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DOI
10.1155/2019/4360643

Publication date
2019

Document Version
Final published version

Published in
Mathematical Problems in Engineering

Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 4360643, 14 pages
https://doi.org/10.1155/2019/4360643

Research Article

Fuzzy Adaptive DSC Design for an Extended Class of MIMO Pure-Feedback Non-Affine Nonlinear Systems in the Presence of Input Constraints

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Received 27 August 2018; Accepted 19 December 2018; Published 2 January 2019

Academic Editor: Sabri Arik

A novel adaptive fuzzy dynamic surface control (DSC) scheme is for the first time constructed for a larger class of (multi-input multi-output) MIMO non-affine pure-feedback systems in the presence of input saturation nonlinearity. First of all, the restrictive differentiability assumption on non-affine functions has been canceled after using the piecewise functions to reconstruct the model for non-affine nonlinear functions. Then, a novel auxiliary system with bounded compensation term is firstly introduced to deal with input saturation, and the dynamic system employed in this work designs a bounded compensation term of tangent function. Thus, we successfully relax the strictly bounded assumption of the dynamic system. Additionally, the fuzzy logic systems (FLSs) are used to approximate unknown continuous systems functions, and the minimal learning parameter (MLP) technique is exploited to simplify control design and reduce the number of adaptive parameters. Finally, two simulation examples with input saturation are given to validate the effectiveness of the developed method.

1. Introduction

In the past several decades, approximation-based adaptive control of nonlinear systems has been attracting much attention, and many significant results have been achieved [1–11]. Among them, the fuzzy logic systems (FLSs) and neural networks (NNs) have been successfully employed to approximate the unknown nonlinear functions. In addition, as a breakthrough in nonlinear control, approximation-based adaptive backstepping control has been extensively introduced to achieve global stability for many classes of nonlinear systems [12–17]. For example, in [12], an adaptive fuzzy control scheme was proposed for a class of nonlinear pure-feedback systems under the framework of backstepping, which requires no *priori* knowledge of the systems dynamic. In [14], an adaptive fuzzy control scheme is presented for a class of pure-feedback nonlinear systems with immeasurable states by utilizing backstepping methodology. Recently, for a class of stochastic nonlinear systems with unknown control direction and unknown dead-zones, an adaptive fuzzy backstepping control method is presented in [17]. However, the problem of "explosion of complexity" caused by repeated differentiations of the virtual control law seriously limits the application of conventional backstepping technique. Thus, the dynamic surface control (DSC) technique has been creatively proposed to avoid this problem effectively by introducing a first-order low-pass filter at each step. Furthermore, compared with strict-feedback systems, pure-feedback systems have a non-affine fashion that the control inputs or variables appear nonlinearly in uncertain systems functions, which leads to the design being more difficult [18, 19]. Moreover, in contrast with SISO pure-feedback nonlinear systems, the control design of MIMO pure-feedback nonlinear systems is, as well known, more complicated due to the couplings among various inputs and outputs [20].

On the other hand, input saturation nonlinearity, as one of the most important input constraints, usually appears in many industrial control systems [21]. In many applications, the input saturation nonlinearity may severely cause degradation of system performance, instability, or even

damage. Consequently, the adaptive control of nonlinear systems in the presence of input saturation nonlinearity has been an active topic and attracted increasing attention in recent years [22–30]. For example, in [22], an adaptive fuzzy controller is constructed for pure-feedback stochastic nonlinear systems to deal with input constraints based on the adjustment of commanded input signal. In [25], an adaptive neural controller is investigated for a class of pure-feedback nonlinear time-varying systems with asymmetric input saturation nonlinearity in combination with the Gaussian error function. Recently, for a class of uncertain nonlinear systems with input saturation constraint and external disturbances, a tracking control scheme is proposed by introducing an auxiliary system in [27]. However, it should be pointed out that, for all above the state-of-the-art schemes [22–30] to work for pure-feedback uncertain nonlinear systems subject to input saturation, the non-affine function is always assumed to be differentiable with respect to control variables or inputs, which is restrictive arising from the fact that non-smooth nonlinearities such as dead zone, backlash, and saturation widely exist in various kinds of practical systems [22–25], which makes the non-affine functions non-differentiable and motives us to explore new methods to overcome this limitation [22].

As a matter of fact, overcoming this limitation is challenging. This is because FLSs approximation errors will inevitably occur while adopting FLSs to approximate unknown systems functions within a compact set, this, in combination with external disturbances, may seriously degrade control performance or even give rise to closed-loop system instability. Additionally, there also exist a large number of fuzzy weights that need to be tuned online, which drastically increases the computational burden [28]. Therefore, a design technique needs to be developed that is able to guarantee that all system trajectories stay in the appropriate compact sets all the time, and the MLP technique needs to be employed to solve the explosion of learning parameters. Based on the aforementioned observations, this paper addresses the control problem for a more general class of MIMO pure-feedback nonlinear systems in the presence of input saturation nonlinearity. What is more, to the best of authors' knowledge, the control design of this huger class of nonlinear systems has not been reported, which is still an open problem with theoretical and applicable significance. The main contributions of this paper are highlighted as follows: (1) it seems that this is the first work that considers both the MIMO non-affine nonlinear systems and input saturation even though some existing works focused on the same topic; (2) to handle input saturation, compared with the auxiliary system $\dot{\wp}_j = -\kappa_j \wp_j + sat(o_j) - o_j$ presented in [27, 30], the dynamic system employed in this work designs a bounded compensation term $\xi_j \tanh \wp_j$, and, thus, the assumption that $\wp_j$ is bounded is cancelled; (3) in contrast to the existing strategies [22–30], we allow the non-affine functions of MIMO input-saturated nonlinear systems to be non-differentiable via the reconstruction of non-affine functions using appropriate piecewise functions, which removes the restrictive differentiability assumption on non-affine functions.

The rest of this paper is organized as follows. Section 2 presents the problem statement and preliminaries. The adaptive controller design is given in Section 3. Section 4 is devoted to stability analysis. In Section 5 simulation results are presented to show the effectiveness of the proposed scheme, followed by the conclusion in Section 6.

2. Problem Statement and Preliminaries

Consider the following MIMO pure-feedback systems [23]:

$$\begin{aligned}
\dot{\chi}_{j,i_j} &= \varphi_{j,i_j}\left(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}\right) + D_{j,i_j}(\chi,t), \\
&\qquad\qquad 1 \le i_j \le \rho_j - 1 \\
\dot{\chi}_{j,\rho_j} &= \varphi_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j}, u_j\left(o_j\right)\right) + D_{j,\rho_j}(\chi,t) \\
y_j &= \chi_{j,1}, \quad j = 1,2,\ldots,m
\end{aligned} \tag{1}$$

where $\chi_{j,i_j} \in R$ is the state of the jth subsystem, $\chi = [\overline{\chi}_{1,\rho_1}^T, \ldots, \overline{\chi}_{j,\rho_j}^T, \ldots, \overline{\chi}_{m,\rho_m}^T]^T \in R^N$ is the state vector of the whole system ($N = \rho_1 + \cdots + \rho_m$), where $\overline{\chi}_{j,\rho_j} = [\chi_{j,1}, \chi_{j,2}, \ldots, \chi_{j,\rho_j}]^T \in R^{\rho_j}$ and ρ_j is the order of the jth subsystem. $\overline{\chi}_{j,i_j} = [\chi_{j,1}, \ldots, \chi_{j,i_j}]^T \in R^{i_j}$, $u_j \in R$ and $y_j \in R$ are the input and output of the jth subsystem, respectively. $\varphi_{j,i_j}(\cdot)$ are unknown non-affine continuous functions, and $D_{j,i_j}(\chi,t)$, $i_j = 1,\ldots,\rho_j$, $j = 1,\ldots,m$ are the unknown external disturbances. $u_j(o_j)$ is the plant input subject to saturation and satisfying [30]

$$u_j\left(o_j\right) = sat\left(o_j\right) = \begin{cases} \operatorname{sign}\left(o_j\right) u_{j,M}, & o_j \ge u_{j,M} \\ o_j, & o_j < u_{j,M} \end{cases} \tag{2}$$

where $u_{j,M}$ is the bound of $u_j(o_j)$, $o_j \in R$ is the input saturation, and $u_j = u_j(o_j)$.

The design objective of this work is to construct a novel dynamic surface controller u_j such that (1) the output tracking error $z_{j,1} = \chi_{j,1} - y_{j,d}$ achieves preselected transient and steady bounds; (2) all signals of system (1) are semiglobally uniformly ultimately bounded (SGUUB); (3) the control input constraint is not violated.

Assumption 1. Define the functions $\psi_{j,i_j}(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}) = \varphi_{j,i_j}(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}) - \varphi_{j,i_j}(\overline{\chi}_{j,i_j}, 0)$. We assume that the functions $\psi_{j,i_j}(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1})$ satisfy

$$\begin{aligned}
\underline{\psi}_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,1i_j} &\le \psi_{j,i_j}\left(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}\right) \\
&\le \overline{\psi}_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,2i_j}, \chi_{j,i_j+1} \ge 0 \\
\underline{\psi}'_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,3i_j} &\le \psi_{j,i_j}\left(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}\right) \\
&\le \overline{\psi}'_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,4i_j}, \chi_{j,i_j+1} < 0
\end{aligned} \tag{3}$$

where $\underline{\psi}_{j,i_j}$, $\overline{\psi}_{j,i_j}$, $\underline{\psi}'_{j,i_j}$, and $\overline{\psi}'_{j,i_j}$ are unknown positive constants; $\hbar_{j,1i_j}$, $\hbar_{j,2i_j}$, $\hbar_{j,3i_j}$, and $\hbar_{j,4i_j}$ are unknown constants. And denote $\chi_{j,\rho_j+1} = u_j$, $\overline{\chi}_{j,\rho_j+1} = [\overline{\chi}^T_{j,\rho_j}, u_j]^T$ for notation conciseness.

Remark 2. In [22–30], the non-affine functions are always assumed to satisfy $\underline{q}_{j,i_j+1} \le \partial\varphi_{j,i_j}(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1})/\partial\chi_{j,i_j+1} \le \overline{q}_{j,i_j+1}$ and $\underline{q}_{j,\rho_j} \le \partial\varphi_{j,i_j}(\overline{\chi}_{j,\rho_j}, u_j)/\partial u_j \le \overline{q}_{j,\rho_j}$ with $\underline{q}_{j,i_j+1} > 0$, $\overline{q}_{j,i_j+1} > 0$, $\underline{q}_{j,\rho_j} > 0$, and $\overline{q}_{j,\rho_j} > 0$ being unknown constants. In fact, this assumption is used to ensure the controllability of system (1). However, the assumption $\underline{q}_{j,\rho_j} \le \partial\varphi_{j,i_j}(\overline{\chi}_{j,\rho_j}, u_j)/\partial u_j \le \overline{q}_{j,\rho_j}$ is too restrictive due to the fact that many kinds of non-smooth nonlinearities (e.g., dead-zone, backlash, or saturation, and so on) extensively exist in control input, leading to the non-differentiability of non-affine functions, even instability of closed-loop systems [10]. Even though some existing works like [16, 19] focus on the same topic, none of them addresses the control problem for both MIMO non-affine systems and input saturation problem. In other words, in this paper, we for the first time investigate a larger class of MIMO nonlinear systems considering both non-differentiable non-affine functions and input saturation.

