BEACH NOURISHMENT: DESIGN PRINCIPLES

ROBERT G. DEAN

Coastal and Oceanographic Engineering Department
University of Florida
Gainesville, Florida

1. Introduction

It is convenient to discuss the physical performance of beach nourishment projects in terms of the cross-shore response (or profile adjustment) and longshore response, i.e. transport of sand out of the area placed. It is also convenient in exploring performance at the conceptual level to utilize idealized considerations and simplified (linearized) equations in some cases. This allows one to obtain a grasp or overview of the importance of the different variables without the problem of being clouded by complications which may be significant at the 10% - 20% level. To simplify our cross-shore considerations, we will use the so- called equilibrium beach profile concept in which the depth h(y) is related to the distance offshore, y, by the scale parameter, A, in the form

$$h(y) = Ay^{2/3} \tag{1}$$

Although this is not a universally valid form, it serves to capture many of the important characteristics of equilibrated beach profiles. To assist in providing an overview of transport in the longshore direction, we will utilize the linearized combined form of the transport and continuity equations first developed by Pelnard Considéré

$$\frac{\partial y}{\partial t} = G \frac{\partial^2 y}{\partial x^2} \tag{2}$$

where x is the longshore distance, t is time, G is a "longshore diffusivity" which depends strongly on the wave height mobilizing the sediment and Eq. (2) is recognized as the "heat conduction equation".

2. Cross-Shore Response

2.1. BEACH WIDTH GAINED VS. SEDIMENT QUALITY

From Fig. 1, it is seen that the scale parameter, A, in Eq. (1) increases with increasing sediment size. Thus, as presented in Fig. 2, a finer sediment will be associated with a milder sloped profile than one composed of coarse sediment. We will denote the native and fill profile scale parameters as A_N and A_F, respectively. The consequence of sand size to beach nourishment is that the coarser the nourishment material, the greater the dry beach width per unit volume placed.

Nourished beach profiles can be designated as "intersecting", "non-intersecting", and "submerged" profiles. Figure 3 presents examples of these. Referring to the top panel in this figure of intersecting profiles, a necessary but not sufficient requirement for intersecting profiles is that the fill material be coarser than the native material. One can see that an advantage of such a profile is that the nourished profile "toes in" to the native profile thereby negating the need for material to extend out to the closure depth. The
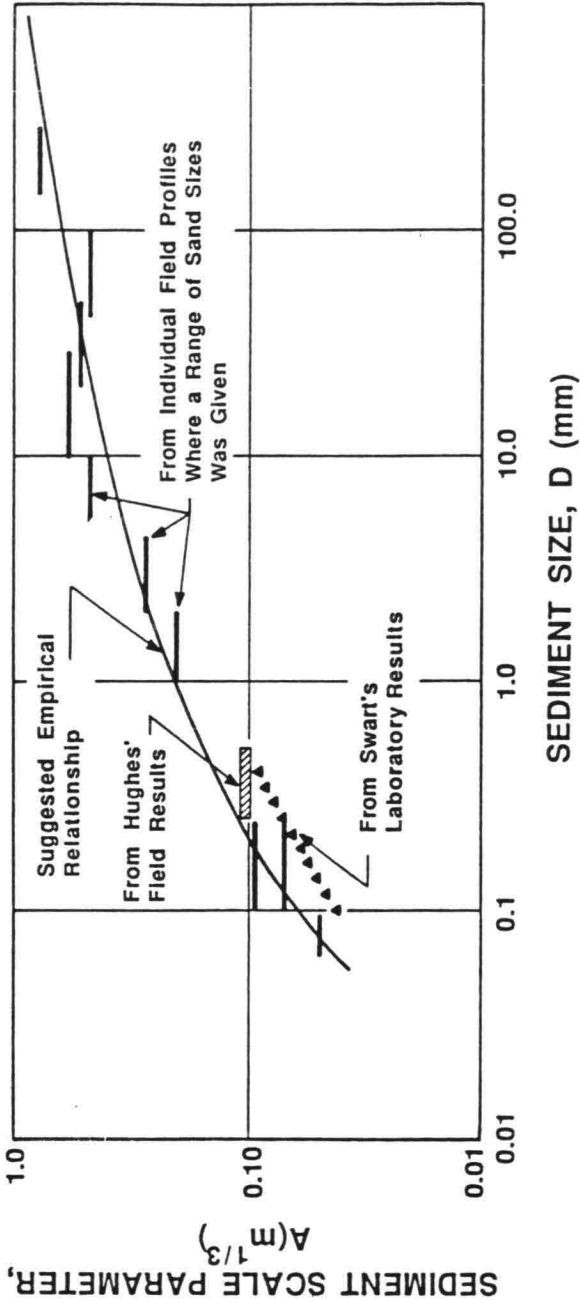
Figure 1.   Beach Profile Factor, A, vs. Sediment Diameter, D, In Relationship
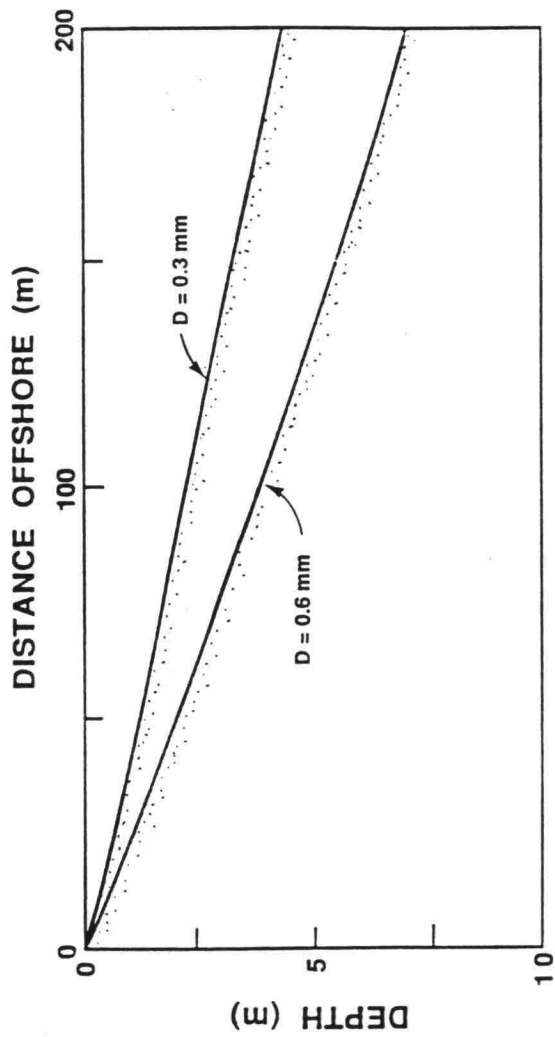$h = Ay^{2/3}$ (Modified from Moore, 1982).

Figure 2, Equilibrium Beach Profiles for Sand Sizes of 0.3 mm and 0.6 mm
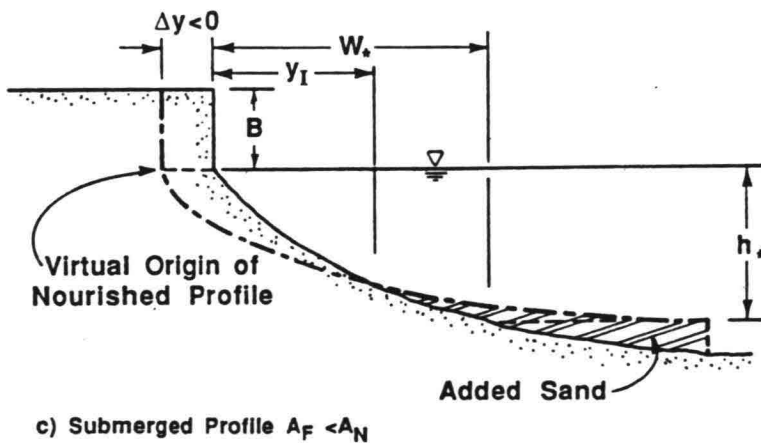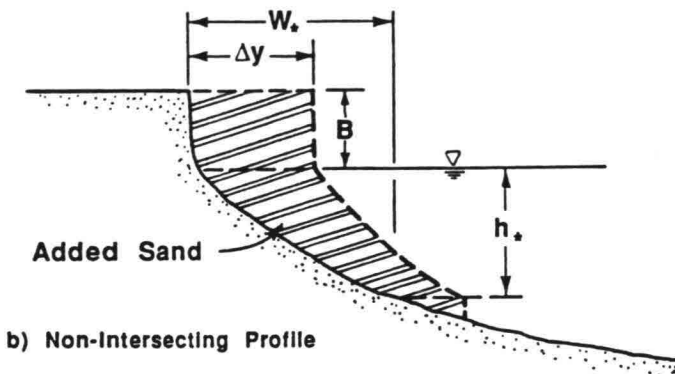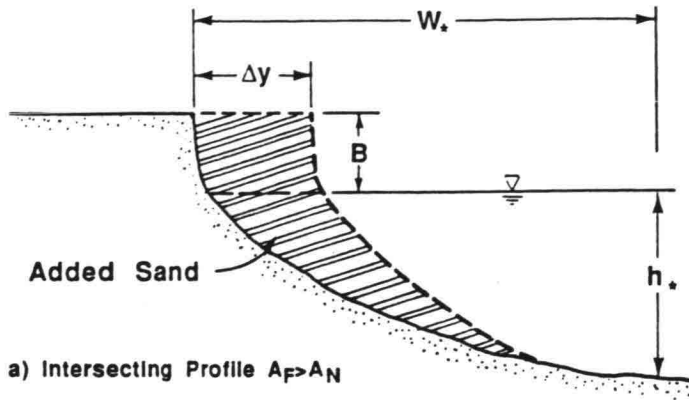A(D = 0.3 mm) = 0.12 $m^{1/3}$; A(D = 0.6 mm) = 0.20 $m^{1/3}$.

Figure 3. Three Generic Types of Nourished Profiles.

second type of profile is one that would usually occur in most beach nourishment projects. Nonintersecting profiles occur if the nourished material grain size is equal to or less than the native grain size. Additionally, this profile always extends out to the closure depth, h.. The third type of profile that can occur is the submerged profile (Fig. 3c) the characteristics of which are shown in greater detail in Fig. 4. This profile type requires the nourished material to be finer than the native. It can be shown that if only a small amount of material is used then all of this material will be mobilized by the breaking waves and moved offshore to form a small portion of the equilibrium profile associated with this grain size as shown in the upper panel. With increasing amounts of fill material, the intersection between the nourished and the original profile moves landward until the intersection point is at the water line. For greater quantities of material, there will be an increase in the dry beach width, Δy, resulting in a profile of the second type described.

Figure 5 illustrates the effect of placing the same volume of four different sized sands. In Fig. 5a, sand coarser than the native is used and a relatively wide beach Δy is obtained. In Fig. 5b, the same volume of sand of the same size as the native is used and the dry beach width gained is less. More of the same volume is required to fill out the milder sloped underwater profile. In Fig. 5c, the placed sand is finer than the native and much of the sand is utilized in satisfying the milder sloped underwater profile requirements. In a limiting case, shown in Fig. 5d, no dry beach is yielded with all the sand being used to satisfy the underwater requirements.

We can quantify the results presented in Fig. 5 for beach widening through nourishment by utilizing equilibrium profile concepts. It is necessary to distinguish two cases. The first is with intersecting profiles such as indicated in Fig. 3a and requires $A_F > A_N$. For this case, the volume placed per unit shoreline length, V_1 associated with a shoreline advancement, Δy, is presented in non-dimensional form as

$$\frac{V_1}{BW_.} = \frac{\Delta y}{W_.} + \frac{3}{5} \frac{h_.}{B} \left(\frac{\Delta y}{W_.}\right)^{5/3} \frac{1}{\left[1 - \left(\frac{A_N}{A_F}\right)^{3/2}\right]^{2/3}} \qquad (3)$$

in which B is the berm height, W. is a reference offshore distance associated with the breaking depth, h., on the original (unnourished) profile, i.e.

