Solution methods for migrating fibre in current telecommunication networks

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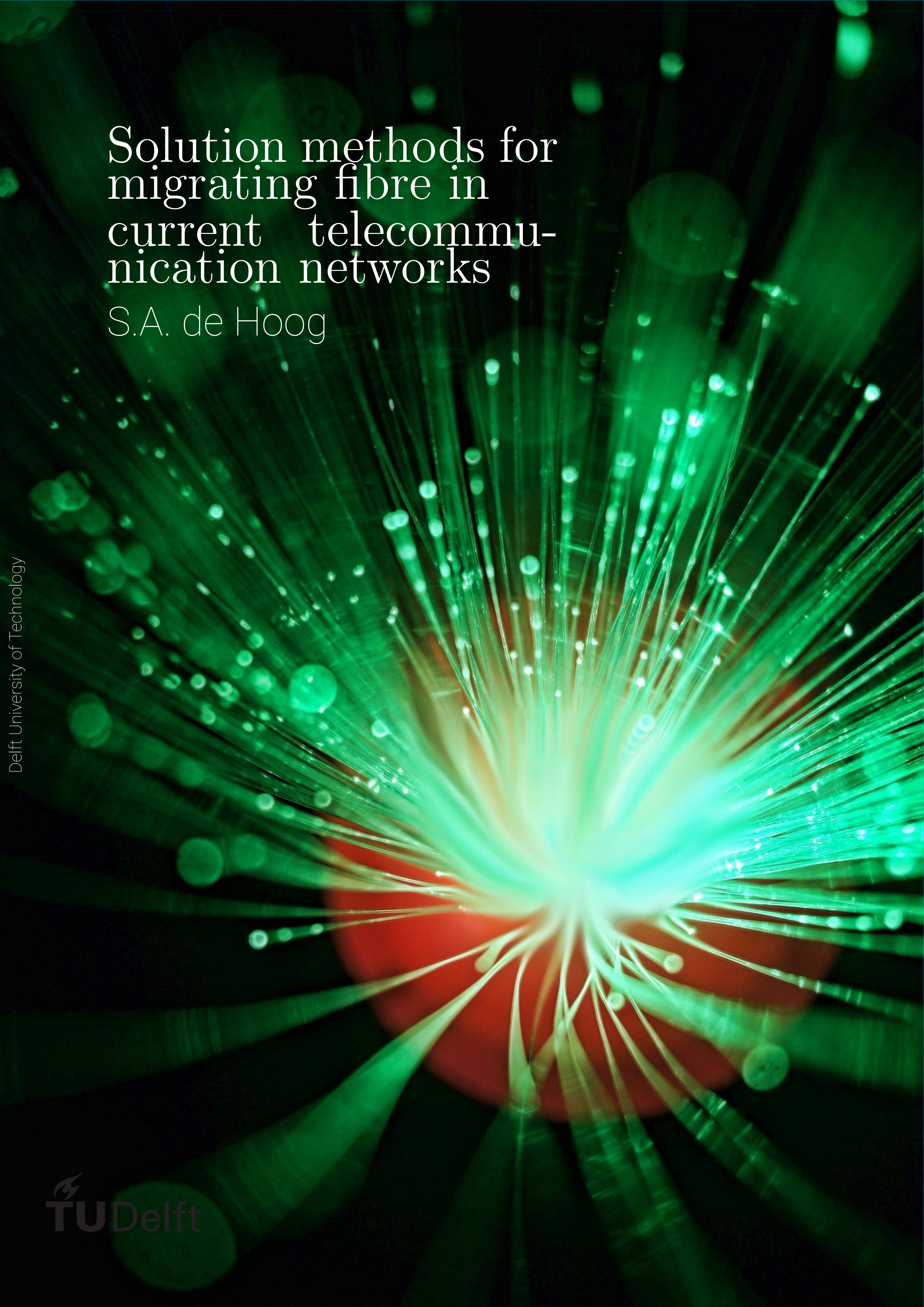

Delft University of Technology

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by

S.A. de Hoog

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Preface

The past 9 months, I have worked on my master thesis, of which the final result is in front of you. Of course this thesis would not have been possible without several people, who I would like to thank here. First of all, I would like to thank Theresia van Essen, my supervisor at the Delft University of Technology. You always motivated me with your enthusiasm during our meetings and corrected parts of text for my thesis an infinite amount of times. I also would like to thank Frank Phillipson, my supervisor at TNO, for offering me a interesting and challenging project and guidance during the project. Furthermore, I would like to thank my friends, for making my time as a student in Delft such an awesome time and providing the necessary relaxation. Last, but definitely not least, I would like to thank my parents. You always supported me to bring out the best out of myself and for offering a shoulder to lean on when needed.

S.A. de Hoog
Delft, November 2017

Contents

1
Introduction

In this thesis, we perform research to obtain a solution method which provides a good migration schedule for the whole of the Netherlands in a fast solution time. By migrating fibre in current networks, the delivered bandwidth will increase and as a result,the requested bandwidth will probably be met. Parts of this research will be published in Phillipson, de Hoog, van Essen [1].
This chapter introduces all the aspects that provide the context and motivation for this graduation project. We start with a short introduction of the company which is the supervisor of this graduation project and describe the background of the field in which the thesis is set. Then, we outline the motivation for this research and present the research questions. Finally, the way in which this study contributes to the field is explained and the structure of the thesis is described.

1.1. Background

This thesis is a collaboration between Delft University of Technology and TNO. Furthermore, it is part of the master's in Applied Mathematics, in relation to the track Computational Science and Engineering with the specialisation Optimisation. The main focus of this thesis is on the field of the migration to Fibre to the Home.

1.1.1. TNO

TNO (The Netherlands Organisation for Applied Scientific Research) is a nonprofit organisation in the Netherlands that focuses on applied science. TNO is a knowledge-based organisation for companies, government bodies and public organisations. Its approximately 3,800 employees work to develop and apply knowledge which makes it the largest research institute in the Netherlands. The focus areas of TNO are Industry, Healthy Living, Urbanisation, Energy, and Defence, Safety & Security. TNO tests and certifies products and services, and issues independent quality evaluations. Recently, TNO has developed methods to discuss the costs and bandwidth for a specific area, based on a limited number of available parameters. As a subsequent step, an expansion of this recent research is needed, to develop a migration plan for the whole of the Netherlands. In this way, telecommunication companies can use this research to determine how to obtain the required bandwidth, at for example, minimal costs.

1.1.2. Fibre

Almost one third of the Dutch households has a fibre optic connection, also called Fibre to the Home (FttH). Fibre is currently the fastest and most reliable Internet technology. It consists of strong and flexible wires that are able to transport light signals over long distances. With such fibre optic connections, information is transmitted from one place to another when pulses of light are sent through an optical fibre and can reach Internet speeds of 500 Mb per second and beyond. The infrastructure of the Internet connection in the Netherlands is divided into three parts, each with its own owner: coax, twisted copper pairs and fibre. The largest possessor of the twisted copper pairs is KPN and for the fibre infrastructure that is Reggefibre, which has been part of KPN since 2014. Both technologies form part of the regulated market, so other companies are allowed to use the copper or fibre network. The coax network, meanwhile, is not on the regulated market. Furthermore, there is only one provider per region in the Netherlands. An example of these companies is Ziggo.

1.2. Research motivation

The Dutch fibre market is growing rapidly. However, it is already possible to reach high Internet speeds via the coax cable. As mentioned above, the coax infrastructure is not part of the regulated market. As a result, telecom operators are forced to upgrade their copper networks in order to also achieve these high speeds. In an ideal situation, all copper in the networks would be replaced by fibre. However, such an operation would be too expensive to complete all at once, so therefore, it would not be a feasible solution. However, more options are available for upgrading a network. For example, the extension of Fibre to the Curb could significantly increase the bandwidth, compared to services offered over a copper cable starting at the Central Office. Furthermore, for this extension, less costs are also involved compared to an extension of Fibre to the Home. On the other hand, the extension to a total fibre network will reach a higher bandwidth compared to an extension of fibre to a curb. When implementing these migration steps, copper will be (partly) replaced by fibre. This research provides a set of models and migration strategies so that more insight can be gained into the advantages of the potential migration steps, the viability of any possible roll out plans and the increase of Internet speeds. In this thesis, we defined two models for the Migration of Fibre problem, i.e., migrating fibre in current networks in order to meet the requested bandwidths. Migrating fibre can be done in periods, which results in a migration path for each location. Such a migration path represents for each period which migration step should be implemented. Moreover, the migration paths are heterogeneous. This means that each location receives a migration path based on the location, so the different migrations paths for all areas in the Netherlands together represent a migration schedule.

1.3. Research questions

From the situation described in Section 1.2, we derive the research question of this thesis. The primary goal of this research is to give an answer to the following research question:

"What is a good strategy for the migration paths of Fibre to the Home for each area of the Netherlands, when described as heterogeneous migration paths consisting of migration steps for each time period in each area?"

In order to answer this question, the following sub-questions are posed:

- Are the Migration of Fibre problems optimally solvable?
- Which (meta)heuristic will provide a good and fast solution?
- What is the scalability of the used solution methods?
- Will placing no restrictions on the budget per time period result in a lower amount of total costs compared to having a fixed budget per time period?

1.4. Contribution made by this work

Various studies has been done on the advantages of the migration to FttH. In Sections 2.2 and 2.3, an overview is given of the available migration models. This research is an extension of previous studies.

Not only a few areas of the Netherlands will get a migration plan; this research will manufacture a migration plan for the whole of the Netherlands. The budgets of telecom companies and the installation capacities of construction companies are also taken into account.

1.5. Structure of this thesis

This thesis is divided into three parts: Introduction, Research and Results & Conclusions. In the first part, an introduction to the research is given. In Chapter 1, background information of TNO and fibre, the goal of this research and the contribution of this research is given. In Chapter 2, an overview of the background of telecommunication technologies and topologies is given, and a description of the related work done in the field of this research.

In the second part, we describe the research performed in this graduation project. In Chapter 3, the model for the Migration of Fibre problem is presented as are the associated definitions and notations that we have used in the remainder of the thesis. In Chapter 4, the first research sub-question is answered. Next, in Chapter 5, we present the heuristic methods which relate to the second research sub-question.

In the third part of this thesis, we provide the results and the conclusions. We present the results based on the test approach described in Chapter 6 and the third and fourth research sub-questions are answered. Furthermore, in Chapter 7, conclusions on the results of this research are given and we provide ideas for possible future work in the field covered by this research.

2
Related Work

There is significant research available in the field of Fibre to the Home (FttH) and digital network technologies. In this chapter, we describe these concepts and give an overview of the research done in these fields.

2.1. Topologies and technologies

The Internet infrastructure consists of a combination of topologies and technologies. Figure 2.1 [2] illustrates the four topologies focused on in our research. The first topology, is Full Copper (FC). Starting from the central office, a network service is offered over a copper cable. The second topology begins with a fibre network that goes to a cabinet and will continue as a copper network. This topology is called Fibre to the Cabinet (FttCab). In the third topology, the fibre network will be extended until the curb. This is known as Hybrid Fibre to the Home or Fibre to the Curb (FttCurb) and the fourth topology has a total fibre network to the home. This latter topology is also called Fibre to the Home (FttH).

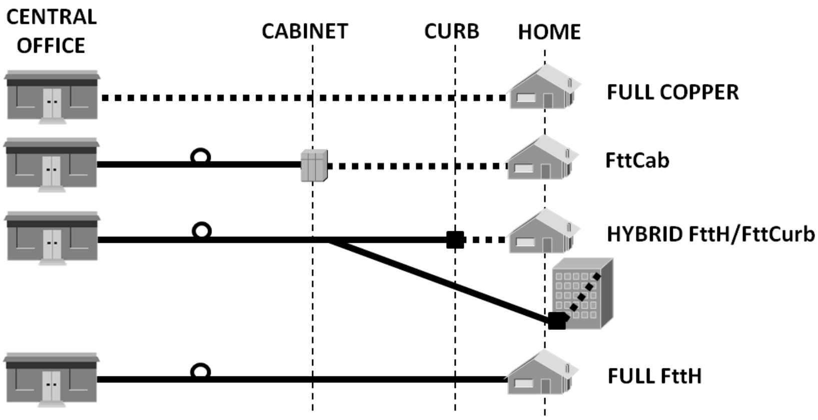

Figure 2.1: Topologies

A number of techniques exist for offering a network service over a copper cable. The best-known technology is ADSL (Asymmetric Digital Subscriber Line). This technology could offer 8 Mb/s over a copper cable with a maximum length of 5 km. The successor of ADSL is VDSL, also known as Very-high-bit-rate Digital Subscriber Line. VDSL can achieve speeds of 80 Mb/s over a copper cable with a maximum length of 1 kilometre. An enhancement to VDSL is VDSL2. The maximum range for VDSL2 is approximately 500 meters, reaching 100 Mb/s. For a 1-kilometre cable, the speed of VDSL2 will be the same as for ADSL2+. The maximum range for ADSL2+ is approximately 1.2 kilometre, reaching 24 Mb. The successor of VDSL2 is V.plus. V.plus can achieve speeds of 300 Mb over a copper cable with a maximum length of 250 metres. The latest technology developed for copper cables is G.Fast. For a cable length of 200 metres, G.Fast is technically feasible up to 500 Mb/s. See Table 2.1 [3] for an overview of all the mentioned and other technologies together with their characteristics.

Technology (over Copper)	(Maximum) distance	Speed per second
ADSL	5 km	8 Mb
ADSL2	2.5 km	12 Mb
ADSL2+	1.2 km	24 Mb
VDSL	1 km	80 Mb
VDSL2	500 m	100 Mb
VVDSL	700 m	100 Mb
V.plus	250 m	300 Mb
G.Fast	200 m	500 Mb

A topology represents the way in which fibre is migrated in the network. The fibre connection starts at the Central Office and continues, for example, to a curb. A technology is the service offered to the remaining part of the network, which consists of copper. An example of a technology and topology combination is VDSL/FttCab, where Internet is offered from the central offices to the cabinet over a fibre connection and then, VDSL is used to offer internet over a copper cable from the cabinet to the home. The bandwidth coverage is determined by the length of the copper network in combination with the technology offered over this copper network. Telecommunication companies have a limited budget for upgrading their telecom networks. One requirement for upgrading is that the bandwidth demand of customers should be met, which is just as important as not exceeding the budget. Therefore, it is necessary to develop a strategy for the migration to Fibre to the Home for a certain time span, for example for 10 to 15 years. A migration path is a combination of topologies and technologies. Figure 2.2 [2] illustrates the possible migration paths. Each node depicted in the graph is a combination of a technology and a topology.

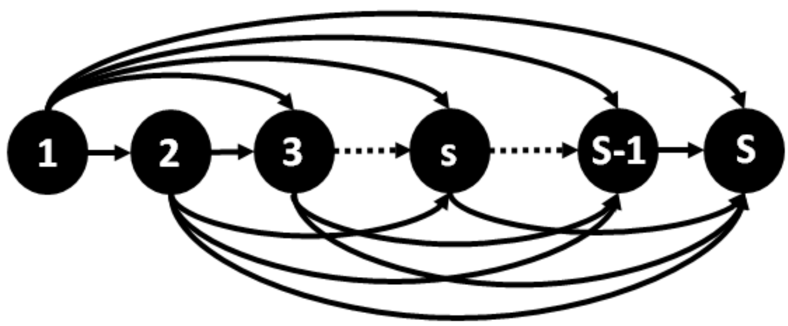

Figure 2.2: Technologies & topologies

The next sections contain an overview of the available models for the Migration of Fibre problem.

2.2. Homogeneous migration

Zhao et al. [4] and Reyes et al. [5] present a generalised migration model for the homogeneous migration to FttH. In a homogeneous migration strategy, one migration path is used for all Central Office areas. The model focuses on the optimal migration path subjected to a fixed target state, namely FttH. The cost function is expressed as:

$$Cost = C_{CAPEX} + C_{OPEX} - C_{RV}.$$

C_{CAPEX} denotes the total investment cost required to introduce a new technology and/or topology in a Central Office area. For example, the purchase cost of land and buildings, network infrastructure, network installation, network software, customers' premises equipment, and project management. C_{OPEX} denotes the operational cost, for example the renting cost for infrastructure, energy consumption, repair costs, operational network planning, pricing and billing, service provision and marketing. The revenues are denoted by C_{RV}. In the model, five different states are taken into account: Greenfield (i.e., users who need complete infrastructure installations), Full Copper, FttC, FttB and FttH. The goal is to find an optimal migration path from the current state to FttH. In the presented case study, a distinction is made between three different scenarios: Rural, Urban and Dense Urban area. The results of the case study showed that migration is not always possible, since for the chosen rural area, the average monthly costs are not compensated by the revenues from the users. In this thesis, we do not take the economic aspects C_{OPEX} and C_{RV} into consideration. However, we likewise focus on the characteristics of different areas in the Netherlands.

