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Near-optimal feedback guidance for low-thrust earth orbit transfers

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Abstract. This research proposes a near-optimal feedback guidance based on nonlinear control for low-thrust Earth orbit transfers. For the numerical simulations, two flight conditions are defined: (i) nominal conditions and (ii) nonnominal conditions that account for the orbit injection errors and the stochastic failures of the propulsion system. Condition (ii) is studied through an extensive Monte Carlo Analysis, to demonstrate the nonlinear feedback guidance's numerical stability and convergence properties. To illustrate the performance under both conditions, an orbit transfer from low Earth orbit to geostationary orbit is considered. Near-optimality of the feedback guidance comes from carefully selecting the nonlinear control gains. Comparison of the transfer with an existing study that uses optimal control reveals that orbit transfers based on feedback orbit control are very close to the optimal solution.

Keywords: Earth Orbit Transfers, Low-Thrust Spacecraft, Feedback Guidance and Control

Introduction

Orbit control is a critical part of spacecraft control design and was extensively studied over the last century. Most studies focus on impulsive transfers. However, near-optimal and nonlinear strategies for low-thrust transfers are becoming popular since they allow compensation of orbital perturbations.

The study of nonlinear and near-optimal feedback guidance for low-thrust spacecraft is a reasonably new topic, with significant publications taking place over the last three decades. An important contribution is due to Gurfil [1], who utilizes nonlinear control with modified equinoctial orbit elements for low-thrust orbit transfers. The study guarantees asymptotic convergence from an initial elliptical orbit to any final elliptical orbit using Gauss's variational equations. Pontani and Pustorino [2] have recently applied nonlinear control strategies to orbit injection and maintenance problems where the control scheme takes advantage of Lyapunov stability combined with LaSalle's invariance principle. Gao [3] presents a linear feedback guidance approach that exhibits near-optimality for low-thrust Earth orbit transfers using orbital averaging. Kluever [4] proposed a simple closed-loop feedback-driven scheme for low-thrust orbit transfers that allows calculating sub-optimal trajectories. Petropoulos [5] developed a simple strategy based on candidate Lyapunov functions for low-thrust orbit transfers while coining the term proximity quotient or Q-Law. There are several other studies based on Q-Law [6, 7], and they focus on mitigating the sub-optimality of this strategy.

This research proposes a near-optimal feedback guidance based on nonlinear control for low-thrust Earth orbit transfers. Both eclipse condition and orbit perturbations (i.e., several Earth gravitational harmonics, solar radiation pressure, aerodynamic drag, and gravitational attraction due to Sun and Moon) are modeled. Two flight conditions are defined: (i) nominal conditions and (ii) nonnominal conditions that account for orbit injection errors and stochastic failures of the propulsion system. An orbit transfer from low Earth orbit to geostationary orbit is considered. The initial and final orbit elements are taken from an existing study on optimal orbit control [8] for the purpose of comparing and demonstrating the near-optimality of the nonlinear feedback guidance.

Nonlinear orbit control using modified equinoctial elements

Orbit elements lead to singularities in the Gauss planetary equations for circular and equatorial orbits. To avoid similar issues, this study utilizes Modified Equinoctial Orbit Elements (MEE), defined as

$$p = a\left(1-e^2\right) \quad l = e\cos\left(\Omega+\omega\right) \quad m = e\sin\left(\Omega+\omega\right) \quad n = \tan\frac{i}{2}\cos\Omega \quad s = \tan\frac{i}{2}\sin\Omega \quad q = \Omega+\omega+\theta_* \tag{1}$$

where $a, e, \Omega, \omega, i, \theta_*$ are semimajor axis eccentricity, right ascension of the ascending node (RAAN), argument of periapsis, inclination, and true anomaly, respectively. Five of them are collected in $\boldsymbol{z}$, defined as $\boldsymbol{z} = \left[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5\right]^T = \left[p \quad l \quad m \quad n \quad s\right]^T$, and subject to the governing equation

