Optimisation Models for Merging Ambulance Regions in the Netherlands

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Source: Piccell (2015). *London ambulance blurred motion - Stockfoto* [Photograph]. Retrieved from https://www.gettyimages.nl.

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Optimisation Models for Merging Ambulance Regions in the Netherlands

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TU Delft

Preface

This bachelor thesis has been written as part of the Bachelor programme in Applied Mathematics at Delft University of Technology and was supervised by dr. ir. J. T. van Essen at the department of optimisation.

The research presents two integer linear programs. With the use of symmetry breaking constraints and a heuristic method, the integer linear programs are applied on a data set containing all data points of the Netherlands. For more information about the programs and the data set that are used, feel free to contact me.

I would like to thank my supervisor dr. ir. J. T. van Essen for her help and support during the whole project. Next to that, I would like to thank dr. ir. R. van der Toorn and drs. E. M. van Elderen for joining my thesis committee and taking the time to read and review the report.

L.J. Zwep
Delft, July 2019

Abstract

This thesis focuses on optimising the Emergency Medical Services (EMS) in the Netherlands. In the current situation, the Netherlands is divided into 24 independent regions in which ambulances operate. These regions can be merged in order to reduce response time and increase efficiency. Different models are presented in which these regions are merged in an optimal way. The thesis starts with an explanation of the current regional system within the Netherlands. Next, two models following from existing literature are discussed. These two models are used as a basis for new models that optimise the merging of regions. Then, the results are discussed, which are in line with the assumption that merging regions leads to a better coverage and a reduction in the number of ambulances needed.

Keywords: ambulances, merging regions, modeling and optimisation, Integer Linear Programming

Contents

1 Introduction

In the case of an emergency call, it is of vital importance that urgent action is taken. The chance that a patient who was involved in an accident will survive depends to a large extent on how quickly an ambulance can arrive [10]. An important aspect of arriving somewhere quickly is starting from the right place. If you start close to the accident, this logically reduces the total travel time. But now the question arises what the best place is to leave from. The goal is to place the ambulance base locations in a way that the biggest population can be reached as soon as possible.

In operations research, the distribution of ambulances is being discussed by multiple scientists. TU Delft and the Center for Mathematics and Computer Science (CWI) worked together to find the optimal placement of the locations of ambulances in the Netherlands [1]. While working on this optimisation, the Netherlands was divided into 25 Regional Ambulance Services (RAVs). A RAV is by law the legal person that is responsible for all actions in the event of an ambulance call, both in the emergency room and when performing ambulance care [2]. The Netherlands currently has 24 RAVs, all of which operate independently of each other.

The optimisation of ambulance locations in the Netherlands is always within these RAVs, because the ambulances do not serve outside their own RAV. However, the response time of calls at the boundary of a region is on average higher than at the centre of a region [14]. Therefore, it would be better to have as few border areas as possible, so equivalently, as few borders as possible. If regions were to work together, more calls could be answered with fewer ambulances. Thus, merging these RAVs saves costs, and more importantly, ensures better care provision.

However, merging RAVs is not desirable from an organisational point of view. The RAVs are separate bodies whose merger would cost a lot of time and money. That is why not all of the Netherlands can suddenly be merged into one RAV.

So, if merging does takes place, it is very important that it is properly determined which RAVs will be merged. This research focuses on this. Therefore, the research question for this research is:
“What is the merger of RAVs within the Netherlands while taking the coverage and efficient deployment of ambulances into account? "
With the sub-questions:

- “What are the disadvantages of merging an RAV?"
- “With a limited number of new RAVs, which existing RAVs must be combined to achieve an optimum?"
- “What are the benefits of this merger in terms of coverage and efficient deployment of ambulances?"

In the report, Chapter 2 discusses the properties of a RAV in more detail, specifically why merging them is hard in practice. In Chapter 3, an overview of mathematical models for optimally locating emergency medical services are presented. Chapter 4 presents the mathematical models developed for this research. In Chapter 5, the results of the research are shown and analysed. At last, in Chapter 6, the research is concluded and discussed.

2

Properties of RAVs

As stated earlier, the RAVs work independently. This means that it is a stand-alone organisation. This has the consequence that merging RAVs entails many disadvantages.

Every RAV has its own form of management. For instance, the RAV Gooi en Vechtstreek is part of the Regio Gooi en Vechtstreek, which has its own Executive and General Board. The RAV Flevoland is part of the Flevoland GGD with a corresponding GGD board [8]. To merge these two RAVs, one or both forms of governance must be adjusted.

In addition, every RAV has its own locations with a staff department for care, policy, scheduling, education, quality, technical management and fleet management. A merge ensures that certain jobs will fall double, which may lead to forced dismissal. In addition, there different control rooms for every RAV. However, the 'Control room of the future' plan stipulates that by 2021 there must be a change from 25 to 10 control rooms, so there is already a need to merge here [12].

Another reason that makes merging problematic is the role of health insurers in ambulance care. In the Netherlands, health insurance companies are responsible for financing ambulance care. The Dutch Healthcare Authority (NZa) specifies how health insurers should distribute the national macro budget [2]. It differs per RAV which health insurers provide payment for ambulance care. Table 2.1 shows an overview of the RAVs and their responsible health insurers. If two RAVs are merged with different health insurers responsible for their budget, then there must be a completely new distribution of the budget and a change in financing. Therefore, this is extra problematic and this must be considered when merging the RAVs.

Finally, there is the problem that the Temporary Ambulance Care Law (TWAZ) states how the RAVs are running. The TWAZ tells us that the RAVs are the only ones allowed to provide ambulance care in their region [16]. Therefore, the merging of RAVs requires adjustments to be made by the ministry, with high costs in terms of time and money due to a lot of bureaucracy.

In summary, there are five points which make a merger between RAVs not desirable from an organisational point of view. Namely:

1. Own management
2. Own office locations and staff
3. Own control rooms
4. Own health insurance companies
5. Determined by law

Despite these factors, it is possible to do merge the RAVs. As can be seen in Table 2.1, RAV 13 is missing. This is because RAVs 11 and 13 have already been merged. In figure 2.1, you can find the current distribution of RAVs in the Netherlands. As can be seen, 11 and 13 are put into the same RAV.

Table 2.1: The 25 RAVs and their responsible health insurers. Source: Nederlandse Zorgautoriteit, *Normoverschrijdingen responstijden ambulances*, 2016

Number	Region	Health insurer 1	Health insurer 2
1	Groningen	Menzis	ZK
2	Fryslan	DFL	VGZ
3	Drenthe	ZK	VGZ
4	IJsselland	ZK	VGZ
5	Twente	Menzis	VGZ
6	Noord- en Oost-Gelderland	Menzis	ZK
7	Gelderland Midden	Menzis	VGZ
8	Gelderland-Zuid	VGZ	CZ
9	Utrecht	ZK	VGZ
10	Noord-Holland-Noord	VGZ	ZK
11	Amsterdam-Waterland	ZK	VGZ
12	Kennemerland	ZK	VGZ
14	Gooi en Vechtstreek	ZK	VGZ
15	Haaglanden	CZ	Menzis
16	Hollands Midden	Z&Z	ZK
17	Rotterdam-Rijnmond	ZK	DSW
18	Zuid-Holland-Zuid	VGZ	CZ
19	Zeeland	CS	VGZ
20	Midden en West-Brabant	VGZ	CZ
21	Brabant-Noord	VGZ	CZ
22	Brabant-Zuidoost	VGZ	CZ
23	Limburg-Noord	VGZ	CZ
24	Limburg-Zuid	CZ	VGZ
25	Flevoland	ZK	VGZ

