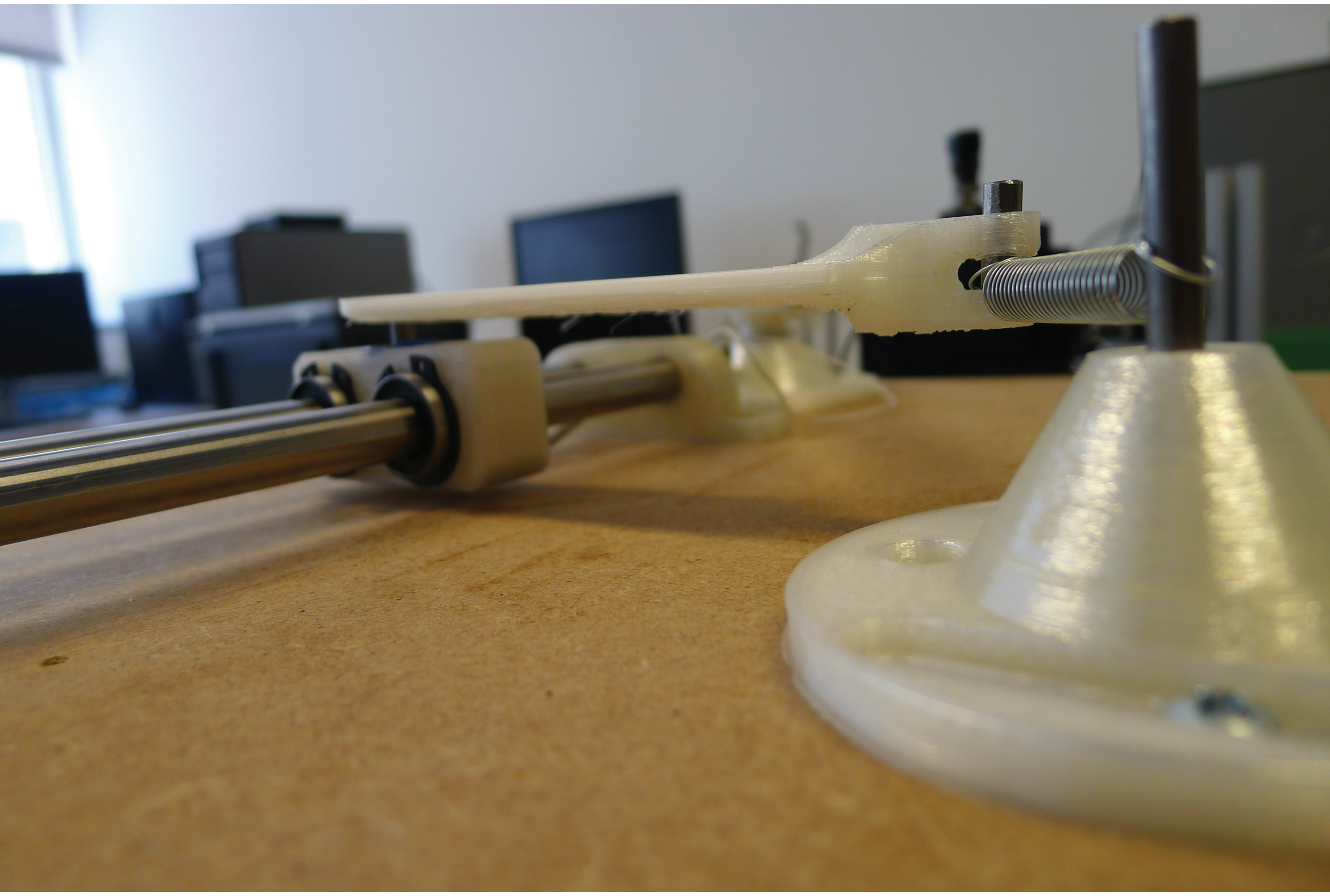
Passing Through Singularity with Elastic Potential Energy

In Crank-Slider Mechanisms



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Passing Through Singularity with Elastic Potential Energy

in Crank-Slider Mechanisms

by

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in partial fulfilment of the requirements for the degree of

Master of Science
in Mechanical Engineering

at the Delft University of Technology,
to be defended publicly on Friday September 1, 2017 at 12:00 noon.

Student number:	4139569	
Project duration:	August 29, 2016 – September 1, 2017	
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Preface

The project is about singularity problems, explained for a classic motion converter, the crank-slider mechanism. This mechanism has been around for centuries and it has been essential in the creation of steam locomotives and combustion engines. With many patents and applications available, it seemed a big challenge to contribute to the development of the crank-slider mechanism. Since I am eager for new challenges, I started my research the 29th of August 2016.

I have always been passionate about solving problems and coming up with solutions that improve our daily life. With a broad range of interests, the study of Mechanical Engineering has been a great match. In the Master program, I focused on biomechanical design. I enjoyed the generation of mechanisms, such as a compliant hole puncher or steerable medical instruments. Therefore, a design related thesis was best suited for me. I am excited to present the work that resembles what I have learned during my journey.

Iori van den Doel

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1

Introduction

The crank-slider mechanism is used to convert translational motion into rotational motion or vice versa. It is present in combustion engines, surface micromachined microengines and many other applications.

Problems emerge when the slider is used as the input. This introduces two singularity points per cycle where no force transmission is possible. The position of the singularity points can be calculated with instant centres, jacobians or other methods[[1]][[2]][[3]]. Throughout history, the singularity problems are considered to be purely geometrical. This point of view rules out the possibilty of providing a solution with springs, as these will not influence the kinematic behaviour.

For the past two centuries, two main solutions have been available for singularity problems in crank-slider mechanisms: Additional actuators with a phase shift or a flywheel connected to the crank. However, it has not been possible to actuate the mechanism with a single rectilinear actuator for low speed applications. Moreover, the use of elastic potential energy has not been used or proven successful for solving singularity problems in crank-slider mechanisms.

The goal of this thesis is to create a proof of concept for actuating the crank-slider mechanism with a single slider as input for low speed applications.

First, a literature review is done to find available manipulation strategies on singularity. The strategies are categorized in six different classifications, made on fundamental differences in force and motion. From the literature review, it is chosen to focus on elastic potential energy to pass through singularity. The second paper proposes a new vision on the singularity problems, based on torque transmission. In addition, a concept is generated, built and experimentally validated. The appendices provide additional information on the concept.

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[2] G. R. Pennock and G. M. Kamthe. Study of dead-centre positions of single-degree-of-freedom planar linkages using assur kinematic chains. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 220(7):1057–1074, 2006.

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2

Literature Review Paper

Singularity Manipulation Strategies in Translation-Rotation Motion Conversion

Singularity Manipulation Strategies in Translation-Rotation Motion Conversion

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***Abstract*—Since the introduction of steam engines, motion conversion has been essential for industrial improvements. In motion converters, problems arise at certain positions when force transmission is not possible, known as singularity points. In this article, different singularity manipulation strategies are discussed for the crank-slider mechanism. Many patents and papers can be found for solving the singularity issues. However, they have not all been structured, categorized, evaluated or proven. Currently the crank-slider is still widely used in piston-cylinder combustion engines and in micro mechatronics. Using available patents and literature, we propose six classes for singularity manipulation strategies, based on fundamental differences. The classification inspires the generation of new concepts. Evaluation of the classification groups enhances the selection process for mechanism design.**

***Keywords*—Crank-slider, singularity, motion conversion, transmission mechanism.**

I. INTRODUCTION

Motion converters exist in many shapes and configurations. They primarily serve to convert a motion from one type to another (e.g. rectilinear to rotary). Given a certain input, the desired output may be different in speed, direction, force, orientation or other characteristics. This article will focus on the translation to rotation motion convertion. The most used translation to rotation motion converter is the crank-slider (CS) mechanism, shown in *Figure 1*.

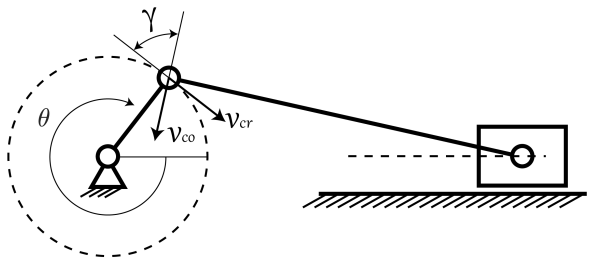


Fig. 1. Schematic representation of the crank-slider mechanism, with crank angle (θ), transmission angle (γ) between the direction of the velocity difference vector of the driving link (v_{co}) and the direction of the absolute velocity vector of the output link (v_{cr}) [1].

The first reported CS was used in 300AD, converting rotational motion of a water driven wheel, into rectilinear reciprocating saw motion to cut rectangular blocks [2]. When the crank is used as input, there are no problems. However, with the introduction of steam engines and cumbustion engines, the slider was used as input. Currently in the field of micro mechatronics, predominantly rectilinear actuators are used such as comb-drives. To create a rotational output, the rectilinear actuators are used in combination with a CS mechanism, since a rotational actuator is not always available (size, power, reliability, etc.) [3].

The reciprocating input motion creates two singularity points per cycle, where no force transmission is possible. The velocity of the slider is zero at the singularity points, corresponding to a crank angle (θ) of 0° and 180°. When the velocity is zero at these points, the instantaneous power input is also zero.

The parameter used in literature for mechanism design to avoid singularity points, is the transmission angle (TA) [4]. In a CS mechanism, the TA (γ) is the smallest angle between the direction of the velocity difference vector of the driving link (v_{co}) and the direction of the absolute velocity vector of the output link (v_{cr}), shown in *figure 1* [1].

The optimal TA is found at 90°. Singularity points are found when the TA is zero. In [1], the limit of the TA in mechanism design is investigated. When transmitting a force, the TA should not be smaller than 40°. This is a rule of thumb, which varies depending on speed, force transmitted and tollerances in the system (e.g. for high speed applications the TA should not be smaller than 45°).

A key problem in mechanism design is selecting the desired method to manipulate singularity points. Multiple articles define different types of singularity, calculated using jacobians, instant centres or other methods [5][6][7][8]. However, these calculation methods do not define how to alter those singularity points or which different manipulation strategies are available. Designers that want to use a mechanism with singularity problems, must find a solution to pass through these points. The search for an alternative is difficult as patents and papers are available to solve the issues, but they have not been structured, categorized, evaluated or proven.

The goal of this article is to: (1) Show the fundamental differences between the available singularity manipulation strategies. (2) Structure them in such a way that research gaps become visible. (3) Find the possibilities and limitations of each class by evaluation on predefined design requirements and criteria.

With the classification based on fundamental differences, designers are supported in finding and evaluating alternatives around singularity problems. Identification of the basic working principle for a (new) manipulation strategy is needed to find the corresponding class. With the overview on the limitations and possibilities of the classes, the designer is able to validate the concept in an early stage. Moreover, researchers and inventors are inspired to find other solutions, once research gaps are indicated.

First, the search method on finding different singularity manipulation strategies is discribed in section 2, followed by the proposed classification method. A set of requirements is provided to differentiate the found concepts. Evaluation is based on the possible application speed criteria. In section 3, an overview of the results is given and discussed, showing an example of each strategy. Section 4 consist of a discussion based on the general implementation of the classification. Conclusions are formulated in the final section.

II. METHOD

A. Search method

A broad literature research is done to collect different solutions for singularity manipulation. The patents and papers in this report claim to solve, overcome or obviate the singularity point in the crank-slider (CS) mechanism. During the research, the following sources were used: Google Scholar, Google Patents, Web of Science, Espacenet.

The patent database of Espacenet consists of an extensive classification. In order to find applicable patents for overcoming the singularity points in a CS mechanism, the proper class should be selected. Class F for is used for patents in mechanical engineering and class Y for new technological development. The available classification of the Espacenet database is on a high perspective, covering many different types of patents. A patent can be found through multiple classes. To cover the patents on singularity manipulation strategies, the following classes in the Espacenet database are used:

- Y10T74/18144: overcomming dead center.
- F16H21/38: with means for temporary energy accumulation e.g. to overcome dead- centre positon.
- F16C3/30: with arrangements for overcoming dead-centres.

Search terms are grouped in keyword sets and combined to find relevant results, shown in the *table* I.

TABLE I. KEYWORDS USED IN RESEARCH, GROUPED IN KEYWORD SETS

Set	Keywords
Mechanism	Transmission, motion converter, crank slider, linkage
Singularity	Dead center position, dead point, toggle point, top dead center, bottom dead center
Application	Steam engine, bicycle, presses, saws, pumps, compressors, micro engine, combustion engine, reciprocating input
Manipulation	Transmission angle, mechanical advantage, instant center

B. Classification

In the available concepts, different working principles are distinguished. In this section, a generalized classification is proposed. The purpose is not to cover all the possible concepts, but to create a tool for dividing different strategies and to guide in choosing a design direction. The classification is based on what is found in literature.

In the singularity points of the crank-slider (CS) mechanism, no force transmission is possible. This is caused by the the kinematics of the mechanism. Two different approaches are possible to manipulate this problem: (1) Apply forces from an external source without changing the kinematics or (2) change the kinematics. The distinction between the classes is made, based on this fundamental difference, shown in *table II*.

TABLE II. CLASSIFICATION FOR SINGULARITY MANIPULATION STRATEGIES

Singularity manipulation strategies			
Kinetic		Kinematic	
Active	Passive	Topology	Geometry

Kinetic
Manipulations that are based on adding forces from external actuators (including human actuation) are *active*. Forces generated on the system, operating without an external input, are *passive*.

Kinematic
Manipulations that have additional links, joints or sliders, create mechanisms with different *topology*. Otherwise, the singularity points are manipulated by changing the *geometry* of the mechanism.

C. Design requirements and criteria

The indication of possibilities and limitations are essential in finding the desired manipulation strategy. First, the *design requirements* are explained. Second, the *design criteria* are introduced.

Design requirements

Passing through singularity

This requirement is based on the force transmission throughout the cycle. The power input must be equal to the required power output [9]:

$$F \cdot v = \frac{T \cdot n \cdot 30}{\pi} \tag{1}$$

With:

$F \ = \ Force \ (N)$

$v \ = \ Velocity \ of \ the \ slider \ (m/s)$

$T \ = \ Torque \ (Nm)$

$n \ = \ Rotational \ speed \ crank \ (RPM)$

In the case of the crank-slider mechanism, the velocity of the slider (v) is zero at the singularity points. This results in a zero power input. The validation of a concept passing through singularity is done when the kinematics are changed in a way that the velocity of the input is non zero at the singularity points or that a different source is used to provide the required output power.

Reversible

Another requirement is based on the possibility to drive the mechanism in both directions. The adjusted mechanism must not contain any joints that allow only motion in a single direction (e.g. ratchet mechanism). The storage and releasement of energy is another cause that defines the reversibility. If the energy is stored (instead of released) in the opposite direction at the singularity point, the concept is not reversible.

Passive

The manipulation strategy does not contain any other actuator (including human actuation), to be passive. For some applications it is not desired to add other actuators. The complexity of the mechanism increases with the amount of actuators.

