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3-D redatuming for breast ultrasound

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ABSTRACT

Breast cancer is the most common form of cancer diagnosed with women. To reduce its mortality rate, early diagnosis is important. In the past, this has led to the introduction of national screening programs using mammography. The disadvantages of mammography (application of ionizing radiation and low detection rate in dense breast) resulted in the demand for an alternative. This demand has led to the development of ultrasonic water bath scanning systems. Those systems scan the breast from all sides and aim for reconstructing the acoustic tissue properties from the measured pressure fields by employing among others full-waveform inversion methods. However, full-wave inversion is computationally expensive, especially in 3-D, and scales almost linear with the size of the spatial domain. To reduce the computational load, we propose a method that reduces the size of the spatial computational domain by back-propagating the field measured on the surface of the 3-D scanning geometry to a surface enclosing a reduced volume. To this end, the measured field is first decomposed into spherical Hankel functions with complex coefficients and subsequently redatumed to a new surface closer by the object. The proposed redatuming method is tested successfully for 3-D synthetic examples.

Keywords: breast ultrasound, redatuming, full-wave inversion

1. INTRODUCTION

Recent research has shown that ultrasound is a promising imaging method for breast cancer diagnosis. Significant amount of works focus on building a water bath scanning system. Some of these systems have been already tested in a clinical setting.[1–3] With a water bath scanner, the breast is scanned from all sides, and transmission and reflection measurements are done simultaneously. Using one full-scan data, three kind of images are generated; reflectivity, speed of sound, and attenuation images. Reflectivity images reveals the echogenicity of breast tissues. Whereas, speed of sound and attenuation images provide quantitative acoustical properties about breast tissues. Consequently, tissue characterization and diagnosis can be done by combining these three images.[4]

It is known that full-wave inversion is the best imaging method to generate accurate speed of sound images.[5] Main challange with full-wave inversion is the comutational cost involved for image reconstruction. The computational cost is proportional with the size of the region of interest to be imaged. To reduce the size of the region of interest, we proposed a redatuming technique which is valid for a 2-D setup.[6] By redatuming the measured pressure field, the region of interest becomes smaller and consequently computational cost of full-wave inversion highly reduces. In this work, we extend this idea to 3-D. We use spherical Hankel functions to describe and back-propagate the measured pressure wave. We show achieved computational gain by comparing the computation time for two cases. First, full-wave inversion is applied for the original region of interest. Second, full-wave inversion is applied to a smaller region of interest that is constructed after redatuming.

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2. THEORY

Consider an unknown object within a finite spatial domain $\mathbb{D}$. A measurement domain $\mathbb{S}$ and a redatuming domain $\mathbb{S}'$ enclose the object. A position in the spatial domain $\mathbb{R}^3$ is denoted in Cartesian coordinates by the vector (x, y, z) and in spherical coordinates by the vector (r, θ, ϕ). All formulations are done in the temporal Fourier domain with angular frequency ω.

The pressure field $p(r, \theta, \phi, \omega)$ measured in $\mathbb{S}$ satisfies the 3-D Helmholtz equation, which reads in spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial p}{\partial\theta}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial^2 p}{\partial\phi^2} + \frac{\omega^2}{c^2}p = 0. \tag{1}$$

This equation can be solved using the separation of variables, hence

$$p(r, \theta, \phi, \omega) = R(r, \omega)\,\Theta(\theta)\,\Phi(\phi). \tag{2}$$

Substituting equation (2) into (1) yields separate equations for $\Gamma(r, \omega)$, $\Theta(\theta)$ and $\Phi(\phi)$,[7] namely

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \left[r^2\frac{\omega^2}{c^2} - n(n+1)\right]R = 0, \tag{3}$$

$$\frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial\Theta}{\partial\theta}\right) + \left[n(n+1) - \left(\frac{m}{\sin(\theta)}\right)^2\right]\Theta = 0, \tag{4}$$

and

$$\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2} = -m^2, \tag{5}$$

with m and n constants.

Equation (3) is known as Bessel's differential equation and has the following solution

$$R_n(r, \omega) = b_{n,1}(\omega)h_n^{(1)}\left(\frac{\omega}{c}r\right) + b_{n,2}(\omega)h_n^{(2)}\left(\frac{\omega}{c}r\right), \tag{6}$$

where the spherical Hankel functions $h_n^{(1)}\left(\frac{\omega}{c}r\right)$ and $h_n^{(2)}\left(\frac{\omega}{c}r\right)$ represent outward and inward propagating waves, respectively. By placing the origin of our coordinate system inside $\mathbb{S}$ and $\mathbb{S}'$ we know that the scattered field is only described by outward propagating waves. Consequently, all coefficients $b_{n,2}(\omega)$ are equal to zero.

