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ORIGINAL ARTICLE

# Modeling and analysis of non-homogenous fabrication/assembly systems with multiple failure modes

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**Abstract** This paper presents an approximate decomposition method for the performance evaluation of non-homogeneous fabrication/assembly (F/A) systems with multiple failure modes, finite buffers, and a fixed assembly proportion. First, we introduce a mixed flow combined with fund flow and material flow to convert an F/A system to a virtual transfer line. Then, we decompose the virtual transfer line into several two-machine lines and establish a continuous decomposition model. To tackle the new emerging characteristics of the F/A system, we propose an F/A decomposition algorithm (FADA) for solving this model and obtain the throughput and buffer level of the F/A system to evaluate system performance. Also, we demonstrate the validity of the proposed model and algorithm by comparing with the results of simulation-based method and completion time approximation (CTA)-based method. Finally, we analyze the impact of several key parameters, including failure rates, repair rates, and buffer capacities, on the performance of the F/A system. The results show that our analytical method outperforms the existing methods and can help production managers to evaluate the system performance, analyze the possible modifications, and further find the best performance improvement of such F/A systems.

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**Keywords** Fabrication/assembly system · Multiple failure modes · Fund flow · Decomposition method · Performance evaluation

## 1 Introduction

Production systems provide manufacturing capacity to convert materials into products. During this conversion process, the related material flow, fund flow, and information flow converge into a production workshop. These production workshops are classified as transfer lines, parallel lines, assembly/disassembly (A/D) systems and closed loop lines. Each line is always composed of machines and buffers. Machines own identical or non-identical processing speed, which corresponds to homogeneous lines and non-homogeneous lines, respectively. Various data on production systems, such as machine status, machine processing speeds, and buffer capacities, can be exactly obtained using new-generation information technologies including internet of things, big data, and cloud manufacturing [1–3]. Based on these data, fast, accurate analytical methods for evaluating factory performance (such as system throughput, and buffer level) can help management to make decisions, such as determining the capacity required to meet production plans, improving throughput, and identifying bottlenecks [4, 5].

Production management and control often suffer from disturbances, such as machine failure, tool breakage, material shortage, and absenteeism. Due to statistical fluctuations and processing dependencies of the production process, these disturbances affect production performance, such as throughput and buffer level. Although such disturbances can be treated as machine failures, it still makes establishing an exact model of an entire production workshop intractable, not to mention performing the corresponding performance analysis.

Traditional research tends to assume that a machine is under a single failure mode, i.e., each machine can only fail in one way. However, real-world machines can experience multiple failure modes, which happen with different frequencies and interrupt production for time periods of different lengths. By considering multiple failure modes rather than single failure mode of machines in production models, it will help to clearly explore the influence mechanism of disturbances on the production management process and further exactly evaluate the performance of production workshops.

Since machines can process at a variety of processing speeds, failure modes, and repair rates, the modeling has to take care of many uncertainties and randomness, which are the main difficulties to solve when modeling production systems. In this paper, we study the performance evaluation of non-homogeneous F/A systems with unreliable machines having multiple failure modes and finite buffers. The F/A system defined in this paper is a special class of A/D systems that are widely used in many electronics manufacturing factories. It consists of an assembly machine with inputs from two proportioned fabrication lines.

To the best of our knowledge, there is no study on the performance evaluation of non-homogeneous fabrication/assembly (F/A) systems with multiple failure modes, finite buffers, and a fixed assembly proportion. Levantesi et al.'s [6] study is the only relevant research that studied homogeneous A/D systems with a special assembly proportion being equal to 1. Specifically, we study non-homogeneous systems in which the machines can operate at different processing speeds. In addition, we extend the special assembly relationship to a general assembly proportion that is known and fixed but can be any real number. Only when the amounts of materials coming from fabrication lines satisfy a fixed assembly proportion can the assembly machine yield final products. The two emerging characteristics make the problem more complex when modeling the F/A systems.

To deal with the characteristics in our proposed F/A systems, a fund flow is introduced to model F/A systems besides the material flow. We explore and find out that the material flow and fund flow in production systems are both one-way flow, but their directions are opposite. Therefore, the fund flow and material flow can be combined together to convert an F/A system to a virtual transfer line. For the virtual transfer line, we present an approximate analytical method based on the decomposition methodology for performance evaluation of such systems. In addition, we define a set of decomposition equations consisting of the processing rate equations, interruption of flow equations, and failure rate equations. These equations are different for fabrication and assembly machines. Furthermore, we introduce a new F/A decomposition algorithm (FADA) for solving these equations. The main contributions of this paper are as follows:

1. We study an F/A system characterized by one-way material flows and two proportioned fabrication lines in the same direction. Therefore, we first introduce a fund flow with an opposite direction from that of the material flow, combine fund flow with material flow together, and further convert such F/A system to a virtual transfer line, which can be decomposed and evaluated by a decomposition method.
2. We propose a continuous decomposition model for non-homogeneous F/A systems with multiple failure modes and develop a decomposition algorithm (FADA) for solving this model. Furthermore, we demonstrate the validity of the proposed model and algorithm by comparing with the results of simulation-based method and completion time approximation (CTA) [7]-based method.
3. We provide an analytical method to help managers to understand the impact of several key parameters, including failure rates, repair rates, and buffer capacities, on system performance. Furthermore, we demonstrate how production managers of such systems may use our method to evaluate the system performance, analyze the possible modifications, and further find the best performance improvement.

The rest of the paper is organized as follows: Section 2 reviews modeling methods of transfer lines and assembly lines. Section 3 contains a description of non-homogeneous F/A systems with multiple failure modes. In Section 4, both the decomposition model and the decomposition equations are proposed based on the mixed fund and material flow. In Section 5, a decomposition algorithm is presented and its effectiveness is analyzed. In Section 6, computational experiments are conducted to analyze the impact of several key parameters. Section 7 concludes the paper.

## 2 Related work

Production system models can be classified into discrete and continuous models [8]. In the discrete models, beginnings and/or completions of operations at the different machines occur at the same time. The behavior of such lines is approximated by a discrete-time Markov chain. While in continuous models, it transfers materials from the upstream buffer to the downstream buffer in a continuous way at a certain rate. The behavior of such lines is approximated by a continuous Markov model. While discrete models are limited to homogeneous lines, continuous models can be applied to both homogeneous lines and non-homogeneous lines. Especially, in a non-homogeneous line, a machine can run at a lower processing speed than its maximum due to disruptions.

For the performance evaluation method of production systems, there exist exact analytical and approximate analytical methods. For very simple systems, exact analytical method can work. Examples are transfer lines with two-machine-one-buffer [9, 10], and parallel lines with two-station-one-buffer [11]. For long and complex lines, approximate analytical methods with high accuracy are developed, including decomposition methods [8, 12–19] and aggregation methods [20–22]. Decomposition methods break an original system into two-machine building blocks and link these building blocks together through a set of derived equations. Aggregation methods handle this problem by replacing the two-machine-one-buffer line by a single equivalent machine and repeatedly aggregating until only one two-machine-one-buffer line remains. There exists a lot of research using decomposition or aggregation methods for modeling different types of systems. The boundaries of what can be modeled both efficiently and accurately are continuously being pushed forwards. Table 1 provides a historic overview of the representative studies on the modeling and performance analysis of production systems along with the types of systems studied.

**Transfer lines** Much research deals with the problem of modeling a transfer line. Transfer lines are by now very well understood, and several representative results can be found in the literature. Gershwin [12] proposed one of the most classic decomposition methods for analyzing a homogeneous transfer line with a single failure mode. Later, Dallery et al. [15] developed the famous Dallery-David-Xie (DDX) algorithm, which is a high-efficiency and robust iterative algorithm compared to the earlier method of Gershwin [12]. Le Bihan and Dalley [23], and Le Bihan and Yves [24] extended DDX using two different decomposition methods in order to solve this issue in the case of homogeneous lines where the orders of magnitude of the different machines' reliability parameter are not at the same level.

Those papers mentioned above all considered the homogenous lines. For non-homogenous lines, Gershwin and Burman [14] presented a decomposition method with exponential repair and failure time distributions. Dallery et al. [8] proposed two approximation methods using a continuous flow model for both homogeneous lines and non-homogeneous lines. Later, Xia et al. [25] extended one of the works in [23, 24] in order to deal with non-homogeneous lines. For multiple failures, Tolio et al. [26] presented an approximate analytical method for an automated flow line with deterministic processing times, finite buffers, and multiple failure modes. Different from Tolio et al. [26], Levantesi et al. [6] proposed a decomposition method for the performance evaluation of production lines with machines having multiple failure modes and different processing times. Tolio et al. [10] later presented an exact analytical model for a two-machine line with multiple failure modes.

**Assembly lines** Compared to the amount of work on the transfer line model, there is relatively few research on the model of Assembly/Disassembly system. Based on the work of Gershwin [12], a decomposition method for evaluating A/D systems with single failure mode was applied in [27, 28]. Later, Levantesi et al. [6] proposed an efficient decomposition method that can also deal with multiple failure modes. This method allows for a better modeling of real-world systems in comparison with existing analytical methods, but it is limited to homogeneous lines. Gershwin [14] proposed a continuous decomposition model for non-homogeneous A/D system with a single failure mode. Xia et al. [25] adopted generalized exponential distributions as repair time distributions and proposed a method to tackle the scenario where the orders of magnitude of machines' reliability parameters (mean times between failures and mean times to repair) are not at the same level. This work is also limited to a single failure mode. In this paper, we propose the first continuous decomposition method dealing with non-homogenous F/A systems with multiple failure modes.

