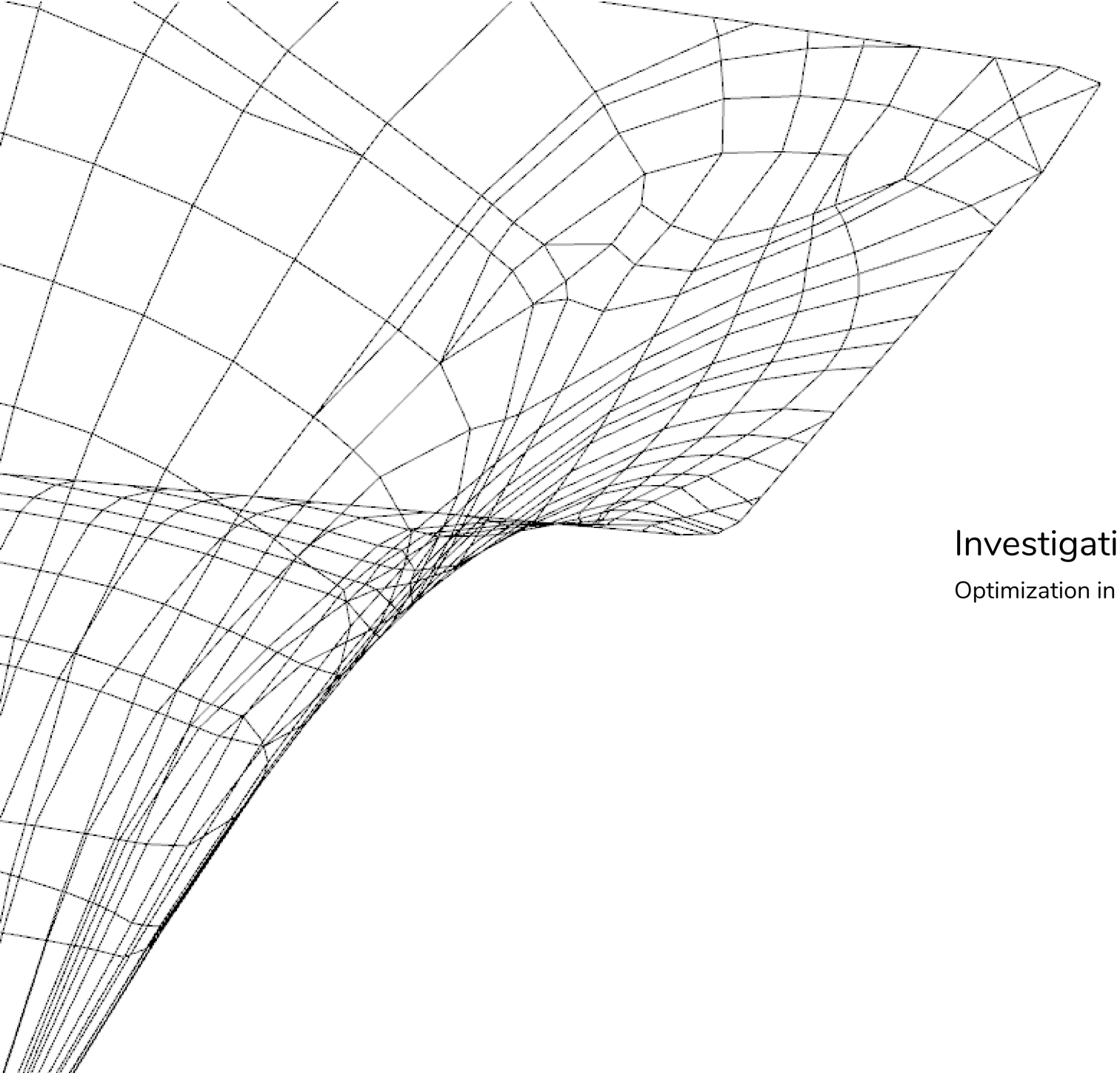
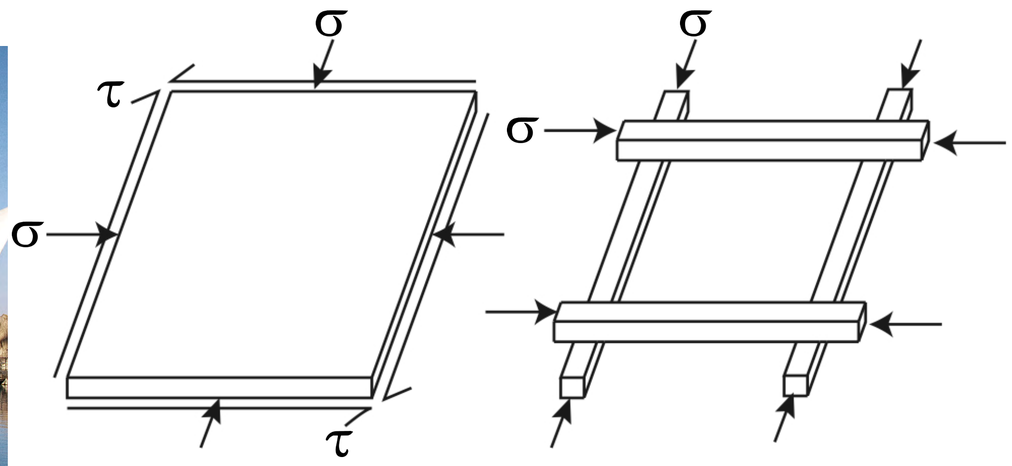


Investigating Principal Stress Lines

Optimization in Gridshell Structures

Michael Cobb
(4510933)

Shell VS Quadrilateral Gridshell



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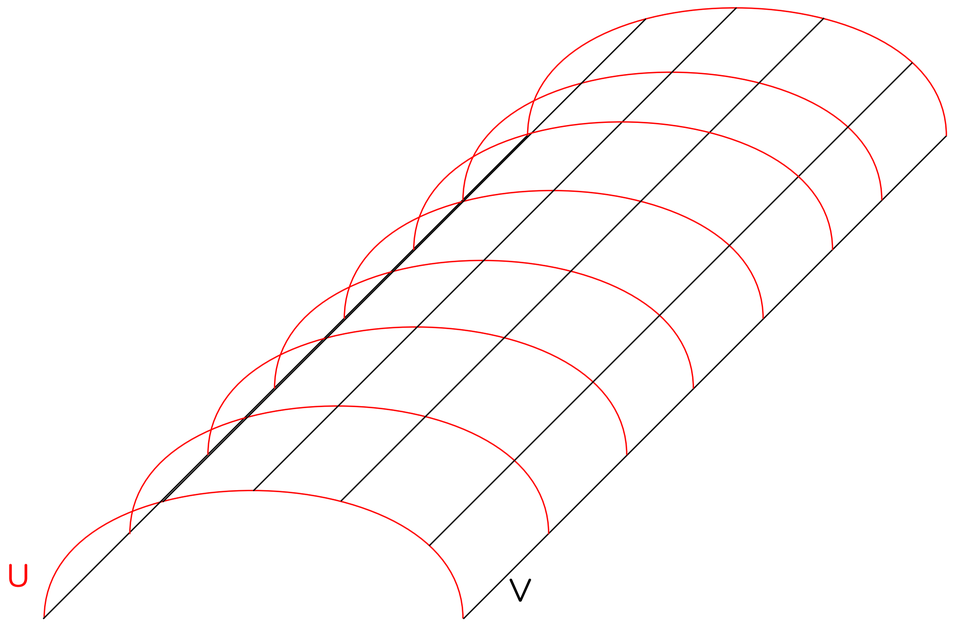
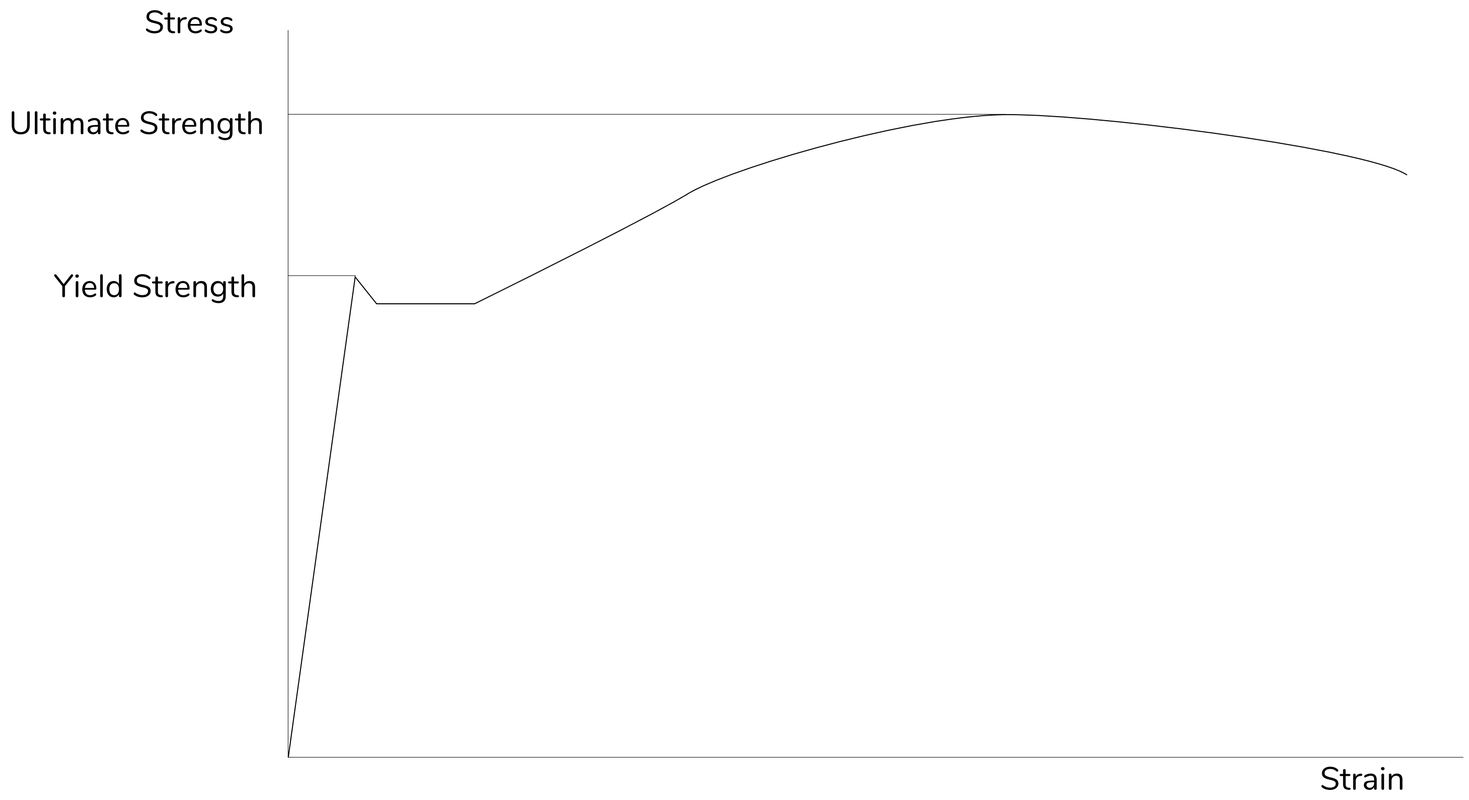


A long span lattice structure emulating the behavior of a shell structure with the majority of load being transmitted in plane through the lattice members with nodes that behave as if they were pinned.

The Two Strengths



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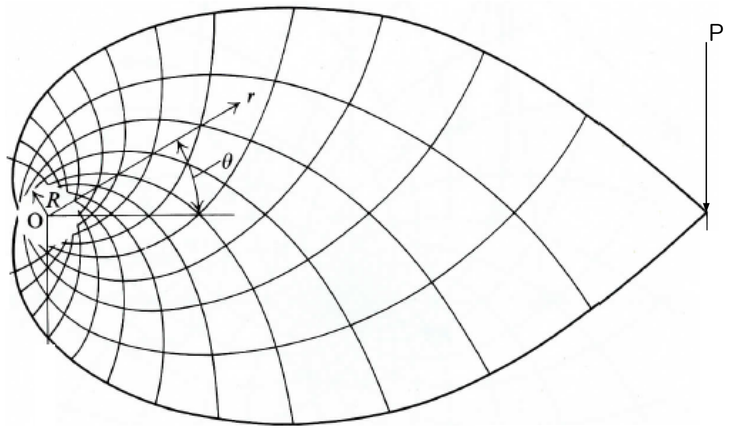
Designing For Ultimate Strength



$$\mathrm{V}_m = \frac{\sum \ell |f|}{\mathrm{P}} = \frac{\delta \mathrm{W}}{\epsilon \mathrm{P}} = \frac{\sum \epsilon \mathrm{F} r \cos\theta}{\epsilon \mathrm{P}} = \frac{\sum \mathrm{F} r \cos\theta}{\mathrm{P}},$$

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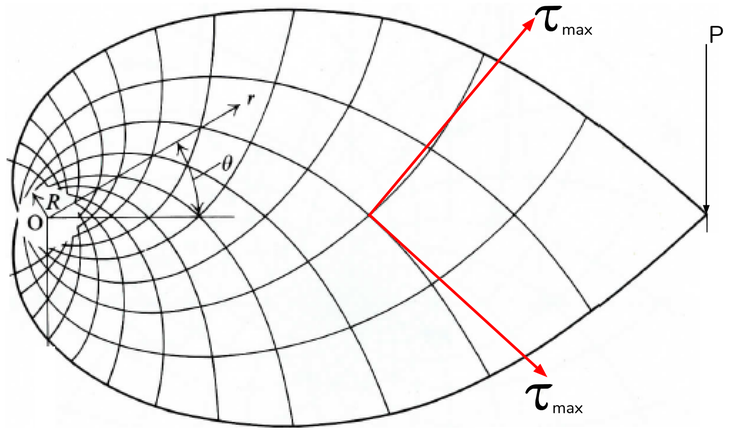
σ_2

Designing For Ultimate Strength



$$\mathrm{V}_m = \frac{\sum \ell |f|}{\mathrm{P}} = \frac{\delta \mathrm{W}}{\epsilon \mathrm{P}} = \frac{\sum \epsilon \mathrm{F} r \cos\theta}{\epsilon \mathrm{P}} = \frac{\sum \mathrm{F} r \cos\theta}{\mathrm{P}},$$

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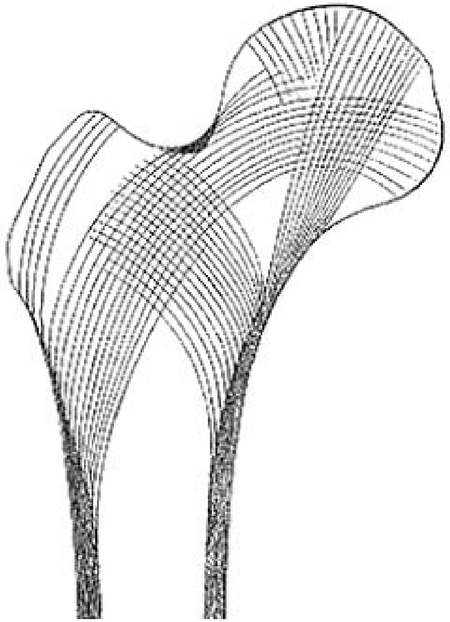
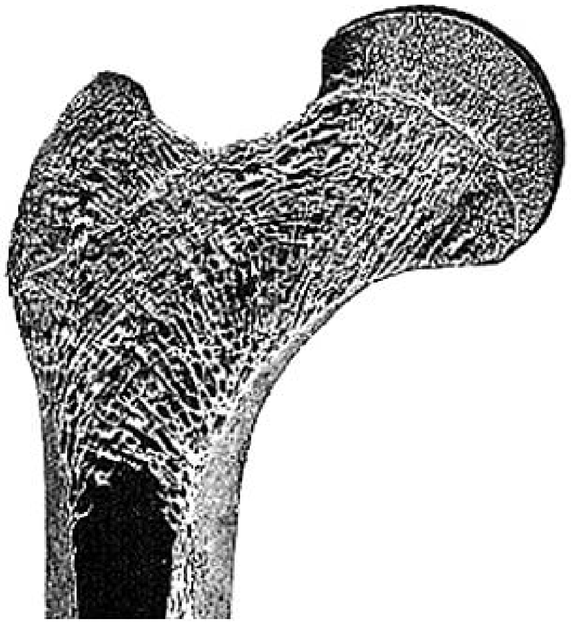
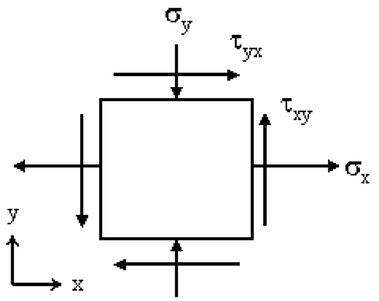
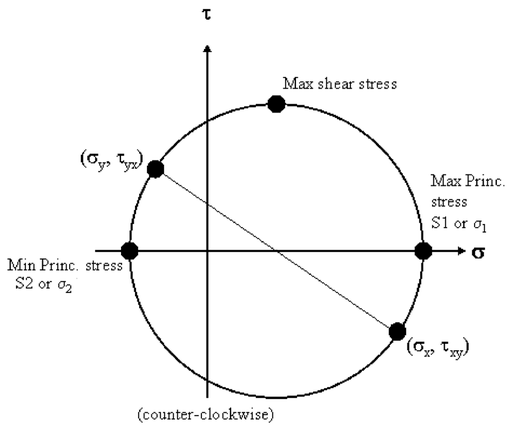
Designing For Yield



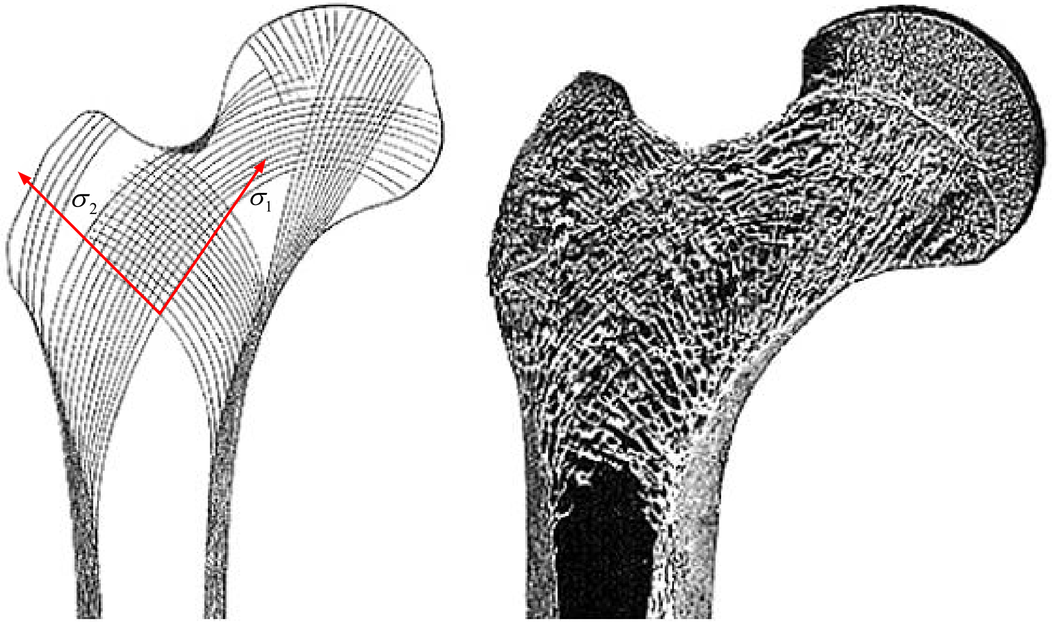


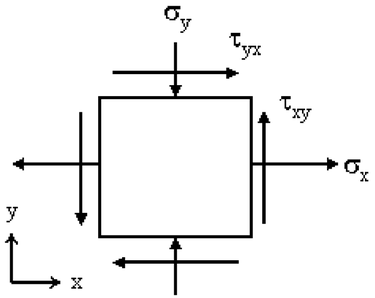


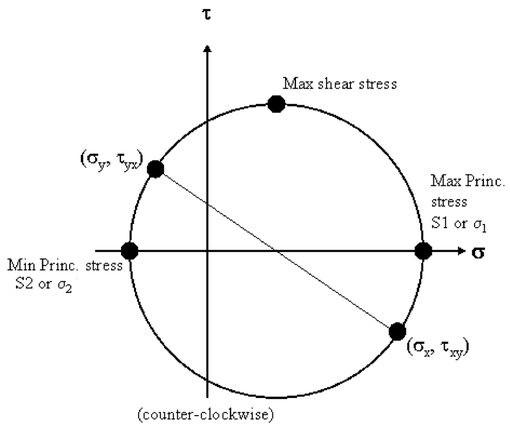


TU Delft

Designing For Yield







$$Shell\ Behavior = \frac{|n|_{max}}{\left(\frac{6|m|_{\max}}{t}\right) + |n|_{max}}$$

$$U_{strain} = \frac{1}{2} V \sigma \epsilon = \frac{1}{2} V E \epsilon^2 = \frac{1}{2} \frac{V}{E} \sigma^2$$

How can the creation of a gridshell by using principle stress directions be used provide a more efficient structure that follows principal stress lines?

How can rod paths be plotted along principal stress streamlines on freeform surfaces?

What form finding methods are suitable for generating an efficient structural form with high percentage shell behavior (no out of plane forces) and low strain energy density(high stiffness)?

Is there a considerable advantage in optimizing a gridshell structure based on principal stress stream lines and an arbitrary generated tessellation?

2 Methods:

1

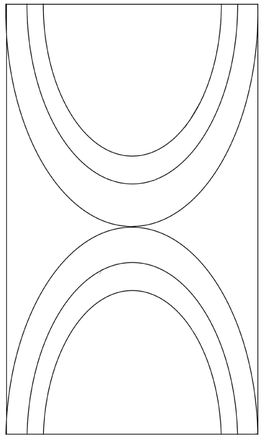
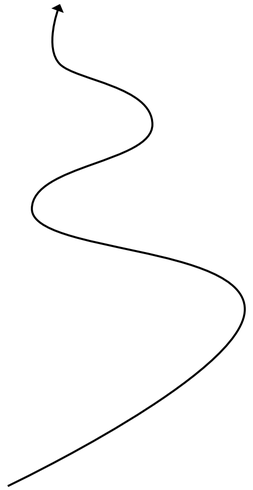


Plate Mechanics

2



Streamline Integrators

Method 1: Plate Mechanics

Twin Surface Shells

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = -p_B \quad (3.14)$$

$$\frac{\partial m_{xx}}{\partial x} + \frac{\partial m_{xy}}{\partial y} = v_x \quad (3.15)$$

$$\frac{\partial m_{yy}}{\partial y} + \frac{\partial m_{xy}}{\partial x} = v_y \quad (3.16)$$

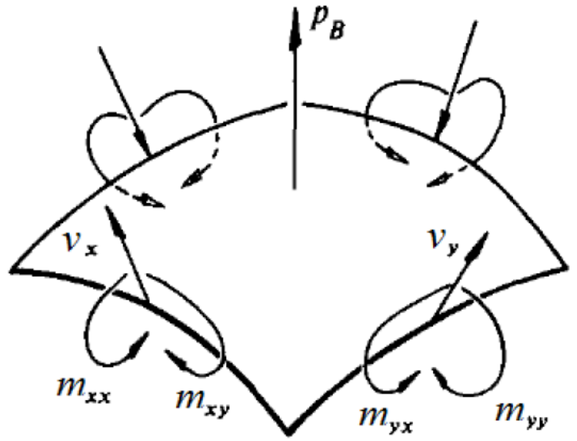


Fig. 3.4 - Equilibrium of bending surface

$$g_S = g_B$$

$$\frac{n_{xx}}{R_1} + \frac{n_{yy}}{R_2} = p_S \quad (3.9)$$

$$\frac{\partial n_{xx}}{\partial x} + \frac{\partial n_{xy}}{\partial y} = 0 \quad (3.10)$$

$$\frac{\partial n_{yy}}{\partial y} + \frac{\partial n_{xy}}{\partial x} = 0 \quad (3.11)$$

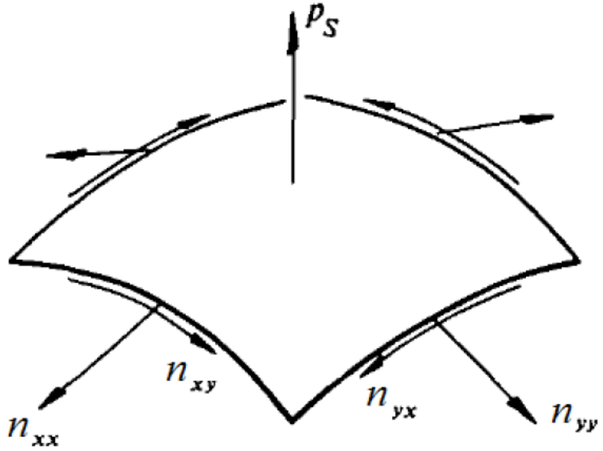
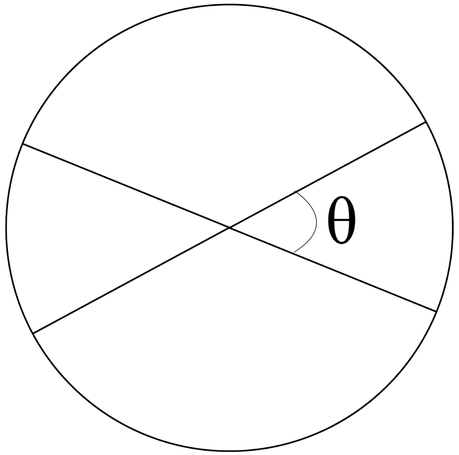
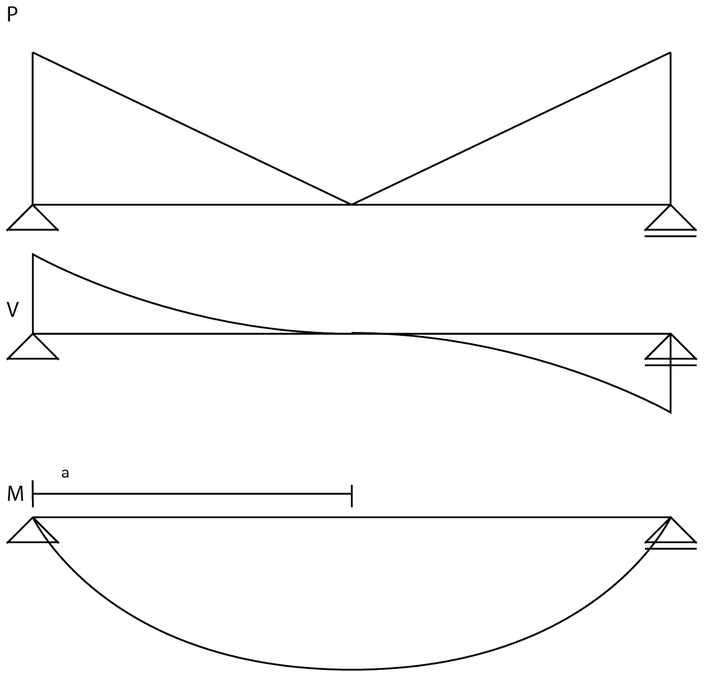


Fig. 3.3 - Equilibrium of stretching surface

$$p_{tot} = p_S + p_B$$

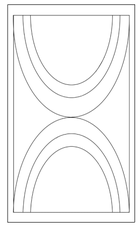
Beam Behavior for Plate Section $d\theta$