Equation (4) is similar to the Legendre differential equation and has a following solution

$$\Theta_n = b_{n,1}(\omega)P_n^m(\cos(\theta)) + b_{n,2}(\omega)Q_n^m(\cos(\theta)), \tag{7}$$

where $P_n^m(\cos(\theta))$ and $Q_n^m(\cos(\theta))$ are associated Legendre functions of the first and second kind.

Equation (5) is a standard second-order differential equation whose solution equals

$$\Phi = ae^{im\phi}, \tag{8}$$

with arbitrary constant a and $-\pi < \phi \leq \pi$.

By combining equations (2), (6), (7) and (8) the solutions of equation (1) are equal to

$$p(r, \theta, \phi, \omega) = \sum_{n=-N}^{N}\sum_{m=-n}^{n} c_{mn}(\omega)h_n^{(2)}\left(\frac{\omega}{c}r\right)Y_n^m(\cos\theta). \tag{9}$$

where Y_m^n equals

$$Y_n^m = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}}P_n^m(\cos\theta)\exp(-im\phi). \tag{10}$$

To find the complex valued coefficients $c_n(\omega)$ of equation (9), the pressure field $p(r, \theta, \phi, \omega)$ is matched to the M measurements $d_m(\omega)$, where $d_m(\omega)$ is the field measured by the m^{th} receiver located on $\mathbb{S}$. Consequently, the unknown coefficients $c_n(\omega)$ are obtained by solving the following system of equations

$$\sum_{n=-N}^{N} \sum_{m=-n}^{n} c_{mn}(\omega) h_n^{(2)} \left(\frac{\omega}{c} r \right) Y_n^m(\cos\theta) = d_m(\omega), \tag{11}$$

for all m.

Once the coefficients $c_n(\omega)$ are found, the field can be computed at any location outside $\mathbb{D}$ using equation (11). In this way, we can redatum the wave field measured at $\mathbb{S}$ to the domain $\mathbb{S}'$.

3. RESULTS

The proposed redatuming method is tested using two synthetic examples. In the first example, we validate the proposed redatuming method by back-propagating the measured pressure field that is generated by an arbitrary source distribution. In the second example, we apply contrast source inversion (CSI)[8] on the measured data before and after redatuming.

