photons get lost in transmission.

In the case of the two-photon state, $|\psi_{\text{multi}}\rangle$, discarding third-order terms, we can lose one photon in each arm, which leads to the following dephased state:

$$\begin{aligned} \rho_{\text{deph}} = \frac{1}{12} \Big(&5|H\rangle_A |V\rangle_B \langle H|_A \langle V|_B + 5|V\rangle_A |H\rangle_B \langle V|_A \langle H|_B \\ &+ |H\rangle_A |H\rangle_B \langle H|_A \langle H|_B + |V\rangle_A |V\rangle_B \langle V|_A \langle V|_B \Big), \end{aligned} \quad (\text{C.7})$$

where, considering that we post-select the measurements with opposite polarizations in the same basis, we will get the aforementioned depolarized state with probability $5\lambda^2 p_{T,A} p_{T,B} (1 - p_{T,B})(1 - p_{T,A})$. Additionally, we can also lose three photons and get a valid measurement due to dark counts with probabilities $3\lambda^2 p_{T,A} (1 - p_{T,A})(1 - p_{T,B})^2 p_{\text{dark}}$ and $3\lambda^2 p_{T,B} (1 - p_{T,B})(1 - p_{T,A})^2 p_{\text{dark}}$.

We can also measure states that give zero fidelity with probabilities $3\lambda^2 (1 - p_{T,A})^2 (1 - p_{T,B})^2 p_{\text{dark}}^2 / 2$, $(1 - \lambda)^2 p_{\text{dark}}^2 / 2$.

Therefore, we can write the final state as in Eq. (8.9), where the parameters A , B and C are given by

$$A = \lambda(1 - \lambda)p_{T,A}p_{T,A}, \quad (\text{C.8})$$

$$B = 5\lambda^2 p_{T,A} p_{T,B} (1 - p_{T,A})(1 - p_{T,B}), \quad (\text{C.9})$$

$$\begin{aligned} C = & \lambda(1 - \lambda) (p_{T,A}(1 - p_{T,B})p_{\text{dark}} + p_{T,B}(1 - p_{T,A})p_{\text{dark}} \\ & + (1 - p_{T,A})(1 - p_{T,B})p_{\text{dark}}^2) \\ & + \lambda^2 (3p_{T,A}(1 - p_{T,A})(1 - p_{T,B})^2 p_{\text{dark}} \\ & + 3p_{T,B}(1 - p_{T,B})(1 - p_{T,A})^2 p_{\text{dark}} \\ & + \frac{3}{2}(1 - p_{T,A})^2(1 - p_{T,B})^2 p_{\text{dark}}^2) \\ & + \frac{1}{2}(1 - \lambda)^2 p_{\text{dark}}^2. \end{aligned} \quad (\text{C.10})$$

C.3. Free-space channel modelling

In this section, we show the models developed for evaluating the free-space channel losses. Firstly, the average photon capture probability as a performance parameter for a downlink channel under a stochastic pointing jitter is justified. Then, a model to accurately compute the average photon capture probability is presented. This model considers the truncation effects on the transmitted single-photon spatial modes. Finally, the atmospheric channel model used for our analysis is explained.

C.3.1. Average photon capture probability

In this section, a demonstration of the average photon-pair capture probability as a valuable parameter to evaluate the performance of quantum entanglement distribution from a satellite is presented.

Consider a Gaussian beam traveling from the transmitter satellite to the ground station. Although in reality the light beam would deviate from an ideal Gaussian shape (due to e.g. atmospheric turbulence and truncation due to finite aperture), this shape can be considered for the present discussion. In App. C.3.2, the effect of the truncation due to the finite transmitter aperture is considered. As seen by the ground station, the transmitter's pointing jitter will stochastically deviate the center of the Gaussian beam from the center of the receiver's aperture. This stochastic deviation will, in turn, create variations of the photon capture probability in the free-space channel.

To evaluate the process of capturing a photon at the ground station under the

stochastic nature of the pointing jitter, the different timescales involved in the system have to be thoroughly assessed. We have three different timescales that have to be considered in a scenario involving a satellite downlink:

1. *Timescale of the link distance variation.* This timescale refers to the characteristic time in which the distance between the transmitter (satellite) and the receiver (ground station) significantly changes. A significant change would be one that has a non-negligible impact on the photon capture probability due to the change in the link distance. This timescale is usually in the order of minutes for the LEO satellite considered, $\tau_D = \mathcal{O}(10^1)$ seconds.
2. *Timescale of the pointing jitter.* This timescale is the correlation time of the pointing jitter process. Due to the nature of the microvibration sources onboard satellites and the limitations of the fine pointing assembly, this is usually in the order of milliseconds, $\tau_{PJ} = \mathcal{O}(10^{-3})$ seconds [81].
3. *Timescale of the photon emission.* The rate at which the single-photons are emitted from the source is also relevant for the discussion. On-demand single-photon sources of interest to this application can work on the order of MHz~GHz [310, 439]. Therefore, the characteristic timescale of this process is $\tau_{SPS} = 1/\nu_{SPS} = \mathcal{O}(10^{-6}) \sim \mathcal{O}(10^{-9})$ seconds.

Considering these timescales, we can see that over a pointing jitter correlation time (milliseconds) the photon capture probability variation due to the distance change can be considered quasi-static. Furthermore, over each realization of the pointing jitter in the order of fractions of milliseconds, at least thousands of photons will be sent due to the single-photon source rate. Hence, in each of these pointing jitter realizations, as the light beam and its position are fixed, the photon capture probability for these thousands of photons will be the same. Hence in a single realization of the pointing jitter that lasts for $\Delta\tau_R$, where $\tau_{SPS} \ll \Delta\tau_R \sim \tau_{PJ}$, the number of photons captured follows a binomial distribution $X \sim B(N, \eta)$ with $N = \Delta\tau_R \nu_{SPS}$ and η is the instantaneous photon capture probability given by the divergence of the light beam and the pointing error value at that realization.

Entangled pair capture probability under pointing jitter

Due to the pointing jitter, the photon capture probability for *each* of the downlink channels varies over time. Consider now the case in which we send a pair of entangled photons. The first photon travels through channel 1, while the second photon travels through channel 2. The photon capture probability η_1 in channel 1, considering pointing jitter is given by a probability density function $f_1(\eta_1)$. In the same way, the photon capture probability in channel 2 (η_2) is given by a probability density function $f_2(\eta_2)$. We will consider that these two stochastic processes are completely uncorrelated. This assumption comes from the fact that, in general, each of the two downlink beams will be controlled by a different fast steering assembly [310].

We will now try to calculate the probability of capturing both entangled photons in their respective stations. In a single realization of these two channels, the probability of channel 1 being under the condition $\in (\eta_1, \eta_1 + d\eta_1)$ is $f_1(\eta_1) d\eta_1$. Hence, the

probability of the photon being captured when the channel is in $\in (\eta_1, \eta_1 + d\eta_1)$ is $\eta_1 f_1(\eta_1) d\eta_1$. An analogous derivation for channel 2 yields $\eta_2 f_2(\eta_2) d\eta_2$. At a certain realization of channels 1 and 2, the probability of capturing both entangled photons is given by $\eta_\infty = \eta_1 \cdot \eta_2$ (recalling that the pointing jitter in these channels is statistically independent). Hence, the probability density function of the probability of both photons being captured is given by [441]

$$f_\infty(\eta_\infty) = \int_{\eta_\infty}^1 f_1(\eta_1) f_2\left(\frac{\eta_\infty}{\eta_1}\right) \frac{1}{\eta_1} d\eta_1, \quad \eta_\infty \in (0, 1]. \quad (\text{C.11})$$

And the average of this probability density function

$$\bar{\eta}_\infty = \bar{\eta}_1 \cdot \bar{\eta}_2 = \int_0^1 f_\infty(\eta_\infty) \cdot \eta_\infty \cdot d\eta_\infty, \quad (\text{C.12})$$

where the first equality is due to the statistical independence of the channels. As mentioned before, over each realization of the pointing jitter, there will be a huge number N of entangled-photon pairs sent through the same channel. Furthermore, considering that each of these entangled pairs is uncorrelated from each other w.r.t. their spatial location, then the probability of capturing k pairs of entangled photons in the ground stations when a total of N are sent through the double downlink channel at condition η_∞ is given by a binomial distribution

$$f(k, N, \eta_\infty) = \binom{N}{k} \eta_\infty^k (1 - \eta_\infty)^{(N-k)}, \quad (\text{C.13})$$

Because each of the entangled-photon pair capture events is independent. These will be the statistics describing the number of photons captured over a specific instant in time. Over a time period Δt_P longer than the pointing jitter characteristic time $\Delta t_P \gg \tau_{PJ}$, the number of photons captured will be given by the weighted sum of binomial distributions. The weighting will be given by the density function $f_\infty(\eta_\infty)$. Hence if in this time period Δt_P the total number of photons emitted by the source is $N_P = \nu_{SPS} \cdot \Delta t_P$, the probability of capturing k entangled photons is given by

$$\begin{aligned} f_P[k, N_P, f_\infty(\eta_\infty), \eta_\infty] &= \int_0^1 f_\infty(\eta_\infty) f(k, N_P, \eta_\infty) d\eta_\infty \\ &= \binom{N_P}{k} \int_0^1 f_\infty(\eta_\infty) \eta_\infty^k (1 - \eta_\infty)^{(N_P-k)} d\eta_\infty. \end{aligned} \quad (\text{C.14})$$

Hence, the number of entangled photons captured over a period Δt_P does no longer follow a binomial distribution. It follows the weighted binomial distribution given in Eq. (C.14), the weighting being given by the statistics of the photon capture probability due to the pointing jitter (and atmospheric effects in case they are considered). Hence, over a single pass of the satellite, the photon capture statistics can be computed by considering each time step t_i where $t_{i+1} - t_i = \Delta t_P$ (where Δt_P is in the timescale of the link distance variation $\sim \tau_D$). In each of these time steps, the entangled-photon pair capture probability is given by η_∞^i , and hence described by the density function

f_{∞}^i . The variation of this density function is due to the variation of the distance (and other considerations) in each of the elementary links.

According to the discussion above, we now have a model to exactly obtain the photon capture probability considering the stochastic disturbances impinging on the channel (pointing jitter and atmospheric effects). The equations above can not be solved analytically, as already the density functions of the photon capture probability of each of the channels, i.e., $f_{1,2}(\eta_{1,2})$, do not have a closed form in the general case.

Our objective is to know if the *average entangled-photon capture probability* $\bar{\eta}_{\infty}$ is a good performance parameter for evaluating the performance of quantum entanglement distribution protocols. As this probability can be obtained by multiplying the average of each of the channels $\bar{\eta}_1$ and $\bar{\eta}_2$, this is very convenient. Remember that the entangled-photon capture probability is averaged out over the pointing jitter statistics in this case. We will consider photons in the spatial Gaussian fundamental mode and pointing jitters given by Gaussian bivariate density functions [275]. We will now numerically demonstrate the statistics obtained for Eq. (C.11) and (C.14).

In Fig. C.1, the results for a Monte Carlo simulation over a second are shown when both satellites are at a 500 km distance from both ground stations. The pointing jitter considered for all the simulations is the one reported in the Micius satellite [81], i.e., $\sigma_{\theta} = 0.47 \mu\text{rad}$. The entangled-photon pair source rate is the one also reported for this satellite 5.9 MHz. In this figure, two scenarios are presented: non-optimized and optimized beams. The non-optimized beam (shown in red color in the figures), refers to a Gaussian beam with a $10 \mu\text{rad}$ full-width divergence approximately equal to the one reported in Ref. [245]. On the other hand, the optimized beam (shown in green color in the figures), refers to a Gaussian beam with a beamwidth w_0 that is 80% of the transmitter's telescope radius of 15 cm, with the truncation effects of the transmitted beam negligible. This gives a divergence of approximately $4 \mu\text{rad}$ for the optimized beam. It can be seen in the figure that the fluctuations over the average are much higher in the optimized case than in the non-optimized case. Although these fluctuations around the average can be very detrimental in classical communication links [473], this is not the case for quantum entanglement distribution. Furthermore, in the bottom of Fig. C.1 the cumulative number of entangled-photon pairs over time is shown, comparing it to the linear fit given by the average entangled-photon capture probability $\bar{\eta}_{\infty} = \bar{\eta}_1 \bar{\eta}_2$. It is shown that almost no difference can be found between the two of them in both the non-optimized and the optimized channels. It is important to mention that the density function shown for η_{∞} in the Monte Carlo simulations corresponds to Eq. (C.14) derived before.

To understand the behavior of the double downlink channel over a longer time period, the approximate time period of the satellite pass is simulated. Maintaining the distance from the two ground stations to the satellite constant and equal, the distribution of the entangled-pairs per second is shown in Fig. C.2. The binomial distribution with the same average entangled-photon capture rate is also shown. For a more realistic assessment, Fig. C.3 shows the simulation considering the distance variation over time from each of the satellites. For this simulation, a typical pass of a satellite at an altitude of 400 km passing over Madrid and Lisbon has been considered, considering at each time the distance from the satellite to the respective ground station

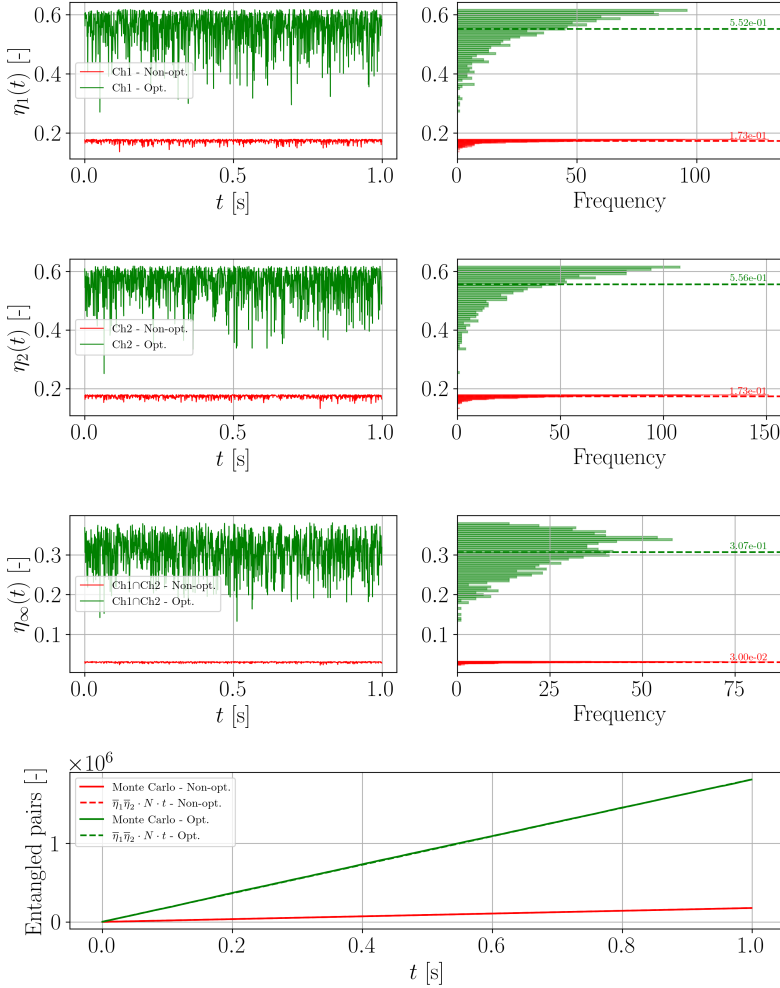


Figure C.1: Monte Carlo simulation of the capture probabilities and the accumulated entangled-pair over a second. The first three rows show the variation over time of the respective capture probabilities and the associated probability distributions. The bottom figure shows the cumulative entangled-photon pairs captured by both ground stations over time.

(Madrid and Lisbon in this case).

By comparing the optimized beam presented is one that tries to maximize (up to the limit in which the truncation effects are negligible) the average photon capture probability for each of the channels. This is done for single-photons in the spatial Gaussian mode by minimizing the width of the beam in the ground station. This maximization of the average, however, comes at the cost of the fluctuations of the capture probability over this average being increased too. Although this last trade-off has to

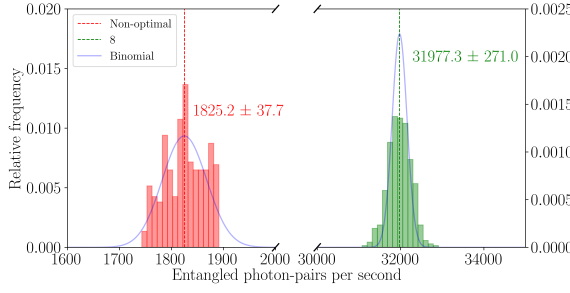


Figure C.2: Probability distribution of the entangled-photon pair capture rate from the satellite to two ground stations at 500 km distance.

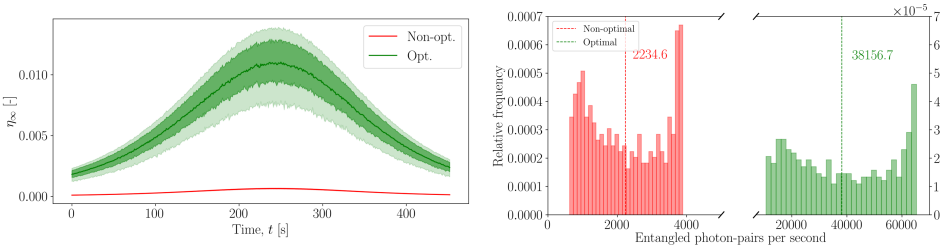


Figure C.3: Entangled-photon pair capture statistics over a realistic satellite pass connecting Madrid and Lisbon. (Left) shows the variation of the entangled-photon pair capture probability over the satellite pass, where the filled areas delimit the 10%-90% and the 25%-75% quantiles. (Right) shows the probability distribution of the entangled-photon pair capture rate over the pass.

be considered in classical channels (captured power instead of capture probability for classical communications), it has been proven in this section that for the dynamical processes considered, the maximization of the average is to be pursued in quantum entanglement distribution channels. The latter should be done even when the fluctuations increase drastically in this optimization, as it results in a larger number of entangled photons captured in both ground stations over a satellite pass.

C.3.2. Effect of clipping and pointing jitter

In the previous section, it has been proven that the maximization of the average capture photon probability for each channel should be pursued in satellite quantum entanglement distribution systems. However, and only for the sake of the demonstration, the single-photon has been considered to be described by the spatial Gaussian fundamental mode. However, trying to maximize the average photon capture probability for each channel means making the spread of this spatial Gaussian mode as small as possible in the ground station. This, in turn, is achieved by collimating the beam as much as possible in the transmitter aperture plane. Hence, this means making the beamwaist w_0 as big as possible in the transmitter aperture plane. It is, however,

important to mention that this trend follows up to a certain w_0 , as the beamwidth at distance z is given by $w_z = w_0 \sqrt{1 + z^2 \lambda^2 / (\pi^2 w_0^4)}$, where λ is the wavelength of the beam. This point is reached for beamwaists w_0 much higher than the apertures of the transmitters considered for the satellite terminal (e.g. the point is reached for $w_0 \approx 35$ cm for a distance of 500 km).

Hence, in principle, making the beamwaist w_0 larger on the satellite aperture plane will create a smaller beamwidth in the ground station location, and therefore a higher average photon capture probability. However, as the beamwaist of the spatial Gaussian mode is increased, the border effects due to the finite size of the transmitter's aperture will start to play a role. The truncation of the Gaussian mode due to the transmitter's aperture will create a loss on the probability of transmitting a single-photon. Furthermore, the truncation of the Gaussian mode will induce a diffraction that will change the divergence of the transmitted mode. The latter will no longer be purely Gaussian due to this diffraction effect.

Optimization of the transmitted beam

All considered, as the truncation of the Gaussian beam is relevant when aiming for beams that have a small far-field beamwidth, a complete optical propagation model is required to accurately calculate the spatial mode associated with the single-photon on the receiver side. By using the model presented in §6, the average photon capture probability, accounting for truncation effects, can be obtained for a given pointing jitter, Gaussian beamwaist, and aperture sizes involved. This model propagates the spatial mode of the single-photon from the transmitter plane to the receiver plane through Fourier optics. Then, the spatial mode of the single-photon in the receiver aperture plane is combined with the pointing jitter statistics to obtain the probability density of the photon capture probability. From this density function, the average photon capture probability for the individual channel can be obtained.

Furthermore, by maximizing this average, an optimization of the transmitted beam can be performed that optimizes the performance of the quantum entanglement distribution system according to App. C.3.1. The results for such an optimization for a distance of 400 km and a ground station aperture radius of 60 cm are presented in Fig. C.4, for different transmitter aperture radii a and transmitter angular pointing jitters. In this manner, the optimal transmitted beam for a given set of parameters is computed, and the maximum average photon capture probability is assigned to the channel. For the set of parameters used in the results of §8, the optimization yields the same truncation ratio w_0/a along the whole satellite pass because the pointing jitter is considered to be the same along the entire orbit (see Fig. 8.3 in §8). However, for other parameters (see top-left in Fig. C.4), the optimum truncation ratio is dependent on the aperture of the telescope. In the latter case, or in the case in which an in-orbit adjustment of the beamwaist is to be done, a beam-divergence control system such as the one in Ref. [454] could be used.

For the operation of a quantum key distribution (QKD) protocol, a classical communication channel is necessary. It is important to mention that the optimization discussed in our manuscript refers only to the quantum beam. For the classical beam, an analogous optimization could be performed, this time attending to the relevant

classical communication performance parameters (e.g., outage probability, see §6).

C.3.3. Atmospheric effects

As the single-photon state propagates through the atmosphere, there are several atmospheric effects that need to be assessed. Firstly, the atmospheric absorption and scattering reduce the probability of the photon being captured by the receiver. This is modeled in our analysis according to Ref. [258]. Secondly, there are several turbulence-induced effects. The stochastic change of the index of refraction due to the atmospheric turbulence will mainly create: beam spread, beam wander, and scintillation. Under the weak turbulence conditions considered in §8 the beam spread and wander effects are negligible in downlink channels [422]. The zenith angle of the downlink is limited to 70° for the simulations, where the effect of beam spread and other atmospheric effects is limited [253].

Concerning the scintillation, there are several things that need to be considered. Assuming weak turbulence and a large diameter of the ground station's telescope, the turbulence-induced scintillation is already diminished [422]. Furthermore, for the results obtained, the ground station is assumed to have an adaptive optics system that counteracts for the variations due to the scintillation on the fiber coupling. An overall detector coupling efficiency of $\gamma = 0.5$ is assumed, which is in agreement with simulations and experimental data in optical downlink channels with adaptive optics and single-mode fiber coupling [254, 255, 442]. This coupling efficiency could be increased by considering free-space coupling into the single-photon detectors [445], or multimode fiber coupling [446, 447] (maintaining the adaptive optics system). Finally, considering the specific application of QKD, previous studies have shown that scintillation has minimal impact on these quantum communication protocols [250].

C.4. Key extraction

The key extraction protocol is performed after the quantum state transmission and reception phase of QKD, leading to a so-called block of detection events. The two communicating parties in the BBM92 configuration transmit the time-tagged information about the (in this case, passively) chosen measurement bases. Basis reconciliation and time filtering allow both parties to discard all events that correspond to mismatched bases or those that fall outside the time window of acceptance. It is important to note that time synchronization, in this case, must be enforced between the receiving parties, i.e., the optical ground stations. Therefore, ground-based solutions, such as the White Rabbit Protocol [522], can be used to achieve synchronization with precision down to tens of picoseconds. At this point, the protocol requires the parties to sacrifice a subset of the remaining events by disclosing the detection outcome in order to calculate the QBER. If the value is below the threshold for security –at 11% for BBM92 [523]– the protocol moves on to the next stage. Depending on the value of the QBER, however, the amount of privacy amplification rounds will vary: for low values of QBER, the expected mutual information between the communicating parties and a potential adversary is low, indicating that the security of the key is high; if the value of the QBER approaches the threshold, more rounds are necessary to reduce the

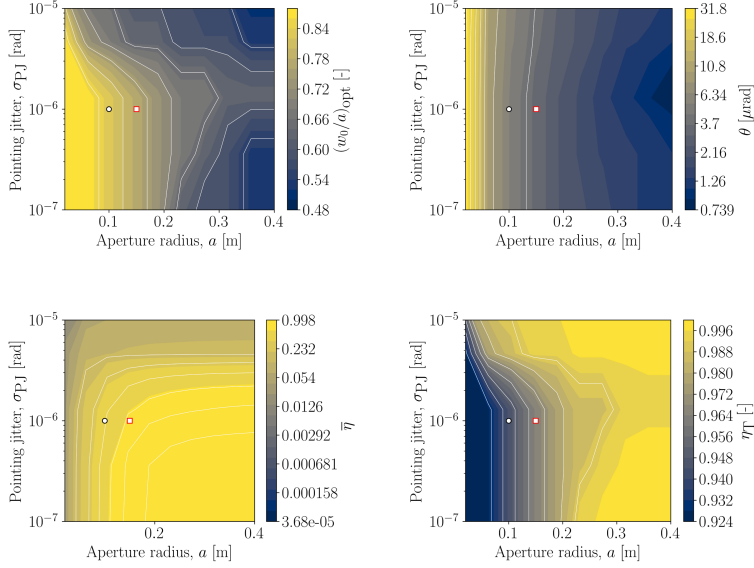


Figure C.4: Optimization of the truncated Gaussian beam for different transmitter aperture sizes a and pointing jitters σ_θ using the full numerical propagation model in §6. Top-left is the optimum beam-waist to aperture ratio. Top-right is the divergence of the resulting optimum beam. Bottom-left is the optimum average photon capture probability (maximum obtainable). Bottom-right is the transmission efficiency of the telescope (i.e. the probability of losing the single-photon on the transmitter's aperture). The circle and square points show the characteristics of the Jinan and Micius satellites, respectively [246, 310].

mutual information and enforce security of the key material.

The factor on the right-hand side of Eq. (8.10) indicates the ratio between the amount of raw detection events before privacy amplification and error correction rounds and the final amount of secure bits output by the protocol. Practical operation of QKD systems requires online computation of the QBER and execution of the subsequent rounds of classical communication that enable producing a key. The complexity of the steps and the fact that they may be executed several times lead to the development of dedicated hardware to tackle this problem efficiently and ensure that it does not represent a bottleneck in the key distribution link. For the analysis presented in this work, several satellite passes have been evaluated over multiple days, which translates into a heavy computational task to execute the classical protocol for each block of raw detection events. Therefore, a look-up table (LUT) was created allowing one to efficiently and accurately estimate the value of the so-called key extraction efficiency (η_{LUT}) given a block length and the corresponding QBER value. In other words:

$$\text{SKR} = R_{\text{final}} \cdot (\max(0, 1 - (1 + \xi) \mathcal{H}(\text{QBER})))$$

$$= R_{\text{final}} \cdot \eta_{\text{LUT}} [L_{\text{initial}}; \text{QBER}]. \quad (\text{C.15})$$

The look-up table was generated by artificially inputting bit strings –of different length and error rate– to a key extraction software implementing the Cascade error correction protocol [524]. For each combination of $[L_{\text{initial}}; \text{QBER}]$, η_{LUT} –presented as a heatmap in Fig. C.5a– outputs a value that can be immediately used to calculate the expected key material generated by the QKD protocol. In Fig. C.5-b, we present the same calculations but considering a static value of $\xi = 1.22$, which approximates the realistic results for large block lengths ($L_{\text{initial}} \geq 10^4$). Due to the limited availability of the satellite during the pass and the dynamic weather conditions that can significantly diminish the key material generated on the ground, the regime of $L_{\text{initial}} \leq 10^4$ is relevant for the analysis of the results and justifies the use of η_{LUT} .

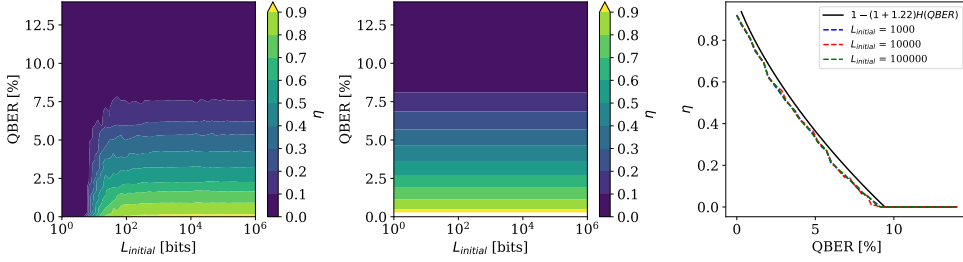


Figure C.5: Key Extraction Efficiency - Look Up Table. The heatmap shows the value of η_{LUT} (Eq. (C.15)) as a function of the block length and the estimated QBER.