## 3 Problem description and assumptions

We focus on an F/A system named $l$, which consists of fabrication machines $M_i$ ,$i = 1, 2, \ldots, a-1, a+1, a+2, \ldots, m$, an assembly machine $M_a$, and finite buffers $B_i$, $i = 1, 2, \ldots, m$, as illustrated in Fig. 1. The goal is to evaluate throughput and buffer level of the F/A system.

Job I flows from the first machine $M_1$, then to the buffer $B_1$, then to the second machine $M_2$, and so forth until they reach the buffer $B_{a-1}$ before the assembly machine $M_a$; job II flows from machine $M_m$, then to the buffer $B_{m-1}$, and so forth until they reach the buffer $B_a$ before the assembly machine $M_a$. Only when the exactly proportioned amounts of both jobs I and II are delivered to the buffer $B_a$ can the assembly machine $M_a$ start to work and yield final products. The assembly machine will be starved if the assembly relation between jobs I and II cannot satisfy the requirement of products. For a fabrication machine, it will be starved if its upstream buffer is empty, but will be blocked if its downstream buffer is full. The first machine of each fabrication line is never starved, and the assembly machine is never blocked. Jobs are never destroyed or rejected at any machine in the system.

We assume that each machine can experience more than one failure mode, i.e., multiple failure modes. Each mode is characterized by a specific time to failure and time to repair. Here, we adopt exponential distribution for modeling the failure and repair times, which are commonly used to represent the machine operations where the downtime of machines is much longer than the cycle time, see literature [5] for details. The machine is named as exponential machine. The lines with exponential machines can be analyzed based on Markov

**Table 1** Selected studies on the modeling and performance analysis on the production systems

| | Production system | | Production system model | | Processing speed | | Failure mode | |
|---|---|---|---|---|---|---|---|---|
| | Transfer line | Assembly line | Discrete model | Continuous model | Homogeneous | Non-homogeneous | Single failure modes | Multiple failure modes |
| Burman [18] | √ | | | √ | | √ | | √ |
| Dallery et al. [15] | √ | | √ | | √ | | √ | |
| Dallery et al. [8] | √ | | | √ | √ | √ | √ | |
| Gershwin et al. [9] | √ | | √ | | √ | | √ | |
| Gershwin [16] | √ | | √ | | √ | | √ | |
| Gershwin [12] | √ | | √ | | √ | | √ | |
| Gershwin [28] | | √ | √ | | √ | | √ | |
| Gershwin [14] | | √ | | √ | √ | | √ | |
| Levantesi [6] | √ | | | √ | | √ | | √ |
| Levantesi [27] | | √ | √ | | √ | | | √ |
| Le Bihan et al. [23] | √ | | | √ | √ | | √ | |
| Le Bihan et al. [24] | √ | | | √ | √ | | √ | |
| Mascolo et al. [28] | | √ | | √ | √ | | √ | |
| Maggio et al. [13] | √ | | √ | | √ | | | √ |
| Tolio et al. [10] | √ | | √ | | √ | | | √ |
| Tolio et al. [26] | √ | | √ | | √ | | | √ |
| Xia et al. [25] | √ | | | √ | | √ | √ | |

process, and further the model with exponential distributions are analytically tractable by solving transition equations. Failure types of machines are assumed to be operation-dependent (ODF) in our study. An ODF failure can occur only when the machine works and is related to the processing speed of the machine, while another failure type named time-dependent failure (TDF) is independent of the status of the machine and can occur even when the machine does not work.

The F/A system is characterized by the specific parameters as follows. The ratio of job I to job II required by the assembly machine is denoted by $\eta$. Buffer $B_i$ has a finite capacity $C_i$. The amount of materials in $B_i$ can be any real number between 0 and $C_i$. For machine $M_i$, the nominal processing rate is denoted by $\mu_i$, which represents the maximum processing rate. $p_{i,z}$ and $r_{i,z}$ represent the failure rate and repair rates of failure mode $z$ of machine $M_i$, respectively. The processing rate of each machine is dependent on the adjacent buffers level. When the upstream buffer is empty or the downstream buffer is full, the machine may be slowed down. Therefore, it holds that $\mu_i(t) < \mu_i$. According to the definition of ODF, the failure rates are dependent on the processing rates. When the machine operates at $\mu_i(t)$, the probability of $M_i$ to break down is $(\mu_i(t)/\mu_i)p_{i,z}\delta t$ during time interval $(t, t+\delta t)$. The repair rates are not affected by the processing rates.

# 4 Decomposition principle of F/A systems

## 4.1 Traditional decomposition of F/A systems

Figure 2 shows the process that traditional decomposition approach applies to an F/A system $l$. Firstly, the assembly

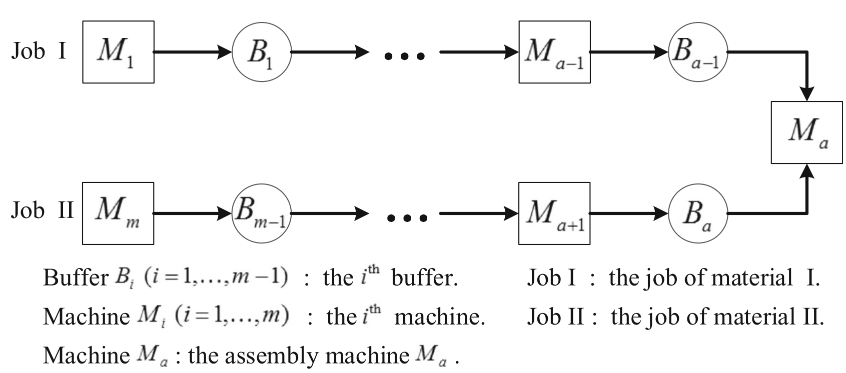



**Fig. 1** An F/A system with multiple failure modes and finite buffers

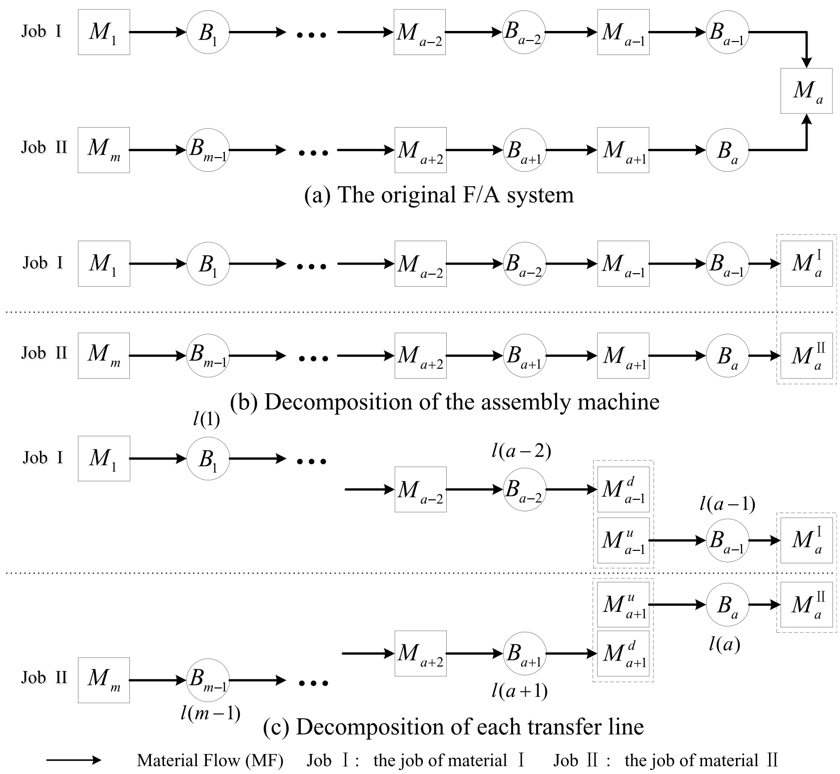



**Fig. 2** Traditional decomposition of an F/A system

machine $M_a$ is decomposed into two machines $M_a^I$ and $M_a^{II}$. Then, the F/A system $l$ is correspondingly divided into two transfer lines. Furthermore, according to the material flow direction, each transfer line is decomposed into a set of two-machine building blocks, i.e., $l(1)$, $l(2)$, $\cdots$, $l(i)$, $\cdots$, $l(m-1)$.

Based on the method proposed by Levantesi et al. [3], these building blocks can be solved if the downstream machine of $l(i-1)$ and the upstream machine of $l(i)$ are decomposed from the same machine, $2 \leq i \leq m-1$. However, this does not occur for the assembly machine. It is because that the downstream machine of $l(a-1)$ is originated from the assembly machine $M_a$, while the upstream machine of $l(a)$ is originated from the fabrication machine $M_{a-1}$. They are not the same machine in the original F/A system. For this special scenario, existing method cannot be applied directly to the F/A system if only considering material flow of the F/A system.