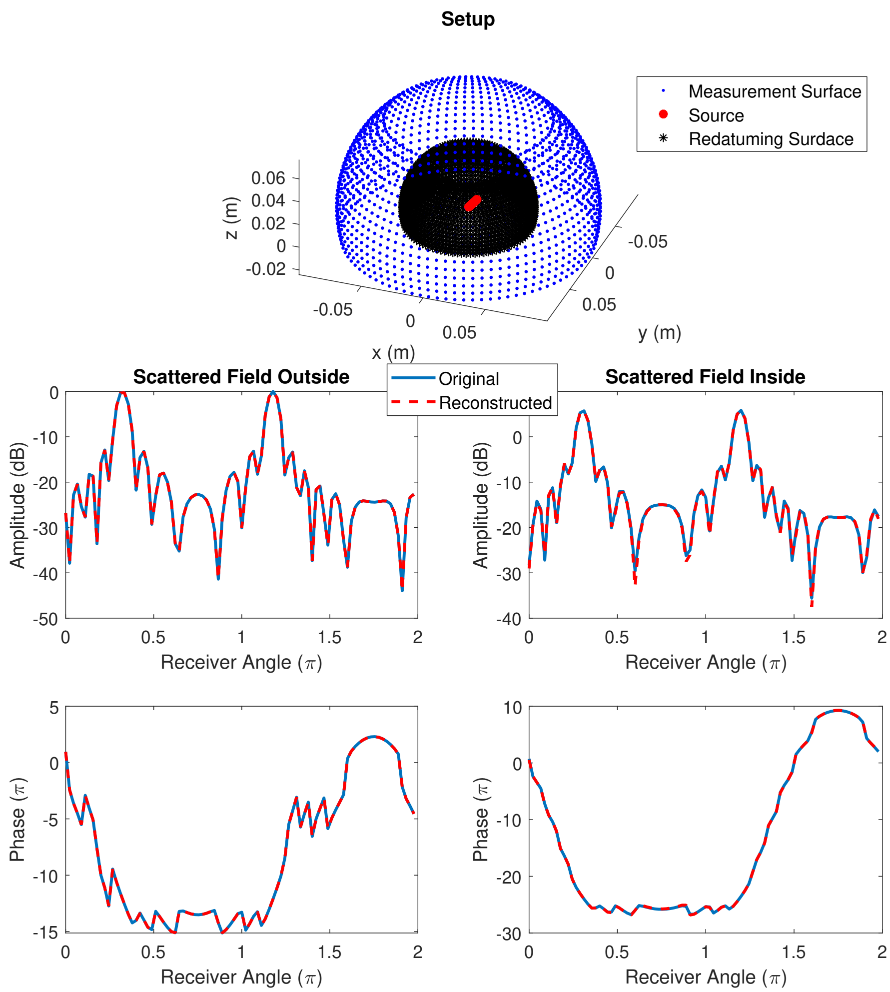

Figure 1. A synthetic example showing results of redatuming method. System geometry is given at top. The amplitudes and phases of the measured (blue) and redatumed (red) wave fields are compared in the graphs below; left the fields at $R_1 = 0.1$ m and right at $R_2 = 0.05$ m.

3.1 Validation

A source is constructed via an arbitrary collection of point sources, see Figure 1. This source generates a pressure field that is measured at two hemispheres with radii R_1 and R_2. Next, the field measured at the outer hemisphere with radius R_1 is decomposed into spherical Hankel functions. This decomposed field is compared with its original measured field, see bottom left image in Figure 1. Next, the field measured at the hemisphere with radius R_1 is redatumed to a hemisphere with radius $R_2 < R_1$. A comparison of the redatumed and measured field at the hemisphere with radius R_2 is shown in the bottom right image. An excellent match between the original and redatumed wave fields is observed for both cases.

3.2 Inversion

The forward problem is solved using a full-wave method[9] and the pressure field is measured at the measurement surface, see Figure 2. There are eight sources distributed equally on the circle with radius $r = 5$ mm in the plane $z = 0$. The central frequency of the signal sent from the source is 1 MHz. As illustrated in Figure 2, 128 receivers are located on the measurement surface. Two spherical objects are placed in the region of interest. One of them has a lower (1485 m/s) and the other one has a higher (1634 m/s) speed of sound value as compared to the homogeneous embedding (1524 m/s).

First, contrast source inversion (CSI) is applied on the pressure field at the measurement surface. Next, CSI is applied on the redatumed pressure field. Reconstruction results for both situations are presented in Figure 3. These images are showing $z = 0$ plane. For both cases, CSI reconstructed the two spherical objects with an accurate location and size. Pixel amount of the region of interest is $42 \times 42 \times 42$ and $32 \times 32 \times 32$ before and after redatuming respectively. Hence, in this particular example, the computational reduction is 55 %. This means that the inversion after redatuming becomes approximately two times faster. Depending on the measurement setup and object of interest, this reduction may vary.

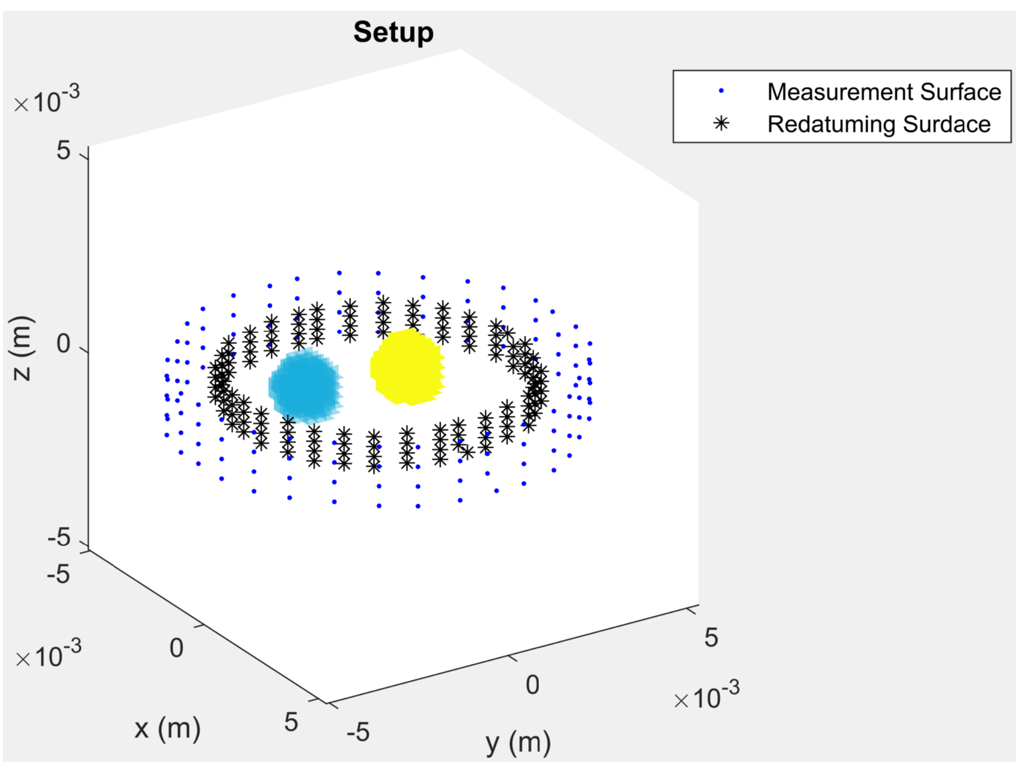

Figure 2. System geometry of the inversion example.