Appendix D

Appendix for Chapter 9

D.1. Pointing jitter statistics

The PDF of the pointing jitter will determine the PDF of the optical power captured by the received aperture. The latter is the one that will determine the communication performance of the system. Hence, it is crucial that the pointing jitter PDF is correctly treated mathematically. Usually, the pointing jitter is given as a function of the angular deviation from the optical axis between the communicating terminals (in μrad or mrad units). To convert this into a PDF in the deviation from the optical axis in the receiver aperture plane, the change of variable $\theta \rightarrow r$ has to be performed, where θ is the angular deviation from the optical axis in the transmitter and r is the radial deviation between the optical axes in the receiver aperture plane. Using probability theory, the change in variables $r = g(\theta)$ from a scalar to scalar given by a monotonic function for the change of variables $g : \mathbb{R} \rightarrow \mathbb{R}$

$$f_R(r) = f_\Theta(g^{-1}(r)) \left| \frac{d}{dr}(g^{-1}(r)) \right|. \quad (\text{D.1})$$

In our case, $g(\theta) = r = z \tan \theta$ where z is the distance between the aperture planes. Given that the PDF of θ will in general be given a Rice distribution,

$$f_\Theta(\theta) = \frac{\theta}{2\pi\sigma^2} \exp\left(\frac{-(\theta^2 + \theta_0^2)}{2\sigma^2}\right) J_0\left(\frac{\theta \theta_0}{\sigma^2}\right), \quad (\text{D.2})$$

the probability density function (PDF) of the radial deviation in the aperture plane will be given by

$$f_R(r) = \frac{\arctan(r/z)}{2\pi\sigma^2} \exp\left(\frac{-(\arctan^2(r/z) + \theta_0^2)}{2\sigma^2}\right) J_0\left(\frac{\arctan(r/z) \theta_0}{\sigma^2}\right) \left(\frac{z}{r^2 + z^2}\right).$$

For the special case in which $\theta_0 = 0$ (the PDF of the angular pointing jitter follows a Rayleigh distribution),

$$f_R(r) = \frac{\arctan(r/z)}{2\pi\sigma^2} \exp\left(\frac{-\arctan^2(r/z)}{2\sigma^2}\right) \left(\frac{z}{r^2 + z^2}\right). \quad (\text{D.3})$$

Small angle approximations $\theta \approx \tan \theta \approx r/z \ll 1$, and hence $\arctan(r/z) \approx r/z - (r/z)^3/3 + \dots$ and $r^2 \ll z^2$. Hence to first order the previous PDF

$$f_R(r) = \frac{r}{2\pi(\sigma z)^2} \exp\left(\frac{-(r^2 + r_0^2)}{2(\sigma z)^2}\right) J_0\left(\frac{r r_0}{(\sigma z)^2}\right), \quad (\text{D.4})$$

where $r_0 = z\theta_0$ and

$$f_R(r) = \frac{r}{2\pi(\sigma z)^2} \exp\left(\frac{-r^2}{2(\sigma z)^2}\right), \quad (\text{D.5})$$

which are the Rice and Rayleigh distributions for the radial deviations. This PDF will be valid in the small angle approximation. Figs. D.1a and D.1b show the difference between the exact PDF and the approximated PDF. It is seen from these figures that the small angle approximation is valid for the orders of magnitude that usually appear in free-space optical and quantum communication ($\sim \mu\text{rad-mrad}$).

D.2. Received power statistics: Gaussian beam

The PDF of the received power in an intersatellite link under pointing errors is going to be defined by the irradiance distribution at the receiver aperture and the PDF of the radial displacement of the center of the field due to the transmitter pointing jitter.

At a given instant of time, the power captured by the receiver aperture is given by the integral of the irradiance at the receiver aperture, hence:

$$P = \int_a^b \int_0^{2\pi} I(r', \phi) r' d\phi dr' = \int \int_{R_{ap}} I(x, y) dx dy, \quad (\text{D.6})$$

where $I(r', \phi)$ is the irradiance pattern in the receiver aperture plane. a and b are the inner and outer radii of the receiver telescope aperture. Considering an azimuthally symmetric irradiance field, $I = I(r')$, the field at a given instant of time will be

$$I(r') = I_c(r' - r) \rightarrow I(x, y) = I_c(x - x_0, y - y_0), \quad (\text{D.7})$$

where I_c is the far-field distribution in the beam-centered situation and r is the radial displacement of the beam center from the optical axis of the receiving telescope. This displacement is characterized by a PDF (see App. D.1). In turn, the received power Eq. (D.6) will also be characterized by a PDF that can in general be obtained by

$$f_{\mathcal{P}}(P) = f_{XY}(g^{-1}(P)) \left| \frac{d}{dP}(g^{-1}(P)) \right|, \quad (\text{D.8})$$

where the several terms appearing in this equation can be written as:

- $f_{XY}(x_0, y_0)$ is the PDF of the transmitter pointing error, which, for the small angle approximation and Gaussian PDF in azimuthal and elevation angles, can be written as

$$f_{XY}(x_0, y_0) = \frac{1}{2\pi(\sigma z)^2} \exp\left(-\frac{(x_0 - x_b)^2 + (y_0 - y_b)^2}{2(\sigma z)^2}\right), \quad (\text{D.9})$$

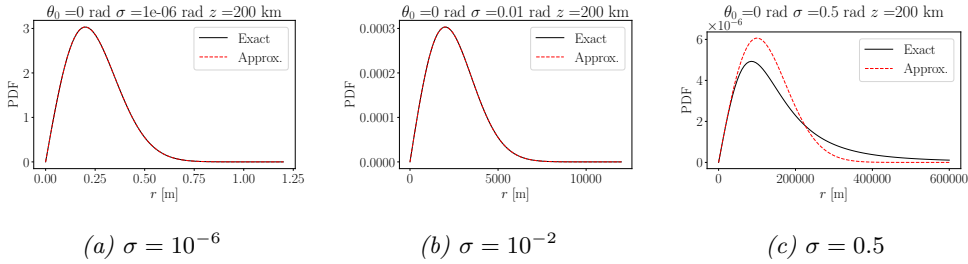


Figure D.1: Comparison between the exact expression in Eq. (D.3) and the small-angle approximation in Eq. (D.5) for the pointing jitter PDF, for different jitter standard deviations. The average errors are (a) 1.2×10^{-12} , (b) 1.2×10^{-9} , and (c) 4.5×10^{-7} .

$$f_{XY}(x_0, y_0) = \frac{1}{2\pi(\sigma z)^2} \exp\left(-\frac{x_0^2 + y_0^2}{2(\sigma z)^2}\right), \quad (\text{D.10})$$

where σ is the standard deviation of the azimuthal and elevation angles, and x_b and y_b are the boresight errors in both axes. The second expression is for zero boresight error. The previous has been obtained considering that the azimuthal and elevation angles are both independent of each other and have the same standard deviation.

- $P = g(x_0, y_0)$ is the function giving the received power as a function of the decentering of the beam. Hence combining Equations (D.6) and (D.7),

$$g(x_0, y_0) = \int \int_{R_{ap}} I_c(x - x_0, y - y_0) dx dy. \quad (\text{D.11})$$

To obtain an explicit expression of the PDF of the received power, this relation has to be inverted to obtain $g^{-1}(P)$. In general, this relation will not be invertible for most of the far-field intensity distributions. Specifically, looking at the usual function describing the PDF of the decenter coordinates in Equations (D.9) and (D.10) we will be interested in inverting the relation and obtaining an explicit form of $(x_0 - x_b)^2 + (y_0 - y_b)^2$ or $x_0^2 + y_0^2$ as a function of the received power P .

In general, any arbitrary function is obtained for $g(x_0, y_0)$ will not be invertible in the sense above stated. Hence, this demonstrates that in general, the explicit expression of the PDF of the received power is not obtainable. However, under certain approximations, some analytical approximations can be obtained in the Gaussian beam case as shown in the next subsections.

D.2.1. Approximation 1: Gaussian beam and $w \gg b$

However, we can check if there is a solution under certain simplifications and assumptions, and if these happen in practical scenarios.

1. The first assumption we are going to make is that the far-field intensity pattern can be fairly described by a Gaussian beam. This is the case for

- (a) Untruncated Gaussian beams without secondary mirror. In this case, this is strictly true in ideal conditions.
- (b) Truncated beam with secondary mirror in the far-field. The diffraction pattern due to the truncation and the obscuration will create several lobes. However, the central lobe (carrying most of the beam power) can usually be fairly well approximated to a Gaussian intensity pattern.

Under this assumption, the far-field intensity field will be given by

$$I_c(x, y) = I_0 \exp \left(\frac{-2(x^2 + y^2)}{w^2} \right), \quad (\text{D.12})$$

where I_0 is the peak intensity at the center of the beam, and w is the beamwidth.

2. The variation of the irradiance $I(x, y)$ across the receiver aperture is negligible at any instant of time. This assumption is applicable under the condition that the beamwidth of the far-field is much larger than the receiver aperture radius:

$$w \gg b, \quad (\text{D.13})$$

this will be the case for relatively large divergence angles needed under relatively large pointing errors in long-distance links. Under this condition, the integral in Eq. (D.11) can be written as:

$$g(x_0, y_0) = I_{c,eq}(x_0, y_0) A, \quad (\text{D.14})$$

where $I_{c,eq}$ is the equivalent irradiance and A is the area of the receiver aperture. The equivalent intensity can be taken as the following of two possibilities:

$$I_{c,eq}(x, y) = I_c(x, y), \quad (\text{D.15})$$

$$I_{c,eq}(x, y) = \begin{cases} I_c(x, y) & \text{for } x^2 + y^2 > b^2, \\ \frac{1}{2} (I_c(x, y) + I_c(x + b, y + b)) & \text{for } x^2 + y^2 < b^2, \end{cases} \quad (\text{D.16})$$

the first approximation is considering that the average irradiance of the far-field across the receiver aperture is given by the irradiance at the optical axis of the receiver aperture. This approach overestimates the received power when more and more as the decentering approaches zero, $x_0, y_0 \rightarrow 0$. The second approximation (Fig. (D.16)) corrects this effect by averaging the irradiance in the previous conditions.

Under these two assumptions, Eq. (D.11) can be calculated as

$$P = g(x_0, y_0) = I_0 \exp \left(\frac{-2(x_0^2 + y_0^2)}{w^2} \right) A, \quad (\text{D.17})$$

yielding to

$$P = g(x_0, y_0) =$$

$$= \begin{cases} I_0 \exp\left(\frac{-2(x_0^2 + y_0^2)}{w^2}\right) A & \text{for } x^2 + y^2 > b^2, \\ \frac{1}{2} I_0 A \left(\exp\left(\frac{-2(x_0^2 + y_0^2)}{w^2}\right) + \exp\left(\frac{-2((x_0+b)^2 + (y_0+b)^2)}{w^2}\right) \right) & \text{for } (x - x_0)^2 + (y - y_0)^2 < b^2, \end{cases} \quad (\text{D.18})$$

for the approximations given by Equations (D.15) and (D.16), respectively. The second expression can not be inverted, so it is no longer useful to obtain the received power PDF. The first of them can be inverted to obtain

$$g^{-1}(P) = x_0^2 + y_0^2 = -\frac{w^2}{2} \ln\left(\frac{P}{I_0 A}\right), \quad (\text{D.19})$$

that finally gives the explicit PDF of the received optical power by using the Eq. (D.8)

$$\begin{aligned} f_{\mathcal{P}}(P) &= f_{XY}(g^{-1}(P)) \left| \frac{d}{dP}(g^{-1}(P)) \right| \\ &= \frac{1}{2\pi(\sigma z)^2} \exp\left(\frac{1}{2(\sigma z)^2} \frac{w^2}{2} \ln\left(\frac{P}{I_0 A}\right)\right) \frac{w^2}{2P} \\ &= \frac{\beta}{\pi} (I_0 A)^{-\beta} P^{\beta-1}, \end{aligned}$$

where $\beta = \frac{w^2}{4(\sigma z)^2}$. We can define the normalized power by dividing the power by the maximum power that can be received in these conditions $h' = P/(I_0 A)$.

$$f_{\mathcal{P}_n}(h') = \frac{1}{C} \beta h'^{\beta-1},$$

where $C = \pi I_0 A$. No limitations have been included for the maximum power that can be received, hence that value of $p \in [0, \infty)$. However, a physical limitation of the system can be applied to convert this equation into a Beta distribution [151]. Under the simplification (2) $w \gg b$ and therefore considering the irradiance across the aperture to be constant, the maximum power that can be received is $P_{max} = I_0 A$. Hence, under this consideration, the normalized power h' can only range in $h' \in [0, 1]$. As the functional form of this distribution is the same as a Beta distribution with constant $\xi = 1$

$$\begin{aligned} f_{\mathcal{P}_n}(h'; \beta, \xi) &= \frac{\Gamma(\beta + \xi)}{\Gamma(\beta)\Gamma(\xi)} h'^{\beta-1} (1 - h')^{\xi-1} \xrightarrow{\xi=1} \\ f_{\mathcal{P}_n}(h'; \beta, 1) &= \frac{\Gamma(\beta + 1)}{\Gamma(\beta)} h'^{\beta-1} = \beta h'^{\beta-1}, \end{aligned}$$

and this one is normalized $\int_0^1 f_{\mathcal{P}_n}(h'; \beta, 1) dh' = 1$, the PDF of the normalized received power follows a Beta distribution

$$\begin{aligned} h' &\sim \text{Beta}(\beta, 1) \rightarrow f_{\mathcal{P}_n}(h') = \beta h'^{\beta-1}, \\ f_{\mathcal{P}_n}(h') &= \left(\frac{\pi w^2}{2A^2}\right)^{\beta-1} \beta h'^{\beta-1} \quad 0 \leq h' \leq \frac{2A^2}{\pi w^2 P_t}. \end{aligned} \quad (\text{D.20})$$

D.2.2. Approximation 2: Gaussian beam and Taylor expansion for the power

A more accurate approach is presented in [175] that considers the variation of the irradiance across the aperture of the detector:

$$f_{\mathcal{P}}(h) = \frac{\gamma^2}{A_0^{\gamma^2}} h^{\gamma^2-1} \quad 0 \leq h \leq A_0, \quad (\text{D.21})$$

$$A_0 = [\text{erf}(v)]^2, \quad v = \frac{\sqrt{\pi}a}{\sqrt{2}w_z}, \quad \gamma = \frac{w_{zeq}}{2\sigma}, \quad w_{zeq} = w_z^2 \frac{\sqrt{\pi}\text{erf}(v)}{2v \exp(-v^2)},$$

The assumptions and applicability for this equation are that (1) the beam is Gaussian, (2) the nominal mean square error is below 10^{-3} for $w_z/b = 6$ and below $1, 3 \times 10^{-2}$ for $w_z/a = 2$ and (3) no secondary mirror obscuration.

Another approach that considers the variation of the irradiance across the aperture of the detector and non-zero boresight error is presented in [119]:

$$f_{\mathcal{P}}(h) = \frac{\gamma^2 \exp\left(\frac{-s^2}{2\sigma_s^2}\right)}{A_0^{\gamma^2}} h^{\gamma^2-1} I_0 \left(\frac{s^2}{\sigma_s^2} \sqrt{\frac{-w_{zeq} \ln \frac{h}{A_0}}{2}} \right), \quad (\text{D.22})$$

$$A_0 = [\text{erf}(v)]^2, \quad v = \frac{\sqrt{\pi}a}{\sqrt{2}w_z}, \quad \gamma = \frac{w_{zeq}}{2\sigma}, \quad w_{zeq} = w_z^2 \frac{\sqrt{\pi}\text{erf}(v)}{2v \exp(-v^2)}, \quad 0 \leq h \leq A_0.$$

It can also be shown (see [154]) that although it is not invertible, the exact value of the received power for a (1) Gaussian beam, without the (2) assumption, is given by Marcum's Q^{Mar} function

$$P = P_t \left[1 - Q^{\text{Mar}} \left(\frac{2d}{w_z}, \frac{2b}{w_z} \right) \right], \quad (\text{D.23})$$

where P_t is the transmitted power and d is the radial error in the aperture plane.

D.2.3. Approximation 3: Gaussian beam and Taylor expansion for the power with receiver secondary mirror

In a similar manner, the received power statistics for a receiver with a secondary mirror can be obtained. The normalized received power $h = P/P_t$ considering a Gaussian beam with a pointing error given by r_0 is

$$h \approx \iint_{\mathcal{S}} \frac{2}{\pi w_z^2} \exp \left(-2 \frac{(x - r_0)^2 + y^2}{w_z^2} \right) dx dy, \quad (\text{D.24})$$

where $\mathcal{S}$ is the approximated aperture of the receiver shown in Fig. D.2. With the Taylor expansion shown in [175] for the approximated squared aperture, the integral approximation above can be computed as

$$h(r_0) \approx \sum_{\substack{l=0 \\ \text{even}}}^{\infty} (B_l - A_l) \left(\frac{\sqrt{2}r_0}{w_z} \right)^l, \quad (\text{D.25})$$

where

$$B_l = \frac{2E}{\sqrt{\pi}} \sum_{\substack{m=l+1 \\ \text{odd}}}^{\infty} \frac{(-1)^{(m-1)/2}}{(m) \left(\frac{m-1}{2}\right)!} \binom{m}{l} \left(\frac{\sqrt{\pi}b}{\sqrt{2}w_z}\right)^{m-l}, \quad (\text{D.26})$$

$$A_l = \frac{2E}{\sqrt{\pi}} \sum_{\substack{m=l+1 \\ \text{odd}}}^{\infty} \frac{(-1)^{(m-1)/2}}{(m) \left(\frac{m-1}{2}\right)!} \binom{m}{l} \left(\frac{\sqrt{\pi}a}{\sqrt{2}w_z}\right)^{m-l}, \quad (\text{D.27})$$

considering only the first two terms of the expansion

$$B_0 = [\text{erf}(v_b)]^2, \quad B_2 = \frac{-2}{\sqrt{\pi}} \text{erf}(v_b) [v_b \text{erf}(-v_b^2)], \quad (\text{D.28})$$

$$A_0 = [\text{erf}(v_a)]^2, \quad A_2 = \frac{-2}{\sqrt{\pi}} \text{erf}(v_a) [v_a \text{erf}(-v_a^2)], \quad (\text{D.29})$$

where

$$v_b = \frac{\sqrt{\pi}b}{\sqrt{2}w_z}, \quad v_a = \frac{\sqrt{\pi}a}{\sqrt{2}w_z}, \quad (\text{D.30})$$

considering only the first two terms of the expansion, we obtain

$$h(r_0) \approx (B_0 - A_0) + (B_2 - A_2) \left(\frac{\sqrt{2}r_0}{w_z}\right)^2, \quad (\text{D.31})$$

that equating to a Gaussian function

$$C \exp\left(-\frac{2r_0^2}{w_{zeq}^2}\right), \quad (\text{D.32})$$

gives

$$(B_0 - A_0) + (B_2 - A_2) \left(\frac{\sqrt{2}r_0}{w_z}\right)^2 = C \exp\left(-\frac{2r_0^2}{w_{zeq}^2}\right). \quad (\text{D.33})$$

Expanding the Gaussian function in a Taylor series to the first two terms gives

$$(B_0 - A_0) + (B_2 - A_2) \left(\frac{2r_0^2}{w_z^2}\right) = C - C \left(\frac{2r_0^2}{w_{zeq}^2}\right), \quad (\text{D.34})$$

$$C = B_0 - A_0, \quad (\text{D.35})$$

$$w_{zeq}^2 = \left| \frac{A_0 - B_0}{B_2 - A_2} \right| w_z^2. \quad (\text{D.36})$$

Hence, the PDF of the received power for a Gaussian beam with a secondary mirror receiver under Rayleigh pointing errors is

$$f_{\mathcal{P}}(h) = \frac{\gamma^2}{C\gamma^2} h^{\gamma^2-1}, \quad (\text{D.37})$$

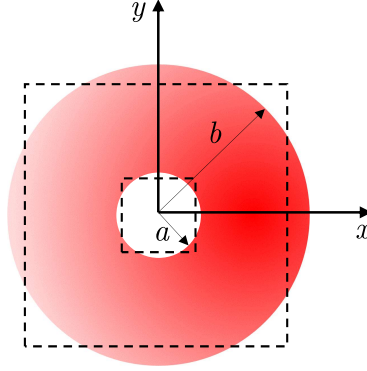


Figure D.2: Approximation of the received power to squared aperture and obscuration

As we did for the case without a secondary mirror, when there is a boresight error, the PDF will be

$$f_{\mathcal{P}}(h) = \frac{\gamma^2 \exp\left(\frac{-s^2}{2\sigma_s^2}\right)}{C\gamma^2} h \gamma^{2-1} I_0 \left(\frac{s^2}{\sigma_s^2} \sqrt{\frac{-w_{zeq} \ln \frac{h}{C}}{2}} \right). \quad (\text{D.38})$$

D.3. Validation of the code

The analytical approximations presented above, are valid under certain approximations that are not always applicable. Indeed, the far-field intensity distributions generated by combining Laguerre-Gaussian beams as proposed in this paper will not have a closed expression in general for the received power PDF. Therefore, it is necessary to numerically compute the received power PDF, from which the ABEP and outage probabilities can be obtained.

Given a PDF of the pointing jitter in the far-field, and by obtaining via the convolution theorem presented in §6-§9 the received power as a function of the decenter of the beam (the Fourier transforms are computed through 2D-fast Fourier transforms), the PDF of the received power can be computed in two ways numerically:

- Monte Carlo simulations
- Numerical derivation and evaluation of

$$f_{\mathcal{P}}(P) = \sum_{k=1}^{n(P)} \left| \frac{d}{dP} g_k^{-1}(P) \right| \cdot f_{\mathcal{R}}(g_k^{-1}(P)). \quad (\text{D.39})$$

Both models have been implemented in the code to compare them and the results are shown in Figs. D.3a-D.3c. For the results presented in §9, the numerical derivation

and evaluation has been performed as the Monte Carlo simulations became very computationally costly as the probabilities that want to be captured get smaller (i.e. to capture probabilities of outage on the order of $\sim 10^{-8}$ at least $\sim 10^9$ simulations have to be performed for each of the points of interest).

In Figs. D.3a the validity of the analytical approximation can be seen. As expected given the assumptions made to obtain this approximation, as the beamwidth in the far-field gets smaller the effect of integrating in an equivalent squared area and expanding in a Taylor series will be larger and larger. Indeed, it can be seen in this figure that for a beamwidth that is approximately 1000 times the aperture of the receiver, $w_z/\sigma \approx 1$ the error is already quite significant.

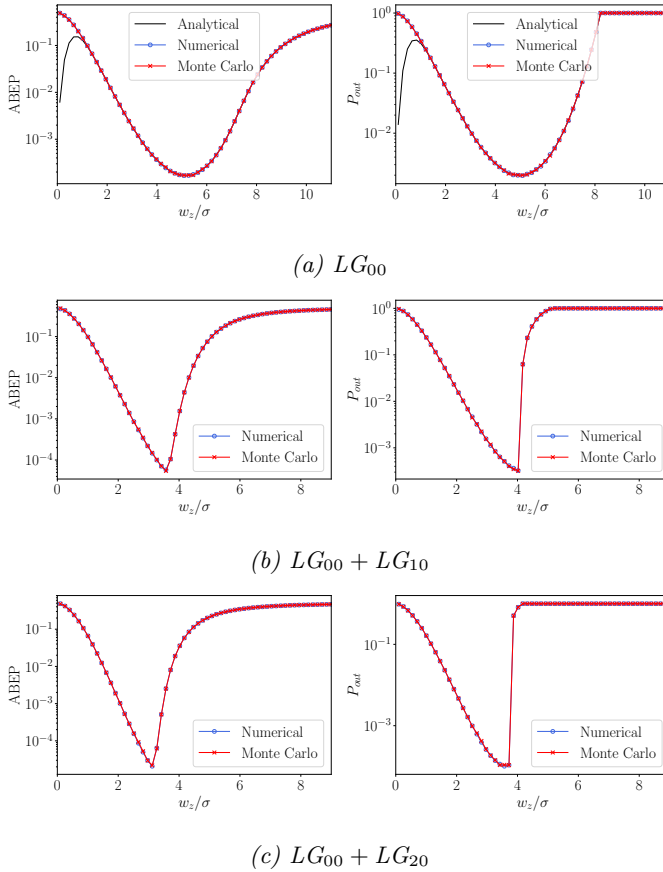


Figure D.3: Comparison of the results obtained with different calculation methods for three beam configurations: (a) fundamental Gaussian beam LG_{00} , (b) combination of LG_{00} and LG_{10} , and (c) combination of LG_{00} and LG_{20} . In all cases, $P_t = 23$ dBm, $b = 0.1$ m, $\sigma = 100$ m, and $a_0 = 0.25$ for the combined beams. The analytical curves correspond to Eq. (D.21).

Appendix E

Appendix for Chapter 10

This appendix provides supplemental information complementary to §10. It begins by presenting the objective and performance of the Gaussian beam cleaning stage. Next, the mathematical model used to generate the simulated beam shapes is described, followed by a discussion of the retrieval of the Gaussian beam at the fiber collimator aperture. The development of the spiral phase plate (SPP) generated beam and its comparison with Laguerre–Gaussian beams are presented in E.4. E.5 introduces the sensitivity analysis of the beam shape with respect to lateral and angular misalignments of the SPP. Finally, an extended set of results is provided, including the measured and simulated beam shapes and the expected average bit error probability.

E.1. Gaussian beam cleaning stage

In order to get closer to the ideal beam shapes proposed theoretically in §9, it is of utmost importance to have a clean Gaussian beam at the input of the system. “Clean” here means a Gaussian beam with as few wavefront aberrations as possible. The 532 nm diode laser source (1 mW power) that was used had quite significant aberrations, which substantially affected the beam shape obtained with the experimental setup. To fix this issue, a beam-cleaning stage was set up using a single-mode fiber (SMF) filtering stage. Although a pinhole solution was also considered for cleaning the wavefront errors of the source laser, a simpler solution is provided by the fiber-filtering stage in this case. Furthermore, in many satellite free-space optical communication transmitter terminals, optical fiber amplifiers and fiber-coupled modulators are used [108, 486, 487]. Hence, this beam-cleaning stage would be provided intrinsically by those systems in a terminal using the proposed beam-shaping optical setup. Therefore, although in the breadboard setup the beam-cleaning stage yields fiber-coupling losses, these losses would be avoided in a fiber-coupled transmitter modulator of a satellite free-space optical communication terminal (and will therefore not be analyzed in this manuscript).

Fig. E.1 shows the shapes of the Gaussian and SPP-generated ($\ell = 2$) beams obtained with and without the fiber beam-cleaning stage. It can be seen that the

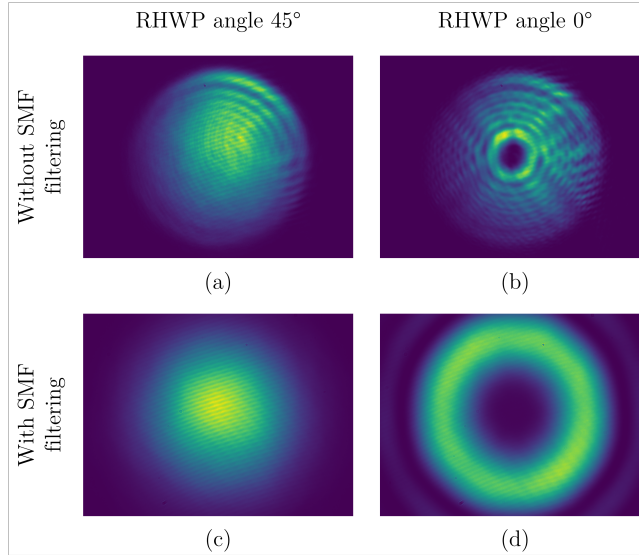


Figure E.1: Experimentally measured beams without and with SMF filtering stage (a) beam outgoing the laser source, (b) beamshaping of the beam in (a) with an $\ell = 2$ SPP, (c) Gaussian beam outgoing the fiber collimator after the SMF filtering stage and (d) beamshaping of the beam in (c) with an $\ell = 2$ SPP. The size differences between the different rows are due to propagation distance and the fiber collimator lens.

aberrations (with a high spherical-aberration component) have a very large impact on the shape of these beams. First, the beam coming from the source (expected to be Gaussian) has several rings around it and is quite radially asymmetric. Although the singularity of the SPP still yields a null intensity at the center, the beam shape obtained for the non-SMF-filtered beam is far from that of a Laguerre–Gaussian beam.

In summary, Fig. E.1 shows the necessary and critical improvement provided by the Gaussian beam-cleaning stage in the proposed optical setup. This stage will allow one to obtain far-field Gaussian and higher-order annular beam superpositions with the desired accuracy (compared to the analytical ideal shapes).

E.2. Model: Fourier optical propagation and Jones matrices

To validate the experimental results, a Fourier optics based simulation tool was developed. The propagation tool uses the Fresnel and Fraunhofer approximations [8, 364] to obtain the field amplitude and phase at any location in the experimental setup. The model not only includes the scalar field numerical calculations given by Fourier optics, but also incorporates the Jones formalism to account for potential polarization phenomena occurring in the system.

The model starts with a given Gaussian beam at the aperture of the fiber collimator (output of the Gaussian beam-cleaning stage). The light is considered to be polarized, and two complex scalar matrices containing the x - and y -polarized fields are propa-

gated through the system. These matrices are affected element-wise by the rotatable half-wave plate (RHWP), implemented through the corresponding Jones matrix [139]. Then, again element-wise, the polarization beam splitter (PBS) is implemented in the model, accounting for the leakage and crosstalk that were previously calibrated.

