TECHNISCHE HOGESCHOOL DELFT
AFDELING DER MARITIEME TECHNIEK
LABORATORIUM VOOR SCHEEPSHYDROMECHANICA

The distribution of the hydrodynamic forces on a heaving and pitching shipmodel in still water

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Reportnr. 22 -P
5th ONR Symposium 1964, Bergen.

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THE DISTRIBUTION OF THE HYDRODYNAMIC FORCES ON A HEAVING AND PITCHING SHIPMODEL IN STILL WATER

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ABSTRACT

Forced oscillation tests are carried out with a segmented shipmodel to investigate the distribution of the hydrodynamic forces along the hull for heaving and pitching motions.

The vertical forces on each of the seven sections of the shipmodel are measured as a function of forward speed and frequency. By using the in-phase and quadrature components of these forces, an analysis is made of their distribution along the length of the shipmodel.

The experimental results are compared with the results of a simple strip theory, taking into account the effect of forward speed.

The comparison shows a satisfactory agreement between theory and experiment.

INTRODUCTION

The calculation of shipmotions in regular head waves by using a strip theory, has been discussed in a number of papers. Recent contributions were given by Korvin-Kroukovsky and Jacobs [1], Fay [2], Watanabe [3] and Fukuda [4].

In these papers the influence of forward speed on the hydrodynamic forces is considered and dynamic cross-coupling terms are included in the equations of motion, which are assumed to describe the heaving and pitching motions.

In earlier work [5] it was shown that a relatively small influence of speed exists on the damping coefficients, the added mass and the exciting forces, at least for the case of head waves and for speeds which are of practical interest. On the other hand, forward speed has an important effect on some of the dynamic cross-coupling coefficients. Although, at a first glance these terms could be regarded as second order quantities, it was pointed out by Korvin-Kroukovsky [1] and also by Fay [2] that they can be very important for the amplitudes and phases of the motions. This has been confirmed in [5] where the coupling terms are neglected in a calculation of the heaving and pitching motions. In this calculation we used coefficients of the motion equations, which were determined by forced oscillation tests. In comparison with the calculation where the cross-coupling terms are included and also in comparison with the measured motions, an important influence is observed, as shown in Fig. 1, which is taken from Ref. [5]. Further analysis showed that the discrepancies between the coupled and uncoupled motions were mainly due to the damping cross-coupling terms.

The influence of forward speed has been discussed to some extent in Vossers' thesis [6]. From a first order slender body theory it was found that the distribution of the hydrodynamic forces along an oscillating slender body is not influenced by forward speed. Vossers concluded that the inclusion of speed dependent damping cross-coupling terms is not in agreement with the use of a

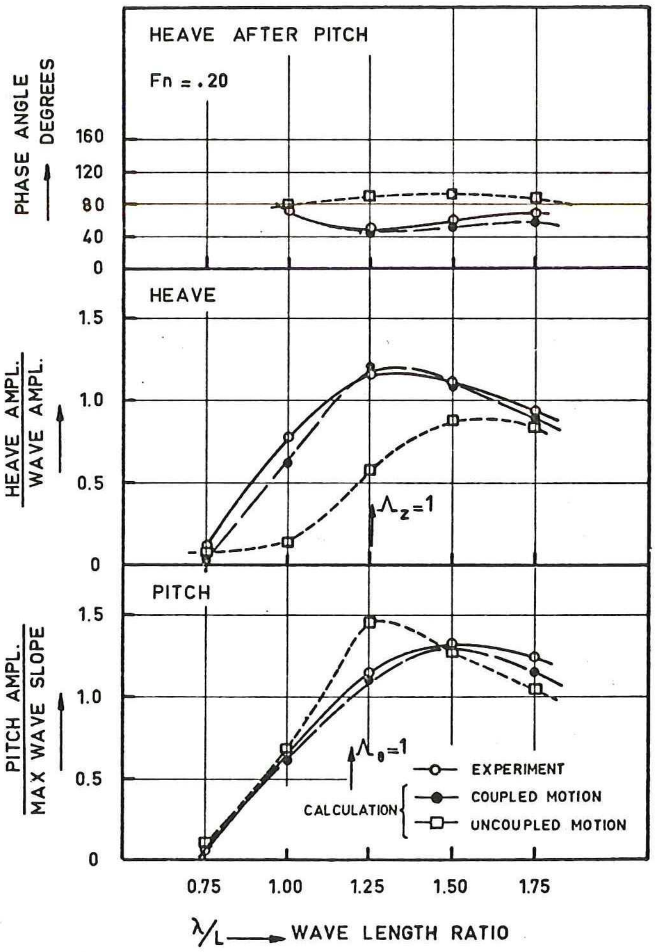

Fig. 1 - Influence of cross-coupling

strip theory. In view of the above mentioned results such a simplification does not hold for actual shipforms.

For symmetrical shipforms at forward speed, it was shown by Timman and Newman [7] that the damping cross-coupling coefficients for heave and pitch are equal in magnitude, but opposite in sign. Their conclusion is valid for thin or slender submerged or surface ships and also for non-slender bodies.

Golovato's work [8] and some of our experiments [5] on oscillating shipmodels confirmed this fact for actual surface ships to a certain extent.

The effects of forward speed are indeed very important for the calculation of shipmotions in waves. The two-dimensional solutions for damping and added mass of oscillating cylinders on a free surface, as given by Grim [9] and Tasai [10] show a very satisfactory agreement with experimental results. When the effects of forward speed can be estimated with sufficient accuracy, such two-dimensional values may be used to calculate the total hydrodynamic forces and moments on a ship, provided that integration over the shiplength is permissible.

In order to study the speed effect on an oscillating shipform in more detail, a series of forced oscillating experiments was designed. The main object of these experiments was to find the distribution of the hydrodynamic forces along the length of the ship as a function of forward speed and frequency of oscillation.

THE EXPERIMENTS

The oscillation tests were carried out with a 2.3 meter model of the Sixty Series, having a block coefficient $C_B = 0.70$. The main dimensions are given in Table 1. The model is made of polyester, reinforced with fibreglass, and consists of seven separate sections of equal length. Each of the sections has two end-bulkheads. The width of the gap between two sections is one millimeter. The sections are not connected to each other, but they are kept in their position by means of stiff strain-gauge dynamometers, which are connected to a longitudinal steel box girder above the model. The dynamometers are sensitive only for forces perpendicular to the baseline of the model.

By means of a Scotch-Yoke mechanism a harmonic heaving or pitching motion can be given to the combination of the seven sections which form the shipmodel. The total forces on each section could be measured as a function of frequency and speed.

A non-segmented model of the same form was also tested in the same conditions of frequency and speed to compare the forces on the whole model with the sums of the section results. A possible effect of the gaps between the sections could be detected in this way. The arrangement of the tests with the segmented model and with the whole model is given in Fig. 2.

The mechanical oscillator and the measuring system is shown in Fig. 3. In principle the measuring system is similar to the one described by Goodman [11]: the measured force signal is multiplied by $\cos \omega t$ and $\sin \omega t$ and after

Table 1
Main Particulars of the Shipmodel

Length between perpendiculars	2.258 m
Length on the waterline	2.296 m
Breadth	0.322 m
Draught	0.129 m
Volume of displacement	0.0657 m^3
Block coefficient	0.700
Coefficient of mid-length section	0.986
Prismatic coefficient	0.710
Waterplane area	0.572 m^2
Waterplane coefficient	0.785
Longitudinal moment of inertia of waterplane	0.1685 m^4
L.C.B. forward of $L_{pp}/2$	0.011 m
Centre of effort of waterplane after $L_{pp}/2$	0.038 m
Froude number of service speed	0.20

integration the first harmonics of the in-phase and quadrature components can be found with distortion due to vibration noise. In some details the electronic circuit differs somewhat from the description in [11]. In particular synchro resolvers are used instead of sine-cosine potentiometers, because they allow higher rotational speeds.

The accuracy of the instrumentation proved to be satisfactory which is important for the determination of the quadrature components, which are small in comparison with the in-phase components of the measured forces.

Throughout the experiments only first harmonics were determined. It should be noted that non-linear effects may be important for the sections at the bow and the stern where the ship is not wall-sided. The forced oscillation tests were carried out for frequencies up to ω = 14 rad/sec and four speeds of advance were considered, namely: Fn = 0.15, 0.20, 0.25 and 0.30. Below a frequency of ω = 3 to 4 rad/sec the experimental results are influenced by wall effect due to reflected waves generated by the oscillating model.

The motion amplitudes of the shipmodel covered a sufficiently large range to study the linearity of the measured values (heave ~4 cm, pitch ~4.6 degrees). An example of the measured forces on section 2, when the combination of the seven sections performs a pitching motion, is given in Fig. 4.