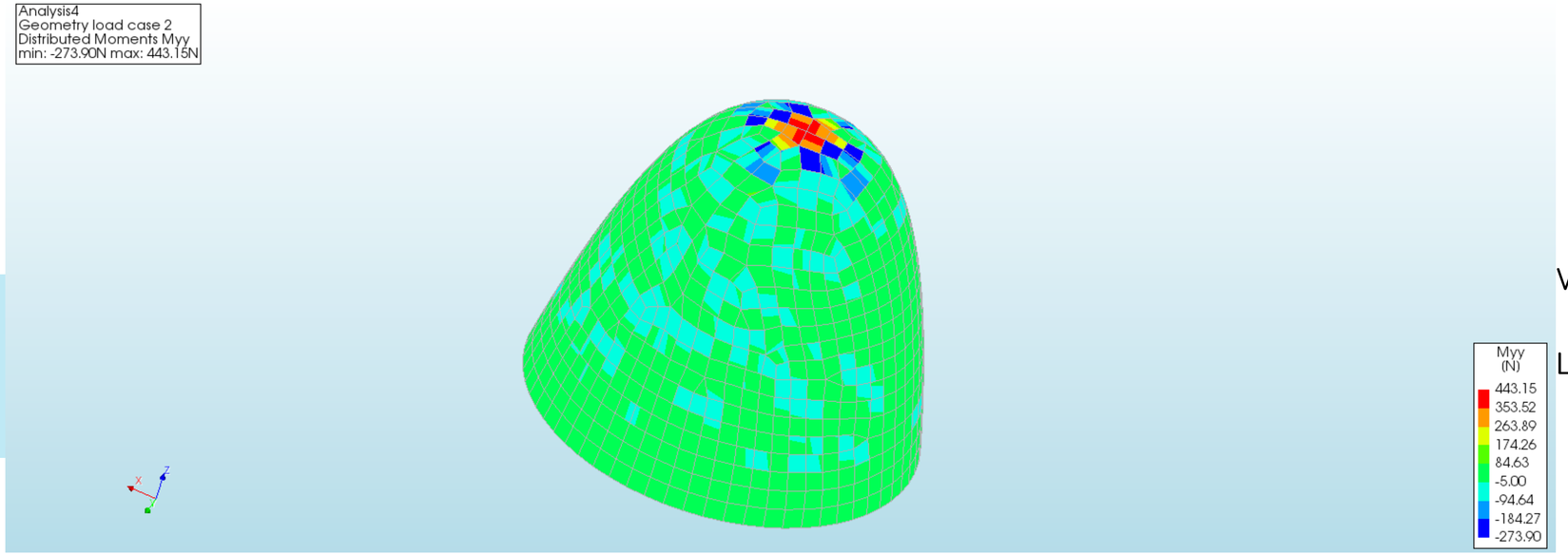




$$M_\theta = -\frac{x^3}{6a} + \frac{a^2}{6}$$

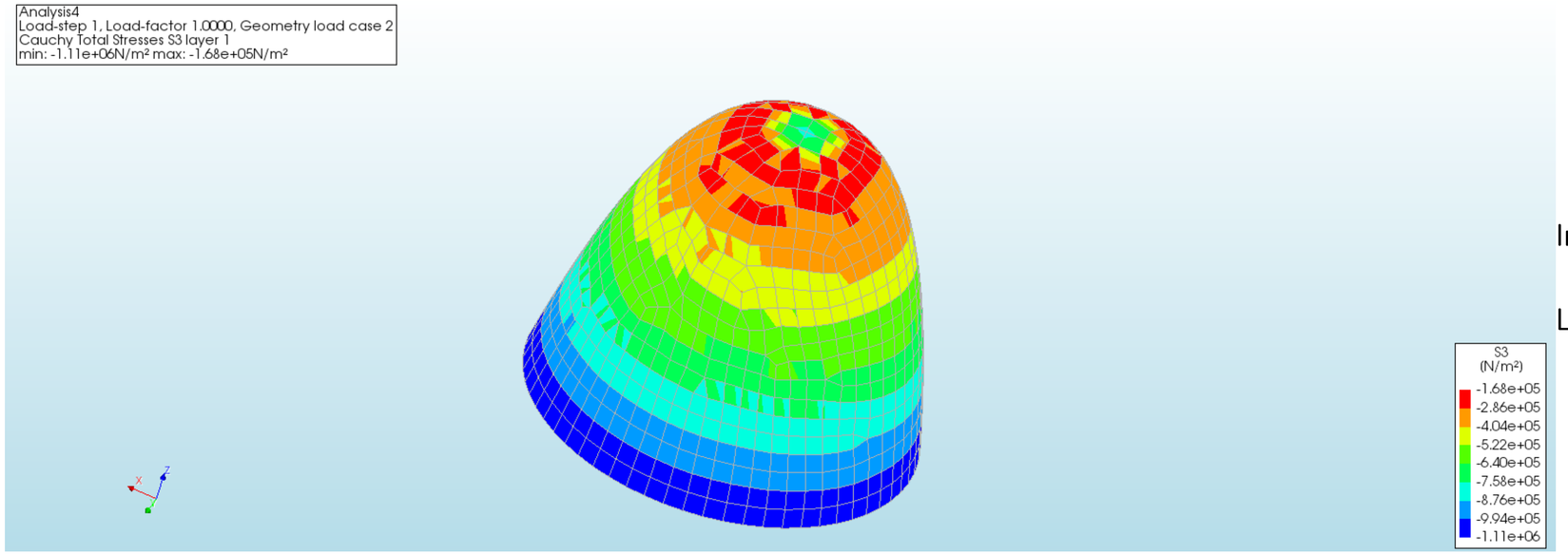
Shell Behavior for Moment Hill of Plate





Worst Case Moment (Myy/T)

Largest Value: 4431.5 N/m



In Plane Force (S3*T)

Largest Value: -1.11e5 N/m

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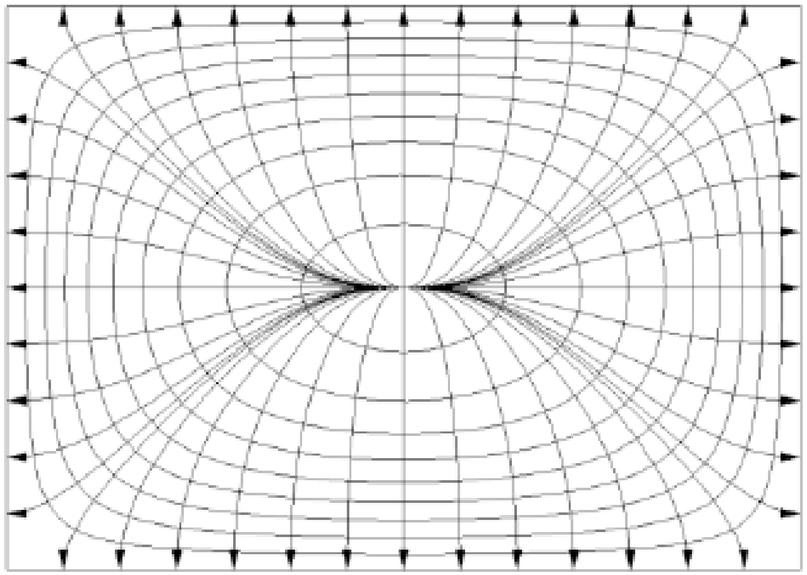
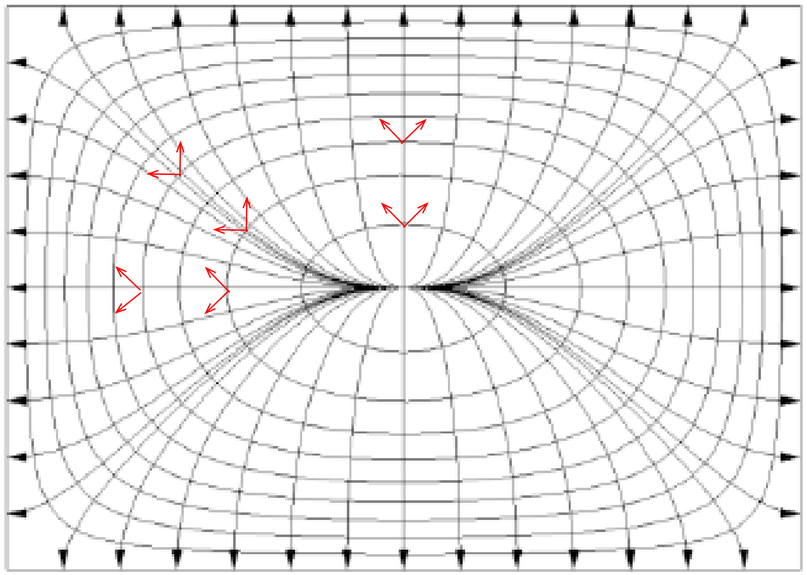
Moment Hill and Gradient Descent



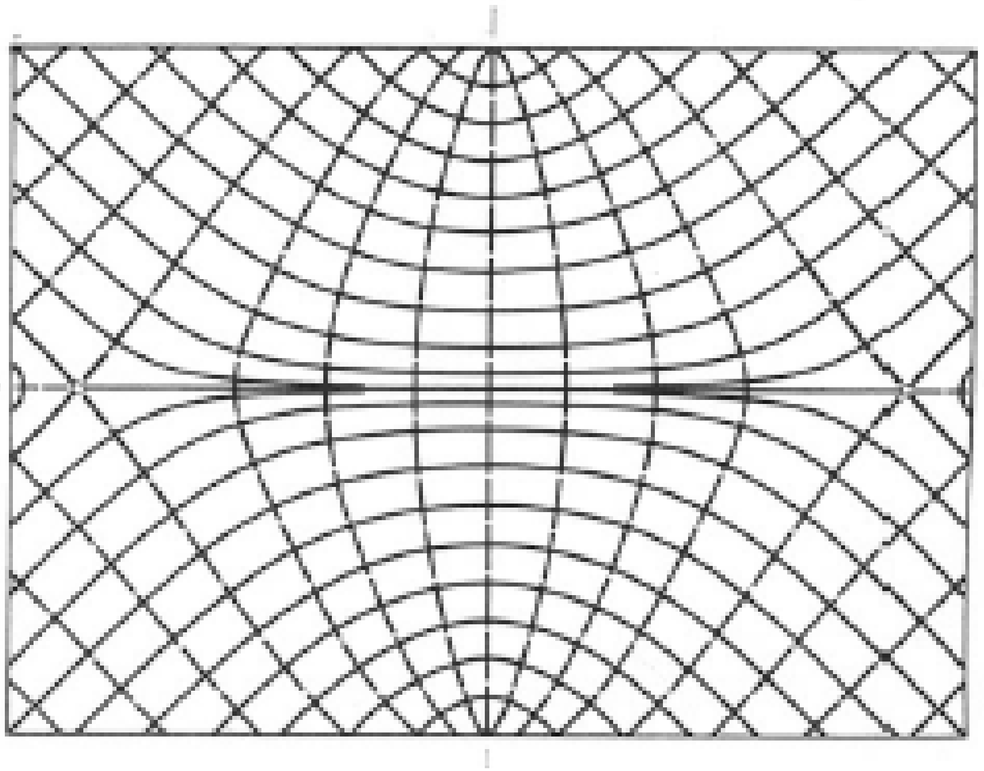
$$v_n = \frac{\partial}{\partial n}(m)$$

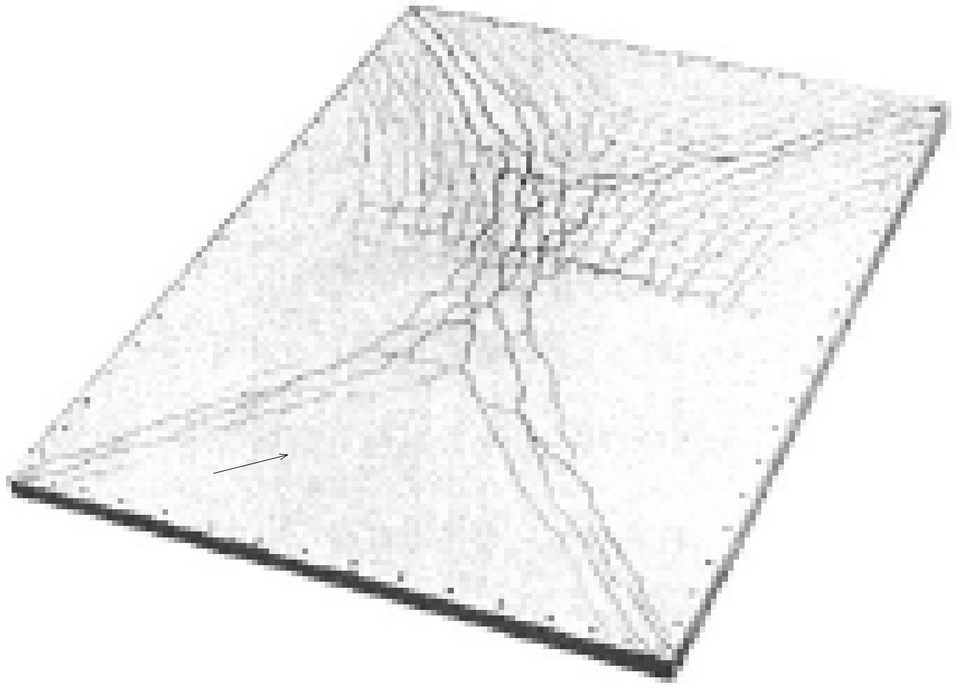
Moment Hill and Gradient Descent



$$v_n = \frac{\partial}{\partial n}(m)$$

Plate Yielding

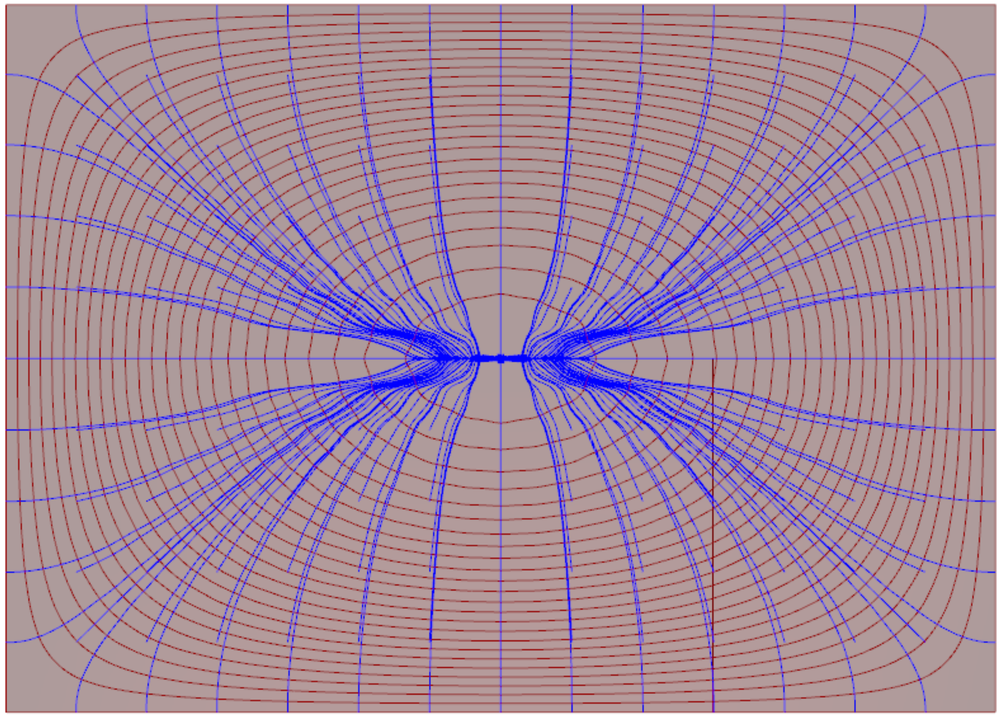


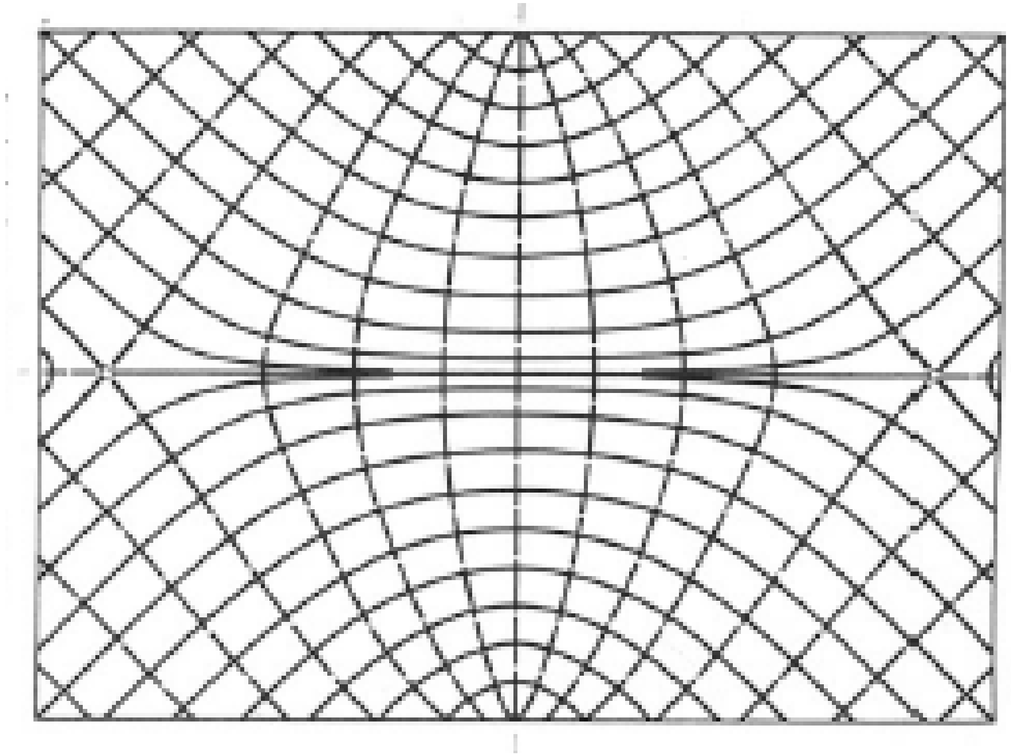


Principal Moment Trajectories Yield Principal Force Directions on the Surface of the Plate

Beranek examined plate yielding behavior

First Estimation: Deflection Surface

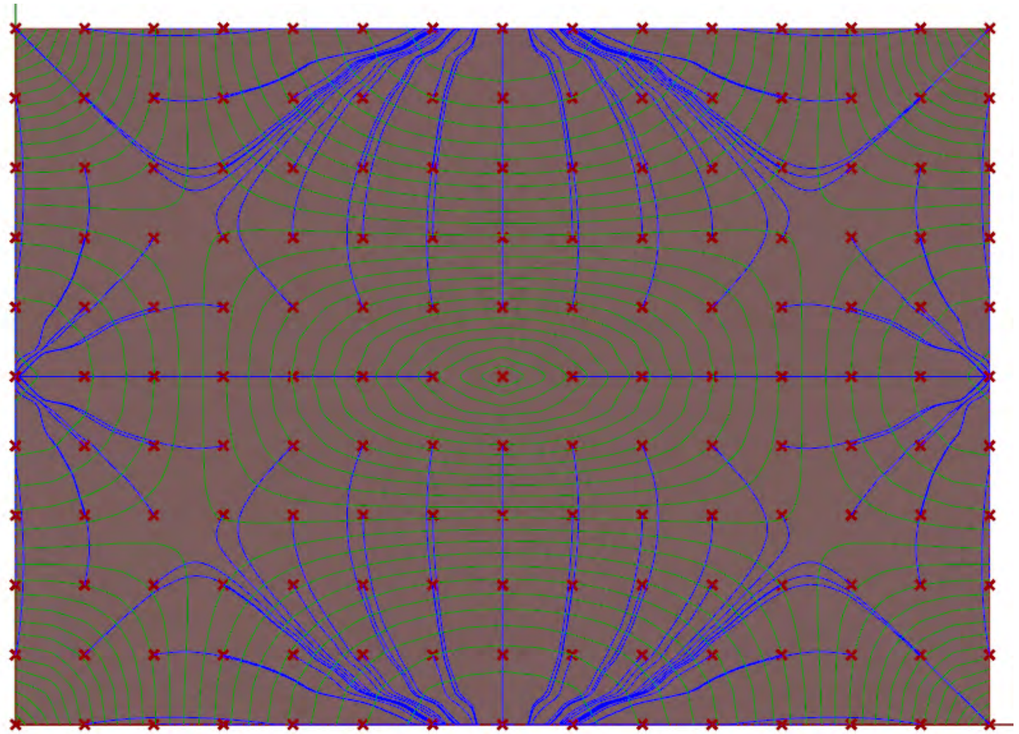




Why didn't this work?

Deflection $\xrightarrow{\frac{d}{dn}}$ Rotation $\xrightarrow{\frac{d}{dn}}$ Moment

Rotation Surface



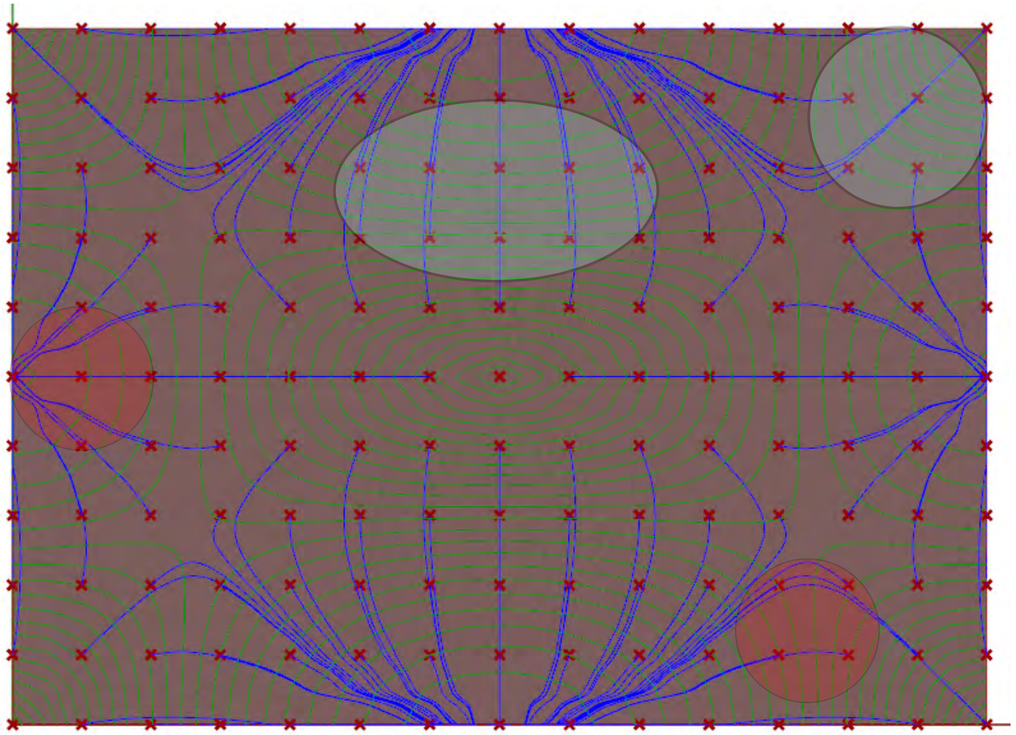
$$\left(\frac{d\varphi}{dt}\right) = n_{tt}$$

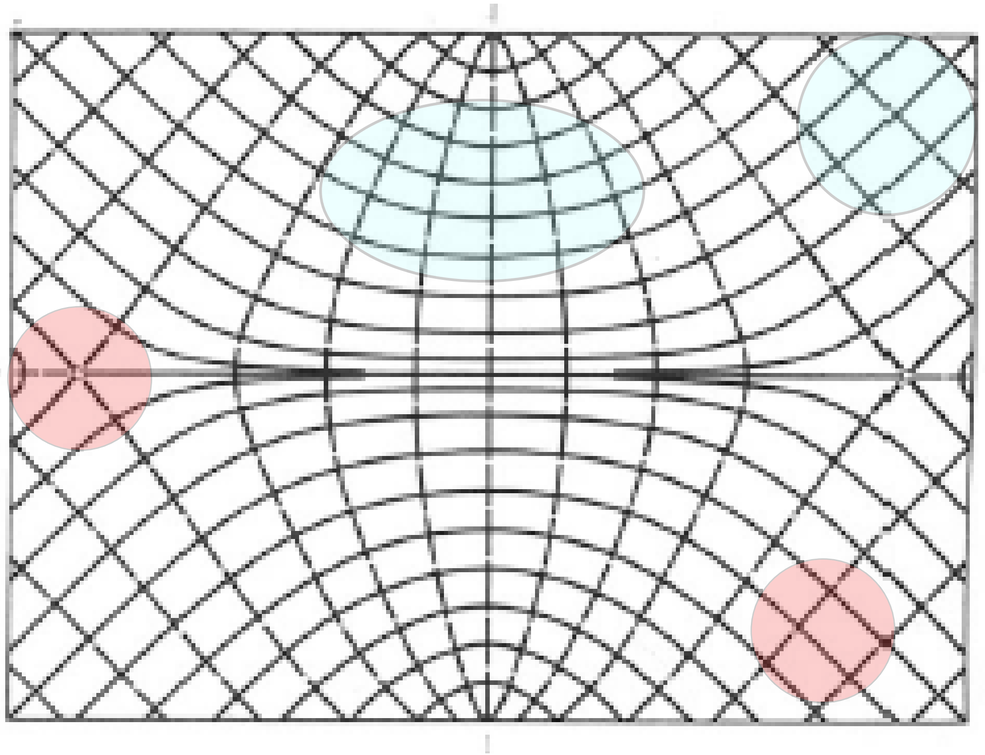
$$\kappa_{xx} = \frac{d\varphi_x}{dx} = K_{xx}$$

$$\varphi_{\max} = \left(\varphi_x^2 + \varphi_y^2\right)^{\frac{1}{2}}$$

Where: $n_{tt} = \frac{\partial \varphi_{\max}}{\partial n}$

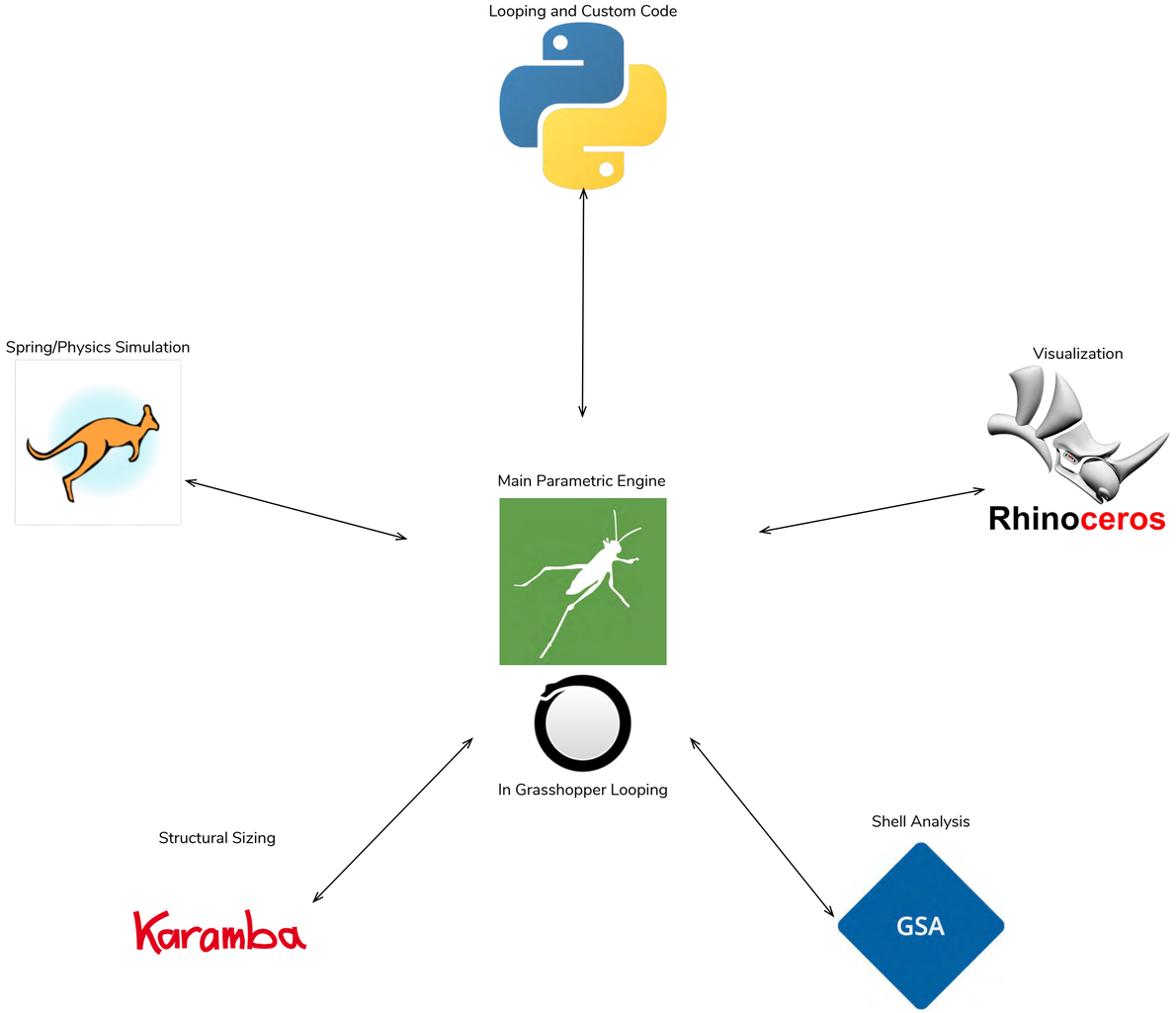
Second Estimation: Rotations Surface





Closer but still some discrepancies: Corners contain high v_y, v_x, m_{xy} and m_{yx} values due to twist. This theory is only applicable to twistless shells and therefore may be causing the discrepancies.