$$W_. = \left(\frac{h_.}{A_N}\right)^{3/2} \qquad (4)$$

and the breaking depth, h. and breaking wave height, H_b are related by

$$h_. = H_b/\kappa$$

OFFSHORE DISTANCE (m)

a) Added Volume = 120 m³ /m

b) Added Volume = 490 m³ /m

c) Added Volume = 900 m³ /m

d) Added Volume = 1660 m³ /m
Case of Incipient Dry Beach
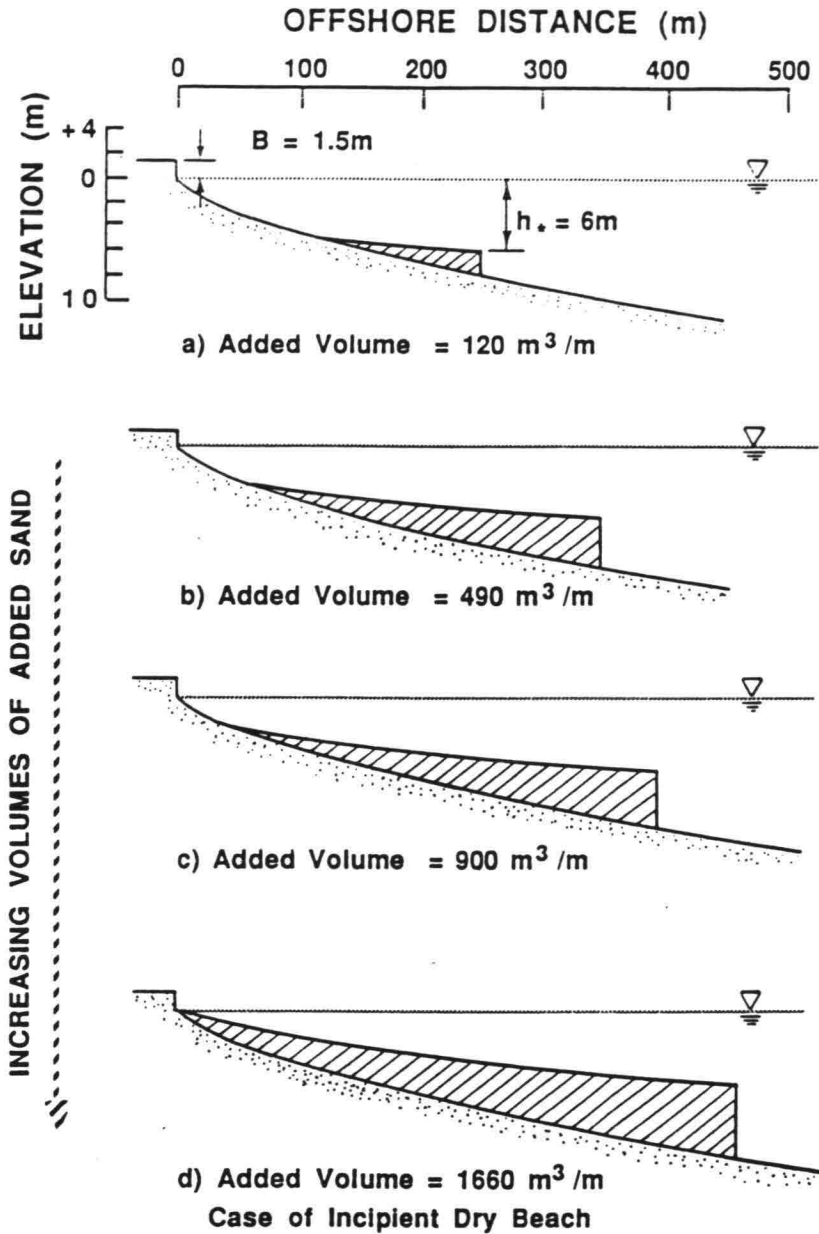
Figure 4   Effect of Increasing Volume of Sand Added on Resulting
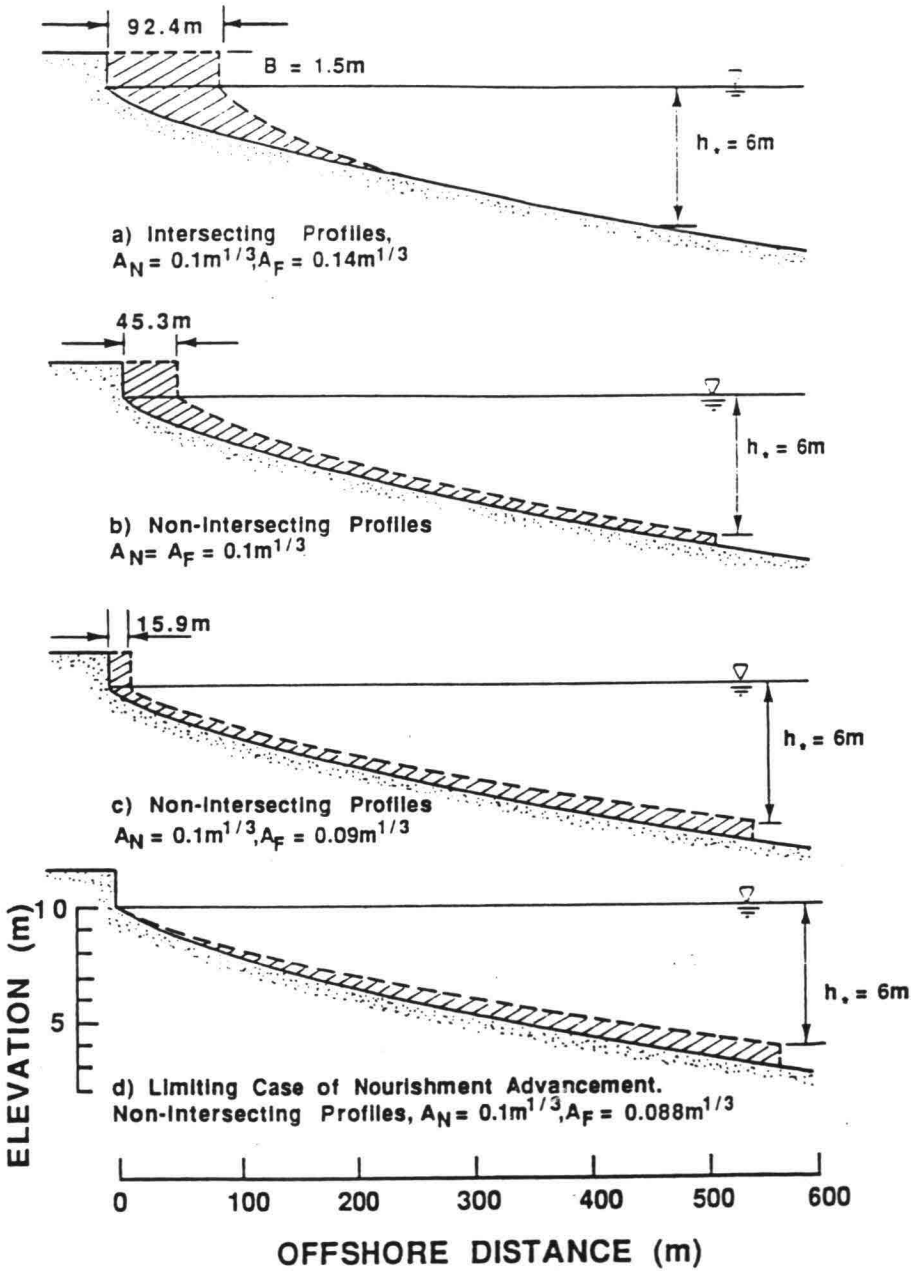Beach Profile. A = 0.1 m¹ᐟ³, A = 0.2 m¹ᐟ³, h. = 6 m, B = 1.5 m.

Figure   5.   Effect of Nourishment Material Scale Parameter, A_F, on Width of
Resulting Dry Beach. Four Examples of Decreasing A_F.

with κ (≈ 0.78), the spilling breaking wave proportionality factor. Figure 6 presents an estimate of h_* around the Florida shoreline.

For non-intersecting profiles, Figures 3b and 5b,c and d, the corresponding volume V_2 in non-dimensional form is

$$\frac{V_2}{W_*B} = \left(\frac{\Delta y}{W_*}\right) + \frac{3}{5}\left(\frac{h_*}{B}\right)\left\{\left[\frac{\Delta y}{W_*} + \left(\frac{A_N}{A_F}\right)^{3/2}\right]^{5/3} - \left(\frac{A_N}{A_F}\right)^{3/2}\right\} \tag{5}$$

It can be shown that the critical value $(\Delta y/W_*)_c$ for intersection/non-intersection of profiles is given by

$$\left(\frac{\Delta y}{W_*}\right)_c = 1 - \left(\frac{A_N}{A_F}\right)^{3/2} \tag{6}$$

with intersection occurring if $\Delta y/W_*$ is less than the critical value.

The critical volume associated with intersecting/non- intersecting profiles is

$$\left(\frac{V}{BW_*}\right)_{cl} = \left(1 + \frac{3}{5}\frac{h_*}{B}\right)\left[1 - \left(\frac{A_N}{A_F}\right)^{3/2}\right] \tag{7}$$

and applies only for $(A_F/A_N) > 1$. Also of interest, the critical volume of sand that will just yield a finite shoreline displacement for non-intersecting profiles $(A_F/A_N < 1)$, is

$$\left(\frac{V}{BW_*}\right)_{c2} = \frac{3}{5}\frac{h_*}{B}\left(\frac{A_N}{A_F}\right)^{3/2}\left\{\frac{A_N}{A_F} - 1\right\} \tag{8}$$

Figure 7 presents these two critical volumes versus the scale parameter ratio A_F/A_N for the special case $h_*/B = 4.0$.

The results from Eqs. (3), (5) and (6) are presented in graphical form in Figs. 8 and 9 for cases of $(h_*/B) = 2$ and 4 respectively. Plotted is the non-dimensional shoreline advancement $(\Delta y/W_*)$ versus the ratio of fill to native sediment scale parameters, A_F/A_N, for various isolines of dimensionless fill volume V' ($= V/W_*B$) per unit length of beach. It is interesting that the shoreline advancement remains more or less constant for $A_F A_N > 1$; for smaller values the additional shoreline width decreases rapidly. For A_F/A_N values slightly smaller than plotted, there is no beach width gain, i.e. as in Fig. 5d.

