

# Train Trajectory Optimization with Signalling Constraints (PPT)

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# Train Trajectory Optimization with Signalling Constraints

# **Delft University of Technology** Pengling Wang, Rob M.P. Goverde, Lei Ma July 20, 2015

# Outline

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- 2 Train Trajectory Optimization
- **3** Train Path Envelope
- 4 Multiple phase train trajectory optimization model
- 5 Train Trajectory Optimization Strategies Dutch Signalling System
- 6 Computational Experiments

## 7 Conclusions

# Introduction

What is the train trajectory optimization?



# Introduction

What is the train trajectory optimization?

- speed trajectory
- energy-saving, on-time, safe, riding comfort...



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# **Train Dynamic Movement Model**

Dynamic constraints:

$$\frac{dv}{ds} = \frac{dv}{dt}\frac{dt}{ds} = \frac{a}{v} = \frac{\theta_1 f - \theta_2 b - R_{train}(v) - R_{line}(s)}{\rho \cdot m \cdot v}$$

$$\frac{dt}{ds} = \frac{1}{v}.$$

$$\theta_1, \theta_2 \in \{0, 1\}$$

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Path constraints:

$$\begin{cases} 0 \le f \le F_{\max} & 100 \\ 0 \le b \le B_{\max} & 100 \\ 0 \le f \cdot v \le P_{\max} & 0 \le v \le V_{\max} \\ A_{\min} \le \frac{dv}{dt} \le A_{\max} & 0 \end{cases}$$

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# **Train Dynamic Movement Model**

Dynamic constraints:

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$$\frac{dt}{ds} = \frac{1}{v}. \qquad \theta_1, \theta_2 \in \{0, 1\}$$



Path constraints:

$$\begin{cases} 0 \leq f \leq F_{\max} \\ 0 \leq b \leq B_{\max} \\ 0 \leq f \cdot v \leq P_{\max} \\ 0 \leq v \leq V_{\max} \\ A_{\min} \leq \frac{dv}{dt} \leq A_{\max} \end{cases}$$

Boundary conditions:

 $v(s_0) = 0, v(s_f) = 0$  $t(s_0) = T_0, t(s_f) = T_f$ 

Objective function:

Minimize 
$$E = \int_{s_0}^{s_f} f \, ds$$

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- optimization approach: Maximum principle
- More constraints should been taken into account:



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- optimization approach: Maximum principle
- More constraints should been taken into account:
  - time constraints



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- optimization approach: Maximum principle
- More constraints should been taken into account:
  - time constraints
  - speed limits, grades and curves



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- optimization approach: Maximum principle
- · More constraints should been taken into account:
  - time constraints
  - speed limits, grades and curves
  - signaling system



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More constraints should been taken into account:

- time constraints
- speed limits, grades and curves
- signaling aspects

In this paper:

- Train Path Envelope
- Multiple phase train trajectory optimization model
- Optimization strategies in consideration of the influence from signaling system



# **Train Path Envelope**

#### Train Path Envelope

a series of time and speed allowances available in real operation



The TPE contains two kinds of targets:

- Mandatory target points, (p, t, v)
- Flexible target windows, (*p*, [*t*<sub>min</sub>, *t*<sub>max</sub>], [*v*<sub>min</sub>, *v*<sub>max</sub>])

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Multiple-phase optimal control model:

- divide the train trajectory into several phases by several linkage points;
- each phase has its own cost function, dynamic model, path constraints and boundary conditions;
- two adjacent phases are linked by linkage conditions.





Multiple-phase optimal control model:

- divide the train trajectory into several phases by several linkage points;
- each phase has its own cost function, dynamic model, path constraints and boundary conditions;
- two adjacent phases are linked by linkage conditions.

The linkage points can be:

the TPE target points



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### boundary conditions:

at mandatory target points:

$$v(s_0^{(r)}) = V_0^{(r)}, \quad t(s_0^{(r)}) = T_0^{(r)}$$
(initial boundaries)  
 $v(s_f^{(r)}) = V_f^{(r)}, \quad t(s_f^{(r)}) = T_f^{(r)}$ (terminal boundaries)

for flexible target windows:

í.

$$V_{0,\min}^{(r)} \le v(s_0^{(r)}) \le V_{0,\max}^{(r)}, \quad T_{0,\min}^{(r)} \le t(s_0^{(r)}) \le T_{0,\max}^{(r)} \text{ (initial boundaries)}$$
  
$$V_{f,\min}^{(r)} \le v(s_f^{(r)}) \le V_{f,\max}^{(r)}, \quad T_{f,\min}^{(r)} \le t(s_f^{(r)}) \le T_{f,\max}^{(r)} \text{ (terminal boundaries)}$$

