Transforming guitars to a violin using Fourier analysis

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Table of contents

Abstract

Nowadays there exists a wide variety of string instruments that produce various sounds. Although all sounds are produced by strings, the sounds of different instruments can be clearly distinguished. The main goal of this research is to transform the sound of one string instrument to the sound of another string instrument. The string instruments that are used for filtering are the acoustic guitar, the electric guitar, the bass guitar, the violin and the viola. To transform the sounds of different instruments the sound is filtered by using Fourier transforms. To transform these sounds accurately the sounds are analysed using Fourier analysis. As sub-goals of this research it is researched how the tones are produced and whether different ways of producing a sound have an impact on the tone. A string produces multiple frequencies which are due to the different possible standing waves on a string. These are called harmonics, the first harmonic produces the fundamental frequency of the tone. The other harmonics are overtones and have a frequency that is a multiple of the fundamental frequency. These produce the timbre of a sound and the difference between the sounds of different string instruments mostly lie in the timbre of a sound. By filtering, some harmonic are enhanced and others reduced or extinguished. This way the timbre of the sound of a different instrument is imitated. Because the relative intensity of harmonics is time dependent multiple filters are used, changing quickly after one another. These filters are produced using a moving window Fourier transform. The timbre of an instrument is dependent on the way a string is played. A note has a different timbre when it is bowed or plucked and whether it is an open or a closed note. Techniques as the flageolet technique can be used to accentuate some overtones whilst repressing others. Also the location where a string is struck has a big effect on the produced timbre. For the filter to work properly the original sound has to contain detectable overtones. This is the case for all instruments but the bass guitar. The filtering of other string instruments works very accurately but can still be distinguished from the sound it is trying to mimic. This could be improved by using even more filters, however creating and implementing these filters takes a lot of computing time. Another explanation why the filtered sound doesn't match perfectly with the sound it is trying to mimic could be that one of the string instruments is not precisely tuned. Finally, the phase of the sound of an overtone could be adjusted so that it corresponds to the sound that is being imitated. This improves the matching of the sounds.

1. Introduction

Music is a big part of our lives. Research shows that people spend an average of 2.5 hours a day listening to it. Music is important to adolescents, and it helps satisfy their emotional needs [1]. Nowadays in modern music there is a wide variety of stringed instruments and specifically string instruments. Stringed instruments have been around for ages. As there is little to no evidence available from prehistoric times it is difficult to date the first instruments. The first string instrument is probably the harp. Some scholars have suggested that the harp was derived from hunting bows. Perhaps old bows where retired and put to recreational use. There are pictures from ancient Egypt and Babylonia that depict these harps. The harp evolves to the lyre. Old Greek mythology credits the god Hermes with the invention of the lyre. A lyre was found in the burial chambers in the ancient Mesopotamian city of Ur. The so called lyre of Ur is accredited to be the oldest surviving instrument, as the city of Ur was founded in the 4th millennium BC. The next instrument is a zither and consists of a flat wooden soundbox with numerous strings stretched across it. Versions of the trapezoidal zither is struck with 2 light hammers and evolve in eastern Europe to the piano. Another instrument from Mesopotamia and Egypt is the plucked lute. The lute is widely used and is spread across the ancient Mediterranean and Asia. With an intermediate step of the fiddle, the lute gives the origin for the violin family and with an intermediate step of the vilhuela, the lute evolves into a guitar. Much later in the 1950s as technology makes remarkable changes the guitar family is expanded with electric guitars [2].

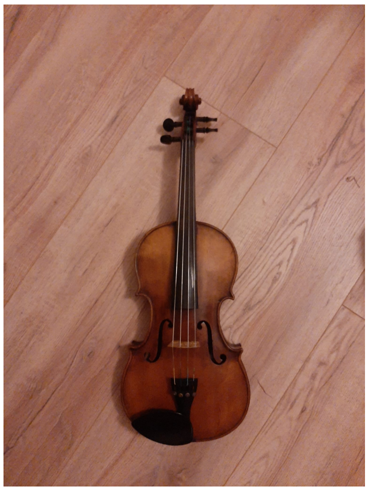
All these different string instruments can be roughly divided into a group of guitars, the violin family and pianos. These different instruments produce a wide variety of music. Although all sounds are produced by strings, the sounds of different instruments can be clearly distinguished. Centuries of development in various directions result in great differences in the sounds. This raises the question of whether it is possible to overcome the differences between the instruments. This leads to the main goal of this research; How can the sound of one string instrument be transformed to another string instrument. Or, to be more precise; How to transform a guitar to a violin.

To transform the sounds of different instruments Fourier transforms are used and filtered.The string instruments that are used for filtering are the acoustic guitar, the electric guitar, the bass guitar, the violin and the viola. In order to better transform the sounds, it is important to get a thorough understanding of the sounds these instruments produce. Using Fourier analysis the different components of sound are analysed. As sub-goals of this research it is researched how the tones are produced and whether different ways of producing a sound have an impact on the tone. These different ways of producing sound have an effect on the timbre of tones and are therefore important to understand for transforming sounds.

The structure of this project is as follows: Chapter 2, Theory, describes the theoretical concept of sound and how it is perceived. It describes properties of sound and musical theory of how sound is produced on string instruments. Different components of the timbre of sound are explained it is explained how these sounds originate from the different harmonics. In chapter 3 the Fourier transform and the Fast Fourier Transform algorithm, which is used to analyse the sound, are discussed. Chapter 4, outlines the experimental setup and how sound is stored and filtered. In Chapter 5, Results and discussion, the most important results of this analysis are presented and these results are interpreted. Chapter 6, Conclusion, contains the most important conclusions of this research.

(a)

(b)



(c)

(d)

(e)

Figure 1: Images of different string instruments, namely the acoustic guitar, the bass guitar, the violin, the electric guitar and the viola.

2. Theory

In the following chapters more and more will be discussed on how sound filters are created. These filters are based on the Fourier analysis of sound. The sounds of different string instruments are recorded and analysed in order to create those filters. However in order to understand that, first a greater understanding is needed about what sound is and how it is produced and perceived. In this chapter the concepts of sound, tempo, frequency and timbre are explained. Afterwards the differences between the string instruments and how they produce sound are discussed in more detail. Fourier analysis is used for analysing and filtering the sound, those will be explained afterwards.

2.1. Sound and hearing

Sound exists of waves that are mechanically created by a moving, or to be more precise by a vibrating object. Sound is created by all kinds of vibrating material and objects, these will be referred to as the source. The physical definition of sound is that it is the transmission of kinetic energy from particles in the source to particles in the medium through which the sound moves. In the case of sound, the medium in which it moves is mostly air and sometimes water. As the kinetic energy from the source is transmitted to the medium, the particles in the medium start to vibrate as well. This causes the pressure of the particles in the medium to fluctuate. Sound travels as a longitudinal wave, meaning that the propagation of energy is parallel to the way the sound moves. The variations of air pressure are detected by their mechanical effect on the tympana (ear drums) of our auditory system. Through a series of small bones the motion of the tympana reach the fluid of a spiral cavity called the cochlea. Here it induces impulses that are sent to the brain. Further details on how these vibrations induce impulses and how those impulses reach our brain is not part of the focus of the research. The important point is that the variations of air pressure are dominant in the hearing process. There are other ways vibrations can be detected. Vibrations can reach the cochlea through the bones in the head if they are in direct contact with the bones. Intense vibrations of low frequencies can be perceived by nerve transducers in other parts of the body. However both of these are not part of the primary sense of hearing. [3] As said before, sound travels as a longitudinal wave. This wave has a certain frequency that is measured in terms of the variation in pressure over a unit of time of air molecules. This means that the frequency is the number of waveform periods, or cycles, that occur in one second. The human sense of hearing can detect waves between 20 Hz and 20 kHz, but it is not as sensitive to the whole width of this spectrum. The sensitivity drops substantially for frequencies below 200 Hz and for frequencies above 10 kHz. Unsurprisingly, these frequencies are reasonably well matched with the human speech. The majority of the frequency content of our speech (approximately 300 Hz - 3 kHz) lies in the spectrum where the human hearing is sensitive. A sound wave has a certain intensity, which relates to its loudness but should not be mistaken for it. The intensity of a sound wave can be measured in various ways. The relative intensity between two sound waves can be measured by comparing the peak-to-peak value (twice the amplitude of a wave). However real life sound rarely consists of a pure sine wave that does not decay over time and therefore it is not easy to define the peak-to-peak value. Another way to define the intensity of a wave is by using the rms-value. This is obtained by squaring the values of a waveform over a given period, taking the average and of these values and squaring them. The intensity of sound can be measured in $\frac{W}{m^2}$. When it comes to sound

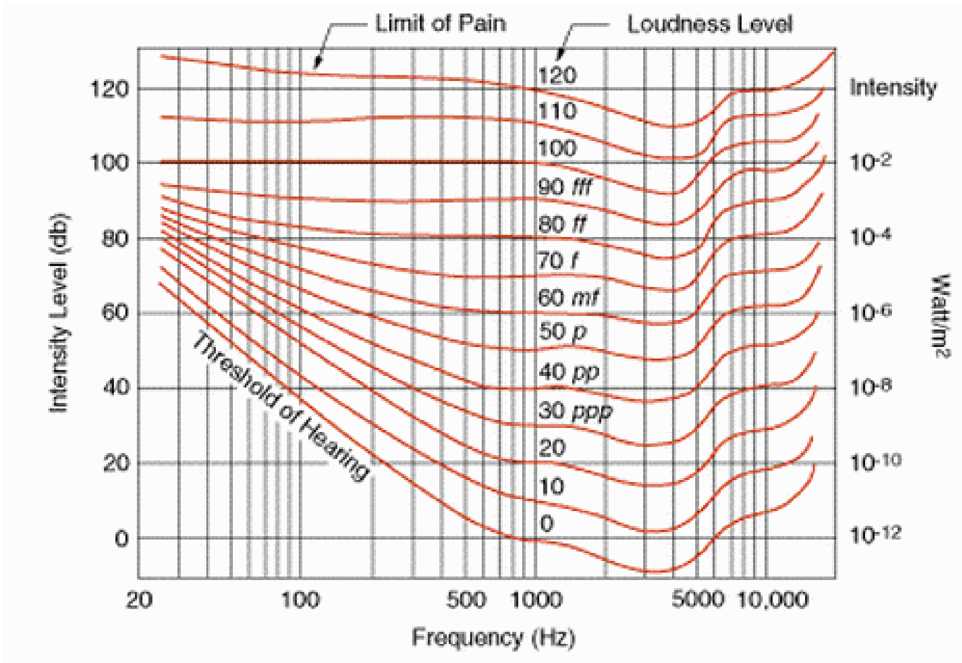


Figure 2: Equal loudness contours. The contours show, for a given dB, how equal loudness levels vary as a function of frequency.

however, most of the time the intensity is measured as a relative intensity on a logarithmic scale in decibel (dB). On this scale, the intensity of a sound is taken relative to the typical intensity where sound is (barely) audible for the human hearing, $I_0 = 10^{-12}$. The sound intensity is then calculated as $L = 10 \log_{10} \frac{I}{I_0}$ dB. The use of the scale where the logarithm is taken of the relative intensity with the minimum audible intensity I_0 is not arbitrary. This reflects that the human hearing seems to perceive sound logarithmic as well. This can be understood in a way that human senses are better in perceiving relative changes than in perceiving absolute changes. Human hearing is much better in detecting relative changes in air pressure than in perceiving absolute changes in air pressure. As said before there is also another distinction to be made when it comes to human hearing. Loudness and intensity relate to each other but are not the same. Intensity can be taken on the logarithmic scale dB relative to $I_0 = 10^{-12}$. The perceived loudness of a sound is measured in dB(A). This scale also accounts for the varying sensitivity of our hearing for sounds of different frequency. It is calculated almost the same but the minimum audible intensity I_0 is specified for each frequency.