The previous homogeneously migration model [4], [5] is fairly complex, whereas telecom operators want to upgrade their network as much as possible and as fast as possible. In order to achieve a migration path towards Fibre to the Home which focuses on these preferences of telecom operators, a cost model is developed in the CELTIC/4GBB project. Phillipson [6] describes the model and presents the results. The model compares two topology migration paths from the cost point of view. The first input of the model is the distance of both the trench and the cable. The trenches are long deep holes dug in the ground, where the cable is laid. Furthermore, the maximum copper distance is essential for the input of the cost model. Both the distances are calculated using a geometric model. In order to use the geometric model, several cost parameters are set up, such as the equipment and installing costs of a cabinet, the digging costs and the costs of breaking and repairing tiles. To clarify the results, an area of 6,000 connections per square kilometre was taken into consideration and two migration paths were compared. The first migration path is from Full Copper to Full Fibre to the Home, using Fibre to the Cabinet and Hybrid Fibre to the Home as intermediate steps. The second migration is from Full Copper directly to Fibre to the Home. The cost model establishes that the first migration path is more expensive compared to the second migration path. However, the addition of the migration time (discounted cash flow) to the model, leads to the second migration being more expensive than the first migration. The inclusion of the discounted cash flow in the model gives a more realistic approach to the migration problem.

The postponement of investments in Fibre to the Home and the use of intermediate steps is cost-effective. Hybrid Fibre to the Home delivers a high bandwidth and is considerably cheaper than direct migration to Fibre to the Home. If after a few years, the investments in the intermediate steps are insufficient, then the costs saved by the postponement of the full migration path will be large enough to compensate for the costs of a total migration path which makes use of the intermediate Fibre to the Cabinet and Hybrid Fibre to the Home steps.

2.3. Restricted heterogeneous migration

In the above-mentioned models, only single or uniform migration paths are considered. Phillipson [2] has defined a Integer Linear Programming (ILP) Problem for the heterogeneous migration problem, where small instances of areas are considered. In a heterogeneous migration strategy, each area of a Central Office receives its own migration path, based on the characteristics of that area. The problem is solved in Excel using Opensolver [7]. To formalise this problem, we use the following notation. The set of combinations of a technology and topology is given by set I. The locations of cabinets in the

Netherlands is given by set L. The set of time periods is given by set T and the distances between the cabinet, curb and houses is given by set D. The ILP has the following two variables and four parameters:

$$x_{ijlt} = \begin{cases} 1 & \text{If migration takes place from } i \in I \text{ to } j \in I \text{ in time period } t \in T \text{ for location } l \in L \\ 0 & \text{Else} \end{cases}$$

$$y_{ilt} = \begin{cases} 1 & \text{If } i \in I \text{ is active in time period } t \in T \text{ and location } l \in L \\ 0 & \text{Else} \end{cases}$$

C_{ijl} = Costs for migrating from $i \in I$ to $j \in I$ for location $l \in L$
R_{ild} = Number of houses reached by $i \in I$ within $d \in D$ meter for location $l \in L$
RT_l = Total number of houses for location $l \in L$
G_{td} = Requested percentage of houses in total area reached within $d \in D$ meter in time period $t \in T$.

The objective function with the corresponding constraints are:

$$\min \sum_{i\in I}\sum_{j\in I}\sum_{l\in L}\sum_{t\in T} C_{ijl}x_{ijlt} \tag{2.1}$$

$$\text{s.t.} \sum_{i\in I}\sum_{j\in I} x_{ijlt} \leq 1 \qquad \forall l \in L, t \in T \tag{2.2}$$

$$\sum_{i\in I} y_{ilt} = 1 \qquad \forall l \in L, t \in T \tag{2.3}$$

$$\frac{1}{2}(y_{jlt} - y_{jlt-1}) - \frac{1}{2}(y_{ilt} - y_{ilt-1}) - \frac{1}{2} \leq x_{ijlt} \quad \forall i, j \in I, l \in L, t \in T \tag{2.4}$$

$$\frac{\sum_{i\in I}\sum_{l\in L} R_{ild} \cdot y_{ilt}}{\sum_{l\in L} RT_l} \geq G_{td} \qquad \forall t \in T, d \in D \tag{2.5}$$

$$x_{ijlt}, y_{ilt} \in \{0,1\} \quad \forall i, j \in I, l \in L, t \in T. \tag{2.6}$$

The objective function minimises the total investment costs, i.e., CapEx. Constraint (2.2) ensures that there is, at most, one migration step per location per time period. Furthermore, Equation (2.3) ensures that each location has one topology and technology combination each time period and Constraint (2.4) creates the migration step. Lastly, Constraint (2.5) makes sure that the demanded bandwidth coverage is provided. In contrast to the models in Section 2.2, the final state of the migration should not necessarily be FttH. The values of C_{ijl} and R_{ild} are hard to obtain in practice and therefore, Phillipson et al. [8] and Phillipson [9] developed an approach for this, using geometric modelling.

3 Problem Formulation

In this chapter, we present two models for developing a migration plan for fibre in the Netherlands. These are a base model and an extended model. The base model is developed by Phillipson [2], which is illustrated in Section 2.3. The latter model is an extended version of the ILP for the (heterogeneous) Migration of Fibre problem, i.e., an extended version of the base model. First, we give an overview of the parameters and variables used. and then, the extended model is presented.

3.1. Definitions

Phillipson's model [2] is the basis of our research, also called the base problem. This model minimises the total investment costs and meets all bandwidth requirements in each migration time period. This model gives a good first insight in the costs related to the bandwidth demand. However, telecommunication companies and investment companies in fibre can not make infinite expenses, so we introduce the budget parameter B_t in the extended model. For each time period $t \in T$, there is a fixed budget. In Section 3.2, we explain this parameter more in debt. Due to the differing infrastructures between rural and urban areas, and because of the limitation in manpower for the contractors, we also introduce the parameter installation capacity IC_t per time period $t \in T$. In the previous chapter, we already mentioned the other parameters and variables, but for the sake of completeness, we display them again here, including the budget and installation capacity parameter.

$$x_{ijlt} = \begin{cases} 1 & \text{If migration takes place from } i \in I \text{ to } j \in I \text{ in time period } t \in T \text{ for location } l \in L \\ 0 & \text{Else} \end{cases}$$

$$y_{ilt} = \begin{cases} 1 & \text{If } i \in I \text{ is active in time period } t \in T \text{ and location } l \in L \\ 0 & \text{Else} \end{cases}$$

C_{ijl}	=	Costs for migrating from $i \in I$ to $j \in I$ for location $l \in L$
B_t	=	Budget of a telecom operator per time period $t \in T$
H_{ijl}	=	Required capacity to migrate from i to $j \in I$ for location l
IC_t	=	Installation capacity per time period $t \in T$
R_{ild}	=	Number of houses reached by $i \in I$ within $d \in D$ meter for location $l \in L$
RT_l	=	Total number of houses for location $l \in L$
G_{td}	=	Requested percentage of houses in total area reached within $d \in D$ meter in time period $t \in T$.

3.2. Heterogeneous migration models

In this section, we give an overview of the base and extended model.

3.2.1. Base problem

The base problem considered in this thesis, as described in Section 2.3, is:

$$\min \sum_{i\in I}\sum_{j\in I}\sum_{l\in L}\sum_{t\in T} C_{ijl}x_{ijlt} \tag{3.1}$$

$$\text{s.t.} \sum_{i\in I}\sum_{j\in I} x_{ijlt} \leq 1 \qquad \forall l \in L, t \in T \tag{3.2}$$

$$\sum_{i\in I} y_{ilt} = 1 \qquad \forall l \in L, t \in T \tag{3.3}$$

$$\frac{1}{2}(y_{jlt} - y_{jlt-1}) - \frac{1}{2}(y_{ilt} - y_{ilt-1}) - \frac{1}{2} \leq x_{ijlt} \quad \forall i,j \in I, l \in L, t \in T \tag{3.4}$$

$$\frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l} \geq G_{td} \qquad \forall t \in T, d \in D \tag{3.5}$$

$$x_{ijlt}, y_{ilt} \in \{0,1\} \quad \forall i,j \in I, l \in L, t \in T. \tag{3.6}$$

3.2.2. Extended model

In the base model, the total costs are minimised. However, in practice, telecommunication companies want to spend their money as efficient as possible and they want to decide in advance what amount of money to spend each year. At the same time, they want to improve their business and have a good balance in their customer satisfaction. For example, customers will switch to another telecom operator, in the case that their original telecom operator lags behind in his technologies. Therefore, the main goal of the extended Migration of Fibre problem is to minimise the difference between the realised bandwidth and the customer's demand. To this end, we introduce the following objective function, also specified by Phillipson [2] :

$$\min \sum_{t\in T}\sum_{d\in D} \max\left(0, G_{td} - \frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l}\right). \tag{3.7}$$

Overcompensating in realised bandwidth, i.e., a realised bandwidth higher than the requested bandwidth in a migration time period for a certain distance, does not influence the objective function, due to taking the maximum. This is because the goal is to fulfil the requested bandwidth in every migration time period as much as possible. As objective function (3.1) is replaced by objective function (3.7), (3.1) should be translated to a constraint. Therefore, the following constraint is added to set a maximum for the costs, related to the budget B_t of a telecom operator per time period, as introduced by Phillipson [2]:

$$\sum_{i\in I}\sum_{j\in I}\sum_{l\in L} C_{ijl}x_{ijlt} \leq B_t, \ \forall t \in T. \tag{3.8}$$

The combination of this objective function (3.7) and constraint (3.8) ensures the good balance between the expenses and the customer satisfaction. Moreover, another aspect could influence the ability of upgrading a network, namely the installation capacity. In the Netherlands, all telecom networks are situated underground. This means that, for example, a lot of streets in a district have to be opened to upgrade the network. However, the district should be still liveable for the residents, so the digging process should be executed in phases. Furthermore, there is manpower needed for this digging process and this is limited to the available employees per time period $t \in T$ of a company that is hired for upgrading the networks. The parameter for this installation capacity per time period is named as IC_t. To make sure that the installation capacity per time period will not be exceeded, the following constraint is added in the extended model:

$$\sum_{i\in I}\sum_{j\in I}\sum_{l\in L} H_{ijl}x_{ijlt} \quad \leq \quad IC_t, \forall t \in T. \tag{3.9}$$

Here, H_{ijl} is the required capacity, i.e., amount of employees, to migrate from topology and technology combination $i \in I$ to combination $j \in I$ for location $l \in L$. To summarise, the formulation of the extended model is:

$$\min \sum_{t\in T}\sum_{d\in D} \max\left(0, G_{td} - \frac{\sum\limits_{i\in I}\sum\limits_{l\in L} R_{ild}\cdot y_{ilt}}{\sum\limits_{l\in L} RT_l}\right) \tag{3.10}$$

$$\text{s.t.} \sum_{i\in I}\sum_{j\in I}\sum_{l\in L} C_{ijl}x_{ijlt} \quad \leq \quad B_t \qquad \forall t \in T \tag{3.11}$$

$$\sum_{i\in I}\sum_{j\in I}\sum_{l\in L} H_{ijl}x_{ijlt} \quad \leq \quad IC_t \qquad \forall t \in T \tag{3.12}$$

$$\sum_{i\in I}\sum_{j\in I} x_{ijlt} \quad \leq \quad 1 \qquad \forall l \in L, t \in T \tag{3.13}$$

$$\sum_{i\in I} y_{ilt} \quad = \quad 1 \qquad \forall l \in L, t \in T \tag{3.14}$$

$$\frac{1}{2}(y_{jlt} - y_{jlt-1}) - \frac{1}{2}(y_{ilt} - y_{ilt-1}) - \frac{1}{2} \quad \leq \quad x_{ijlt} \qquad \forall i, j \in I, l \in L, t \in T \tag{3.15}$$

$$x_{ijlt}, y_{ilt} \quad \in \quad \{0,1\} \qquad \forall i, j \in I, l \in L, t \in T. \tag{3.16}$$

In order to investigate the research sub-question "Will placing no restrictions on the budget per time period, result in a lower amount of total costs compared to having a fixed budget per time period?", the budget constraint should be replaced by:

$$\sum_{i\in I}\sum_{j\in I}\sum_{l\in L}\sum_{t\in T} C_{ijl}x_{ijlt} \quad \leq \quad B, \tag{3.17}$$

where B is the total budget for the migration plan.

3.2.3. Present value of migration costs

In the base and extended model, the migration costs are time-independent. In practise, the value of money will decrease in time, due to the interest on money that companies receive. Furthermore, it is beneficial to postpone investments, because then the telecom company could also postpone loans for the investments and this saves interests. In economics, this is also called the present value. In this way, we make the migration costs time-dependent. The migration costs for time period $t \in T$ are described by Kieso, et al. [10] as:

$$C_{ijlt} = \frac{C_{ijl}}{(1+R)^n}, \qquad \forall i, j \in I, l \in L, t \in T. \tag{3.18}$$

where R is the interest rate and n is the number of years from $t = 1$ until time period $t \in T$. For example, when $t = 1$ represents 2017 and $t = 2$ represents 2020, then n should be equal to 3 for the calculating of C_{ijl2}. We use both the values C_{ijl} and C_{ijlt} in our research. If there is nothing specified for the costs, then C_{ijl} is used.

4

Computational Complexity

In this chapter, we discuss the complexity of the Migration of Fibre problems. First, we discuss the definitions of the classes P, NP and NP-hard. After that, it is shown that the base and extended model are both NP-hard.

4.1. Definitions

A complexity class contains a set of problems which have the same complexity. The class P is the class of problems that can be solved in polynomial time. For every optimisation problem, it is possible to define a closely related recognition problem. This is a problem that can be answered by *yes* or *no*. The class of non-deterministic polynomially solvable problems, NP, contains problems for which the answer of the related recognition problem can be verified in polynomial time. In order to classify the complexity of a problem, it is effective to take advantage of the reduction method. Assume that A_1 and A_2 are recognition problems. In polynomial time, A_1 reduces to A_2, whenever there exists a polynomial time algorithm Q_1 that makes a polynomial number of times use of a polynomial time algorithm Q_2 for A_2, which could verify if an instance I_1 of A_1 leads to an answer *yes* [11]. The main consequence is that if it is possible to get a solution for A_2 in polynomial time, then this is also possible for A_1. A problem A is called NP-complete if it is possible to polynomially reduce a problem in NP to A. Then, it is clear that A is as hard as any problem in NP, but based on only this, we are unable to argue that $A \in NP$. If a recognition problem is NP-complete, then the corresponding decision problem is NP-hard. In the next sections, this is shown for the base and extended Migration of Fibre problem.

4.2. Complexity of the base problem

The Single Source Capacitated Facility Location Problem (SSCFLP) is NP-hard and can be reduced to the base model of the Migration of Fibre problem. In this problem, a number of facilities should be located, whereby each customer is fully assigned to a facility at minimum cost, such that the demand of each customer is served and a facility does not supply more than his capacity. This can be described as follows [12]:

$$\min \sum_{i\in Q}\sum_{j\in J} V_{ij}x_{ij} + \sum_{j\in J} F_j y_j \tag{4.1}$$

$$\text{s.t.} \sum_{j\in J} x_{ij} = 1 \qquad \forall i \in Q \tag{4.2}$$

$$x_{ij} \leq y_j \qquad \forall i \in Q, j \in J \tag{4.3}$$

$$\sum_{i\in Q} K_i x_{ij} \leq S_j y_j \qquad \forall j \in J \tag{4.4}$$

$$x_{ij}, y_i \in \{0,1\} \qquad \forall i \in Q, j \in J, \tag{4.5}$$

where I is the set of customers, J is the set of facilities, V_{ij} are the costs for assigning customer $i \in I$ to facility $j \in J$ and F_j are the costs for opening facility $j \in J$. Furthermore, K_i is the demand of customer $i \in I$ and S_j is the capacity of location $j \in J$. It holds that variable x_{ij} is equal to 1 if customer $i \in I$ is assigned to facility $j \in J$. Otherwise, this variable is equal to 0. The variable y_j is equal to 1 if facility $j \in J$ is opened. Otherwise, this variable is equal to 0.

