CHALMERS UNIVERSITY OF TECHNOLOGY
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THE INFLUENCE OF WATER DEPTH ON THE MIDSHIP BENDING MOMENTS OF A SHIP MOVING IN LONGITUDINAL REGULAR HEAD WAVES

by

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CONTENTS

ABSRACT

The heaving and pitching motions and the midship bending moments of a T-2 Tanker model moving in longitudinal regular head waves of shallow water are calculated by Watanabe's strip theory[1],[2],[3],[4],[5]. The results are represented in in Figures and the depth effect is discussed.

NOMENCLATURE

a,b,c,d,e,g	coefficients of heave equation
A,B,C,D,E,G	coefficients of pitch equation
A_W	waterplane area
B(x)	beam of a section
C_m	midship bending moment coefficient
F_a	exciting force amplitude
g	gravity constant
G_o	center of gravity (C.G.)
h	water depth
$\bar{h}$	wave amplitude
H	half-beam draft ratio
i	longitudinal rad. of gyration in % of L
I_W	moment of waterplane area
I_{yy}	moment of inertia of the ship about y-axis
L	length between perpendiculars
m_o	midship bending moment at time t
m_a	amplitude of midship bending moment
M_a	exciting moment amplitude
N	sectional heave damping coefficient
t	time
T	draft
$\bar{T}$	mean draft of a section
V	ship velocity
∇	displacement volume
W	suffix designating wave
x,y,z	body coordinates
X,Y,Z	space coordinates

β	fullness coefficient of a section
$\varepsilon_{\zeta W}$	phase difference between heave and wave.
$\varepsilon_{\psi W}$	" " " pitch and wave.
$\varepsilon_{\psi\zeta}$	" " " pitch and heave.
ε_{FW}	" " " exciting force and wave.
ε_{MW}	" " " exciting moment and wave.
ε_{mW}	" " " midship bending moment and wave.
ζ	heave at time t
ζ_a	heave amplitude
ζ_W	wave elevation at time t
λ	wave length
ν	wave number (ω^2/g)
ν_o	shallow water wave number
ρ	water density
ψ	pitch at time t
ψ_a	pitch amplitude
ω	circular frequency
ω_e	circular frequency of encounter

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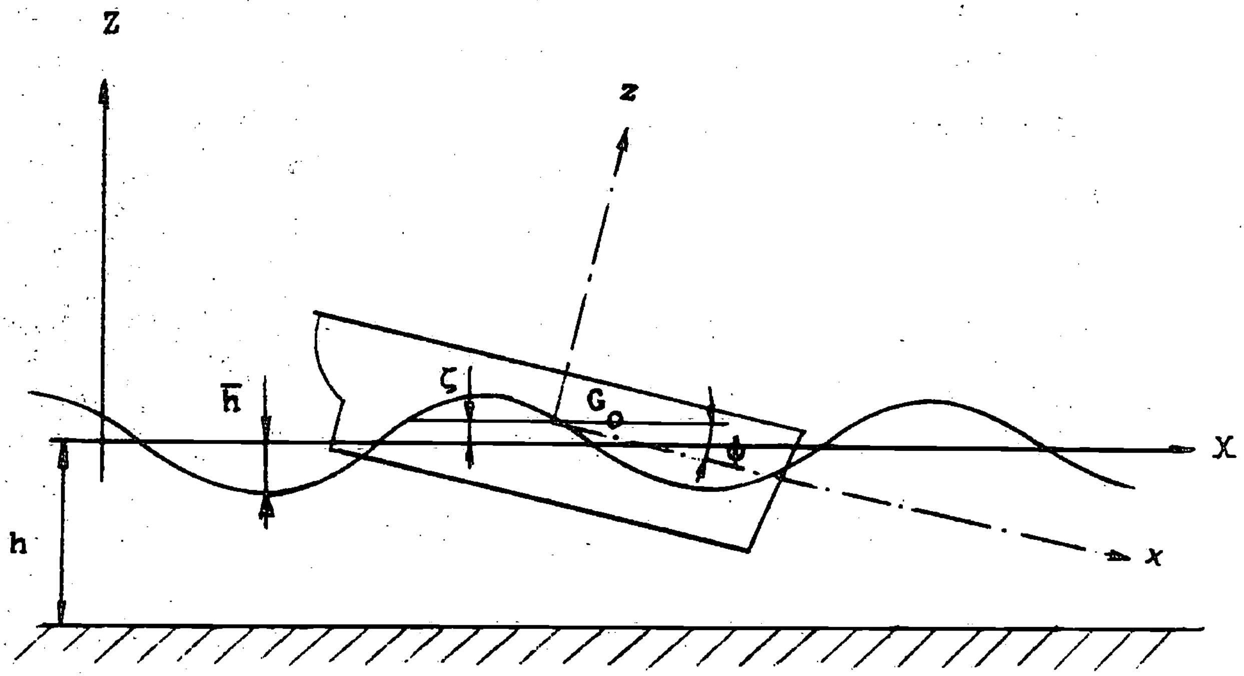
INTRODUCTION

By applying Watanabe's strip method [1] ,[2] , [3] , [4] , [5] , the heaving and pitching motions as well as the midship bending moments of a T-2 Tanker model moving in regular head waves of shallow water are calculated and the effects of water depth are discussed.

It is revealed by the calculation that the midship bending moments are increased, while the motions heave and pitch are remarkably damped by the depth effect.

DEFINITION OF SHIP MOTIONS AND WAVES

The coordinate systems here utilized are space- and body-coordinate system O-XYZ and G_o-xyz respectively. X-axis lies on the undisturbed water surface and Z-axis points vertically upward. x-axis is longitudinal passing through the center of gravity G_o of the ship, while y- and z-axis point port and upward, respectively. The coordinate system G_o-xyz coincides with the system O-XYZ at the initial rest condition. We follow the convention of right-handed coordinate system.



Assuming only heaving and pitching motions of a ship at the speed V in a longitudinally oncoming wave system, we describe the surface wave as follows

$$\zeta_W = \bar{h} \cos (\nu_o x + \omega_e t) \qquad \text{.........(1)}$$

, where $\bar{h}$ wave amplitude

ν_o shallow water wave number, i.e. $(\frac{\omega^2}{g} = \nu_o \tanh \nu_o h)$

ω_e circular frequency of encounter i.e. $(\omega + \nu_o V)$

The heaving and pitching motions of the ship corresponding to the wave defined above are then expressed by

$$\left.\begin{aligned} \zeta &= \zeta_a \cos\,(\omega_e t + \varepsilon_{\zeta w}) \\ \psi &= \psi_a \cos\,(\omega_e t + \varepsilon_{\psi w}) \end{aligned}\right\} \quad \ldots\ldots\ldots(2)$$

respectively, where ζ_a, ψ_a are heave and pitch amplitudes and $\varepsilon_{\zeta w}$, $\varepsilon_{\psi w}$ phase differences between heave and wave and pitch and wave, respectively.