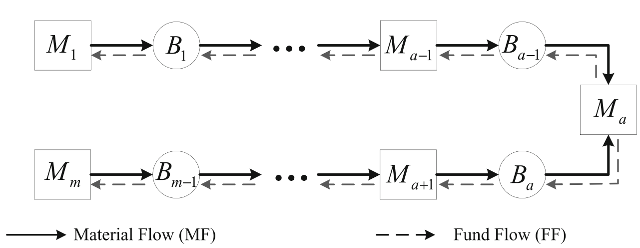



**Fig. 3** Flow directions of material flow (*solid*) and fund flow (*dashed*) in an F/A system

### 4.2 The proposed decomposition of F/A systems

We introduce the fund flow besides the material flow to model the F/A system. We observe that, when materials flow from upstream machine to buffer, or from buffer to downstream machine, the funds paying for these materials flow in the opposite direction. The material flow and fund flow are both one-way flow, but their directions are opposite, as shown in Fig. 3.

**Definition 1** *Fund flow.* A fund flow (FF) for a given material flow (MF) is such that

1. $\overrightarrow{\mu(FF)} = -\overrightarrow{\mu(MF)}$ (opposite processing rate)
2. $x(FF) = C - x(MF)$ (complementarity in buffer level)
3. $C(FF) = C(MF) = C$ (conformity in buffer capacity)

where $\overrightarrow{\mu}$ is a vector representing the processing rate, $x$ is a real variable representing the buffer level, and $C$ is a real constant representing the buffer capacity.

In Definition 1, condition (1) ensures that the processing rates of the fund flow and the material flow are the same, but their directions are opposite. Conditions (2) and (3) together ensure that when a buffer is full with fund, its material level is zero, and when the buffer is full with materials, its fund level is zero. The underlying assumption is that when a machine is starved of fund, it is blocked with materials.

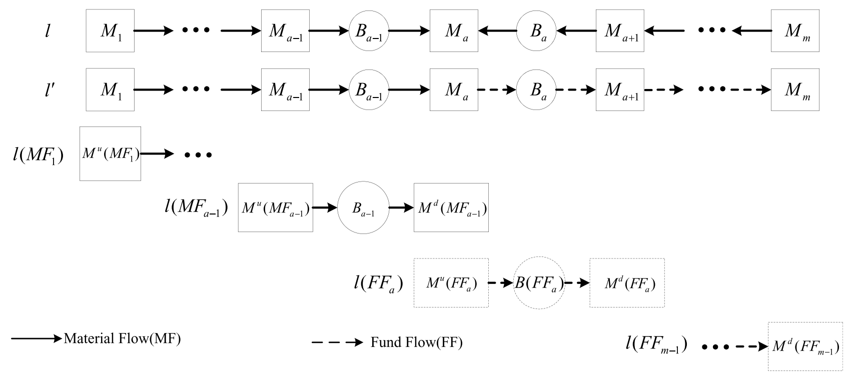



**Fig. 4** The proposed decomposition of an F/A system

We introduce the fund flow for modeling the F/A system. Firstly, we convert the F/A system $l$ to a virtual transfer line $l'$. Specifically, the MF of job I inputted into assembly machine and the FM of job II outputted from assembly machine are chosen and combined together to form a virtual transfer line $l'$, as shown in Fig. 4. Secondly, the virtual transfer line $l'$ is decomposed into $m-1$ two-machine building blocks, including the material-flow building blocks $l(MF_i)$, $i \in \{1, \ldots, a-1\}$ and the fund-flow building blocks $l(FF_i)$, $i \in \{a, \ldots, m-1\}$. Based on the performance measures of two-machine building blocks, we can evaluate that of F/A systems.

The lines that we construct for modeling the material flow are symmetrical to those we construct for the fund flows. Two lines are called symmetrical when one can be converted to the other by changing the upstream machine into the downstream machine and vice versa.

### 4.3 Parameters of virtual transfer lines

For convenience, we denote these building blocks of virtual transfer lines as

$$l(\mathrm{X}) = \{l(X_1), l(X_2), \cdots, l(X_{m-1})\}$$

where

$$X_i = \begin{cases} MF_i, i \in \{1, \ldots, a-1\} \\ FF_i, i \in \{a, \ldots, m-1\} \end{cases}$$

Based on the definition of $X_i$, all the downstream machines of $l(X_{i-1})$ and the upstream machines of $l(X_i)$ can be decomposed from the machine $M_i$. Specially, for the assembly machine $M_a$, $l(MF_{a-1})$ represents the material-flow building block with the assembly machine $M_a$ as downstream machine, and $l(FF_a)$ represents the fund-flow building block with the assembly machine $M_a$ as upstream machine. Thus, these building blocks of virtual transfer lines can be solved based on the decomposition method proposed by Levantesi et al. [6].

The machines in the original F/A systems are called physical machines or real machines, and the machines in two-machine lines or building blocks are called pseudo-machines. Parameters of pseudo-machines are denoted as follows.

#### *4.3.1 Pseudo-machine parameters*

For each building block, there are two pseudo-machines with one buffer in between. The downstream pseudo-machine $M^d(X_{i-1})$ of building block $l(X_{i-1})$ and the upstream pseudo-machine $M^u(X_i)$ of building block $l(X_i)$ refer to the same physical machine $M_i$ ($i = 1, 2, \ldots, m$) in the original F/A system.

For the upstream machine $M^u(X_i)$, $\mu^u(X_i)$ represents the nominal processing rate that is not affected by the downstream machine. $p^u_{i,z}(X_i)$ and $r^u_{i,z}(X_i)$ respectively indicate the real failure and repair rates caused by the physical machine $M_i$. Correspondingly, $p^u_{k,z}(X_i)$ and $r^u_{k,z}(X_i)$, for $k = 1, 2, \ldots, i-1$ and $z = 1, 2, \ldots, Z_k$, respectively indicate the virtual failure and repair rates from any of the machines that are upstream of $M_i$. Thus, $p^u_{k,z}(X_i)$ and $r^u_{k,z}(X_i)$ can model the starvation of $M_i$ in the original F/A system.

Similarly, for the downstream machine $M^d(X_{i-1})$, $\mu^d(X_{i-1})$ denotes the nominal processing rate, and $p^d_{i,z}(X_{i-1})$ and $r^d_{i,z}(X_{i-1})$ indicate the real failure and repair rates caused by machine $M_i$. For $k = i+1, \ldots, m$ and $k = 1, \ldots, Z_k$, the virtual failure and repair rates $p^d_{k,z}(X_{i-1})$ and $r^d_{k,z}(X_{i-1})$ refer to the failure caused by the machines downstream of $M_i$, which models the blockage of $M_i$ in the original system.

#### *4.3.2 Pseudo-machine states*

The upstream machine $M^u(X_i)$ in two-machine building blocks can be in one of three possible states: (1) working with the efficiency $E^u(X_i)$, (2) down in real failure modes $D^u_{i,z}(X_i)$ or in virtual failure modes $V^u_{k,z}(X_i)$, for $z = 1, \ldots, i-1$, and (3) blocked with probability $Block_{k,z}(X_i)$, for $k = i+1, \ldots, m$. Similarly, the downstream machine $M^d(X_{i-1})$ in two-machine building blocks also has three states: (1) working with the efficiency $E^d(X_{i-1})$, (2) down in real failure mode $D^d_{i,z}(X_{i-1})$ or virtual failure mode $V^d_{k,z}(X_{i-1})$, for $k = i+1, \ldots, m$, and (3) starved with possibility $Starve_{k,z}(X_{i-1})$, for $k = 1, \ldots, i-1$.

## 5 Decomposition modeling of F/A systems

### 5.1 Performance measures of two-machine building blocks

The exact performance analysis of two-machine building blocks is explained in detail by Tolio et al. [10]; here, it is briefly restated below for readers' convenience.

Let $(x, a_u, a_d) \in Q$, $x \in [0, C]$ denote the state of a two-machine building block, where $x$ represents the amount of materials in the buffer; $a_u \in \{1, u_1, \ldots, u_Z\}$ and $a_d \in \{1, d_1, \ldots, d_{Z'}\}$ represent the states of the upstream machine $M^u$ and the downstream machine $M^d$, respectively. Function $f() : Q \to [0, 1]$ describes the probability of a two-machine building block being in one of the internal states $(0 < x < C)$. Function $p() : Q \to [0, 1]$ describes the probability of a two-machine building block being in one of the boundary states ($x = 0$, $x = C$). We require that

$$1 = \int_0^C \left[ f(x,1,1) + \sum_{e=1}^{Z} f(x,u_e,1) + \sum_{f=1}^{Z'} f(x,1,d_f) + \sum_{e=1}^{Z} \sum_{f=1}^{Z'} f(x,u_e,d_f) \right] dx$$
$$+ \sum_{x \in \{0,C\}} \sum_{e=1}^{Z} p(x,1,1) + p(x,u_e,1) + \sum_{f=1}^{Z'} p(x,1,d_f) + \sum_{e=1}^{Z} \sum_{f=1}^{Z'} p(x,u_e,d_f)$$

Moreover, we have that

$$0 = \int_0^C \left[ p(x,1,1) + \sum_{e=1}^{Z} p(x,u_e,1) + \sum_{f=1}^{Z'} p(x,1,d_f) + \sum_{e=1}^{Z} \sum_{f=1}^{Z'} p(x,u_e,d_f) \right] dx$$
$$+ \sum_{x \in \{0,C\}} \sum_{e=1}^{Z} f(x,1,1) + f(x,u_e,1) + \sum_{f=1}^{Z'} f(x,1,d_f) + \sum_{e=1}^{Z} \sum_{f=1}^{Z'} f(x,u_e,d_f)$$