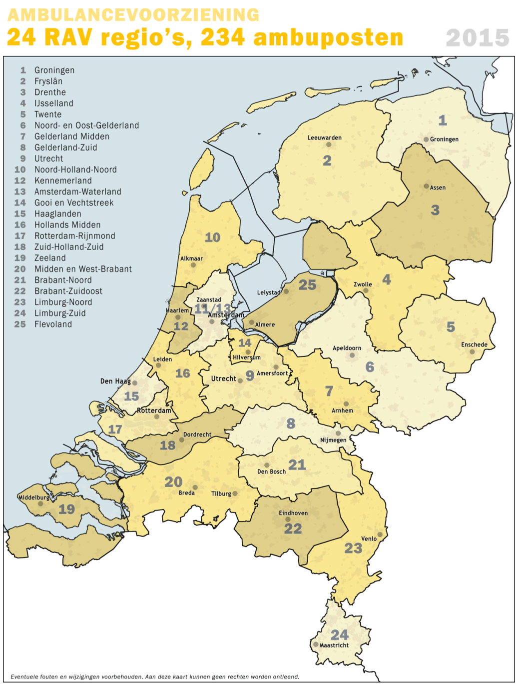

Figure 2.1: Map of the current 24 RAVs in the Netherlands. Source: J. W. van Aalst (2015). *Kaart van de RAV-regio's, 2015* [map]. Retrieved from http://www.imergis.nl

3

Literature overview of ambulance location models

In this chapter, two of the most frequently used ambulance location models are given as a basis for the research. Xueping et al. [17] states that all the models made for ambulance covering can be divided into three broad groups. First, we have covering models, which guarantee the coverage within a given time standard. Second, we have p-median models, which minimise the total mean distance to all the demand locations. Third, we have p-centre models, which minimise the maximal distance regarding all demand locations.

This literature overview considers two of the basic covering models. We only discuss the models that are used in this research. A broader overview of ambulance location models can be found in the review paper of Xueping et al. [17] and the comparison paper by Van den Berg et al. [15].

In order to introduce the mathematical models, it is of importance to introduce some notation.

Notation	Type	Meaning
I	Set	All demand locations
J	Set	All potential base locations
J_i	Set	All $j \in J$ such that $t_{ji} < r$
t_{ji}	Parameter	Travel time from $j \in J$ to $i \in I$
r	Parameter	Maximal time to reach a demand location i
p	Parameter	Amount of ambulances that can be placed
d_i	Parameter	Weight of location i
x_j	Binary variable	1 when there is an ambulance at base location j 0 otherwise
y_i	Binary variable	1 when demand location i is covered 0 otherwise

3.1. Maximum Covering Location Problem (MCLP)

Having this notation clear, we can present the first model, the Maximal Covering Location Problem (MCLP). This is a model designed in 1974 by Church and ReVelle [4] and later discussed in an abundance of literature ([1], [14], [17], [15]). MCLP is a model which maximises the population that can be reached within a predetermined time limit given p ambulances.

$$
\begin{aligned}
\text{Maximise} \quad & \sum_{i \in I} d_i y_i \\
\text{such that} \quad & \sum_{j \in J_i} x_j \geq y_i \qquad \forall i \in I \\
& \sum_{j \in J} x_j = p \\
& y_i, x_j \in \{0,1\} \qquad \forall i \in I, j \in J
\end{aligned}
\tag{3.1}
$$

The objective function maximises the number of locations covered while taking their weights into account. The first constraint makes sure that y_i only gets value 1 if there is an ambulance placed within a travelling time less than r. The second constraint makes sure that exactly p ambulances are placed.

3.2. Maximum Expected Covering Location Problem (MEXCLP)

A major disadvantage of MCLP is that it assumes that at an ambulance is always available. This is of course not something that could be assumed as ambulances depart from their base location when they respond to a call. That is why Daskin [6] introduced the Maximum Expected Covering Location Problem (MEXCLP) in 1983. This model considers how busy the ambulances are. To this end, they introduce the parameter q which is the probability that an ambulance is busy. The binary variable y_{ir} now indicates whether demand location i is covered by at least r ambulances. Then, the expected coverage with r ambulances becomes $E_r = 1 - q^r$. This is equal to the probability of at least one success in r independent Bernoulli experiments [14]. The marginal coverage of the r^{th} ambulance is then $E_r - E_{r-1} = q^{r-1}(1-q)$. The model also considers the possibility to place more than one ambulance at a base location. Consequently, x_j changes from a binary variable into an integer variable.Therefore, the model is as follows.

$$\begin{array}{lll}
\text{Maximise} & \sum_{i\in I}\sum_{r=1}^{p} d_i(1-q)q^{r-1}y_{ir}, & \\
\text{such that} & \sum_{j\in J_i} x_j \geq \sum_{r=1}^{p} y_{ir}, & \forall i \in I \\
& \sum_{j\in J} x_j \leq p, & \\
& x_j \in \mathbb{N}, & \forall j \in J \\
& y_{ir} \in \{0,1\}, & \forall i \in I, r = 1,\dots,p.
\end{array} \tag{3.2}$$

MEXCLP is preferred over other models such as MCLP but also Maximum Availability Location Problem (MALP) [11] and Double Standard Model (DSM) [9] due to better coverage, quicker response time, and for most of the models, also shorter computation time [15].

So, from many models, two models have been discussed, MCLP and MEXCLP. MCLP is a basic model where there are only a few constraints and where many assumptions are made in advance. This makes it an easy model, but not a realistic one. A more realistic model is MEXCLP, because it considers how busy the ambulances are. These models are used often in the literature and both form a good basis for an extension.

4

Merging ambulance regions problem

Now that the basic covering models have been explained, we are able to reconstruct these models into models that can solve the given problem. This chapter presents four different models: two based on MCLP and two based on MEXCLP, which all give a solution to the merging of ambulance regions problem. There are two different formulations presented for each of the problems: one based on putting RAVs into main regions and one based on directly merging the RAVs.

4.1. MCLP

As stated above, two of the models are based on MCLP. This is due to the fact that this model is really basic, so it does not have a lot of constraints at prior. This is because it has a lot of assumptions taken into account, which makes it easy to modify the model itself. Next to that, the computation time of MCLP is significant low [15], which is preferable as there is quite an amount of data used as input.

4.1.1. First formulation

The objective of the considered problem is to merge the RAVs in the Netherlands optimally, under the condition that there is still a pre-determined number of RAVs left. This condition is necessary, due to the fact that the optimal solution will otherwise always be to merge all the RAVs into one. This would not be preferred due to the reasons given in Chapter 2.

MCLP has no features at all that have to do with regions. To this end, some new parameters and variables have to be introduced. First, there is the set containing all the RAVs, which is called K. Every demand location $j \in J$ is pre-assigned to a RAV $k \in K$. If two RAVs are merged, they are put into the same so called main region. The set H contains all the main regions $h \in H$.

The objective function of the model does not need any changes compared to the original MCLP model. It is still the aim to cover as much demand locations as possible, so the objective function is:

$$\text{Maximise} \quad \sum_{i \in I} d_i y_i. \tag{4.1}$$

The first constraint does need some change. In MCLP, a demand location is covered if a base location has an ambulance which can reach the demand location within r minutes. For the new model, this is not always true. This is only true if the demand location and the base location are in the same main region $h \in H$. Hence, there is a need for a variable which shows whether that is true. Define:

$$l_{ijh} = \begin{cases} 1, & \text{if potential base location } j \in J \text{ and demand location } i \in I \text{ are both in main region } h \in H \\ 0, & \text{otherwise} \end{cases}$$

So, the first constraint becomes the following:

$$\sum_{h \in H} \sum_{j \in J_i} x_j l_{ijh} \geq y_i, \quad \forall i \in I. \tag{4.2}$$

This constraint contains a multiplication of two decision variables, and thus, it is not linear. This will be dealt with in Section 4.3. To ensure that l_{ijh} only gets value 1 if potential base location $j \in J$ and demand location $i \in I$ are both in main region $h \in H$, some more constraints need to be added. To check if a demand location or potential base location is in a certain main region, there is the need to know in which main region each RAV is. In consequence, it is necessary to introduce another binary variable. This time to check whether RAV $k \in K$ is in main region $h \in H$.

$$b_{kh} = \begin{cases} 1, & \text{if RAV } k \text{ is in main region } h \\ 0, & \text{otherwise} \end{cases}$$

But it is also important to know in which RAV each demand and potential base location is. This information is known and is used as input. Therefore, the parameters a_{jk} and α_{ik} are introduced. a_{jk} is 1 if potential base location j is in RAV k and 0 in all the other cases. α_{ik} is 1 if demand location i is in RAV k, and 0 otherwise. Now, it is possible to set up the constraints such that l_{ijh} takes the correct value.

$$\begin{aligned} \sum_{k \in K} a_{jk} b_{kh} \geq l_{ijh}, &\quad \forall i \in I, j \in J, h \in H \\ \sum_{k \in K} \alpha_{ik} b_{kh} \geq l_{ijh}, &\quad \forall i \in I, j \in J, h \in H. \end{aligned} \tag{4.3}$$

These constraints only work in combination with the objective function and the first constraint. Without the objective function, l_{ijh} could easily be chosen 0 for all i, j and h. But because the objective is to maximise y_i, and y_i can only be 1 if l_{ijh} is 1, constraints (4.3) work.