Remark 3. From (3), there exist functions $\ell_{j,1i_j}(\overline{\chi}_{j,i_j+1})$ and $\ell_{j,2i_j}(\overline{\chi}_{j,i_j+1})$ taking values in $[0, 1]$ and satisfying

$$\begin{aligned}
&\psi_{j,i_j}\left(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}\right)\\
&\quad = \left(1 - \ell_{j,1i_j}\left(\overline{\chi}_{j,i_j+1}\right)\right)\left(\underline{\psi}_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,1i_j}\right)\\
&\qquad + \ell_{j,1i_j}\left(\overline{\chi}_{j,i_j+1}\right)\left(\overline{\psi}_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,2i_j}\right),\\
&\qquad\qquad \chi_{j,i_j+1} \ge 0\\
&\psi_{j,i_j}\left(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}\right)\\
&\quad = \left(1 - \ell_{j,2i_j}\left(\overline{\chi}_{j,i_j+1}\right)\right)\left(\underline{\psi}'_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,3i_j}\right)\\
&\qquad + \ell_{j,2i_j}\left(\overline{\chi}_{j,i_j+1}\right)\left(\overline{\psi}'_{j,i_j}\chi_{j,i_j+1} + \hbar_{j,4i_j}\right),\\
&\qquad\qquad \chi_{j,i_j+1} \ge 0
\end{aligned} \tag{4}$$

To make the control design succinct, define the functions $Q_{j,i_j}(\overline{\chi}_{j,i_j+1})$ and $\Delta_{j,i_j}(\overline{\chi}_{j,i_j+1})$ as

$$\begin{aligned}
&Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\\
&= \begin{cases}
\left(1 - \ell_{j,1i_j}\left(\overline{\chi}_{j,i_j+1}\right)\right)\underline{\psi}_{j,i_j} + \ell_{j,1i_j}\left(\overline{\chi}_{j,i_j+1}\right)\overline{\psi}_{j,i_j}, & \chi_{j,i_j+1} \ge 0\\
\left(1 - \ell_{j,2i_j}\left(\overline{\chi}_{j,i_j+1}\right)\right)\underline{\psi}'_{j,i_j} + \ell_{j,2i_j}\left(\overline{\chi}_{j,i_j+1}\right)\overline{\psi}'_{j,i_j}, & \chi_{j,i_j+1} < 0
\end{cases}\\
&\Delta_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\\
&= \begin{cases}
\left(1 - \ell_{j,1i_j}\left(\overline{\chi}_{j,i_j+1}\right)\right)\hbar_{j,1i_j} + \ell_{j,1i_j}\left(\overline{\chi}_{j,i_j+1}\right)\hbar_{j,2i_j}, & \chi_{j,i_j+1} \ge 0\\
\left(1 - \ell_{j,2i_j}\left(\overline{\chi}_{j,i_j+1}\right)\right)\hbar_{j,3i_j} + \ell_{j,2i_j}\left(\overline{\chi}_{j,i_j+1}\right)\hbar_{j,4i_j}, & \chi_{j,i_j+1} < 0
\end{cases}
\end{aligned} \tag{5}$$

Using (5), we can model the non-affine terms $\psi_{j,i_j}(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1})$ as

$$\begin{aligned}
\psi_{j,i_j}\left(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1}\right) &= Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\chi_{j,i_j+1}\\
&\quad + \Delta_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)
\end{aligned} \tag{6}$$

In view of (5), it can be known that

$$\begin{aligned}
&0 < \underline{Q}_{j,i_j} \le Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right) \le \overline{Q}_{j,i_j},\\
&0 \le \left|\Delta_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\right| \le \hbar^*_{j,i_j}
\end{aligned} \tag{7}$$

where $\underline{Q}_{j,i_j} = \min_{i_j=1,2,\ldots,\rho_j}\{\underline{\psi}_{j,i_j}, \overline{\psi}_{j,i_j}, \underline{\psi}'_{j,i_j}, \overline{\psi}'_{j,i_j}\}$, $\overline{Q}_{j,i_j} = \max_{i_j=1,2,\ldots,\rho_j}\{\underline{\psi}_{j,i_j}, \overline{\psi}_{j,i_j}, \underline{\psi}'_{j,i_j}, \overline{\psi}'_{j,i_j}\}$ and $\hbar^*_{j,i_j} = \max\{|\hbar_{j,1i_j}| + |\hbar_{j,2i_j}| + |\hbar_{j,3i_j}| + |\hbar_{j,4i_j}|\}$. According to (6) and the definition of $\psi_{j,i_j}(\overline{\chi}_{j,i_j}, \chi_{j,i_j+1})$, system (1) can be rewritten as

$$\begin{aligned}
\dot{\chi}_{j,i_j} &= \varphi_{j,i_j}\left(\overline{\chi}_{j,i_j}, 0\right) + Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\chi_{j,i_j+1}\\
&\quad + \Delta_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right) + D_{j,i_j}(\chi, t),\\
&\qquad\qquad i_j = 1, 2, \ldots, \rho_j - 1\\
\dot{\chi}_{j,\rho_j} &= \varphi_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j}, 0\right) + Q_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j+1}\right)u_j\\
&\quad + \Delta_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j+1}\right) + D_{j,\rho_j}(\chi, t)\\
y_j &= \chi_{j,1}
\end{aligned} \tag{8}$$

Assumption 4. The reference signal $y_{j,d}(t)$ is continuous and available, and there exists a positive constant P_{j0} such that $\Omega_{j0} := \{(y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}) : (y_{j,d})^2 + (\dot{y}_{j,d})^2 + (\ddot{y}_{j,d})^2 \le P_{j0}\}$.

Assumption 5. For $i_j = 1, \ldots, \rho_j$, $j = 1, \ldots, m$, there exist unknown positive constants D^*_{j,i_j} satisfying $|D_{j,i_j}(\chi, t)| \le D^*_{j,i_j}$.

Lemma 6 (see [8]). *Consider the first-order dynamical system:*

$$\dot{\Upsilon}(t) = -\lambda\Upsilon(t) + b\tau(t) \tag{9}$$

with $\lambda > 0$, $b > 0$ and $\tau(t)$ a positive function. Then, for any given bounded initial condition $\Upsilon(t_0) \ge 0$, the inequality $\Upsilon(t) \ge 0$, $(\forall t \ge 0)$ holds.

Lemma 7 (see [17]). *For any $\gamma \in R$ and $\forall \aleph > 0$, the hyperbolic tangent function* $\tanh(\cdot)$ *fulfills*

$$0 \le |\gamma| - \gamma \tanh\left(\frac{\gamma}{\aleph}\right) \le 0.2785\aleph$$
$$0 \le \gamma \tanh\left(\frac{\gamma}{\aleph}\right) \tag{10}$$

The fuzzy logic systems (FLSs) are employed as function approximator. Construct FLSs with the following IF-THEN rules:

$$R_l : \text{If } \chi_1 \text{ is } F_1^l \text{ and } \ldots \text{ and } \chi_n \text{ is } F_n^l$$
$$\text{THEN } y \text{ is } B^l, \quad l = 1, 2, \ldots, N. \tag{11}$$

where $\chi = [\chi_1, \chi_2, \ldots, \chi_n]^T \in R^n$ and y are input and output of the FLSs. Based on the singleton fuzzifier, product inference, and center average defuzzifier, the FLSs can be formulated as

$$y(\chi) = \frac{\sum_{i=1}^{N} \max_{y \in R} u_{B^i}(y) \prod_{j=1}^{n} u_{F_j^i}(\chi_j)}{\sum_{i=1}^{N} \left[\prod_{j=1}^{n} u_{F_j^i}(\chi_j)\right]} \tag{12}$$

where $u_{F_j^i}(\chi_j)$ and $u_{B^i}(y)$ are the membership of F_j^i and B^i, respectively. Let

$$\overline{\phi}_i(\chi) = \frac{\prod_{j=1}^{n} u_{F_j^i}(\chi_j)}{\sum_{i=1}^{N} \left[\prod_{j=1}^{n} u_{F_j^i}(\chi_j)\right]} \tag{13}$$

where $\overline{\Theta}_i = \max_{y \in R} u_{B^i}(y)$, $\Theta = [\overline{\Theta}_1, \overline{\Theta}_2, \ldots, \overline{\Theta}_N]^T$, and $\phi(\chi) = [\overline{\phi}_1(\chi), \overline{\phi}_2(\chi), \ldots, \overline{\phi}_N(\chi)]^T$. Then, the FLSs can be expressed as follows:

$$y(\chi) = \Theta^T \phi(\chi) \tag{14}$$

Lemma 8 (see [23]). *On a compact set Ω_χ, if $f(\chi)$ is a continuous function, for any given constant $\varpi^* > 0$, then there exist FLSs $y(\chi)$ such that*

$$\sup_{\chi \in \Omega_\chi} |f(\chi) - y(\chi)| \le \varpi^* \tag{15}$$

3. Fuzzy Adaptive Controller Design

In this section, an adaptive fuzzy controller is proposed for a larger class of MIMO pure-feedback nonlinear systems (1) utilizing the DSC technique. To start, consider the following change of coordinates:

$$z_{j,1} = \chi_{j,1} - y_{j,d},$$
$$z_{j,i_j} = \chi_{j,i_j} - \nu_{j,i_j}, \quad i_j = 2, 3, \ldots, \rho_j - 1, \tag{16}$$
$$z_{j,\rho_j} = \chi_{j,\rho_j} - \nu_{j,\rho_j} - \xi_j \tanh \wp_j$$

where $z_{j,1}$ is the output tracking error, ν_{j,i_j} is the output of the first-order filter with s_{j,i_j-1} as the input, ξ_j is a positive design parameter, and $\wp_j$ is a dynamic system defined as

$$\dot{\wp}_j = \frac{\cosh^2 \wp_j}{\xi_j}\left(-\kappa_j \tanh \wp_j + sat\left(o_j\right) - o_j\right), \quad \wp_j(0) = 0 \tag{17}$$

where $\kappa_j > 0$ is a design parameter.

Remark 9. It has to be noted that, compared with the existing work [27, 30], a novel auxiliary system is proposed, and the dynamic system employed in this brief designed a bounded compensation term $\xi_j \tanh \wp_j$ to cope with input saturation problem. Therefore, the restrictive bounded assumption of the dynamic system has been deleted.

Since $\varphi_{j,i_j}(\overline{\chi}_{j,i_j}, 0)$, $i_j = 1, \ldots, \rho_j$ are unknown continuous functions, we use fuzzy logic systems (FLSs) to approximate them as follows:

$$\varphi_{j,i_j}\left(\overline{\chi}_{j,i_j}, 0\right) = \Theta_{j,i_j}^T \phi\left(\overline{\chi}_{j,i_j}\right) + \varpi_{j,i_j}, \quad i_j = 1, \ldots, \rho_j \tag{18}$$

where ϖ_{j,i_j} is the approximation error and satisfies $|\varpi_{j,i_j}| \le \varpi_{j,i_j}^*$ with $\varpi_{j,i_j}^* > 0$ being an unknown constant.

Define

$$\Phi_{j,i_j} = \underline{Q}_{j,i_j}^{-1} \left\|\Theta_{j,i_j}\right\|^2, \quad i_j = 1, \ldots, \rho_j \tag{19}$$

where Φ_{j,i_j} are unknown constants and $\widehat{\Phi}_{j,i_j}$ is the estimate of Φ_{j,i_j} with $\widehat{\Phi}_{j,i_j} = \Phi_{j,i_j} - \widetilde{\Phi}_{j,i_j}$.

