MODELLING THE DYNAMIC OUT-OF-PLANE TWO-WAY BENDING SEISMIC RESPONSE OF UNREINFORCED MASONRY WALLS

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Abstract: *The collapse of walls in the out-of-plane (OOP) direction is a common failure mechanism in existing unreinforced masonry (URM) buildings when subjected to seismic excitation. Such local mechanisms also prevent the realisation of the full in-plane seismic capacity of URM buildings. Among OOP failures, a distinction can be made between (i) one-way bending which occurs walls without side supports, and (ii) two-way bending which occurs in walls that have at least one vertical and one horizontal edge supported. This paper proposes a single-degree-of-freedom model for modelling the dynamic behaviour of URM walls subjected to OOP seismic excitation and undergoing two-way bending. The model considers two distinct phases: (i) initial elastic and (ii) post cracking, transitioning instantaneously between the two once the force required to crack the wall has been exceeded. The model's complete behaviour (i.e. in both phases) is described by six independent parameters, which can be calculated analytically. Post cracking, the wall is treated as a system composed of rigid blocks, and the wall resistance is calculated as the superimposition of three separate contributions: (i) the bilinear elastic rigid block rocking, (ii) the elastoplastic friction, and (iii) a bilinear degrading component taking into account strength and stiffness degradation of the wall. The proposed model is then calibrated against experimental results from incremental dynamic testing of full-scale walls. For this, two walls with different failure modes under horizontal bending, i.e. line and stepped failure are considered. Calibration of the model shows excellent agreement with test results, successfully capturing their behaviour from cracking to collapse. The calibrated models were employed to evaluate behaviour factors under both natural and induced seismicity, resulting in values close to two for both walls.*

1 Introduction

Recent trends in earthquake engineering, especially in the context of seismic risk analysis require the development of computationally light models which are still capable of predicting collapse in a sufficiently accurate manner. With the high vulnerability of unreinforced masonry (URM) constructions to seismic loading especially in out-of-plane (OOP) direction being well known, several analytical models have been developed for the OOP behaviour of URM walls (Doherty et al., 2002; Godio and Beyer, 2019; Lam et al., 2003; Sorrentino et al., 2008). Recent developments in this field also include more advanced methods for identifying and considering collapse mechanisms (Funari et al., 2021; Mehrotra and DeJong, 2018). However, these models study the OOP response of URM by considering the wall to be behaving as an assemblage of rigid or semirigid blocks. This is done by assuming an initial cracked state of the wall, given that the associated calculations are inapplicable for un-cracked walls. Such an assumption (i.e. to consider the wall to be in an initial cracked state) can be considered conservative for vertically spanning walls exhibiting one-way vertical bending behaviour, where the initial cracking resistance is controlled by the bond strength of masonry and in a majority of cases is significantly lower than the force required to trigger a kinematic mechanism. In the case of two-way spanning walls, evidence from the first incremental dynamic experimental campaigns on full scale walls carried out by Graziotti *et al.* (Graziotti et al., 2019b) and Sharma *et al.* (Sharma et al., 2020) have shown that such an assumption can be overly conservative. The initial cracking resistance of the tested full-scale masonry panels was significantly higher than the force associated with any kinematic mechanism and therefore such walls should not be implicitly considered in an already cracked state. A procedure that takes into account the initial un-cracked state of two-way spanning walls is thus proposed in this study to model their dynamic OOP behaviour. The proposed model marks an improvement on state-of-the-art works by Vaculik and Griffith (Vaculik and Griffith, 2017a) who also proposed OOP load-displacement models of two-way spanning URM walls but did not account for their pre-cracking behaviour. A summary of the experiments used as a reference for this study is first presented in section 2. The proposed model is then described in section 3, along with recommendations to calculate all parameters governing the behaviour of the model analytically. The calibration of the proposed model and the comparison of its performance against the reference experiments is discussed in section 4. The calibrated models are subsequently employed to estimate behaviour factors for conducting the seismic assessment of URM walls in the OOP direction in section 5.

2 Reference Experiments

The reference experiments adopted in this paper are two single leaf full-scale URM walls tested incrementally dynamically by Graziotti *et al.* (Graziotti et al., 2019b). These tests were part of a larger experimental campaign that aimed to assess the vulnerability of unreinforced buildings under the action of induced seismicity in Groningen, the Netherlands (Graziotti et al., 2019a; Jafari et al., 2017; Rots et al., 2017) . Both the walls have the same dimensions, i.e. a length of 4 m and height of 2.75 m. They were also tested with the same boundary conditions, with bottom and lateral edges restrained but the top edge kept free. Fully interlocking 1 m long return walls were constructed along the vertical edges. They were designed to be full moment restraints, thereby functioning as if the restrained edges were completely fixed. What characterizes and makes each wall unique is the adopted unit-mortar combination: 1) wall CS-000-RF was constructed in calcium silicate (CS) brick while 2) wall CL-000-RF was constructed in clay brick (CL) masonry. When these two masonry types are tested under pure horizontal bending, CS brick masonry exhibits vertical line crack and can be therefore classified as a “Weak Unit- Strong Joint” URM typology; on the opposite, saw-toothed stepped cracks develop in CL masonry, being it then classified as a “Strong Unit-Weak Joint” typology.