2.2. EFFECTS OF SEA LEVEL RISE ON BEACH NOURISHMENT QUANTITIES

Recently developed future sea level scenarios based on assumed fossil fuel consumption and other relevant factors have led to concern over the viability of the beach
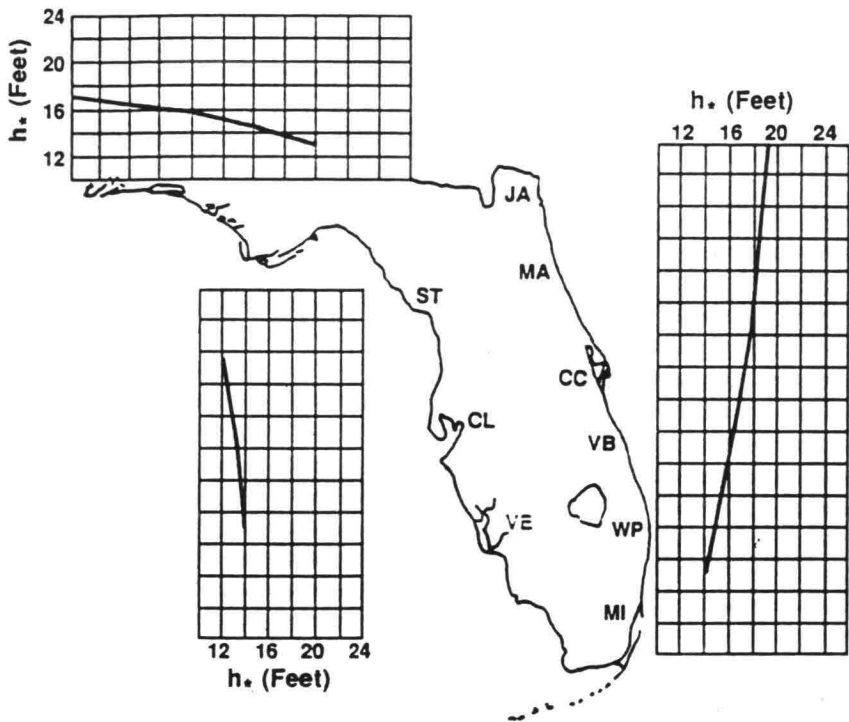
Figure 6 Recommended Distribtuion of h*, Along the Sandy Shoreline of Florida.
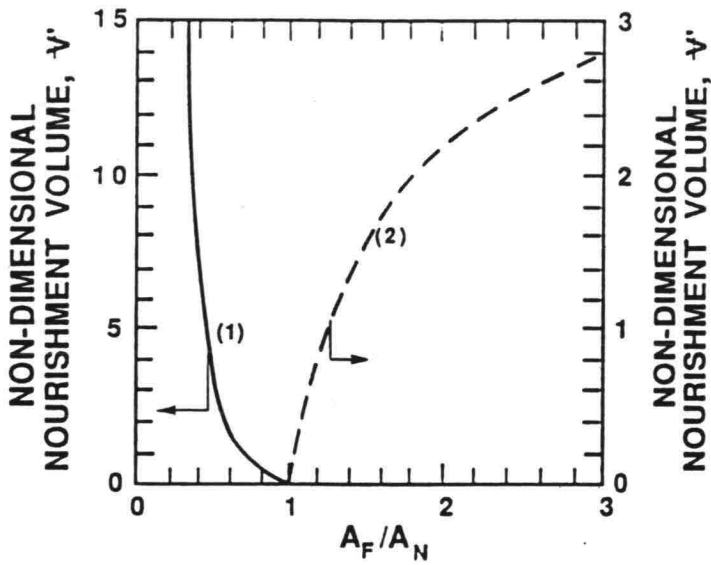
Figure   7. (1) Volumetric Requirement for Finite Shoreline Advancement
            (Eq. 2.8); (2) Volumetric Criterion for Intersecting Profiles
            (Eq. 2.7). Variation with A_F/A_N. Results Presented for $h_*/B = 4.0$
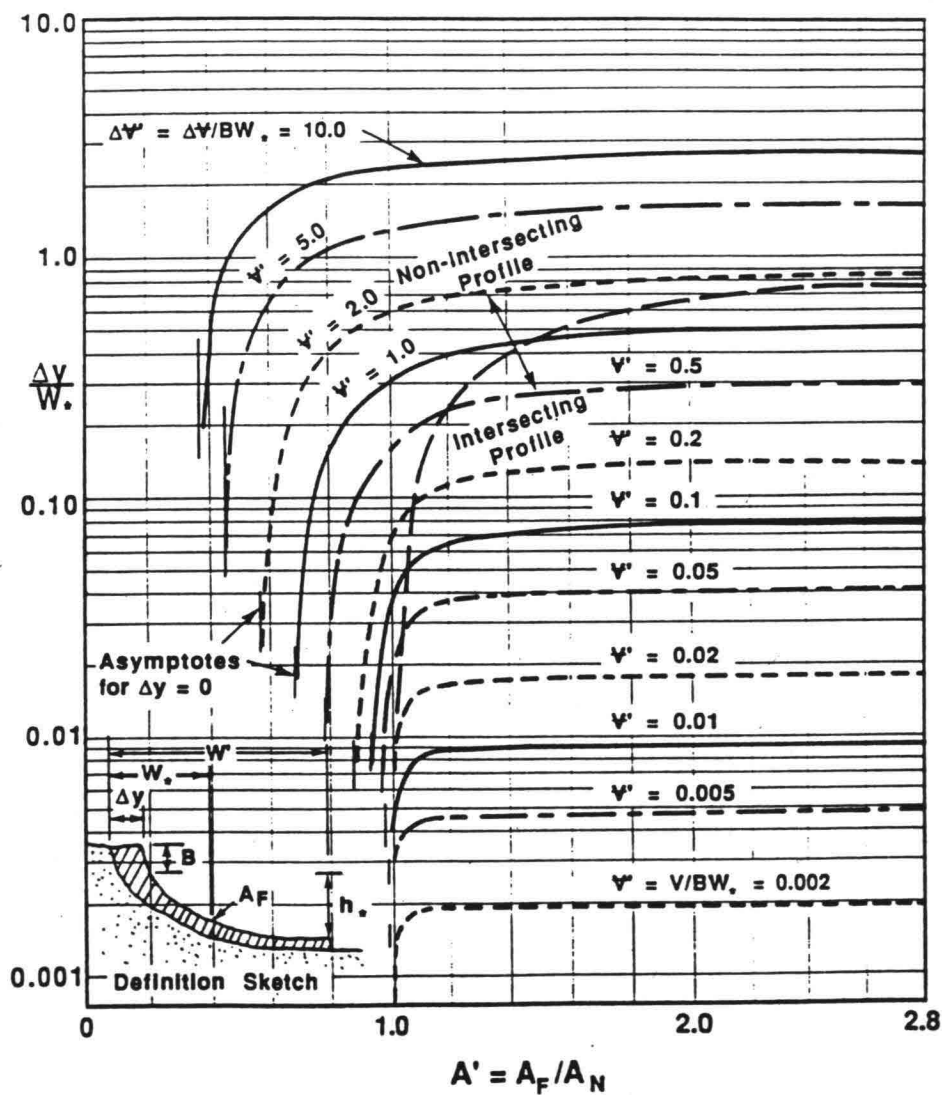
Figure 8. Variation of Non-Dimensional Shoreline Advancement
$\Delta y/W_*$ With A' and Ψ^2. Results Shown for $h_*/B = 2.0$
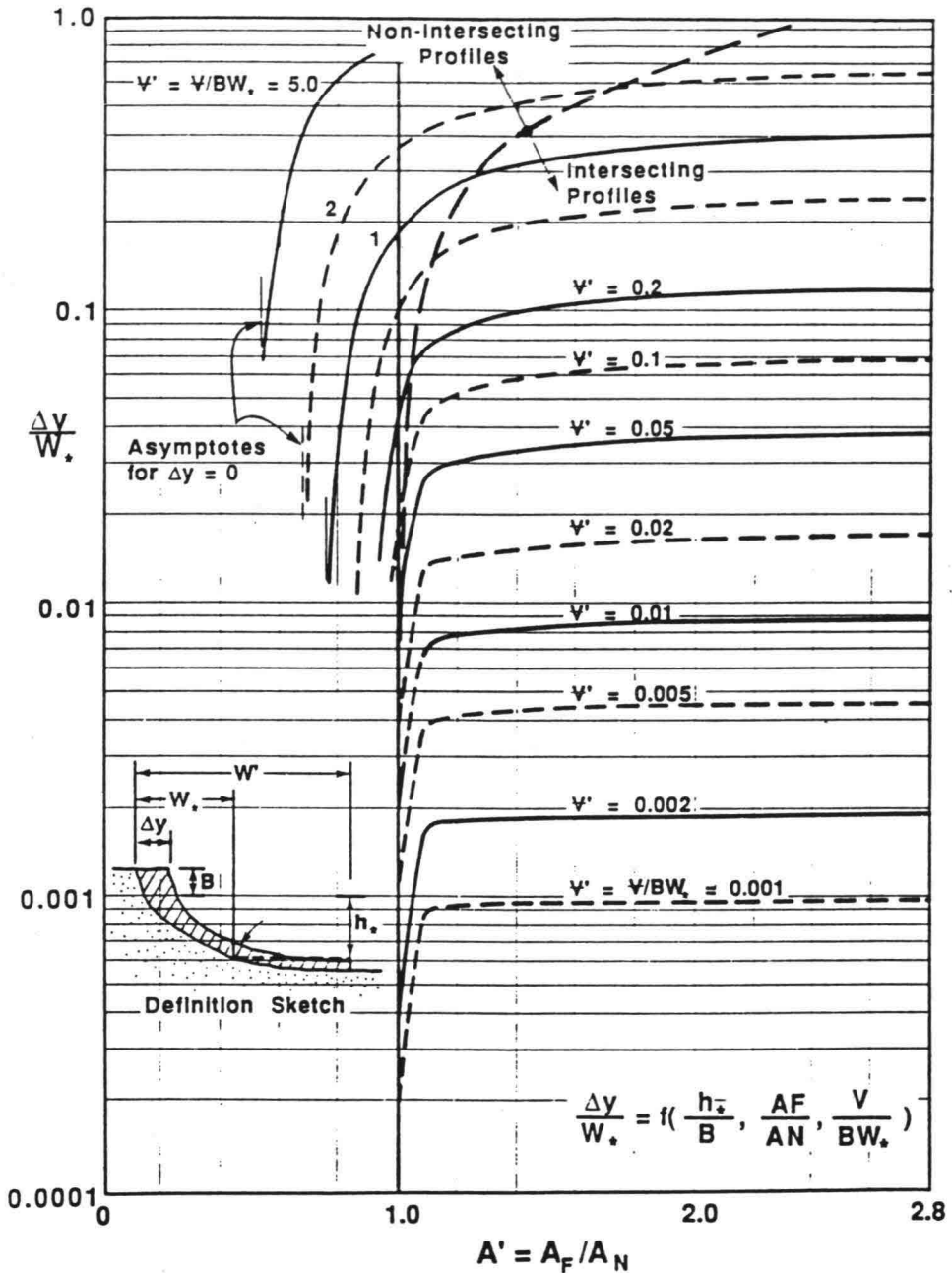
Figure 9. Variation of Non-Dimensional Shoreline Advancement $\Delta y/W_*$ With A' and Ψ'. Results Shown for $h_*/B = 4.0$

nourishment option. First, in the interest of objectivity, it must be said that the most extreme of the scenarios published by the Environmental Protection Agency (EPA) which amounts to over 3 m. by the year 2100 are extremely unlikely. While it is clear that worldwide sea level has been rising over the past century and is highly likely to increase in the future, the future rate is very poorly known. Moreover, probably at least 20 to 40 years will be required before our confidence level of future sea level rise rates will improve substantially. Within this period, it will be necessary to assess the viability of beach restoration on a project-by-project basis in recognition of possible future sea level increases. Presented below is a basis for estimating nourishment needs for the scenario in which there is no sediment supply across the continental shelf and there is a more-or-less well-defined seaward limit of sediment motion; in the second case the possibility of onshore sediment transport will be discussed.

2.2.1. Case I - Nourishment Quantities for the Case of No Onshore Sediment Transport

Bruun's Rule (1962) is based on the consideration that there is a well-defined depth limit of sediment transport. With this assumption, the only response possible to sea level rise is seaward sediment transport. Considering the shoreline change Δy, to be the superposition of recession due to sea level rise Δy_S and the advancement due to beach nourishment, Δy_N,

$$\Delta y = \Delta y_s + \Delta y_N \tag{9}$$

and, from Bruun's Rule

$$\Delta y_s = - S \, \frac{W_*}{h_* + B} \tag{10}$$

in which S is the sea level rise, W_* is the distance from the shoreline to the depth, h_*, associated with the seaward limit of sediment motion and B is the berm height. Assuming that compatible sand is used for nourishment (i.e. $A_F = A_N$)

$$\Delta y_N = \frac{V}{h_* + B} \tag{11}$$

and V is the beach nourishment volume per unit length of beach. Therefore

$$\Delta y_N = \frac{1}{(h_* + B)} \, [V - SW_*] \tag{12}$$

The above equation can be expressed in rates by,

$$\frac{dy}{dt} = \frac{1}{(h_*+B)} \left[\frac{dV}{dt} - W_* \frac{dS}{dt} \right] \tag{13}$$

where dS/dt now represents the rate of sea level rise and dV/dt is the rate at which nourishment material is provided. It is seen from Eq. (13) that in order to maintain the shoreline stable due to the effect of sea level rise the nourishment rate dV/dt is related to the rate of sea level rise dS/dt by

$$\frac{dV}{dt} = W_* \frac{dS}{dt} \tag{14}$$

Of course, this equation only applies to cross-shore mechanisms and therefore does not recognize any background erosion, or longshore transport (so-called "end losses"). It is seen that W_* behaves as an amplifier of material required. Therefore, it is instructive to explore the nature of W_* and it will be useful for this purpose to consider an equilibrium profile given by

$$h = Ay^{2/3}$$

in which A is the scale parameter presented in Fig. 1. Using the spilling breaking wave approximation

$$h_* = \frac{H_b}{\kappa} = A \ W_*^{2/3}$$

then

$$W_* = \left(\frac{H_b}{\kappa A} \right)^{3/2} \tag{15}$$

i.e. W_* increases with breaking wave height and with decreasing A (or sediment size).

2.2.2. Case II - Nourishment Quantities for the Case of Onshore Sediment Transport

Evidence is accumulating that in some locations there is a substantial amount of onshore sediment transport. Dean (1987) has noted the consequences of the assumption of a "depth of limiting motion" in allowing only offshore transport and proposed instead that if this assumption is relaxed, onshore transport can occur leading to a significantly different response to sea level rise. Recognizing that there is a range of sediment sizes in the active profile and adopting the hypothesis that a sediment particle of given hydraulic characteristics is in equilibrium under certain wave conditions and at a

particular water depth, if sea level rises, then our reference particle will seek equilibrium which requires landward rather than seaward transport as resulting from the Bruun Rule. Figure 10 summarizes some of the elements of this hypothesis.

Turning now to nourishment requirements in the presence of onshore sediment transport, the conservation of cross-shore sediment yields

$$\frac{\partial Q}{\partial y} = \frac{\partial h}{\partial t} + \text{sources} - \text{sinks} \tag{16}$$

in which h is the water depth referenced to a _fixed_ vertical datum and the sources could include natural contributions such as hydrogenous or biogenous components, and suspended deposition or human related contributions, i.e. beach nourishment. Sinks could include removal of sediment through suspension processes. Eq. (16) can be integrated seaward from a landward limit of no transport to any location, y

$$Q(y) - \int_0^y (\text{sources} - \text{sinks}) dy = \int_0^y \frac{\partial h}{\partial t} \, dy \tag{17}$$

If only natural processes are involved and there are no gradients of longshore sediment transport, the terms on the left hand side of Eq. (17) represent the net rate of increase of sediment deficit as a function of offshore distance, y. For y values greater than the normal width, W_*, of the zone of active motion, the left hand side can be considered as representing the "ambient" deficit rate due to cross-shore sediment transport resulting from long-term disequilibrium of the profile and source and sink terms.

In attempting to apply Eq. (17) to the prediction of profile change and/or nourishment needs under a scenario of increased sea level rise, it is reasonable to assume that over the next several decades the ambient deficit rate (or surplus) of sediment within the active zone will remain constant. However, an increased rate of sea level rise will cause an augmented demand which can be quantified as $W_*\left[\left(\frac{dS}{dt}\right)-\left(\frac{dS}{dt}\right)_0\right]$ in which $\left(\frac{dS}{dt}\right)_0$ is the reference sea level change rate during which time the ambient demand rate is established. Thus the active zone sediment deficit rate will be

$$\text{New Deficit Rate} = \left[\int_0^{W_*} \frac{\partial h}{\partial t} \, dy\right]_0 + W_*\left[\left(\frac{dS}{dt}\right) - \left(\frac{dS}{dt}\right)_0\right] - \frac{dV}{dt} \tag{18}$$

in which dV/dt represents the nourishment rate and the subscript "0" on the bracket represents the reference period before increased sea level rise. In order to decrease the deficit rate to zero, the required nourishment rate is

$$\frac{dV}{dt} = \left[\int_0^{W_*} \frac{\partial h}{\partial t} \, dy\right]_0 + W_*\left[\left(\frac{dS}{dt}\right) - \left(\frac{dS}{dt}\right)_0\right] \tag{19}$$

POSSIBLE MECHANISM OF SEDIMENTARY EQUILIBRIUM

Increased Sea Level

Original Sea Level

Q_s

S
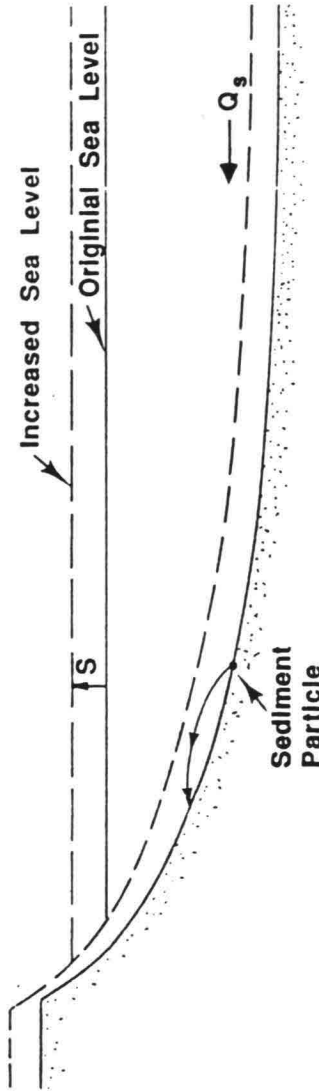
Sediment
Particle

"Subjected to a Given Statistical Wave Climate, A Sediment
Particle of a Particular Diameter Is In Statistical Equilibrium
When In a Given Water Depth"

Thus When Sea Level Increases, Particle Moves Landward

Figure   10.   Possible Mechanism of Sedimentary Equilibrium (After Dean, 1987).