Design criteria

Application speed of the crank-slider mechanism differ greatly from low speed (≤ 1 RPM) in a watch, to high speed (> 100 RPM) in combustion engines. The design criteria are based on the application speed, as it influences the feasibility of the manipulation strategy. For high speed applications, a low transmission angle ($< 45°$) is not recommended, as already explained in section I. For low speed applications, the TA should not be lower than ($< 40°$). Additionally, manipulation strategies dependent on the velocity will not work for both domains.

III. Results

Table III provides an overview of the available singularity manipulation strategies. Patent numbers are added to indicate the deviation and development within each class. The patents shown in the overview are not covering all the available concepts. However, using the patents enables comparison to classify new concepts or inspire the creation of new concepts. Each class is discussed in the following section.

A. Kinetic manipulation strategies

Active

Actuator force

This class includes all manipulations that involve an additional actuator. The most used concept in combustion engines today is an additional actuated slider, connected to the crank, with a phase shift of 90°, shown in *table III(a)*. However, it requires an additional control unit and infrastructure to power the actuator, increasing the complexity of the entire system. The motion is reversible by changing the order of actuation between the two sliders. Throughout the cycle, a high transmission angle ($> 45°$) is possible depending on the configuration (phase shift between the sliders). This enables the mechanism to pass through singularity. The additional actuator works for high and low speed applications. However, if one of the actuators fails, it is impossible to actuate the crank. Other patents are available that use human actuation, a lever as additional actuator, or change the arrangement between two sliders [10][11][12][13][14] [15].

Passive

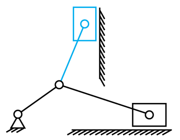
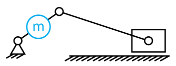
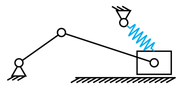
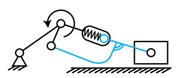
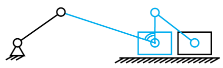
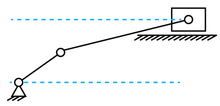
Dynamic force

The first recorded and implemented solution to pass through singularity, is the use of dynamic forces (a flywheel connected to the crank), shown in *table III(b)*. The flywheel is a proven concept that enables passing through the singularity points, for high speed applications. The potential kinetic energy (E_{ke}) is a function of the mass (m) and the velocity (v), described by the following formula:

$$E_{ke} = \frac{1}{2} m \cdot v^2 \tag{2}$$

The potential kinetic energy stored in the flywheel should be higher than the required work to pass through singularity. The quadratic relation of velocity with kinetic energy makes dynamic forces unusable for low speed applications. The use of dynamic forces is considered as passive. However, the flywheel is undesirable during the start up phase for requiring more energy to create motion. In practise, another source is used to get the mechanism up to speed, questioning the passivity of the system. For a low speed output, the dynamic forces combined with a high speed input require an extra transmission. The flywheel allows for reversible actuation, as it is able to rotate in both directions. The concept adds extra mass to system, which in many cases is undesired.

TABLE III. CLASSIFICATION OF SINGULARITY MANIPULATION STRATEGIES, INCLUDING (A) ACTUATOR FORCE, (B) DYNAMIC FORCE (C) ELASIC FORCE, (D) ADAPTIVE STRUCTURES, (E) ADDITIONAL LINK AND (F) OFF - CENTER.

Singularity manipulation strategies					
Kinetic			Kinematic		
Active	Passive		Topology		Geometry
a Actuator force	b Dynamic force	c Elastic force	d Adaptive structures	e Additional link	f Off- center
					
W.G. Garman (US 3693463) [6] G. Marquet (US 2391725) [11] F. Munzinger (US 3853014) [12] C. Phillips (US 725285) [14] D.C. Slaght (US 1681910) [19] J.H. Towers (US 1753485) [24]		C. Tisell (US 2480273) [22] J.H. Towers (US 1636395) [23]	A.H. Gurley (US 919855) [8] J.P. Kelly (US 1906614) [10] L. Robertson (US 1782746) [16] C.S. Sarkar (US 2167314) [17] E.M. Trammel (US 2233857) [25]	A.J. Bohman (US 1420236) [2] J. Calcaterra (US 1430491) [3] W.H. Cords (US 2368412) [4] J.T. Hale (US 1574573) [9] P.H. Schneider (US 2858702) [18]	

Elastic force
In this class, a part of the actuation energy is stored in elastic potential energy (e.g. extending a spring). The elastic potential energy (E_{sp}) is a function of the spring constant (k) and the displacement (x), described by the following formula:

$$E_{sp} = \frac{1}{2} k \cdot x^2 \tag{3}$$

The elastic potential energy is used when the power input is insufficient, enabling the mechanism to pass through singularity. The manipulation strategy of using potential elastic energy is not depending on speed, allowing the use for high and low speed applications. However, within this class it is not possible to create a reversible mechanism. The concept must release energy when the input is insufficient. If the motion is reversed, the energy releasement and storage are interchanged. Requiring energy for storage, when the input power is already insufficient to supply the required output power. In other words, the transition point of the spring should be overcome before the singularity point is reached. In the transition point, the spring changes from storing energy in providing energy. When changing the direction of actuation, the singularity point is reached when energy is still stored in the spring. No actuators are necessary for using elastic potential energy. Therefore, the manipulation strategy is passive.

The Towers patent is generalized and shown in *table III(c)* as an example [16]. In the Towers patent, the spring is connected between the slider and ground, making it unable to pass through the singularity points. In this case, energy is stored in the spring during the unfavourable regions, when the transmission angle (TA) is low. Energy should be released in these regions, not stored. The mechanism will not be able to reach the singularity points (TA is zero), which are equal to the transition points of the spring. The transition points need to be overcome before the singularity point is reached.

The Tisell patent creates a translational motion, using two shields connected to the crank with a sliding contact [17]. The motion of the shields are in a 90° phase shift with the input slider. Both shields have an individual spring with the transition points at 90° and 270° crank angle. The concept is inable to pass through the singularity in this configuration. The transition point is overcome before the singularity point, in both springs. However, the stored energy is completely released at the singularity point. More energy is required to pass through the singularity. In addition, the sliding contact introduces wear problems over time. Moreover, the patent requires many additional parts.

B. *Kinematic manipulation strategies*

Topology

Addaptive structures
In this class, extra joints and links are added. One of these joints has variable degrees of freedom (DOF). Such a joint is also used to create adaptive structures (or reconfigurable mechanisms). An example of an adaptive structure is shown in the schematic representation of the Trammel patent, *table III(d)* [18]. The blue revolute joint (allowing rotation), combined with the black slot (allowing translation), create a variable DOF that is controlled with the spring. The spring is not used to store energy to pass through singularity. The spring is only a switch that enables sliding of the blue joint when the pretension force is overcome. The revolute joint at the end of the crank is added with a ratchet, only allowing rotation in the direction of the arrow. At the singularity point, the slider provides the threshold force to overcome the spring and slide the blue joint inwards. Changing the kinematics, with the ratchet and blue link, increases the transmission angle. In this way, a force is transmitted from the slider to the crank in singularity position.

The problem in adaptive structures is the configuration in which the DOF is released [19][20][21][22]. This occurs at the singularity point. To enable changes in kinematics, the mechanism should first approach the singularity point, travelling through the unfavourable region with low TA. The slider itself is not able to provide the required energy to move through this region. Therefore, it is not possible to pass through singularity with adaptive structures. The motion is not reversible with adaptive structures, as parts are included that only allow rotation in a single direction. The additional links, sliders and joints increase the complexity of the system. A part of the low transmission angle region is not affected with this solution, solving only part of the problem and making it unfavourable for high and low speed applications.

Additional link
In this class, an extra link (or linkage) is added with an extra constraint [23][24][25][26]. The Calcaterra patent adds a passive slider in between the crank and input slider, shown in *table III(e)*[27]. The additional linkage is able to shift the position of the singularity point and change the velocity profile. By changing the position of singularity, the region with a low transmission angle is altered. The region could be minimized at one of the singularity points. However, this results by an increased region at the other singularity point. In the shifted singularity point, the velocity of the slider is again equal to zero. Therefore, the mechanism is inable to pass through singularity. The shifted singularity points, relative to the initial singularity points, are shown in *figure 2*. This manipulation strategy might be used in combination with other strategies to extend the design space. The motion is reversible. The additional links and sliders increase the complexity of the system.

Geometry
Off-center slider
In this class, only the geometry of the crank-slider mechanism is adjusted. An example is changing the lengths of the links or shifting the slider off-center, shown in *table III(f)*. The change in geometry, shifts the singularity position, shown in *figure 2*, and influences the velocity profile. Adjusting the length of the links decreases the region with a low TA at one singularity point. As consequence, the region with a low TA increases at the other singularity point. In the shifted singularity point, the velocity of the slider is again equal to zero. Therefore, it is not possible to pass through singularity with this strategy. When the position of the singularity points is altered, a non symmetrical transmission is created. The motion is reversible.

An overview of the design requirements and design criteria evaluation on singularity manipulation strategies is shown in *table IV*. Manipulation strategies which satisfy the design requirements, are indicated with ✓. The design criteria are based on the lowest TA in the cycle. In low speed applications, the TA should be higher than 40 °, for a good (+) transmission. For high speed applications, higher than 45°. For a transmission with medium (+−) performance, the TA should be higher than 30°. If it is lower than 30°, the transmission is considered poor (−). Dynamic forces are not taken into account with the TA. Therefore, the evaluation is based on the ability to store energy. This is poor for low speeds ($\leq$ 1 RPM) and good for high speeds ($>$ 100 RPM). The ○ indicates the potential for a manipulation strategy to be used in a certain application speed, when no working concept is found in literature.

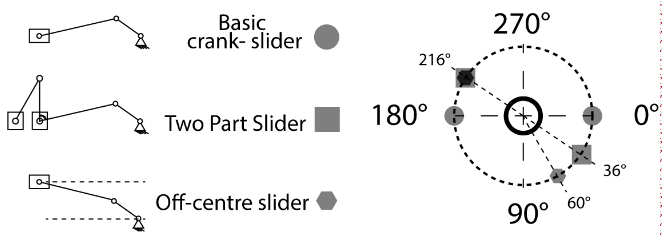


Fig. 2. The shift of singularity position for the off-center slider and the additional link concept, in perspective with the basic crank-slider mechanism.

TABLE IV. DESIGN REQUIREMENTS AND EVALUATION CRITERIA OVERVIEW ON SINGULARITY MANIPULATION STRATEGIES.

	Design requirements			Design criteria	
	Passing through	Reversible	Passive	Low speed	High speed
Actuator force	✓	✓		+	+
Dynamic force	✓	✓	✓	−	+
Elastic force	✓		✓	○	○
Adaptive structures			✓	−	−
Additional link		✓	✓	−	−
Off- center		✓	✓	−	−

IV. DISCUSSION

The proposed classification for singularity manipulation strategies in crank-slider mechanisms is based on fundamental differences between the strategies. The same fundamental differences are present for manipulation strategies in other mechanisms with singularity problems. Therefore, the proposed classification could be implemented in other motion converters with singularity problems. However, the solutions might be different and other classes might be added.

The kinematic manipulation strategies will not provide a solution that will pass through singularity. This is shown for crank-slider mechanisms. However, the shifting of the singularity position might be sufficient in other mechanisms to solve the problem. Therefore, the results should not be used directly for other mechanisms, without further research. However, the kinetic manipulation strategies should be able to provide a solution in any type of mechanism with singularity problems.

The design requirements are indicating the general differences between the classes. However, requirements can be added to the overview, creating a more detailled differentiation. The overview in *table IV* is not showing all the limitations and possibilities that kinematic strategies have. Velocity profiles generate a deeper understanding of each concept. However, a small change in the geometry changes the velocity profile, within a single concept, significantly. Therefore, the creation of a generalized velocity profiles is not valuable.

V. Conclusion

In this study, different singularity manipulation strategies for the crank-slider mechanism (existing in prior art) are introduced, classified and evaluated. A set of design requirements are introduced to validate the possibility of passing through singularity.

It was found that existing singularity manipulation strategies belong in one of the six classes, based on their fundamental differences. Moreover, it was shown that not all classes are able to pass through singularity. Strategies that change the kinematics, are only able to shift the singularity points. The evaluation on application speeds, show the potential for new concepts with elastic potential energy, as no working concepts are found in literature.

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3

Design Paper

Passing Through Singularity with Elastic Potential Energy in Crank-Slider Mechanisms

Passing Through Singularity with Elastic Potential Energy in Crank-Slider Mechanisms

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***Abstract*—Since the introduction of steam engines, motion conversion has been essential for industrial improvements. In the conversion of rectilinear motion into rotational motion, problems arise at certain positions where force transmission is not possible, known as singularity points. This paper proposes the use of elastic potential energy to overcome the singularity problems in crank-slider mechanisms. This novel method is demonstrated by using a single spring without extra links or linkages. Solutions were found by connecting a translational spring between the coupler link and the ground. A contour plot indicates variations on the concept for different connection points with the coupler link. Moreover, the theoretical model shows a transmission of at least 40% of the maximum transmitted torque through the full cycle motion. In addition, an experiment is conducted for validation of the proposed modelling. Conversion of low speed translation to rotation is applicable in multiple fields, such as micro mechatronics, energy harvesters and bicycles.**

Keywords*—Elastic potential energy, crank-slider, singularity, motion conversion, transmission mechanisms.

I. Introduction

The history of crank-slider mechanisms starts in 300AD. The first reported application is the motion conversion of a water driven wheel into a reciprocating saw motion, to cut rectangular blocks [1]. A schematic representation of the crank-slider mechanism is shown in *figure 1*. Crank-slider mechanisms are used to convert rotational motion into reciprocating rectilinear motion or vice versa. When the rotational motion is used as input, the mechanism works without problems. However, with the introduction of steam engines, it became necessary to create a rotational output, using the reciprocating motion as input.

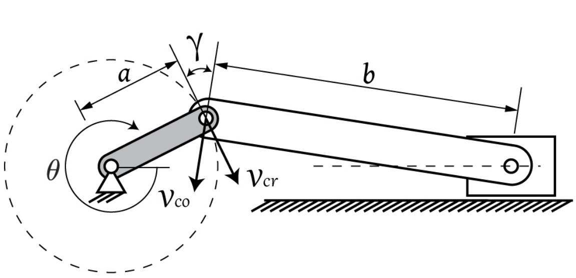

Fig. 1. Schematic representation of the crank-slider mechanism, with crank length a, coupler length b, crank angle θ, transmission angle (γ) between the direction of the velocity difference vector of the coupler link (v_{co}) and the direction of the absolute velocity vector of the crank (v_{cr}) [2].

The reciprocating input motion creates two singularity points per cycle, where no force transmission is possible. The velocity of the slider is zero at the singularity points, corresponding to a crank angle (θ) of 0° and 180°, shown in *figure 2*. When the velocity is zero, the instantaneous power input is also zero.