For the reduction of the SSCFLP to the base problem, firstly, one instance of the base problem is given. Assume there is one location l, one time period $t \in T$ and one distance $d \in D$. By this, the ILP of the base problem can be reduced to:

$$\min \sum_{i=i_0}\sum_{j\in I} C_{ij}x_{ij} \tag{4.6}$$

$$\text{s.t.} \sum_{j\in I} x_{ij} \leq 1 \qquad \forall\, i = i_0 \tag{4.7}$$

$$\sum_{j\in I} y_j = 1 \tag{4.8}$$

$$\frac{1}{2}(y_j - Y_j) - \frac{1}{2}(y_i - Y_i) - \frac{1}{2} \leq x_{ij} \qquad \forall i = i_0, j \in I \tag{4.9}$$

$$\frac{\sum_{j\in I} R_j \cdot y_j}{RT} \geq G \tag{4.10}$$

$$x_{ij}, y_j \in \{0,1\} \qquad \forall\, i = i_0, j \in I. \tag{4.11}$$

Here, i_0 denotes the start state, namely the combination i of a technology and a topology at time $t = 0$. So, Y_i is equal to 1 for i equal to the technology and topology combination at time period $t = 0$ and zero otherwise. Note that Constraint (2.4) does not force that $x_{ij} = 1$ for $i = j$ when no migration takes place. However, having $x_{ij} = 1$ for $i = j$ does not affect the objective function, because the migration costs for migrating to the same technology and topology combination i are zero, as there is actually no migration happening. As a result, we can change Constraint (2.4) to:

$$x_{ij} \geq Y_i + y_j - 1, \qquad \forall\, i = i_0, j \in I. \tag{4.12}$$

In this equation, it is forced that $x_{ij} = 1$ for $i = j$ when no migration takes place. We know that $Y_{i_0} = 1$, because i_0 is the start state, thus Constraint (4.12) can be changed to:

$$x_{ij} \geq y_j, \qquad \forall\, i = i_0, j \in I. \tag{4.13}$$

In the base model, Constraint (4.7) has got an inequality sign and not a equality sign due to the fact that there is no time period $t = -1$ before the start state, so there is no migration possible from $t = -1$ to $t = 0$, and thus, for the start state $t = 0$ it holds that $x_{ijl0} = 0$ for all $i, j \in I$ and $l \in L$. In the

ILP described above, we only have one time period $t \in T$, so we can change the inequality sign in Constraint (4.7) to an equality sign:

$$\sum_{j \in I} x_{ij} = 1, \qquad \forall\, i = i_0. \tag{4.14}$$

As a result of Constraint (4.8) and (4.14) , we can flip the inequality sign in Constraint (4.12), because this does not affect the relation between the y_j and $x_{i_0 j}$, which should be equal to each other:

$$x_{ij} \leq y_j, \qquad \forall i = i_0, j \in I. \tag{4.15}$$

From Constraint (4.8), we know that exactly one technology and topology combination $j \in I$ should be active in the considered time period. We can replace this constraint by adding the following part to the objective function:

$$\sum_{j \in I} U_j y_j, \tag{4.16}$$

where it holds that $U_j = U$ for all $j \in I$ and $U > \max_{\forall i, j \in I} C_{ij}$. As a result, the ILP becomes:

$$
\begin{aligned}
\min \sum_{i=i_0} \sum_{j \in I} C_{ij} x_{ij} + \sum_{j \in I} U_j y_j & & & & (4.17)\\
\text{s.t.} \sum_{j \in I} x_{ij} &= 1 & & \forall\, i = i_0 & (4.18)\\
x_{ij} &\leq y_j & & \forall\, i = i_0, j \in I & (4.19)\\
\frac{\sum_{j \in I} R_j \cdot y_j}{RT} &\geq G & & & (4.20)\\
x_{ij}, y_i &\in \{0,1\} & & \forall\, i, j \in I. & (4.21)
\end{aligned}
$$

From Constraint (4.18), (4.19) and the fact that only one technology and topology combination $j \in I$ could be active, it follows that we can remove the sum in Constraint (4.20), by adding $x_{i_0 j}$ at the right side of the inequality sign. This is because it must hold that the active technology and topology combination j after migrating fulfills the bandwidth demand G. This results in the following ILP:

$$
\begin{aligned}
\min \sum_{i=i_0} \sum_{j \in I} C_{ij} x_{ij} + \sum_{j \in I} U_j y_j & & & & (4.22)\\
\text{s.t.} \sum_{j \in I} x_{ij} &= 1 & & \forall\, i = i_0 & (4.23)\\
x_{ij} &\leq y_j & & \forall\, i = i_0, j \in I & (4.24)\\
\frac{R_j}{RT} \cdot y_j &\geq \sum_{i=i_0} G_i x_{i_0 j} & & \forall\, j \in I & (4.25)\\
x_{ij}, y_i &\in \{0,1\} & & \forall\, i, j \in I, & (4.26)
\end{aligned}
$$

where $G_i = G$ for all $i \in I$.
The values C_{ij}, U_j, $\frac{R_j}{RT}$ and G_i for all $i, j \in I$ correspond to the SSCFLP values V_{ij}, F_j, S_j and K_i for all $i \in Q$ and $j \in J$, respectively. Moreover, i_0 is the set of customers Q and the set of facilities J is equal to the set I of technology and topology combinations. This shows that the SSCFLP is a special case of the base problem and leads to the conclusion that the base problem is at least as hard as the SSCFLP. The SSCFLP is NP-hard [12], and thus, the base problem is also NP-hard.

4.3. Complexity of the extended problem

The Multiple Constraint Knapsack Problem is NP-hard and can be reduced to the extended model of the Migration of Fibre problem. In this problem, a set of items, each with a weight and value, could be packed once into a knapsack. The objective is to determine which item to include in the knapsack, to maximise the total profit and without exceeding the knapsack constraints. This can be described as follows [13]:

$$\max \sum_{i \in I} P_i y_i \tag{4.27}$$

$$\text{s.t.} \sum_{i \in I} A_{ji} y_i \leq W_j \qquad \forall j \in M \tag{4.28}$$

$$y_i \in \{0, 1\} \qquad \forall i \in I, \tag{4.29}$$

where the sets of items is given by set I and the set of knapsack constraints is given by set M with corresponding capacities W_j with $j \in M$. The required capacity of item i for knapsack constraint j is A_{ji} with $j \in M, i \in I$. The value of item i is denoted by P_i and y_i is equal to 1 if item i is in the knapsack and otherwise this variable is equal to 0.

Similarly, for the reduction of the Multiple Constraint Knapsack problem to the extended problem, firstly, one instance of the extended problem is given. Assume there is one location $l \in L$, one time period $t \in T$ and one distance $d \in D$. By this, the IP (presented in Chapter 3) is reduced to:

$$\min \max \left(0, G - \sum_{i \in I} \frac{R_i}{RT} \cdot y_i \right) \tag{4.30}$$

$$\text{s.t.} \sum_{i \in I} \sum_{j \in I} C_{ij} x_{ij} \leq B \tag{4.31}$$

$$\sum_{i \in I} \sum_{j \in I} H_{ij} x_{ij} \leq IC \tag{4.32}$$

$$\sum_{i \in I} \sum_{j \in I} x_{ij} \leq 1 \tag{4.33}$$

$$\sum_{i \in J} y_i = 1 \tag{4.34}$$

$$\frac{1}{2}(y_j - Y_{j_0}) - \frac{1}{2}(y_i - Y_{i_0}) - \frac{1}{2} \leq x_{ij} \qquad \forall i, j \in I \tag{4.35}$$

$$x_{ij}, y_i \in \{0, 1\} \qquad \forall i, j \in I, \tag{4.36}$$

Again, i_0 denotes again the start state, namely the combination i of a technology and a topology at time $t = 0$. The objective function is a max-min function. However, it is possible to modify the objective function to a maximisation function. Since there is only one location, the objective function can be changed to maximising the bandwidth for this location. The new objective function is defined as:

$$\max \sum_{i \in I} \frac{R_i}{RT} \cdot y_i. \tag{4.37}$$

Furthermore, the amount of variables can be reduced. This is possible, because there is only one location, one time period and the start state is known. Therefore, x_{ij} can be replaced by y_i. As a result, C_{ij} and H_{ij} are respectively changed to C_i and H_i, and Constraint (4.33) and (4.35) become superfluous. Without loss of generality, the equality sign in Constraint (4.34) can be changed to a "less than or equal to" sign, because the optimal solution will never be $y_i = 0$, for all $i \in I$, due to the used objective function and positive values of $\frac{R_i}{RT}$. Furthermore, it holds that $C_i = 0$ and $H_i = 0$ for

$i \in I$ equal to the start state. Summarising, the described instance of the Migration of Fibre problem becomes:

$$\max \sum_{i \in I} \frac{R_i}{RT} \cdot y_i \tag{4.38}$$

$$\text{s.t.} \sum_{i \in I} C_i y_i \leq B \tag{4.39}$$

$$\sum_{i \in I} H_i y_i \leq IC \tag{4.40}$$

$$\sum_{i \in I} y_i \leq 1 \tag{4.41}$$

$$y_i \in \{0,1\} \quad \forall i \in I. \tag{4.42}$$

The budget B, installation capacity IC and 1 correspond to the knapsack capacities W_1, W_2 and W_3, respectively. Furthermore, C_i corresponds to A_{1i} for all $i \in I$, H_i corresponds to A_{2i} for all $i \in I$, and A_{3i} is equal to 1 for all $i \in I$. Lastly, $\frac{R_i}{RT}$ is equal to P_i for all i, thus the Multiple Constraint Knapsack problem is a specific case of the extended problem. This leads to the conclusion that the extended problem is at least as hard as the Multiple Constraint Knapsack problem. The Multiple Constraint Knapsack problem is NP-hard [13], thus, the extended problem is also NP-hard.

5
Solution Methods

The computation times for solving the Migration of Fibre problems (base and extended problem) have to be of a reasonable magnitude, regardless of the input of the models. The reason for this is that the telecommunication companies should be able to run the optimisation model in a few minutes. After obtaining a migration plan, the company has to consider whether the migration plan is performable. The performability of a migration schedule is, for example, based on the fact that it is not desirable that too much networks in an area are upgraded at the same too time, whereby residents can not reach there houses, because all access roads are closed. If it is not a performable plan, they should be able to create a new migration plan. Furthermore, in Section 4.2, it is shown that the Migration of Fibre problems are NP-hard. For these two reasons, heuristic methods are developed to obtain a good solution in an acceptable computation time. A heuristic method is a procedure that is likely to discover a good and feasible solution, but not necessarily an optimal solution. In this chapter, we present the different heuristic solution methods used in this research. First of all, the model used for the exact solution method is described.

5.1. Exact solution method

By first creating an optimal solution by means of an exact solution method, the quality of the developed heuristics can be assessed. The base model used for creating an optimal exact solution is described in Section 2.3. The exact solution method for the base model is solving the ILP with an available solver.
Furthermore, the objective function of the extended model, as defined in Section 3.2.2,

$$\min \sum_{t\in T} \sum_{d\in D} \max\left(0, G_{td} - \frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l}\right), \tag{5.1}$$

minimises the gap between the required and the realised bandwidth per time period. However, this a non-linear function. Linearisation of the objective function would make the extended model easier to solve. For this, we introduce the variable z_{td} with $t \in T, d \in D$. Furthermore, the following constraints for z_{td} are added to the model:

$$z_{td} \geq 0 \qquad \forall d \in D, t \in T \tag{5.2}$$

$$z_{td} \geq G_{td} - \frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l} \qquad \forall d \in D, t \in T. \tag{5.3}$$

As a result, the objective function can be changed to:

$$\min \sum_{t\in T}\sum_{d\in D} z_{td}. \tag{5.4}$$

Moreover, z_{td} does not have to be integer. Summarising, the MILP formulation used to obtain an optimal solution is:

$$
\begin{aligned}
&\min \sum_{t\in T}\sum_{d\in D} z_{td} && && && (5.5)\\
\text{s.t.} &\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C_{ijl}x_{ijlt} &\leq\;& B_t && \forall t\in T && (5.6)\\
&\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} H_{ijl}x_{ijlt} &\leq\;& IC_t && \forall t\in T && (5.7)\\
&\sum_{i\in I}\sum_{j\in J} x_{ijlt} &\leq\;& 1 && \forall l\in L, t\in T && (5.8)\\
&\sum_{i\in I} y_{ilt} &=\;& 1 && \forall l\in L, t\in T && (5.9)\\
&\frac{1}{2}(y_{jlt}-y_{jlt-1})-\frac{1}{2}(y_{ilt}-y_{ilt-1})-\frac{1}{2} &\leq\;& x_{ijlt} && \forall i,j\in I, l\in L, t\in T && (5.10)\\
&z_{td} &\geq\;& G_{td}-\frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l} && \forall d\in D, t\in T && (5.11)\\
&z_{td} &\geq\;& 0 && \forall d\in D, t\in T && (5.12)\\
&x_{ijlt}, y_{ilt} &\in\;& \{0,1\} && \forall i,j\in I, l\in L, t\in T. && (5.13)
\end{aligned}
$$

The exact solution method for the extended model is solving the MILP with an available solver. Note that due to the NP-hardness of the Migration of Fibre problem, the computation time for the exact solution method using large data sets might be significantly large. Considering the usability of the model for the telecommunication companies, it is necessary to develop methods which create good solutions in a reasonable computation time. These are described in the next sections.

5.2. Per year optimisation

The first heuristic method to create a (feasible) migration schedule is a per year optimisation, where for each time period, an exact solution is created. The model for the base problem is:

$$
\begin{aligned}
&\min \sum_{i\in I}\sum_{j\in I}\sum_{l\in L} C_{ijl}x_{ijl} && && && (5.14)\\
\text{s.t.} &\sum_{i\in I}\sum_{j\in I} x_{ijl} &\leq\;& 1 && \forall l\in I && (5.15)\\
&\sum_{i\in I} y_{il} &=\;& 1 && \forall l\in L && (5.16)\\
&\frac{1}{2}(y_{jl}-Y_{jl})-\frac{1}{2}(y_{il}-Y_{il})-\frac{1}{2} &\leq\;& x_{ijl} && \forall i,j\in I, l\in L && (5.17)\\
&\frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{il}}{\sum_{l\in L} RT_l} &\geq\;& G_d && \forall d\in D. && (5.18)
\end{aligned}
$$

The model for the extended model is:

$$\min \sum_{d \in D} z_d \tag{5.19}$$

$$\text{s.t.} \sum_{i \in I} \sum_{j \in I} \sum_{l \in L} C_{ijl} x_{ijl} \leq B \tag{5.20}$$

$$\sum_{i \in I} \sum_{j \in I} \sum_{l \in L} H_{ijl} x_{ijl} \leq IC \tag{5.21}$$

$$\sum_{i \in I} \sum_{j \in I} x_{ijl} \leq 1 \qquad \forall l \in L \tag{5.22}$$

$$\sum_{i \in I} y_{il} = 1 \qquad \forall l \in L \tag{5.23}$$

$$\frac{1}{2}(y_{jl} - Y_{jl}) - \frac{1}{2}(y_{il} - Y_{il}) - \frac{1}{2} \leq x_{ijl} \qquad \forall i, j \in I, l \in L \tag{5.24}$$

$$z_d \geq G_d - \frac{\sum_{i \in I} \sum_{l \in L} R_{ild} \cdot y_{il}}{\sum_{l \in L} RT_l} \qquad \forall d \in D \tag{5.25}$$

$$z_d \geq 0 \qquad \forall d \in D \tag{5.26}$$

$$x_{ijl}, y_{il} \in \{0,1\} \qquad \forall i, j \in I, l \in L. \tag{5.27}$$

Here, Y_{il} denotes the technology and topology combination $i \in I$ in the migration schedule of the previous migration period for location $l \in L$. The expected computation time of this method is shorter than the expected computation time of the exact method, but it could still be too large for practise.

5.3. Problem-based heuristic

The second heuristic we use to obtain a good solution in an acceptable computation time is based on the restrictions of the Migration of Fibre problem. The main characteristic of the base problem is the requested bandwidth percentage and the two main characteristics of the extended problem are the budget and the installation capacities. The heuristic starts with a solution in which the technology and topology combination in each time period is equal to the start time period, i.e., the current situation. For the base mode, the heuristic starts in the first time period that has to be upgraded to meet the requested bandwidth and upgrades the locations with the largest profit. For the extended model, the heuristic starts in the first time period that still could be upgraded, i.e., where there is budget and installation capacity left and upgrades the locations with the largest profit. The profit of an upgrade is the ratio of the amount of bandwidth the upgrade will deliver and the corresponding upgrade costs (and required installation capacity). In an ideal situation, the realised bandwidth is as high as possible and the investments costs (and required installation capacity) are as low as possible. For the base model it holds that in case that enough locations are upgraded to meet the constraints for that time period, the heuristic continues with the same procedure in the next time period. For the extended model it holds that in case that no more locations can be upgraded for that time period, the heuristic continues with the same procedure in the next time period. Using this procedure, a feasible solution is constructed for the base model and solution for the extended model is still feasible. In this way, the quality of the solution is guaranteed. Next, we explain how this is implemented for the base and extended problems.

5.3.1. Implementation

For the implementation of the problem-based heuristic, we distinguish the base problem and the extended problem. For both the problems, a total profit matrix is made. For the base problem, the profit is based on the following ratio:

$$\frac{R_{jl}}{C_{ijl}}, \tag{5.28}$$

where R_{jl} is the matrix containing the mean values of R_{jld} over all the distances $d \in D$. For the extended problem, the profit is based on the following ratio:

$$\frac{R_{jl}}{\frac{C_{ijl}}{B_t} + \frac{H_{ijl}}{IC_t}}. \tag{5.29}$$

By dividing C_{ijl} and H_{ijl} respectively by B_t and IC_t, the influence of the migration costs and required installation capacities are equivalent. Moreover, the profit matrix shows for each possible upgrade per location what the corresponding profit ratio is per time period. After this matrix is made, the following steps are followed:

1. Construct a migration schedule in which the technology and topology combinations in each time period are equal to the start time period, i.e., there are no migration upgrades in this schedule.

2. Select the lowest time period $t \in T$ which has not been upgraded yet and which has to be upgraded (base model: requested bandwidth constraint) or which could be upgraded (extended model: there is budget and installation capacity left over).

3. Using the total profit matrix, a profit matrix is made for the current situation. This is a matrix containing the profits for the selected time period $t \in T$ and the technology and topology combination $i \in I$ of the previous time period $(t-1) \in T$.