These models may assist in evaluating the vulnerability of various shoreline systems to increased rates of sea level rise. As an example, for Florida, long-term trend estimates of dS/dt over the last 60 or so years are 0.3 mm/year although there is considerable variability in the year-to-year values of sea level changes, including interannual increases and changes which can amount of 40 times the annual trend value.

3. Planform Evolution of Beach Nourishment Projects

To a community that has allocated substantial economic resources to nourish their beach, there is considerable interest in determining how long those beaches can be expected to last. Prior to addressing this question, we will develop some tools.

3.1. THE LINEARIZED EQUATION OF BEACH PLANFORM EVOLUTION

The linearized equations for beach planform evolution were first combined and applied by Pelnard Consideré in 1956. The combined equation is the result of the sediment transport equation and the equation of continuity.

3.1.1. Governing Equations

3.1.1.1. Transport Equation - Utilizing the spilling breaker assumption, the equation for longshore sediment transport has been presented as

$$Q = \frac{K}{8} \frac{H_b^{5/2}\sqrt{g/\kappa}}{(1-p)(s-1)} \frac{\sin 2\theta_b}{2} \tag{20}$$

in which p is the sediment porosity (≈ 0.35-0.40) and s is the sediment specific gravity (≈ 2.65). Equation (20) will later be linearized by considering the deviation of the shoreline planform from the general shoreline alignment to be small. Referring to Fig. 11, denoting μ as the azimuth of the general alignment of the shoreline as defined by a baseline, β as the azimuth of an outward normal to the shoreline, α_b as the azimuth of the direction <u>from which</u> the breaking wave originates, then

$$Q = \frac{K H_b^{5/2}\sqrt{g/\kappa}}{8(1-p)(s-1)} \frac{\sin 2(\beta - \alpha_b)}{2} \tag{21}$$

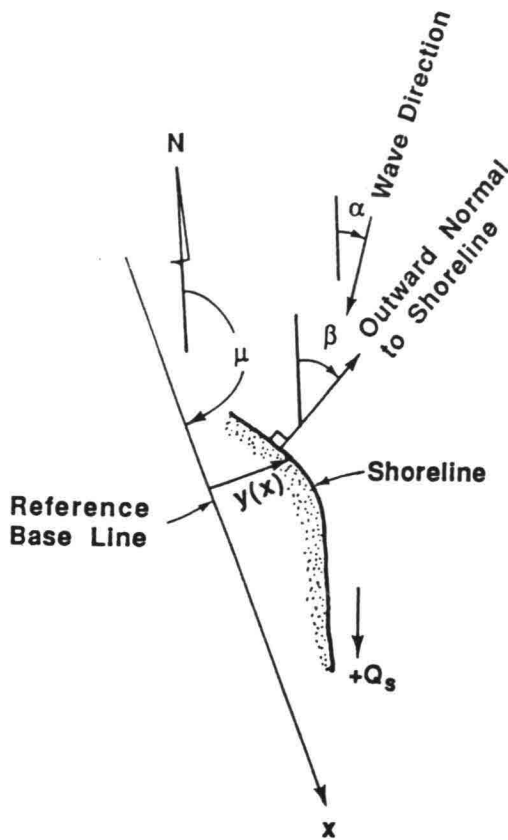where $\beta = \mu - \pi/2 - \tan^{-1}(\partial y/\partial x)$.

Figure 11. Definition Sketch.

3.1.1.2. Equation of Sediment Conservation - The one-dimensional equation of sediment conservation is

$$\frac{\partial y}{\partial t} + \frac{1}{(h_*+B)} \frac{\partial Q}{\partial x} = 0 \tag{22}$$

3.1.2. Combined Equation of Beach Planform Evolution

Differentiating with respect to x, the equation of longshore sediment transport, Eq. (21), we find

$$\frac{\partial Q}{\partial x} = \frac{K\, H_b^{5/2}\sqrt{g/\kappa}}{8(1-p)(s-1)}\, \cos2(\beta-\alpha_b)\, \frac{\partial \beta}{\partial x} \tag{23}$$

Recalling the definition of β and linearizing

$$\beta = \mu - \frac{\pi}{2} - \tan^{-1}\left(\frac{\partial y}{\partial x}\right) \approx \mu - \frac{\pi}{2} - \frac{\partial y}{\partial x} \tag{24}$$

and considering the wave approach angle $(\beta - \alpha_b)$ to be small such that $\cos 2(\beta - \alpha_b) \approx 1$, the final result is

$$\frac{\partial Q}{\partial x} = -\frac{K\, H_b^{5/2}\sqrt{g/\kappa}}{8(1-p)(s-1)}\, \frac{\partial^2 y}{\partial x^2} \tag{25}$$

Combining Eqs. (22) and (25), a single equation describing the planform evolution for a shoreline which is initially out of equilibrium is obtained as

$$\frac{\partial y}{\partial t} = G\, \frac{\partial^2 y}{\partial x^2} \tag{26}$$

where

$$G \equiv \frac{K\, H_b^{5/2}\sqrt{g/\kappa}}{8(s-1)(1-p)(h_*+B)} \tag{27}$$

The parameter G may be considered as a "shoreline diffusivity" with dimensions of (length)^2/time. Field studies have documented the variation of K with sediment size, D, as presented in Fig. 12. A detailed evaluation demonstrates that a more appropriate expression for G can be developed and expressed in terms of deep water conditions
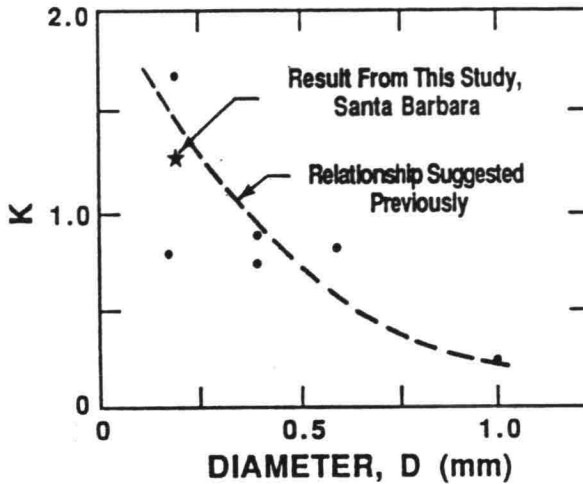
Figure   12. Plot of K vs. D. Results of Present and Previous
Studies (Modified From Dean, 1978).

$$G = \frac{K \, H_0^{2.4} C_{G_0}^{1.2} g^{0.4} \cos^{1.2}(\beta_0 - \alpha_0) \cos 2(\beta_0 - \alpha_*)}{8(s-1)(1-p)C_* \kappa^{0.4}(h_* + B)\cos(\beta_0 - \alpha_*)} \qquad (28)$$

where the subscript "0" denotes deep water conditions and C_* is the wave celerity in water depth, h_*. Figure 13 presents estimates of G around the Florida peninsula and Figs. 14 and 15 present estimates of effective deep water wave height and period.

It is recognized that the form of Eq. (26) is the heat conduction or diffusion equation for which a number of analytical solutions are available. Several of these will be explored in the next section.

It is of interest to know approximate values of the shoreline diffusivity, G. It is seen that G depends strongly on H_b, and secondarily on $(h_* + B)$ and κ. Table 1 presents values of G for various wave heights in several systems of units where it is noted that the reference wave height is the breaking wave height.
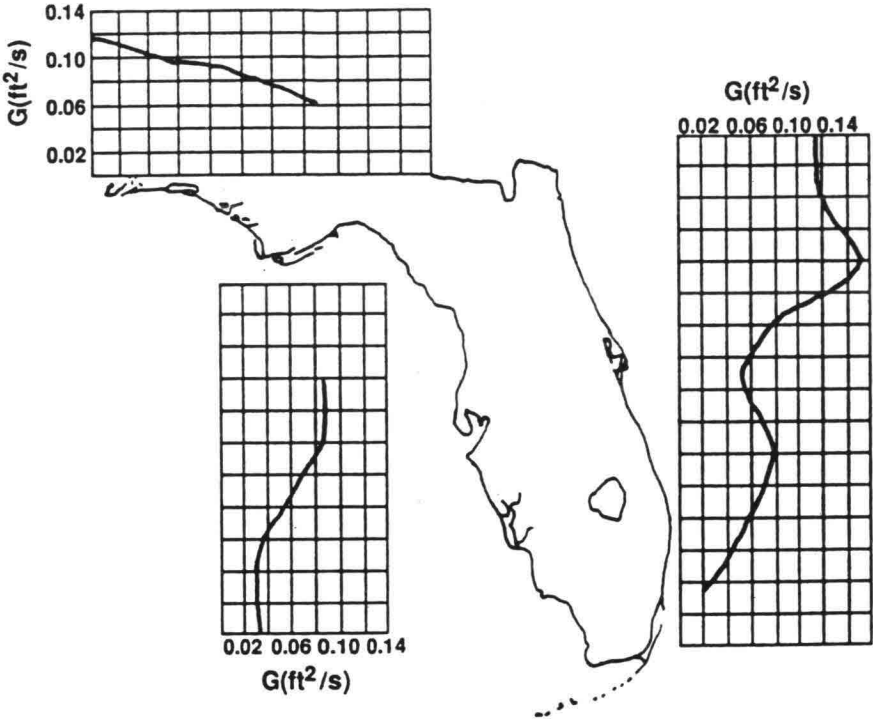
Figure   13.   Approximate Estimates of G(ft²/s) Around the Sandy
Beach Shoreline of the State of Florida. Based on
the Following Values: K = 0.77, g = 32.2 ft/sec²,
S = 2.65, p = 0.35, κ= 0.78, h, From Fig. 8., B Estimates
Ranging from 6 to 9 ft, H_0 from Fig. 23, T from Fig. 24.

Figure   14.   Recommeneded Values of Effective Deep Water Wave Height, H_0, Along Florida's Sandy Shoreline.

Figure   15.   Recommended Values of Effective Wave Period, T, Along
              Florida's Sandy Shoreline.

Table 1: Values of G for Representative Wave Heights

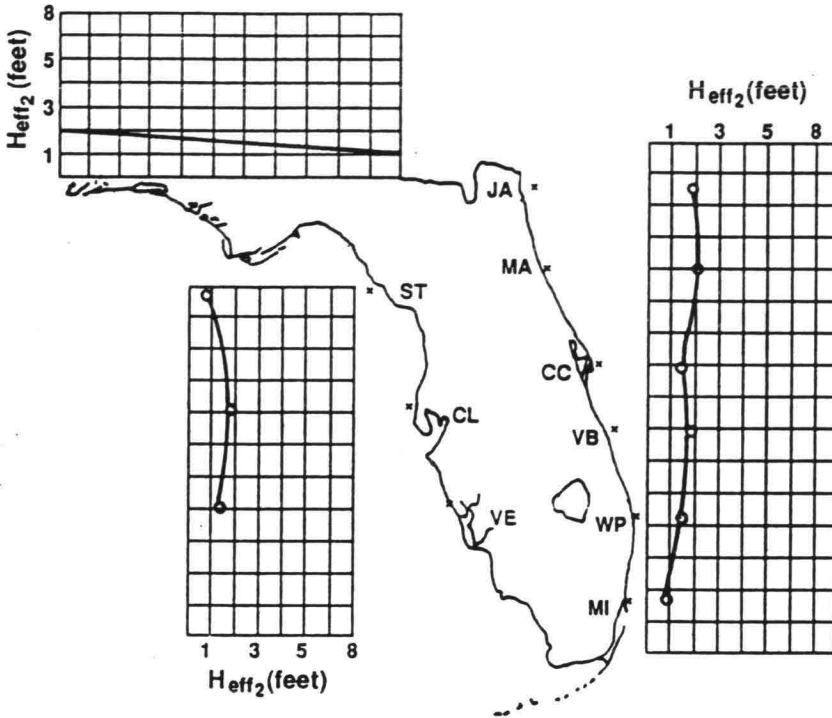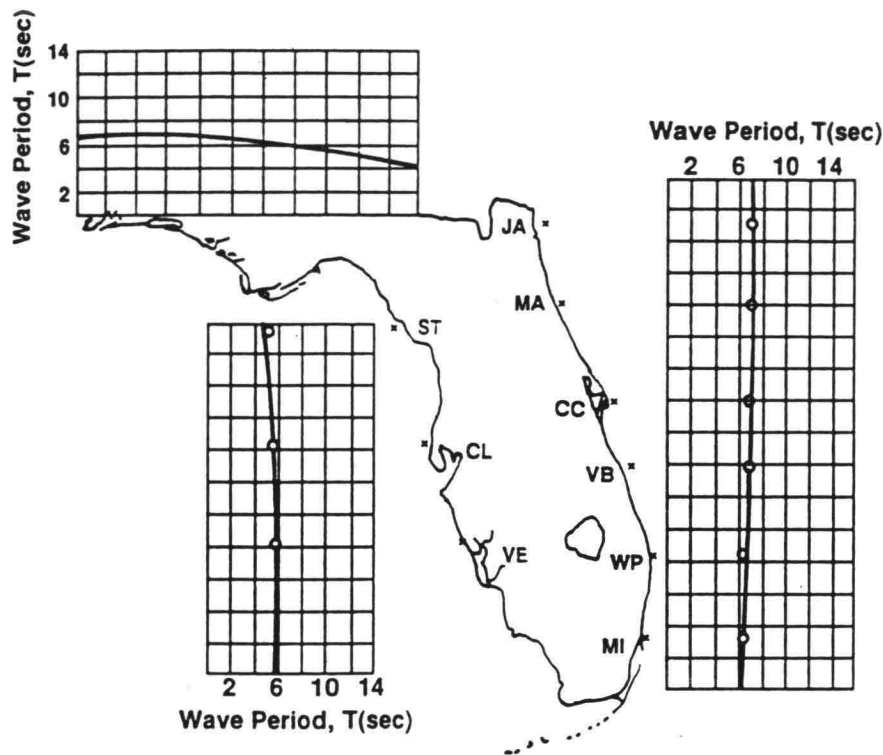| H_b	Value of G in			
(ft.)	ft^2/s	mi^2/yr	m^2/s	km^2/yr
1	0.0214	0.0242	0.00199	0.0626
2	0.121	0.14	0.0112	0.354
5	1.19	1.4	0.111	3.50
10	6.8	7.6	0.628	19.79
20	38.2	43.2	3.55	111.9

Note: In this table the following values have been employed: K = 0.77, κ = 0.78, g = 32.2 ft/s^2 = 9.81 m/s^2, s = 2.65, p = 0.35, h$_*$+B = 27 ft = 8.2 m.