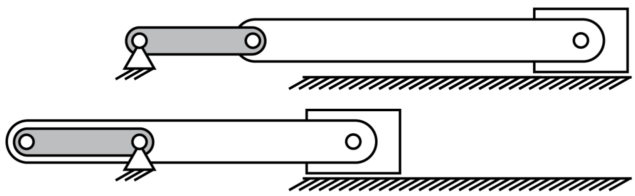

Fig. 2. Both singularity positions in a crank-slider mechanism. Top figure shows the singularity at crank angle (θ) of 0°. Bottom figure at θ of 180°.

With the consideration of friction, a margin is defined to avoid singularity problems in mechanisms [3]. For mechanism design, the transmission angle is used to define the margin. This should not be smaller than 40° [2]. In crank-slider mechanisms, the transmission angle is the smallest angle between the direction of the velocity difference vector of the coupler link (v_{co}) and the direction of the absolute velocity vector of the crank link (v_{cr}), shown in *figure 1*. However, even without friction, there are problems in the force transmission. Therefore, an additional definition of singularity problems in motion converters needs to be made.

Nowadays, crank-slider mechanisms are widely used to convert motion. Larger applications, such as combustion engines use the rectilinear motion of the piston as the input. In the field of micro mechatronics, predominantly rectilinear actuators are used such as comb-drives. Rotational actuators are not always available due to the size, power, reliability, etc. [4]. Therefore, it is still necessary to use a crank-slider mechanism in combination with the rectilinear actuators to create a rotational output.

In prior art, two main solutions are used to avoid the singularity problems in crank-slider mechansims:

- A flywheel connected to the crank is able to store kinetic energy and is used when the input energy is insufficient. This solution only works for high speed applications. Furthermore, the additional weight of the flywheel is in many cases undesirable.

- An additional actuator with a phase shift [5]. This requires a control unit, additional infrastructure and additional space. It also increases the complexity of the system.

Currently it is not possible to actuate a crank-slider mechanism with the input of a single slider in low speed applications. In the past 200 years, many other patents are filed to solve, overcome or obviate the singularity points [6]. However, no alternative is available for low speed applications with a single actuator. Furthermore, from a literature review (see *Chapter 2*), it is concluded that changing the kinematics will not result in the ability to create a force transmission in the full cycle motion. Thus, the energy to pass the crank-slider mechanism through its singularity points should come form an alternative source.

The main goal of this paper is to create a non-zero force transmission throughout the full cycle motion of a crank-slider mechanism, using elastic potential energy by including a single spring to the system without adding extra links or linkages. The main goal is divided into four subgoals: (1) Propose a new margin to avoid the singularity problems based on torque at the output. (2) Propose a new method for solving singularity problems, using elastic potential energy. (3) Find a solution map with a single spring in crank-slider mechanisms. (4) Experimental validation of the theoretical model.

The impact of creating a rotational output with a single rectilinear input is relevant on lots of applications, such as:

(a) In micro mechatronics, surface micromachined micro-engines are used to actuate optical switching elements, electrical switches and micropositioners [4]. The microengine is currently operated by two comb-drives with a phase shift of 90°, shown in *figure 3*. Creating a transmission with only a single rectilinear input is more compact, cheaper, less complex (no control unit necessary) and increase the reliability.

(b) Energy harvesting in mechanical watches is done with a rotating mass. By creating a rotational output with a single rectilinear input, it is possible to harvest energy from translational movements.

(c) Elastic potential energy could be used in cycling to assist the crank in regions where less muscles are available. Creating a force transmission through the full cycle motion also enhances cycling with one leg.

First a deeper understanding of the singularity problems is given in *section II*. Second, a set of design requirements is made, after which a general method is proposed to pass through singularity with elastic potential energy. Kinematic conditions are formulated that must be complied with to create a solution. The concept is shown in *section II.F*, followed with the theoretical model and a contour plot of all the possible solutions. The fabrication and experimental setup of a chosen concept are presented in *section III*. The obtained results are shown in *section IV*, and discussed in *section V*. In the final section, the conclusions are formulated.

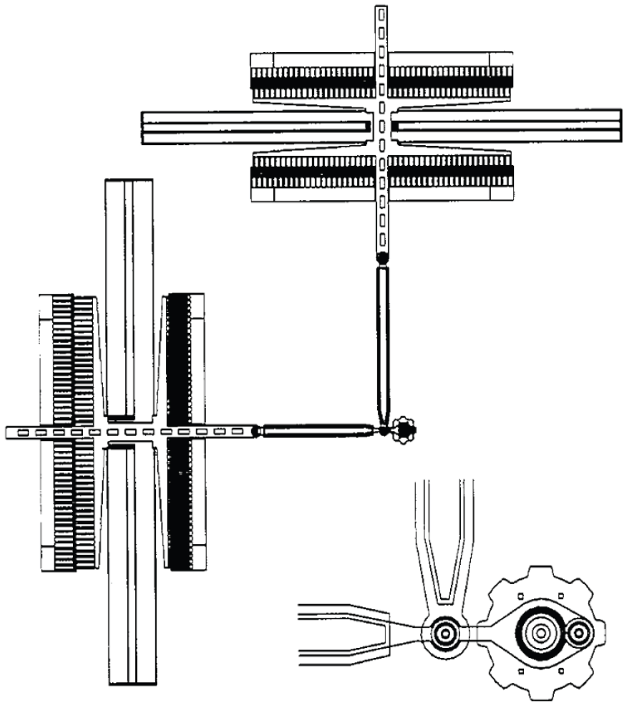

Fig. 3. Schematic overview of a surface micromachined microengine [4].

II. METHOD

A. Singularity problems

The kinematics of a transmission mechanism define the singularity positions. In case of the crank-slider mechanism, the kinematics are based on the geometry of the links and the elevation of the slider. The transmission is defined by the ratio λ between the length of the crank a and the length of the coupler link b, indicated in *figure 1*.

$$\lambda = \frac{b}{a} \tag{1}$$

The ratio λ is commonly used between 2.5 and 6 [7]. To illustrate the problem, the force transmission of a crank-slider mechanism (with $\lambda = 6$) is normalized and shown in *figure 4*. The input force at the slider (F) is considered constant. The transmitted torque at the output (T_i) is calculated with the displacement of the slider (x_{sl}) and the angle of the crank (θ) in the following formula:

$$T_i(\theta) = F \, \frac{d(x_{sl})}{d\theta} \tag{2}$$

The problems become evident when a required output torque is considered and plotted in *figure 4* (40% of the maximum transmitted torque, shown as black dashed line). The problem is not only limited to the singularity points.

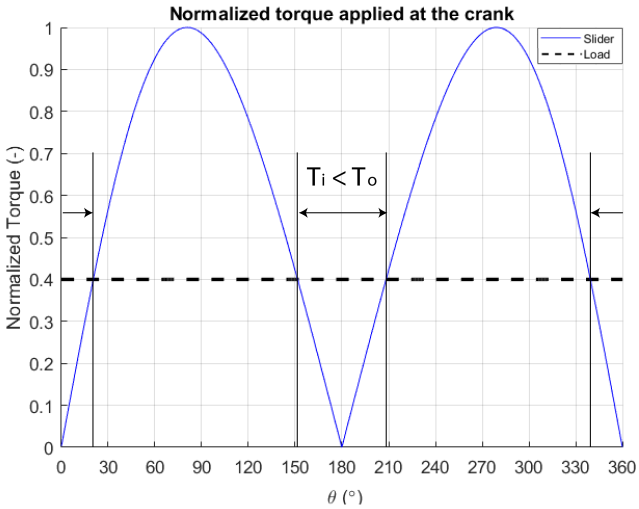

Fig. 4. The force transmission of a crank-slider mechanism ($\lambda = 6$). The blue line indicates the normalized transmitted torque at the crank from a constant input force at the slider. The dashed line indicates an output load that is 40% of the maximum output torque. In the region between the arrows, the transmitted torque (T_i) is smaller than the required torque output (T_o).

The regions around these points are also causing problems. The main issue starts where the power input of the slider is insufficient. The slider is not able to transmit the required torque at the output ($T_i < T_o$). The input force of the slider will result in a lower torque between 151°-208° and 339°-20°, indicated as the region between the arrows. The problematic regions are smaller, if the required output torque is reduced. Therefore, the singularity problem is defined as: ”*The region where the transmitted torque from the input is smaller than the required torque output, caused by the kinematics in combination with the required output load.*”

The difference in torque must be provided by another source to create a solution. The alternative source can be expressed as a force acting on the crank, shown in *figure 5*. When the combined force of the alternative source and the slider is resulting in a higher torque than the required output torque, the singularity problems are solved. Therefore, passing through singularity is defined as having a force transmission at every point of the cycle. The drivable load is determined by the minimum torque in the full cycle motion.

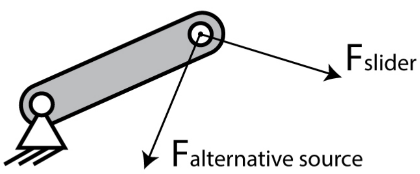

Fig. 5. An indication of forces acting on the crank, from the slider and an alternative source.

B. Design requirements

Without adding other actuators, the concept must be passive. The alternative power source described in the previous section must be generated by the input slider itself. Dynamic forces can not be used for low speed applications. Drivable mechanism is defined as creating a minimum output torque through the full cycle motion of at least 40% of the maximum torque, shown in *figure 4*. Theoretically, the highest possible constant output torque is 62.8% of the maximum torque (with $\lambda = 6$), equal to the average transmitted torque (T_{avg}):

$$T_{avg} = \frac{F * 2a}{\pi} \tag{3}$$

C. Passing through Singularity with Elastic Potential Energy

Elastic potential energy is used to manipulate the force transmission in crank-slider mechanisms. Storing energy in a spring results in a negative torque on the crank and the release of energy results in a positive torque.

The region where energy must be stored is determined by the kinematics. The force transmission in crank-slider mechanisms show two peaks (*figure 4*). The regions around the peaks have a high transmission angle. The torque from the input is higher than the required output torque. Therefore, the excessive torque in these regions must be stored. In general, the peaks are located around crank angles of 90° and 270°. The ideal region for storing energy around a crank angle of 270° is indicated in *figure 6(a)*. The transition point is located where the ideal region for storing energy ends.

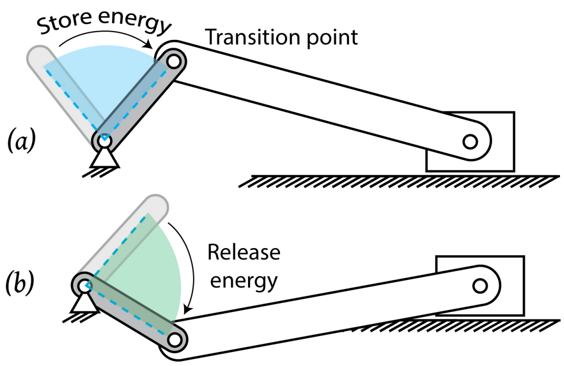

Fig. 6. (a) The region with a high transmission angle, ideal for storing energy. The transition point is where the ideal region for storing energy ends. (b) The region with a low transmission angle. Energy must be provided by another medium (e.g. spring) to pass through the region.

The part where energy must be released from the spring is around the singularity points (θ of 0° and 180°). Here, the torque from the input is lower than the required output torque. The region around a crank angle of 0° is shown in *figure 6(b)*.

D. Kinematic conditions

The ideal storing and releasing of energy parts of the cycle are indicated at the crank-slider mechanism in *figure 6*. The challenge is to match these parts with the elastic potential energy cycle (EPEC) of a spring. The EPEC of a translational spring is determined by the change in relative distance of the connection points, referred to as the extension cycle. An example of the extension cycle for a translational spring is shown in *figure 7*. The spring must be extended in the same part of the cycle as indicated in *figure 6(a)*, to store energy. Likewise, the spring must be contracted in the corresponding part of the cycle, indicated in *figure 6(b)*, to release energy. With a rotational spring, the EPEC is determined by the change in relative angle between two links.

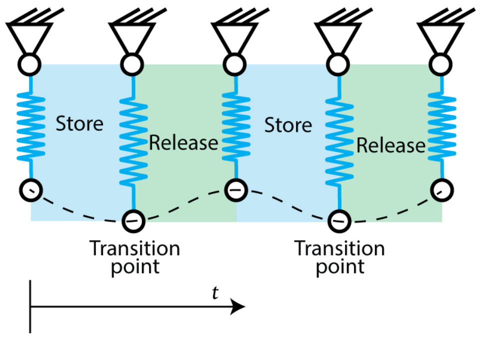

Fig. 7. Example of the extension cycle of a translational spring with respect to time (t). The cycle is divided in 4 parts: 2 energy store parts (blue), 2 energy release parts (green). The transition points are where the spring changes from storing energy into releasing energy.

The placement of a spring in the crank-slider mechanism is critical for the feasability of a concept. To create a working concept, the following two kinematic conditions must be satisfied: (1) The EPEC of a spring must have two transition points; one for each singularity point. The transition point is where the spring changes from storing energy into releasing energy. With two transition points, the elastic potential energy cycle is divided into four parts; two release of energy parts and two energy storing parts. (2) The singularity points must be in the energy release parts of the cycle.

E. Trajectory search

With the stated kinematic conditions known, the placement of the spring is investigated to find the desired elastic potential energy cycle. Without the use of additional links or linkages, the spring must be connected between the available links (crank, slider or coupler) or a link and the ground.

Every point on the crank-slider mechanism travels through a trajectory during the motion conversion. The shape of the trajectory determines the possibility of providing two transition points for translational springs. For rotational springs, the relative angle between the links is used.

Rotational springs
The rotational springs can be applied in the three revolute joints of the crank-slider mechanism. The two joints that are connected to the crank, do not provide any transition point, as the relative angle is just increasing. In this case, energy is only stored or only released in the spring.

The relative angle between the coupler link and the slider provides two transition points. However, it is not possible to have both singularity points in the energy release part of the rotational spring.

Crank
Any point on the crank is travelling in a circlulair motion. A circular trajectory is limited to provide only one transition point for translational springs. Therefore, connecting a translational spring to any point on the crank, will not result in a possible solution to pass through singularity.

Slider
Any point on the slider is travelling in a rectilinear motion. The line trajectory is able to provide two transition points for translational springs. However, the singularity points are at the same positions as the transition points. When the singularity points are not in the energy release part of the cycle, the energy is stored in the unfavourable region with a low transmission angle. Therefore, the mechanism is not able to pass through singularity by connecting a translational spring to the slider.

Coupler link
Connection points on the coupler link have a great variety of trajectories (circular, line, elipse, etc.). Excluding circular trajectories, most of the trajectories provide two transition points. Moreover, many solutions are found (*section II.F*) in which the singularity points are located in the release of energy parts of the spring. Therefore, crank-slider mechanisms are able to pass through singularity by connecting a translational spring between the ground and specific points on the coupler link.

F. Possible solutions

All the connection points of the coupler link are investigated with a grid search. The grid search is conducted with the generalized coördinates [length c, angle δ]. The configuration ($\theta = 0$) of the crank-slider mechanism ($\lambda = 6$) is used as a reference. The revolute joint of the slider is equalized with the origin ($x = 0$, $y = 0$). The grid is normalized to the length of the coupler link and shown in *figure 8*.