None of the walls show any damage below accelerations up to 1g, indicating significant cracking strength. This large resistance needs to be accounted for while carrying out seismic assessments, being in favour of the approach proposed in this paper. Additionally, the effect of the distinct URM material typologies (“Weak Unit-Strong Joint” or “Strong Unit-Weak Joint”) was clearly perceived in the failure mechanisms of the walls under dynamic excitation. Due to the formation of line cracks passing through bricks along the vertically restrained edges, CS-000-RF eventually exhibited one-way bending behaviour. The cracks were vertical (as opposed to stepped or saw-toothed), providing negligible frictional resistance. In the case of CL-000-RF, two-way bending behaviour was observed following the formation of the crack pattern required for the failure mechanism to develop, with significant frictional resistance contributions from the stepped cracks.

Figure 1: Failure mechanisms of walls CS-000-RF and CL-000-RF observed experimentally.

3 The Proposed Model

The proposed model functions in two phases:1) initial elastic phase and 2) post cracking phase. The initial elastic phase is completely defined by coordinates of the points corresponding to the achievement of the peak strength: F_{cr} (peak strength) and u_{cr} (the displacement at which the peak strength is attained) (Figure 2a). Once F_{cr} is exceeded, the model immediately switches to the post-cracking phase, where the resistance of the walls is modelled via the superposition of three contributing sources:

- A bilinear elastic component, which accounts for rocking (F_o, u_o, $u_{o,f}$) (Figure 2b);
- An elastoplastic component, which takes into account the torsional frictional resistance (F_{fr}, u_{fr}) that develops on the cracked masonry bed joints (Figure 2c) ;
- A bilinear component which unloads with secant stiffness, which accounts for the degradation of both the wall strength and stiffness under two-way bending excitation (F_{deg}, u_{deg}, $u_{deg,f}$) (Figure 2d).

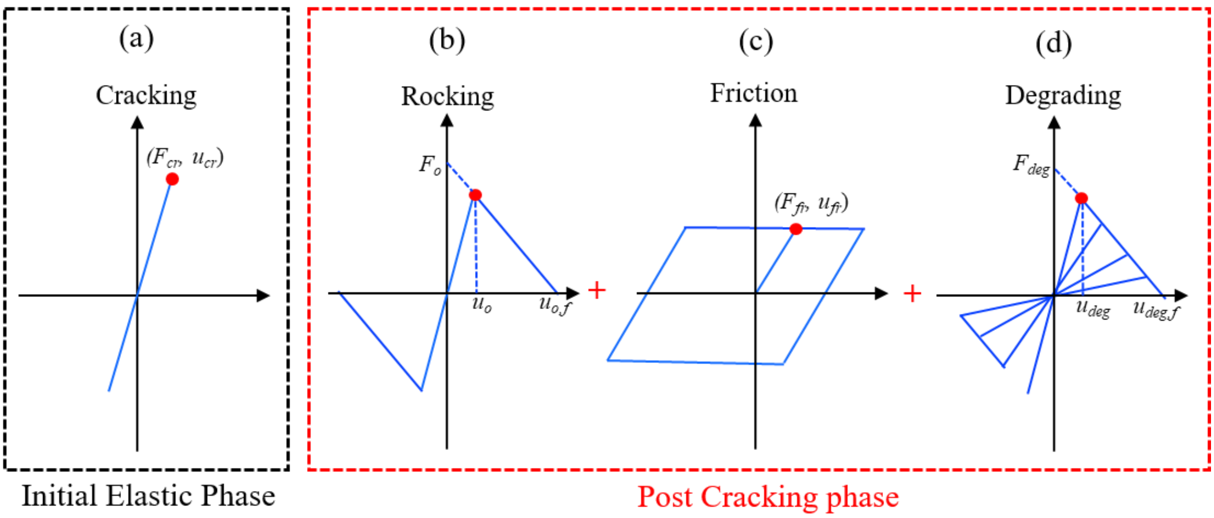

Figure 2: Schematic representation of the proposed model.

