Shells and arches

Developing a new method to calculate shells and arches through graphic statics

P5 presentation
Rik Rozendaal

Main mentor: Andrew Borgart
Mentors: Thijs Welman, Joris Smits

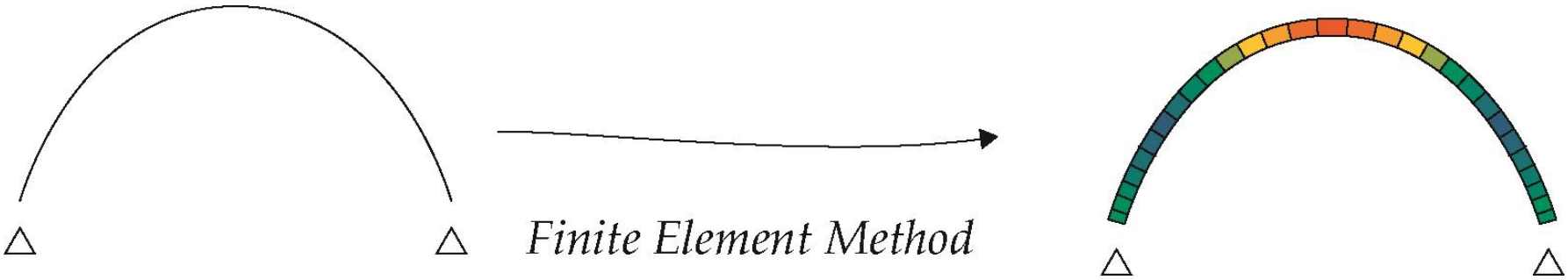
Inhoud

- Introductie
- Literatuur
- Onderzoek
- Conclusies

Introductie

L'Oceanogràfic designed by Felix Candela, 2003 (Gabaldón, 2010)

Introductie



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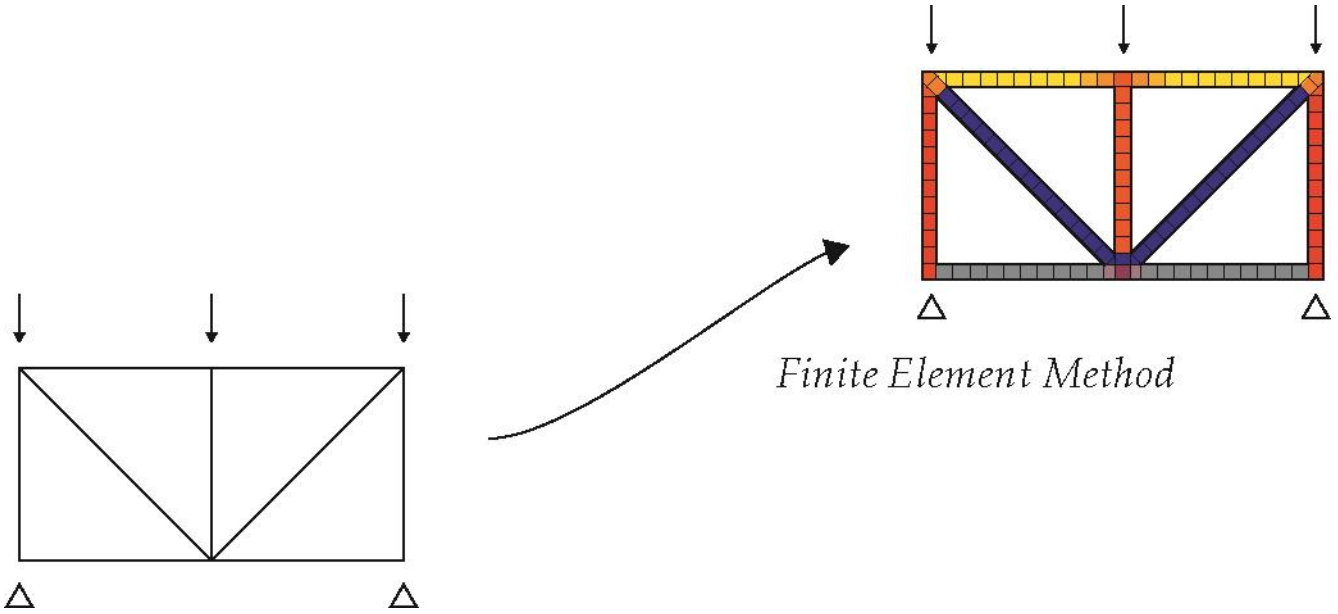

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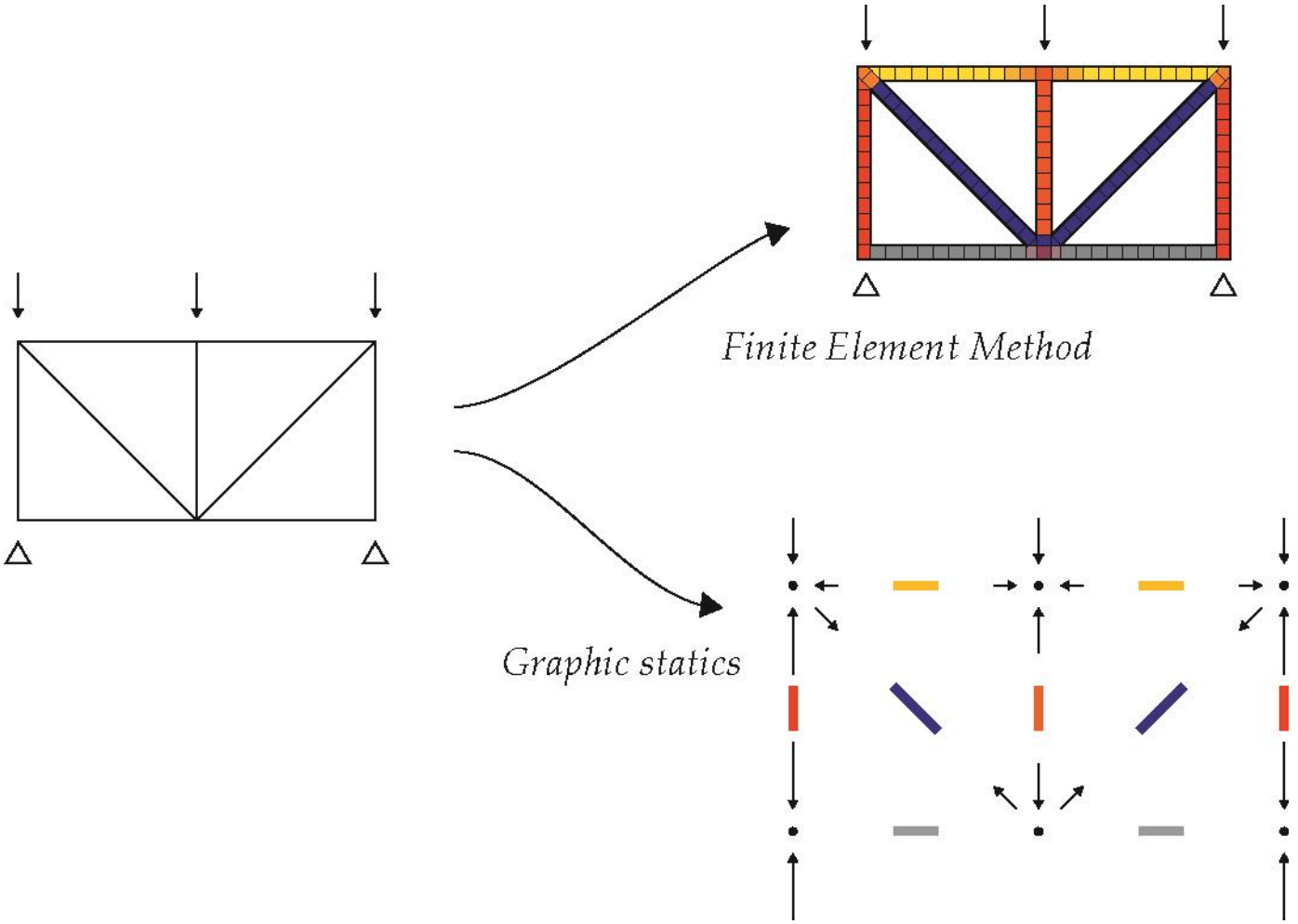

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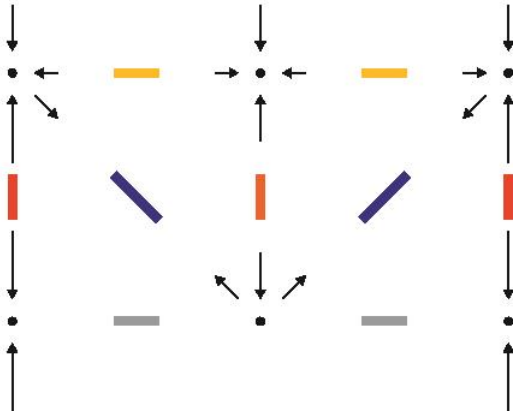
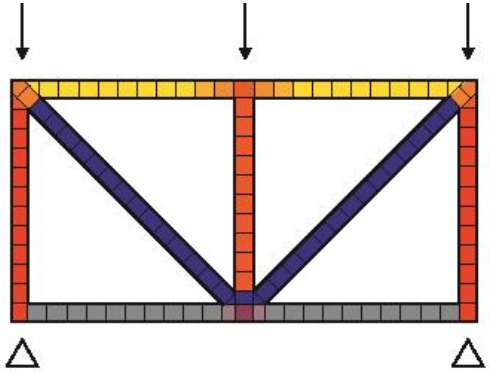
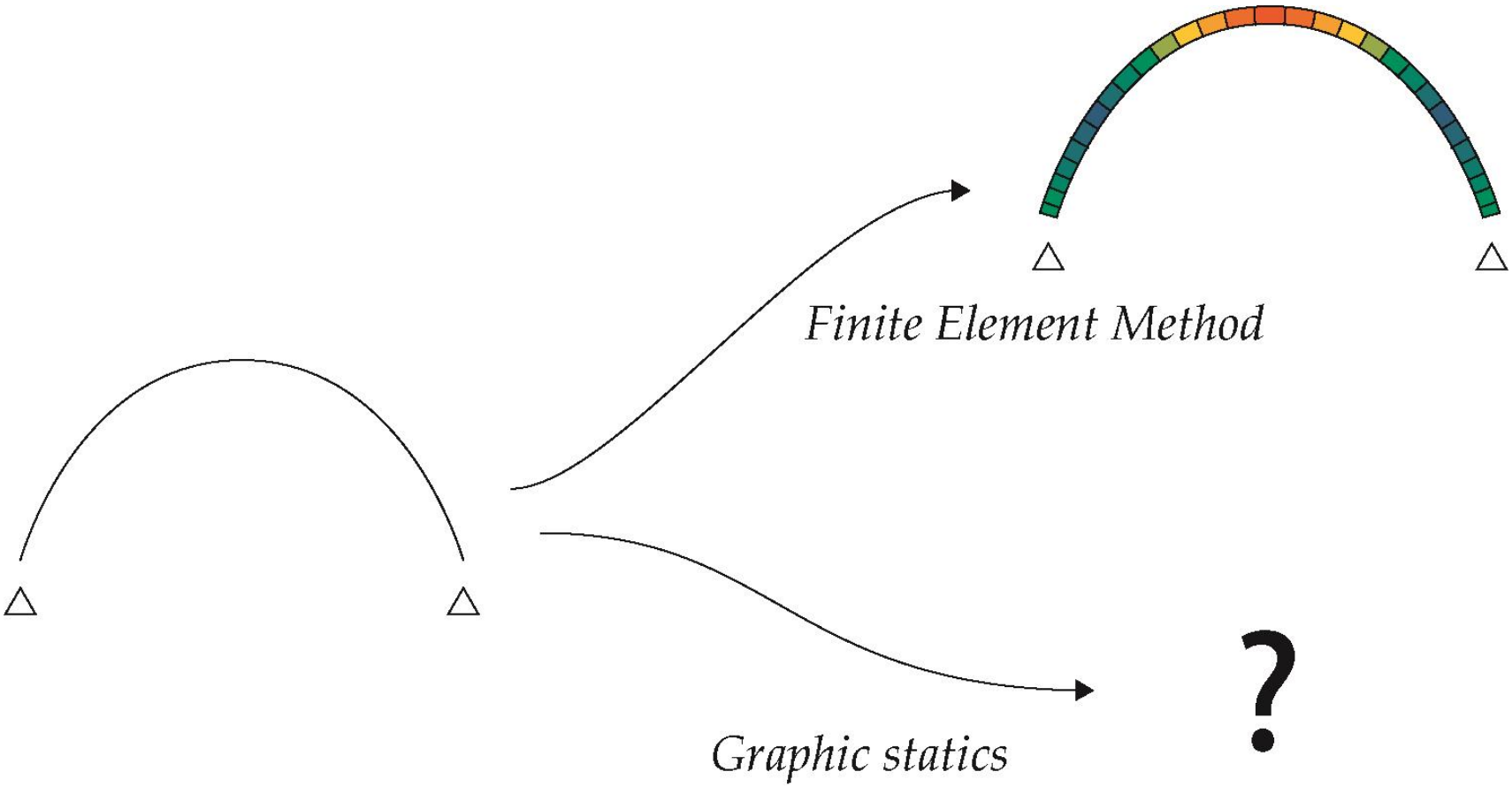




Constructie onbekend	Constructie uitgewerkt
Begin ontwerpproces	Einde ontwerpproces
Handberekeningen	FEM-programma

Introductie



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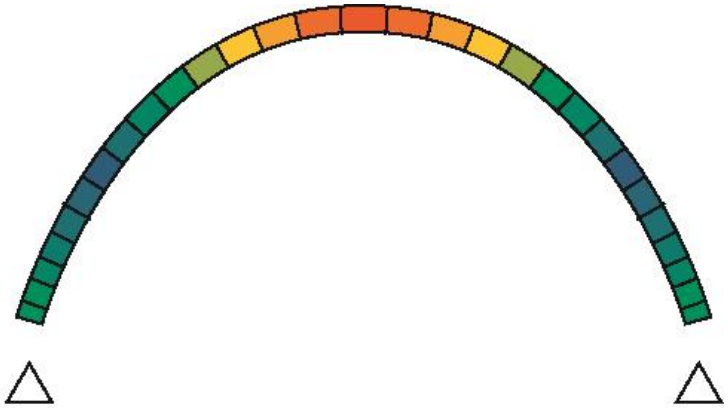


- Methode geeft geen inzicht in werking van boog of schaal
- Eerder in ontwerpproces is rekenmethode gewenst

Introductie

Onderzoeksvraag

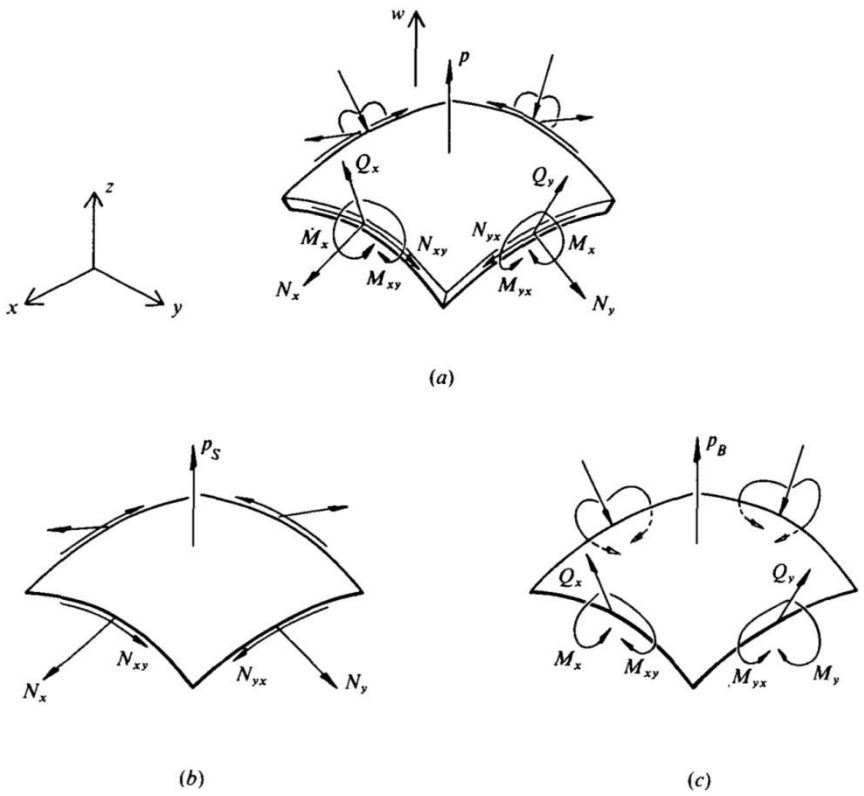
How can the structural performance of a shell structure be calculated in such a way that the relation between the geometry and the structural performance is shown?