The length c is varying from $0 - 2b$. The angle δ of the extension of the coupler link is varying from $0° - 360°$. For each connection point of the coupler link the trajectory is calculated. The position of each component is modelled in MATLAB, corresponding to the crank angle (θ varying from $0° - 360°$).

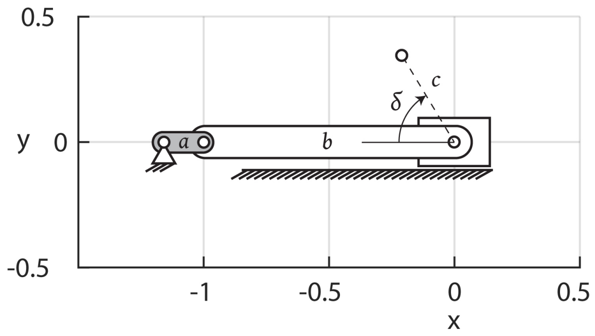

Fig. 8. Reference configuration ($\theta = 0$) of the crank-slider mechanism ($\lambda = 6$). The grid is normalized to the length of the coupler link. The generalized coördinates [length c, angle δ] are used for the grid search.

The ground connection of the spring is placed on the midpoint of the line between the transition points. With the position of the connection points, the extension of the spring (x_{sp}) is determined.

The spring characteristics (unstretched length, stiffness) are based on the required output load and the extension cycle of the spring. The spring constant (k_{sp}) is determined from the difference between the maximum (X_{max}) and minimum (X_{min}) distance of the connection points.

$$k_{sp} = \frac{2 \cdot W_s}{(X_{max} - X_{min})^2} \tag{4}$$

With W_s as the amount of energy (work) required to move the load over the largest unfavourable region. In case of the example in *section II.A*, the largest unfavourable region is 57° wide. This calculation ensures that enough energy can be stored in the spring to move the mechanism through singularity. The unstretched length of the spring is equal to the minimal distance between the connection points, to eliminate pretension in the system.

With the extension cycle and spring characteristics known, the force transmission of the spring is calculated:

$$T_{sp}(\theta) = -\frac{1}{2}\, k_{sp}\, \frac{d(x_{sp}^2)}{d\theta} \tag{5}$$

The torque from the spring (T_{sp}) is negative in the store of energy part, as the extension of the spring is increasing. In contrast, the torque is positive in the release of energy part.

The sum of the torques, from the input force and the spring force, represents the total torque transmission. The minimum in the total torque transmission is equal to the maximum drivable load. The maximum drivable load of the connection points with the coupler link are used to create a contour plot, shown in *figure 9*. The red colour indicates the highest possible (*equation 3*) maximum drivable load of 0.628 with respect to the maximum input of the slider. Dark blue indicates the presence of singularity points, as the force transmission is equal to zero.

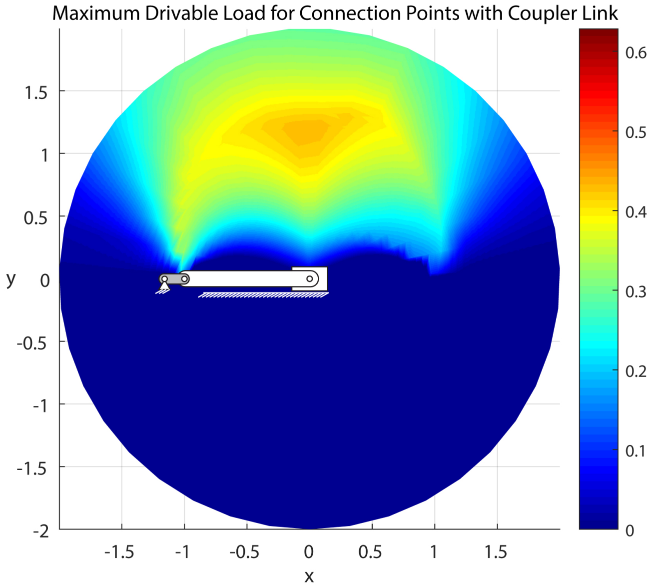

Fig. 9. Contour plot to indicate the maximum drivable load for different connection points with the coupler link. Dark blue indicates the presence of singularity points with a force transmission equal to zero. Red indicates the highest possible (*equation 3*) maximum load of 0.628 with respect to the maximum torque of the slider.

The contour plot is calculated with the crank rotating clockwise. The connection points below the coupler link ($y < 0$) have a mirrored trajectory from the points above the coupler link ($y > 0$). The shape of these trajectories provide two transition points. However, the singularity points are in the store of energy parts of the spring. Therefore, no possible solutions are found below the crank slider mechanism. If the crank is rotating counter clockwise, the contour plot will be mirrored.

The possible solutions that satisfy the design requirement (maximum drivable load of 0.4), are in the range of the yellow to red colour of the contour plot, as shown in the colourbar of *figure 9*.

A single concept is chosen from the available variations that are shown in the contour plot. The concept with the shortest length for the extension of the coupler link ($c = 0.7b$), with an angle ($\delta = 0°$), is used to minimize the footprint of the mechanism.

G. The concept

The coupler link is extended to create a connection point with a specific trajectory, shown in *figure 10*. The placement of the ground connection as explained in *section II.F* combined with this trajectory, results in an unstretched length of the spring equal to zero. Therefore, the connection with the ground is placed on the perpendicular bisector of the transition points. More details about the placement of the ground connection is discussed in *Appendix A*. The height from the midpoint is equal to the unstretched length of the spring. In this case, the unstretched length is chosen to be a quarter of the width of the trajectory.

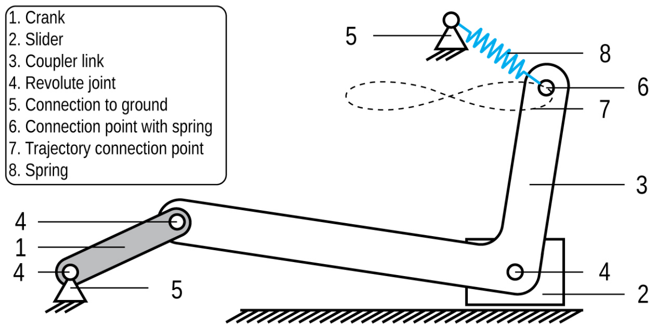

Fig. 10. Schematic representation of crank-slider mechanism with extended coupler link and spring connected to the ground.

The part of the cycle where energy is stored is shown in *figure 11(a)*. The spring and the trajectory of the connection point are isolated to create a detailed picture. The motion starts with the spring at the minimal extension, from which it is extended until the transition point. At the transition point, the spring is maximally extended and switches from storing energy into releasing energy.

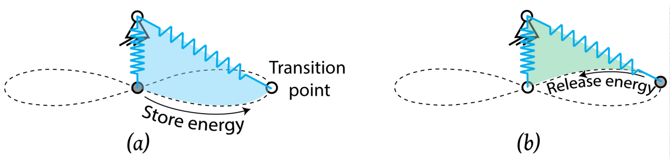

Fig. 11. (a) The part of the trajectory where the spring is extended to store energy until the transition point. This region is corresponding to the region with a high transmission angle at the crank. (b) The part of the trajectory where the spring is contracted to release energy. This region is corresponding to the region with a low transmission angle at the crank. The released energy is used in the mechanism to pass through singularity.

The motion where energy is released from the spring is indicated in *figure 11(b)*. The singularity point is positioned halfway the energy release part. At the end of the release part, the mechanism moved beyond the unfavourable region. In this way, the crank is actuated in a clockwise direction with only a single slider as input. The cycle continues with a mirrored motion of the store and release of energy parts, to pass through the second singularity point. The extension of the spring is normalized and shown in *figure 12*.

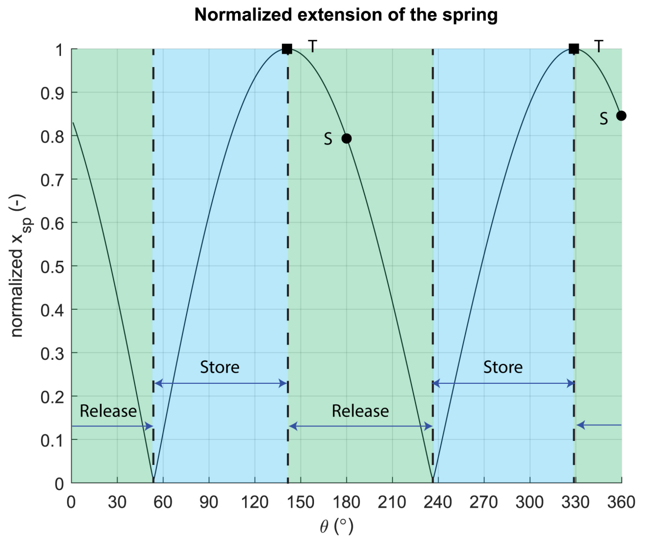

Fig. 12. Example of the normalized extension (x_{sp}) of a spring with respect to the crank angle (θ). The cycle is divided in 4 parts: 2 energy store parts (blue), 2 energy release parts (green). The dots (S) indicate the singularity points. The squares (T) indicate the transition points.

Theoretical model

Figure 14 shows the torque from the slider (blue), calculated with *equation 2*. The torque from the spring (green) is calculated with *equation 4*. The sum of the torques represents the total (red) force transmission of the system. The dashed line indicates the maximum constant load that can be driven. With the shown concept, the maximum drivable load is 40% of the maximum transmitted torque.

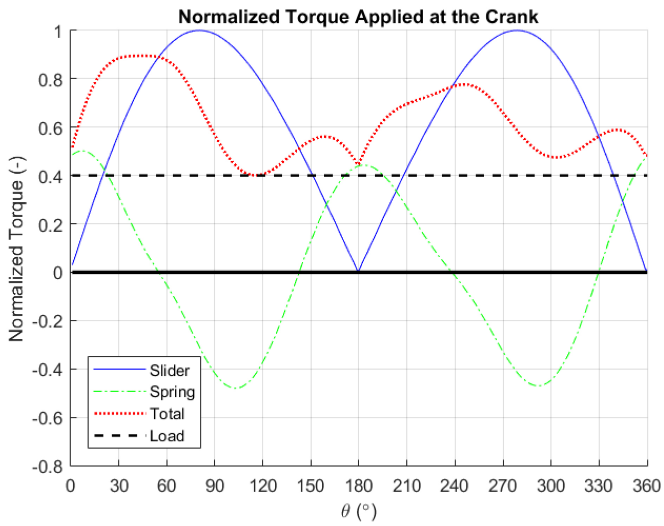

Fig. 14. Transmission overview of the crank-slider mechanism with extended coupler link and spring connected to the ground, normalized for the maximum transmitted torque of the slider. The torque applied from the slider (blue), spring (green) and combined (red). The dashed line is indicating the maximum drivable load.

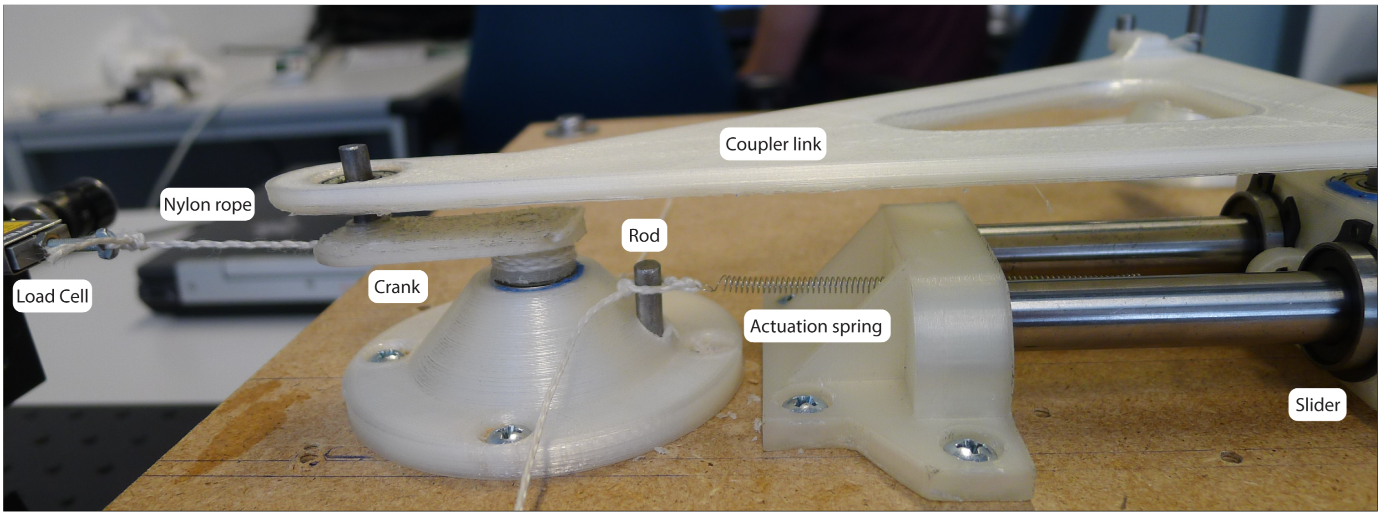

Fig. 13. Front view of the experimental test setup of the crank-slider mechanism ($\theta = 180°$). The load cell connected with the nylon rope to the crank to measure the torque output of the mechanism. The actuation spring connected between the rod and the slider.

The force transmission of the concept is calculated with the velocity of the crank, the slider and the connection point of the spring. The method is based on calculations in force transmissions for gears. In the crank slider mechanism, the change in position is used to calculate the velocity difference. The considered speed is low (crank speed $\leq$ 1 RPM), with minimal accelerations. Therefore, it is assumed to neglect the dynamic forces.

III. Fabrication and Experimental Setup

The concept is fabricated for experimental validation of the proposed modelling. First, the fabrication method is shown. Second, the experimental setup is explained.

A. Fabrication

The force transmission in crank-slider mechanisms is independent of the scale. For mechanism design, the characteristics of the spring are based on the actuation force and required output load. In case of the experiment, an off-the-shelf spring is used with the dimensions and characteristics shown in *table* I. The dimensions of the crank-slider mechanism and the loads are based on these values. In other words, the spring characteristics are dominant for the design of the experiment.

TABLE I. Dimensions and characteristics of the spring used to store elastic potential energy in the experiment.

Spring dimensions and characteristics				
Stiffness	Natural length	Wire Thickness	Diameter	Number of windings
56.8 N/m	20 mm	0.5 mm	8 mm	26

With the dimensions and characteristics of the spring known, the crank-slider mechanism is scaled. The dimensions for the experimental model are shown in *table II*. The crank-slider mechanism is built on a wooden MDF plate that is used as base. The individual parts are designed and 3D printed in PLA (polylactic acid). A small pin is added to the crank, which can be used in combination with a rope to measure the exerted torque during the motion, shown in *figure 13*.

Bearings are used for moving parts to reduce friction in the system. Rotational ball bearings are used for the revolute joints. The rectilinear guiding of the slider is done with two parallel rectilinear bearings, mounted in a 3D printed part. The bearings are guided by two hardened shafts. The slider is added with a hook on each side for actuation in both directions.

