2025 23rd European Control Conference (ECC)
June 24-27, 2025. Thessaloniki, Greece

A Feedback-Based Optimization Algorithm with Designed Gain Matrix

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***Abstract*— In this paper, we propose a gradient projection algorithm aimed at improving the transient performance of feedback-based optimization (FO) for linear dynamical systems. Our approach leverages a specifically designed gain matrix, replacing the usual scalar step size to enhance trajectory efficiency and reduce oscillations. By solving a semi-definite programming, we select the gain matrix to trade off between convergence rate and oscillation minimization. Compared to the standard FO algorithms, our method demonstrates improved transient performance in numerical simulations and in turn faster convergence.**

I. Introduction

In various engineering applications, such as power networks [1], [2], communication networks [3], [4], robotics control [5] and wind farm control [6], [7], it is often necessary to regulate a dynamical system to a steady-state that is the solution of a target optimization problem. However, the presence of unknown and unmeasurable disturbance terms in the real physical system makes it impossible to compute the optimal solution explicitly via a traditional feedforward optimization procedure. Consequently, it is desirable for control inputs to be able to adapt in real time based on the instantaneous output feedback measured from the system. Feedback-based optimization (also known as real-time or autonomous optimization in the literature) has emerged as an effective framework that aims at addressing these challenges [8], [9], [10].

The key idea of feedback optimization is to implement the traditional optimization algorithms within a feedback loop with the original dynamical system, using real-time measurements of the output to replace the steady output, thus eliminating the requirement on perfect knowledge of the steady-state input-output mapping. This approach has been extensively studied both theoretically and practically in recent years. For instance, [8] analyzes the stability of the feedback gradient descent flow and applies it to the power systems. Similarly, [9] combines the Proportional-Integral (PI) controller to design a feedback gradient flow to find the optimal steady state of a linear time-invariant system. Furthermore, [10] proposes feedback primal-dual algorithms for a time-varying optimization problem, establishing some tracking properties of the optimal trajectory. Building upon these developments, the algorithm is further generalized to address stochastic steady state optimization problems for linear systems affected by stochastic disturbance with time-varying distributions [11]. To address scalability and privacy issues in large-scale network systems, decentralized implementations of feedback optimization have been studied in [12], [13] and [14], among others, demonstrating convergence. Additionally, inspired by zeroth-order optimization, model-free feedback optimization algorithms have been proposed in [15] even for nonlinear dynamical systems. For a more comprehensive review of the recent efforts in feedback optimization, we refer the interested reader to [16].

While most of the aforementioned studies focus on continuous-time dynamics, our work addresses discrete-time linear dynamical systems. Previous works on discrete-time dynamics have primarily concentrated on the asymptotic stability of proposed methods, that is, steady-state behaviour [17] and [18], neglecting the transient performance of trajectories. However, in real engineering systems, it is often desirable to improve the transient performance while achieving the optimal steady state. For example, in power systems, minimizing the oscillations of the closed-loop dynamics is crucial to ensure system stability and prevent excessive wear and tear on mechanical components. Oscillations might in fact lead to increased operational costs and decreased system reliability. Although feedback-optimizing MPC [19], [20] and economic MPC [21] could potentially address transient performance through repeated online optimization, these methods typically involve significant computational complexity and often lack straightforward theoretical guarantees.

In this paper, we propose a feedback gradient projection algorithm to improve the transient performance in regulating a discrete-time linear dynamical system. By using a gain matrix instead of a scalar step size parameter, our method offers greater flexibility in enhancing trajectory performance. Notably, adaptive and heterogeneous step sizes have demonstrated improvements in the numerical performance of optimization algorithms on specific test cases [22]. Our results show that this approach can also effectively reduce the oscillations in feedback optimization. This is particularly beneficial in applications where fast convergence and reduced oscillations are critical. Specifically, our technical contributions are summarized as follows:

- We propose a novel feedback optimization algorithm to find the optimal steady-state of a linear dynamical system with a quadratic objective function, and demonstrate its linear convergence via a linear matrix inequality technique.
- We provide a method to choose the gain matrix by solving a semidefinite programming and apply the proposed method to power systems. The numerical results illustrate that our algorithm achieves smaller oscillations compared to the standard feedback projected gradient algorithm.