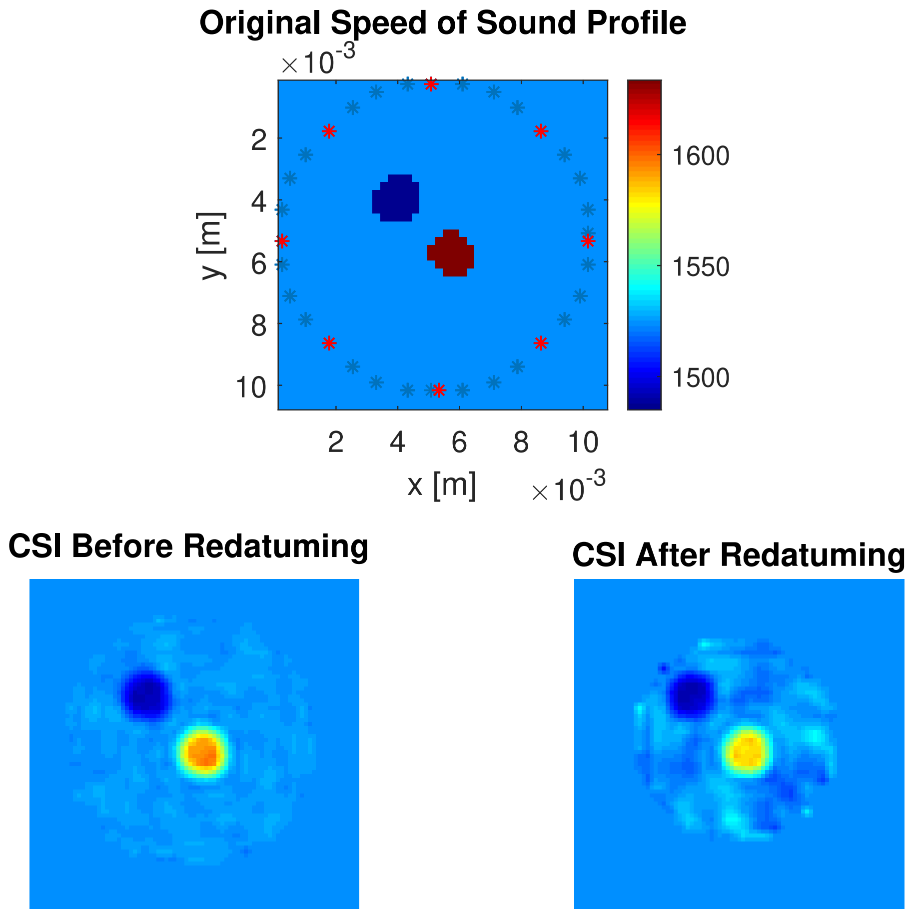

Figure 3. Reconstruction results of CSI. System geometry is given at top. Red dots are the sources, and blue dots are the receivers located at $z = 0$ plane. Colorbar indicates speed of sound in m/s. CSI results before and after redatuming are given at the bottom images.

4. CONCLUSION

A new redatuming method based on Hankel function decomposition is presented for a 3-D environment. The method aims to reduce the spatial computational domain and hence the computation time for 3-D full-wave inversion. Results with two synthetic examples show that proposed method is accurate and efficient. For the first example shown in this work, the radius of the hemisphere is reduced from $R_1 = 0.1$ m to $R_2 = 0.05$ m. This reduces the size of the domain of interest and hence the computational load for inversion by a factor eight. For the second example, a full-wave inversion method is applied to the measured and redatumed data. Similar reconstruction results are obtained with redatumed case as compared to the normal situation.

In this work, we only focus on the computational gain that is achieved with redatuming. In practice, redatuming brings more advantages. First, with the ability of redatuming to any point in the embedding, the measurement geometry can be altered. This can be useful to apply different kinds of imaging and inversion methods optimized for a specific geometry. Second, redatuming acts as a filter on the pressure field so it can be useful to remove noise in the measurement. Third, it may lead to an improved convergence as is observed with solving the forward problem.[10] Finally, the particle velocity can be computed with this method which can be useful for inversion of mass density.[11]

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