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Modified Semi-Analytical Method for 3D Slope Reliability

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Abstract: An improved method for the reliability analysis of 3D slopes has been proposed based on the semi-analytical method of Vanmarcke (1977). Comparing the predicted responses of an idealised 3D slope obtained by the more general, albeit computationally intensive, random finite element method (RFEM), and the original semi-analytical method showed that the latter gives unconservative estimates of the probability of failure. Three significant areas were identified as requiring improvement in the simpler method. These were corrected by: (i) a correction factor to reduce the overestimation of end-resistance in 3D failures, i.e. reducing the impact of conservative geometric assumptions; (ii) a correction factor to correct for overestimating the average shear strength on the failure plane, which is found to be lower than the average shear strength for the entire slope; and (iii) an alternative relationship for the expected failure length for intermediate values of the spatial correlation length of the shear strength. The proposed modified semi-analytical method gives substantially improved results that are comparable to RFEM, while retaining the simplicity of the original method.

Keywords: Analytical; finite element analysis; random fields; slope stability; spatial variability; three dimensional.

1 Introduction

The inherent nature of soil is to be spatially variable (Phoon and Kulhawy 1999) due to a combination of various geological, environmental and physico-chemical processes, among others. The presence of this heterogeneity has been shown to have a significant influence on computations of geotechnical performance (for example, Hicks and Onisiphorou (2005)). Several reliability-based analysis methods have been developed to account for the uncertainties associated with soil heterogeneity. Of particular interest is the random finite element method (RFEM) (Fenton and Griffiths 2008), which has proven to be an effective and versatile method. RFEM has been applied to 2D slope reliability analysis (Hicks and Samy 2002; Griffiths et al. 2009; among others) based on the simplifying assumption that the spatially varying parameters are correlated over an infinite distance in the third dimension. More recent research has indicated a need for 3D slope reliability analysis (Hicks and Spencer 2010; Huang et al. 2013; Hicks et al. 2014; Xiao et al. 2016), although only a limited amount of research has been done, due (at least in part) to the large computational requirements (especially for 3D RFEM).

Vanmarcke (1977; 1980) developed a method for 3D reliability assessments of slopes, which gives a quick and convenient solution by making certain (important) simplifying assumptions. Li et al. (2015) and Varkey et al. (2019) compared the performance of this method with that of RFEM for reliability predictions of an idealised 3D slope, for cohesive and c–ϕ soils, respectively, and have highlighted those instances in which the two methods give similar results, as well as those instances in which there are significant differences. Varkey et al. (2019) identified three main assumptions in Vanmarcke's method that resulted in the differences and proposed a modified Vanmarcke method to correct for them. The proposed method gives substantially improved results for a range of possible levels of anisotropy of the heterogeneity in the shear strength and for a range of cross-sectional geometries. This paper further investigates the effectiveness of the modified Vanmarcke method in predicting 3D slope responses.

2 Random Finite Element Method

In the context of finite element analysis, the mechanical response of a system is approximated by the spatial discretisation of the geometry. RFEM combines finite elements with random fields (i.e. mathematical representations of the spatial variability of parameters) within a Monte Carlo framework. In this paper, the discretisation of random fields for both the shear strength variables (c and ϕ) is carried out by Local Average Subdivision (LAS) (Fenton & Vanmarcke 1990). The random fields are generated using the Markov covariance function:

$$\beta_{\mathrm{M}} = \sigma^2 \exp\left(-\frac{2\tau_z}{\theta_z} - \sqrt{\left(\frac{2\tau_x}{\theta_x}\right)^2 + \left(\frac{2\tau_y}{\theta_y}\right)^2}\right) \tag{1}$$

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where σ^2 is the variance, θ_x, θ_y and θ_z are the scales of fluctuation (i.e. spatial correlation distances), and τ_x, τ_y and τ_z are the lag distances in the respective directions.

The separation of the vertical (z) correlation structure from the two horizontal (x and y) directions was done to model the long-term depositional characteristic in soil. See Hicks and Samy (2002), and Hicks and Spencer (2010), for the approach used in generating anisotropic random fields. Following the random field generation, the field values are mapped to the Gauss points of a finite element mesh, and the boundary value problem is analysed by the finite element method. In this paper, the strength reduction method is used to determine the factor of safety (FS) of the slope in each realisation, and multiple realisations have been performed to generate a distribution of FS.