Now, the following step is to limit the number of ambulances that can be placed in each main region. It is useful to introduce parameter p_k, which specifies how many ambulances are available in RAV $k \in K$. The following constraint ensures that the number of ambulances per main region $h \in H$ is at most the total number of ambulances in all RAVs $k \in K$ which are part of main region $h \in H$.

$$\sum_{k \in K} \sum_{j \in J} a_{jk} b_{kh} x_j \leq \sum_{k \in K} p_k b_{kh}, \quad h \in H. \tag{4.4}$$

This constraint is also not linear, so this will also be changed in Section 4.3.

When there is a coverage of 100%, the RAVs can be merged randomly, because it will be optimal in all cases. This means that two RAVs who are not adjacent might be merged together. This is a situation that should be prohibited, because it is inefficient to merge RAVs who are not adjacent. To prohibit this, a new parameter is defined. This parameter makes use of the subscript $k_1 k_2$ to show that there are two different RAVs $k_1, k_2 \in K$ taken into account.

$$t_{k_1 k_2} = \begin{cases} 1, & \text{if RAVs } k_1 \text{ and } k_2 \text{ are adjacent} \\ 0, & \text{otherwise} \end{cases}$$

Then by adding a new constraint on b_{kh}, the situation described above will not occur. So, the following constraint is added to the model.

$$b_{k_1 h} + b_{k_2 h} - 1 \leq t_{k_1 k_2}, \quad \forall k_1 \in K, k_2 \in K, h \in H. \tag{4.5}$$

Lastly, there are two constrains regarding the main regions to make the model complete. Each RAV should be assigned to exactly one main region. At last, it is needed to limit the number of RAVs that can be put in a main region. This is because of the condition that is presented at the beginning of the chapter. To ensure this, s is presented, which gives the number of RAVs that can be put into each main region. Now the last two constraints are as follows.

$$\begin{aligned} \sum_{h \in H} b_{kh} = 1, &\quad \forall k \in K \\ \sum_{k \in K} b_{kh} \leq s, &\quad \forall h \in H. \end{aligned} \tag{4.6}$$

Thus, the complete model is the following.

$$
\begin{array}{lll}
\text{Maximise} & \sum\limits_{i\in I} d_i y_i, & \\
\text{such that} & \sum\limits_{h\in H}\sum\limits_{j\in J_i} x_j l_{ijh} \geq y_i, & \forall i \in I \\
& \sum\limits_{k\in K} a_{jk} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum\limits_{k\in K} \alpha_{ik} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum\limits_{k\in K}\sum\limits_{j\in J} a_{jk} b_{kh} x_j \leq \sum\limits_{k\in K} p_k b_{kh}, & \forall h \in H \\
& b_{k_1 h} + b_{k_2 h} - 1 \leq t_{k_1 k_2}, & \forall k_1 \in K, k_2 \in K, h \in H \\
& \sum\limits_{h\in H} b_{kh} = 1, & \forall k \in K \\
& \sum\limits_{k\in K} b_{kh} \leq s, & \forall h \in H \\
& y_i, b_{kh}, x_j, l_{ijh} \in \{0,1\}, & \forall i \in I, j \in J, k \in K, h \in H.
\end{array}
\tag{4.7}
$$

4.1.2. Symmetry breaking constraints

There can be put an equal number of RAVs in all the main regions. This has as consequence that there is total symmetry regarding the main regions [7]. For example, if you have four RAVs that need to be merged into two main regions, then RAVs one and two can be put in main region one and RAVs three and four in main region two. But in essence this is the same as putting RAVs one and two in main region two and RAVs three and four in main region one. All this symmetry takes up a lot of computation time and should be reduced [13]. Therefore, two symmetry breaking constraints were added to the model to reduce computation time.

The first symmetry breaking constraint is used to solve the problem described above. If we only allow to assign RAVs with index k greater than or equal to the index of main region h, then, in our example, RAVs one and two can never be put into main region two, because RAV one should always be put into main region one and if it is being merged with RAV two they will stay in the first main region.

$$
b_{kh} \quad \forall (k,h) : k \geq h \tag{4.8}
$$

The second symmetry breaking constraint is based on the constraint presented by Denton et al. [7] in their paper about optimal allocation of surgery blocks. In their article, they create a model in which they assign surgeries to different operating rooms. This can be translated to assigning RAVs to main regions. We explain this constraint by means of an example. Say that the first four RAVs are put into the first two main regions. Then, the fifth RAV can either be put into one of the two main regions which already have other RAVs in it, or it will be put into another, new main region. If the last case happens, we assume that the fifth region is put into the third main region, and not in main region 4, 5, etcetera. This is forced with the following constraint:

$$
\begin{aligned}
&b_{53} \leq b_{22} + b_{32} + b_{42}, \\
&b_{54} \leq b_{33} + b_{43}, \\
&b_{55} \leq b_{44}.
\end{aligned}
\tag{4.9}
$$

So b_{55} can only get value 1 if b_{44} has value 1, because then all the RAVs are put into four different main regions and so the fifth RAV can be put in a fifth main region. If this was not the case, and some RAVs were put together, then the fifth RAV could never be in the fifth main region because then a main region is being skipped. To make this general, it uses the first symmetry breaking constraint and it results in:

$$
b_{kh} \leq \sum_{u=h-1}^{k-1} b_{u,h-1}, \quad \forall (k,h) : k \geq h. \tag{4.10}
$$

4.1.3. Second formulation

Continuing, another formulation for the problem is presented. This formulation models the same problem through a different approach. This formulation does not introduce main regions, but directly determines if two RAVs are merged. To this end, some new notation is needed.

First of all, a decision variable is needed which states whether two RAVs are merged. The decision variable is as follows:

$$f_{k_1 k_2} = \begin{cases} 1, & \text{if RAVs } k_1 \in K \text{ and } k_2 \in K \text{ are merged} \\ 0, & \text{otherwise.} \end{cases}$$

Again, we use MCLP as a basis. This is because the calculation time of MCLP is one of the quickest for all static ambulance location models [15]. Thus, the objective function stays the same as in Section 4.1.1:

$$\text{maximise} \quad \sum_{i \in I} d_i y_i. \tag{4.11}$$

Then, the first constraint has to make sure that location $i \in I$ is only covered, if there is an ambulance at potential base location $j \in J$ which can reach demand location $i \in I$ within r minutes and $i \in I$ and $j \in J$ are in the same newly merged region. So, to this end, it is needed to know if j is in some k_1 that is merged with k_2 which i is in. This leads to the following constraint.

$$\sum_{k_1 \in K} \sum_{k_2 \in K} \sum_{j \in J_i} a_{jk_1} x_j f_{k_1 k_2} \alpha_{ik_2} \geq y_i, \quad \forall i \in I. \tag{4.12}$$

This constraint has a multiplication of x_j and $f_{k_1 k_2}$, and thus, it is not linear. This will be resolved in Section 4.3.