Literatuur

- Druklijnen en –vlakken
- Gelijke oppervlakte methode

Literatuur

Druklijnen en –vlakken



(Calladine, 1977)

Literatuur

Wat is een druklijn?

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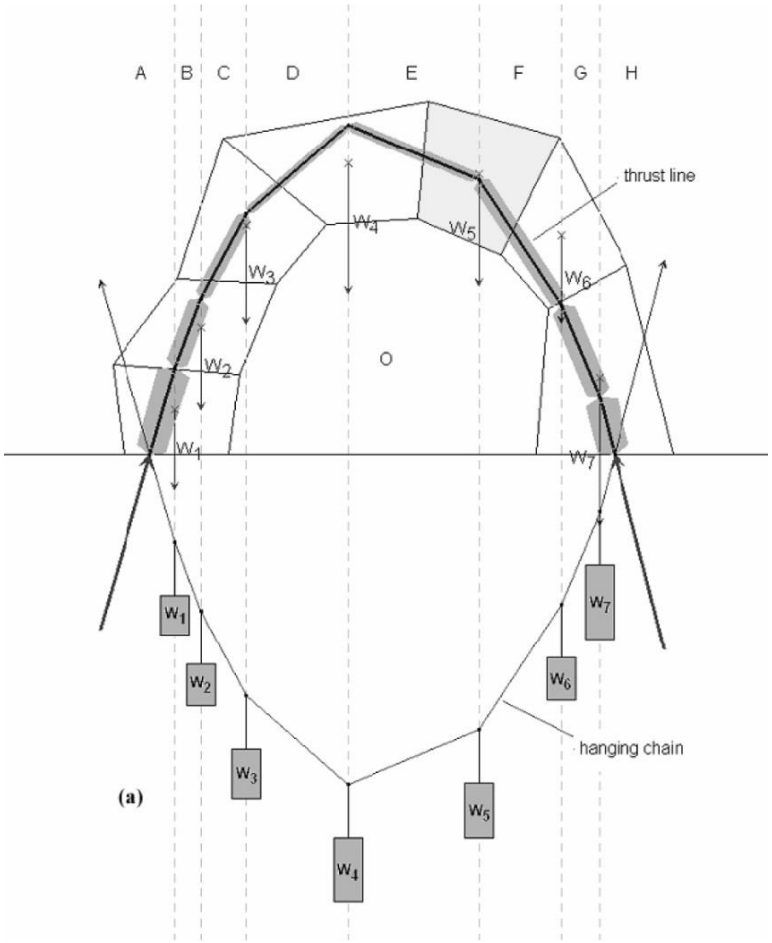

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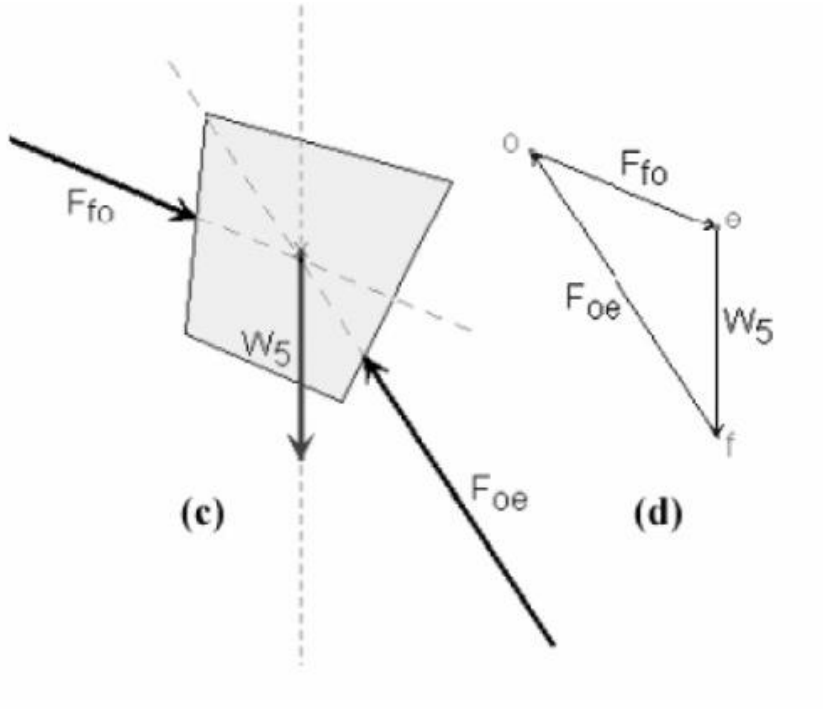

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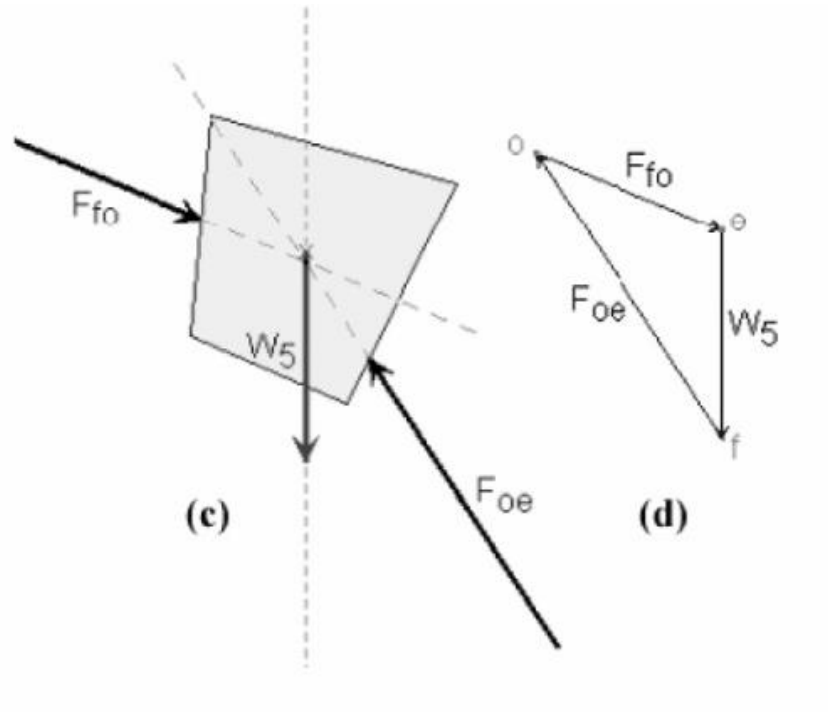

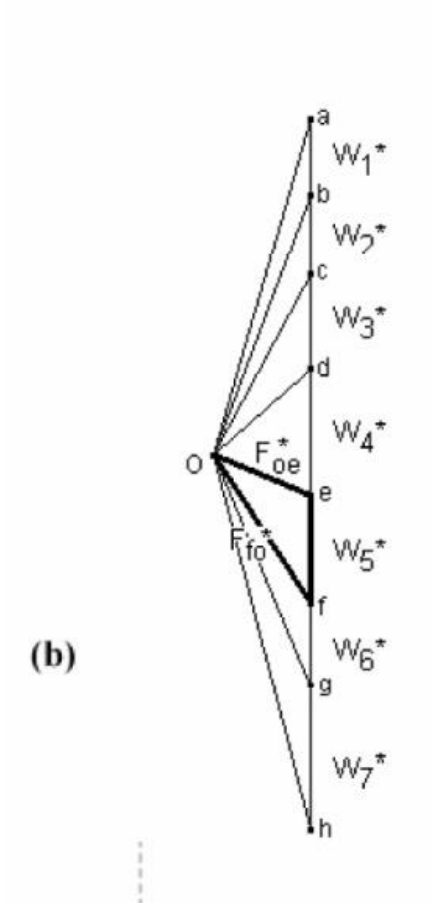

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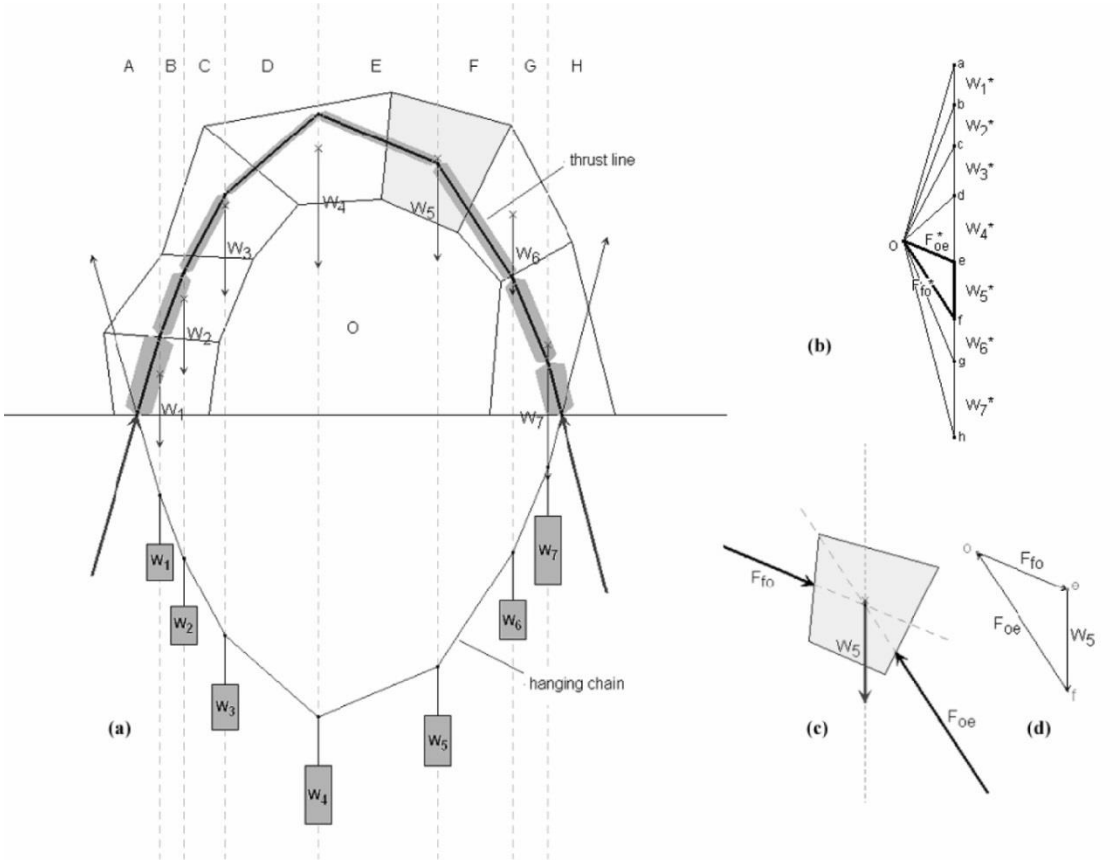

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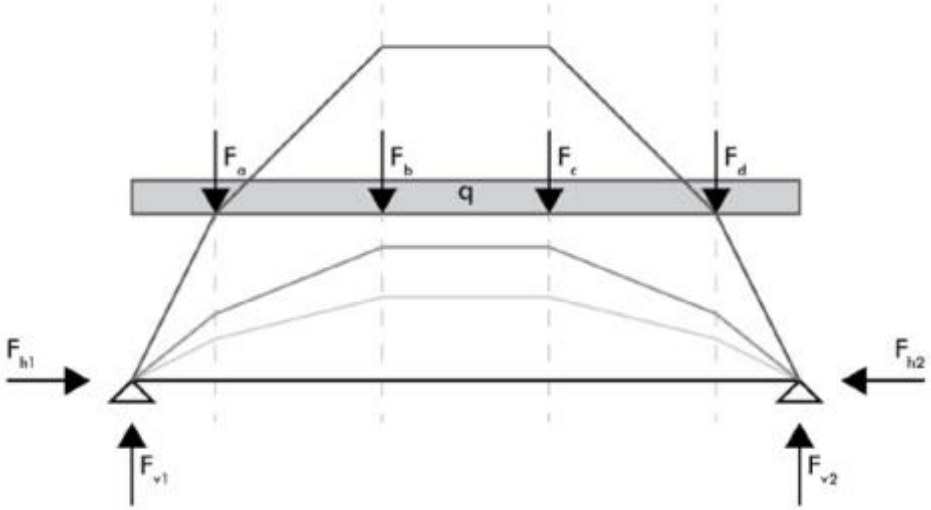

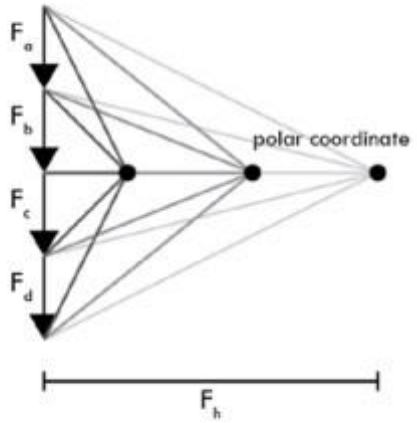

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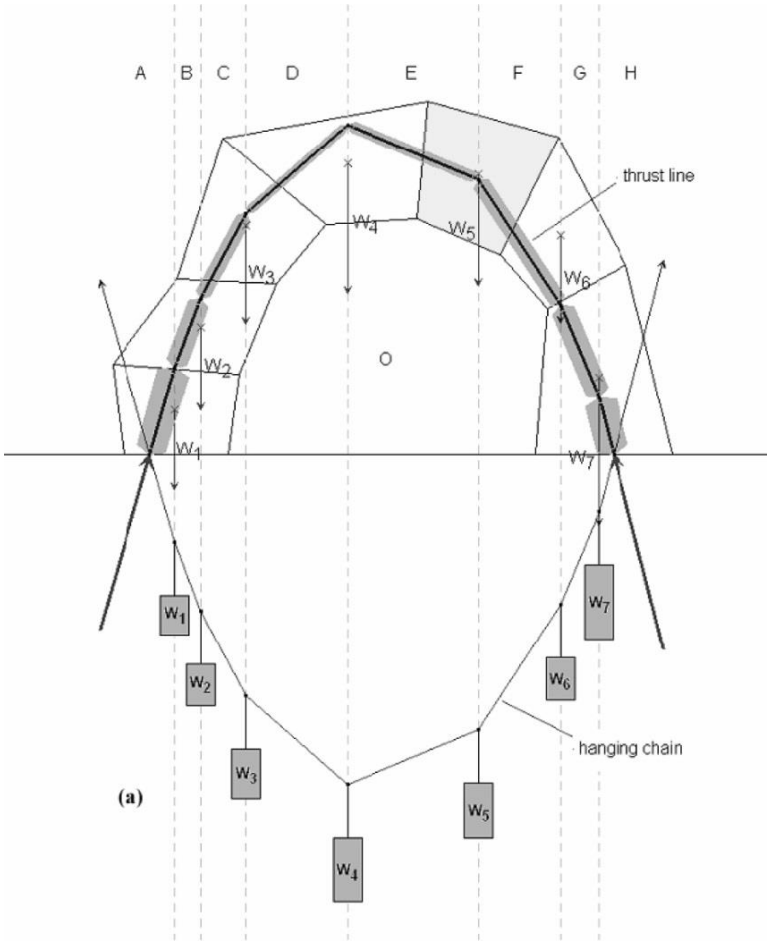

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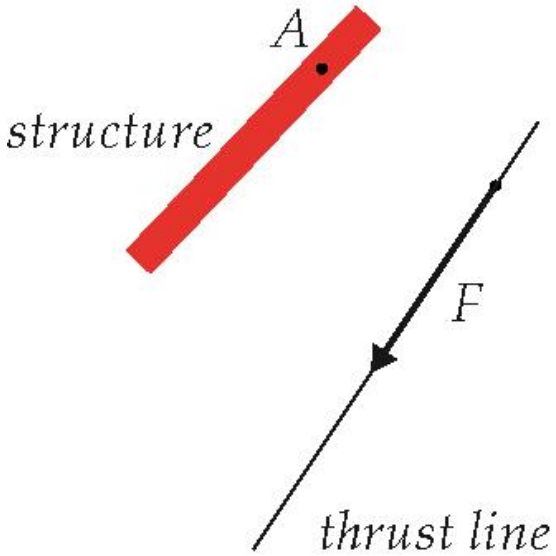