3 Vanmarcke's Method and Its Shortcomings

Vanmarcke (1977; 1980) developed a method for 3D reliability assessments, by extending a 2D circular failure arc to a 3D cylindrical failure surface with resisting end-sections within a probabilistic framework. The load (due to self-weight) and cross-sectional characteristics were assumed to be constant along the slope axis, and only the uncertainty due to the natural variability of the soil strength (s) mobilised along the failure surface was considered. Based on the equilibrium of moments about a centre of rotation, the 3D FS of the slope (see Fig. 1) is given by

$$F_b = \frac{(s_b L_a b)r + R_e}{(Wb)a} \tag{2}$$

where s_b is the averaged shear strength along a failure surface of length b, L_a is the length of the cross-sectional failure arc, r is the lever arm of the resisting moment about the centre of rotation, W is the weight per unit length of the sliding mass and a is the lever arm of the centre of gravity of the sliding mass about the same centre of rotation. R_e $(= 2s_e Ar')$ is the resisting moment of the end-sections, where s_e is the shear strength over the two end-sections, A is the area of each end-section and r' is the effective rotation arm for the end sections.

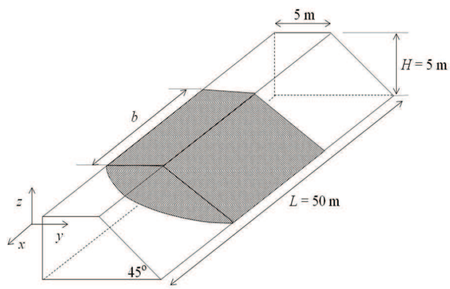


Figure 1. Failure mass within a 3D slope (based on Vanmarcke (1977)).

Assuming a deterministic overturning moment, and by neglecting any variance in the end-resistance, the mean and standard deviation (denoted by a bar and tilde, respectively, above the random variable) of the 3D FS are given by

$$\overline{F_b} = \frac{(\overline{s_b} L_a b)r + 2(\overline{s_e} A)r'}{(Wb)a} \tag{3}$$

$$\widetilde{F_b} = \frac{(\widetilde{s_b} L_a b)r}{(Wb)a} \tag{4}$$

Following Vanmarcke's assumption of $\overline{s_b} = \overline{s_e} = \bar{s}$ for a stationary random field of s, and assuming $r' = r$, Eqs. (3) and (4) simplify to

$$\overline{F_b} = \bar{F}\left(1 + \frac{d}{b}\right) \tag{5}$$

$$\widetilde{F_b} = \Gamma(L_a)\Gamma(b)V_s\bar{F} \tag{6}$$

where $\bar{F}$ is the mean plane strain FS, $d\ (= 2\,A/L_a)$ is the effective width of the end-sections, V_s is the coefficient of variation of the point shear strength, and $\Gamma(L_a)$ and $\Gamma(b)$ are the reduction factors relating to the standard deviation along the failure arc and failure length, respectively. $\Gamma(b)$ is given by

$$\begin{aligned} &\Gamma(b) = \sqrt{\theta_{\mathrm{h}}/b}; && b > \theta_{\mathrm{h}} \\ &\Gamma(b) = 1; && b \leq \theta_{\mathrm{h}} \end{aligned} \qquad (7)$$

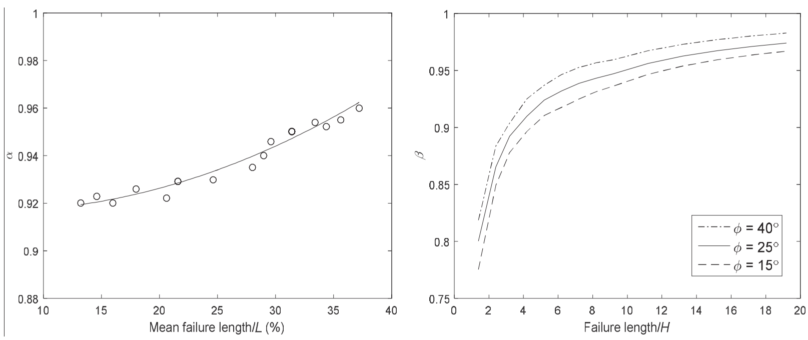
and $\Gamma(L_a)$ is found by replacing b with L_a and θ_h by the equivalent scale of fluctuation θ_e (see Li et al. (2015) for the procedure to obtain θ_e) in Eq. (7). Vanmarcke (1977) proposed the following equation for the expected failure length:

$$\begin{aligned} &b = b_c = \frac{\bar{F}}{\bar{F}-1}d; && b_c > \theta_{\mathrm{h}} \\ &b = \theta_{\mathrm{h}}; && b_c \leq \theta_{\mathrm{h}} \end{aligned} \qquad (8)$$

where b_c is the critical failure length which maximises the probability of failure centred at a specific location.