The behaviour of the model is thus governed by a total of 10 input parameters namely: F_{cr}, u_{cr}, F_o, u_o, $u_{o,f}$, F_{fr}, u_{fr}, F_{deg}, u_{deg} and $u_{deg,f}$. However, this reduces to 6 input parameters: F_{cr}, u_{cr}, F_o, $u_{o,f}$, F_{fr}, and $u_{deg,f}$ due to the assumption that the model shifts instantaneously from the initial elastic phase to post cracking phase on the attainment of F_{cr}. To ensure numerical continuity between these two phases, this assumption translates to:

$$u_{cr} = u_o = u_{fr} = u_{deg}$$
$$F_{deg} = F_{cr} - F_o - F_{fr} \tag{1}$$

Physically, equation 1 is tantamount to letting the strength and stiffness degradation of URM, rocking and friction along cracked joints start occurring once the strength of the panel has been exceeded and a failure mechanism has been formed. This is not only reasonable but was also experimentally observed in both the reference experimental campaign Graziotti *et al.* (Graziotti et al., 2019b) as well as incremental dynamic testing reported in Sharma *et al.* (Sharma et al., 2020).

In order to estimate u_{cr} (and consequently u_o, u_{fr}, u_{deg}), it has been demonstrated in Sharma *et al.*(Sharma et al., 2020) that an analytical formulation based on the theory of plates can be used to calculate the initial stiffness as well as cracking displacement of two-way spanning URM walls under OOP loading. To calculate F_{cr}, F_o, u_o and F_{fr} a failure mechanism needs to be postulated for the wall *a priori*. This can be done based on the boundary conditions and geometry of the wall as per the recommendations provided by Lawrence and Marshall (Lawrence and Marshall, 2000), which have been incorporated in the building codes of Australia (Council of Standards Australia, 2001) and the Netherlands (NEN, 2020). These recommendations, which are based on the virtual work method, can be subsequently used to calculate also F_{cr}. The improved formulations to calculate the moment capacity of cracks proposed by Willis (Willis, 2004), Vaculik (Vaculik, 2012) and Sharma (Sharma et al., 2021) are employed. For the remaining input parameters controlling the rocking and frictional components in Figure 2, the mechanics-based formulas developed in Vaculik and Griffith (Vaculik and Griffith, 2017b) can be adopted. An overview of the failure modes for which formulations for the input parameters of the model have been already developed are provided in Figure 3. The K1Y mechanism with both sides supported is postulated for both CS-000-RF and CL-000-RF based on their similar geometry and boundary conditions, confirmed also by experimental observations (Figure 1).

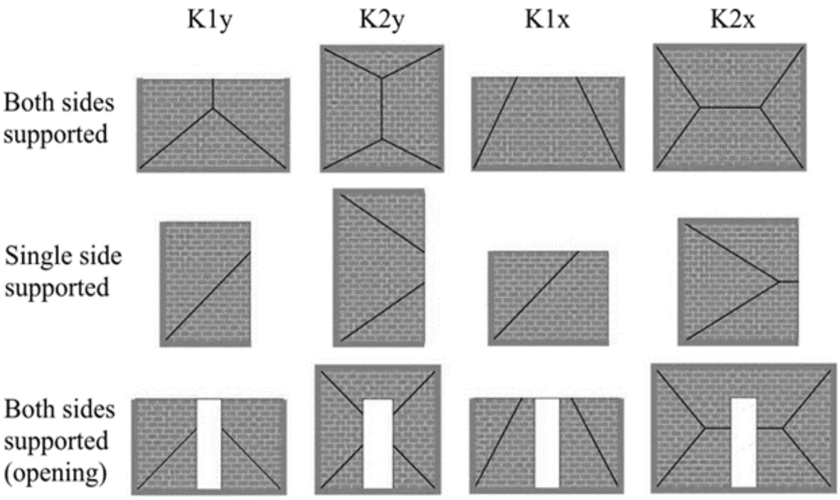

Figure 3: Failure mechanisms considered by the proposed model (Vaculik and Griffith, 2017b).

4 Calibrated Numerical vs. Experimental Performance

The proposed model was used to perform non-linear time-history analyses and calibrated against the tests performed on the two full-scale walls mentioned in section 2. The reference experimental campaigns involved incremental dynamic testing, i.e. series of input motions of gradually increasing intensity were applied on each specimen to fully exploit their load-carrying capacity and allow comprehensive characterisation of their dynamic behaviour. In general, after observations of damage, incremental testing was resumed again from a lower value of intensity associated with the used input motions. The full testing sequences can be found in Graziotti *et al.*. Considering the length of the testing sequence (more than 22 runs in both cases), in this paper only runs of the testing sequence in which damage occurred and the tests preceding and succeeding such runs were simulated.