THE COUPLED EQUATIONS AND COEFFICIENTS

The coupled equations of heave and pitch of a ship moving in longitudinal regular waves [1] , [2] are written in the form

$$\left.\begin{aligned} a\ddot{\zeta} + b\dot{\zeta} + c\zeta - d\ddot{\psi} - e\dot{\psi} - g\psi &= F_a \cos\,(\omega_e t + \varepsilon_{FW}) \\ A\ddot{\psi} + B\dot{\psi} + C\psi - D\ddot{\zeta} - E\dot{\zeta} - G\zeta &= M_a \cos\,(\omega_e t + \varepsilon_{FW}) \end{aligned}\right\} \quad \ldots.(3)$$

The coefficients on the left-hand sides of the above equations are

$$a = \rho\nabla + \int_L m'' \, dx$$

$$b = \int_L N \, dx$$

$$c = 2\rho g \int_L y_w dx$$

$$d = \int_L m'' \, x \, dx +$$

$$e = \int_L N x \, dx - V\int_L m'' \, dx$$

$$g = 2\rho g \int_L y_w x \, dx - V\int_L N \, dx$$

$$A = I_{yy} + \int_L m'' x^2 \, dx$$

$$B = \int_L N x^2 \, dx$$

$$C = 2\rho g \int_L y_w x^2 \, dx - VE$$

$$D = \int_L m'' x \, dx +$$

$$E = \int_L N x \, dx + V\int_L m'' \, dx$$

$$G = 2\rho g \int_L y_w x \, dx$$

, where ρ water density

g gravity constant

∇ displacement volume

y_w half-breadth of a section on the calm water-line

I_{yy} longitudinal moment of inertia of the ship's mass about G_c-y-axis

m'' sectional added mass of unit thickness for heave

N sectional heave damping coefficient of unit thickness

The exciting forces and moments on the right-hand sides of the equations (3) are represented in the form

$$F_a \begin{Bmatrix} \cos \varepsilon_{FW} \\ \sin \varepsilon_{FW} \end{Bmatrix} = 2\rho g \bar{h} \int_L y_w \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \cos \nu_o x \\ \sin \nu_o x \end{Bmatrix} dx$$

$$- \omega \bar{h} \, (\omega + \nu_o V) \int_L m'' \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \cos \nu_o x \\ \sin \nu_o x \end{Bmatrix} dx$$

$$\mp \omega \bar{h} \int_L N \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \sin \nu_o x \\ \cos \nu_o x \end{Bmatrix} dx$$

$$M_a \begin{Bmatrix} \cos \varepsilon_{MW} \\ \sin \varepsilon_{MW} \end{Bmatrix} = \bar{h} \int_L (\omega^2 m'' - 2\rho g y_w) \, x \, \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \cos \nu_o x \\ \sin \nu_o x \end{Bmatrix} dx$$

$$\pm \omega \bar{h} \int_L \left(N - V \frac{dm''}{dx} \right) x \, \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \sin \nu_o x \\ \cos \nu_o x \end{Bmatrix} dx$$

,where h — water depth

$\overline{T}$ — mean draft of a section

$\varepsilon_{FW}, \varepsilon_{MW}$ — phase differences between exciting force and wave and exciting moment and wave, respectively

Sectional values of added mass and damping coefficient m" and N for heave are obtained from [6] . In the case of deep water these values are obtained from [12]. If $h \to \infty$ then ν_o and $\frac{\cosh \nu_o (h-\overline{T})}{\cosh \nu_o h}$ are replaced by ν and $e^{-\nu T}$, respectively.

THE MIDSHIP BENDING MOMENTS

The preceeding discussions are on the calculations of the heaving and pitching motions of a ship moving in regular head waves of shallow water. By making use of the motions calculated above we obtain the midship bending moments in the following form:

$$m_o = m_a \cos (\omega_e t + \varepsilon_{mW}) \qquad \ldots\ldots\ldots\ldots (4)$$

, where m_a and ε_{mW} are the midship bending moment amplitude and the phase difference between wave and bending moment respectively. Assuming that C.G. lies at the midship section the sine and cosine components of the amplitude are written as follows:

$$\left\{\begin{matrix} m_a \cos \varepsilon_{mW} \\ m_a \sin \varepsilon_{mW} \end{matrix}\right\} = \left\{\begin{matrix} \zeta_a \cos \varepsilon_{\zeta W} \\ \zeta_a \sin \varepsilon_{\zeta W} \end{matrix}\right\} [-2\rho g \int y_W \, x dx + \omega_e^2 (\int m'' x dx + \int \frac{W}{g} x dx)]$$

$$+ \left\{\begin{matrix} \zeta_a \sin \varepsilon_{\zeta W} \\ \zeta_a \cos \varepsilon_{\zeta W} \end{matrix}\right\} [\pm \omega_e (\int N \, x dx + V \int m'' \, dx)]$$

$$+\begin{Bmatrix}\psi_a \cos\varepsilon_{\psi W}\\ \psi_a \sin\varepsilon_{\psi W}\end{Bmatrix}\left[\,2\rho g\int y_W\, x^2 dx - V\int N\, x dx - V^2\int m''\, dx\; - \right.$$

$$\left. -\omega_e^2\left(\int m''\, x^2 dx + \int \frac{w}{g}\, x^2 dx\right)\right]$$

$$+\begin{Bmatrix}\psi_a \sin\varepsilon_{\psi W}\\ \psi_a \cos\varepsilon_{\psi W}\end{Bmatrix}\left[\,\mp\int N\, x^2 dx\,\right]$$

$$+\bar{h}\int\left(-\omega^2 m'' + 2\rho g y_W\right)\times\frac{\cosh\nu_0(h-\bar{T})}{\cosh\nu_0 h}\begin{Bmatrix}\cos\nu_0 x\\ \sin\nu_0 x\end{Bmatrix}dx$$

$$\mp\,\omega\bar{h}\int\left(N - V\frac{dm''}{dx}\right)\times\frac{\cosh\nu_0(h-\bar{T})}{\cosh\nu_0 h}\begin{Bmatrix}\sin\nu_0 x\\ \cos\nu_0 x\end{Bmatrix}dx$$

, where the integral is taken either between -L/2 and 0 or between +L/2 and 0 and $\frac{w}{g}$ designates mass per unit lenght along the ship lenght.

DIMENSIONLESS REPRESENTATION

In representing the calculated results, the following non-dimensional forms are used:

h / T	depth parameter
λ / L	wave length to ship length ratio
$V / \sqrt{gL}$	Froude Number
$\zeta_a / \bar{h}$	heave amplitude ratio
$\psi_a / \nu_0 \bar{h}$	pitch amplitude ratio
$C_m = m_a / \rho g \bar{h} B L^2$	midship bending moment coefficient

, where λ is a wave length.

CALCULATION AND DISCUSSION

For the numerical calculations we adopt a model of T-2 Tanker having the following particulars.