Method 2:
Discrete Computation and Streamline Mapping



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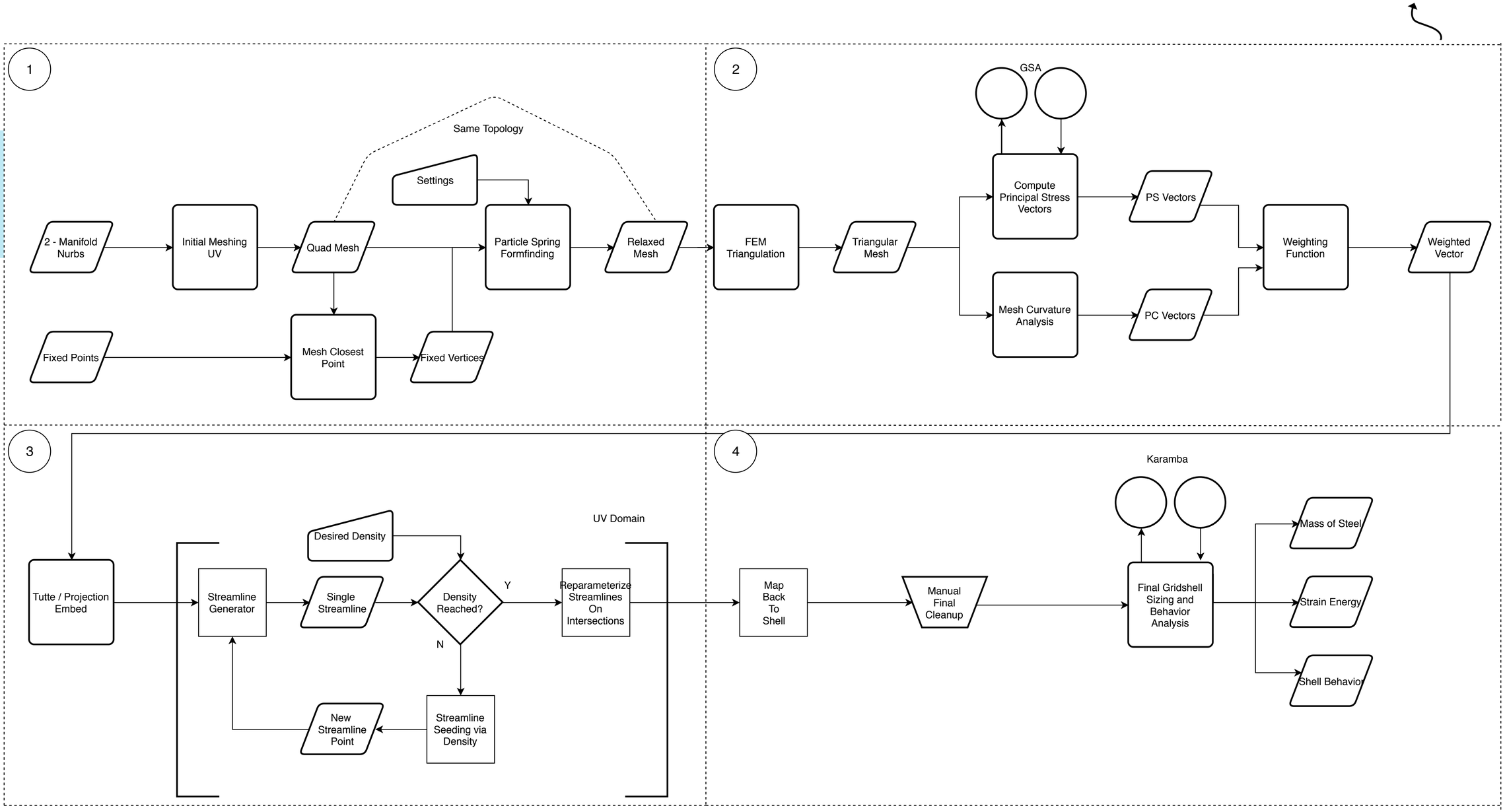
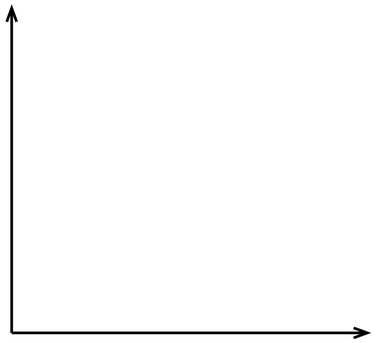
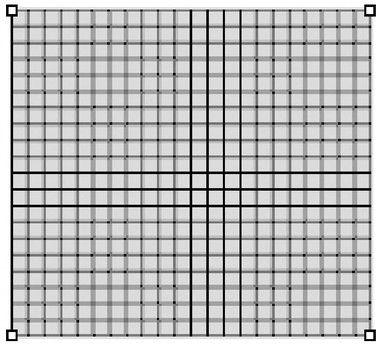
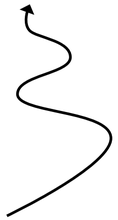

1
2 - Manifold Nurbs
Initial Meshing UV
Quad Mesh
Same Topology
Settings
Particle Spring Formfinding
Relaxed Mesh
Fixed Points
Mesh Closest Point
Fixed Vertices
2
FEM Triangulation
Triangular Mesh
GSA
Compute Principal Stress Vectors
PS Vectors
Mesh Curvature Analysis
PC Vectors
Weighting Function
Weighted Vector
3
Tutte / Projection Embed
Streamline Generator
Desired Density
Single Streamline
Density Reached?
Y
N
UV Domain
Reparameterize Streamlines On Intersections
New Streamline Point
Streamline Seeding via Density
4
Map Back To Shell
Manual Final Cleanup
Karamba
Final Gridshell Sizing and Behavior Analysis
Mass of Steel
Strain Energy
Shell Behavior

TU Delft



2 Curve Directional Loft





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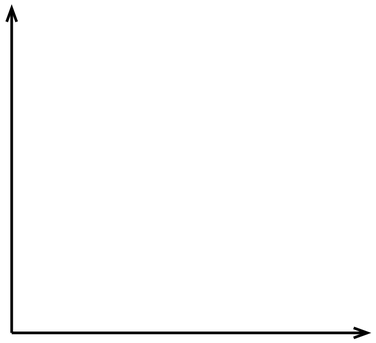
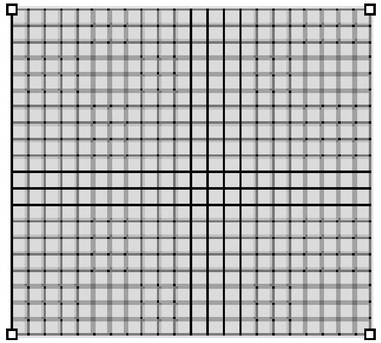
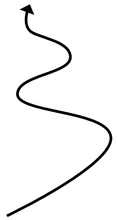
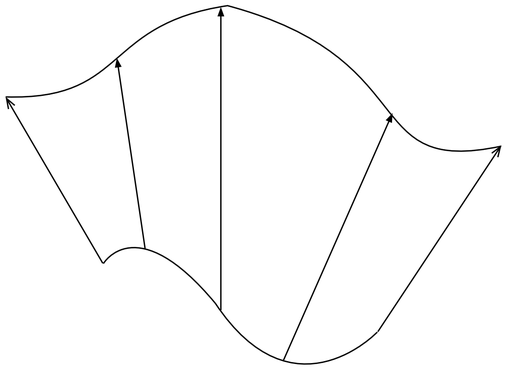


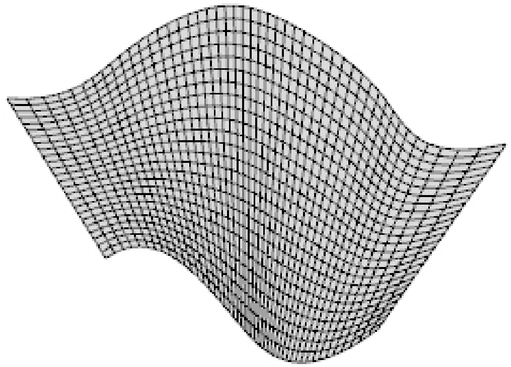
2 Curve Directional Loft







Tween Between 2 Curves



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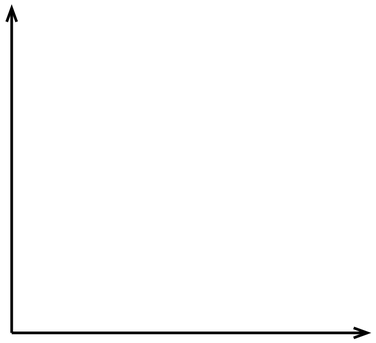
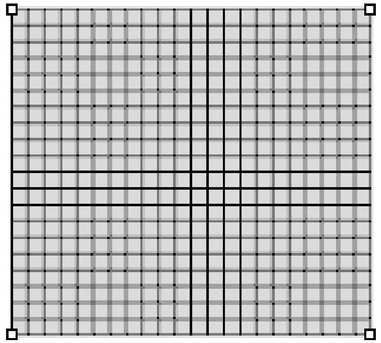
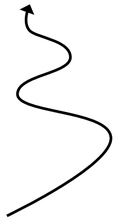
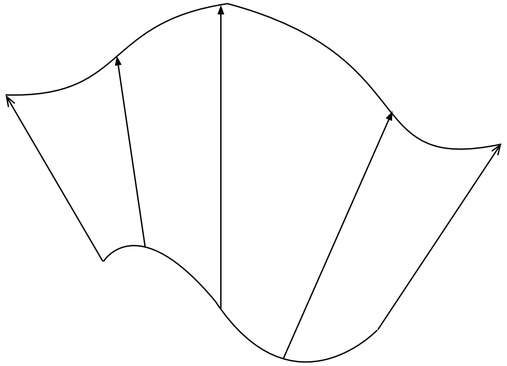


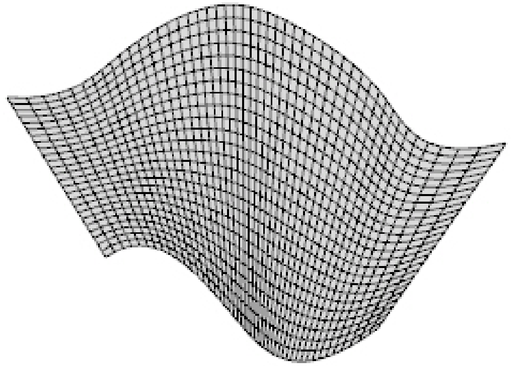
2 Curve Directional Loft

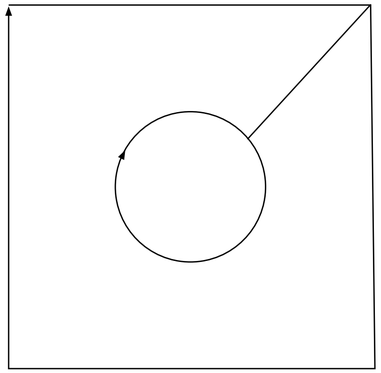




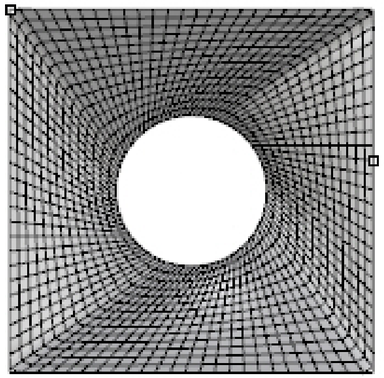


Tween Between 2 Curves

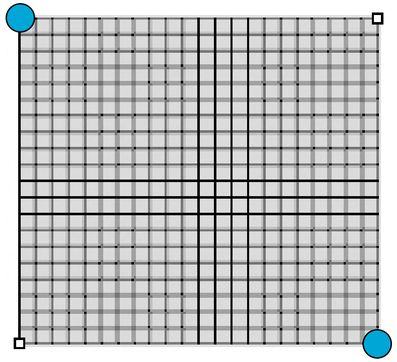




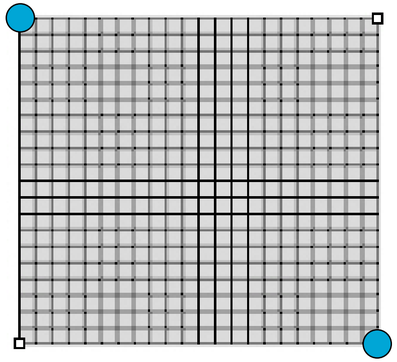
Sweep Along Rail



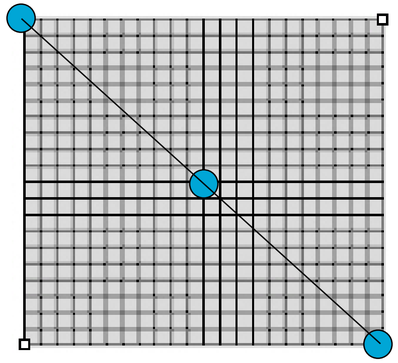
TU Delft



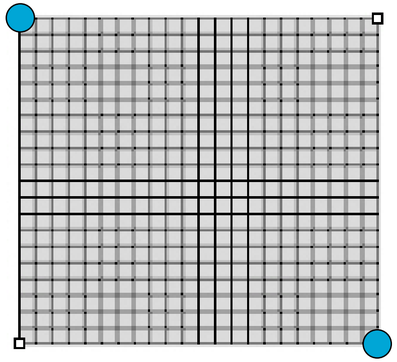
Not Enough Supports



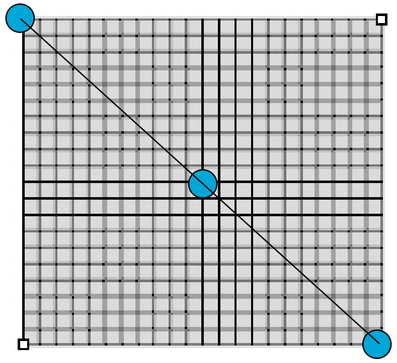
Not Enough Supports



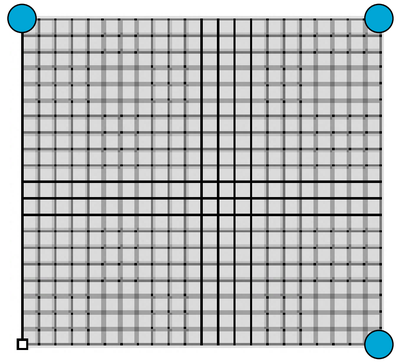
Co-linear Supports



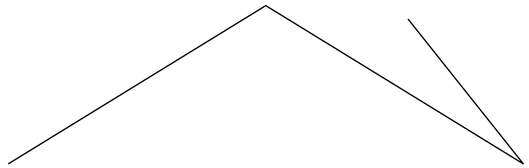
Not Enough Supports

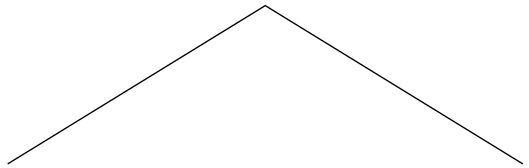


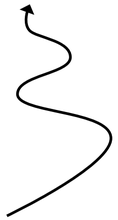
Co-linear Supports



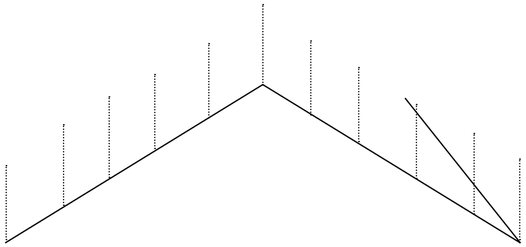
Properly placed Supports

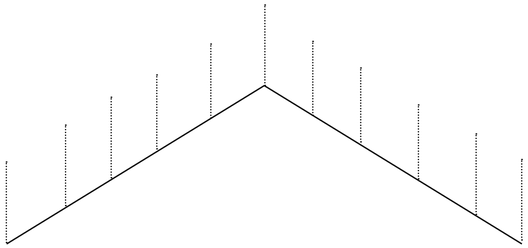


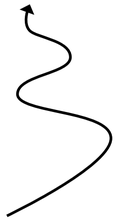




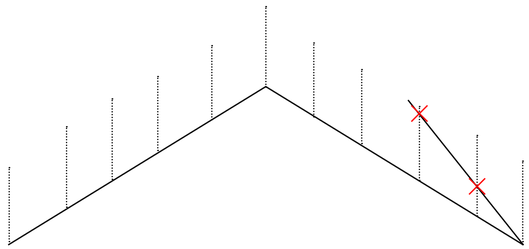
Define Initial Shape

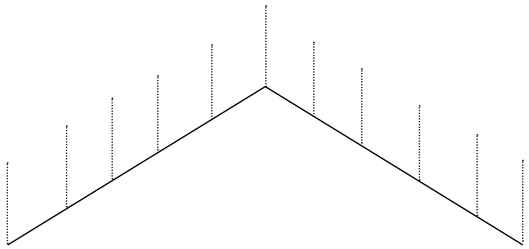






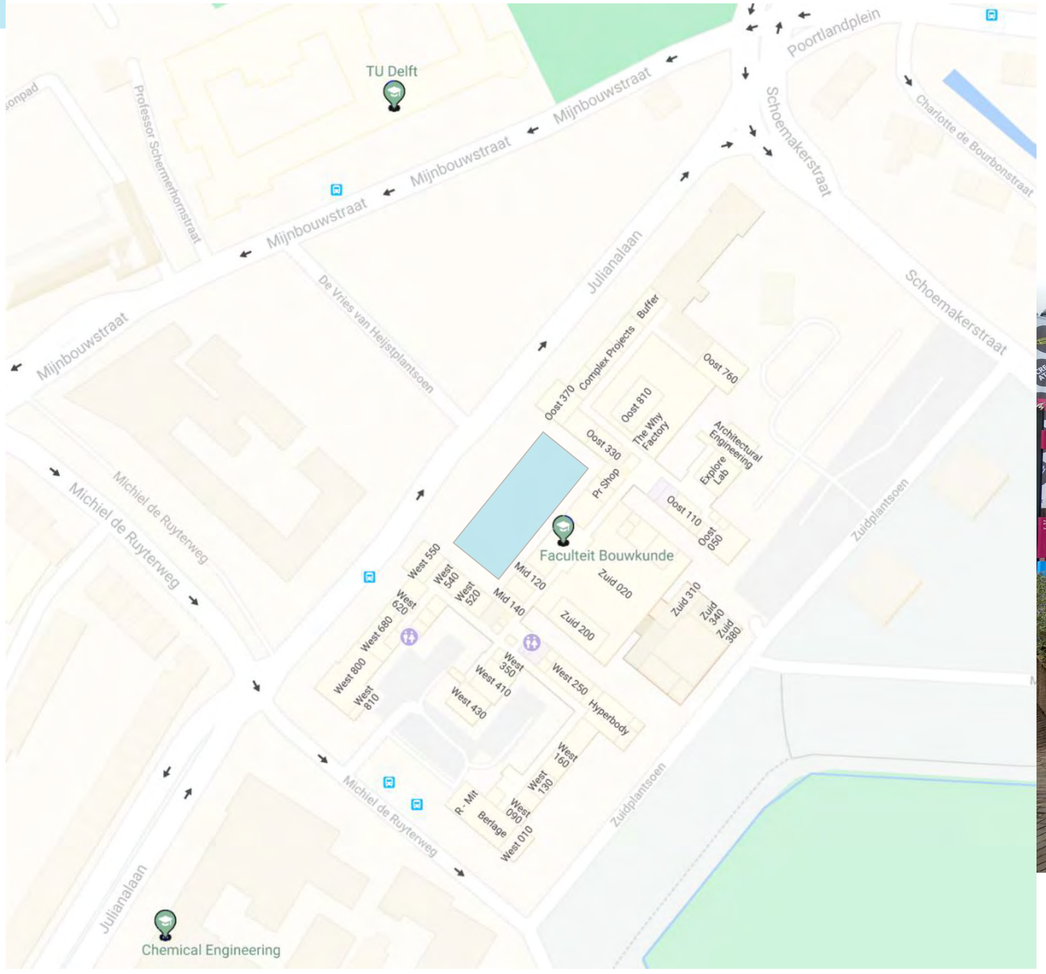
Extrapolate UV Points Vertically



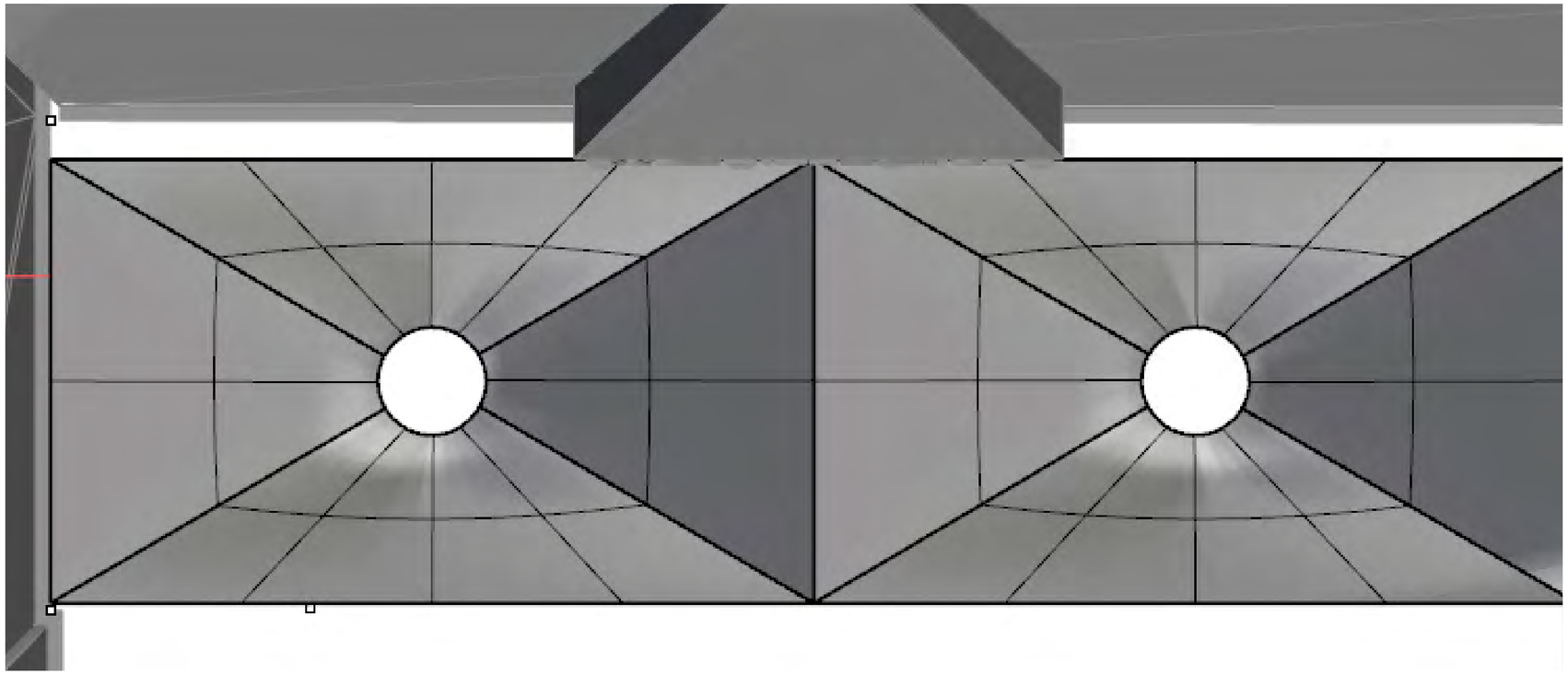


Any Curve/Surface Intersection?

Design Location

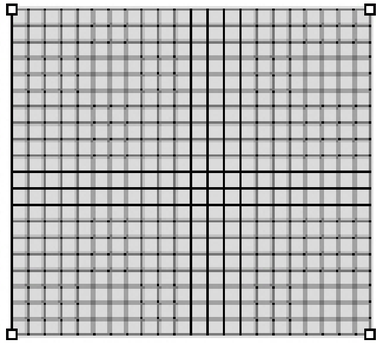


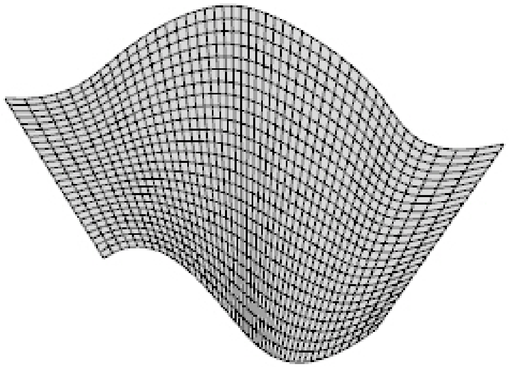
Base in Plan

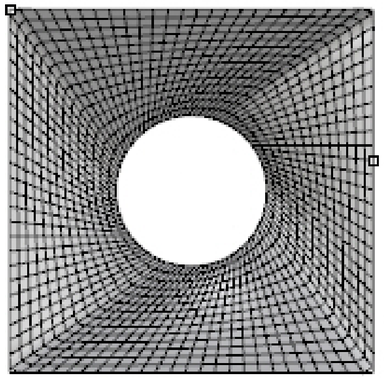


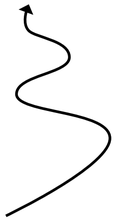
Base In Section

3 Test Structures For Form Finding

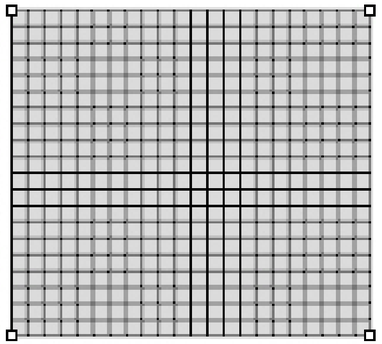


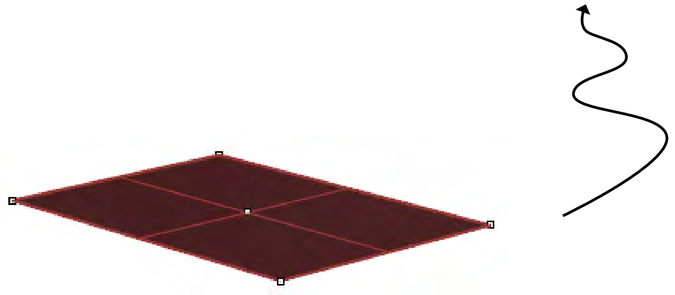




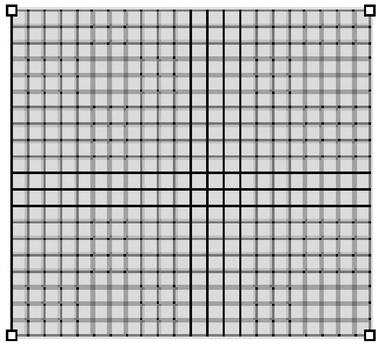


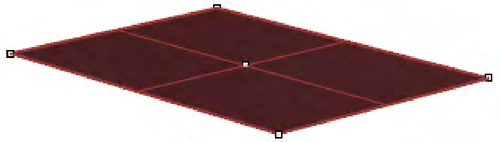
4 Point Square

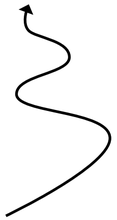




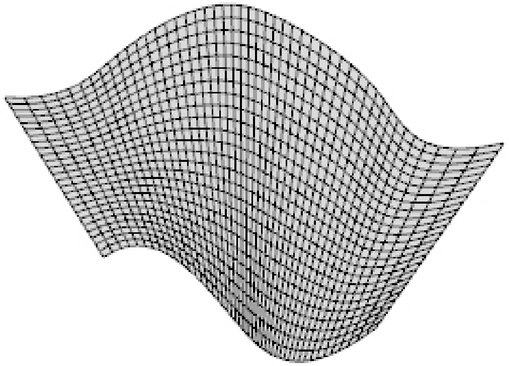
4 Point Square

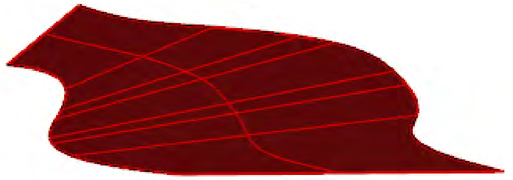






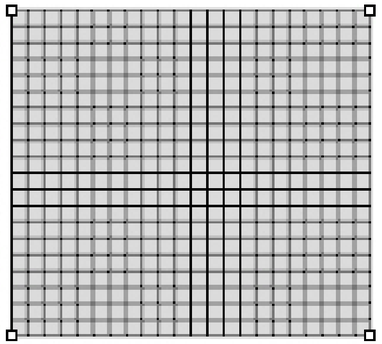
Lofted Structure Line Supports

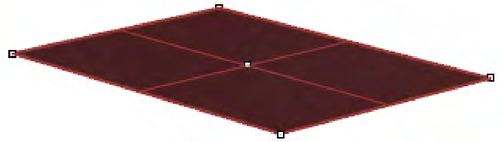


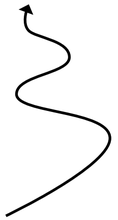


TU Delft

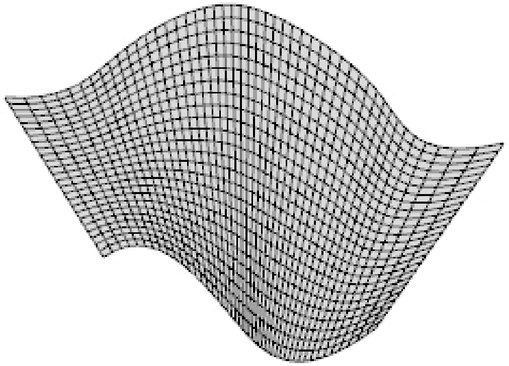
4 Point Square

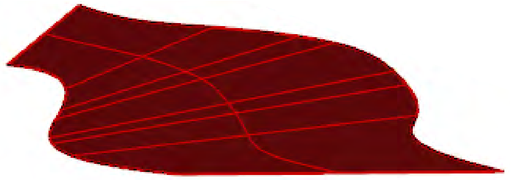




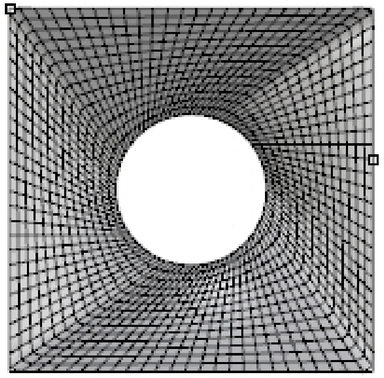


Lofted Structure Line Supports





Rotational Structure With Mixed Height Supports



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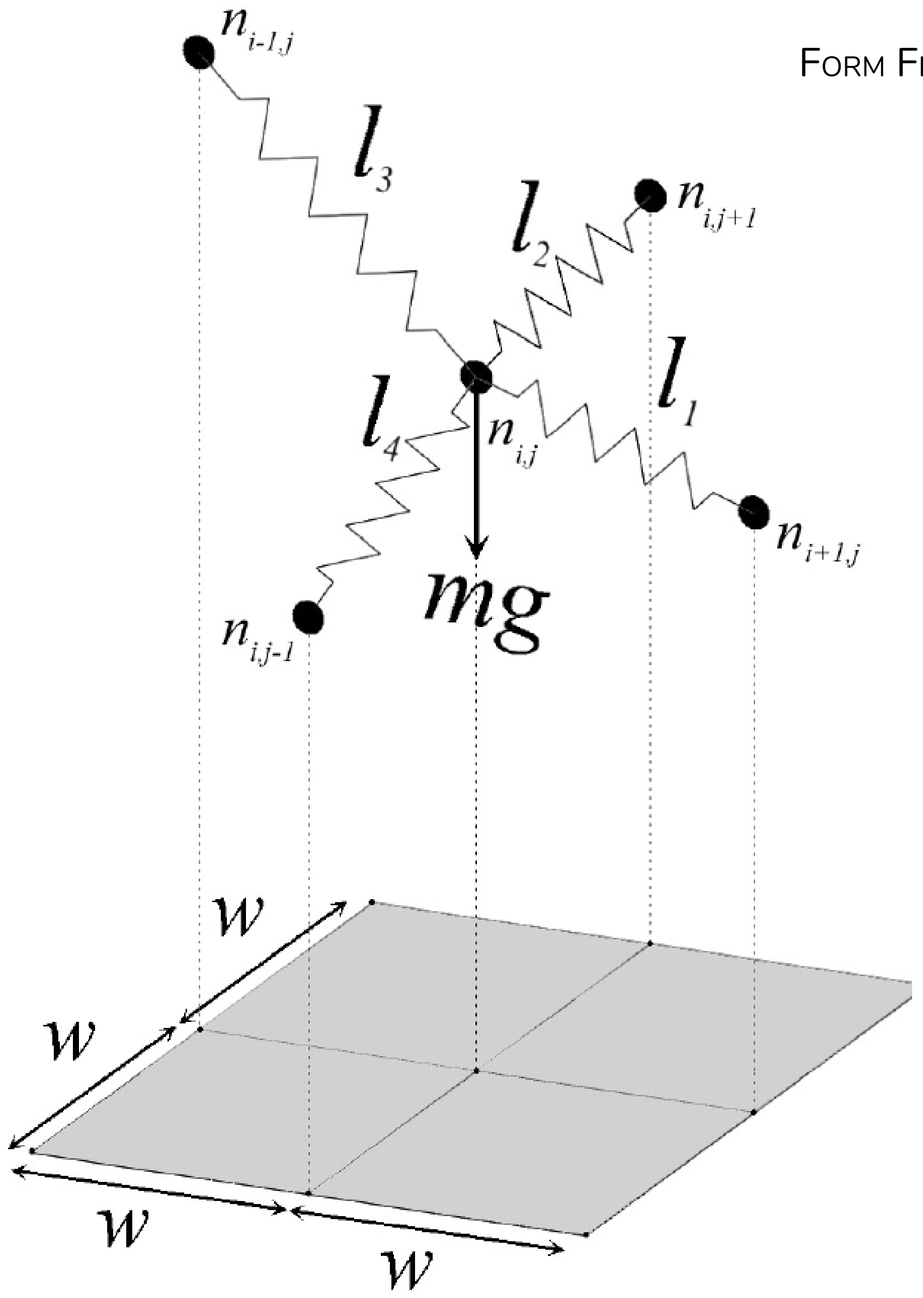
Form Finding Theory