Li et al. (2015) and Varkey et al. (2019) carried out a detailed comparison of the performance of Vanmarcke's method relative to that obtained by RFEM, for slopes in cohesive and c–ϕ soils, respectively, and with the geometry shown in Fig. 1. It was observed that there is a large difference in FS predicted by the two methods at small θ_{h} (relative to the slope dimensions) due to the differences in predicted failure length coupled with an exaggerated influence of the cylinder ends in Vanmarcke's method. In contrast, at very large θ_{h} the two methods converged to the same FS as the 2D solution. In total, Varkey et al. (2019) identified three reasons behind the differences in FS, as follows:

1. Overestimating the contribution from the end-resistance due to geometric assumptions in Vanmarcke's method.
2. No account of failure being attracted to weaker zones in Vanmarcke's method, resulting in higher FS for intermediate values of θ_{h}.
3. Critical failure length predicted by Vanmarcke's method not coinciding with the length of potentially unstable zones.



(a) (b)

Figure 2. Calibration curves for the correction factors: (a) α and (b) β.

4 Modifications Proposed to Vanmarcke's Method

Varkey et al. (2019) proposed a modified Vanmarcke method which includes the following three changes:

1. Correction factor β to correct for the overestimated end-resistance.
2. Correction factor α to account for the attraction of failure to weaker zones.
3. An alternative relationship for the expected failure length ($b = 2H + \theta_{\mathrm{h}}/2$, where H is the slope height) for intermediate values of θ_{h}.

Calibration curves for the two correction factors are plotted in Fig. 2 (where L is the slope length); see Varkey et al. (2019) for details. These suggest that, for very long embankments, $\alpha \approx 0.92$ and $0.85 \leq \beta \leq 0.92$ may be reasonable first approximations. The modified equation for the mean 3D FS by the modified Vanmarcke method is then given by

$$\overline{F_b} = \bar{F}\left(\alpha + \frac{d}{b}\right)\beta \qquad (9)$$

5 Results and Discussion

A slope that is 50 m long in the third dimension, with the geometry shown in Fig. 1, has been analysed by Vanmarcke's method (VM), the modified Vanmarcke's method (MVM) and RFEM. The finite element model was meshed by 4000, 20-node hexahedral elements, which were 0.5 m deep and 1 m × 1 m in plan (except along the slope face), and used 2 × 2 × 2 Gaussian integration. The mesh was fixed at the base, with rollers on the back face preventing movement perpendicular to the face, and rollers on the two end-faces allowing movement only in the vertical direction (see Hicks and Spencer (2010) and Hicks and Li (2018) for an explanation and investigation of these boundary conditions).

The soil parameter values are listed in Table 1, and a normal distribution was considered appropriate for both c and ϕ (given the low coefficients of variation). The vertical scale of fluctuation was taken to be 1 m for all analyses, whereas a wide range of θ_h was considered. Based on the mean values of the shear strength parameters listed in Table 1, using finite elements $\bar{F}$ was found to be 1.4 and d was computed to be 2.58 m based on the failure geometry. These derived parameters were used to compute Vanmarcke's solution (Eqs. (5–8)) and modified Vanmarcke's solution (Fig. 2 and Eqs. 6, 7 and 9), while a total of 500 Monte Carlo realisations were carried out to make predictions using RFEM.

The failure lengths obtained by VM and MVM, and the mean discrete failure lengths obtained by RFEM, are plotted in Fig. 3. For each RFEM realisation, the discrete failure lengths were calculated from the number of continuously linked elements, in the row of elements directly above the slope toe, in which out-of-face displacements were greater than a threshold value (for details of the procedure see Hicks et al. (2014)). For this investigation, the threshold displacement was calibrated to be 37% of the maximum computed out-of-face displacement. Fig. 3 shows that the failure lengths predicted by MVM have been improved compared to those predicted by VM.

Table 1. Parameter values.

Parameter	Mean	Standard deviation	θ_v	θ_h
Cohesion, c	10 kPa	2 kPa	1 m	1 to 10^4 m
Friction angle, ϕ	25°	5°	1 m	1 to 10^4 m
Dilation angle	0°	-	-	-
Young's modulus	1×10^5 kPa	-	-	-
Poisson's ratio	0.3	-	-	-
Unit weight	20 kN/m^3	-	-	-

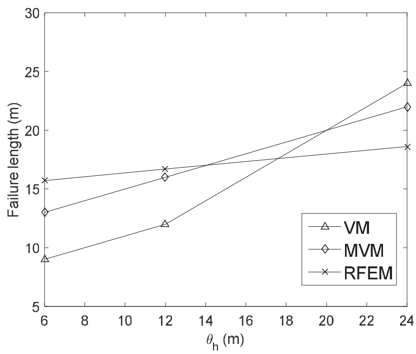


Figure 3. Failure lengths obtained by the 3 methods.