In figure 2 it can be seen how loudness in dB(A) and intensity in dB differ. Sounds with the same intensity but different frequencies are perceived as louder or less loud depending on their frequency. A sound of 20 dB is not detected when it has a frequency of 200 Hz or less, and a sound of 0 dB is perceived as approximately 10 dB when it has a frequency of 3000-4000 Hz. This and other effects make our perception and experience of sound very complex. It is worth here to also make a notion about the phase of a sound wave. For a long time it was believed that people could not tell that there was a phase-shift in the sound. Such a phase-shift would have a negligible effect on the perception of sound, the human auditory system was considered "Phase deaf". Recent studies however have disproved this and shown that humans can detect changes in the phase spectrum. The phase spectrum of a sound effects how the timbre of a sound is perceived. Nonetheless the human hearing can only detect these changes in some cases. The sensibility to changes in the phase spectrum greatly depends on the type of sound and its frequency [4]. There is much more to be discussed on how the human auditory system detects sound waves and converts them to impulses. There is also much more to say about how those impulses are perceived. For example, it is believed that sound with a pitch of around 4000 Hz is perceived as louder because it resonates in our middle ear canal. To frame the research, the main part of the research will be focused on the pitch and timbre of sound [5].

2.2. Properties of sound

After having gained some basic understanding as to what sound is, it is time to look into what kind of properties sound has. It comes from a source, that source transmits energy to a medium through which the sound travels. Sound is a longitudinal wave which has a certain range of frequencies. First we will categorise complex sound and make more and more distinctions.

2.2.1. Travelling wave

Because sound is a wave it also has the basic properties of a wave. It has a certain amplitude which correlates to the amount of energy and the intensity of the sound. A sound wave has a frequency and a wavelength. The period of time in which a wave completes a cycle is called the period and at a specific time and position the wave has a specified phase. The velocity at which this wave propagates is dependent on the medium through which it travels. The speed of sound in an ideal gas is given by $v = \sqrt{\frac{\gamma RT}{M}}$ where γ is the adiabatic index, R is the gas constant, T the absolute temperature and M the molecular mass of the gas. In a fluid the speed of sound is different. Then it is given by $v = \sqrt{\frac{B}{\rho}}$, where B is the bulk modulus of the fluid and ρ its density [6]. The velocity of a wave is also given by $v = f/\lambda$, thus sound with a higher frequency has a shorter wavelength.

To examine a travelling wave on a string some properties of the string have to be taken into account. Consider a uniform string with linear density $\mu(\frac{kg}{m})$ that is stretched with tension T (in Newtons). There is a net force dF applied on segment ds of the string to restore it to its equilibrium position. This force equals the difference between the y components of the tension on both sides of the string. Then this segment is calculated as

$$dF_y = (T\sin(\theta))_{x+dx} - (T\sin(\theta))_x, \tag{1}$$

where θ is the angle of the direction of the string and the x axis. By applying Taylor expansion to the second order on the first term of equation (1) this changes to

$$dF_y = [(T\sin(\theta)_x + \frac{\partial(T\sin(\theta))}{\partial x}dx] - (T\sin(\theta))_x = \frac{\partial(T\sin(\theta))}{\partial x}dx. \tag{2}$$

Because we are considering a small displacement in y, $\sin(\theta)$ can be replaced by $\tan(\theta)$ which in turn is equal to $\frac{\partial y}{\partial x}$. Then the restoring force is calculated as

$$dF_y = \frac{\partial(T\frac{\partial y}{\partial x})}{\partial x}dx = T\frac{\partial^2 y}{\partial x^2}. \tag{3}$$

The net force from equation (3) is used in the Newton's second law of motion. The mass of the line segment is μds and since dy is small, ds can be approximated with dx. The equation of motion then becomes

$$\mu\frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2}, \frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}. \tag{4}$$

Equation (4) can be solved using the general solution from d'Alembert:

$$y = f_1(ct - x) + f_2(ct + x), \tag{5}$$

where f_1 and f_2 represent waves that are travelling to the right and to the left, respectively. Both waves in this equation have velocity c [3].

2.2.2. Rhythm, pitch and timbre

The first important distinction to be made is about how frequent a sound repeats itself. Rhythm, pitch and the timbre of a sound are all related to different frequencies within a sound and how often they repeat themselves. Rhythm and pitch are most alike of those properties. The distinction between those properties is whether or not we can easily distinguish the different impulses of a specific sound. In music for example when an instrument produces the same note a few times following each other the human hearing can distinguish those impulses. On the other hand, when we can't tell the difference between different impulses it is called the pitch of a sound. A violin is perhaps the best example for this; While it is played continuously in one bowing of the violin there is no distinction to be made between different impulses. Although the rhythm is of great importance in music, the analysis of the sound of string instruments is focused on pitch and timbre. To discuss pitch we take a pure tone as an example. The pitch of a sound is determined by the frequency of a sound wave, thus by the frequency of the tone. This is perceived as the so called "height" of a tone. However in reality sounds rarely consist of a pure tone. Usually there are various waves of different frequency combined in one sound wave. How these tones of different frequencies are produced and relate to each other will be discussed in more detail in section 2.3.2. In the case of string instruments, different string instruments produce sound with a different timbre. Sounds with a different timbre can have the same 'main' frequency, that is the lowest frequency produced and mostly that main frequency will be heard. A fuller timbre can make a sound more interesting. Some timbres are perceived as 'full' or 'interesting' and other can be perceived as 'thin' or 'empty'. These terms sound subjective but have everything to do with very detectable overtones.

2.2.3. Distinguishing different sounds

The human hearing is quite good in recognising the difference between frequencies. This recognition is also by determining the relative frequency instead of the absolute frequency. It is said that most people can hear the difference in frequency if this differs by $0,03\%$. Some people are able to even identify and recreate a given musical note without a reference tone. This is a rare ability though and is called having perfect pitch. However, when two sounds with a small difference in frequency are played together then those sounds are perceived together. Mathematically, this means the following;

$$\sin((f + \Delta f)t) + \sin((f - \Delta f)t) = 2\sin(ft)\cos(\Delta ft), \tag{6}$$

with $f + \delta f$ the frequency of the first source and $f - \delta f$ the sound of the second source [7]. The sound that is perceived has a frequency that is the mean of the original frequencies and has an amplitude of $2\cos(\Delta ft)$. Therefore the intensity of the sound fluctuates, this phenomenon is called the beating of a sound.

2.2.4. Consonance and dissonance

Different notes can go well together or clash with each other. Notes that sound good when played together are called consonant. A chord, which is a concordance of multiple notes, that is built up of consonances sounds pleasant and "stable". Conversely, when a chord has dissonance in it it may feel "unstable" or unpleasant and harsh. Obviously whether sound is perceived as pleasant or unpleasant is partly a matter of opinion but there is a clear concept behind consonance and dissonance which is crucial in music and our tuning system. In ancient Greece the Greek scholar Pythagoras already discovered the concept of consonance and dissonance. Pythagoras divided a string into 2 parts by a placing a movable block between the bridges where the string was held. Then the ratio of the frequency of both parts of the string is equal to the ratio of the length of both parts of the string. As a simple rule, when the ratio can be expressed as a fraction of two low integers $R = \frac{m}{n}$ the tones are consonant. In that case the waveform of the higher frequency harmonic lines up with the other harmonic after a few periods. That has the effect that the phase relation of the higher frequency is time dependent on the lower frequency. The resulting waveform is stationary with the repeat time of the wave that is shorter. This makes the tone relatively easy to recognise and to analyse for the brain. Of course there are more tones that have a non-integer ratio between the frequencies. The human brain perceives these tones as less special and harder to recognise. Recent fMRI studies have shown that the human brain in fact has separate centres for analysing consonant and dissonant tones. This would explain why consonant tones are perceived as pleasant and dissonant as unpleasant. This plays a key role in musical theory, as dissonance is used to build op tension in a musical play [8].

2.3. Muscial theory

2.3.1. Standing wave

Now let's go into more detail about how a sound is created by zooming in on string instruments. The excitation of a string sends energy to both sides of the string. That energy travels in the form of a travelling wave. If the string would be infinitely long those travelling waves would move to opposing sides and not meet each other again. However strings are not infinitely long, a string on an instrument is fixed at both sides. When a travelling wave meets a fixed point it is reversed with opposite phase. The travelling waves from both directions meet each other and interfere. When two waves of identical frequency interfere with one another while travelling in opposite directions it creates a standing wave. A standing wave is characterised by locations on the string where the string doesn't move and locations on the string where there is a maximum displacement from the equilibrium. Such a place where the string has no displacement from the equilibrium is called a node and a location with maximum displacement is called an anti-node. The amount of nodes and anti-nodes and their location on the string are specified for each standing wave [9] [10].

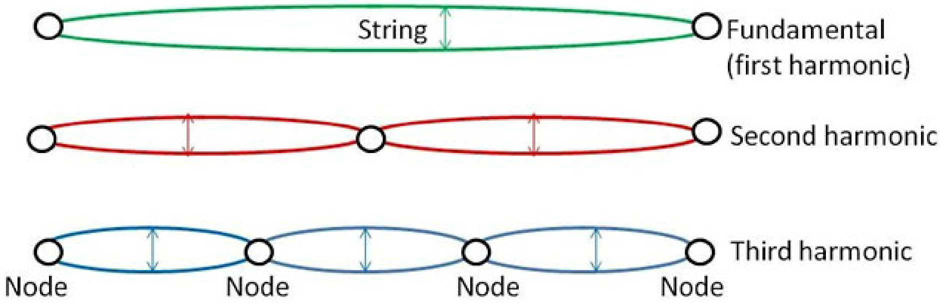


Figure 3: The first 3 harmonics of a string that is bound on both sides.

2.3.2. Harmonics

A mode of a standing wave that is possible on a string is called a harmonic. The simplest type harmonic is the first harmonic and can be seen in figure 3.