3.2. ANALYTICAL SOLUTIONS FOR BEACH PLANFORM EVOLUTION

Examples which will be presented and discussed include: (1) the case of a narrow strip of sand protruding a distance, Y, from the general shoreline alignment, and (2) a rectangular distribution of sand extending into the ocean which could provide a reasonably realistic representation of a beach nourishment project.

3.2.1. A Narrow Strip of Sand Extending into the Ocean

Consider the case of a narrow strip of sand extending a distance, Y into the ocean and of width Δx such that m = YΔx, Fig. 16. The total area of the sand is designated m and the solution for this initial condition and the differential equation described by Eq. (26) is the following

$$y(x,t) = \frac{m}{\sqrt{4\pi Gt}} \exp\left(-\frac{x^2}{4Gt}\right) \tag{29}$$

which is recognized as a normal distribution with increasing standard deviation or "spread" as a function of time. Figure 17 shows the evolution originating from the initial strip configuration. Examining Eq. (29), it is seen that the important time parameter is Gt. The quantity, G, which is the constant in Eq. (27) serves to hasten the evolution toward an unperturbed shoreline. In Eq. (29) it is seen that the quantity, G, is proportional to the wave height to the 5/2 power which provides some insight into the significance of wave height in remolding beach planforms which are initially out of equilibrium.
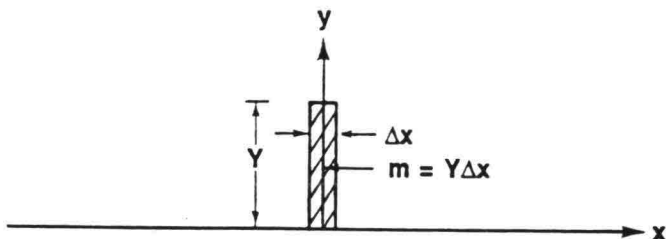
11-41

Figure 16. Initial Beach Planform. Narrow Strip of Sand
Extending From Unperturbed Shoreline.

It is interesting that, contrary to intuition, as the planform evolves it remains symmetric and centered about the point of the initial shoreline perturbation even though waves may arrive obliquely. Intuition would suggest that sediment would accumulate on the updrift side and perhaps erosion would occur on the downdrift side of the perturbation. It is recalled that the solution described in Fig. 17 applies only for the case of small deviations of the shoreline from the original alignment and may be responsible for the difference between the linear solution and intuition.

For purposes of the following discussion, we recover one of the nonlinearities removed from the definition of the "constant" G from Eqs. (23) and (26)

$$G = \frac{K H_b^{5/2} \sqrt{g/\kappa}}{8(s-1)(1-p)(h_* + B)} \cos2(\beta - \alpha_b) \tag{30}$$

and it is seen that if the difference between the wave direction and the shoreline orientation exceeds 45°, then the quantity, G, will be negative. Examining the results presented earlier, it is clear that if this should occur then it is equivalent to "running the equation backwards" in time. That is, if we were to commence with a shoreline which had a perturbation represented by a normal distribution then rather than smoothing out, the perturbation would tend to grow, with the ultimate planform being a very narrow distribution exactly as was our initial planform! In fact, regardless of the initial distribution one would expect the shoreline to grow into one or more accentuated features. Shorelines of this type ($\cos 2(\beta - \alpha_b)$ less than zero) can be termed "unstable" shorelines and may provide one possible explanation for certain shoreline features including cuspate forelands.

$$y(x,t) = \frac{m}{\sqrt{4\pi Gt}} \, e^{-x^2/4Gt}$$
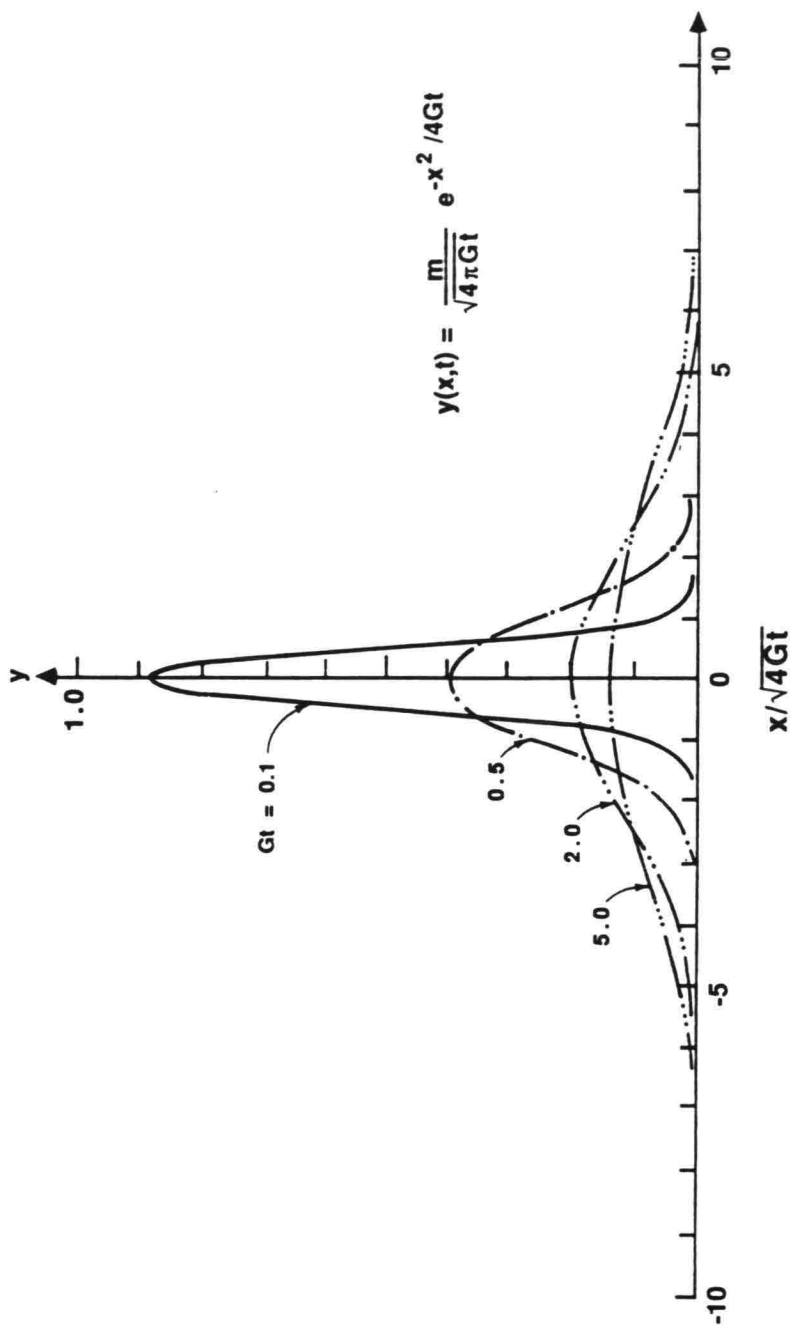
Figure   17.   Evolution of an Initially Narrow Shoreline Protuberence.

3.2.2. Initial Shoreline of Rectangular Planform

Consider the initial planform presented in Fig. 18 with a longshore length, ℓ, and extending into the ocean a distance, Y. This planform might represent an idealized configuration for a beach restoration program and thus its evolution is of considerable interest to coastal engineers, especially in interpreting and predicting the behavior of such projects.

It is seen that in a conceptual sense it would be possible to consider the problem of interest to be a summation of the narrow small strip planforms presented in the previous example. In fact, this is the case and since Eq. (26) is linear, the results are simply a summation or linear superposition of a number of normal distributions. The analytic solution for this initial planform can be expressed in terms of two error functions as

$$y(x,t) = \frac{Y}{2}\left\{ \text{erf}\left[\frac{\ell}{4\sqrt{Gt}}\left(\frac{2x}{\ell} + 1\right)\right] - \text{erf}\left[\frac{\ell}{4\sqrt{Gt}}\left(\frac{2x}{\ell} - 1\right)\right]\right\} \qquad (31)$$

where the error function "erf{}" is defined as

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-u^2}du \qquad (32)$$

and here u is a dummy variable of integration. This solution is examined in Fig. 18 where it is seen that initially the two ends of the planform commence spreading out and as the effects from the ends move towards the center, the planform distribution becomes more like a normal distribution. There are a number of interesting and valuable results that can be obtained by examining Eq. (31). First, it is seen that the important parameter is

$$\frac{\ell}{\sqrt{Gt}} \qquad (33)$$

where ℓ is the length of the rectangle and G is the parameter in the diffusion equation as discussed earlier. If the quantity $\left(\frac{\ell}{\sqrt{Gt}}\right)$ is the same for two different situations, then it is clear that the planform evolutions are also the same. Examining this requirement somewhat further, if two nourishment projects are exposed to the same wave climate but have different lengths, then the project with the greater length would tend to last longer. In fact, the longevity of a project varies as the square of the length, thus if Project A with a shoreline length of one mile "losses" 50 percent of its material in a period of 2 years, Project B subjected to the same wave climate but with a length of 4 miles would be expected to lose 50 percent of its material from the region where it was placed in a period of 32 years. Thus the project length is very significant to its performance.
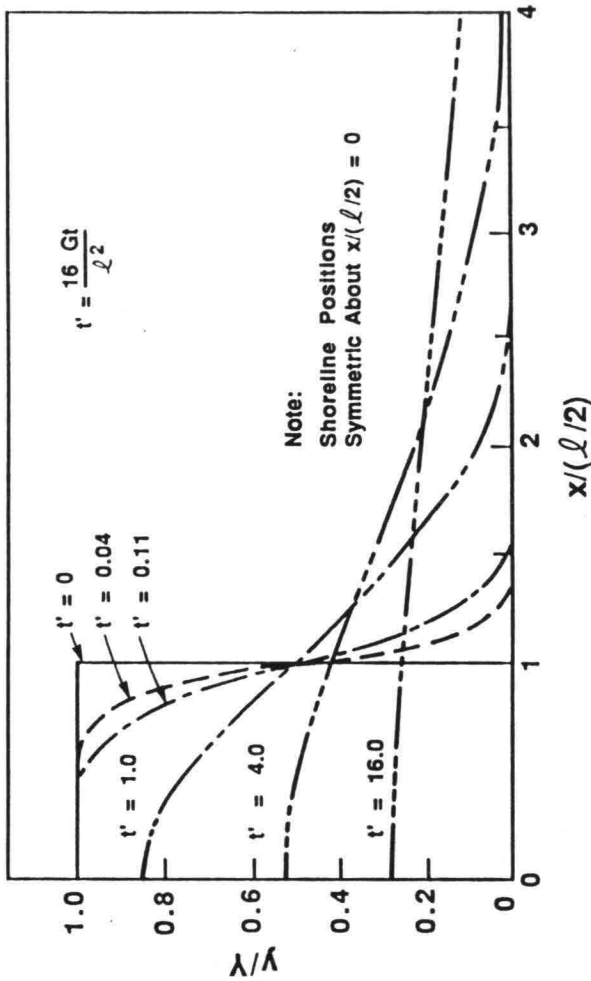
Figure   18.   Evolution of an Initially Rectangular Beach Planform on an Otherwise Straight Beach. Only One-Half of Project is Shown.

$$t' = \frac{16\ Gt}{\ell^2}$$

Note:
Shoreline Positions
Symmetric About x/(ℓ/2) = 0

Considering next the case where two projects are of the same length but located in different wave climates, it is seen that the G factor varies with the wave height to the 5/2 power. Thus if Project A is located where the wave height is 3 m and loses 50 percent of its material in a period of 2 years then Project B with a similarly configured beach planform located where the wave height is 1 m would be expected to lose 50 percent of its material in 18 years.

Figure 19 shows a specific example of beach evolution and Fig. 20 presents results in terms of the proportion of sediment remaining in front of the beach segment where it was placed as a function of time. These results are illustrated for several examples of combinations of wave height and project lengths. As an example of the application of Fig. 20, a project of 4 miles length in a location where the wave height is 3 m would lose 60 percent of its material in 7 years and a second project in a location where the wave height is 1.5 m and the project length is 16 miles would lose only 10 percent of its material in a period of 40 years. Figure 20 was developed based on the solution presented in Eq. (31).