The processing rates of the upstream and downstream machines in a two-machine building block are given by $\mu_u$ and $\mu_d$, respectively. Different from discrete models, in continuous lines, if $\mu_u > \mu_d$, the buffer can become full. In this case, both the upstream and downstream machines can still work at the rate of $\mu_d$, but the upstream machine $M^u$ is partially blocked. Thus, for the upstream machine $M^u$, its efficiency $E_u$ and blockage *Block* are respectively given by

$$E_u = prob[a_u = 1, x < C] + \frac{\mu_d}{\mu_u} prob[a_u = 1, a_d = 1, x = C]$$
$$= \sum_{f=1}^{Z'} \left[ p(0,1,d_f), p(0,1,1) + \int_0^C \left[ f(x,1,d_f) + f(x,1,1) \right] dx \right] + \frac{\mu_d}{\mu_u} p(C,1,1) \tag{1}$$

$$Block = \sum_{f=1}^{Z'} p(C,1,d_f) + (1 - \mu_d/\mu_u) p(X,1,1) \tag{2}$$

Similarly, for the downstream machine $M^d$, its efficiency $E_d$ and starvation *Starve* are respectively given by

$$E_d = prob[a_d = 1, x > 0] + \frac{\mu_u}{\mu_d} prob[a_u = 1, a_d = 1, x = 0]$$
$$= \sum_{e=1}^{Z} \left[ p(C,u_e,1), p(C,1,1) + \int_0^C [f(x,u_e,1) + f(x,1,1)] dx \right] + \frac{\mu_u}{\mu_d} p(0,1,1) \tag{3}$$

$$Starve = \sum_{e=1}^{Z} p(0,u_e,1) + (1 - \mu_u/\mu_d) p(0,1,1) \tag{4}$$

Since no product waste in our model, the amounts of input and output materials to both machines in the two-machine building block are identical. The throughputs of the upstream and downstream machines are thus given by

$$P_u = P_d, \quad P_d = \mu_d E_d, \quad P_u = \mu_u E_u \tag{5}$$

The amount of materials in the buffer is given by

$$\overline{x} = \int_0^C x\left[\sum_{e=1}^{Z} f(x, u_e, 1) + \sum_{f=1}^{Z'} f(x, 1, d_f) + \sum_{e=1}^{Z}\sum_{f=1}^{Z'} f(x, u_e, d_f)\right]dx + \int_0^C x[f(x, 1, 1)]dx + C\left[p(C, 1, 1) + \sum_{e=1}^{Z} p(C, u_e, 1)\right] + C\left[\sum_{f=1}^{Z'} p(C, 1, d_f) + \sum_{e=1}^{Z}\sum_{f=1}^{Z'} p(C, u_e, d_f)\right] \tag{6}$$

Let $D_{u,e}$ denote the real failure probability of the upstream machine $M^u$ in failure mode $e$, and $D_{d,f}$ denote the real failure probability of the downstream machine $M^d$ in failure mode $f$. $D_{u,e}$ and $D_{d,f}$ are given by

$$D_{u,e} = \sum_{f=1}^{Z'}\left[p(0, u_e, d_f) + \int_0^C f(x, u_e, d_f)dx\right] + p(0, u_e, 1) + \int_0^C f(x, u_e, 1)dx, \quad e = 1, \ldots, Z \tag{7}$$

$$D_{d,f} = \sum_{e=1}^{Z}\left[p(X, u_e, d_f) + \int_0^C f(x, u_e, d_f)dx\right] + p(C, 1, d_f) + \int_0^C f(x, 1, d_f)dx, \quad f = 1, \ldots, Z' \tag{8}$$

## 5.2 Decomposition equations of virtual transfer lines

We now derive the decomposition equations for the processing rates, the real and virtual failure rates of each pseudo-machine. There are two key problems that need to be tackled. Firstly, when the machines operate at different rates, the throughput of fast machines can be affected by that of slow machines. This slowing down phenomenon should be reflected in the processing rate equations. Secondly, according to the definition of ODF, when the processing rate of a machine changes, both the real and virtual failure probabilities of each machine will change. In order to reflect this, we must define processing rate equations, interruption of flow equations, and failure rate equations.

### 5.2.1 Processing rate equations

Since there exist starvation and blockage, the processing rates of pseudo-machines are not equal to their nominal rates in general. Instead, we derive processing rate equations describing the relationship between pseudo-machines $M^d(X_{i-1})$ and $M^u(X_i)$, based on the conservation of flows.

#### 5.2.1.1 Processing rate equations of fabrication machines

For the fabrication machine $M_i$, $i = 2, \ldots, a-1, a+1, \ldots, m-1$, both building blocks $l(X_{i-1})$ and $l(X_i)$ come from the same transfer line. Therefore, the outflow provided by previous building block $l(X_{i-1})$ must be equal to the inflow transferred to the next building block $l(X_i)$.

$$P(X_i) = \mu^u(X_i)E^u(X_i) = P(X_{i-1}) \tag{9}$$

$$P(X_i) = \mu^u(X_i)E^u(X_i) = P(X_{i-1}) \tag{10}$$

$$P(X_i) = \mu^u(X_i)E^u(X_i) = P(X_{i-1}) \tag{11}$$

$$P(X_{i-1}) = \mu^d(X_{i-1})E^d(X_{i-1}) = P(X_i) \tag{12}$$

#### 5.2.1.2 Processing rate equations of assembly machines

The upstream machine $M^u(FF_a)$ of fund-flow building block $l(FF_a)$ and downstream machine $M^d(MF_{a-1})$ of material-flow building block $l(MF_{a-1})$ refer to the same assembly machine $M_a$. Since the assembly proportion of jobs I and II is $\eta$, we obtain:

$$P(FF_a) = \mu^u(FF_a)E^u(FF_a) = P(MF_{a-1})/\eta \tag{13}$$

$$P(MF_{a-1}) = \mu^d(MF_{a-1})E^d(MF_{a-1}) = \eta P(FF_a) \tag{14}$$

### 5.2.2 Interruption of flow equations

The interruption of flow equations describes the failure propagation between adjacent building blocks and are used to derive the failure rates of the pseudo-machines $M^d(X_{i-1})$ and $M^u(X_i)$, $i = 1, 2, \ldots, m$. Based on the fact that the failure frequency equals to the repair frequency in both real and virtual failure modes [4], we derive the corresponding interruption of flow equations.

For the fabrication machine $M_i$, $i = 1, \ldots, a-1, a+1, \ldots, m-1$, a set of equations for the real failure modes and virtual failure modes of the pseudo-machine $M^d(X_{i-1})$ and $M^u(X_i)$ are derived as follows

$$p^u_{i,z}(X_i)E^u(X_i) = r_{i,z}D^u_z(X_i), \quad z = 1, \ldots, Z_i \tag{15}$$

$$p^u_{k,z}(X_i)E^u(X_i) = r_{k,z}V^u_{k,z}(X_i), \quad k = 1, \ldots, i-1, \quad z = 1, \ldots, Z_k \tag{16}$$

$$p^d_{i,z}(X_{i-1})E^d(X_{i-1}) = r_{i,z}D^d_z(X_{i-1}), \quad z = 1, \ldots, Z_i \tag{17}$$

$$p^d_{k,z}(X_{i-1})E^d(X_{i-1}) = r_{k,z}V^d_{k,z}(X_{i-1}), \quad k = i+1, \ldots, m, \quad z = 1, \ldots, Z_k \tag{18}$$

For the assembly machine $M_a$, it is the link between building blocks $l(MF_{a-1})$ and $l(FF_a)$ (see Section 3 for the details).

Considering that the amounts of materials coming from the two fabrication lines must satisfy a fixed proportion $\eta$, we establish the following equations as follows.

$$p^u_{a,z}(FF_a)E^u(FF_a) = r_{a,z}D^u_z(FF_a), \quad z = 1,\dots,Z_a \tag{19}$$

$$p^u_{k,z}(FF_a)E^u(FF_a) = r_{k,z}V^u_{k,z}(FF_a), \quad k = 1,\dots,a{-}1, \quad z = 1,\dots,Z_k \tag{20}$$

$$p^d_{a,z}(MF_{a-1})E^d(MF_{a-1})\eta = r_{a,z}D^d_z(MF_{a-1}), \quad z = 1,\dots,Z_a \tag{21}$$

$$p^d_{k,z}(MF_{a-1})E^d(MF_{a-1})\eta = r_{k,z}V^d_{k,z}(MF_{a-1}), \quad k = a+1,\dots,m, \quad z = 1,\dots,Z_k \tag{22}$$

*5.2.3 Failure rate equations*

Regarding the real failure modes, $M^u(X_i)$ and $M^d(X_{i-1})$ must be the same. It is because that the pseudo-machines $M^u(X_i)$ and $M^d(X_{i-1})$ refer to the same physical machine $M_i$, for $i = 1, \dots, m$. Hence, the failure rate equations hold:

$$D^u_z(X_i) = D^d_z(X_{i-1}), \quad z = 1,\dots,Z_i \tag{23}$$

Regarding the virtual failure, we have two following equations. Let $V^u_{k,z}(X_i)$ represent the virtual failures of machine $M^u(X_i)$ caused by the upstream machine $M_k$, and $Starve_{k,z}(X_{i-1})$ represent the starvation possibility caused by the same machine. The following equation holds

$$V^u_{k,z}(X_i) = Starve_{k,z}(X_{i-1}), \quad k = 1,\dots,i{-}1, \quad z = 1,\dots,Z_k \tag{24}$$