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This work was partially supported by the NWO under research project Online Optimization for Offshore Wind Farms.

Notations. We denote $\mathbb{R}^n$ as the n-dimensional real Euclidean space. For a column vector $x \in \mathbb{R}^n$ (matrix $A \in \mathbb{R}^{m\times n}$, $x^\top (A^\top)$ denotes its transpose. For a symmetric matrix A, $A \succ (\succcurlyeq) 0$ denotes that A is a positive (semi)definite matrix. Denote $\mathrm{proj}_{\mathcal{U}}$ as the Euclidean projection operator on a set $\mathcal{U}$, i.e., $\mathrm{proj}_{\mathcal{U}}(v) \triangleq \arg\min_{u\in\mathcal{U}}\{\|u-v\|\}$. For a multi-variable function $f(x)$, denote $\nabla f(x)$ as the gradient.

II. PROBLEM FORMULATION AND PROPOSED FEEDBACK OPTIMIZATION ALGORITHM

A. Problem Setup

Consider a discrete-time linear time-invariant dynamical system:

$$\begin{cases} x(k+1) &= Ax(k) + Bu(k) + Ew \\ y(k) &= Cx(k) + Du(k) + Fw \end{cases}, \tag{1}$$

where at each time step k, $x(k) \in \mathbb{R}^n$ represents the system state, $u(k) \in \mathbb{R}^p$ represents the control variable, $y(k) \in \mathbb{R}^m$ represents the measured output. We assume the system matrices (A, B, C, D) are perfectly known, whereas the exogenous disturbance $w \in \mathbb{R}^d$ is unknown and unmeasurable. For this system, we denote by $S_\mathrm{u} := C(I-A)^{-1}B + D$ and $S_\mathrm{w} := C(I-A)^{-1}E+F$ the transfer function matrices from u to y and from w to y, respectively. Then for a constant control input $\bar{u}$, in steady state we have

$$\bar{y} = S_\mathrm{u}\bar{u} + S_\mathrm{w}w. \tag{2}$$

In this paper, the goal is to optimize over the steady state to ensure that the system settles at a desirable operating point. In many engineering applications, such as the economic dispatch problem in power systems, the objective function can be formulated as a quadratic function. Specifically, we consider a constrained quadratic optimization problem over the steady state of system (1):

$$\begin{cases} \min\limits_{\bar{u},\bar{y}} \frac{1}{2}\bar{y}^\top Q\bar{y} + \frac{1}{2}\bar{u}^\top R\bar{u} + b^\top \bar{u} \\ \text{s.t.} \quad \bar{y} = S_\mathrm{u}\bar{u} + S_\mathrm{w}w \\ \qquad \bar{u} \in \mathcal{U} \end{cases}, \tag{3}$$

where $\mathcal{U}$ is a convex and compact set, and Q and R are positive definite matrices. The optimal solution of this optimization problem (u^*, y^*) is usually called the optimal steady state of the system in (1).

To solve the problem in (3), standard feedforward optimization methods substitute (2) into the cost function term $\frac{1}{2}\bar{y}^\top Q\bar{y}$ and transform it into a quadratic programming problem

$$\begin{cases} \min\limits_{\bar{u}} \frac{1}{2}\bar{u}^\top S_\mathrm{u}^\top QS_\mathrm{u}\bar{u} + \frac{1}{2}\bar{u}^\top R\bar{u} + (b^\top + w^\top S_\mathrm{w}^\top QS_\mathrm{u})\bar{u} \\ \text{s.t.} \quad \bar{u} \in \mathcal{U} \end{cases}, \tag{4}$$

which can be further solved via gradient-based optimization algorithms. Unfortunately, while the system matrices are known, these feedforward methods require knowing the value of disturbance w, which is hardly available in many practical scenarios. This motivates the development of feedback optimization (FO) algorithms, as described in the Introduction section.