The first step towards using the proposed model consists of converting the walls into single-degree-of-freedom (SDOF) systems associated with a lumped effective mass (M_{eff}) and displacement (Δ_{eff}). This can be done as per recommendations by Vaculik and Griffith (Vaculik and Griffith, 2008). Incremental dynamic analyses of the SDOF systems representing the walls were then carried out by solving the equation of motion (equation 2).

The Newmark 'linear acceleration' integration scheme in its non-iterative formulation version is used (Newmark, 1959).

$$M_{eff} \cdot \ddot{u}(t) + C(t) \cdot \dot{u}(t) + f(\dot{u}(t)) = M_{eff} \cdot \ddot{u}_g(t) \tag{2}$$

In equation 2: t represents time, u, $\dot{u}, \ddot{u}$ represent the equivalent displacement, velocity and acceleration associated with the SDOF system representing the wall; $f(u,t)$ represents the wall restoring force which depends on which phase the model is in (Figure 2), while $\ddot{u}_g$ refers to the ground acceleration, i.e. in this context the applied input motion. The damping coefficient, *C(t)* was calculated assuming a constant damping ratio (ξ) acting on the initial system circular frequency (ω_1) of the SDOF. A damping ratio of 0.05 is associated with the model in the initial elastic phase. In the post cracking phase, updated values *of* ξ are calibrated, i.e. chosen to give a good fit with experimental data. The controlling parameters of the proposed numerical model were first calculated as per analytical recommendations summarised in section 3 and then calibrated to match the numerical results with experimental measurements in terms of:

- Force-displacement hysteresis (Figure 4);
- Incremental dynamic testing (IDT) curves: plotting PGA and peak displacement in all tests up to collapse (Figure 5);
- Displacement time-histories in various tests in the incremental dynamic testing sequence (Figure 6 and Figure 7). For more information on the individual tests, the reader is referred to the detailed testing sequence reported in Graziotti *et al.* (Graziotti et al., 2019b).

The excellent performance of the calibrated model is evident from the comparison of numerical and experimental force-displacement hysteresis and IDT curves. A very good agreement is also observed between numerical and experimental displacement time histories. Specifically, the model can also capture collapse with good accuracy (Figure 6 and Figure 7). The parameters used to obtain the calibrated response are reported in Table 1. It is of interest to note that these input values are simply assigned at the beginning of the testing sequence. Post-cracking, the stiffness of the degrading component is automatically updated starting from the value at the end of the previous run of the testing sequence.

Table 1: Calibrated controlling parameters of the proposed model.

	T_{SDOF}	F_{cr}	u_{cr}	F_o	$u_{o,f}$	F_{fr}	F_{deg}	$u_{deg,f}$	ξ
	[s]	**[kN]**	**[mm]**	**[kN]**	**[mm]**	**[kN]**	**[kN]**	**[mm]**	**[-]**
CS-000-RF	0.099	24.8	5.5	0.91	102	0.3	28.95	30	0.0235
CL-000-RF	0.0845	29.12	4.4	3.48	204	3.78	22.33	200	0.0205