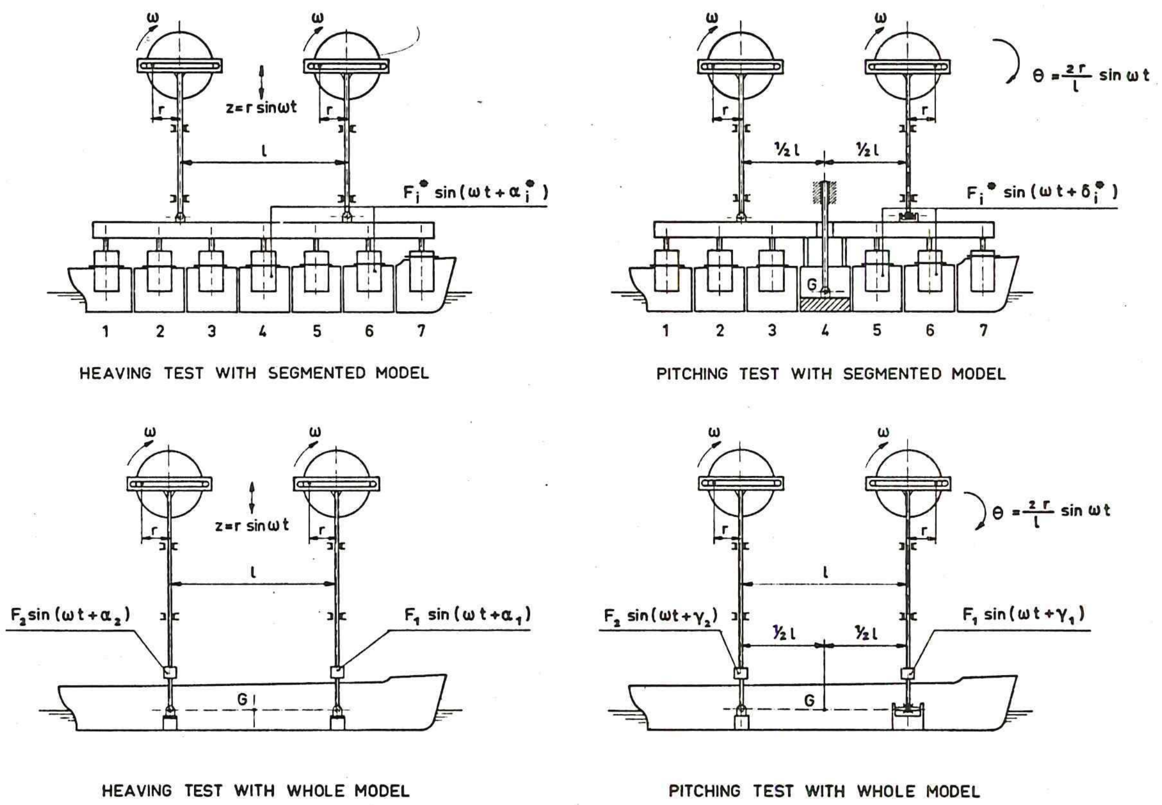

Fig. 2 - Arrangement of oscillation tests

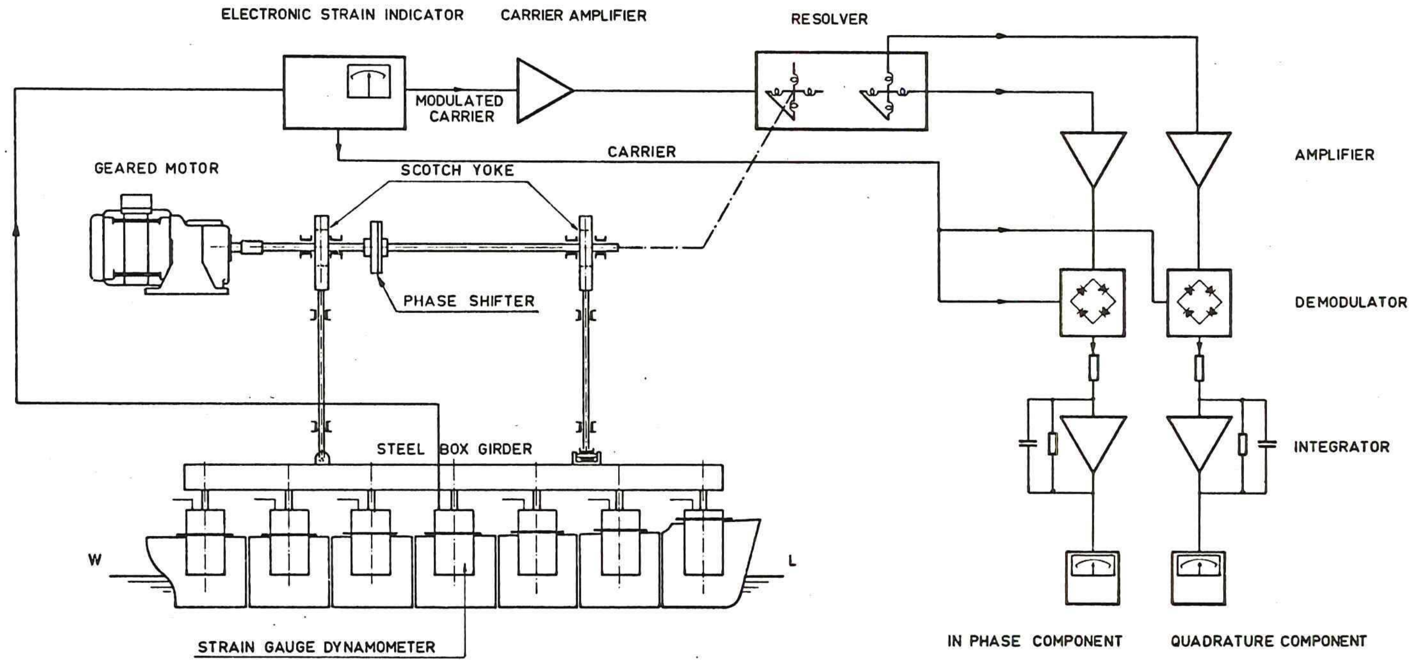

Fig. 3 - Principle of mechanical oscillator and electronic circuit

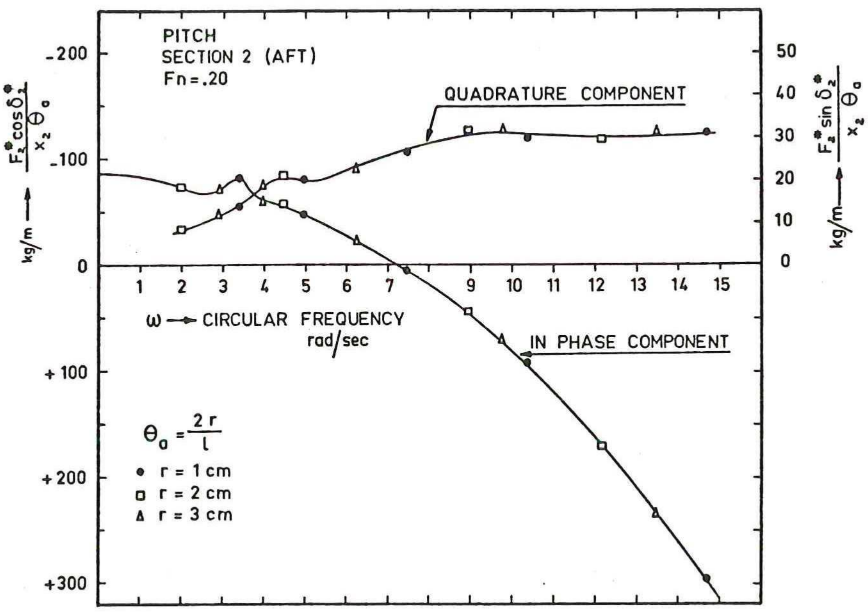

Fig. 4 - Components of force on section 2, pitching motion

PRESENTATION OF THE RESULTS

Whole Model

It is assumed that the force F and the moment M acting on a forced heaving or pitching shipmodel can be described by the following equations:

Heave

$$\left.\begin{aligned} (a+\rho\nabla)\,\ddot{z}_o + b\dot{z}_o + cz_o &= F_z \sin(\omega t + \alpha) \\ D\ddot{z}_o + E\dot{z}_o + Gz_o &= -M_z \sin(\omega t + \beta) \end{aligned}\right\} . \qquad (1)$$

Pitch

$$\left.\begin{aligned} \left(A + k_{yy}^2 \rho\nabla\right)\ddot{\theta} + B\dot{\theta} + C\theta &= M_\theta \sin(\omega t + \gamma) \\ d\ddot{\theta} + e\dot{\theta} + g\theta &= -F_\theta \sin(\omega t + \delta) \end{aligned}\right\} . \qquad (2)$$

For a given heaving motion $z_o = z_a \sin \omega t$, it follows that:

$$\left.\begin{aligned} b &= \frac{F_a \sin \alpha}{z_a \omega} \qquad & E &= \frac{-M_z \sin \beta}{z_a \omega} \\ a &= \frac{cz_a - F_z \cos \alpha}{z_a \omega^2} - \rho\nabla \qquad & D &= \frac{gz_a + M_z \cos \beta}{z_a \omega^2} \end{aligned}\right\} . \qquad (3)$$

Similar expressions are valid for the pitching motion. The determination of the damping coefficients b and B and the damping cross-coupling coefficients e and E is straightforward: for a given frequency these coefficients are proportional to the quadrature components of the forces or moments for unit amplitude of motion. For the determination of the added mass, the added mass moment of inertia, a and A, and the added mass cross-coupling coefficients d and D it is necessary to know the restoring force and moment coefficients c and C, and the statical cross-coupling coefficients g and G.

The statical coefficients can be determined by experiments as a function of speed at zero frequency. For heave the experimental values show very little variation with speed; they were used in the analysis of the test results.

In the case of pitching there is a considerable speed effect on the restoring moment coefficient C. C decreases approximately 12% when the speed increases from Fn = 0.15 to 0.30. This reduction is due to a hydrodynamic lift on the hull when the shipmodel is towed with a constant pitch angle. Obviously this lift effect also depends on the frequency of the motion. Consequently, the coefficient of the restoring moment, as determined by an experiment at zero frequency, may differ from the value at a given frequency.

As it is not possible to measure the restoring moment and the statical cross-coupling as a function of frequency, it was decided to use the calculated values at zero speed. This is an arbitrary choice, which affects the coefficients of the acceleration terms: for harmonic motions a decrease of C by ΔC results in an increase of A by $\Delta C/\omega^2$ when C is used in the calculation.

The results for the whole model are given in the Figs. 5 and 6. The results for the heaving motion were already published in [13]; they are presented here for completeness.