Let $V^d_{k,z}(X_{i-1})$ represent the virtual failures of machine $M^d(X_{i-1})$ caused by the downstream machine $M_k$, and $Block_{k,z}(X_i)$ represent the blockage possibility caused by the same machine. The equation is as follows:

$$V^d_{k,z}(X_{i-1}) = Block_{k,z}(X_i), \quad k = i+1,\dots,m, \quad z = 1,\dots,Z_k \tag{25}$$

### 5.3 Formulas of virtual transfer lines

*5.3.1 Processing rate*

From Eqs. (11) and (12), the processing rate of $M^u(X_i)$ and $M^d(X_{i-1})$ can be derived

$$\mu^u(X_i) = \frac{P(X_{i-1})}{E^u(X_i)} \tag{26}$$

$$\mu^d(X_{i-1}) = \frac{P(X_i)}{E^d(X_{i-1})} \tag{27}$$

Also, from Eqs. (13) and (14), the processing rate of $M^u(FF_a)$ and $M^d(MF_{a-1})$ can be obtained:

$$\mu^u(FF_a) = \frac{P(MF_{a-1})}{\eta E^u(FF_a)} \tag{28}$$

$$\mu^d(MF_{a-1}) = \frac{\eta P(FF_a)}{E^d(MF_{a-1})} \tag{29}$$

*5.3.2 Real failure rate*

For the fabrication machine $M_i$, $i = 1, \dots, a-1, a+1, \dots, m-1$, substituting Eq. (23) into Eqs. (15) and (17), the following formulas are obtained:

$$p^u_{i,z}(X_i) = \frac{D^d_z(X_{i-1})}{E^u(X_i)}r_{i,z}, \quad z = 1,\dots,Z_i \tag{30}$$

$$p^d_{i,z}(X_{i-1}) = \frac{D^u_z(X_i)}{E^d(X_{i-1})}r_{i,z}, \quad z = 1,\dots,Z_i \tag{31}$$

Similarly, for the assembly machine $M_a$, these equations follow:

$$p^u_{a,z}(FF_a) = \frac{D^d_z(MF_{a-1})}{E^u(FF_a)}r_{a,z}, \quad z = 1,\dots,Z_a \tag{32}$$

$$p^d_{a,z}(MF_{a-1}) = \frac{D^u_z(FF_a)}{\eta E^d(MF_{a-1})}r_{a,z}, \quad z = 1,\dots,Z_a \tag{33}$$

*5.3.3 Virtual failure rate*

Substituting Eq. (24) into Eq. (16) obtains

$$p^u_{k,z}(X_i) = \frac{Starve_{k,z}(X_{i-1})}{E^u(X_i)}r_{k,z}, \quad k = 1,\dots,i{-}1, \quad z = 1,\dots,Z_k \tag{34}$$

Substituting Eq. (25) into Eq. (18) obtains

$$p^d_{k,z}(X_{i-1}) = \frac{Block_{k,z}(X_i)}{E^d(X_{i-1})}r_{k,z}, \quad k = i+1,\dots,m, \quad z = 1,\dots,Z_k \tag{35}$$

The following equations also hold for the assembly machine $M_a$:

$$p^u_{k,z}(FF_a) = \frac{Starve_{k,z}(MF_{a-1})}{E^u(FF_a)} r_{k,z}, \quad k = 1, \ldots, a-1, \quad z = 1, \ldots, Z_k \tag{36}$$

$$p^d_{k,z}(MF_{a-1}) = \frac{Block_{k,z}(FF_a)}{\eta E^d(MF_{a-1})} r_{k,z}, \quad k = a+1, \ldots, m, \quad z = 1, \ldots, Z_k \tag{37}$$

Both in the case of real and virtual failure modes, the repair rates of pseudo-machines are identical to those of the corresponding physical machines from the original line. This follows from the fact that other machines and buffers do not affect machines that are under repair.

## 6 Algorithm development and validation

The DDX algorithm proposed by Dallery [8, 15] is widely used for solving decomposition equations of a transfer line. The algorithm has proven to be efficient, fast, and reliable. For the virtual transfer line, we use an adaptation of the DDX algorithm for obtaining the performance measures. We call our adaptation the F/A decomposition algorithm or FADA for short.

### 6.1 Steps of the algorithm

**Step 1 initialization** The F/A system $l$ is decomposed into material-flow building blocks and fund-flow building blocks $l(X_i)$, $i = 1, \ldots, m-1$. The parameters of the pseudo-machines are initialized to the corresponding values of the physical machines in the original F/A system.

$$\begin{aligned}
\mu^u(X_i) &= \mu_i \\
p^u_{i,z}(X_i) &= p_{i,z}, \quad z = 1, \ldots, Z_i \\
p^u_{k,z}(X_i) &= p_{k,z}, \quad k = 1, \ldots, i-1, \quad z = 1, \ldots, Z_k \\
\mu^d(X_i) &= \mu_{i+1} \\
p^d_{i+1,z}(X_i) &= p_{i+1,z}, \quad z = 1, \ldots, Z_{i+1} \\
p^d_{k,z}(X_i) &= p_{k,z}, \quad k = i+2, \ldots, m, \quad z = 1, \ldots, Z_k
\end{aligned}$$

**Step 2** For $i = 2, \ldots, m-1$, updating parameters of the upstream pseudo-machines $M^u(X_i)$ based on the results of the downstream two-machine building block $l(X_{i-1})$ and then evaluating the two-machine building block $l(X_i)$.

**Step 2.1** For $i = 2, \ldots, a-1, a+1, \cdots, m-1$,

$$\begin{aligned}
\mu^u(X_i) &= \frac{P(X_{i-1})}{E^u(X_i)} \\
p^u_{i,z}(X_i) &= \frac{D^d_z(X_{i-1})}{E^u(X_i)} r_{i,z}, \quad z = 1, \ldots, Z_i \\
p^u_{k,z}(X_i) &= \frac{Starve_{k,z}(X_{i-1})}{E^u(X_i)} r_{k,z}, \quad k = 1, \ldots, i-1, \quad z = 1, \ldots, Z_k
\end{aligned}$$

**Step 2.2** For $i = a$,

$$\begin{aligned}
\mu^u(FF_a) &= \frac{P(MF_{a-1})}{\eta E^u(FF_a)} \\
p^u_{a,z}(FF_a) &= \frac{D^d_z(MF_{a-1})}{E^u(FF_a)} r_{a,z}, \quad z = 1, \ldots, Z_a \\
p^u_{k,z}(FF_a) &= \frac{Starve_{k,z}(MF_{a-1})}{E^u(FF_a)} r_{k,z}, \quad k = 1, \ldots, a-1, \quad z = 1, \ldots, Z_k
\end{aligned}$$

**Step 3** For $i = m-1, \ldots, 2$, updating parameters of the downstream pseudo-machines $M^d(X_{i-1})$ based on the results of the upstream two-machine building block $l(X_i)$ and then evaluating the two-machine building block $l(X_{i-1})$.

**Step 3.1** For $i = m-1, \ldots, a+1, a-1, \ldots, 2$,

$$\begin{aligned}
\mu^d(X_{i-1}) &= \frac{P(X_i)}{E^d(X_{i-1})} \\
p^d_{i,z}(X_{i-1}) &= \frac{D^u_z(X_i)}{E^d(X_{i-1})} r_{i,z}, \quad z = 1, \ldots, Z_i \\
p^d_{k,z}(X_{i-1}) &= \frac{Block_{k,z}(X_i)}{E^d(X_{i-1})} r_{k,z}, \quad k = i+1, \ldots, m, \quad z = 1, \ldots, Z_k
\end{aligned}$$

**Step 3.2** For $i = a$,

$$\begin{aligned}
\mu^d(MF_{a-1}) &= \frac{\eta P(FF_a)}{E^d(MF_{a-1})} \\
p^d_{a,z}(MF_{a-1}) &= \frac{D^u_z(FF_a)}{\eta E^d(MF_{a-1})} r_{a,z}, \quad z = 1, \ldots, Z_a \\
p^d_{k,z}(MF_{a-1}) &= \frac{Block_{k,z}(FF_a)}{\eta E^d(MF_{a-1})} r_{k,z}, \quad k = a+1, \ldots, m, \quad z = 1, \ldots, Z_k
\end{aligned}$$

**Step 4 convergence conditions** Steps 3 and 4 are repeated alternatively until $\varepsilon$ is smaller than $10^{-5}$, where $\varepsilon = \max_i (P(X_i) - P(X_{i-1}))$, for $i = 1, \ldots, m-1$.

