

## Perpendicular Transport through Magnetic Multilayers

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(Received 29 June 1992)

A theory of the perpendicular transport of electrons through metallic multilayers based on the Landauer-Büttiker formalism is presented and applied to the magnetoresistance of antiferromagnetically coupled magnetic multilayers. The contributions of contact potential, interface roughness, and bulk impurity scattering to the spin-selective transmission or spin-valve effect are described by a simple closed formula which, in the absence of spin-flip scattering, unites previous approaches.

PACS numbers: 72.10.Fk, 73.40.Jn, 73.50.Jt, 75.50.Rr

The “giant magnetoresistance” or “spin-valve” effect in antiferromagnetically coupled magnetic multilayers [1] has great potential for magnetic recording applications [2]. The effect is caused by the reorientation of the spins under an applied magnetic field from an antiferromagnetic to a ferromagnetic configuration. The sample geometry for transport experiments is mostly chosen such that the current flows parallel to the multilayer planes [1–5]. The “perpendicular” configuration, where the potential drops normal to the interfaces, has been under scrutiny only recently [6–9]. The most complete theoretical description so far is the linear response (Kubo) formalism by Levy and co-workers [3, 4, 6]. The experimental results for the parallel configuration are well described by this theory if a number of disposable parameters are introduced [5]. For infinite superlattices Zhang and Levy [6] predicted an increase of the spin-valve effect in the perpendicular as compared to the parallel geometry, which was subsequently verified by experiment [7]. However, the perpendicular transport experiments through microstructured samples by Gijs [9] allow investigation of just a few layers, a case which cannot be treated by the theory of [6]. The thermodynamic formalism for the perpendicular magnetoresistance by Johnson [8] does not suffer from this drawback, but oversimplifies the important impurity and interface roughness scattering.

The present paper addresses the perpendicular transport in metallic magnetic multilayers. It is shown that the Landauer-Büttiker (LB) scattering formalism [10], established mainly for the transport properties of semiconductor nanostructures [11], is well suited for this problem, since finite size effects are included naturally. The difficult problem of the charge and magnetization redistribution in inhomogeneous systems induced by an applied bias, discussed in [8] but ignored in [6], is integrated out [12]. The results are simple and physically transparent, and previous theories of the perpendicular magnetoresistance [6, 8] are recovered as special limiting cases. In the following the ballistic regime is treated first, followed by a discussion of the effects of imperfections.

For temperatures  $k_B T \ll E_F$  the Landauer conductance formula reads

$$G = \frac{e^2}{h} \text{Tr } \mathbf{t} \mathbf{t}^\dagger = \frac{e^2}{h} \sum_{nm,s} |t_{nm,s}|^2, \quad (1)$$

expressing the transport properties of the sample in terms of the  $t$  matrices, whose elements are the scattering amplitudes  $t_{nm,s}$  between the modes  $n$  and  $m$  with spin  $s$  at the Fermi energy  $E_F$  of two perfect leads. The leads are connected to the contacts which are in thermodynamic equilibrium. The neglect of inelastic and spin-flip scatterings in Eq. (1) is allowed when the conductance is limited by a narrow region which is shorter than the inelastic and spin-flip relaxation lengths. Consider a multilayer of a nonmagnetic metal  $A$  and a metal  $B$ , which may be ferromagnetic, sandwiched between a substrate and a capping layer of  $A$  (Fig. 1). Mesoscopic sample dimensions are much larger than the typical Fermi wavelengths which are of the order of a lattice constant. Then the incoming and outgoing states  $n, m$  are Bloch waves at the Fermi energy, which in the following will be approximated by plane waves. The conductance of the sample without insertions is limited by its finite cross section  $S$  to (two spin channels)

$$G_0 = \frac{2e^2}{h} \frac{S k_F^2}{4\pi}, \quad (2)$$

where  $k_F$  is the Fermi wave vector. The charge transfer at heterointerfaces aligns the Fermi energies to that of the leads. The relative shifts of the band bottoms can be described by a Kronig-Penney potential. Figure 1 illustrates this potential landscape for spin-up and spin-down electrons in an antiferromagnetically coupled multilayer. The reflection of electrons at the step potentials decreases the conductance below  $G_0$  even for a perfect multilayer. In the free electron model the transmission through ideal multilayers is easily calculated exactly [13], but interference fringes average out efficiently in Eq. (1) even in a regime where quantum effects are important. Therefore a semiclassical approximation is justified, in which the transmission is zero for modes with kinetic energy