Step j, 1. Differentiating $z_{j,1}$ along with (16) yields

$$\dot{z}_{j,1} = \varphi_{j,1}\left(\overline{\chi}_{j,1}, 0\right) + Q_{j,1}\left(\overline{\chi}_{j,2}\right)\chi_{j,2} + \Delta_{j,1}\left(\overline{\chi}_{j,2}\right) + D_{j,1}(\chi, t) - \dot{y}_{j,d} \tag{20}$$

Consider the following quadratic Lyapunov function candidate:

$$V_{z_{j,1}} = \frac{1}{2} z_{j,1}^2 \tag{21}$$

Invoking (7), (20), and Assumption 5, we have

$$\dot{V}_{z_{j,1}} \le z_{j,1}\varphi_{j,1}\left(\overline{\chi}_{j,1}, 0\right) + Q_{j,1}\left(\overline{\chi}_{j,2}\right)\chi_{j,2} z_{j,1} - z_{j,1}\dot{y}_{j,d} + \left|z_{j,1}\right| \hbar_{j,1}^* + \left|z_{j,1}\right| D_{j,1}^* \tag{22}$$

Substituting (18) into (22) gives

$$\dot{V}_{z_{j,1}} \le z_{j,1}\Theta_{j,1}^T \phi\left(\overline{\chi}_{j,1}\right) + Q_{j,1}\left(\overline{\chi}_{j,2}\right)\chi_{j,2} z_{j,1} - z_{j,1}\dot{y}_{j,d} + \left|z_{j,1}\right| \underline{Q}_{j,1} \eta_{j,1}^* \tag{23}$$

where $\eta_{j,1}^* = \underline{Q}_{j,1}^{-1}(\varpi_{j,1}^* + \hbar_{j,1}^* + D_{j,1}^*)$. In view of Young's inequality, we can further have

$$\begin{aligned}\dot{V}_{z_{j,1}} \le{}& Q_{j,1}\left(\overline{\chi}_{j,2}\right)\chi_{j,2}z_{j,1} \\ &+ \frac{z_{j,1}^2\left\|\Theta_{j,1}\right\|^2}{2a_{j,1}^2}\phi^T\left(\overline{\chi}_{j,1}\right)\phi\left(\overline{\chi}_{j,1}\right) + \frac{a_{j,1}^2}{2} \\ &+ \left|z_{j,1}\right|\underline{Q}_{j,1}\eta_{j,1}^* - z_{j,1}\dot{y}_{j,d}\end{aligned} \tag{24}$$

where $a_{j,1}$ is positive constant.

Then, construct the virtual control law $s_{j,1}$ and parameters adaptation laws $\widehat{\eta}_{j,1}$ and $\widehat{\Phi}_{j,1}$ as

$$\begin{aligned}s_{j,1} ={}& -c_{j,1}z_{j,1} - \frac{\widehat{\Phi}_{j,1}z_{j,1}}{2a_{j,1}^2}\phi^T\left(\overline{\chi}_{j,1}\right)\phi\left(\overline{\chi}_{j,1}\right) \\ &- \widehat{\eta}_{j,1}\tanh\left(\frac{z_{j,1}}{\upsilon_{j,1}}\right) \\ &- \alpha_{j,1}\dot{y}_{j,d}\tanh\left(\frac{z_{j,1}\dot{y}_{j,d}}{\upsilon_{j,1}}\right)\end{aligned} \tag{25}$$

$$\dot{\widehat{\eta}}_{j,1} = \xi_{j,1}z_{j,1}\tanh\left(\frac{z_{j,1}}{\upsilon_{j,1}}\right) - \sigma_{j,1}\xi_{j,1}\widehat{\eta}_{j,1} \tag{26}$$

$$\dot{\widehat{\Phi}}_{j,1} = \frac{\beta_{j,1}z_{j,1}^2}{2a_{j,1}^2}\phi^T\left(\overline{\chi}_{j,1}\right)\phi\left(\overline{\chi}_{j,1}\right) - \sigma_{j,1}\beta_{j,1}\widehat{\Phi}_{j,1} \tag{27}$$

where $c_{j,1} > 0$, $\beta_{j,1} > 0$, $\xi_{j,1} > 0$, $\sigma_{j,1} > 0$, $a_{j,1} > 0$, $\upsilon_{j,1} > 0$ and $\alpha_{j,1} \ge \underline{Q}_{j,1}^{-1}$ are design parameters, and $\widehat{\eta}_{j,1}$ is the estimate of $\eta_{j,1}^*$.

Remark 10. Note that (26) and (27) satisfy Lemma 6. Thus, by choosing $\widehat{\eta}_{j,1}(0) \ge 0$ and $\widehat{\Phi}_{j,1}(0) \ge 0$, one has $\widehat{\eta}_{j,1}(t) \ge 0$ and $\widehat{\Phi}_{j,1}(t) \ge 0$ for $\forall t \ge 0$. Furthermore, since the initial conditions $\widehat{\eta}_{j,1}(0)$ and $\widehat{\Phi}_{j,1}(0)$ are selected by control law designer, we choose $\widehat{\eta}_{j,1}(0) \ge 0$ and $\widehat{\Phi}_{j,1}(0) \ge 0$.

In line with the DSC technique, introduce variable $\nu_{j,2}$. Let $s_{j,1}$ pass through a first-order filter with time constant $\iota_{j,2}$ to obtain $\nu_{j,2}$ as

$$\iota_{j,2}\dot{\nu}_{j,2} + \nu_{j,2} = s_{j,1}, \quad \nu_{j,2}(0) = s_{j,1}(0) \tag{28}$$

Define the filter error $e_{j,2} = \nu_{j,2} - s_{j,1}$, which yields $\dot{\nu}_{j,2} = -(e_{j,2}/\iota_{j,2})$ and

$$\begin{aligned}\dot{e}_{j,2} ={}& -\frac{e_{j,2}}{\iota_{j,2}} \\ &+ \Xi_{j,2}\left(z_{j,1}, z_{j,2}, e_{j,2}, \overline{\widehat{\Phi}}_{j,1}, \overline{\widehat{\eta}}_{j,1}, y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right)\end{aligned} \tag{29}$$

where $\Xi_{j,2}(\cdot)$ is the introduced continuous function.

By $\chi_{j,2} = z_{j,2} + \nu_{j,2}$ and $e_{j,2} = \nu_{j,2} - s_{j,1}$, we have

$$\chi_{j,2} = z_{j,2} + s_{j,1} + e_{j,2} \tag{30}$$

Noting that $\Phi_{j,1} = \underline{Q}_{j,1}^{-1}\|\Theta_{j,1}\|^2$ and $\alpha_{j,1} \ge \underline{Q}_{j,1}^{-1}$, and substituting (25) and (30) into (24), we can further obtain

$$\begin{aligned}\dot{V}_{z_{j,1}} \le{}& Q_{j,1}\left(\overline{\chi}_{j,2}\right)\left(z_{j,2} + e_{j,2}\right)z_{j,1} - c_{j,1}\underline{Q}_{j,1}z_{j,1}^2 \\ &+ \left|z_{j,1}\dot{y}_{j,d}\right| + \frac{a_{j,1}^2}{2} - \underline{Q}_{j,1}\widehat{\eta}_{j,1}z_{j,1}\tanh\left(\frac{z_{j,1}}{\upsilon_{j,1}}\right) \\ &- z_{j,1}\dot{y}_{j,d}\tanh\left(\frac{z_{j,1}\dot{y}_{j,d}}{\upsilon_{j,1}}\right) \\ &+ \frac{\underline{Q}_{j,1}\widetilde{\Phi}_{j,1}z_{j,1}^2}{2a_{j,1}^2}\phi^T\left(\overline{\chi}_{j,1}\right)\phi\left(\overline{\chi}_{j,1}\right) \\ &+ \left|z_{j,1}\right|\underline{Q}_{j,1}\eta_{j,1}^*\end{aligned} \tag{31}$$

Take the following Lyapunov function candidate:

$$V_{j,1} = V_{z_{j,1}} + \frac{\underline{Q}_{j,1}\widetilde{\eta}_{j,1}^2}{2\xi_{j,1}} + \frac{\underline{Q}_{j,1}\widetilde{\Phi}_{j,1}^2}{2\beta_{j,1}} \tag{32}$$

where $\widetilde{\eta}_{j,1} = \eta_{j,1}^* - \widehat{\eta}_{j,1}$ and $\widetilde{\Phi}_{j,1} = \Phi_{j,1} - \widehat{\Phi}_{j,1}$ are the estimates of $\eta_{j,1}$ and $\Phi_{j,1}$, respectively.

It follows from (31) that the time derivative of $V_{j,1}$ is

$$\begin{aligned}\dot{V}_{j,1} \le{}& Q_{j,1}\left(\overline{\chi}_{j,2}\right)\left(z_{j,2} + e_{j,2}\right)z_{j,1} - c_{j,1}\underline{Q}_{j,1}z_{j,1}^2 + \frac{a_{j,1}^2}{2} \\ &- \frac{\underline{Q}_{j,1}\widetilde{\Phi}_{j,1}}{\beta_{j,1}}\left[\dot{\widehat{\Phi}}_{j,1} - \frac{\beta_{j,1}z_{j,1}^2}{2a_{j,1}^2}\phi^T\left(\overline{\chi}_{j,1}\right)\phi\left(\overline{\chi}_{j,1}\right)\right] \\ &+ \left[\left|z_{j,1}\dot{y}_{j,d}\right| - z_{j,1}\dot{y}_{j,d}\tanh\left(\frac{z_{j,1}\dot{y}_{j,d}}{\upsilon_{j,1}}\right)\right] \\ &- \frac{\underline{Q}_{j,1}\widetilde{\eta}_{j,1}}{\xi_{j,1}}\left[\dot{\widehat{\eta}}_{j,1} - \xi_{j,1}z_{j,1}\tanh\left(\frac{z_{j,1}}{\upsilon_{j,1}}\right)\right] \\ &+ \underline{Q}_{j,1}\eta_{j,1}^*\left[\left|z_{j,1}\right| - z_{j,1}\tanh\left(\frac{z_{j,1}}{\upsilon_{j,1}}\right)\right]\end{aligned} \tag{33}$$

Applying (26), (27), and Lemma 7, one has

$$\begin{aligned}\dot{V}_{j,1} \le{}& Q_{j,1}\left(\overline{\chi}_{j,2}\right)\left(z_{j,2} + e_{j,2}\right)z_{j,1} - c_{j,1}\underline{Q}_{j,1}z_{j,1}^2 \\ &+ \underline{Q}_{j,1}\sigma_{j,1}\left(\widetilde{\eta}_{j,1}\widehat{\eta}_{j,1} + \widetilde{\Phi}_{j,1}\widehat{\Phi}_{j,1}\right) + \frac{a_{j,1}^2}{2} \\ &+ 0.2785\underline{Q}_{j,1}\eta_{j,1}^*\upsilon_{j,1} + 0.2785\upsilon_{j,1}\end{aligned} \tag{34}$$

Step j, i_j $(2 \le i_j \le \rho_j - 1,\ j = 1, \ldots, m)$. Similar to the design method in Step j, 1 differentiating z_{j,i_j} along with (16) yields

$$\begin{aligned}\dot{z}_{j,i_j} ={}& \varphi_{j,i_j}\left(\overline{\chi}_{j,i_j}, 0\right) + Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\chi_{j,i_j+1} \\ &+ \Delta_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right) + D_{j,i_j}(\chi, t) - \dot{\nu}_{j,i_j}\end{aligned} \tag{35}$$

Consider the following quadratic Lyapunov function candidate:

$$V_{z_{j,i_j}} = \frac{1}{2} z_{j,i_j}^2 \tag{36}$$

In view of Young's inequality and using (35), we can obtain the time derivative of (36) as

$$\begin{aligned}\dot{V}_{z_{j,i_j}} \le{}& Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\chi_{j,i_j+1} z_{j,i_j} \\ &+ \frac{z_{j,i_j}^2 \left\|\Theta_{j,i_j}\right\|^2}{2a_{j,i_j}^2}\phi^T\left(\overline{\chi}_{j,i_j}\right)\phi\left(\overline{\chi}_{j,i_j}\right) + \frac{a_{j,i_j}^2}{2} \\ &+ \left|z_{j,i_j}\right| \underline{Q}_{j,i_j}\eta_{j,i_j}^* - z_{j,i_j}\dot{\nu}_{j,i_j}\end{aligned} \tag{37}$$

where $\eta_{j,i_j}^* = \underline{Q}_{j,i_j}^{-1}(\varpi_{j,i_j}^* + \hbar_{j,i_j}^* + D_{j,i_j}^*)$, and a_{j,i_j} is positive constant.