B. Feedback Gradient Projection Algorithm

To tackle the challenge of imperfect knowledge of the system disturbance, feedback optimization leverages real-time measurements of the output to avoid the knowledge of the input-output mapping in (2), thereby eliminating explicit dependence on the disturbance w. While some first-order and zeroth-order FO algorithms have been developed in the literature, they often neglect the transient performance and may fall short of efficiency for the specific problem formulation in (3). To address this issue, we propose the following feedback gradient projection algorithm:

Algorithm 1 Feedback Gradient Projection Algorithm

Offline Phase:

1) Compute transfer function $S_\mathrm{u} = C(I-A)^{-1}B + D$
2) Design positive definite gain matrix G

Online Phase (at each time step k):

3) Output measurement:
$$y(k) = Cx(k) + Du(k) + Fw \tag{5}$$
4) System dynamics:
$$x(k+1) = Ax(k) + Bu(k) + Ew \tag{6}$$
5) Input update:
$$u(k+1) = \mathrm{proj}_{\mathcal{U}}^{G^{-1}}(u(k) - G(S_\mathrm{u}^\top Qy(k) + Ru(k) + b)) \tag{7}$$

In Algorithm 1, The generalized projection $\mathrm{proj}_{\mathcal{U}}^{G^{-1}}$ over $\mathcal{U}$ is defined as

$$\mathrm{proj}_{\mathcal{U}}^{G^{-1}}(x) = \arg\min_{u\in\mathcal{U}}(u-x)^\top G^{-1}(u-x).$$

This approach provides greater flexibility compared to the traditional feedback projected gradient algorithm because Algorithm 1 includes several common algorithms as special cases: If $G = \alpha I$, with $\alpha > 0$, it reduces to the traditional feedback projected gradient method; if $G = (R + S_\mathrm{u}^\top QS_\mathrm{u})^{-1}$, which is the inverse of the Hessian matrix of the objective function in (4), it reduces to the feedback projected Newton method.

Remark 1 If $\mathcal{U}$ is a box constraint and G is a diagonal matrix, the generalized projection is equivalent to the Euclidean projection onto $\mathcal{U}$. This equivalence means that each component of u is projected independently onto a scalar interval, making the projection computationally efficient while retaining the benefits of using a non-scalar gain matrix for improved transient performance.

III. CONVERGENCE ANALYSIS

This section presents our convergence result for the dynamics in Algorithm 1 using a linear matrix inequality (LMI) technique. For simplicity, we assume a box structure for $\mathcal{U}$ in (3) as in [23].

A. Main Results

Lemma 1 *Let f be a convex and continuously differentiable objective function, $\mathcal{U}$ be a box constraint, and G be a diagonal positive definite matrix. Then u^* is a solution of the optimization problem $\min_{u\in\mathcal{U}} \ f(u)$ if and only if*

$$u^* = \mathrm{proj}_{\mathcal{U}}\left(u^* - G\nabla f(u^*)\right). \tag{8}$$

See Appendix for the proof.

Lemma 2 *Let $(\bar{x}, \bar{u})$ be an equilibrium point of the dynamics in (5)-(7) and let (u^*, y^*) be the optimal steady state, i.e., a solution to (3) . Then we have $\bar{u} = u^*$ and $y^* = S_{\mathrm{u}}\bar{u} + S_{\mathrm{w}}w$.*

Proof: According to the definition, $(\bar{x}, \bar{u})$ satisfies

$$\bar{x} = A\bar{x} + B\bar{u} + Ew \tag{9a}$$
$$\bar{u} = \mathrm{proj}_{\mathcal{U}}\left(\bar{u} - G\left(S_{\mathrm{u}}^\top Q\bar{y} + b + R\bar{u}\right)\right) \tag{9b}$$
$$\bar{y} = C\bar{x} + D\bar{u} + Fw \tag{9c}$$

Therefore, substituting the input-output steady state mapping $\bar{y} = S_{\mathrm{u}}\bar{u} + S_{\mathrm{w}}w$ into (9b) derives that $\bar{u}$ fulfils the fixed-point relation

$$\bar{u} = \mathrm{proj}_{\mathcal{U}}\left(\bar{u} - G\left(S_{\mathrm{u}}^\top Q(S_{\mathrm{u}}\bar{u} + S_{\mathrm{w}}w) + b + R\bar{u}\right)\right).$$