TABLE II. The dimensions of each part, used in the experimental model.

Dimensions of the experimental model	
Part	**Length [m]**
Crank (a)	0.03
Coupler link (b)	0.18
Extension (c)	0.126

B. Experimental Setup

The experimental validation is done with a force-displacement measurement. The reaction forces are measured at the crank to calculate the output torque, with an actuated slider as input.

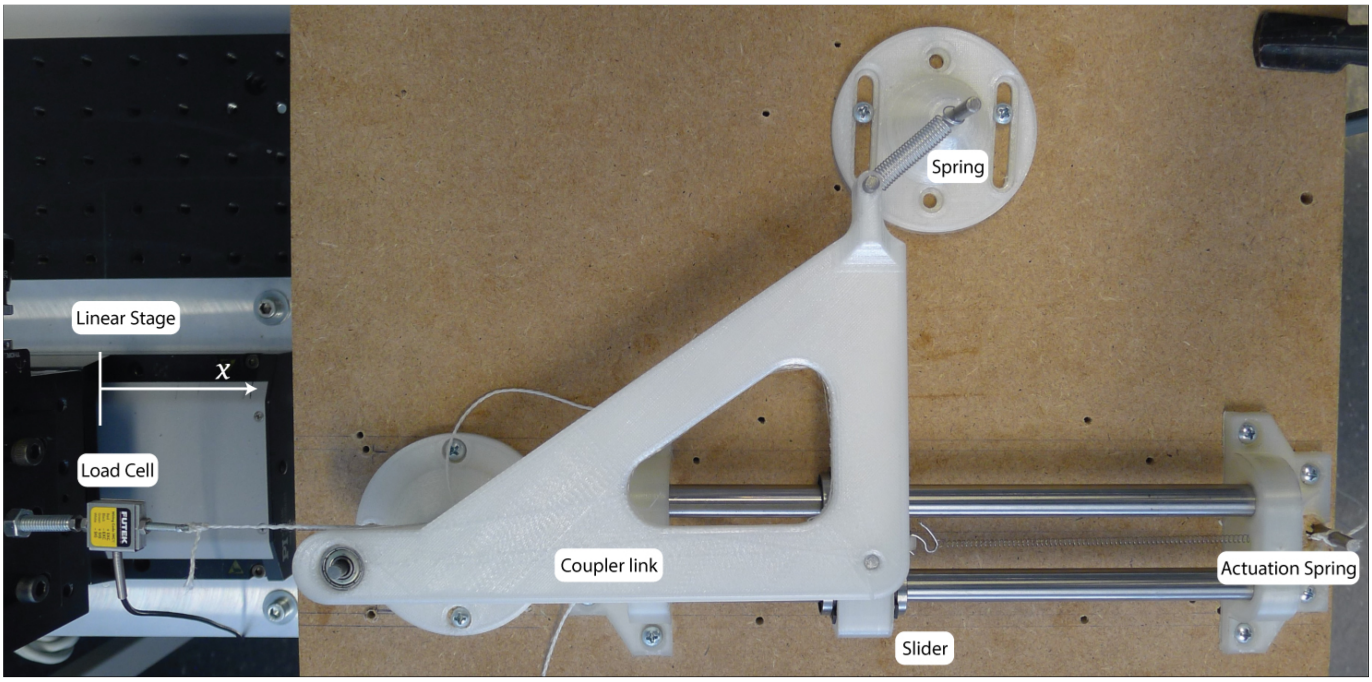

Fig. 15. Top view of the experimental test setup with the crank-slider mechanism ($\theta = 180°$). With the rectilinear stage, load cell, actuation spring, crank-slider mechanism with extended coupler link and spring connected to the ground. The rectilinear stage allows for motion in the x-direction, creating a clockwise rotation of the crank.

The reaction forces are measured by a load cell connected to the crank with a nylon rope (t = 1 mm). The nylon rope is wound around the pin (r = 7 mm). The output torque is calculated with the total radius (pin and rope combined) and the reaction forces. The load cell is mounted on a rectilinear stage that controls the position of the mechanism. The position of the stage is measured with a distance sensor. The stage is moving in x direction, shown in *figure 15*, allowing for clockwise rotation. The measured travelled distance of the stage is converted to an angular displacement of the crank.

In the theoretical model a constant input force is used at the slider. In the experiment, due to friction problems, it is chosen to directly actuate the slider with springs, shown in *figure 15*. Two springs are used that are activated, one at a time. This is done manually by connecting one end of the spring with a rod and disconnecting the other spring. The actuation with springs does not result in a constant input force. Therefore, the experiment is different from the theoretical model, explained in the *section II.G*. However, the actuation with springs can be implemented in the theoretical model. This updated model is used for validation.

IV. Results

Figure 16 shows the updated theoretical model and measured results. The transmitted torque, indicated in blue, is not equal for both strokes of the slider as two different springs with different pretension are used. It can be seen that the measured results are not starting at a crank angle of zero ($\theta = 0°$). The offset is created to maintain tension on the rope and prevent backwards rotation. If the measurement is started in the singularity point, the direction of motion is unstable.

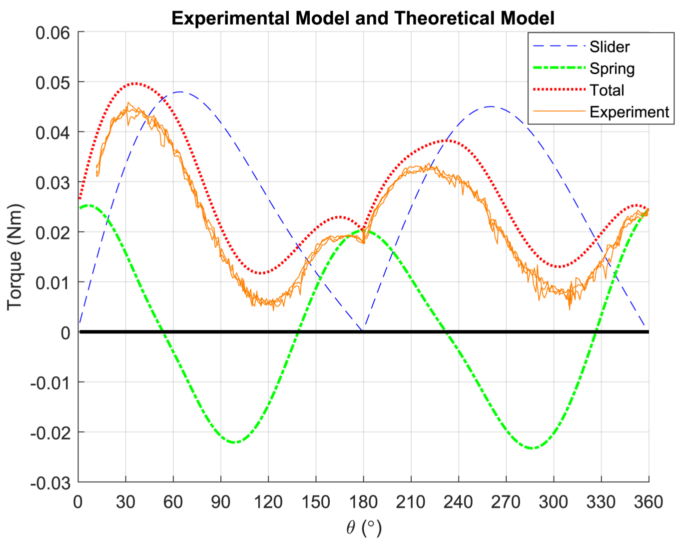

Fig. 16. The torque-angular displacement graph of the experiment. The blue line is indicating the torque from the spring input at the slider. The green line is the torque from the spring. The red line is indicating the theoretical sum of the input and spring. 3 measured values are plotted on top of each other.

At the other singularity point ($\theta = 180°$), a small vertical line is observable. In this position, the actuation at the slider is changed manually from one side to the other. During this process, a small additional torque is transmitted to the crank. However, the minimum transmission at this point is of interest, not the maximum value. In the singularity points, the force from the slider is not transmitted to the crank. The only torque present at this position is caused by the spring that is connected to the coupler link.

V. Discussion

The solution is shown for a crank-slider mechanism (with $\lambda = 6$), and can be scaled to any size or force. However, the ratio λ influences the kinematics and subsequently the force transmission charactersitics (e.g. maximum transmitted torque), explained in *appendix B*. Therefore, the contour plot is not universal for each ratio λ.

The lowest torque in the theoretical model of *figure 14* satisfies the design requirement of 40% of the maximum transmitted torque. From this concept an experimental setup is built for validation of the modelling. Due to friction problems, a different type of actuation is chosen (with springs). An updated theoretical model is made and presented in *figure 16*. The maximum drivable load of the updated theoretical model, is lower than the concept with constant force input of *figure 14* . This is because the torque from the input is significantly lower between $90° - 140°$. Furthermore, the spring stiffness is too high, in contrast to the input force. Basically, the spring is too strong and stores more energy than necessary. The experimental setup was not designed for the actuation by springs on the slider.

Differences are present between the updated theoretical model and the experiment. Friction is not taken into account for the updated theoretical model. This explains the downward shift of the measured torque. However, the experimental results are showing a very similar trend. Therefore, the modelling proposed in this paper is validated by the experimental measurements. Furthermore, at every point in the mechanism a force transmission is present. This proves the method of using elastic potential energy to overcome singularity problems in crank-slider mechanisms. The noise in the signal is caused by a stick-slip motion of the mechanism, due to the friction.

The principle of using elastic potential energy could also be used in other transmission mechanisms with singularity problems, if both the kinematic conditions are satisfied. The kinematic conditions are stated to solve the singularity problems in general. The method that is applied for the crank-slider mechanism in this paper might inspire the creation of solutions in other mechanisms with singularity problems.

VI. Conclusions

Classically, the problems of singularity are avoided with a margin based on friction in the mechanism. In this paper, a margin of torque transmission has been proposed.

It was found that a passive concept using elastic potential energy is able to overcome the singularity problems. It was shown that solutions are available using a single spring without extra links or linkages.

It was shown that the elastic potential energy cycle of the spring must have two transition points. Moreover, the singularity points must be in the release of energy part of the spring. These two kinematic conditions must be satisfied to create solutions for passing through singularity with elastic potential energy.

It was illustrated theoretically that a drivable load of at least 40% of the maximum output torque is possible, during a full cycle motion when the slider is used as input in crank-slider mechanisms. Besides, the contour plot was shown for multiple variations of connecting a translational spring between the coupler link and the ground. The experimental measurements have validated the modelling proposed in this paper.

This paper presents the first solution of creating a full cycle rotation with a single rectilinear actuator for low speed applications. The proof of concept enables the next step towards a simple and compact product in microengines. Furthermore, a mass travelling in a translational motion is now accessible for harvesting energy.

References

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4

Conclusions

Classically, the problems of singularity are avoided with a margin based on friction in the mechanism. It was found that a margin based on torque transmission also causes problems, even without the presence of friction.

It was proposed that existing singularity manipulation strategies belong in one of the six classifications, based on their fundamental differences. Moreover, it was shown that not all classifications are able to pass through singularity. Strategies that change the kinematics, are only able to shift the singularity points.

A passive concept using elastic potential energy was found that is able to overcome the singularity problems. It was shown that solutions are available using a single spring without extra links or linkages.

It was shown that the elastic potential energy cycle of a spring must have two transition points, where the spring changes from storing energy into releasing energy. Moreover, the singularity points must be in the release of energy part of the spring. These two kinematic conditions must be satisfied to create solutions for passing through singularity with elastic potential energy.

It was illustrated theoretically that a drivable load of at least 40% of the maximum output torque is possible, during a full cycle motion when the slider is used as input in crank-slider mechanisms. It was shown that multiple variations are available for connecting a translational spring between the coupler link and the ground. The experimental results have validated the proposed modelling.

This thesis presents the first solution of creating a full cycle rotation with a single rectilinear actuator for low speed applications. The proof of concept enables the next step towards a simple and compact product in microengines. Furthermore, a mass travelling in a translational motion is now accessible for harvesting energy.

5

Future Work and Recommendations

Using elastic potential energy to pass through singularity is proven in this thesis for crank-slider mechanisms. However, singularity problems are also present in other mechanisms (e.g. double slider mechanism). Therefore, it is recommended to investigate these mechanisms for specific trajectories that supply two transition points. The relative angle between links could also be used to provide two transition points. The method used for crank-slider mechanisms is specified with two kinematic conditions. The same kinematic conditions must be satisfied for solving similar singularity problems in other motion converters.

This research is focussed on providing a proof of concept for passing through singularity with elastic potential energy. A general search is done to find the trajectories possible of satisfying the kinematic conditions. In this search, the spring constant, unstretched length and placement of the spring are calculated, based on the dimensions of the trajectory. However, this might not provide the highest possible drivable load. Therefore, it is recommended to optimize the maximum drivable load in the crank-slider mechanism with a translational spring connected to the coupler link. This specific optimization is outside the scope of this research. From the optimization a GUI could be built that automatically generates the optimized solution by adjusting a couple of parameters, provided by the user.

The experimental measurements show a similar trend of the theoretical model, with a downward shift caused by the friction in the mechanism. In the theoretical model an approximation of the friction could be made. This would increase the correlation between the two models. However, the friction is different for each application. The theoretical model is applicable for any scale of crank-slider mechanism. The experimental model is made with 3D printed parts that have large tolerances. An improved measurement setup could be built according to the optimized theoretical model to find the experimental maximum drivable load.

The next step in mechanism design is the conversion of the concepts into compliant mechanisms. In micro mechatronics, it is not possible (yet) to produce translational coil springs with surface micromachining. Therefore, an alternative concept must be found to store elastic potential energy using leaf springs. A benefit of compliant mechanisms is the possibility of creating a monolthic design, a mechanism manufactured out of one piece..

Appendices

A

Ground Connection Spring

Once a trajectory is found that is able to provide two transition points for a translational spring, the placement of the ground connection needs to be chosen. The placement of the ground connection for the spring is discussed in this chapter. In the case of the crank-slider mechanism, many different trajectories can be found. A method is proposed to determine the transition points. As an example, four different trajectories are shown in *figure A.1*.

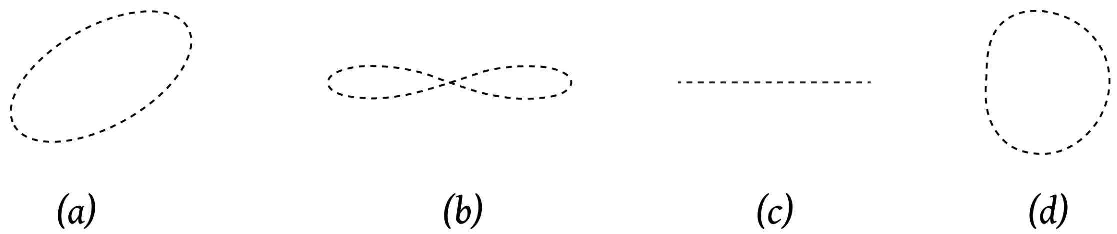


Figure A.1: Four different shapes of trajectory that provide two transition points. With (a) ellipse shape, (b) infinite sign shape, (c) line and (d) wedge shape.

The first step in choosing the ground connection is finding the transition points (T). In the trajectory, the points with the largest relative distance, are considered to be the transition points. A line between the transition points provides a midpoint (M) on which the ground connection can be placed. This is shown in *figure A.2*.

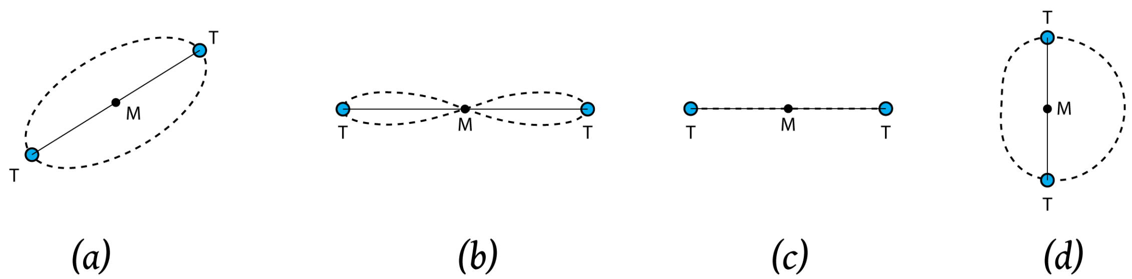


Figure A.2: Four different shapes of trajectory that provide two transition points. With (a) ellipse shape, (b) infinite sign shape, (c) line and (d) wedge shape. The transition points (T) are indicated with the blue dots. A line between the transition points provides the midpoint (M).