$$\boldsymbol{f}_{ij,z} = k_s \cdot \left(\frac{(\boldsymbol{z}^t_{i+1,j} - \boldsymbol{z}^t_{i,j}) - (\boldsymbol{z}^0_{i+1,j} - \boldsymbol{z}^0_{i,j})}{|\boldsymbol{z}^0_{i+1,j} - \boldsymbol{z}^0_{i,j}|} + \frac{(\boldsymbol{z}^t_{i,j+1} - \boldsymbol{z}^t_{i,j}) - (\boldsymbol{z}^0_{i,j+1} - \boldsymbol{z}^0_{i,j})}{|\boldsymbol{z}^0_{i,j+1} - \boldsymbol{z}^0_{i,j}|} + \frac{(\boldsymbol{z}^t_{i,j-1} - \boldsymbol{z}^t_{i,j}) - (\boldsymbol{z}^0_{i,j-1} - \boldsymbol{z}^0_{i,j})}{|\boldsymbol{z}^0_{i,j-1} - \boldsymbol{z}^0_{i,j}|} + \frac{(\boldsymbol{z}^t_{i-1,j} - \boldsymbol{z}^t_{i,j}) - (\boldsymbol{z}^0_{i,j-1} - \boldsymbol{z}^0_{i,j})}{|\boldsymbol{z}^0_{i-1,j} - \boldsymbol{z}^0_{i,j}|} \right) - m\boldsymbol{f}_g$$

$$\boldsymbol{f}_{ij,(x,y)} = k_s \cdot \left(\frac{(\boldsymbol{n}^t_{i+1,j} - \boldsymbol{n}^t_{i,j}) - (\boldsymbol{n}^0_{i+1,j} - \boldsymbol{n}^0_{i,j})}{|\boldsymbol{n}^0_{i+1,j} - \boldsymbol{n}^0_{i,j}|} + \frac{(\boldsymbol{n}^t_{i,j+1} - \boldsymbol{n}^t_{i,j}) - (\boldsymbol{n}^0_{i,j+1} - \boldsymbol{n}^0_{i,j})}{|\boldsymbol{n}^0_{i,j+1} - \boldsymbol{n}^0_{i,j}|} + \frac{(\boldsymbol{n}^t_{i,j-1} - \boldsymbol{n}^t_{i,j}) - (\boldsymbol{n}^0_{i,j-1} - \boldsymbol{n}^0_{i,j})}{|\boldsymbol{n}^0_{i,j-1} - \boldsymbol{n}^0_{i,j}|} + \frac{(\boldsymbol{n}^t_{i-1,j} - \boldsymbol{n}^t_{i,j}) - (\boldsymbol{n}^0_{i-1,j} - \boldsymbol{n}^0_{i,j})}{|\boldsymbol{n}^0_{i-1,j} - \boldsymbol{n}^0_{i,j}|} \right)$$

4 Connected Springs

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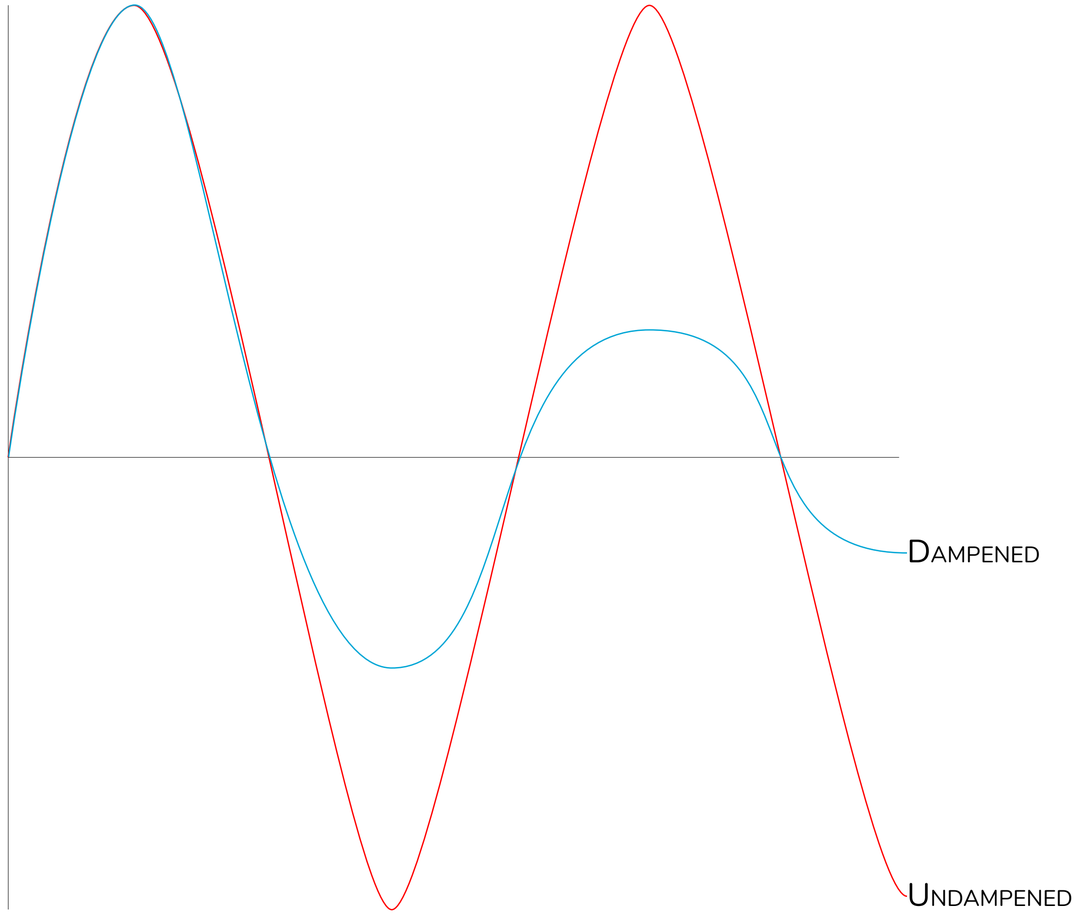
Form Finding Theory

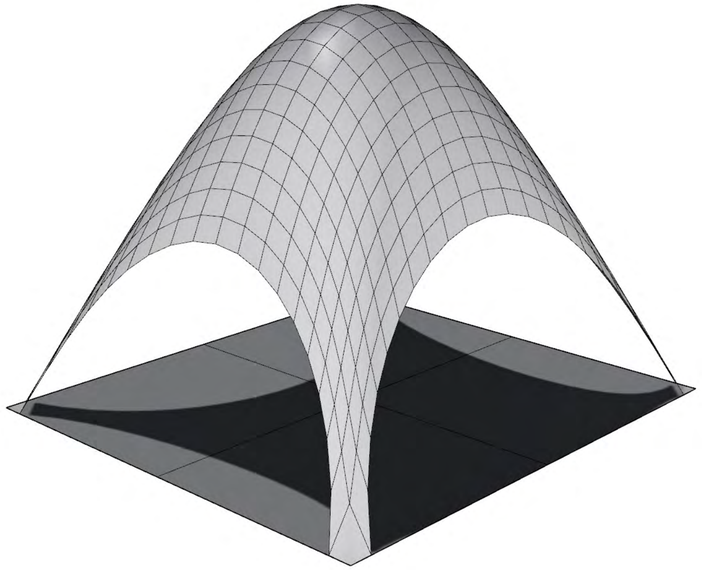


$$\boldsymbol{u}_{ix}^{t+\frac{\Delta t}{2}} = A \times \boldsymbol{u}_{ix}^{t-\frac{\Delta t}{2}} + B \times \left(\frac{\Delta t}{m_i}\right) \boldsymbol{f}_{ix}^{t}$$

$$where\ A = \frac{1-\frac{C}{2}}{1+\frac{C}{2}}, B = \frac{1+A}{2}, C = damping\ factor$$

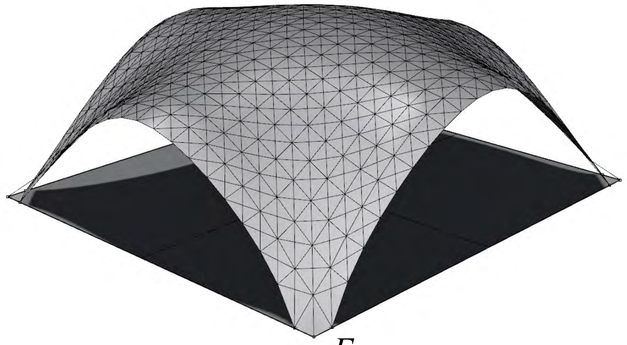
Viscous Damping

Quad - No Shear Strength



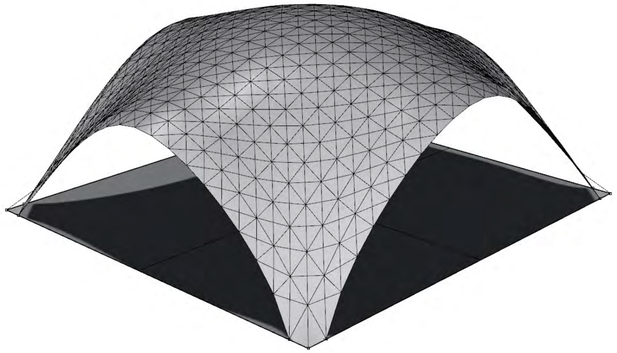
D = 0

Diagonals With Same Strength as Quads



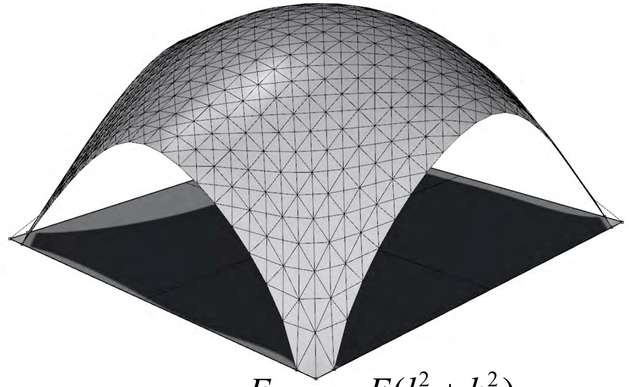
$$k_d = \frac{E}{2}$$

Diagonals with Strength Based on Geometry



$$k_d = \frac{(l - l_0)\sqrt{2}}{2}$$

Diagonals Based on Poisson's Ratio



$$k_d = \frac{E}{2(1+\upsilon)} = \frac{E(l_0^2 + h_0^2)}{4 l_0 h_0 (1+\upsilon)}$$

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Quad Form Finding Structure

Strain Energy Density (kJ/m$^{3)}$

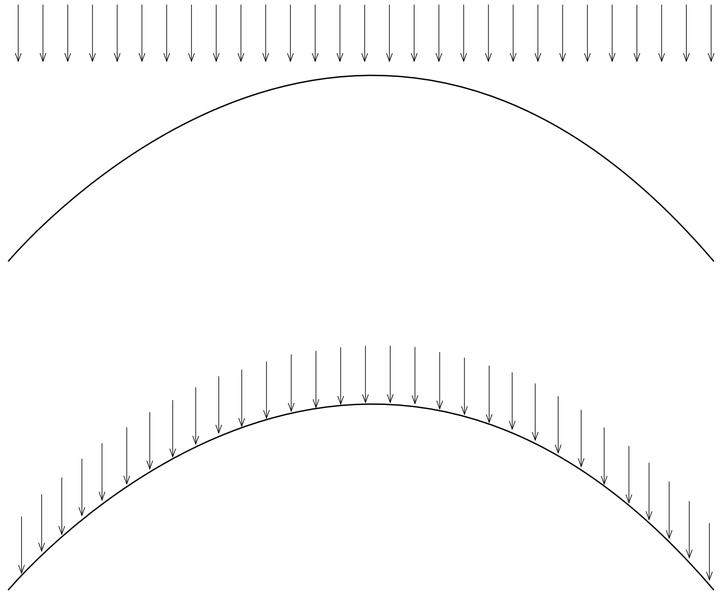
	4 Point	Line Supports	MHS
Quads	5.77e-2	**1.33e-3**	**1.06e-3**
Diagonals Same Strength as Quads	2.10e-2	2.16e-3	1.22e-3
Diagonals based on Geometry	1.81e-2	1.94e-3	1.26e-3
Diagonals Based on Poisson's Ratio	**1.58e-2**	1.74e-3	1.29e-3

Mean Shell Behavior score

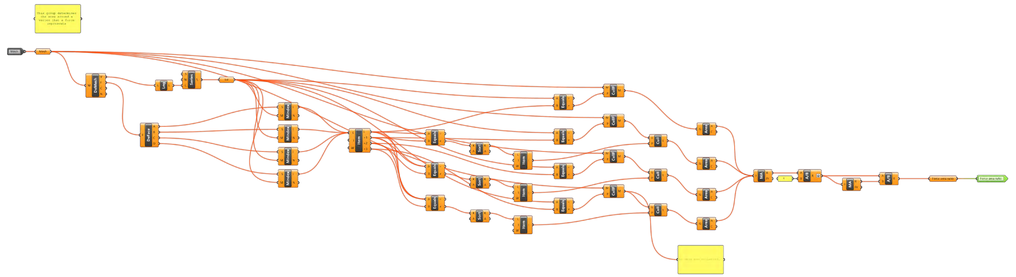
	4 Point	Line Supports	MHS
Quads	93	96.8	91.5
Diagonals Same Strength as Quads	**97.2**	96.7	92.2
Diagonals based on Geometry	88.4	96.8	91.8
Diagonals Based on Poisson's Ratio	90	96.9	91.9

Force Area Ratio

$$P = G \times A \neq G \times A_{\perp}$$

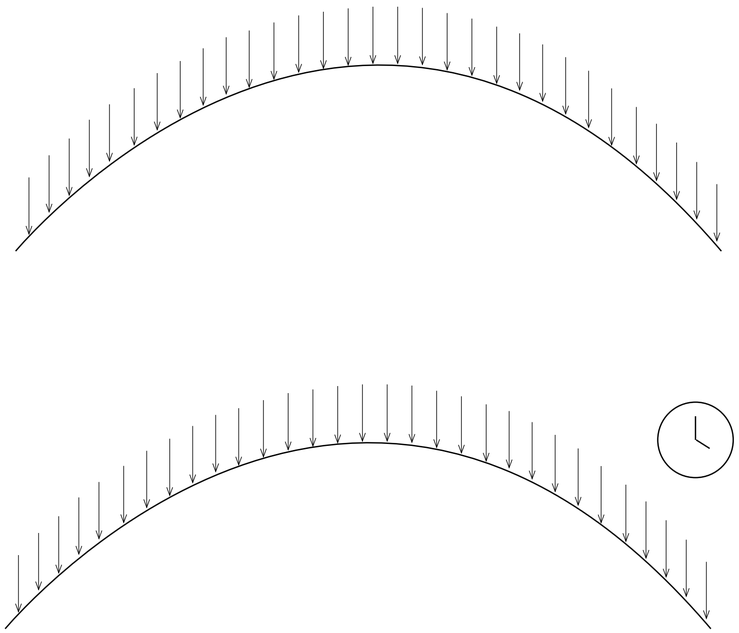


	4 Point	Line Supports	MHS
Non FAR Strain Energy Density	2.32e-2	1.26e-3	1.39e-3
FAR Strain Energy Density	2.34e-2	1.17e-3	1.13e-3
% Improvement	-0.82%	7.18%	18.5%

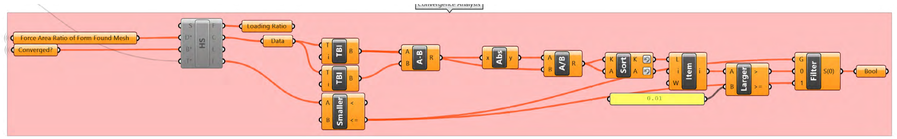


F.A.R. Looping

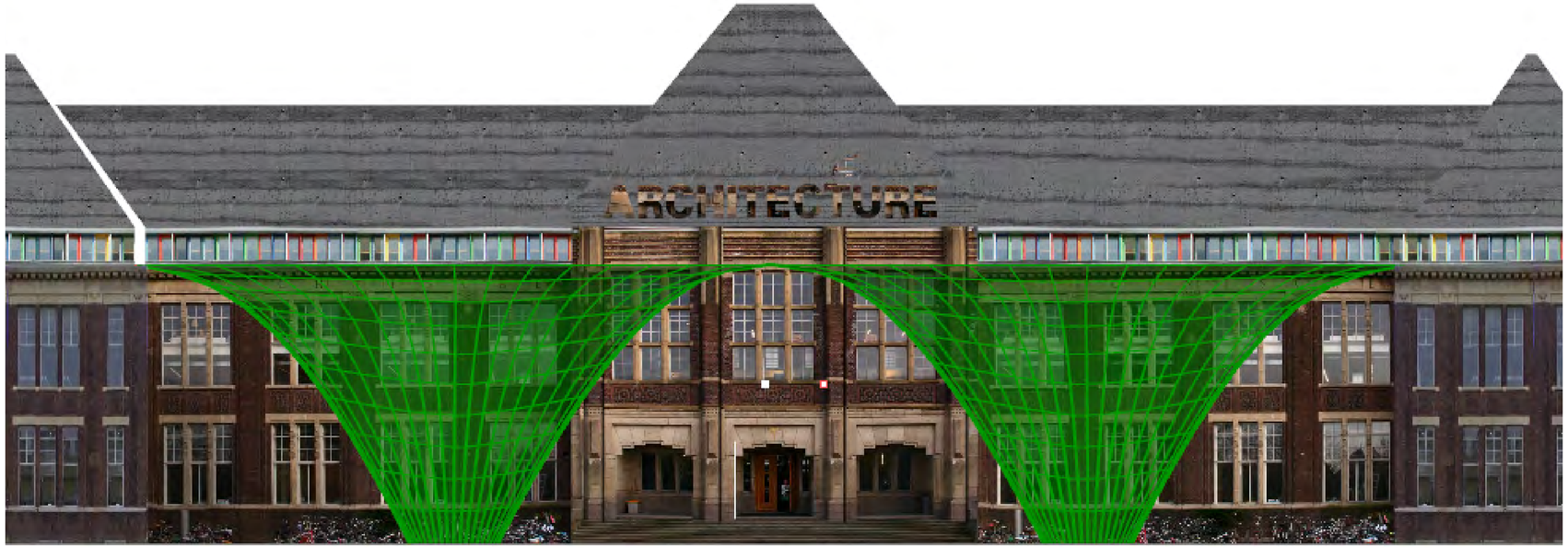
$$C < 0.01 = \frac{\text{mean}\left|A_{i,j} - A_{i-1,j}\right|}{A_{i-1,j}}$$



	4 Point	Line Supports	MHS
FAR Stain Energy Density	2.34e-2	1.17e-3	1.13e-3
Looped Strain Energy Density	2.35e-2	1.19e-3	1.14e-3
% Improvement	-0.42	-1.7%	-0.88%



Form Found Section



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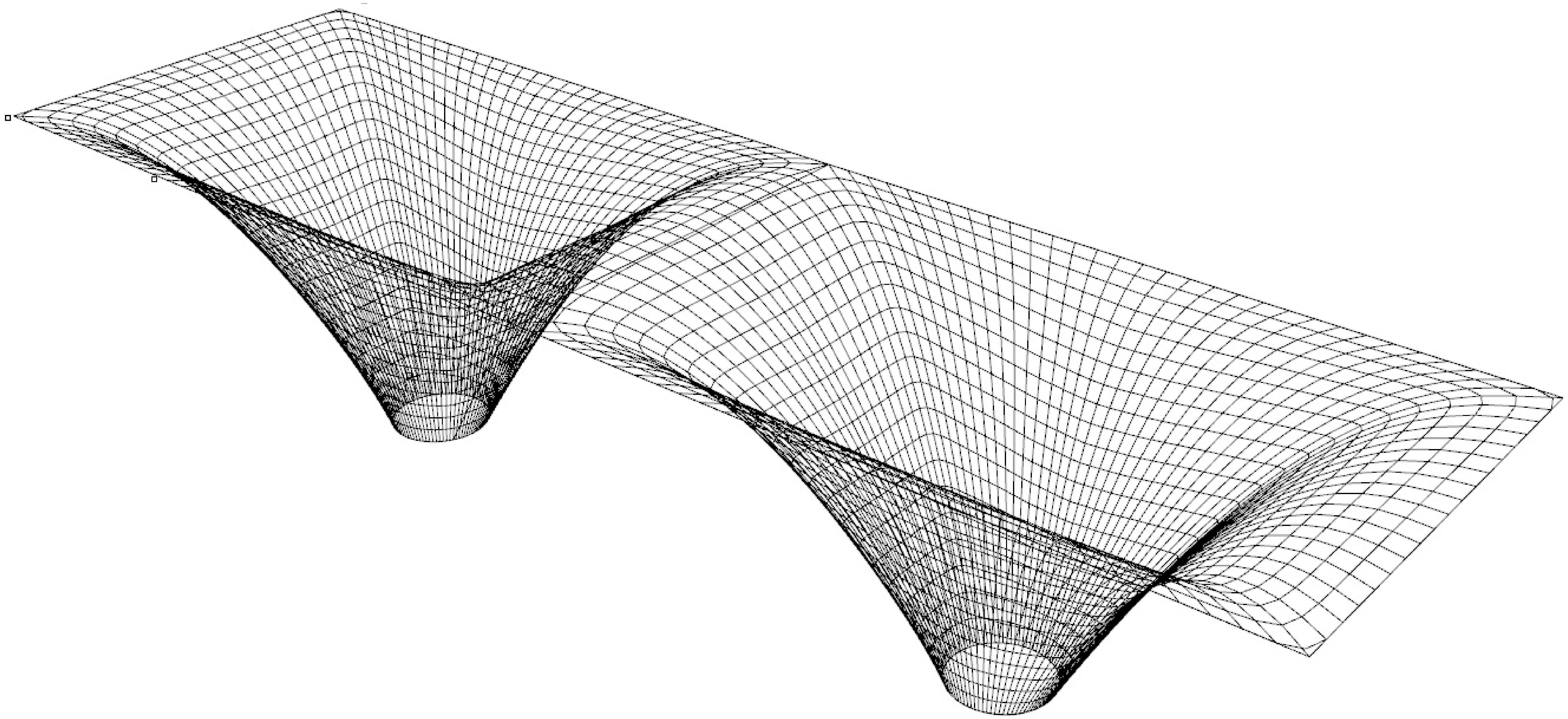
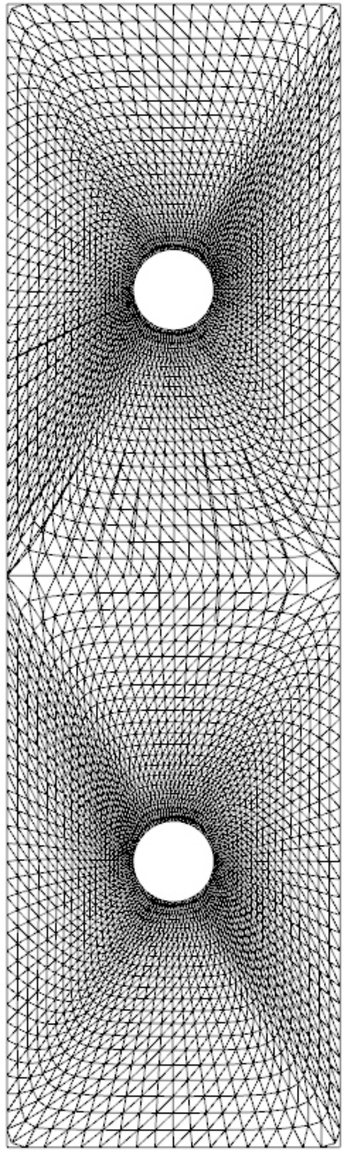
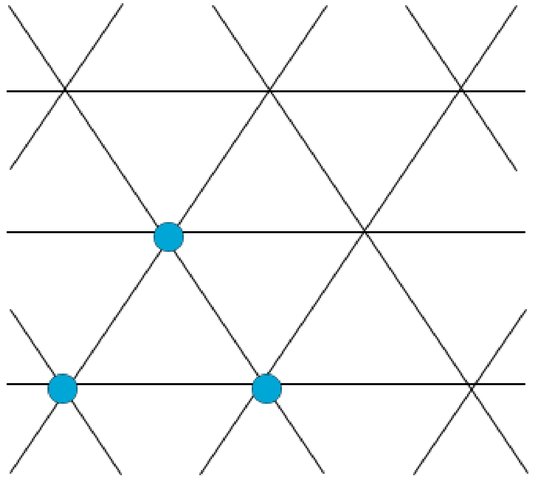
Form Found Design 3d View