Furthermore, applying Lemma 1 to problem (4) with $\nabla f(u^*) = S_{\mathrm{u}}^\top Q(S_{\mathrm{u}}u^* + S_{\mathrm{w}}w) + b + Ru^*$ implies that the optimal solution satisfies

$$u^* = \mathrm{proj}_{\mathcal{U}}\left(u^* - G\left(S_{\mathrm{u}}^\top Q(S_{\mathrm{u}}u^* + S_{\mathrm{w}}w) + b + Ru^*\right)\right).$$

As a result, we can get $u^* = \bar{u}$ by the uniqueness of the solution. The conclusion follows from (9a) and (9c). ■

In order to give the main convergence result, we first define the following system-related matrix:

$$R_1 = R + S_{\mathrm{u}}^\top QD. \tag{10}$$

Theorem 1 *Assume that $\mathcal{U}$ in (3) has a box structure, i.e., $\mathcal{U} = \prod_{i=1}^{p}[u_{\min,i}, u_{\max,i}]$. Furthermore, suppose that there exists $\gamma \in (0, 1)$ such that the linear matrix inequality*

$$\begin{bmatrix} I & \mathcal{A} \\ \mathcal{A}^\top & \gamma I \end{bmatrix} \succcurlyeq 0, \ \text{with } \mathcal{A} = \begin{bmatrix} A & B \\ -GS_u^\top QC & I - GR_1 \end{bmatrix} \tag{11}$$

has a solution for positive definite diagonal matrix G. Then the control sequence generated by algorithm (1) with gain matrix G linearly converges to the optimal solution u^*.

Proof: By substituting the expression for $y(k)$ from (5) into the update formula (7) for $u(k)$, we derive

$$\begin{aligned} &u(k+1) \\ &= \mathrm{proj}_{\mathcal{U}}\left(u(k) - G\left(S_{\mathrm{u}}^\top Qy(k) + b + Ru(k)\right)\right) \\ &= \mathrm{proj}_{\mathcal{U}}\Big(u(k) - G\Big(S_{\mathrm{u}}^\top Q(Cx(k) + Du(k) + Fw) \\ &\qquad + Ru(k)\Big) - Gb\Big) \\ &= \mathrm{proj}_{\mathcal{U}}\big((I - GR_1)u(k) - GS_{\mathrm{u}}^\top QCx(k) \\ &\qquad - G(S_{\mathrm{u}}^\top QFw + b)\big), \end{aligned}$$

where R_1 is defined by (10).

At the same time, the state update equation (6) can be written as

$$x(k+1) = \mathrm{proj}_{\mathbb{R}^n}\left(Ax(k) + Bu(k) + Ew\right).$$

Combining these two equations, we can equivalently express (6) and (7) in the following compact form

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} &= \mathrm{proj}_{\mathbb{R}^n\times\mathcal{U}}\left(\begin{bmatrix} A & B \\ -GS_{\mathrm{u}}^\top QC & I - GR_1 \end{bmatrix}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix}\right. \\ &\qquad \left. + \begin{bmatrix} Ew \\ -G(S_{\mathrm{u}}^\top QFw + b) \end{bmatrix}\right) \\ &= \mathrm{proj}_{\mathbb{R}^n\times\mathcal{U}}\left(\mathcal{A}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix}\right. \\ &\qquad \left. + \begin{bmatrix} Ew \\ -G(S_{\mathrm{u}}^\top QFw + b) \end{bmatrix}\right), \end{aligned}$$

with $\mathcal{A}$ defined in (11).