Length between perpendiculars	(L)	3.066 m
Beam	(B)	0.415 m
Draft	(T)	0.183 m
Displacement volume	(∇)	0.1725 m^3
Block coefficient	(C_B)	0.741
Radius of gyration	(% of L)	0.23

Station	B(x)	β(x)	H(x)	w/g
1	0.168	0.442	0.459	6.15
3	0.351	0.749	0.959	12.62
5	0.406	0.911	1.115	20.01
7	0.415	0.960	1.134	24.65
9	0.415	0.980	1.134	22.82
11	0.415	0.980	1.134	24.71
13	0.415	0.980	1.134	24.74
15	0.397	0.961	1.085	21.79
17	0.311	0.871	0.850	13.27
19	0.109	0.837	0.298	5.00

,where B(x) beam of a section

β(x) fullness coefficient of a section

H(x) half-beam draft ratio

w/g mass in kg per L/10 (see Fig. 1)

The calculations are carried out for the following speeds, waves and depths.

F_n= 0.0, 0.1, 0.2

λ/L= 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.3, 1.5, 1.7, 2.0.

h/T= ∞ , 10.0, 4.0, 2.5, and 1.5

In the calculation the following assumptions are made:

1. Although trim and parallel sinkage are produced they are not considered.
2. C.G. lies at midship section.

The Heave and Pitch Amplitudes together with the phase differences with respect to the waves are illustrated in Fig. 2-7. In general the motions are remarkably damped as the depth decreases. This tendency is more significant as the Froude Number increases.

The Midship Bending Moments together with the phase difference with respect to the waves are illustrated in Fig. 8-10. The bending moments are generally increased as the depth decreases. This tendency is quite opposite to the above mentioned motions. Probably it is partly caused by the decrease of inertial bending moments due to the damped motions. The double peaks are nearing each other as the depth decreases. This is probably caused by the delayed position of the peaks of heaving motions.