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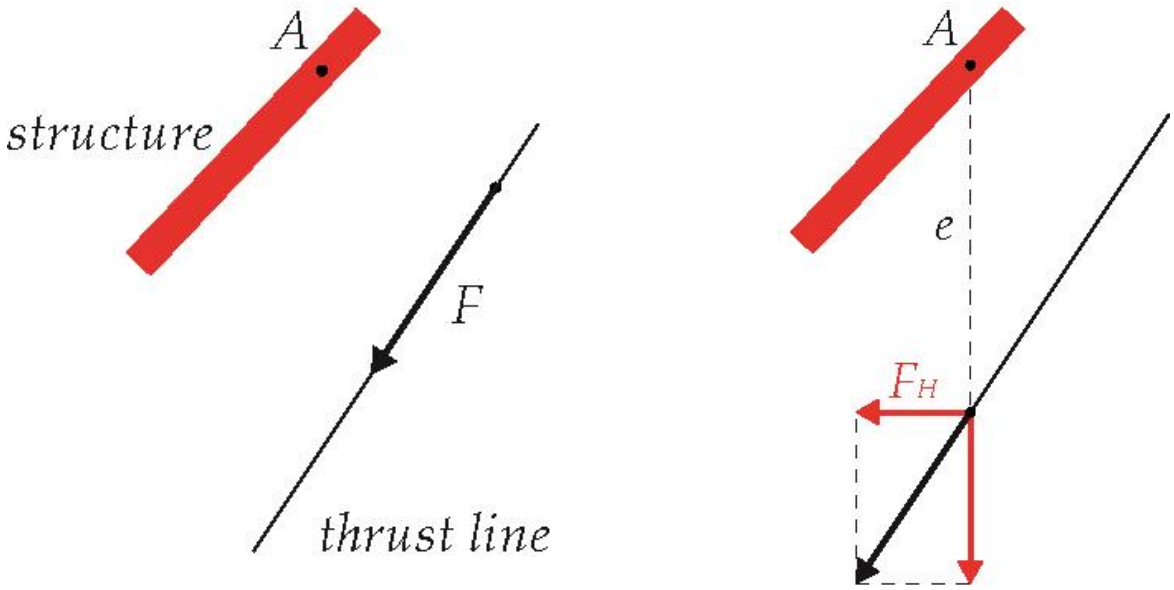

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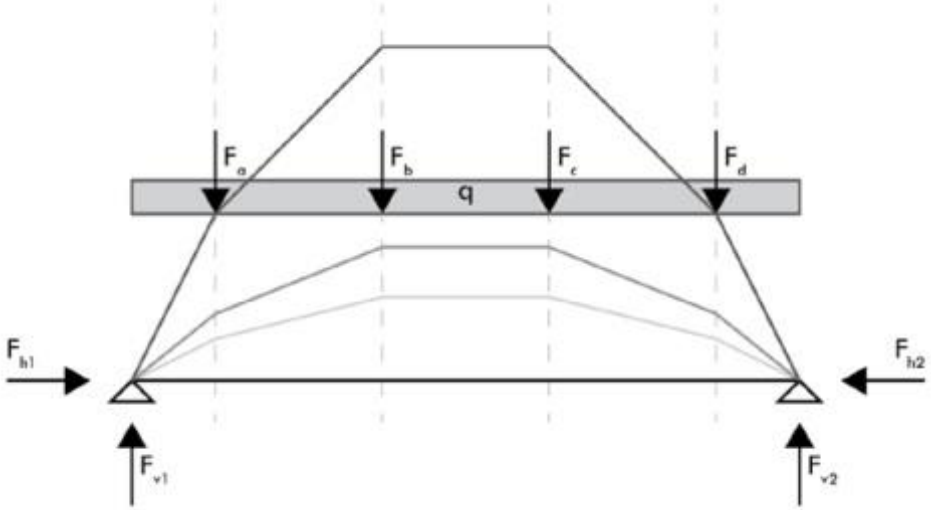
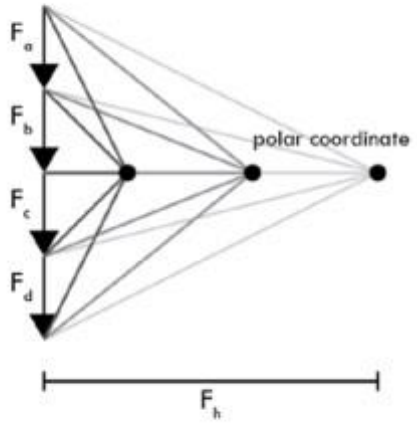
Welke druklijn is de juiste?





Literatuur

Welke druklijn is de juiste?

Complementary energy

Literatuur

Welke druklijn is de juiste?

$$E_{c;N} = N^2 l$$

$$E_{c;M} = \frac{12}{t^2} M^2 l$$

Complementary energy method

Literatuur

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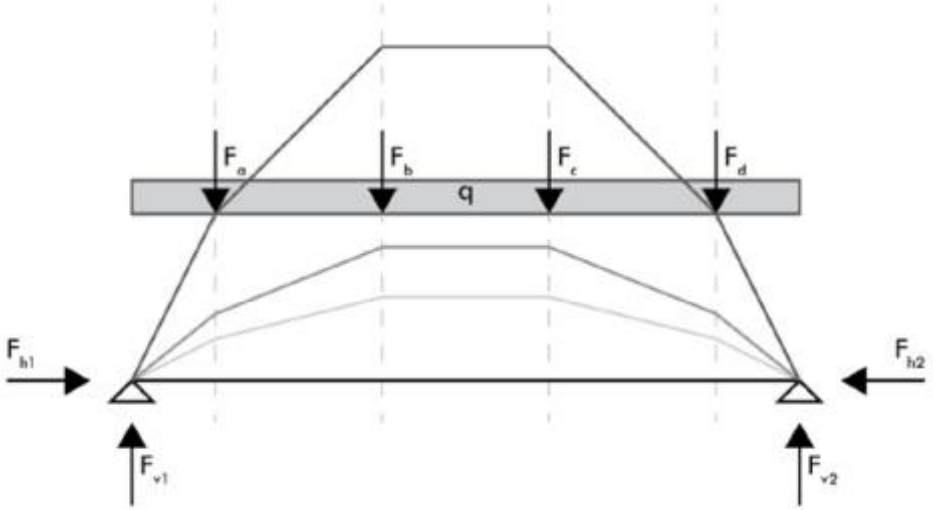

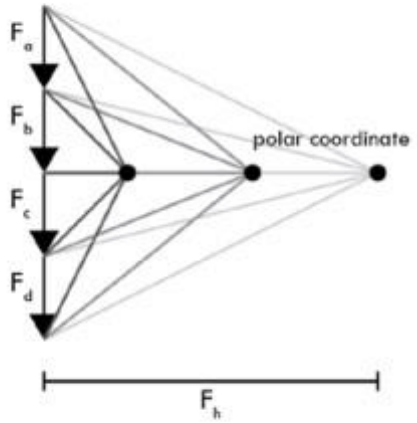

$$E_c = N^2 l + \frac{12}{t^2} M^2 l$$

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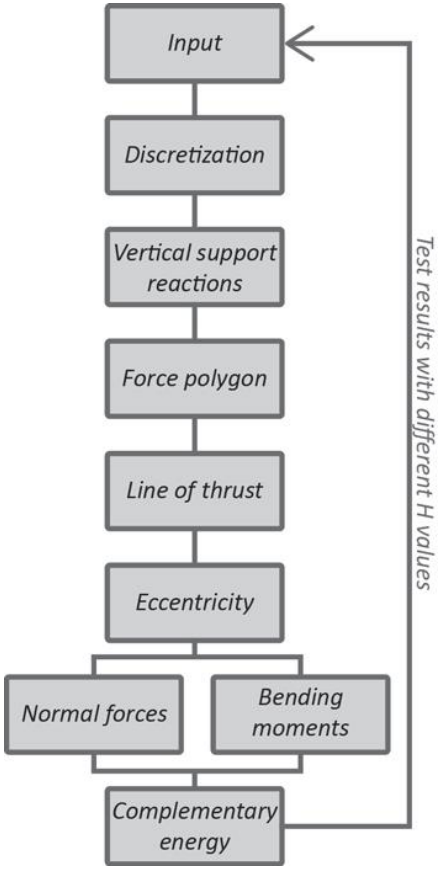

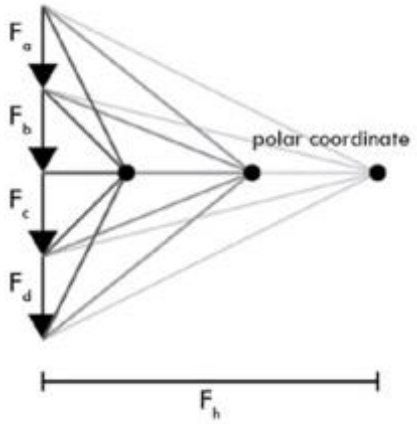

(van Dijk, 2014)

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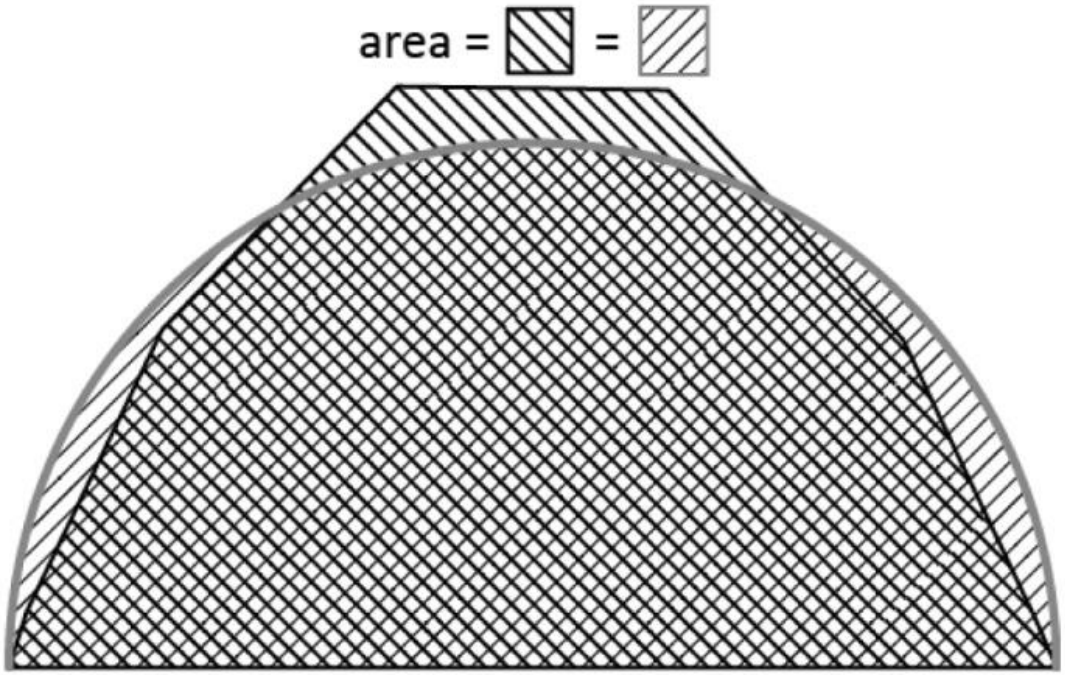

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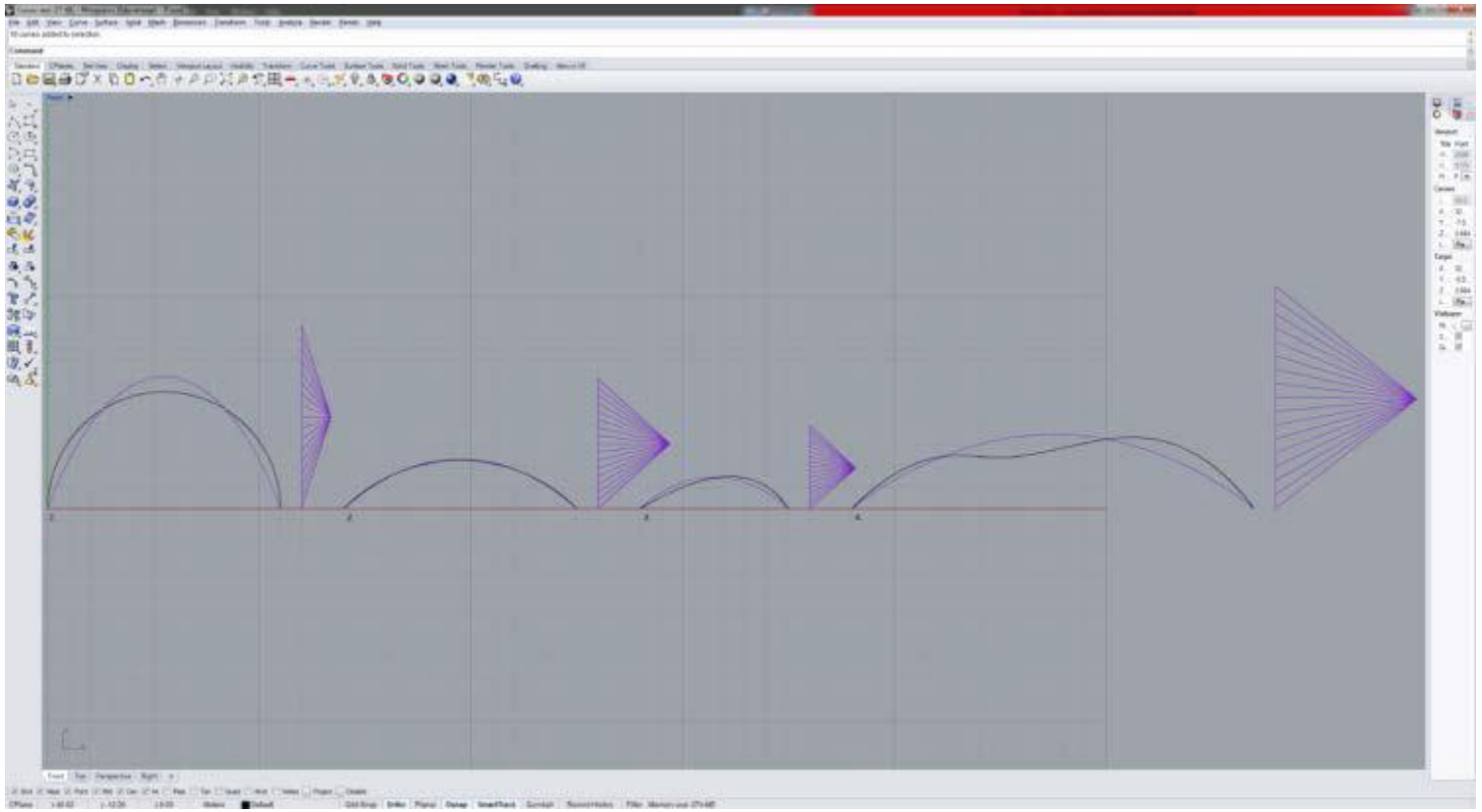

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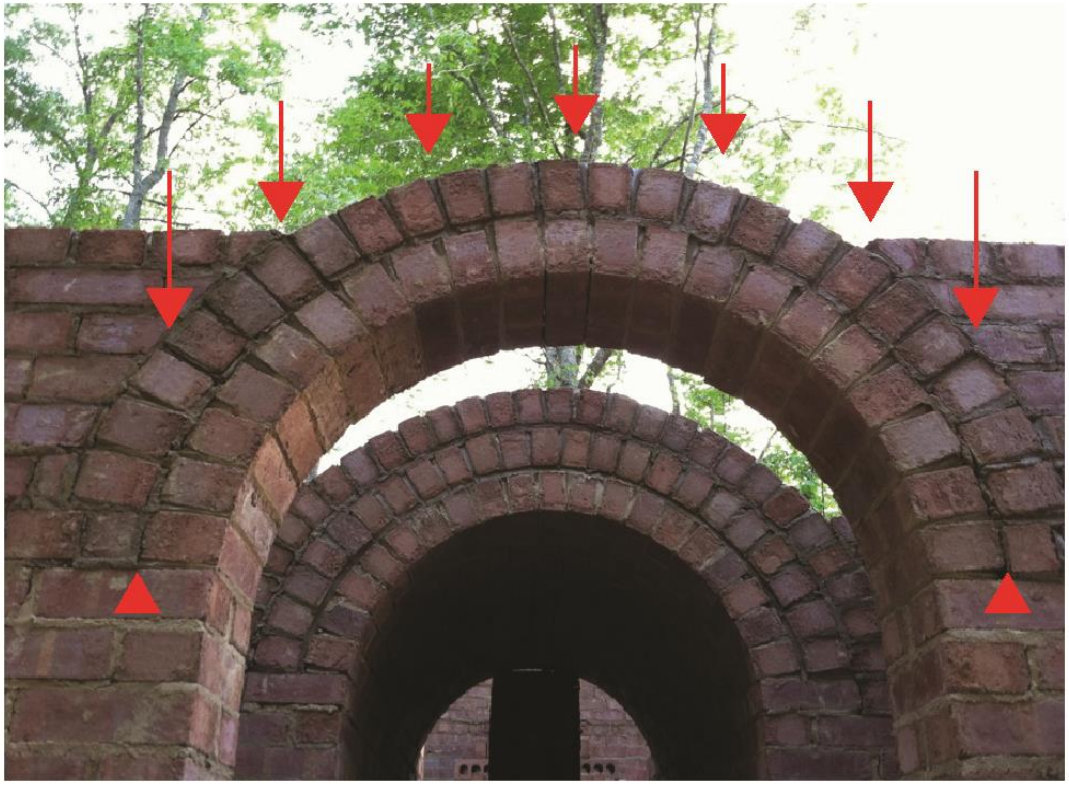