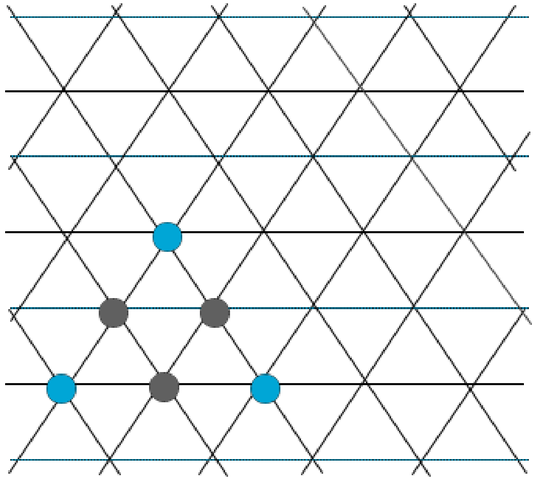


Refining Mesh for FEA



Loop Subdivision





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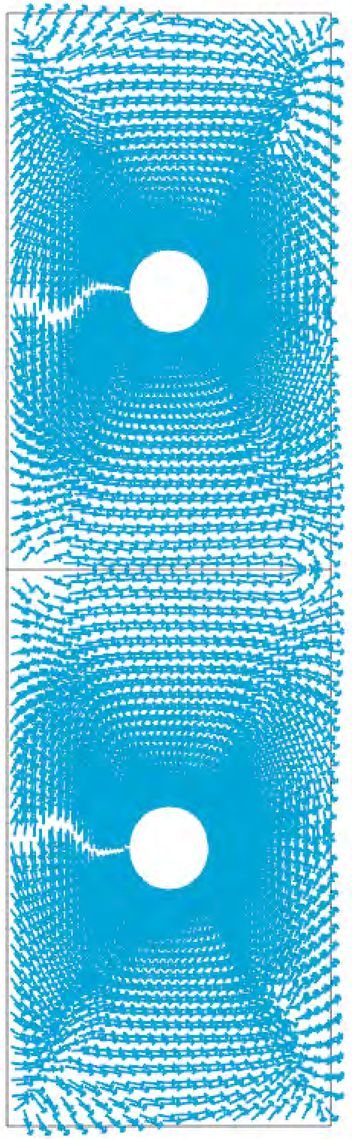
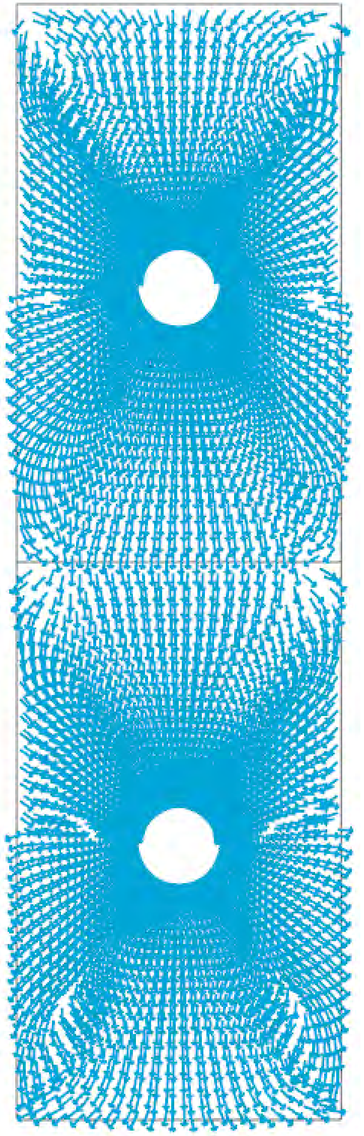
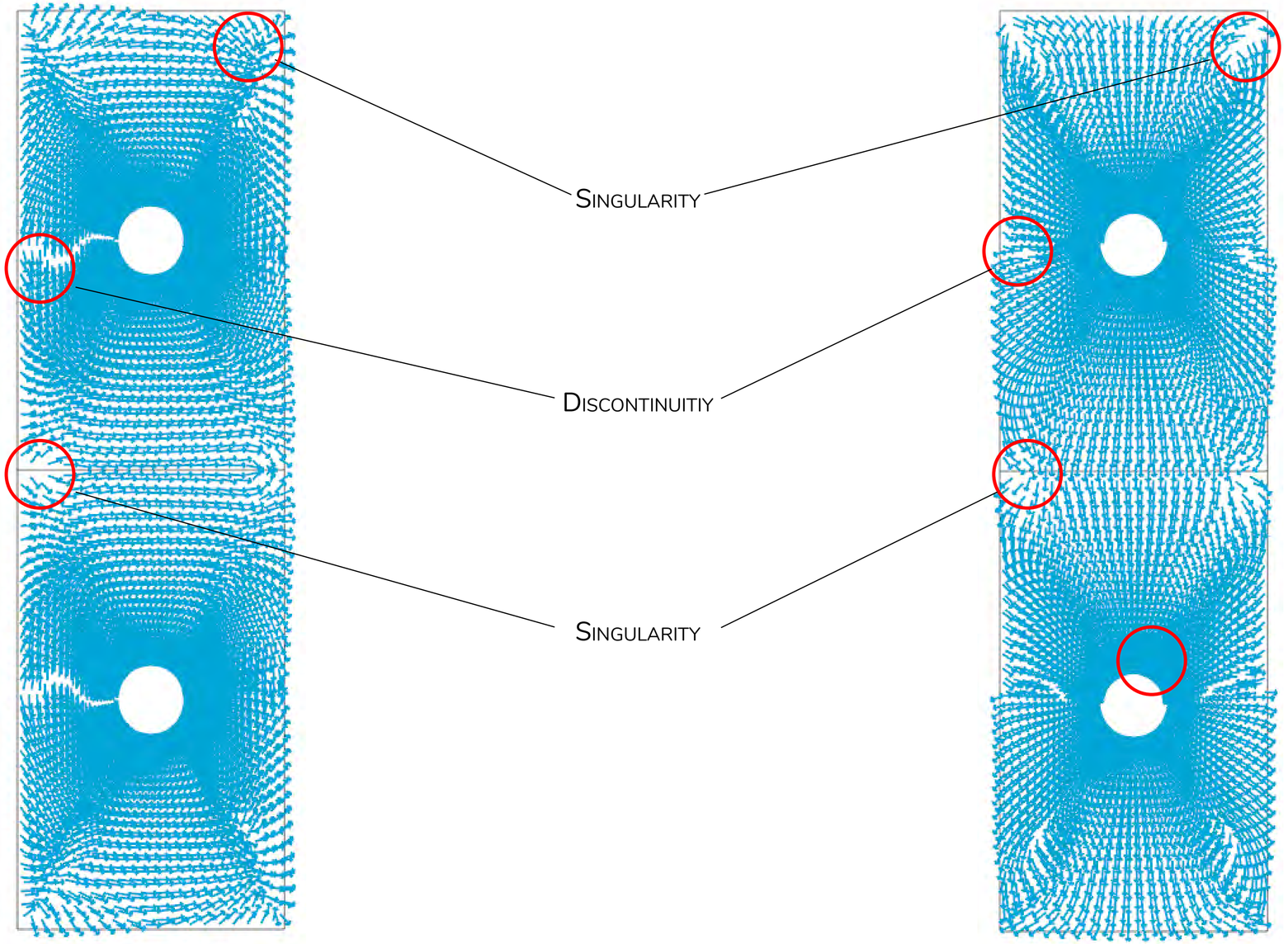
Principal Stress Vectors

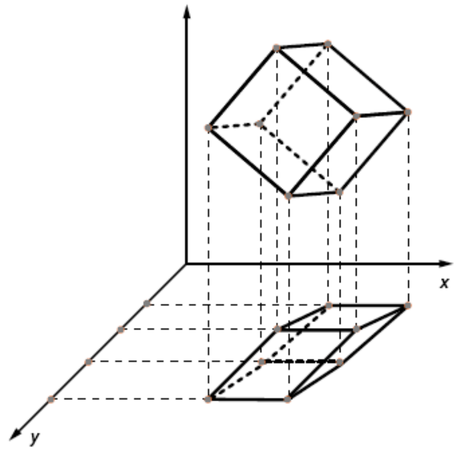




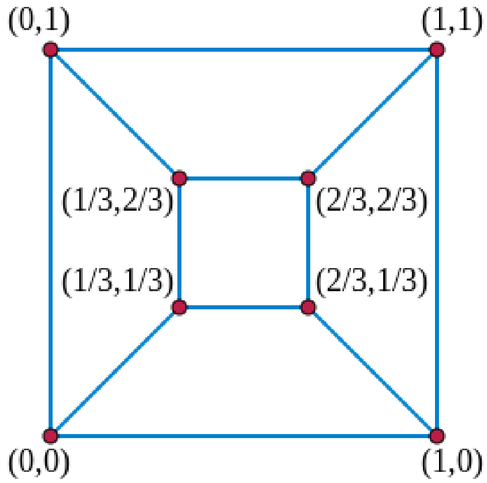
Principal Stress Vectors



Mesh Flattening / Embedding



N(x,y,z) → N(x,y)



$$v_{i,j} = \sum v_n$$

Where $v_{i,j}$ is the mesh vertex index location and

$$v_n = v_{i+1,j}, v_{i,j+1}, v_{i-1,j}, v_{i,j-1}$$

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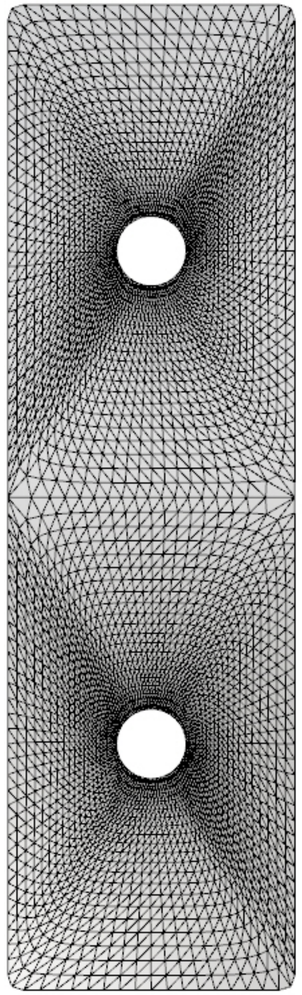
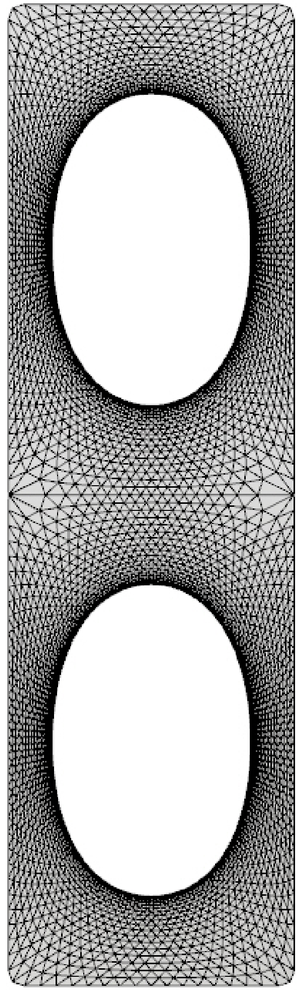
Flattening Methods



Projection



Tutte Embed

	Maximum Difference in Face Area	Minimum Difference in Face Area	Mean Difference in Face Area	Median Difference in Face Area
XY Projection	0.197	1.65e-7	0.018	0.003
Tutte Embedding	0.205	3.5e-5	0.101	0.099

Streamline Integrator - Gaussian Weighting

Generalized Gaussian Smoothing

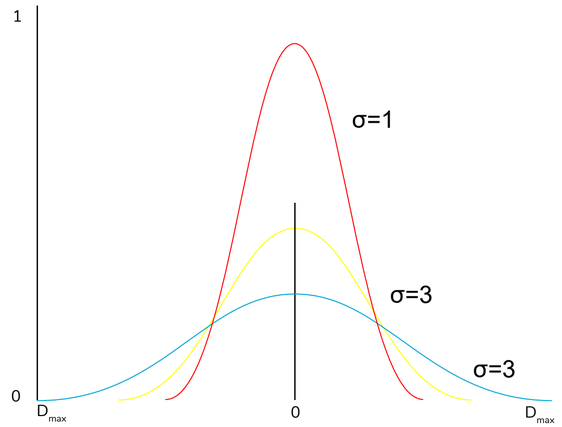
$$g(x) = ce^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Adapted Gaussian Smoothing

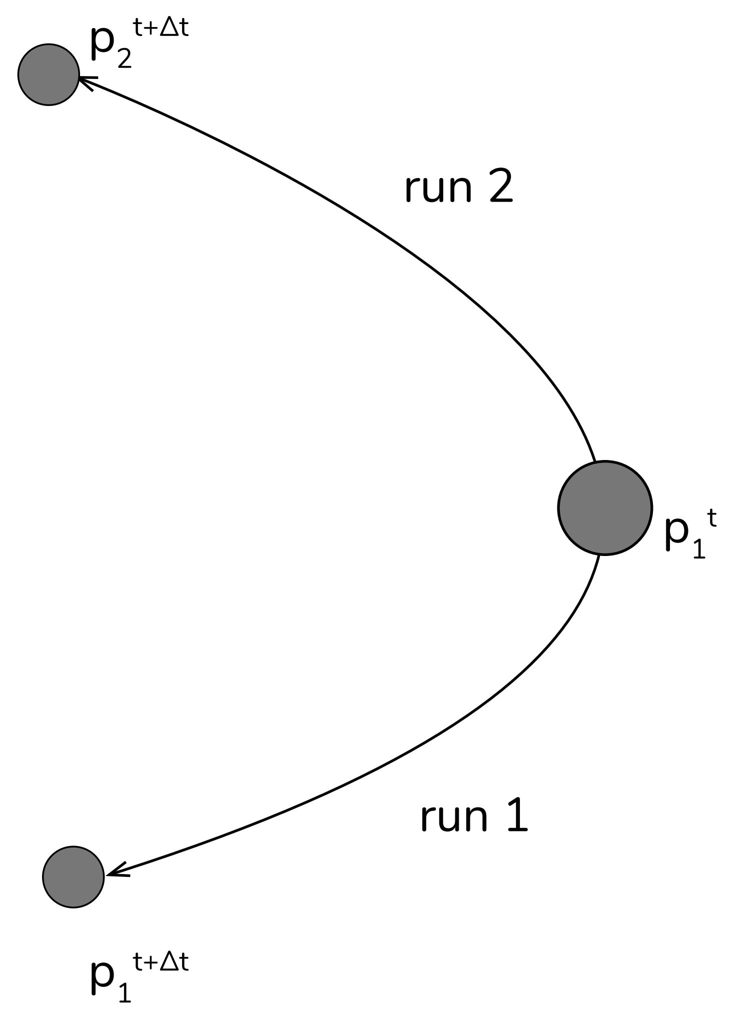
$$w_j = e^{\frac{-(d_{i,j}-\mu)^2}{2\sigma^2}}$$

$$w_{sum} = \sum_{j=1}^{n} e^{\frac{-(d_{i,j}-\mu)^2}{2\sigma^2}}$$

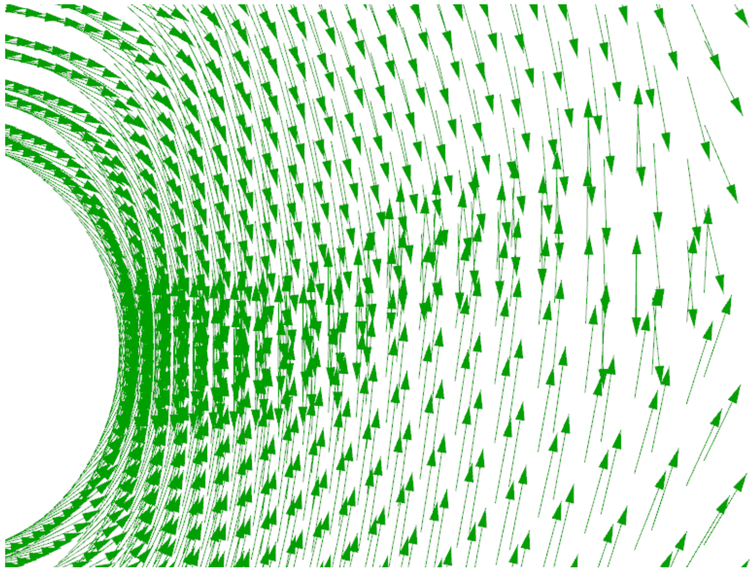
$$\boldsymbol{\sigma_{1,i}} = \sum_{j=1}^{n} \boldsymbol{\sigma_{1,j}} * \frac{w_j}{w_{sum}}$$