Results for the Sections

The components of the forces on each of the seven sections were determined in the same way as for the whole model. As only the forces and no moments on the sections were measured two equations remain for each section:

Heave

$$(a^* + \rho\nabla^*)\,\ddot{z}_o + b^*\dot{z}_o + c^* z_o = F_z^* \sin(\omega t + \alpha^*) ,$$

Pitch (4)

$$(d^* + \rho\nabla^* x_i)\,\ddot{\theta} + e\dot{\theta} + g\theta = -F_\theta^* \sin(\omega t + \delta^*) ,$$

where $\rho\nabla^* x_i$ is the mass-moment of the section i with respect to the pitching axis. The star (*) indicates the coefficients of the sections. The section coefficients divided by the length of the sections give the mean cross-section coefficients, thus:

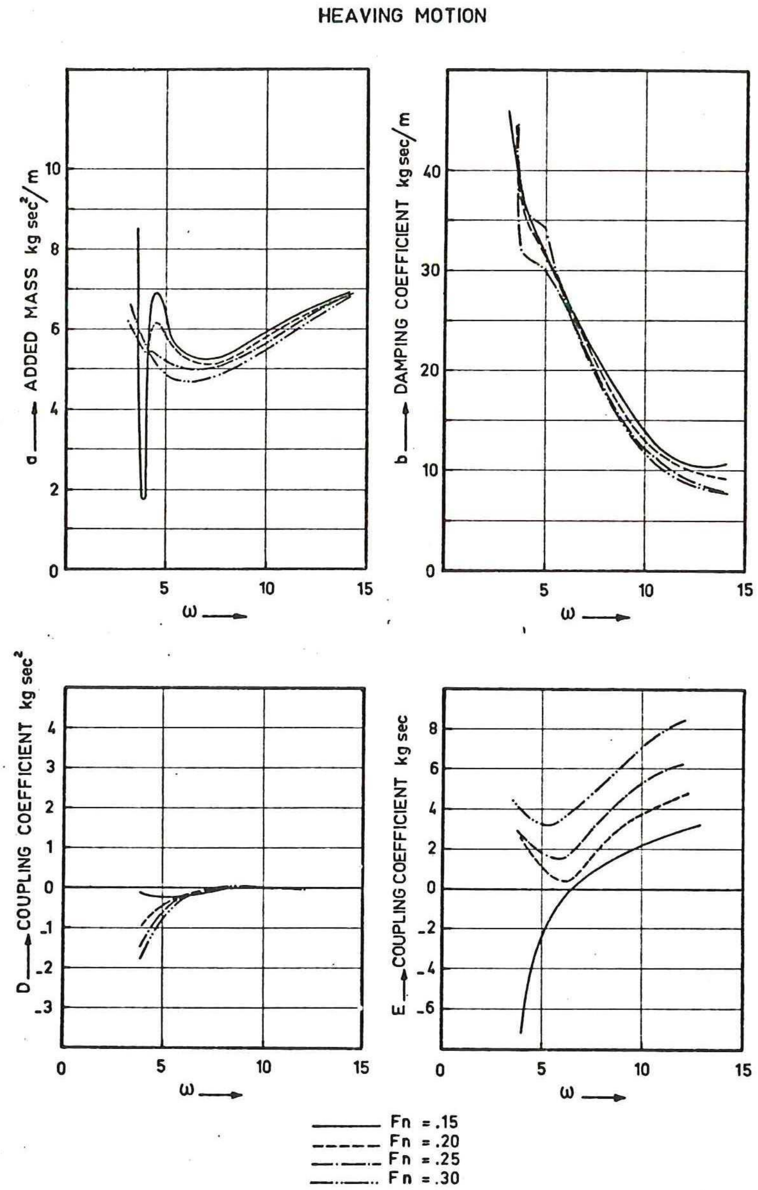

Fig. 5 - Experimental results for whole model

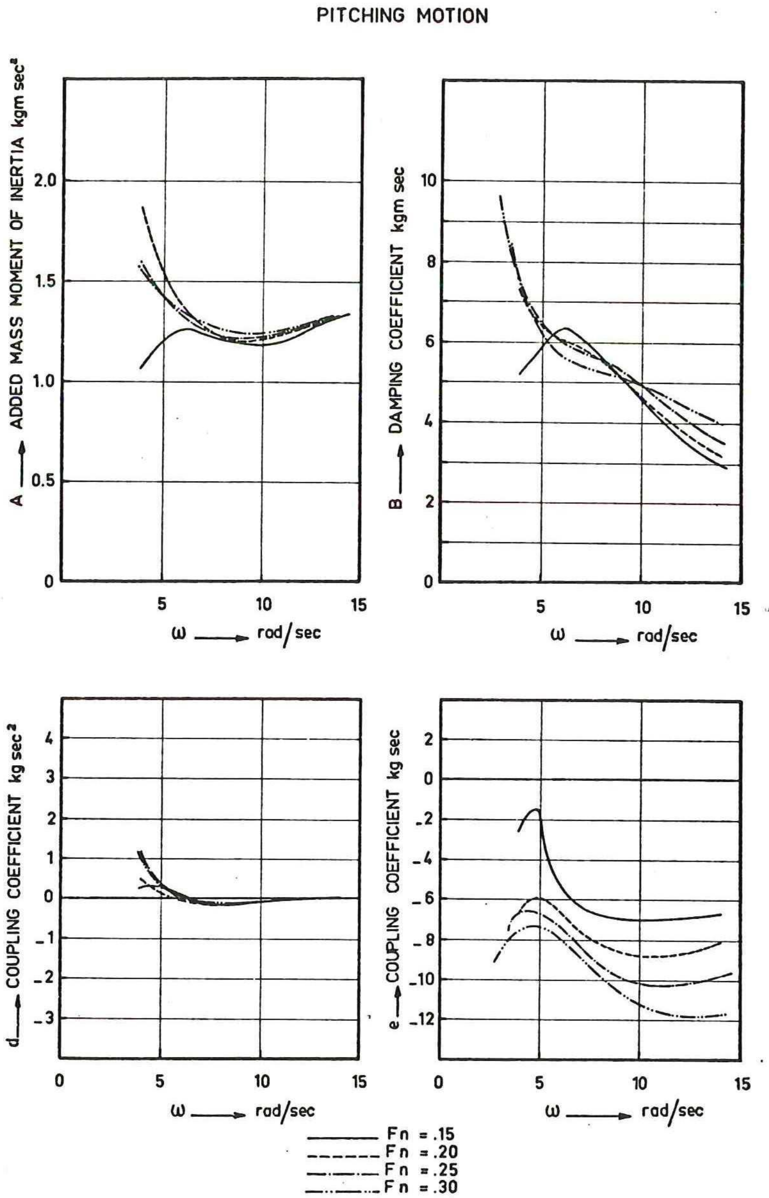

Fig. 6 - Experimental results for whole model

$$\frac{a^*}{L_{pp}/7} = \bar{a}' ,$$

and so on. Assuming that the distributions of the cross-sectional values of the coefficients a', b', etc., are continuous curves, these distributions can be determined from the seven mean cross-section values. In the Figs. 7, 8, 9 and 10 the distributions of the added mass a, the damping coefficient b and the cross-coupling coefficients d and e are given as a function of speed and frequency. Numerical values of the section results, a^*, b^*, etc., are summarized in the Tables 2, 3, 4 and 5.

Table 2
Added Mass for the Sections and the Whole Model
kg sec ^{2}/m

Fn = 0.15

ω rad/sec	a^*							a	
	1	2	3	4	5	6	7	Sum of Sections	Whole Model
4	-1.21	0.59	-	0.54	0.87	0.41	-0.17	-	1.84
6	0.31	0.66	1.08	1.38	1.26	0.65	0.02	5.36	5.37
8	0.24	0.60	1.09	1.37	1.28	0.76	0.10	5.44	5.26
10	0.20	0.69	1.29	1.48	1.34	0.85	0.14	5.99	5.91
12	0.18	0.78	1.40	1.60	1.45	0.90	0.17	6.48	6.39

Fn = 0.20

4	0.59	0.83	1.29	1.59	1.15	0.22	-0.27	5.40	5.63
6	0.32	0.65	1.00	1.40	1.23	0.64	0	5.24	5.19
8	0.21	0.55	1.08	1.38	1.21	0.75	0.12	5.30	5.18
10	0.19	0.65	1.23	1.49	1.33	0.83	0.14	5.86	5.78
12	0.20	0.77	1.37	1.60	1.45	0.88	0.17	6.44	6.32

Fn = 0.25

4	0.86	1.09	1.26	1.66	1.20	0.16	-0.32	5.91	4.99
6	0.33	0.65	1.01	1.38	1.19	0.55	-0.02	5.09	4.89
8	0.20	0.54	1.03	1.39	1.26	0.68	0.08	5.18	5.13
10	0.18	0.62	1.19	1.48	1.34	0.77	0.12	5.70	5.65
12	0.20	0.76	1.37	1.60	1.45	0.83	0.16	6.37	6.21

Fn = 0.30

4	0.70	0.91	1.49	1.58	1.07	-0.10	-0.22	5.43	5.59
6	0.25	0.44	1.15	1.39	1.07	0.45	0.07	4.82	4.51
8	0.16	0.42	1.14	1.45	1.08	0.58	0.13	4.96	4.93
10	0.15	0.55	1.26	1.47	1.22	0.68	0.17	5.50	5.48
12	0.17	0.69	1.41	1.57	1.35	0.81	0.19	6.19	6.18