By applying the Schur complement, (11) is equivalent to $\mathcal{A}^\top\mathcal{A} \preccurlyeq \gamma I$. Since $\mathrm{proj}_{\mathcal{U}}$ is firmly nonexpansive in the Euclidean norm, the composite operator $\mathrm{proj}_{\mathbb{R}^n\times\mathcal{U}} \circ \mathcal{A}$ is contractive. Therefore, by the Banach Fixed-Point Theorem, which guarantees convergence of iterates to a unique fixed point under a contractive mapping, and by invoking Lemma 2, we conclude that the algorithm converges linearly to the optimal steady state. ■

Remark 2 For a general convex and compact constraint set $\mathcal{U}$, we can similarly derive a sufficient condition for the linear convergence by using the fact that the generalized projection operator $\mathrm{proj}_{\mathbb{R}^n\times\mathcal{U}}^{G^{-1}}$ is firmly nonexpansive in G^{-1}-norm. Specifically, we can derive the following LMI for G^{-1}:

$$\begin{bmatrix} \mathbf{G}^{-1} & \mathcal{A}_1 \\ \mathcal{A}_1^\top & \gamma\mathbf{G}^{-1} + (\mathcal{A}_1 + \mathcal{A}_1^\top) - I \end{bmatrix} \succcurlyeq 0,$$

where $\mathbf{G} := \begin{bmatrix} I & 0 \\ 0 & G \end{bmatrix}$ and $\mathcal{A}_1 := \begin{bmatrix} I - A & B \\ S_{\mathrm{u}}^\top QC & R_1 \end{bmatrix}$ is a system-related matrix independent of G. Furthermore, we can derive a more easily verifiable condition to ensure that this LMI has a solution for G, that is, $\mathcal{A}_1$ satisfies $\lambda_{\min}(\mathcal{A}_1 + \mathcal{A}_1^\top) > \lambda_{\max}(\mathcal{A}_1^\top\mathcal{A}_1)$ or $\lambda_{\min}(\mathcal{A}_1 + \mathcal{A}_1^\top) + \lambda_{\max}(\mathcal{A}_1^\top\mathcal{A}_1) < 2$.

B. Choice of the gain matrix

From Theorem 1, to guarantee the linear convergence of Algorithm 1, we need to solve an LMI to find a feasible gain matrix, where an appropriate objective function with this LMI constraint may yield a gain matrix that enhances the transient performance of the algorithm. Specifically, consider the following semi-definite programming (SDP) problem:

$$\begin{cases} \min\limits_{g,\gamma} & \mu_1 \mathrm{trace}(\mathrm{diag}\{g\}) + (1-\mu_1)\gamma \\ \text{s.t.} & \begin{bmatrix} I & \mathcal{A} \\ \mathcal{A}^\top & \gamma I \end{bmatrix} \succcurlyeq 0, \\ & 0 < \gamma < 1, \\ & g \in \mathbb{R}^p \geq 0 \end{cases}, \tag{12}$$

where $\mathrm{diag}\{g\}$ denotes a diagonal matrix with diagonal elements taken from the elements of the vector g. The relationship between gain matrix G and oscillatory behavior can be understood through the system's response to gradient-based updates of the control input in (7). Intuitively speaking, in Algorithm 1, smaller values in G result in more conservative input adjustments at each iteration. These gradual gradient-based corrections allow the system state to approach its optimal steady state without significant overshooting, thereby reducing oscillations in the trajectory. At the same time, this improvement in oscillation behavior typically leads to slower convergence speed. Therefore, we introduce γ to control the convergence speed in (12). Consequently, by tuning the weight parameter μ_1, the proposed objective function is able to balance the trade-off between oscillations and convergence speed.

To illustrate the numerical performance of this method, we consider a simple test example with randomly selected system matrices. Fig. 1 shows that the proposed algorithm can reduce the transient oscillations in the trajectory without lowering the convergence rate compared to the traditional feedback projected gradient algorithm.

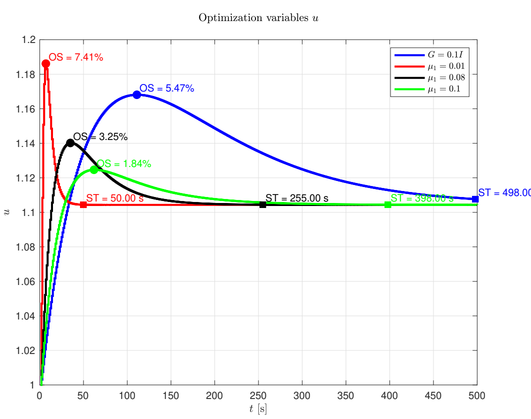

Fig. 1. Numerical example illustrating the trade-off between oscillation and convergence rate. We conducted the simulations on a system with matrices (A, B, C, D, E, F) having random entries with $n = m = 1$, $p = 2$. The weight matrices in the objective function of (3) is set as $Q = I$ and $R = 0.1I$ (OS := overshooting percentage := $\frac{\max(u) - u^*}{u^*}$, ST := settling time, which is defined as the time at which the trajectory first enters and remains within a 0.01% margin of the steady state value following the peak value).