It is possible to develop an analytical expression for the proportion of sand, M(t), remaining in the location placed, as defined by

$$M(t) = \frac{1}{Y\ell}\int_{-\ell/2}^{\ell/2} y(x,t)dx \tag{34}$$

to yield

$$M(t) = \frac{2\sqrt{Gt}}{\ell\sqrt{\pi}}\left(e^{-(\ell/2\sqrt{Gt})^2} - 1\right) + \text{erf}\left(\frac{\ell}{2\sqrt{Gt}}\right) \tag{35}$$

which is plotted in Fig. 21 along with the asymptote for small times

$$M(t) = 1 - \frac{2}{\sqrt{\pi}}\frac{\sqrt{Gt}}{\ell} \tag{36}$$

which appears to fit reasonably well for

$$\sqrt{Gt}/\ell < 0.5 \tag{37}$$

A useful approximation for estimating the "half-life" of a project is obtained by noting that $M = 0.5$ for $\sqrt{Gt}/\ell = 0.46$. Thus the half-life, t_{50}, is

$$t_{50} = (0.46)^2 \frac{\ell^2}{G} = 0.21 \frac{\ell^2}{G} \tag{38}$$

in which all variables are in consistent units. A more readily applied form is developed from Eq. (27) as
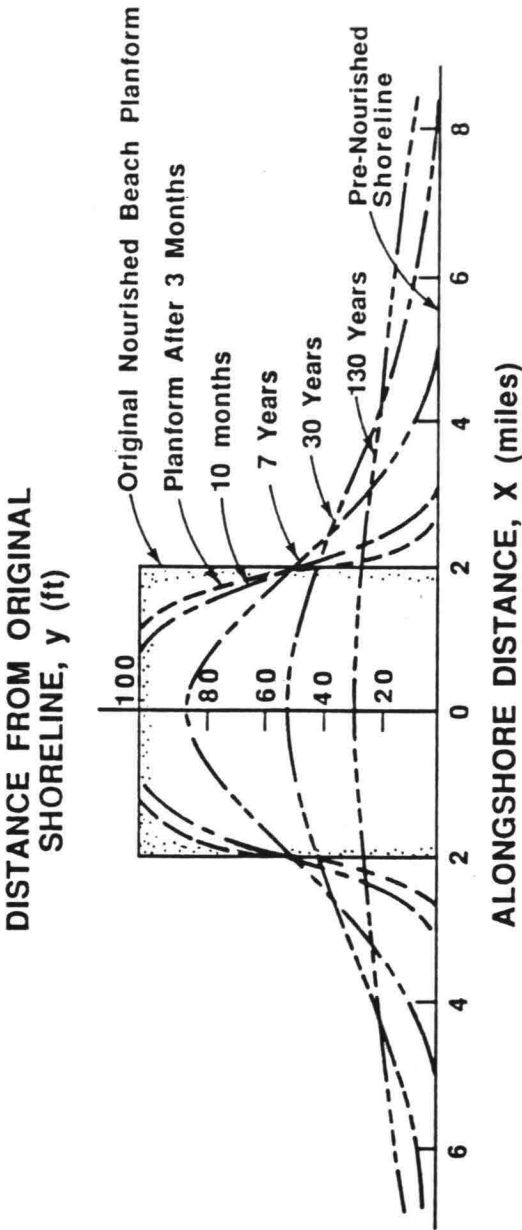
Figure 19.   Example of Evolution of Initially rectangular Nourished Beach Planform.
Example for Project Length, l, of 4 Miles and Effective Wave Height, H,
of 2 feet and Initial Nourished Beach Width of 100 Feet.
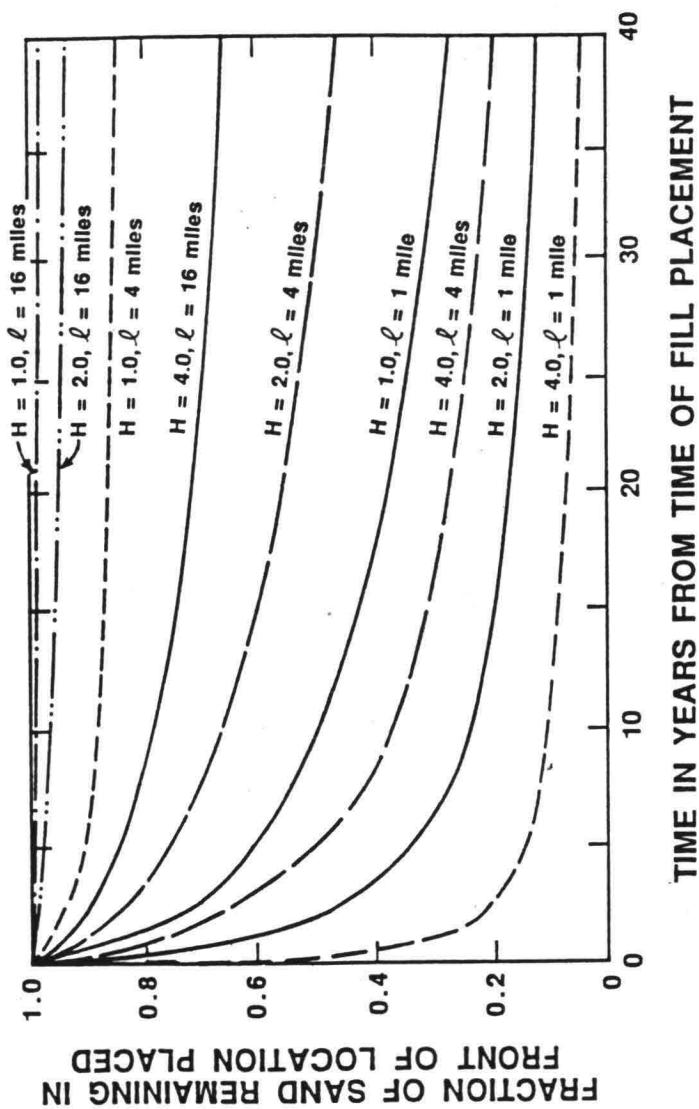
Figure   20.   Fraction of Material Remaining in Front of Location Placed for
                Several Wave Height, H, and Project Lengths, ℓ . Effect of
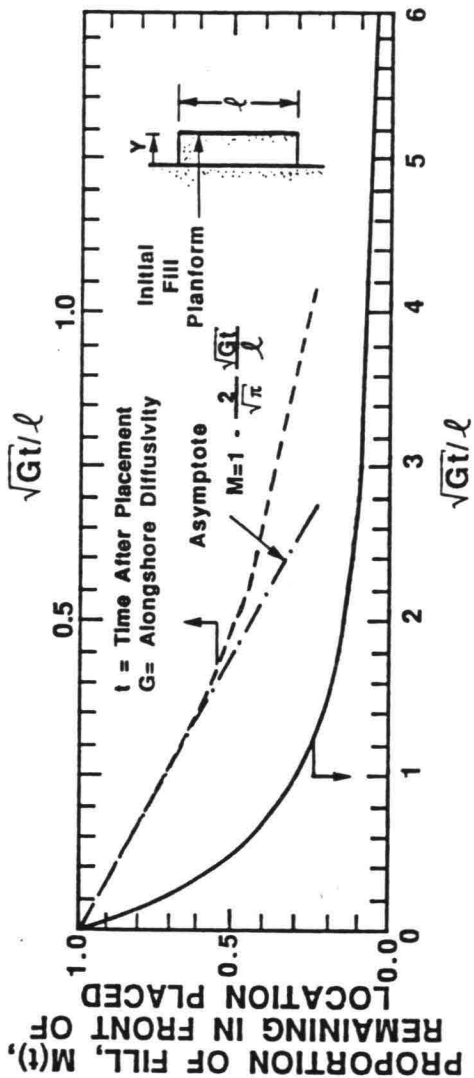   Longshore Transport.

Figure 21. Percentage of Material Remaining In Region Placed vs. the Parameter $\sqrt{Gt}/\ell$.

$$t_{50} = 8.7 \frac{\ell^2}{H_b^{5/2}}$$

(39a)

where t_{50} is in years, ℓ in miles and H_b is the breaking wave height in ft or

$$t_{50} = 0.172 \frac{\ell_2}{H_b^{5/2}}$$

(39b)

where t_{50} is still in years, ℓ is in kilometers and H_b is in meters.

3.3. VARIOUS FACTORS AFFECTING PERFORMANCE OF BEACH NOURISHMENT PROJECTS

3.3.1. Effect on Retention of Setting Back the Fill Ends from Project Boundaries

As noted earlier, there is an understandable interest by a community or other entity which is funding a project in retaining the sand within their boundaries as long as practical. One approach to this concern would be to install retaining or stabilization structures near the ends of the fill. A second would be to simply set-back the limits of the fill from the project boundaries with the understanding that the sand would soon "spread out". Omitting the details, Fig. 22 presents results for relative end set-backs Δ/ℓ = 0, 0.2 and 0.5. It is seen that the effects are greatest early in the project life (say $\sqrt{Gt}/\ell$ = 0.6 or 0.8) where a set back $\Delta/\ell = 0.5$ would increase the percent material retained from 42% to 73%.

3.3.2. Effect of Ends on a Beach Fill

It is somewhat interesting to evaluate the effect on longevity of providing a fillet at the two ends of a fill which is otherwise rectangular in planform. Basing the longevity on the retention of sand within the placed planform, it is interesting that tapered-end planforms have a substantially greater longevity than rectangular planforms. The reasons are apparent by examining Fig. 19. The loss rates of a rectangular planform fill are higher over the first increment of time than over the same increment of time but later in the project history. It is seen from Fig. 19 that the evolution of the planform occurs with the early changes occurring where the planform changes are the most extreme. This is not surprising when one recalls that the governing equation (Eq. (26)) is the heat conduction equation and that the fill planform is equivalent to a temperature distribution above background of the same form in an infinitely long rod. Returning again to the
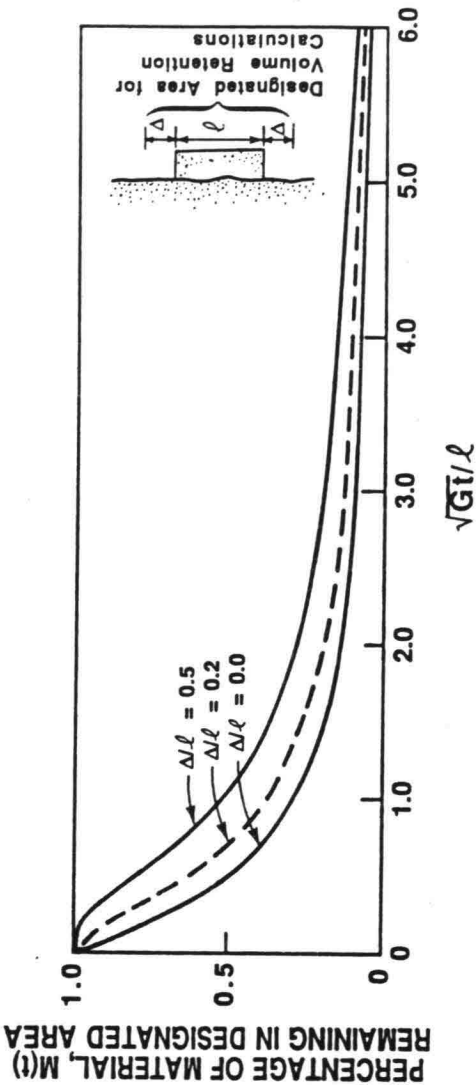
Figure   22.   Percentage of Material Remaining in Designated Area of Length, $\ell + 2\Delta$. Rectangular Beach Fill of Length, ℓ.

tapered end planform, which approximates the evolved rectangular planform at a later stage, the evolution of the tapered end fill at an early stage approximates that of a rectangular fill at a later stage.

Figures 23 and 24 present calculated evolutions for rectangular and tapered end planforms, respectively and Table 2 summarizes the cumulative losses from the region placed over the first five years. It is seen that the tapered end fills have reduced the end losses by about 33%.

Table 2: Comparison of Cumulative Percentage Losses from Rectangular and Tapered Fill Planforms (G = 0.02 ft^2/sec; ℓ = 3 miles; Y = 55 ft)

Years After Placement	Cumulative Percentage Losses With	
	Rectangular Planform	Rectangular Planform With Triangular Fillets
1	5.7	2.4
2	9.5	4.6
3	11.8	6.6
4	13.8	8.3
5	15.5	9.8