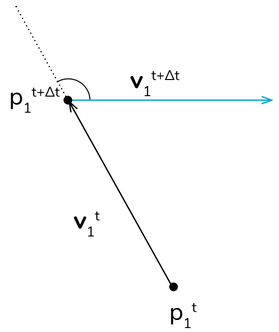
Streamline Integrator - Step Function



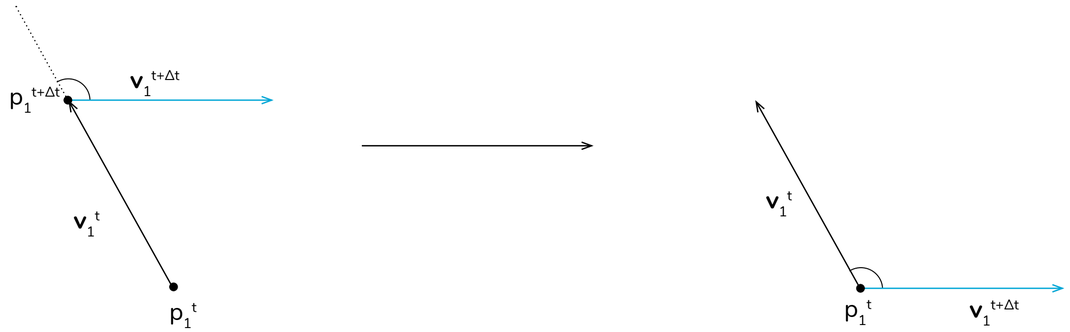
Streamline Integrator - Vector Reverse



Streamline Integrator - Vector Reverse

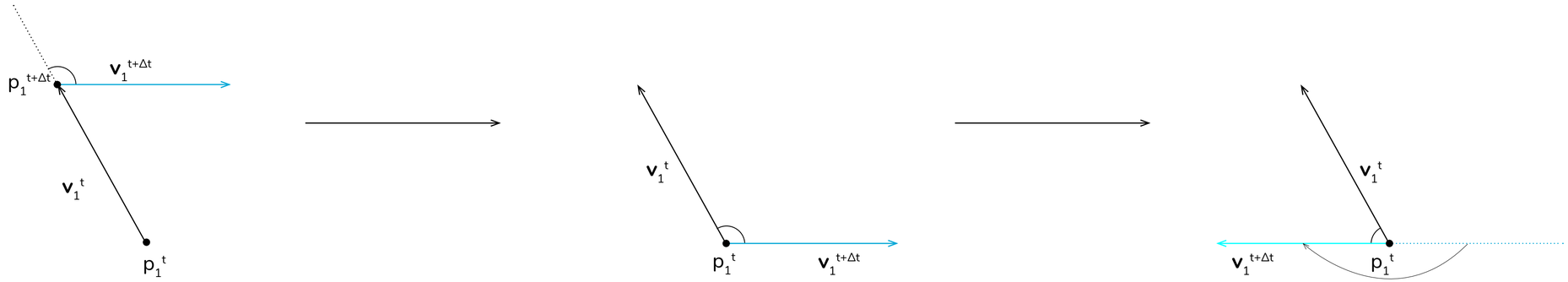


Streamline Integrator - Vector Reverse



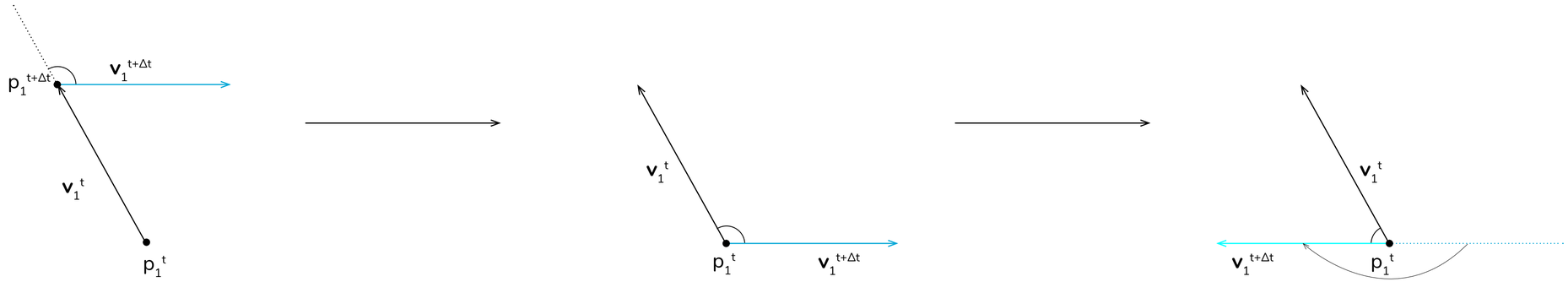
$$v_1^{t} \cdot v_1^{t+\Delta t} < 0$$

Streamline Integrator - Vector Reverse

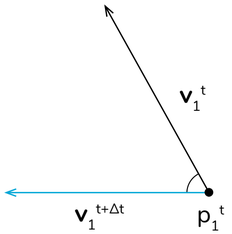


$$v_1^{t} \cdot v_1^{t+\Delta t} < 0$$

Streamline Integrator - Vector Reverse



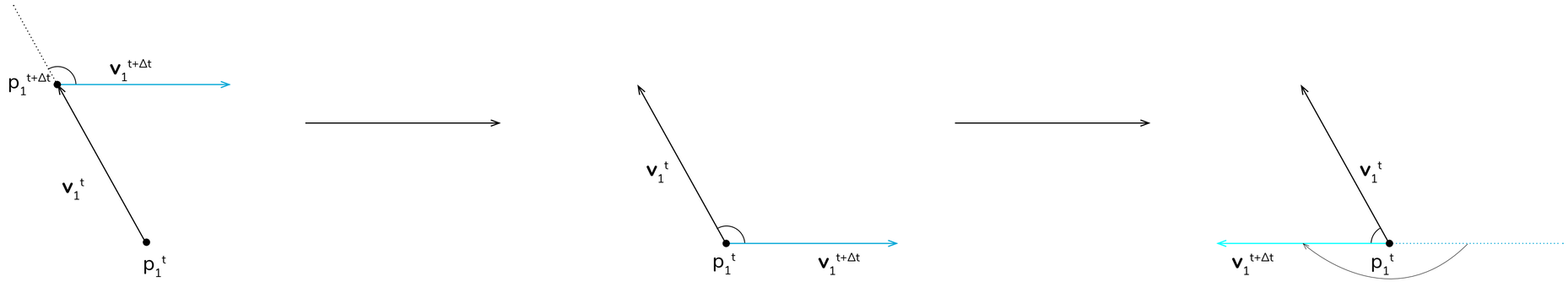
$$v_1^{t} \cdot v_1^{t+\Delta t} < 0$$



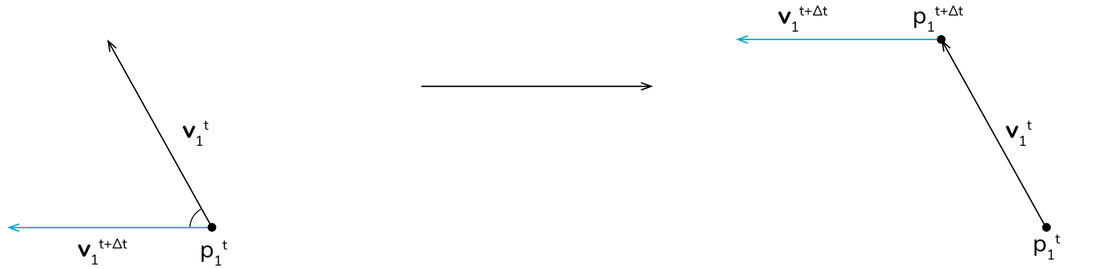
$$v_1^{t} \cdot v_1^{t+\Delta t} > 0$$

TU Delft

Streamline Integrator - Vector Reverse

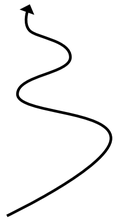


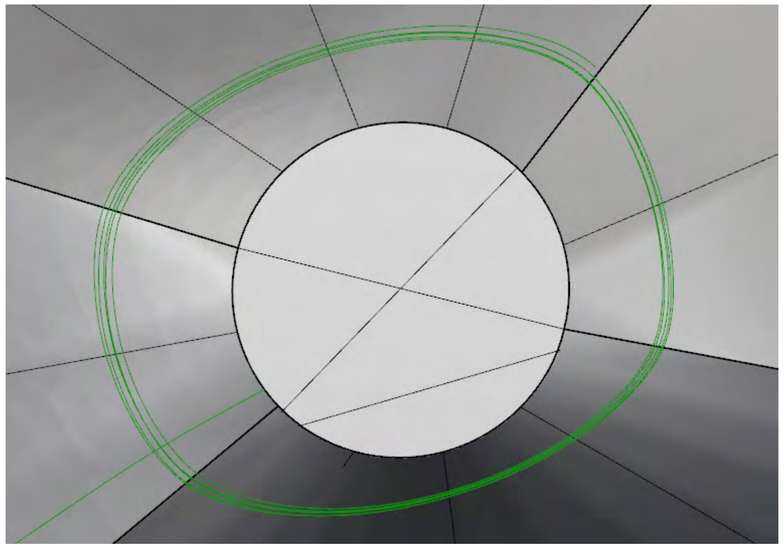
$$v_1^{t} \cdot v_1^{t+\Delta t} < 0$$



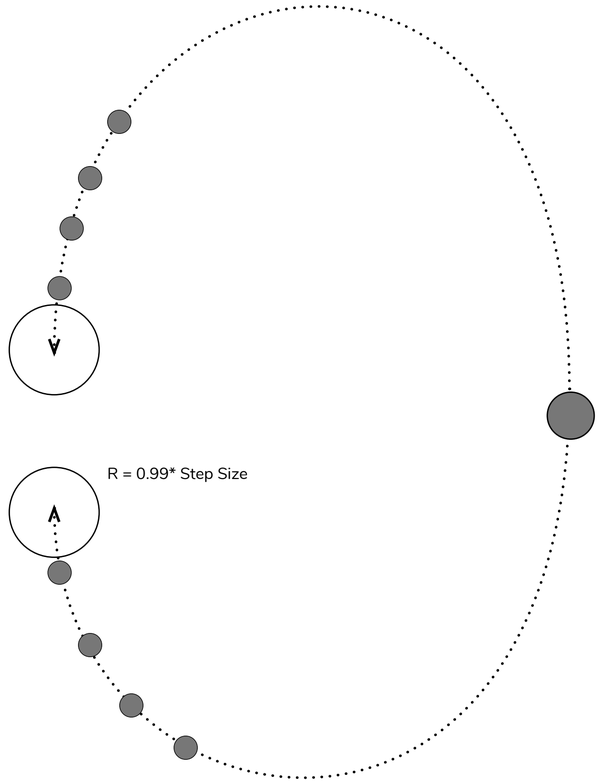
$$v_1^{t} \cdot v_1^{t+\Delta t} > 0$$

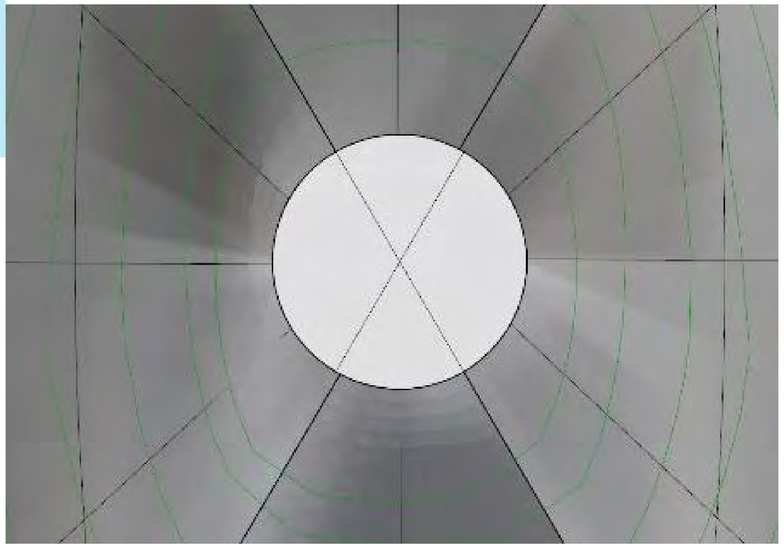
Streamline Integrator - Looping





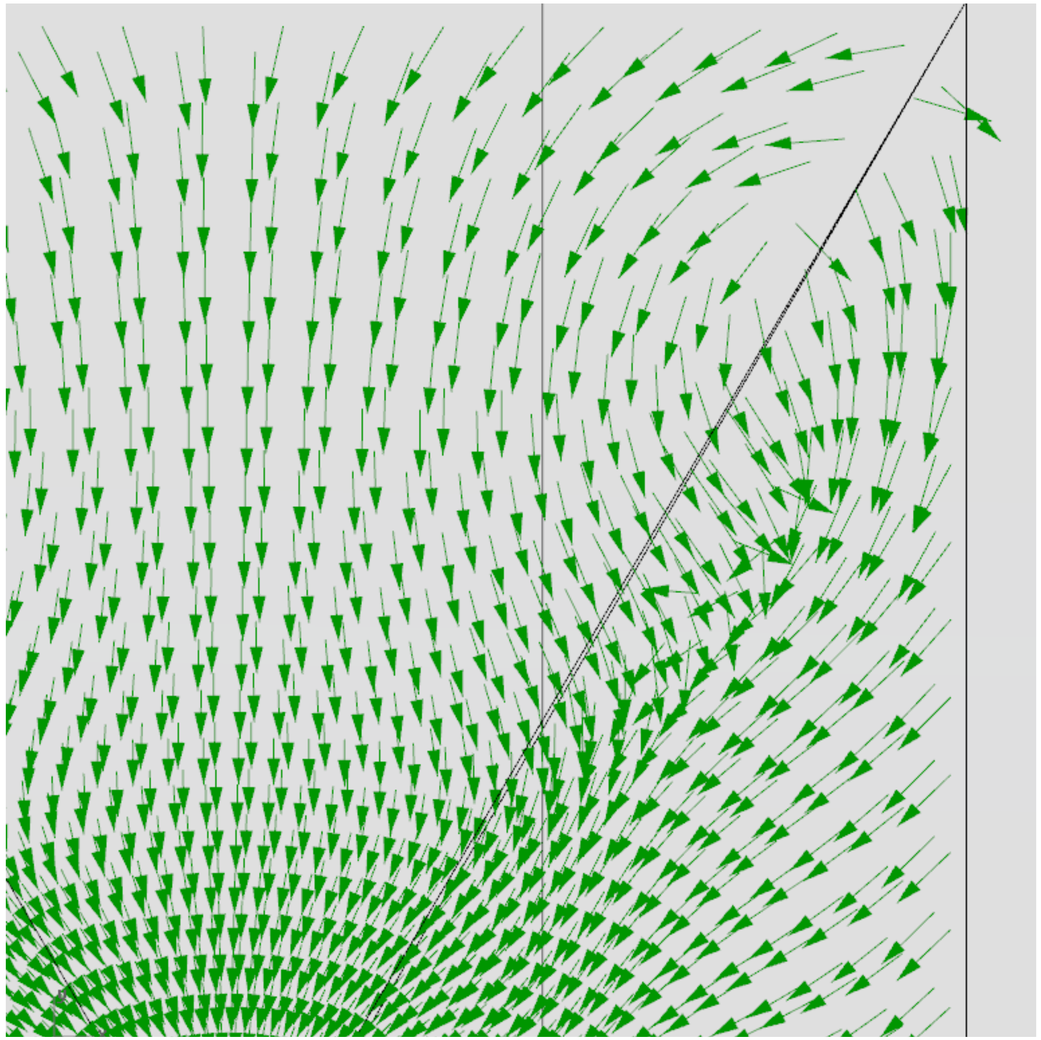
Karamba



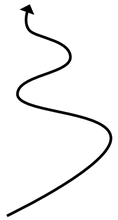


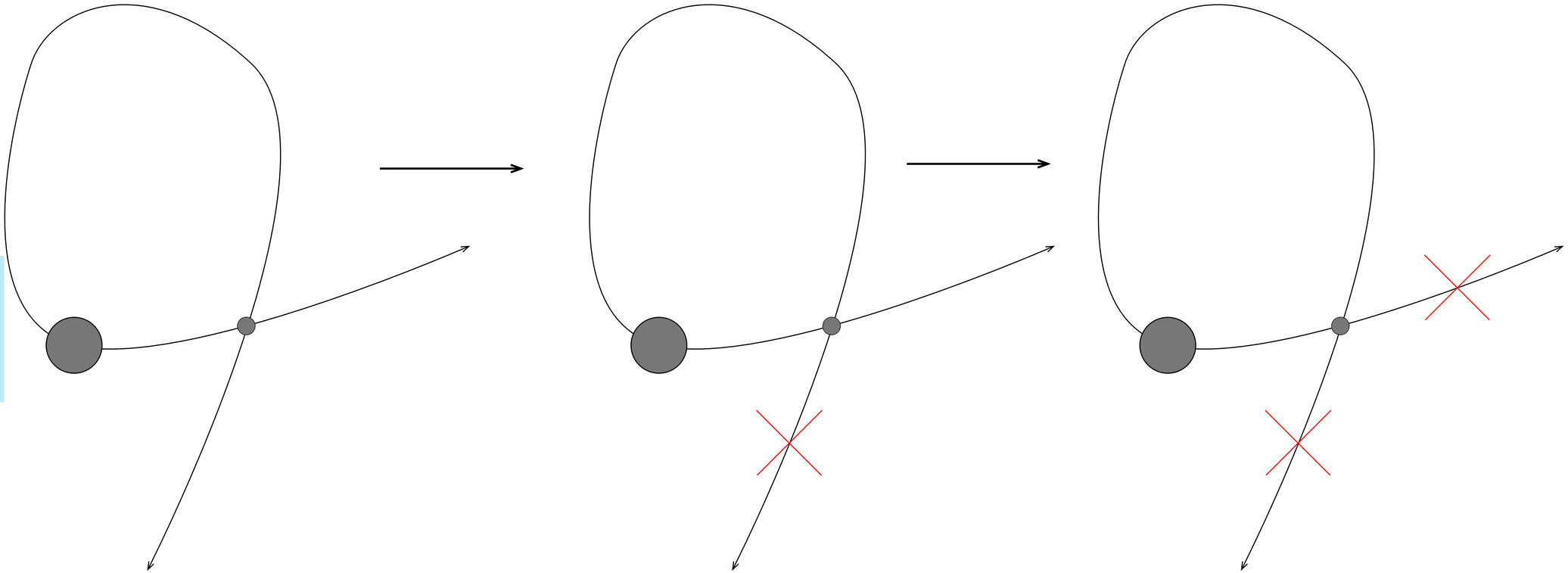
Custom

Streamline Integrator - Looping



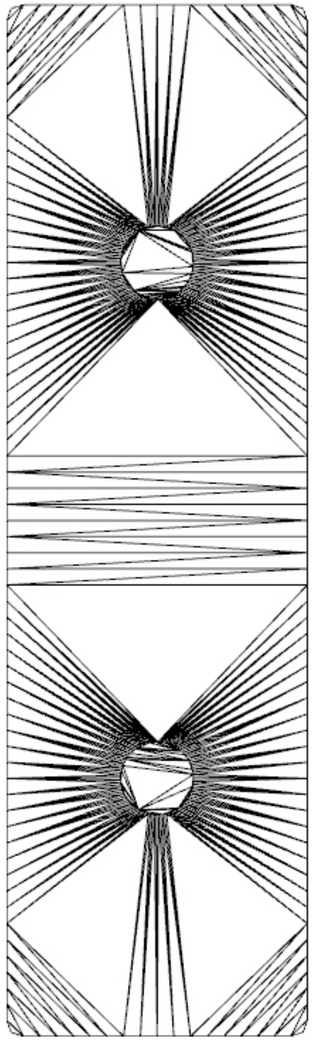
Streamline Integrator - Looping

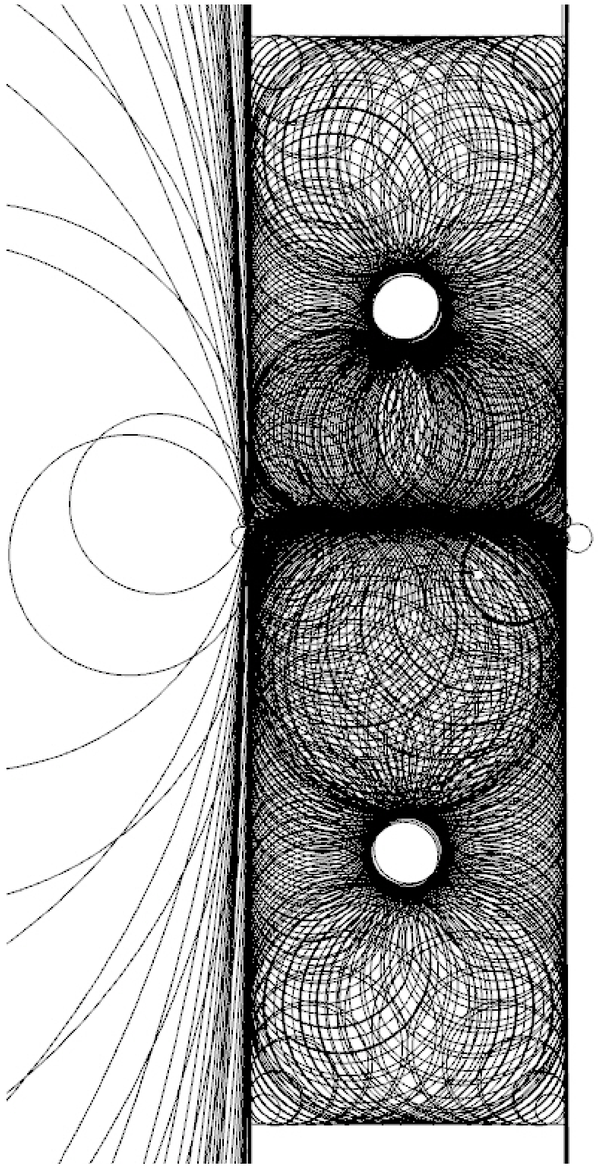




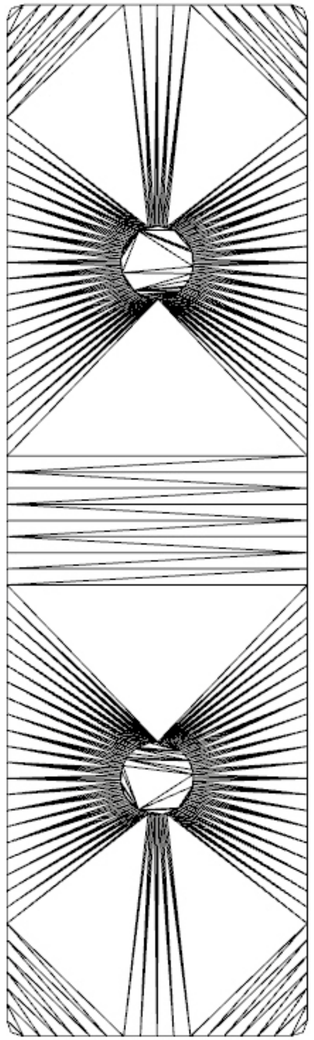
EVENLY SPACED STREAMLINE SEEDING

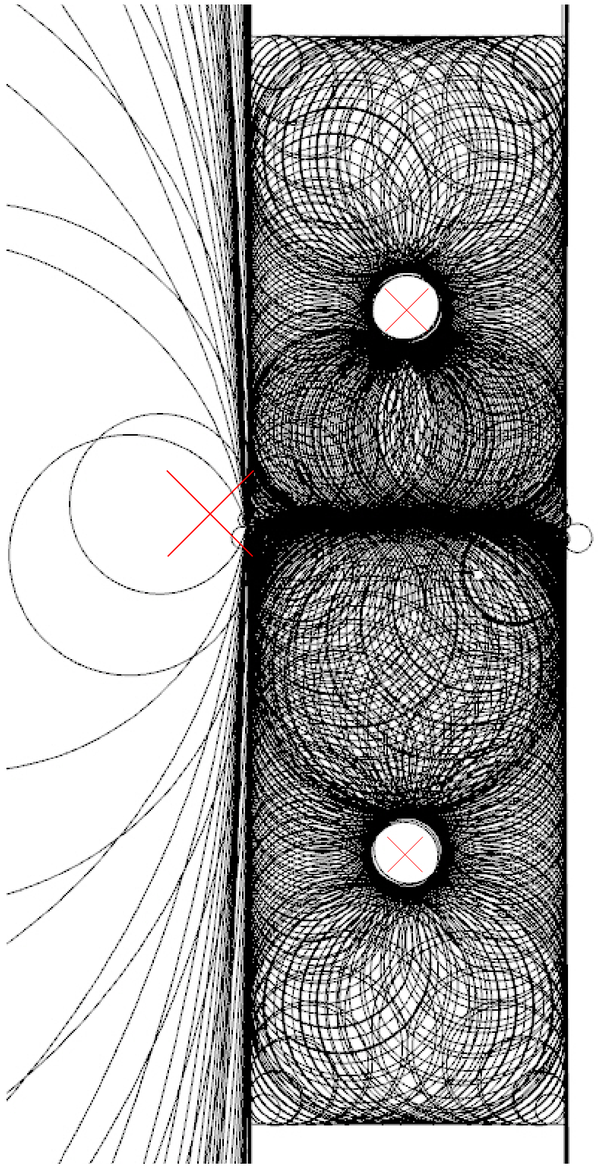
Delaunay



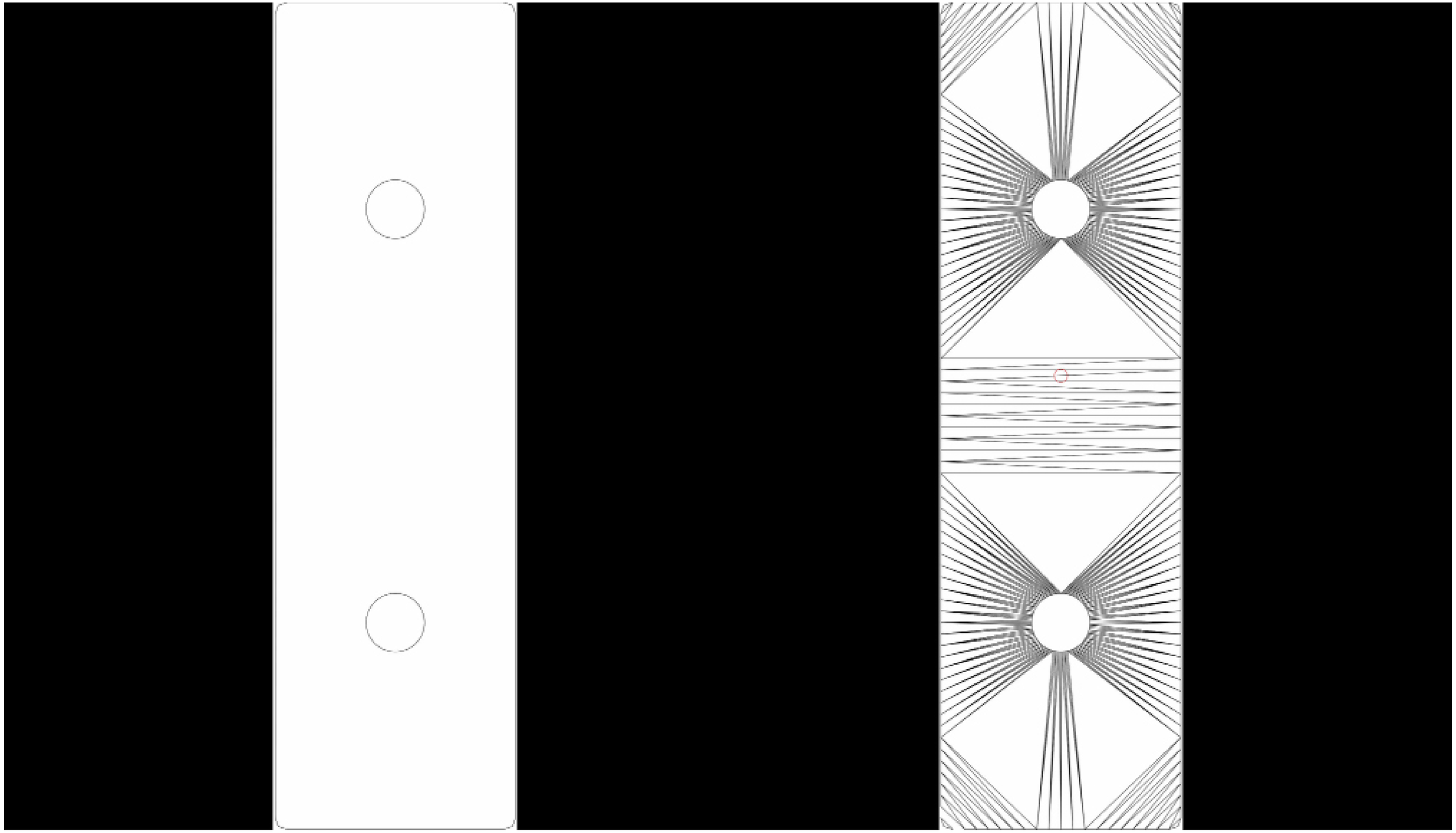


EVENLY SPACED STREAMLINE SEEDING
On Mesh?

