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## SRME and estimation of primaries by sparse inversion: a hybrid approach

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### Summary

Recently, a new approach to multiple removal has been introduced: estimation of primaries by sparse inversion (EPSI). Although based on the same relationship between primaries and multiples as in surface-related multiple elimination (SRME), it involves quite a different process. Instead of the traditional prediction and subtraction of multiples, in EPSI the unknown primaries are the parameters of a large-scale inversion process. The downside is its long calculation times, involving the equivalent of about 100-200 SRME processes. For improving the accuracy and efficiency in multiple removal a hybrid SRME+EPSI is proposed, which makes use of the strongest points of both SRME and EPSI methodologies. It appears that the final result is better than either the SRME or the EPSI algorithm alone and where the calculation time is limited to the equivalent of 10-20 SRME processes.

### Introduction

With the introduction of surface-related multiple elimination (SRME) (Berkhout, 1982; Verschuur et al., 1992; Berkhout and Verschuur, 1997; Weglein et al., 1997; Biersteker, 2001), a complete new approach to multiple removal was developed: the multiples could be predicted without any prior knowledge of the subsurface. All of the required information was embedded in the seismic data, because of the physical relationship between primaries and multiples. However, there also limitations to the SRME approach. First, the adaptive subtraction is usually based on minimum energy, which is not always a good assumption (see e.g. Nekut and Verschuur, 1998). Second, it needs the reconstruction of missing offsets. Especially in the case of shallow water, the reconstruction of the near offsets is not trivial, whereas it has a large impact on the quality of the predicted multiples (Verschuur, 2006).

Therefore, recently a new approach to multiple removal was developed by van Groenestijn and Verschuur (2009a): estimation of primaries by sparse inversion (EPSI). The main difference with SRME is that the two-stage processing method, being prediction and adaptive subtraction, is replaced by a full waveform inversion process: the primary reflection events are the unknowns in this algorithm and are parameterized in a suitable way. In van Groenestijn and Verschuur (2009a) the adopted parameterization consists of band-limited spikes and an effective source wavelet. Baardman et al. (2010) discussed a refinement, where the wavelet was made time-variant in order to include the change of the observed seismic wavelet in case of complex propagation effects (fine layering,

dispersion) and absorption. Lin and Herrmann (2009, 2011) redefined EPSI in the curvelet domain. Savels et al. (2011) have shown various applications to complex synthetic and field datasets.

One of the main advantages of the EPSI method is that the adaptive subtraction, involved in SRME, is avoided. Instead, in EPSI the full input data is explained, being the sum of the estimated primaries and their associated surface multiples. The new objective function – the difference between the input data and the estimated primaries plus their multiples - will truly go to zero. Furthermore, missing data can be estimated together with the primaries, such that the method has great virtue in the situation of shallow water (van Groenestijn and Verschuur, 2009b).

Even when EPSI presents several advantages over traditional SRME, its difficulty to pick up deep low-impedance reflectors and its computational time cost are limitations in the current EPSI algorithm. These problems are not present in SRME, where results can be obtained fast, and whose output presents all the desired reflection structures, even for the deepest interfaces in models.

For overcoming this issues a hybrid SRME-EPSI approach is proposed, in which the primaries resulting from a quick and coarse EPSI algorithm (we will call it ‘greedy EPSI’) will serve as initial primary guess for the SRME iterations. In addition, multiple subtraction can be done in such a way that the original primaries found in the greedy-EPSI process are conserved. This prevents distortion of the primaries already found by EPSI.

### Review of the theory of SRME and EPSI

In the detail-hiding operator notation for 2D data (Berkhout, 1982) a bold quantity represents a pre-stack data volume for one frequency; columns represent monochromatic shot records and rows represent monochromatic common receiver gathers. With the use of this notation we can express the upgoing data at the surface,  $\mathbf{P}$ , as:

$$\mathbf{P} = \mathbf{X}_0 \mathbf{S} + \mathbf{X}_0 \mathbf{R}^\top \mathbf{P}, \quad (1)$$

where the primary impulse responses,  $\mathbf{X}_0$ , multiplied with the source properties,  $\mathbf{S}$ , equal the primaries:  $\mathbf{P}_0 = \mathbf{X}_0 \mathbf{S}$ . Note that what is called ‘primaries’ in this paper actually refers to all events that did not reflect at the surface, which also includes internal multiples. The matrix multiplication of  $\mathbf{X}_0$  with the reflection operator at the surface,  $\mathbf{R}^\top$ , and the total data results in the surface multiples,  $\mathbf{M} = \mathbf{X}_0 \mathbf{R}^\top \mathbf{P}$ .

