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Bandgap modulation in active metamaterial beams through feedback control

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Abstract – **Incorporating actively implemented resonators within elastic piezoelectric metastructures presents a unique approach for vibration attenuation, enabling the creation of tuneable low-frequency bandgaps. Through feedback control, we enhance the compactness of these metastructures by integrating resonator dynamics internally. We study the influence of varying the cross-section of the base substrate and the arrangement of transducers on bandgap generation. This influence is captured by the changes in the electromechanical coupling and stiffness of the metastructure, which appear directly in the formulas for bandgap edge frequencies in ideal conditions. This relationship is illustrated with numerical examples for realistic metastructures with a finite number of transducers. Our focus is on metastructures with sensors and actuators, employing feedback control techniques for resonator implementation as an alternative to shunt circuits. When a bandgap is generated in a finite metastructure, its edge frequencies can be calculated in closed form using the assumption of an infinite number of transducers of infinitesimal length distributed along the structure.**

I. Introduction

In the realm of elastic metamaterials, a bandgap denotes a specific frequency range wherein the structure effectively mitigates the propagation of vibrations. In this paper, we study the creation of bandgaps in piezoelectric metastructures in sensor/actuator configuration. Such configuration offers an attractive alternative to shunt circuits commonly used for bandgap generation in structures with piezoelectric transducers. While the use of active feedback systems is well-researched for resonance attenuation, their application for bandgap creation is underexplored. The bandgap can be created by actively implementing resonant dynamics in the feedback loop [1, 2]. The control elements can be seen as generalized stiffness, which makes it easy to relate to passive mechanical solutions. Frequency domain tuning techniques based on experimental data can be used to design the controller. This removes the burden of meticulous modelling of minute details of the structure to capture the dynamics vital to designing the feedback loop. Moreover, adaptive systems, self-sensing and other advanced control architectures can be adopted.

So far, the basic beam structures with rectangular cross-sections and attached piezoelectric transducers have been explored for generating bandgaps through resonant dynamics in feedback systems. We attempt to enhance the compactness of such metastructure by considering the actuators inside of the beam. A comparative analysis of three different cases based on the location of piezo actuators with hollow and solid sections of the beam was studied. The effect of the altered flexural rigidity and electro-mechanical coupling has been accounted for, and the resulting changes in the edge frequencies bandgap are shown. To this end, Section II. presents the model of the considered structure and numerical analysis. The conclusions of the paper are given in Section III.

II. Model and numerical analysis

Considering piezoelectric bimorph cantilever beams of length L with a rectangular cross-section, both solid and hollow sections made from two continuous piezoelectric layers poled in the transverse direction, the estimation of the bandgap region boundaries based on the assumption of an infinite number of transducers is carried out using the formulation provided in [3, 2]. This beam-based metastructure is analyzed for three different configurations

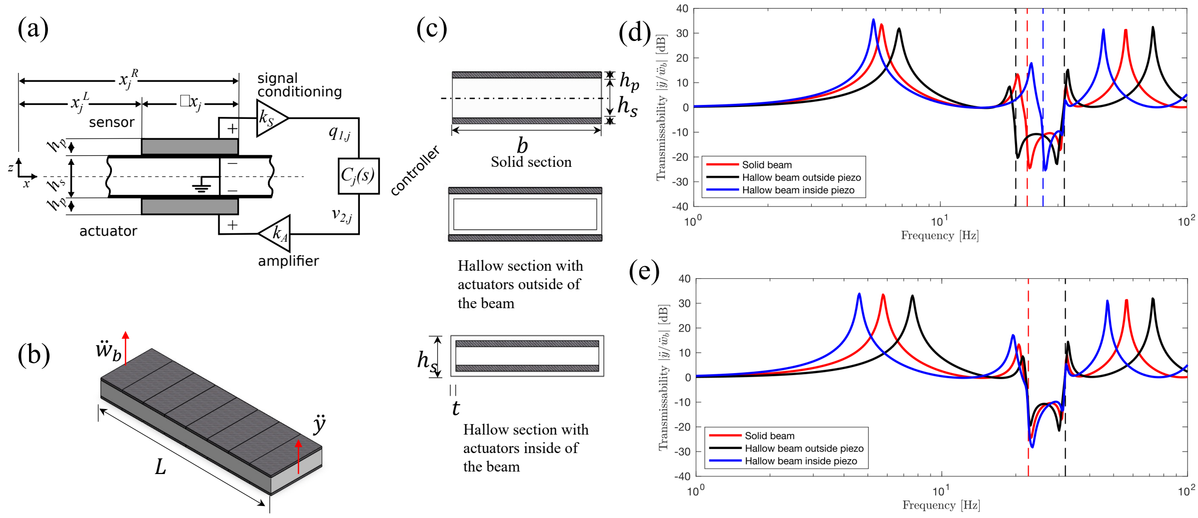

Fig. 1: (a) Schematic representation of a single unit cell. (b) Finite metastructure excited by base vibration $\ddot{w}_b$ and with tip acceleration $\ddot{y}$. (c) Considered cross-sections of the metamaterial beam. (d) Transmissibility $\ddot{y}(s)/\ddot{w}_b(s)$ of finite metastructures with different cross-sections controller with the same controller. Vertical dashed lines in the same colours show the predicted bandgap region boundaries based on the infinite number of transducers assumption. (e) Controllers are adjusted to compensate for cross-section effects to obtain identical bandgap widths.