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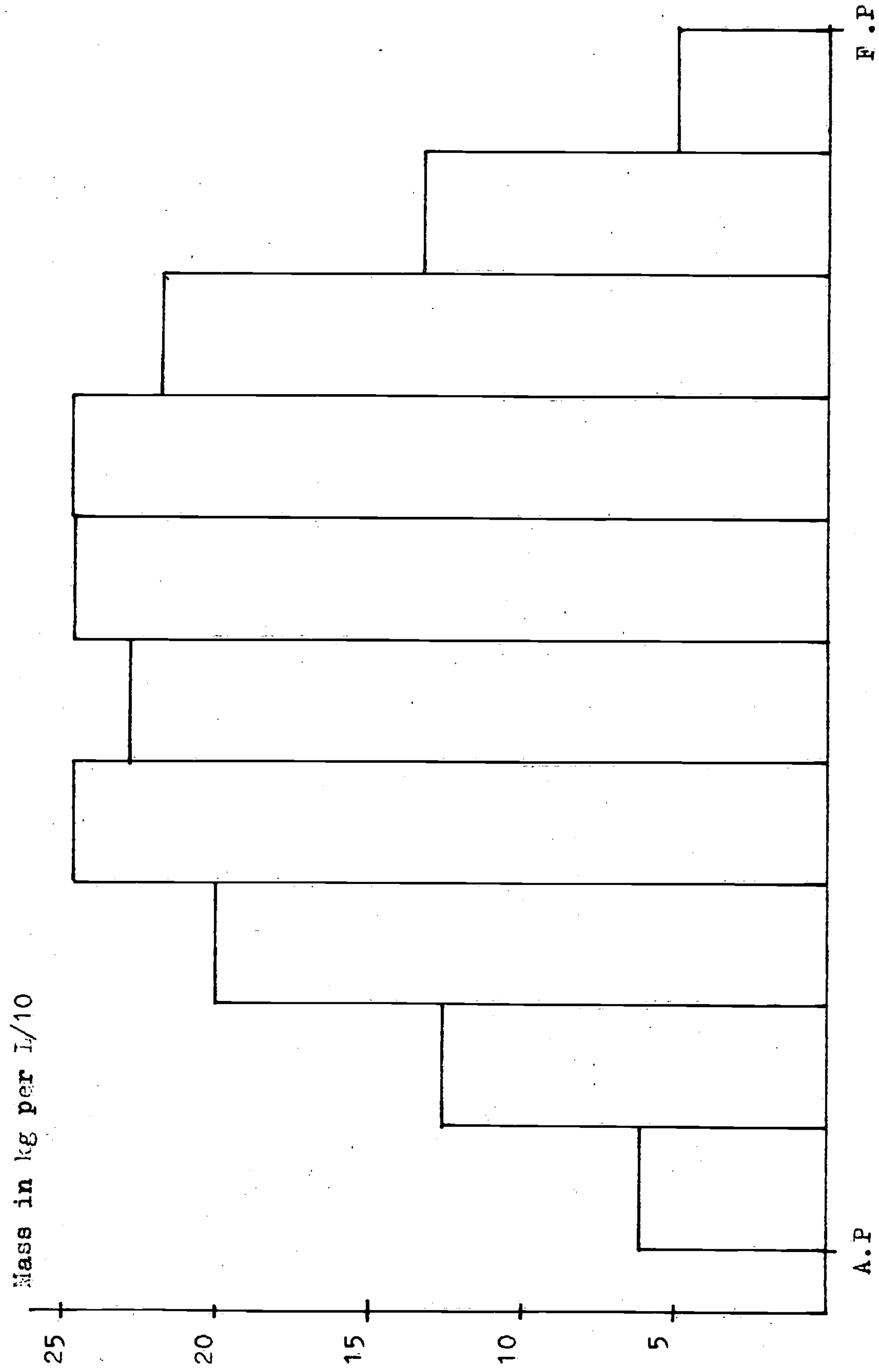
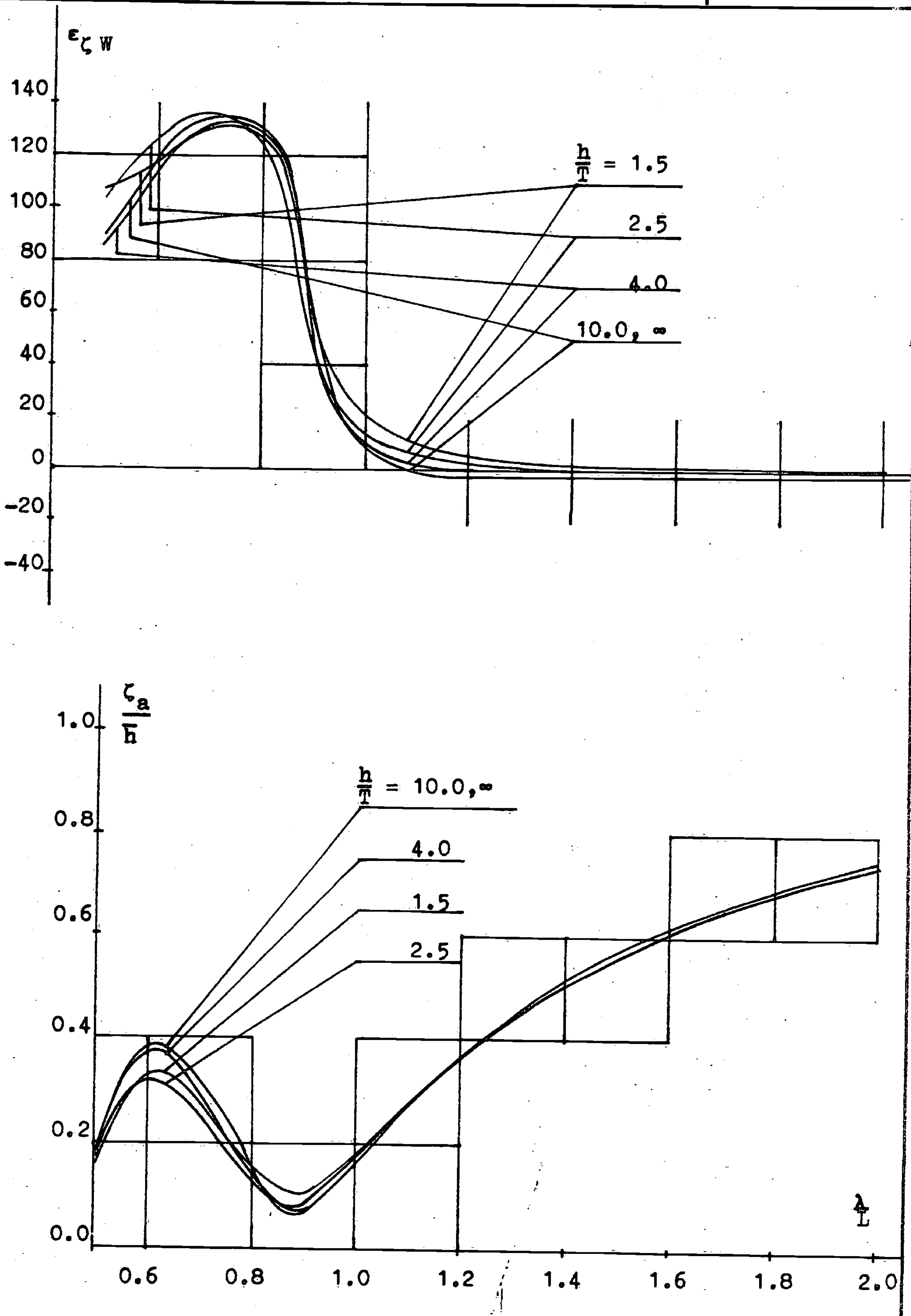
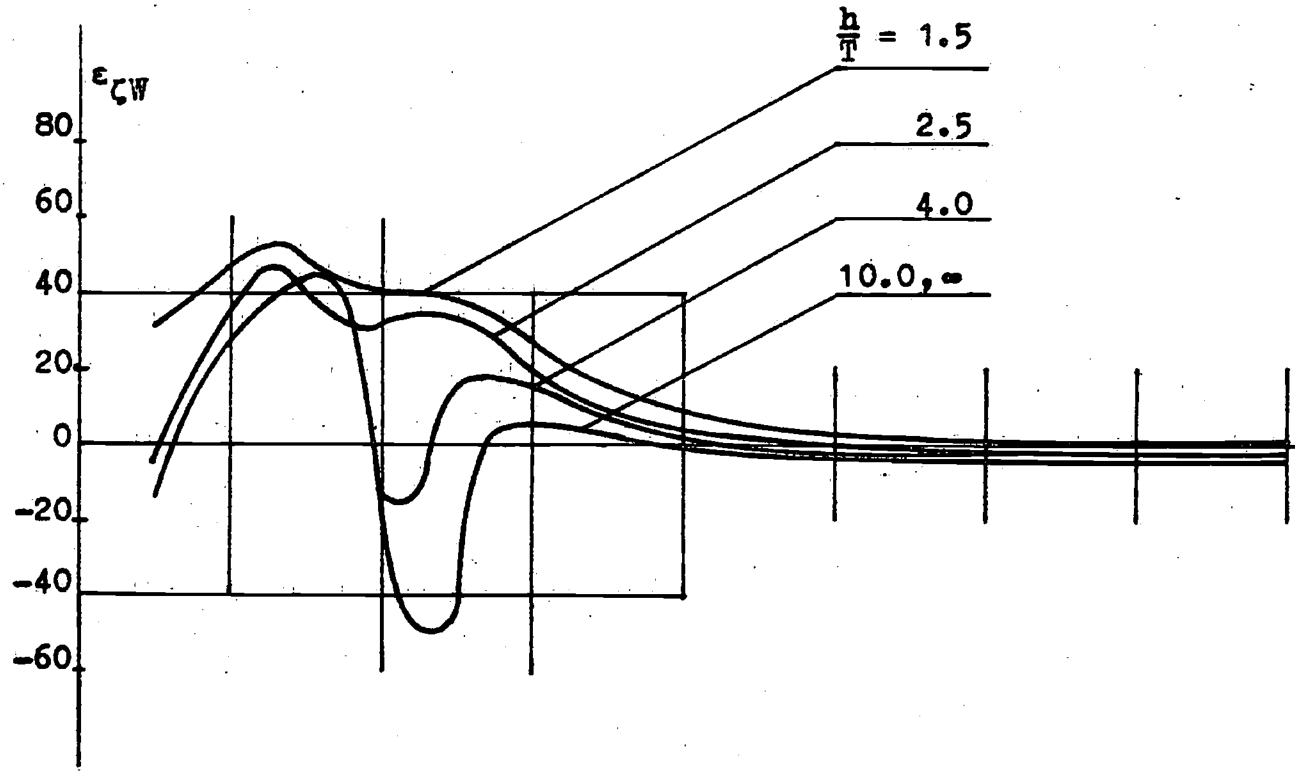
CHALMERS TEKNISKA HÖGSKOLA	**Mass Distribution of** T-2 Tanker Model	**Fig. 1** CTH - SH Report 45