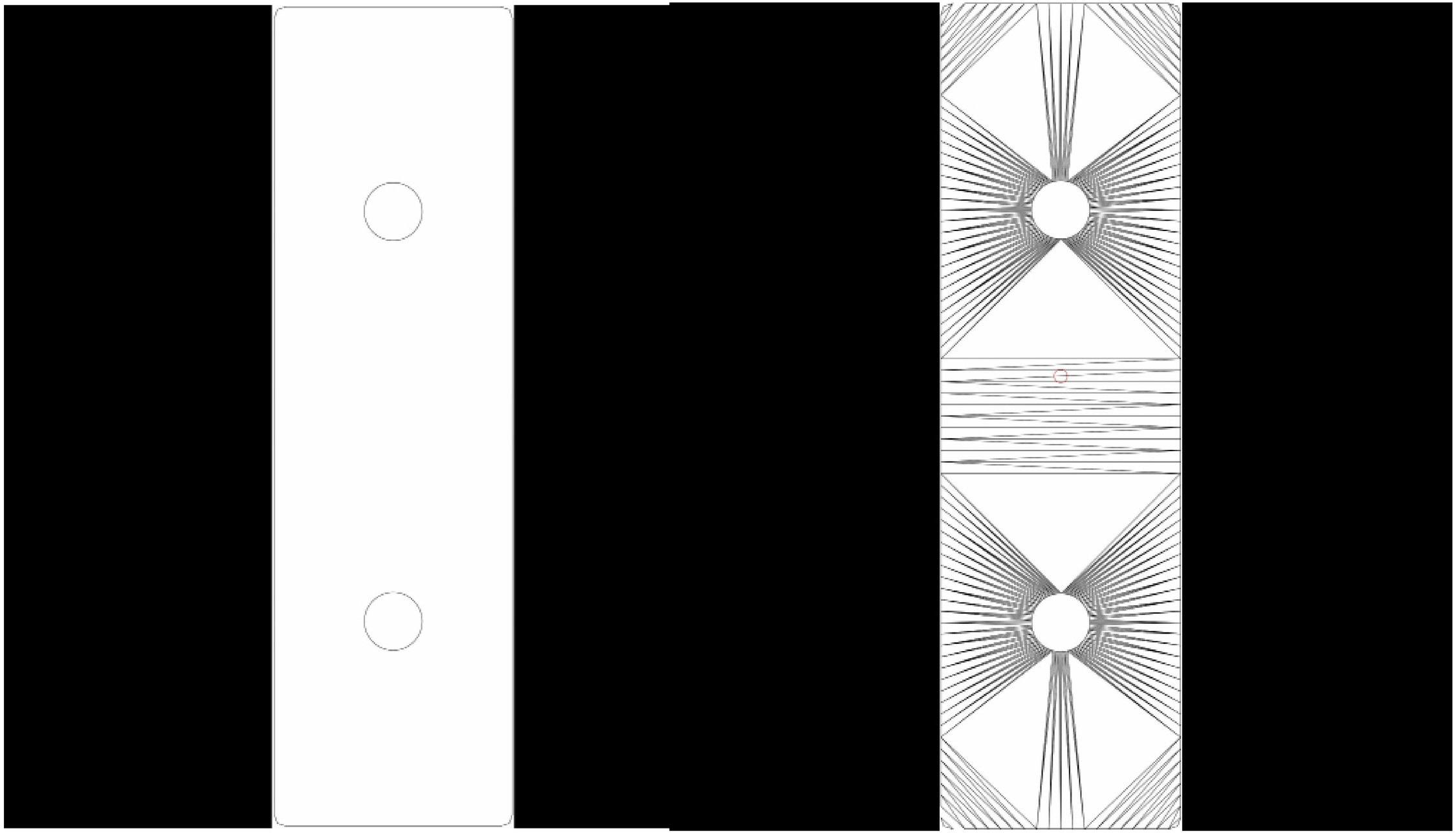




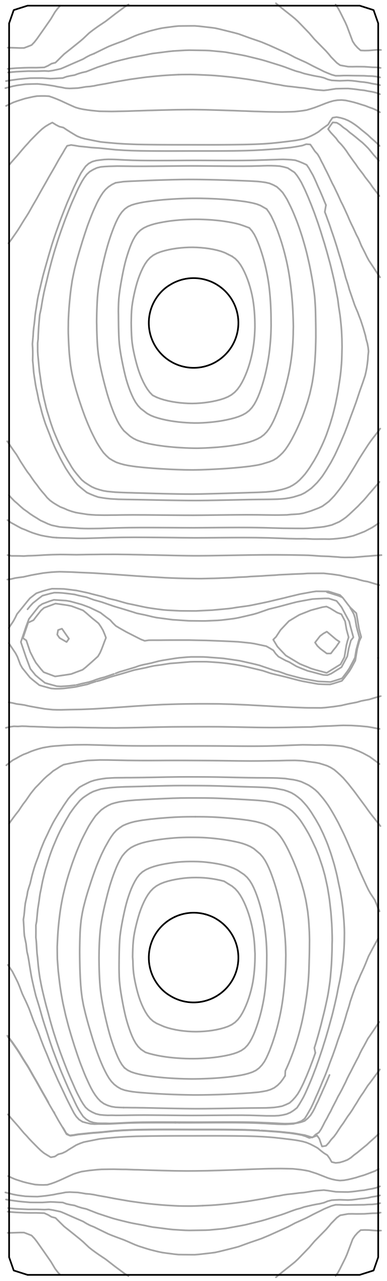
Streamline Generation - U

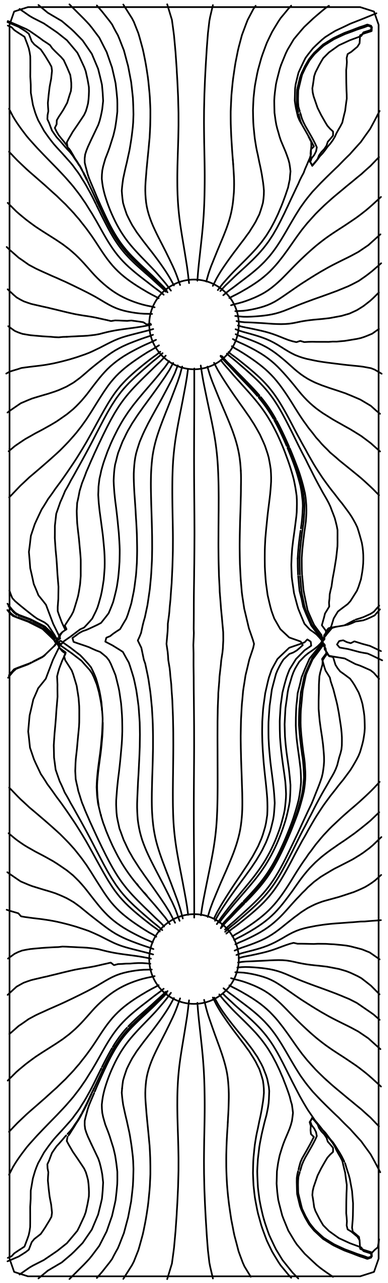


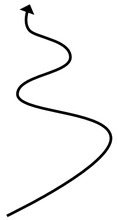
Streamline Generation - V



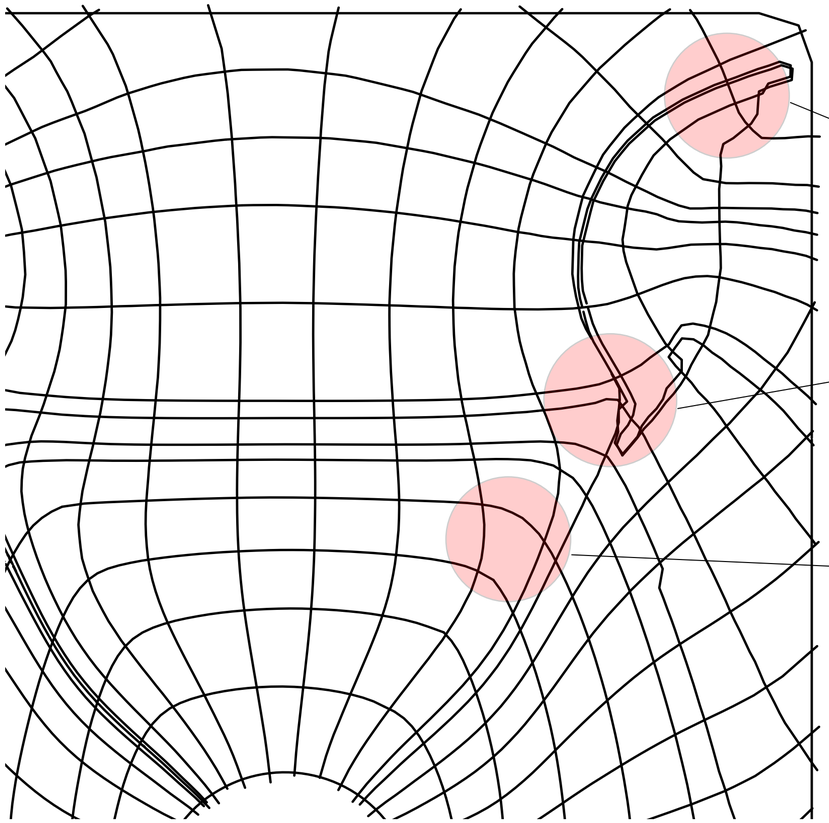
Full Density Streamlines







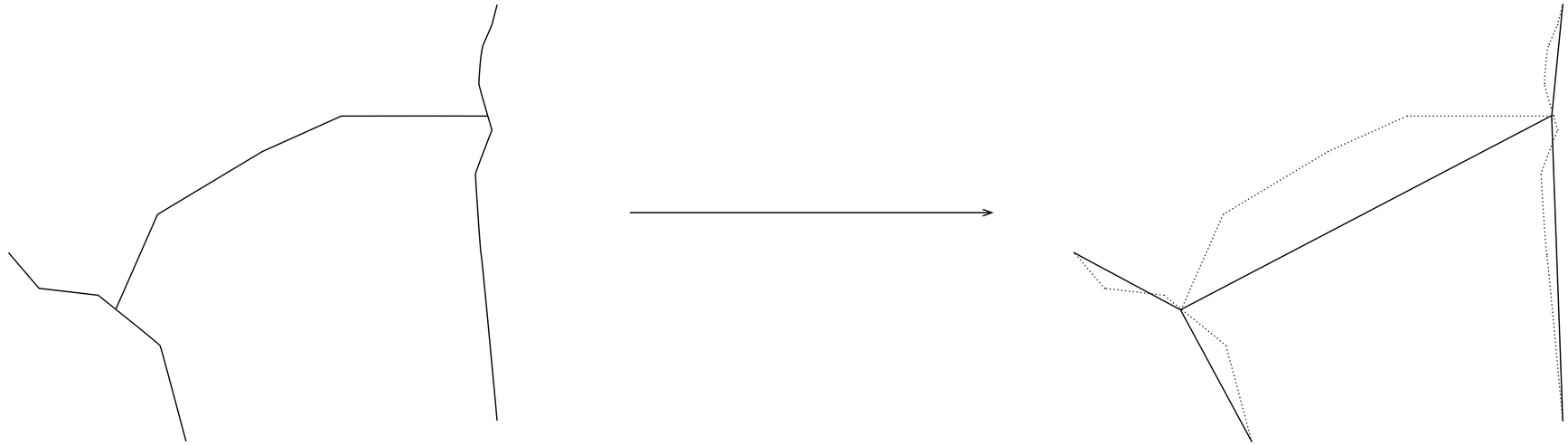
Continuing Issues



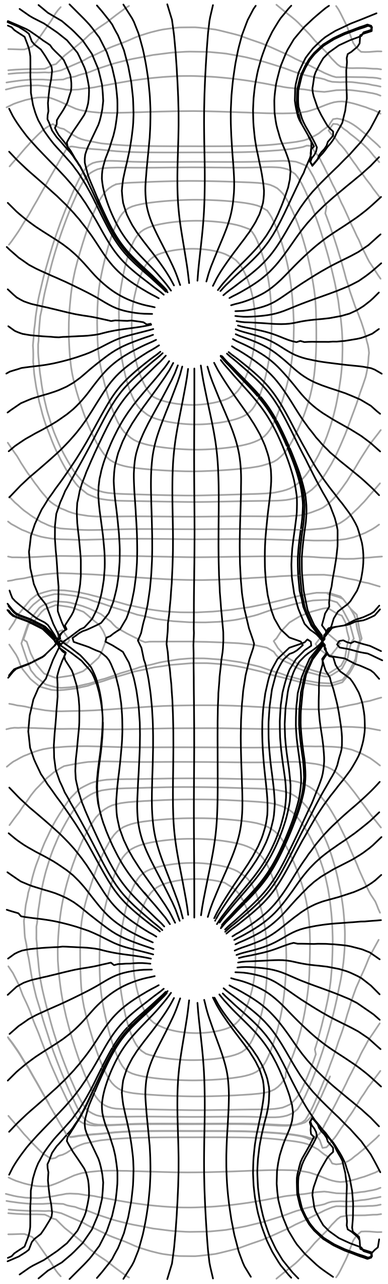
Too High Density

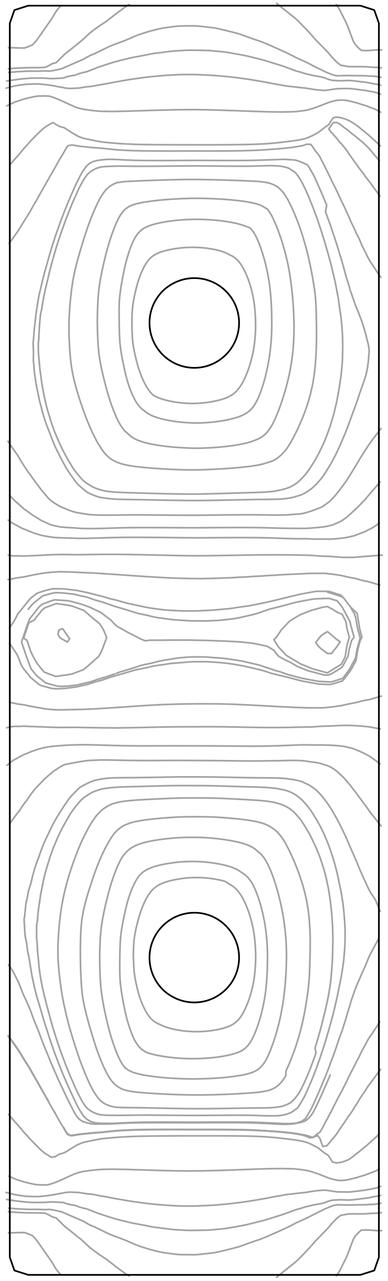
Not Straight Bars

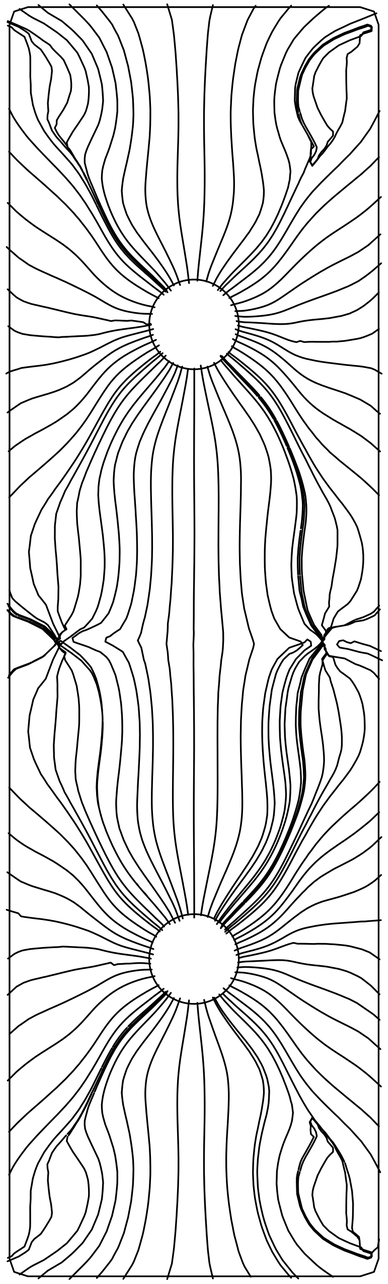
Streamline Discretization

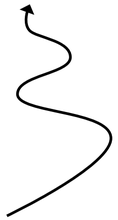


Streamline Discretization

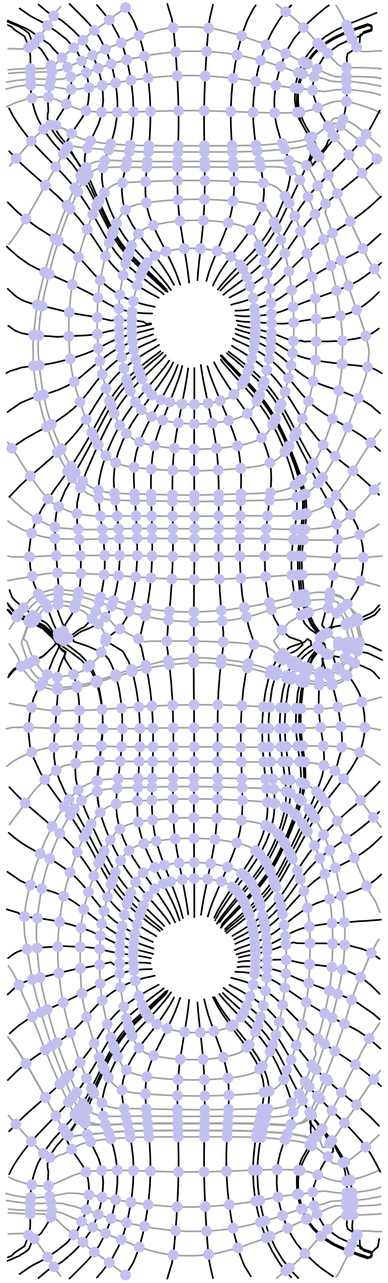


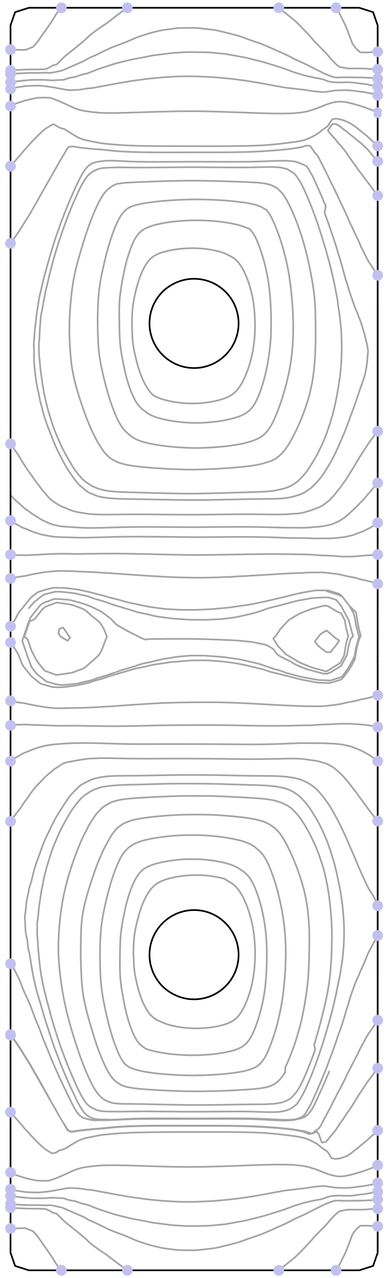


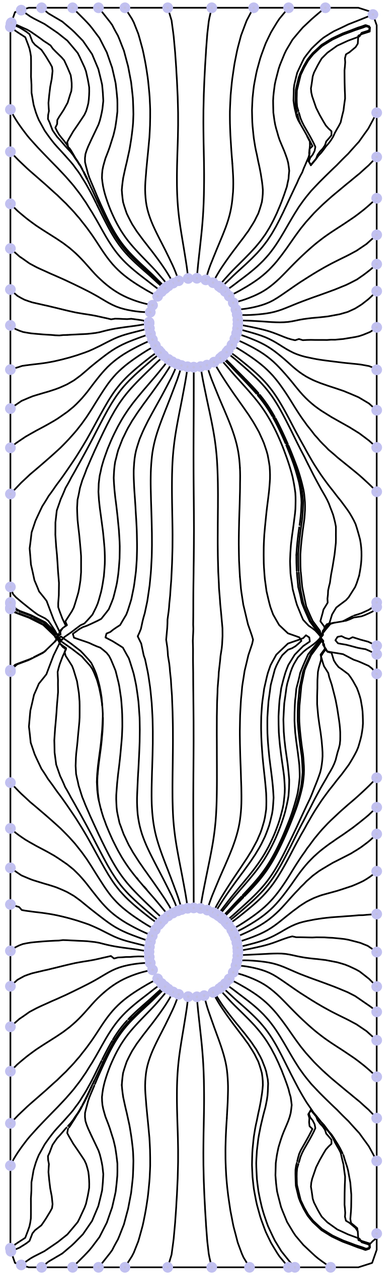


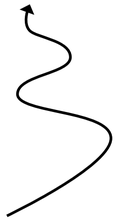


Streamline Discretization

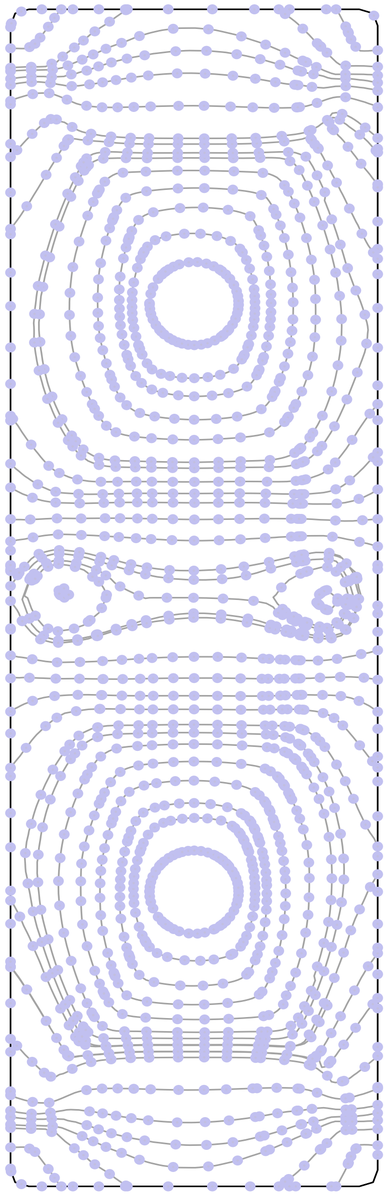


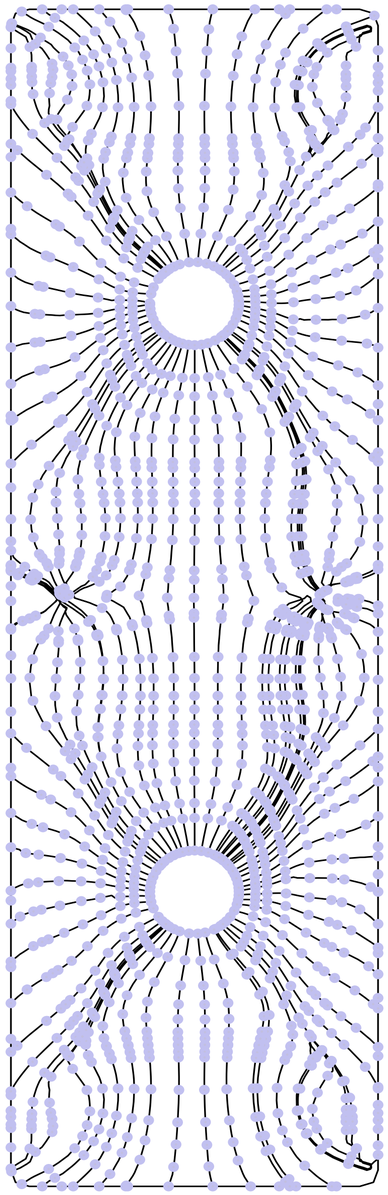


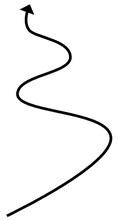




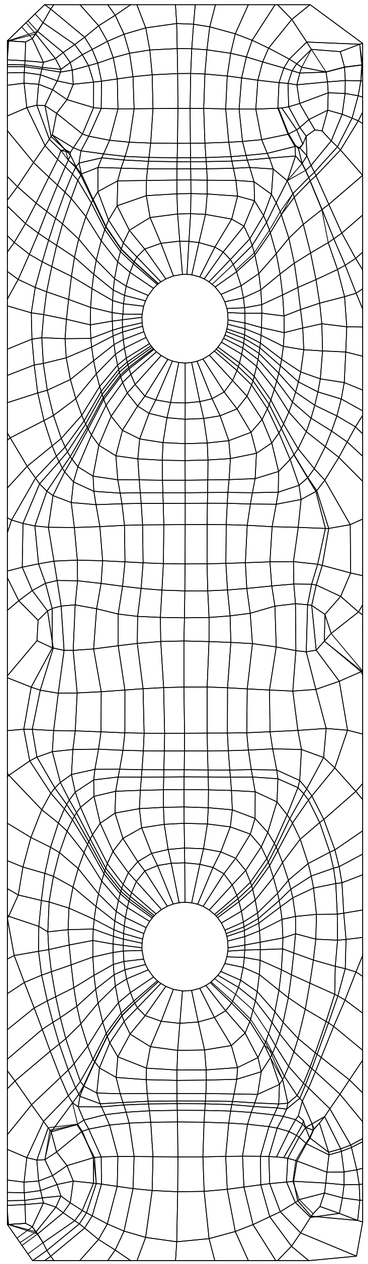
Streamline Discretization





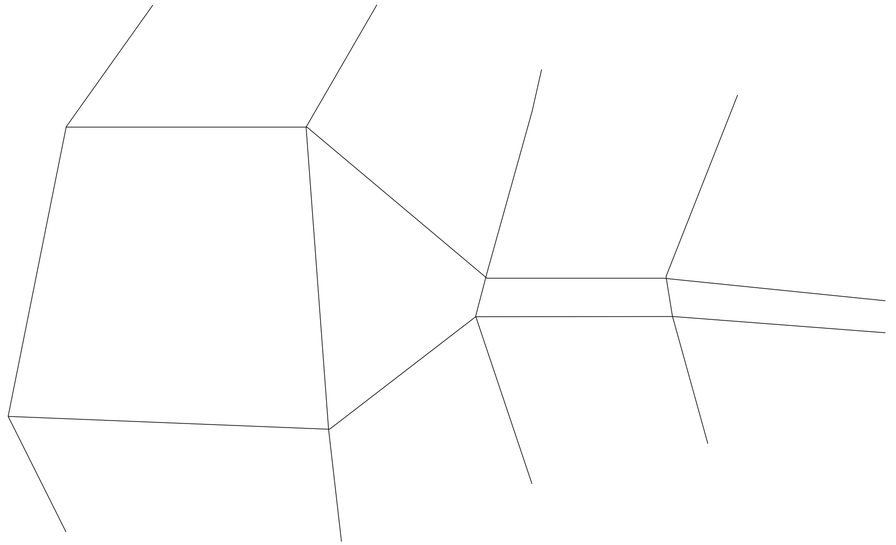


Streamline Discretization

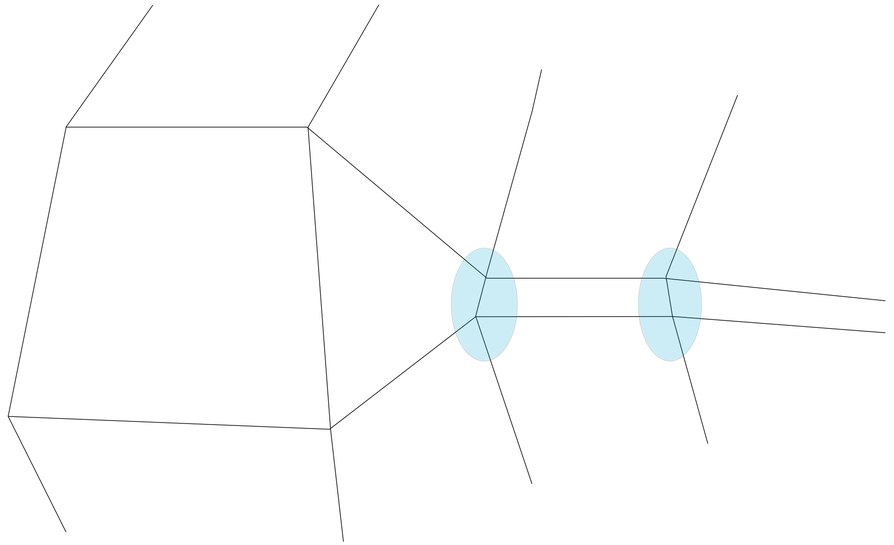


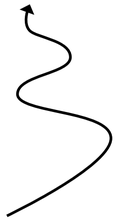
TU Delft

Automatic Cleanup Using Convergence Distance

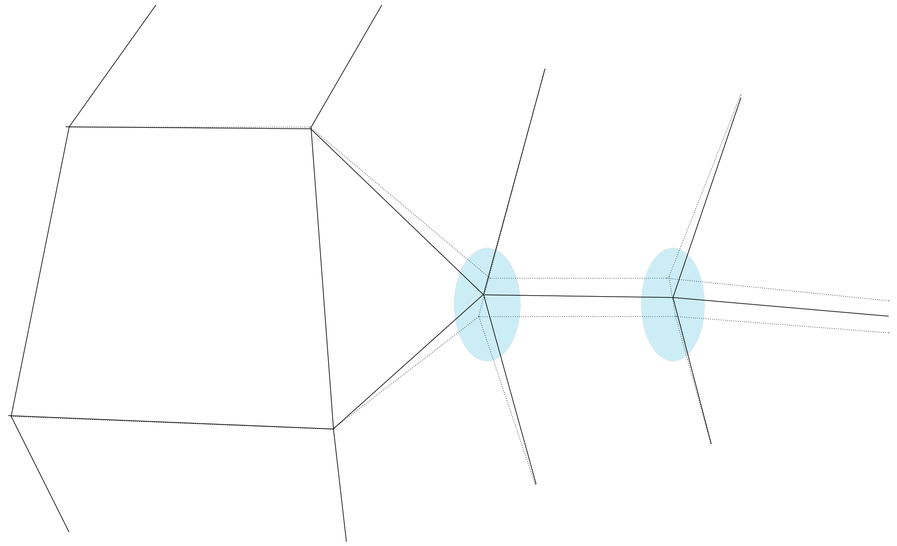


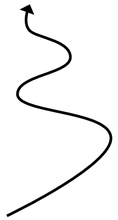
Automatic Cleanup Using Convergence Distance



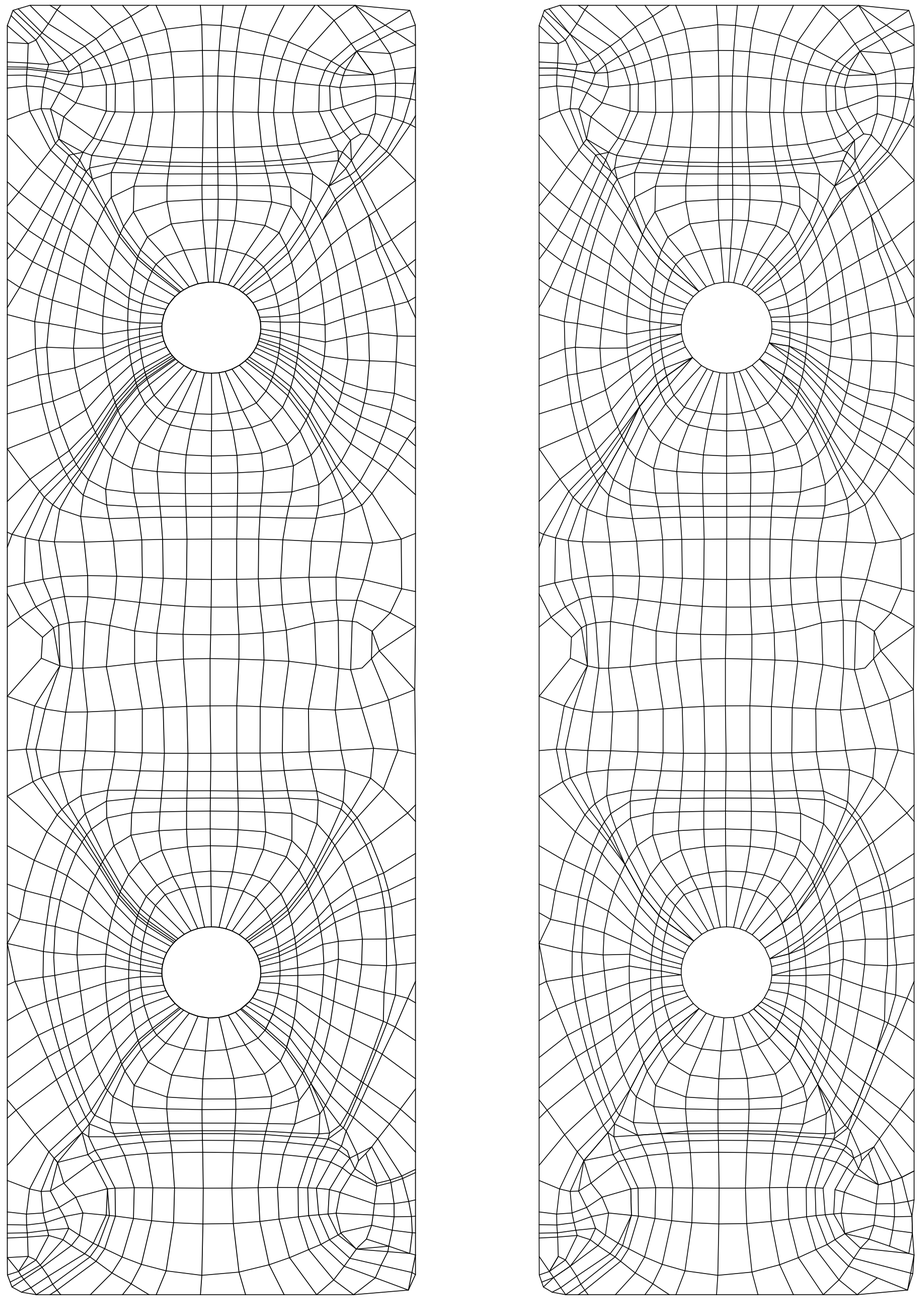


Automatic Cleanup Using Distance Convergence

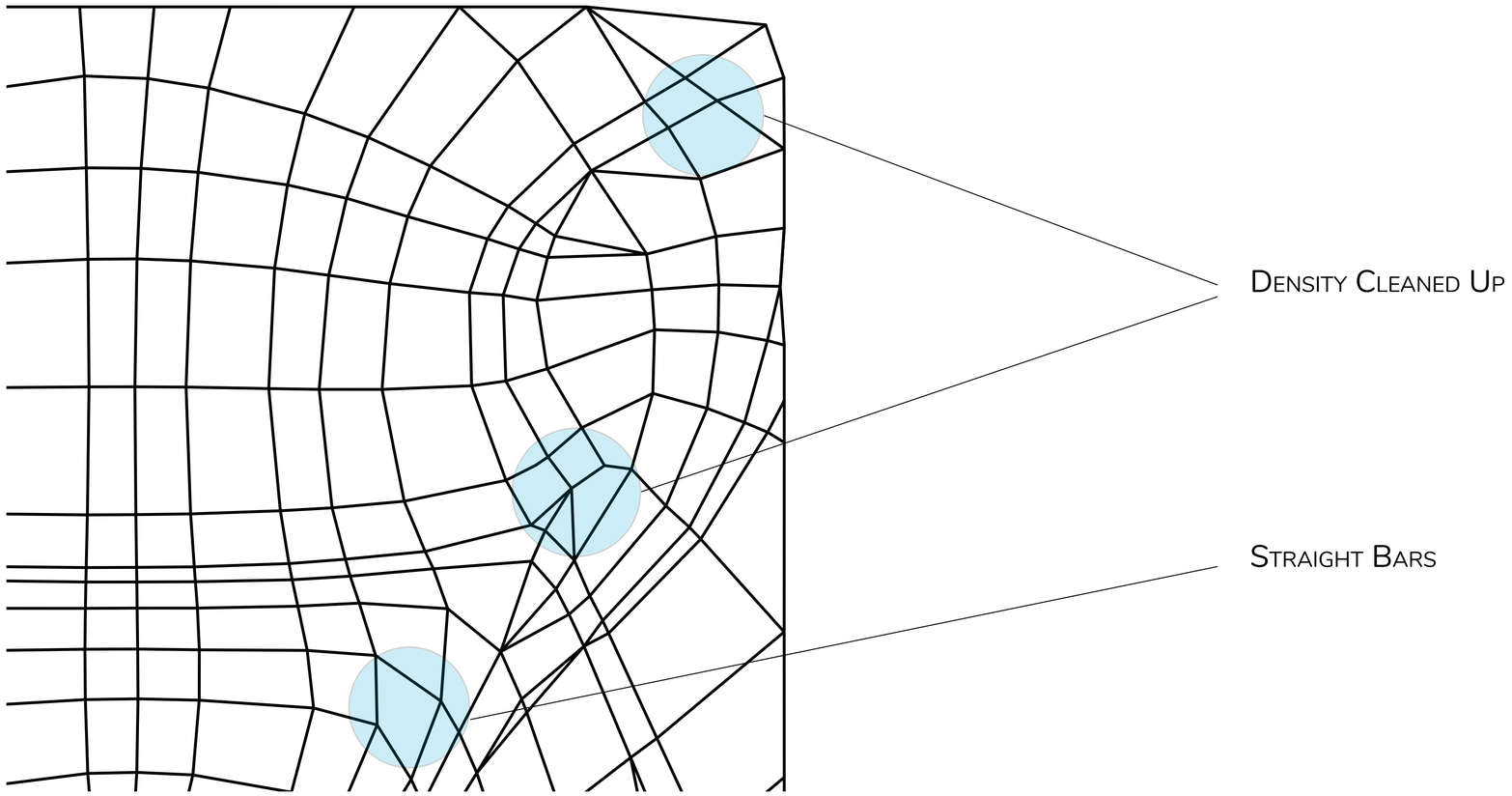




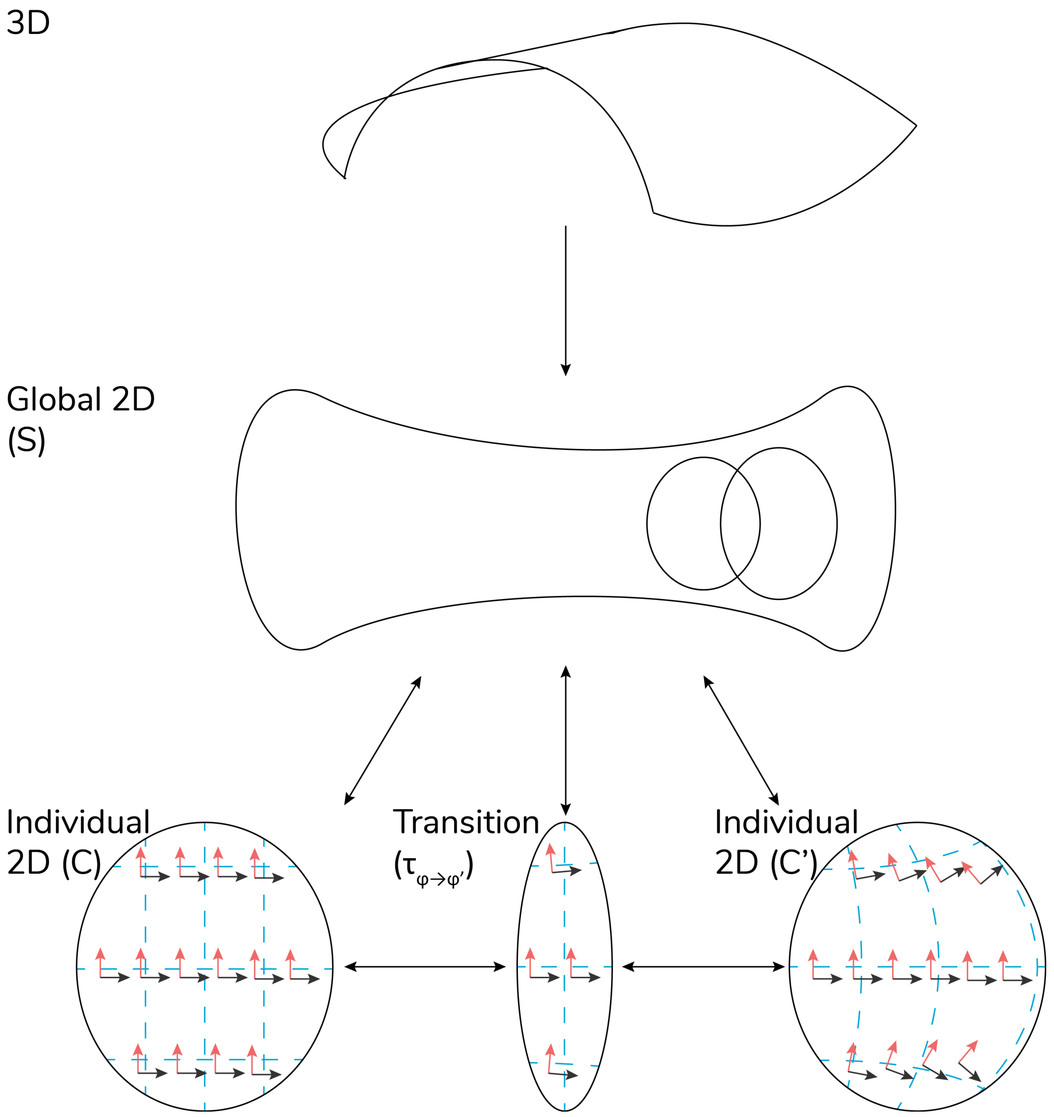
Hand Cleanup Vs Distance Convergence



Re-examining the Singularity Areas

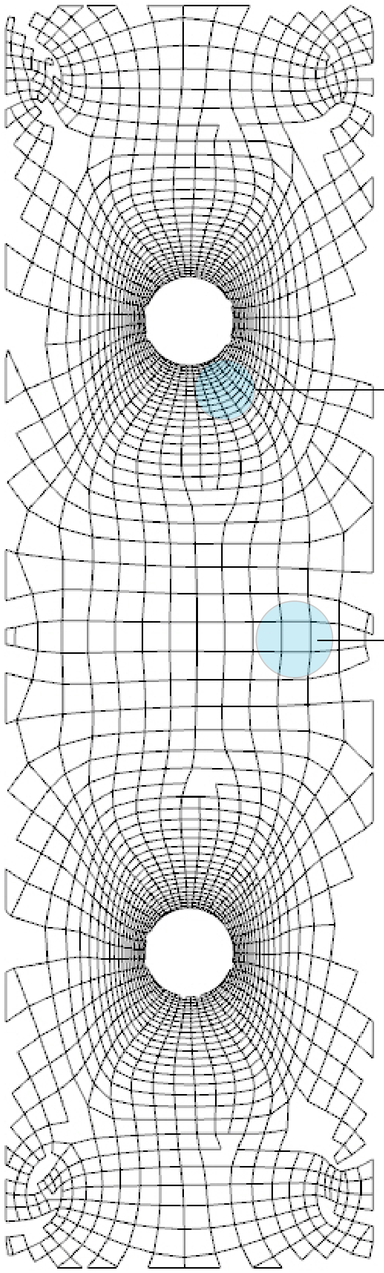


Periodic Global Parameterization



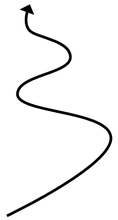
$$F = \int_S \left(\left\| \nabla \theta^T - \omega \vec{K} \right\|^2 + \left\| \nabla \phi^T - \omega \vec{K}^{\perp} \right\|^2 \right) dS$$

Periodic Global Parameterization

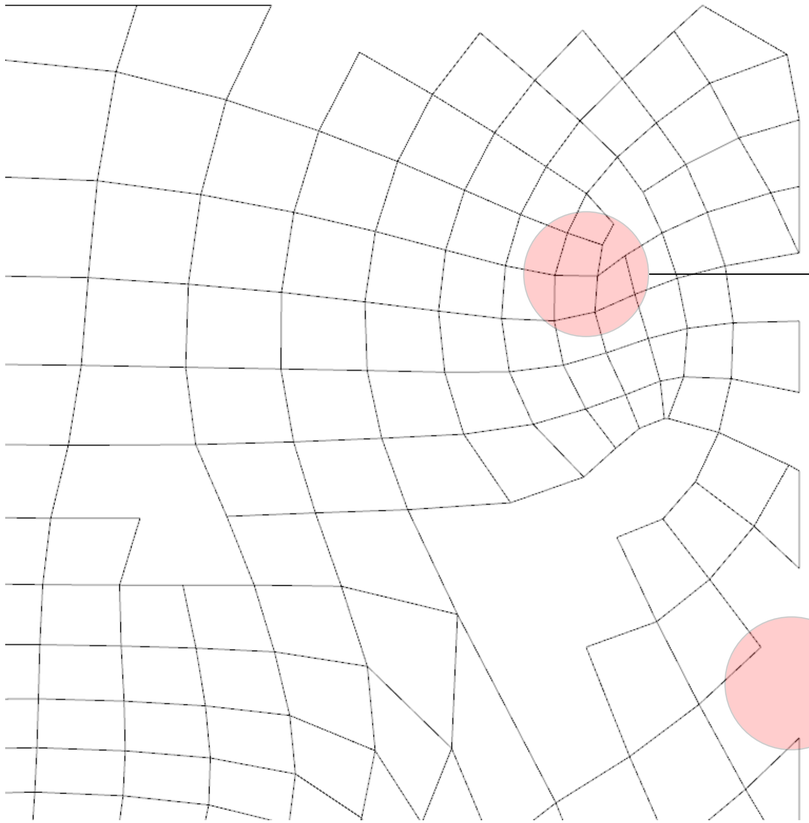


Consistent Density in 3D Space

Consistent Quads Throughout Entire Structure



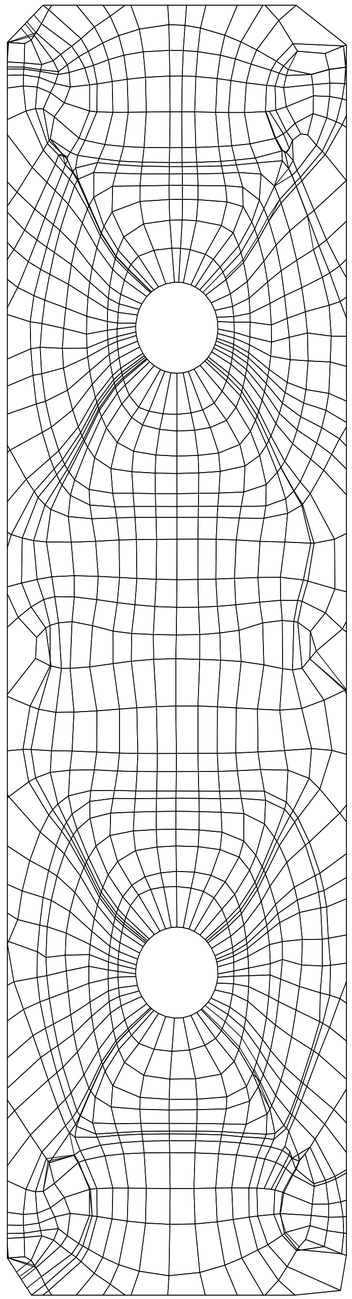
Periodic Global Parameterization - Shortcomings