For a string without stiffness, the wavelength of the fundamental is twice the length of the string. How the frequency of a harmonic changes with the stiffness of a string is specified in 2.3.3 but for now assume there is no stiffness. In the middle of the string there is a node. The sound that the first harmonic produces is referred to as the keynote of a string. The second harmonic has a node in the middle of the string as can also be seen in figure 3. The length of the string is equal to the wavelength of the standing wave. The frequency is calculated by equation (7).

$$f = v/\lambda \tag{7}$$

Both modes have the same velocity v but the wavelength of the first harmonic is twice as big. Thus the second harmonic has twice the frequency of the keynote. This is called the first overtone. The third harmonic has 2 nodes and gives the second overtone that has thrice the frequency of the keynote. This goes on and on. The keynote is heard and the overtones combined give a sound its timbre. A full set of overtones usually makes a sound more interesting and "full". The same string on an instrument can also create another, higher keynote. As the frequency is calculated with equation (7), this would imply that either speed of sound increases or the wavelength λ decreases. Because the speed of sound is determined by the medium through which it travels it is not possible to alter. Decreasing the wavelength of the sound is possible. This is done by using a smaller part of the string by holding a finger somewhere on the string. By clamping the string it effectively makes the string shorter. The effective wavelength decreases as well and the frequency increases. When a string is used without putting your finger on it it is called an "open" string or an "open" note. In turn when a note is created by clamping the string at a specific location it is called a "fretted" note. On guitars the different locations to create these fretted notes are specified and are called a fret [11].

2.3.3. Vibrations of a stiff string

Thus far we have assumed a string has no stiffness. In that case the frequency of strings would only depend on the length and the different strings of most instruments would all produce a sound of the same frequency. Strings in real life however have a certain stiffness and mass density. When a part of a string is displaced from its equilibrium position then a force applies

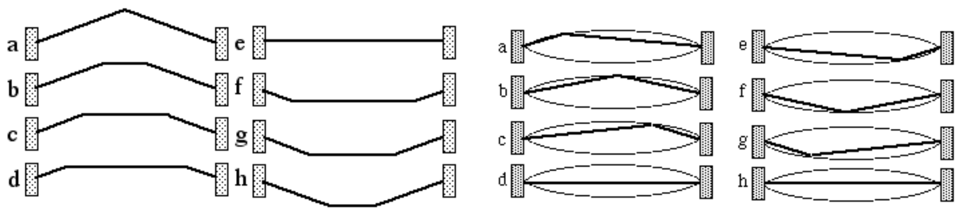


Figure 4: Time analysis of the motion of a string plucked at its midpoints in images a-h to the left. On the right side a time analysis of a string that is bowed at the left fixed point in a downward direction.

to return the string to its equilibrium position. This restoring force is partly due to the tension on the string and partly due to the stiffness of the string. This alters the equation of motion that was used for travelling waves (equation 4). The new equation of motion for a string now equals

$$\mu\frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2} - ESK^2\frac{\partial^4 y}{\partial x^4}, \tag{8}$$

where μ is the mass per unit length, T is the tension, E is the Young's modulus, S is the cross-sectional areas and K is the radius of gyration. This equation is difficult to solve but when the stiffness of the string is sufficiently small then the mode frequencies can be written as

$$f_n = nf_1^\circ(1 + \beta + \beta^2 + \frac{n^2\pi^2}{8}\beta^2), \tag{9}$$

where f_1° is the fundamental frequency without stiffness and $\beta = \frac{2K}{L}\sqrt{\frac{ES}{T}}$. Because of the last term in equation (9) the different frequencies are not exact multiples of each other anymore. However for string instruments with only a few strings this term is sufficiently small and thus this problem is negligible. This problem does arise for the piano and it produces beating as is described in section 2.2.2 between the overtones of a note and the same note in higher octaves. To tackle the problem the ratios in octaves are stretched to slightly above 2:1 [3].

2.3.4. Plucking and bowing

A string is triggered in different ways in different string instruments. Instruments of the violin family are played with a bow and strings of instruments like guitars are plucked. When a string is excited by plucking the vibration (at $t = 0$) can be considered to be a combination of several modes of vibration. For example when a string is struck in the middle it can be seen as a combination of the odd-numbered harmonics [3]. When the string is released 2 waves start to move in opposite directions as can be seen on the left in figure 4. Both waves travel to the fixed point of the string and return with opposite phase. The original modes of vibration at the release of the string quickly get out of phase and the shape of the string changes rapidly after the plucking. As energy transfers to other modes, the sound of the string changes and becomes more mellow. Energy is lost and after some time the sound dies out. Bowing the string on the other hand has a "continuous energy" (as long as the bowing continues). The string does not just vibrate back and forth. Instead the string forms 2 straight lines with a sharp bend at the intersection. This bend travels on the curved path, as can be seen in figure 4 on the right. In this figure the string is bowed in a downwards direction at the left side of the string. The action of the bow on the string is often described as a stick and slip action. The string "sticks" to the horse hairs of the bow by its stationary friction and moves down until the force of the tension in the string exceeds the stationary friction. Then the string "slips", as it slips the dynamic friction is way less than the stationary friction. Until that point in the string made a half cycle, then it sticks to the bow again, thus repeating the process [12] .

2.3.5. Tuning system

Music notes of particular frequencies have been named according to tuning systems. The tuning system that is used in western culture exists of 12 semitones. After those 12 semitones you get the first tone again, only then it is an octave higher. That means that the frequency is twice the frequency of the first tone. Not all of those semitones are used in a scale. Most scales only use 7 musical tones. The reason that those scales have 7 semitones is that those semitones are consonant. The first scale was invented by the Greeks, again by Pythagoras. In his experiment he mostly liked the tones with a frequency ratio of $R = \frac{3}{2}$ or $R = 2$. By dividing or multiplying 1 by $\frac{3}{2}$ and 2 he gets 7 tones in the interval from 1 to 2 (2 not included because this is an octave higher). In that way all the tones are consonant together. A notion here can be made that on a string instrument the first overtone has the ratio $R = 2$ with the fundamental and the second overtone has the ratio $R = \frac{3}{2}$ with the first overtone and are therefore consonant. These scales have been modified to make the scale consequent up to higher octaves. Still the "just scale", otherwise knows as "harmonic tuning" makes use of tones that are consonant together. Nowadays the mostly used tuning system is the Equal temperament. This scale is a slight modification of the just scale and is slightly less consonant in some octaves but works better in higher octaves. The equal temperament is defined by setting the A_4 note to a frequency $f = 440$ Hz. The 4 means that it refers to the musical note A in the fourth octave. The frequency of a note in this octave that is m semitones higher is then simply calculated with $f = 440 \cdot 2^{\frac{m}{12}}$ [13].

2.3.6. The power of timbre

The keynote produced by a string primarily is what we perceive. The overtones produce a rich or thin timbre which makes the sound more interesting. The timbre of an instrument is dependent on the ratio of the overtones. It is also referred to as the tone colour. That name reveals that is has a subjective nature, because how exactly is the colour of a tone defined. The overtones however have a bigger impact on the sound. It makes all the difference between the various string instruments. Some instruments have strings that produce a frequency that is way higher than another instrument, but also when different instruments play a tone of the same frequency their sound is very different. Instruments with sharper sound like a trumpet and a trombone have stronger odd overtones while even overtones tend to make sound warmer and softer. The overtones are also used to identify the keynote. It has been found that the keynote is still heard even when it is removed from the sound. Even when the first overtone is removed as well the human hearing still identifies the keynote, even though the sound is completely different. In case of this psychoacoustic phenomenon the keynote is also called the Tartini tone. This principle is used in choir singing. Different singers can produce a pitch to produce the various overtones and so the illusion of the keynote is created although non of the singers can reach that. This is called overtone singing or throat singing. In non western culture this throat singing is, unintentionally, used in the meditation of Tibetian monks. The overtones are created by monks who sing very slowly and deeply while meditating [14] [15].

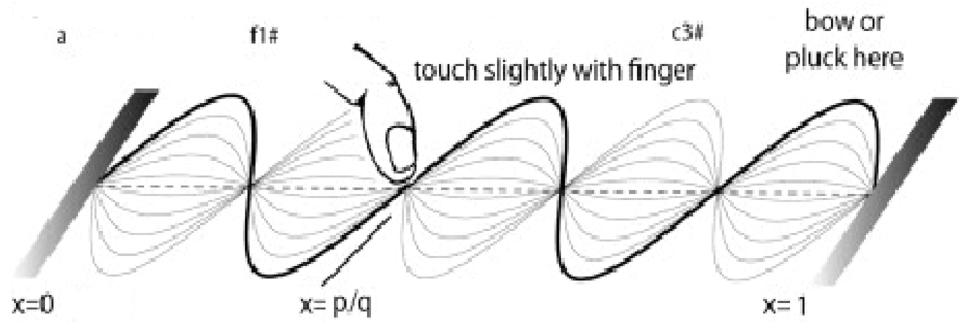


Figure 5: The flageolet technique by slight finger touch is demonstrated on an a string with $f = 220$ Hz. The audible frequency amounts to a frequency $f = 1100$ Hz.

2.3.7. Playing the overtones

So the overtones define a large part of a sound. While playing a string instrument there are ways to change which overtones are contained in the sound. The technique flageolet is common to elicit specific harmonics on all stringed instruments like guitars, pianos and the violin family. Flageolet itself literally means overtone and this technique is used to reach high overtones. A string is divided into 2 parts by the slight touch of the finger. It is important to note here that the string is only touched slightly and is not clamped. The slight touch prevents the string from moving at that specific point and therefore creates an artificial node. When the string is touched at $0 < \frac{p}{q} < 1$ ratio of the string, the overtone with the frequency $f = f_{keynote} * \frac{q}{gcd(p,q-p)}$ is audible. Gcd denotes greatest common divisor. In the example of figure 5 the string is touched at $\frac{p}{q} = \frac{3}{5}$. The string is an a string with frequency $f = 220$ Hz. This means that the overtone with frequency $f = 220$ Hz$\cdot\frac{5}{gcd(3,2)} = 1100$ Hz is audible. If the snare was clamped at that point it would produce a sound of $f = \frac{5}{3} \cdot 220$ Hz ≈ 367 Hz [16].
Besides the flageolet technique, another way of suppressing certain overtones is predicted in the literature. By striking a string at the point 1/n of the string all nth harmonics can be suppressed. This is because in this way an artificial antinode is created at a certain place on the string. All overtones that should have a node in their standing wave at that location are thus suppressed [3].

2.4. String instruments

2.4.1. Acoustic guitar

The modern classical guitar with 6 strings and its most recent ancestor is the vilhuela, an instrument played in the 16th century in Spain. Since then there have been a lot of modification on the instrument, mostly on the body of the guitar.

An overview of a guitar can be seen in figure 6. The strings of the guitar stretch along the neck and are fixed at one point on the head of the guitar. The tension on the strings can be increased by the tuning machine. The other fixed point of the guitar is the bridge. An acoustic guitar can be considered as a system of coupled vibrators. By plucking one string the bridge and the top plate begin to vibrate a little as well. In turn those transfer energy to the air cavity, ribs and the back plate. Sound is radiated efficiently through the air cavity and the sound hole. The strings of a guitar are tuned to E_2, A_2, D_3, G_3, B_3 and E_4 with frequencies

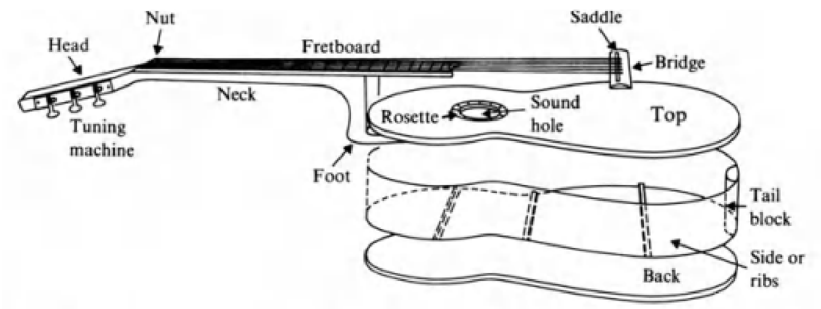


Figure 6: A schematic overview of a classic acoustic guitar

$f = 82, 110, 147, 196, 246$ and 330 Hz. It has quite a sound range as one string is 2 octaves higher than the "lowest" string [17].