Now it is of course of importance that $f_{k_1 k_2}$ is working correctly. $f_{k_1 k_2}$ should possess the transitive property. So, if k_1 is merged to k, and k is merged with k_2, then k_1 and k_2 are also merged. The next constraint makes sure $f_{k_1 k_2}$ is working the way it should.

$$f_{k_1 k} + f_{k k_2} - 1 \leq f_{k_1 k_2}, \quad \forall k, k_1, k_2 \in K. \tag{4.13}$$

After that, the number of RAVs that are merged should be limited. Otherwise, the same problem as earlier can occur, namely that all the RAVs are merged into one RAV. This is not desired, so it needs to be restricted with a constraint. Also, in this model, s is the number of RAVs that can maximally be merged into one another. Now, the next constraint prohibits that this number goes over this limit.

$$\sum_{k_2 \in K} f_{k_1 k_2} \leq s, \quad \forall k_1 \in K. \tag{4.14}$$

Also with this model, it needs to be prevented that, with a coverage of 100%, two RAVs which are not adjacent are getting merged. By adding a new constraint which only allows $f_{k_1 k_2}$ to be 1 if $t_{k_1 k_2}$ is 1, this cannot happen. So, the following constraint is added to the model.

$$f_{k_1 k_2} \leq t_{k_1 k_2}, \quad \forall k_1 \in K, k_2 \in K. \tag{4.15}$$

Then there is the need to put a constraint on the number of ambulances that can be used. For every group of merged RAVs, the total number of ambulances should be less than or equal to the sum of the number of ambulances available for the RAVs assigned to this merged group. So, the constraint on the left-hand side should sum all the ambulances that are used in the merged RAVs and the right-hand side should sum the number of ambulances that are available in all the merged RAVs. So, the constraint becomes the following:

$$\sum_{k_1 \in K} \sum_{j \in J} a_{jk_1} f_{k_1 k_2} x_j \leq \sum_{k_1 \in K} p_{k_1} f_{k_1 k_2} \quad k_2 \in K \tag{4.16}$$

At last, $f_{k_1 k_2}$ needs two more constraints to work properly. First, $f_{k_1 k_2}$ must possess the symmetry property. If k_1 is assigned to k_2, then k_2 is also assigned to k_1. Second, every RAV is merged to itself. So, for all k the value of f_{kk} is 1. These two constraints are added to make the model complete.

$$\begin{aligned} f_{k_1 k_2} &= f_{k_2 k_1} & \forall k_1 \in K, k_2 \in K \\ f_{kk} &= 1 & \forall k \in K \end{aligned} \tag{4.17}$$

Taking that all into account, the following model is presented:

$$\begin{array}{lll}
\text{Maximise} & \sum\limits_{i\in I} d_i y_i, & \\
\text{such that} & \sum\limits_{k_1\in K}\sum\limits_{k_2\in K}\sum\limits_{j\in J_i} a_{jk_1} x_j f_{k_1k_2}\alpha_{ik_2} \geq y_i, & \forall i \in I \\
& f_{k_1k} + f_{kk_2} - 1 \leq f_{k_1k_2}, & \forall k_1, k_2, k \in K \\
& \sum\limits_{k_2\in K} f_{k_1k_2} \leq s, & \forall k_1 \in K \\
& f_{k_1k_2} \leq t_{k_1k_2}, & \forall k_1 \in K, k_2 \in K \\
& \sum\limits_{k_1\in K}\sum\limits_{j\in J} a_{jk_1} f_{k_1k_2} x_j \leq \sum\limits_{k_1\in K} p_{k_1} f_{k_1k_2}, & \forall k_2 \in K \\
& f_{k_1k_2} = f_{k_2k_1}, & \forall k_1, k_2 \in K \\
& f_{kk} = 1, & \forall k \in K \\
& y_i, x_j, f_{k_1k_2} \in \{0,1\}, & \forall i \in I, j \in J, k_1, k_2 \in K.
\end{array} \tag{4.18}$$

4.2. MEXCLP

As stated before, MEXCLP preforms better than MCLP on a few aspects. First of all, it is more realistic, because it takes the probability into account that the ambulance is not present at its base location. Also, double and triple coverage, which is the number of demand locations that is being covered by respectively two and three ambulances, is better than with MCLP [15]. That is why we transform the models presented in Sections 4.1.1 and 4.1.3, into models with MEXCLP as basis instead of MCLP.

4.2.1. First formulation

MCLP and MEXCLP have different objective functions, as described in Chapter 3. So, the objective function becomes the one of MEXCLP. Also, the y_i changes into y_{ir} as defined in Section 3.2. Next to that, the parameter p is all the ambulances that are available, i.e., $p = \sum\limits_{k\in K} p_k$. The objective function becomes the following.

$$\text{Maximise} \quad \sum_{i\in I}\sum_{r=1}^{p} d_i(1-q)q^{r-1} y_{ir}. \tag{4.19}$$

The first constraint becomes a combination of the model presented in Section 4.1.1 (left hand side) and MEXCLP as presented in Section 3.2 (right hand side). This is due to the fact that the condition under which a demand location i is covered stays the same in the MEXCLP version, but now y_i is changed to y_{ir}. Therefore, we need to sum over all the covering ambulances, as a demand location $i \in I$ can be covered with more than just one ambulance. Taking this into account, the first constraint becomes:

$$\sum_{h\in H}\sum_{j\in J_i} x_j l_{ijh} \geq \sum_{r=1}^{p} y_{ir}, \quad \forall i \in I. \tag{4.20}$$

This constraint is also not linear. This is being resolved in Section 4.3. Lastly, it is important to note that x_j is changed from a binary variable into an integer variable. The consequences of this are discussed later in Section 4.3.3. All the other constraints are the same as the one with MCLP as basis. The complete model is as follows:

$$\begin{array}{lll}
\text{Maximise} & \sum\limits_{i\in I}\sum\limits_{r=1}^{p} d_i(1-q)q^{r-1} y_{ir}, & \\
\text{such that} & \sum\limits_{h\in H}\sum\limits_{j\in J_i} x_j l_{ijh} \geq \sum\limits_{r=1}^{p} y_{ir}, & \forall i \in I, \\
& \sum\limits_{k\in K} a_{jk} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum\limits_{k\in K} \alpha_{ik} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum\limits_{k\in K}\sum\limits_{j\in J} a_{jk} b_{kh} x_j \leq \sum\limits_{k\in K} p_k b_{kh}, & h \in H
\end{array} \tag{4.21}$$

$$
\begin{array}{ll}
b_{k_1 h} + b_{k_2 h} - 1 \leq t_{k_1 k_2}, & \forall k_1 \in K, k_2 \in K, h \in H \\
\sum\limits_{h \in H} b_{kh} = 1, & \forall k \in K \\
\sum\limits_{k \in K} b_{kh} \leq s, & \forall h \in H \\
x_j \in \mathbb{N} & \forall j \in J \\
y_{ir}, b_{kh}, l_{ijh} \in \{0,1\} & \forall i \in I, k \in K, h \in H, r = 1, \ldots, p
\end{array}
$$