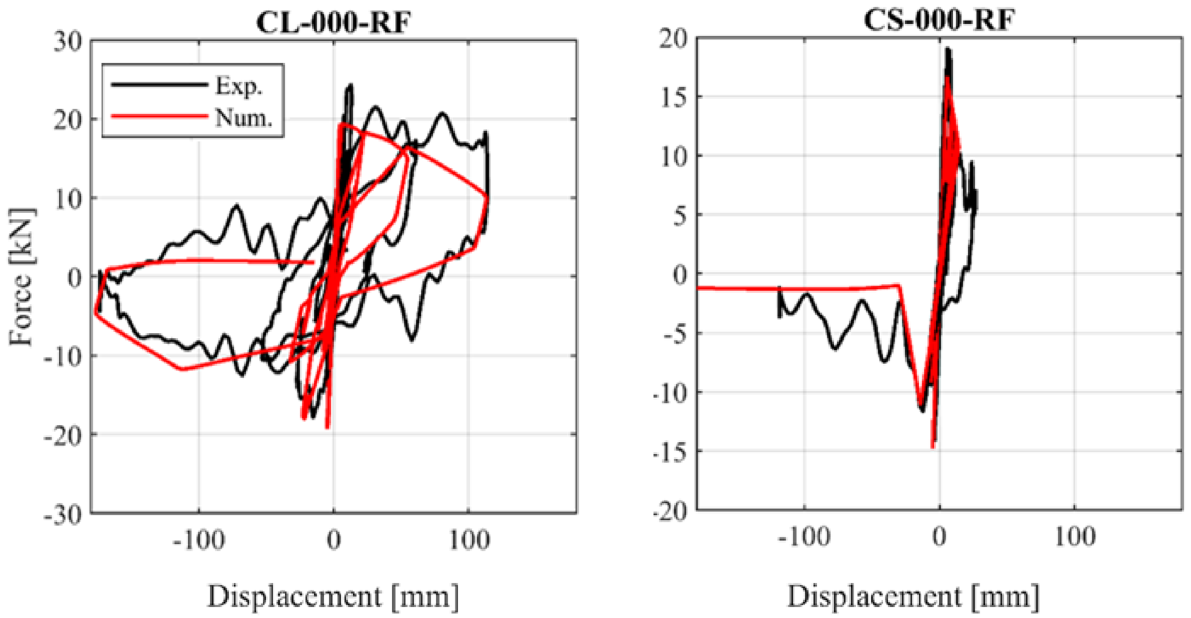

Figure 4: Comparison of numerical and experimental force-displacement hysteresis

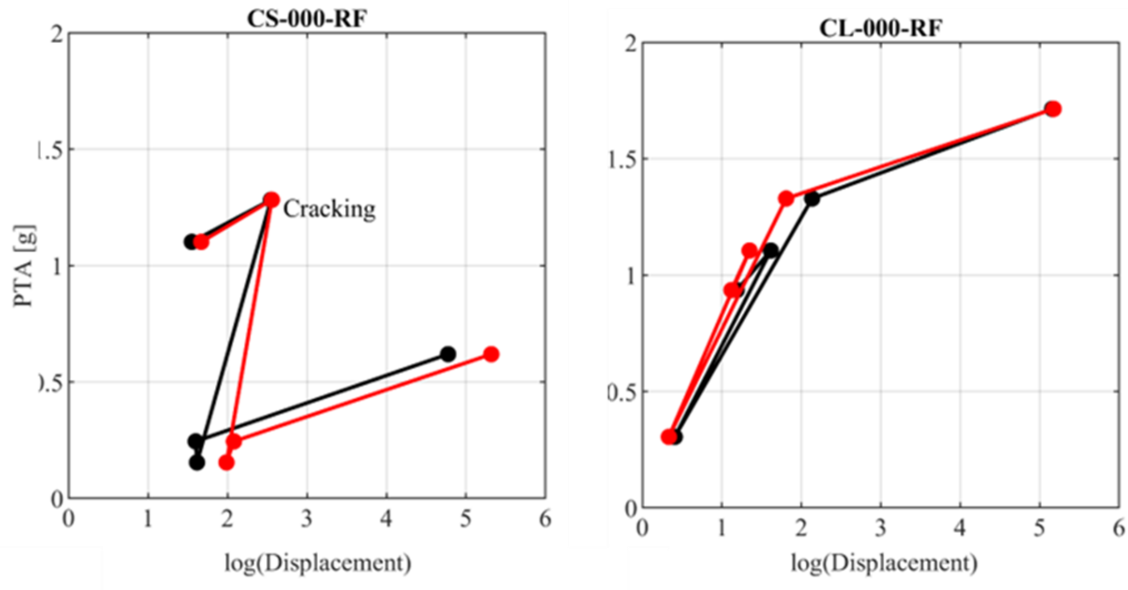

Figure 5: Comparison of numerical and experimental behaviour in terms of PGA and peak wall displacement (displacements plotted in a log scale to allow clearer visualization).