3.3.3. Project Downdrift of a Partial or Complete Littoral Barrier

In this case the project is located downdrift of a partial or complete littoral barrier, such as a jettied inlet. We will denote the net longshore transport as Q_0 and the bypassed quantities as $F\,Q_0$ ($0 < F < 1$), see Fig. 25. In this case, the fraction remaining, $M_2(t)$, is

$$M_2(t) = \frac{\int_0^\ell V(x,t)dx}{V_o \ell} \tag{40}$$

and can be shown to be

$$M_2(t) = \mathrm{erf}\left(\frac{\ell}{\sqrt{Gt}}\right) + \frac{1}{\sqrt{\pi}}\,\frac{\sqrt{Gt}}{\ell}\left(e^{-(\ell/\sqrt{Gt})^2} - 1\right) - \frac{(1-F)Q_o t}{V_o \ell} \tag{41}$$
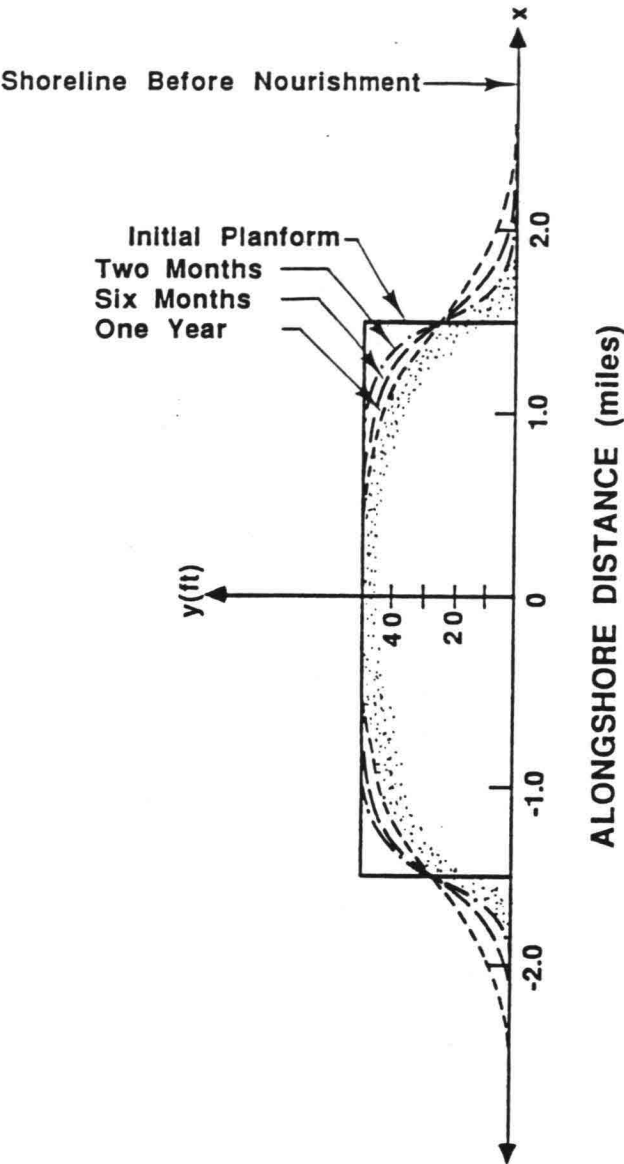
Figure 23. Calculated Evolution of a Rectangular Planform Beach Nourishment Project. Planforms Presented for Initial Conditions and 1,2 and 5 Years After Placement.
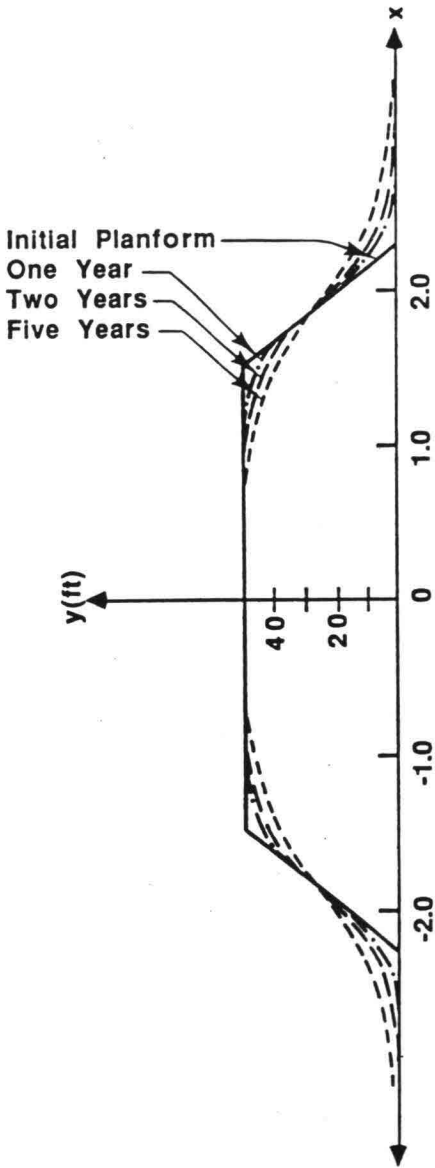
Figure 24. Calculated Evolution of a Rectangular Planform with Triangular End
Fillets. Planforms Presented for Initial Conditions and 1,2 and 5
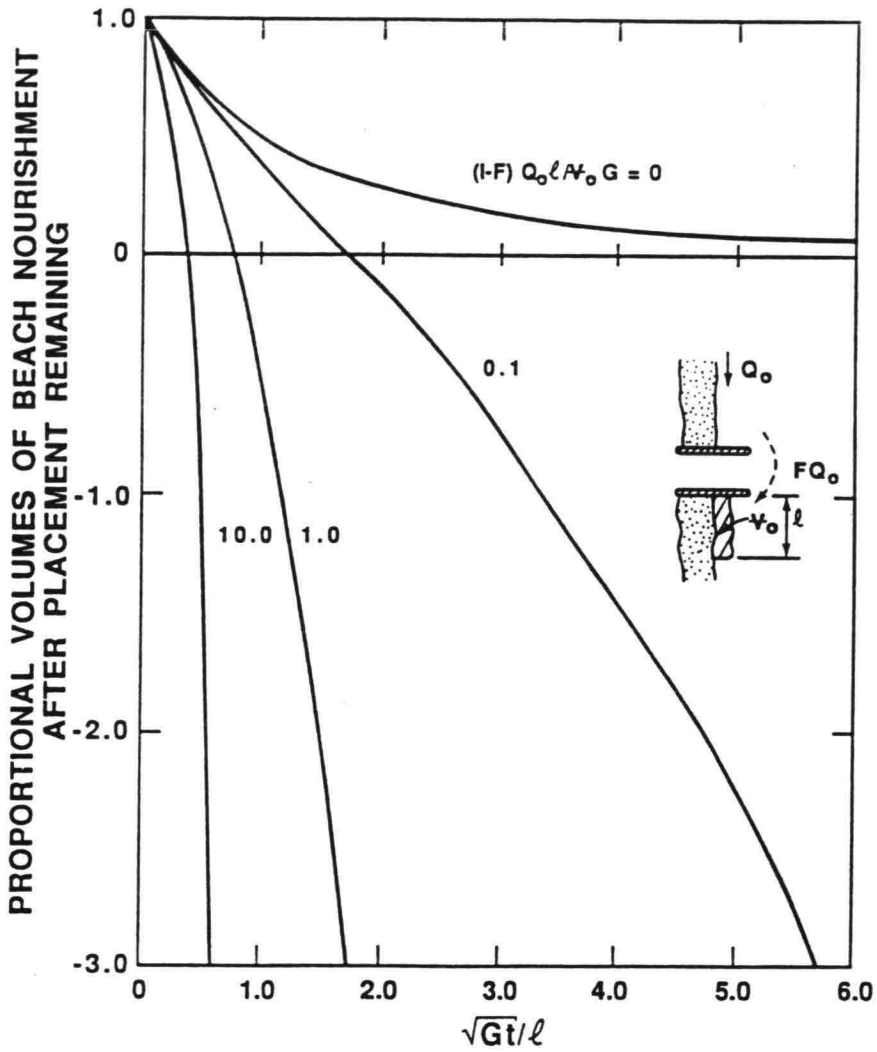Years After Placement.

Figure 25. Proportional Volumes of Beach Nourishment Remaining After Placement vs, $\sqrt{Gt}/\ell$ and $(1-F)Q_0/V_0 G$.

in which V_o is the volume placed. Eq. (41) is presented vs $\sqrt{Gt}/\ell$ in Fig. 25 for various values of (1-F) $Q_o\ell/V_oG$. This latter parameter represents the ratio of longshore transport losses due to a bypassing deficit to those losses resulting from the anomalous planform.

3.4. A CASE EXAMPLE - BETHUNE BEACH

In 1985, shorefront property owners in Bethune Beach, Volusia County, FL applied for a permit to construct two segments of armoring. The Governor and Cabinet of Florida initially deferred a decision requesting that consideration be given to utilizing the same funds for beach nourishment. The two segment lengths were 925 ft and 3,850 ft, as presented in Fig. 26. The designation beside each segment (e.g. VO 353) is the identifier given by the Division of Beaches and Shores to the permit application. The cost of the revetments was about $200 per foot which at a nourishment cost of $6 per cubic yard would purchase approximately 33 cubic yards per front foot or a total of 160,000 cubic yards for the two segments combined.

Rather simple numerical modeling was carried out using Eqs. (26) and (27) with monthly averaged wave heights as determined by the University of Florida's wave gage at nearby Marineland, FL. The results of this numerical modeling are presented in Figs. 26 and 27. Figure 26 presents the planform evolution after one month and one year. It is seen in accordance with earlier discussions, that due to the relative short lengths of these segments, the sand spreads out rapidly in an alongshore direction. Figure 26 presents, as a function of time, the volume of sand remaining in front of the two segments where the nourishment would have been placed.

4. Damage Reduction Due to Beach Nourishment

The concept of reduction in storm damage by beach nourishment will be illustrated by two approaches. First, data collected and summarized by Shows (1978) documented the relationship between average damage costs suffered by a structure as a function of the proximity of that structure to the shoreline set-back line in Bay County. The set-back line is approximately parallel to the shoreline. Figure 28 presents these results for 540 structures in Bay County following Hurricane Eloise in 1975. The horizontal axis is the structure location relative to the set-back line which is more or less parallel to the shoreline. Relative to beach nourishment, the two most significant features of Fig. 28 are: (1) the steeply rising damage function with proximity to the set-back line (or shoreline), and (2) the possibility of displacing the damage function seaward by beach nourishment which would translate the curve in Fig. 28 horizontally to the left by the width of beach added. As a second illustration consider the situation in Fig. 28 which corresponds to a profile off Sand Key, Florida. A peak storm tide of 11 ft and an offshore breaking wave height of 20 ft will be assumed for purposes of this example. These conditions are believed to be reasonably representative of a 100 year return period. Considering the

OFFSHORE
DISTANCE (ft)
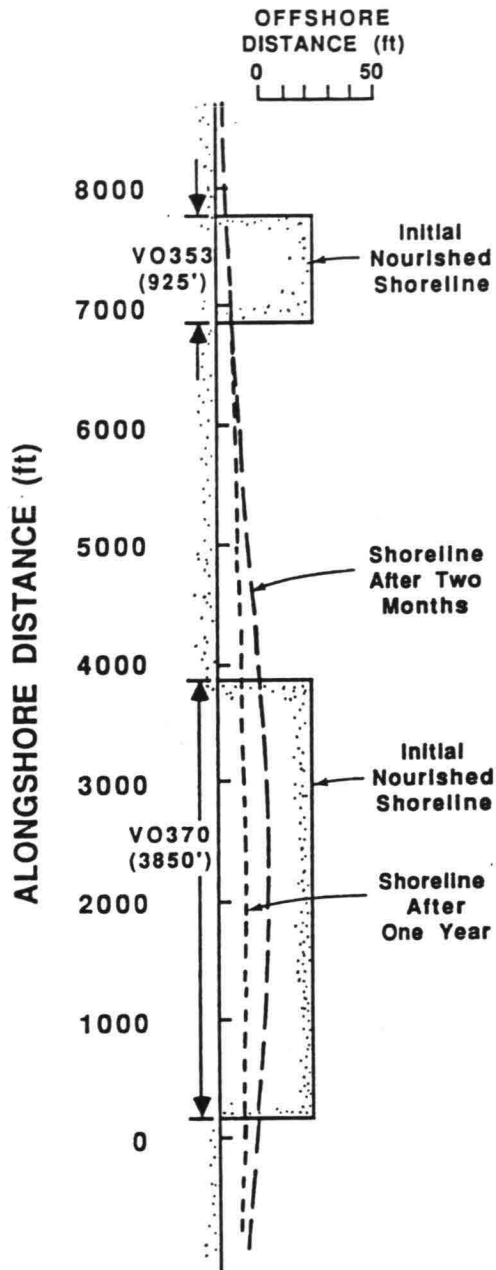
Figure 26. Initial and Subsequent Planforms of Nourished
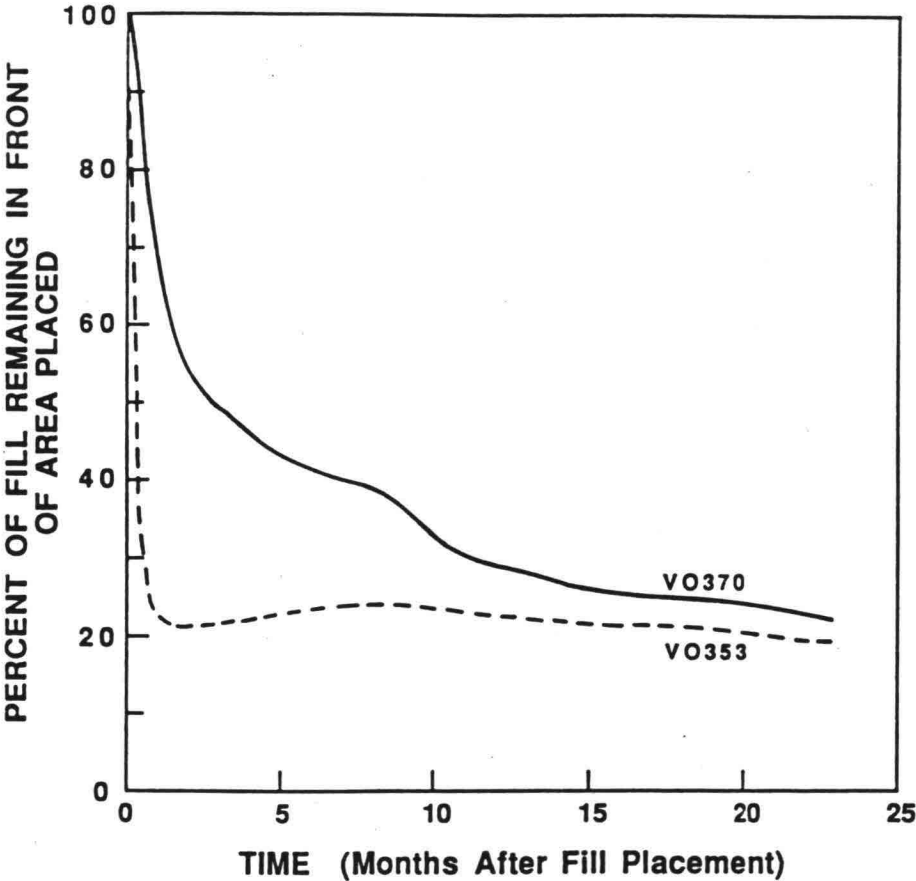Beach. Bethune Beach, Florida, Example.