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From equation 1 it can be derived that surface multiples can be predicted by a multidimensional convolution of the primaries with the data:

$$\mathbf{M} = \mathbf{P}_0 \mathbf{A} \mathbf{P}, \quad (2)$$

where  $\mathbf{A} = \mathbf{S}^{-1} \mathbf{R}^\dagger$  is the surface operator. Iterative SRME (Berkhout and Verschuur, 1997) estimates the primaries according to:

$$\mathbf{P}_{0,i+1} = \mathbf{P} - \mathbf{A}_{i+1} \mathbf{P}_{0,i} \mathbf{P}, \quad (3)$$

where  $i$  represents the iteration number. Usually,  $\mathbf{A}$  is replaced by an angle-independent approximation  $A(\omega)\mathbf{I}$  and the iterations are initiated by  $\mathbf{P}_{0,1} = \mathbf{P}$ . Since there are more unknowns,  $\mathbf{P}_{0,i+1}$  and  $A_{i+1}$ , than knows,  $\mathbf{P}$ , in equation 3 an extra constraint is needed. Typically it is assumed that the primaries have minimum energy (the L2 norm). This constraint is used when  $A_{i+1}$  is estimated as a filter that matches the predicted multiples,  $\mathbf{M}_i = \mathbf{P}_{0,i} \mathbf{P}$ , to the input data in the time domain, resulting in the new primary estimation,  $\mathbf{P}_{0,i+1}$  (Verschuur and Berkhout, 1997). The minimum energy norm often leads to a satisfactory subtraction result, but does not work properly in all cases (see e.g. Nekut and Verschuur, 1998). Guitton and Verschuur (2004) and van Groenestijn and Verschuur (2008) have shown that other minimization norms, like the L1 norm or a sparseness norm, can lead to different, and sometimes better, subtraction results.

To describe the EPSI algorithm (van Groenestijn and Verschuur, 2009a) we should again consider equation 1. If we take  $\mathbf{S}(\omega) = S(\omega)\mathbf{I}$  (meaning assuming a constant source wavelet for all shots) and we assume the surface reflectivity to be a scalar  $R^\dagger$  (being approximately  $-1$ ) we get:

$$\mathbf{P} = \mathbf{X}_0 \mathbf{S} + \mathbf{X}_0 R^\dagger \mathbf{P}. \quad (4)$$

Through full waveform inversion we try to estimate the unknown, multidimensional primary impulse response  $\mathbf{X}_0$  and source wavelet  $S$  such that the primaries  $\mathbf{X}_0 \mathbf{S}$  together with the surface multiples  $\mathbf{X}_0 R^\dagger \mathbf{P}$  can explain the total upgoing data  $\mathbf{P}$ . The unknown dataset  $\mathbf{X}_0$  is parameterized in the time domain with spikes. The difference between the total upgoing data  $\mathbf{P}$  and the estimated primaries and multiples,  $\mathbf{X}_0 \mathbf{S} + \mathbf{X}_0 R^\dagger \mathbf{P}$ , is the residual  $\mathbf{V}$ :

$$\mathbf{V} = \mathbf{P} - \mathbf{X}_0 \mathbf{S} - \mathbf{X}_0 R^\dagger \mathbf{P}. \quad (5)$$

The EPSI algorithm drives the residual  $\mathbf{V}$  to zero. This is done in an iterative way where the primary impulse response data volume  $\mathbf{X}_0$  is built up slowly during the iteration process in the time domain. In this way the adaptive subtraction is avoided and interference between primaries and multiples is better handled.

### Analysis of SRME and EPSI in shallow water

It is common knowledge that the effectiveness of the SRME process is limited for shallow water if the water bottom reflectivity is strong. Usually, in this case a combination with predictive deconvolution and/or parabolic Radon multiple removal is adopted (Verschuur, 2006). The main reason is that SRME requires all offsets available up to zero offset. The more shallow the water, the more difficult the process of interpolating the near offsets becomes, because this is usually based on NMO-corrected data, yielding strong stretching artifacts for the shallow events.

In the EPSI process, the reconstruction of the near offsets has been included in the inversion process, such that actually the near offsets are interpolated based on the multiples that are present in the data. NMO-correction is not involved here (van Groenestijn and Verschuur, 2009a).

In the following examples, the effects of shallow water on SRME and EPSI are demonstrated. Two different models are used for synthetic data generation, the only difference between the models being the sea floor depth (around 200m for the deep model, and 90 m for the shallow model). The two models are depicted in figures 1 and 2.