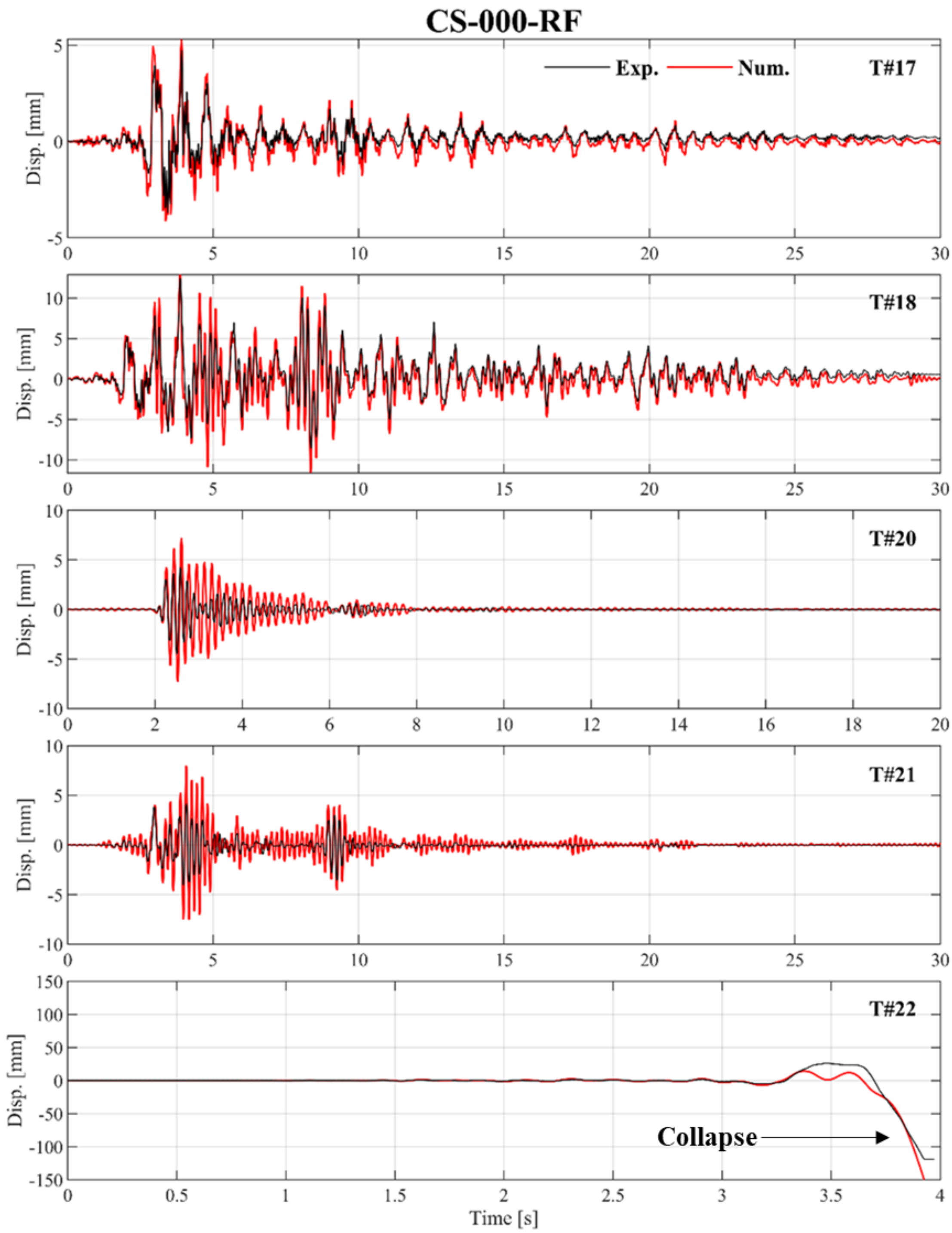

Figure 6: Comparison of numerical and experimental displacement time histories for wall CS-000-RF.