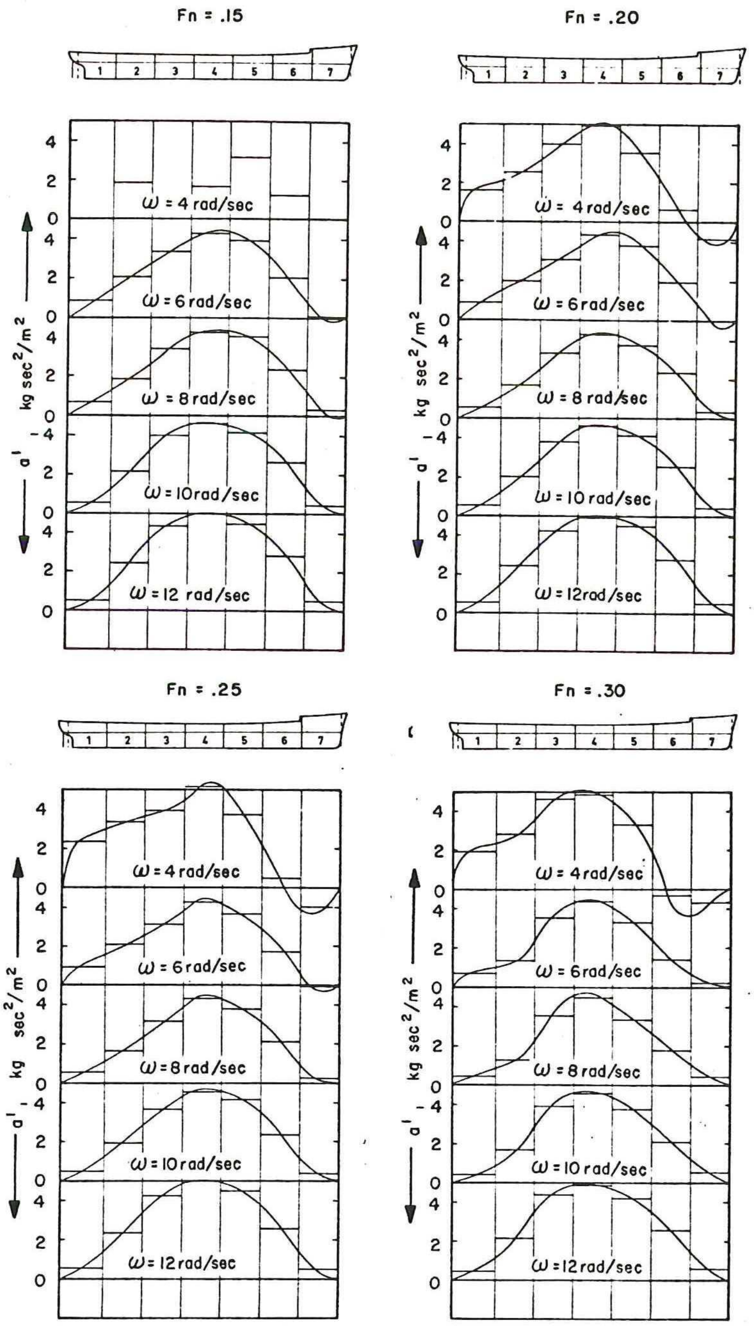

Fig. 7 - Distribution of a over the length of the shipmodel

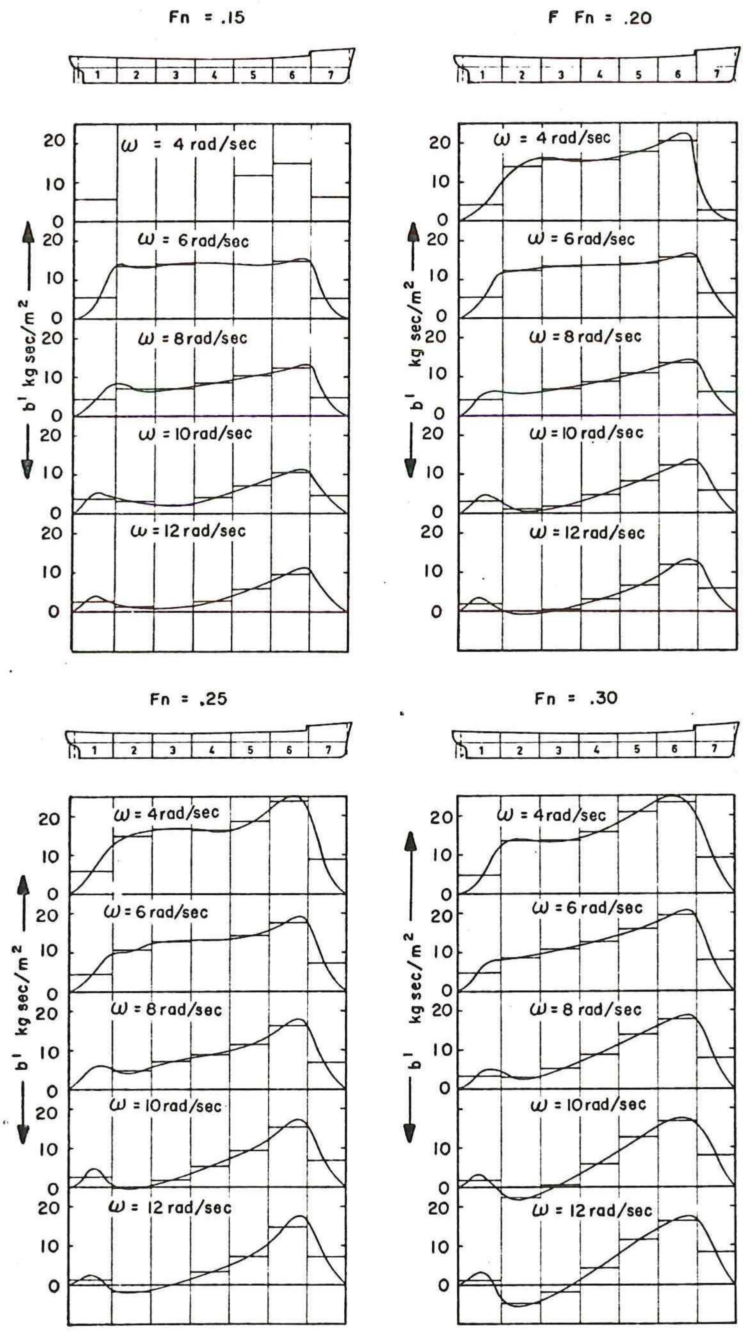

Fig. 8 - Distribution of b over the length of the shipmodel

In Fig. 8 it is shown that the distribution of the damping coefficient b depends on forward speed and frequency of oscillation. The damping coefficient of the forward part of the shipmodel increases when the speed is increasing. At the same time a decrease of the damping coefficient of the afterbody is noticed. For high frequencies negative values for the cross-sectional damping coefficients are found.

Table 3
Damping Coefficients for the Sections and the Whole Model
kg sec/m

ω rad/sec	b*							b	
	1	2	3	4	5	6	7	Sum of Sections	Whole Model
Fn = 0.15									
4	2.03	9.78	-	5.78	3.80	4.80	2.00	-	35.63
6	1.82	4.42	4.55	4.58	4.52	4.78	1.67	26.34	26.53
8	1.61	2.31	2.26	2.75	3.35	3.94	1.53	17.75	17.49
10	1.36	1.08	0.76	1.39	2.36	3.43	1.49	11.87	11.63
12	0.95	0.47	0.44	0.87	1.89	3.09	1.50	9.21	8.54
Fn = 0.20									
4	1.53	4.53	5.08	5.05	5.73	6.63	2.50	31.05	31.33
6	1.95	3.95	4.32	4.45	4.52	5.07	2.07	26.33	26.15
8	1.50	1.91	2.25	2.81	3.49	4.38	1.94	18.28	17.78
10	1.10	0.37	0.62	1.54	2.70	4.01	1.90	12.24	12.14
12	0.74	-0.15	0.21	1.01	2.18	3.84	1.93	9.76	9.03
Fn = 0.25									
4	2.13	4.80	5.38	5.20	5.98	7.63	2.85	33.97	35.88
6	1.97	3.43	4.17	4.23	4.62	5.68	2.35	26.45	27.63
8	1.48	1.58	2.28	2.83	3.68	5.21	2.19	19.25	18.75
10	0.95	-0.06	0.60	1.68	3.00	4.96	2.20	13.33	12.69
12	0.52	-0.56	-0.03	1.03	2.63	4.74	2.29	10.62	9.78
Fn = 0.30									
4	1.78	4.40	4.40	5.15	6.78	7.60	2.98	33.09	38.10
6	1.75	2.77	3.50	4.10	5.18	6.32	2.55	26.17	28.45
8	1.21	0.99	1.70	2.81	4.50	5.73	2.51	19.45	20.40
10	0.64	-0.87	0.17	1.88	4.07	5.42	2.59	13.90	13.95
12	0.42	-0.56	-0.63	1.37	3.72	5.28	2.66	11.26	10.42

The added mass distribution, as shown in Fig. 7, changes very little with forward speed but there is a shift forward of the distribution curve for increasing frequencies.

Negative values for the cross-sectional added mass are found for the bow sections at low frequencies. For higher frequencies the influence of frequency becomes very small.

Table 4
Added Mass Cross-Coupling Coefficients
for the Sections and the Whole Model
kg sec^2

Fn = 0.15

ω rad/sec	d*							d	
	1	2	3	4	5	6	7	Sum of Sections	Whole Model
4	-	-	-	-	+0.59	+0.28	-	-	-
6	-0.42	-0.47	-0.33	+0.02	+0.46	+0.57	+0.13	-0.04	+0.09
8	-0.27	-0.44	-0.40	-0.01	+0.38	+0.50	+0.13	-0.11	-0.16
10	-0.19	-0.43	-0.40	-0.01	+0.37	+0.49	+0.15	-0.02	-0.10
12	-0.19	-0.45	-0.40	-0.01	+0.40	+0.51	+0.15	+0.01	-0.04

Fn = 0.20

4	-0.57	-0.67	-	-	-	+0.78	+0.32	-	-
6	-0.39	-0.52	-0.34	+0.01	+0.46	+0.59	+0.13	-0.06	-0.06
8	-0.24	-0.45	-0.40	-0.01	+0.39	+0.51	+0.11	-0.09	-0.14
10	-0.20	-0.45	-0.40	-0.01	+0.38	+0.51	+0.13	-0.04	-0.08
12	-0.20	-0.47	-0.41	-0.01	+0.40	+0.53	+0.14	-0.02	-0.03

Fn = 0.25

4	-0.62	-0.59	-0.01	+0.12	+0.72	+0.86	+0.21	+0.69	+0.15
6	-0.39	-0.50	-0.32	+0.02	+0.46	+0.59	+0.13	-0.01	0.00
8	-0.23	-0.48	-0.40	-0.01	+0.39	+0.52	+0.14	-0.07	-0.13
10	-0.18	-0.46	-0.42	-0.01	+0.38	+0.51	+0.13	-0.05	-0.08
12	-0.20	-0.46	-0.42	-0.01	+0.40	+0.51	+0.15	-0.03	-0.05