4. The upgrade with the highest ratio in the matrix, made in the previous step, is selected and is carried out in the migration schedule. Also the subsequent time periods of this location get the same upgrade.

5a. (Base model) Repeat step 4. until the migration schedule for the selected time period meets the required bandwidth constraint. For the base problem, this is the bandwidth constraint, and in this way, the migration schedule up to the selected time period has became a feasible schedule.

5b. (Extended model) Repeat step 4. until as much locations as possible are upgraded and the solution still meets the budget and installation capacity constraint. In this way, the migration schedule is still a feasible schedule.

6a (Base model) Repeat step 2 until 5, until every time period $t \in T$ is upgraded as much as needed, and then, the migration schedule feasible.

6b (Extended model) Repeat step 2 until 5, until every time period $t \in T$ is upgraded as much as possible, without losing feasibility.

Note that the two last steps of the problem based heuristic are dependent of the type of the model, i.e., the base or extended model.

5.4. Simulated annealing

The meta-heuristic used in this research is Simulated Annealing (SA). A meta-heuristic is a general solution method that provides general structures and strategy guidelines for developing a specific

heuristic method. SA is a stochastic algorithm which searches for a global optimum and avoids getting stuck in local, non-global optimum [14]. It is based on a heating and cooling process and simulates the energy changes in a system subjected to a cooling process until it converges to an equilibrium state.
From an initial solution s_0, the SA algorithm generates a random neighbour during each iteration. A neighbour is a (feasible) solution obtained by performing an operation on the current solution. If this neighbour is a better solution than the current solution, related to the corresponding values of the objective function, the neighbour solution will be accepted and becomes the new current solution. If this is not the case, the neighbour will be accepted with a certain probability, which depends on the current temperature. This probability is the Boltzmann probability:

$$P(\text{acceptance}) = e^{-\frac{|f(s')-f(s)|}{T}}, \tag{5.30}$$

where $|f(s') - f(s)|$ denotes the difference ΔE between the objective function value of the generated neighbour s' and the objective function value of the current state s. T denotes the temperature. After M_{max} iterations of the algorithm, the temperature T decreases, whereby the probability of acceptance also decreases. The probability of acceptance also depends on how the worse the neighbour solution is, i.e., the worse the neighbour solution, the lower the chance of acceptance. A cooling schedule $g(T)$ defines for each step k of the algorithm the temperature T_k. Due to the possibility of accepting worse solutions, the algorithm can escape an inferior local optimum. The algorithm stops after a predefined amount of iterations N_{max}. The following overview summarises the algorithm of SA [15]:

Algorithm 1 Simulated annealing algorithm

```
Input: Cooling schedule g(T) and data
s = s_0  (initial solution)
T = T_start  (starting temperature)
N = 0
while N < N_max do
    M = 0
    while M < M_max do
        Generate a random neighbour s'
        ΔE = f(s') − f(s)
        if ΔE ≤ 0 for minimisation problem | ΔE ≥ 0 for maximisation problem then
            s = s'  (accept the neighbour solution)
        else
            Accept s' with probability e^(−|ΔE|/T)
            s = s' if s' is accepted
        end
        save s' and f(s') if s' is accepted
        M = M + 1
        N = N + 1
    end
    T = g(T)
end
Output: Saved solutions s' and corresponding objective function values f(s')
```

5.4.1. Implementation

The first step of our implementation of SA is to obtain a good initial solution. To create an initial solution, the problem-based heuristic as described in Chapter 5.3 is used. Solutions are presented as a matrix of which the rows represent the locations, the columns represent the migration time periods and the elements of the matrix represent the technology and topology combination for the corresponding location and time period. The technology and topology combinations are ranged from 1 until k, with k the total amount of combinations. Furthermore, combination 1 provides the smallest bandwidth and combination k provides the largest bandwidth. The objective function for a solution s of the base model is described as:

$$f(s) = \sum_{i \in I} \sum_{j \in I} \sum_{l \in L} \sum_{t \in T} C_{ijl} x_{ijlt}. \tag{5.31}$$

The objective function for a solution s of the extended model is described as:

$$f(s) = \sum_{t \in T} \sum_{d \in D} \max \left(0, G_{td} - \frac{\sum_{i \in I} \sum_{l \in L} R_{ild} \cdot y_{ilt}}{\sum_{l \in L} RT_l} \right). \tag{5.32}$$

The goal of Simulated Annealing is to find a solution with the lowest possible objective value.

Simulated Annealing also needs a temperature scheme. This scheme defines for each step of the algorithm the temperature T. First, we set an initial temperature T_{start}:

$$T_{\text{start}} = \frac{f(s_0) - f(s_{\text{worst}})}{\ln(0.05)}. \tag{5.33}$$

Here, $f(s_0)$ is the objective function value of the start solution and $f(s_{\text{worst}})$ is the objective function value of the worst neighbour of s_0. Consequently, the neighbour with the maximum possible decrease in objective function value is accepted with probability 0.05.

Furthermore, we define the geometric cooling schedule as:

$$g(T) = \alpha T \text{ , with } 0 < \alpha < 1. \tag{5.34}$$

We apply this scheme after each M_{max} iterations. Previous research showed that α should be between 0.5 and 0.99 [15].

The stop condition is defined as that the algorithm will stop after N_{max} iterations.

In each iteration of the algorithm, a neighbourhood solution will be created, using the current solution. A neighbourhood solution of a feasible solution is defined as follows. There are three operations possible to create a feasible neighbour solution. First, choose a random number. If the selected number is smaller than $\frac{1}{3}$, then operation 1 is performed, if the number is smaller than $\frac{2}{3}$ and bigger than $\frac{1}{3}$, then operation 2 is performed and otherwise, operation 3 is performed. By operation 1, a location is upgraded in a time period and, if possible, an other location is downgraded in the same time period. By operation 2, a location will be upgraded in a timed period and by operation 3, a location will be downgraded in a time period. The operations are specified as follows:

1. A location is randomly chosen. If the selected location contains already the best possible technology and topology combination in each migration time period, reselect the location randomly, until upgrading in at least one of the migration time periods of this location is possible. Then select a migration time period randomly, where upgrading the technology and topology combination for this location is possible and upgrade the selected location for the selected time period. With upgrading a network, we mean that we add 1 to the corresponding entry in the solution matrix. If needed, some of the following time periods for this location should also be

increased by 1, such that the migration steps for the location form a row of non-descending entries.
If it is possible to downgrade an other location in the selected time period, select randomly an other location and check if it is possible to downgrade this location in the selected time period. With downgrading a network, we mean that we subtract 1 from the corresponding entry. If the technology and topology combination for this location and time period is already as low as possible, then reselect the location randomly. This is repeated until a location is found where a downgrade is still possible in the selected time period and then the location in this time period is downgraded. In addition, if needed, some of the previous time periods for this location should be decreased by 1, such that the migration steps for the location form a row of non-descending entries. If it is not possible to downgrade an other location in the selected time period, no additional steps are performed.

2. A location is randomly chosen. If the selected location contains already the best possible technology and topology combination in each migration time period, reselect the location randomly, until upgrading in at least one of the migration time periods of this location is possible. Then, select a migration time period randomly, where upgrading the technology and topology combination for this location is possible and upgrade the selected location for the selected time period. With upgrading a network, we mean that we add 1 to the corresponding entry in the solution matrix. If needed, some of the following time periods for this location should also be increased by 1, such that the migration steps for the location form a row of non-descending entries.

3. A location is randomly chosen. If the selected location contains already the worst possible technology and topology combination in each migration time period, reselect the location randomly, until downgrading in at least one of the migration time periods of this location is possible. Then, select a migration time period randomly, where downgrading the technology and topology combination for this location is possible and downgrade the selected location for the selected time period. With downgrading a network, we mean that we subtract 1 from the corresponding entry in the solution matrix. If needed, some of the previous time periods for this location should be decreased by 1, such that the migration steps for the location form a row of non-descending entries.

For the extended problem, we added a small extension to operation 2 and 3, to increase the chance of creating a solution which is feasible:

2. Check the feasibility of the adapted solution. If it is infeasible, i.e., it does not meet the budget and/or installation capacity constraint, then also perform operation 3 in the selected time period. In this case, a location is upgraded and an other location is downgraded in the same time period.

3. Check the feasibility of the adapted solution. After operation 3 is performed, i.e., a location is downgraded in a time period, it is possible that the costs for the next time period becomes higher and exceeds the budget for this next time period. If the created neighbour solution is infeasible, i.e., it does not meet the budget and/or installation capacity constraint, then also perform operation 2 in the selected time period. In this case, a location is upgraded and an other location is downgraded in the same time period.

For the base problem, check if the new solution is feasible, i.e., it meets the bandwidth constraint. For the extended problem, if an extension of operation 2 or 3 is performed, also a check has to be performed: it is checked whether or not the new solution meets the budget and installation capacity constraints. If it does not meet these constraints, the adapted solution is rejected and the procedure of the operation is started again, using the unadapted solution. This is repeated until a solution is found which meets the constraints. We call this adapted solution a neighbour. All the solutions which can be formed by using one of the operations to adapt the current solution, form the neighbourhood of the current solution. Additionally, during each iteration, the neighbour will be compared with the current solution. It will be accepted if it is better than the current solution and otherwise it will be accepted with the Boltzmann probability.

6
Computational Results

In this research, several solution methods are used to obtain a good migration plan for the migration of fibre in the Netherlands. The solution methods are implemented in MATLAB and all computations are done on a laptop with an Intel ® Core^{TM} vPro^{TM} 2,40 GHz with 8 GB RAM memory. Firstly, an overview is given of the data which is used for creating migration plans using the different solution methods. Subsequently, we display the results of the exact solution method for the base and extended model. Furthermore, the results are given when having the present value for the migrations costs, when having a total budget and when the migration constraint is adjusted. Then, the results are given for the other solution methods. These results are compared to the solutions of the exact solution methods, to determine the quality of the solution methods. The scalability of the solution methods is based on the quality of the solution found by the different solutions methods and the corresponding computation times of these methods.

6.1. Data

To illustrate the scalability of the solution methods, five real-life areas containing different amounts of cabinets and houses are selected. The time-span of the migration schedule and the amount of possible technology and topology combinations are three moments in time and three combinations, respectively. In Table 6.1, an overview is given of the characteristics of the areas, where area A is the same as used in Phillipson [2].

Area	No. of cabinets	No. of houses
A	40	18,550
B	180	58,842
C	496	44,151
D	874	433,092
E	26,164	6,352,365

TABLE 6.1. AREA CHARACTERISTICS

For example, for area A the number of locations $l \in L$, i.e., the number of cabinets, is equal to 40. Furthermore, for area A it holds that $\sum_{l \in L} RT_l = 18,550$. Area E consists of the first four areas and some additional areas. The requested bandwidth per time period is assumed to be the same for every area. The percentages G_{td} can be found in Table 6.2. For example, in the second migration period, the goal is to have 85% of the houses connected within 600 meter. To obtain the number of premises R_{ild} reached by technology and topology combination $i \in I$ within $d \in D$ meters at location $l \in L$, we used geometric modelling, described by Phillipson et al. [8] and Phillipson [9]. The start state, i.e., the

Time period	Distance [d in meter]	Requested bandwidth percentage
1	200	0.20
1	400	0.40
1	600	0.70
2	200	0.30
2	400	0.70
2	600	0.85
3	200	0.40
3	400	0.85
3	600	0.85

TABLE 6.2. REQUESTED BANDWIDTH

current technology and topology combination is a Full Copper/ADSL network ($i = 1$) for all locations. For creating more realistic values for the migration costs, we also used the present value of migration costs (see Section 3.2.3). These are calculated using the formula:

$$C_{ijlt} = \frac{C_{ijl}}{(1+R)^n}, \qquad \forall i, j \in I, l \in L, t \in T. \tag{6.1}$$

where R is the interest rate and n is the number of years from the starting year of time period $t = 1$ until time period $t \in T$. We use both the values C_{ijl} and C_{ijlt} in our research. We used $R \approx 0.06$ for testing the different solution methods and we used the time periods 2017-2019, 2020-2022 and 2023-2025. In this case, n is equal to 3 for computing C_{ijl2} and $n = 6$ for computing C_{ijl3}.

Furthermore, we made the following assumptions for the budget B_t, installation capacity IC_t and required migration capacity H_{ijl}, because we have no real-life data for these three parameters. We set the budget for a time period $t \in T$ equal to 90% of the migration costs of the optimal solution of the base model resulting from solving the per year optimisation method for that time period. Thus,

$$B_t = 0.9 \cdot \sum_{i\in I}\sum_{j\in I}\sum_{l\in L} C_{ijl}x_{ijlt}, \qquad \forall t \in T, \tag{6.2}$$

where x_{ijlt} is the schedule of the base model resulting from solving the per year optimisation method. Furthermore, because there is no data available for the required capacity for migrating H_{ijl}, we assumed that $H_{12l} = 2$, $H_{13l} = 4$ and $H_{23l} = 3$ for every location $l \in L$. For example, the amount of employees needed for migrating a location from technology and topology combination 1 to combination 2 is two, so we do not distinguish locations. From this, we assumed that the installation capacity for a time period $t \in T$ is 90% of the needed migration capacity of the optimal solution of the base model resulting from solving the per year optimisation method of that time period. Thus,

$$IC_t = 0.9 \cdot \sum_{i\in I}\sum_{j\in I}\sum_{l\in L} H_{ijl}x_{ijlt}, \qquad \forall t \in T, \tag{6.3}$$

where x_{ijlt} is the schedule of the base model resulting from solving the per year optimisation method.

In the next sections, results are given per solution method. For these results, the data described above is used. Per method, for the base and extended model, an overview is given of the runtime of the methods for the five areas and of the quality of the created migration schedule. The quality of the schedule is based on the divergence between the corresponding costs (base model) or realised bandwidth (extended model) created by a solution method and the costs/realised bandwidth of the schedule created by the exact solution method. Furthermore, a comparison is made between the solution methods.

6.2. Exact solution

In this section, we discuss the used solvers for obtaining a solution using the exact solution method. Furthermore, an overview is given of the results of the exact solution methods and the results when using the present value for migration costs, a total budget and when the migration constraint is adjusted.

6.2.1. Solvers

To determine the quality of the solutions resulting from solving the different solution methods, first the value of the optimal solutions of the base model and extended model for the different areas are determined. For this, three different solvers are used, because the type of the solver can affect the runtime for obtaining an optimal solution and we want to obtain a solution as fast as possible. The first solver used is 'OpenSolver' in Excel, in combination with COIN-OR [7]. Furthermore, CPLEX Optimizer [17] in combination with MATLAB and the standard solver IntLinProg in MATLAB are used. In Table 6.3, an overview is given of the computation times using these solvers for the base problem.

Area	COIN-OR	CPLEX Optimizer	IntLinProg
A	7.5	6.8	110.8
B	36.1	9.2	> 8 hours
C	396.3	5.1	> 8 hours
D	912.3	9.7	> 8 hours
E	> 8 hours	10,804.0	> 8 hours

TABLE 6.3. COMPUTATION TIME OF EXACT SOLVERS FOR THE BASE PROBLEM [SECONDS]

The runtime as noted in this table is the time for building the problem plus the time for solving the problem. As seen in the table, the CPLEX Optimizer solver provides the best results related to the runtimes. Therefore, the solver we used for further (exact solution) results is CPLEX Optimizer.

In Table 6.4, an overview is given of the optimal objective function values of the base and extended model for the five areas and the corresponding computation times for obtaining an optimal solution.

Area	Obj. val. [€] Base	Runtime [sec.] Base	Obj. val. Extended	Runtime [sec.] Extended
A	2,963,578	6.8	0.070	11.2
B	2,224,871	9.2	0.025	7.5
C	3,993,816	5.1	0.003	18.9
D	15,466,701	9.7	0.035	29.7
E	807,770,027	10,804.0	0.015	8,296.7

TABLE 6.4. RESULTS EXACT SOLUTIONS FOR THE BASE AND EXTENDED PROBLEM

As seen in Table 6.4, the NP-hardness of the base and extended model affects the computation time for area E. This computation time is approximately 3 hours for the base model and 2.3 hours for the extended model. However, this is not a reasonable runtime for the telecom operators to obtain a migration schedule. The operator prefers to obtain a schedule in a few minutes, checks the feasibility of the plan and when necessary, adds some extra restrictions to obtain a new schedule and checks again the feasibility.

6.2.2. Present value for migration costs

Additionally, we are interested in the effect of making the migration costs time-dependent, i.e., C_{ijlt}. In Table 6.5, an overview is given of the optimal objective function values of the base model for the five areas and the corresponding computation times for obtaining an optimal solution, where the costs are present values. Replacing the costs and budget by their present values in the extended model, gives the same solutions and results as in Table 6.4.