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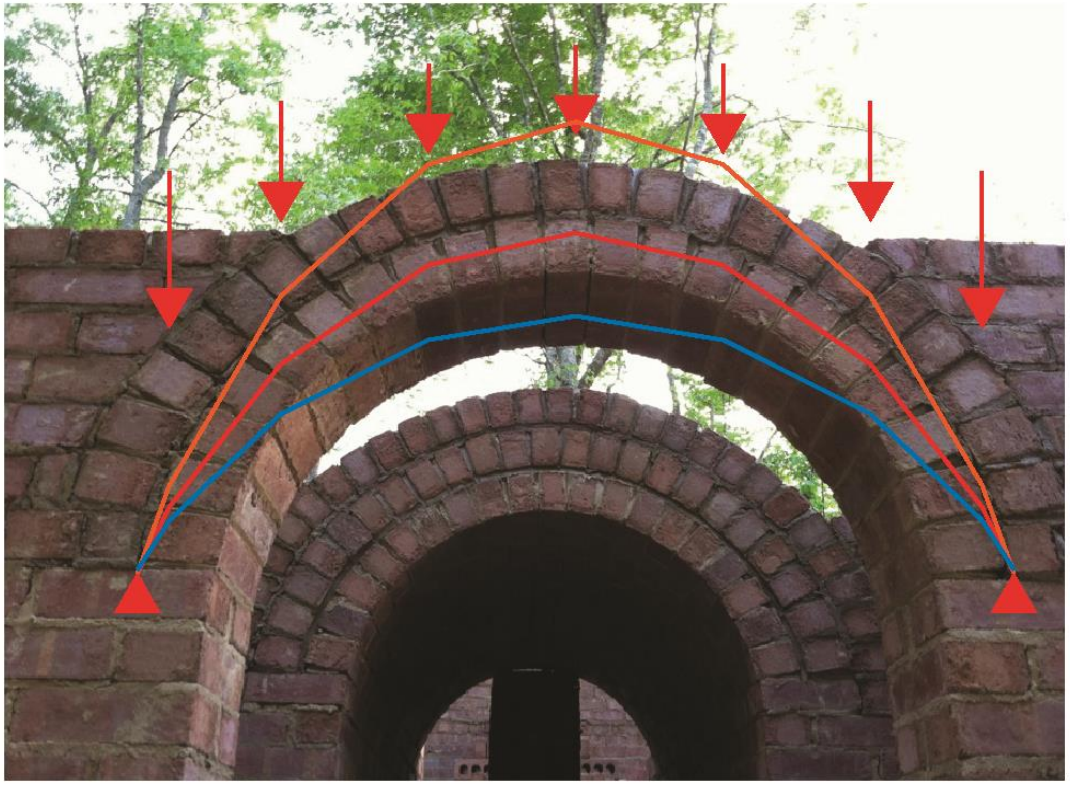

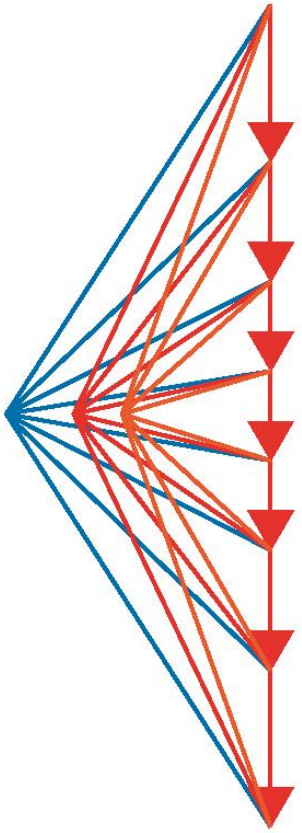

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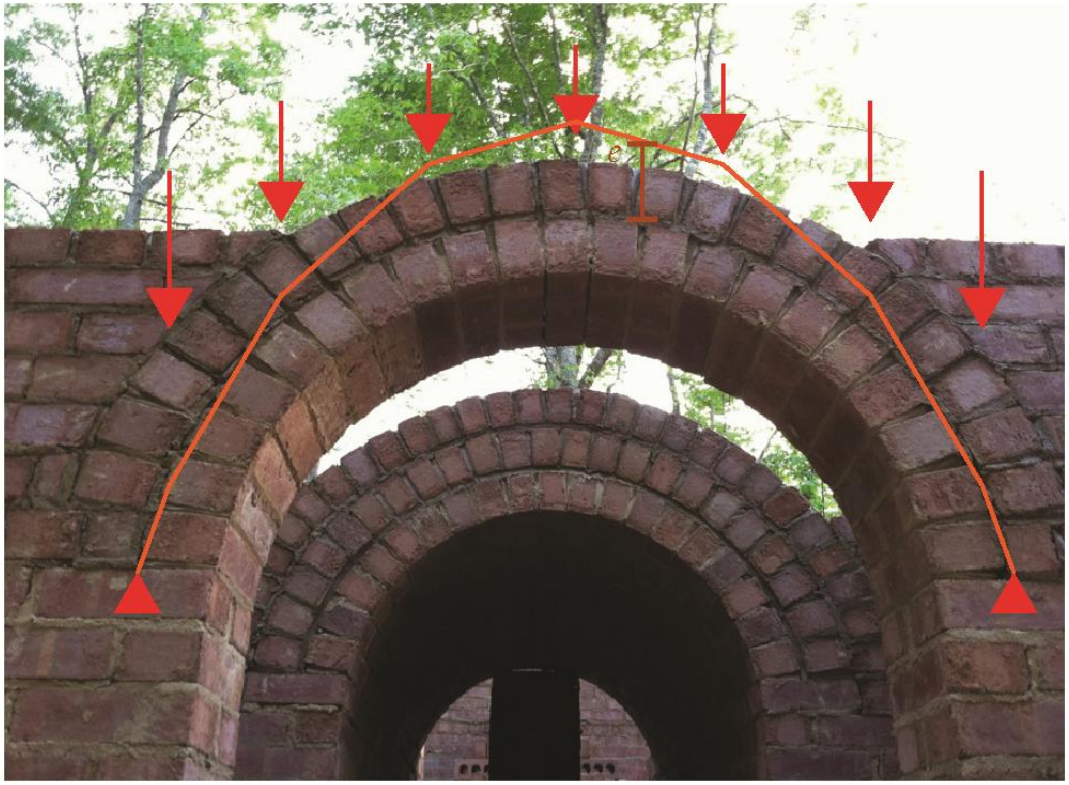
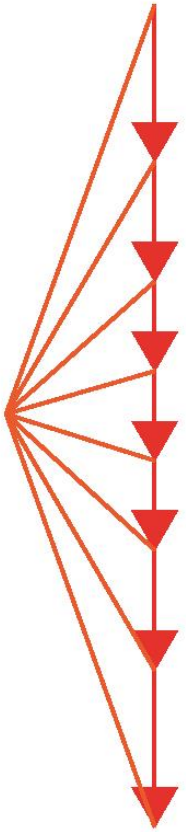


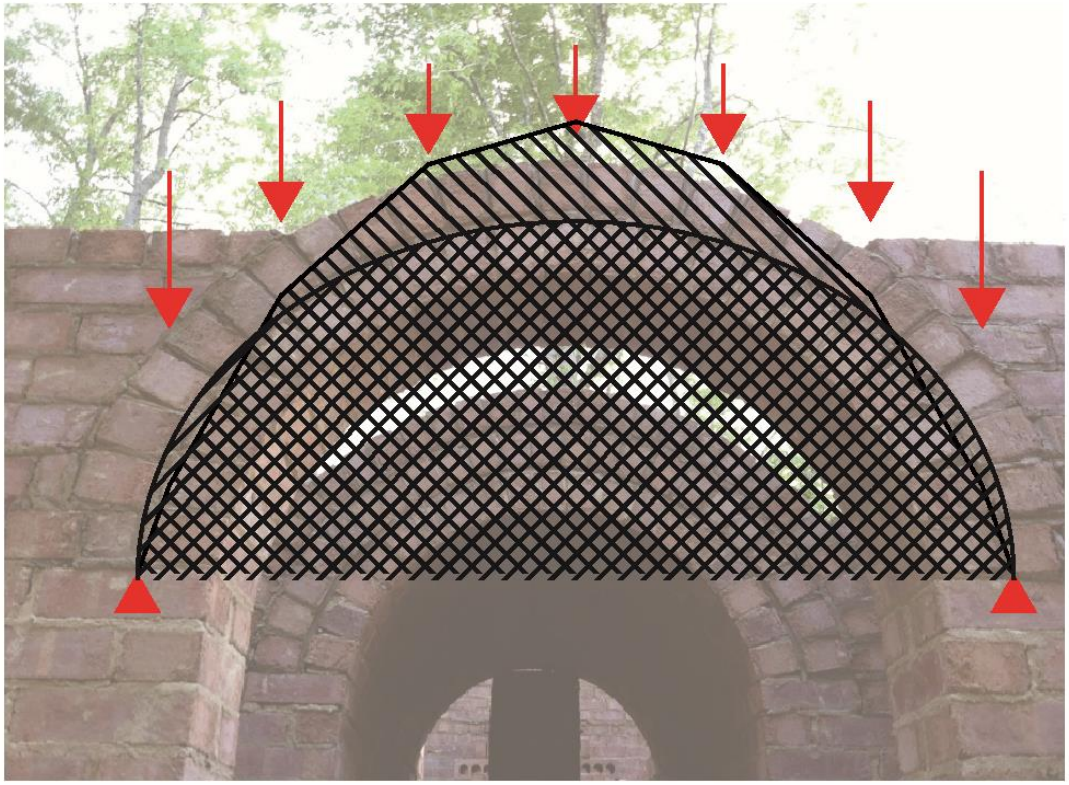


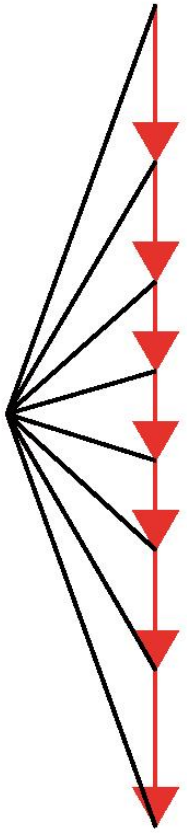
$$E_c = \left(N^2 + \frac{12}{t^2}M^2\right) l$$

min $E_c \rightarrow$ *correct thrust line*

Literatuur

Wat kun je met de juiste druklijn?





Onderzoeksvraag

How can the structural performance of a shell structure be calculated in such a way that the relation between the geometry and the structural performance is shown?

Onderzoeksvraag

How can the structural performance of a shell structure be calculated in such a way that the relation between the geometry and the structural performance is shown?

- *Can the method of equal areas be proven for arches*
- *Which methods can be made applicable to shell structures?*
- *How can this calculation method be translated into a computational algorithm and modelled in a 3D visualization program?*

Onderzoek

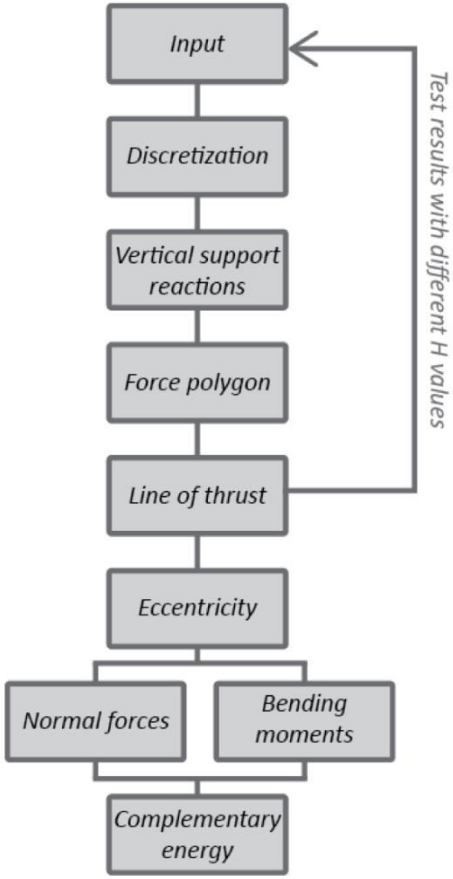
- Onderzoek naar gelijke oppervlakte methode
- Ontwikkeling nieuwe methode

Onderzoek

Gelijke oppervlakte methode

Onderzoek

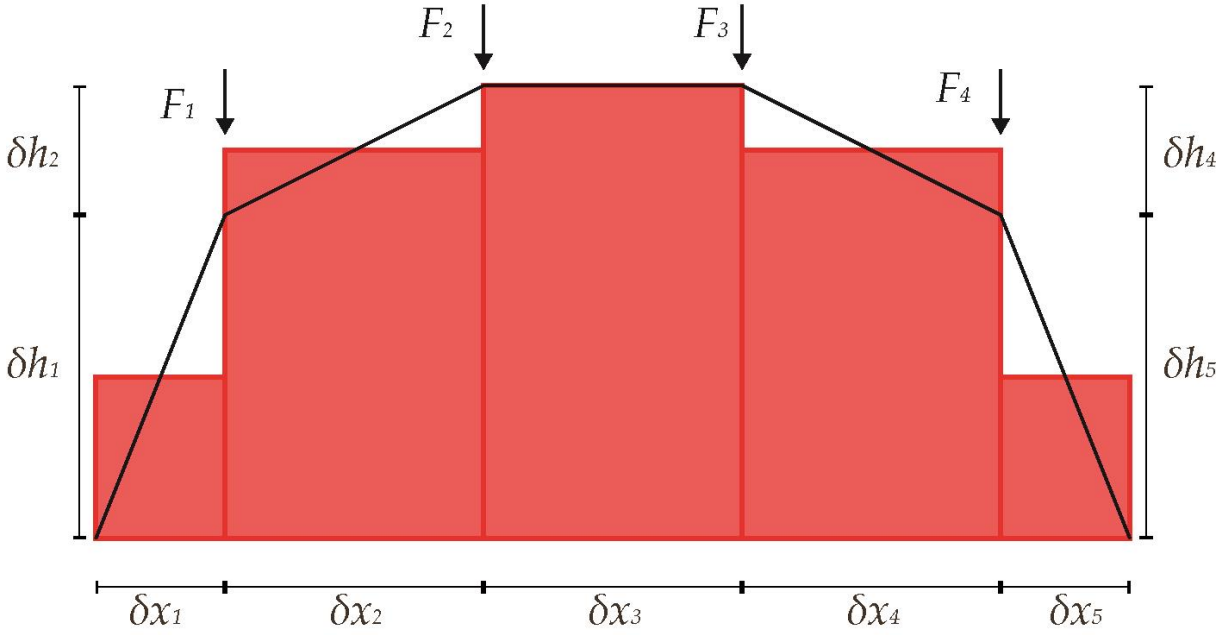
Gelijke oppervlakte methode



(van Dijk, 2014)

Onderzoek

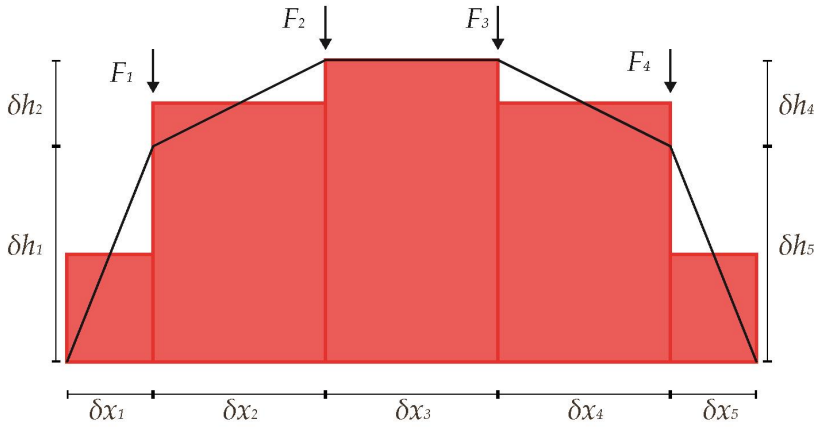
Gelijke oppervlakte methode



Onderzoek

Gelijke oppervlakte methode

$$A_{str} =$$
$$\delta x_1 \cdot \frac{1}{2} \cdot \delta y_1$$
$$+ \delta x_2 \left(\delta y_1 + \frac{1}{2} \cdot \delta y_2 \right)$$
$$+ \delta x_3 \left(\delta y_1 + \delta y_2 + \frac{1}{2} \cdot \delta y_3 \right)$$
$$+ \ \ldots$$
$$+ \delta x_n \left(\delta y_1 + \ldots + \delta y_{n-1} + \frac{1}{2} \cdot \delta y_n \right)$$