The resulting average production $P$ and average buffer levels $\bar{x}$ are provided as output of the algorithm.

$$P = P_d\left(p^u_{m-1,z}(X_{m-1}), p^d_{m-1,z}(X_{m-1}), \mu^u(X_{m-1}), \mu^d(X_{m-1}), r_{m-1,z}, r_{m,z}, N_{m-1}\right)$$
$$x_i = x_i\left(p^u_{i,z}(X_i), p^d_{i,z}(X_i), \mu^u(X_i), \mu^d(X_i), r_{i,z}, r_{i+1,z}, N_i\right), i = 1, \ldots, m-1$$

**Table 2** Parameter settings for three-machine F/A systems

| Experiment 1 | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $p$ | 0.0125 | 0.0050 | 0.0200 |
| $r$ | 0.0600 | 0.0500 | 0.2000 |
| $\mu(group1)$ | 1.6667 | 1.1111 | 0.9000 |
| $\mu(group2)$ | 1.1111 | 0.9000 | 1.6667 |
| $\mu(group3)$ | 0.9000 | 1.6667 | 1.1111 |

**Table 3** Parameter settings for five-machine F/A systems

| Experiment 2 | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $p$ | 0.0125 | 0.0050 | 0.0200 | 0.01 | 0.01 |
| $r$ | 0.0600 | 0.0500 | 0.2000 | 0.1 | 0.08 |
| $\mu$ | 1.1111 | 1.6667 | 1.000 | 1.428 | 1.25 |

### 6.2 Validation of the algorithm

We evaluate the quality of our decomposition model and algorithm by comparing our results with simulation-based method and CTA [7]-based method. Our decomposition model and algorithm for F/A systems is implemented in Matlab. The simulation-based method builds a simulation model in Plant Simulation by Siemens PLM Software to obtain the statistical results. CTA-based method transforms the originally non-homogeneous line into an approximately equivalent homogeneous line. All models are run on an Intel(R) Pentium(R) CPU G640 @ 280GHz with 2GB RAM under Windows 7.

Two criteria are used for evaluating the performance of methods: the average throughput and the average buffer level of each buffer, defined as follows.

$$\epsilon_{PR} = \left(\frac{PR_{decomposition} - PR_{Simulation}}{PR_{Simulation}}\right) \times 100\% \tag{38}$$

$$\epsilon_{x_i} = \left(\frac{x_{i_{decomposition}} - x_{i_{Simulation}}}{C_i}\right) \times 100\% \tag{39}$$

We evaluate the methods on three-machine F/A systems and five-machine F/A systems, with different assembly proportions for materials from the two fabrication lines. For the three-machine F/A systems, machines $M_1$ and $M_3$ processes jobs I and II, respectively, while machine $M_2$ assembles them together. Regarding the buffer capacity, we set that $C_1 = 15$ and $C_2 = 20$. We choose three different assembly proportions of jobs I and II, $\eta = 1$, $\eta = 1/2$, and $\eta = 1/3$. For each assembly proportion, there are three cases considering different processing rates, $\mu$. Hence, we have nine cases in total for three-machine F/A systems. The specific parameter settings are shown in Table 2.

For the five-machine F/A systems, machines $M_1$ and $M_2$ process job I, and machines $M_5$ and $M_4$ process job II, machine $M_3$ is the assembly machine. Buffer capacities are $C_1 =$ 15, $C_2 = 20$, $C_3 = 10$, and $C_4 = 15$. Assembly proportions are $\eta = 1$ and $\eta = 2$. The parameter settings for the five-machine F/A systems are shown in Table 3.

For each case, we run our algorithm until convergence as described in Section 5.1. This takes at most several minutes to output the performance measures. Meanwhile, we set the simulation run for 100 h. The resulting average system throughputs and average buffer levels are compared in Tables 4 and 5.

From Tables 4 and 5, we draw the following conclusions:

1. The average throughput calculated by our decomposition method is rather accurate than that by the CTA-based method for the non-homogenous F/A systems with multiple failure modes. In all experiments, the relative deviations of average throughput are less than 5%, of which only two are greater than 1%, and it appears that the CTA-based method does not, in general, provide accurate results.
2. The average buffer levels obtained from our decomposition method are more accurate than those obtained from CTA-based method. For three-machine F/A systems, the maximum relative deviations of these levels are no more than 7.56%. For five-machine F/A systems, the maximum relative deviations are 33.57%. In these cases, however, the deviations obtained using the CTA-based method are even worse.

Overall, the comparison results indicate that our decomposition method is accurate.

## 7 Performance analysis of F/A systems

We choose case 4 as the testing benchmark to analyze the impact of several key parameters on the system performance, the parameter setting refers to the first three rows of Table 2 in Section 6.2 besides $\eta = 1/2$. The results are shown in Figs. 5 and 6. In this testing case, the fabrication machine $M_3$ is identified as the bottleneck machine because its processing rate is smallest. Buffer $B_2$ is behind the bottleneck machine and is called the bottleneck buffer.

We perform four experiments and report the average throughput and buffer level to measure the impact of the failure rates and repair rates of different machines on the F/A system. Our analysis of the results demonstrates how production managers can use our method to evaluate the system performance, analyze the possible modifications, and further find the best performance improvement. Specifically,

**Table 4** Comparative results of experiment 1

| $\eta$ | Case no. | Method | $x_1$ | $x_2$ | $PR$ |
|---|---|---|---|---|---|
| $\eta = 1$ | Case 1 | Plant simulation | 12.6212 | 3.4946 | 0.7900 |
| | | CTA | 14.5087 | 1.9100 | 0.4408 |
| | | Error (%) | 12.58 | −7.92 | −44.20 |
| | | FADA | 13.5924 | 3.3480 | 0.7867 |
| | | Error (%) | 6.47 | −0.73 | −0.42 |
| | Case 2 | Plant simulation | 10.6764 | 18.6545 | 0.7664 |
| | | CTA | 11.7872 | 19.8902 | 0.4294 |
| | | Error (%) | 7.41 | 6.18 | −43.97 |
| | | FADA | 11.4518 | 19.5667 | 0.7617 |
| | | Error (%) | 5.17 | 4.56 | −0.61 |
| | Case 3 | Plant simulation | 0.5912 | 17.2206 | 0.7325 |
| | | CTA | 0.2606 | 18.7729 | 0.4000 |
| | | Error (%) | −2.20 | 7.76 | −45.39 |
| | | FADA | 0.8513 | 18.3165 | 0.7263 |
| | | Error (%) | 1.73 | 5.48 | −0.84 |
| $\eta = 1/2$ | Case 4 | Plant simulation | 13.8541 | 0.5714 | 0.4057 |
| | | CTA | 14.8762 | 0.4434 | 0.2209 |
| | | Error (%) | 6.81 | −0.64 | −45.56 |
| | | FADA | 14.5964 | 0.6510 | 0.4038 |
| | | Error (%) | 4.95 | 0.40 | −0.48 |
| | Case 5 | Plant simulation | 11.1642 | 5.7470 | 0.7027 |
| | | CTA | 11.8040 | 18.9455 | 0.4286 |
| | | Error (%) | 4.27 | 65.99 | −39.01 |
| | | FADA | 12.2981 | 5.7788 | 0.6986 |
| | | Error (%) | 7.56 | 0.16 | −0.58 |
| | Case 6 | Plant simulation | 12.2448 | 0.8280 | 0.4984 |
| | | CTA | 10.6579 | 0.3649 | 0.3350 |
| | | Error (%) | −10.58 | −2.32 | −0.70 |
| | | FADA | 13.3019 | 0.8453 | 0.4949 |
| | | Error (%) | 7.05 | 0.09 | −0.70 |
| $\eta = 1/3$ | Case 7 | Plant simulation | 14.3222 | 0.2627 | 0.2712 |
| | | CTA | 14.9472 | 0.2490 | 0.1473 |
| | | Error (%) | 4.17 | −0.07 | −45.68 |
| | | FADA | 14.8162 | 0.3516 | 0.2706 |
| | | Error (%) | 3.29 | 0.44 | −0.21 |
| | Case 8 | Plant simulation | 13.0207 | 1.4277 | 0.4890 |
| | | CTA | 12.1679 | 12.7941 | 0.4126 |
| | | Error (%) | −5.69 | 56.83 | −15.63 |
| | | FADA | 13.9222 | 1.5225 | 0.4858 |
| | | Error (%) | 6.01 | 0.47 | −0.66 |
| | Case 9 | Plant simulation | 13.7848 | 0.3136 | 0.3339 |
| | | CTA | 14.0903 | 0.1560 | 0.2238 |
| | | Error (%) | 2.04 | −0.79 | −32.98 |
| | | FADA | 14.3355 | 0.3799 | 0.3334 |
| | | Error (%) | 3.67 | 0.33 | −0.16 |

1. The impact of failure rates of the fabrication machines on system performance is demonstrated by decreasing the failure rates of the fabrication machines $M_1$ (from 0.0125 to 0.0025) and $M_3$ (from 0.0200 to 0.0100). Meanwhile, the buffer capacities of $B_1$ and $B_2$ are increased from 1 to 150. The data is listed in Table 6 and the results are shown in Figs. 5 and 6.
2. The data for the experiments on the impact of failure rate (from 0.0050 to 0.0010) of the assembly machine $M_2$ is listed in Table 7, and the results are shown in Figs. 7 and 8.
3. We do not show results of modifying the repair rates of the fabrication machines and the assembly machine since those have nearly the same effect (somewhat smaller) as modifying the failure rates.

### 7.1 Modifying failure rates of fabrication machines and buffer capacities

In experiment 3, we focus on the fabrication machines to analyze the impact of failure rates and buffer capacities on the system performance. We decrease the failure rate of machine $M_1$ gradually from 0.0125 to 0.0125–0.0025 × 4 = 0.0025. Meanwhile, the buffer capacity $C_1$ is increased from 1 to 150. Similarly, the failure rate of machine $M_3$ is decreased gradually from 0.0200 to 0.0200–0.0025 × 4 = 0.0100, and the buffer capacity $C_2$ is increased from 1 to 150. The parameters for the experiment are shown in Table 6.

Fig. 5 shows the impact of the increasing buffer capacities on the average throughput under different failure rates of the fabrication machines. Figure 6 shows the impact of the increasing buffer capacities on the average buffer level of the system under different failure rates of the fabrication machines.