$$\dot{\boldsymbol{z}} = \mathbf{G}(\boldsymbol{z}, x_6)\boldsymbol{a} \tag{2}$$

The last element is $x_6 = q$. The $\boldsymbol{a}$ term in (2) includes the projections of both perturbing and thrust acceleration onto the LVLH-frame. The explicit expression of $\mathbf{G}$ and the governing equation for x_6 are reported in [2]. Two (constant) parameters identify the characteristics of the low-thrust propulsion system: $u_T^{(max)} = T_{max}/m_0$ and c, where T_{max}, m_0, and c denote respectively maximum thrust magnitude, initial mass, and effective exhaust velocity. As a result, letting $x_7 = m/m_0$ (where m is the instantaneous mass), one obtains $\dot{x}_7 = -u_T/c$, with $u_T = T/m_0$. Thus, $\boldsymbol{a}_T = \boldsymbol{u}_T/x_7$ defines the instantaneous thrust acceleration. The term $\boldsymbol{a}$ includes two contributions, $\boldsymbol{a} = \boldsymbol{a}_T + \boldsymbol{a}_P$, where the $\boldsymbol{a}_P$ term refers to the perturbing acceleration. For this study, four types of orbital perturbations are considered: (a) the Earth gravitational harmonics (with $\left|J_{l,m}\right| > 10^{-6}$), (b) solar radiation pressure, (c) third-body attraction due to the Sun and the Moon, and (d) aerodynamic drag. The drag is modeled by assuming a reference surface area of $23.569\ \mathrm{m}^2$ and ballistic coefficient equal to $0.0576\ \mathrm{m}^2/\mathrm{kg}$. In addition, the solar radiation pressure is modeled using a fully reflective surface area where the reflective coefficient is equal to 2. In the end, $\boldsymbol{x} = \left[\boldsymbol{z}^T \quad x_6 \quad x_7\right]^T = \left[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7\right]^T$ identifies the complete state vector in compact form, whereas $\boldsymbol{u}_T$ is the control vector.

Nonlinear orbit control allows identifying a feedback law that can drive the spacecraft toward the desired orbit while ensuring global asymptotic stability. For the problem at hand, the target set, associated with the final orbit, is $\boldsymbol{\psi} = \left[x_1 - p_d \quad {x_2}^2 + {x_3}^2 - {e_d}^2 \quad {x_4}^2 + {x_5}^2 - \tan^2\left(i_d/2\right)\right]^T$, where subscript d denotes the desired value of the respective variable. The feedback law

$$\boldsymbol{u}_T = -u_T^{(max)} \frac{x_7\left(\boldsymbol{b}+\boldsymbol{a}_P\right)}{\max\left\{u_T^{(max)}, \left|x_7\left(\boldsymbol{b}+\boldsymbol{a}_P\right)\right|\right\}}, \text{ with } \boldsymbol{b} = \mathbf{G}^T\left(\frac{\partial\boldsymbol{\psi}}{\partial\boldsymbol{z}}\right)^T \mathbf{K}\boldsymbol{\psi} \text{ and } \mathbf{K} = \mathrm{diag}\left\{k_1, k_2, k_3\right\} \tag{3}$$

is proven to enjoy quasi global stability [2], using the Lyapunov direct method, in conjunction with the LaSalle's principle. However, the choice of the three gains $\left(k_1, k_2, k_3\right)$ plays a crucial role for the purpose of speeding up convergence to the target set. This study proposes and applies a gain selection method composed of two sequential steps:

Step 1. Exhaustive table search that includes different gain combinations; each gain is changed with increment by $10^{0.1}$, in the interval $1 \le k_i \le 10^6$.

Step 2. Using the values found at step 1, the native "fminsearch" MATLAB routine, which employs a Nelder-Mead simplex algorithm, is used.

The preceding two steps are completed for different initial orbits, associated with identical values of semimajor axis, eccentricity, RAAN, and argument of perigee, and different initial inclinations. The propulsion parameters for the gain optimization process are assumed to be $c = 30$ km/s and $u_T^{(\max)} = 10^{-4} g_0$ with $g_0 = 9.8065$ m/s^2.

Numerical results

The near-optimal feedback guidance proposed in this study is tested under nominal and nonnominal conditions. For both cases, initial and final orbit elements and the propulsion parameters are taken from an existing study focusing on optimal orbit control [8]. The final orbit is geostationary, whereas the initial orbit is circular, with $a_0 = 6927$ km and $i_0 = 28.5°$. The propulsion parameters are $c = 32.361$ km/s and $u_T^{(\max)} = 3.348 \cdot 10^{-4}$ m/s^2, and they characterize a low-thrust propulsion system. The gain values are selected from the preceding systematic study and are $k_1 = 0.9722$, $k_2 = 1056$, and $k_3 = 967$. For the transfer time and final mass ratios reported in this section, the following criteria are used to indicate the end of the transfer:

$$|p - p_d| \leq 10 \text{ km} \qquad e \leq 0.005 \qquad i \leq 0.5° \tag{4}$$

Nominal Conditions

This subsection reports the numerical results under nominal conditions and compares the proposed nonlinear feedback guidance with the existing optimal solution. Figure 1 shows the near-optimal transfer path, with eclipse arcs (where propulsion is unavailable) highlighted in blue.