This numerical example demonstrates the potential to enhance the overall performance of feedback-based optimization algorithms by selecting an appropriate gain matrix. This approach is particularly useful in applications requiring both fast convergence and minimal oscillations, such as power systems and wind farm control.

IV. APPLICATION TO POWER SYSTEMS AND NUMERICAL SIMULATIONS

A. Economic Dispatch Problem

In this section, we consider the economic dispatch problem in power systems, modeled as the feedback optimization framework (3), to illustrate the efficiency of the proposed FO algorithm. Consider a power system with r buses and s lines. Following [10], we use a linearized swing equation represented as

$$\begin{cases} \dot{\xi} = \bar{A}\xi + \bar{B}p^c + \bar{E}w \\ p_l = \bar{C}\xi \end{cases}, \tag{13}$$

with the system matrices defined as

$$\bar{A} = \begin{bmatrix} 0 & I \\ -M^{-1}Y & -M^{-1}D \end{bmatrix},\ \bar{B}_u = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}$$

$$\bar{B}_w = \begin{bmatrix} 0 \\ B_2 \end{bmatrix},\ \bar{C} = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix}.$$

In this LTI system, the state $\xi := \begin{bmatrix} \theta \\ \omega \end{bmatrix}$ includes the generator's phase angle θ_i and frequencies ω_i. The input p^c denotes the power generation setpoints, w represents the uncontrollable power injections, and the output p^l is the line power. Here, Y is the grid's admittance matrix, $B_1 \in \{0, 1\}^{n \times m}$ and $B_2 \in \{0, 1\}^{n \times m}$ select the controllable and uncontrollable power injections, respectively. $\mathbf{C}$ is the matrix that maps voltage angles θ to line powers p_l. $M = \mathrm{diag}\{m_1, \ldots, m_n\}$ is the rotational inertial matrix, and $D = \mathrm{diag}\{d_1, \ldots, d_n\}$ is the damping matrix incorporating friction coefficients and primary control gains of the generators.

Note that $\bar{A}$ has a zero eigenvalue, making the system in (13) marginally stable. To address this, an appropriate coordinate transformation $x(t) = T^\top \xi(t)$ is introduced to get the reduced system

$$\begin{cases} \dot{x} = Ax + Bu + Ew \\ y = Cx \end{cases}, \tag{14}$$

with $u := p^c$, $A := T^\top \bar{A} T$, $B := T^\top \bar{B}$, $E := T^\top \bar{E}$, and $C := \bar{C}T$.

In order to further represent (14) as a discrete-time dynamical system, we adopt the Tustin discretization method. The corresponding discrete-time state-space model is

$$\begin{cases} x(k+1) & = A_{\mathrm{d}}x(k) + B_{\mathrm{d}}u(k) + E_{\mathrm{d}}w \\ y(k) & = C_{\mathrm{d}}x(k) \end{cases}. \tag{15}$$

We further consider optimizing an economic cost while simultaneously controlling frequency and line congestion. Namely, we consider the following economic dispatch problem:

$$\begin{cases} \min\limits_{u,y} \ u^\top H u + c^\top u \\ \text{s.t.} \ \ y = h(u, w) \\ \qquad u_{\min} \le u \le u_{\max} \\ \qquad p_{\min} \le y \le p_{\max} \end{cases}, \tag{16}$$

where $y = h(u, w)$ denotes the steady-state input-output mapping of the dynamic model (15). To formulate (16) in the form of (3), we use soft constraints to address the constraints on the output by considering the augmented objective function

$$\Phi(u, y) = u^\top H u + c^\top u + \sum_i \lambda_i \left((p_{\min} - y_i)^2 + (y_i - p_{\max})^2 \right),$$

where λ_i represents the penalty parameters.