Figure 27. "Loss" of Beach Fill From Infront of Area Placed as a Result
of Longshore Transport. Bethune Beach, Florida, Example.

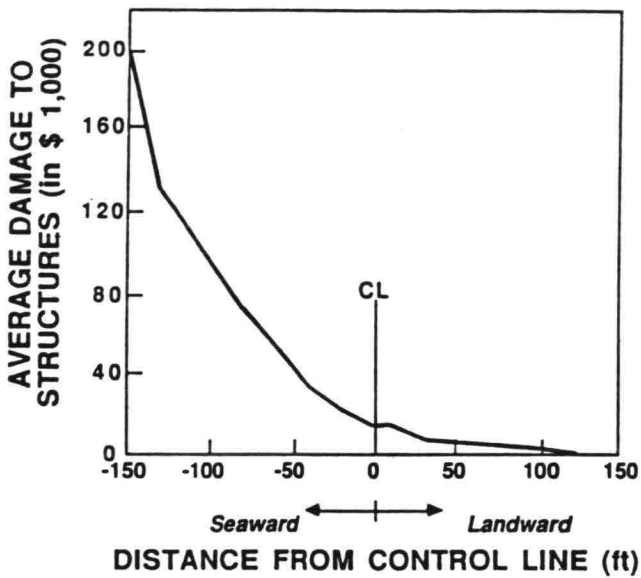Figure   28.   Damage to Structure in Relation to its Location
              with Control Line (Resulting From Study of 540
              Structures in Bay County After Hurricane Eloise,
              by Shows, 1978).

pre-nourishment condition and utilizing the breaking wave model reported by Dally, Dean and Dalrymple (1985), the wave height distribution is presented in Fig. 29. Considering now a beach nourishment project which advances the shoreline gulfward a distance of 40 ft, the wave height distribution is as presented in Fig. 29. Table 3 summarizes the wave height at the seawall for the original and nourished conditions and also presents a measure of the damage potential for the two cases with and without nourishment. In these results the damage potential is considered to be proportional to the cube of the wave height. The presence of the nourishment project reduces the damage potential by nearly a factor of four!

Table 3: Summary of Wave Height and Damage Potential Reduction at Seawall with Beach Nourishment Project*

Case	Wave Heights (ft)	Damage Potential αH^3
Without Nourishment	4.5	90
With Nourishment	2.9	24

*Refer to Fig. 29.

There are various general approaches to developing estimates of damage reduction due to beach nourishment. One approach is to attempt to carry out a structure-by-structure damage analysis due to a storm of a certain severity as characterized by a storm tide, wave height and duration. The damage due to many such storms weighted by their probability of occurrence can then be combined to yield the total expected damage. A second approach and that which will be employed here is to recognize that during a particular storm, it is appropriate to consider (1) relative alongshore uniformity of wave attack, and (2) a representative proportional damage as a function of storm severity and beach width, W.

Having demonstrated qualitatively the damage reduction due to beach nourishment, we will proceed to a formalized procedure, making assumptions and simplifying as necessary.

The methodology will assume that a proportional structural damage curve is available as a function of storm return period, T_R, and additional beach width, w. Curves of this type would be site specific depending on the location of the existing structure relative to the shoreline, and the design and quality of the structures. Figure 30 presents one example of such a set of relationships. The cumulative probability, $P(T_R)$ of encountering a storm of return period T_R in any given year is
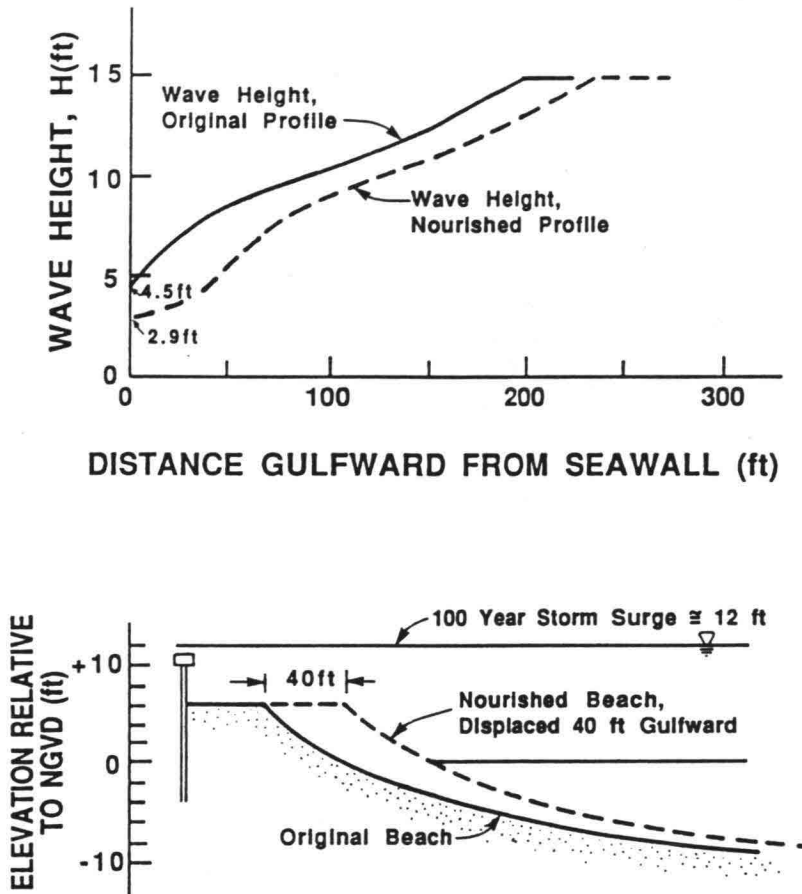
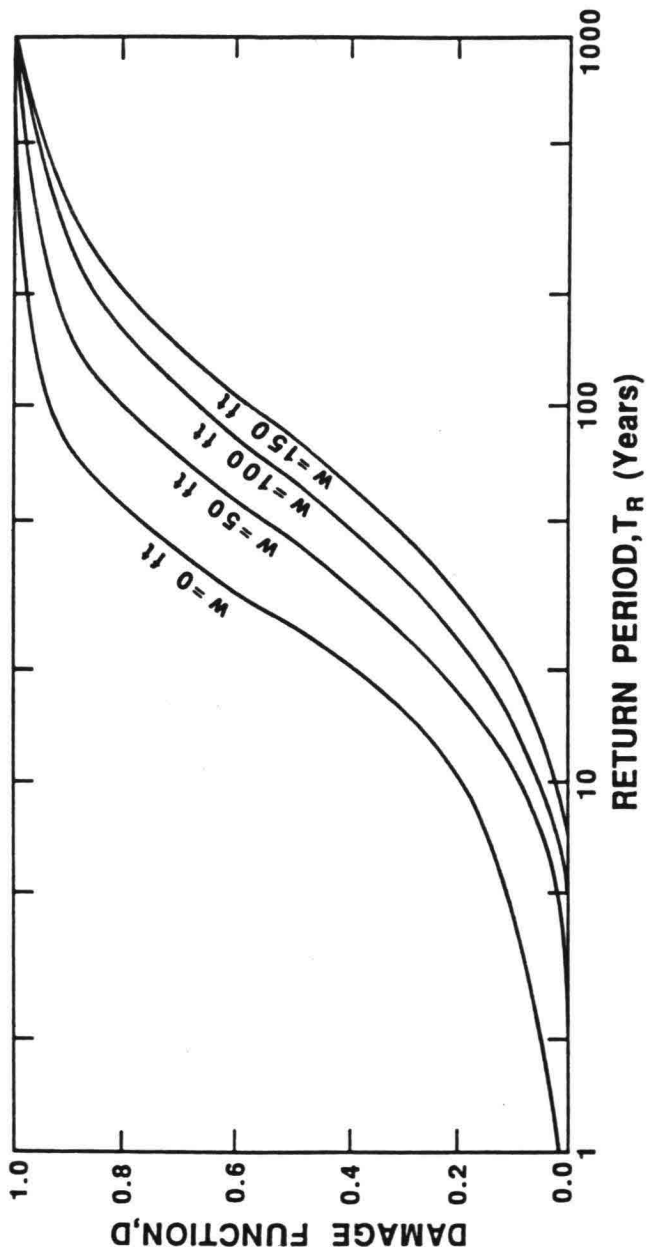Figure 29. Wave Height Reduction at Seawall Due to Presence of a Beach Nourishment Project.

Figure 30. Assumed Damage Function, D, for Various Beach Widths, w, and Storm Return Periods, T_R.

$$P(T_R) = \frac{1}{T_R} \tag{42}$$

The information presented in Fig. 30 can be developed with varying degrees of realism through Monte Carlo simulation methodology such that the result is applied directly and easily. One approach is to assume that the damage from one storm is repaired prior to the occurrence of a succeeding storm. The present worth damage factor, F(w,I,J) in a period of J years, depends on the interest rate, I, the maintained beach width, w, and represents the ratio of present worth of all damage values over the J year to the present structure value.

This method obviously embodies many approximations, but does provide a rational framework for a very complex problem. One realization of the present worth damage factor for storms over the next J years if the beach width is maintained constant can be shown to be

$$F^K(w,I,J) = \sum_{j=1}^{J} D(w,T_R) \frac{1}{(1+I)^j} \tag{43}$$

Here the superscript K denotes the Kth realization and the selection of the J storms is carried out through Monte Carlo simulation in accordance with the cumulative probability distribution, $P(= 1/T_R)$. Thus, in addition to the most probable damage, it is possible to develop probability distributions of the present worth damage factor.

Table 4 presents the values of the average present worth damage factor $\bar{F}(w,I,\infty)$ for all future damages and constant beach width, w. As expected, for the higher interest rates, the present worth values are less. Of relevance is that the greatest incremental benefits occur for the beaches that are initially the most narrow, i.e. for the situation in which the structures are in greatest jeopardy. This reinforces the earlier statement that sand transported from a nourishment project that widens adjacent beaches should be recognized as a financial benefit to rather than a loss from that project.

A somewhat more realistic approach would be to recognize that due to erosional processes, it would be necessary to renourish every j_* years during which the beach would narrow from w_o to w' at an annual recession rate, r,

$$r = \frac{w_o - w'}{j_*} \tag{44}$$

For this case, one realization of the present worth damage function, $F(w_o, j_*, r, I, J)$, is determined as

$$F^K(w_o, j_*, r, I, J) = \sum_{n=0}^{J/j_*} \sum_{j=nj_*}^{(n+1)j_* - 1} D\left\{ \left[w_o - r \left(j - nj_* \right) \right] T_R \right\} \frac{1}{(1+I)^j} \tag{45}$$

Table 4: Present Worth Damage Factor, F(w,I) as a Function of Interval Considered and Beach Width

Interest Rate, I	Present Worth Damage Factor, F(w,I), for Various Beach Widths, w			
	w = 0 ft	w = 50 ft	w = 100 ft	w = 150 ft
6%	1.84	0.89	0.59	0.37
8%	1.39	0.56	0.44	0.27
10%	1.07	0.49	0.44	0.27

Each of the inner summations represents the contributions to the present worth damage factor during one nourishment interval. Damage reductions employing Eq. (42) can assist in identifying the optimal renourishment interval, j_*.

SYMBOLS

A   Sediment scale parameter
A'   Non-dimensional sediment scale parameter
B   Berm height
$_b$ Subscript denoting breaking
C   Wave celerity
C_G Wave group velocity
D   Sediment diameter or damage function
$_F$ Subscript signifying "fill"
G   Longshore diffusivity
g   Gravitational constant
H_b Breaking wave height
H_o Deep water wave height
h   Water depth
h_* Depth of limiting motion
I   Interest rate
j_* Renourishment interval in years
K   Sediment transport factor
^{K} Superscript denoting K^{th} realization
m   Sand area for idealized initial strip distribution
$_N$ Subscript signifying "native"
n   Summation index
p   In place porosity of sediment

Q   Cross-shore sediment transport rate
r   Recession rate
S   Sea level rise
s   Relative specific gravity of sediment to water in which it is immersed.
T_R Storm return period
t   Time
V Volume of sand added in nourishment project or volume of sand remaining
V' Non-dimensional volume
W   Beach width
W.   Width of equilibrium profile
x   Longshore distance
Y   Initial nourished beach width for idealized initial rectangular planform distribution
y   Distance offshore
α_b Azimuth of breaking wave direction. Taken as direction <u>from</u> which wave originates
$\alpha.$ Azimuth of wave at depth h., same directional convention as above
β Azimuth of outward normal of shoreline
β_o Azimuth of outward normal of baseline
λ Ratio of breaking wave height to breaking depth
μ Azimuth of baseline
ℓ Length of initial idealized beach nourishment project

REFERENCES

BRUUN, P. (1962) "Sea Level Rise as a Cause of Shore Erosion", *ASCE Journal of Waterways and Harbors Division*, Vol. 1, p. 116-130.

DALLY, W.R., DEAN R.G., AND DALRYMPLE, R.A. (1985) "Wave Height Variation Across Beaches of Arbitrary Profile", *Journal of Geophysical Research*, Vol. 90, No. C6, pp. 11917-11927.

DEAN, R.G. (1987) "Additional Sediment Input to the Nearshore Region", *Shore and Beach*, Vol. 55, Nos. 3-4, p. 76-81.

SHOWS, E.W. (1978) "Florida's Coastal Setback Line - An Effort to Regulate Beachfront Development", Vol. 4, Nos. 1/2, *Coastal Zone Management Journal*, p. 151-164.