The second step in determining the placement of the ground connection is based on the unstretched length (l_n) of the spring. The minimal distance between the connection points of the spring must not be smaller than l_n to prevent undesired interference between the spring and the mechanism. If the trajectory is intersecting with the midpoint (in case of (b) and (c)), the ground connection can be placed on the perpendicular bisector of the transition points, in y-direction, as shown in *figure A.3*. It is recommended to equalize the l_n with the minimal distance between the connection points to reduce pretension in the spring, as this would increase the friction in the joints.

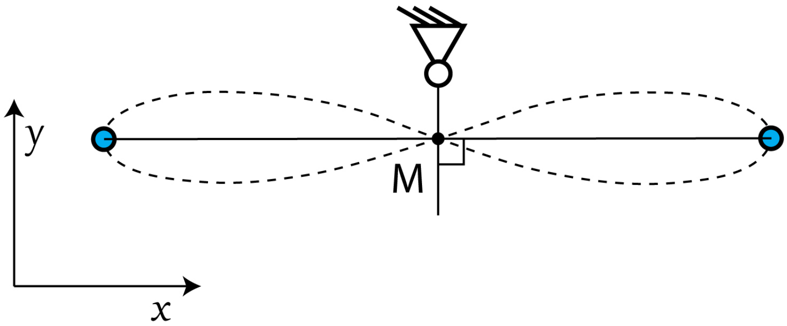


Figure A.3: The ground connection of the spring on the perpendicular bisector of the transition points.

The placement of the ground connection can also be moved sideways, in x-direction, as shown in *figure A.4*. In this case, the spring is extended more for the region around the first singularity point (S1). The limit for the placement of the ground connection is determined by the singularity points. If the ground connection is placed on or beyond the limit, the singularity points will not be in the release of energy part of the spring. Therefore, the second kinematic condition is not satisfied and the mechanism is not able to pass through singularity.

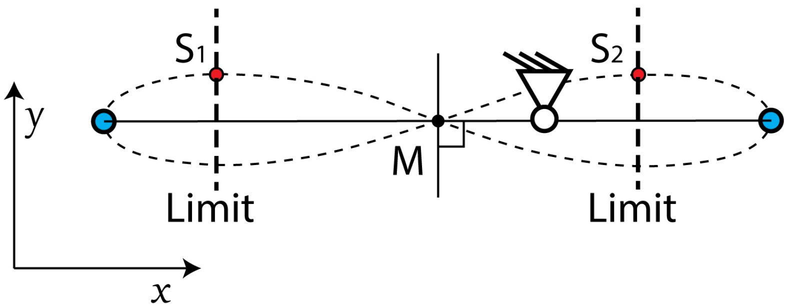


Figure A.4: The ground connection of the spring moved sideways, in x-direction, with limits defined by the singularity points (S1 and S2).

B

Influence of geometry

The ratio λ between the length of the coupler link (b) and the length of the crank (a) influences the kinematics of the crank-slider mechanism. The kinematics are determining both the trajectory of the connection points with the coupler link and the velocity of each part. With the new velocity profiles, the corresponding force transmission is calculated. Moreover, a grid search is done for different ratio λ to investigate the general implementation of the contour plot in *chapter 3*.

$$T_i(\theta) = F\,\frac{d(x_{sl})}{d\theta} \tag{B.1}$$

$$\lambda = \frac{b}{a} \tag{B.2}$$

This chapter first shows the influence of the ratio λ for the transmitted torque at the crank, shown in *figures B.1 - B.6*. The ratio varies in six steps from 2.5 to ∞. In the figures, the theoretical highest possible constant output torque is plotted. This is equal to the average transmitted torque (T_{avg}):

$$T_{avg} = \frac{F * 2a}{\pi} \tag{B.3}$$

Second, the adjusted contour plots are shown in *figures B.7 - B.12*, in the same six steps. The colourbar is adjusted to the theoretical highest possible constant output torque for each ratio, indicated in red. Dark blue indicates the presence of singularity points, as the force transmission is equal to zero.

In the contour plots, the same shape appears for every ratio. For the higher ratio's, the found maximum drivable loads are higher. This is because the range for singularity problems is smaller.

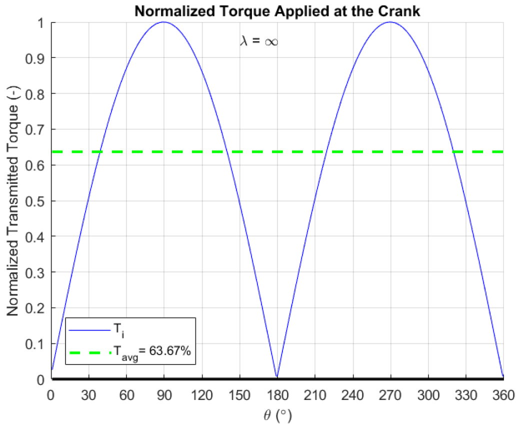


Figure B.1: The force transmission of a crank-slider mechanism ($\lambda = \infty$).

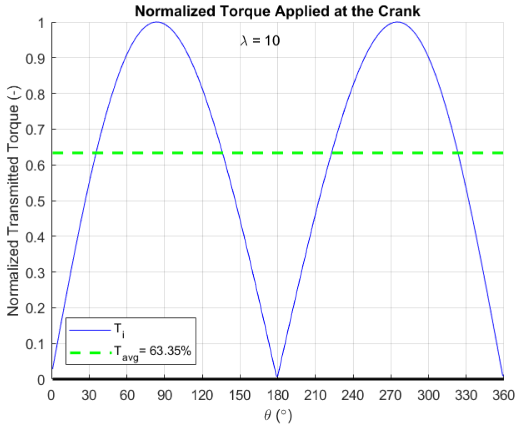


Figure B.2: The force transmission of a crank-slider mechanism ($\lambda = 10$).

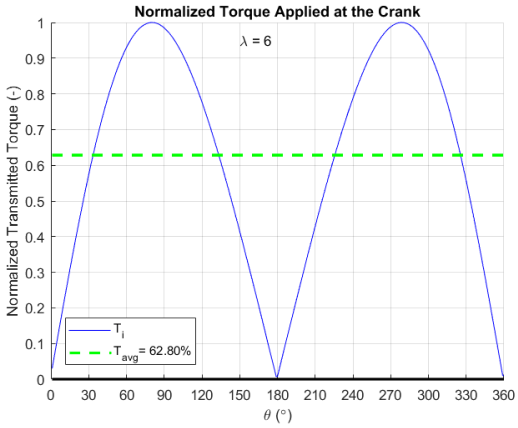


Figure B.3: The force transmission of a crank-slider mechanism ($\lambda = 6$).

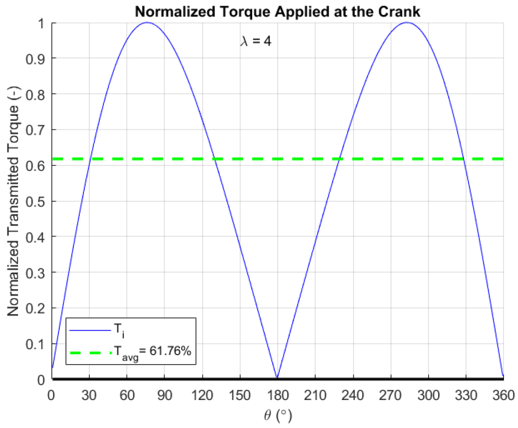


Figure B.4: The force transmission of a crank-slider mechanism ($\lambda = 4$).

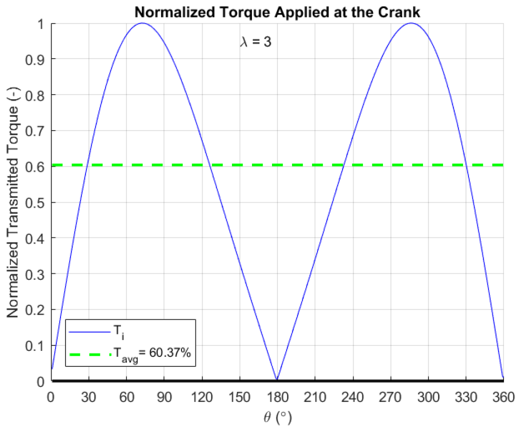


Figure B.5: The force transmission of a crank-slider mechanism ($\lambda = 3$).

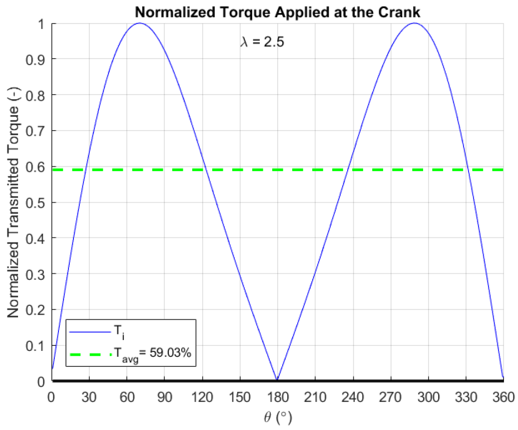


Figure B.6: The force transmission of a crank-slider mechanism ($\lambda = 2.5$).

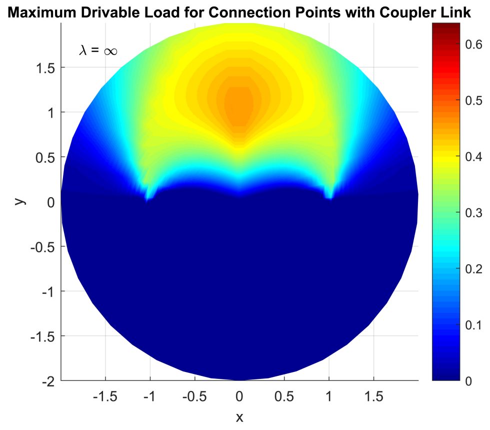


Figure B.7: Contour plot to indicate the maximum drivable load for different connection points with the coupler link. ($\lambda = \infty$)

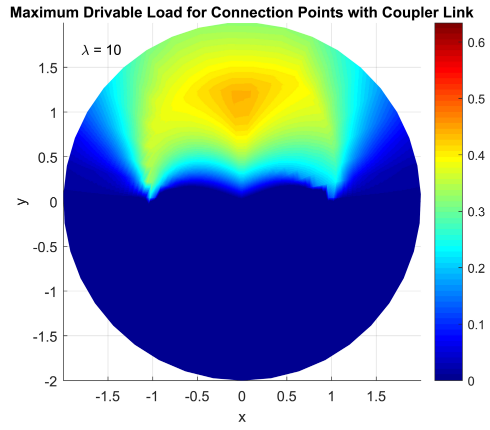


Figure B.8: Contour plot to indicate the maximum drivable load for different connection points with the coupler link. ($\lambda = 10$)

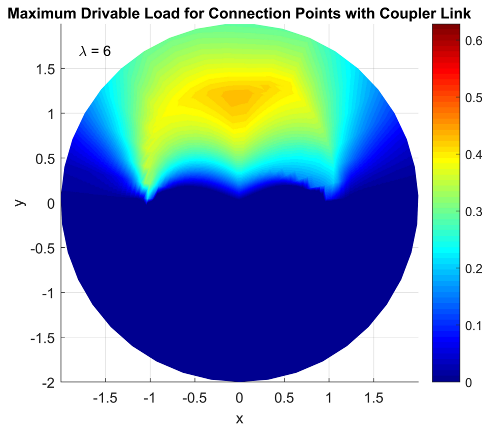


Figure B.9: Contour plot to indicate the maximum drivable load for different connection points with the coupler link. ($\lambda = 6$)

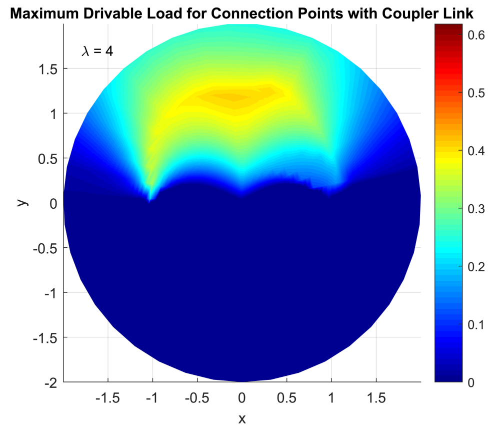


Figure B.10: Contour plot to indicate the maximum drivable load for different connection points with the coupler link. ($\lambda = 4$)

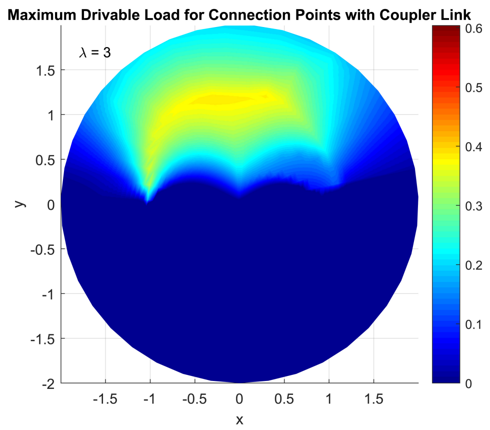


Figure B.11: Contour plot to indicate the maximum drivable load for different connection points with the coupler link. ($\lambda = 3$)

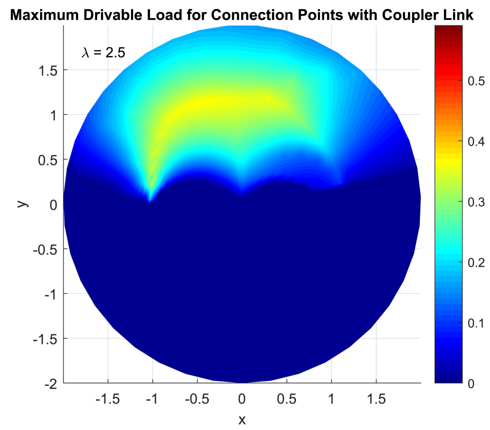


Figure B.12: Contour plot to indicate the maximum drivable load for different connection points with the coupler link. ($\lambda = 2.5$)

C

Measurement Setup

The design of the measurement setup is shown in *figure C.1* with actuation by gravitational forces on mass one (m1) and mass two (m2). The coupler link is extended and connected with a spring to the ground. The force sensor (load cel) is connected to the crank by a rope that is wound around a pin. The displacement of the force sensor (x) is measured to create a torque-angular displacement curve.

The measurement is started with a slight offset ($\theta = 10°$) to ensure clockwise rotation. If the crank is starting in the singularity point, the direction of motion is unstable. The actuation is done with m1 = 0.13 kg and m2 = 0 kg for the first 180 °. At this singularity point, the weights are manually changed: m1 = 0 kg and m2 = 0.13 kg.