Onderzoek

Gelijke oppervlakte methode

$$
\begin{aligned}
A_{th} = & \\
& \delta x_1 \cdot \frac{1}{2} \cdot \frac{F_{V1} \cdot \delta x_1}{F_H} \\
& + \delta x_2 \left(\frac{F_{V1} \cdot \delta x_1}{F_H} + \frac{1}{2} \cdot \frac{F_{V2} \cdot \delta x_2}{F_H} \right) \\
& + \delta x_3 \left(\frac{F_{V1} \cdot \delta x_1}{F_H} + \frac{F_{V2} \cdot \delta x_2}{F_H} + \frac{1}{2} \cdot \frac{F_{V3} \cdot \delta x_3}{F_H} \right) \\
& + \ \ldots \\
& + \delta x_n \left(\frac{F_{V1} \cdot \delta x_1}{F_H} + \ldots + \frac{F_{V(n-1)} \cdot \delta x_{n-1}}{F_H} + \frac{1}{2} \cdot \frac{F_{Vn} \cdot \delta x_n}{F_H} \right)
\end{aligned}
$$

Onderzoek

Gelijke oppervlakte methode

$A_{th} = A_{str}$

Onderzoek

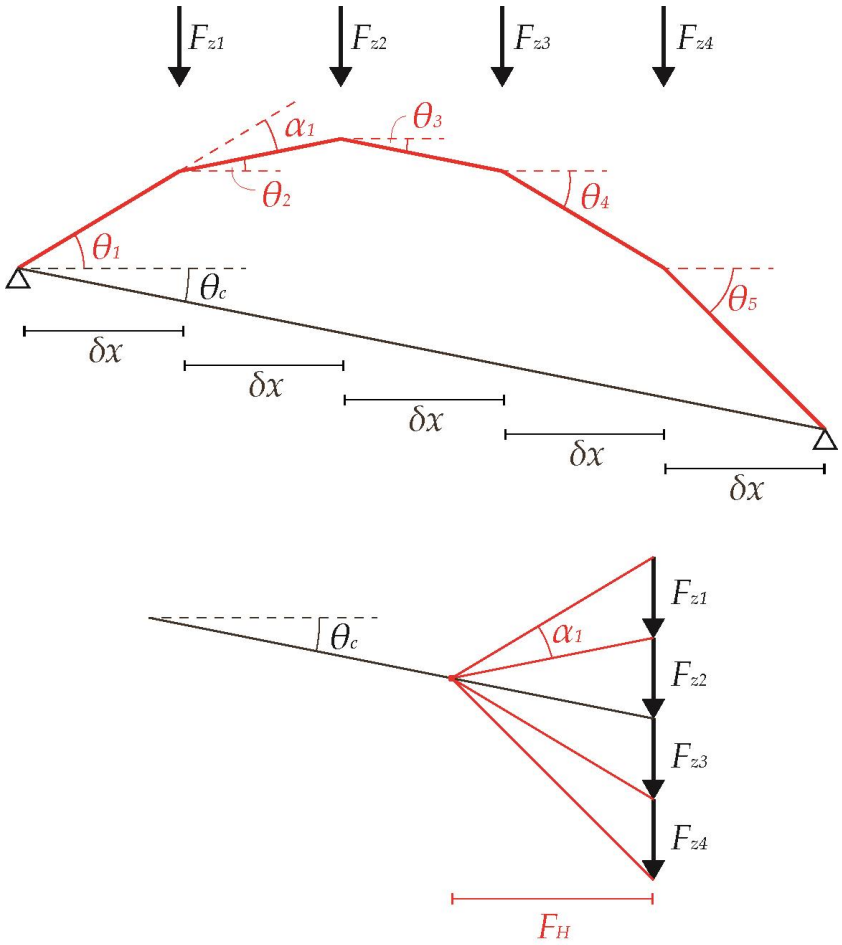
Gelijke oppervlakte methode

$$A_{th} = A_{str}$$

$$F_H = \frac{\delta x_1 \cdot \frac{1}{2} \cdot F_{V1} \cdot \delta x_1 + \delta x_2 \left(F_{V1} \cdot \delta x_1 + \frac{1}{2} \cdot F_{V2} \cdot \delta x_2 \right) + \ldots + \delta x_n \left(F_{V1} \cdot \delta x_1 + \ldots + F_{V(n-1)} \cdot \delta x_{n-1} + \frac{1}{2} \cdot F_{Vn} \cdot \delta x_n \right)}{\delta x_1 \cdot \frac{1}{2} \cdot \delta y_1 + \delta x_2 \left(\delta y_1 + \frac{1}{2} \cdot \delta y_2 \right) + \delta x_3 \left(\delta y_1 + \delta y_2 + \frac{1}{2} \cdot \delta y_3 \right) + \ldots + \delta x_n \left(\delta y_1 + \ldots + \delta y_{n-1} + \frac{1}{2} \cdot \delta y_n \right)}$$

Onderzoek

Gelijke oppervlakte methode



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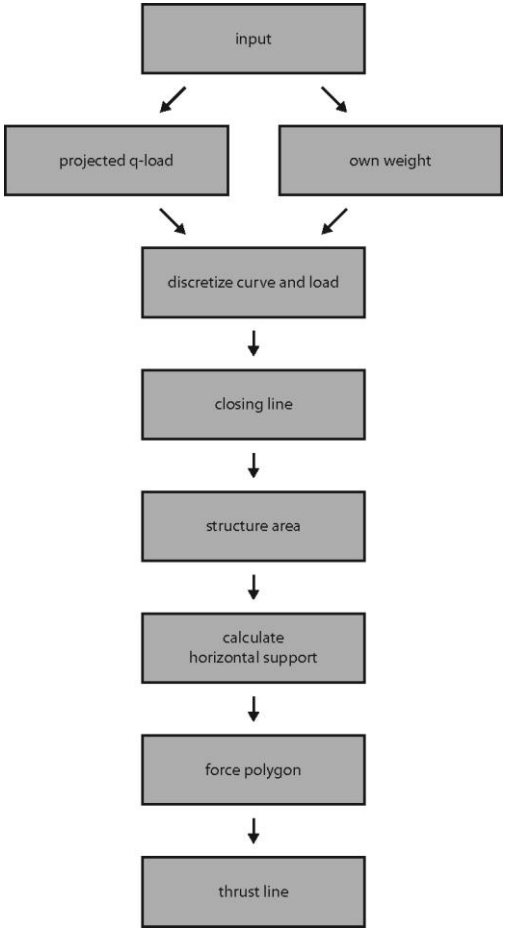

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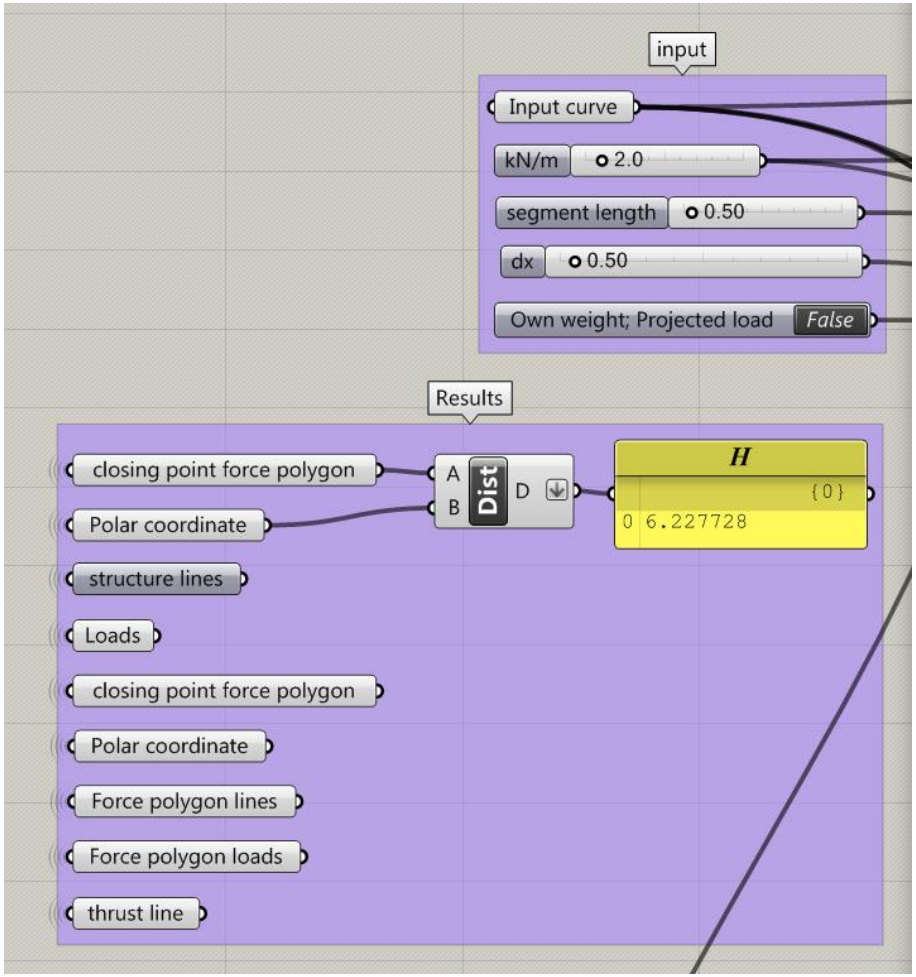

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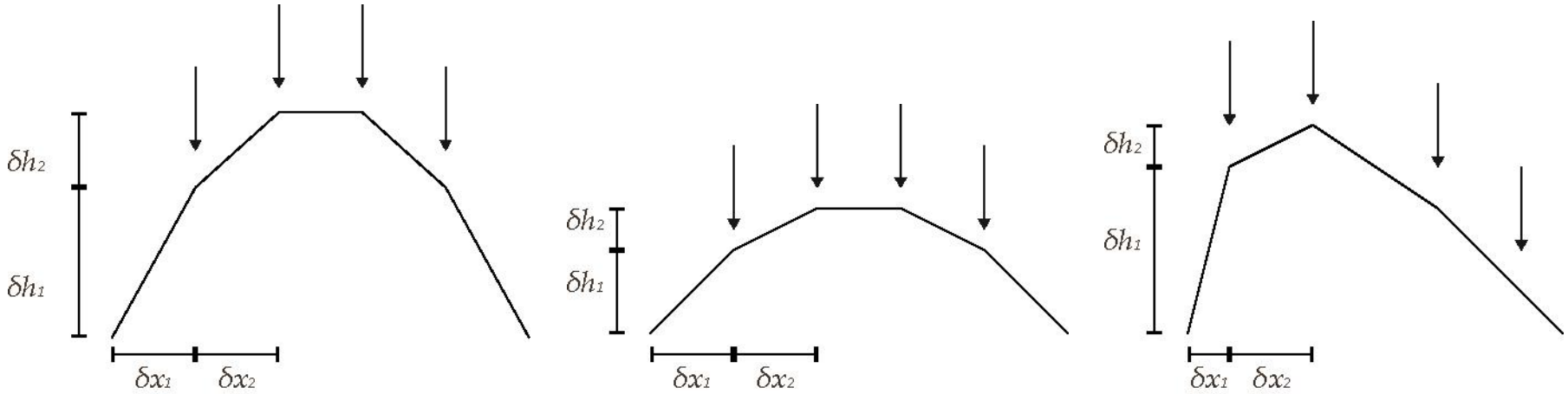

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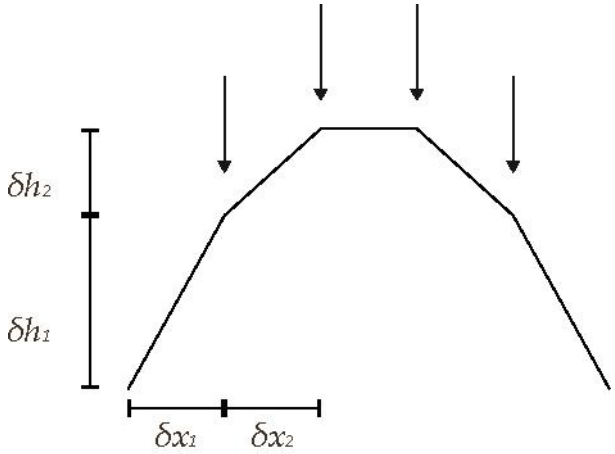
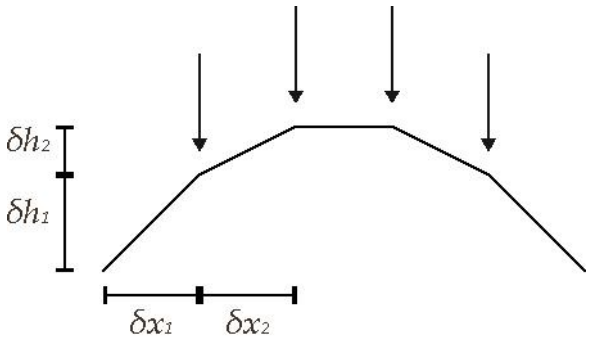
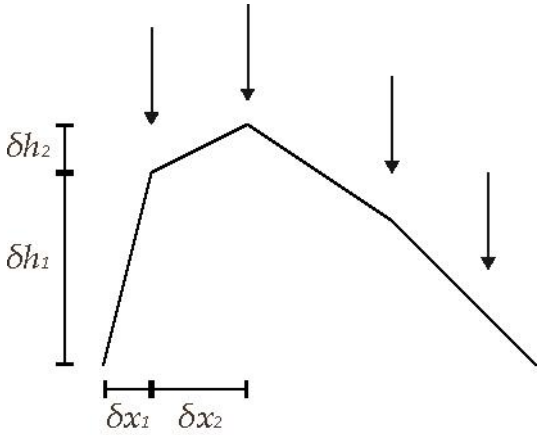


method	F_H
FEM	2,22
eq A	2,22
min E	2,22



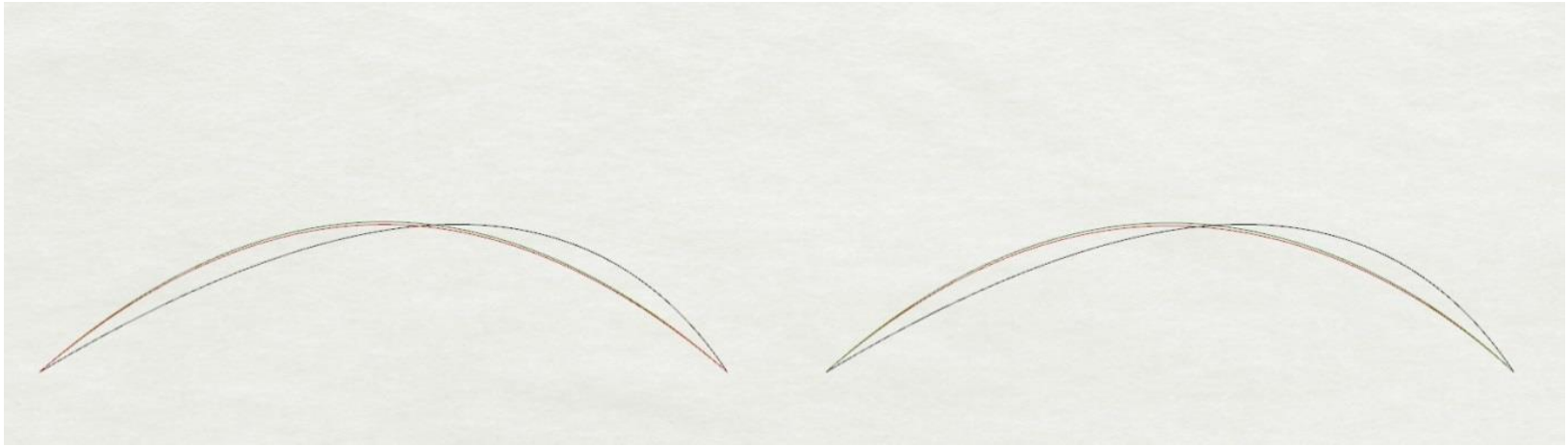
method	F_H
FEM	3,99
eq A	4,00
min E	4,00