Area	Obj. val. [€]	Runtime [sec.]
	Base	Base
A	2,642,231	10.2
B	1,629,356	11.8
C	2,837,715	14.1
D	11,481,366	26.4
E	614,666,000	15,545.8

TABLE 6.5. RESULTS EXACT SOLUTIONS FOR THE BASE PROBLEM, PRESENT VALUE COSTS

To analyse the effect of using the present value for the migration costs more in depth, in Figure 6.1, the corresponding costs for the optimal migration plan for area E are depicted per migration time period. The realised and requested bandwidths of area E for the base model are depicted in Figure 6.2. The two schedules differ from each other. As seen in these two figures, in the first migration period, there are no investments costs made in the case of time-dependent costs. This means, that the current network already meets the required bandwidth constraint. Therefore, we know that there are multiple optimal solutions in the case of time-independent costs. The investment costs in the first time period are unnecessary and also could be made a later time period. The realised bandwidth in the final time period is roughly the same, as seen in Figure 6.2.

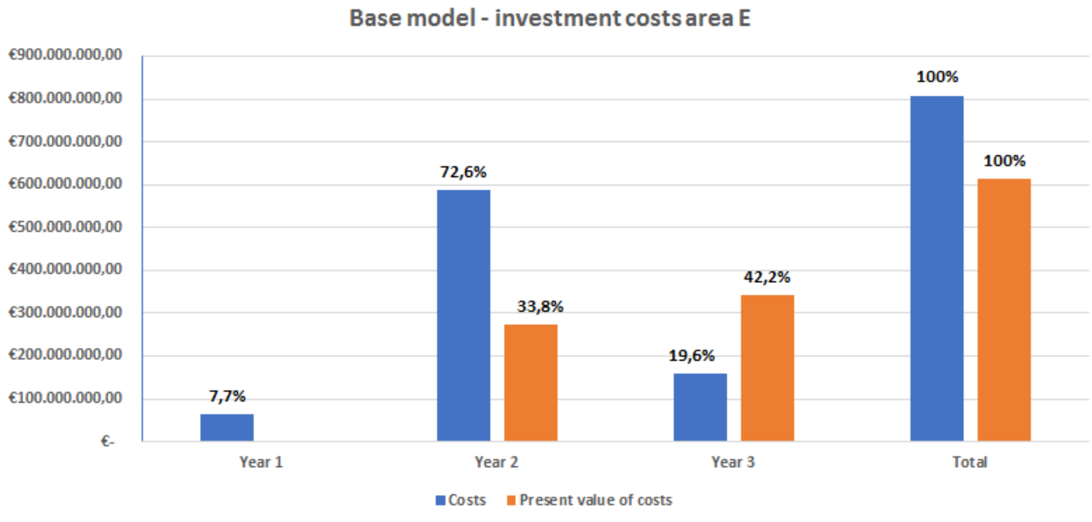

FIGURE 6.1. INVESTMENT COSTS

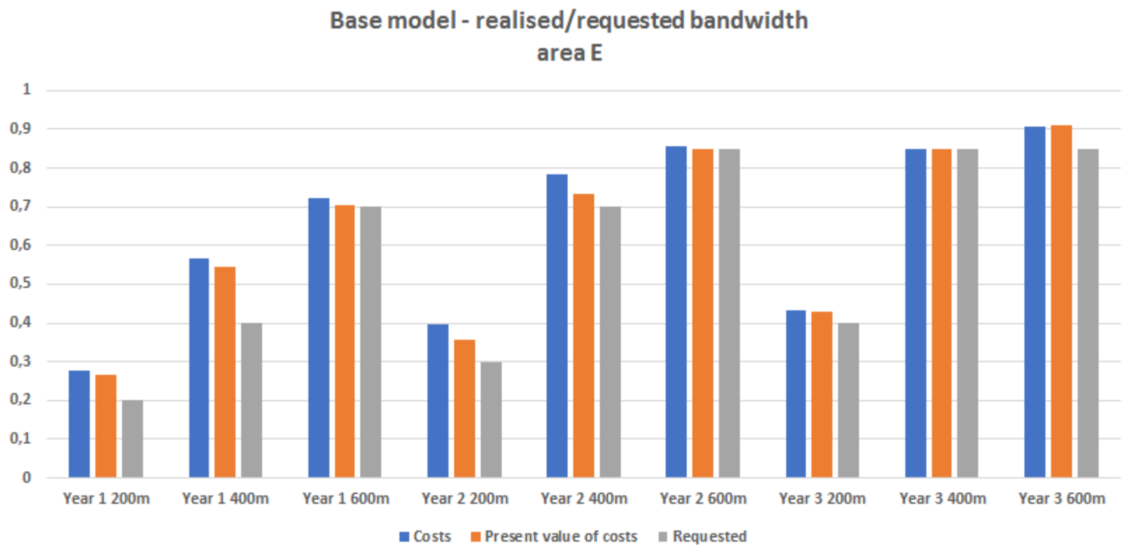

FIGURE 6.2. BANDWIDTH

6.2.3. Total budget

In the previous results, the budget B_t is dependent on migration period $t \in T$. However, we are also interested in the effect of having one budget for the total migration period. We assume that $B = \sum_{t \in T} B_t$, where B_t is defined as in Equation (6.2). In Table 6.6, an overview is given of the exact solutions for the extended model, using a total budget, compared with the exact solutions, using a budget per period.

Area	Obj. val. B	Dif. obj. val. B_t	Total costs B [€]	Dif. costs B_t [€]	Runtime [sec.]
A	0	-0.070	3,251,149	10,603	24.2
B	0.016	-0.010	2,002,191	2,440	6.7
C	0	-0.003	3,893,915	0	16.0
D	0.003	-0.032	13,919,628	1,109	14.3
E	0	-0.015	810,841,014	-40,964	11,260.8

TABLE 6.6. RESULTS EXACT SOLUTIONS OF EXTENDED MODEL, USING TOTAL BUDGET

These results show, for example, that for area A there is spent 10,603 euro more over the total migration period than when using a budget per time period, but the bandwidth demand is reached in every time period $t \in T$ and $d \in D$. This means that there is reached 0.070 bandwidth more when using a total budget B instead of a per period budget B_t. Furthermore, we see that for area C, the costs over the total migration period are the same, meanwhile the bandwidth demand is totally met. Also the bandwidth demand of area E is totally met when using a total budget, and even less money has been spent in total. Therefore, it is more efficient to set a budget for the total migration period, instead of a budget per migration time period.

6.2.4. Adjusted migration constraint

The migration constraint in the base and extended model, as mentioned before, is:

$$\frac{1}{2}(y_{jlt} - y_{jlt-1}) - \frac{1}{2}(y_{ilt} - y_{ilt-1}) - \frac{1}{2} \leq x_{ijlt}, \qquad \forall i, j \in I, l \in L, t \in T. \tag{6.4}$$

However, this constraint is quite complicated to interpret. Without loss of generality, we can change this constraint to:

$$y_{ilt-1} + y_{jlt} - 1 \leq x_{ijlt}, \qquad \forall i, j \in I, l \in L, t \in T. \tag{6.5}$$

The only difference between Constraint (6.4) and (6.5) is that in this latter constraint, it is forced that $x_{ijlt} = 1$ for $i = j$ when no migration takes place. However, this has no effect on the optimal solutions obtained for the base or extended model, because the migration costs for migrating to the same technology and topology combination are zero. Moreover, we are interested whether or not this simplified migration constraint also reduces the computation time of obtaining an exact solution. In Table 6.7, an overview is given of these runtimes.

Area	Runtime [sec.] Adjusted - Base	Runtime [sec.] Base	Runtime [sec.] Adjusted - Extended	Runtime [sec.] Extended
A	7.7	6.8	8.2	11.2
B	2.8	9.2	7.4	7.5
C	5.7	5.1	18.2	18.9
D	10.1	9.7	15.8	29.7
E	> 12 hours	10,804.0	6,734.6	8,296.7

TABLE 6.7. RUNTIMES, WITH ADJUSTED MIGRATION CONSTRAINT - EXACT SOLUTIONS

Comparing these runtimes with Table 6.4, we see that the adjusted migration constraint has got a positive effect on the runtime for the extended model.

6.2.5. Limited runtime

The runtime for obtaining a solution resulting from solving the ILP or MILP for small areas is short. Unfortunately, this is not the case for larger areas, like area E. Therefore, we are interested in the obtained solution after solving the ILP and MILP in a predefined limited runtime. The runtime for obtaining a solution resulting from solving the ILP or MILP is a combination of the runtime for building the model and the time for solving the model. The building time in MATLAB for the base problem of area E is $6,575$ seconds and the building time for the extended problem of area E is $5,991$ seconds. For both models, we set the limited solving time to 600 seconds and 900 seconds. In Table 6.8, an overview is given of the obtained solutions resulting from solving the ILP and MILP when the solving time is limited. As can be seen in this table, the objective function value of the solution obtained for the base problem is not close to the objective function value of the optimal solution. However, the objective function value of the solution obtained for the extended problem is nearly the same as the optimal solution.

Limited solving time	Obj. val. [€] Base	Obj. val. exact [%] Base	Obj. val. Extended	Obj. val. exact [%] Extended
600	3,067,303,470	280 %	0.015	0.005%
900	3,067,303,470	280 %	0.015	0.005%

TABLE 6.8. RESULTS EXACT SOLUTIONS WHEN LIMITED SOLVING TIME - AREA E

6.3. LP-relaxation

To create a lower bound for the objective function values, we relaxed the ILP of the base model and the MILP of the extended model. In an ideal situation, the optimal solution found by the LP-relaxation is also a feasible solution for the original ILP and MILP, i.e., x_{ijlt} and y_{ilt} are both binary for all $i, j \in I, l \in I$ and $t \in T$. Additionally, the expected runtime for obtaining an optimal solution for an LP-relaxation is lower than for obtaining an optimal solution for an ILP or MILP. However, in the case that the solution by the LP-relaxation is a feasible solution to the original ILP and MILP, then the runtime will be the same. Lastly, for the LP-relaxations it holds that $0 \leq x_{ijlt} \leq 1$ and $0 \leq y_{ilt} \leq 1$ for all $i, j \in I, l \in L$ and $t \in T$.

Area	Obj. val. [€] Base	Runtime [sec.] Base	Obj. val. Extended	Runtime [sec.] Extended
A	92,883	1.6	0	5.2
B	0	2.4	0	7.5
C	0	4.4	0	12.0
D	0	7.4	0	19.9
E	0	6,598	0	7,948.5

TABLE 6.9. RESULTS OF THE LP-RELAXATIONS

In Table 6.9, an overview is given of these lower bounds. As seen in this table, except for the base problem of area A, the lower bound for the original base and extended model has no added value. This is because, without loss of generality, we already knew that 0 is the lower bound for the objective function value of the base and extended model. The reason for this, is the fact that it might hold that $\sum_{i\in I}\sum_{j\in I} x_{ijlt} = 0$ for all $l \in L$ and $t > 1$. Striking is that the runtime for obtaining a solution for area E using the LP-relaxation for both models is close to the runtime of obtaining a solution for area E using the exact solution method for both models. This is because the runtime stated in all table of this chapter include the runtime for building the problem and is not only the runtime for solving the problem. The building time in MATLAB for the base problem of area E is 6,575 seconds and the building time for the extended problem of area E is 5,991 seconds. Furthermore, the optimal solutions resulting from solving the LP-relaxations are not feasible for the base and extended models, i.e., x_{ijlt} and y_{ilt} are not binary for all $i, j \in I, l \in I$ and $t \in T$. However, we can make the LP-relaxation stronger by using the adjusted migration constraint Constraint (6.5) instead of Constraint (6.4), see Table 6.10.

Area	Obj. val. [€] Base	Gap exact Base	Runtime [sec.] Base	Obj. val. Extended	Gap exact Extended	Runtime [sec.] Extended
A	1,815,207	38.7%	2.1	0.049	30.3%	6.1
B	887,976	60.1%	2.4	0.014	44.7%	7.5
C	1,361,341	65.9%	3.3	0	100%	9.9
D	6,862,030	55.6%	5.4	0.024	31.3%	13.8
E	295,087,519	63.5%	4,494.7	0.009	40.9%	5,991.0

TABLE 6.10. RESULTS OF THE LP-RELAXATIONS - ADJUSTED MIGRATION CONSTRAINT

Gap exact is the percentage of the difference between the objective function value of the solution resulting from solving the exact solution method and the objective function value of the solution resulting from solving the LP-relaxation. As seen in this table, the solution found by the LP-relaxation of the base problem of area A is closer to the objective function value of the exact solution than the solution found by the LP-relaxation of the base problem of area C, namely a gap of 38.7% and 65.9%, respectively. In the case of the adapted migration constraint, none of the solutions resulting from solving the LP-relaxations of the base problem are equal to 0. This is because the fact that it might not hold that $\sum_{i\in I}\sum_{j\in I} x_{ijlt} = 0$ for all $l \in L$ and $t > 1$. Unfortunately, the solutions resulting from solving the LP-relaxations, using the adjusted migration constraint, are still not feasible for the original ILP and MILP i.e., x_{ijlt} and y_{ilt} are not binary for all $i, j \in I, l \in I$ and $t \in T$. However, because the LP-relaxation is strengthened by the adjusted migration constraint and the results in Table 6.7, we replace the migration constraint by the adjusted migration constraint in the extended model for the per year optimisation method, which results are described in the next section.

6.4. Per year optimisation

In this section, we discuss the results for the per year optimisation method. For each time period, an exact solution is obtained. An overview of the results is given in Table 6.11.

Area	Obj. value [€] Base	Gap exact Base	Runtime [sec.] Base	Obj. value Extended	Gap exact Extended	Runtime [sec.] Extended
A	3,646,141	23.0%	17.5	0.084	19.5%	26.8
B	2,224,871	0%	16.3	0.025	0%	23.6
C	4,088,637	2.4%	19.6	0.010	325.8%	26.8
D	15,466,701	0%	21.2	0.035	0%	29.6
E	900,982,160	11.5%	1,181.3	0.019	24.7%	1,045.7

TABLE 6.11. RESULTS OF PER YEAR OPTIMISATION

The gap is determined as the objective function value of the solution resulting from solving the exact solution method minus the objective function value of the solution resulting from solving the per year method, divided by this latter objective function value. Especially the largest area, E, has got a significantly lower runtime compared to the runtime for obtaining a solution using the exact solution method. Striking is the enormous gap for the solution of the extended model of area C.

6.5. Problem-based heuristic

In this section, we discuss the results for the problem-based heuristic. An overview of the results is given in Table 6.12.

Area	Obj. val. [€] Base	Gap exact Base	Runtime [sec.] Base	Obj. val. Extended	Gap exact Extended	Runtime [sec.] Extended
A	3,144,668	6.1%	3.3	0.268	280.7%	3.5
B	2,890,151	29.9%	4.1	0.094	270.4%	4.0
C	5,176,185	29.6%	6.4	0.050	1,743.9%	6.0
D	18,932,626	22.4%	8.9	0.111	217.4%	8.5
E	905,425,985	12.1%	700.1	0.123	708.2%	630.4

TABLE 6.12. RESULTS OF THE PROBLEM-BASED HEURISTIC

As seen in this table, for the base model, the problem-based heuristic does not perform well for area B, C and D, having gaps of more than 20%. However, the runtime of approximately 11 minutes for area E is much smaller than the runtime of the exact solution method for area E, resulting in a deviation of 12.1% relative to the solution of the exact solution method. Unfortunately, the performance of the problem-based heuristic for the extended model is not good. The objective function values of the solutions of all areas have got a deviation of more than 200% relative to the objective function values of the solutions of the exact solution method.

6.6. Simulated annealing

In this section, we discuss the results of Simulated Annealing (SA). The parameters we used for SA are given in Table 6.13. Here, T_{start} is the initial temperature, $M_{\max}$ is the number of iterations per temperature, $N_{\max}$ is the total amount of iterations and α is the reduction factor of the temperature. The initial solution of SA is the solution resulting from solving the problem-based heuristic, because the runtime of the problem-based heuristic is significantly smaller than the runtime of the per year optimisation method.

Parameter	Base model	Extended model
T_{start}	3,316,808	0.000567
M_{max}	50	50
N_{max}	100,000	100,000
α	0.95	0.95

TABLE 6.13. PARAMETER VALUES

In the next subsections, a variation of parameter values is used and compared with the defined parameter values in Table 6.13. This, to gain insight in the influence of the parameters on the progression of the objective function values during the iterations of SA and to find the best parameter values for each model. For these results, we used the data set of area E, because we are the most interested in large data sets and the other areas are already included in area E.

Initial temperature

In Figure 6.3 and 6.4, the progress of the objective function value during the iterations of SA is depicted, using the parameters in Table 6.13. As can be seen, this progression for the base model does converge to a solution, but this is a worse solution than the start solution. This is caused by the start temperature, which is relatively high.