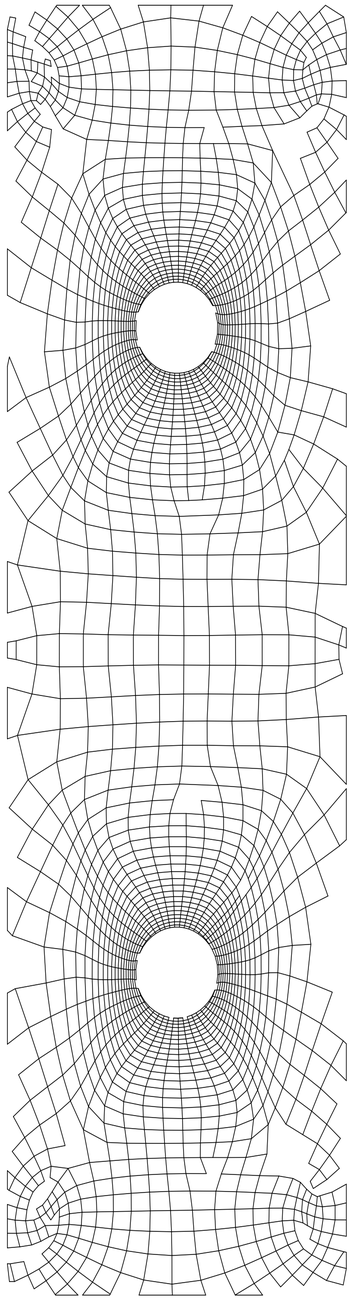


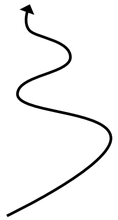
Aborts Around Singularities

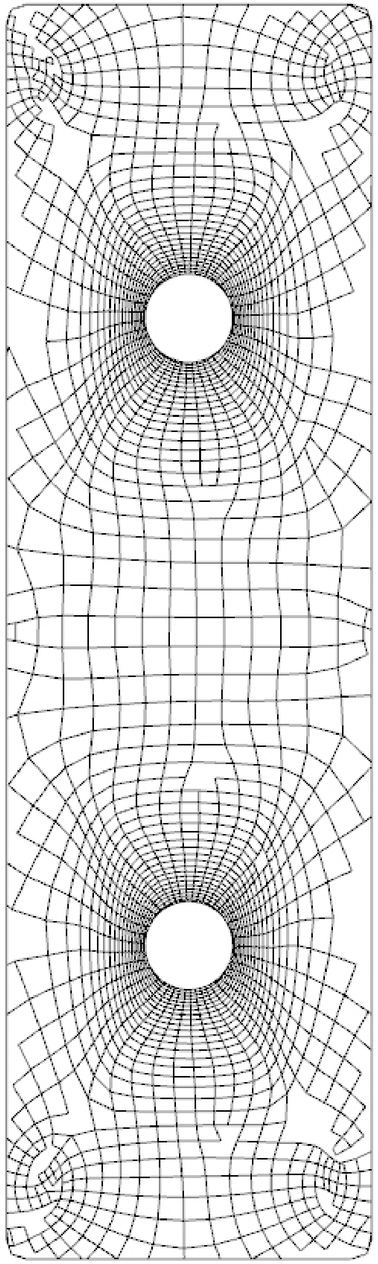
Does Not Remember Boundary

Discretized Streamline vs Periodic Global Parameterization

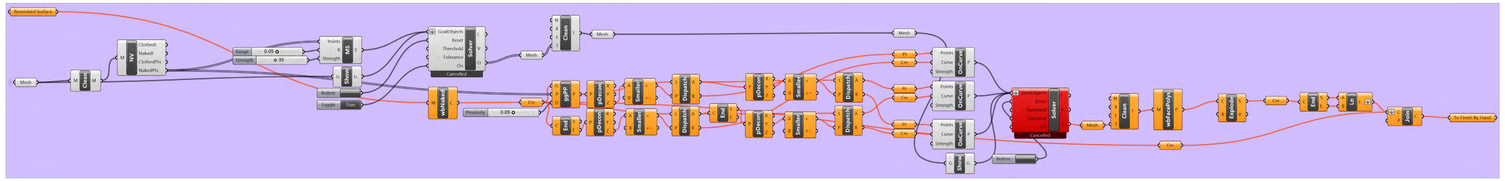








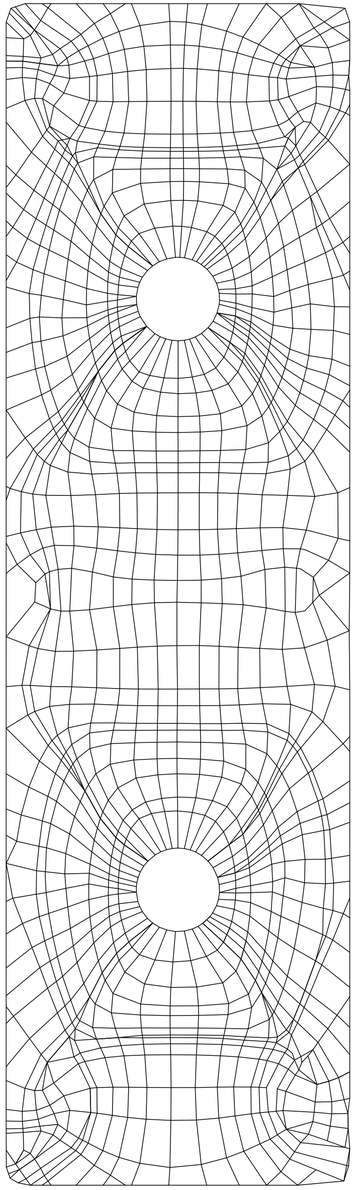
Parameterization Cleanup



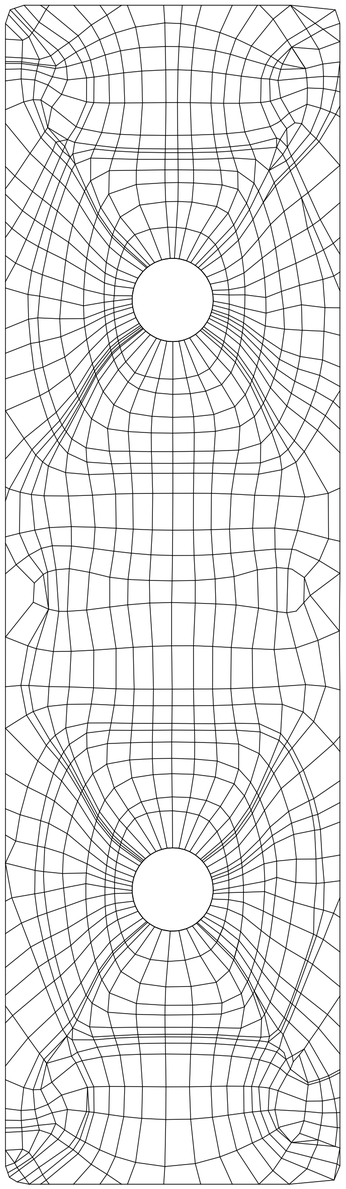
Reintroduce Outline

Pull Mesh Edges to Boundary

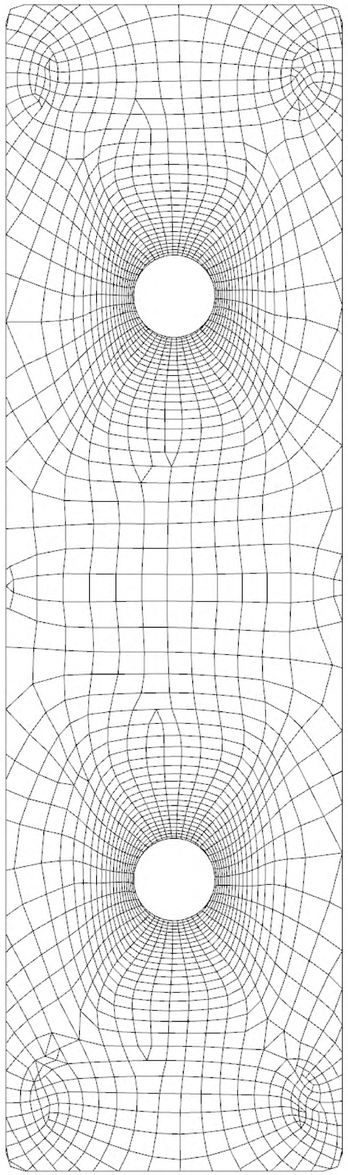
Hand Cleanup and Comparison



Streamline with automated convergence

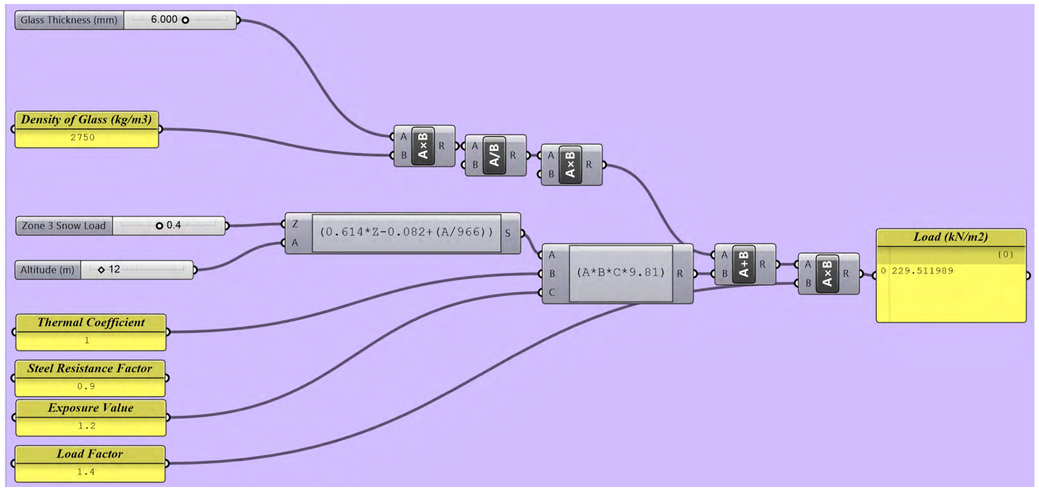


Streamline Finished By Hand

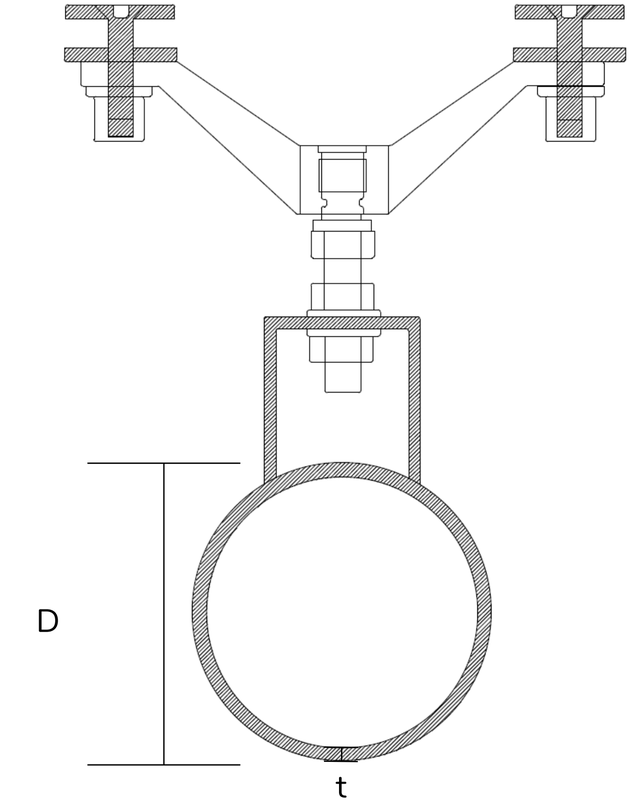


PGP Finished By Hand

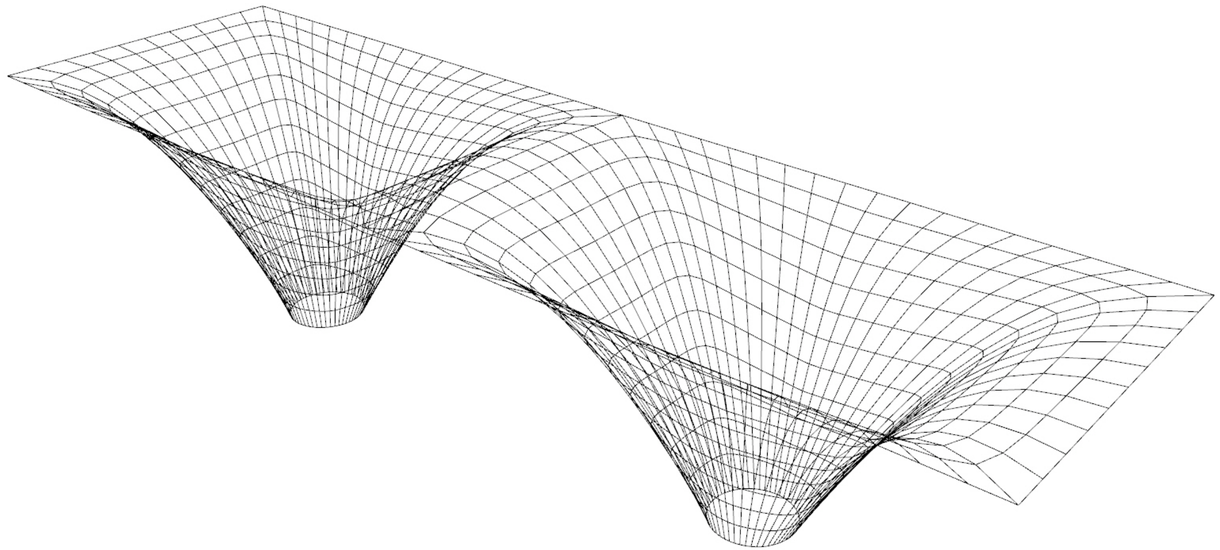
Loading and Bar Shape



Eurocode EN-1991: Actions on Structures



UV Base Directions (Expected Optimum)



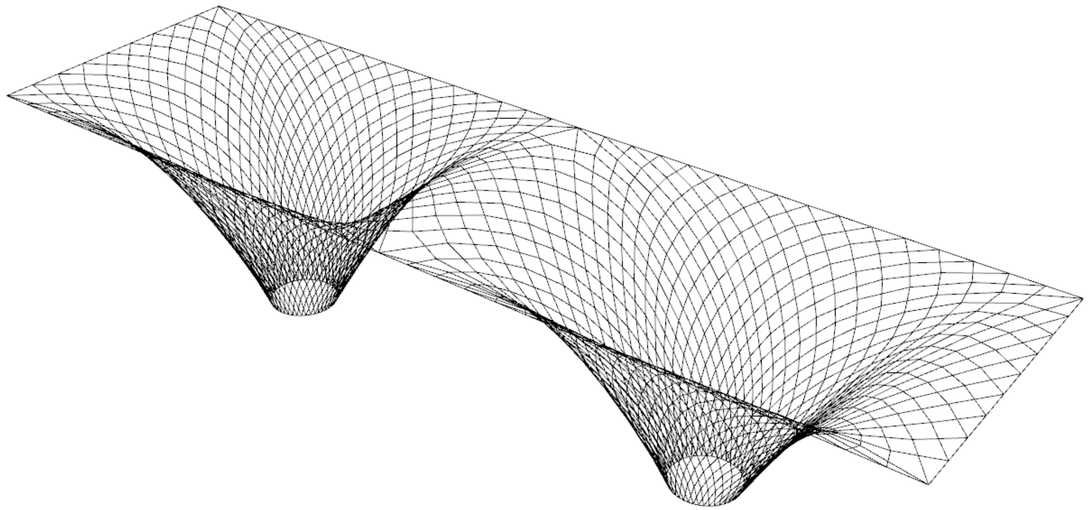
Strain Energy(kJ): 2527

Shell Behavior Percentage: 80.83%

Mass(kg): 368794

TU Delft

Diagonals

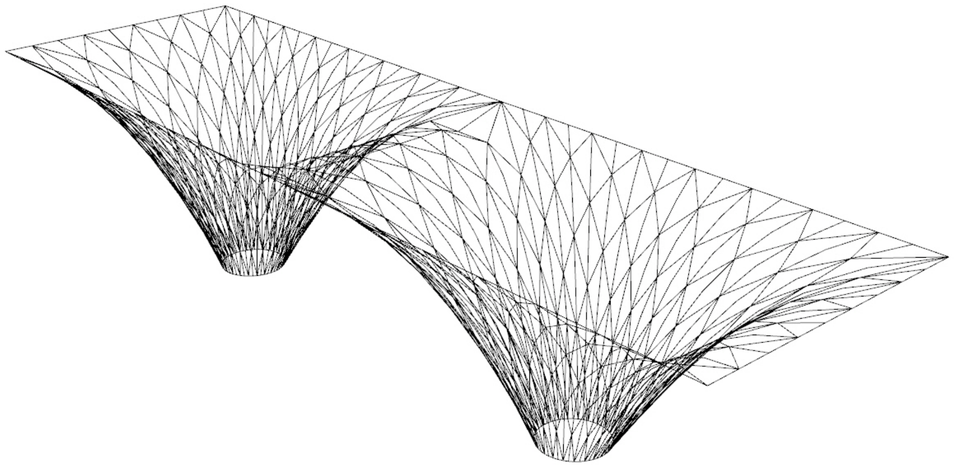


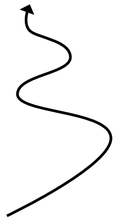
Strain Energy(kJ): 2043

Shell Behavior Percentage: 87.36%

Mass(kg): 719257

Combined Diagonals and V Direction





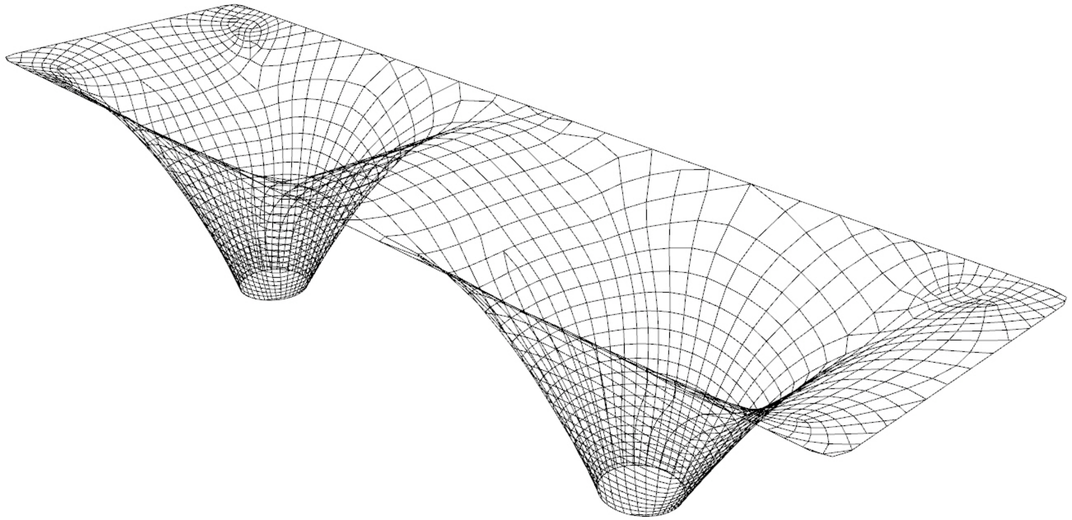
Strain Energy(kJ): 2535

Shell Behavior Percentage: 75.15%

Mass(kg): 669424

TU Delft

Periodic Global Parameterization

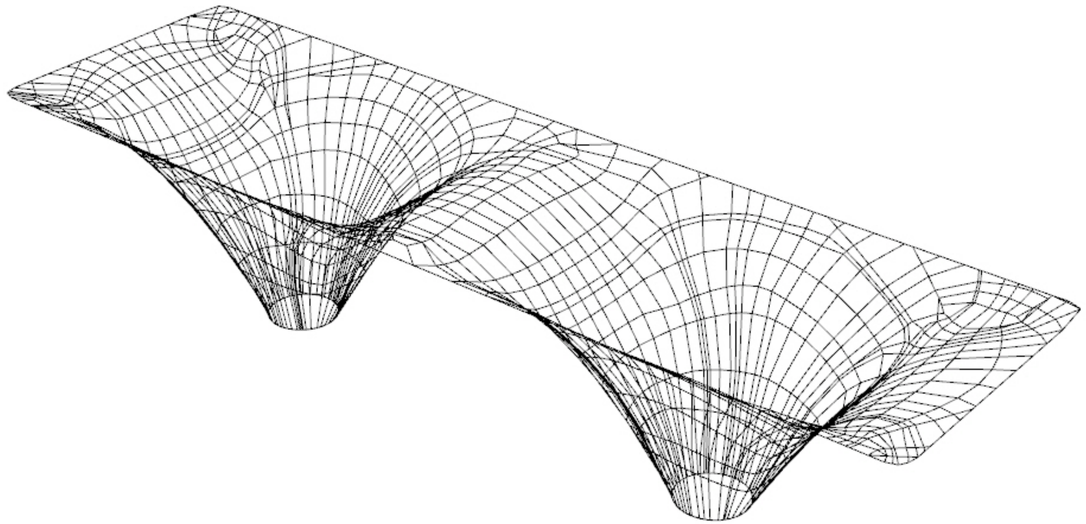


Strain Energy(kJ): 2281

Shell Behavior Percentage: 86.15%

Mass(kg): 517174

Streamline



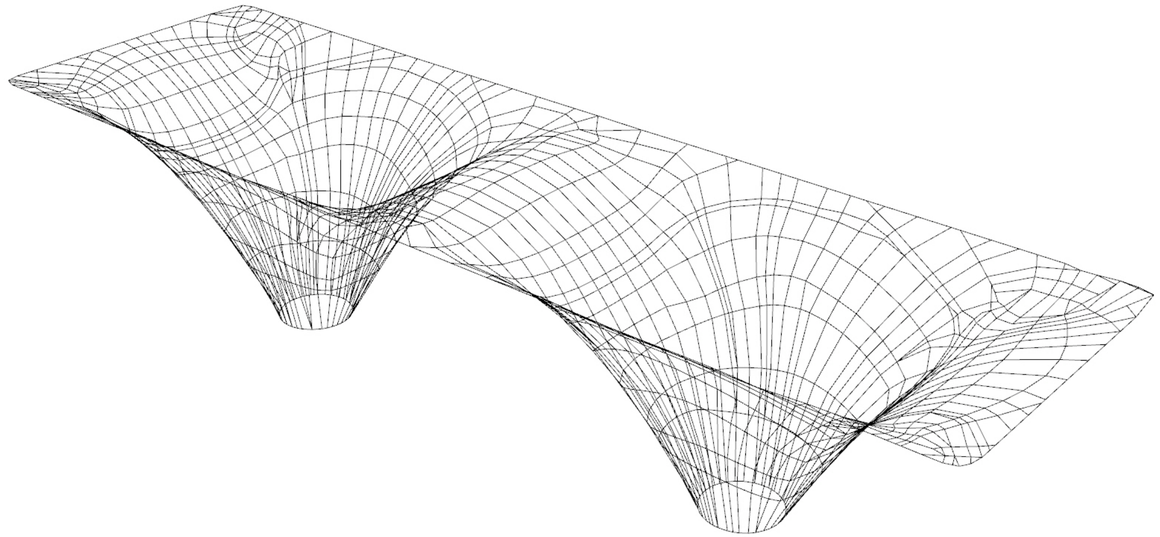
Strain Energy(kJ): 2135

Shell Behavior Percentage: 78.94%

Mass(kg): 509424

TU Delft

Streamline With Automated Convergence



Strain Energy(kJ): 2139

Shell Behavior Percentage: 78.92%

Mass(kg): 509488

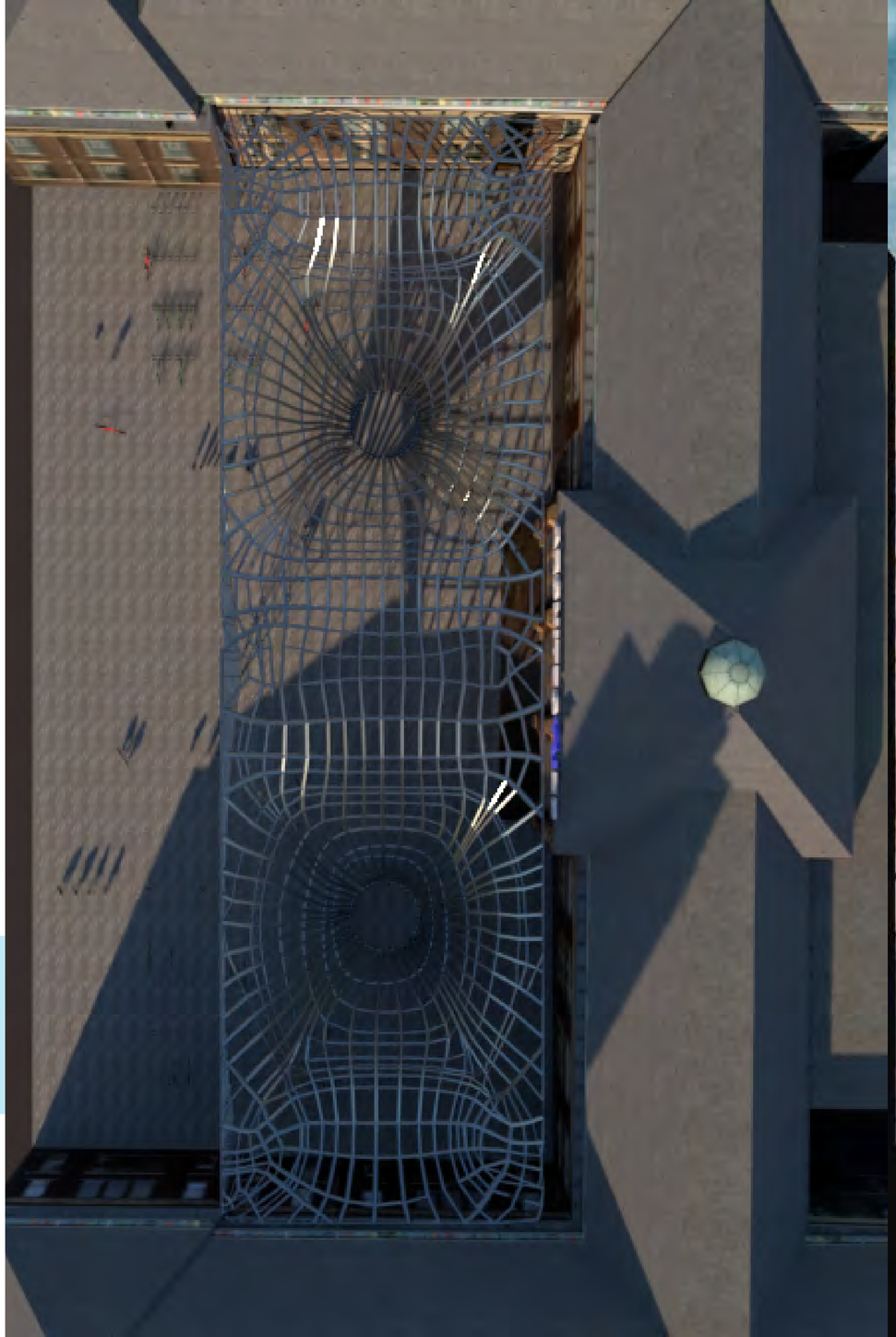
TU Delft

Results Summary

$$Score = \frac{\frac{mass_{lowest}}{mass} + \frac{strain\_energy_{lowest}}{strain\_energy} + \frac{shell\_behavior}{shell\_behavior_{highest}}}{3}$$

Type	Mass (kg)	Strain Energy (kJ)	Mean Shell Behavior	Score
Base 1 (UV)	368794	2527	0.8083	**0.911**
Base 2 (Diagonals)	719527	2043	0.8736	**0.838**
Base 3 (Combined)	669424	2535	0.7515	**0.739**
Streamline	509424	2135	0.7894	**0.861**
Parameterization	517174	2281	0.8615	**0.865**
Streamline Automated Convergence	509488	2139	0.7892	**0.860**


ARCHITECTURE




ARCHITECTURE

How can rod paths be plotted along principal stress streamlines on freeform surfaces?

What form finding methods are suitable for generating an efficient structural form with high percentage shell behavior (no out of plane forces) and low strain energy density(high stiffness)?

Is there a considerable advantage in optimizing a gridshell structure based on principal stress stream lines and an arbitrary generated tessellation?

How can rod paths be plotted along principal stress streamlines on freeform surfaces?

Streamline Method Generates A Viable Topology Generation for Non-Standard Patterning

- Allows for Topologies which do not require UV strips.
- Currently Still Requires Cleanup By Hand
- Singularities Can Create Major Problems In Areas As Vectors Converge
- Future Work: Principal Curvature Vectors can be weighted and integrated into the analysis for planarity constraints

Plate Analysis Deemed Most Likely Elegant Long Term Solution

Creates clearer and more generalized solutions not reliant on a vector step function if a surface can be created to generate isolevels and principal ascent

What form finding methods are suitable for generating an efficient structural form with high percentage shell behavior (no out of plane forces) and low strain energy density(high stiffness)?

	4 Point	Line Supports	MHS
FAR Strain Energy Density	2.34e-2	1.17e-3	1.13e-3
Mean Shell Behavior	93	96.8	91.5

Is there a considerable advantage in optimizing a gridshell structure based on principal stress stream lines and an arbitrary generated tessellation?

Type	Mass (kg)	Strain Energy (kJ)	Mean Shell Behavior	Score
Base 1	368794	2527	0.8083	**0.911**
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Base 3 (Combined)	669424	2535	0.7515	**0.739**
Streamline	509424	2135	0.7894	**0.861**
Parameterization	517174	2281	0.8615	**0.865**
Cleaned Up	517174	2281	0.8615	**0.865**

$$Score = \frac{\frac{mass_{lowest}}{mass} + \frac{strainenergy_{lowest}}{strainenergy} + \frac{shellbehavior}{shellbehavior_{highest}}}{3}$$


ARCHITECTURE


THANK YOU