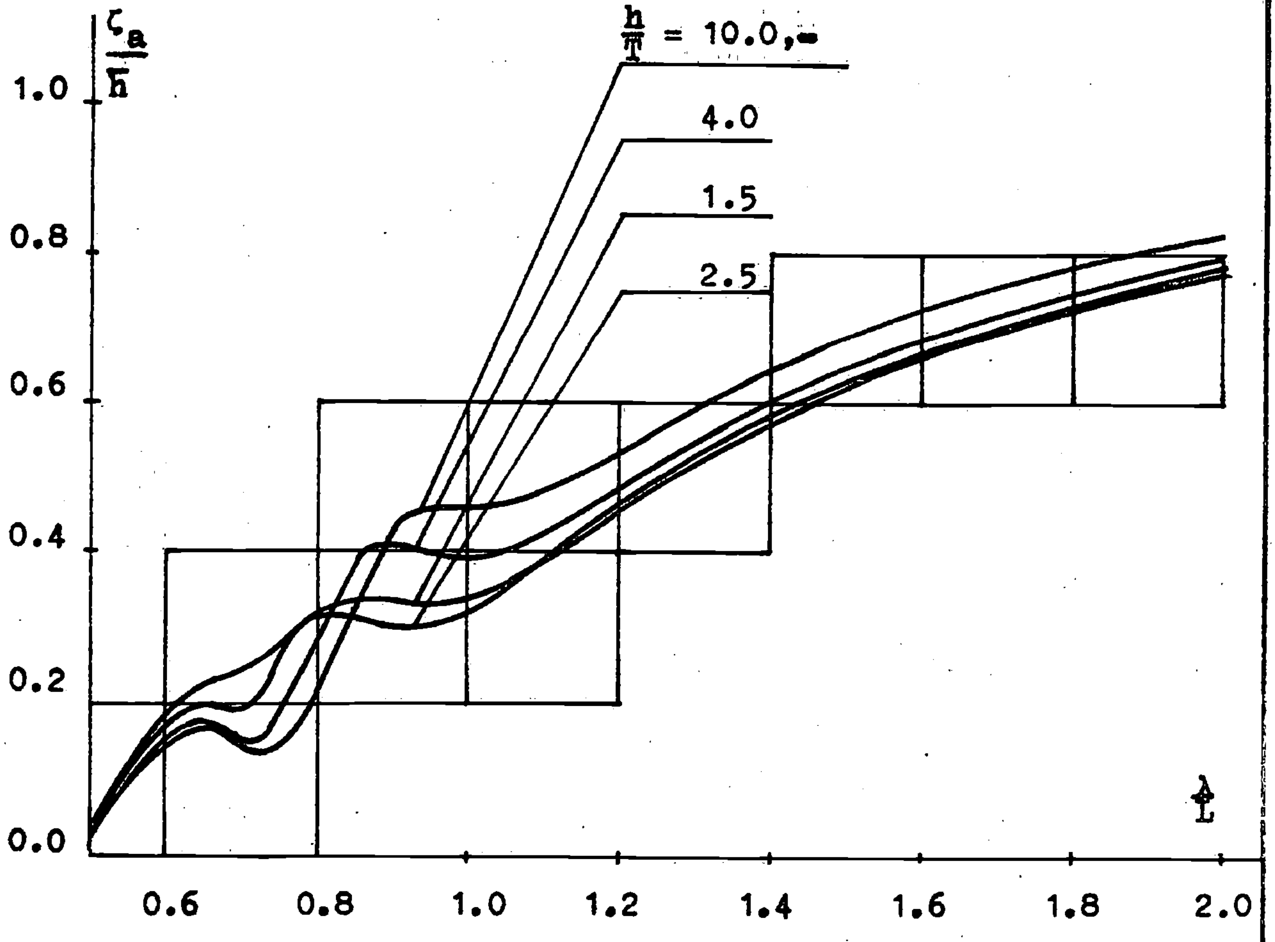


CHALMERS TEKNISKA HÖGSKOLA	Heave Amplitude Ratio at F_n = 0.0	Fig. 2 CTH - SH Report 45

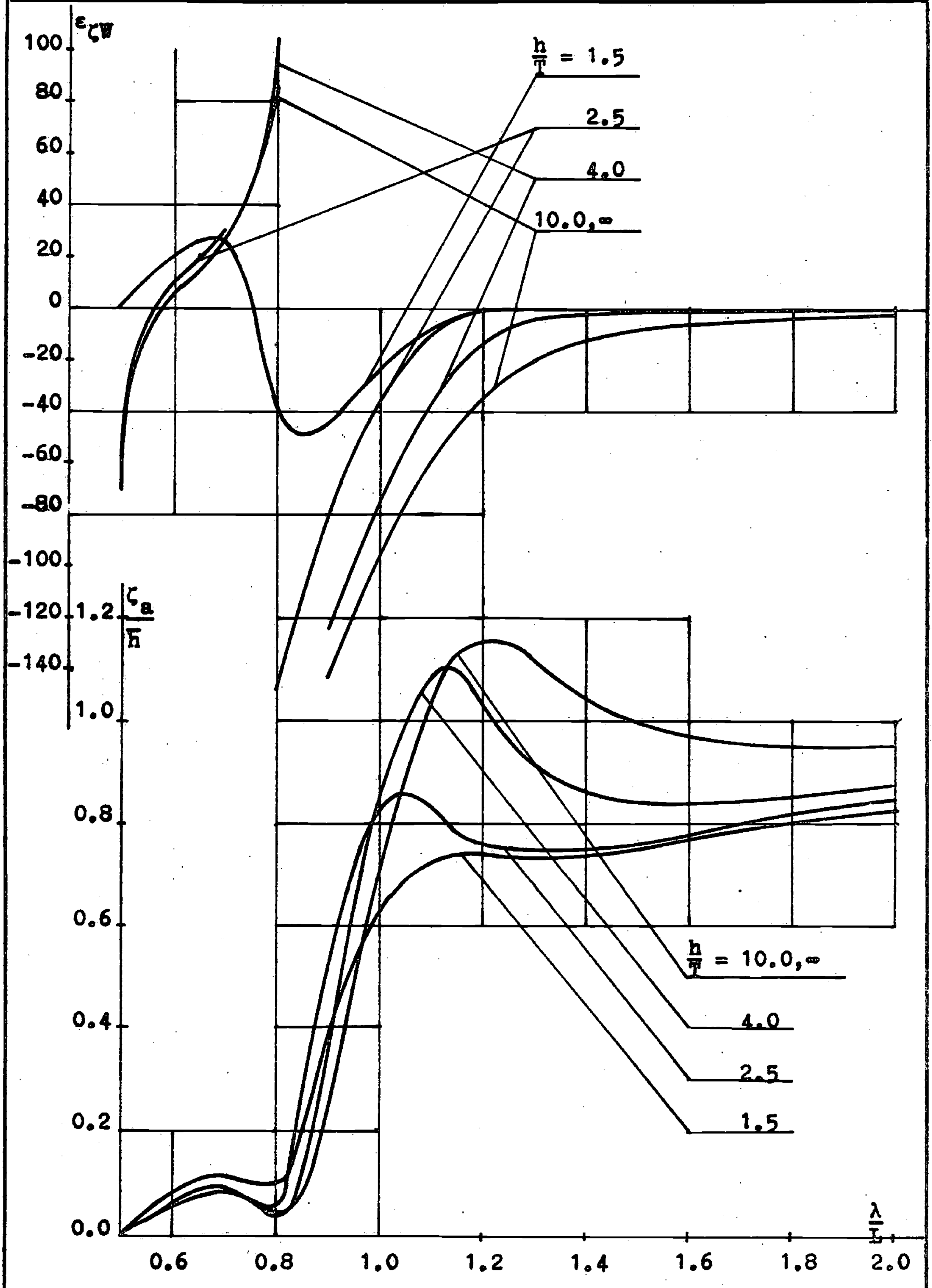


CHALMERS TEKNISKA HÖGSKOLA	Heave Amplitude Ratio at $F_n = 0.1$	Fig. 3
		CTH - SH
		Report 45