4.2.2. Second formulation

Also for the second formulation of the model, a transformation to a model with MEXCLP as basis is made. In this case, the objective function becomes the same as in Section 4.2.1. Also, the same adjustment is made for the first constraint. So, taking the left side of the MCLP model and the right side of the MEXCLP model. Thus, the full model is:

$$
\begin{array}{lll}
\text{Maximise} & \sum\limits_{i \in I} \sum\limits_{r=1}^{p} d_i (1-q) q^{r-1} y_{ir}, & \\
\text{such that} & \sum\limits_{k_1 \in K} \sum\limits_{k_2 \in K} \sum\limits_{j \in J_i} a_{jk_1} f_{k_1 k_2} x_j \alpha_{ik_2} \geq \sum\limits_{r=1}^{p} y_{ir} & \forall i \in I \\
& f_{k_1 k} + f_{k k_2} - 1 \leq f_{k_1 k_2}, & \forall k, k_1, k_2 \in K \\
& \sum\limits_{k_2 \in K} f_{k_1 k_2} \leq s, & \forall k_1 \in K \\
& f_{k_1 k_2} \leq t_{k_1 k_2}, & \forall k_1, k_2 \in K \\
& \sum\limits_{k_1 \in K} \sum\limits_{j \in J} a_{jk_1} f_{k_1 k_2} x_j \leq \sum\limits_{k_1 \in K} p_{k_1} f_{k_1 k_2}, & \forall k_2 \in K \\
& f_{k_1 k_2} = f_{k_2 k_1}, & \forall k_1, k_2 \in K \\
& f_{kk} = 1, & \forall k \in K \\
& x_j \in \mathbb{N}, & \forall j \in J \\
& y_i, f_{k_1 k_2} \in \{0,1\}, & \forall i \in I, k_1, k_2 \in K.
\end{array}
\tag{4.22}
$$

4.3. Linearisation

Note that the resulting models described above are all non-linear. This means that the regular methods for solving LP, like the branch and bound method, cannot be applied. This makes that non-linear programs by its very nature are more difficult to solve [3]. Therefore, there are methods to make a non-linear program linear to solve it more easily. In order to do this, a new decision variable is introduced. This variable represents the product of the two decision variables which made the model non-linear. This has to be done separately for all four models. By replacing the multiplication of the two decision variables with a new decision variable, the model becomes linear.

4.3.1. First formulation MCLP

For the first formulation of the MCLP model, there are two multiplications of two binary decision variables. To fix this, c_{hkj} and m_{ijh} are replacing $b_{kh} x_j$ and $x_j l_{ijh}$, respectively, which are defined in the following manner.

$$
m_{ijh} = \begin{cases} 1, & \text{if an ambulance is positioned at potential base location } j \in J \\ & \text{which is in the same main region } h \in H \text{ as location } i \in I \\ 0, & \text{otherwise} \end{cases}
$$

$$
c_{hkj} = \begin{cases} 1, & \text{if an ambulance is positioned at potential base location } j \in J \\ & \text{in RAV } k \in K \text{ assigned to main region } h \in H \\ 0, & \text{otherwise} \end{cases}
$$

Following, some constraints need to be added such that the new variables are behaving the same as the multiplication of the two variables it is replacing. This means that, for example, c_{hkj} can be one if and only if both b_{kh} and x_j are one. If either b_{kh}, x_j or both are zero, c_{hkj} must be zero. By constraining c_{hkj} to be smaller than both b_{kh} and x_j, this will always be the case. When both b_{kh} and x_j are one, c_{hkj} needs to get the value one. The first two constraints make sure that c_{hkj} will not be bigger than one. But now, there is a need that c_{hkj} cannot be zero if both b_{kh} and x_j are one. By adding the constraint that c_{hkj} must be more than the sum of the variables, minus one, we force c_{hkj} such that $c_{hkj} \geq 1$ and $c_{hkj} \leq 1$, leading to $c_{hkj} = 1$. Using these three conditions, c_{hkj} behaves the same as $b_{kh} \cdot x_j$. Following are the constraints for c_{hkj} as well as for m_{ijh}.

$$\begin{array}{ll}
c_{hkj} \leq b_{kh}, & \forall j \in J, k \in K, h \in H \\
c_{hkj} \leq x_j, & \forall j \in J, k \in K, h \in H \\
c_{hkj} \geq b_{kh} + x_{jk} - 1, & \forall j \in J, k \in K, h \in H \\
m_{ijh} \leq x_j, & \forall i \in I, j \in J, h \in H \\
m_{ijh} \leq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
m_{ijh} \geq l_{ijh} + x_j - 1, & \forall i \in I, j \in J, h \in H.
\end{array} \tag{4.23}$$

Taking these constraints into account, the full linear model of the first formulation with MCLP is:

$$\begin{array}{lll}
\text{Maximise} & \sum_{i \in I} d_i y_i, & \\
\text{such that} & \sum_{h \in H} \sum_{j \in J_i} m_{ijh} \geq y_i, & \forall i \in I \\
& \sum_{k \in K} a_{jk} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum_{k \in K} \alpha_{ik} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum_{k \in K} \sum_{j \in J} a_{jk} c_{hkj} \leq \sum_{k \in K} p_k b_{kh}, & h \in H \\
& b_{k_1 h} + b_{k_2 h} - 1 \leq t_{k_1 k_2}, & \forall k_1 \in K, k_2 \in K, h \in H \\
& \sum_{h \in H} b_{kh} = 1, & \forall k \in K \\
& \sum_{k \in K} b_{kh} \leq s, & \forall h \in H \\
& c_{hkj} \leq b_{kh}, & \forall j \in J, k \in K, h \in H \\
& c_{hkj} \leq x_j, & \forall j \in J, k \in K, h \in H \\
& c_{hkj} \geq b_{kh} + x_{jk} - 1, & \forall j \in J, k \in K, h \in H \\
& m_{ijh} \leq x_j, & \forall i \in I, j \in J, h \in H \\
& m_{ijh} \leq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& m_{ijh} \geq l_{ijh} + x_j - 1, & \forall i \in I, j \in J, h \in H \\
& y_i, b_{kh}, x_j, c_{hkj}, l_{ijh}, m_{ijh} \in \{0,1\}, & \forall i \in I, j \in J, k \in K, h \in H.
\end{array} \tag{4.24}$$

4.3.2. Second formulation MCLP

For the second formulation of the model with MCLP used as basis, the same problem arises: the model is not linear as there is a multiplication of x_j and $f_{k_1 k_2}$. Because both variables are binary, this can be fixed the same way as in Section 4.3.1. So, the following decision variable is introduced.

$$n_{jk_1k_2} = \begin{cases} 1, & \text{if potential base location } j \text{ in RAV } k_1 \text{ has an ambulance and } k_1 \text{ and } k_2 \text{ are} \\ & \text{within the same merged region} \\ 0, & \text{otherwise} \end{cases}$$

Then, by adding the following constraints, $n_{jk_1k_2}$ is replacing $x_j f_{k_1k_2}$ and it results in a linear model.

$$\begin{array}{ll}
n_{jk_1k_2} \leq f_{k_1k_2}, & \forall k_1, k_2 \in K, j \in J \\
n_{jk_1k_2} \leq x_j, & \forall k_1, k_2 \in K, j \in J \\
n_{jk_1k_2} \geq f_{k_1k_2} + x_{jk} - 1, & \forall k_1, k_2 \in K, j \in J.
\end{array} \tag{4.25}$$

So, the linear model is as follows.

$$\begin{array}{lll}
\text{Maximise} & \sum_{i \in I} d_i y_i, & \\
\text{Such that} & \sum_{k_1 \in K} \sum_{k_2 \in K} \sum_{j \in J_i} a_{jk_1} n_{jk_1k_2} \alpha_{ik_2} \geq y_i, & \forall i \in I
\end{array} \tag{4.26}$$

$$
\begin{array}{ll}
f_{k_1k} + f_{kk_2} - 1 \leq f_{k_1k_2}, & \forall k, k_1, k_2 \in K \\
\sum_{k_2 \in K} f_{k_1k_2} \leq s, & \forall k_1 \in K \\
t_{k_1k_2} \leq f_{k_1k_2}, & \forall k_1 \in K, k_2 \in K \\
\sum_{k_1 \in K} \sum_{j \in J} a_{jk_1} n_{jk_1k_2} \leq \sum_{k_1 \in K} p_{k_1} f_{k_1k_2}, & k_2 \in K \\
f_{k_1k_2} = f_{k_2k_1}, & \forall k_1, k_2 \in K \\
f_{kk} = 1, & \forall k \in K \\
n_{jk_1k_2} \leq f_{k_1k_2}, & \forall k_1, k_2 \in K, j \in J \\
n_{jk_1k_2} \leq x_j, & \forall k_1, k_2 \in K, j \in J \\
n_{jk_1k_2} \geq f_{k_1k_2} + x_{jk} - 1, & \forall k_1, k_2 \in K, j \in J \\
y_i, x_j, f_{k_1k_2}, n_{jk_1k_2} \in \{0,1\}, & \forall i \in J, j \in J, k_1, k_2 \in K.
\end{array}
$$