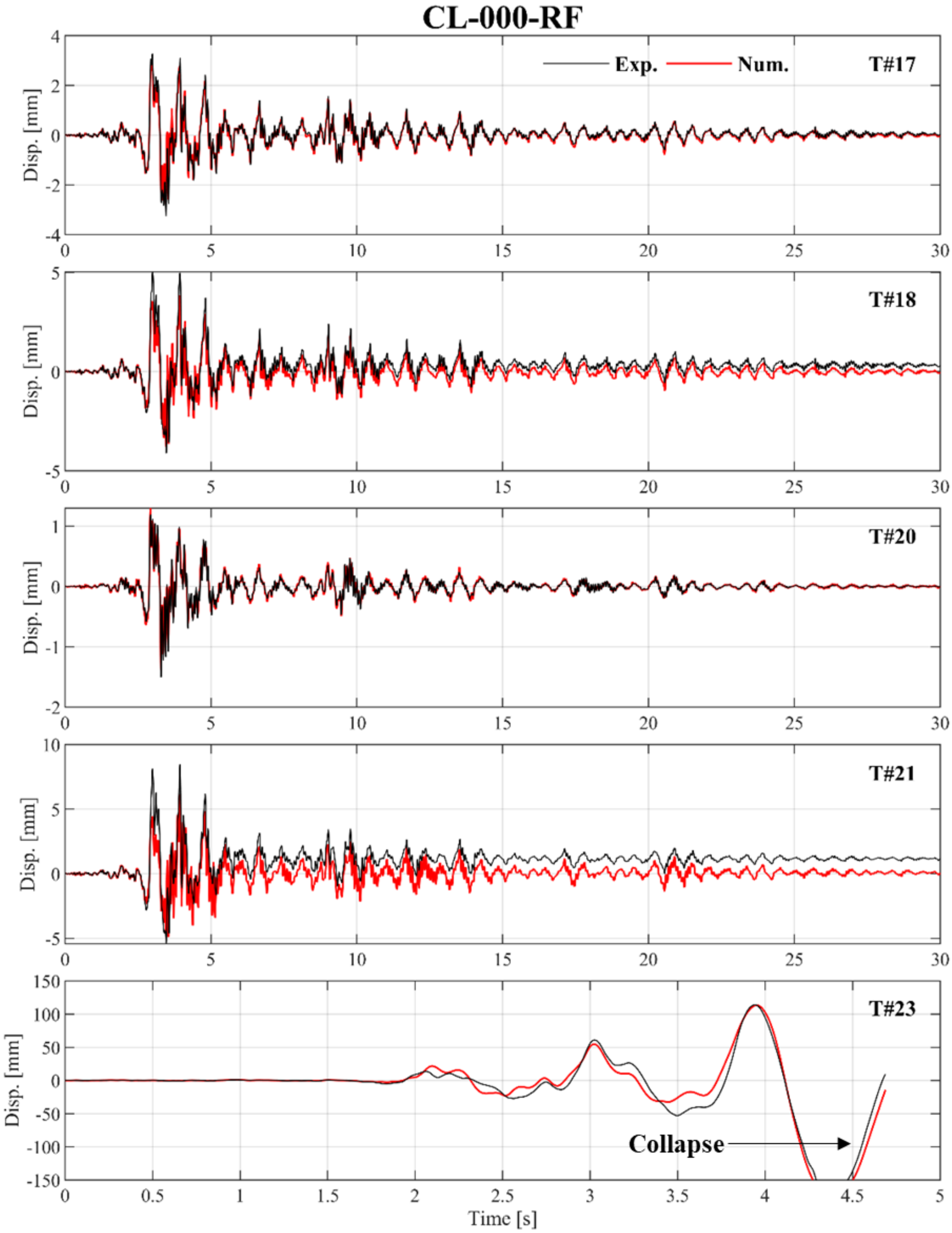

Figure 7: Comparison of numerical and experimental displacement time histories for wall CL-000-RF

5 Preliminary Estimation of Behaviour Factors

The calibrated numerical models were used to conduct a preliminary estimation of behaviour factors for the seismic assessment of the OOP two-way bending seismic evaluation of URM walls. This was also carried out to investigate the effect of the distinctly different failure mechanisms exhibited by both walls (highlighted in section 2/Figure 1) on their seismic vulnerability. Here, behaviour factors refer to factors that can be used to scale down the seismic demand obtained from an elastic acceleration response spectrum when conducting assessments using linear elastic approaches. These factors are used to account for inelastic response in a simplified manner at the ultimate state or collapse. These behaviour factors were estimated by performing incremental dynamic analyses (IDA), which involves subjecting the numerical models to ground motion records scaled to multiple levels of intensity. To calculate behaviour factors from IDA, two critical points on the IDA curves were identified: 1) The acceleration at which deviation from linear elastic behaviour occurred (a_{ldev}) and 2) The acceleration at which a sudden increase in displacements, highly non-proportional to the increase in input excitation and indicating potential collapse, was observed ($a_{collapse}$). The values of behaviour factors reported in this section are calculated as $a_{collapse}$ / a_{ldev} (Figure 8). Behaviour factors from both models of CS-000-RF and CL-000-RF were derived from IDA's considering 22 + 22 human activity induced seismicity and tectonic seismicity ground motion records. The induced seismicity records were taken from the Groningen database (Bommer et al., 2016) while the tectonic seismicity were taken from the NGA database (Chiou et al., 2008). These results in summarised in Table 2. 5th percentile values of around 1 are calculated for both walls under tectonic seismicity ground records. For induced seismicity records, the corresponding 5th percentile behaviour factors are slightly higher, with values of 1.37 and 1.56 for CS-000-RF and CL-000-RF, respectively. Median behavioural factors of approximately 2 are calculated for both walls in the two scenarios considered.