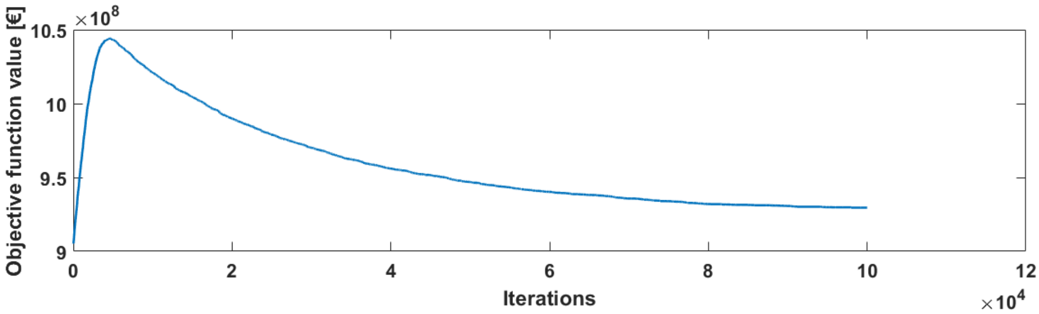

FIGURE 6.3. OBJECTIVE FUNCTION VALUE - BASE MODEL

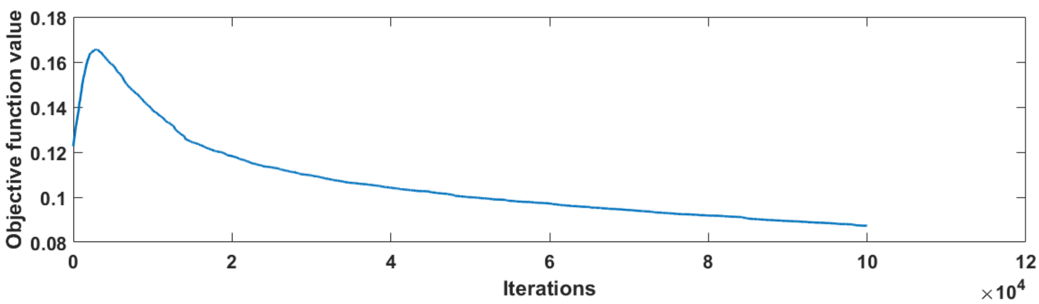

FIGURE 6.4. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL

The start temperature is determined by:

$$T_{start} = \frac{f(s_0) - f(s_{worst})}{\ln(0.05)}, \tag{6.6}$$

as described in Section 5.4.1. Replacing $f(s_{worst})$ by the average of the objective function values of all worse neighbour solutions, also provides a start temperature which is too high. The reason for this relatively high start temperature are outliers in investments costs for some locations. As a result, SA accepts solutions which are actually too worse to accept in the first 5,000 iterations of the base model which results in SA deviating too much from the start solution. This also happens in the first 3,000 iterations of SA for the extended model. Therefore, we changed the predefined start temperature for the base model to $T_{\text{start}} = 7,000$ and for the extended model to $T_{\text{start}} = 0.00001$. These selected values for the start temperature are based on preliminary experiments. In Figure 6.5 and 6.6, the progress of the objective function values for area E during the iterations of SA is illustrated, using the parameters in Table 6.14.

Parameter	Base model	Extended model
T_{start}	7,000	0.00001
$M_{\max}$	50	50
$N_{\max}$	100,000	100,000
α	0.95	0.95

TABLE 6.14. PARAMETER VALUES

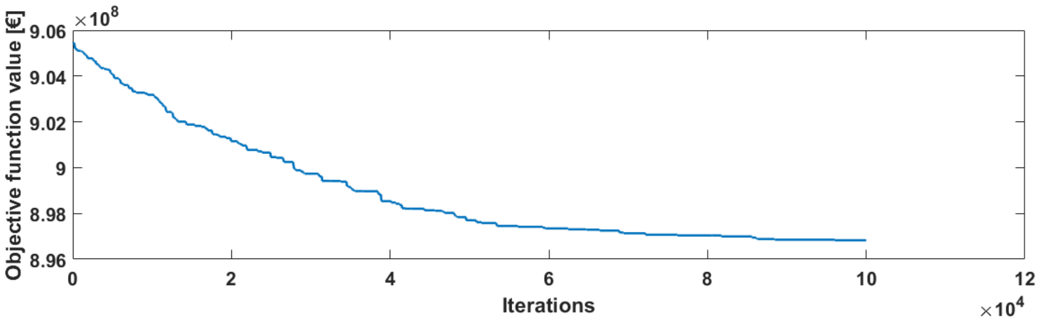

FIGURE 6.5. OBJECTIVE FUNCTION VALUE - BASE MODEL

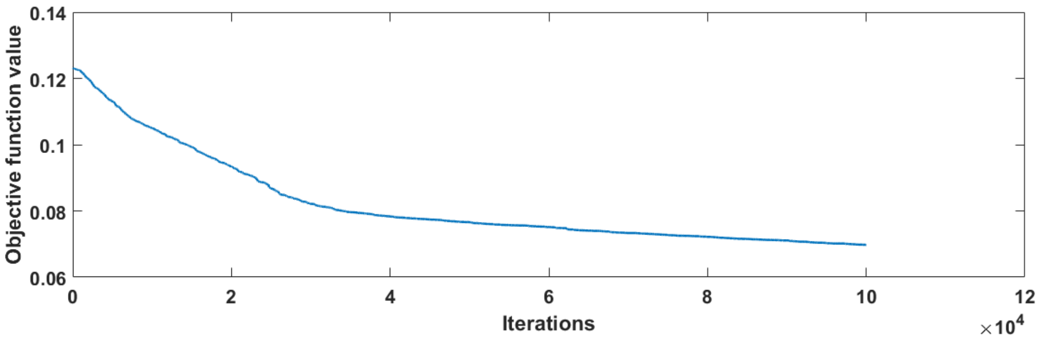

FIGURE 6.6. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL

We set the start temperature for the base model to $T_{\text{start}} = 7,000$ and for the extended model to $T_{\text{start}} = 0.00001$. However, these were not the start temperatures using Equation (6.6), so we compare these results with other start temperatures. For the base model, the results of setting the start temperature equal to $T_{\text{start}} = 6,500$ and $T_{\text{start}} = 7,500$ are depicted in Figures 6.7, 6.8, 6.9 and in Table 6.15. The objective function values of the solutions obtained with these two start temperature are slightly worse compared to the solution obtained with $T_{\text{start}} = 7,000$. The runtime when using $T_{\text{start}} = 6,500$ decreased a bit, but this does not compensate for the better objective function value of the obtained solution when using $T_{\text{start}} = 7,000$. Therefore, $T_{\text{start}} = 7,000$ is preferred for the base model.

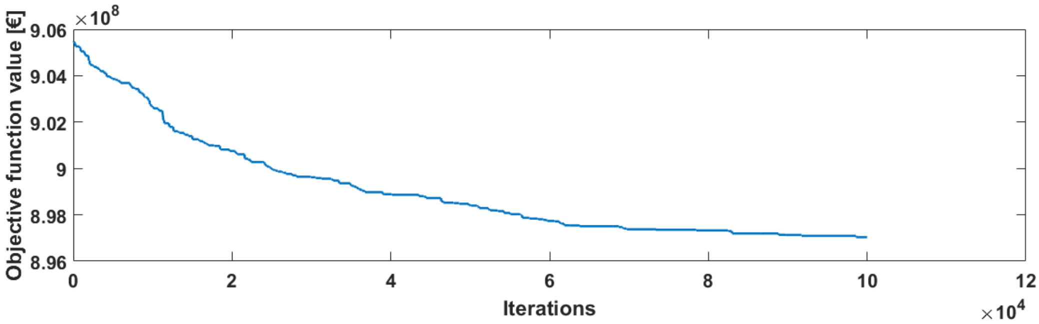

FIGURE 6.7. OBJECTIVE FUNCTION VALUE - BASE MODEL - $T_{\text{START}} = 6,500$

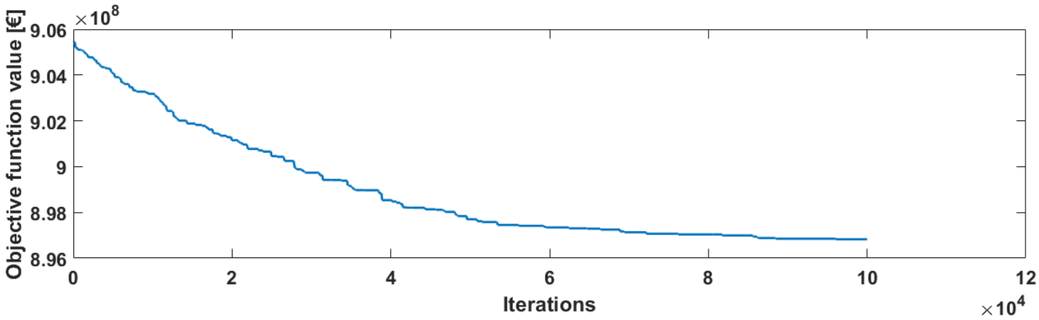

FIGURE 6.8. OBJECTIVE FUNCTION VALUE - BASE MODEL - $T_{\text{START}} = 7,000$

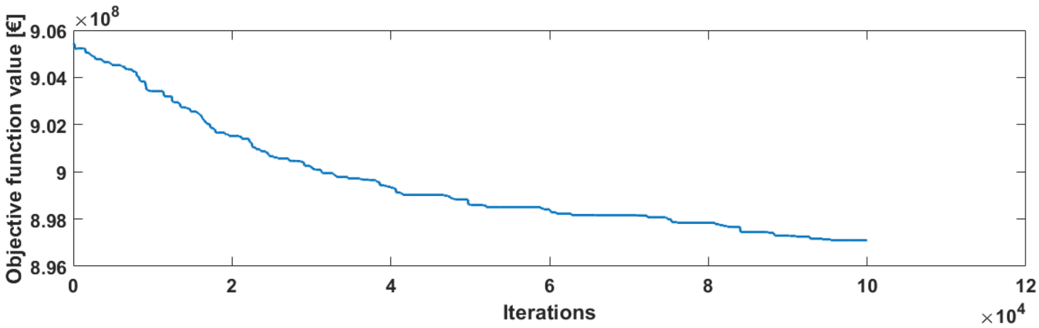

FIGURE 6.9. OBJECTIVE FUNCTION VALUE - BASE MODEL - $T_{\text{START}} = 7,500$

For the extended model, we compared the results with start temperatures $T_{\text{start}} = 0.000005$ and $T_{\text{start}} = 0.00005$. These results are depicted in Figures 6.10, 6.11, 6.12 and in Table 6.16. In the case of a high start temperature, $T_{\text{start}} = 0.00005$, too much worse solutions are accepted in the first 1,500 iterations

T_{start}	Obj. val. [€] Base	Gap exact Base	Runtime [sec.] Base
6,500	897,038,206	11.1%	1,452.1
7,000	896,824,637	11.0%	1,535.0
7,500	897,101,275	11.1%	1,551.3

TABLE 6.15. RESULTS OF DIFFERENT INITIAL TEMP. - AREA E

of SA, which results in a convergence to a less good solution when using $T_{start} = 0.000001$. The runtime when using $T_{start} = 0.000005$ decreases a bit, but that does not compensate for the better result when using $T_{start} = 0.00001$. Concluding, $T_{start} = 0.00001$ is preferred for the extended model.

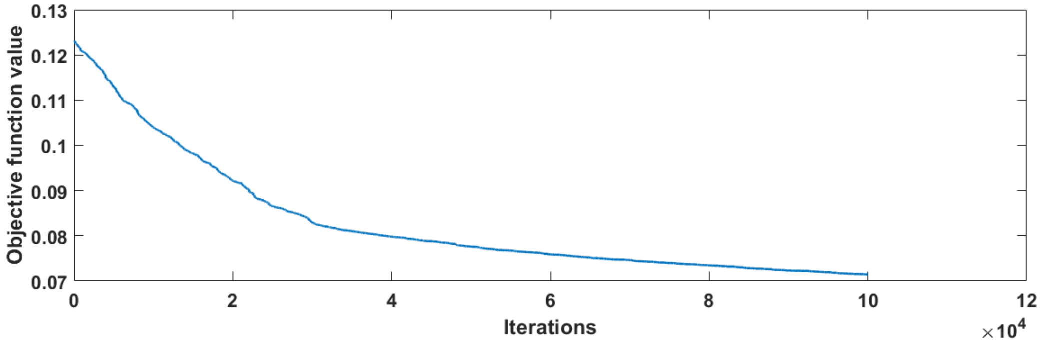

FIGURE 6.10. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - $T_{START} = 0.000005$

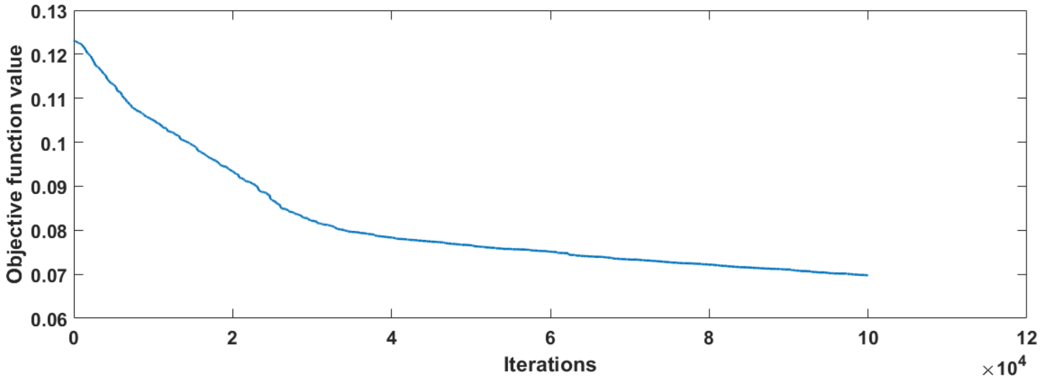

FIGURE 6.11. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - $T_{START} = 0.00001$

T_{start}	Obj. val. Extended	Gap exact Extended	Runtime [sec.] Extended
0.000005	0.071	373.3%	2,670.2
0.00001	0.070	366.7%	2,774.2
0.00005	0.074	393.3%	2,594.5

TABLE 6.16. RESULTS OF DIFFERENT INITIAL TEMP. - AREA E

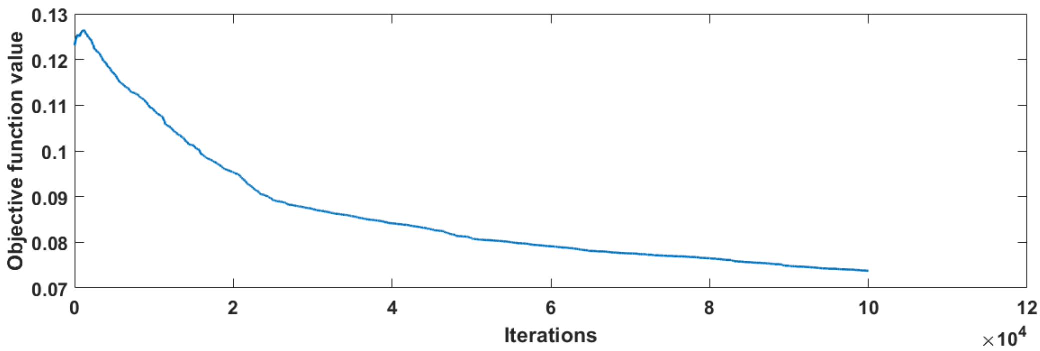

FIGURE 6.12. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - $T_{\text{START}} = 0.00005$

Number of iterations per temperature

Our next variation is the variation in number of iterations per temperature. Originally, we chose $M_{\text{max}} = 50$ for both the base and extended model. In Figures 6.13, 6.14, 6.15 and Table 6.17, there is an overview of the results using different amount of iterations per temperature for the base model, i.e., $M_{\text{max}} = 25$, $M_{\text{max}} = 50$ and $M_{\text{max}} = 75$. As seen in these figures, when $M_{\text{max}} = 75$, more worse neighbour solutions are accepted in the first 3,500 iterations and also over all iterations. This results in a better objective function value of the obtained solution, but it also results in a longer runtime for obtaining a solution. However, note that the objective function value curve is situated lower during the algorithm when $M_{\text{max}} = 75$. This means that when $M_{\text{max}} = 75$ and we interrupt SA after 40,000 iterations, SA always obtains a better solution then when using a lower M_{max}. Therefore, $M_{\text{max}} = 75$ is preferred for the base model.