4.3.3. First formulation MEXCLP

The problem of non-linearity is again encountered in the models with MEXCLP as basis. But this time, it is not two binary variables that are multiplied, but a binary variable and an integer variable. The linearisation of this models is done by the method from the paper of Coelho [5]. To this end, boundaries for x_j need to be found. These are quite clear, because a base location can have at least zero ambulances and at most $p = \sum_{k \in K} p_k$ ambulances, if it owns all the ambulances over all the RAVs. So, the boundaries are $0 \leq x_j \leq p$. Now that these boundaries are defined, it is possible to replace $x_j l_{ijh}$ with m_{ijh} and $b_{kh} x_j$ with c_{hkj}. The method works as follows: first, two constraints are constructed to ensure that m_{ijh} is smaller than $x_j l_{ijh}$. If one of the two is zero, m_{ijh} also has to be zero. So m_{ijh} has to be smaller than or equal to both x_j and l_{ijh}. But it can be maximally p, namely if x_j is p and l_{ijh} is one. So, it has to be smaller than not just l_{ijh}, but $p \cdot l_{ijh}$. Then it is for sure smaller than the product of the two variables. Now the third constraint make sure that m_{ijh} becomes large enough. So, if x_j and l_{ijh} are both greater than zero, the value of m_{ijh} becomes the value of x_j. This could be done by making m_{ijh} greater than or equal to $x_j - (1 - l_{ijh}) \cdot p$. But now, if l_{ijh} is zero, then m_{ijh} can be less than zero. To prohibit this, an extra constraint needs to be added in which m_{ijh} has to be equal than or greater to zero. So, the constraints for m_{ijh} are:

$$
\begin{array}{ll}
m_{ijh} \leq p \cdot l_{ijh}, & \forall i \in I, j \in J, h \in H \\
m_{ijh} \leq x_j, & \forall i \in I, j \in J, h \in H \\
m_{ijh} \geq x_j - (1 - l_{ijh}) \cdot p, & \forall i \in I, j \in J, h \in H \\
m_{ijh} \geq 0, & \forall i \in I, j \in J, h \in H.
\end{array}
\tag{4.27}
$$

This could be done equivalently for c_{hkj}. Resulting, the linear version of the model becomes:

$$
\begin{array}{lll}
\text{Maximise} & \sum_{i \in I} \sum_{r=1}^{p} d_i (1-q) q^{r-1} y_{ir}, & \\
\text{such that} & \sum_{h \in H} \sum_{j \in J_i} m_{ijh} \geq \sum_{r=1}^{p} y_{ir}, & \forall i \in I \\
& \sum_{k \in K} a_{jk} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum_{k \in K} \alpha_{ik} b_{kh} \geq l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& \sum_{k \in K} \sum_{j \in J} a_{jk} c_{hkj} \leq \sum_{k \in K} p_k b_{kh}, & h \in H \\
& b_{k_1h} + b_{k_2h} - 1 \leq t_{k_1k_2}, & \forall k_1 \in K, k_2 \in K, h \in H \\
& \sum_{h \in H} b_{kh} = 1, & \forall k \in K \\
& \sum_{k \in K} b_{kh} \leq s, & \forall h \in H \\
& c_{hkj} \leq p \cdot b_{kh}, & \forall j \in J, k \in K, h \in H \\
& c_{hkj} \leq x_j, & \forall j \in J, k \in K, h \in H \\
& c_{hkj} \geq x_j - (1 - b_{kh}) \cdot p, & \forall j \in J, k \in K, h \in H \\
& c_{hkj} \geq 0, & \forall j \in J, k \in K, h \in H \\
& m_{ijh} \leq p \cdot l_{ijh}, & \forall i \in I, j \in J, h \in H \\
& m_{ijh} \leq x_j, & \forall i \in I, j \in J, h \in H \\
& m_{ijh} \geq x_j - (1 - l_{ijh}) \cdot p, & \forall i \in I, j \in J, h \in H \\
& m_{ijh} \geq 0, & \forall i \in I, j \in J, h \in H
\end{array}
\tag{4.28}
$$

$$\begin{aligned}
&x_j, m_{ijh} \in \mathbb{N}, && \forall i \in I, j \in J, h \in H \\
&y_{ik}, b_{kh}, c_{hkj}, l_{ijh}, \in \{0,1\}, && \forall i \in I, j \in J, k \in K, h \in H.
\end{aligned}$$

4.3.4. Second formulation MEXCLP

For the fourth and last model, the same technique is used as in Section 4.3.3 and the model becomes the following.

$$\begin{aligned}
\text{Maximise} \quad & \sum_{i \in I} \sum_{r=1}^{p} d_i (1-q) q^{r-1} y_{ir}, && \\
\text{such that} \quad & \sum_{k_1 \in K} \sum_{k_2 \in K} \sum_{j \in J_i} a_{jk_1} n_{jk_1k_2} \alpha_{ik_2} \geq \sum_{r=1}^{p} y_{ir}, && \forall i \in I, \\
& f_{k_1k} + f_{kk_2} - 1 \leq f_{k_1k_2}, && \forall k_1 \in K, k_2 \in K \\
& \sum_{k_2 \in K} f_{k_1k_2} \leq s, && \forall k_1 \in K \\
& t_{k_1k_2} \leq f_{k_1k_2}, && \forall k_1 \in K, k_2 \in K \\
& \sum_{k_1 \in K} \sum_{j \in J} a_{jk_1} n_{jk_1k_2} \leq \sum_{k_1 \in K} p_{k_1} f_{k_1k_2}, && k_2 \in K \\
& f_{k_1k_2} = f_{k_2k_1}, && \forall k_1 \in K, k_2 \in K \\
& f_{kk} = 1, && \forall k \in K \\
& n_{jk_1k_2} \leq p \cdot f_{k_1k_2}, && \forall j \in J, k_1, k_2 \in K \\
& n_{jk_1k_2} \leq x_j, && \forall j \in J, k_1, k_2 \in K \\
& n_{jk_1k_2} \geq x_j - (1 - f_{k_1k_2}) p, && \forall j \in J, k_1, k_2 \in K \\
& n_{jk_1k_2} \geq 0, && \forall j \in J, k_1, k_2 \in K \\
& x_j, n_{jk_1k_2} \in \mathbb{N}, && \forall j \in J, k_1, k_2 \in K \\
& y_i, x_j, f_{k_1k_2} \in \{0,1\}, && \forall i \in I, j \in J, k_1, k_2 \in K.
\end{aligned} \tag{4.29}$$

5

Results

In this chapter, an overview is given for all the results that were collected while performing tests with the models described in Chapter 4. It starts off with a comparison of the different models on a small data set, to set out the performances of the different models. Based on this comparison, two of the four models are chosen to continue the research with. These two models are used to perform tests with the data set containing all the data from the Netherlands and these results are presented.

5.1. Comparison on small data set

To verify that the models work and to compare them with one another, they were tested on a small data set. The models were implemented in Python 3 and solved with CPLEX 12.9.0 on an Intel Core i3-2310M CPU @ 2.10GHz 2.10 GHz with 4.00 GB RAM. The data consists of eight demand locations in a total of four regions. The travel time in seconds between all the demand locations is given in Table 5.1. The division of the demand locations in the four regions can be found in Table 5.2.