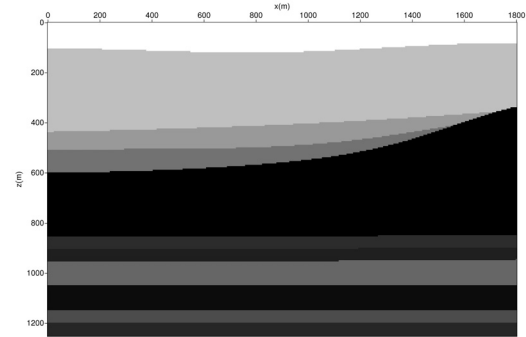


Fig 1. Shallow water model, with a water bottom around 90 m.

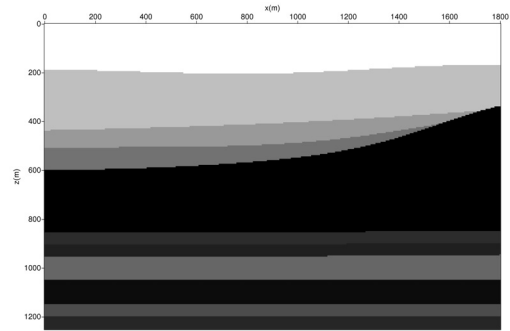


Fig 2. Deep water model, with a water bottom around 200 m.

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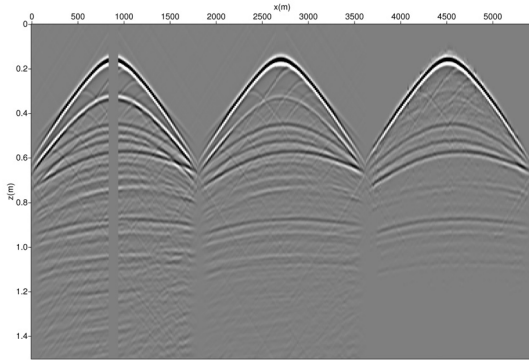


Fig 3. Input data, SRME primaries, and EPSI primaries for the shallow sea floor model.

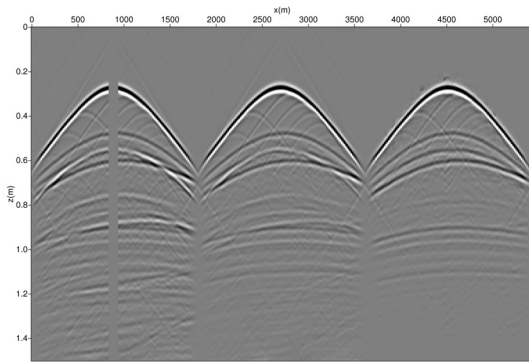


Fig 4. Input data, SRME primaries, and EPSI primaries for the deep sea floor model.

As we can see in figures 3 and 4, EPSI is able to pick more details in the primaries and its outputs look clearer than with SRME, the latter showing some residual multiples. As EPSI does not rely on adaptive subtraction, no multiple leakage is found in the estimated primaries. The difference becomes more dramatic in the shallow case, where surface multiples rely more on the missing near offsets. At the same time the near offset interpolation of the sea floor reflector is more confident in the EPSI result.

### Hybrid SRME and EPSI

Even though EPSI presents significant advantages over SRME, there are still some issues about the current EPSI implementation that can be improved. One example is correctly finding deep, low-impedance reflections. Once EPSI starts, it picks up the most prominent reflections in reflection data and constructs the primary response by inversion. But if we have deep low-impedance reflectors in our model, the corresponding primary will be very low amplitude, meaning that EPSI – who tends to concentrate more on the stronger primaries – could have problems

resolving such a weak reflection, especially if the algorithm has already converged to the first predominant primaries. Another issue about EPSI is the computational cost required to run it. One average EPSI iteration takes about two times an SRME process, and often tens of iterations are needed to cover the full data set. For large datasets this may become a serious limitation.

In order to overcome these drawbacks, we introduce a hybrid version of SRME and EPSI such that the combined outcome would reinforce the weak points of each method. The idea is to use a fast version of EPSI (‘greedy EPSI’, shown in figures 5 and 6 for our two models, requiring only 10 iterations) to extract the strongest primaries using large windows during iterations. Then this result – including its reconstructed near offsets - is introduced in SRME as initial primary guess, which can be improved during one or two SRME iterations.

In order to avoid damaging the original primary estimation done by the greedy EPSI step, an additional constraint is added to SRME: multiple subtraction should be done in such a way that the original primaries already found in the greedy EPSI step are conserved. This will prevent multiple distorting the primaries during subtraction and reads:

$$| \mathbf{P} - A \mathbf{M}_{0,i} - \mathbf{P}_{0,e} |^2 = \text{minimum} , \quad (6)$$

where  $\mathbf{P}_{0,e}$  are the primaries from the greedy EPSI process,  $\mathbf{M}_{0,i}$  are the SRME-predicted multiples, and  $A$  is the surface operator that is estimated. Note that  $A$  is again estimated as a short convolution filter in the time domain (Verschuur and Berkhou, 1997).