Take the virtual control law s_{j,i_j} and parameters adaptation laws $\widehat{\eta}_{j,i_j}$ and $\widehat{\Phi}_{j,i_j}$ as

$$\begin{aligned}s_{j,i_j} = {}& -c_{j,i_j} z_{j,i_j} - \frac{\widehat{\Phi}_{j,i_j} z_{j,i_j}}{2a_{j,i_j}^2}\phi^T\left(\overline{\chi}_{j,i_j}\right)\phi\left(\overline{\chi}_{j,i_j}\right) \\ &- \widehat{\eta}_{j,i_j}\tanh\left(\frac{z_{j,i_j}}{\upsilon_{j,i_j}}\right) \\ &+ \alpha_{j,i_j}\dot{\nu}_{j,i_j}\tanh\left(\frac{z_{j,i_j}\dot{\nu}_{j,i_j}}{\upsilon_{j,i_j}}\right)\end{aligned} \tag{38}$$

$$\dot{\widehat{\eta}}_{j,i_j} = \xi_{j,i_j} z_{j,i_j}\tanh\left(\frac{z_{j,i_j}}{\upsilon_{j,i_j}}\right) - \sigma_{j,i_j}\xi_{j,i_j}\widehat{\eta}_{j,i_j} \tag{39}$$

$$\dot{\widehat{\Phi}}_{j,i_j} = \frac{\beta_{j,i_j} z_{j,i_j}^2}{2a_{j,i_j}^2}\phi^T\left(\overline{\chi}_{j,i_j}\right)\phi\left(\overline{\chi}_{j,i_j}\right) - \sigma_{j,i_j}\beta_{j,i_j}\widehat{\Phi}_{j,i_j} \tag{40}$$

The design process of parameters is similar to Step j, 1. Then, let s_{j,i_j} pass through a first-order filter with time constant ι_{j,i_j+1} as follows:

$$\iota_{j,i_j+1}\dot{\nu}_{j,i_j+1} + \nu_{j,i_j+1} = s_{j,i_j}, \quad \nu_{j,i_j+1}(0) = s_{j,i_j}(0) \tag{41}$$

Define $e_{j,i_j+1} = \nu_{j,i_j+1} - s_{j,i_j}$, it yields $\dot{\nu}_{j,i_j+1} = -(e_{j,i_j+1}/\iota_{j,i_j+1})$ and

$$\begin{aligned}\dot{e}_{j,i_j+1} = {}& -\frac{e_{j,i_j+1}}{\iota_{j,i_j+1}} \\ &+ \Xi_{j,i_j+1}\left(\overline{z}_{j,i_j+1}, \overline{e}_{j,i_j+1}, \overline{\widehat{\Phi}}_{j,i_j}, \overline{\widehat{\eta}}_{j,i_j}, y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right)\end{aligned} \tag{42}$$

where $\overline{z}_{j,i_j+1} = [z_{j,1}, \ldots, z_{j,i_j+1}]^T$, $\overline{e}_{j,i_j+1} = [e_{j,2}, \ldots, e_{j,i_j+1}]^T$, $\overline{\widehat{\Phi}}_{j,i_j} = [\widehat{\Phi}_{j,1}, \ldots, \widehat{\Phi}_{j,i_j}]^T$, $\overline{\widehat{\eta}}_{j,i_j} = [\widehat{\eta}_{j,1}, \ldots, \widehat{\eta}_{j,i_j}]^T$ and $\Xi_{j,i_j+1}(\cdot)$ is a continuous function.

According to $\chi_{j,i_j+1} = z_{j,i_j+1} + \nu_{j,i_j+1}$ and $e_{j,i_j+1} = \nu_{j,i_j+1} - s_{j,i_j}$, one reaches

$$\chi_{j,i_j+1} = z_{j,i_j+1} + s_{j,i_j} + e_{j,i_j+1} \tag{43}$$

Substituting (38) and (43) into (37) results in

$$\begin{aligned}\dot{V}_{z_{j,i_j}} \le{}& Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\left(z_{j,i_j+1} + e_{j,i_j+1}\right) z_{j,i_j} \\ &- c_{j,i_j}\underline{Q}_{j,i_j} z_{j,i_j}^2 + \frac{a_{j,i_j}^2}{2} \\ &- \underline{Q}_{j,i_j}\widehat{\eta}_{j,i_j} z_{j,i_j}\tanh\left(\frac{z_{j,i_j}}{\upsilon_{j,i_j}}\right) \\ &- z_{j,i_j}\dot{\nu}_{j,i_j}\tanh\left(\frac{z_{j,i_j}\dot{\nu}_{j,i_j}}{\upsilon_{j,i_j}}\right) \\ &+ \frac{\underline{Q}_{j,i_j}\widetilde{\Phi}_{j,i_j} z_{j,i_j}^2}{2a_{j,i_j}^2}\phi^T\left(\overline{\chi}_{j,i_j}\right)\phi\left(\overline{\chi}_{j,i_j}\right) + \left|z_{j,i_j}\dot{\nu}_{j,i_j}\right| \\ &+ \left|z_{j,i_j}\right|\underline{Q}_{j,i_j}\eta_{j,i_j}^*\end{aligned} \tag{44}$$

Consider the following Lyapunov function candidate:

$$V_{j,i_j} = V_{z_{j,i_j}} + \frac{\underline{Q}_{j,i_j}\widetilde{\eta}_{j,i_j}^2}{2\xi_{j,i_j}} + \frac{\underline{Q}_{j,i_j}\widetilde{\Phi}_{j,i_j}^2}{2\beta_{j,i_j}} \tag{45}$$

where $\widetilde{\eta}_{j,i_j} = \eta_{j,i_j}^* - \widehat{\eta}_{j,i_j}$ and $\widetilde{\Phi}_{j,i_j} = \Phi_{j,i_j} - \widehat{\Phi}_{j,i_j}$.

Noting $\widetilde{\eta}_{j,i_j} = \eta_{j,i_j}^* - \widehat{\eta}_{j,i_j}$ and following the same way as Step j, 1 give rise to

$$\begin{aligned}&\dot{V}_{j,i_j} \\ &\le Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\left(z_{j,i_j+1} + e_{j,i_j+1}\right) z_{j,i_j} - c_{j,i_j}\underline{Q}_{j,i_j} z_{j,i_j}^2 \\ &\quad + \frac{a_{j,i_j}^2}{2} \\ &\quad - \frac{\underline{Q}_{j,i_j}\widetilde{\Phi}_{j,i_j}}{\beta_{j,i_j}}\left[\dot{\widehat{\Phi}}_{j,i_j} - \frac{\beta_{j,i_j} z_{j,i_j}^2}{2a_{j,i_j}^2}\phi^T\left(\overline{\chi}_{j,i_j}\right)\phi\left(\overline{\chi}_{j,i_j}\right)\right] \\ &\quad - \frac{\underline{Q}_{j,i_j}\widetilde{\eta}_{j,i_j}}{\xi_{j,i_j}}\left[\dot{\widehat{\eta}}_{j,i_j} - \xi_{j,i_j} z_{j,i_j}\tanh\left(\frac{z_{j,i_j}}{\upsilon_{j,i_j}}\right)\right] \\ &\quad + \left[\left|z_{j,i_j}\dot{\nu}_{j,i_j}\right| - z_{j,i_j}\dot{\nu}_{j,i_j}\tanh\left(\frac{z_{j,i_j}\dot{\nu}_{j,i_j}}{\upsilon_{j,i_j}}\right)\right] \\ &\quad + \underline{Q}_{j,i_j}\eta_{j,i_j}^*\left[\left|z_{j,i_j}\right| - z_{j,i_j}\tanh\left(\frac{z_{j,i_j}}{\upsilon_{j,i_j}}\right)\right]\end{aligned} \tag{46}$$

Applying (39), (40), and Lemma 7 yields

$$\begin{aligned}\dot{V}_{j,i_j} \le{}& Q_{j,i_j}\left(\overline{\chi}_{j,i_j+1}\right)\left(z_{j,i_j+1}+e_{j,i_j+1}\right)z_{j,i_j}\\&+0.2785\upsilon_{j,i_j}+0.2785\underline{Q}_{j,i_j}\eta^*_{j,i_j}\upsilon_{j,i_j}\\&+\underline{Q}_{j,i_j}\sigma_{j,i_j}\left(\widetilde{\eta}_{j,i_j}\widehat{\eta}_{j,i_j}+\widetilde{\Phi}_{j,i_j}\widehat{\Phi}_{j,i_j}\right)\\&-c_{j,i_j}\underline{Q}_{j,i_j}z^2_{j,i_j}+\frac{a^2_{j,i_j}}{2}\end{aligned}\tag{47}$$

Step j,ρ_j $(j=1,\ldots,m)$. Similar to the former design process, we can obtain

$$\begin{aligned}\dot{z}_{j,\rho_j} ={}& \varphi_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j},0\right)+Q_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j+1}\right)u_j\\&+\Delta_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j+1}\right)+D_{j,\rho_j}(\chi,t)-\dot{\nu}_{j,\rho_j}\\&+\kappa_j\tanh\wp_j-sat\left(o_j\right)+o_j\end{aligned}\tag{48}$$

For $|u_j|=|sat(o_j)|\le u_{j,M}$, there exists a continuous function $\mathbb{Q}_{j,\rho_j}(\overline{\chi}_{j,\rho_j})$ such that

$$\begin{aligned}\left|Q_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j+1}\right)u_j\right|&=\left|Q_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j},sat\left(o_j\right)\right)sat\left(o_j\right)\right|\\&\le\mathbb{Q}_{j,\rho_j}\left(\overline{\chi}_{j,\rho_j}\right)\end{aligned}\tag{49}$$

Consider a compact set $\Omega_{j,\rho_j}:=\{\sum_{l=1}^{\rho_j}(\underline{Q}_{j,l}\widetilde{\eta}^2_{j,l}/\xi_{j,l}+\underline{Q}_{j,l}\widetilde{\Phi}^2_{j,l}/\beta_{j,l})+z^2_{j,1}+\sum_{l=2}^{\rho_j}(e^2_{j,l}+z^2_{j,l})\le 2\omega_{j,\rho_j}\}$. It can be seen from (43) that all the variables of $\overline{\chi}_{j,\rho_j}$ are included in the compact set $\Omega_{j,\rho_j}\times\Omega_{j0}$. Thus, $\mathbb{Q}_{j,\rho_j}(\overline{\chi}_{j,\rho_j})=\mathbb{Q}_{j,\rho_j}(\overline{\chi}_{j,\rho_j-1},z_{j,\rho_j}+s_{j,\rho_j-1}+e_{j,\rho_j})$ have maximums on $\Omega_{j,\rho_j}\times\Omega_{j0}$. There exist unknown positive constants $\mathbb{Q}^*_{j,\rho_j}$ such that $\mathbb{Q}_{j,\rho_j}(\overline{\chi}_{j,\rho_j})\le\mathbb{Q}^*_{j,\rho_j}$.

Choosing the quadratic function $V_{z_{j,\rho_j}}$ as $V_{z_{j,\rho_j}}=z^2_{j,\rho_j}/2$, it gives

$$\begin{aligned}\dot{V}_{z_{j,\rho_j}}\le{}&\Theta^T_{j,\rho_j}\phi\left(\overline{\chi}_{j,\rho_j}\right)z_{j,\rho_j}+\left|z_{j,\rho_j}\right|\eta^*_{j,\rho_j}-z_{j,\rho_j}\dot{\nu}_{j,\rho_j}\\&+o_jz_{j,\rho_j}\end{aligned}\tag{50}$$

where $\eta^*_{j,\rho_j}=\varpi^*_{j,\rho_j}+\hbar^*_{j,\rho_j}+D^*_{j,\rho_j}+\mathbb{Q}^*_n+\kappa_j+u_{j,M}$.