2.4.2. Electric guitars

Electric guitars produce sound in a slightly different way. As the name suggests the main difference is that these guitars are electrified. An electric guitar is equipped with an electromagnetic pickup that consists of a coil with a permanent magnet. The steel strings of the guitar vibrates and causes changes in the magnetic flux through the coil. This produces an electric current which is sent to an amplifier. Vibrations of the body of the guitar are relatively unimportant compared to the acoustic guitar. There is also a relatively small energy transfer from the strings to the body of a guitar. The vibrations of a string of an electric guitar is sustained longer. The string-induced change in the magnetic field is not very strong. That is why the coil is susceptible to picking up a 60 Hz hum from electric wires. The humbucking mode is used to counter this problem. It combines 2 coils wound in opposite direction to cancel the effect of electric wires. An electric guitar has multiples coils and most can switch which are used or if the guitar uses single coil or humbucking mode. Pickups that are farther from the bridge of the guitar are more sensitive to higher harmonics and pickups that are closer to the bridge produce a stronger fundamental. An important type of electric guitar is the bass guitar. The bass guitar has 4 strings with the notes E_1, A_1, D_2 and G_2 with frequencies $f = 41, 49, 73$ and 98 Hz [3].

2.4.3. Violin

The origin of the violin is unclear, the most probable explanation is that the instrument gradually developed from other string instruments throughout the Middle Ages. They developed in two families, "violins of the leg" and "violins of the arm". The violin has been a subject for scientific research for ages. In the 1800s Herman von Helmholtz discovered the stick-slip motion as is discussed in 2.3.7. He deduced the sawtooth waveform of the string displacement which is now commonly referred to as the Helmholtz motion. The violin has 4 strings that are very tightly stretched. The total tension on the four strings is 220 N which causes a strong force downward on the bow of 90 N. This is approximately equivalent to a little more than a weight of 9 kg. The strings produce the notes G_3, D_4, A_4 and E_5 with frequencies $f = 196, 294, 440$ and 596 Hz. As the vibrational behavior of the body plays a key role in the sound quality of most string instruments, so it does for the violin. In the last 50 to 60 years it has been found that the normal modes of a violin are mainly determind by the coupled motion of the top and the back plate, together with the air it encloses [3].

2.4.4. Viola

The viola is tuned a perfect fifth below the violin (with the notes C_3, G_3, D_4 and A_4. It is the alto member of the violin family. Nevertheless it is a very different instrument, not a scaled-up violin. The dimensions of the viola are 15% greater and its principal resonances lie 20 to 40% below that of a violin. The main resonances of the instrument of the main body and the air lie between the open snares instead of at the same frequency like the violin. The lowest notes played on the D and A strings (the strings with the higher frequency) produce more power than those same notes played on the C and G string. That is because of that the different frequency of the body resonance [3].

3. Fourier analysis

3.1. Fourier series

The basic idea behind a Fourier series is that every function that is periodic can be represented by a series of sine and cosine functions. When a certain function has a period $T = 2\pi$, then it is sufficient to represent the behaviour on the interval $[-2\pi, 2\pi]$. More general, for functions with periodic boundary conditions on the interval [-L,L] the Fourier series S(x) is obtained by using the following expression;

$$S(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi x}{L}) + \sum_{n=1}^{\infty} b_n \sin(\frac{n\pi x}{L}) \tag{10}$$

with constants a_n and b_n this Fourier series doesn't necessarily converge. However, when this series converges then the Fourier coefficients a_n and b_n are found by using the orthogonality relation between the sines and cosines. As a result a_n and b_n are calculated using the following relations:

$$\begin{aligned} a_0 &= \frac{1}{2L} \int_{-L}^{L} f(x)dx, \\ a_n &= \frac{1}{L} \int_{-L}^{L} f(x) \cos(\frac{n\pi x}{L})dx, \\ b_n &= \frac{1}{L} \int_{-L}^{L} f(x) \sin\frac{n\pi x}{L})dx. \end{aligned} \tag{11}$$

To be able to use the convergence of this Fourier series to a certain function the convergence theorem 1 is used from Haberman [18]. This theorem is also often referred to as Fouriers theorem.

Theorem 1 *If f(x) is piecewise smooth on the interval $-L \leq x \leq L$,*
then the Fourier series of f(x) converges to
1. to the periodic extension of f(x), where the periodic extension is continuous;
2 to the average of the 2 limits,

$$\frac{1}{2}[f(x^+) + f(x^-)]$$

where the periodic extension has a jump discontinuity

A function $f(x)$ is pieciewise smooth at an interval $-L \leq x \leq L$ when the interval of the function can divided into finitely many intervals on which f and $\frac{df}{dx}$ are continuous. The periodic extension of $f(x)$ is obtained by extending the function $f(x)$ at interval $-L \leq x \leq L$ with $f(x) = f(x + n \cdot 2L)$ and $n \in \mathbb{Z}$. If $f(x)$ is a periodic function with period $2L$ then these it is equal to its periodic extension. $f(x^+)$ and $f(x^-)$ denote one-sided limits which are defined as

$$f(x^+) = \lim_{x' \uparrow x} f(x'), \quad f(x^-) = \lim_{x' \downarrow x} f(x'). \tag{12}$$

The Fourier sine series is the Fourier series where the coefficients $a_n = 0, n \in \mathbb{Z}$. By an extension of theorem 1, for piecewise smooth functions $f(x)$, the Fourier sine series of $f(x)$ is continuous and converges to $f(x)$ for $0 \leq x \leq L$ if and only if $f(x)$ is continuous and both $f(0) = 0$ and $f(L) = 0$. As a string is also fixed at both ends, the Fourier sine series is used to represent the standing waves on a string.

3.2. Fourier transform

In a sound wave it is unknown beforehand which frequencies of waves are present in the signal and what the relative amplitude is of the frequencies that are present. To find these frequencies and their relative amplitude a more generalised version of the Fourier series is used, known as the Fourier transform. The input of the Fourier transform is a function f which denotes the measurable signal in the time-domain. The Fourier transform equals

$$C(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx. \tag{13}$$

The output $C(\omega)$ of the Fourier transform is a function in the frequency domain and gives a continuous spectrum of the frequencies contained in $f(x)$. $C(\omega)$ usually consists of complex values. The absolute value of the function at a certain frequency represents the amplitude of that frequency and the argument represents the phase of that frequency. The Fourier transform in equation (13) is possible when the input of the Fourier transform is a continuous function $f(x)$. Although the original sound signal is a continuous, the sound is measured with discrete points. Thus the signal that is measured is a discrete function $f(x)$. The discrete fourier transform (DFT) is calculated as with

$$\mathcal{F}(x) = \sum_{n=0}^{N-1} x(n) e^{-i\omega nk} \tag{14}$$

where $x(n)$ are the discrete points of the input signal, $\mathcal{F}(x)$ is the Fourier transform and $\omega = \frac{2\pi}{n}$.

3.3. Fast Fourier Transform

The discrete Fourier transform could have been essential in the analysis of sounds. This method has the problem that its computation takes too much time. DFT analysis only became an important research tool after the introduction of the Fast Fourier Transform (FFT) in 1965 by Cooley and Tukey. This algorithm decreases the amount of operations required drastically [19]. Where the DFT uses $\mathcal{O}(2N^2)$ computations for a signal of N points, the FFT uses $\mathcal{O}(2Nlog_2N)$ computations. For small N this difference is not important but for signals with a lot of samples the amount of computations is dropped with several orders of magnitude. To get an idea of the difference between the amount of computations, when $N = 60000$ (which is used in this research) this implies that the DFT needs 3750 as many computations as the FFT. The FFT approximates $C(\omega)$ by taking equidistant points from the input signal f [18]. When N is a power of 2 the discrete Fourier transform can be computed by an FFT using the Danielson-Lanczos lemma. When N is not a power of two, a transform can still be performed on sets of points corresponding to the prime factors of N. When N is not a power of 2 the computation is however slowed down. Later more complicated fast Fourier transforms have been introduced that are even faster then the Cooley-Tukey FFT. The basic idea of their FFT is to break up the transform of length N into two transforms of length $\frac{N}{2}$. This uses the identity that

$$\begin{aligned}\sum_{n=0}^{N-1} a_n e^{-2\pi i n \frac{k}{N}} &= \sum_{n=0}^{\frac{N}{2}-1} a_{2n} e^{-2\pi i (2n)\frac{k}{N}} + \sum_{n=0}^{\frac{N}{2}-1} a_{2n+1} e^{-2\pi i (2n+1)\frac{k}{N}} \\ &= \sum_{n=0}^{\frac{N}{2}-1} a_n^{even} e^{-2\pi i (2n)\frac{2k}{N}} + e^{-2\pi i \frac{k}{N}} \sum_{n=0}^{\frac{N}{2}-1} a_n^{odd} e^{-2\pi i (2n+1)\frac{2k}{N}}\end{aligned} \tag{15}$$

which is sometimes referred to as the Danielson-Lanczos lemma [20].

3.4. Nyquist frequency

As sound of various frequencies is measured by instruments with a finite sampling rate it is important to note that is not possible to measure a spectrum of infinite frequencies. With an instrument with a certain sampling rate frequencies are only measured up until the Nyquist frequency. The Nyquist frequency is equal to half of the sampling frequency. Signals above this frequency are misinterpreted as a signal of a lower frequency. This phenomenon is called aliasing. In figure 7 it is illustrated how aliasing occurs. The blue line is a signal with a

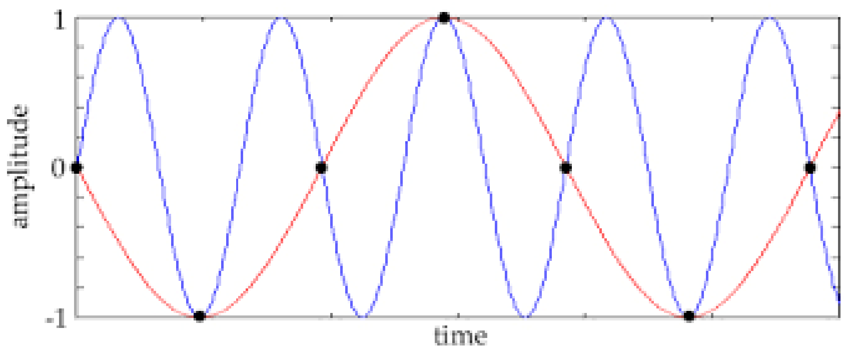


Figure 7: Different waveforms are illustrated in the time domain. The blue dots represent moments in time when the signal is sampled.

frequency that is higher than the Nyquist frequency. As it is sampled less than twice per period, there is no difference measured between the red and the blue wave at the sampled points. The blue waveform is misinterpreted as the red waveform. In order to prevent aliasing the sampling rate has to be higher than twice the maximum frequency. Of course it is not known beforehand which frequencies are to be measured. Another way is to use an anti-aliasing filter which filters out the frequencies that are higher than the Nyquist frequency. Because of the same reason enough samples have to be taken to make out the difference between different frequencies to avoid aliasing [21].