B. Numerical Simulations

We use the IEEE 9-bus power system test case to simulate the proposed feedback optimization algorithm. The test case includes three controllable generators whose power setpoints are viewed as the input and three uncontrollable loads as output. The parameter in the dynamics are chosen as randomized values for M and D with mean 10 and 0.1 p.u., respectively. Furthermore, we consider a nominal disturbance of $w = (0.9, 1, 1.25)$ p.u. and impose a line flow limit of 2.5 p.u. on all lines.

1) Optimal Control Variables and Line Powers: In the economic dispatch problem (16), we set the parameters to $H = 0.1I$ and $c = [-0.08, -0.12, -0.1]^\top$. The penalty parameters are set to $\lambda_i = 1$ for all i. Additionally, the power setpoints are bounded by $u_{\min} = 0$ p.u. and $u_{\max} = 3$ p.u..

With accurate information of the disturbance w, the optimal operating points can be calculated by applying a traditional feedforward algorithm to (4). Under the given parameters, the optimal setpoints are denoted as u^*. Fig. 2 and Fig. 3 show the trajectories of the control variables and the line powers. These figures illustrate that, without access to the disturbance information, our feedback-based optimization algorithm can still converge to the optimal steady state by utilizing the real-time measurements of output.

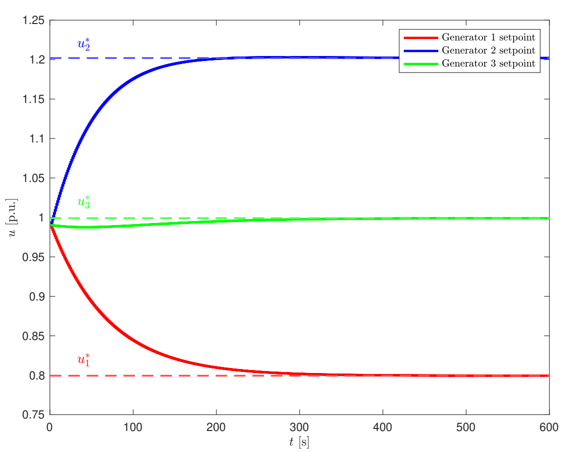

Fig. 2. Generator's power setpoints u_i [p.u.]

2) Influence of Weight Parameter μ_1: Our primary interest is to reduce the oscillation of feedback-based optimization algorithms by choosing an appropriate gain matrix. In this section, we compare the performance of Algorithm 1 to the standard feedback-based projected gradient method with a scalar step size $\alpha \equiv 0.5$ and study the influence of weight parameter μ_1 in the SDP problem (12).

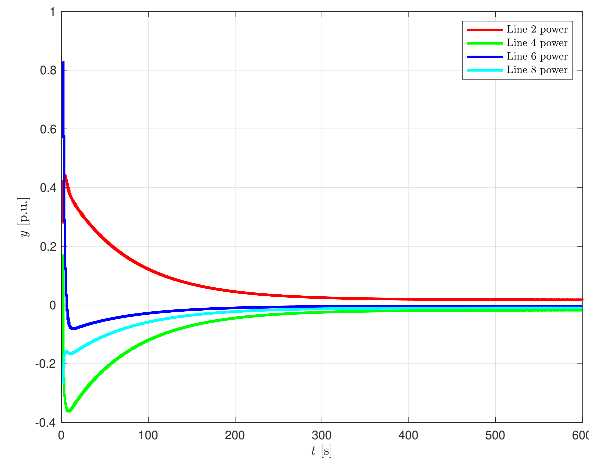

Fig. 3. Line power flow for selected lines [p.u.]