In the following section we will see some examples of SRME, EPSI and the hybrid method, applied in the two synthetic models already described. This will allow comparison between the different methodologies.

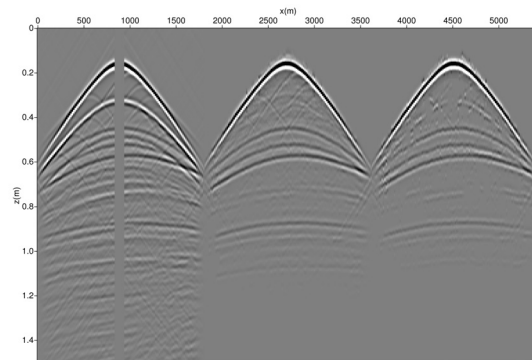


Fig 5. Input data, regular EPSI primaries and greedy EPSI primaries for the shallow sea floor model. Note that the greedy EPSI does not fully pick up all primaries.

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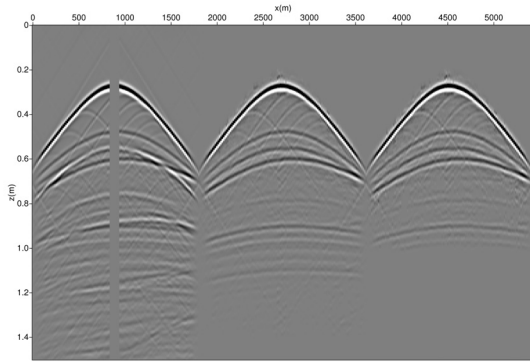


Fig 6. Input data, regular EPSI primaries and greedy EPSI primaries for the deep sea floor model.

### Analysis of hybrid SRME-EPSI

Figures 7 and 8 compare the regular EPSI and the hybrid SRME-EPSI primaries. As we can see from the figures, the hybrid method presents more information in the deeper part than EPSI, showing more details with better illumination. In the shallow part of data the results are quite similar. Nevertheless there is a great difference in computational efficiency between current EPSI and the hybrid model. Also note again that the hybrid SRME-EPSI result is much better than the standard SRME results that were shown in Figures 4 and 5.

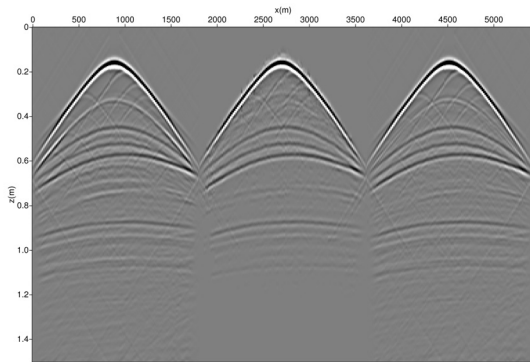


Fig 7. SRME primaries, EPSI primaries, and hybrid SRME-EPSI primaries for the shallow deep sea floor model.

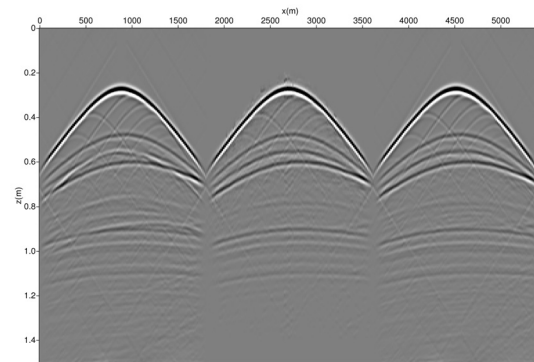


Fig 8. SRME primaries, EPSI primaries, and hybrid SRME-EPSI primaries for the deep sea floor model.

### Conclusions

We have reviewed the EPSI method for primary estimation that has advantages over the traditional SRME method. In EPSI adaptive subtraction (which can produce distorted primaries) is avoided, and replaced by a full waveform inversion process, in which the complete data is used to estimate the primaries, allowing near-offset reconstruction to be included in the process. This method, however, sometimes has difficulties to pick up the low-amplitude, deeper reflections, and requires much more computational power than SRME.

In order to extract the best of each methodology in one single algorithm we propose a hybrid SRME-EPSI method, in which the initial primary estimation is done by a coarse and fast version of EPSI, and the subsequent refinement is done by SRME. Multiple leakage is avoided by including an additional ‘primary-saving’ constraint in the subtraction process. Some examples on synthetic data with shallow water demonstrate the advantages of the hybrid method.

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