Fn = 0.30

4	-0.62	-0.61	+0.13	+0.08	+0.64	+0.93	+0.20	+0.75	+1.09
6	-0.29	-0.47	-0.36	+0.01	+0.43	+0.59	+0.21	+0.12	+0.01
8	-0.21	-0.47	-0.44	-0.01	+0.38	+0.53	+0.16	-0.06	-0.11
10	-0.19	-0.46	-0.44	-0.02	+0.38	+0.51	+0.15	-0.07	-0.10
12	-0.20	-0.46	-0.44	-0.02	+0.39	+0.52	+0.16	-0.05	-0.06

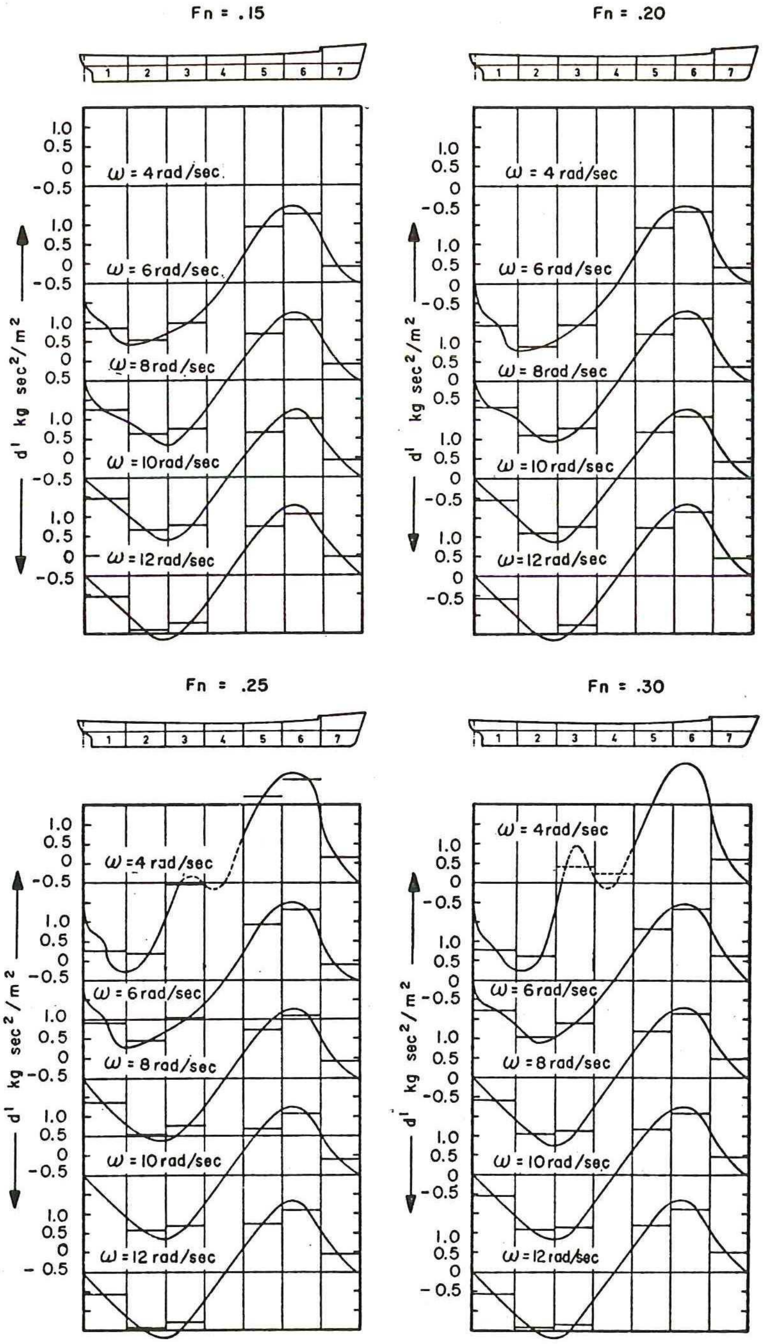

Fig. 9 - Distribution of d' over the length of the shipmodel

Distribution of Hydrodynamic Forces on a Shipmodel

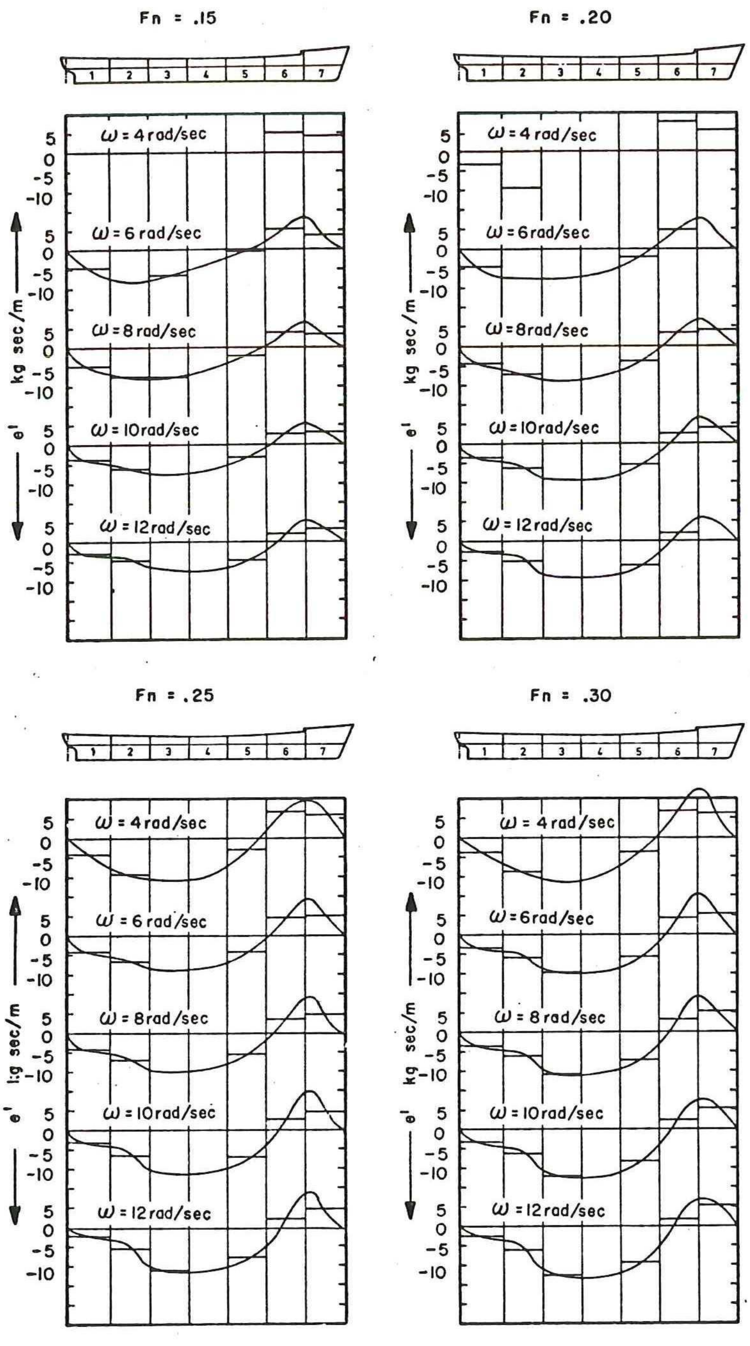

Fig. 10 - Distribution of e over
the length of the shipmodel

Table 5
Damping Cross-Coupling Coefficients for the Sections and the Whole Model
kg sec

ω rad/sec	e*							e	
	1	2	3	4	5	6	7	Sum of Sections	Whole Model
Fn = 0.15									
4	-	-	-	-	+1.63	+1.34	-	-	-2.43
6	-1.65	-2.58	-2.12	-1.19	-0.09	+1.70	+1.21	-4.72	-5.32
8	-1.71	-2.49	-2.45	-1.81	-0.68	+1.20	+1.09	-6.84	-6.75
10	-1.40	-2.01	-2.43	-2.10	-1.21	+0.88	+1.05	-7.22	-7.04
12	-1.07	-1.55	-2.28	-2.39	-1.52	+0.63	+1.05	-7.13	-6.88
Fn = 0.20									
4	-1.22	-3.07	-	-	-	+2.39	+1.77	-	-6.63
6	-1.68	-2.43	-2.40	-2.06	-0.68	+1.52	+1.42	-6.31	-6.65
8	-1.59	-2.36	-2.83	-2.50	-1.25	+1.11	+1.32	-8.10	-8.23
10	-1.29	-2.04	-3.02	-2.87	-1.75	+0.82	+1.29	-8.86	-8.86
12	-0.98	-1.65	-2.99	-2.97	-2.06	+0.61	+1.30	-8.74	-8.75
Fn = 0.25									
4	-1.52	-3.04	-3.47	-3.03	-0.96	+2.16	+1.91	-7.95	-6.70
6	-1.50	-2.21	-2.85	-2.66	-1.36	+1.47	+1.61	-7.50	-7.38
8	-1.50	-2.26	-3.21	-2.97	-1.79	+1.11	+1.51	-9.11	-9.30
10	-1.22	-2.14	-3.56	-3.39	-2.27	+0.86	+1.49	-10.23	-10.18
12	-0.85	-1.81	-3.66	-3.58	-2.53	+0.66	+1.47	-10.30	-10.31
Fn = 0.30									
4	-1.37	-2.82	-3.61	-3.06	-1.22	+2.19	+1.98	-7.91	-7.55
6	-1.23	-1.93	-3.16	-3.06	-1.84	+1.43	+1.72	-8.07	-7.95
8	-1.30	-1.96	-3.55	-3.42	-2.32	+1.03	+1.67	-9.85	-9.81
10	-1.19	-2.06	-3.94	-3.90	-2.70	+0.76	+1.67	-11.36	-11.25
12	-0.91	-1.97	-4.08	-4.19	-2.97	+0.56	+1,69	-11.87	-11.84

The distribution of the damping cross-coupling coefficient e varies with speed and frequency as shown in Fig. 10. From Fig. 9 it can be seen that the added mass cross-coupling coefficient depends very little on speed. For higher frequencies the influence of frequency is small.