According to Young's inequality, one has

$$\begin{aligned}\dot{V}_{z_{j,\rho_j}}\le{}&\frac{z^2_{j,\rho_j}\left\|\Theta_{j,\rho_j}\right\|^2}{2a^2_{j,\rho_j}}\phi^T\left(\overline{\chi}_{j,\rho_j}\right)\phi\left(\overline{\chi}_{j,\rho_j}\right)+\frac{a^2_{j,\rho_j}}{2}\\&+o_jz_{j,\rho_j}+\left|z_{j,\rho_j}\right|\eta^*_{j,\rho_j}-z_{j,\rho_j}\dot{\nu}_{j,\rho_j}\end{aligned}\tag{51}$$

where a_{j,ρ_j} is positive constant.

Similarly, construct the actual control law o_j and the adaptation laws $\widehat{\eta}_{j,\rho_j}$ and $\widehat{\Phi}_{j,\rho_j}$ as

$$\begin{aligned}o_j={}&-c_{j,\rho_j}z_{j,\rho_j}-\frac{\widehat{\Phi}_{j,\rho_j}z_{j,\rho_j}}{2a^2_{j,\rho_j}}\phi^T\left(\overline{\chi}_{j,\rho_j}\right)\phi\left(\overline{\chi}_{j,\rho_j}\right)\\&-\widehat{\eta}_{j,\rho_j}\tanh\left(\frac{z_{j,\rho_j}}{\upsilon_{j,\rho_j}}\right)\\&-\alpha_{j,\rho_j}\dot{\nu}_{j,\rho_j}\tanh\left(\frac{z_{j,\rho_j}\dot{\nu}_{j,\rho_j}}{\upsilon_{j,\rho_j}}\right)\end{aligned}\tag{52}$$

$$\dot{\widehat{\eta}}_{j,\rho_j}=\xi_{j,\rho_j}z_{j,\rho_j}\tanh\left(\frac{z_{j,\rho_j}}{\upsilon_{j,\rho_j}}\right)-\sigma_{j,\rho_j}\xi_{j,\rho_j}\widehat{\eta}_{j,\rho_j}\tag{53}$$

$$\dot{\widehat{\Phi}}_{j,\rho_j}=\frac{c_{j,\rho_j}z^2_{j,\rho_j}}{2a^2_{j,\rho_j}}\phi^T\left(\overline{\chi}_{j,\rho_j}\right)\phi\left(\overline{\chi}_{j,\rho_j}\right)-\sigma_{j,\rho_j}\beta_{j,\rho_j}\widehat{\Phi}_{j,\rho_j}\tag{54}$$

The design process of parameters is also similar to Step j,i_j and Step $j,1$. Take the following Lyapunov function candidate:

$$V_{j,\rho_j}=V_{z_{j,\rho_j}}+\frac{\widetilde{\eta}^2_{j,\rho_j}}{2\xi_{j,\rho_j}}+\frac{\widetilde{\Phi}^2_{j,\rho_j}}{2\beta_{j,\rho_j}}\tag{55}$$

where $\widetilde{\eta}_{j,\rho_j}=\eta^*_{j,\rho_j}-\widehat{\eta}_{j,\rho_j}$ and $\widetilde{\Phi}_{j,\rho_j}=\Phi_{j,\rho_j}-\widehat{\Phi}_{j,\rho_j}$.

Following the same way as the former steps gives

$$\begin{aligned}\dot{V}_{j,\rho_j}\le{}&\sigma_{j,\rho_j}\left(\widetilde{\eta}_{j,\rho_j}\widehat{\eta}_{j,\rho_j}+\widetilde{\Phi}_{j,\rho_j}\widehat{\Phi}_{j,\rho_j}\right)-c_{j,\rho_j}z^2_{j,\rho_j}\\&+0.2785\upsilon_{j,\rho_j}+0.2785\eta^*_{j,\rho_j}\upsilon_{j,\rho_j}+\frac{a^2_{j,\rho_j}}{2}\end{aligned}\tag{56}$$

4. Stability Analysis

The main stability results of the MIMO pure-feedback nonlinear systems (1) are presented.

Theorem 11. *Supposing that Assumptions 1, 4, and 5 hold and the above proposed design procedure is employed to MIMO pure-feedback nonlinear systems described by (1), for* $\widehat{\Phi}_{j,i_j}(0)\ge 0$, $\widehat{\eta}_{j,i_j}(0)\ge 0$, $\forall\omega>0$ *and* $V_j(0)\le\omega$, *there exist design parameters* c_{j,i_j}, β_{j,i_j}, ξ_{j,i_j}, σ_{j,i_j}, a_{j,i_j}, υ_{j,i_j}, α_{j,i_j} *and* ι_{j,i_j} *such that*

(1) $V_j(t)\le\omega$ *for* $\forall t>0$, *and hence all of the signals in the closed-loop systems remain semiglobally uniformly ultimately bounded;*

(2) the output tracking error $z_{j,1}$ *is such that* $\lim_{t\to\infty}|z_{j,1}(t)|\le\Delta_{j,1}$, *where* $\Delta_{j,1}$ *is a positive constant depending on the design parameters. Furthermore, the whole system output tracking error* $z_1=[z_{1,1},\ldots,z_{m,1}]^T$ *satisfies* $\lim_{t\to\infty}\|z_1(t)\|\le\Delta_1$ *with* Δ_1 *a positive constant that relies on the design parameters;*

(3) the dynamic system $\wp_j$ *is bounded, and the control input constraint is not violated.*

Proof. Choose the following Lyapunov function candidate for the whole systems:

$$V = \sum_{j=1}^{m} \overline{V}_j \tag{57}$$

where $\overline{V}_j = (1/2)\sum_{i_j=1}^{\rho_j-1}(z_{j,i_j}^2 + \underline{Q}_{j,i_j}\widetilde{\eta}_{j,i_j}^2/\xi_{j,i_j} + \underline{Q}_{j,i_j}\widetilde{\Phi}_{j,i_j}^2/\beta_{j,i_j} + e_{j,i_j+1}^2) + (1/2)(z_{j,\rho_j}^2 + \widetilde{\eta}_{j,\rho_j}^2/\xi_{j,\rho_j} + \widetilde{\Phi}_{j,\rho_j}^2/\beta_{j,\rho_j})$.

According to (34), (47), and (56), we can obtain the time derivative of $\overline{V}_j$ as

$$\begin{aligned}
\dot{\overline{V}}_j &\le \sum_{i_j=1}^{\rho_j-1}\overline{Q}_{j,i_j}\left(\left|z_{j,i_j+1}\right| + \left|e_{j,i_j+1}\right|\right)\left|z_{j,i_j}\right| \\
&\quad - \sum_{i_j=1}^{\rho_j-1} c_{j,i_j}\underline{Q}_{j,i_j} z_{j,i_j}^2 + \frac{a_{j,\rho_j}^2}{2} \\
&\quad + \sigma_{j,\rho_j}\left(\widetilde{\eta}_{j,\rho_j}\widehat{\eta}_{j,\rho_j} + \widetilde{\Phi}_{j,\rho_j}\widehat{\Phi}_{j,\rho_j}\right) + 0.2785\eta_{j,\rho_j}^* \upsilon_{j,\rho_j} \\
&\quad + \sum_{i_j=1}^{\rho_j-1}\left(\frac{a_{j,i_j}^2}{2} + 0.2785\underline{Q}_{j,i_j}\eta_{j,i_j}^*\upsilon_{j,i_j} + 0.2785\upsilon_{j,i_j}\right) \\
&\quad + \sum_{i_j=1}^{\rho_j-1}\underline{Q}_{j,i_j}\sigma_{j,i_j}\left(\widetilde{\eta}_{j,i_j}\widehat{\eta}_{j,i_j} + \widetilde{\Phi}_{j,i_j}\widehat{\Phi}_{j,i_j}\right) - c_{j,\rho_j} z_{j,\rho_j}^2 \\
&\quad + \sum_{i_j=1}^{\rho_j-1}\left(-\frac{e_{j,i_j+1}^2}{\iota_{j,i_j+1}} + \left|e_{j,i_j+1}\Xi_{j,i_j+1}(\cdot)\right|\right) \\
&\quad + 0.2785\upsilon_{j,\rho_j}
\end{aligned} \tag{58}$$

Using the following inequalities

$$\begin{aligned}
\widetilde{\eta}_{j,i_j}\widehat{\eta}_{j,i_j} &\le \frac{\eta_{j,i_j}^{*2}}{2} - \frac{\widetilde{\eta}_{j,i_j}^2}{2}, \\
\widetilde{\Phi}_{j,i_j}\widehat{\Phi}_{j,i_j} &\le \frac{\Phi_{j,i_j}^2}{2} - \frac{\widetilde{\Phi}_{j,i_j}^2}{2}
\end{aligned} \tag{59}$$

we can arrive at

$$\begin{aligned}
\dot{\overline{V}}_j &\le -\sum_{i_j=1}^{\rho_j-1} c_{j,i_j}\underline{Q}_{j,i_j} z_{j,i_j}^2 - \frac{1}{2}\sum_{i_j=1}^{\rho_j-1}\underline{Q}_{j,i_j}\sigma_{j,i_j}\left(\widetilde{\eta}_{j,i_j}^2 + \widetilde{\Phi}_{j,i_j}^2\right) \\
&\quad + \sum_{i_j=1}^{\rho_j-1}\overline{Q}_{j,i_j}\left(\left|z_{j,i_j+1}\right| + \left|e_{j,i_j+1}\right|\right)\left|z_{j,i_j}\right| - c_{j,\rho_j} z_{j,\rho_j}^2 \\
&\quad + \sum_{i_j=1}^{\rho_j-1}\left(-\frac{y_{j,i_j+1}^2}{\iota_{j,i_j+1}} + \left|e_{j,i_j+1}\Xi_{j,i_j+1}(\cdot)\right|\right) + C_{j,0} \\
&\quad - \frac{1}{2}\sigma_{j,\rho_j}\left(\widetilde{\eta}_{j,\rho_j}^2 + \widetilde{\Phi}_{j,\rho_j}^2\right)
\end{aligned} \tag{60}$$

where $C_{j,0} = \sum_{i_j=1}^{\rho_j-1}(a_{j,i_j}^2/2 + 0.2785\underline{Q}_{j,i_j}\eta_{j,i_j}^*\upsilon_{j,i_j} + 0.2785\upsilon_{j,i_j} + \underline{Q}_{j,i_j}\sigma_{j,i_j}(\eta_{j,i_j}^{*2} + \Phi_{j,i_j}^2)/2) + 0.2785\eta_{j,\rho_j}^*\upsilon_{j,\rho_j} + a_{j,\rho_j}^2/2 + \sigma_{j,\rho_j}(\eta_{j,\rho_j}^{*2} + \Phi_{j,\rho_j}^2)/2 + 0.2785\upsilon_{j,\rho_j}$.