This principle explains why the industry standard, a standard that is used for example on CD's, is a sampling rate of 44100 Hz, as the human ear can hear frequencies up to around 20 kHz.

4. Experimental Method

Now we have a greater understanding about the concepts of sound and how these are created and perceived. In this chapter the experimental method of this research will be discussed. First up it will be discussed how the sounds of various string arguments are measured. This data will be stored in wav files and will be read out in jupyter notebook. The data will be analysed and on the basis of that analysis a filter is created.

4.1. Experimental setup

In order to compare a broad scala of different instruments the sounds of five string instruments were measured, namely the acoustic guitar, electric guitar, bass guitar, violin and viola. From all those instruments multiple notes were measured, open and fretted notes. The open strings of these instruments cover a wide spectrum of notes. To compare the different tones on all instruments the notes of G_3, B_3 and E_4 with frequencies $f = 196, 246$ and 330 Hz are measured on almost all instruments. Not on all instruments, because unfortunately not all instruments can reach those frequencies. The bass guitar can just barely reach the B_3 note. The sounds of all string instruments but the electric guitar are measured by the same the device; the audio system of a Pocophone f1. The sounds are recorded on two channels (stereo) and with a sample width of 2. The sample frequency is set to $f = 48$ kHz.

The bass guitar is a Fender jazz bass. The bass guitar is tuned using an electronic tuner. The length of the strings vary between 86 and 90 cm. An amplifier is used with all the filtering knobs (bass, mid, treble) set to neutral. The temperature in the room is around $17°C$. The bass guitar is plucked at the "normal" location between the neck and the bridge, in the middle of the string and at a third and fourth part of a string, taken from the bridge.

The acoustic guitar has a Parlor shape. The strings have a length of 63 cm and are tuned with an electronic tuner. The temperature in the room is around $22°C$. Like the bass guitar the strings are plucked at various locations; Between neck and bridge, in the middle of the string and at a third and fourth part of the string taken from the bridge. There are also measurements taken where the flageolet technique is used.

The sound of a violin is measured from a full measure violin. The strings are tuned using an electronic tuner. The temperature in the room is around $18°C$. Sound was measured that was created by bowing the strings and also by plucking the strings. The effect of the flageolet technique is measured for some strings.

The viola was tuned beforehand. The temperature in the room is around $18°C$ as well. Sound is measured from every string by bowing and by plucking.

4.1.1. Setup electric guitar

The electric guitar Fender Lead III is used in the experiment. The length of the strings are 645 mm. There are measurements with two different pickup elements. One of them is centred 35 mm from the bridge and the other is centred 140 mm from the bridge. The coils rows are 18 mm apart. Both pickup elements can either be switched to humbucking mode or single coil. As measurements equipment the guitar is connected to a linear mixing desk directly. No amplifier is used and all the knobs of the track (filters) are set to neutral. The sound is recorded with Ardour with a Juli@ soundcard on a linux PC on a single channel with sample width 3. The sample frequency is set to $f = 96$ kHz. The guitar is tuned using an electronic tuner. At the time of measuring the temperature in the room is about $12°C$

4.2. WAV files

WAV (or WAVE) files are one of the older type file formats that are used today and its file extension is .wav . It is a subset of Microsoft's Resourche interchange File Format (RIFF) and is specified for storing digital audio files. With WAV, the value of each sample is directly encoded as its bit value, varying between -1 and 1. The data stored in WAV files is not compressed. In that way no information is lost of the original audio, however the files are very large. The header of a WAV file is 44 bytes long and contains information about the file. The parameters of a WAV file are [22]:

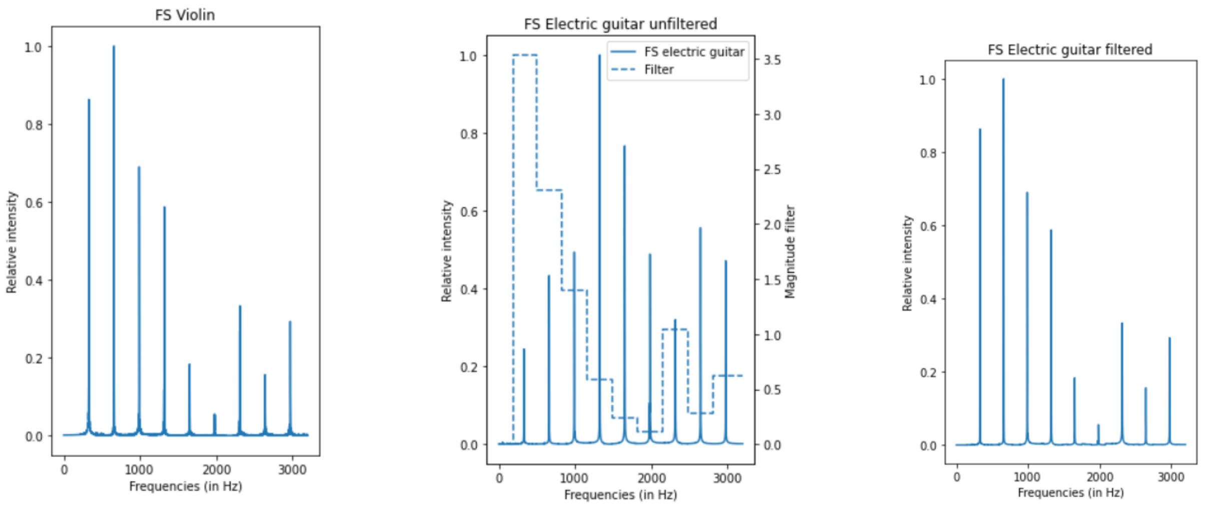
- Sample rate or frame rate: This specifies how many samples there are in the time frame of one second. Industry standard is 44100 Hz but 48 kHz and 96 kHz are popular as well.
- nchannels; This specifies the amount of channels in which the data is stored. 1 is mono and 2 is stereo. When there are n channels the data is not stored in separate arrays but in one array where each element is a list of n elements.
- Sample width: This the number of bytes that is used to store the information of one sample. A byte contains 8 bits which in turn have 2 possible states. Thus n bytes have 2^8n possible states. The values it contains vary between -1 and +1 and thus n bytes are specified with an accuracy of 2^{8n-1}.
- The total number of frames that are stored in the file.

4.3. Analysis

The data is extracted from the WAV file and read out in jupyter notebook with python 3. To analyse this data the numerical algorithm FFT which is explained in 3.2 is used. As input for this algorithm around 40 to 60 thousand samples are given, which is more than twice the frequency range that we are interested in (see 3.4). This algorithm gives the Fourier spectrum of a sound. The peaks in this spectrum are the frequencies present in the sound. These peaks are located and used for further analysis of the signal and to create a filter. To locate the small peaks of frequencies that are barely present the original peaks are magnified by a factor of 10^4.

4.3.1. Creating the filter

The filter is created based on the Fourier spectrum of the sound that will be filtered and on the Fourier spectrum of the sound that is imitated. As it is easier to explain with a concrete example, take the case where a filter is created to change the sound of an electrical guitar to the sound of a violin. The peaks in the Fourier spectra of both sounds are located. Every peak in the spectrum of the guitar is matched with a peak from the spectrum of the violin, if there is one of course. If there is no peak of that frequency in the spectrum of the violin, the magnification factor of the filter is 0. If a peak is found of that frequency in the spectrum of the violin, The magnification factor $M = \frac{I_{peakviolin}}{I_{peakguitar}}$.



(a) (b) (c)

Figure 8: A part of the normalised Fourier spectrum is illustrated in plot (a), in plot (b) part of the normalised Fourier spectrum of an electric guitar is shown. The dashed line is the filter that is based on the spectra. (c) is the filtered Fourier spectrum of an electric guitar.

This filter can be seen in figure 8 In figure 8a the Fourier spectrum of the violin is illustrated. The intensities of the peaks are normalised. In figure 8b the Fourier spectrum of the electric guitar is illustrated. As described on the basis of both peaks the magnifying factors are calculated. The magnification factors form the filter, which is the dashed line in the same plot.

4.3.2. Filtering

Finally after the filter is created the original sound of the electric guitar is filtered. The filter consists of magnification factors, each peak in the Fourier spectrum of the original sound has its own magnification factor. When filtering the signal, the peaks of the overtones of the original sound are multiplied by their magnification factors, thus creating peaks of different heights. The result of these multiplication is the new filtered Fourier spectrum of the electric guitar and is illustrated in figure 8c. This modified Fourier spectrum can be transformed with the inverse Fourier transform (IFFT). This returns a sound in the time domain that has as many samples as the original unfiltered sound signal (and is therefore of the same time duration).

The sound that is being imitated however changes rapidly changes over time, so the filter should change rapidly as well. For that reason there is a multitude of filters used together for dynamic and time dependent filtering. As the creation of a filter requires a lot of samples, it is not enough to take the next 40 to 60 thousand samples. As some original sounds are only audible for 150 thousand samples, this would imply that only 3 different filters could be used. To solve this problem filters are created with a moving window.

The operation of the moving window is illustrated in figure 9. A filter is created as described and filters an X amount of samples. These samples are from a "window" of samples. An IFFT is used to transform these samples back to the time domain. A few samples dX are taken after the IFFT. Those are collected and are part of the new signal. After this whole operation the original window is moved dX samples and the procedure repeats itself. This way a large number of filters can be used and every few samples are filtered with a new filter.

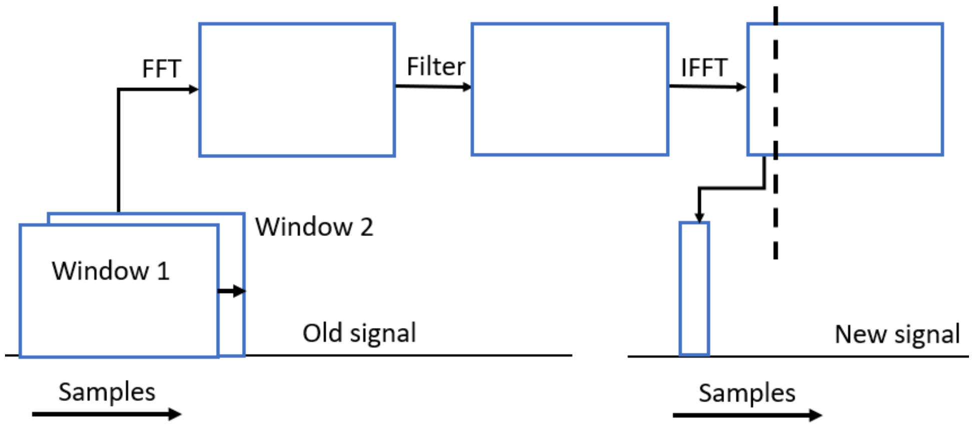


Figure 9: A systematic overview how a moving window is used to produce a new signal.