At this stage, after the PBS, the complex matrices corresponding to the x and y polarization states describe the system for each of the paths (Gaussian and SPP path). The mirrors are also modeled, accounting for polarization effects due to reflection. The effect of propagation of the field from the collimator to the SPP is implemented by propagating each of these matrices with Fourier optics from the collimator aperture to the aperture of the SPP. At this point, the complex matrices corresponding to the Laguerre–Gaussian path are multiplied by a $e^{i\ell\varphi}$ phase factor accounting for the helical wavefront induced by the SPP with topological order ℓ .

Before propagating to the plane at which the camera is located, the effects of the polarization beam combiner (PBC) are included element-wise in a similar manner to how this was done for the PBS. Finally, the four matrices (two polarizations for the two paths) are combined into two complex matrices (two polarizations), and each of these is propagated to the far-field plane with Fresnel or Fraunhofer propagation, depending on the distance of interest.

E.3. Retrieval of Gaussian beam profile after the collimator

Using the propagation tool described above, the experimental results can be compared with the numerical predictions provided by Fourier propagation and Jones polarization models. From the datasheet of the fiber collimator, the beam waist w_0 of the Gaussian beam at the aperture of the collimator was obtained. This beam waist was then used as an input parameter in the developed model, and the resulting simulations were compared with the experimental images.

A mismatch between the simulated and experimental results is evident in Fig. E.2. Although the overall beam shapes agree well, the simulation does not correctly reproduce the size of the beams observed experimentally.

By examining several experiments and running additional simulations, it was found that the source of this mismatch was an incorrect assumption in the model. The hypothesis was that the Gaussian beam exiting the collimator has its beam waist at the aperture of the collimator –hence, that the wavefront is planar at the aperture; see Fig. E.3(a). However, it was found that this is not the case, and that the wavefront at the aperture of the collimator is instead slightly diverging (see Fig. E.3(b)). By performing a grid search over the beam waist w_0 and the radius of curvature of the wavefront R_{col} at the aperture of the collimator, it was found that the following values are consistent with the experimental results:

$$w_0 = 1.19 \text{ mm}, \quad R_{\text{col}} = 12 \text{ m} \quad \text{at the aperture of the collimator.} \quad (\text{E.1})$$

These values were obtained by simulating the far-field irradiance (at the camera plane) for different values of the beam waist and the radius of curvature of the Gaussian beam at the aperture of the collimator (see Fig. E.4). The results of these simulations

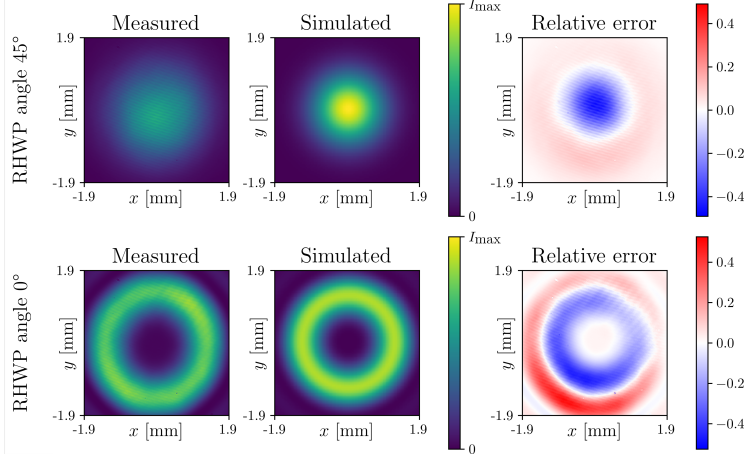


Figure E.2: Comparison between the experimental and simulated beams before the Gaussian beam retrieval. All the fields are normalized to unit power and the relative error is the normalized subtraction of the experimental and simulated beams.

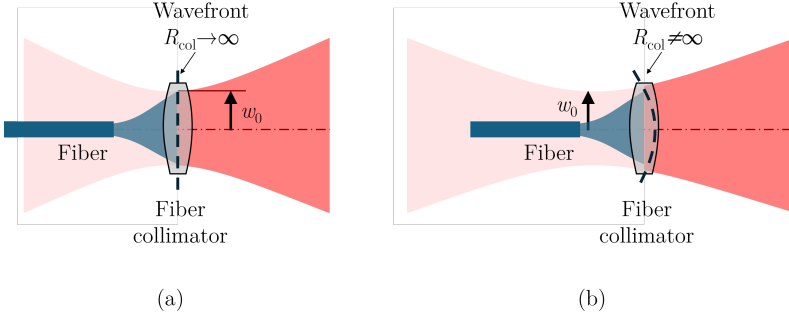


Figure E.3: Illustration of the Gaussian beam exiting the fiber collimator (a) Gaussian beam waist at the aperture of the collimator and (b) Gaussian beam slightly diverging at the aperture of the collimator.

were then compared with the experimentally obtained images to obtain the $R^2 = 1 - \iint |I_{\text{sim}} - I_{\text{exp}}|^2 dx dy$ values.

The simulations were performed in two extreme cases: when the beam is purely Gaussian (RHWP angle at 45°) and when it is purely SPP-generated with topological order $\ell = 2$ (RHWP angle at 0°). It can be seen in the figures that the maximum R^2 is obtained for the values corresponding to those given above, for both the Gaussian and SPP-generated cases. Similar results were obtained for the SPP-generated $\ell = 1$ beam shapes.

After including this initial divergence of the Gaussian beam exiting the fiber collimator ((E.1)), very good agreement between the experimental and simulated irradiance fields is obtained for all combinations of the Gaussian and SPP-generated beams,

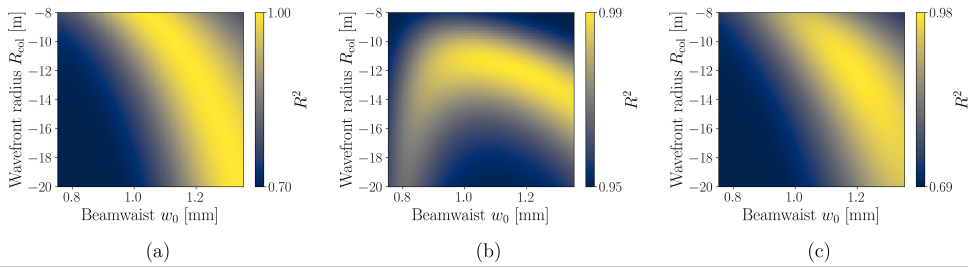


Figure E.4: R^2 values of the (a) Gaussian and (b) SPP-generated beam shapes as a function of the beamwaist w_0 and the radius of curvature R of the Gaussian beam at the aperture of the collimator. (c) is the product of (a) and (b).

as shown in Fig. E.5. In this way, the retrieval of the Gaussian beam parameters at the aperture of the collimator was successfully achieved.

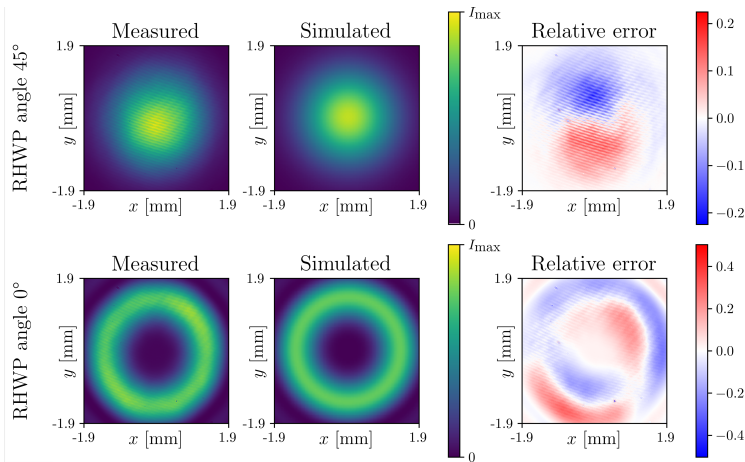


Figure E.5: Comparison between the experimental and simulated beams after the Gaussian beam retrieval with the values obtained in Eq. (E.1). All the fields are normalized to unit power and the relative error is the normalized subtraction of the experimental and simulated beams.

E.4. Development of the annular beam and far-field equation

A SPP imparts a helicoidal wavefront to a beam propagating through it. It is often assumed [389, 525] that when a Gaussian beam passes through an SPP of topological order ℓ (and radial order p), the resulting field in the far-field ($z \rightarrow \infty$) is a Laguerre–Gaussian (LG) beam with topological order ℓ and radial order $p = 0$. This statement, however, is not strictly correct: a Gaussian beam incident on an SPP does not generate an exact LG mode.

As discussed in Refs. [474, 488], SPPs generate beams carrying orbital angular

momentum (OAM) with topological order ℓ . In this sense, since these beams carry OAM and exhibit an annular intensity profile, they closely resemble LG beams with topological order ℓ . Moreover, their decomposition in the LG modal basis yields a dominant contribution from the corresponding $\text{LG}_{\ell,0}$ mode. Nevertheless, the far-field profile of an SPP-generated beam is not exactly that of a Laguerre–Gaussian beam of order ℓ .

In this section, we first present the field resulting from a Gaussian beam passing through an SPP and analyze its propagation. We then provide an analytical derivation of the corresponding far-field distribution and compare it with that of ideal Laguerre–Gaussian beams.

E.4.1. Development of the annular beam

To study the formation of the annular beam, the field along the SPP arm of the setup (with the Gaussian reference path blocked) was measured and numerically simulated at several distances from the SPP. Fig. E.6 shows the evolution of the beam profile at experimentally accessible propagation distances. Although the phase singularity introduced by the SPP creates an intensity null at the beam center—giving rise to an annular structure—the ring is not fully developed immediately after the SPP. Instead, the annular profile becomes more pronounced as the propagation distance increases.

E.4.2. Far-field equation derivation

As discussed above, the beam generated by an SPP is not an exact LG mode. Moreover, even if one were to approximate it as such, its transverse size would still differ from that of an LG beam with the same topological order ℓ and beam waist w_0 as the incident Gaussian beam. In particular, the SPP-generated beam exhibits a larger beam width than an ideal $\text{LG}_{\ell,0}$ mode under the same initial conditions.

Fig. E.7 illustrates these differences in beam shape and size. In Fig. E.7(a), the free-space propagation of a Gaussian beam is shown. If a Gaussian beam incident on an SPP were to generate an ideal Laguerre–Gaussian beam, its subsequent propagation would be as depicted in Fig. E.7(b). In this hypothetical case, both the Gaussian beam in (a) and the LG beam in (b) would share the same beam width $w(z)$ at a distance z from the beam-waist (or SPP) plane.

In reality, however, the situation corresponds to Fig. E.7(c): the beam produced by the SPP is neither an exact LG mode nor does it preserve the same beam width as in the idealized case. Instead, the SPP-generated beam expands more rapidly, resulting in a larger transverse size than that of the corresponding LG beam shown in Fig. E.7(b).

To prove the previous, we will compare the analytical expression of the LG beam and the analytical expression of the far-field propagation of an SPP-generated annular beam. Consider first the analytical equation for the LG beam of topological order ℓ and radial order p

$$u_{\ell}^p(r, \phi, z) \propto \frac{1}{w(z)} \left(\frac{r\sqrt{2}}{w(z)} \right)^{|\ell|} \exp\left(-\frac{r^2}{w^2(z)}\right) \mathcal{L}_p^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right) \quad (\text{E.2})$$

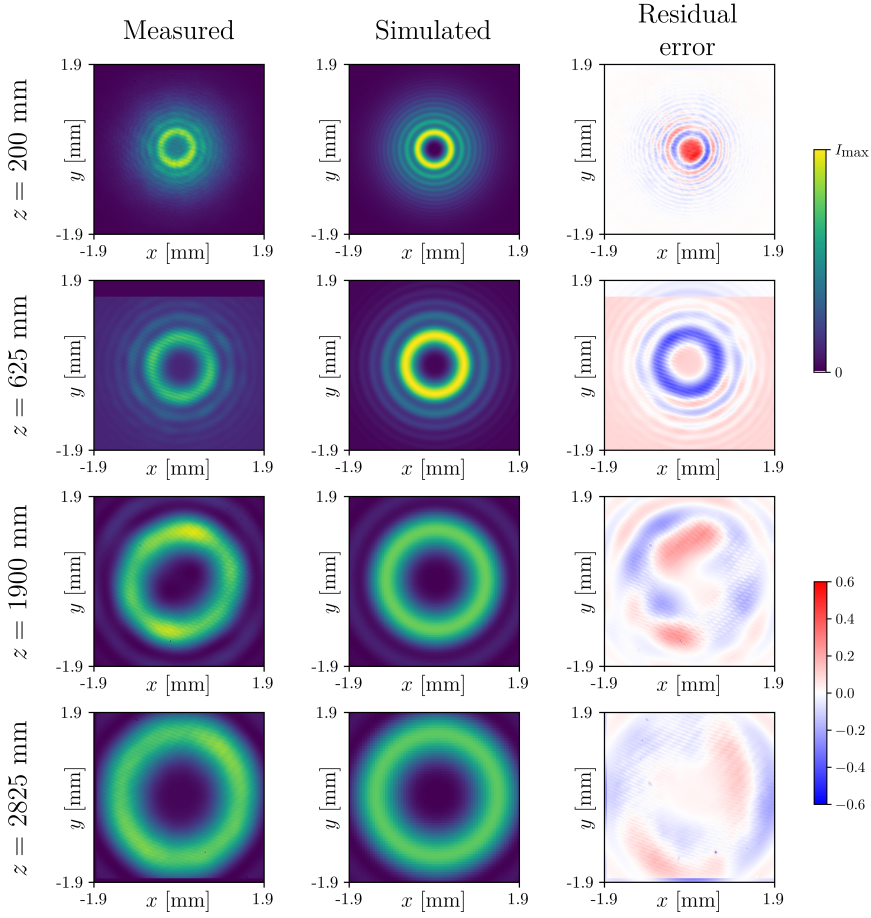


Figure E.6: Development of the SPP-generated annular beam with $\ell = 2$ at different distances z from the SPP.

$$\times \exp\left(-ik \frac{r^2}{2R(z)}\right) \exp(-i\ell\phi) \exp(i\psi(z)), \quad (\text{E.3})$$

where $\mathcal{L}_p^{|\ell|}$ are the generalized Laguerre polynomials and

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2} \quad \text{Beamwidth,} \quad (\text{E.4})$$

$$R(z) = z \left[1 + \left(\frac{z_R}{z}\right)^2 \right] \quad \text{Wavefront radius of curvature,} \quad (\text{E.5})$$

$$\psi(z) = (|\ell| + 2p + 1) \arctan\left(\frac{z}{z_R}\right) \quad \text{Gouy phase.} \quad (\text{E.6})$$

In the idealized case of a Laguerre–Gaussian beam, the far-field beam width $w(z)$

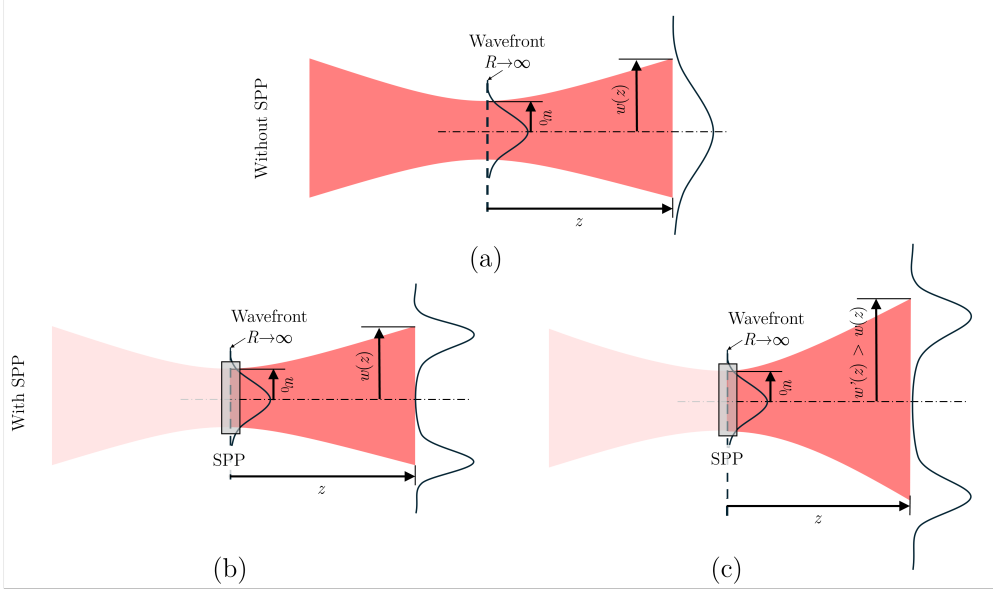


Figure E.7: Illustration of the SPP-generated beam propagation, (a) Gaussian beam through vacuum, (b) illustration of the LG beam that could be expected to be generated by an SPP, and (c) illustration of the SPP-generated beam.

at a distance z is given by Eq. (E.4), as illustrated in Fig. E.7(b). To demonstrate that the beam generated by an SPP is neither an exact Laguerre–Gaussian mode nor characterized by this same beam width $w(z)$, we analytically evaluate the far-field propagation of a Gaussian beam impinging on an SPP placed at the beam-waist plane.

Specifically, we perform a Fraunhofer (far-field) propagation of a Gaussian field modified by the phase profile of the SPP. In polar coordinates, the Fraunhofer diffraction integral for a field $U(\rho', \theta')$ in the source plane is given by

$$U_{\text{FF}}(\rho, \theta, z) = \frac{e^{ikz} e^{ik \frac{\rho^2}{2z}}}{i\lambda z} \int_0^{2\pi} d\theta' \int_0^\infty \rho' d\rho' U(\rho', \theta') e^{-i \frac{2\pi}{\lambda z} \rho \rho' \cos(\theta - \theta')}. \quad (\text{E.7})$$

In the case of interest, the field $U(\rho', \theta')$ is given by a Gaussian beam in its beamwaist impinged by a helicoidal phase of topological order ℓ (the phase induced by the SPP)

$$U(\rho', \theta') \propto e^{-\rho'^2/w_0^2} e^{i\ell\theta'}. \quad (\text{E.8})$$

Combining both, we have

$$\begin{aligned} U_{\text{FF}}(\rho, \theta, z) &\propto \frac{e^{ikz} e^{ik \frac{\rho^2}{2z}}}{i\lambda z} \int_0^{2\pi} d\theta' \int_0^\infty \rho' d\rho' C e^{-\rho'^2/w_0^2} e^{i\ell\theta'} e^{-i \frac{2\pi}{\lambda z} \rho \rho' \cos(\theta - \theta')} \\ &\propto \frac{e^{ikz} e^{ik \frac{\rho^2}{2z}}}{i\lambda z} \int_0^\infty \rho' d\rho' C e^{-\rho'^2/w_0^2} \int_0^{2\pi} d\theta' e^{i\ell\theta'} e^{-i \frac{2\pi}{\lambda z} \rho \rho' \cos(\theta - \theta')}. \end{aligned} \quad (\text{E.9})$$

Using the Jacobi-Anger expansion,

$$e^{iz \cos \theta} \equiv \sum_{n=-\infty}^{\infty} i^n \mathcal{J}_n(z) e^{in\theta},$$

where $\mathcal{J}_n(z)$ is the Bessel function of the first kind of order n , we have that the integral over θ' in Eq. (E.9) can be solved as

$$\begin{aligned} \int_0^{2\pi} d\theta' e^{i\ell\theta'} e^{-i\frac{2\pi}{\lambda z} \rho \rho' \cos(\theta-\theta')} &= \int_0^{2\pi} d\theta' e^{i\ell\theta'} \left(\sum_{n=-\infty}^{\infty} i^n \mathcal{J}_n \left(-\frac{2\pi \rho \rho'}{\lambda z} \right) e^{in(\theta-\theta')} \right) \\ &= \sum_{n=-\infty}^{\infty} \left[i^n e^{in\theta} \mathcal{J}_n \left(-\frac{2\pi \rho \rho'}{\lambda z} \right) \int_0^{2\pi} d\theta' e^{i(\ell-n)\theta'} \right] \\ &= \sum_{n=-\infty}^{\infty} \left[i^n e^{in\theta} \mathcal{J}_n \left(-\frac{2\pi \rho \rho'}{\lambda z} \right) 2\pi \delta_{\ell,n} \right] \\ &= 2\pi i^\ell e^{i\ell\theta} \mathcal{J}_\ell \left(-\frac{2\pi \rho \rho'}{\lambda z} \right), \end{aligned}$$

Introducing this result in Eq. (E.9), we have that

$$\begin{aligned} U_{\text{FF}}(\rho, \theta, z) &\propto 2\pi i^\ell e^{i\ell\theta} \frac{e^{ikz} e^{ik\frac{\rho^2}{2z}}}{i\lambda z} \int_0^\infty \rho' d\rho' e^{-\rho'^2/w_0^2} \mathcal{J}_\ell \left(-\frac{2\pi \rho \rho'}{\lambda z} \right) \\ &\propto 2\pi i^\ell e^{i\ell\theta} \frac{e^{ikz} e^{ik\frac{\rho^2}{2z}}}{i\lambda z} \frac{\pi^{3/2} \rho w_0^3}{4\lambda z} e^{-\frac{\pi^2 \rho^2 w_0^2}{2\lambda^2 z^2}} \\ &\quad \times \left(\mathcal{I}_{\frac{\ell-1}{2}} \left(\frac{\pi^2 \rho^2 w_0^2}{2z^2 \lambda^2} \right) - \mathcal{I}_{\frac{\ell+1}{2}} \left(\frac{\pi^2 \rho^2 w_0^2}{2z^2 \lambda^2} \right) \right) \\ &\propto (-i)^\ell e^{i\ell\theta} e^{ikz} e^{ik\frac{\rho^2}{2z}} e^{-\frac{\pi^2 \rho^2 w_0^2}{2\lambda^2 z^2}} \\ &\quad \times \underbrace{\frac{\rho}{2i\lambda^2 z^2} \left[\mathcal{I}_{\frac{\ell-1}{2}} \left(\frac{\pi^2 \rho^2 w_0^2}{2\lambda^2 z^2} \right) - \mathcal{I}_{\frac{\ell+1}{2}} \left(\frac{\pi^2 \rho^2 w_0^2}{2\lambda^2 z^2} \right) \right]}_{\text{}}, \quad (\text{E.10}) \end{aligned}$$

where $\mathcal{I}_n(x)$ is the modified Bessel function of the first kind of order n . This analytical propagation expression does not share the same functional form in polar coordinates as the LG beams given in Eq. (E.2). It can be shown that, in the limit $z \rightarrow \infty$, the underbraced factor in Eq. (E.10) scales as ρ^ℓ , which suggests an apparent correspondence between the SPP-generated beam and a Laguerre–Gaussian beam. However, approximating only this factor is not mathematically rigorous, since the accompanying exponential term $\exp(-\pi^2 \rho^2 w_0^2 / 2\lambda^2 z^2)$ must be treated on the same footing.

This analysis, therefore, demonstrates that a Gaussian beam incident on an SPP does not result in a LG beam in the far-field. It is also important to note that the observed discrepancy cannot be attributed to a simple rescaling of the radial coordinate. This can be understood from the fact that there is no known relation

between the underbraced factor in Eq. (E.10) and the $r^{|\ell|}$ factor appearing in Eq. (E.2). To illustrate this discrepancy, a direct comparison between the SPP-generated beam and the corresponding LG beam is shown in Fig. E.8.

To generate an exact LG beam using a SPP, an additional transmission screen would need to be placed at the aperture plane of the SPP. This transmission screen would require a position-dependent transmission function matching the desired LG mode. In this case, the field at the combined aperture of the SPP and transmission screen would be

$$U(\rho', \theta') \propto e^{-\rho'^2/w_0^2} e^{i\ell\theta'} \rho'^{|\ell|}, \quad (\text{E.11})$$

for a radial index $p = 0$. In this expression, the first term represents the incident Gaussian beam, the second term corresponds to the helical phase imposed by the SPP, and the third term describes the transmission screen necessary to produce an exact LG mode of order $p = 0$ and topological order ℓ . For our beam-shaping application, however, the use of such a transmission screen would introduce additional power losses, which would be detrimental to the overall performance of the system.

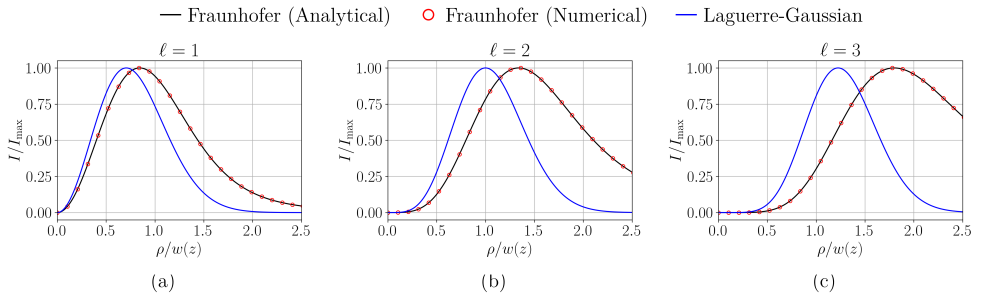


Figure E.8: Comparison of the SPP-generated far-field beam obtained through Fraunhofer propagation in Eq. (E.10) and the LG beam in Eq. (E.2) for $\ell = 1, 2, 3$ and $p = 0$. The results obtained for the Fraunhofer propagation through numerical simulations using the fast Fourier transform are also shown for completeness.

E.5. Spiral phase plate alignment sensitivity analyses

For a potential future implementation of the proposed beam-shaping technique in a satellite optical terminal, it is of utmost importance to assess how sensitive the resulting beam profile is to misalignment between the SPP and the incident Gaussian beam. In this section, we investigate two types of misalignment: lateral and angular.

A lateral misalignment corresponds to a transverse displacement of the SPP center with respect to the center of the incident Gaussian beam. An angular misalignment, on the other hand, refers to a tip-tilt of the SPP relative to the propagation direction of the incoming Gaussian beam.

E.5.1. Lateral misalignment of the SPP

First, we analyze the sensitivity of the beam shape to a lateral displacement of the SPP with respect to the incident Gaussian beam. Fig. E.9 shows the simulated and measured results for a lateral offset of $200\ \mu\text{m}$. In addition to the very good agreement between simulation and experiment, the impact of the misalignment is clearly visible. The lateral displacement breaks the axial symmetry of the aligned beam and causes the optical power to accumulate predominantly in a single lobe.

This effect must be avoided, as it would be highly detrimental to communication performance. In particular, pointing jitter in a free-space optical link is typically axially symmetric, and concentrating the power into a single lobe directs it toward a region of lower capture probability in the far-field.

Fig. E.10 shows box plots of the relative errors for different values of lateral misalignment. For an incident Gaussian beam width of $w(z) \approx 1.2\ \text{mm}$, the beam shape is seen to be sensitive to displacements at the micrometer level. This implies that lateral alignment tolerances below this order of magnitude are required. The exact tolerance will depend on both the size of the incident Gaussian beam and the topological order of the SPP.

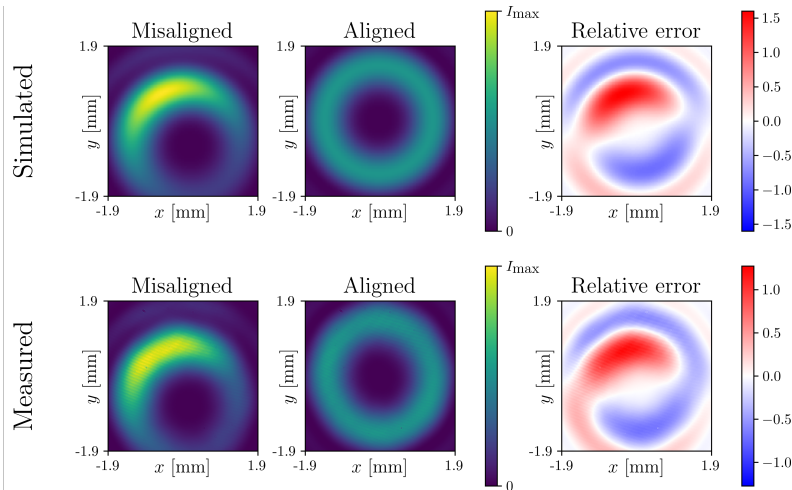


Figure E.9: Effect of lateral misalignment of the SPP. The figure shows the simulated and experimental results for a $200\ \mu\text{m}$ displacement.