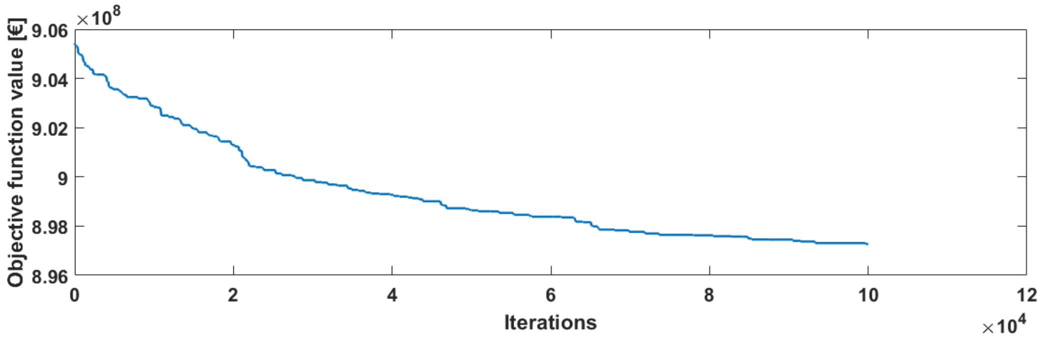

FIGURE 6.13. OBJECTIVE FUNCTION VALUE - BASE MODEL - $M_{\text{MAX}} = 25$

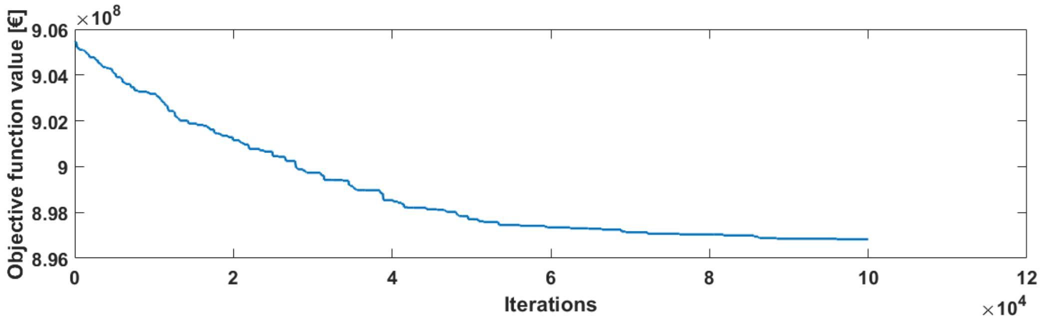

FIGURE 6.14. OBJECTIVE FUNCTION VALUE - BASE MODEL - $M_{\text{MAX}} = 50$

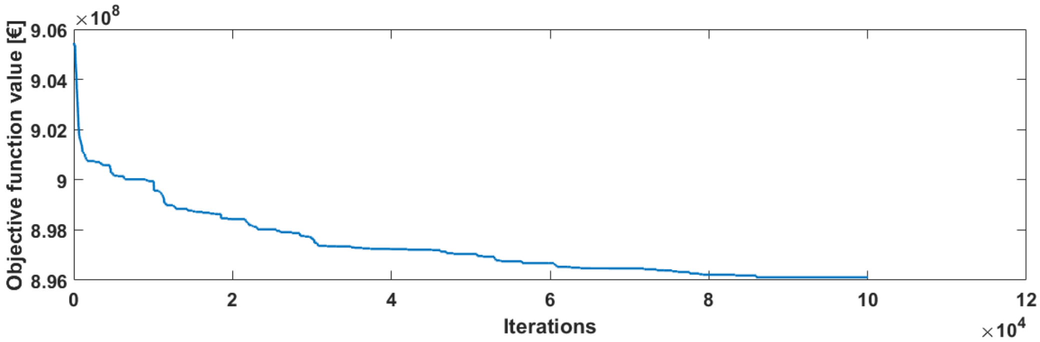

FIGURE 6.15. OBJECTIVE FUNCTION VALUE - BASE MODEL- $M_{MAX} = 75$

M_{max}	Obj. val. [€] Base	Gap exact Base	Runtime [sec.] Base
25	897,271,086	11.1%	1,541.5
50	896,824,637	11.0%	1,535.0
75	896,112,886	10.9%	1,715.6

TABLE 6.17. RESULTS OF DIFFERENT AMOUNT OF ITERATIONS PER TEMP. - AREA E

For the extended model, we also compared the results with start temperatures $M_{max} = 25$, $M_{max} = 50$ and $M_{max} = 75$. These results are depicted in Figures 6.16, 6.17, 6.18 and in Table 6.18. As seen in the table, when $M_{max} = 75$, the runtime for obtaining a solution is the smallest. This is probably because the algorithm is able to create a feasible neighbour solution in a shorter time. Also, the objective function value of the obtained solution is the best, compared with the situation that $M_{max} = 25$ of $M_{max} = 50$. In conclusion, $M_{max} = 75$ is also preferred for the extended model.

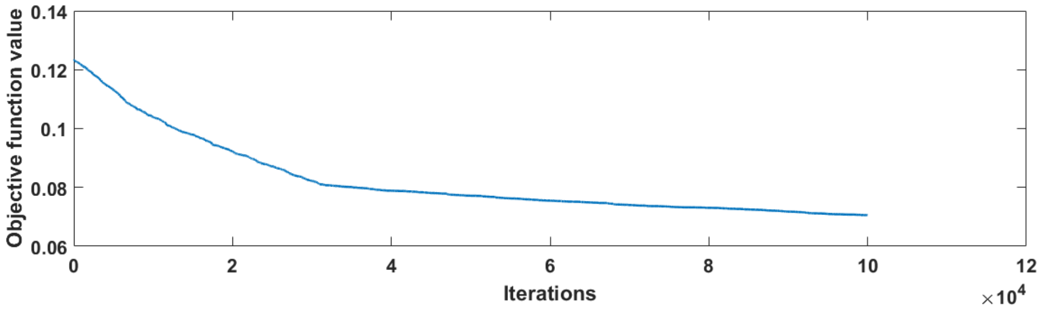

FIGURE 6.16. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL- $M_{MAX} = 25$

M_{max}	Obj. val. Extended	Gap exact Extended	Runtime [sec.] Extended
25	0.071	373.3%	2,521.2
50	0.070	366.7%	2,774.2
75	0.067	346.7%	2,470.1

TABLE 6.18. RESULTS OF DIFFERENT AMOUNT OF ITERATIONS PER TEMP. - AREA E

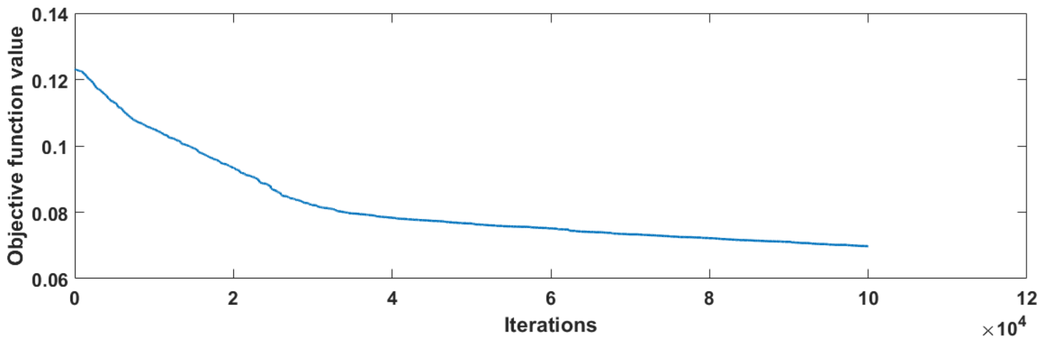

FIGURE 6.17. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - $M_{MAX} = 50$

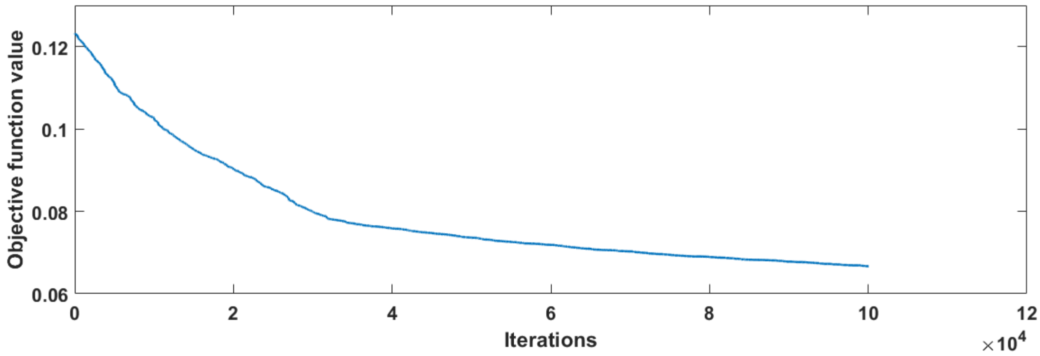

FIGURE 6.18. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL- $M_{MAX} = 75$

Total amount of iterations

We set the total amount of iterations to $N_{\max} = 100,000$, because for this amount of iterations, SA converges to a local optimum. However, the corresponding runtimes for obtaining a solution are approximately 25 minutes for the base problem and 45 minutes for the extended problem. As we have mentioned before, the operator prefers to obtain a schedule in a few minutes. Therefore, we are also interested in the objective function values of solutions resulting from solving SA when $N_{\max} = 10,000$ and $N_{\max} = 50,000$. These results are depicted in Figures 6.19 and 6.20, and in Table 6.19 and 6.20. As seen in these pictures, the gradient of the objective function value curve at 10,000 iterations is significant larger than the gradient of the objective function value curve at 50,000 iterations. This means than during the first part of the algorithm, the efficiency of obtaining better neighbour solution is larger. For the base problem, Figure 6.19 shows that after approximately 877 seconds, the gradient of the objective function value curve significantly decreases. This would be a good point to stop SA, for obtaining a better solution than the initial solution and more in the direction of a reasonable runtime for the telecom operators. For the extended problem, this point would be around 35,000 iterations, which is after approximately 900 seconds. Moreover, note that the objective function value curve of the extended model is still not perfectly converged after 100,000 iterations. To conclude, in the case we want to obtain a solution which has the best objective function value, we prefer $N_{\max} = 100,000$ for both models. In the case we want to obtain a solution which has a good objective function value in a reasonable runtime, we prefer $N_{\max} = 50,000$ for the base model and $N_{\max} = 30,500$ for the extended model.

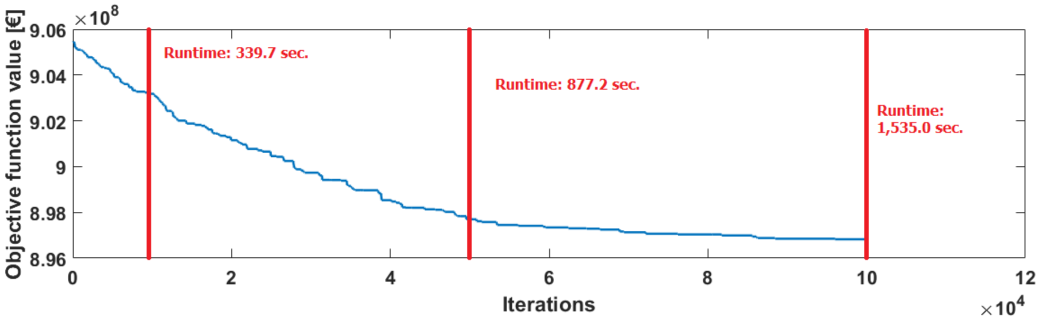

FIGURE 6.19. OBJECTIVE FUNCTION VALUE - BASE MODEL - N_{MAX}

N_{max}	Obj. val. Base	Gap exact Base	Runtime [sec.] Base
10,000	903,434,858	11.8%	339.7
50,000	898,379,820	11.2%	877.2
100,000	896,824,637	11.0%	1,535.0

TABLE 6.19. RESULTS OF DIFFERENT TOTAL AMOUNT OF ITERATIONS - AREA E

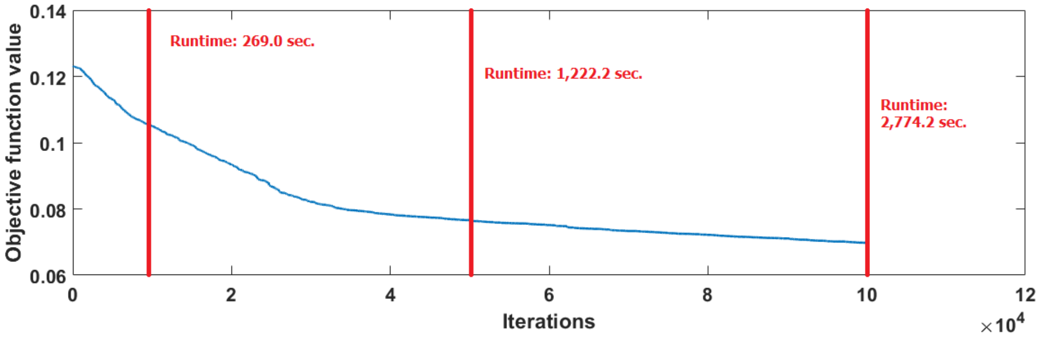

FIGURE 6.20. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - N_{MAX}

N_{max}	Obj. val. Extended	Gap exact Extended	Runtime [sec.] Extended
10,000	0.105	600.1%	269.0
50,000	0.079	426.7%	1,222.2
100,000	0.070	366.7%	2,774.2

TABLE 6.20. RESULTS OF DIFFERENT TOTAL AMOUNT OF ITERATIONS - AREA E

Reduction factor of temperature

The last variation we discuss is a variation in the reduction factor of the temperature. We set this reduction factor to $\alpha = 0.95$ for both models and we compared the results with a reduction factor of $\alpha = 0.90$ and $\alpha = 0.99$. For the base model, the results are depicted in Figures 6.21, 6.22 and 6.23, and in Table 6.19. As seen in this table, the runtimes and objective function values of the obtained solutions are almost the same. Therefore, we prefer the α which obtained a solution have the best objective function value, i.e., $\alpha = 0.95$.

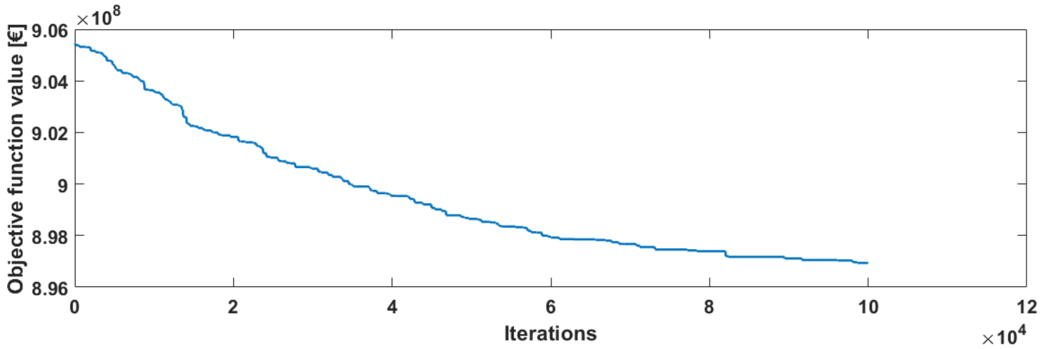

FIGURE 6.21. OBJECTIVE FUNCTION VALUE - BASE MODEL - $\alpha = 0.90$

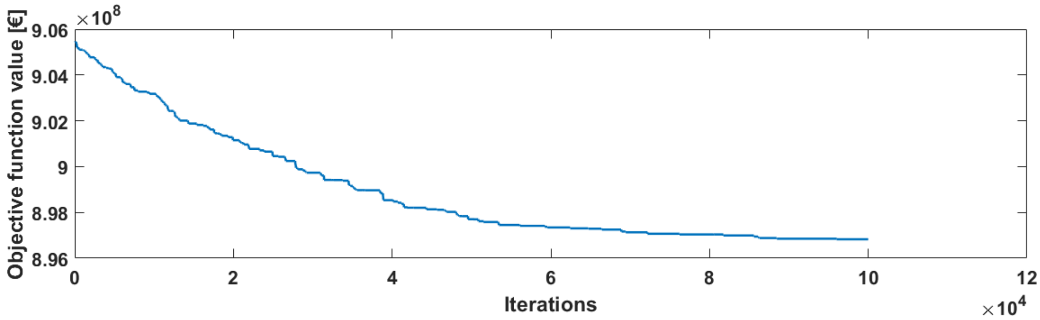

FIGURE 6.22. OBJECTIVE FUNCTION VALUE - BASE MODEL - $\alpha = 0.95$

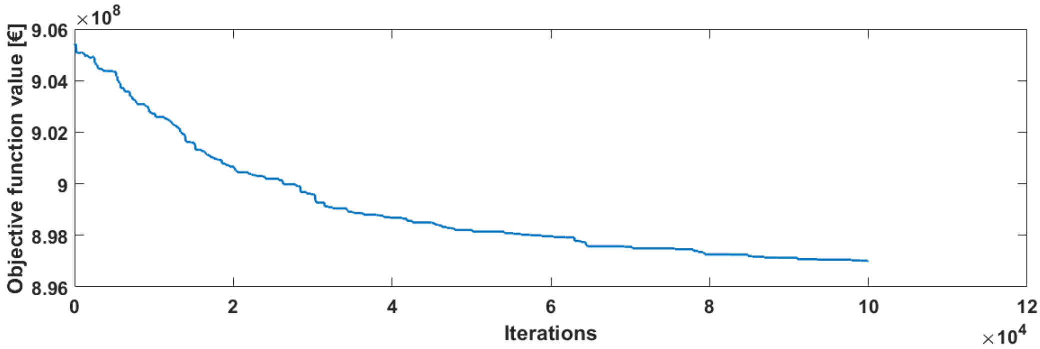

FIGURE 6.23. OBJECTIVE FUNCTION VALUE - BASE MODEL - $\alpha = 0.99$

α	Obj. val. Base	Gap exact Base	Runtime [sec.] Base
0.90	896,938,380	11.0%	1,505.0
0.95	896,824,637	11.0%	1,535.0
0.99	896,983,567	11.0%	1,567.6

TABLE 6.21. RESULTS OF DIFFERENT REDUCTION FACTORS - AREA E

For the extended model, the results are depicted in Figures 6.24, 6.25 and 6.26, and in Table 6.20. The runtime for obtaining a solution is the lowest when $\alpha = 0.90$ and $\alpha = 0.99$. However, the runtime for obtaining a solution when $\alpha = 0.95$ is a bit larger, but the objective function value is also better. Therefore, also for the extended model we prefer $\alpha = 0.95$.