From Figs. 5 and 6, we make the following conclusions:

1. With the same failure rate, a proper increase of the two buffers capacities can increase the throughput of the entire F/A system as shown in Fig. 5a, b. But, the performance cannot continue to be better with the increase of the buffer capacity. Furthermore, the influence of an increase of the bottleneck buffer capacity is greater than that of the non-bottleneck buffer capacity.
2. Decreasing the failure rate of the machine $M_1$ and $M_3$ can increase the throughput of the system, as shown in Fig. 5a, b, respectively. In addition, with a large capacity for the non-bottleneck buffer $B_1$, the effect of the failure rate of non-bottleneck machine $M_1$ on the system throughput can be neglected in Fig. 5a. By contrast, with a large capacity of the bottleneck buffer $B_2$, the negative influence of the failure rate of the bottleneck machine $M_3$ on the

**Table 5** Comparative results of experiment 2

| $\eta$ | Case no. | Method | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $PR$ |
|---|---|---|---|---|---|---|---|
| $\eta = 1$ | Case 10 | Plant simulation | 6.4959 | 13.6143 | 8.9908 | 12.5240 | 0.8390 |
| | | CTA | 8.7578 | 19.0425 | 9.2859 | 13.3255 | 0.5356 |
| | | Error (%) | 15.08 | 27.14 | 2.95 | 5.34 | −36.16 |
| | | FADA | 8.7048 | 17.2156 | 8.8147 | 13.0564 | 0.8279 |
| | | Error (%) | 14.73 | 18.01 | −1.76 | 3.55 | −1.33 |
| $\eta = 2$ | Case 11 | Plant simulation | 6.1736 | 13.2584 | 9.8683 | 14.7256 | 0.4258 |
| | | CTA | 0.6677 | 1.4034 | 9.7288 | 14.6284 | 0.3008 |
| | | Error (%) | −36.71 | −59.28 | −1.40 | −0.65 | −29.35 |
| | | FADA | 1.1376 | 13.9255 | 9.7146 | 14.6196 | 0.4443 |
| | | Error (%) | −33.57 | 3.34 | −1.54 | −0.71 | 4.35 |

system throughput remains intact, as shown in Fig. 5b.

3. With the increase of buffer capacity of $B_1$, the decrease of the failure rate of $M_1$ has little impact on the average buffer level of $B_1$ (Fig. 6a). But, the average buffer level of $B_2$ is decreased when $B_1$ has a small capacity (Fig. 6c). In addition, with the increase of buffer capacity of $B_2$, a decrease of the failure rate of $M_3$ can decrease the average buffer level of $B_1$ (see Fig. 6b) and increase the average buffer level of $B_2$ (Fig. 6d).

In summary, the results show that the throughput of the system can be increased by decreasing the failure rate of the bottleneck machine, along with a proper increase of the bottleneck buffer.

## 7.2 Modifying failure rates of the assembly machine and buffer capacities

In experiment 4, we focus on the assembly machine to analyze the impact of the failure rates and buffer capacities on the system performance. The failure rates of machine $M_2$ are decreased gradually from 0.0050 to 0.0050–0.0010 × 4 = 0.0010. The buffer capacities of $C_1$ are increased from 1 to 150 when the buffer capacity of $B_2$ stays the same. Similarly, the buffer capacities of $C_2$ are increased from 1 to 150 when the buffer capacity of $B_1$ stays the same. The related parameters for experiment 4 are given in Table 7.

Fig. 7 shows the impact of the increasing capacities of $B_1$ and $B_2$ on the average throughput of the system under different failure rates of the assembly machine. Fig. 8 shows the impact of the increasing bottleneck buffer capacity of $B_2$ on

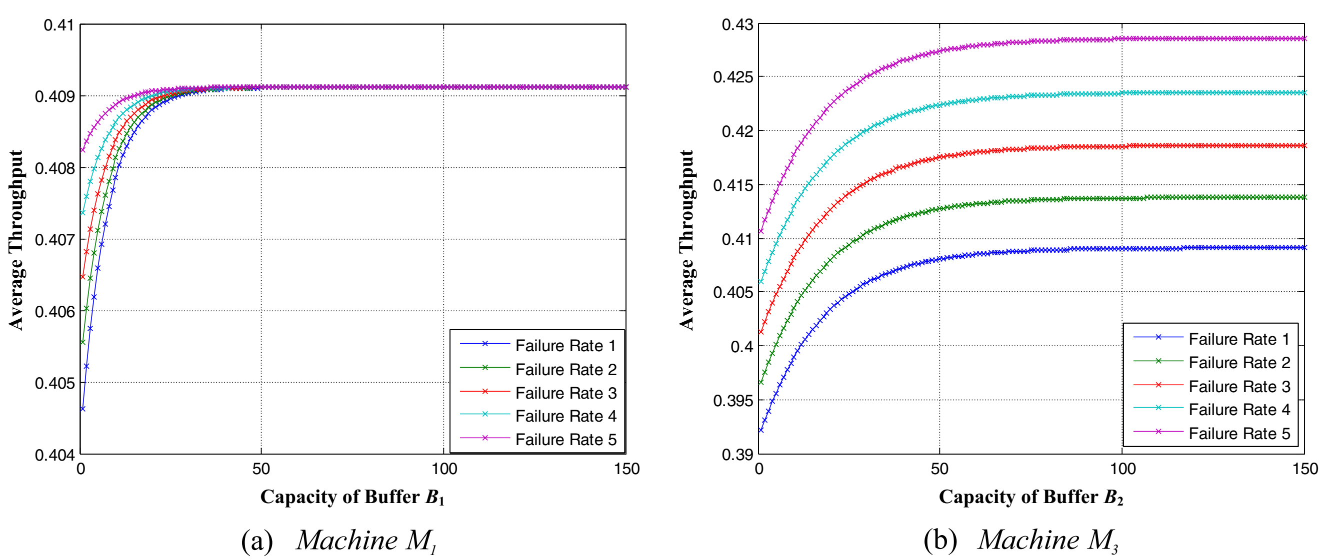
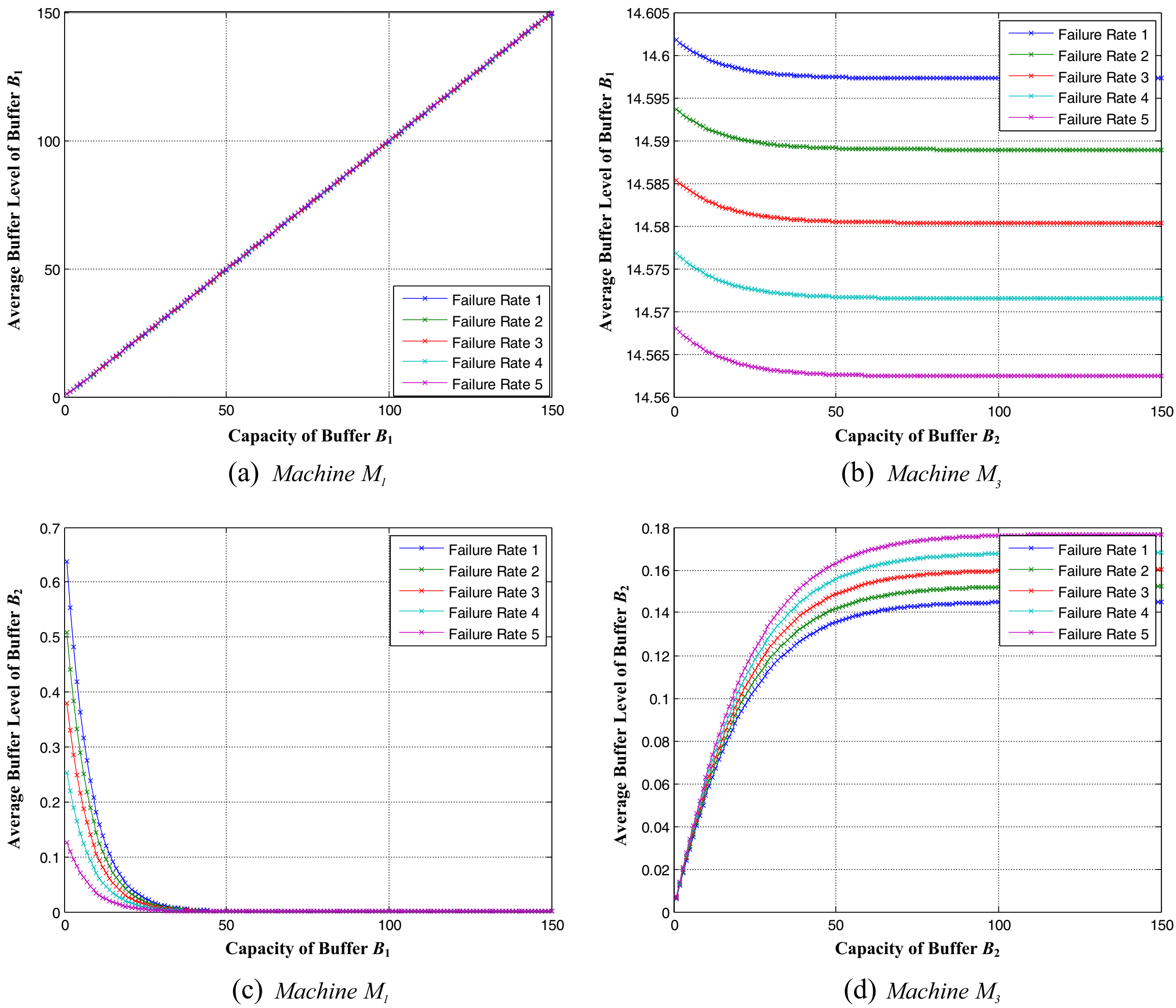



**Fig. 5** Impact of increasing buffer capacities on the average throughput under different failure rates of the fabrication machines




(a) *Machine* $M_1$ (b) *Machine* $M_3$

(c) *Machine* $M_1$ (d) *Machine* $M_3$

**Fig. 6** Impact of increasing corresponding buffer capacities on the average buffer level under different failure rates of the fabrication machines

average buffer level of the system under different failure rates of the assembly machine.