E.5.2. Angular misalignment of the SPP

Similar to the lateral case, a sensitivity analysis of the beam shape with respect to angular misalignment of the SPP is presented in Fig. E.12. It is evident that the alignment requirements for this degree of freedom are much more relaxed, as noticeable distortions of the beam shape only appear for angular displacements on the order of $\sim 20^\circ$ (see Fig. E.11). Although such large tilt angles could not be imposed with high accuracy in the experimental setup, good agreement was nevertheless found between

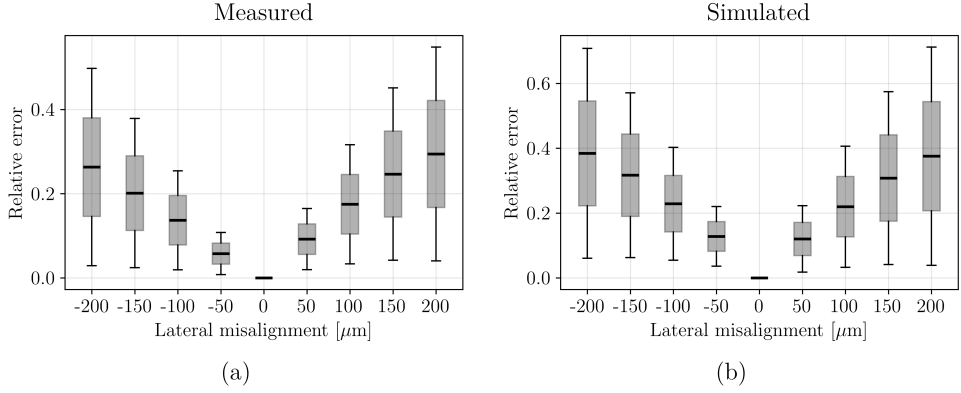


Figure E.10: Relative error bars obtained for different lateral displacement values. The error bars are obtained by evaluating the statistics of the relative errors shown in Fig. E.9 for each of the considered misalignments.

measurements and simulations at small misalignment angles.

Overall, these results demonstrate that the SPP is highly resilient to angular misalignments. Consequently, significantly greater attention must be devoted to controlling lateral alignment in optical setups of this type, as it represents the dominant source of beam shape degradation.

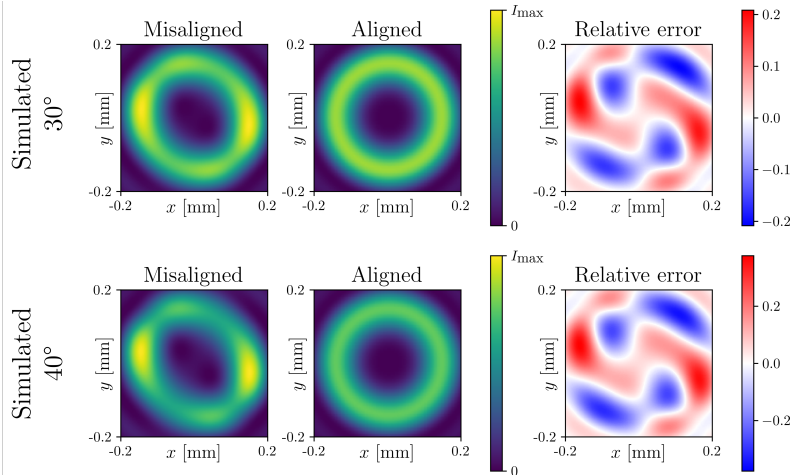


Figure E.11: Simulated results of the SPP-generated beam under angular misalignments of 30° and 40°.

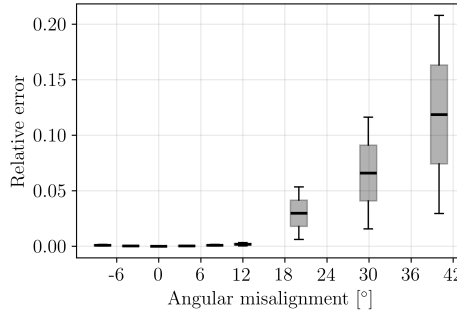


Figure E.12: Simulated results of the relative error bars as a function of the angular misalignment angle.

E.6. Extended results

E.6.1. Beamshapes

In this section, we present an extended set of figures analogous to those shown in Fig. 2 of §10. Figs. E.13–E.15 compare the measured and simulated beam profiles at the camera plane for different topological orders of the SPP and for different RHPW angles.

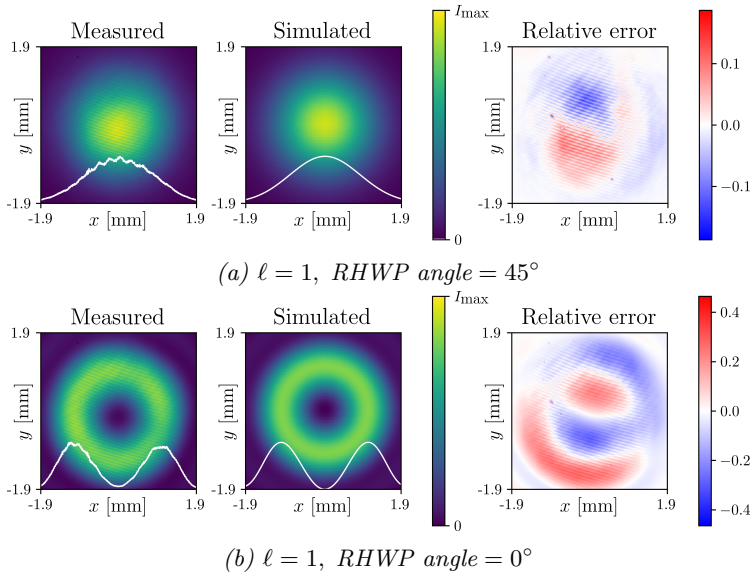


Figure E.13: Comparison of measured and simulated results for SPP $\ell = 1$ at the camera plane.

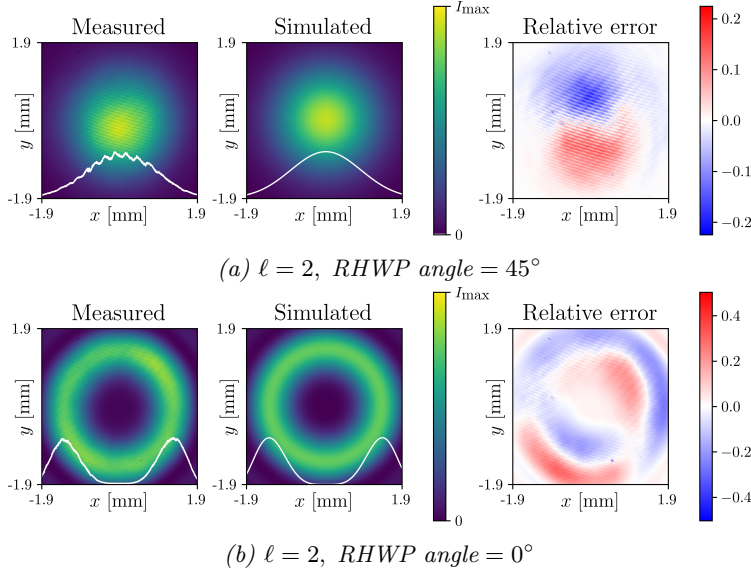


Figure E.14: Comparison of measured and simulated results for SPP $\ell = 2$ at the camera plane.

E.6.2. Intersatellite optical communication

In this section, we present the results for the average bit error probability (ABEP) of the generated beams, following an approach analogous to that used for the outage probability analysis in §10. We first assess the communication performance of the system by comparing the results obtained using the beam shapes at the camera plane, both from simulations and from experiments (for this purpose, the mathematical model presented in §9.2 is used). These beam profiles are not yet fully developed and are partially clipped due to the finite size of the camera sensor. Although these conditions do not correspond to the expected far-field performance at the receiver plane, the close agreement between experimental and simulated results demonstrates that the numerical model accurately reproduces the measured beam shapes. This, in turn, justifies extrapolating the simulated beams to the far-field in order to predict the performance at the receiver aperture.

Fig. E.16 reports the optimal ABEP values obtained with these camera-plane beam shapes, allowing both the beam size and the relative contributions of the Gaussian and SPP-generated annular components to vary (through different RHPW angles). The same system parameters as in Tab. 9.1 are used, with the pointing jitter set to 100 m. From the results presented in Fig. E.16, it can be seen that the performances derived from the experimental and simulated beam shapes are in good agreement. The data in each row refers to a superposition of the Gaussian with an SPP-generated beam of ℓ topological order, e.g., $\ell = \{0\}$ is only the Gaussian beam and $\ell = \{0, 1\}$ is the superposition of a Gaussian beam with a SPP-generated beam of topological order 1.

Given this strong agreement between measured and simulated beam profiles at the

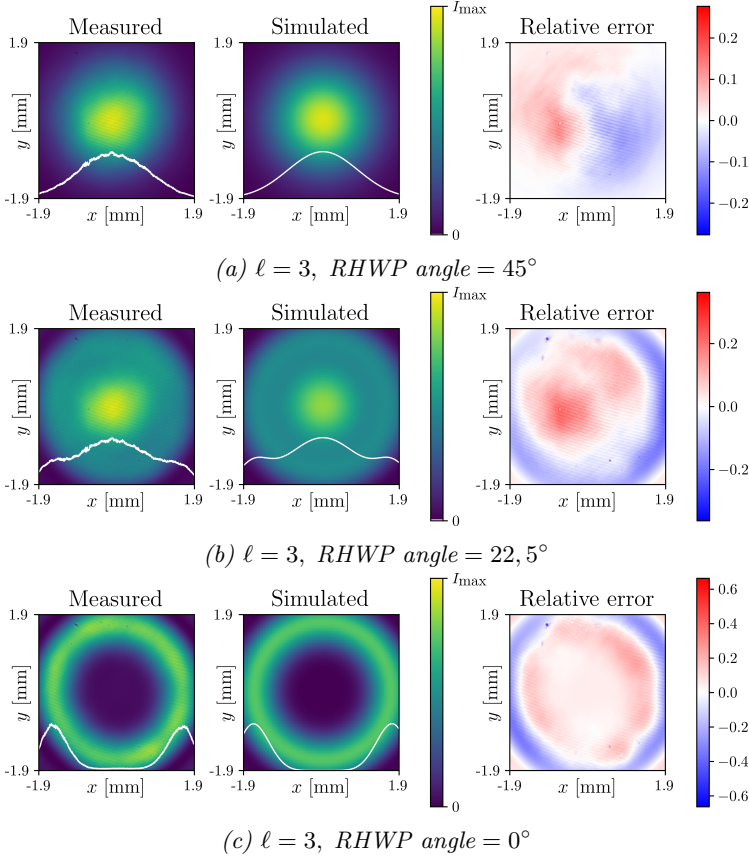


Figure E.15: Comparison of measured and simulated results for SPP $\ell = 3$ at the camera plane.

camera plane, the numerical beams can be reliably propagated to the far-field to estimate the expected real-world performance at the receiver. These far-field beam shapes are both fully developed and no longer clipped. Two approaches are considered for this propagation: a numerical Fraunhofer propagation using a fast Fourier transform and an analytical far-field expression for the SPP-generated beams (see Eq. (E.10)). In addition, we also analyze the performance obtained when using ideal Laguerre–Gaussian beams instead of the SPP-generated annular beams, as originally proposed in §9. The corresponding ABEP results for these far-field beam shapes are reported in Fig. E.17. The results presented here exhibit the same trends as those discussed for the outage probability in §10.

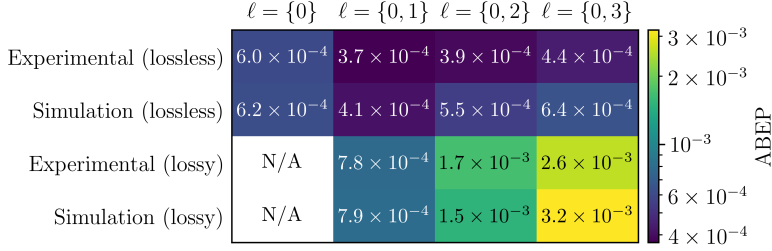


Figure E.16: Average bit error probability (ABEP) performance for the measured and simulated beam shapes at the camera plane (clipped by the camera size, and not fully developed beam shapes).

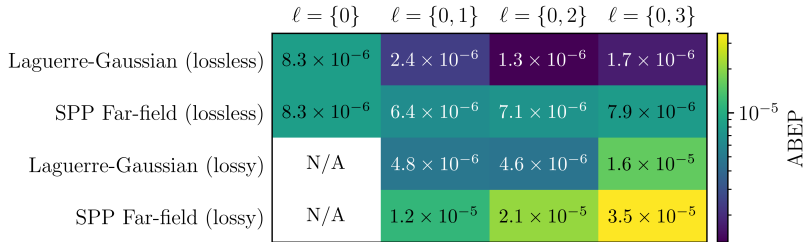


Figure E.17: Average bit error probability (ABEP) performance for LG beam shapes and far-field beam shapes obtained with an SPP.

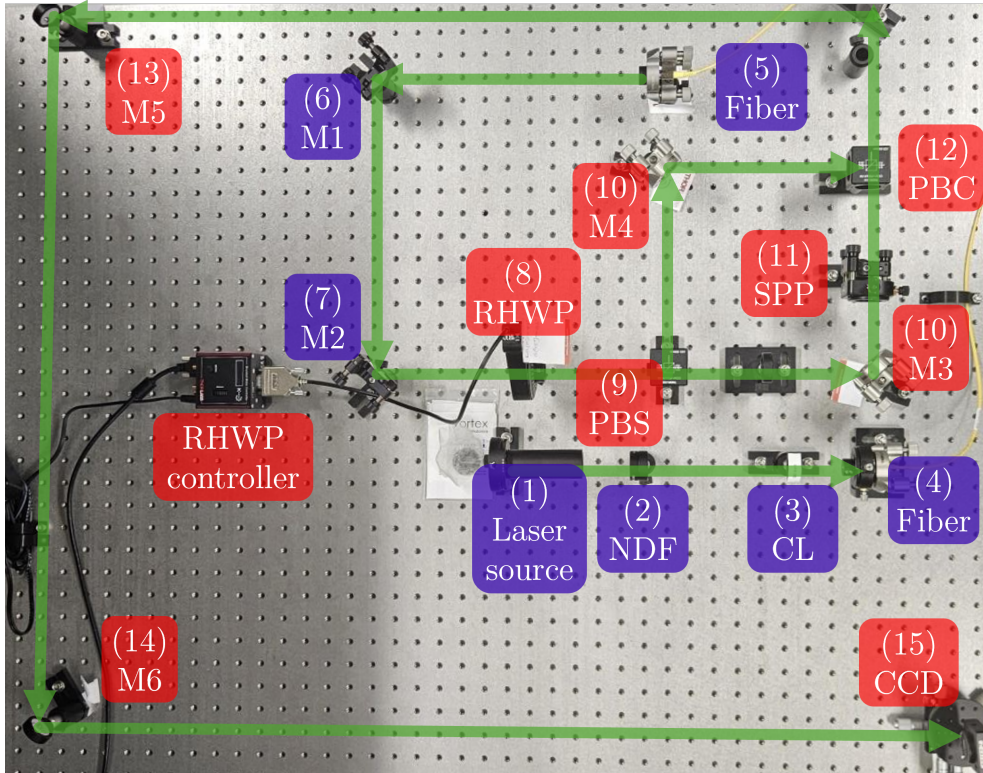


Figure E.18: Experimental setup in the laboratory. The numbered elements indicate the sequence of light propagation through the system. The blue and red colors correspond to the Gaussian beam cleaning and beam shaping stages in Fig. 10.1. Neutral density filter (NDF), coupling lens (CL), mirror (M), rotatable half-wave plate (RHWP), polarization beam splitter/combiner (PBS/PBC), and charge-coupled device (CCD).

Appendix F

Appendix for Chapter 11

This appendix provides supplemental information complementary to §11.

F.1. Variational analysis of the functional $\mathcal{S}[I(\rho)]$

In this section, the variational approach is applied to functional $\mathcal{S}[I]$ to find the optimum $I(x, y)$ far-field irradiance. It is found that implementing the subsidiary conditions and implementing the Weierstrass-Erdmann corner conditions for allowing discontinuous solutions in $I(x, y)$ does not solve the variational problem posed.

F.1.1. Formulation in $\rho(P)$

For a monotonical function $g(\rho)$ the condition Eq. (11.6) becomes

$$\mathcal{F}_P(P) = \left| \frac{d}{dP} \rho(P) \right| \cdot f_R(\rho).$$

And then assuming the small aperture approximation Eq. (11.4), we can directly solve the functional, by transforming problem to one dealing with the functional $\mathcal{S}[\rho] = \int_0^{P_{th}} \mathcal{F}_P[\rho, f_R] dP$ where $\rho(P)$ is the inverse of the function of the power collected as a function of the radial coordinate $P = g(\rho)$. Considering that we have a Rayleigh distribution for the pointing jitter radially $f_R(\rho)$,

$$\mathcal{F}_P = -\rho' \cdot \frac{\rho}{2\pi\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right),$$

given that as $dg/d\rho < 0$, then $\rho' = d\rho/dP < 0$. The Euler-Lagrange equation associated with the variational problem is

$$\frac{\partial \mathcal{F}_P}{\partial \rho} - \frac{\partial}{\partial P} \left(\frac{\partial \mathcal{F}_P}{\partial \rho'} \right) = 0.$$

Inputing the previous

$$\frac{\partial}{\partial \rho} \left[-\rho' \cdot \frac{\rho}{2\pi\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \right] - \frac{\partial}{\partial P} \frac{\partial}{\partial \rho'} \left[-\rho' \cdot \frac{\rho}{2\pi\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \right] = 0,$$
~~$$\rho' \cdot \frac{(\rho^2 - \sigma^2)}{\sigma^4} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) + \rho' \cdot \frac{(\sigma^2 - \rho^2)}{\sigma^4} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) = 0.$$~~

So the solution can not be found in this way.

F.1.2. Subsidiary condition

To narrow down the problem, the normalization condition given by the conservation of energy can be included through Lagrange multipliers. The normalization condition is

$$\frac{2\pi}{A} \int_{\mathbb{R}^+} g(\rho) \rho \, d\rho = P_0,$$

where $g(\rho)$ is the inverse of the $\rho(P)$ function. For a monotonical function $g(\rho)$ this can be written in terms of its inverse as

$$\int_{\mathbb{R}^+} \rho^2(P) \, dP = \frac{P_0 A}{\pi}, \quad (\text{F.1})$$

The augmented functional can be written as

$$\begin{aligned} \tilde{S} &= \int_0^{P_{\text{th}}} \mathcal{F}_P[\rho, \rho', P] dP + \lambda \left(\int_0^\infty \rho^2(P) \, dP - \frac{P_0 A}{\pi} \right) \\ &= \int_0^{P_{\text{th}}} \mathcal{F}_P[\rho, \rho', P] dP + \lambda \int_0^{P_{\text{th}}} \rho^2(P) \, dP + \lambda \left(\int_{P_{\text{th}}}^\infty \rho^2(P) \, dP - \frac{P_0 A}{\pi} \right). \end{aligned}$$

The variational $\rho \rightarrow \rho + \delta\rho$ is only performed in the region $P \in [0, P_{\text{th}}]$, therefore the variation of the augmented functional becomes

$$\delta\tilde{S} = \int_0^{P_{\text{th}}} \left(\frac{\partial \mathcal{F}_P}{\partial \rho} - \frac{\partial}{\partial P} \frac{\partial \mathcal{F}_P}{\partial \rho'} \right) \delta\rho \, dP + \lambda \int_0^{P_{\text{th}}} \left(\frac{\partial \mathcal{G}}{\partial \rho} - \frac{\partial}{\partial P} \frac{\partial \mathcal{G}}{\partial \rho'} \right) \delta\rho \, dP,$$

where $\mathcal{G} = \rho^2$. This results in the Euler equation

$$\left(\frac{\partial \mathcal{F}_P}{\partial \rho} - \frac{\partial}{\partial P} \frac{\partial \mathcal{F}_P}{\partial \rho'} \right) + \lambda \left(\frac{\partial \mathcal{G}}{\partial \rho} - \frac{\partial}{\partial P} \frac{\partial \mathcal{G}}{\partial \rho'} \right) = 0, \quad P \in [0, P_{\text{th}}].$$

Given that the first factor is zero, and substituting the expression for $\mathcal{G}$ we have

$$\rho(P) = 0, \quad P \in [0, P_{\text{th}}].$$

On the other hand, the function $\rho(P)$ in the region $P \in (P_{\text{th}}, \infty)$ will have to be so that it satisfies the condition given in Eq. (F.1).

F.1.3. Weierstrass-Erdman corner conditions

The Weierstrass-Erdmann corner conditions can be considered when allowing discontinuous solutions for the function of the functional being solved in the variational problem [490]. In the case of interest, these conditions are

$$\left. \frac{\partial \mathcal{L}}{\partial \rho'} \right|_{P=P_{\text{th}}-0} = \left. \frac{\partial \mathcal{L}}{\partial \rho'} \right|_{P=P_{\text{th}}+0}$$

$$\left(\mathcal{L} - \rho' \frac{\partial \mathcal{L}}{\partial \rho'} \right) \Big|_{P=P_{\text{th}}-0} = \left(\mathcal{L} - \rho' \frac{\partial \mathcal{L}}{\partial \rho'} \right) \Big|_{P=P_{\text{th}}+0},$$

where $\mathcal{L} = \mathcal{F}_P - \lambda \mathcal{G}$ is the augmented Lagrangian. This conditions become

$$\cancel{\left. -\frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \right|_{P=P_{\text{th}}-0}} = \cancel{\left. -\frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \right|_{P=P_{\text{th}}+0}},$$

$$\cancel{\left[-\rho' \cdot \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) + \rho' \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \right] \Big|_{P=P_{\text{th}}-0}} = \cancel{\left[-\rho' \cdot \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) + \rho' \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \right] \Big|_{P=P_{\text{th}}+0}},$$

in our case. So this also does not provide the condition that $\rho(P)$ has to fulfill to minimize the functional $\mathcal{S}$.

F.2. Proofs for §11.3

Proof 1. Here is a simple argument showing that you cannot have a collected finite power at any displacement (x_0, y_0) of the beam in the real bidimensional plane

$$g(x_0, y_0) = \iint_{\mathbb{R}^2} I(x, y) \mathcal{A}(x - x_0, y - y_0) dx dy \geq P_{\text{th}} \quad \forall (x_0, y_0) \in \mathbb{R}^2,$$

while at the same time having a finite total power of the beam

$$P_0 = \iint_{\mathbb{R}^2} I(x, y) dx dy < \infty.$$

1. Partition the plane into infinitely many disjoint translates of the annular aperture. Let $\{(x_0^{(k)}, y_0^{(k)})\}_{k=1}^{\infty}$ be a collection of centers so that the sets

$$\mathcal{A}_k = \{(x, y) : a < (x - x_0^{(k)})^2 + (y - y_0^{(k)})^2 < b\},$$

are pairwise disjoint. (You can do this because the plane admits a packing of disjoint congruent annuli, the argument is also valid for other kinds of aperture shapes)

2. By hypothesis each translate carries at least P_{th} power

$$\iint_{\mathcal{A}_k} I(x, y) dx dy = g(x_0^{(k)}, y_0^{(k)}) \geq P_{\text{th}}.$$

3. Since the $\mathcal{A}_k$ are disjoint, the total power is at least the sum over all k

$$P_0 = \iint_{\mathbb{R}^2} I(x, y) dx dy \geq \sum_{k=1}^{\infty} \iint_{\mathcal{A}_k} I(x, y) dx dy \geq \sum_{k=1}^{\infty} P_{\text{th}}.$$

But $\sum_{k=1}^{\infty} P_{\text{th}} = \infty$ (since $P_{\text{th}} > 0$), so this forces

$$P_0 = \infty,$$

contradicting the finiteness assumption $P_0 < \infty$.

Hence, it is impossible to have

$$g(x_0, y_0) \geq P_{\text{th}} > 0 \quad \text{for all } (x_0, y_0),$$

because that would force the total beam power P_0 to be infinite.

Proof 2. Here we wish to prove that the bidimensional integral of the function $g(x_0, y_0)$ defined in Eq. (11.3) is finite,

$$G = \iint_{\mathbb{R}^2} g(x_0, y_0) dx_0 dy_0 < \infty.$$

1. Write G as an iterated integral

$$G = \iint_{\mathbb{R}^2} \left[\iint_{\mathbb{R}^2} I(x, y) \mathcal{A}(x - x_0, y - y_0) dx dy \right] dx_0 dy_0.$$

2. By Fubini (all functions are nonnegative), swap the order of integration

$$G = \iint_{\mathbb{R}^2} I(x, y) \left[\iint_{\mathbb{R}^2} \mathcal{A}(x - x_0, y - y_0) dx_0 dy_0 \right] dx dy.$$

3. Observe that

$$\iint_{\mathbb{R}^2} \mathcal{A}(x - x_0, y - y_0) dx_0 dy_0 = \iint_{\mathbb{R}^2} \mathcal{A}(u, v) du dv = |\mathcal{A}| < \infty,$$

where we have changed variables $u = x - x_0$, $v = y - y_0$. By hypothesis $|\mathcal{A}|$ (the area of the aperture) is finite.

4. Hence

$$G = \iint_{\mathbb{R}^2} I(x, y) |\mathcal{A}| dx dy = |\mathcal{A}| \iint_{\mathbb{R}^2} I(x, y) dx dy = |\mathcal{A}| P_0 < \infty.$$

Since $0 < P_0 < \infty$ and $0 < |\mathcal{A}| < \infty$, it follows that $0 < G < \infty$.

Proof 3. Here we wish to prove that among all nonnegative $g(x_0, y_0)$ with fixed total mass

$$\iint_{\mathbb{R}^2} g(x_0, y_0) dx_0 dy_0 = G,$$

the one that maximizes the area

$$A = |\{(x_0, y_0) : g(x_0, y_0) > P_{\text{th}}\}|,$$

is a circular flat-top function of height P_{th} . Moreover, the maximum area that can be covered is a circular area A_{max} with radius r_0

$$A_{\text{max}} = \frac{G}{P_{\text{th}}} \implies \text{radius } r_0 = \sqrt{\frac{A_{\text{max}}}{\pi}} = \sqrt{\frac{G}{\pi P_{\text{th}}}}.$$

1. For any nonnegative measurable g one has the identity

$$\iint_{\mathbb{R}^2} g(x, y) dx dy = \int_0^\infty |\{(x, y) : g(x, y) > t\}| dt.$$

In particular, split the integral at $t = P_{\text{th}}$:

$$G = \int_0^{P_{\text{th}}} |\{g > t\}| dt + \int_{P_{\text{th}}}^\infty |\{g > t\}| dt.$$

Since $\{g > t\} \supseteq \{g > P_{\text{th}}\}$ for all $t < P_{\text{th}}$, we get a lower bound

$$\int_0^{P_{\text{th}}} |\{g > t\}| dt \geq \int_0^{P_{\text{th}}} A dt = P_{\text{th}} A,$$

where $A := |\{g > P_{\text{th}}\}|$. Also the second integral is nonnegative. Hence

$$G \geq P_{\text{th}} A \implies A \leq \frac{G}{P_{\text{th}}}.$$

So no function can have $|\{g > P_{\text{th}}\}|$ larger than G/P_{th} .

2. Achieving the bound by a circ function. To see that the bound is attained, simply take

$$g(x, y) = \begin{cases} P_{\text{th}}, & x^2 + y^2 \leq r_0^2, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{F.2})$$

and choose r_0 so that its total mass is G :

$$\iint_{\mathbb{R}^2} g dx dy = \iint_{x^2 + y^2 \leq r_0^2} P_{\text{th}} dx dy = P_{\text{th}} (\pi r_0^2) \stackrel{!}{=} G \implies r_0^2 = \frac{G}{\pi P_{\text{th}}}.$$

Then $\{g > P_{\text{th}}\}$ is exactly the disk of radius r_0 , whose area is

$$\pi r_0^2 = \pi \frac{G}{\pi P_{\text{th}}} = \frac{G}{P_{\text{th}}}.$$

Proof 4. Below is a direct way to see that there is no nonnegative, compactly supported function $I(x, y)$ whose finite-aperture convolution

$$g(x_0, y_0) = \iint_{\mathbb{R}^2} I(x, y) \mathcal{A}(x - x_0, y - y_0) dx dy,$$

is exactly the sharp top-hat of radius r_0 and height P_{th} . Furthermore, the exact solution in the small aperture approximation for the farfield irradiance I is obtained.

1. Let $\mathcal{A}(u, v)$ be aperture function of a finite aperture, say a disk of radius a

$$\mathcal{A}(u, v) = \begin{cases} 1, & u^2 + v^2 \leq a^2, \\ 0, & \text{otherwise.} \end{cases}$$

Then g is the two-dimensional convolution of I with $\mathcal{A}$.