method	F_H
FEM	1,70
eq A	1,89
min E	1,70

Onderzoek

Gelijke oppervlakte methode

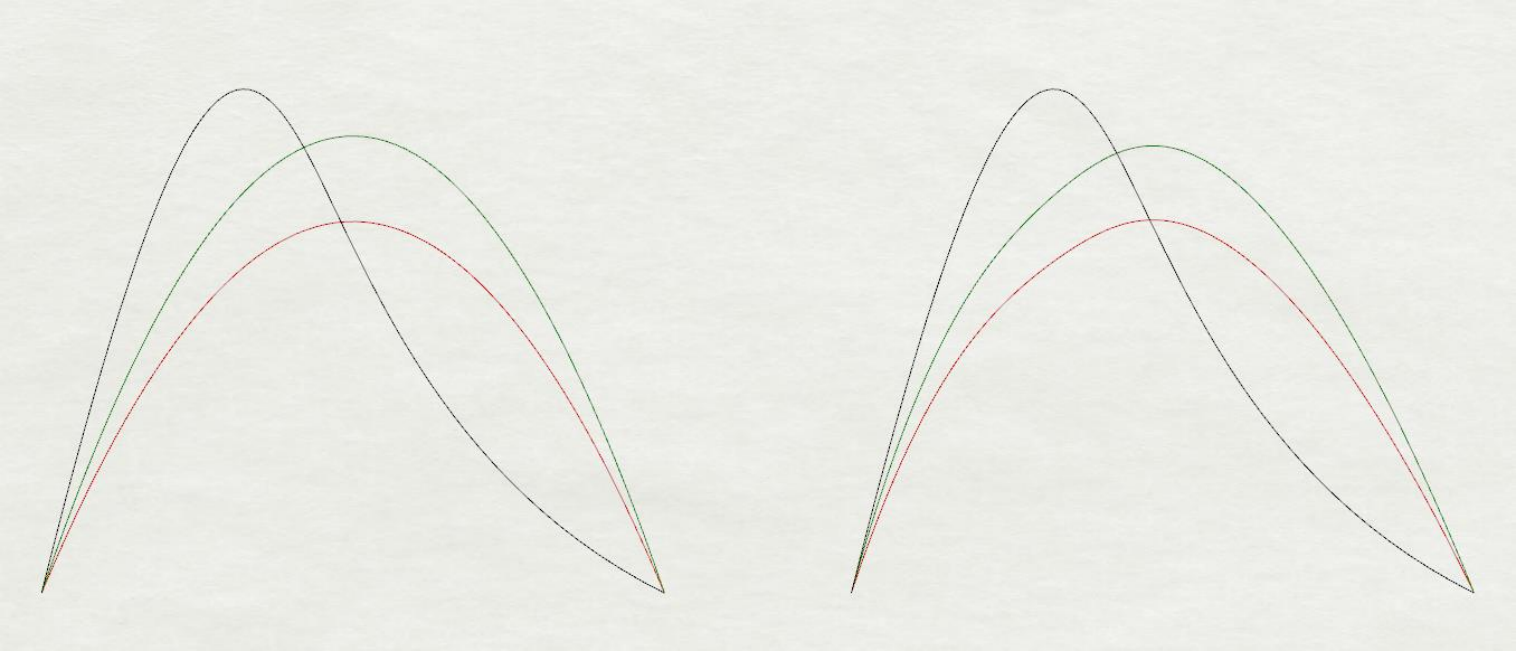


own weight	F_H	
min. E_c	8,54	
equal A	8,71	1,99 %

projected load	F_H	
min. E_c	8,03	
equal A	8,17	1,74%

Onderzoek

Gelijke oppervlakte methode



own weight	F_H	
min. E_c	6,48	
equal A	7,77	19,91 %

projected load	F_H	
min. E_c	3,41	
equal A	4,35	27,57 %

Onderzoek

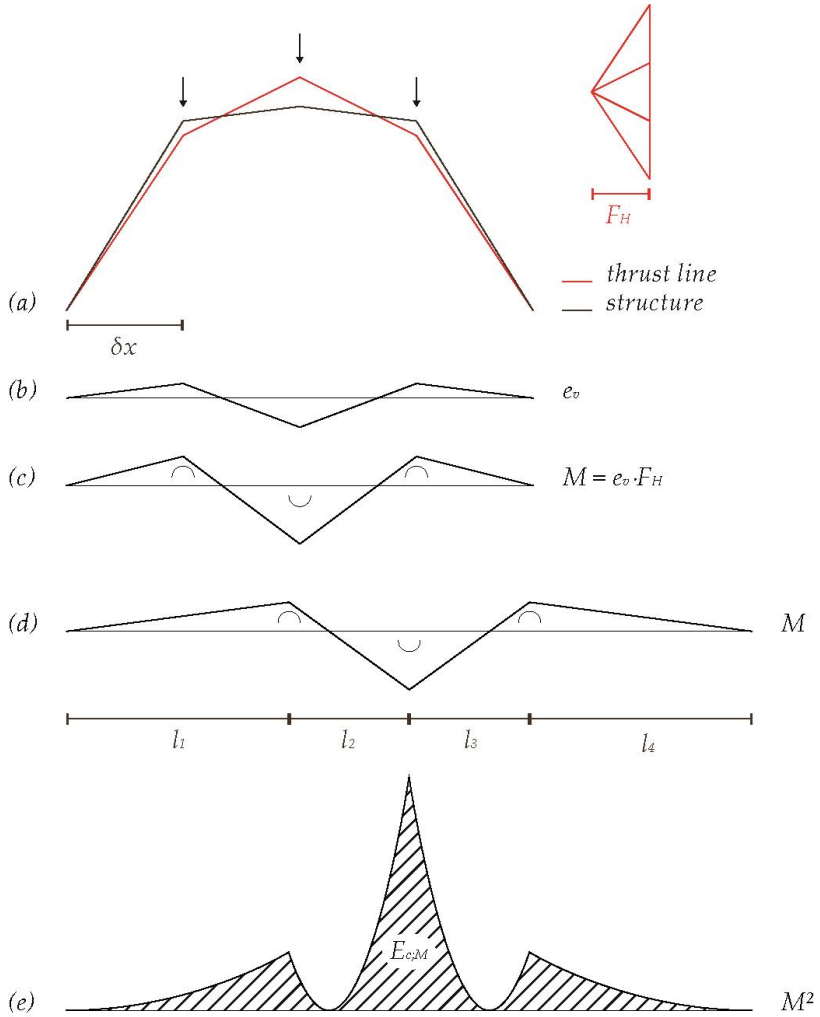
Gelijke oppervlakte methode

Onderzoek

Ontwikkelen nieuwe methode

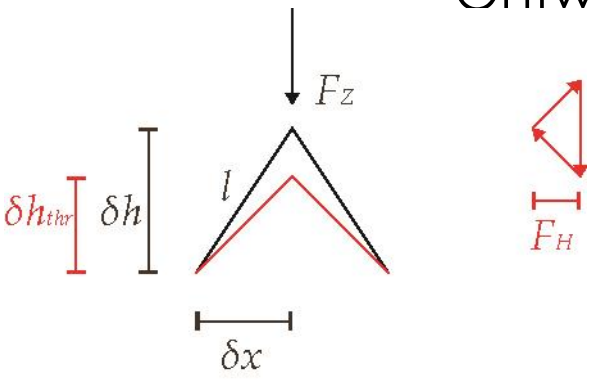
Onderzoek

Ontwikkelen nieuwe methode



Onderzoek

Ontwikkelen nieuwe methode



$$\delta h \cdot \delta x = \delta h_{thr} \cdot \delta x$$

$$\delta h_{thr} = \frac{1}{2}\frac{F_z}{F_H}\delta x$$
$$E_{c;M} = M^2 l$$
$$e_V = h - h_{thr}$$
$$e_V = \frac{\delta h - \delta h_{thr}}{l} x \text{ for domain } [0, l]$$
$$e_V = \left(\delta h - \frac{1}{2}\frac{F_z}{F_H}\delta x\right)\frac{x}{l}$$
$$M = \left(F_H \delta h - \frac{1}{2}F_z \delta x\right)\frac{x}{l}$$
$$M^2 = \left(F_H \delta h - \frac{1}{2}F_z \delta x\right)^2 \frac{x^2}{l^2} \text{ for domain } [0, l]$$

$$E_{c;M} = \int_0^l \left(F_H\, \delta h - \frac{1}{2}F_z\, \delta x\right)^2 \frac{1}{l^2}x^2\ \delta l$$
$$= \frac{1}{3}\delta h^2 l\, {F_H}^2 - \frac{1}{3}F_z\, \delta x\, \delta h\, l\, F_H + \frac{1}{12}{F_z}^2 \delta x^2 l$$

$$E'_{c;M} = \frac{2}{3}\delta h^2 l\, F_H - \frac{1}{3}F_z\, \delta x\, \delta h\, l$$

$$\frac{2}{3}\delta h^2 l\, F_H - \frac{1}{3}F_z\, \delta x\, \delta h\, l = 0$$
$$\frac{2}{3}\delta h^2 l\, F_H = \frac{1}{3}F_z\, \delta x\, \delta h\, l$$
$$2\, \delta h\, F_H = F_z\, \delta x$$
$$\delta h = \frac{1}{2}\frac{F_z}{F_H}\, \delta x$$

$$\delta h = \delta h_{thr}$$

Onderzoek

Ontwikkelen nieuwe methode

Onderzoek

Ontwikkelen nieuwe methode

$$E_c = N^2 l + \frac{12}{t^2} M^2 l$$

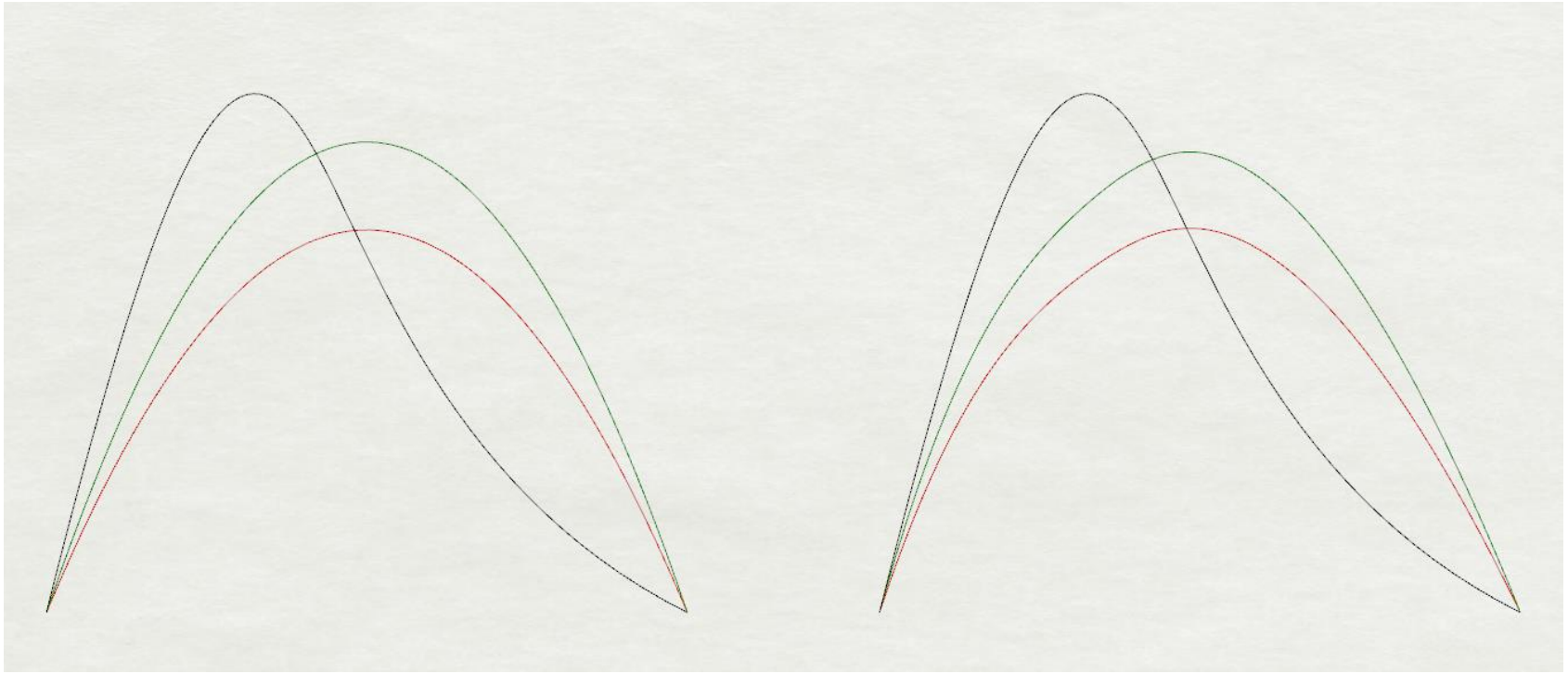
Onderzoek

Ontwikkelen nieuwe methode

$$E_c = \cancel{M^2 l} + \frac{12}{t^2} M^2 l$$

Onderzoek

Ontwikkelen nieuwe methode

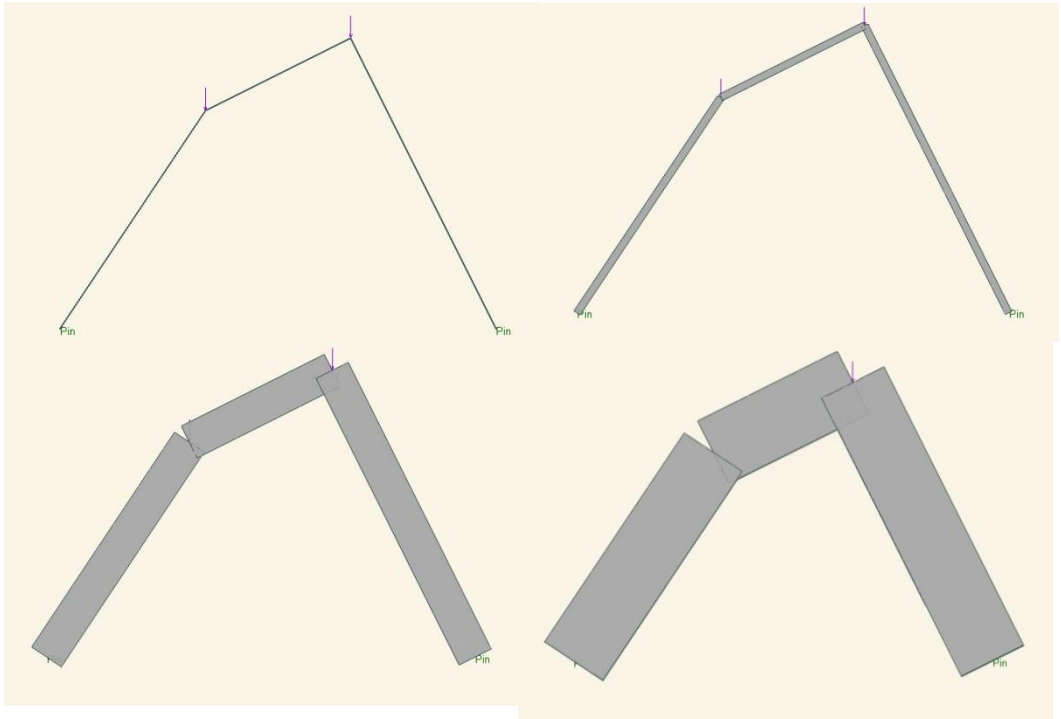


own weight	F_H	
min. E_c	6,48	
min. $E_{c;M}$	6,48	0,00 %
equal A	7,77	19,91 %

projected load	F_H	
min. E_c	3,41	
min. $E_{c;M}$	3,41	0,00 %
equal A	4,35	27,57 %