As a check on the accuracy of the measurements the sum of the results for the sections were compared with the results for the whole model. The following relations were analysed:

$$\Sigma a^* = a \int_L d'\,x\,dx = A$$

$$\Sigma b^* = b \int_L e'\,x\,dx = B$$

$$\Sigma d^* = d \int_L a'\,x\,dx = D$$

$$\Sigma e^* = e \int_L b'\,x\,dx = E.$$

The results are shown in Fig. 11 for a Froude number Fn = 0.20. For the other Speeds a similar result was found. A numerical comparison is given in the Tables 2, 3, 4 and 5. It may be concluded that the section results are in agreement with the values for the whole model. No influence of the gaps between the sections could be found.

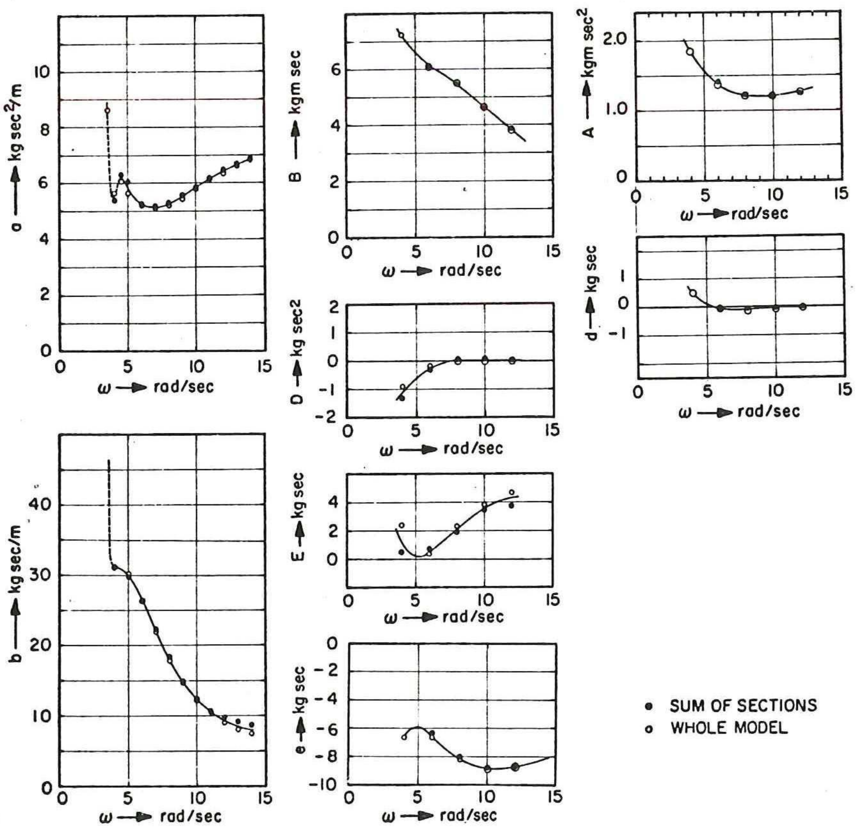

Fig. 11 - Comparison of the sums of section results and the whole model results for Froude number Fn = 0.20

ANALYSIS OF THE RESULTS

The experimental values for the hydrodynamic forces and moments on the oscillating shipmodel will now be analysed by using the strip theory, taking into account the effect of forward speed. For a detailed description of the strip theory the reader is referred to [1], [2] and [3]. For convenience a short description of the strip theory is given here. The theoretical estimation of the hydrodynamic forces on a cross-section of unit length is of particular interest with regard to the measured distributions of the various coefficients along the length of the shipmodel.

Strip Theory

A right hand coordinate system $x_o y_o z_o$ is fixed in space. The z_o-axis is vertically upwards, the x_o-axis is in the direction of the forward speed of the vessel and the origin lies in the undisturbed water surface. A second right hand system of axis xyz is fixed to the ship. The origin is in the centre of gravity. In the mean position of the ship the body axis have the same directions as the fixed axis.

Consider first a ship performing a pure harmonic heaving motion of small amplitude in still water. The ship is piercing a thin sheet of water, normal to the forward speed of the ship, at a fixed distance x_o from the origin.

At the time t a strip of the ship at a distance x from the centre of gravity is situated in the sheet of water. From $x_o = Vt + x$ it follows that $\dot{x} = -V$, where V is the speed of the ship.

The vertical velocity of the strip with regard to the water is $\dot{z}_o$, the heaving velocity. The oscillatory part of the hydromechanical force on the strip of unit length will be

$$F_H' = -\frac{d}{dt}(m'\dot{z}_o) - N'\dot{z}_o - 2\rho g y z_o ,$$

where m' is the added mass and N' is the damping coefficient for a strip of unit length and y is the half width of the strip at the waterline. Because

$$\frac{dm'}{dt} = \frac{dm'}{dx} \times \dot{x} ,$$

it follows that

$$F_H' = -m'\ddot{z}_o - \left(N' - V\frac{dm'}{dx}\right)\dot{z}_o - 2\rho g y z_o . \qquad (5)$$

For the whole ship we find, because

$$\int_L \frac{dm'}{dx}\, dx = 0:$$

$$F_H = -\left(\int_L m'dx\right)\ddot{z}_o - \left(\int_L N'dx\right)\dot{z}_o - \rho g A_w z_o \tag{6}$$

where A_w is the waterplane area. The moment produced by the force on the strip is given by

$$M'_H = -xF'_H = (xm')\ddot{z}_o + \left(N'x - Vx\frac{dm'}{dx}\right)\dot{z}_o + 2\rho gxyz_o\,. \tag{7}$$

Because

$$\int_L x\frac{dm'}{dx}\,dx = -m\,,$$

we find for the whole ship

$$M_H = \left(\int_L x\,m'dx\right)\ddot{z}_o + \left(\int_L N'x\,dx + Vm\right)\dot{z}_o + \rho g S_w z_o \tag{8}$$

where S_w is the statical moment of the waterplane area.

For a pitching ship the vertical speed of the strip at x with regard to the water will be $-x\dot{\theta} + V\theta$, and the acceleration is $-x\ddot{\theta} + 2V\dot{\theta}$. The vertical force on the strip will be

$$F'_p = -\frac{d}{dt}m'(-x\dot{\theta} + V\theta) - N'(-x\dot{\theta} + V\theta) - 2\rho gyx\theta\,,$$

or

$$F'_p = m'x\ddot{\theta} + \left(N'x - 2Vm' - xV\frac{dm'}{dx}\right)\dot{\theta} + \left(2\rho g\,yx + V^2\frac{dm'}{dx} - N'V\right)\theta\,. \tag{9}$$

The total hydromechanical force on the pitching ship will be

$$F_p = \left(\int_L m'x\,dx\right)\ddot{\theta} + \left(\int_L N'x\,dx - Vm\right)\dot{\theta} + \left(\rho g S_w - V\int_L N'dx\right)\theta\,. \tag{10}$$

The moment produced by the force on the strip is given by

$$M'_p = -xF'_p = -m'x^2\ddot{\theta} - \left(N'x^2 - 2Vm'x - x^2V\frac{dm'}{dx}\right)\dot{\theta} - \left(2\rho gyx^2 + V^2x\frac{dm'}{dx} - N'Vx\right)\theta\,. \tag{11}$$

The total moment on the pitching ship will be

$$M_p = -\left(\int_L m'x^2dx\right)\ddot{\theta} - \left(\int_L N'x^2dx\right)\dot{\theta} - \left(\rho g I_w - V^2m - V\int_L N'x\,dx\right)\theta\,, \tag{12}$$

because

$$\int_L x^2 V \frac{dm'}{dx} dx = -2V \int_L m'x\,dx .$$

A summary of the expressions for the various coefficients for the whole ship according to the notation in Eqs. (1) and (2) is given in Table 6.

Table 6
Coefficients for the Whole Ship
According to the Strip Theory

$a = \int_L m'dx$	$d = \int_L m'x\,dx + \frac{Vb}{\omega^2}$
$b = \int_L N'dx$	$e = \int_L N'x\,dx - Vm$
$c = \rho g A_w$	$g = \rho g S_w$
$A = \int_L m'x^2\,dx + \frac{VE}{\omega^2}$	$D = \int_L m'x\,dx$
$B = \int_L N'x^2\,dx$	$E = \int_L N'x\,dx + Vm$
$C = \rho g I_w$	$G = \rho g S_w$

(13)

For the cross-sectional values of the coefficients similar expressions can be derived from the Eqs. (5) to (12). For the comparison with the experimental results two of these expressions are given here, namely:

$$b' = N' - V\frac{dm'}{dx} ,$$

$$e' = N'x - 2Vm' - xV\frac{dm'}{dx} . \quad (14)$$

Also it follows that

$$A = \int d'x\,dx$$

and (15)

$$B = \int e'x\,dx .$$

Comparison of Theory and Experiment

For a number of cases the experimental results are compared with theory. First of all the damping cross-coupling coefficients are considered. From Eqs. (13) it follows that:

$$E = \int_L N'x\,dx + Vm$$
$$e = \int_L N'x\,dx - Vm\,. \qquad (16)$$

The first term in both expressions is the cross-coupling coefficient for zero forward speed. For a fore and aft symmetrical ship this term is equal to zero. For such a ship the resulting expressions are equal in magnitude but have opposite sign, which is in agreement with the result found by Timman and Newman [7]. The experiments confirm this fact as shown in Fig. 13 where e and E are plotted on a base of forward speed as a function of the frequency of oscillation. The magnitude of the speed dependent parts of the coefficients is equal within very close limits. Extrapolation to zero speed shows that the e and E lines intersect in one point which should represent the zero speed cross-coupling coefficient.

Using Grim's two-dimensional solution for damping and added mass at zero speed [9] the coefficients e and E were also calculated according to the Eqs. (16). The distribution of added mass and damping coefficient for zero speed is given in Fig. 12 and the calculated damping cross-coupling coefficients are shown in Fig. 13.