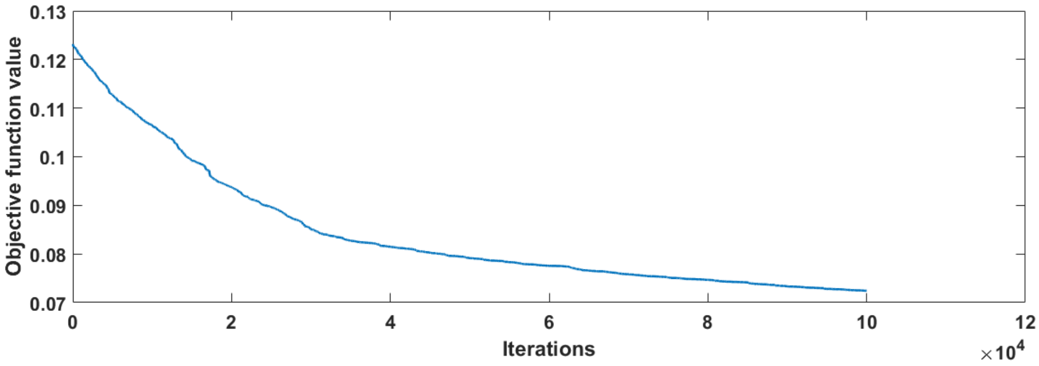

FIGURE 6.24. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - $\alpha = 0.90$

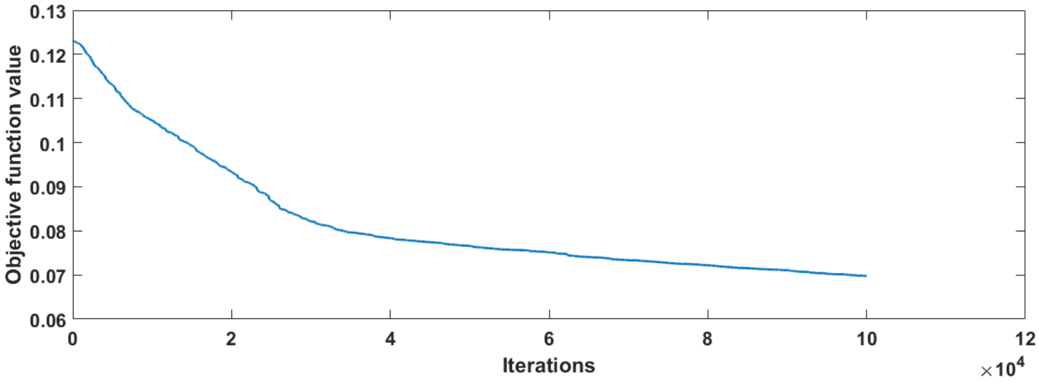

FIGURE 6.25. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - $\alpha = 0.95$

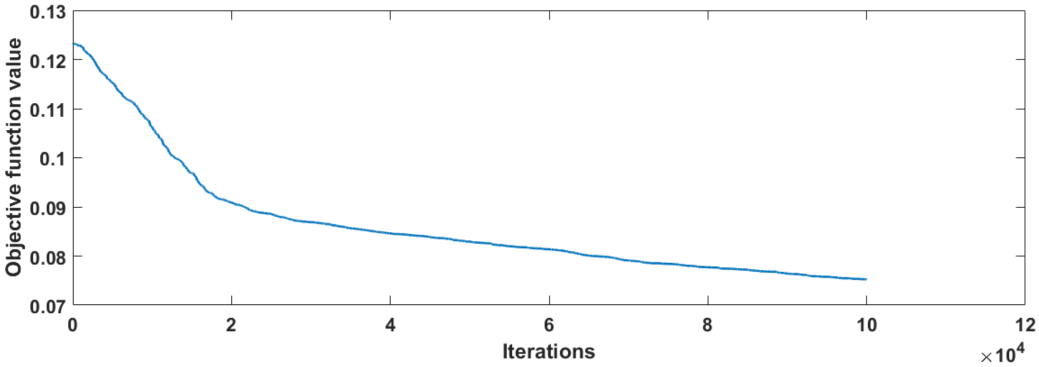

FIGURE 6.26. OBJECTIVE FUNCTION VALUE - EXTENDED MODEL - $\alpha = 0.99$

α	Obj. val. Extended	Gap exact Extended	Runtime [sec.] Extended
0.9	0.072	380.2%	2,590.0
0.95	0.070	366.7%	2,774.2
0.99	0.075	401.0%	2,575.9

TABLE 6.22. RESULTS OF DIFFERENT REDUCTION FACTORS. - AREA E

Best solutions

To conclude, we chose the values in Table 6.23 for obtaining a solution having the best objective function value.

Parameter	Base model	Extended model
T_{start}	7,000	0.00001
$M_{\max}$	75	75
$N_{\max}$	100,000	100,000
α	0.95	0.95

TABLE 6.23. BEST PARAMETER VALUES

Furthermore, an overview of the results of all areas for the base model is given in Table 6.24 and for the extended model in Table 6.25. In these tables, also an overview is provided of the objective function value of the solutions resulting from using SA compared with the objective function value of the solutions resulting from using the exact solution method and the per year optimisation method.

Area	Obj. val. [€] Base	Gap exact Base	Gap per year opt. Base	Runtime [sec.] Base
A	2,991,264	0.9%	-18.0%	122,6
B	2,501,698	12.4%	12.4%	125,4
C	4,922,463	23.3%	20.4%	116,8
D	18,324,132	18.5%	18.5%	192,5
E	896,112,886	10.9%	-0.5%	1,715.6

TABLE 6.24. RESULTS OF BEST SOLUTIONS - SA

Area	Obj. val. Extended	Gap exact Extended	Gap per year opt. Extended	Runtime [sec.] Extended
A	0.196	181.0%	133.3%	34,4
B	0.056	124.3%	124.0%	77,3
C	0.018	500.2%	80.0%	66,5
D	0.065	85.7%	85.7%	136,4
E	0.067	346.7%	252.6%	2,470.1

TABLE 6.25. RESULTS OF BEST SOLUTIONS - SA

The solution obtained by SA for area A in the base model is nearly the optimal. The obtained solution is even better than the solution resulting from solving the per year optimisation method. Furthermore, for area E in the base model, the solution obtained by SA is better than the solution obtained by the per year optimisation method, but 10% worse than the optimal solution. Moreover, the objective function values of the solutions obtained by using SA for the other three areas, also deviate on average 18% from the objective function values of the optimal solutions. In addition, the objective function values obtained from using SA for the extended model are not good. These objective function values deviate on average 245% from the objective function values of the optimal solutions. This behaviour can be due to the bad objective function values of the solutions obtained from solving the problem-based heuristic, which are the initial solutions for SA.

6.7. Comparison between methods

In this section, we compare the runtimes of obtaining a solution resulting from using the different solution methods and we compare the objective function values of these solutions with the optimal solution. For the base model, an overview of these results are given in Table 6.26 and for the extended model, the results are given in Table 6.27. For small areas, i.e., A, B, C and D, the exact solution method is the best method to obtain a solution, which provides an optimal solution. For bigger instances, the exact solution method is not a good method to use, due to the large runtimes for obtaining the optimal solution. For the base model, the problem-based heuristic gives the best performance, i.e. the quality of the obtained solution related to the runtime, for bigger instances. This is because, although the objective function value of the solution for area E resulting from using SA is slightly better, the runtime for obtaining a solution resulting from using the problem-based heuristic is significantly lower.

Area	Runtime exact [sec.]	Per year opt. [%]	Runtime [sec.]	P.-b. heur. [%]	Runtime [sec.]	SA [%]	Runtime [sec.]
A	6.8	23.0	17.5	6.1	3.3	0.9	122.6
B	9.2	0	16.3	29.9	4.1	12.4	125.4
C	5.1	2.4	19.6	29.6	6.4	23.3	116.8
D	9.7	0	21.2	22.4	8.9	18.5	192.5
E	10,804.0	11.5	1,181.3	12.1	700.1	10.9	1,715.6

TABLE 6.26. OBJECTIVE FUNCTION VALUE DIFFERENCES AND RUNTIME, RELATIVE TO EXACT SOLUTION METHOD - BASE MODEL

For the extended model, the per year optimisation method gives the best performance for bigger instances. However, the objective function value for the solution of area E resulting from using the per year optimisation method still deviates much from the optimal solution, namely 24.7%. Furthermore, as described in Section 6.2.5, the objective function value of the solution obtained by the exact solution method with a limited runtime of 6,591 seconds, of which 5,991 seconds building time, deviates 0.005% from the optimal solution. Because of this long building time, the exact solution method with a limited runtime is not the best method for larger instances when using MATLAB, but it might be when using other programming software.

Area	Runtime exact [sec.]	Per year opt. [%]	Runtime [sec.]	P.-b. heur. [%]	Runtime [sec.]	SA [%]	Runtime [sec.]
A	11.2	19.5	26.8	280.7	3.5	181.0	34.4
B	7.5	0	23.6	270.4	4.0	124.3	77.3
C	18.9	325.8	26.8	1,743.9	6.0	500.2	66.5
D	29.7	0	29.6	217.4	8.5	85.7	136.4
E	8,296.7	24.7	1,045.7	708.2	630.4	346.7	2,470.1

TABLE 6.27. OBJECTIVE FUNCTION VALUE DIFFERENCES AND RUNTIME, RELATIVE TO EXACT SOLUTION METHOD - EXTENDED MODEL

7
Conclusions

In this thesis, we defined two models for developing a migration plan for fibre in current telecommunication networks in the Netherlands. As described in Chapter 3, the base model minimises the migration costs, whereby the bandwidth requirements are met for each migration period. The extended model minimises the difference between the required bandwidth and the realised bandwidth, whereby the migration costs might not exceed the budget and the required capacity for the migration schedule might not exceed the installation capacity. In Chapter 4, we proved that both models are NP-hard. Based on these proofs, we expected a long runtime for obtaining an optimal solution. Therefore, we developed three heuristic methods to obtain a good migration in a reasonable runtime, namely a per year optimisation method, a problem-based heuristic and Simulated Annealing.
In this chapter, the conclusions are described and some recommendations for future research are given.

7.1. Conclusions

The research question of this thesis was:

"What is a good strategy for the migration paths of Fibre to the Home for each area of the Netherlands, when described as heterogeneous migration paths consisting of migration steps for each time period in each area?"

In order to answer this question, we proposed four sub-questions, which will be answered below.

1. *Are the Migration of Fibre problems optimally solvable?*

   Both the base model and extended model are NP-hard. The largest data set we used for testing the solution methods, area E, is comparable with the Netherlands. For this large instance, the exact solution method obtained the optimal solution within 3 hours for the base model and within 2.5 hours for the extended model. This means that for our test data, the Migration of Fibre problems are optimally solvable. However, for these test results, we assumed that only three technology and topology combinations $i \in I$ exist. Due to the NP-hardness, the runtime for obtaining the optimal solution can grow exponentially when considering a larger set of technology and topology combinations.

2. *Which (meta)heuristic will provide a good and fast solution?*

   For smaller instances, the runtime for obtaining an optimal solution using the exact solution method is reasonable. The heuristic which provides a good and fast solution for small instances of the base model and the extended model is the per year optimisation method. Also for bigger instances of the extended model, the per year optimisation method is the best to use when using MATLAB. However, the exact solution method with a limited runtime might be a good method when using other programming software. For bigger instances of the base model, the problem-based heuristic is better to use.

3. *What is the scalability of the used solution methods?*

   We define the scalability of the solution methods as the quality of the solution found by the different solutions methods and the corresponding runtimes for obtaining a solution with these methods. The per year optimisation method is the best scalable method in terms of the quality of the objective function values of the obtained solutions. However, the runtime for small areas is relative large, compared to the runtime of the exact solution, whereas the runtime for large areas is significantly smaller than the runtime for the exact solution method.
   The problem based heuristic is the best scalable method in terms of the runtime for obtaining a solution. The ratio between the amount of variables and runtime is much smaller than the ratio for the exact solution method. However, the quality of the solution obtained by using the problem-based heuristic is not as good as the solution obtained by using the exact solution method, especially for the extended model.
   SA has got a good scalability in terms of the runtime for obtaining a solution. However, the number of iterations directly affects the quality of the obtained solution and when setting this number of iterations extremely high, the objective function value of the obtained solution does not come close to the objective function value of the optimal solution.

4. *Will placing no restrictions on the budget per time period result in a lower amount of total costs compared to having a fixed budget per time period?*

   This is not always the case. The results show that the total costs can be larger when using a total budget instead of a fixed budget per period, but as positive side effect, this automatically means that the difference between the requested bandwidth and the realised bandwidth becomes smaller. Moreover, money is spent more efficient, because the total costs are still under budget, while a higher bandwidth is realised then when using a fixed budget per period. The results also show that it is possible that a total budget instead of a budget per period results in a lower amount of costs, and meanwhile, in a higher realised bandwidth. To conclude, it is more effective to have no restrictions on the budget per time period.

7.2. Future research

In this research, we assumed that there are three available technology and topology combinations. Furthermore, we assumed that a migration plan consists of three migration periods and a period consists of three years. In reality, a lot more technology and topology combinations are available and also having more migration periods can extensively affect the scalability of the used solution methods. Therefore, future research can be done on investigating the effect of more available combinations and more migration periods.

The installation capacity is one of the parameters of the extended model. In this research, we assumed that this is the amount of employees needed for migrating fibre in a location. The corresponding constraint is quite basic, and therefore, it is mandatory to develop a more realistic way of defining the installation capacity constraint.

The quality of the solutions obtained from using the problem-based heuristic is not good, especially for the extended mode. However, we recommend to look for better performing problem-based heuristics, or a data-based heuristic, because we expect that these heuristics will have the best runtime. During this research, a simple data-based heuristic is developed for the extended model, see Appendix A.

We also expect that the performance of SA can be improved. When creating neighbour solutions, three operations are possible and during an upgrade or downgrade, the technology and topology combination of a location becomes one combination better or worse. Further research can be done to improve this heuristic. For example, by replacing this fixed upgrade or downgrade from one to a random amount of upgrades of downgrades for a location. Furthermore, in this research, the initial solution of SA is the solution obtained by using the problem-based heuristic. The quality of these solutions is, as mentioned before, not good. This might be the reason that SA does not converge to the optimal solution. Therefore, it is mandatory to research the effect of the initial solution on the convergence of SA. For example, the initial solution could also be the solution obtained from using the per year optimisation method or from solving the ILP for the base model, or the MILP for the extended model, with a predefined maximum runtime.

Bibliography

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A

Data-based heuristic for the extended model

The results, which are illustrated in Chapter 6, show that the problem-based heuristic for the extended model, illustrated in Section 5.3, creates solutions which are not good. More about this can be found in the corresponding chapter. Therefore, we developed a data-based heuristic for the extended model, which is based on data analysis. The data where this data-based heuristic is based on, is described in Section 6.1. Solutions (migration schedules) are presented as a matrix of which the rows represent the locations, the columns represent the migrations periods and the elements of the matrix represent the technology and topology combination for the corresponding location and time period. The heuristic is structured as follows:

1. Create a schedule in which the technology and topology combination in each period of a location is equal to the start state.

2. Create the matrix R_{jl}, which contains the mean values of R_{jld} over all the distances $d \in D$ for all technology and topology combinations $j \in I$ and locations $l \in L$.

3. Go along each location. If $R_{1l} \leq R_{2l} \leq R_{3l}$, then if $\frac{C_{13l}}{R_{3l}} > \frac{C_{12l}}{R_{2l}}$, set all migration periods after the start period equal to 2, else if $\frac{C_{13l}}{R_{3l}} \leq 550$, set all migration periods after the start period equal to 3.

4. If the realised bandwidth is not larger than or equal to the requested bandwidth for each period $t \in T$ and distance $d \in D$, create a ratio matrix based on ratio (5.29). Focus on the locations which not have an upgrade yet and upgrade all periods of the location which has got the highest ratio. Repeat this, until the realised bandwidth is larger than the requested bandwidth for each period $t \in T$ and distance $d \in D$ or until each location has got an upgrade.

5. If the solution is not feasible, i.e., the solution does not meets the budget constraint and/or the installation capacity constraint, downgrade the location having the smallest ratio to the corresponding technology and topology combination in the corresponding period. Repeat this, until a feasible solution is found.

In Table A.1, an overview is given of the results. The results obtained by using the data-based heuristic are better than the results obtained by using the problem-based heuristic, except for area A.

Area	Obj. val. Extended	Gap exact [%] Extended	Gap p.-b. heur. [%] Extended	Runtime [sec.] Extended
A	0,351	401.4%	31.0%	6,1
B	0,064	156.0%	-31.9%	7,1
C	0,027	800.0%	-46.0%	10,0
D	0,067	91.4%	-39.6%	13,7
E	0,122	713.3%	-0.8%	1,424.8

TABLE A.1. RESULTS OF DATA-BASED HEURISTIC