From Figs. 7 and 8, we make the following conclusions:

1. The system throughput is increased by decreasing failure rate of the assembly machine $M_2$ see Fig. 7a, b. However, this effect seems to level off when increasing the capacity of $B_2$, see Fig. 7b. This occurs because a disturbance of the assembly machine $M_2$ can only affect the bottleneck machine $M_3$ and thus the throughput of the entire system, when the capacity of the bottleneck buffer $B_2$ is small. Hence, the disturbance of the assembly machine $M_2$ can be eliminated by increasing the capacity of the bottleneck buffer $B_2$.

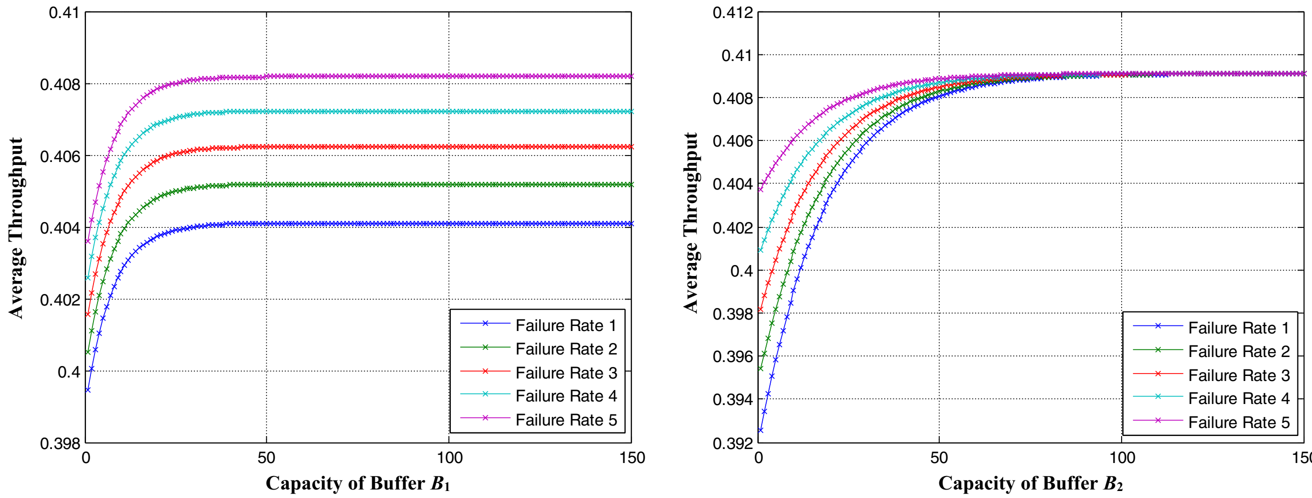
2. With the increase of the buffer capacity of $B_2$, the decrease of the failure rate of $M_2$ can decrease the average buffer

**Table 6** Parameter setting for the fabrication machine failure experiment

| Experiment 3 | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $p$ | 0.0125/(0.0125~0.0025) | 0.0050 | 0.0200/(0.0200~0.0100) |
| $r$ | 0.0600 | 0.0500 | 0.2000 |
| $\mu$ | 1.6667 | 1.1111 | 0.9000 |

**Table 7** Parameter setting for the assembly machine failure experiment

| Experiment 4 | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $p$ | 0.0125 | 0.0050/(0.0050~0.0010) | 0.0200 |
| $r$ | 0.0600 | 0.0500 | 0.0200 |
| $\mu$ | 1.6667 | 1.1111 | 0.9000 |




(a) *Buffer $B_1$* (b) *Buffer $B_2$*

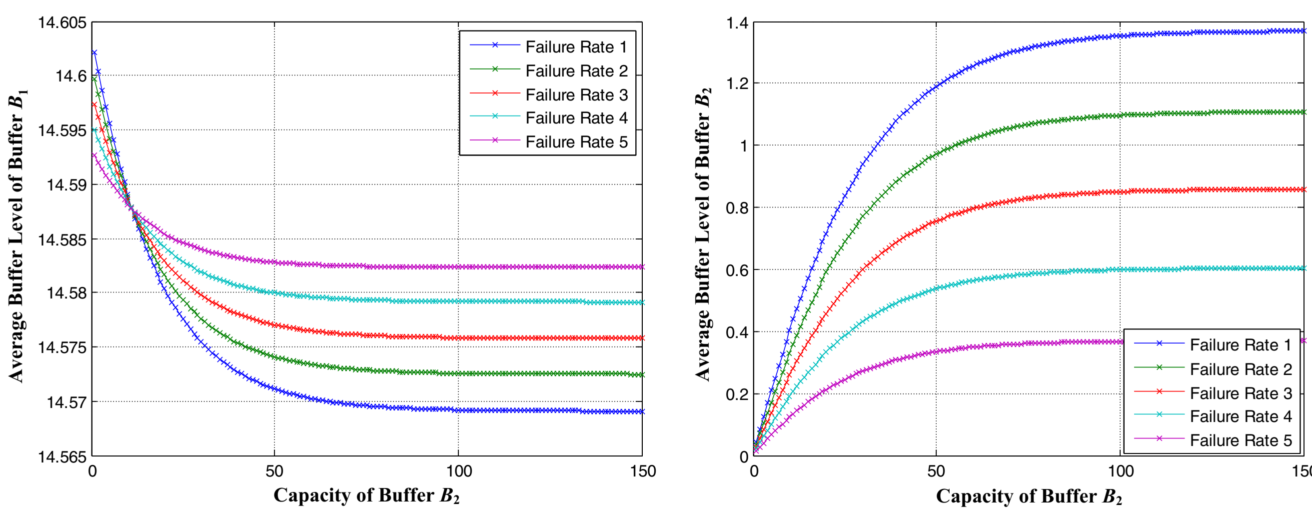
**Fig. 7** Impact of the increasing buffer capacities on the average throughput of system under different failure rates of the assembly machine

level of $B_1$ firstly and increase the average buffer level of $B_1$ then (Fig. 8a). Because buffer $B_1$ is affected by the bottleneck machine $M_3$ and the assembly machine $M_2$. Moreover, with the increase of the buffer capacity of $B_2$, the decrease of the failure rate of $M_2$ can decrease the average buffer level of $B_2$, as shown in Fig. 8b, since the buffer level $B_2$ is affected by the assembly machine $M_2$.

In summary, the system throughput can be increased by decreasing the failure rate of the assembly machine. Less obvious, however, is that this is only useful when the capacity of the bottleneck buffer is small.

## 8 Conclusions

Fast and accurate performance evaluation of the production systems can aid the decision making of helping production managers. When possible, the most effective way to evaluate this performance is to establish an analytical model. The machines in a production system, however, can process materials in a variety of speeds and with different failure and repair rates. Such uncertainties and randomness are, however, very difficult to model fully analytically. The typical solution to this problem is to find approximate models. In this paper, we propose such a continuous decomposition model in order to




(a) *Bottleneck buffer $B_2$* (b) *Bottleneck buffer $B_2$*

**Fig. 8** Impact of the increasing buffer capacities on the average buffer level of bottleneck buffer under different failure rates of the assembly machine evaluate the performance of non-homogeneous fabrication/assembly systems with multiple failure modes. In our method, the original F/A system is converted to a virtual transfer line based on a newly introduced fund flow besides the commonly used material flow. We derive the processing rate equations, interruption of flow equations, and failure rate equations that connect these two-machine building blocks and develop a new decomposition algorithm for solving these equations to evaluate system performance.

The validity of the proposed continuous model for F/A systems is analyzed by comparing its performance on 11 different experiment cases with the well-known CTA-based method, adapted for non-homogeneous problems. The results show that our method is more accurate. In detail, in all 11 experiment cases, the relative deviations of the average system throughput resulting from our method are less than 5%, of which only two are greater than 1%. In experimental settings with two fabrication machines and one assembly machine, the relative deviation of the average buffer level is no more than 7.56%. In the more difficult setting with four fabrication machines and one assembly machine, the relative deviation of the average buffer level obtained by our method is better than that by the CTA-based method.

Production managers can use our decomposition method to evaluate the system performance, analyze the possible modifications, and further find the best performance improvement of a production system. Intuitively, the overall system throughput would be increased by either the enlargement of buffer capacities or the reduction of repair and failure rates. However, this may contradict the results demonstrated by our experiments that the overall system throughput has not always been increased by these activities. Furthermore, we investigate the impact of several key parameters, including failure rates, repair rates, and buffer capacities, on the performance of the F/A system, and provide an analytical method to help managers discover such relationships.

In future work, we would like to extend our model to more general assembly/disassembly systems and other machine reliability models. Another interesting problem is to study lean buffer capacity provided the desired throughput is satisfied.

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