2. In the Fourier domain, convolution becomes multiplication. Denoting by $\tilde{I}(k_x, k_y)$, $\tilde{\mathcal{A}}(k_x, k_y)$, $\tilde{g}(k_x, k_y)$ the Fourier transforms,

$$\tilde{g}(k_x, k_y) = \tilde{I}(k_x, k_y) \tilde{\mathcal{A}}(k_x, k_y).$$

Hence any candidate I must satisfy

$$\tilde{I}(k_x, k_y) = \frac{\tilde{g}(k_x, k_y)}{\tilde{\mathcal{A}}(k_x, k_y)}.$$

3. But $\tilde{g}$ is the transform of a disk of radius r_0 and height P_{th} :

$$g(x_0, y_0) = \begin{cases} P_{\text{th}}, & x_0^2 + y_0^2 \leq r_0^2, \\ 0, & \text{otherwise,} \end{cases}$$

so

$$\tilde{g}(k) = P_{\text{th}} \pi r_0^2 \frac{2\mathcal{J}_1(|k| r_0)}{|k| r_0}, \quad |k| = \sqrt{k_x^2 + k_y^2}.$$

Likewise $\tilde{\mathcal{A}}(k) = \pi a^2 (2\mathcal{J}_1(|k| a)/(|k| a))$. Therefore

$$\tilde{I}(k) = P_{\text{th}} \frac{r_0}{a} \frac{\mathcal{J}_1(|k| r_0)}{\mathcal{J}_1(|k| a)}. \quad (\text{F.3})$$

4. The inverse Fourier transform (the Hankel transform of the first kind) of this ratio is a function in polar coordinates given as

$$I(\rho) = \int_0^\infty \tilde{I}(k) \mathcal{J}_1(k\rho) \rho d\rho, \quad (\text{F.4})$$

which in fact blows up and changes sign (because of the zeros of $\mathcal{J}_1$), so one cannot get a strictly nonnegative, finite-supported $I \geq 0$.

5. However, for small aperture sizes $a \rightarrow 0$ we can expand the $\tilde{\mathcal{A}}(k)$ as

$$\lim_{a \rightarrow 0} \tilde{\mathcal{A}}(k) = \frac{a^2}{2} + \mathcal{O}(a^4). \quad (\text{F.5})$$

Hence, $\tilde{I}(k)$ can be written as

$$\lim_{a \rightarrow 0} \tilde{I}(k) = \frac{2P_{\text{th}}r_0}{a^2} J_1(|k| r_0). \quad (\text{F.6})$$

Yielding the inverse Hankel transform

$$I(\rho) = \begin{cases} \frac{2P_{\text{th}}r_0}{a^2}, & \rho \leq r_0, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{F.7})$$

which is the flat-top beam. The normalization of the obtained flat-top might not be correct due to the $a \rightarrow 0$ approximation. To correctly normalize, the total power normalization condition can be included.

Hence, there is no nonnegative function I of finite support whose convolution with a finite aperture $\mathcal{A}$ is exactly the sharp flat-top function of radius r_0 . Equivalently, the deconvolution kernel $\tilde{g}/\tilde{\mathcal{A}}$ has poles (zeros of $\tilde{\mathcal{A}}$) and forces sign-changes. In practice, one can only approximate the step-function g by choosing $\tilde{I} = \tilde{g}/\tilde{\mathcal{A}}$ through numerical evaluation.

F.3. Ring flat-top beam

The reader could be wondering if a ring type flat-top of the kind

$$I(\rho) = \begin{cases} \frac{P_{\text{th}}}{A}, & r_1 < \rho < r_2, \\ 0, & \text{else,} \end{cases}$$

would be better than the flat-top, where the r_1 and r_2 are the inner and outer radius of the ring and the power of the beam is P_0 . To show that in this case the optimum is found when $r_1 = 0$, hence a flat-top beam, we obtain the value of $\mathcal{S}$ for this shape in a similar manner to what was done for the flat-top beam

$$\mathcal{S} = \int_0^{r_1} f_{\text{R}}(\rho) d\rho + \int_{r_2}^{\infty} f_{\text{R}}(\rho) d\rho,$$

that under the condition that the power of the beam is P_0 we have

$$\pi(r_2^2 - r_1^2) \frac{P_{\text{th}}}{A} = P_0 \rightarrow r_2 = \sqrt{\frac{P_0}{\pi P_{\text{th}}}} + r_1,$$

and then the performance can be written as a function of r_1 as

$$\mathcal{S}(r_1) = \int_0^{r_1} f_{\text{R}}(\rho) d\rho + \int_{\sqrt{\frac{P_0}{\pi P_{\text{th}}}} + r_1}^{\infty} f_{\text{R}}(\rho) d\rho = \frac{1}{2\pi} \left[1 + e^{-\frac{r_1^2}{2\sigma^2}} \left(e^{-\frac{P_0}{2\pi P_{\text{th}} \sigma^2}} - 1 \right) \right].$$

Now we have to find the r_1 at which $\mathcal{S}$ is minimum so

$$\begin{aligned}\frac{d\mathcal{S}(r_1)}{dr_1} &= 0 \\ &= -\frac{r_1}{2\pi\sigma^2} \left[e^{-\frac{r_1^2}{2\sigma^2}} \left(e^{-\frac{P_0}{2\pi P_{\text{th}}\sigma^2}} - 1 \right) \right],\end{aligned}$$

which has solutions at $r_1 = 0$ and $r_1 \rightarrow \infty$. It can be seen that the extrema at r_1 is a minimum by the second derivative test

$$\left. \frac{d^2\mathcal{S}(r_1)}{dr_1^2} \right|_{r_1=0} = \frac{1 - e^{-\frac{P_0}{2\pi P_{\text{th}}\sigma^2}}}{2\pi\sigma^2} > 0,$$

because $P_0/(2\pi P_{\text{th}}\sigma^2) > 0$ and $\sigma^2 > 0$. Hence, we see that the ring flat-top of the type analysed here, converges to the flat-top at its maximum performance.

List of acronyms

- ABEP** average bit error probability 112, 113, 116, 117, 164, 173, 178–185, 198, 233, 243, 246–248, 280, 296–298
- ADCS** attitude determination and control system 55, 57
- APD** avalanche photodetector 73, 74, 78
- BEP** bit error probability 113, 114, 232
- BER** bit error rate 33, 47, 49, 50, 53, 54
- BS** beam splitter 33, 148, 152
- BSM** Bell-state measurement 74, 126, 127, 129–132, 135, 136, 141, 142, 252
- CCD** charge-coupled device 33, 34
- CNOT** controlled-NOT 132, 138
- COG** center of gravity 243
- CPA** coarse pointing assembly 34, 56
- CZ** controlled-Z 131, 137
- DFT** discrete Fourier transform 100
- DM** dichroic mirror 71
- EDFA** erbium-doped fiber amplifier 40, 57
- EPS** entangled-photon source 72, 78, 79
- FDA** finite difference analysis 42
- FEA** finite element analysis 42, 43, 45, 46
- FFT** fast Fourier transform 109
- FPG** fast pulse generator 71
- FRF** frequency response function 42
- FSM** fast steering mirror 34, 41, 56, 58, 59
- FSOC** free-space optical communications 25, 27, 32–48, 50–61, 164, 165, 167, 171, 184, 188, 189, 192, 193, 195

- GEO** geostationary orbit 40, 69, 70, 104, 116, 119, 122, 185
- GS** Gerchberg-Saxton 215–218
- HG** Hermite-Gaussian 213, 217–219
- HWP** half-wave plate 148, 152
- LEO** low Earth orbit 40, 69, 70, 76, 104, 116, 119, 122, 146, 147, 178, 185
- LG** Laguerre-Gaussian 188, 191, 194, 195, 213, 217–219
- MI** maximum irradiance 243
- OAM** orbital angular momentum 34–36, 50, 53, 80–82, 87, 172, 231
- OLR** earth orbital long-wavelength radiation 39, 40
- OOK IM/DD** on-off keying intensity modulation direct detection 113, 114, 173
- OPL** optical path length 44, 45
- PAT** pointing, acquisition and tracking 33, 37, 38, 59
- PBC** polarization beam combiner 175, 189, 299
- PBS** polarization beam splitter 33, 34, 54, 148, 152, 174–176, 189, 285, 299
- PDF** probability density function 41, 42, 49, 50, 52, 55, 111–113, 115, 116, 118, 165–170, 172, 173, 178, 179, 184, 198, 200–203, 205, 245, 273–275, 277, 279, 280
- PER** polarization extinction ratio 41, 54, 55, 83, 86
- PIC** photonic integrated circuits 72
- PL** pump laser 71
- PSD** power spectral density 37, 38, 42, 56
- QBER** quantum bit error rate 65, 73, 82–85, 231, 259, 269–271
- QKD** quantum key distribution 23, 24, 26, 29, 63–87, 126, 145–148, 150, 152, 154, 156, 157, 227, 231, 234, 235, 268–271
- QWP** quarter wave plate 33
- RHWP** rotatable half-wave plate 71, 174–177, 188–190, 192, 194, 285, 286, 295, 296, 299
- RQWP** rotatable quarter-wave plate 71
- SAM** spin angular momentum 34–36
- SEA** statistical energy analysis 43
- SKR** secret key rate 65, 84, 85, 146, 147, 149, 153, 155, 156, 234
- SNR** signal to noise ratio 49, 50
- SNSPS** superconducting-nanowire single-photon detectors 73, 74, 76, 78
- SPD** single-photon detector 71
- SPDC** spontaneous parametric down-conversion 72, 78, 127, 146, 149, 259, 260

SPP spiral phase plate 174, 175, 177, 187–189, 191–195, 283–298

STOP structural, thermal, and optical performance 32, 46, 48

VPN virtual private network 146, 147, 156

WCP weak coherent pulses 72, 78, 79, 231

WFE wavefront error 33, 41, 43–47, 50–55, 57–60

Bibliography

- [1] J. W. Bradbury, *Principles of animal communication*. Sinauer Associates, 2011.
- [2] G. Marconi, *Wireless telegraphic communication*, 1909.
- [3] E. Hecht, *Optics*. Boston: Pearson Education, Inc, 2017.
- [4] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*. Pergamon Press, 1975.
- [5] P. Kok and B. W. Lovett, *Introduction to Optical Quantum Information Processing*. Cambridge: Cambridge University Press, 2010.
- [6] H. Goldstein et al., *Classical mechanics*. Noida: Pearson India Education Services, 2016.
- [7] J. D. Jackson, *Classical Electrodynamics*. Hoboken, NY: Wiley, 2009.
- [8] J. W. Goodman, *Introduction to Fourier optics*. Englewood, Colo.: Roberts & Co., 2005.
- [9] E. Fradkin, *Quantum Field Theory: An Integrated Approach*. Princeton (N. J.): Princeton University Press, 2021.
- [10] M. Bandres and J. Gutierrez Vega, “Ince-Gaussian beams”, *Optics letters*, vol. 29, pp. 144–6, Feb. 2004.
- [11] A. Aiello, “Field theory of monochromatic optical beams: I. Classical fields”, *Journal of Optics*, vol. 22, no. 1, p. 014 001, Dec. 2019.
- [12] A. Aiello, “Field theory of monochromatic optical beams: II. Classical and quantum paraxial fields”, *Journal of Optics*, vol. 22, no. 1, p. 014 002, Jan. 2020.
- [13] A. Aiello and J. P. Woerdman, “Exact quantization of a paraxial electromagnetic field”, *Physical Review A*, vol. 72, no. 6, p. 060 101, Dec. 2005.
- [14] L. Allen et al., “Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes”, *Physical Review A*, vol. 45, no. 11, pp. 8185–8189, Jun. 1992.
- [15] R. B. Ash, *Information Theory*. Dover Publications, 1990.
- [16] J. V. Stone, *Information theory: a tutorial introduction*. Sheffield, UK: Sebtel Press, 2015.
- [17] G. Benenti et al., *Principles Of Quantum Computation And Information: A Comprehensive Textbook*. World Scientific, Dec. 2018.
- [18] M. A. Nielsen, *Quantum Computation and Quantum Information*. Cambridge ; New York: Cambridge University Press, Jan. 2004.
- [19] C. E. Shannon, “A mathematical theory of communication”, *The Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, Jul. 1948.
- [20] A. Holevo, “The capacity of the quantum channel with general signal states”, *IEEE Transactions on Information Theory*, vol. 44, no. 1, pp. 269–273, Jan. 1998.
- [21] B. Schumacher and M. D. Westmoreland, “Sending classical information via noisy quantum channels”, *Physical Review A*, vol. 56, no. 1, pp. 131–138, Jul. 1997.
- [22] D. Main et al., “Distributed quantum computing across an optical network link”, *Nature*, vol. 638, no. 8050, pp. 383–388, Feb. 2025.

- [23] R. Horodecki et al., “Quantum entanglement”, *Reviews of Modern Physics*, vol. 81, no. 2, pp. 865–942, Jun. 2009.
- [24] R. L. Rivest et al., “A Method for Obtaining Digital Signatures and Public-Key Cryptosystems”, *Communications of the ACM*, vol. 21, no. 2, 1977.
- [25] P. Shor, “Algorithms for quantum computation: Discrete logarithms and factoring”, in *Proceedings 35th Annual Symposium on Foundations of Computer Science*, Nov. 1994, pp. 124–134.
- [26] R. Babbush et al., *Securing Elliptic Curve Cryptocurrencies against Quantum Vulnerabilities: Resource Estimates and Mitigations*, Mar. 2026.
- [27] M. Cain et al., *Shor’s algorithm is possible with as few as 10,000 reconfigurable atomic qubits*, Mar. 2026.
- [28] C. H. Bennett and G. Brassard, “Quantum cryptography: Public key distribution and coin tossing”, *Theoretical Computer Science, Theoretical Aspects of Quantum Cryptography – celebrating 30 years of BB84*, vol. 560, pp. 7–11, Dec. 2014.
- [29] C. H. Bennett et al., “Quantum cryptography without Bell’s theorem”, *Physical Review Letters*, vol. 68, no. 5, pp. 557–559, Feb. 1992.
- [30] A. K. Ekert, “Quantum cryptography based on Bell’s theorem”, *Physical Review Letters*, vol. 67, no. 6, pp. 661–663, Aug. 1991.
- [31] J. D. Hidary, *Quantum computing: an applied approach*. Cham, Switzerland: Springer, 2021.
- [32] R. Zhang et al., “4x100-Gb/s PAM-4 FSO Transmission Based on Polarization Modulation and Direct Detection”, *IEEE Photonics Technology Letters*, vol. 31, no. 10, pp. 755–758, May 2019.
- [33] K. Zou et al., “High-capacity free-space optical communications using wavelength- and mode-division-multiplexing in the mid-infrared region”, *Nature Communications*, vol. 13, no. 1, p. 7662, Dec. 2022.
- [34] G. Xie et al., “Performance metrics and design considerations for a free-space optical orbital-angular-momentum-multiplexed communication link”, *Optica*, vol. 2, no. 4, pp. 357–365, Apr. 2015.
- [35] W. W. Peterson and E. J. Weldon, *Error-correcting Codes*. MIT Press, 1972.
- [36] P. van Loock, “Quantum Communication with Continuous Variables”, *Fortschritte der Physik*, vol. 50, no. 12, pp. 1177–1372, 2002.
- [37] V. C. Usenko et al., *Continuous-variable quantum communication*, Jan. 2025.
- [38] H. Wang et al., “Scalable photonic quantum technologies”, *Nature Materials*, pp. 1–15, Aug. 2025.
- [39] N. Gisin and R. Thew, “Quantum communication”, *Nature Photonics*, vol. 1, no. 3, pp. 165–171, Mar. 2007.
- [40] I. Marcikic et al., “Time-bin entangled qubits for quantum communication created by femtosecond pulses”, *Physical Review A*, vol. 66, no. 6, p. 062308, Dec. 2002.
- [41] J. M. Lukens and P. Lougovski, “Frequency-encoded photonic qubits for scalable quantum information processing”, *Optica*, vol. 4, no. 1, pp. 8–16, Jan. 2017.
- [42] A. Khodadad Kashi and M. Kues, “Frequency-bin-encoded entanglement-based quantum key distribution in a reconfigurable frequency-multiplexed network”, *Light: Science & Applications*, vol. 14, no. 1, p. 49, Jan. 2025.
- [43] G. Molina-Terriza et al., “Twisted photons”, *Nature Physics*, vol. 3, no. 5, pp. 305–310, May 2007.

- [44] N. Bent et al., “Experimental Realization of Quantum Tomography of Photonic Qudits via Symmetric Informationally Complete Positive Operator-Valued Measures”, *Physical Review X*, vol. 5, no. 4, p. 041 006, Oct. 2015.
- [45] H. Bechmann-Pasquinucci and W. Tittel, “Quantum cryptography using larger alphabets”, *Physical Review A*, vol. 61, no. 6, p. 062 308, May 2000.
- [46] X. Cheng et al., “High-dimensional time-frequency entanglement in a singly-filtered biphoton frequency comb”, *Communications Physics*, vol. 6, no. 1, p. 278, Sep. 2023.
- [47] K. Edamatsu, “Entangled Photons: Generation, Observation, and Characterization”, *Japanese Journal of Applied Physics*, vol. 46, no. 11R, p. 7175, Nov. 2007.
- [48] M. Badás et al., “Opto-thermomechanical phenomena in satellite free-space optical communications: Survey and challenges”, *Optical Engineering*, vol. 63, no. 4, p. 041 206, Oct. 2023.
- [49] H. Kaushal and G. Kaddoum, “Optical Communication in Space: Challenges and Mitigation Techniques”, *IEEE Communications Surveys and Tutorials*, vol. 19, no. 1, pp. 57–96, Jan. 2017.
- [50] A. Jahid et al., “A contemporary survey on free space optical communication: Potentials, technical challenges, recent advances and research direction”, *Journal of Network and Computer Applications*, vol. 200, p. 103 311, Apr. 2022.
- [51] A. Könsgen and A. Förster, “Current state and future challenges in deep space communication: A survey”, *IT - Information Technology*, vol. 63, no. 4, pp. 219–234, 2021.
- [52] H. Hemmati et al., “Deep-Space Optical Communications: Future Perspectives and Applications”, *Proceedings of the IEEE*, vol. 99, no. 11, pp. 2020–2039, Nov. 2011.
- [53] A. Vats et al., “Free Space Optical Communication: Laser Sources, Modulation Schemes and Detection Techniques”, Feb. 2013.
- [54] M. Dinu et al., “Qualification and performance of a high-efficiency laser transmitter for deep-space optical communications”, in *Free-Space Laser Communications XXXIV*, vol. 11993, SPIE, Mar. 2022, pp. 129–140.
- [55] W. Shao et al., “Terabit FSO communication based on a soliton microcomb”, *Photonics Research*, vol. 10, no. 12, pp. 2802–2808, Dec. 2022.
- [56] A. Bekkali et al., “New Generation Free-Space Optical Communication Systems With Advanced Optical Beam Stabilizer”, *Journal of Lightwave Technology*, vol. 40, no. 5, pp. 1509–1518, Mar. 2022.
- [57] A. Turpin et al., “Free Space Optical Polarization De-multiplexing and Multiplexing by means of Conical Refraction”, *Optics letters*, vol. 37, pp. 4197–4199, Oct. 2012.
- [58] S. Benedetto and P. Poggiolini, “Theory of polarization shift keying modulation”, *IEEE Transactions on Communications*, vol. 40, no. 4, pp. 708–721, Apr. 1992.
- [59] X. Tang et al., “Free-space optical communication employing polarization shift keying coherent modulation in atmospheric turbulence channel”, in *2010 7th International Symposium on Communication Systems, Networks & Digital Signal Processing (CSNDSP 2010)*, Jul. 2010, pp. 615–620.
- [60] Y. Zhao et al., “Circular polarization shift-keying modulation based on orbital angular momentum division multiplexing in free space optical communication”, *Optics Communications*, vol. 475, p. 126 165, Nov. 2020.

- [61] Z.-Y. Chen et al., "Use of polarization freedom beyond polarization-division multiplexing to support high-speed and spectral-efficient data transmission", *Light: Science & Applications*, vol. 6, no. 2, e16207–e16207, Feb. 2017.
- [62] J. Wu et al., "Polarization-maintaining design for satellite-based quantum communication terminals", *Optics Express*, vol. 28, no. 8, pp. 10 746–10 759, Apr. 2020.
- [63] K. B. Doyle et al., "Stress birefringence modeling for lens design and photonics", in *International Optical Design Conference 2002*, vol. 4832, SPIE, Dec. 2002, pp. 436–447.
- [64] H. Hemmati, *Near-Earth Laser Communications*. CRC, 2009.
- [65] A. M. Yao and M. J. Padgett, "Orbital angular momentum: Origins, behavior and applications", *Advances in Optics and Photonics*, vol. 3, no. 2, pp. 161–204, Jun. 2011.
- [66] G. Gibson et al., "Free-space information transfer using light beams carrying orbital angular momentum", *Optics Express*, vol. 12, no. 22, pp. 5448–5456, Nov. 2004.
- [67] A. E. Willner et al., "Orbital Angular Momentum Beams for High-capacity Communications", *Journal of Lightwave Technology*, pp. 1–17, 2022.
- [68] B. Naimullah et al., "Comparison of wavelength propagation for Free Space Optical Communications", in *International Conference of Electronic Design*, Jan. 2009, pp. 1–5.
- [69] T. Plank et al., "Wavelength-selection for high data rate Free Space Optics (FSO) in next generation wireless communications", in *2012 17th European Conference on Networks and Optical Communications*, Jun. 2012, pp. 1–5.
- [70] R. Miglani, "Analysis of FSO Communication Links for Mid And Far Infrared Wavelengths", *International Journal of Scientific & Engineering Research*, vol. 4, no. 7, p. 5, 2013.
- [71] T. Liu et al., "Investigation of the wavelength selection for the free space optical communication system", in *2018 Asia Communications and Photonics Conference (ACP)*, Oct. 2018, pp. 1–3.
- [72] Y. Lu et al., "A Review of the Space Environment Effects on Spacecraft in Different Orbits", *IEEE Access*, vol. 7, pp. 93 473–93 488, 2019.
- [73] F. Berghmans et al., "An Introduction to Radiation Effects on Optical Components and Fiber Optic Sensors", in *Optical Waveguide Sensing and Imaging*, Dec. 2007, pp. 127–165.
- [74] S. Girard et al., "Recent advances in radiation-hardened fiber-based technologies for space applications", *Journal of Optics*, vol. 20, no. 9, p. 093 001, Aug. 2018.
- [75] S. Arnon and N. Kopeika, "Laser satellite communication network-vibration effect and possible solutions", *Proceedings of the IEEE*, vol. 85, no. 10, pp. 1646–1661, Oct. 1997.
- [76] C. Dennehy and O. S. Alvarez-Salazar, "A Survey of the Spacecraft Line-of-Sight Jitter Problem", in *AAS Annual Guidance and Control Conference*, 2019.
- [77] L. You et al., "Superconducting nanowire single photon detection system for space applications", *Optics Express*, vol. 26, no. 3, pp. 2965–2971, Feb. 2018.
- [78] N. Jedrich et al., "Cryo Cooler Induced MicroVibration Disturbances to the Hubble Space Telescope", *NASA STI/Recon Technical Report N*, Jan. 2002.
- [79] D. Yu et al., "On-Orbit Measurement and Analysis of the Micro-vibration in

- a Remote-Sensing Satellite”, *Advances in Astronautics Science and Technology*, vol. 1, no. 2, pp. 191–195, Dec. 2018.
- [80] X. Tang et al., “Overview of Earth Observation Satellite Platform Microvibration Detection Methods”, *Sensors*, vol. 20, no. 3, p. 736, Jan. 2020.
- [81] X. Wang et al., “Angular micro-vibration of the Micius satellite measured by an optical sensor and the method for its suppression”, *Applied Optics*, vol. 60, no. 7, pp. 1881–1887, Mar. 2021.
- [82] M. E. Wittig et al., “In-orbit measurements of microaccelerations of ESA’s communication satellite Olympus”, in *Free-Space Laser Communication Technologies II*, vol. 1218, SPIE, Jul. 1990, pp. 205–214.
- [83] S. Dyne et al., “Satellite mechanical health monitoring”, in *IEE Colloquium on Advanced Vibration Measurements, Techniques and Instrumentation for the Early Prediction of Failure*, May 1992, pp. 4/1–4/8.
- [84] S. Dyne et al., “The vibration environment on a satellite in orbit”, in *IEE Colloquium on High Accuracy Platform Control in Space*, Jun. 1993, pp. 12/1–12/6.
- [85] M. Toyoshima et al., “In-orbit measurements of spacecraft microvibrations for satellite laser communication links”, *Optical Engineering*, vol. 49, no. 8, p. 083604, Aug. 2010.
- [86] J. Johnston and E. Thornton, “Thermally Induced Dynamics of Satellite Solar Panels”, *Journal of Spacecraft and Rockets - J SPACECRAFT ROCKET*, vol. 37, pp. 604–613, Sep. 2000.
- [87] M. Oda et al., “Measurement of Satellite Solar Array Panel Vibrations Caused by Thermal Snap and Gas Jet Thruster Firing”, in *Recent Advances in Vibrations Analysis*, IntechOpen, Sep. 2011.
- [88] M. Toyoshima and K. Araki, “In-orbit measurements of short term attitude and vibrational environment on the Engineering Test Satellite VI using laser communication equipment”, *Optical Engineering*, vol. 40, no. 5, pp. 827–832, May 2001.
- [89] D. González-Bárcena et al., “Selection of time-dependent worst-case thermal environmental conditions for Low Earth Orbit spacecrafts”, *Advances in Space Research*, vol. 70, no. 7, pp. 1847–1868, Oct. 2022.
- [90] J. Hansen et al., “Earth’s Energy Imbalance and Implications”, *Atmospheric Chemistry and Physics Discussions*, vol. 11, May 2011.
- [91] C. C. C. Service, *Surface albedo 10-daily gridded data from 1981 to present*, 2018.
- [92] C. C. C. Service, *Earth’s radiation budget from 1979 to present derived from satellite observations*, 2020.
- [93] D. G. Gilmore, *Spacecraft thermal control handbook. Volume I, Fundamental technologies*. Aerospace Press, 2002.
- [94] R. Jimenez, “Effects of natural environment charged particle heating on the design and performance of spacecraft cryogenic components”, in *24th Thermophysics Conference*, ser. Fluid Dynamics and Co-located Conferences, American Institute of Aeronautics and Astronautics, Jun. 1989.
- [95] H. Dang et al., “Development of 2-K Space Cryocoolers for Cooling the Superconducting Nanowire Single Photon Detector”, *IEEE Transactions on Applied Superconductivity*, vol. 29, no. 5, pp. 1–4, Aug. 2019.
- [96] D. Hengeveld et al., “Hot and Cold-Case Orbits for Robust Thermal Control”, *Journal of Spacecraft and Rockets - J SPACECRAFT ROCKET*, vol. 46, pp. 1249–1260, Nov. 2009.

- [97] B. J. Anderson et al., “Guidelines for the Selection of Near-Earth Thermal Environment Parameters for Spacecraft Design”, Tech. Rep. NASA/TM-2001-211221, 2001.
- [98] C. G. Justus et al., “Simple Thermal Environment Model (STEM) User’s Guide”, NTRS Research Center: Marshall Space Flight Center (MSFC), Tech. Rep. NAS 1.15:211222, Oct. 2001.
- [99] L. Yenchesky et al., “Optomechanical design for cubesat laser infrared crosslinks”, in *AIAA Scitech 2019 Forum*, 2019.
- [100] H. Zhang et al., “On-orbit thermal deformation prediction for a high-resolution satellite camera”, *Applied Thermal Engineering*, vol. 195, p. 117 152, Aug. 2021.
- [101] S.-P. Chen, “Investigations of free space and deep space optical communication scenarios”, *CEAS Space Journal*, vol. 14, no. 2, pp. 357–364, 2022.
- [102] C. E. DeVoe et al., “Optical overview and qualification of the LLCD space terminal”, in *International Conference on Space Optics — ICSO 2014*, vol. 10563, SPIE, Nov. 2017, pp. 115–123.
- [103] A. Biswas et al., “Mars laser communication demonstration: What it would have been”, in *Free-Space Laser Communication Technologies XVIII*, vol. 6105, SPIE, Mar. 2006, p. 610 502.
- [104] W. Hart et al., “Requirements Development and Management on the Psyche Project”, in *2019 IEEE Aerospace Conference*, Mar. 2019, pp. 1–14.
- [105] C. Y. Han and J.-M. Choi, “Thermal analysis of spacecraft propulsion system and its validation”, *KSME International Journal*, vol. 18, no. 5, pp. 847–856, May 2004.
- [106] V. Ashoori et al., “Heat Generation and Removal in Solid State”, in *An Overview of Heat Transfer Phenomena*, IntechOpen, Oct. 2012.
- [107] H. Noda et al., “Thermal analyses for initial operations of the soft x-ray spectrometer onboard the Hitomi satellite”, *Journal of Astronomical Telescopes, Instruments, and Systems*, vol. 4, no. 1, p. 011 202, Oct. 2017.
- [108] A. Carrasco-Casado et al., “Development and Space-Qualification of a Miniaturized CubeSat’s 2-W EDFA for Space Laser Communications”, *Electronics*, vol. 11, no. 15, p. 2468, Jan. 2022.
- [109] E. Haddad et al., “Space qualification of a 10W single-mode PM optical amplifiers in the 1.5- μ m region”, in *International Conference on Space Optics — ICSO 2020*, vol. 11852, SPIE, Jun. 2021, pp. 820–830.
- [110] C. Campanella et al., “Theoretical Investigation of Thermal Effects in High Power Er³⁺/Yb³⁺ Codoped Double-Clad Fiber Amplifiers for Space Applications”, *Physica Status Solidi A*, vol. 216, no. 3, 2019.
- [111] C. M. Schieler et al., “200 Gbps TBIRD CubeSat downlink: Pre-flight test results”, in *Free-Space Laser Communications XXXIV*, H. Hemmati and B. S. Robinson, Eds., San Francisco, United States: SPIE, Mar. 2022, p. 34.
- [112] K. Riesing et al., “Pointing, acquisition, and tracking for the TBIRD CubeSat mission: System design and pre-flight results”, in *Free-Space Laser Communications XXXIV*, H. Hemmati and B. S. Robinson, Eds., San Francisco, United States: SPIE, Mar. 2022, p. 6.
- [113] P. d. V. Pereira et al., “Characterization of laser thermal loading on microelectromechanical systems-based fast steering mirror in vacuum”, *Optical Engineering*, vol. 59, no. 5, p. 056 109, May 2020.
- [114] P. L. Rall et al., “Simulation and compensation of thermal lensing in optical systems”, *Optics Express*, vol. 30, no. 21, pp. 38 643–38 662, Oct. 2022.