of piezo placements while maintaining identical outer cross-sectional dimensions for all cases. Specifically, Case I, II, and III correspond to the arrangement of piezoelectric elements positioned outside of the solid section beam, outside of the hollow section beam, and inside the hollow section beam, respectively, as illustrated in Fig. 1 (c). The electrodes are segmented, forming S evenly spaced transducer pairs on opposite sides of the beam, such that transducers in one layer have the role of sensors and, in the other, act as actuators. Each transducer has length $\Delta x_j = L/S$. The electrode layers and bonding layers are treated as having negligible thickness. The role of the metastructures is to prevent the transmission of vibration through the structure at a targeted bandgap range of frequencies. This can be evaluated by measuring transmissibility between the base excitation acceleration $\ddot{w}_b$ and the acceleration at the tip of the cantilever $\ddot{y}$. The voltage $v_{2,j}(t)$ applied to the jth actuator is generated by an external amplifier with amplification factor k_A. The charge $q_{1,j}(t)$ measured at the jth sensor is proportional to the difference of slopes at the extremities of the transducer $\Delta w'_j$, with a factor dependent on the signal conditioning circuit k_S.

The system can be modelled as described in [2]. Using an assumed-modes type expansion with N modes, the transverse displacement of the beam is expanded as $w(x,t) = \sum_{r=1}^{N} \phi_r(x)\eta_r(t)$ where $\eta_r(t)$ are the modal weighting and $\phi_r(t)$ are the normalized mode shapes of the beam for a given set of boundary conditions (at short circuit), corresponding to resonance frequency ω_r. The relationships between the signals of interest can be represented in the Laplace domain as

$$\left(s^2 + \omega_r^2\right) H_r(s) - k_A \vartheta \sum_{j=1}^{S} V_{2,j}(s)\Delta\phi'_{r,j} = -\ddot{w}_b(s) \int_0^L m\phi_r(x)dx \tag{1}$$

$$Q_{1,j}(s) = k_S \vartheta \sum_{r=1}^{N} \Delta\phi'_{r,j} H_r(s) \tag{2}$$

$$\ddot{y}(s) = \ddot{w}_b(s) + \sum_{r=1}^{N} \phi_r(x_T) s^2 H_r(s), \tag{3}$$

where $\Delta\phi'_{r,j} = \left(\frac{\mathrm{d}\phi_r}{\mathrm{d}x}\right)_{x_j^L}^{x_j^R}$, $\vartheta = \bar{e}_{31} b\,(h_s + h_p)$ and, with some abuse of notation, $Q_{1,j}(s), V_{2,j}(s), \ddot{y}(s), \ddot{w}_b(s), H_r(s)$ denote the Laplace transforms of $q_{1,j}(t), v_{2,j}(t), \ddot{y}(t), \ddot{w}_b(t), \eta_r(t)$.

The electromechanical coupling factor ϑ, flexural rigidity EI, and mass per unit length m depend on the beam's cross-section geometry and material properties. For the cross-sections presented in Fig. 1(c), the ϑ and EI reduce if the actuators are placed inside of the beam. This is due to the fact that the moment of inertia decreases as the distance of the piezo from the neutral axis is reduced.

Controller dynamics describe the relationship between the actuator and sensor signals

$$V_{2,j}(s) = C_j(s)Q_{1,j}(s). \tag{4}$$

A suitable controller for bandgap generation in piezoelectric metastructures that can be applied in practice is positive position feedback (PPF)

$$C_j(s) = \frac{g_j}{s^2/\omega_c^2 + 2s\zeta_c/\omega_c + 1}. \tag{5}$$

The bandgap width obtained in a finite metastructure with the considered controller dynamics can be calculated in closed form, assuming that infinitely many infinitesimally small transducer pairs are applied [2]. The results obtained with such an approximation are accurate if a sufficient number of transducers is used. If the gains of all controllers are the same $g_j = g_k = g$ the bandgap will appear in the range of frequencies

$$\omega_c\sqrt{1-k_c} < \omega < \omega_c, \quad k_c = g\frac{k_S k_A \vartheta^2}{EI}\Delta x_j \tag{6}$$

Figure 1 (d) presents the transmissibility of a metastructure with different cross-sections, consisting of $S = 4$ unit cells with sensors and actuators. While the same controller dynamics are implemented in all the cases, the obtained bandgap ranges differ. This can be explained by the differences in the beam cross-section parameters and eq. (6). The use of a hollow beam results in a lowering of the stiffness of the structure. In consequence, the lower edge frequency of the bandgap decreases, and a wider bandgap is obtained. However, moving the transducer inside the beam results in a decrease of ϑ and a narrower bandgap range. The upper bandgap edge is independent of cross-section parameters and remains fixed. Note that edge frequency predictions obtained with the assumption of an infinite number of transducers are accurate, as indicated by the vertical lines, while only 4 unit cells are used in the structure. In the case presented in Fig. 1 (e), the controller gains are related to the static responses of the structure to compensate for the influence of the beam cross-section. As a result, bandgaps of the same width are obtained for all the considered beam cross-sections.

III. Conclusion

This work aimed to report on the influence of the cross-section of a piezoelectric metastructure on the generated bandgap. In the considered cases, the changes influenced the electromechanical coupling factors and structure stiffness, which led to a shift in the edge frequencies of the bandgap. Fine-tuning the beam cross-section to widen the bandgap is a well-suited approach in high-performance applications. This effect can be compensated if desired by adjusting the controller gains to obtain the same bandgap range irrespective of the beam parameters. The obtained results highlight the influence of the design parameters on the obtained bandgap and are relevant for the practical design of metastructures.

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