For the tests, several parameters can be chosen. The number of regions that could be merged with each other was chosen to be three. This means that either three regions can be merged with each other and one is left alone, the regions are divided two by two or none of the regions is merged. For generality, all the demand locations have weight one. All the regions are adjacent to each other, and every region has one ambulance available, i.e., $p_k = 1$ for all $k \in K$. r is three minutes and q is chosen to be 0.5 for all the four ambulances. In Table 5.3, the results for the test can be found. The first row gives the single coverage. This is the percentage of demand locations which get covered by one ambulance. The double coverage has the same notion but now for two ambulances. Then, the two and four minute threshold is given, which is the percentage of the demand locations which are covered within two- and four-minutes. Also, the merger of the regions is stated. Lastly, the calculation time for each of the models in both formulations is given.

Table 5.1: Travel time in seconds between all the data points in the test data set

	1011	1012	1013	1014	1015	1016	1017	1018
1011		123	371	411	242	256	198	161
1012	112		338	365	180	145	129	273
1013	360	333		193	268	352	443	506
1014	400	359	184		287	370	469	546
1015	231	174	259	285		165	253	377
1016	245	139	343	368	161		162	357
1017	196	124	434	467	249	157		273
1018	159	268	497	544	366	353	269	

Table 5.2: The four regions in the small data set and their possessing demand locations

1	2	3	4
1011	1012	1013	1015
1016	1018	1014	1019
1017			

Table 5.3: Results on different criteria for the small data test

Description criterion	MCLP	MEXCLP
Single coverage	87.5 %	87.5 %
Double coverage	25%	62.5 %
Four min threshold	100 %	100 %
Two min threshold	62.5%	62.5 %
Merged regions	(1,2,3) and (4)	(1,2,4) and (3)
Calculation time First formulation	0:00:01.82	0:00:01.91
Calculation time Second formulation	0:00:00.80	0:00:00.90

5.1.1. Calculation time

The results for the calculation time that are stated in Table 5.3 for the first formulation are the once with both the symmetry breaking constraints taken into account. These were about 30% as quick as in the model without symmetry breaking constraints. If only the first constraint was added, the model was approximately 20% as quick as the original model. It was not possible to check the calculation time if only the second symmetry breaking constraint was taken into account, because it uses the first constraint.

5.1.2. Results

The results show that, despite the symmetry breaking constraints, the second formulation can be solved to optimality a lot quicker than the first formulation. This is why we use the second formulation in the remainder of this thesis. Furthermore, the double coverage is much better for the models with MEXCLP as basis, as was expected. The choice of RAVs results in the difference in double coverage. Lastly, there is a small difference in the computation time between the models with a MCLP basis and a MEXCLP basis.

5.2. Data

Tests with two models were executed on a data set containing information about the whole Netherlands to see the extent to which the merging improves the efficiency of the ambulance care. In order to do the research, the Netherlands had to be divided into small areas. The small areas that are used for this research are the four-digit postal code zones of the Netherlands. There are in total 3990 four-digit postal codes areas in the Netherlands. For every area, it is assumed that multiple ambulances can be placed there and that from there an ambulance call could be executed. So, each demand location is also a potential base location, i.e., $J = I$. The distances between all the four-digit postal codes were provided by my supervisor J. T. van Essen. These distances are based on a data set with all the driving distances in the Netherlands from Google Maps, in which a factor of approximately 0.96 is multiplied because the ambulances can go faster than the regular traffic. Other data was collected from the paper by Van den Berg et al. [15]: the number of ambulances that are available for each RAV and the busy fraction for each RAV. This data can be found in Table 5.4. For MCLP, another number of ambulances is used. The reason for this is explained in Section 5.3. For the weights d_i, we use the number of people living in each four-digit postal code area $i \in I$.

In the Netherlands, an ambulance call should be present at the demand location within 15 minutes. From this 15 minutes, 3 minutes are taken for dispatch and chute time. So, to reach a demand location within 15 minutes, 3 minutes are subtracted to make the model realistic. this leaves you with 12 minutes' travel time. This is the same as is taken at the research from van den Berg et al. [15], so these results can be compared with the results in this paper.

RAV	Ambulances available	Ambulances used for MCLP	Busy fraction
1	15	8	0.18
2	18	9	0.10
3	13	7	0.17
4	12	6	0.16
5	11	6	0.20
6	14	7	0.21
7	8	4	0.27
8	10	5	0.20
9	15	8	0.30
10	9	5	0.22
11	16	8	0.38
12	8	4	0.31
14	4	2	0.23
15	12	6	0.39
16	10	5	0.30
17	12	6	0.45
18	8	4	0.22
19	18	9	0.09
20	16	8	0.27
21	9	5	0.26
22	9	5	0.28
23	10	5	0.21
24	7	4	0.36
25	8	4	0.20
Total	272	140	

Table 5.4: Number of ambulances used for the test runs with MCLP and MEXCLP and the busy fraction

Figure 5.1: The resulting separation of the RAVs. Source: J. W. van Aalst (2015). *Kaart van de RAV-regio's, 2015* [map]. Retrieved from http://www.imergis.nl, with own edit

5.2.1. Heuristic method

During the creation of the model, a space limit was reached. To this end, a heuristic is used. The Netherlands is partitioned into two groups of 12 RAVs. The separation line runs straight through the middle of the Netherlands. The two groups in which the Netherlands is divided is as follows: the RAVs 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 14 and 25 are put together and the RAVs 8, 9, 15, 16, 17, 18, 19, 20, 21, 22, 23 and 24 are together. The partition of the RAVs is also made visual in Figure 5.1 with a red line separating the two parts.

This partition is made upon the results of different tests with the MCLP model in smaller regions. The RAVs were partitioned into blocks of four, and within these blocks merges of two RAVs were carried out. First, it was looked at what was visually a logical separation. As can be seen in Figure 5.1, the areas in the middle part are 7, 8, 9, 11, 12,14, 15 and 16. The other RAVs are so far north or south that it was clear in which part they belong. So, a test was done with the four RAVs 10, 11, 12, 25, then 9, 14, 15, 16 and lastly 5, 6, 7, 8. Two results of the tests were important in this: first, 16 and 9 were put together and 14 and 15 were left alone. Apparently, the merger of only RAVs 16 and 9 outperforms the two duos you can make with it. This gave the incentive to put RAVs 9 and 16 in one region and RAV 14 in the other one. Another interesting result was that 7 and 8 were put together, but this resulted in a coverage of only 97.3%, which was one of the lowest of all quarters that were tested. This lead to the assumption that of the places where a separation needed to be made, it is logical to do it between RAVs 7 and 8 and not for example between RAVs 9 and 16. This was reason to separate the Netherlands in the way that is seen in Figure 5.1. After that, it was looked at the distribution of the ambulances. Both parts have exactly half of the ambulances with this distribution. This was the deciding factor to choose the separation as stated above.

5.3. MCLP

To start collecting results, tests with the MCLP model using the second formulation were performed. The model was implemented in Python 3 and solved with CPLEX 12.9.0 on the computer described in Section 5.1 and on an Intel Core i7-6600U CPU @ 2.10GHz 2.606 GHz 2.80 CHz with 8.00 GB RAM. When using the model with MCLP basis, the single coverage of the Netherlands is on average 100% [15]. So, when running this program, the RAVs are put together at random, because it will neither improve nor worsen the result. Merging the RAVs is used to improve the efficiency, so if areas were to be merged, less ambulances are needed in order to get a result as high as without the merge. That is why it is chosen to do the research with half the number of ambulances that is actually available. This to show how much the efficiency improves by merging the RAVs. In Table 5.4 the number of ambulances per RAV can be found.

Due to the fact that a heuristic is used, we cannot state that the found results are optimal. Especially in the middle RAVs, it could be better to merge over the border of the separation. To this end, the results are not claimed to be optimal, but the results can be compared to the current situation.