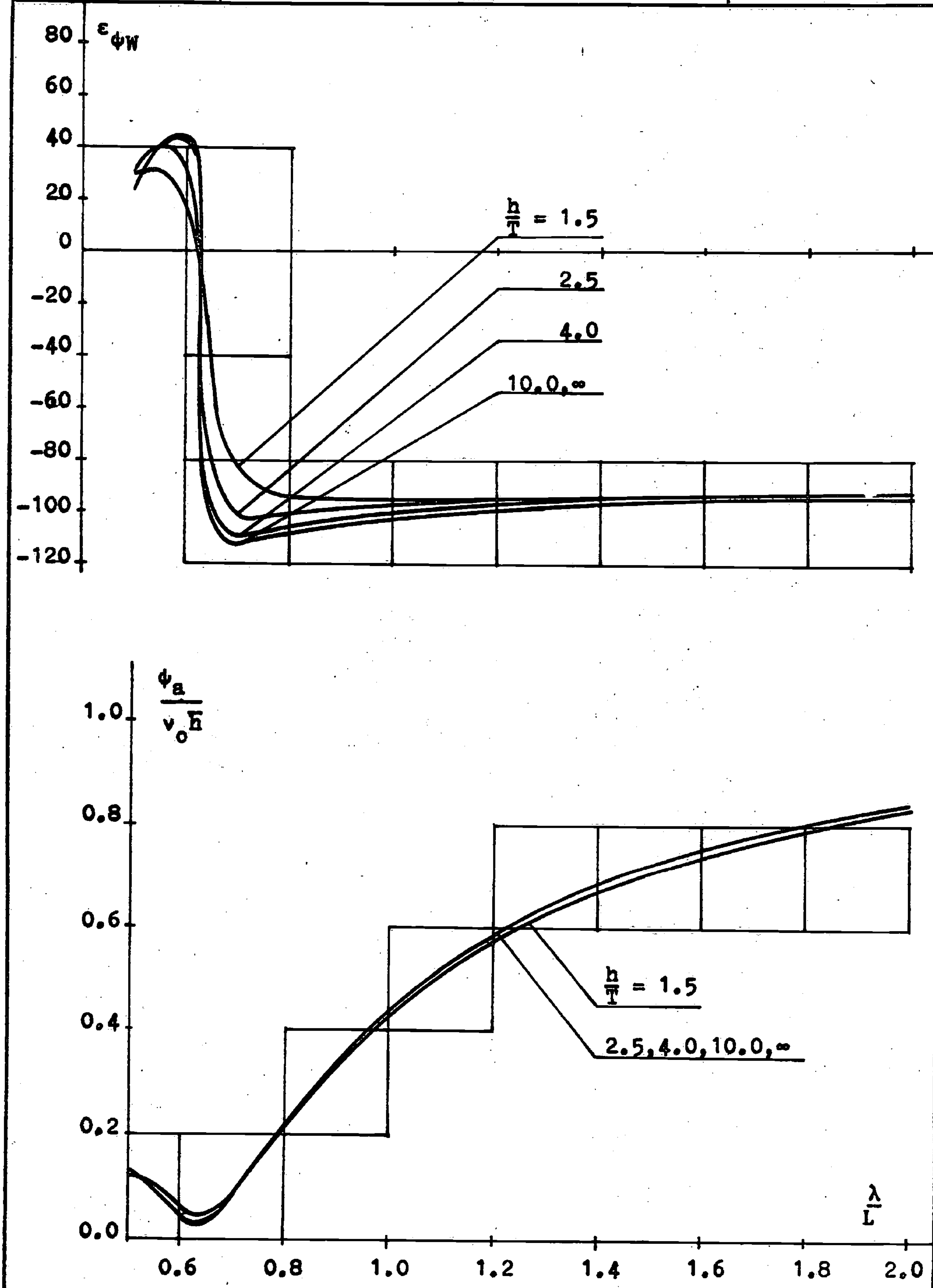




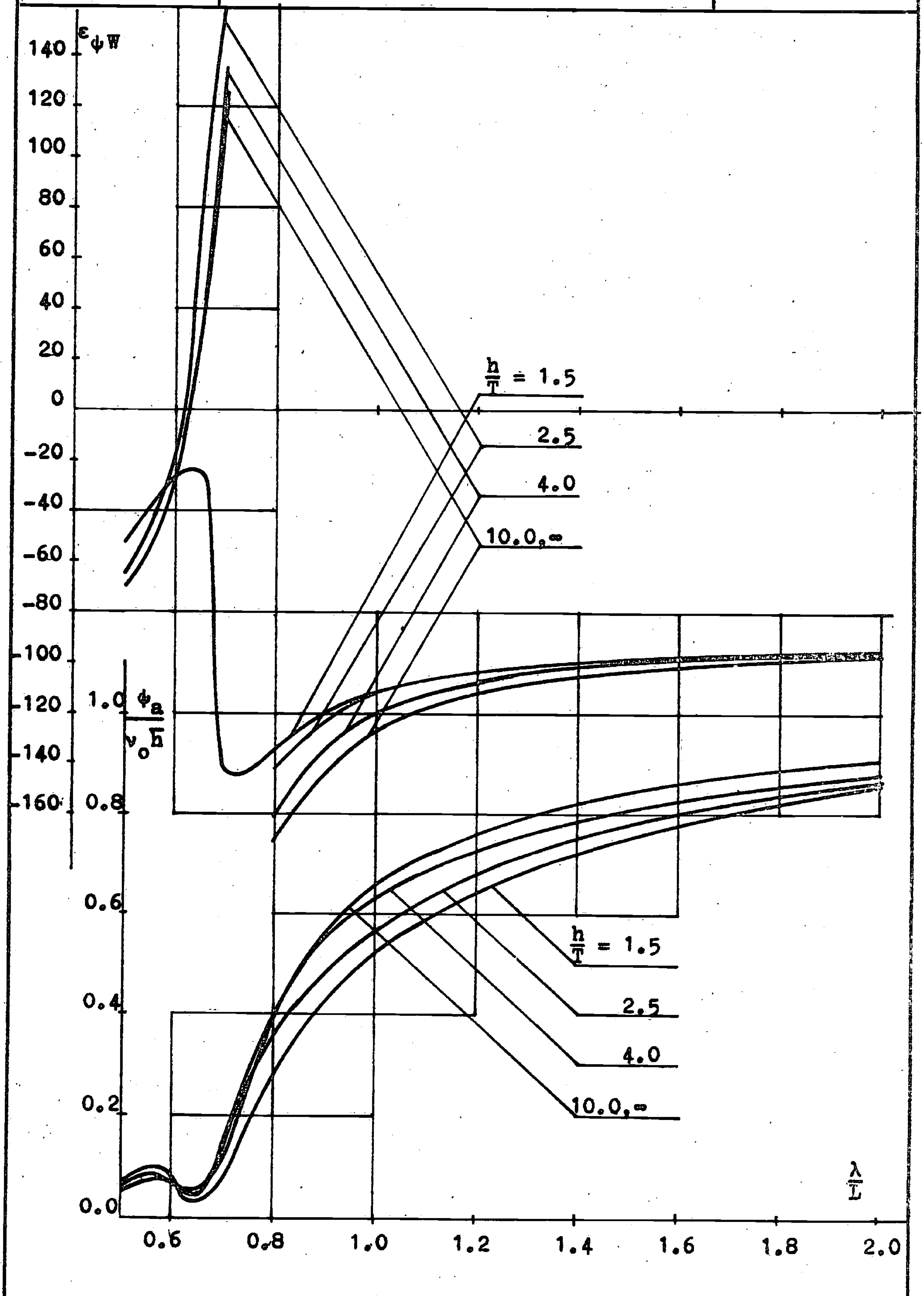
CHALMERS TEKNISKA HÖGSKOLA	Heave Amplitude Ratio at F_n = 0.20	Fig. 4 CTH - SH Report 45