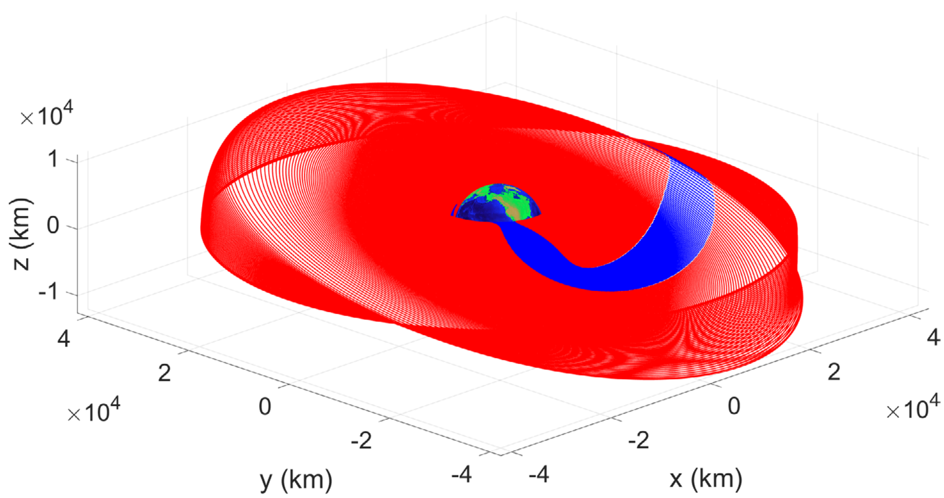

Figure 1: Cartesian motion of the spacecraft in the ECI frame (blue lines indicate eclipse)

At the beginning of the transfer, the perturbing acceleration is higher than the thrust acceleration. However, the perturbing term quickly decays to low values as the osculating radius increases. Using the criteria defined by (4), the transfer time is $t_f = 228.2$ days with a final mass ratio, $x_{7f} = 0.8245$. This result is compared to the optimal solution found in [8], which considers shadowing (but neglects orbit perturbations). The optimal path is completed in 215.9 days, with final mass ratio $x_{7f} = 0.8394$. Therefore, with the proposed nonlinear feedback approach, the transfer time is only 5.71% higher, and the final mass ratio is 1.78% lower than the optimal solution. Hence, this demonstrates that nonlinear feedback control can generate a transfer path very close to the optimal, minimum-time solution.

Monte Carlo Analysis

This subsection concentrates on nonnominal flight conditions, which account for the orbit injection errors and the stochastic failures of the propulsion system. The propulsion parameters, initial and final orbits, and gain values are the same as in the nominal case. Orbit injection errors are modeled by randomizing the initial orbit elements, using a uniform distribution. More specifically, the perigee and apogee radii (r_p and r_a) and inclination i have uniform distribution in the following ranges: $r_p = [350, 549]$ km $+ R_E$ and $r_a = [549, 750]$ km $+ R_E$ (where R_E is the Earth radius), and $i = [22.5, 34.5]$ deg . Moreover, RAAN, argument of perigee, and true anomaly have uniform distribution in the entire range of definition. The stochastic failure of the propulsion system is modeled by specifying the starting point of failure and its duration. These two stoachastic variables, denoted respectively with t_{fail} and t_{dur} have uniform distribution as well, i.e. $t_{fail} = [1, 100]$ days and $t_{dur} = [5, 20]$ days .

In spite of initial errors at orbit injection and stochastic propulsion failures, feedback control successfully drives the spacecraft to the desired orbit. Table 1 reports the Monte Carlo Analysis's statistical results, based on 1000 simulations, and compares the proposed feedback guidance and existing optimal solutions. These results testify to the excellent stability properties of feedback control as well as to the effectiveness of the gain selection method.

Table 1: Statistical results of the Monte Carlo Analysis and comparison with the optimal solution

	Mean	Std. Dev.	Optimal Solution
t_f (days)	236.41	10.51	215.94
x_7	0.8234	0.0063	0.8394

Concluding Remarks

This paper proposes and applies a near-optimal feedback guidance strategy based on nonlinear orbit control to low-thrust Earth orbit transfers. Feedback guidance utilizes some fundamental principles of Lyapunov stability theory and LaSalle's invariance principle. A novel gain selection strategy that involves an exhaustive table search and a numerical optimization algorithm provides near-optimality of the optimal paths traveled through feedback guidance. Two different flight conditions are considered: *(i) nominal conditions and (ii) nonnominal conditions that account for the orbit injection errors and the stochastic failures of the propulsion system. The numerical results* testify to the excellent stability properties of feedback control, as well as to the effectiveness of the gain selection method, even in nonnominal flight conditions.

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