Onderzoek

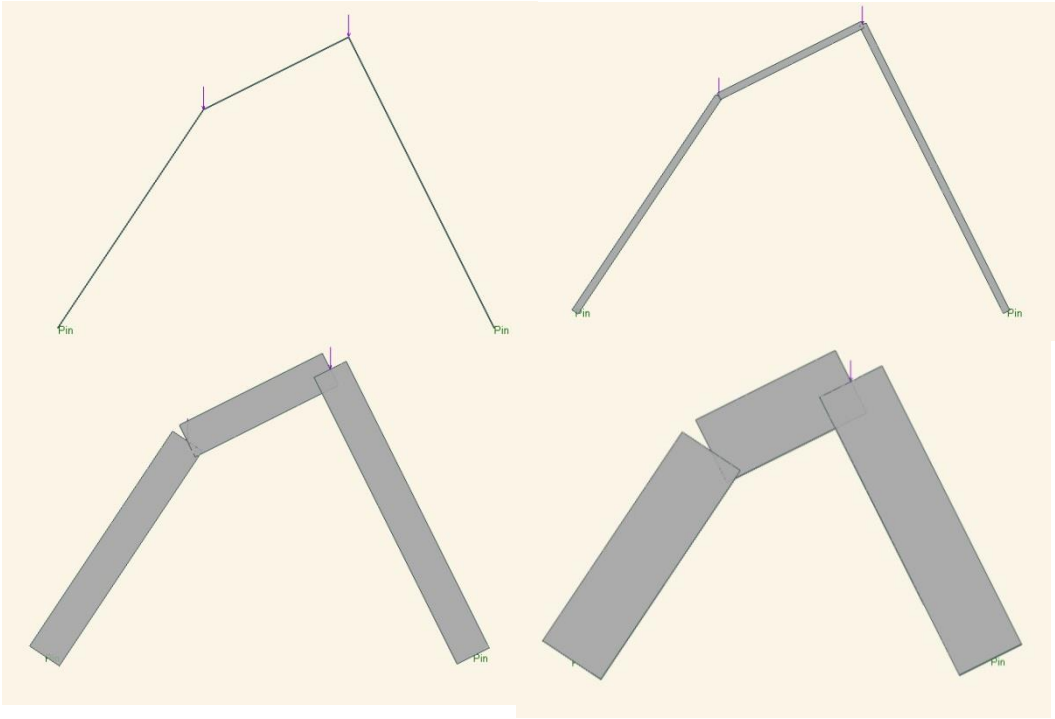
Ontwikkelen nieuwe methode



$$E_{c;M} = \frac{12}{t^2} M^2 l$$

Onderzoek

Ontwikkelen nieuwe methode



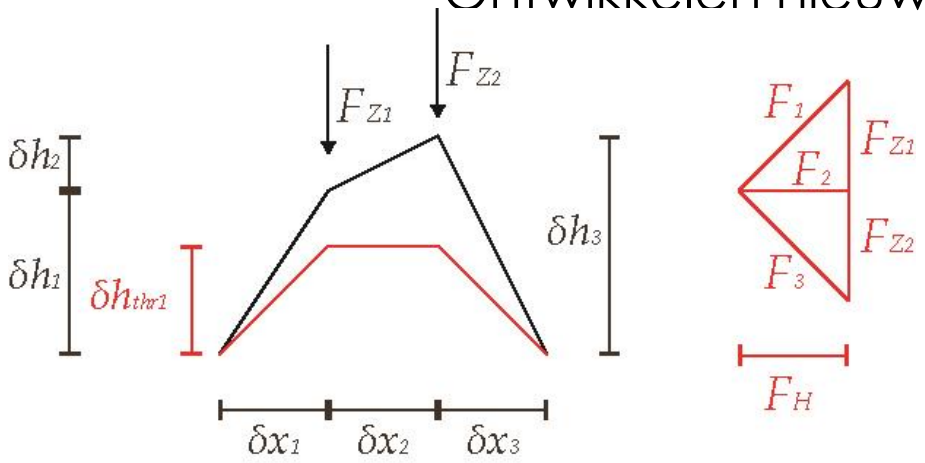
$$E_{c;M} = \frac{12}{t^2} M^2 l$$

Direct method	F_H	
t undefined	1118	

FEM	F_H	
t = 0,01	1118	0,00%
t = 0,1	1118	0,00%
t = 0,5	1114	0,36%
t = 1,00	1102	1,45%

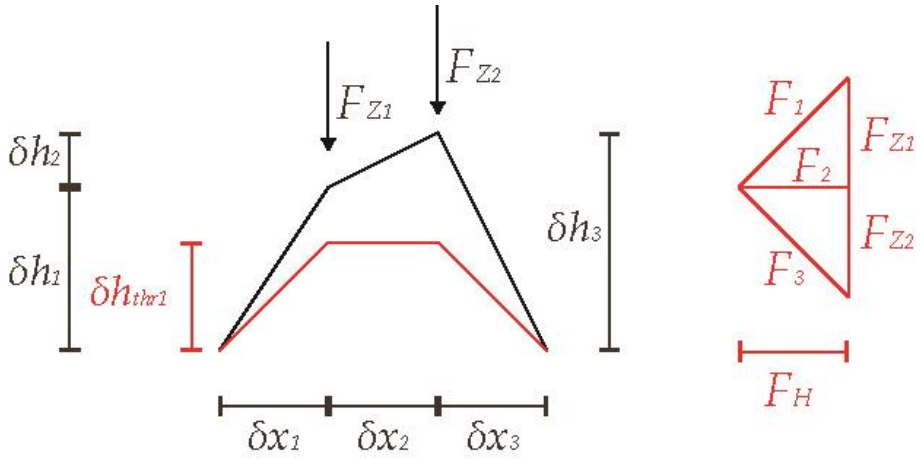
Onderzoek

Ontwikkelen nieuwe methode



Onderzoek

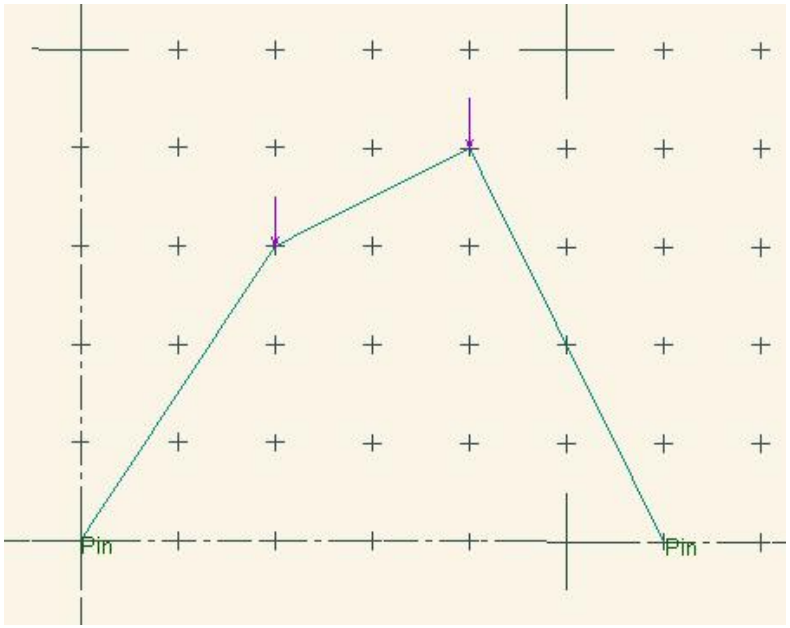
Ontwikkelen nieuwe methode



$$F_H = \frac{F_z\,\delta x\left(\frac{2}{3}\delta h_1 l_1 + 2\,\delta h_1 l_2 + \delta h_2 l_2 + \delta h_1 l_3 + \delta h_2 l_3 + \frac{1}{3}\delta h_3 l_3\right)}{\frac{2}{3}\delta {h_1}^2 l_1 + \frac{2}{3}\delta {h_2}^2 l_2 + 2\,\delta h_1 \delta h_2 l_2 + 2\,\delta {h_1}^2 l_2 + \frac{2}{3}\delta {h_3}^2 l_3 + 4\,\delta h_1 \delta h_2 l_3 + 2\,\delta {h_1}^2 l_3 + 2\,\delta {h_2}^2 l_3 + 2\,\delta h_1 l_3 + 2\,\delta h_2 l_3}$$

Onderzoek

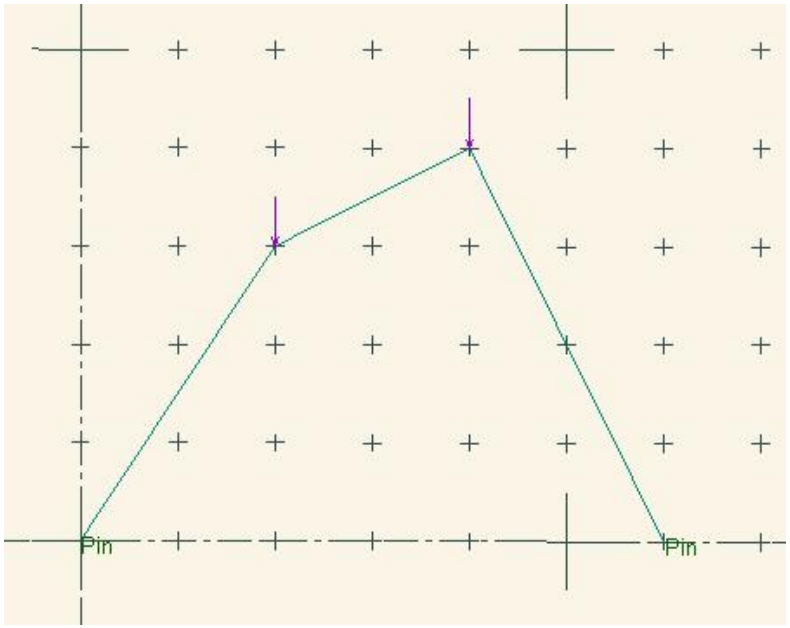
Ontwikkelen nieuwe methode



	F_H	
FEM	1117	
min. $E_{c;M}$	1118	0,09 %

Onderzoek

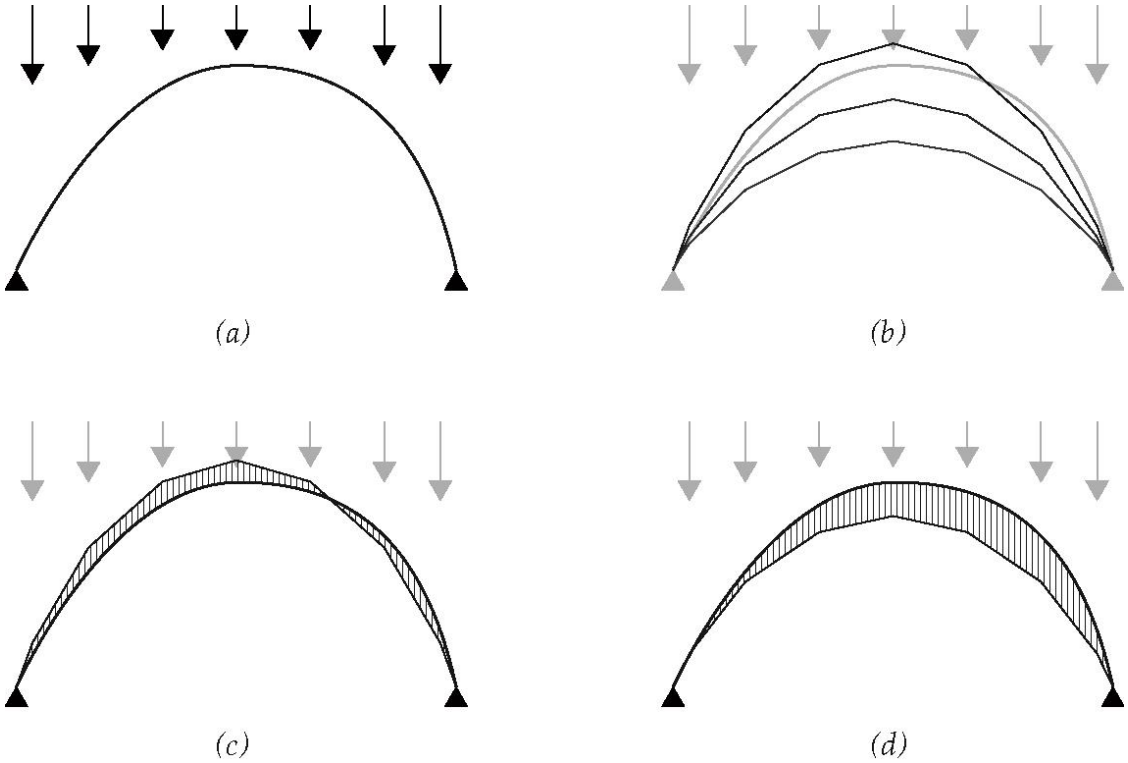
Ontwikkelen nieuwe methode



	F_H	
FEM	1117	
min. $E_{c;M}$	1118	0,09 %

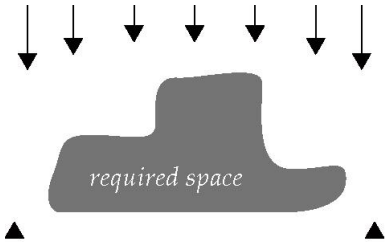
Onderzoek

Toepassing nieuwe methode

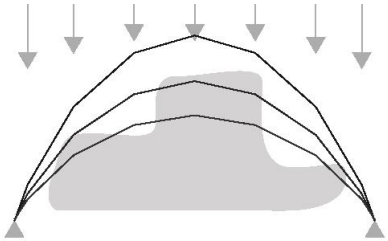


Onderzoek

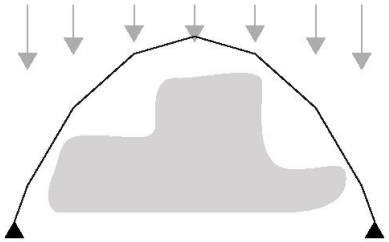
Toepassing nieuwe methode



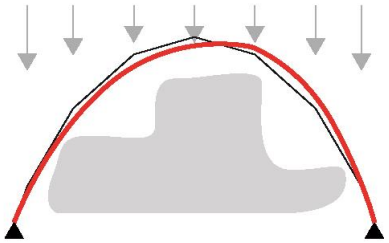
(a)



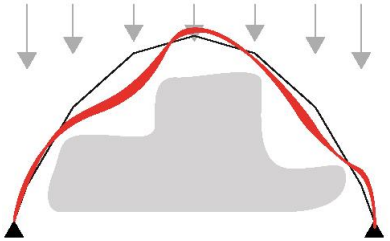
(b)



(c)



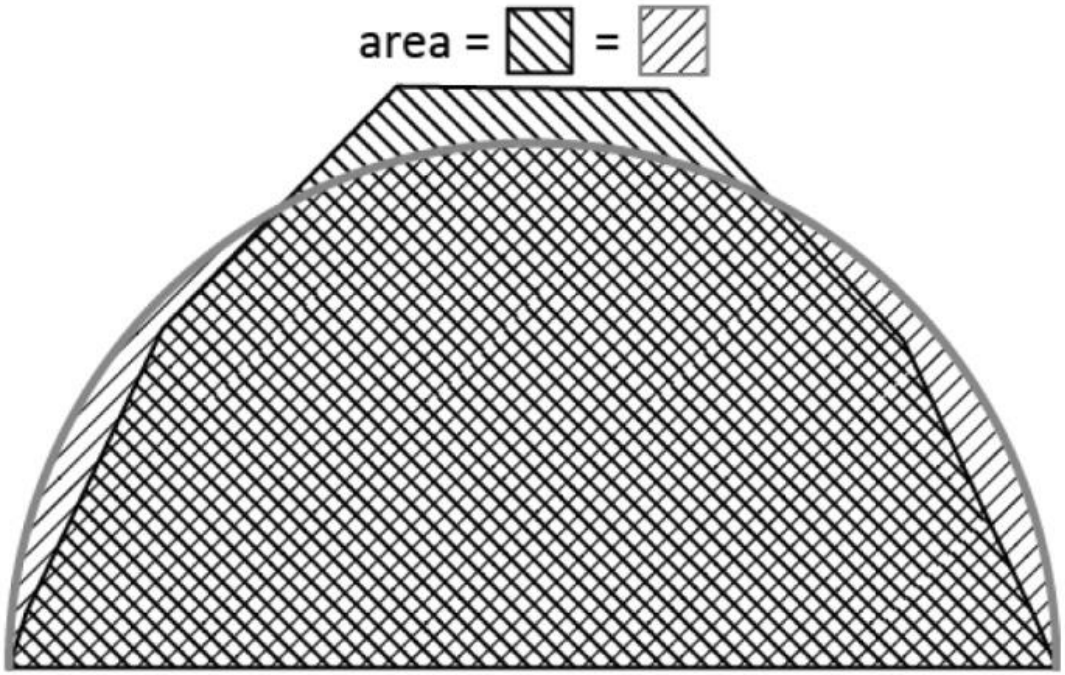
(d)



(e)

Conclusies

Can the method of equal areas be proven for arches?