CHALMERS TEKNISKA HÖGSKOLA	Pitch Amplitude Ratio at $F_n = 0.0$	Fig. 5 CTH - SH Report 45



CHALMERS TEKNISKA HÖGSKOLA	Pitch Amplitude Ratio at $F_n = 0.1$	Fig. 6 CTH - SH Report 45



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Pitch Amplitude Ratio at $F_n = 0.2$

Fig. 7
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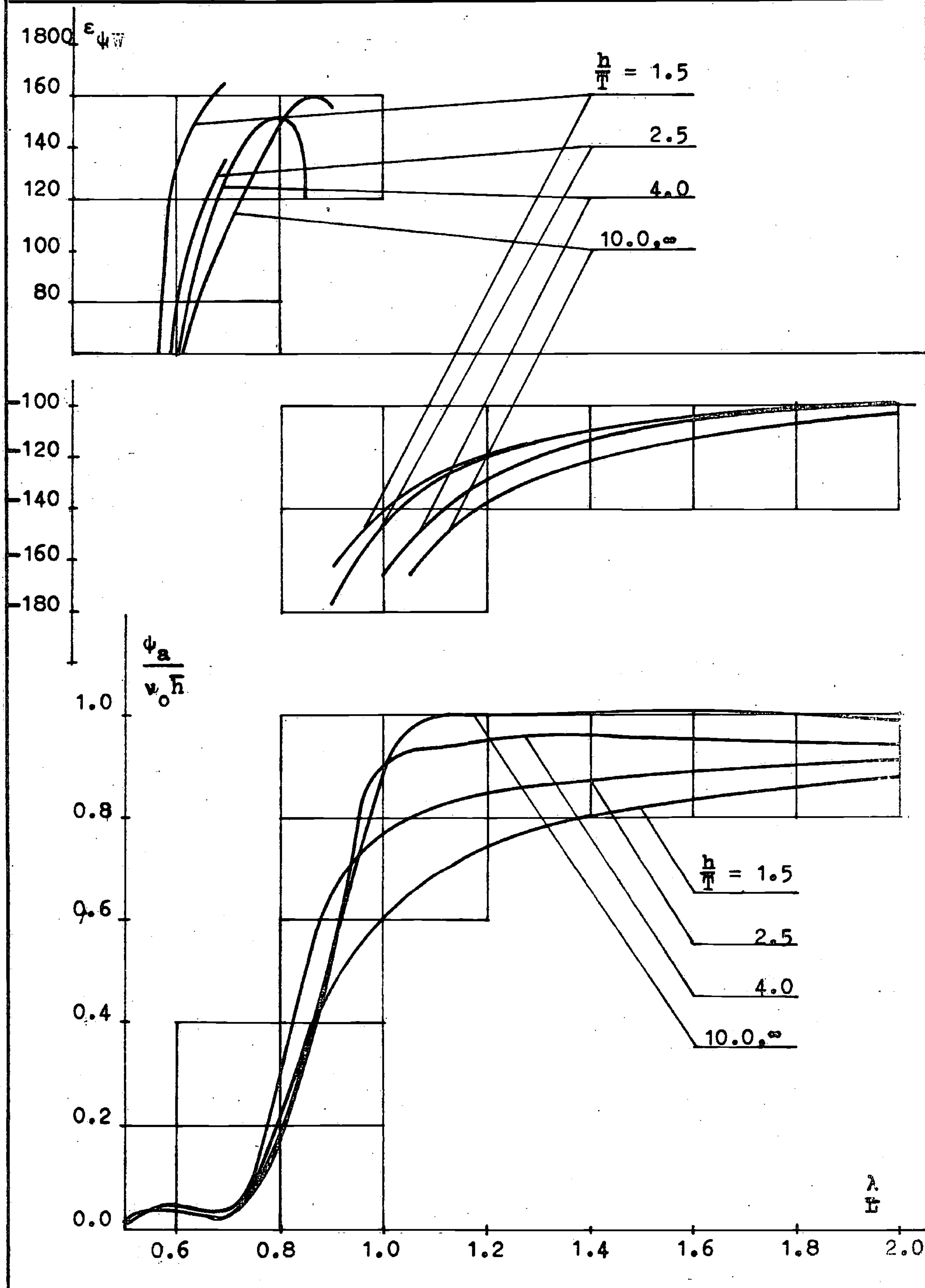