5. Results and discussion

In this chapter, the different measurements of the various stringed instruments are analysed. Because of each instrument the sounds of several strings are measured, it is not possible to show all the different recordings and how they change over time. Therefore, certain effects are analysed and how they occur on some instruments. To understand the aspects of a sound, we will first look in detail at the Fourier spectra of various sounds and into effects that have an effect on these.. The spectra of the sounds of all instruments will be compared for the tones they can all produce. Then the difference between open notes and fretted notes and the difference between bowing and plucking a string will be analysed. The flageolet technique and plucking a string in different locations will be examined and how these affect the overtones. Finally the filters of these different sounds are examined.

5.1. Analysis of Fourier spectra

5.1.1. The Fourier spectra of string instruments

First, the Fourier spectra of all sounds are compared. Since the bass guitar has a sound with a much lower frequency, this instrument does not reach the E_3 tone of frequency $f = 330$ Hz. However, all instruments do reach the tones B_3 ($f = 246$ Hz) and G_3 ($f = 196$ Hz). The Fourier spectra of all instruments with keynote B_3 are illustrated in figure 10. As the bass guitar had strings of a lower frequency it plays a fretted note. The violin and the viola both play fretted notes as well because they don't have a B string. These fretted notes are played on the G_2, G_3 and G_3 string, respectively.

In figure 11 the Fourier spectra of all instruments are illustrated of the G_3 note. These sounds are (except for the bass guitar) all produced from open strings. As can be seen, the spectra of different sounds are vastly different. The acoustic guitar had a wide variety of overtones that play a role in the timbre of the sound.

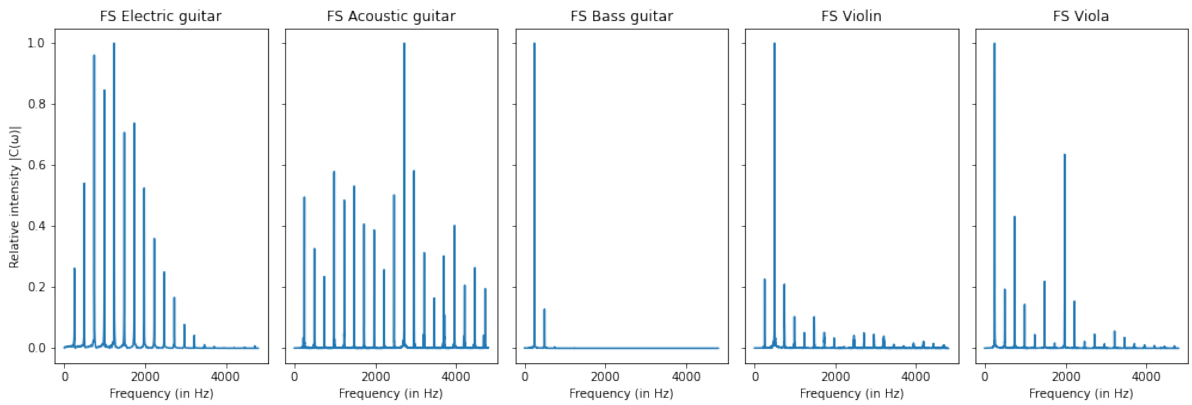


Figure 10: The normalised Fourier spectra of different string instruments with keynote B_3 with frequency $f = 246$ Hz. A frequency range of $f = 0 - 5$ kHz is shown. Electrical guitar with pickup 1 on humbucking mode. The bass guitar, violin and viola play fretted notes on the G_2, G_3 and G_3 string, respectively. The other instruments play open notes. The guitars are plucked and the violin family bowed.

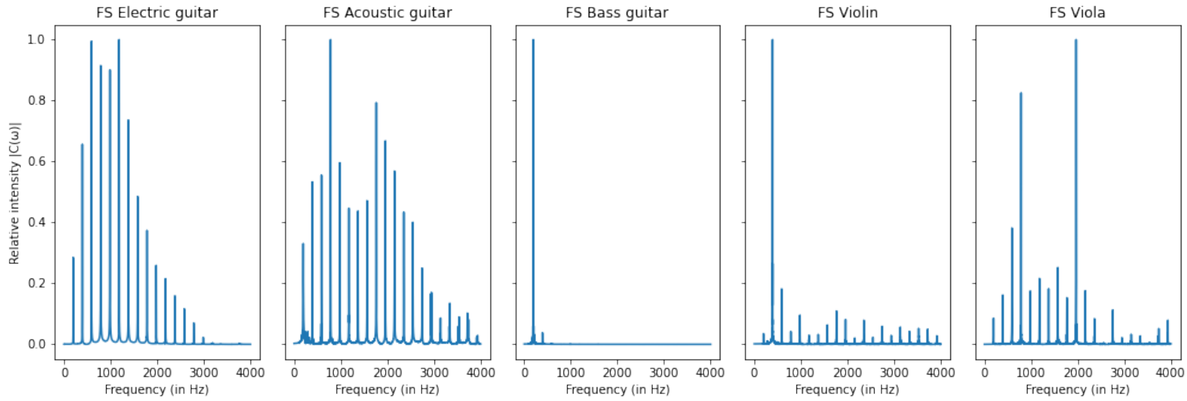


Figure 11: The normalised Fourier spectra of different string instruments with keynote G_3 with frequency $f = 196$ Hz. A frequency range of $f = 0 - 4$ kHz is shown. Electrical guitar with pickup 1 on humbucking mode. The bass guitar plays a fretted note on the G_2 string, the other instruments play open notes. The guitars are plucked and the violin family bowed.

Except for one overtone that is more dominant, most overtones have around the same intensity. This gives a full timbre. In the sound of the electric guitar also multiple overtones play a role. An important difference here is that the sound of the acoustic guitar also contains a lot of overtones of higher frequencies whereas the relative intensity of the overtones of the electric guitar rapidly decays. Above a frequency of 3 kHz the overtones play a small to insignificant role. The bass guitar on the other hand has almost no overtones. The spectrum of the B_3 note has one overtone that plays a role and in the spectrum of the G_3 note even the first overtone is barely visible. The entire sound consists almost exclusively of the fundamental frequency. The violin and the viola have more overtones that contribute to the timbre of the sound, although less overtones than the spectrum of the electric and acoustic guitar. The violin is dominated in the G_3 and B_3 note by the first overtone. Other overtones, however present, have a relative intensity below the $0,2$. The spectrum of the viola is most alike the spectrum of the violin. Difference between the spectra is that overtones of the viola make a greater contribution to the timbre and have a greater relative intensity.

5.1.2. Open versus fretted notes

When comparing the sounds of different instruments, it is important to bear in mind that an instrument does not always produce a certain note in exactly the same way. Striking a string slightly differently or bowing in a different way alters the sound. For instance, a note on an instrument can sound very different on an open string than on a closed string. Thus it is important to take the difference between open and fretted notes into account. Although these different methods all produce the same keynote the tone colour varies.

This difference can be seen clearly in the Fourier spectra of the notes. As an example, figure 12 shows the Fourier spectra of the E_3 note as played on an acoustic guitar. In the first plot this note is played on an open string, next it is played on the closed G string and next to that on the closed B string. The spectrum of the open string has a lot of overtones with strong relative intensity. It also has overtones of higher frequencies that the spectra of the fretted notes. The spectra of these fretted notes also vary a lot. The E note on the G string mostly looks like it has

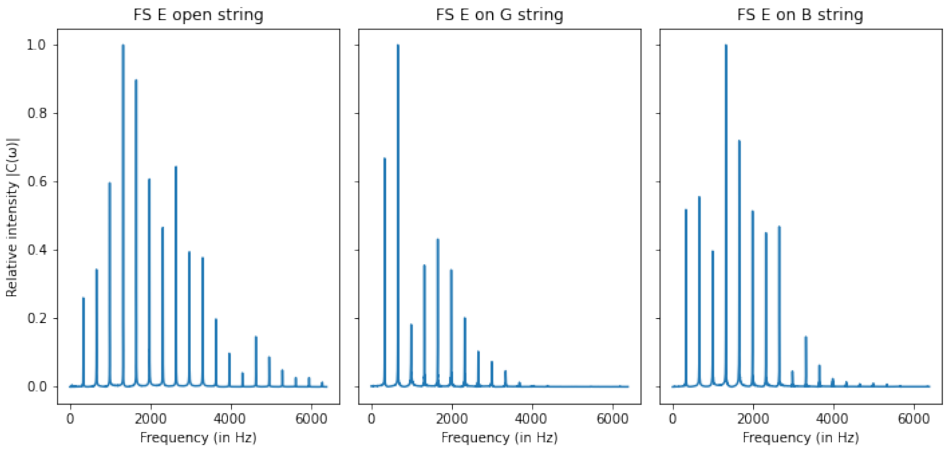
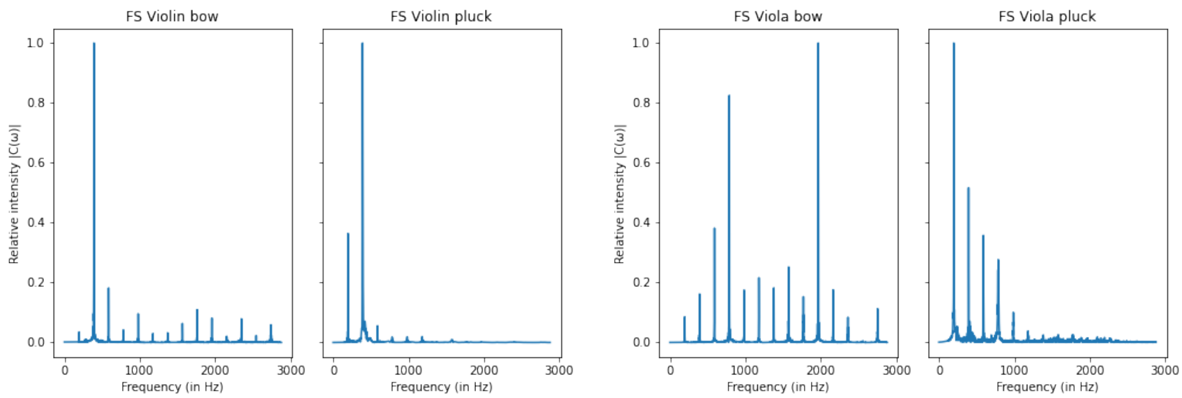


Figure 12: The normalised Fourier spectra of the acoustic guitar. The note E_4 of frequency $f = 330$ Hz is played open on the E string, fretted on the G string and fretted on the B string. A frequency range of $f = 0 - 6$ kHz is illustrated.

a thinned out timbre relative to the open note. The fretted note has close to no overtones above 4000 Hz. The fretted note on the B string has a fuller timbre. It is noticeable from the spectrum that there are overtones with greater relative intensity. Also there are peaks to be seen above the frequency of 4000 Hz, although these are relatively small.

5.1.3. Plucking and bowing

The way a string is struck can also make a difference to the sound of an instrument. Similarly, whether a string is bowed or plucked has a great deal of influence on the sound. Bowing a string provides it with a continuous source of energy. As only the violin and the viola are bowed, this effect will be studies on those instruments. In figure 13 the different sound spectra of plucking and bowing is illustrated for both the violin and the viola.



(a) (b)

Figure 13: The normalised Fourier spectra of the violin in (a) and the viola in (b) are illustrated when the instruments are plucked and bowed at keynote G_3 with frequency $f = 196$ Hz. A frequency range of $f = 0 - 3$ kHz is illustrated.