5.3.1. Duo merging

So, for the first test, it was tested what happens if there could be maximally two RAVs merged with each other. The next results were found: for the upper half, RAVs 1 and 3, 2 and 4, 5 and 6, 10 and 12 and 11 and 25 were merged together. RAVs 7 and 14 were left alone. This could also be seen graphically in Figure 5.2. The total coverage was 97.90%. Over all the 12 RAVs, 71 ambulances were used. For the lower half, RAVs 8 and 18, 15 and 17, 9 and 16, 19 and 20, 21 and 22 and 23 and 24 are merged. This can also be seen in Figure 5.2. The total single coverage was 99.19%. A total of 69 ambulances were used in the 12 regions. The results in coverage and the number of ambulances used for this research and for the research of Van den Berg et al. [15] can be found in Table 5.5

	Coverage	Ambulances used
Upper half	97.90%	71
Lower half	99.19%	69
Total	98.55%	140
Without merge	100%	272

Table 5.5: Results of the test with the MCLP model

Figure 5.2: The resulting division of RAV's after the test with the MCLP model. Source: J. W. van Aalst (2015). *Kaart van de RAV-regio's, 2015* [map]. Retrieved from http://www.imergis.nl, with own edit

If the merges are compared to the health insurances in Section 2.1, all the merged RAVs share at least one responsible health insurance, except for RAVs 15 and 17. Overall merging could thus be done fairly well with this merging result.

5.4. MEXCLP

To continue, tests were done with the model with MEXCLP (second formulation) as basis. In this case, it is not needed to half the number of ambulances, because it does not have a 100% coverage to begin with. The number of ambulances per RAV is now the number that is actually available in this RAV. The number of ambulances can be found in the second column of Table 5.4, along with the busy fraction for each RAV, which is in the fourth column of that table.

For this part of the research, the Netherlands is partitioned in the same way as in the previous section. Due to this, it cannot be said that the results are optimal.

5.4.1. Duo merging

First, it was calculated what would happen if maximally two RAVs were merged. The results for this test are the following: for the upper half, the RAVs 1 and 3, 4 and 5, 6 and 7, 10 and 11 and 14 and 25 were merged in duos. RAVs 2 and 12 were left alone. The single coverage was 99.74% and the double coverage 92.3%. For the lower half, the regions 8 and 21, 15 and 17, 9 and 16, 19 and 20 and 22 and 23 were merged. RAVs 18 and 24 were not merged. These merges are presented in Figure 5.3. There was a single coverage of 99.61%. The double coverage was 94.74%. All available ambulances are used.

If this is compared to the coverage that was found in the research without merging the RAVs, we see that the coverage is much higher. In Table 5.6, the results for single, double and triple coverage for the MEXCLP were shown, both with and without merging. The results in the table are the average coverage of the total coverage of the upper half and the lower half of the Netherlands.

	Without merge	With merge
Single coverage	99.4 %	99.7 %
Double	91.4 %	93.52 %
Triple	59.1 %	68.71 %

Table 5.6: Results for the MEXCLP model with maximal two RAVs merged

For the health insurance responsibility for the new regions, the same situation has occurred as in Section 5.3. Due to the fact that MEXCLP provides a more profound research result, it is chosen to perform the rest of the tests only with MEXCLP and not the MCLP model.

5.4.2. Trio merging

Continuing, it is examined what happens if three RAVs could be merged into one another. This does not mean that the 24 RAVs are merged into 8 regions, but it is reviewed whether it is the best to merge a RAV with one or two other RAVs or to be left alone. The results are as follows: For the upper half, the RAVs 1, 2 and 3, but also, 4, 5 and lastly, 6, 10, 11 and 12 were merged in trios. The RAVs 14 and 25 and 8 and 21 were merged in a duo. The RAVs 24 and 13 were not merged. The single coverage is 99.73% and the double coverage 93.46%. For the lower half of the Netherlands, RAVs 8, 9 and 18 were merged in a trio. RAVs 15 and 17, 19 and 20 and 21, 22 and 23 were merged in duos. RAVs 16 and 24 were not merged. The single coverage is 99.73% and the double coverage 95.31%. The merging of RAVs is shown in Figure 5.4. The average single, double and triple coverage for MEXCLP with and without merging is shown in Table 5.7.

	Without merge	With merge
Single coverage	99.4 %	99.73 %
Double coverage	91.4 %	94.38 %
Triple coverage	59.1 %	69.93 %

Table 5.7: Results for the MEXCLP model with maximal three RAVs merged

For the health insurance responsibility for the new regions, the same situation has occurred as in Section 5.3.

Figure 5.3: The resulting division of RAV's after the test with MEXCLP model with maximally two RAVs merged. Source: Source: J. W. van Aalst (2015). *Kaart van de RAV-regio's, 2015* [map]. Retrieved from http://www.imergis.nl, with own edit

5.4.3. Quartet merging

Lastly, the results were collected for the case that four RAVs could be merged. But remarkably, the results were precisely the same as for the tests in which maximal three RAVs would be put together. Apparently, it does not matter whether to merge the RAVs, which were not yet merged, into another trio.

5.4.4. All results

Table 5.8 shows all the results of the different tests with the MEXCLP model in second formulation.

	Without merge	With duo merge	With triple merge
Single coverage	99.4 %	99.7 %	99.73 %
Double coverage	91.4 %	93.52 %	94.38 %
Triple coverage	59.1 %	68.71 %	69.93 %

Table 5.8: All average results for the MEXCLP model

Figure 5.4: The resulting division of RAV's after the test with MEXCLP model with maximal three RAVs merged. Source: J. W. van Aalst, *Kaart van de RAV-regio's, 2015*— www.imergis.nl, with own edit

6
Conclusion and discussion

In this thesis, the different aspects of merging Regional Ambulance Services (RAVs) in the Netherlands have been studied. RAVs are the regions in which ambulances serve. To increase the efficiency and provide better ambulance care, the RAVs can be merged. We first looked at why it is complicated to merge the RAVs. After that, we constructed four different models which determined, in case of a merge, which RAVs should be put together. These models were tested on a data set containing all the data points from the Netherlands and the results were presented.

If we look at the results, we start off with the results gained from this research by the MCLP model and compare these with the results gained from the paper by Van den Berg et al. [15]. In this comparison, we see that with a bit more than halve the number of RAVs, half of the ambulances are needed to perform approximately the same. The average coverage is 98.55% for the model where RAVs were merged. So, the coverage is 1.45 percentage point less even although only 51.47% of the number of ambulances were used. So, by changing from 24 to 13 RAVs, you need approximately halve of the ambulances to perform similar. In this research, it has not been invested how much ambulances were needed in the model with merger to get the 100% coverage with MCLP. This would have been interesting and is something that can be looked at in future research.

As expected, we see that using the MEXCLP models that allow merging results in a higher coverage than the one where no merging took place. Especially on the triple coverage, the model is doing better. For the single coverage, the difference is less big. So apparently, merging the RAVs does mainly mean that the demand locations are covered by more ambulances, not many more demand locations are covered that were not covered before.

Of course, the results cannot be claimed to be optimal, because a heuristic is used. This is partly because of a lack of good resources and partly in the way the program was written. In future research, if better resources are available and the code is improved, the results with 24 RAVs can be calculated all at once and the heuristic would not be necessary. Furthermore, for future research, there are still a lot of different tests that would be interesting to investigate, like allowing to merge more than four RAVs with each other. In addition, it could be investigated what happens if you force every RAV to be merged with at least one other RAV, so no RAV is left alone. Also for future research, the model could be changed into one with another basis, for example one of the ARTM and MEXCLP combined models presented by Van den Berg et al. [15]. In this research, it is not taken into account how quick the ambulance arrives. A location is either covered if it is reached within 15 minutes or it is not. But of course, in real life it is also important to arrive as quick as possible. This could also be interesting to investigate further.

In the research, the separation of the 24 RAVs into two groups of 12 could have been investigated more profoundly. Because of a lack of time, the separation was based on just one distribution of foursomes. It would have been better to check it with different combinations and with different sizes.

Overall, merging RAVs increases the overall coverage and asks less ambulances to fulfil the demand. So, it can help the ambulance care in the whole Netherlands. On the other hand, merging is organisational not preferred, so it would be recommended to merge at most two RAVs together, because then a good balance is found between increasing health care and realistic implementation.

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