- [115] K. Dobek, "Thermal lensing: Outside of the lasing medium", *Applied Physics B*, vol. 128, no. 2, p. 18, Jan. 2022.
- [116] Y. Yang et al., "Influence of localized distortion in lenses on Strehl ratio in lasercom", *Optik*, vol. 124, no. 23, pp. 6415–6418, Dec. 2013.
- [117] A. Calvi et al., *ECSS-E-HB-32-26A Spacecraft Mechanical Loads Analysis Handbook*. Feb. 2013.
- [118] M. Toyoshima et al., "Optimum intersatellite link design in the presence of random pointing jitter for free-space laser communication systems", in *Free-Space Laser Communication Technologies XIV*, vol. 4635, SPIE, Apr. 2002, pp. 95–102.
- [119] F. Yang et al., "Free-Space Optical Communication with Nonzero Boresight Pointing Errors", *IEEE Transactions on Communications*, vol. 62, no. 2, pp. 713–725, Feb. 2014.
- [120] F. D. Iv and G. Nadorff, "Rigid body movements of optical elements due to opto-mechanical factors", in *Optical Modeling and Performance Predictions II*, vol. 5867, SPIE, Aug. 2005, pp. 137–148.
- [121] V. L. Genberg et al., "Integrated modeling of jitter MTF due to random loads", in *Optical Modeling and Performance Predictions V*, vol. 8127, SPIE, Sep. 2011, pp. 157–165.
- [122] S.-B. Chen et al., "Simulating and Testing Microvibrations on an Optical Satellite Using Acceleration Sensor-Based Jitter Measurements", *Sensors*, vol. 19, no. 8, p. 1797, Apr. 2019.
- [123] K. B. Doyle, "Structural line-of-sight jitter analysis for MLCD", in *New Developments in Optomechanics*, vol. 6665, SPIE, Sep. 2007, pp. 138–149.
- [124] P. Klocek, *Handbook of infrared optical materials* (Optical engineering). New York: CRC Press, 1991, vol. 30.
- [125] S. Appel and J. Wijker, *Simulation of Thermoelastic Behaviour of Spacecraft Structures: Fundamentals and Recommendations*. Springer Nature, Aug. 2021.
- [126] H.-S. Yang et al., "Three-shell-based lens barrel for the effective athermalization of an IR optical system", *Applied Optics*, vol. 50, no. 33, pp. 6206–6213, Nov. 2011.
- [127] E. Segato et al., "Method for studying the effects of thermal deformations on optical systems for space application", *Applied Optics*, vol. 50, no. 18, pp. 2836–2845, Jun. 2011.
- [128] K. B. Doyle et al., *Integrated optomechanical analysis*. 2012.
- [129] C. B. Burroughs et al., "An introduction to statistical energy analysis", *The Journal of the Acoustical Society of America*, vol. 101, no. 4, pp. 1779–1789, Apr. 1997.
- [130] M. Remedias and G. Aglietti, "Modeling micro-vibrations transmission in spacecraft structures", in *62nd International Astronautical Congress 2011, IAC 2011*, vol. 7, Jan. 2011, pp. 5682–5690.
- [131] R. M. Waxler and G. Cleek, "The effect of temperature and pressure on the refractive index of some oxide glasses", *Journal of Research of the National Bureau of Standards Section A: Physics and Chemistry*, vol. 77A, no. 6, p. 755, Nov. 1973.
- [132] Y.-f. Tsay et al., "Theory of the Temperature Derivative of the Refractive Index in Transparent Crystals", *Physical Review B*, vol. 8, no. 6, pp. 2688–2696, Sep. 1973.
- [133] G. Ghosh, "Model for the thermo-optic coefficients of some standard optical glasses", *Journal of Non-Crystalline Solids*, vol. 189, no. 1, pp. 191–196, Aug. 1995.

- [134] T. Bååk, "Thermal Coefficient of Refractive Index of Optical Glasses", *Journal of the Optical Society of America*, vol. 59, no. 7, pp. 851–857, Jul. 1969.
- [135] C. J. Parker and W. A. Popov, "Experimental Determination of the Effect of Temperature on Refractive Index and Optical Path Length of Glass", *Applied Optics*, vol. 10, no. 9, pp. 2137–2143, Sep. 1971.
- [136] J. R. Rogers, "Passive athermalization: Required accuracy of the thermo-optical coefficients", in *International Optical Design Conference 2014*, vol. 9293, SPIE, Dec. 2014, pp. 352–360.
- [137] G. Michels and V. Genberg, "Analysis of thermally loaded transmissive optical elements", in *Proceedings Volume 8127, Optical Modeling and Performance Predictions V*, Sep. 2011.
- [138] Y. C. Yiu and A. R. Meyer, "Computation of optical errors in transparent optical elements due to three-dimensional photoelastic effect", in *Advances in Optical Structure Systems*, vol. 1303, SPIE, Oct. 1990, pp. 206–216.
- [139] E. Collett, *Field Guide to Polarization*. 1000 20th Street, Bellingham, WA 98227-0010 USA: SPIE, Sep. 2005.
- [140] E. Cotte et al., "Study of stress birefringence for 193-nm immersion photomasks", in *Photomask and Next-Generation Lithography Mask Technology XII*, vol. 5853, SPIE, Jun. 2005, pp. 10–19.
- [141] H. Xiao and Z. Fan, "Imaging quality evaluation of aerodynamically heated optical dome using ray tracing", *Applied Optics*, vol. 49, no. 27, pp. 5049–5058, Sep. 2010.
- [142] M. Huang, "Stress Effects on the Performance of Optical Waveguides", *International Journal of Solids and Structures*, vol. 40, pp. 1615–1632, Jul. 2003.
- [143] K. B. Doyle and W. M. Bell, "Thermoelastic wavefront and polarization error analysis of a telecommunication optical circulator", in *Current Developments in Lens Design and Optical Systems Engineering*, vol. 4093, SPIE, Oct. 2000, pp. 18–27.
- [144] V. Genberg et al., "Integrating optical, mechanical, and test software (with applications to freeform optics)", in *Optifab 2017*, vol. 10448, SPIE, Oct. 2017, pp. 151–158.
- [145] J. Moreno Raso et al., "Optomechanical design of the Laser Launch Telescope for the GTC adaptive optics system", vol. 11871, 2021.
- [146] A. Haber et al., "Modeling and system identification of transient STOP models of optical systems", *Optics Express*, vol. 28, p. 39250, Dec. 2020.
- [147] J. Heijmans et al., "Combined Optomechanical analysis for modern optical instruments", in *Proceedings Volume 10705, Modeling, Systems Engineering, and Project Management for Astronomy VIII*, Jul. 2018, p. 89.
- [148] D. Driscoll et al., "Deep space optical communications (DSOC) beam expander design and engineering", in *Free-Space Laser Communications XXXII*, vol. 11272, SPIE, Mar. 2020, pp. 110–119.
- [149] I. U. Zaman and O. Boyraz, "Impact of receiver architecture on small satellite optical link in the presence of pointing jitter", *Applied Optics*, vol. 59, no. 32, pp. 10177–10184, Nov. 2020.
- [150] C.-C. Chen and C. Gardner, "Impact of random pointing and tracking errors on the design of coherent and incoherent optical intersatellite communication links", *IEEE Transactions on Communications*, vol. 37, no. 3, pp. 252–260, Mar. 1989.
- [151] K. Kiasaleh, "On the probability density function of signal intensity in free-

- space optical communications systems impaired by pointing jitter and turbulence”, *Optical Engineering*, vol. 33, no. 11, pp. 3748–3757, Nov. 1994.
- [152] H. Trung, “Performance of UAV-to-Ground FSO Communications with APD and Pointing Errors”, *Applied System Innovation*, vol. 4, p. 65, Sep. 2021.
- [153] K. Peppas and C. Datsikas, “Average Symbol Error Probability of General-Order Rectangular Quadrature Amplitude Modulation of Optical Wireless Communication Systems Over Atmospheric Turbulence Channels”, *Optical Communications and Networking, IEEE/OSA Journal of*, vol. 2, pp. 102–110, Mar. 2010.
- [154] T. Song et al., “Impact of Pointing Errors on the Error Performance of Intersatellite Laser Communications”, *Journal of Lightwave Technology*, vol. 35, no. 14, pp. 3082–3091, Jul. 2017.
- [155] L. Li et al., “Orbital-angular-momentum-multiplexed free-space optical communication link using transmitter lenses”, *Applied Optics*, vol. 55, no. 8, pp. 2098–2103, Mar. 2016.
- [156] M. T. Dabiri et al., “Advancing OAM-Based FSO Systems: Tackling Pointing Errors for Next-Generation Space and Terrestrial Links”, *IEEE Photonics Journal*, pp. 1–14, 2025.
- [157] X. Liu et al., “Propagation Characteristics of Hermite–Gaussian Beam under Pointing Error in Free Space”, *Photonics*, vol. 9, no. 7, p. 478, Jul. 2022.
- [158] Y. Yang et al., “Influence of wavefront aberrations on bit error rate in inter-satellite laser communications”, *Optics Communications*, vol. 284, no. 12, pp. 3065–3069, Jun. 2011.
- [159] K. E. Nevin et al., “Optomechanical design and analysis for the LLCD space terminal telescope”, in *Optical Modeling and Performance Predictions V*, vol. 8127, SPIE, Sep. 2011, pp. 146–156.
- [160] D. Zhao et al., “Optimization design and performance test of optical antenna for laser communication”, in *9th International Symposium on Advanced Optical Manufacturing and Testing Technologies: Micro- and Nano-Optics, Catenary Optics, and Subwavelength Electromagnetics*, vol. 10840, SPIE, Jan. 2019, pp. 292–301.
- [161] M. Toyoshima, “Maximum fiber coupling efficiency and optimum beam size in the presence of random angular jitter for free-space laser systems and their applications”, *JOSA A*, vol. 23, no. 9, pp. 2246–2250, Sep. 2006.
- [162] Y. Chen et al., “Coupling efficiency of a plane wave to few-mode fiber in the presence of beam random angular jitter”, *Journal of the Optical Society of America B*, vol. 38, no. 12, F8–F15, Dec. 2021.
- [163] W. Liu et al., “Free space optical communication performance analysis with focal plane based wavefront measurement”, *Optics Communications*, vol. 309, pp. 212–220, Nov. 2013.
- [164] Y. Liu et al., “The Effect of Spatial Mode Distribution on Coupling Efficiency of Single-Mode Fiber: Theoretical Analysis and Experimental Verification”, *Applied Sciences*, vol. 9, no. 16, p. 3296, Jan. 2019.
- [165] X. Xue et al., “Thermal vacuum optical performance test system for space laser communication terminal”, in *Third International Conference on Photonics and Optical Engineering*, vol. 11052, SPIE, Jan. 2019, pp. 289–296.
- [166] L. Li et al., “Micro-vibration suppression methods and key technologies for high-precision space optical instruments”, *Acta Astronautica*, vol. 180, pp. 417–428, Mar. 2021.

- [167] T. Hughes et al., "Design Optimization of Reaction Wheel Assembly Characteristics to Reduce Spacecraft Vibrations", in *AIAA SciTech Forum*, 2022.
- [168] C. Dennehy and O. S. Alvarez-Salazar, "Spacecraft Micro-Vibration: A Survey of Experiences, Potential Solutions, and Some Lessons Learned", Technical Memorandum 20180006315, 2018, p. 26.
- [169] C. Liu et al., "Recent advances in micro-vibration isolation", *Mechanical Systems and Signal Processing*, vol. 56-57, pp. 55–80, May 2015.
- [170] T. T. Hyde and L. P. Davis, "Vibration reduction for commercial optical intersatellite communication links", in *Smart Structures and Materials 1998: Smart Structures and Integrated Systems*, vol. 3329, SPIE, Jul. 1998, pp. 94–105.
- [171] X. Jiao et al., "Advances in spacecraft micro-vibration suppression methods", *Progress in Aerospace Sciences*, vol. 138, p. 100 898, Apr. 2023.
- [172] Y. Kaymak et al., "A Survey on Acquisition, Tracking, and Pointing Mechanisms for Mobile Free-Space Optical Communications", *IEEE Communications Surveys & Tutorials*, vol. 20, no. 2, pp. 1104–1123, 2018.
- [173] E. Miller et al., "Fine pointing and tracking concepts for optical intersatellite links", in *2017 IEEE International Conference on Space Optical Systems and Applications (ICSOS)*, Nov. 2017, pp. 208–213.
- [174] M. S. Islam and O. Boyraz, "Large Aperture and Wide Field of View Meta-Receiver for Free Space Optical Communications", in *Conference on Lasers and Electro-Optics*, Optica Publishing Group, May 2022.
- [175] A. A. Farid and S. Hranilovic, "Outage Capacity Optimization for Free-Space Optical Links With Pointing Errors", *Journal of Lightwave Technology*, vol. 25, no. 7, pp. 1702–1710, Jul. 2007.
- [176] P. X. Do et al., "Numerical and analytical approaches to dynamic beam waist optimization for LEO-to-GEO laser communication", *OSA Continuum*, vol. 3, no. 12, pp. 3508–3522, Dec. 2020.
- [177] V. Mai and H. Kim, "Beam Size Optimization and Adaptation for High-Altitude Airborne Free-Space Optical Communication Systems", *IEEE Photonics Journal*, vol. PP, pp. 1–1, Feb. 2019.
- [178] S. Arnon et al., "Performance limitations of free-space optical communication satellite networks due to vibrations: Direct-detection digital mode", in *10th Meeting on Optical Engineering in Israel*, vol. 3110, SPIE, Sep. 1997, pp. 357–368.
- [179] A. K. Majumdar, *Advanced Free Space Optics (FSO): A Systems Approach* (Springer series in optical sciences volume 186). New York: Springer, 2015.
- [180] J. Miao et al., *Spacecraft thermal control technologies* (Space Science and Technologies). Singapore: Springer, 2021.
- [181] J. Meseguer et al., *Spacecraft Thermal Control*. Elsevier, Aug. 2012.
- [182] M. Lesniewski and T. Kryszczynski, "Athermalization of optical systems", in *Tenth Polish-Czech-Slovak Optical Conference: Wave and Quantum Aspects of Contemporary Optics*, vol. 3320, SPIE, Jan. 1998, pp. 297–306.
- [183] A. Ahmad, *Handbook of optomechanical engineering*. CRC Press, 2017.
- [184] S. K. S.-U. Rahman et al., "High performance all metal telescope for satellite based laser communication terminals", in *Sensors and Systems for Space Applications XV*, vol. 12121, SPIE, Jun. 2022, pp. 153–167.

- [185] H. T. Eyyuboglu et al., “Flat topped beams and their characteristics in turbulent media”, *Optics Express*, vol. 14, no. 10, pp. 4196–4207, May 2006.
- [186] N. Kumar and V. Khandelwal, “Compensation of wavefront aberration using oppositional-breeding artificial fish swarm algorithm in free space optical communication”, *Journal of Optics*, Aug. 2022.
- [187] R. E. Morgan et al., “On-orbit operations summary for the Deformable Mirror Demonstration Mission (DeMi) CubeSat”, in *Adaptive Optics Systems VIII*, vol. 12185, SPIE, Aug. 2022, pp. 2423–2432.
- [188] N. Nabavi and T. J. Hall, “Symmetric signal and local oscillator polarization diverse coherent optical receiver”, *Optics Express*, vol. 24, no. 3, pp. 2391–2405, Feb. 2016.
- [189] V. V. Mai and H. Kim, “Non-Mechanical Beam Steering and Adaptive Beam Control Using Variable Focus Lenses for Free-Space Optical Communications”, *Journal of Lightwave Technology*, vol. 39, no. 24, pp. 7600–7608, Dec. 2021.
- [190] P. F. McManamon and A. Ataei, “Progress and opportunities in optical beam steering”, in *Quantum Sensing and Nano Electronics and Photonics XVI*, vol. 10926, SPIE, May 2019, pp. 109–129.
- [191] Z. Wang et al., “Liquid crystal spatial light modulator based non-mechanical beam steering system fractional-order model”, *Optics Express*, vol. 30, no. 7, pp. 12 178–12 191, Mar. 2022.
- [192] H. Guo et al., “Research review on the development of liquid crystal optical phased array device on the application of free space laser communication”, in *9th International Symposium on Advanced Optical Manufacturing and Testing Technologies: Meta-Surface-Wave and Planar Optics*, vol. 10841, SPIE, Jan. 2019, pp. 69–89.
- [193] M. V. Gorkunov et al., “Non-Mechanical Multiplexed Beam-Steering Elements Based on Double-Sided Liquid Crystal Metasurfaces”, *Photonics*, vol. 9, no. 12, p. 986, Dec. 2022.
- [194] R. Mahon et al., “Two dimensional thermo-optic beam steering using a silicon photonic optical phased array”, in *Free-Space Laser Communication and Atmospheric Propagation XXVIII*, vol. 9739, SPIE, Mar. 2016, pp. 224–230.
- [195] D. A. Goldman et al., “MOEMS-Based Lens-Assisted Beam Steering for Free-Space Optical Communications”, *Journal of Lightwave Technology*, vol. 41, no. 9, pp. 2675–2690, May 2023.
- [196] F. Ishola et al., “Conceptual design and analysis of a compact liquid crystal on silicon non-mechanical optical beam steering antenna for lean platforms”, in *Free-Space Laser Communications XXXV*, vol. 12413, SPIE, Mar. 2023, pp. 160–171.
- [197] Z. Wang et al., “On-chip integration of metasurface-doublet for optical phased array with enhanced beam steering”, *Nanophotonics*, Jan. 2023.
- [198] D. Renner et al., “Integrated Photonics Technologies for Sensing and Free Space Communication”, in *Applications of Lasers for Sensing and Free Space Communications*, Optica Publishing Group, Sep. 2019, LW3B.2.
- [199] C.-Y. Hsu et al., “Free-Space Applications of Silicon Photonics: A Review”, *Micromachines*, vol. 13, no. 7, p. 990, Jul. 2022.
- [200] V. Billault et al., “Robust free space optical communication receiver based on a spatial mode demultiplexer and a silicon photonic coherent combining circuit”, in *Free-Space Laser Communication*

- tions XXXIV, vol. 11993, SPIE, Mar. 2022, pp. 16–20.
- [201] M. Milanizadeh et al., “Separating arbitrary free-space beams with an integrated photonic processor”, *Light: Science & Applications*, vol. 11, no. 1, p. 197, Jul. 2022.
- [202] G. N. Tzintzarov et al., “Integrated Silicon Photonics for Enabling Next-Generation Space Systems”, *Photonics*, vol. 8, no. 4, p. 131, Apr. 2021.
- [203] S. Piacentini et al., “Space Qualification of Ultrafast Laser-Written Integrated Waveguide Optics”, *Laser & Photonics Reviews*, vol. 15, no. 2, p. 2 000 167, 2021.
- [204] R. T. Logan and D. Basuita, “Photonic components for spacecraft fiber optic datalinks and free-space optical communications terminals”, in *International Conference on Space Optics — ICSSO 2020*, vol. 11852, SPIE, Jun. 2021, pp. 1754–1761.
- [205] V. V. Corbacho et al., “Review on thermal and mechanical challenges in the development of deployable space optics”, *Journal of Astronomical Telescopes, Instruments, and Systems*, vol. 6, no. 1, p. 010 902, Mar. 2020.
- [206] N. Devaney, “Active Optics for Space Telescopes”, in *Optical Design and Fabrication 2019 (Freeform, OFT) (2019)*, paper OW4A.4, Optica Publishing Group, Jun. 2019, OW4A.4.
- [207] N. Devaney et al., “Development of a prototype active optics system for future space telescopes”, *Applied Optics*, vol. 57, no. 22, E101–E106, Aug. 2018.
- [208] M. Laslandes et al., “Mirror actively deformed and regulated for applications in space: Design and performance”, *Optical Engineering*, vol. 52, May 2013.
- [209] D. Grosse et al., “Adaptive Optics for Satellite Imaging and Earth Based Space Debris Manoeuvres”, in *Proc. 7th European Conference on Space Debris*, Darmstadt, Germany,, Darmstadt, Germany, ESA Space Debris Office, 2017.
- [210] X. Ke and P. Wu, *Adaptive Optics Theory and Its Application in Optical Wireless Communication: 237*. Springer, Feb. 2022.
- [211] M. Al Rifaie et al., “Advances in mechanical metamaterials for vibration isolation: A review”, *Advances in Mechanical Engineering*, vol. 14, no. 3, p. 16 878 132 221 082 872, Mar. 2022.
- [212] J. Y. Yoon et al., “Pendulum-type elastic metamaterial for reducing the vibration of a space tether”, *Acta Astronautica*, vol. 162, pp. 359–366, Sep. 2019.
- [213] G. Ruffato et al., “Test of mode-division multiplexing and demultiplexing in free-space with diffractive transformation optics”, *Optics Express*, vol. 25, no. 7, pp. 7859–7868, Apr. 2017.
- [214] S. V. Karpeev et al., “Free-Space Transmission and Detection of Various Polarized Near-IR Beams Using Standard Communication Systems with Embedded Singular Phase Structures”, *Sensors*, vol. 22, no. 3, p. 890, Jan. 2022.
- [215] K. J. Kataja et al., “Diffractive optical element in free-space data transmission systems”, in *Diffractive/Holographic Technologies and Spatial Light Modulators VII*, vol. 3951, SPIE, Mar. 2000, pp. 94–101.
- [216] P. Blattner et al., “Diffractive optics for compact space communication terminals”, *Journal of Modern Optics*, vol. 43, Jul. 1996.
- [217] S.-Y. Yu et al., “Design of a combined functional phase-only diffractive element in intersatellite optical communications”, in *Optical Transmission, Switching, and Subsystems III*, vol. 6021, SPIE, Dec. 2005, pp. 926–933.

- [218] J. Jia et al., “Superresolution technology for reduction of the far-field diffraction spot size in the laser free-space communication system”, *Optics Communications*, vol. 228, no. 4, pp. 271–278, Dec. 2003.
- [219] J. Wang and C. Xue, “Athermalization and thermal characteristics of multilayer diffractive optical elements”, *Applied Optics*, vol. 54, no. 33, pp. 9665–9670, Nov. 2015.
- [220] M. Piao et al., “Optimization method of multilayer diffractive optical elements with consideration of ambient temperature”, *Applied Optics*, vol. 57, no. 30, pp. 8861–8869, Oct. 2018.
- [221] A. Arbabi and A. Faraon, “Advances in optical metalenses”, *Nature Photonics*, pp. 1–10, Dec. 2022.
- [222] M. S. Islam et al., “Metalens wide-angle receiver for free space optical communications”, in *Current Developments in Lens Design and Optical Engineering XXII*, R. B. Johnson et al., Eds., San Diego, United States: SPIE, Aug. 2021, p. 11.
- [223] H. Tan et al., “A Free-Space Orbital Angular Momentum Multiplexing Communication System Based on a Metasurface”, *Laser & Photonics Reviews*, vol. 13, no. 6, p. 1800278, 2019.
- [224] T. Dinter et al., “Metasurface Measuring Twisted Light in Turbulence”, *ACS Photonics*, vol. 9, no. 9, pp. 3043–3051, Sep. 2022.
- [225] F. Cheng et al., “Mechanically tunable focusing metamirror in the visible”, *Optics Express*, vol. 27, no. 11, pp. 15 194–15 204, May 2019.
- [226] Z. Guanxing et al., “Reconfigurable metasurfaces with mechanical actuations: Towards flexible and tunable photonic devices”, *Journal of Optics*, vol. 23, no. 1, p. 013 001, Dec. 2020.
- [227] T. Lewi et al., “Thermal tuning capabilities of semiconductor metasurface resonators”, *Nanophotonics*, vol. 8, no. 2, pp. 331–338, Feb. 2019.
- [228] J. Young, “Advanced Concepts for Small Satellite Thermal Control”, in *Small Satellite Conference*, Aug. 2021.
- [229] R. C. Consolo and S. K. S. Boetcher, “Advances in spacecraft thermal control”, in *Advances in Heat Transfer*, Elsevier, Jun. 2023.
- [230] I. Garmendia et al., “Design and Fabrication of a Phase Change Material Heat Storage Device for the Thermal Control of Electronics Components of Space Applications”, *Aerospace*, vol. 9, no. 3, p. 126, Feb. 2022.
- [231] P. Kuruba et al., “The future of thermal stability in small satellites using aerogel and graphene”, in *AIP Conference Proceedings*, vol. 2297, Dec. 2020.
- [232] M. Delkowski et al., “Multifunctional Nanostructures with Controllable Band Gap Giving Highly Stable Infrared Emissivity for Smart Thermal Management”, *ACS Nano*, vol. 17, no. 2, pp. 1335–1343, Jan. 2023.
- [233] L. Anderson et al., “CubeSat active thermal control in support of advanced payloads: The active thermal architecture project”, in *CubeSats and SmallSats for Remote Sensing V*, vol. 11832, SPIE, Aug. 2021, p. 1183 203.
- [234] L. de Forges de Parny et al., “Satellite-based quantum information networks: Use cases, architecture, and roadmap”, *Communications Physics*, vol. 6, no. 1, pp. 1–17, Jan. 2023.
- [235] V. Scarani et al., “The security of practical quantum key distribution”, *Reviews of Modern Physics*, vol. 81, no. 3, pp. 1301–1350, Sep. 2009.
- [236] Y. Huang et al., “Vacuum Beam Guide for Large Scale Quantum Networks”,

- Physical Review Letters*, vol. 133, no. 2, p. 020 801, Jul. 2024.
- [237] R. Bedington et al., “Progress in satellite quantum key distribution”, *npj Quantum Information*, vol. 3, no. 1, 2017.
- [238] S. Goswami and S. Dhara, “Satellite-Relayed Global Quantum Communication without Quantum Memory”, *Physical Review Applied*, vol. 20, no. 2, p. 024 048, Aug. 2023.
- [239] S. Goswami et al., *Satellites promise global-scale quantum networks*, May 2025.
- [240] S. Wehner et al., “Quantum internet: A vision for the road ahead”, *Science*, vol. 362, no. 6412, eaam9288, 2018.
- [241] E. Diamanti et al., “Practical challenges in quantum key distribution”, *npj Quantum Information*, vol. 2, no. 1, p. 16 025, Nov. 2016.
- [242] F. Xu et al., “Secure quantum key distribution with realistic devices”, *Reviews of Modern Physics*, vol. 92, no. 2, p. 025 002, May 2020.
- [243] J. Sidhu et al., “Advances in space quantum communications”, *IET Quantum Communication*, vol. 2, no. 4, pp. 182–217, 2021.
- [244] T. Jennewein et al., “QEYSSat 2.0—white paper on satellite-based quantum communication missions in Canada”, *Canadian Journal of Physics*, vol. 103, no. 4, pp. 328–376, Apr. 2025.
- [245] C.-Y. Lu et al., “Micius quantum experiments in space”, *Reviews of Modern Physics*, vol. 94, no. 3, p. 035 001, Jul. 2022.
- [246] Y. Li et al., “Microsatellite-based real-time quantum key distribution”, *Nature*, pp. 1–8, Mar. 2025.
- [247] R. Bedington et al., “Nanosatellite experiments to enable future space-based QKD missions”, *EPJ Quantum Technology*, vol. 3, no. 1, pp. 1–10, Dec. 2016.
- [248] L. Paccard et al., “The Role of the Satellite in Quantum Information Networks”, in *International Conference on Space Optics — ICSO 2024*, Jul. 2025, p. 192.
- [249] H. Podmore et al., “Quantum and classical laser communication terminals on the QEYSSat mission”, in *International Conference on Space Optics — ICSO 2024*, vol. 13699, SPIE, Jul. 2025, pp. 1551–1562.
- [250] J. H. Shapiro, “Scintillation has minimal impact on far-field Bennett-Brassard 1984 protocol quantum key distribution”, *Physical Review A*, vol. 84, no. 3, p. 032 340, Sep. 2011.
- [251] M. Toyoshima et al., “Polarization measurements through space-to-ground atmospheric propagation paths by using a highly polarized laser source in space”, *Optics Express*, vol. 17, no. 25, pp. 22 333–22 340, Dec. 2009.
- [252] C. Ivan et al., “Impact of Turbulence in Long Range Quantum and Classical Communications”, *Physical review letters*, vol. 109, Jul. 2012.
- [253] D. Vasylyev et al., “Satellite-mediated quantum atmospheric links”, *Physical Review A*, vol. 99, no. 5, p. 053 830, May 2019.
- [254] V. Marulanda Acosta et al., “Analysis of satellite-to-ground quantum key distribution with adaptive optics”, *New Journal of Physics*, vol. 26, no. 2, p. 023 039, Feb. 2024.
- [255] M. T. Gruneisen et al., “Modeling satellite-Earth quantum channel downlinks with adaptive-optics coupling to single-mode fibers”, *Optical Engineering*, vol. 56, no. 12, p. 126 111, Dec. 2017.
- [256] C. Simmons et al., “Dawn and dusk satellite quantum key distribution using time- and phase-based encoding and polarization filtering”, *Optica Quantum*, vol. 2, no. 5, pp. 381–389, Oct. 2024.