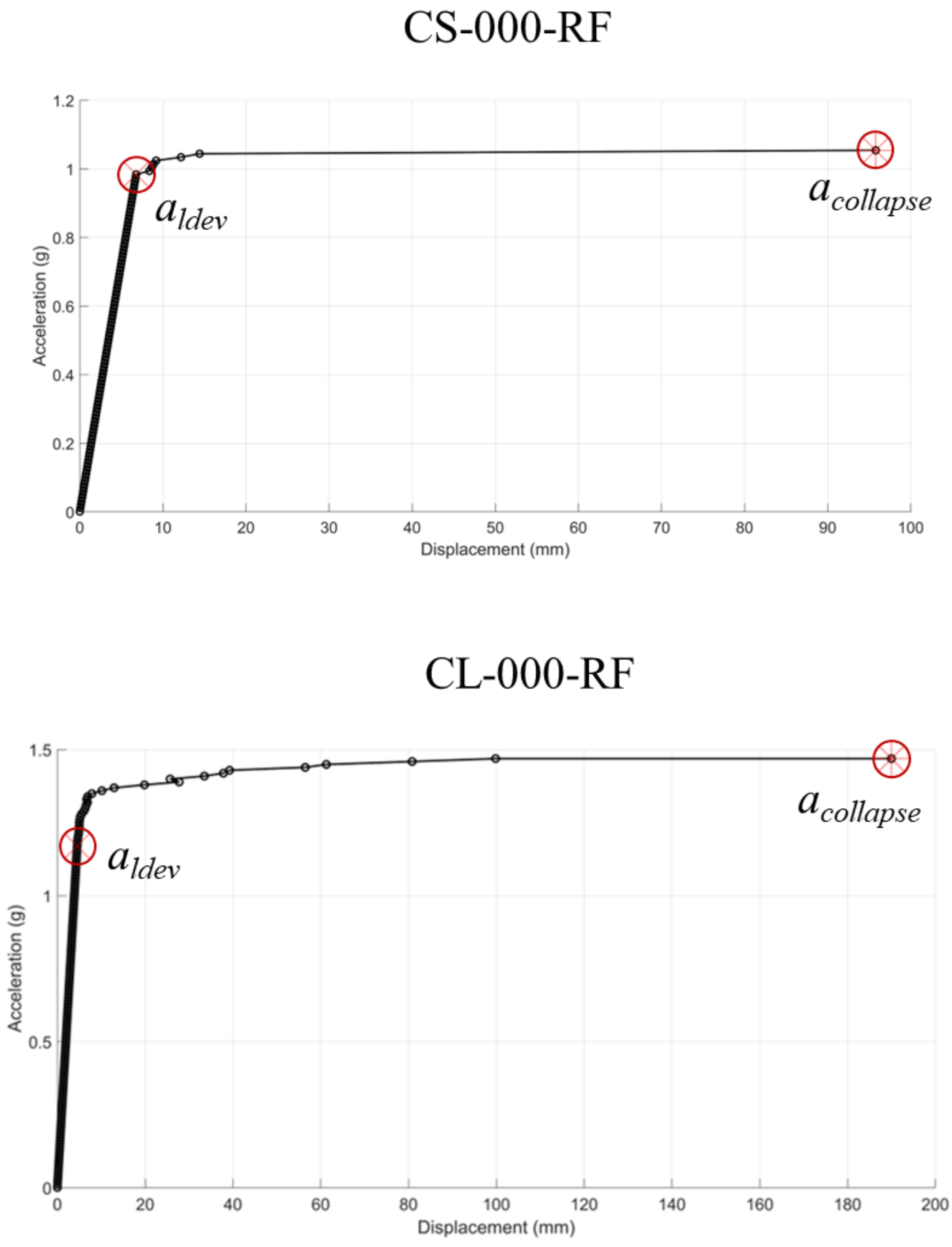

Figure 8: Estimation of behaviour factors from incremental dynamic analyses under tectonic seismicity.

Table 2: Summary of estimation of behaviour factors.

	Induced Seismicity		**Natural Seismicity**	
	CS-000-RF	CL-000-RF	CS-000-RF	CL-000-RF
5th Percentile	1.37	1.56	1.09	1.15
Median	1.93	2.01	1.98	2.33

6 Concluding Remarks

This paper proposes a single-degree-of-freedom model for modelling the dynamic out-of-plane response of unreinforced masonry walls under two-way bending. The model considers two distinct phases: an initial elastic phase and a post-cracking phase. The behaviour of the model is controlled by ten parameters, which reduces to six independent parameters upon adopting the assumption that there is an immediate transition between the two phases. The transition between the two phases occurs when the cracking resistance of the wall is overcome. Most of the controlling parameters of the model can be calculated analytically with mechanics-based formulae. Calibration of the model showed excellent agreement with the results of incremental dynamic tests, successfully capturing their behaviour from cracking to collapse. The calibrated model was then used to carry out a preliminary estimation of behaviour factors which can be used to account for inelastic behaviour when carrying out linear elastic seismic assessments of these walls.

Future work should address calibrating the proposed model against dynamic experiments in the literature on walls tested under other boundary conditions. These calibrated models should then be used to refine the estimates of behaviour factors. In this regard, focus should be placed on comparing the seismic vulnerability of:

1. Walls constructed in "Weak Unit-Strong Joint" masonry, which exhibits a vertical crack passing through the brick units and head joints under pure horizontal bending, with respect to "Strong Unit-Weak Joint" (SU-WJ) masonry, which exhibits a stepped crack passing through head joints and half a bed joint under pure horizontal bending.
2. Walls under one-way bending compared to two-way bending excitation.

There is also a need to further develop and refine the analytical recommendations for calculating parameters for the component of the proposed model that accounts for the strength and stiffness degradation of URM walls under two-way bending excitation.

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