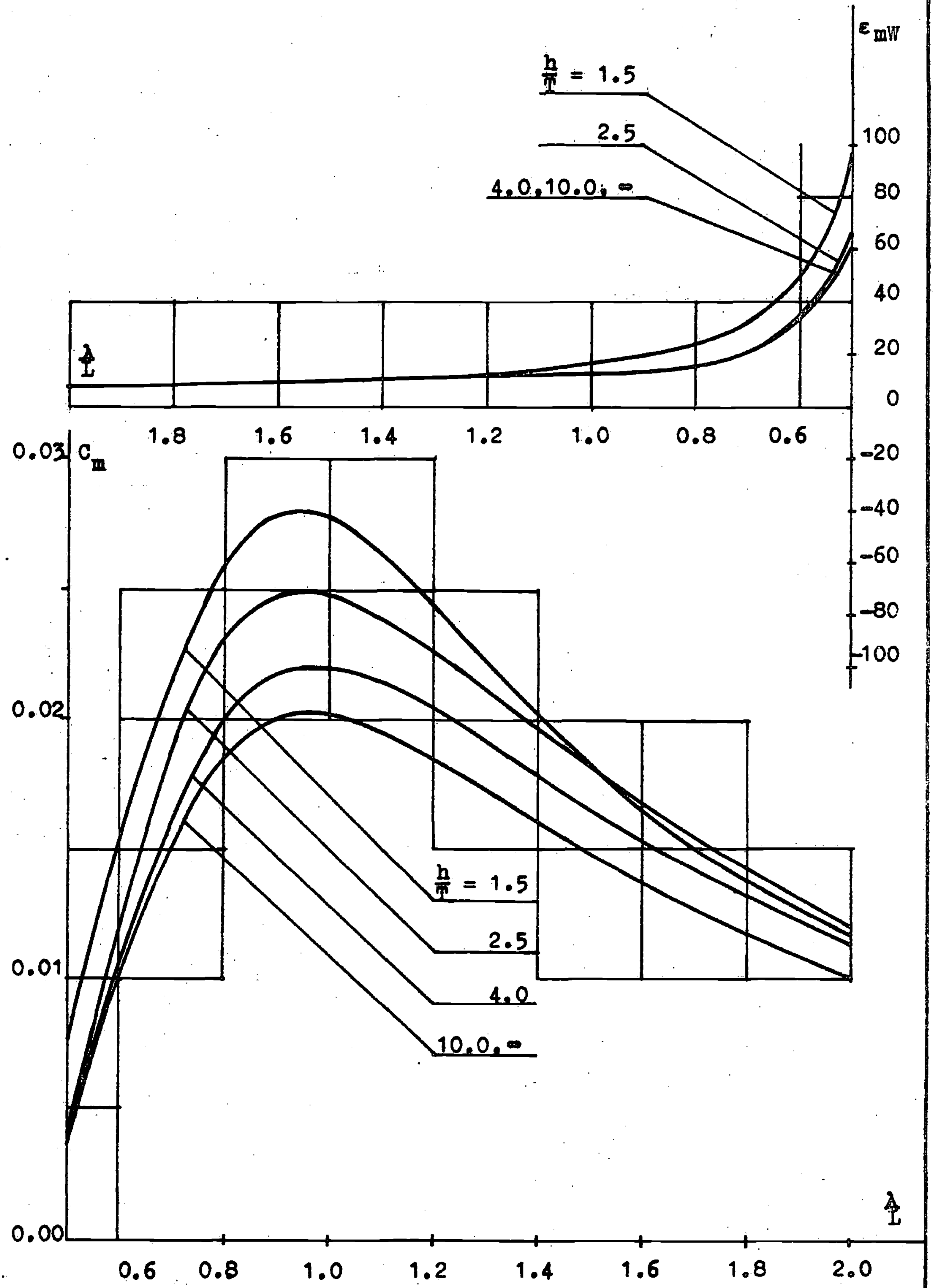
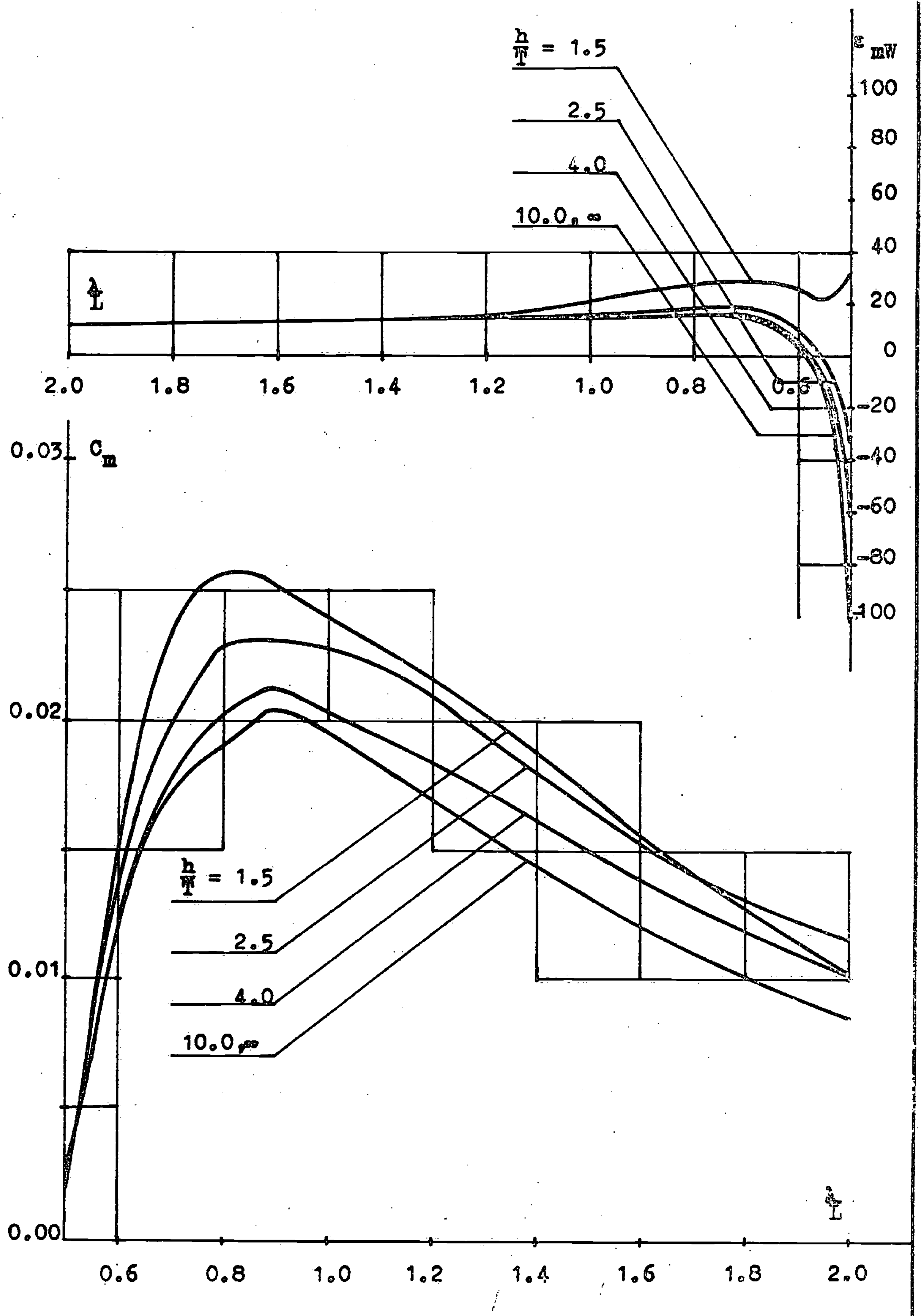


CHALMERS TEKNISKA HÖGSKOLA	Midship Bending Moment at $F_n = 0.0$	Fig. 8 CTH - SH Report 45



CHALMERS TEKNISKA HÖGSKOLA	Midship Bending Moment at F_n = 0.1	Fig. 9 CTH - SH Report 45



CHALMERS TEKNISKA HÖGSKOLA	Midship Bending Moment at $F_n = 0.2$	Fig. 10 CTH -SH Report 45