- [257] A. Kržič et al., “Adjustable spatial filter for optimal free-space quantum communication round the clock”, *Optics Express*, vol. 31, no. 20, pp. 32 362–32 372, Sep. 2023.
- [258] D. L. Bakker et al., “A Best-Path Approach to the Design of a Hybrid Space–Ground Quantum Network with Dynamic Constraints”, *Photonics*, vol. 11, no. 3, p. 268, Mar. 2024.
- [259] M. Polnik et al., “Scheduling Space-to-Ground Optical Communication Under Cloud Cover Uncertainty”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 57, no. 5, pp. 2838–2849, Oct. 2021.
- [260] L. de Forges de Parny et al., “Towards a Global Scale Quantum Information Network: A Study Applied to Satellite-Enabled Distributed Quantum Computing”, *Entropy*, vol. 27, no. 11, p. 1166, Nov. 2025.
- [261] D. Gottesman et al., “Longer-Baseline Telescopes Using Quantum Repeaters”, *Physical Review Letters*, vol. 109, no. 7, p. 070 503, Aug. 2012.
- [262] P. Xu et al., “Satellite testing of a gravitationally induced quantum decoherence model”, *Science*, vol. 366, no. 6461, pp. 132–135, Oct. 2019.
- [263] J. P. Covey et al., “Probing Curved Spacetime with a Distributed Atomic Processor Clock”, *PRX Quantum*, vol. 6, no. 3, p. 030 310, Jul. 2025.
- [264] J. Borregaard and I. Pikovski, “Testing quantum theory on curved spacetime with quantum networks”, *Physical Review Research*, vol. 7, no. 2, p. 023 192, May 2025.
- [265] M. Mohageg et al., “The deep space quantum link: Prospective fundamental physics experiments using long-baseline quantum optics”, *EPJ Quantum Technology*, vol. 9, no. 1, pp. 1–76, Dec. 2022.
- [266] M. Mohageg et al., “Towards satellite tests combining general relativity and quantum mechanics through quantum optical interferometry: Progress on the deep space quantum link”, *EPJ Quantum Technology*, vol. 12, no. 1, pp. 1–13, Dec. 2025.
- [267] A. Belenchia et al., “Quantum physics in space”, *Physics Reports*, vol. 951, pp. 1–70, 2022.
- [268] F. Nothlawala et al., *Orbital angular momentum of entangled photons as a probe for relativistic effects*, Aug. 2025.
- [269] G. B. Xavier et al., “Energy-time and time-bin entanglement: Past, present and future”, *npj Quantum Information*, vol. 11, no. 1, p. 129, Jul. 2025.
- [270] S. Cocchi et al., “Time-bin encoding quantum key distribution in free-space horizontal links during nighttime and daytime”, *Optica Quantum*, vol. 3, no. 4, pp. 346–350, Aug. 2025.
- [271] N. Tagliavacche et al., “Frequency-bin entanglement-based Quantum Key Distribution”, *npj Quantum Information*, vol. 11, no. 1, p. 60, Apr. 2025.
- [272] A. Scott et al., “The QEYSSAT mission: On-orbit demonstration of secure optical communications network technologies”, in *Environmental Effects on Light Propagation and Adaptive Systems III*, vol. 11532, SPIE, Sep. 2020, pp. 71–76.
- [273] A. Carrasco-Casado et al., “LEO-to-ground polarization measurements aiming for space QKD using Small Optical TrAnsponder (SOTA)”, *Optics Express*, vol. 24, no. 11, p. 12 254, May 2016.
- [274] S. Vinet, “Towards satellite-assisted quantum communication with reconfigurable networks and frequency-bin qubits”, Ph.D. dissertation, University of Waterloo, Sep. 2025.
- [275] V. D. Tubío et al., *Satellite-assisted quantum communication with single pho-*

- ton sources and atomic memories, Nov. 2024.
- [276] J. Meyer et al., “Analogy of free-space quantum key distribution using spatial modes of light: Scaling up the distance and the dimensionality”, *Optics Letters*, Vol. 50, Issue 10, pp. 3297–3300, May 2025.
- [277] V. D’Ambrosio et al., “Complete experimental toolbox for alignment-free quantum communication”, *Nature Communications*, vol. 3, no. 1, p. 961, Jul. 2012.
- [278] G. Vallone et al., “Free-Space Quantum Key Distribution by Rotation-Invariant Twisted Photons”, *Physical Review Letters*, vol. 113, no. 6, p. 060 503, Aug. 2014.
- [279] D. Dequal et al., “Feasibility of satellite-to-ground continuous-variable quantum key distribution”, *npj Quantum Information*, vol. 7, no. 1, pp. 1–10, Jan. 2021.
- [280] D. Orsucci et al., *Assessment of practical satellite quantum key distribution architectures for current and near-future missions*, Apr. 2024.
- [281] S. Bose et al., “Purification via entanglement swapping and conserved entanglement”, *Physical Review A*, vol. 60, no. 1, pp. 194–197, Jul. 1999.
- [282] H.-K. Lo et al., “Decoy State Quantum Key Distribution”, *Physical Review Letters*, vol. 94, no. 23, p. 230 504, Jun. 2005.
- [283] J. Yin et al., “Satellite-to-Ground Entanglement-Based Quantum Key Distribution”, *Physical Review Letters*, vol. 119, no. 20, p. 200 501, Nov. 2017.
- [284] C. Liorni et al., “Satellite-based links for quantum key distribution: Beam effects and weather dependence”, *New Journal of Physics*, vol. 21, no. 9, p. 093 055, Sep. 2019.
- [285] B. Huttner et al., “Long-range QKD without trusted nodes is not possible with current technology”, *npj Quantum Information*, vol. 8, no. 1, pp. 1–5, Sep. 2022.
- [286] K. Boone et al., “Entanglement over global distances via quantum repeaters with satellite links”, *Physical Review A*, vol. 91, no. 5, p. 052 325, May 2015.
- [287] C. Liorni et al., “Quantum repeaters in space”, *New Journal of Physics*, vol. 23, no. 5, p. 053 021, May 2021.
- [288] M. Gündoğan et al., “Proposal for spaceborne quantum memories for global quantum networking”, *npj Quantum Information*, vol. 7, no. 1, pp. 1–10, Aug. 2021.
- [289] J. Wallnöfer et al., “Simulating quantum repeater strategies for multiple satellites”, *Communications Physics*, vol. 5, no. 1, pp. 1–8, Jun. 2022.
- [290] M. Gündoğan et al., “Time-delayed single satellite quantum repeater node for global quantum communications”, *Optica Quantum*, vol. 2, no. 3, pp. 140–147, Jun. 2024.
- [291] N. Sangouard et al., “Quantum repeaters based on atomic ensembles and linear optics”, *Reviews of Modern Physics*, vol. 83, no. 1, pp. 33–80, Mar. 2011.
- [292] J.-W. Ji et al., *A global quantum network with ground-based single-atom memories in optical cavities and satellite links*, Jul. 2025.
- [293] K. Heshami et al., “Quantum memories: Emerging applications and recent advances”, *Journal of Modern Optics*, vol. 63, no. 20, pp. 2005–2028, Nov. 2016.
- [294] C. Simon et al., “Quantum memories”, *The European Physical Journal D*, vol. 58, no. 1, pp. 1–22, May 2010.
- [295] Y. Lei et al., “Quantum optical memory for entanglement distribution”, *Optica*, vol. 10, no. 11, pp. 1511–1528, Nov. 2023.
- [296] A. Alvaro et al., “GARBO QKD GEO: Challenges and architecture”, in *Inter-*

- national Conference on Space Optics — ICSO 2024*, vol. 13699, SPIE, Jul. 2025, pp. 1527–1540.
- [297] S. Khatri et al., “Spooky action at a global distance: Analysis of space-based entanglement distribution for the quantum internet”, *npj Quantum Information*, vol. 7, no. 1, pp. 1–15, Jan. 2021.
- [298] L.-J. Wang et al., “Experimental authentication of quantum key distribution with post-quantum cryptography”, *npj Quantum Information*, vol. 7, no. 1, p. 67, May 2021.
- [299] D. J. Bernstein et al., *Post-Quantum Cryptography*. Springer Science & Business Media, Feb. 2009.
- [300] H. Takenaka et al., “Satellite-to-ground quantum-limited communication using a 50-kg-class microsatellite”, *Nature Photonics*, vol. 11, no. 8, pp. 502–508, Aug. 2017.
- [301] K. Günthner et al., “Quantum-limited measurements of optical signals from a geostationary satellite”, *Optica*, vol. 4, no. 6, pp. 611–616, Jun. 2017.
- [302] J. Yin et al., “Experimental quasi-single-photon transmission from satellite to earth”, *Optics Express*, vol. 21, no. 17, pp. 20 032–20 040, Aug. 2013.
- [303] D. Dequal et al., “Experimental single-photon exchange along a space link of 7000 km”, *Physical Review A*, vol. 93, no. 1, p. 010 301, Jan. 2016.
- [304] Z. Tang et al., “Generation and Analysis of Correlated Pairs of Photons aboard a Nanosatellite”, *Physical Review Applied*, vol. 5, no. 5, p. 054 022, May 2016.
- [305] A. Villar et al., “Entanglement demonstration on board a nano-satellite”, *Optica*, vol. 7, no. 7, pp. 734–737, Jul. 2020.
- [306] A. Lenart et al., “Comparing a radiation damage model for avalanche photodiodes through in-situ observation of CubeSat based devices”, *Communications Physics*, vol. 8, no. 1, p. 118, Mar. 2025.
- [307] S.-K. Liao et al., “Satellite-to-ground quantum key distribution”, *Nature*, vol. 549, no. 7670, pp. 43–47, Sep. 2017.
- [308] S.-K. Liao et al., “Satellite-Relayed Intercontinental Quantum Network”, *Physical Review Letters*, vol. 120, no. 3, p. 030 501, Jan. 2018.
- [309] Y.-A. Chen et al., “An integrated space-to-ground quantum communication network over 4,600 kilometres”, *Nature*, vol. 589, no. 7841, pp. 214–219, Jan. 2021.
- [310] J. Yin et al., “Satellite-based entanglement distribution over 1200 kilometers”, *Science*, vol. 356, no. 6343, pp. 1140–1144, Jun. 2017.
- [311] T. Hiemstra et al., “The European satellite-based QKD system EAGLE-1”, in *Free-Space Laser Communications XXXVII*, vol. 13355, SPIE, Mar. 2025, pp. 216–222.
- [312] T. D. Jennewein, “QEYSSat: Canada’s first quantum communication satellite”, in *Photonics for Quantum 2024*, vol. PC13106, SPIE, Aug. 2024, PC1310604.
- [313] T. Jaeken et al., “Emulation of satellite up-link quantum communication with entangled photons”, *Optica Quantum*, vol. 3, no. 5, pp. 452–458, Oct. 2025.
- [314] A. Anwar et al., “Development of compact entangled photon-pair sources for satellites”, *Applied Physics Letters*, vol. 121, no. 22, p. 220 503, Nov. 2022.
- [315] G. Xu et al., “Satellite-to-ground optical communication systems under orbital deviations and atmospheric turbulence: Channel modeling and performance analysis”, *Optics Express*, vol. 33, no. 9, pp. 18 912–18 927, May 2025.
- [316] G. Amaral et al., “Towards the integration of a PIC-based entangled

- photon-pair source and an ensemble-based quantum memory compatible with the telecommunication C-band”, in *International Conference on Space Optics — ICSO 2024*, vol. 13699, SPIE, Jul. 2025, pp. 1541–1550.
- [317] A. Alvarez-Herrero et al., “Conceptual optical design for CARAMUEL payload: A quantum key distribution system from a GEO satellite”, in *Proc. SPIE*, vol. 12633, SPIE, Jul. 2023, pp. 72–87.
- [318] A. Lohrmann et al., “Classical clock synchronization for quantum communications using the quantum channel”, *Applied Optics*, vol. 62, no. 32, pp. 8567–8573, Nov. 2023.
- [319] C. Harney et al., “End-To-End Capacities of Hybrid Quantum Networks”, *Physical Review Applied*, vol. 18, Jul. 2022.
- [320] X.-Y. Luo et al., *Entangling quantum memories over 420 km in fiber*, Apr. 2025.
- [321] T. Chakraborty et al., “Towards a spectrally multiplexed quantum repeater”, *npj Quantum Information*, vol. 11, no. 1, p. 3, Jan. 2025.
- [322] P. M. Burdekin et al., “Enhancing quantum memories with light–matter interference”, *Optica*, vol. 12, no. 9, pp. 1514–1521, Sep. 2025.
- [323] Y. Yu et al., “Entanglement of two quantum memories via fibres over dozens of kilometres”, *Nature*, vol. 578, no. 7794, pp. 240–245, Feb. 2020.
- [324] D. Lago-Rivera et al., “Telecom-heralded entanglement between multimode solid-state quantum memories”, *Nature*, vol. 594, no. 7861, pp. 37–40, Jun. 2021.
- [325] E. Da Ros et al., “Proposal for a long-lived quantum memory using matter-wave optics with Bose-Einstein condensates in microgravity”, *Physical Review Research*, vol. 5, no. 3, p. 033 003, Jul. 2023.
- [326] F. Bussi eres et al., “Prospective applications of optical quantum memories”, *Journal of Modern Optics*, vol. 60, no. 18, pp. 1519–1537, Oct. 2013.
- [327] B. J. Brown et al., “Quantum memories at finite temperature”, *Reviews of Modern Physics*, vol. 88, no. 4, p. 045 005, Nov. 2016.
- [328] J. Morales et al., “Optical transmitter for time-bin encoding quantum key distribution”, *Journal of the Optical Society of America B*, vol. 40, no. 4, p. C15, Apr. 2023.
- [329] A. Tomaello et al., “Link budget and background noise for satellite quantum key distribution”, *Advances in Space Research*, vol. 47, no. 5, pp. 802–810, Mar. 2011.
- [330] M. Er-long et al., “Background noise of satellite-to-ground quantum key distribution”, *New Journal of Physics*, vol. 7, pp. 215–215, Oct. 2005.
- [331] W.-Q. Cai et al., “Free-space quantum key distribution during daylight and at night”, *Optica*, vol. 11, no. 5, pp. 647–652, May 2024.
- [332] P. Trinh et al., “Statistical verifications and deep-learning predictions for satellite-to-ground quantum atmospheric channels”, *Communications Physics*, vol. 5, p. 225, Sep. 2022.
- [333] Y. C. Tan et al., “Radiation tolerance of opto-electronic components proposed for space-based quantum key distribution”, *Journal of Modern Optics*, vol. 62, no. 20, pp. 1709–1712, Nov. 2015.
- [334] M. Yang et al., “Spaceborne, low-noise, single-photon detection for satellite-based quantum communications”, *Optics Express*, vol. 27, no. 25, pp. 36 114–36 128, Dec. 2019.

- [335] B. A. Wilson et al., “Radiation-Induced Dark Counts for Silicon Single-Photon Detectors in Space”, *Physical Review Applied*, vol. 16, no. 6, p. 064 049, Dec. 2021.
- [336] Z.-Y. Chen et al., “Radiation effect on silicon photonics chips for space quantum key distribution”, *Optics Express*, vol. 32, no. 2, pp. 2015–2028, Jan. 2024.
- [337] M. Yang et al., “In-orbit dark count rate performance and radiation damage high-temperature annealing of silicon avalanche photodiode single-photon detectors of the Micius satellite”, *Optics Express*, vol. 32, no. 7, pp. 12 601–12 608, Mar. 2024.
- [338] H. Y. Lim et al., *Thermo-mechanical design for a miniaturized quantum light source on board the SpooQy-1 CubeSat*, Jun. 2020.
- [339] C. Bonato et al., “Influence of satellite motion on polarization qubits in a Space-Earth quantum communication link”, *Optics Express*, vol. 14, no. 21, pp. 10 050–10 059, Oct. 2006.
- [340] P. Arteaga-Díaz et al., “Modeling the effect of steering mirrors on polarization for free-space quantum key distribution”, *Optik*, vol. 265, p. 169 434, Sep. 2022.
- [341] P. K. Dahal and D. R. Terno, “Polarization rotation and near-Earth quantum communications”, *Physical Review A*, vol. 104, no. 4, p. 042 610, Oct. 2021.
- [342] S. Murk et al., “Gravity-induced birefringence in spherically symmetric spacetimes”, *Physical Review D*, vol. 111, no. 4, p. 044 001, Feb. 2025.
- [343] C.-W. Chou et al., “Functional Quantum Nodes for Entanglement Distribution over Scalable Quantum Networks”, *Science*, vol. 316, no. 5829, pp. 1316–1320, Jun. 2007.
- [344] X.-Q. Zhu et al., “Transportable single-crystal silicon ultra-stable cavity toward space applications”, *Optica*, vol. 12, no. 9, pp. 1342–1349, Sep. 2025.
- [345] M. T. Dabiri et al., *Physical Limits of Entanglement-Based Quantum Key Distribution over Long-Distance Satellite Links*, Jun. 2025.
- [346] M. T. Dabiri et al., *On the Impact of Tracking Inaccuracy in Space-Based Quantum Key Distribution: A Stochastic Geometric Approach*, May 2025.
- [347] W. Yan et al., “Iso-propagation vortices with OAM-independent size and divergence toward future faster optical communications”, *Advanced Photonics*, vol. 6, no. 3, p. 036 002, May 2024.
- [348] R. Ursin et al., “Entanglement-based quantum communication over 144 km”, *Nature Physics*, vol. 3, no. 7, pp. 481–486, Jul. 2007.
- [349] J.-P. Bourgoin et al., “A comprehensive design and performance analysis of low Earth orbit satellite quantum communication”, *New Journal of Physics*, vol. 15, no. 2, p. 023 006, Feb. 2013.
- [350] Z.-D. Hu et al., “Impacts of low-order aberrations on capacity of orbital-angular-momentum quantum states”, *Communications in Theoretical Physics*, 2023.
- [351] A. Pilan Zanoni et al., “Thermal performance of a radiatively cooled system for quantum optomechanical experiments in space”, *Applied Thermal Engineering*, vol. 107, pp. 689–699, Aug. 2016.
- [352] M. Toyoshima et al., “Polarization-Basis Tracking Scheme in Satellite Quantum Key Distribution”, *International Journal of Optics*, vol. 2011, no. 1, p. 254 154, 2011.
- [353] S.-H. Yin et al., “Polarization compensation for entanglement-based quantum key distribution”, *Optics Express*, vol. 33, no. 18, pp. 38 431–38 439, Sep. 2025.

- [354] S. Chatterjee et al., “Polarization bases compensation towards advantages in satellite-based QKD without active feedback”, *Communications Physics*, vol. 6, no. 1, p. 116, May 2023.
- [355] J. F. Fitzsimons, “Private quantum computation: An introduction to blind quantum computing and related protocols”, *npj Quantum Information*, vol. 3, no. 1, p. 23, Jun. 2017.
- [356] R. Van Meter and S. J. Devitt, “The Path to Scalable Distributed Quantum Computing”, *Computer*, vol. 49, no. 9, pp. 31–42, Sep. 2016.
- [357] P. Kómár et al., “A quantum network of clocks”, *Nature Physics*, vol. 10, no. 8, pp. 582–587, Aug. 2014.
- [358] X. Guo et al., “Distributed quantum sensing in a continuous-variable entangled network”, *Nature Physics*, vol. 16, no. 3, pp. 281–284, Mar. 2020.
- [359] L.-Z. Liu et al., “Distributed quantum phase estimation with entangled photons”, *Nature Photonics*, vol. 15, no. 2, pp. 137–142, Feb. 2021.
- [360] D. E. Bruschi et al., “Testing the effects of gravity and motion on quantum entanglement in space-based experiments”, *New Journal of Physics*, vol. 16, no. 5, p. 053 041, May 2014.
- [361] G. Giusfredi, *Physical Optics*. 2019.
- [362] M. Lax et al., “From Maxwell to paraxial wave optics”, *Physical Review A*, vol. 11, no. 4, pp. 1365–1370, Apr. 1975.
- [363] H. Urbach et al., “On the modeling of optical systems containing elements of different scales”, *Journal of Modern Optics*, vol. 58, no. 5-6, pp. 496–508, Mar. 2011.
- [364] J. D. Schmidt, *Numerical Simulation of Optical Wave Propagation with Examples in MATLAB*. SPIE, Jul. 2010.
- [365] I. Bialynicki-Birula, “V Photon Wave Function”, in *Progress in Optics*, E. Wolf, Ed., vol. 36, Elsevier, Jan. 1996, pp. 245–294.
- [366] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*. Cambridge University Press, 1995.
- [367] M. Badás et al., “Seidel optical aberrations and optimum truncated Gaussian beams on intersatellite free space optical communications”, *Optics Express*, Jul. 2025.
- [368] H. Du et al., “A new statistical fading model for loss factor induced by angle-of-arrival fluctuations in free space optical communication”, *Optics Express*, vol. 32, no. 14, pp. 24 926–24 946, Jul. 2024.
- [369] Y. Yang et al., “Effects of localized deformation induced by reflector antenna on received power”, *Optics Communications*, vol. 282, no. 3, pp. 396–400, Feb. 2009.
- [370] W. Xie et al., “Received power analysis due to antenna deformation based on wavelet in inter-satellite laser communication links”, *Optik*, vol. 123, no. 8, pp. 670–674, Apr. 2012.
- [371] Y. Zhu et al., “Average bit-error rate analysis of an inter-satellite optical communication system under the effect of perturbations”, *Optics Express*, vol. 32, no. 21, pp. 36 796–36 810, Oct. 2024.
- [372] M. Toyoshima et al., “Mutual alignment errors due to the variation of wave-front aberrations in a free-space laser communication link”, *Optics Express*, vol. 9, no. 11, pp. 592–602, Nov. 2001.
- [373] J. Sun et al., “Mutual alignment errors due to wave-front aberrations in intersatellite laser communications”, *Applied Optics*, vol. 44, no. 23, pp. 4953–4958, Aug. 2005.
- [374] Y. Yang et al., “Mutual alignment errors due to localized distortion in free-space laser communication links”, *Optics Com-*

- munications*, vol. 281, no. 17, pp. 4180–4187, Sep. 2008.
- [375] L. Tan et al., “Pointing and tracking errors due to localized deformation in intersatellite laser communication links”, *Optics Express*, vol. 16, no. 17, pp. 13 372–13 380, Aug. 2008.
- [376] Y. Yang et al., “Pointing and tracking errors due to localized distortion induced by a transmission-type antenna in intersatellite laser communications”, *Applied Optics*, vol. 48, no. 4, pp. 786–791, Feb. 2009.
- [377] A. Yoshida, “Spherical aberration in beam optical systems”, *Applied Optics*, vol. 21, no. 10, pp. 1812–1816, May 1982.
- [378] V. N. Mahajan, “Uniform versus Gaussian beams: A comparison of the effects of diffraction, obscuration, and aberrations”, *J. Opt. Soc. Am. A*, vol. 3, no. 4, pp. 470–485, Apr. 1986.
- [379] P. J. Cronin et al., “High-aperture diffraction of a scalar, off-axis Gaussian beam”, *Journal of the Optical Society of America A*, vol. 17, no. 9, pp. 1556–1564, Sep. 2000.
- [380] E. Cojocaru et al., “Astigmatism together with longitudinal focal shift in off-axis optical systems”, *Applied Optics*, vol. 29, no. 28, pp. 4208–4211, Oct. 1990.
- [381] S. C. Biswas and J.-E. Villeneuve, “Diffraction of a laser beam by a circular aperture under the combined effect of three primary aberrations”, *Applied Optics*, vol. 25, no. 13, pp. 2221–2232, Jul. 1986.
- [382] Y. Yang et al., “Research on Bit Error Rate in the Presence of Local Wavefront Aberration in Intersatellite Laser Communications”, *Journal of Lightwave Technology*, vol. 29, no. 19, pp. 2893–2898, Oct. 2011.
- [383] K. Niu and C. Tian, “Zernike polynomials and their applications”, *Journal of Optics*, vol. 24, no. 12, p. 123 001, Nov. 2022.
- [384] H. Rabal et al., “Diffraction by a Tilted Aperture”, *Optica Acta: International Journal of Optics*, vol. 32, no. 11, pp. 1309–1311, Nov. 1985.
- [385] K. Patorski, “Fraunhofer Diffraction Patterns of Titled Planar Objects”, *Optica Acta: International Journal of Optics*, vol. 30, no. 5, pp. 673–679, May 1983.
- [386] K. Matsushima et al., “Fast calculation method for optical diffraction on tilted planes by use of the angular spectrum of plane waves”, *Journal of the Optical Society of America A*, vol. 20, pp. 1755–62, Oct. 2003.
- [387] Y. Hu et al., “Diffraction modeling between arbitrary non-parallel planes using angular spectrum rearrangement”, *Optica*, vol. 12, no. 1, pp. 39–45, Jan. 2025.
- [388] P. F. I. Scott et al., “Real-time photon beam localization methods using high-resolution imagers and parallel processing using a reconfigurable system”, *Optical Engineering*, vol. 48, no. 7, p. 073 601, Jul. 2009.
- [389] M. Badás et al., “On the optimum far-field irradiance distribution using Laguerre-Gaussian beams for intersatellite free-space optical communications”, *Optics Express*, vol. 32, no. 18, pp. 31 597–31 620, Aug. 2024.
- [390] R. D. Nelson et al., “Experimental Verification Of The Pointing Error Distribution Of An Optical Intersatellite Link”, in *Free-Space Laser Communication Technologies*, vol. 0885, SPIE, May 1988, pp. 132–142.
- [391] M. Badás et al., “Impact of transmitter wavefront errors and pointing jitter on intersatellite free space optical communications”, in *Computational Optics 2024*, vol. 13023, SPIE, Jun. 2024, p. 1 302 302.