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The linkage points can be:

- Target positions of the TPE
- Critical points of speed limits or gradients and curves



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#### cost function:

$$J^{(r)} = \int_{s_0^{(r)}}^{s_f^{(r)}} f^{(r)} \, ds$$

#### dynamic model:

$$\frac{dv^{(r)}}{ds} = \frac{\theta_1 f^{(r)} - \theta_2 b^{(r)} - R_{train}(v^{(r)}) - R_{line}^{(r)}(s)}{\rho \cdot m \cdot v^{(r)}}$$
$$\frac{dt^{(r)}}{ds} = \frac{1}{v^{(r)}}$$

#### path constraints:

$$\begin{cases} 0 \le f^{(r)} \le F_{\max} \\ 0 \le b^{(r)} \le B_{\max} \\ 0 \le f^{(r)} \cdot v^{(r)} \le P_{\max} \\ 0 \le v^{(r)} \le V_{\max}^{(r)} \\ A_{\min} \le \frac{dv^{(r)}}{dt^{(r)}} \le A_{\max} \end{cases}$$

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### linkage conditions:

$$\begin{split} s_f^{(r)} &- s_0^{(r+1)} = 0, \\ v(s_f^{(r)}) &- v(s_0^{(r+1)}) = 0, \\ t(s_f^{(r)}) &- t(s_0^{(r+1)}) = 0. \end{split}$$



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- Gauss Pseudospectral methods can be applied for solving multiple-phase optimization problems.
- The optimization objective is to minimize the sum of the cost functions of all phases.

Solver:

- GPOPS
- PROPT
- DIDO



# Next Subsection

- 5 Train Trajectory Optimization Strategies Dutch Signalling System



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# **Dutch Signalling System**

### Example of Dutch signalling system (a):



### Example of Dutch signalling system (b):



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Two cases of information about the signaling system available:

- Case I: Limited information about the signal aspect ahead only.
- Optimization strategy is to rapidly respond to signaling aspects.

#### Green:

 calculate the optimal trajectory from the current position to the next timetable point



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#### Yellow:



Decelerating curve

Curve I

#### Yellow 8:



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- Decelerating curve
- Curve II

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- Case II: Full information about the entire train trajectory of the preceding train
- Optimization strategy: Green wave policy

•  $t(p_s) \ge T_{p_s,\min}$ ,  $T_{p_s,\min}$  is the predicted time that the signal changes from yellow to green.



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If the remaining running time is insufficient,

- increase the remaining running time The time boundary condition of the arrival event is changed from  $t(s_f^{(r)}) = T_f^{(r)}$  to  $T_f^{(r)} \le t(s_f^{(r)}) \le T_f^{(r)} + T_{add}$ .
- the cost function is designed as

$$J^{(r)} = t(s_f^{(r)}) + \omega \int_{s_0^{(r)}}^{s_f^{(r)}} f^{(r)} ds$$

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# **Computational Experiments - Data**

#### Infrastructure: from Htnc to CI



Train: one Intercity, one Sprinter (Local train), and the Sprinter train runs ahead of the Intercity.





# **Computational Experiments - Results**

the train trajectories of the IC train with four different departure headways after the SPR train at station *Htnc*. (solid line–Case I, dashed line–Case II)

#### Headway 120 s



#### Headway 140 s



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# **Computational Experiments - Results**

	Headway [s]	Energy Consumption [J]	Running time [s]	Delay [s]
	120	3.3609×10 <sup>8</sup>	285	45
	140	1.1878×10 <sup>8</sup>	282	42
'	160	1.7640×10 <sup>8</sup>	255	15
	180	0	240	0
	120	1.8624×10 <sup>8</sup>	283	43
n	140	1.4541×10 <sup>8</sup>	263	23
"	160	1.4652×10 <sup>8</sup>	243	3
	180	0	240	0

Table: Results of the IC train operation optimization for different departure headways.



# Conclusions

- Train path envelope is a useful formulation of the time constraints for the train operation.
- The multiple-phase optimal control model and the Gauss Pseudospectral Method can been used for the train trajectory optimization problem.
- The influences from the signalling system on train operations should be taken into consideration. More information about signaling system and green wave policy result in better optimal solutions.



# Thank you for listening!