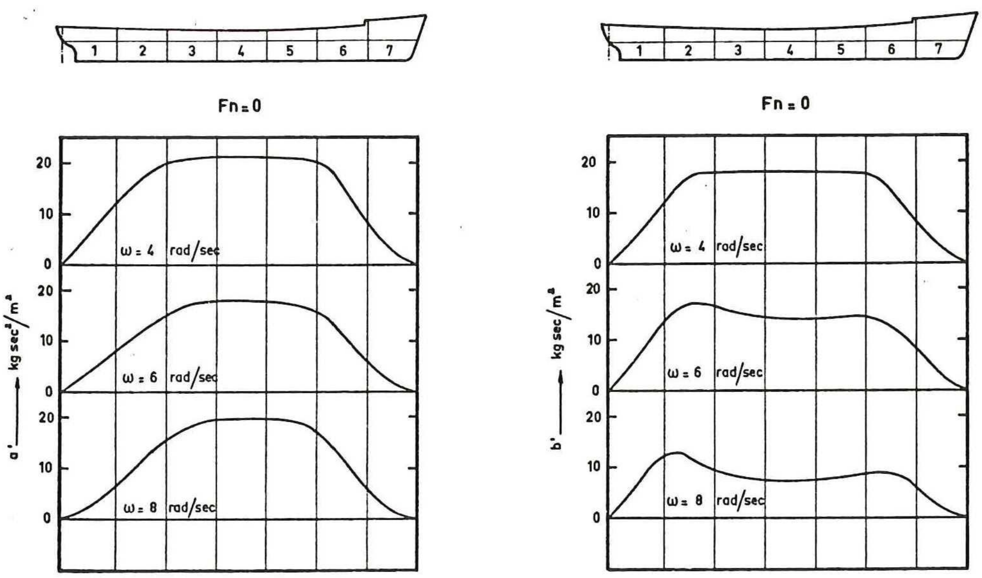

Fig. 12 - Calculated distribution of a and b for zero speed

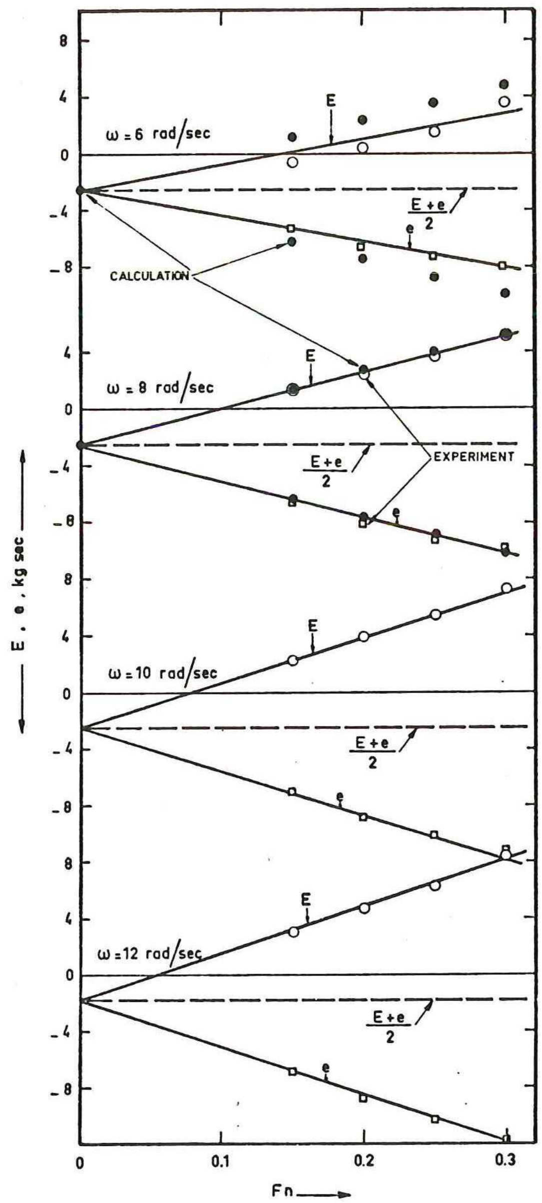

Fig. 13 - Comparison of calculated and measured values for e and E

The calculated values are in line with the experimental results. The natural frequencies for pitch and heave are respectively ω = 7.0/6.9 rad/sec and in this important region the calculation of the damping cross-coupling coefficients is quite satisfactory. The zero speed case will be studied in the near future by oscillating experiments in a wide basin to avoid wall influence.

Another comparison of theory and experiment concerns the distribution along the length of the shipmodel of the damping coefficient and of the damping cross-coupling coefficient e. From Eq. (14):

$$b' = N' - V \frac{dm'}{dx} ,$$

$$e' = N'x - 2Vm' - xV \frac{dm'}{dx} .$$

Again using Grim's two-dimensional values for N' and m', these distributions could be calculated. An example is given in Fig. 14. Also in this case the agreement between the calculation and the experiment is good. For high speeds negative values of the cross-sectional damping in the afterbody can be explained on the basis of the expression for b', because in that region dm'/dx is a positive quantity.

Finally the values for the coefficients A, B, a and b for the whole model, as given by the Eqs. (13) were calculated and compared with the experimental results. Figure 15 shows that the damping in pitch is over-estimated for low frequencies. The other coefficients agree quite well with the experimental results.

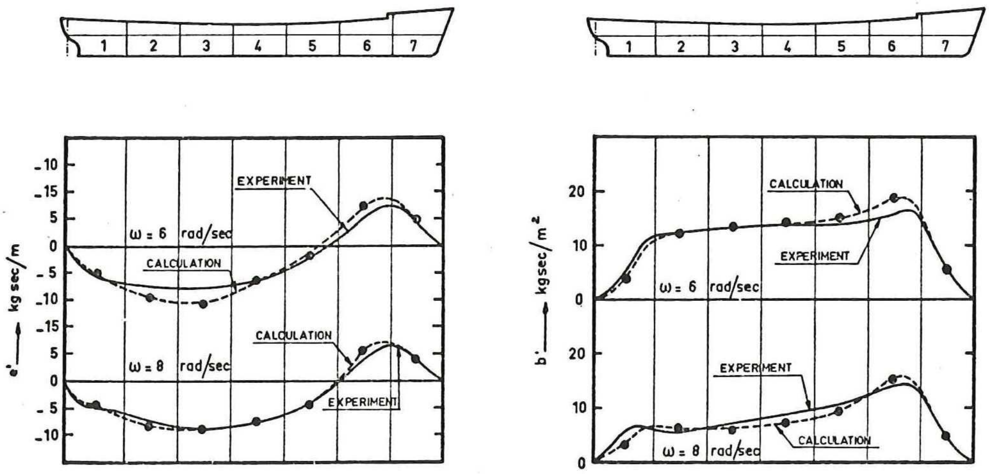

Fig. 14 - Comparison of the calculated distribution of e and b with experimental values for Froude number 0.20

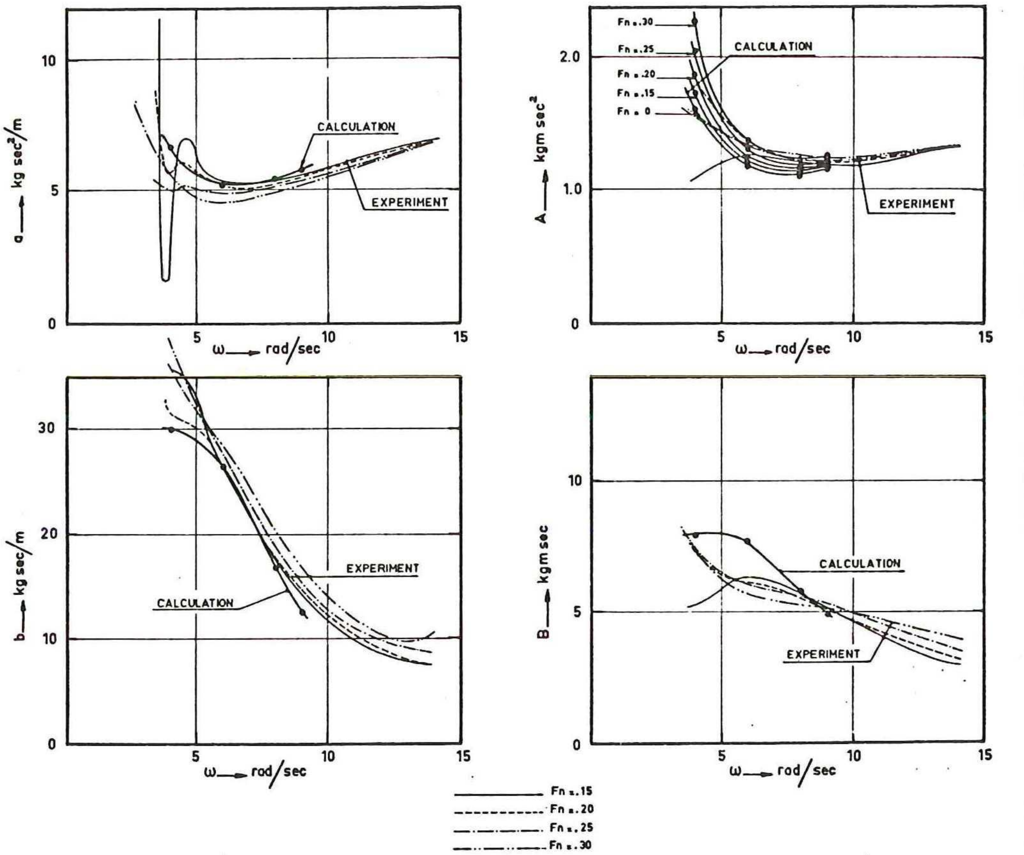

Fig. 15 - Comparison of calculated and measured values for a, b, A and B (whole model)

LIST OF SYMBOLS

a ... g, A ... G	coefficients of the motion equations (hydromechanical part),
$a^*\ldots g^*$, $A^*\ldots G^*$	the same for a section of the ship,
$a'\ldots g'$, $A'\ldots G'$	the same for a cross-section of the ship,
C_B	Block coefficient,
Fn	Froude number
F_z, F_θ	amplitude of vertical force on a heaving or pitching ship,

F_H, F_p oscillatory part of the hydromechanical force on a heaving or pitching ship,

g acceleration of gravity,

k_{yy} longitudinal radius of inertia of the ship,

L_{pp} length between perpendiculars,

M_z, M_θ amplitude of moment on a heaving or pitching ship,

M_H, M_p oscillatory part of the hydromechanical moment on a heaving or pitching ship,

m' added mass of a cross-section (zero speed),

N' damping coefficient of a cross-section (zero speed),

t time,

V forward speed of ship,

$x\ y\ z$ right hand coordinate system, fixed to the ship,

x_o, y_o, z_o right hand coordinate system, fixed in space,

z_o vertical displacement of ship,

x_i distance of centre of gravity of a section to the pitching axis,

$\alpha, \beta, \gamma, \delta$ phase angles,

θ pitch angle,

ρ density of water,

ω circular frequency,

∇ volume of displacement of ship, and

∇^* volume of displacement of section.