- [392] J. B. Thomas, *Introduction to Probability*. Springer, 1986.
- [393] H. T. Yura, "Optimum truncation of a Gaussian beam in the presence of random jitter", *JOSA A*, vol. 12, no. 2, pp. 375–379, Feb. 1995.
- [394] S. B. Alexander, *Optical communication receiver design* (SPIE tutorial texts in optical engineering v. TT 22). Bellingham, Wash., USA : London, UK: SPIE Optical Engineering Press ; Institution of Electrical Engineers, 1997.
- [395] A. A. Farid and S. Hranilovic, "Link reliability, range and rate optimization for free-space optical channels", in *2009 10th International Conference on Telecommunications*, Jun. 2009, pp. 19–23.
- [396] R. R. Hayes, "Fading statistics for inter-satellite optical communication", *Applied Optics*, vol. 36, no. 30, pp. 8063–8068, Oct. 1997.
- [397] F. Heine et al., "Status on laser communication activities at Tesat-Spacecom", in *Free-Space Laser Communications XXXV*, vol. 12413, SPIE, Mar. 2023, pp. 83–93.
- [398] K. Cahoy et al., "The CubeSat Laser Infrared Crosslink Mission (CLICK)", in *International Conference on Space Optics — ICSSO 2018*, N. Karafolas et al., Eds., Chania, Greece: SPIE, Jul. 2019, p. 33.
- [399] F. Kenny and N. Devaney, "Beam propagation simulations for LISA in the presence of telescope aberrations", *Classical and Quantum Gravity*, vol. 38, no. 3, p. 035 010, Dec. 2020.
- [400] E. Wolf, "The diffraction theory of aberrations", *Reports on Progress in Physics*, vol. 14, no. 1, p. 95, Jan. 1951.
- [401] V. Domínguez Tubío et al., "Satellite-assisted quantum communication with single photon sources and atomic memories", *Physical Review Research*, vol. 8, no. 1, p. 013 099, Jan. 2026.
- [402] S. Pirandola et al., "Advances in quantum cryptography", *Adv. Opt. Photon.*, vol. 12, no. 4, pp. 1012–1236, Dec. 2020.
- [403] H. Buhrman and H. Röhrig, "Distributed Quantum Computing", in *Mathematical Foundations of Computer Science 2003*, B. Rovan and P. Vojtáš, Eds., ser. Lecture Notes in Computer Science, Berlin, Heidelberg: Springer, 2003, pp. 1–20.
- [404] Z. Tang et al., "Quantum Networks for Resilient Power Grids: Theory and Simulated Evaluation", *IEEE Transactions on Power Systems*, vol. 38, no. 2, pp. 1189–1204, 2023.
- [405] B. C. Nichol et al., "An elementary quantum network of entangled optical atomic clocks", *Nature*, vol. 609, no. 7928, pp. 689–694, Sep. 2022.
- [406] W. K. Wootters and W. H. Zurek, "A single quantum cannot be cloned", *Nature*, vol. 299, no. 5886, pp. 802–803, Oct. 1982.
- [407] L. -. Duan et al., "Long-distance quantum communication with atomic ensembles and linear optics", *Nature*, vol. 414, no. 6862, pp. 413–418, Nov. 2001.
- [408] S. Muralidharan et al., "Ultrafast and Fault-Tolerant Quantum Communication across Long Distances", *Phys. Rev. Lett.*, vol. 112, no. 25, p. 250 501, Jun. 2014.
- [409] W. J. Munro et al., "Quantum communication without the necessity of quantum memories", *Nature Photonics*, vol. 6, no. 11, pp. 777–781, Nov. 2012.
- [410] J. Yin et al., "Entanglement-based secure quantum cryptography over 1,120 kilometres", *Nature*, vol. 582, no. 7813, pp. 501–505, Jun. 2020.
- [411] C. Schimpf et al., "Quantum dots as potential sources of strongly entangled photons: Perspectives and challenges for

- applications in quantum networks”, *Applied Physics Letters*, vol. 118, no. 10, p. 100 502, Mar. 2021.
- [412] K. C. Chen et al., “Zero-Added-Loss Entangled-Photon Multiplexing for Ground- and Space-Based Quantum Networks”, *Phys. Rev. Appl.*, vol. 19, no. 5, p. 054 029, May 2023.
- [413] Q. Zhang et al., “Demonstration of a scheme for the generation of “event-ready” entangled photon pairs from a single-photon source”, *Phys. Rev. A*, vol. 77, no. 6, p. 062 316, Jun. 2008.
- [414] J. McKeever et al., “Deterministic Generation of Single Photons from One Atom Trapped in a Cavity”, *Science*, vol. 303, no. 5666, pp. 1992–1994, 2004.
- [415] D. Bluvstein et al., “A quantum processor based on coherent transport of entangled atom arrays”, *Nature*, vol. 604, no. 7906, pp. 451–456, Apr. 2022.
- [416] A. W. Young et al., “Half-minute-scale atomic coherence and high relative stability in a tweezer clock”, *Nature*, vol. 588, no. 7838, pp. 408–413, Dec. 2020.
- [417] H. Levine et al., “Parallel Implementation of High-Fidelity Multiqubit Gates with Neutral Atoms”, *Phys. Rev. Lett.*, vol. 123, no. 17, p. 170 503, Oct. 2019.
- [418] Z. Fu et al., “High-fidelity entanglement of neutral atoms via a Rydberg-mediated single-modulated-pulse controlled-phase gate”, *Phys. Rev. A*, vol. 105, no. 4, p. 042 430, Apr. 2022.
- [419] S. J. Evered et al., “High-fidelity parallel entangling gates on a neutral-atom quantum computer”, *Nature*, vol. 622, no. 7982, pp. 268–272, Oct. 2023.
- [420] W. Huie et al., “Multiplexed telecommunication-band quantum networking with atom arrays in optical cavities”, *Phys. Rev. Res.*, vol. 3, no. 4, p. 043 154, Dec. 2021.
- [421] J. P. Covey et al., “Quantum networks with neutral atom processing nodes”, *npj Quantum Information*, vol. 9, no. 1, p. 90, Sep. 2023.
- [422] L. C. Andrews and R. L. Phillips, *Laser Beam Propagation Through Random Media*. SPIE, Sep. 2005.
- [423] S. D. Barrett and P. Kok, “Efficient high-fidelity quantum computation using matter qubits and linear optics”, *Phys. Rev. A*, vol. 71, no. 6, p. 060 310, Jun. 2005.
- [424] C. Cabrillo et al., “Creation of entangled states of distant atoms by interference”, *Phys. Rev. A*, vol. 59, no. 2, pp. 1025–1033, Feb. 1999.
- [425] E. Deist et al., “Superresolution Microscopy of Optical Fields Using Tweezer-Trapped Single Atoms”, *Phys. Rev. Lett.*, vol. 128, no. 8, p. 083 201, Feb. 2022.
- [426] H. Levine et al., “Dispersive optical systems for scalable Raman driving of hyperfine qubits”, *Phys. Rev. A*, vol. 105, no. 3, p. 032 618, Mar. 2022.
- [427] Y. Li and J. D. Thompson, “High-Rate and High-Fidelity Modular Interconnects between Neutral Atom Quantum Processors”, *PRX Quantum*, vol. 5, no. 2, p. 020 363, Jun. 2024.
- [428] D. Jaksch et al., “Fast Quantum Gates for Neutral Atoms”, *Phys. Rev. Lett.*, vol. 85, no. 10, pp. 2208–2211, Sep. 2000.
- [429] S. Jandura and G. Pupillo, “Time-Optimal Two- and Three-Qubit Gates for Rydberg Atoms”, *Quantum*, vol. 6, p. 712, May 2022.
- [430] O. A. Collins et al., “Multiplexed Memory-Insensitive Quantum Repeaters”, *Phys. Rev. Lett.*, vol. 98, no. 6, p. 060 502, Feb. 2007.
- [431] E. Brattich et al., “Measurements of aerosols and charged particles on the BEXUS18 stratospheric balloon”, *An-*

- nales *Geophysicae*, vol. 37, no. 3, pp. 389–403, Jun. 2019.
- [432] N. Gisin et al., “Quantum cryptography”, *Rev. Mod. Phys.*, vol. 74, no. 1, pp. 145–195, Mar. 2002.
- [433] M. Vitorica et al., “Entanglement purification on quantum networks”, *Phys. Rev. Res.*, vol. 5, no. 3, p. 033171, Sep. 2023.
- [434] V. Domínguez Tubío et al., “Quantum key distribution in the Iberian Peninsula”, *Optica Quantum*, vol. 4, no. 1, pp. 22–30, Feb. 2026.
- [435] O. Alia et al., “Quantum-safe 10 Gbps Site-to-Site IPsec VPN Tunnel over 46 km Deployed Fibre”, in *2024 Optical Fiber Communications Conference and Exhibition (OFC)*, Mar. 2024, pp. 1–3.
- [436] R. Rojo, *Telefónica y Vithas prueban el blindaje frente a ataques cuánticos en dos hospitales*, Feb. 2025.
- [437] A. E. Siegman, *Lasers*. University Science Books, 1986.
- [438] *Free Open-Source Weather API Open-Meteo.com*.
- [439] J.-M. Merolla et al., “High-performance 1560 nm entangled photon source for high secure key rates QKD satellite-based communications”, in *International Conference on Space Optics — ICSSO 2022*, vol. 12777, SPIE, Jul. 2023, pp. 897–908.
- [440] X. Ma et al., “Quantum key distribution with entangled photon sources”, *Physical Review A*, vol. 76, no. 1, p. 012307, Jul. 2007.
- [441] V. K. Rohatgi and A. K. M. E. Saleh, *An Introduction to Probability and Statistics*. John Wiley & Sons, Incorporated, 2001.
- [442] M. Chen et al., “Experimental demonstration of single-mode fiber coupling over relatively strong turbulence with adaptive optics”, *Applied Optics*, vol. 54, no. 29, p. 8722, Oct. 2015.
- [443] M. T. Gruneisen et al., “Adaptive-Optics-Enabled Quantum Communication: A Technique for Daytime Space-To-Earth Links”, *Physical Review Applied*, vol. 16, no. 1, p. 014067, Jul. 2021.
- [444] E. Barker et al., “Guide to IPsec VPNs”, National Institute of Standards and Technology, Gaithersburg, MD, Tech. Rep. NIST SP 800-77r1, Jun. 2020, NIST SP 800-77r1.
- [445] A. S. Mueller et al., “Free-space coupled superconducting nanowire single-photon detector with low dark counts”, *Optica*, vol. 8, no. 12, pp. 1586–1587, Dec. 2021.
- [446] D. Zheng et al., “Free-space to few-mode-fiber coupling under atmospheric turbulence”, *Optics Express*, vol. 24, no. 16, pp. 18739–18744, Aug. 2016.
- [447] P. V. Trinh et al., “Experimental verification of fiber coupling characteristics for FSO downlinks from the International Space Station”, *Optics Express*, vol. 31, no. 5, p. 9081, Feb. 2023.
- [448] H. G. Sandalidis et al., “Optical Wireless Communications With Heterodyne Detection Over Turbulence Channels With Pointing Errors”, *Journal of Lightwave Technology*, vol. 27, no. 20, pp. 4440–4445, Oct. 2009.
- [449] S. Huang and M. Safari, “Free-Space Optical Communication Impaired by Angular Fluctuations”, *IEEE Transactions on Wireless Communications*, vol. 16, no. 11, pp. 7475–7487, Nov. 2017.
- [450] T. Tolker-Nielsen and J.-C. Guillen, “SILEX: The First European Optical Communication Terminal in Orbit”, *ESA Bulletin*, vol. 96, 1998.
- [451] J. Cheng, *The Principles of Astronomical Telescope Design*. Springer, 2009.
- [452] V. S. R. Gudimetla and J. F. Riker, “Moment-matching method for extracting beam jitter and boresight in experiments with satellites of small physical

- cross section”, *Applied Optics*, vol. 46, no. 23, pp. 5608–5616, Aug. 2007.
- [453] M. Toyoshima and K. Araki, “Far-field pattern measurement of an onboard laser transmitter by use of a space-to-ground optical link”, *Applied Optics*, vol. 37, no. 10, pp. 1720–1730, Apr. 1998.
- [454] A. Carrasco-Casado et al., “Prototype Development and Validation of a Beam-Divergence Control System for Free-Space Laser Communications”, *Frontiers in Physics*, vol. 10, May 2022.
- [455] A. E. Willner et al., “Free-space mid-IR communications using wavelength and mode division multiplexing”, *Optics Communications*, p. 129 518, Apr. 2023.
- [456] A. E. Willner and C. Liu, “Perspective on using multiple orbital-angular-momentum beams for enhanced capacity in free-space optical communication links”, *Nanophotonics*, vol. 10, no. 1, pp. 225–233, 2021.
- [457] A. Willner et al., “Utilizing Structured Modal Beams in Free-Space Optical Communications for Performance Enhancement”, *IEEE Journal of Selected Topics in Quantum Electronics*, pp. 1–13, 2023.
- [458] X. Zhong et al., “Influence of Finite Apertures on Orthogonality and Completeness of Laguerre-Gaussian Beams”, *IEEE Access*, vol. 6, pp. 8742–8754, 2018.
- [459] M. J. Padgett et al., “Divergence of an orbital-angular-momentum-carrying beam upon propagation”, *New Journal of Physics*, vol. 17, no. 2, p. 023 011, Feb. 2015.
- [460] J. W. Lee et al., “Solar background noise mitigation using the orbital angular momentum mode in vertical FSO downlink transmissions”, *Optics Express*, vol. 29, no. 21, p. 33 312, Oct. 2021.
- [461] A. Trichili et al., “Encoding information using Laguerre Gaussian modes over free space turbulence media”, *Optics Letters*, vol. 41, no. 13, pp. 3086–3089, Jul. 2016.
- [462] L. Zhu et al., “Free-space optical communication with quasi-ring Airy vortex beam under limited-size receiving aperture and atmospheric turbulence”, *Optics Express*, vol. 29, no. 20, pp. 32 580–32 590, Sep. 2021.
- [463] D. Yang et al., “Underwater entanglement propagation of auto-focusing Airy beams”, *Optics Express*, vol. 32, no. 4, p. 4887, Feb. 2024.
- [464] S. Li and J. Wang, “Adaptive free-space optical communications through turbulence using self-healing Bessel beams”, *Scientific Reports*, vol. 7, no. 1, p. 43 233, Feb. 2017.
- [465] S. Shang et al., “Fiber coupling efficiency of a Bessel-Gaussian beam received by a Cassegrain antenna under atmospheric turbulence”, *Applied Optics*, vol. 61, no. 23, pp. 6871–6878, Aug. 2022.
- [466] C. Lyu et al., “Generation of diffraction-free Bessel beams based on combined axicons”, *Optics & Laser Technology*, vol. 164, p. 109 548, Sep. 2023.
- [467] J. Yang et al., “Generation and free-space transmission characterization of bottle vortex beam”, *Optics Express*, vol. 31, no. 25, pp. 41 094–41 104, Dec. 2023.
- [468] F. Saad et al., “Propagation behavior of a generalized Hermite cosh-gaussian laser beam through marine environment”, *Optical and Quantum Electronics*, vol. 56, no. 1, p. 130, Dec. 2023.
- [469] N. K. Fontaine et al., “Hermite-Gaussian mode multiplexer supporting 1035 modes”, in *2021 Optical Fiber Communications Conference and Exhibition (OFC)*, Jun. 2021, pp. 1–3.
- [470] J. Wang et al., “Terabit free-space data transmission employing orbital angular momentum multiplexing”, *Nature Photonics*, vol. 14, no. 11, pp. 1031–1037, Nov. 2020.

- tonics*, vol. 6, no. 7, pp. 488–496, Jul. 2012.
- [471] H. Huang et al., “100 Tbit s free-space data link enabled by three-dimensional multiplexing of orbital angular momentum, polarization, and wavelength”, *Optics Letters*, vol. 39, no. 2, pp. 197–200, Jan. 2014.
- [472] F. M. Dickey et al., Eds., *Laser Beam Shaping Applications*. CRC, Oct. 2018.
- [473] H. Hemmati, *Deep Space Optical Communications*. John Wiley & Sons, Jun. 2006.
- [474] M. Massari et al., “Fabrication and characterization of high-quality spiral phase plates for optical applications”, *Applied Optics*, vol. 54, no. 13, pp. 4077–4083, May 2015.
- [475] G. Ruffato et al., “Spiral phase plates for the generation of high-order Laguerre-Gaussian beams with non-zero radial index”, in *Complex Light and Optical Forces IX*, vol. 9379, SPIE, Mar. 2015, pp. 11–21.
- [476] W. Pan et al., “Generation of High-Purity Laguerre–Gaussian Beams by Spatial Filtering”, *IEEE Photonics Journal*, vol. 11, no. 3, pp. 1–10, Jun. 2019.
- [477] M. Caño-García et al., “Dynamic multi-level spiral phase plate generator”, *Scientific Reports*, vol. 8, no. 1, p. 15 804, Oct. 2018.
- [478] K. M. Dorney et al., “Controlling the polarization and vortex charge of attosecond high-harmonic beams via simultaneous spin–orbit momentum conservation”, *Nature Photonics*, vol. 13, no. 2, pp. 123–130, Feb. 2019.
- [479] K. Toyoda et al., “Using Optical Vortex To Control the Chirality of Twisted Metal Nanostructures”, *Nano Letters*, vol. 12, no. 7, pp. 3645–3649, Jul. 2012.
- [480] J. Sun et al., “Concealing with Structured Light”, *Scientific Reports*, vol. 4, no. 1, p. 4093, Feb. 2014.
- [481] J. E. Nordholt et al., “Polarization tracking system for free-space optical communication, including quantum communication”, Los Alamos National Laboratory (LANL), Los Alamos, NM (United States), Tech. Rep. 9,866,379, Jan. 2018.
- [482] Y. Lian et al., “OAM beam generation in space and its applications: A review”, *Optics and Lasers in Engineering*, vol. 151, p. 106 923, Apr. 2022.
- [483] Y. Shen et al., “Optical skyrmions and other topological quasiparticles of light”, *Nature Photonics*, vol. 18, no. 1, pp. 15–25, Jan. 2024.
- [484] S. Gao et al., “Paraxial skyrmionic beams”, *Physical Review A*, vol. 102, no. 5, p. 053 513, Nov. 2020.
- [485] M. Badás et al., “Annular beams for reliable intersatellite optical communications”, *Optics Letters*, Jun. 2026.
- [486] J. d. Plooy et al., “CubeCAT: In-orbit results and the future of DTE LCT”, in *Small Satellites Systems and Services Symposium (4S 2024)*, vol. 13546, SPIE, Mar. 2025, pp. 366–377.
- [487] R. Rüdtenklau et al., “In-orbit demonstration of acquisition and tracking on OSIRIS4CubeSat”, *Optics Express*, vol. 32, no. 23, pp. 41 188–41 200, Nov. 2024.
- [488] G. Ruffato et al., “Generation of high-order Laguerre–Gaussian modes by means of spiral phase plates”, *Optics Letters*, vol. 39, no. 17, pp. 5094–5097, Sep. 2014.
- [489] M. Badás et al., *Super-Gaussian approximations for optimum far-field irradiance in intersatellite optical communications: Coherent and incoherent beam shaping*, Jun. 2026.

- [490] I. M. Gelfand et al., *Calculus of Variations*. Courier Corporation, Jan. 2000.
- [491] X. Deng et al., “Pure-phase plates for super-Gaussian focal-plane irradiance profile generations of extremely high order”, *Optics Letters*, vol. 21, no. 24, p. 1963, Dec. 1996.
- [492] K. Gillen-Christandl et al., “Comparison of Gaussian and super Gaussian laser beams for addressing atomic qubits”, *Applied Physics B*, vol. 122, no. 5, p. 131, Apr. 2016.
- [493] P. Buske et al., “Advanced beam shaping for laser materials processing based on diffractive neural networks”, *Optics Express*, vol. 30, no. 13, pp. 22 798–22 816, Jun. 2022.
- [494] Y. Hayasaki et al., “Automatic generation of a holographically shaped beam in an actual optical system for use in material laser processing”, *Optics Express*, vol. 31, no. 2, pp. 1982–1991, Jan. 2023.
- [495] G. Dong et al., “Super-Gaussian mirror for high-field-seeking molecules”, *Physical Review A*, vol. 72, no. 3, p. 031 605, Sep. 2005.
- [496] X. Jiang et al., “Uniform irradiation of adjustable target spots in high-power laser driver”, *Applied Optics*, vol. 50, no. 27, pp. 5213–5220, Sep. 2011.
- [497] J. Deng et al., “Fresnel diffraction and small-scale self-focusing of a phase modulated and spectrally dispersed laser beam”, *Optics & Laser Technology*, vol. 45, pp. 56–61, Feb. 2013.
- [498] C. Jacquard et al., “Enhancing LEO satellite uplink performance through spatial mode diversity-based scintillation mitigation”, in *International Conference on Space Optics — ICSO 2024*, vol. 13699, SPIE, Jul. 2025, pp. 1355–1365.
- [499] G. Zhang et al., “Beam Shaping by Mode Superposition With Coefficient Optimization”, *IEEE Photonics Journal*, vol. 16, no. 4, pp. 1–5, Aug. 2024.
- [500] A. Billaud et al., “10 Gbps free space optical communication link using Multi-Plane Light Conversion turbulence mitigation”, in *International Conference on Space Optics — ICSO 2022*, vol. 12777, SPIE, Jul. 2023, pp. 2040–2054.
- [501] Y. Zhang and N. K. Fontaine, *Multi-Plane Light Conversion: A Practical Tutorial*, May 2023.
- [502] M. Badás et al., “Metalens for inter-satellite free-space optical communications”, in *International Conference on Space Optics — ICSO 2024*, vol. 13699, SPIE, Jul. 2025, pp. 424–435.
- [503] D. Wang et al., “Variable shape or variable diameter flattop beam tailored by using an adaptive weight FFT-based iterative algorithm and a phase-only liquid crystal spatial light modulator”, *Optics Communications*, vol. 285, no. 24, pp. 5044–5050, Nov. 2012.
- [504] L. Burger et al., “Implementation of a spatial light modulator for intracavity beam shaping”, *Journal of Optics*, vol. 17, no. 1, p. 015 604, Dec. 2014.
- [505] S. Li et al., “Using the spatial light modulator as a binary optical element: Application to spatial beam shaping for high-power lasers”, *Applied Optics*, vol. 57, no. 24, pp. 7060–7064, Aug. 2018.
- [506] Y. Xu et al., “Fabrication-integrated design for diffractive optical elements”, *Optica*, vol. 12, no. 2, pp. 228–238, Feb. 2025.
- [507] P. Buske et al., “Diffractive neural networks with polynomial phase masks for laser beam shaping with quasi-continuous diffractive optical elements”, *Optics Express*, vol. 33, no. 10, pp. 21 040–21 054, May 2025.
- [508] J. J. Yang and M. R. Wang, “Analysis and optimization on single-zone binary

- flat-top beam shaper”, *Optical Engineering*, vol. 42, no. 11, pp. 3106–3113, Nov. 2003.
- [509] L. Taylor, “The phase retrieval problem”, *IEEE Transactions on Antennas and Propagation*, vol. 29, no. 2, pp. 386–391, Mar. 1981.
- [510] J. R. Fienup, “Phase retrieval algorithms: A comparison”, *Applied Optics*, vol. 21, no. 15, p. 2758, Aug. 1982.
- [511] J. R. Fienup, “Phase retrieval algorithms: A personal tour [Invited]”, *Applied Optics*, vol. 52, no. 1, pp. 45–56, Jan. 2013.
- [512] Y. Shechtman et al., “Phase Retrieval with Application to Optical Imaging: A contemporary overview”, *IEEE Signal Processing Magazine*, vol. 32, no. 3, pp. 87–109, May 2015.
- [513] C. Guo et al., “A review of iterative phase retrieval for measurement and encryption”, *Optics and Lasers in Engineering*, 3DIM-DS 2015: Optical Image Processing in the context of 3D Imaging, Metrology, and Data Security, vol. 89, pp. 2–12, Feb. 2017.
- [514] O. Ripoll et al., “Review of iterative Fourier-transform algorithms for beam shaping applications”, *Optical Engineering*, vol. 43, no. 11, pp. 2549–2548, Nov. 2004.
- [515] S. N. Dixit et al., “Designing fully continuous phase screens for tailoring focal-plane irradiance profiles”, *Optics Letters*, vol. 21, no. 21, p. 1715, Nov. 1996.
- [516] Y. Lin et al., “Distributed phase plates for super-Gaussian focal-plane irradiance profiles”, *Optics Letters*, vol. 20, no. 7, p. 764, Apr. 1995.
- [517] J. A. Marozas, “Fourier transform-based continuous phase-plate design technique: A high-pass phase-plate design as an application for OMEGA and the National Ignition Facility”, *JOSA A*, vol. 24, no. 1, pp. 74–83, Jan. 2007.
- [518] S. N. Dixit et al., “Kinoform phase plates for focal plane irradiance profile control”, *Optics Letters*, vol. 19, no. 6, p. 417, Mar. 1994.
- [519] J. S. Liu and M. R. Taghizadeh, “Iterative algorithm for the design of diffractive phase elements for laser beam shaping”, *Optics Letters*, vol. 27, no. 16, p. 1463, Aug. 2002.
- [520] O. Bryngdahl, “Geometrical transformations in optics*”, *JOSA*, vol. 64, no. 8, pp. 1092–1099, Aug. 1974.
- [521] D. Giggenbach and A. Shrestha, “Atmospheric absorption and scattering impact on optical satellite-ground links”, *International Journal of Satellite Communications and Networking*, vol. 40, no. 2, pp. 157–176, 2022.
- [522] J. Serrano et al., “The White Rabbit Project”, Kobe, Japan, Nov. 2009.
- [523] E. Waks et al., “Security of quantum key distribution with entangled photons against individual attacks”, *Physical Review A*, vol. 65, no. 5, p. 052310, Apr. 2002.
- [524] T. Attema et al., “Optimizing the decoy-state BB84 QKD protocol parameters”, *Quantum Information Processing*, vol. 20, no. 4, p. 154, Apr. 2021.
- [525] R. S. R. Ribeiro et al., “Generation of Laguerre Gaussian beams using spiral phase diffractive elements fabricated on optical fiber tips using focused ion beam milling”, in *Complex Light and Optical Forces X*, vol. 9764, SPIE, Mar. 2016, pp. 86–91.

Curriculum Vitæ

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Education

- 2022-2026 **PhD in Optical and Quantum Satellite Communications**
Delft University of Technology (TUDelft)
Delft, the Netherlands
Thesis: *Beyond Gaussian beams:
 optical and quantum satellite communications*
Promotor: Dr. Jérôme Loicq
Co-promotors: Dr. ir. Jasper Bouwmeester, Dr. Pierre Piron
- 2023-2026 **MSc in Theoretical Physics**
National University of Distance Education (UNED)
Madrid, Spain
Thesis: *Relativistic quantum friction and dynamic Casimir effect
 in the vectorial electromagnetic field*
Supervisor: Dr. Pablo Rodríguez-López
- 2019-2023 **BSc in Physics**
National University of Distance Education (UNED)
Madrid, Spain
- 2018-2019 **MSc in Aerospace Dynamics**
Cranfield University
Cranfield, United Kingdom
Double degree programme alongside the MSc in Mechanical Engineering
Thesis: *Compressibility corrections in the vortex lattice method*
Supervisor: Dr. Mudassir Lone
- 2017-2019 **MSc in Mechanical Engineering**
University of the Basque Country (UPV/EHU)
Bilbao, Spain
- 2013-2017 **BSc in Mechanical Engineering**
University of the Basque Country (UPV/EHU)
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Experience

- 2026-Present **Postdoctoral Research Associate**
University of Cambridge, Cambridge (United Kingdom)
Satellite optical and quantum communications
- 2020-2022 **Space thermo-mechanical engineer**
Added Value Solutions, Vitoria (Spain)
Development of space hardware for the ATHENA telescope and
the gripper for the Mars Sample Return mission.

Awards

- 2018 **Excellence Training Scholarship**
- 2017 **Prize in the National Olympiad of Theory of Machines and Mechanisms**

List of Publications

Journal papers

 Indicates publications whose contents are included, in whole or in part, in this thesis.

10. G. Castro do Amaral, D. L. Bakker, **M. Badás**, S. Loarte, R. Saathof, B. Perlingeiro, D. van der Vuurst, C. Esteve, L. de Forges de Parny, *A long-distance quantum entanglement distribution network with ground and space connectivity*, In preparation.
9. **M. Badás**, P. Rodríguez-López, *Relativistic quantum friction and dynamic Casimir effect for a detector-plane system in the vectorial electromagnetic vacuum*, In preparation.
-  8. **M. Badás**, Y. van Gennip, P. Piron, J. Bouwmeester, J. Loicq, *Super-Gaussian approximations to optimum far-field irradiance in intersatellite optical communications: coherent and incoherent beam shaping*, Submitted for publication in **Journal of Optics**, preprint available at DOI: 10.48550/arXiv.2606.15362.
-  7. **M. Badás**, E. Pauwels, P. Piron, J. Bouwmeester, J. Loicq, *Annular beams for reliable intersatellite optical communications*, **Optics Letters** **51**, 14 (2026), DOI: 10.1364/OL.600449.
6. J. van der Zwet, **M. Badás**, C. Ayas, M. Langelaar, *Thermo-mechanical topology optimization to mitigate specific Zernike aberrations for laser satellite communication*, Submitted for publication in **Optics and Laser Technology**.
-  5. V. Domínguez[†], **M. Badás**[†], D. L. Bakker, G. Castro do Amaral, D. López, J. Borregaard, *Quantum key distribution in the Iberian Peninsula*, **Optica Quantum** **4**, 1 (2026), DOI: 10.1364/OPTICAQ.581903.
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-  4. V. Domínguez, **M. Badás**, J. van Dam, A. S. Sørensen, J. Borregaard, *Satellite-assisted quantum communication with single photon sources and atomic memories*, **Physical Review Research** **8**, 1 (2026), DOI: 10.1103/z1vn-11m5.
-  3. **M. Badás**, P. Piron, J. Bouwmeester, R. Saathof, J. Loicq, *Seidel optical aberrations and optimum truncated Gaussian beams on intersatellite free space optical communications*, **Optics Express** **33**, 16 (2025), DOI: 10.1364/OE.565652.

2. **M. Badás**, P. Piron, J. Bouwmeester, J. Loicq, *On the optimum far-field irradiance distribution using Laguerre-Gaussian beams for intersatellite free-space optical communications*, **Optics Express** **32**, 18 (2024), DOI: 10.1364/OE.533250.
1. **M. Badás**, P. Piron, J. Bouwmeester, J. Loicq, J. Kuiper, E. Gill, *Opto-thermo-mechanical phenomena in satellite free-space optical communications: survey and challenges*, **Optical Engineering** **63**, 4 (2023), DOI: 10.1117/1.OE.63.4.041206.

Selected conference contributions

2. **M. Badás**, J. Algera, J. Bouwmeester, P. Piron, J. Loicq, *Metalens for intersatellite free-space optical communications*, International Conference in Space Optics 2024, SPIE Proc. **13699**, 424–435 (2025), DOI: 10.1117/12.3075404.
1. **M. Badás**, J. Bouwmeester, P. Piron, J. Loicq, *Impact of transmitter wavefront errors and pointing jitter on intersatellite free space optical communications*, Computational Optics 2024, SPIE Proc. **13023** (2024), DOI: 10.1117/12.3021617.