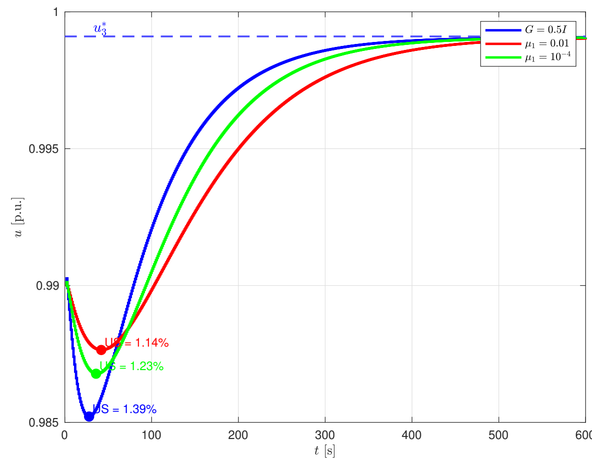

Fig. 4. Comparison of Generator 3's control variables (US := undershooting percentage, which is defined as $\frac{u^* - \min(u)}{u^*}$)

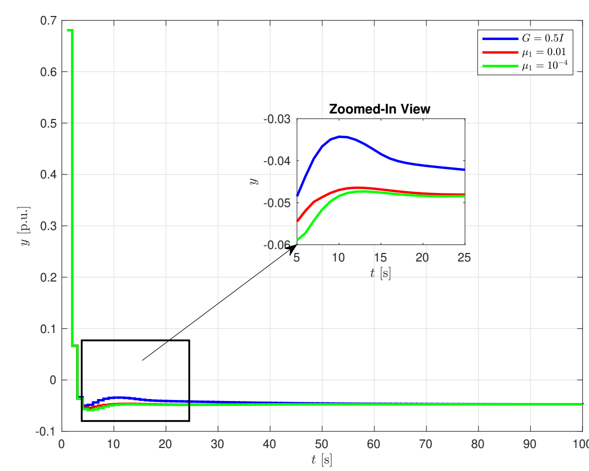

Fig. 5. Comparison of power flow on Line 2 between scalar step size and designed gain matrix

Fig. 4 shows the system behavior of generator 3's control variable, and Fig. 5 shows the line powers. These results illustrate that the proposed Algorithm 1, combined with an appropriate μ_1 in (12), can reduce the oscillations of certain control variables. Moreover, by choosing μ_1, we can balance the trade-off between oscillation and convergence rate of the algorithm.

V. CONCLUSION

This paper demonstrates the potential of feedback optimization framework in enhancing transient performance of optimizing the steady-state for linear dynamic system. We design a Semidefinite Programming method (SDP) to choose a gain matrix to choose a gain matrix, replacing the traditional scalar step size. This approach provides greater flexibility, allowing for smoother trajectories and faster convergence to the optimal steady state. We theoretically prove the convergence of the algorithm but also numerically show improvements in reducing oscillations. Future work may extend this approach to more complex nonlinear systems and explore its potential in other engineering applications.

APPENDIX

Proof of Lemma 1. According to the optimality condition, u^* is an optimal solution if and only if

$$(u-u^*)^\top \nabla f(u^*) \geq 0, \quad \forall u \in \mathcal{U}. \tag{17}$$

For all $v \in \mathbb{R}^p, u \in \mathcal{U}$, by the definition of $\mathrm{proj}_{\mathcal{U}}^{G^{-1}}$ and applying (17) with

$$f(u) := \frac{1}{2}(u-v)^\top G^{-1}(u-v),$$

we get the following well-known property for the generalized projection

$$\left(\mathrm{proj}_{\mathcal{U}}^{G^{-1}}(v) - v\right)^\top G^{-1}\left(\mathrm{proj}_{\mathcal{U}}^{G^{-1}}(v) - u\right) \leq 0. \tag{18}$$

Combining (18) and Lemma 2.2 of [24], we get that (17) is equivalent to

$$u^* = \mathrm{proj}_{\mathcal{U}}^{G^{-1}}\left(u^* - G\nabla f(u^*)\right).$$

From the definition of projection, it is easy to see that under the special structures of $\mathcal{U}$ and G,

$$\mathrm{proj}_{\mathcal{U}}^{G^{-1}}(v) = \mathrm{proj}_{\mathcal{U}}(v), \ \forall v \in \mathbb{R}^p,$$

which yields (8).

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