By completion of squares, one has

$$\begin{aligned}
\left|e_{j,i_j+1}\Xi_{j,i_j+1}(\cdot)\right| &\le \frac{e_{j,i_j+1}^2\Xi_{j,i_j+1}^2(\cdot)}{2k_{j,1}} + \frac{k_{j,1}}{2} \\
\overline{Q}_{j,i_j}\left|e_{j,i_j+1}\right|\left|z_{j,i_j}\right| &\le \frac{\overline{Q}_{j,i_j}^2 e_{j,i_j+1}^2 k_{j,2}}{2} + \frac{z_{j,i_j}^2}{2k_{j,2}} \\
\overline{Q}_{j,i_j}\left|z_{j,i_j+1}\right|\left|z_{j,i_j}\right| &\le \frac{\overline{Q}_{j,i_j} z_{j,i_j}^2}{2} + \frac{\overline{Q}_{j,i_j} z_{j,i_j+1}^2}{2}
\end{aligned} \tag{61}$$

with $k_{j,1}$ and $k_{j,2}$ being positive constants. Then, we can further rewrite (60) as

$$\begin{aligned}
\dot{\overline{V}}_j &\le -\sum_{i_j=1}^{\rho_j-1} c_{j,i_j}\underline{Q}_{j,i_j} z_{j,i_j}^2 - \frac{1}{2}\sum_{i_j=1}^{\rho_j-1}\underline{Q}_{j,i_j}\sigma_{j,i_j}\left(\widetilde{\eta}_{j,i_j}^2 + \widetilde{\Phi}_{j,i_j}^2\right) \\
&\quad - c_{j,\rho_j} z_{j,\rho_j}^2 + \sum_{i_j=1}^{\rho_j-1}\overline{Q}_{j,i_j}\left(\frac{z_{j,i_j}^2}{2} + \frac{z_{j,i_j+1}^2}{2}\right) \\
&\quad + \sum_{i_j=1}^{\rho_j-1}\left(\frac{z_{j,i_j}^2}{2k_{j,2}}\right) + \frac{\left(\rho_j - 1\right)k_{j,1}}{2} + \sum_{i_j=1}^{\rho_j-1}\left(-\frac{e_{j,i_j+1}^2}{\iota_{j,i_j+1}}\right. \\
&\quad \left. + \frac{e_{j,i_j+1}^2\Xi_{j,i_j+1}^2(\cdot)}{2k_{j,1}} + \frac{\overline{Q}_{j,i_j}^2 e_{j,i_j+1}^2 k_{j,2}}{2}\right) + C_{j,0} - \frac{1}{2} \\
&\quad \cdot \sigma_{j,\rho_j}\left(\widetilde{\eta}_{j,\rho_j}^2 + \widetilde{\Phi}_{j,\rho_j}^2\right)
\end{aligned} \tag{62}$$

Then, it can be known from [16] that $|\Xi_{j,i_j+1}(\cdot)|$ has a maximum M_{j,i_j+1} on the compact set $\Omega_{j,i_j+1} \times \Omega_{j0}$. Let $1/\iota_{j,i_j+1} = M_{j,i_j+1}^2/(2k_{j,1}) + \overline{Q}_{j,i_j}^2 k_{j,2}/2 + \omega_j$ with ω_j being a positive constant. Setting $c_{j,i_j} = \underline{Q}_{j,i_j}^{-1}(\overline{Q}_{j,i_j} + 1/(2k_{j,2}) + \vartheta_j)$, $c_{j,\rho_j} = \overline{Q}_{j,\rho_j} + 1/(2k_{j,2}) + \vartheta_j$ with ϑ_j being any positive constant, one has

$$\begin{aligned}
\dot{\overline{V}}_j &\le -\sum_{i_j=1}^{\rho_j}\vartheta_j z_{j,i_j}^2 - \frac{1}{2}\sum_{i_j=1}^{\rho_j-1}\underline{Q}_{j,i_j}\sigma_{j,i_j}\left(\widetilde{\eta}_{j,i_j}^2 + \widetilde{\Phi}_{j,i_j}^2\right) + C_j \\
&\quad - \frac{1}{2}\sigma_{j,\rho_j}\left(\widetilde{\eta}_{j,\rho_j}^2 + \widetilde{\Phi}_{j,\rho_j}^2\right) - \sum_{i_j=1}^{\rho_j-1}\omega_j e_{j,i_j+1}^2
\end{aligned} \tag{63}$$

where $C_j = C_{j,0} + (\rho_j - 1)k_{j,1}/2$. Noting (32), (45), (55), and (57), it yields

$$\dot{\overline{V}}_j \le -\lambda_j \overline{V}_j + C_j \tag{64}$$

where $\lambda_j = \min_{i_j=1,\ldots,\rho_j}\{2\omega_j, 2\vartheta_j, \sigma_{j,i_j}\xi_{j,i_j}, \sigma_{j,i_j}\beta_{j,i_j}\}$. Note that C_j/λ_j can be made arbitrarily small by decreasing c_{j,i_j}, β_{j,i_j},

and ξ_{j,i_j} and meanwhile increasing $\sigma_{j,i_j}, a_{j,i_j}, \upsilon_{j,i_j}, \alpha_{j,i_j}$ and ι_{j,i_j}. Hence we can have $C_j/\lambda_j \le \omega$ by appropriately choosing the design parameters. It follows from $C_j/\lambda_j \le \omega$ and (64) that $\dot{\overline{V}}_j \le 0$ on the level set $\overline{V}_j = \omega$. Therefore, all the signals of the closed-loop systems are SGUUB. The property (1) of Theorem 11 is proved.

Solving (64) shows

$$\overline{V}_j(t) \le \overline{V}_j(0) + \Sigma \tag{65}$$

with $\Sigma = C_j/\lambda_j$ a positive constant. According to (21), (36), and (57), we have $\sum_{i_j=1}^{\rho_j} z_{j,i_j}^2/2 \le \overline{V}_j$. Using the first inequality in (65), the following inequality holds:

$$\lim_{t\longrightarrow\infty} \left|z_{j,1}\right| \le \lim_{t\longrightarrow\infty} \sqrt{2\overline{V}_j} \le \sqrt{2\Sigma} = \Delta_{j,1} \tag{66}$$

Now let us consider the Lyapunov function candidate for the whole systems as $V = \sum_{j=1}^{m} \overline{V}_j$. From (65), it can be derived that

$$\dot{V} \le \sum_{j=1}^{m} \left[-\lambda_j \overline{V}_j + C_j\right] \le -\mathfrak{R}V + \Pi \tag{67}$$

where $\mathfrak{R} = \min\{\lambda_1, \ldots, \lambda_m\}$ and $\Pi = \sum_{j=1}^{m} C_j$. Then, we further have

$$V(t) \le [V(0) - \Gamma] z^{-\mathfrak{R}t} + \Gamma \tag{68}$$

where $\Gamma = \Pi/\mathfrak{R}$ is a positive constant.

Similarly, we have $\lim_{t\longrightarrow\infty} V(t) \le \Gamma$,which leads to

$$\lim_{t\longrightarrow\infty} \left\|z_1(t)\right\| \le \lim_{t\longrightarrow\infty} \sqrt{2V(t)} \le \sqrt{2\Gamma} = \Delta_1 \tag{69}$$

Noting that the size of Δ_1 depends on the design parameters c_{j,i_j}, β_{j,i_j}, ξ_{j,i_j}, σ_{j,i_j}, a_{j,i_j}, υ_{j,i_j}, α_{j,i_j} and ι_{j,i_j}. Thus, by appropriately online-tuning the design parameters, the tracking error z_1 can be regulated to a neighborhood of the origin as small as desired and property (2) of Theorem 11 is proved.

Furthermore, for input saturation, there exists a nonnegative scalar $\mathfrak{T}_j$ to satisfy $|\Lambda_j| \le \mathfrak{T}_j$ with $\Lambda_j = sat(o_j) - o_j$ and $\kappa_j > \mathfrak{T}_j$. Choosing the Lyapunov function candidate quadratic function $V_{\wp_j}$ as $V_{\wp_j} = \mathbb{C}\wp_j^2/2$, we can obtain

$$\begin{aligned}\dot{V}_{\wp_j} &= \cosh^2\wp_j\left(-\kappa_j\wp_j\tanh\wp_j + \wp_j\left(sat\left(o_j\right) - o_j\right)\right)\\ &\le \cosh^2\wp_j\left(\kappa_j\left|\wp_j\right| - \kappa_j\wp_j\tanh\wp_j - \kappa_j\left|\wp_j\right|\right.\\ &\left.+\ \mathfrak{T}_j\left|\wp_j\right|\right) \le \cosh^2\wp_j\left(0.2785\kappa_j - \left(\kappa_j - \mathfrak{T}_j\right)\left|\wp_j\right|\right)\end{aligned} \tag{70}$$

If $|\wp_j| > 0.2785\kappa_j/(\kappa_j - \mathfrak{T}_j)$, we have $\dot{V}_{\wp_j} < 0$. Therefore, $\wp_j$ will lie in the compact set $\{\wp_j||\wp_j| \le 0.2785\kappa_j/(\kappa_j - \mathfrak{T}_j)\}$ $(\forall t \ge 0)$ and property (3) of Theorem 11 is proved. This completes the proof. □

5. Simulation Analysis

In this section, two simulation examples are given to show validity of the proposed method in this paper.

Example 1. Consider the MIMO non-affine nonlinear uncertain systems as follows:

$$\begin{aligned}\dot{\chi}_{1,1} &= \varphi_{1,1}\left(\chi_{1,1}, \chi_{1,2}\right) + D_{1,1}\left(\chi, t\right)\\ \dot{\chi}_{1,2} &= \varphi_{1,2}\left(\chi_{1,1}, \chi_{1,2}, u_1\left(o_1\right)\right) + D_{1,2}\left(\chi, t\right)\\ \dot{\chi}_{2,1} &= \varphi_{2,1}\left(\chi_{2,1}, \chi_{2,2}\right) + D_{2,1}\left(\chi, t\right)\\ \dot{\chi}_{2,2} &= \varphi_{2,2}\left(\chi_{2,1}, \chi_{2,2}, u_2\left(o_2\right)\right) + D_{2,2}\left(\chi, t\right)\\ y_1 &= \chi_{1,1}\\ y_2 &= \chi_{2,1}\end{aligned} \tag{71}$$

where $\varphi_{1,1}(\chi_{1,1}, \chi_{1,2}) = 0.2\chi_{1,1}\chi_{1,2}/5 + \chi_{1,1} + \chi_{1,2}$, $\varphi_{1,2}(\chi_{1,1}, \chi_{1,2}, u_1(o_1)) = 0.3\chi_{1,2}\chi_{1,1} + 0.25\chi_{1,1} + 0.5u_1(o_1)$, $\varphi_{2,1}(\chi_{2,1}, \chi_{2,2}) = 0.5\chi_{2,1}\chi_{2,2} + \chi_{2,2} + \chi_{2,1}$ and $\varphi_{2,2}(\chi_{2,1}, \chi_{2,2}, u_2(o_2)) = 0.8\chi_{2,1}^2\chi_{2,2} + \chi_{2,2} + 0.5u_2(o_2)$, $D_{1,1}(\chi, t) = 0.5\sin(t)\cos(0.2t^3)^2 + \chi_{1,1}^2\chi_{2,1}\chi_{2,2}$, $D_{1,2}(\chi, t) = 0.5u_1 + 0.3\cos(t)^3\sin(t) + \chi_{1,2}^2 + \chi_{1,2}\chi_{2,1}$, $D_{2,1}(\chi, t) = 0.2\sin(t^2)\cos(0.5t)^2 + \chi_{1,1}\chi_{2,1}\chi_{1,2}^2$ and $D_{2,2}(\chi, t) = 0.5\cos(0.2t^2)\sin(5t) + \chi_{2,2}^2 + \chi_{2,1}^2$. $u_1(o_1)$ and $u_2(o_2)$ are defined as follows:

$$\begin{aligned}u_1\left(o_1\right) &= sat\left(o_1\right)\begin{cases}\text{sign}\left(o_1\right)5.5, & \left|o_1\right| \ge 5.5\\ o_1, & \left|o_1\right| < 5.5\end{cases}\\ u_2\left(o_2\right) &= sat\left(o_2\right)\begin{cases}\text{sign}\left(o_2\right)4.5, & \left|o_2\right| \ge 4.5\\ o_2, & \left|o_2\right| < 4.5\end{cases}\end{aligned} \tag{72}$$

It can be known that the existence of input saturation nonlinearity implies that non-affine functions $\varphi_{1,2}(\chi_{1,1}, \chi_{1,2}, u_1(o_1))$ and $\varphi_{2,2}(\chi_{2.1}, \chi_{2.2}, u_2(o_2))$ are non-differentiable. In this case, the existing approaches cannot be used. However, Assumption 1 in this paper is still satisfied which means that the scheme proposed here is able to deal with the control design difficulty in spite of the input saturation nonlinearity.