Table 2. Expected failure lengths, corresponding correction factors and mean 3D FS calculated by MVM.

θ_h (m)	b (m)	α	β	$\overline{F_b}$
6	13	0.935	0.870	1.380
12	16	0.950	0.880	1.369
24	22	0.970	0.910	1.385

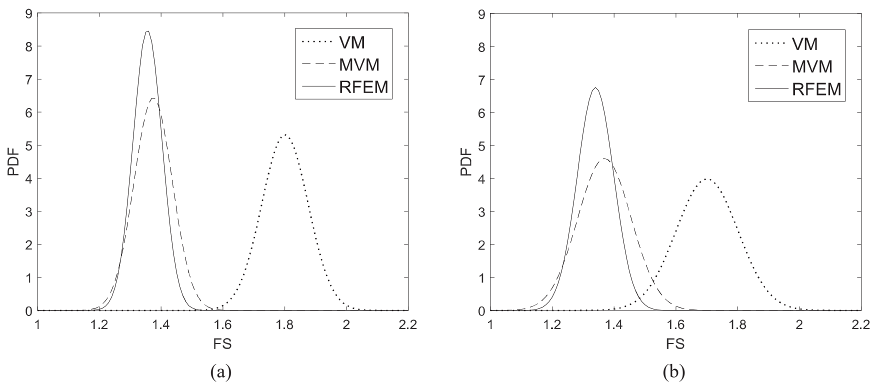


Figure 4. PDF of 3D FS obtained by the 3 methods for: (a) $\theta_h = 6$ m and (b) $\theta_h = 12$ m.

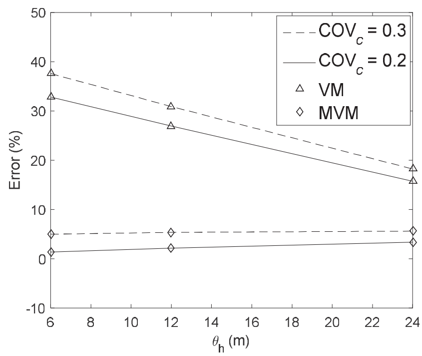


Figure 5. Error in mean 3D FS by VM and MVM with respect to mean 3D FS by RFEM

The expected failure lengths, corresponding correction factors and mean 3D FS calculated by MVM for different values of θ_h are summarised in Table 2. Fig. 4(a–b) shows the probability density functions (PDFs) of FS obtained by the various methods with $\theta_h = 6$ m and $\theta_h = 12$ m. Note that the PDFs are obtained by computing the mean and standard deviation of the 3D FS by the three methods and assuming a normal distribution for FS. The figure shows that the PDFs obtained by MVM are substantially improved compared to those obtained by VM. The significant improvement in predicting the mean FS by MVM is due to the improvement in predicting the failure length and including the correction factors, α and β. The small remaining error in the mean FS in the MVM analyses may be attributed to a slightly overestimated α (see Varkey et al. (2019) for details). The slight improvement in predicting the standard deviation of 3D FS by MVM is also due to the improvement in predicting the failure length. However, the standard deviation of FS by MVM has not improved as significantly as the mean. This may be attributed to the approximate form of the variance reduction factor used in Vanmarcke's method (Eq. (7)) compared to the variance reduction factor derived from the covariance function (Eq. (1)) used in the RFEM model in this paper.

Fig. 5 shows the percentage errors in mean 3D FS obtained by VM and MVM with respect to the mean 3D FS by RFEM for the same problem (base case). Also plotted in the figure are the corresponding percentage errors obtained with a coefficient of variation (COV) for *c* of 0.3, while the rest of the parameters are the same as those of the base case. This shows that the mean FS computed by MVM has an error < 6% (relative to the mean FS computed by RFEM) and is substantially better than the mean FS computed by VM (with an error of approximately 15–38%, and a tendency for larger errors at lower θ_h). Varkey et al. (2019) carried out a detailed comparison for various cases with different properties and cross-sectional geometries to the base case, and found that the mean FS computed by MVM had an error < 8% while that obtained by VM had an error of approximately 15–50% (relative to the mean FS computed by RFEM).

6 Conclusions

A modified semi-analytical method for slope reliability has been proposed (Varkey et al. 2019) based on Vanmarcke's (1977) method. Three significant areas needing improvement were identified based on a numerical investigation. These were corrected by an alternative relationship for the expected failure length and a modified equation for the mean FS that utilises two correction factors. Calibration curves for the correction factors are provided. The expected failure lengths and PDFs of the 3D FS by the modified Vanmarcke method were in good agreement with those obtained by RFEM. The results show that the proposed method gives substantially improved results while retaining the simplicity of the original method.

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