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* * *

DISCUSSION

E. V. Lewis
Webb Institute of Naval Architecture
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This is a noteworthy paper in an important series by Professor Gerritsma and his colleagues that is of vital importance to ship motion theory. This continuing work has been characterized by unerring choice of the right research subjects and by extraordinary experimental skill. The results have served to clarify the so-called "strip theory" of ship motion calculations and to provide step by step confirmation of the different elements of the theory. Thus the tremendous power of this comparatively simple approach to the problems of ship motions is being reinforced and the value of the pioneering insight of Korvin-Kroukovsky and others confirmed.

It may not be generally realized that this type of experiment, in which forces on seven different sections are measured, is of unusual difficulty, not only because of the many simultaneous readings to be taken, but in the need for accurate determination of in-phase and out-of-phase force components in spite of extraneous noise. The authors have mastered this difficult problem.

The particular value of the resulting research is in showing that when the ship velocity terms are included, excellent predictions of the longitudinal distribution of damping forces are obtained. Furthermore, the nature of the cross-coupling coefficients, E and e, has been clarified by the demonstration that they should be equal at zero speed and differ only by the term $\pm Vm$ at forward speeds. (Incidentally, m is not defined, but is apparently equal to $-a$.)

Incidental features of the paper are simplifications in the coefficients, which are not immediately obvious. It is mentioned that

$$\int x^2 V \frac{dm'}{dx}\, dx = -2V \int m' x\, dx ,$$

which makes the B coefficient, Eq. (13), much simpler than given in [1]. Also

$$\int x \frac{dm'}{dx}\, dx = \int x\, dm' = \int m'\, dx = -m \; (= a) ,$$

and therefore the e coefficient is also simplified [Eq. (13)]. Hence, the simple relationship between e and E emerges in Eq. (16) and Fig. 13.

It is hoped that this important work strengthening the strip theory approach will be continued, including oscillation tests at zero speed and restrained tests in waves. My congratulations to the authors for a beautiful piece of research.

* * *

DISCUSSION

J. N. Newman
David Taylor Model Basin
Washington, D.C.

First of all let me congratulate the authors on yet another in the series of excellent papers which we have come to expect from Delft.

Certainly one of the most valuable results obtained recently is the very simple forward speed correction to the strip theory, as outlined in the strip theory paragraph, and the correlation of this theory with experiments. It would seem that all important speed effects are taken into account simply by replacing the time derivative in a fixed coordinate system by that for a moving coordinate system, or

$$\frac{d}{dt} \rightarrow \frac{\partial}{\partial t} - V \frac{\partial}{\partial x} .$$

As a result, the added mass coefficient contributes both to the acceleration and velocity terms of the equations of motion, since

$$\frac{d}{dt} (m' \dot{z}_o) \rightarrow m' \ddot{z}_o - V \frac{dm'}{dx} \dot{z}_o .$$

However this process seems rather arbitrary; why not repeat it for the second time derivative, so that

$$F'_H = -\frac{d^2}{dt^2} m' z_o - \frac{d}{dt} N' z_o - 2\rho g y z_o$$

$$= -m' \ddot{z}_o - \left(N' - 2V \frac{dm'}{dx}\right) \dot{z}_o - \left(2\rho g y + V^2 \frac{d^2 m'}{dx^2} - V \frac{dN'}{dx}\right) z_o \; ?$$

It is clear from the experimental results that too much cross-coupling would result, and thus that the last equation is ridiculous both in appearance and in practical utility, but I am left wondering why the equation used in the paper is so much better. Is it possible to give any rational explanation for this?

Finally, since Professor Vossers is not here to defend himself, let me point out that, in general, forward speed will have an effect on the distribution of hydrodynamic forces along an oscillating slender body. Vossers reached the opposite conclusion only for the special case of high frequencies of encounter and very slow speeds.

* * *

DISCUSSION OF THE PAPERS BY GERRITSMA AND BEUKELMAN AND BY VASSILOPOULOS AND MANDEL

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The paper by Vassilopoulos and Mandel rigorously examined seakeeping theory, with valuable emphasis on practical ship design. The paper by Gerritsma and Beukelman contains significant experimental results and a clear concise strip theory, thus relating theory and physical phenomena. However, the paper by Vassilopoulos and Mandel agrees only partially with Gerritsma and Beukelman, and with Korvin-Kroukovsky.

The papers were examined by this discusser with the following results:

1. Complete agreement exists as to (a) which motion derivatives appear in each coefficient, and (b) the appearance of velocity dependent terms arising purely from the mechanics of a fixed axis system.

2. Disagreement exists as to the importance of the effect of forward speed on strip theory, but this is the only point of disagreement.

This disagreement led to different evaluations of some motion derivatives. Direct comparison of the coefficients in the two papers does not reveal all disagreement, because of the cancellation of terms due to strip theory by terms due to the mechanics of a fixed axis system. The disagreement in the strip theory specifically arose in two ways: (1) Gerritsma and Beukelman consider sectional added mass to be a function of time, as suggested by Korvin-Kroukovsky. This is a "three-dimensional correction" and is justified experimentally by a velocity dependence in the b' term for the three-dimensional end sections of Gerritsma and Beukelman's model. (2) Gerritsma and Beukelman consider the distance x, between the body axis origin and the hypothetical sheet of water, to be a function of time. This is independent of dimensionality. The second difference is confusing; for Vassilopoulos and Mandel do implicitly take x as function of time when converting from movable to fixed axes, but do not when applying the strip theory.

The strip theory of Gerritsma and Beukelman was re-derived, eliminating these disagreements. The results agreed completely with those of Vassilopoulos and Mandel. Application of integrals quoted by Gerritsma and Beukelman showed agreement between that paper and Korvin-Kroukovsky. This therefore showed no errors in Korvin-Kroukovsky's work, only disagreement with Vassilopoulos and Mandel as to the role of forward speed on the strip theory. Conversion of Gerritsma and Beukelman results to a movable axis system revealed no difficulties, but clearly showed which speed terms result from mechanics and which from strip theory.

The differences, therefore, are seen to be completely a result of a different assumption of the importance of forward speed on strip theory, independent of what axis system is used. The assumption of Gerritsma and Beukelman seems to be justified by experiment. The derivation of the equations of motion by Vassilopoulos and Mandel, due to Abkowitz, seems the most rigorous and satisfying. However, the evaluation of the motion derivatives by Gerritsma and Beukelman, due in part to Korvin-Kroukovsky, seems to yield better results.

This discusser therefore feels it most practical to use the former work to study the mathematics of motion and the latter to evaluate the motion derivatives.

* * *

REPLY TO THE DISCUSSION BY E. V. LEWIS

J. Gerritsma and W. Beukelman
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Delft, Netherlands

The authors are grateful to have Professor Lewis' comments on their paper.

The definition of m, which is omitted in the paper, is given by

$$\int_L m'dx = m = a .$$

It should be noted that

$$\int_L x\,dm' = -\int_L m'dx$$

and not

$$\int_L x\,dm' = \int_L m'dx ,$$

as suggested by Professor Lewis.

The work reported in this paper was recently extended for the zero forward speed case.

These tests were carried out in a wide basin to avoid wall influence, due to reflected waves. The results support the conclusions of the present paper.

Within the very near future the restrained tests in waves with the segmented model will be carried out in our Laboratory. The results will be compared with calculated values.

* * *

Distribution of Hydrodynamic Forces on a Shipmodel

REPLY TO THE DISCUSSION BY J. N. NEWMAN

J. Gerritsma and W. Beukelman
Technological University
Delft, Netherlands

For a fully submerged slender body of revolution in unsteady motion, the total hydrodynamic force on a transverse section is equal to the negative time rate of change of fluid momentum. By taking the time derivative in the moving body axis system the expression

$$\frac{d}{dt}(m'\dot{z}_o) = m'\ddot{z}_o - V\frac{dm'}{dx}\dot{z}_o ,$$

is found.

For the surface ship, it is assumed that the flow over the submerged portion of the ship is similar to the flow over the lower half of a fully submerged body with circular cross sections.

Corrections are then necessary for the shape of the sections and for free surface effects. It is assumed that these corrections are introduced by using Grim's values for the sectional damping and added mass coefficients of cylinders having ship-like cross sections oscillating at a free surface. It is admitted that this assumption is more or less intuitive and it was clearly necessary that the assumptions being made had to be verified by experiments, as shown in the paper.

The authors cannot give a similar physical interpretation of the procedure put forward in Dr. Newman's discussion; they have therefore no rational explanation why such an approach is not successful. In addition, the result would certainly not agree with the experiments.

Vossers' results are discussed too shortly in our paper, and the authors are grateful to Dr. Newman for his additional comments.

However, for the actual ship form, as tested in our case, the forward speed effect cannot be neglected, even at quite low speeds, say $Fn = 0.15$.

For pitch, the method, as given in our paper, is valid for such combinations of forward speed and frequency that the motion of the ship in the stationary sheet of water does not depart too much from a harmonic motion (see Ref. [2]).

* * *