According to Theorem 11, the virtual control laws and actual control laws are constructed as

$$\begin{aligned}s_{1,1} &= -8z_{1,1} - \frac{\widehat{\Phi}_{1,1}z_{1,1}}{2\times 0.25^2}\phi^T\left(\chi_{1,1}\right)\phi\left(\chi_{1,1}\right)\\ &\quad - 5\dot{y}_{1,d}\tanh\left(\frac{z_{1,1}\dot{y}_{1,d}}{0.25}\right) - \widehat{\eta}_{1,1}\tanh\left(\frac{z_{1,1}}{0.25}\right)\\ s_{2,1} &= -6z_{2,1} - \frac{\widehat{\Phi}_{2,1}z_{2,1}}{2\times 0.1^2}\phi^T\left(\chi_{2,1}\right)\phi\left(\chi_{2,1}\right)\\ &\quad - 2\dot{y}_{2,d}\tanh\left(\frac{z_{2,1}\dot{y}_{2,d}}{0.1}\right) - \widehat{\eta}_{2,1}\tanh\left(\frac{z_{2,1}}{0.1}\right)\end{aligned}$$

$$
\begin{aligned}
o_1 &= -8z_{1,2} - \frac{\widehat{\Phi}_{1,2}z_{1,2}}{2\times 0.25^2}\phi^T(\chi_{1,2})\phi(\chi_{1,2}) \\
&\quad - 2\dot{\nu}_{1,2}\tanh\left(\frac{z_{1,2}\dot{\nu}_{1,2}}{0.5}\right) - \widehat{\eta}_{2,1}\tanh\left(\frac{z_{2,1}}{0.5}\right) \\
o_2 &= -12z_{2,2} - \frac{\widehat{\Phi}_{2,2}z_{2,2}}{2\times 0.1^2}\phi^T(\chi_{2,2})\phi(\chi_{2,2}) \\
&\quad - 2\dot{\nu}_{2,2}\tanh\left(\frac{z_{2,2}\dot{\nu}_{2,2}}{0.2}\right) - \widehat{\eta}_{2,2}\tanh\left(\frac{z_{2,2}}{0.2}\right)
\end{aligned} \tag{73}
$$

with adaptive laws

$$
\begin{aligned}
\dot{\widehat{\eta}}_{1,1} &= 2z_{1,1}\tanh\left(\frac{z_{1,1}}{0.25}\right) - 0.15\times 2\widehat{\eta}_{1,1}, \\
\dot{\widehat{\Phi}}_{1,1} &= \frac{2z_{1,1}^2}{2\times 0.25^2}\phi^T(\overline{\chi}_{1,1})\phi(\overline{\chi}_{1,1}) - 0.15\times 2\widehat{\Phi}_{1,1} \\
\dot{\widehat{\eta}}_{2,1} &= 2z_{2,1}\tanh\left(\frac{z_{2,1}}{0.1}\right) - 0.1\times 2\widehat{\eta}_{2,1}, \\
\dot{\widehat{\Phi}}_{2,1} &= \frac{z_{2,1}^2}{2\times 0.1^2}\phi^T(\overline{\chi}_{2,1})\phi(\overline{\chi}_{2,1}) - 0.1\widehat{\Phi}_{2,1} \\
\dot{\widehat{\eta}}_{1,2} &= 1.5z_{1,2}\tanh\left(\frac{z_{1,2}}{0.5}\right) - 0.15\times 1.5\widehat{\eta}_{1,2}, \\
\dot{\widehat{\Phi}}_{1,2} &= \frac{2z_{1,2}^2}{2\times 0.25^2}\phi^T(\overline{\chi}_{1,2})\phi(\overline{\chi}_{1,2}) - 0.15\times 2\widehat{\Phi}_{1,2} \\
\dot{\widehat{\eta}}_{2,2} &= 1.5z_{2,2}\tanh\left(\frac{z_{2,2}}{0.2}\right) - 0.1\times 1.5\widehat{\eta}_{2,2}, \\
\dot{\widehat{\Phi}}_{2,2} &= \frac{z_{2,2}^2}{2\times 0.1^2}\phi^T(\overline{\chi}_{2,2})\phi(\overline{\chi}_{2,2}) - 0.1\widehat{\Phi}_{2,2}
\end{aligned} \tag{74}
$$

where $z_{1,1} = \chi_{1.1} - y_{1,d}$, $z_{1,2} = \chi_{1.2} - \nu_{1,2}$, $z_{2,1} = \chi_{2,1} - y_{2,d}$ and $z_{2,2} = \chi_{2,2} - \nu_{2,2}$. Let the initial conditions be $[\chi_{1,1}(0), \chi_{1,2}(0), \chi_{2,1}(0), \chi_{2,2}(0)]^T = [0.5, 0.2, 0.35, 0.15]^T$, $\widehat{\Phi}_{1,1}(0) = \widehat{\Phi}_{1,2}(0) = \widehat{\Phi}_{2,1}(0) = \widehat{\Phi}_{2,2}(0) = 0$ and $\widehat{\eta}_{1,1}(0) = \widehat{\eta}_{1,2}(0) = \widehat{\eta}_{2,1}(0) = \widehat{\eta}_{2,2}(0) = 0$. The simulation results are provided in Figures 1–5.

From Figure 1, we can see that the outputs y_1 and y_2 track the desired trajectories $y_{1,d}$ and $y_{2,d}$ with small tracking error. Figure 2 shows that the proposed scheme works well with bounded system inputs, and the response curves of adaptive parameters $\widehat{\Phi}_{1,1}$, $\widehat{\eta}_{1,1}$, $\widehat{\Phi}_{1,2}$, $\widehat{\eta}_{1,2}$, $\widehat{\Phi}_{2,1}$, $\widehat{\eta}_{2,1}$, $\widehat{\Phi}_{2,2}$, and $\widehat{\eta}_{2,2}$ are depicted in Figure 3. From Figure 4, it can be seen that the bounds for $z_{1,1}$, $z_{1,2}$, and $z_{2,2}$ are not overstepped. Finally, Figure 5 is given to explain the boundedness of states $\chi_{1,1}$, $\chi_{1,2}$, $\chi_{2,1}$, and $\chi_{2,2}$.

Example 2. Consider the following two inverted pendulums systems composed of spring and damper connections. The

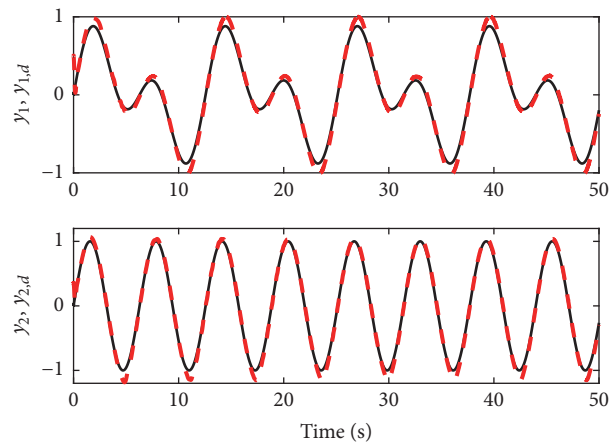

Figure 1: Outputs y_1 and y_2 and desired trajectories $y_{1,d}$ and $y_{2,d}$.

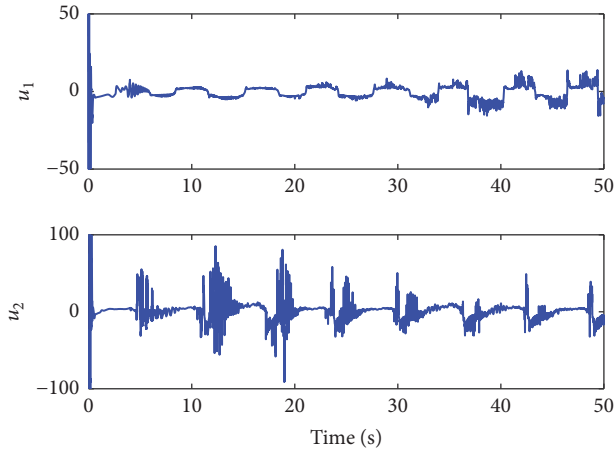

Figure 2: System inputs u_1 and u_2.

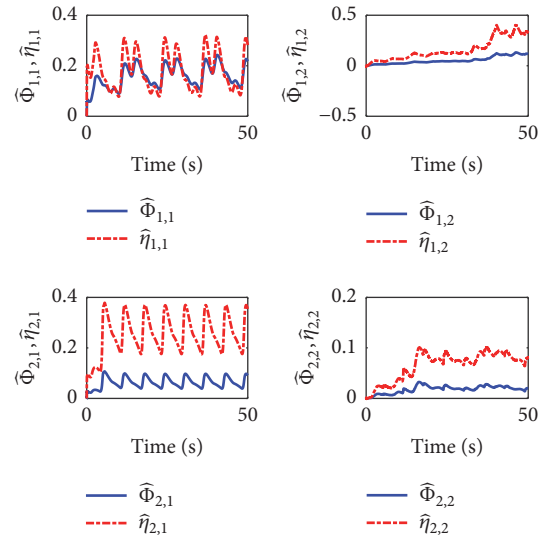

Figure 3: Adaptation parameters $\widehat{\Phi}_{1,1}$, $\widehat{\eta}_{1,1}$, $\widehat{\Phi}_{1,2}$, $\widehat{\eta}_{1,2}$, $\widehat{\Phi}_{2,1}$, $\widehat{\eta}_{2,1}$, $\widehat{\Phi}_{2,2}$, and $\widehat{\eta}_{2,2}$.

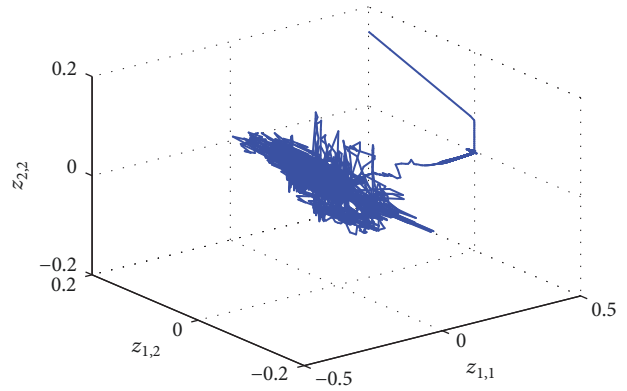

FIGURE 4: Phase portrait of $z_{1,1}$, $z_{1,2}$, and $z_{2,2}$.

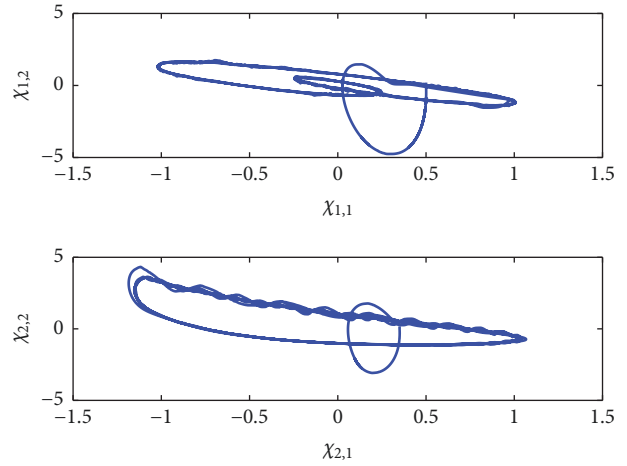

FIGURE 5: Phase portrait of states $\chi_{1,1}$, $\chi_{1,2}$, $\chi_{2,1}$ and $\chi_{2,2}$.