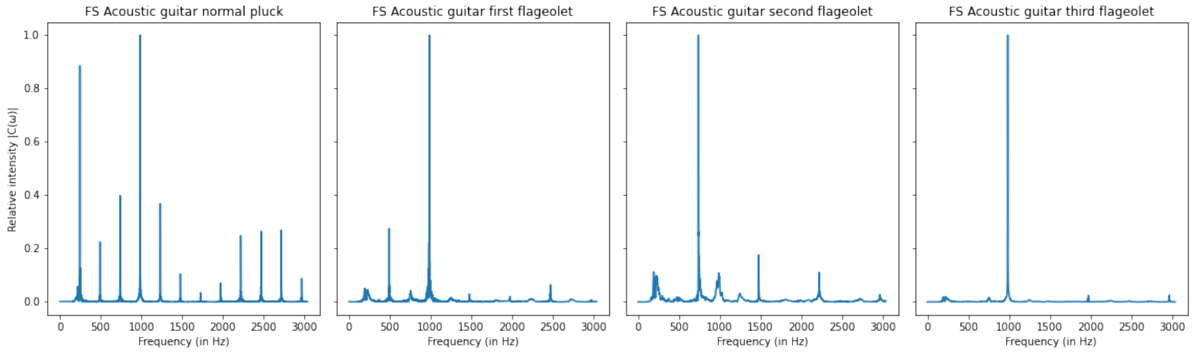


Figure 14: The normalised Fourier spectra of the acoustic guitar when it is plucked and when it is plucked in the first 3 flageolet modes at keynote B_3 with frequency $f = 246$ Hz. A frequency range of $f = 0 - 3$ kHz is illustrated.

The note that is displayed is the same for all these figures and is G_3 ($f = 196$ Hz). For the violin the difference between plucking and bowing is the greatest as can be seen in figure 13a. Where the timbre of the bowed violin has a lot of overtones which are relatively small but present, the timbre of the plucked violin shows virtually no overtones of higher frequencies. In figure 13b the same effect can be seen for the difference between the bowed and plucked viola. In the sound spectrum of the viola, more overtones are visible, but there are also fewer overtones there than in the sound spectrum of the bowed viola. Higher overtones exist but are relatively small compared to the overtones of a bowed viola. The difference between bowing and plucking in both instruments is probably due to the continuous influx of energy when bowing a string. Higher frequencies that lose their energy to distortion from lower frequencies remain present. To add to this, the sound that is produced by plucking a violin or viola quickly dies out compared to the guitars. This could be due to the higher tension on the strings or because less vibrational energy is lost to the surroundings or the body of the instrument.

5.1.4. The flageolet technique

As described earlier, it is possible to influence exactly which overtones you play. The first possible way is to use the flageolet technique. How this technique works exactly can be read in section 2.3.7. Here we show the effect that this technique has on the acoustic guitar and on the violin.

In figure 14 the effect of the flageolet technique on the acoustic guitar is illustrated on note B_3 (246 Hz). In the first plot the normal Fourier spectrum is shown where this technique is not used. In the second plot the first flageolet mode is depicted. The spectrum is a bit messy with some small but broad peaks. These broad peaks are also present in the second flageolet mode. The flageolet essentially gives the string a new fundamental frequency. The first mode would then have a fundamental frequency of $f = 492$ Hz. The frequencies that are multiples of that frequency should be the most visible. In the second flageolet mode the multiples of frequency $f = 738$ Hz are clear in the spectrum. However the peaks at $f = 246$ Hz and $f = 984$ Hz are also still present. Thus can be concluded that the other frequencies on the acoustic guitar are repressed but not completely suppressed. In the last plot the third flageolet mode is illustrated, this spectrum has a clear peak at $f = 984$ Hz. The multiples of that frequency are visible

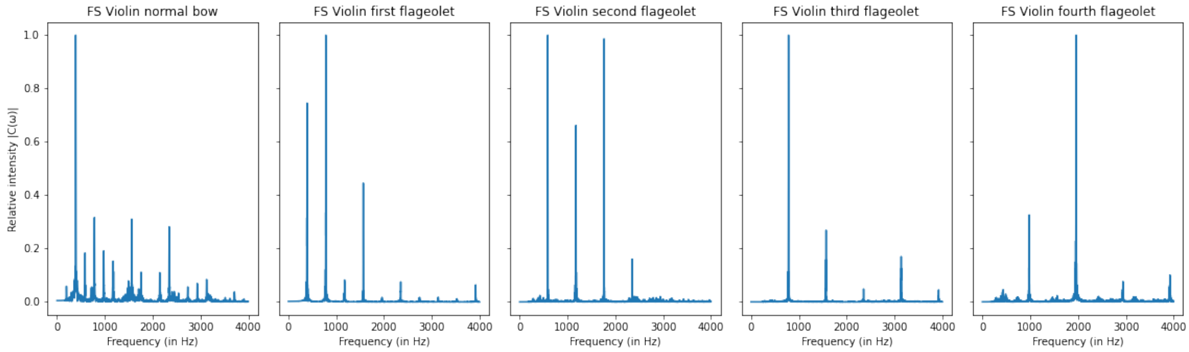


Figure 15: The normalised Fourier spectra of the violin when it is bowed and when it is bowed in the first 4 flageolet modes at keynote G_3 with frequency $f = 196$ Hz. A frequency range of $f = 0 - 4$ kHz is illustrated.

but small. As in the other plots, there is some distortion of small peaks around overtones that should have been suppressed. These are again respressed but not totally suppressed. A possible explanation for this is that the energy of the vibrations of the string is transferred through the bridge to the body of the acoustic guitar. This transfer of energy is relatively large for an acoustic guitar. because the body of the guitar vibrates more, other modes of the guitar can also be triggered to a certain extent. Or perhaps some frequencies, like $f = 1000$ Hz are slightly preferred by the sound box of a guitar.

In figure 15 a bowed violin and the first 4 flageolet modes are shown for the note G_3 ($f = 196$ Hz). Where the spectrum of the acoustic guitar that is plucked without the flageolet technique has sharp peaks, the spectrum of the violin is messy and gives and shows distortion at the bottom of the peaks. At the same time, the peaks in the spectra of the flageolet technique are very sharp and well defined. The continuous influx of energy by the bowing makes it possible to determine the exact location where the string should be touched for the maximum effect of the flageolet technique. The first flageolet mode essentially looks like the spectrum with fundamental frequency $f = 392$ Hz. Even the multiples of this fundamental frequency without large peaks have a small point that is visible. However this spectrum has some overtones of higher frequency which are clearly present and audible in the sound. The second flageolet mode has four clear peaks at frequencies that are multiples of $f = 588$ Hz. After the fourth peak is some small distortion. The third mode has clear peaks as well and the same applies for the last mode. However the last mode has some distortion throughout the spectrum. These spectra give the impression that they are the Fourier spectra of a note with a different, higher, fundamental tone. This especially has a large effect on the perception of the sound. The sound that is perceived is also "higher" with each flageolet mode, in accordance with the new perceived fundamental frequency. The continuous influx of energy is probably the main reason that the right position for the flageolet technique can be found more accurately than with the acoustic guitar. Another reason may be that the shorter strings have less deviation from the equilibrium point. This may also contribute to finding the exact location more precisely.

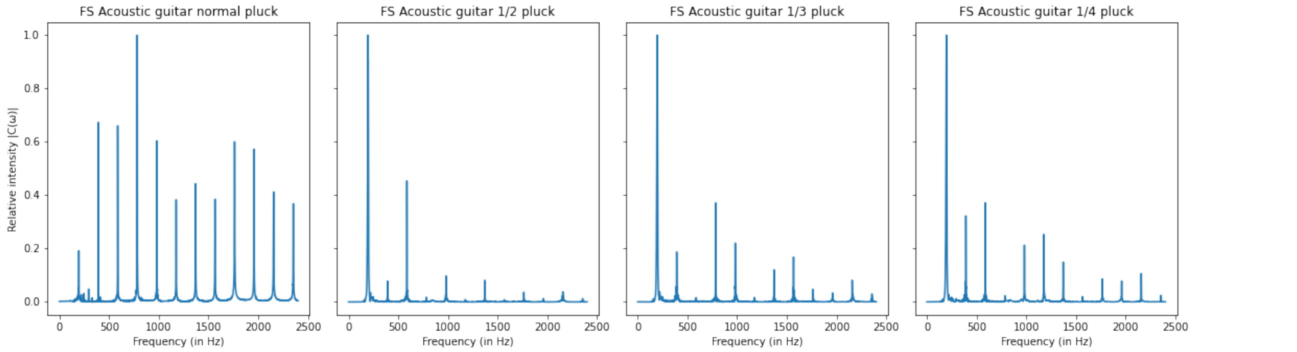


Figure 16: The normalised Fourier spectra of the acoustic guitar when it is plucked next to the bridge, in the middle of the string and at a 1/3 and 1/4 of the string. The keynote G_3 with frequency $f = 196$ Hz. A frequency range of $f = 0 - 2.5$ kHz is illustrated.

5.1.5. Locations of plucking

There is also another way in which playing the guitar can affect the overtones of the sound produced, as described in section 2.3.7. The string is also sensitive to the location where it is struck. The effect is greatest when the string is struck at location $\frac{1}{n}$, $n \in \mathbb{Z}$.

To analyse this, the Fourier spectra of the sounds of an acoustic guitar and a bass guitar are included, with the instruments being played at specific points. The Fourier spectra of the acoustic guitar with keynote G_3 with frequency ($f = 196$ Hz) is shown in figure 16. The "normal" Fourier spectrum has various overtones with similar peaks. In the second plot mostly the fundamental frequency and the second overtone dominate the spectrum. It is clear in the spectrum that all the even harmonics are close to cancelled. Only the first overtone (second harmonic) still has a clear peak. The uneven peaks are small compared to the fundamental frequency but notable. The effect is perhaps more clear to see in the third plot. Groups of two peaks repeat each other with some interval as every third peak is almost cancelled. Only the ninth harmonic still has an undeniable peak. The suppression of every third peak is quite clear. The last plot also clearly show the same effect. There every fourth peak is cancelled. Groups of three peaks follow each other on some interval here as well.

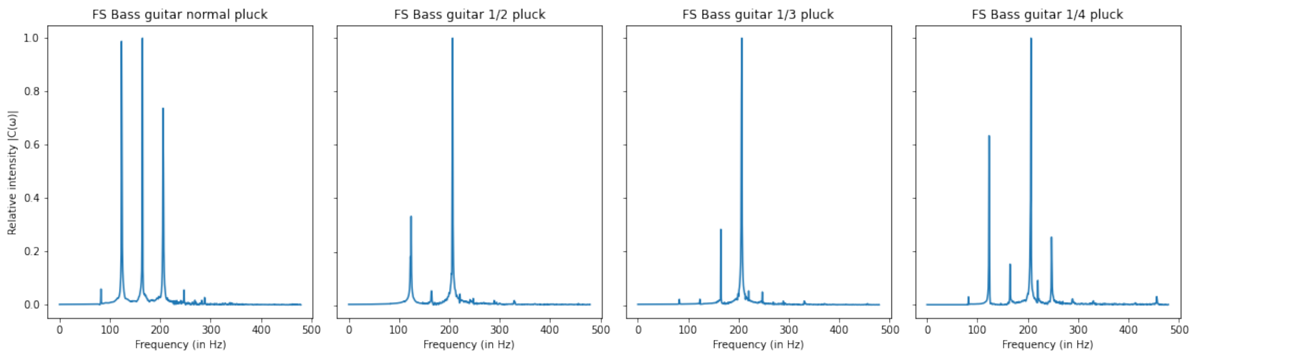


Figure 17: The normalised Fourier spectra of the acoustic guitar when it is plucked next to the bridge, in the middle of the string and at a 1/3 and 1/4 of the string. The keynote E_1 with frequency $f = 41$ Hz. A frequency range of $f = 0 - 500$ Hz is illustrated.