It was found that the friction in the pulleys was reducing the actuation force at the slider significantly. To compensate for the friction, the weights could be increased with 70 grams. However, it is chosen to directly actuate the slider with springs, so no pulleys have to be used. This does not result in a constant input force. The theoretical model is adjusted and used for the experimental validation.

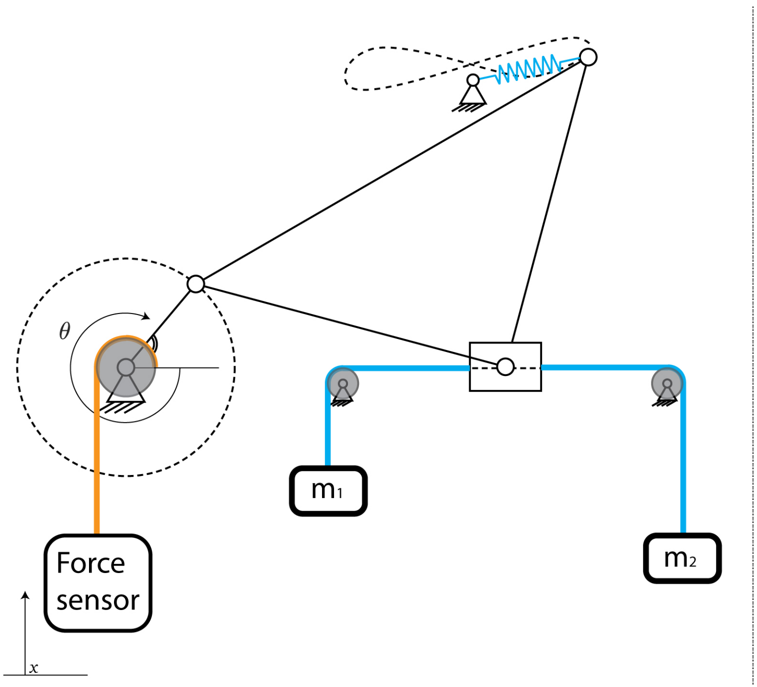


Figure C.1: Original design of measurement setup with actuation by gravitational forces on mass one (m1) and mass two (m2).

The crank, coupler link, slider and holders are 3D printed in PLA (polylactic acid) with the Ultimaker 2. The design of these parts is shown in *figures C.2 and C.3*

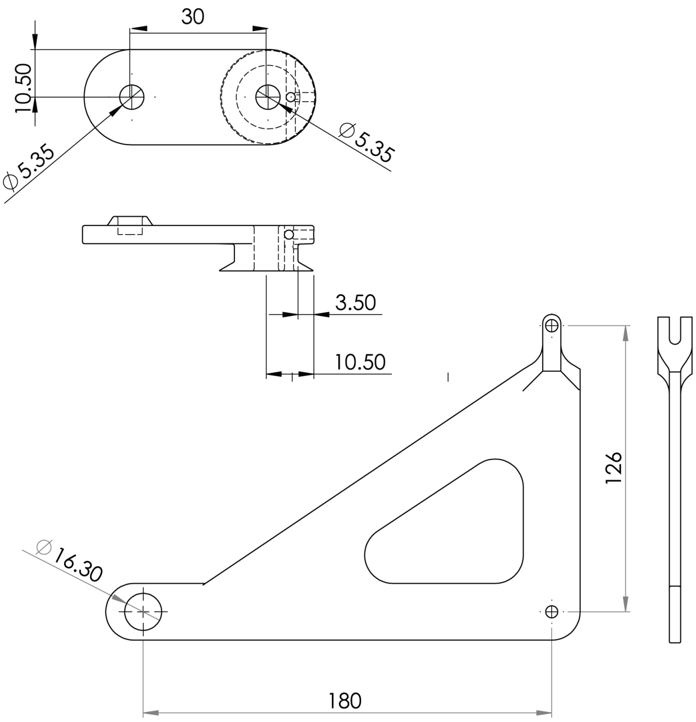


Figure C.2: The design of the crank with added pin on top. The design of the coupler link on the bottem.

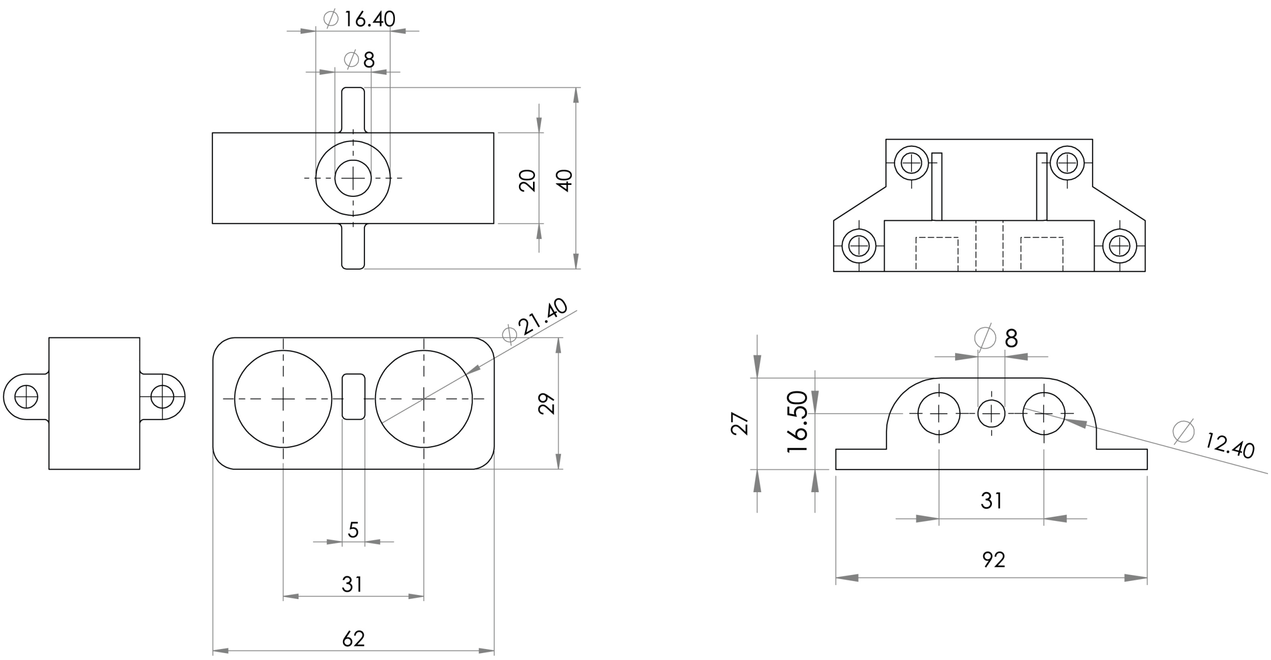
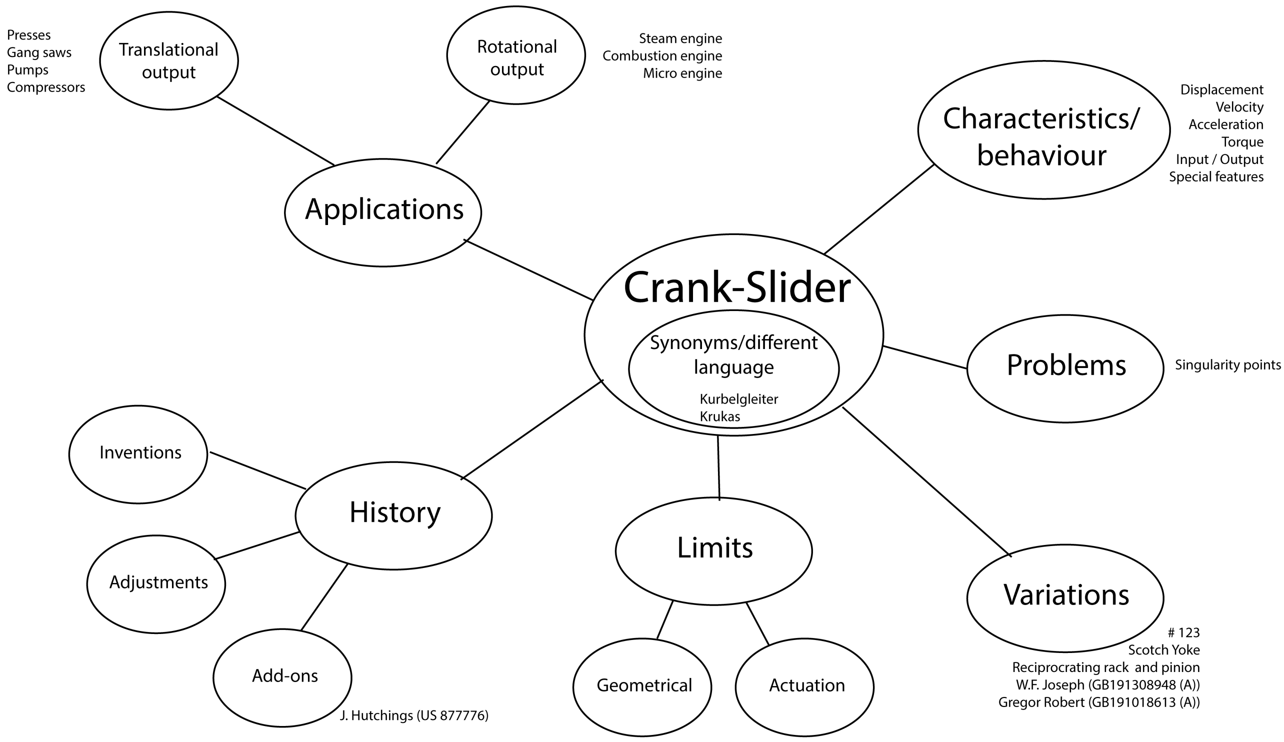


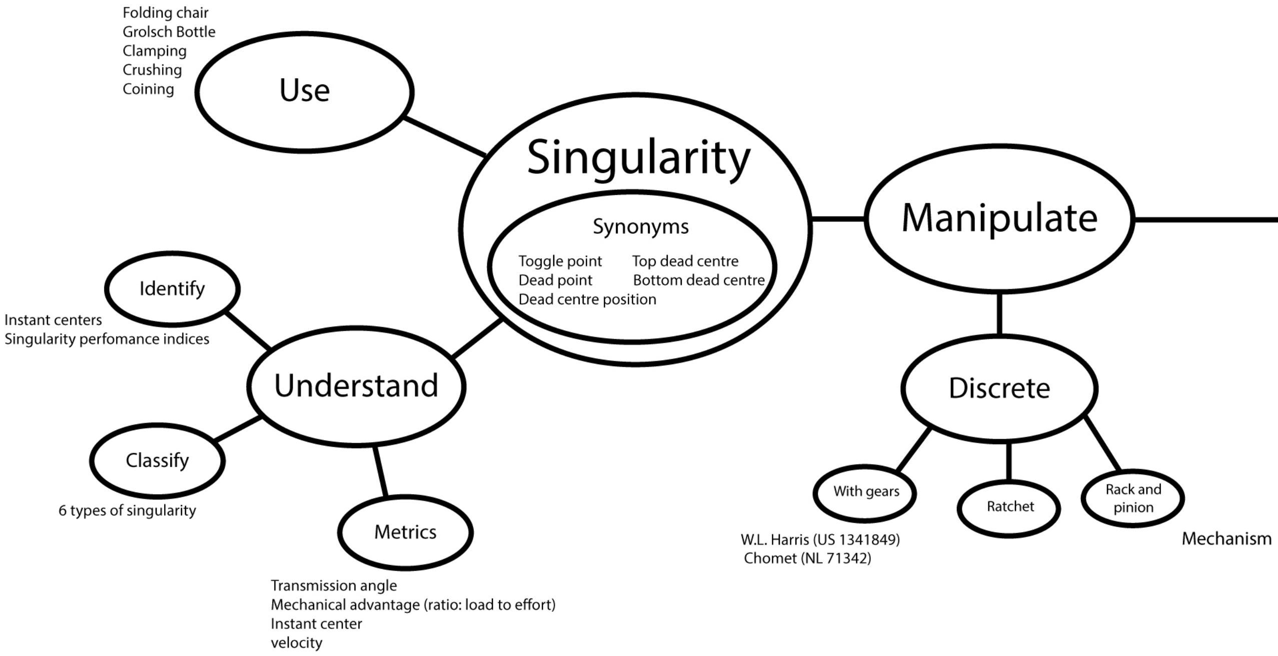
Figure C.3: The design of the slider on the left. The design of the holder on the right.

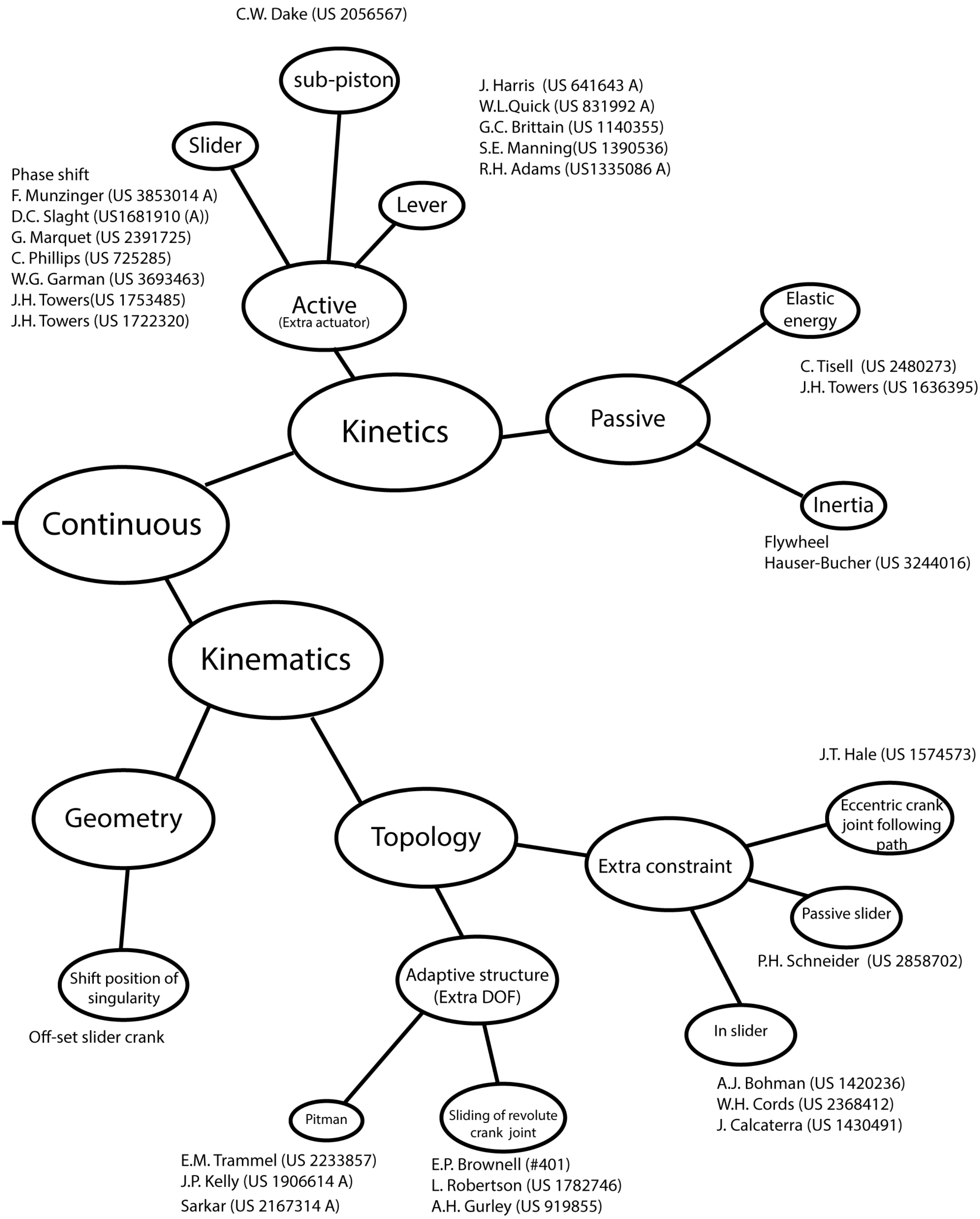
D

Singularity Mind Map

During the literature study, two mind map are constructed for the subjects "crank-slider mechanism" and "singularity". Both mind maps enhanced the literature research. The second mind map is the first categorization of the found literature. From the singularity mind map the classification is made, explained in the literature review paper, *Chapter 2*.