(van Dijk, 2014)

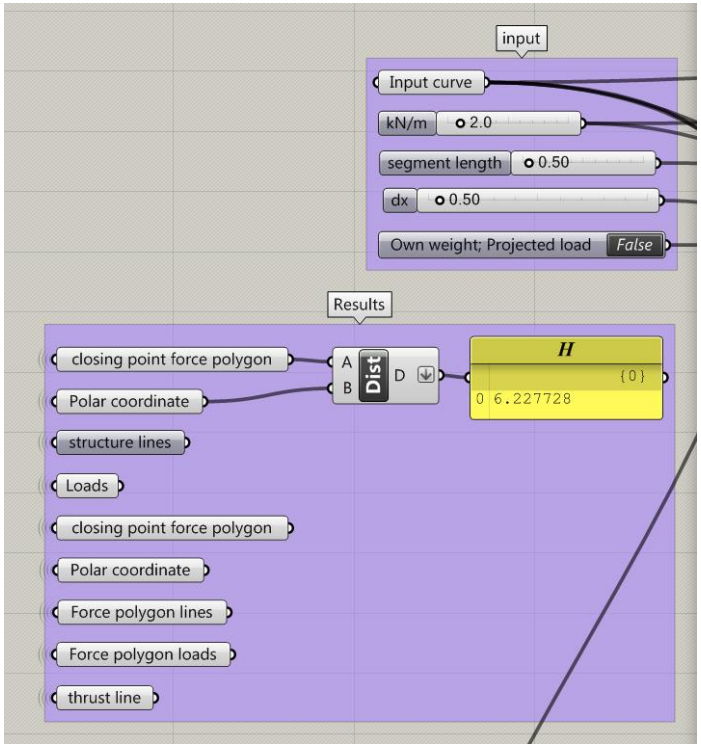
Conclusies

Which methods can be made applicable to shell structures?

L'Oceanogràfic designed by Felix Candela, 2003 (Gabaldón, 2010)

Conclusies

How can this calculation method be translated into a computational algorithm and modelled in a 3D visualization program?



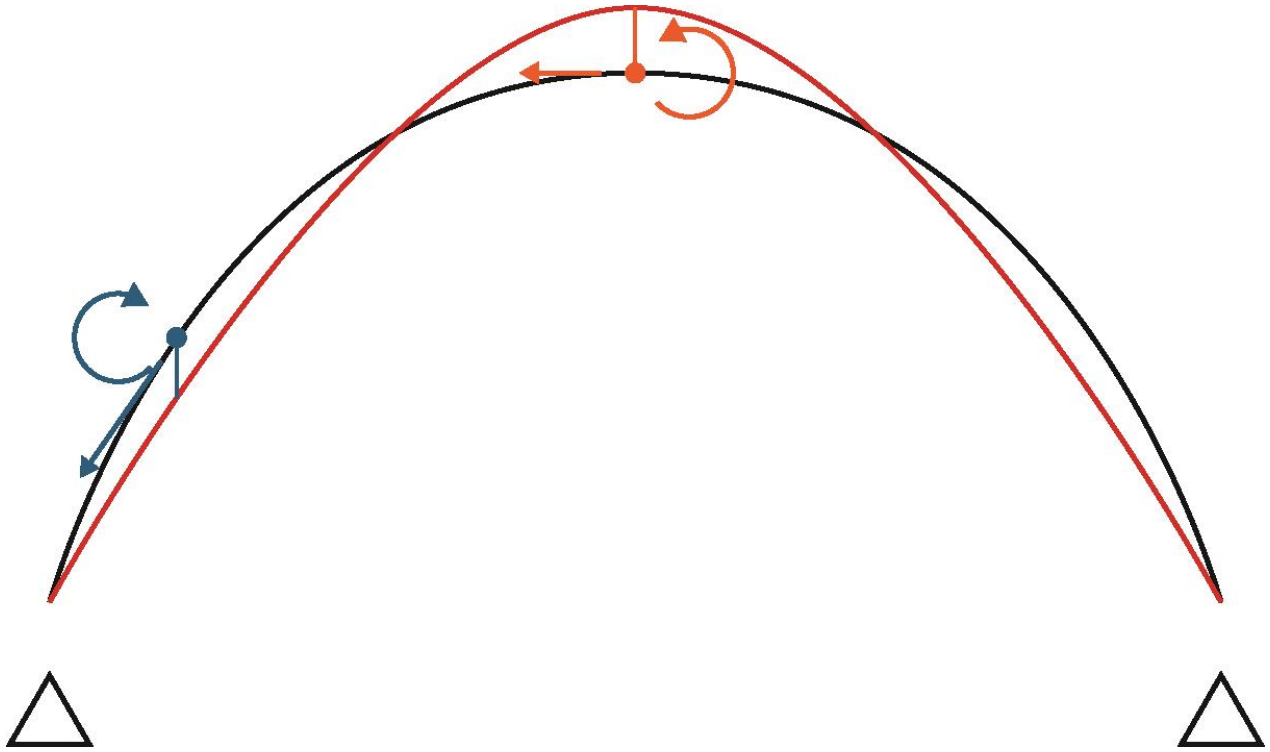
Conclusies

How can the structural performance of a shell structure be calculated in such a way that the relation between the geometry and the structural performance is shown?

$$F_H = \frac{F_z\ \delta x\ \left(\frac{2}{3}\delta h_1 l_1 + 2\ \delta h_1 l_2 + \delta h_2 l_2 + \delta h_1 l_3 + \delta h_2 l_3 + \frac{1}{3}\delta h_3 l_3\right)}{\frac{2}{3}\delta {h_1}^2 l_1 + \frac{2}{3}\delta {h_2}^2 l_2 + 2\ \delta h_1 \delta h_2 l_2 + 2\ \delta {h_1}^2 l_2 + \frac{2}{3}\delta {h_3}^2 l_3 + 4\ \delta h_1 \delta h_2 l_3 + 2\ \delta {h_1}^2 l_3 + 2\ \delta {h_2}^2 l_3 + 2\ \delta h_1 l_3 + 2\ \delta h_2 l_3}$$

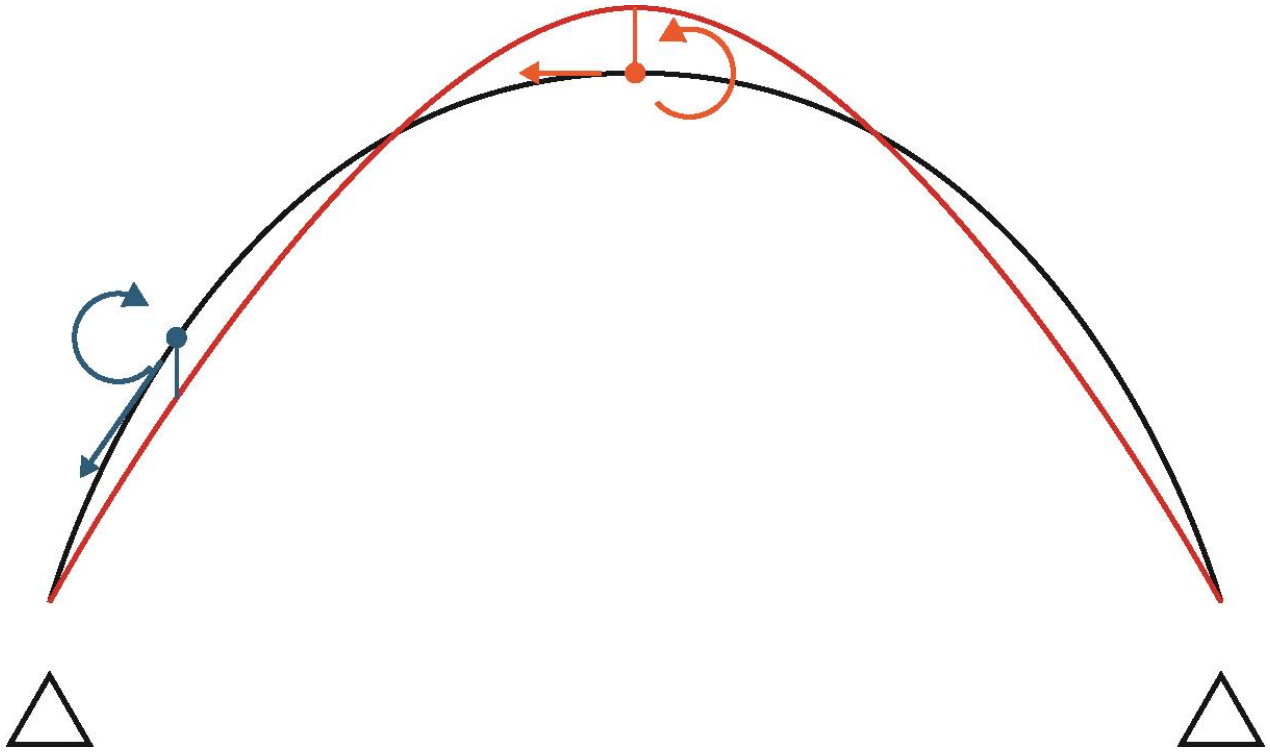
Conclusies

How can the structural performance of a shell structure be calculated in such a way that the relation between the geometry and the structural performance is shown?

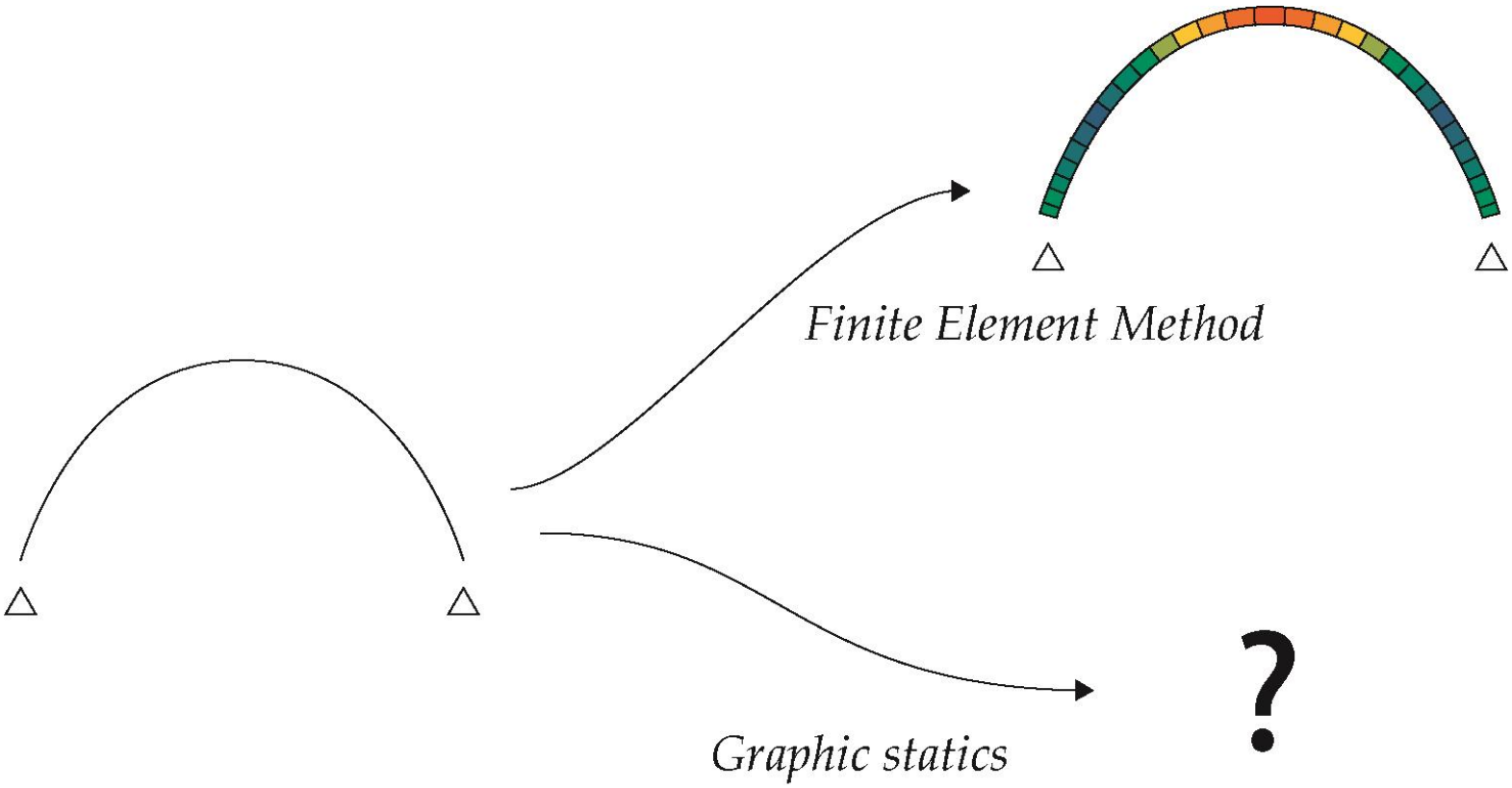


Conclusies

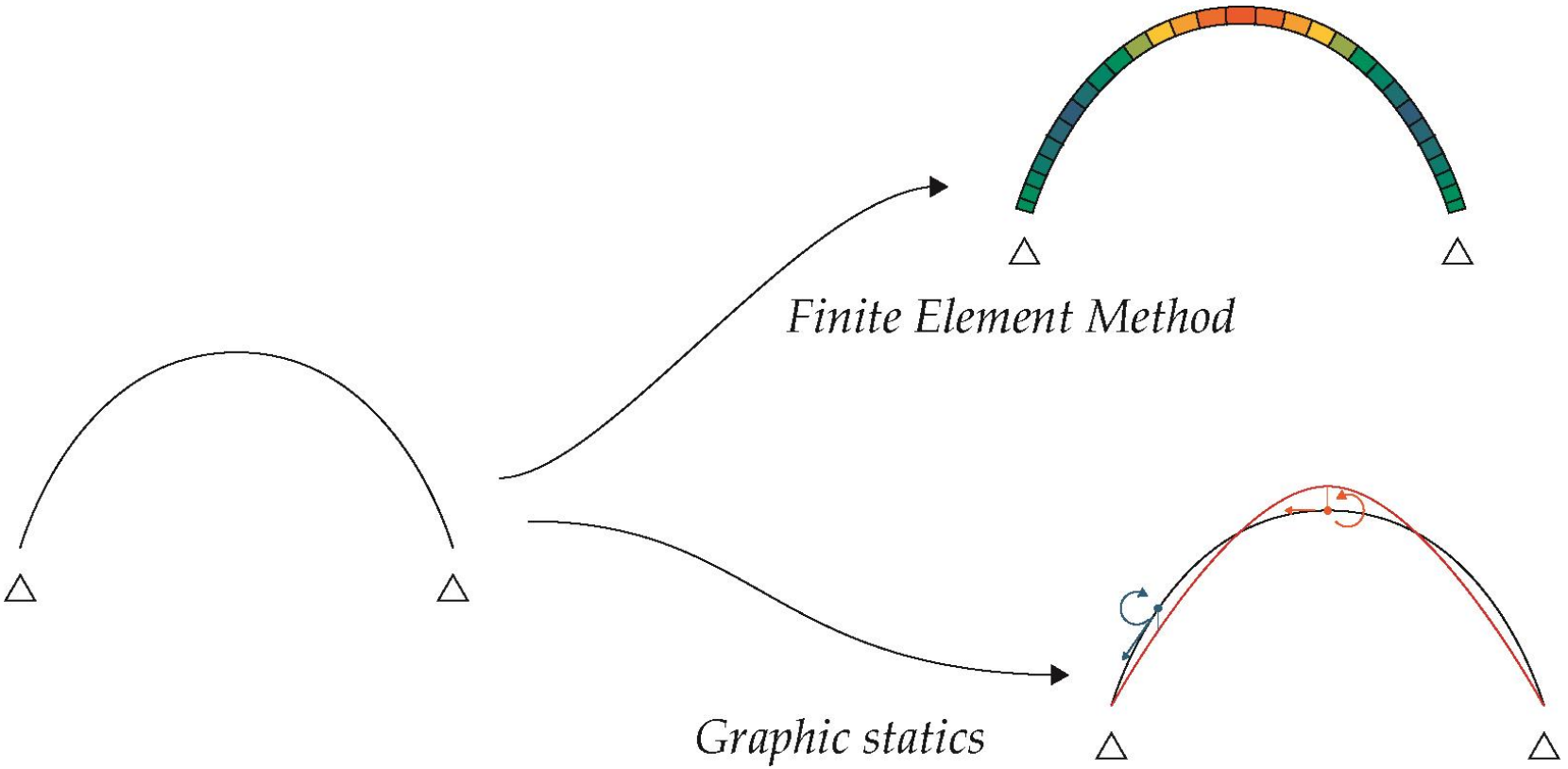
How can the structural performance of a shell structure be calculated in such a way that the relation between the geometry and the structural performance is shown?



Conclusies



Conclusies



Aanbevelingen

- Methode uitbreiden naar constructies van meer dan drie staven
- Methode vertalen naar schaalconstructies