The effect of the locations where a string is struck is also clearly illustrated by the example of the bass guitar in figure 17 with keynote E_1 ($f = 41$ Hz). The Fourier spectrum of the instrument only has a few peaks. As can be seen in the first plot, three peaks are clearly visible and on both sides of those peaks a small peak can be detected. In the second plot all the even harmonics are suppressed. The large peak in the middle is still detectable but the small peaks on both sides are complete gone. In the third plot all the third harmonics are suppressed and thus the large peak at $f = 123$ Hz is barely visible. In the last plot the effect is also visible but the fourth harmonic at $f = 164$ Hz is still visible. The peak is repressed less than it was when all the second harmonics were repressed in the second plot.

An important observation to make about the Fourier spectrum of the bass guitar is that the fundamental frequency of the string is missing. Despite the fact that this frequency is missing, it is still perceived, in contrast to the effect in the flageolet technique. This also clearly shows that the overtones are very decisive for a sound. In addition to determining the timbre, they can also determine the fundamental of a sound.

These results combined clearly show that the location where a string is struck has a great influence on the overtones present in the sound. Probably these are some extreme cases where certain overtones are almost completely suppressed. As described in the theory, this is because certain harmonic vibrations together approach the original deviation of the string. By striking the string at the point $\frac{1}{n}$, all nth harmonic vibrations are then exactly suppressed. According to Fourier's theory (1) however, any function can be approximated by a combination of sines. This means that also every initial deflection of the string can be obtained with a certain combination of harmonic vibrations, regardless the location where the string is struck. This means that the location where the string is struck everywhere has a great influence on the harmonic vibrations in the sound. Striking the string at a different location results in a different set of harmonics and therefore produces a sound with a different timbre.

A few aspects that influence the sound of stringed instruments have now been discussed. As has been shown, all these aspects influence the sound produced. Therefore it is not possible to speak about one exact sound that a certain string from an instrument with a certain fundamental tone produces. Sound of an instrument is dependent on how and where a certain note is produced.

5.2. Filters

By now, the aspects in Fourier spectra that influence the sound have been analysed. These aspects are the building blocks of the signal. By using the knowledge of these building blocks, the sound is now filtered with the aim of transforming the sound of one string instrument into the sound of another string instrument.

This subsection shows for which sounds this process works well and for which sounds it works less well and where this originates from. An example of how a sound is filtered successfully is shown in figure 18. In the left plot the Fourier spectrum of the original electric guitar is shown. This is a measurement of keynote B_3 with frequency $f = 246$ Hz in single coil mode where the first pickup is used and the guitar is struck at pick up 1. The aim is to filter this sound in such a way that it mimics the sound of a violin where the same keynote is played by bowing the instrument. As explained in section 4.3.1 a filter is created based on the height of the peaks from the guitar spectrum and the height of the peaks from the violin spectrum. The filter and its magnification factor are also illustrated in the first plot of figure 18 with the

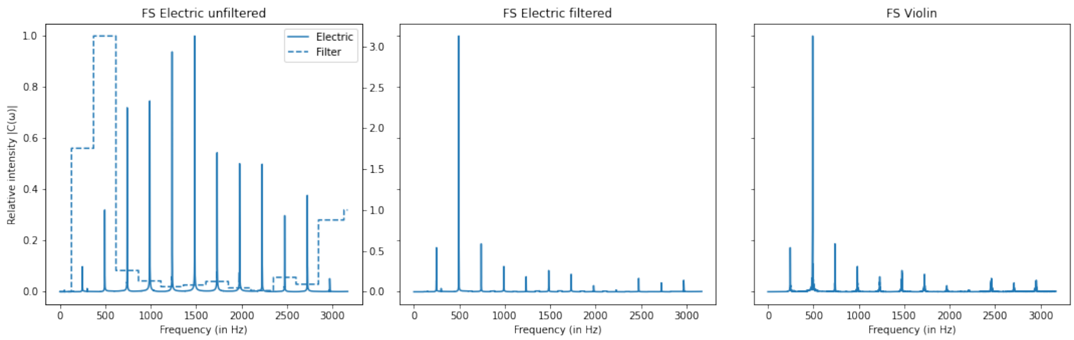


Figure 18: The filtering of the sound of an electric guitar of a B_3 with frequency $f = 246$ Hz to the sound of a violin is shown. The first plot shows the Fourier spectrum of the unfiltered sound together with the magnifying filter. The second plot illustrates the filtered sound and the third plot is the Fourier spectrum of the sound that is imitated, namely a violin. A frequency range of $f = 0 - 3000$ Hz is illustrated.

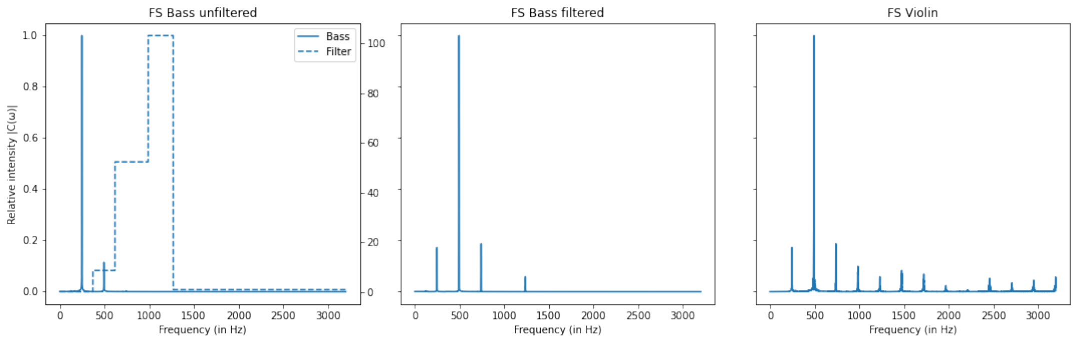


Figure 19: The filtering of the sound of a bass guitar of a B_3 with frequency $f = 246Hz$ to the sound of a violin is shown. The first plot shows the Fourier spectrum of the unfiltered sound together with the magnifying filter. The second plot illustrates the filtered sound and the third plot is the Fourier spectrum of the sound that is imitated, namely a violin. A frequency range of $f = 0 - 3000$ Hz is illustrated.

dashed line. Based on that filter, the Fourier spectrum is filtered. The filtered Fourier spectrum is showed in the second plot. The Fourier spectrum of the violin that is mimicked is shown in the third plot. Clearly the second and third plot are very much alike. When these spectra are examined very closely some slight differences can be found. The peaks in the Fourier spectrum of the violin are a little broader but otherwise the spectra match very accurately.

The filter works for almost all cases as accurately as in this example. The filtered sound also is perceived as if it was produced by the instrument it is trying to mimic. Although a notion has to be made that it does not sound completely the same as the sound it is trying to imitate. Before evaluating what the possible causes might be, first an example is shown where this way of creating a filter and filtering is not so accurate.

This way of creating and filtering does not work well for the bass guitar. This is illustrated in figure 19. Most of the setup here is the same as in the last example. The instrument which sound will be imitated is the violin at keynote B_3 ($f = 246$ Hz). In the first plot is the Fourier spectrum of the bass guitar. This spectrum only contains 2 peaks. The filter is shown with the dashed line. The magnification factor in this case is way larger, up to a 100. This filter seems to have four different levels. There are some very high levels for frequencies where no peaks can be found. The reason for that is that very small peaks are found and that is how the third and fifth harmonic of the bass guitar are located. These are greatly magnified and as a result the Fourier spectrum of the filtered signal even has a visible peak at that location. Still the filtered signal only has four peaks in the Fourier spectrum. For that reason the Fourier spectrum is very different than the Fourier spectrum of the violin. The produced sound is for that reason quite different from the sound of the violin. This way of filtering can only create peaks when there is a small peak detectable, otherwise the filter is unable to produce an overtone at that frequency.

It is evident that the filtering in figure 19 is less than ideal, but also the sound created in figure 18 is distinguishable from the sound of the violin. Also when the moving window technique is used the filtered sound is alike but not quite the same as the sound of the violin. One way to improve sound quality is to use even more filters. However, this has only a limited effect. Tests have been carried out where up to 200 filters have been used. Since sounds of most string instruments (when plucked) last 3 seconds, this means that for every 15 milliseconds of sound a new filter is used. The ideal case is of course that filters could be used for each sample. However, calculating such an operation takes a lot of time and computing power of the computer that calculates it. It is also doubtful whether this is much more accurate than using a new filter every 15 milliseconds. Another possibility why the sounds do not fully match is that both instruments are not precisely tuned. Perhaps one instrument is off by a few Hertz. The signals are then still filtered properly because the code recognises that the harmonic vibrations have approximately the same frequency. However, the end result of the filtering will sound slightly different. A final and important possibility is that the phase effect is not included in the filtering. The peaks from the Fourier spectra are only scaled based on their relative intensity. The phase of a peak in the Fourier spectrum is not considered in this process. As explained earlier in section 3.2, the Fourier spectrum actually consists of complex numbers where the norm is correlated with the intensity and the argument of these numbers with the phase of that frequency. The norm is now increased or decreased but the argument and thus the phase is not changed. This while research has shown that people are not phase deaf. By adjusting this, the filtered sound will more accurately mimic the sound that is imitated and these sounds may no longer be distinguishable.

6. Conclusion

In this research various string instruments have been analysed and filtered. To conclude the analysis a sound is defined by its fundamental frequency together with the overtones. Different string instruments have vastly different Fourier spectra where overtones are present in different ratios. Even when using the same instrument the same notes vary a lot between open and fretted notes or when a string is bowed or plucked. Techniques as the flageolet technique can be used to accentuate some overtones whilst repressing other overtones. The Fourier spectrum of a sound is also very dependent on the place where a string is struck. These effects combined make it impossible to speak of one exact sound of an instrument. Nonetheless it is possible to filter the sound of a string instrument to imitate the sound of another string instrument. One condition for this is that the original sound has detectable overtones. This causes some tones of the bass guitar to be improperly filtered.

The filtering of other string instruments works very accurately but is still distinguishable from the sound it is trying to mimic. One way to improve this is to use even more filters. However, calculating such an operation takes a lot of time and computing power of the computer that calculates it. It is also doubtful whether this will provide much improvement compared to the current situation (where a new filter is used every 15 milliseconds). Another possibility why the sounds do not fully match is that both instruments are not precisely tuned. Finally, the phase of the sound of an overtone could be adjusted so that it corresponds to the sound that is being imitated. This improves the matching of the sounds.

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