Folding chair
Grolsch Bottle
Clamping
Crushing
Coining
Use
Singularity
Synonyms
Toggle point
Dead point
Dead centre position
Top dead centre
Bottom dead centre
Identify
Instant centers
Singularity perfomance indices
Understand
Classify
6 types of singularity
Metrics
Transmission angle
Mechanical advantage (ratio: load to effort)
Instant center
velocity
Manipulate
Discrete
With gears
W.L. Harris (US 1341849)
Chomet (NL 71342)
Ratchet
Rack and pinion
Mechanism


C.W. Dake (US 2056567)
sub-piston
J. Harris (US 641643 A)
W.L.Quick (US 831992 A)
G.C. Brittain (US 1140355)
S.E. Manning(US 1390536)
R.H. Adams (US1335086 A)
Slider
Lever
Phase shift
F. Munzinger (US 3853014 A)
D.C. Slaght (US1681910 (A))
G. Marquet (US 2391725)
C. Phillips (US 725285)
W.G. Garman (US 3693463)
J.H. Towers(US 1753485)
J.H. Towers (US 1722320)
Active
(Extra actuator)
Elastic energy
C. Tisell (US 2480273)
J.H. Towers (US 1636395)
Kinetics
Passive
Inertia
Flywheel
Hauser-Bucher (US 3244016)
Continuous
Kinematics
J.T. Hale (US 1574573)
Eccentric crank joint following path
Geometry
Topology
Extra constraint
Passive slider
P.H. Schneider (US 2858702)
Shift position of singularity
Off-set slider crank
Adaptive structure
(Extra DOF)
In slider
A.J. Bohman (US 1420236)
W.H. Cords (US 2368412)
J. Calcaterra (US 1430491)
Pitman
Sliding of revolute crank joint
E.M. Trammel (US 2233857)
J.P. Kelly (US 1906614 A)
Sarkar (US 2167314 A)
E.P. Brownell (#401)
L. Robertson (US 1782746)
A.H. Gurley (US 919855)

E

Matlab Code

The Matlab Code to compute the theoretical model and to implement the measured data, shown in the *Design Paper, Chapter 3*.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                   %
%           Crank-Slider mechanism                  %
%               G.J. vd Doel                        %
%           Last update: 27-07-2017                 %
%                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc, clear all, close all

%% Parameters
% Constants
G = 9.81 ;                                  % Gravitational acceleration (m/s^2)
RPM = 30/pi;                                % Transform radians per second to RPM(-)

% Geometry
a = 3e-2 ;                                  % Length of crank (m)
b = 6*a ;                                   % Length of coupler (m)
c = 0.126;                                  % Length of Spring link (m)
alpha = 0;                                  % Angle of Spring link (degrees)
r_load = 7.35e-3 ;                          % Radius of load cilinder (m)
gamma = b/a ;                               % Ratio between crank and coupler (-)

% Spring
k_sp =  56.8;                               % Spring constant [N/m]
sp_gnd = [0.178  0.149];                    % Chosen spring connection with groud [x_pos y_pos]
sp_n = 0.0143;                              % Natural length spring(m)
k2 = 18.1;                                  % Spring constant[N/m]
sp_v2 = 0.0488;                             % pretention(m)
k3 = 18.8;                                  % Spring constant[N/m]
sp_v3 = 0.0467;                             % pretention(m)

% weights = 0.210 ;                         % Weight on the input(kg)
% F_w = 0.7;                                % Measured resistance with 200 gram
% req_act_force = weights*G - F_w;          % Actuation force on the slider

% Vectors
theta = linspace((2*pi)/360,2*pi,360);    % Angle vector of crank [rad]
diff_theta = deg2rad(1)*ones(359);        % Angle vector difference of crank [rad]
theta_d = theta * (180/pi) ;              % Angle of crank [deg]
theta_sing = 56;                          % Degrees that need to be overcome (degrees)

% Loads
m_load = 0.2249;                          % Mass of load that is lifted(kg)
```

```
%% Load measurement files
% Complete data
data_raw0 = load('17 05 23 12 01 35 springs_complete8.txt')
data_raw1 = load('17 05 23 12 13 06 springs_complete10.txt')
data_raw2 = load('17 05 23 12 21 44 springs_complete11.txt')

F0 = data_raw0(:,4)*7.35e-3;
x0 = data_raw0(:,5)/(2*pi*7.35)*360+10;

F1 = data_raw1(:,4)*7.35e-3;
x1 = data_raw1(:,5)/(2*pi*7.35)*360+10;

F2 = data_raw2(:,4)*7.35e-3;
x2 = (data_raw2(:,5)/(2*pi*7.35)*360)+10;

%% Calculations
% Geometry
phi = asin((1/gamma)*sin(theta)) ;            % Angle of coupler [rad]
phi_d = phi * (180/pi) ;                      % Angle of coupler [deg]
TA =180 -(phi_d + theta_d);                   % Transmission angle coupler-crank[deg]

% Load torque
T_load = r_load * m_load* G ;                 % Torque of load that is lifted(Nm)
T_friction = F_r * T_load ;                   % Assumed Friction in system(Nm)
T = T_load + T_friction ;                     % Total Torque that has to be actuated(Nm)

% Work
W_sing = T * deg2rad(theta_sing)   ;          % Total amount of work needed through sing(J)
T_total = T * 2*pi;                           % Total amount of work for one revolution(J)

% Average input and output force
W_degree = T_total / 360 ;                    % Amount of torque for one degree(J)
avg_force = T_total/(4*a);                    % Average force at slider with perfect(N)
T_degree = W_degree/r_load;

% Position of slider
x = a*cos(theta) + b*cos(phi) +a -b ;
x_plot = x +b;
y_plot = zeros(1,360);

% Position of crank joint
x_crank = a*cos(theta);
y_crank = a*-sin(theta);

% Position of spring link
x_sp_link = x + c*-sin(phi+deg2rad(alpha)) + b -a ;
y_sp_link = (c*cos(deg2rad(alpha)+phi)) ;

% Actuation by springs
for z1 = 1:179
req_act_force(z1) = ((x(z1))+sp_v3)*k3;
end

for z2 = 180:360
req_act_force(z2) = (0.06-x(z2)+sp_v2)*k2;
end

% Extension of the spring(m)
for p = 1:length(theta)
sp_ext(p) = sqrt(((sp_gnd(1)-x_sp_link(p))^2)+((sp_gnd(2)-y_sp_link(p))^2))-sp_n;
if sp_ext(p) < 0
sp_ext(p)=0;
else
end
end

% Spring constant
% k_sp = W_sing*2/(max(sp_ext)^2);  =

% Energy in spring
E_sp = 0.5*k_sp*sp_ext.^2;
```

```
%% Velocity and Force calculations
% Calculate the required force with the amount of work
for i = 1:length(theta)-1
diff(i) = x(i)-x(i+1);
F_req(i) = abs((T * diff_theta(i))/diff(i));
T_sp(i)= 0.5*k_sp*((sp_ext(i)^2)-(sp_ext(i+1)^2))*(180/pi);
end

F_req= [F_req F_req(359)];
T_sp = [T_sp T_sp(359)];


%% Torque at crank for different forces
for l = 1:359
T_min_deg(l) = abs(avg_force * (x(l)-x(l+1)))*(180/pi);
T_req_deg(l) = abs(req_act_force(l) * (x(l)-x(l+1)))*(180/pi);
end

T_min_deg = [T_min_deg T_min_deg(359)];
T_req_deg = [T_req_deg T_req_deg(359)];

% Trajectory geometry
max_x_tr = max(x_sp_link)-min(x_sp_link);
max_y_tr = max(y_sp_link)-min(y_sp_link);
Trajectory_geometry = [max_x_tr max_y_tr];

% Total amount of torque at crank
T_total_crank = T_req_deg + T_sp;

% Maximum force on sensor
F_sensor_act = max(T_req_deg)/r_load;
F_sensor_act_spring = max(T_total_crank)/r_load;

% Minimal sensor displacement
x_sensor = r_load *2*pi;


%% plot Data
% reference
desired = 90 * ones(length(phi)) ;
desired_1 = -90 * ones(length(phi));

% limits
limit_up = 40 * ones(length(phi)) ;
limit_up2 = 140 * ones(length(phi));
limit_down = -40 * ones(length(phi)) ;
limit_down2 = -140 * ones(length(phi));

avg_force_plot = avg_force * ones(length(phi));
limit_force = (req_act_force)* ones(length(phi));



%% Plot Transmission angle
figure
hold on
plot(theta_d,TA,'b',theta_d,desired,'g',theta_d,desired_1,'g')
plot(theta_d,limit_up,'--k',theta_d,limit_up2,'--k')
plot(theta_d,limit_down,'--k',theta_d,limit_down2,'--k') xlabel('\theta (rad)')
ylabel('TA (degrees)')
title('Relation between crank angle \theta and transmission angle (TA) ')
axis([0 360 -180 180])
set(gca,'XTick', 0:30:360);
grid on
```

```
%% Plot Torque at crank
figure
hold on
plot(theta_d,T_req_deg,'--b')
plot(theta_d,T_sp,'-.g','LineWidth',1.5)
plot(theta_d,T_total_crank,':r','LineWidth',1.5)
plot(x0,F0,'color',[1 .5 0])
plot(x1,F1,'color',[1 .5 0])
plot(x2,F2,'color',[1 .5 0])
plot(theta_d,zeros(1,360),'-k','LineWidth',2)
axis([0 360 -0.03 0.06])
set(gca,'XTick', 0:30:360);
xlabel('\theta (\circ)')
ylabel('Torque (Nm)')
title('Experimental Model and Theoretical Model')
legend('Slider','Spring','Total','Experiment')
grid on


%% Plot Extension of the spring
figure
hold on
plot(theta_d,sp_ext,'g')
axis([0 360 0 0.10])
set(gca,'XTick', 0:10:360);
xlabel('\theta (rad)')
ylabel('x_{sp} (m)')
title('Extension of the spring')
legend('ln = 0.04, Offset =-0.056, k=27.9')
grid on


%% Plot trajectory alone
figure
hold on
plot (x_sp_link,y_sp_link,'+','LineWidth',1)
plot(sp_gnd(1),sp_gnd(2),'.','LineWidth',2)
plot(x_sp_link(40),y_sp_link(40),'o',x_sp_link(133),y_sp_link(133),'o')
plot(x_sp_link(226),y_sp_link(226),'o',x_sp_link(313),y_sp_link(313),'o')
axis([-0.08 0.6  -0.08 0.6]);
set(gca,'DataAspectRatio',[1 1 1])
grid on


%% Animation
figure
hold on
plot (x_plot,y_plot)
plot (x_sp_link,y_sp_link)
x_circle=a.*cos(theta);
y_circle=a.*sin(theta);
plot(x_circle,y_circle,'-.k','LineWidth',2)
plot(sp_gnd(1),sp_gnd(2),'+','LineWidth',2)
axis([-0.6 0.6  -0.6 0.6]);
set(gca,'DataAspectRatio',[1 1 1])
grid on
```

Reflection

In the Bachelor and Master of Mechanical Engineering, many projects are done in groups. It is very different to do everything yourself, which requires a large amount of discipline. Working on the thesis by myself challenged me to stay motivated and to set deadlines. By working in a structured way, the overall process went fairly smooth. The planning was updated regularly to keep a detailed plan for each week. This helped in working efficiently and reduced the time spent on things that were unnecessary. Everytime, I asked myself *why* I was doing something. Although the question was not always easy to answer, it kept me on the right track.

The structured way of working also implied a thorough literature search. During this search, it was sometimes difficult to understand the working principle of each patent, but I enjoyed finding so many different patents. The entire literature research was very usefull in choosing the design approach for the thesis. Moreover, it was a great inspiration for the generation of concepts.

In the process, I was aware of the busy schedule for both my supervisors. To respect their precious time, I prepared each meeting carefully. With short presentations, it was possible to get the most out of it. The prepared presentations, together with notes helped in the finalisation of the thesis.

Building the experimental setup took more time than initially planned. I underestimated the influence of friction on the measurement. A lot of fine tuning was done to obtain the final results.

I planned to start writing the thesis during the entire design process. However, the parts I wrote on forehand were not usable for the design paper. Most of the thesis was written after the research was done. This is something I would definitely do differently next time. The process of writing was very time consuming and lacks diversity.

It was important to keep a good balance between study, work and side activities. I worked in a restaurant for one or two days a week. It was refreshing to be in a total different environment and work with my hands, in contrast to sitting behind a desk. In addition, I occasionally surfed and worked out in the gym. This has been essential in staying motivated, being persistant and focused.

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Acknowledgements

I would like to thank my supervisors Just and Davood for their time and great counselling during the project. The positive support always increased my motivation. It was a pleasure to work together for the past year.

I also want to thank my parents for mental en financial support, I could not have done this without you. Thanks Sandra Kaandorp for coaching me through the last part of the project. Thanks to Jo Creemers for always providing good advice and a nice dinner when I didn't felt like cooking. Thanks to my sister Myrthe, for calling me regularly from Belgium to provide a listening ear when I needed it.

A special thanks to the technicians of PME: Harry Jansen and Patrick van Holst for the great help in setting up the experimental setup. The measurement setup would not be as smooth if it wasn't for you. Thanks Jos van Driel from the MEETSHOP, for installing and preparing all the sensors.

I would like to thank my friends: Naphur van Apeldoorn, IJsbrand de Lange, Wilco Vreugdenhil, Timo Aaldriks and Marleen Vos for the great feedback during the project.

Thanks to Steven van 't Klooster for pushing me in the gym and keeping me in shape. The weekly excersize helped me to focus during the project and empty my head when it was necessary. Thanks to other friends and family for always supporting me. It has been a great journey.