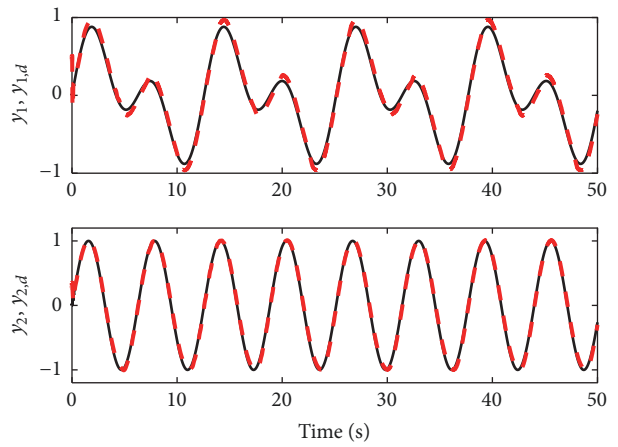

FIGURE 6: Outputs y_1 and y_2 and reference signals $y_{1,d}$ and $y_{2,d}$.

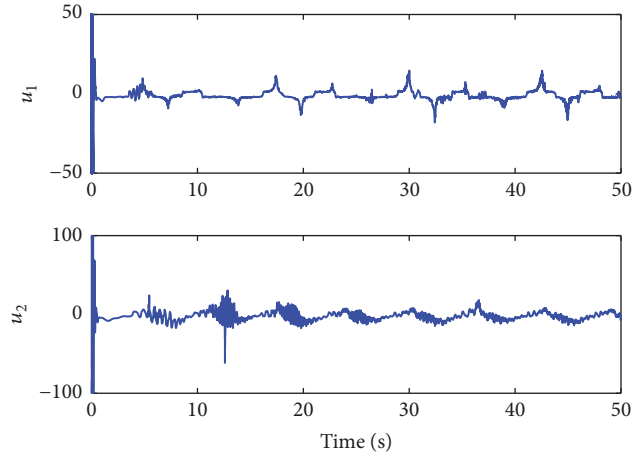

FIGURE 7: Control inputs u_1 and u_2.

pendulum angle and angular velocity were controlled using the torque inputs generated by a servomotor at each base. The dynamic equations can be described as follows [6]:

$$J_1\ddot{\theta}_1 = m_1 gr\sin\theta_1 - 0.5Fr\cos(\theta_1 - \theta) - T_{f1} + u_1$$
$$J_2\ddot{\theta}_2 = m_2 gr\sin\theta_2 - 0.5Fr\cos(\theta_2 - \theta) - T_{f2} + u_2 \quad (75)$$

where θ_1 and θ_2 are the angular positions, $J_1 = 1$ kgm^2 and $J_2 = 1$ kgm^2 are the moments of inertia, $m_1 = 2$ kg and $m_2 = 2.5$ kg are the masses, $r = 0.5$ m, $F = k(p - l) + b\dot{p}$ denotes the force applied by the spring and damper at the connection points, and p is the distance between the connection points as follows:

$$p = \sqrt{d^2 + dr(\sin\theta_1 - \sin\theta_2) + \frac{r^2}{2}[1 - \cos(\theta_2 - \theta_1)]} \quad (76)$$

where $d = 0.5$ m, $k = 150$ N/m and $b = 1$ N sec/m. The relative angular position θ can be defined as

$$\theta = \tan^{-1}\left(\frac{(r/2)(\cos\theta_2 - \cos\theta_1)}{d + (r/2)(\sin\theta_1 - \sin\theta_2)}\right) \quad (77)$$

and T_{fi} $(i = 1, 2)$ are assumed to be a LuGre friction model defined as

$$T_{fi} = s_0\dot{\varepsilon}_i + s_1\dot{\varepsilon}_i + s_2\dot{\theta}_i$$
$$\dot{\varepsilon}_i = \dot{\theta}_i - \frac{s_0\left|\dot{\theta}_i\right|}{T_c + (T_s - T_c)\exp\left(-\left|\dot{\theta}/\dot{\theta}_2\right|^2\right)} \quad (78)$$

where $s_0 = 1$ Nm, $s_1 = 1$ Nm sec, $s_2 = 1$ Nm sec, $\dot{\theta}_s = 0.1$ rad/sec, $T_s = 2$ Nm and $T_c = 1$ Nm.

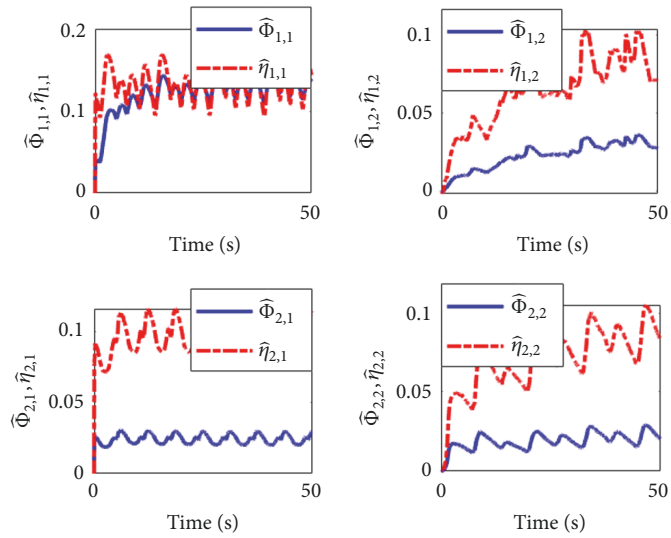

FIGURE 8: Adaptive parameters $\widehat{\Phi}_{1,1}$, $\hat{\eta}_{1,1}$, $\widehat{\Phi}_{1,2}$, $\hat{\eta}_{1,2}$, $\widehat{\Phi}_{2,1}$, $\hat{\eta}_{2,1}$, $\widehat{\Phi}_{2,2}$, and $\hat{\eta}_{2,2}$.

Defining $\chi_{1,1} = \theta_1$, $\chi_{1,2} = \dot{\theta}_1$, $\chi_{2,1} = \theta_2$ and $\chi_{2,2} = \dot{\theta}_2$, system (75) can be rewritten in the following form:

$$\begin{aligned}
\dot{\chi}_{1,1} &= \varphi_{1,1}(\chi_{1,1}, \chi_{1,2}) + D_{1,1}(\chi, t)\\
\dot{\chi}_{1,2} &= \varphi_{1,2}(\chi_{1,1}, \chi_{1,2}, u_1(o_1)) + D_{1,2}(\chi, t)\\
\dot{\chi}_{2,1} &= \varphi_{2,1}(\chi_{2,1}, \chi_{2,2}) + D_{2,1}(\chi, t)\\
\dot{\chi}_{2,2} &= \varphi_{2,2}(\chi_{2,1}, \chi_{2,2}, u_2(o_2)) + D_{2,2}(\chi, t)\\
y_1 &= \chi_{1,1}\\
y_2 &= \chi_{2,1}
\end{aligned} \tag{79}$$

where $\varphi_{1,1}(\cdot) = (1 - e^{-1-\chi_{1,1}})/(1 + e^{-1-\chi_{2,1}}) + \chi_{1,2}$, $\varphi_{1,2}(\cdot) = g_{1,2}(m_1 gr \sin x_{1,1} - 0.5Fr\cos(\chi_{1,1} - \theta) - T_{f1}) + g_{1,2}u_1(o_1)$, $g_{1,2} = 1/J_1$, $\varphi_{2,2}(\cdot) = g_{2,2}u_2(o_2) + g_{2,2}(m_2 gr \sin x_{2,1} - 0.5Fr\cos(\chi_{2,1} - \theta) - T_{f2})$, $g_{2,2} = 1/J_2$, $\varphi_{2,1}(\cdot) = (\chi_{2,1} + \chi_{2,2})/(1 + \chi_{2,1}) + \chi_{2,2}$; $D_{1,1}(\chi, t) = 0.5\sin(\chi_{1,2}\chi_{2,1}\chi_{2,2})\cos(0.2t)$, $D_{2,1}(\chi, t) = 2 \times \sin(\chi_{1,1}\chi_{1,2}\chi_{2,1}^2)$, $D_{1,2}(\chi, t) = 0.3\cos(\chi_{1,2}^2 + \chi_{1,1}\chi_{2,1})$ and $D_{2,2}(\chi, t) = \cos(\chi_{2,1}^2 + \chi_{2,2}^2 + \chi_{1,1})(\sin(t))^2$. Moreover, $u_1(o_1)$ and $u_2(o_2)$ are described by the following:

$$\begin{aligned}
u_1(o_1) &= sat(o_1)\begin{cases}\text{sign}(o_1)\,4, & |o_1| \geq 4\\ o_1, & |o_1| < 4\end{cases}\\
u_2(o_2) &= sat(o_2)\begin{cases}\text{sign}(o_2)\,6.5, & |o_2| \geq 6.5\\ o_2, & |o_2| < 6.5\end{cases}
\end{aligned} \tag{80}$$

It can be seen that the non-affine functions are non-differentiable with respect to u_1 and u_2. In simulation, choose the desired reference trajectories as $y_{1,d} = 0.5(\sin t + \sin(0.5t))$ and $y_{2,d} = \sin(t)$; the virtual control laws, actual control laws, and adaption laws are provided by (25), (52) and (26)-(27), (53)-(54) with design parameters $c_{1,1} = c_{1,2} = 6, c_{2,1} = c_{2,2} = 9$, $a_{1,1} = a_{1,2} = 0.2$, $a_{2,1} = a_{2,2} = 0.5$, $\alpha_{1,1} = \alpha_{2.1} = 4$, $\alpha_{1,2} = \alpha_{2,2} = 2$, $\upsilon_{1,1} = \upsilon_{1,2} = 0.25$, $\upsilon_{2,1} = \upsilon_{2,2} = 0.1$, $\beta_{1,1} = \beta_{1,2} = 1.5$, $\beta_{2,1} = \beta_{2,2} = 2$, $\sigma_{1,1} = \sigma_{1,2} = 0.15$, $\sigma_{2,1} = \sigma_{2,2} = 0.25$, $\xi_{1,1} = 2$, $\xi_{1,2} = \xi_{2,2} = 2.5$, $\xi_{2,1} = 1.5$. Let the initial conditions be $[\chi_{1,1}(0), \chi_{1,2}(0), \chi_{2,1}(0), \chi_{2,2}(0)]^T = [0.3, 0.2, 0.5, 0.45]^T$, $\widehat{\Phi}_{1,1}(0) = \widehat{\Phi}_{1,2}(0) = \widehat{\Phi}_{2,1}(0) = \widehat{\Phi}_{2,2}(0) = 0$ and $\hat{\eta}_{1,1}(0) = \hat{\eta}_{1,2}(0) = \hat{\eta}_{2,1}(0) = \hat{\eta}_{2,2}(0) = 0$. The simulation results are provided in Figures 6–8.

As can be seen in Figure 6, the system outputs track the desired trajectories, perfectly. Figures 7-8 illustrate the system inputs and adaptive parameters, from which, we can see that the fairly good tracking performance is obtained.

6. Conclusion

This work for the first time proposes fuzzy adaptive dynamic surface control design for a larger class of MIMO non-affine nonlinear systems in the presence of input saturation. To overcome the design difficulty of input saturation, a novel auxiliary system with bounded compensation term has been proposed, and a bounded compensation term of tangent function is designed in this paper. Thanks to this design, we successfully relax the strictly bounded assumption of the dynamic system. SGUUB stability of the closed-loop systems is rigorously proved by combining Lyapunov theory and invariant set theory.

Data Availability

The simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant no. 71601183).

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