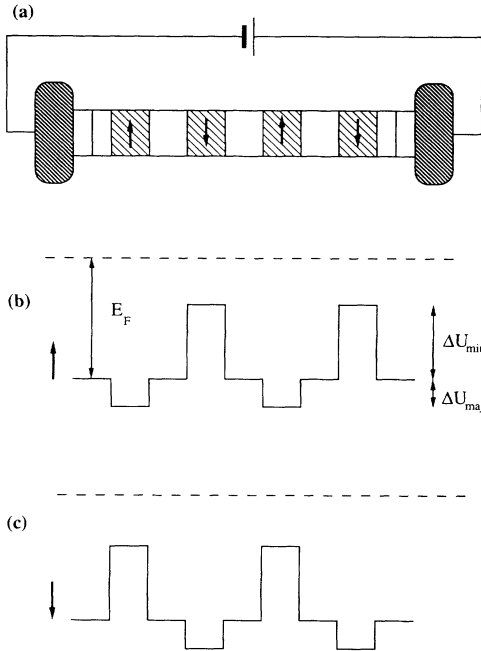


FIG. 1. (a) Schematic configuration of a typical sample considered in this paper.  $N$  double layers of a normal/ferromagnetic metal composite are assumed to limit the conductance. As a mathematical construct ideal leads are inserted between the contacts and the sample. The contacts are in thermodynamic equilibrium. (b),(c) The potential profiles as seen by spin-up and spin-down electrons when the magnetic layers are antiferromagnetically coupled.

normal to the layers smaller than the barrier potential  $\Delta U = U_B - U_A$  (cf. Fig. 1), assuming  $B$  to be nonmagnetic for the moment. The *contact conductance* thus obtained reads

$$G_{\text{con}} = G_0 \left[ 1 - \frac{\max\{0, \Delta U\}}{E_F} \right]. \quad (3)$$

It is clear that the contact conductance is different when ferromagnetic layers  $B$  are coupled ferromagnetically (F) or antiferromagnetically (AF). A very simple expression for the magnetoconductance  $\Delta G_{\text{con}} \equiv G_{\text{con}}^{\text{F}} - G_{\text{con}}^{\text{AF}}$  results, which depends only on the barrier potentials of minority and majority spin channels  $\Delta U_{\text{maj}} < \Delta U_{\text{min}}$  and the Fermi energy of the leads:

$$\frac{\Delta G_{\text{con}}}{G_0} = \frac{\max\{0, \Delta U_{\text{min}}\} - \max\{0, \Delta U_{\text{maj}}\}}{2E_F}. \quad (4)$$

$\Delta G_{\text{con}}$  is a direct measure of spin polarization of the transmitted (and, with opposite sign, reflected) electrons caused by the AF  $\rightarrow$  F transition, which justifies the term "spin valve" or "spin filter." In this model a sizable effect may be expected for the Cu/Co system: The electronic structure of the Co majority spin system is similar to that of copper, causing a small  $\Delta U_{\text{maj}}$ , while the density of the minority spin electrons is smaller, i.e.,  $\Delta U_{\text{min}} > 0$ . Using the parameters of Inoue, Oguri, and Maekawa [14]

for the Co/Cu system ( $\Delta U_{\text{min}} \approx 0.65$  eV,  $\Delta U_{\text{maj}} \approx 0$  eV,  $E_F \approx 9.4$  eV)  $\Delta G_{\text{con}}/G_0 \approx 0.04$  is obtained. On the other hand, Eq. (4) approximately vanishes for the Cr/Fe system on a Cr substrate because the potential steps are small or negative.

These results are equivalent to those of Johnson [8] in the limit of vanishing spin-flip scattering. In particular, the experiments by Johnson and Silsbee [15] are simply explained by the increased collection efficiency of a ferromagnetic contact for the spin-polarized electron gas. On the other hand, the present problem is much simpler than that of the (spin-polarized) tunnel conductance, which is complicated by tunneling matrix elements [16]. The tunneling experiments teach us, however, that band structure effects should be important also in the present case. Therefore, the  $\Delta U$ 's should be interpreted as phenomenological parameters which effectively account for Fermi surfaces and umklapp processes, which can be included in the formalism if necessary.

In the above calculations only the first two layers contribute to the magnetoconductance because the layer structure was assumed to be perfect. In the presence of interface roughness (IR) and bulk impurities (BI) a more efficient spin filtering is expected, since any scattering process (also inelastic and spin-flip) will cause additional selective backscattering at the next interface. The IR is modeled by short-range scattering potentials  $\gamma_{\text{IR}}$  which are randomly distributed over the interface with density  $n_{\text{IR}}$ . Starting from exact expressions similar to those given by, e.g., Cahay, McLennan, and Datta [17], the configurationally averaged transmission probabilities  $\langle \text{tt}^\dagger \rangle$  for a single rough interface can be obtained with arbitrary precision. By, e.g., summation of repeated Born scatterings one obtains

$$\langle \text{tt}^\dagger \rangle_{k_{\parallel}, k_{\perp}} \equiv T_{k_{\parallel}}^{(1)} \delta_{k_{\parallel}, k_{\perp}} = \frac{\sqrt{k_{\perp}^A k_{\perp}^B}}{\bar{k}_{\perp} + M_{\text{IR}}} \delta_{k_{\parallel}, k_{\perp}}, \quad (5)$$

where  $k_{\parallel}, k_{\perp}$  are the parallel and normal components of the wave vector and

$$M_{\text{IR}} = n_{\text{IR}} \left( \frac{m\gamma_{\text{IR}}}{\hbar} \right)^2 \left( \frac{1}{S} \sum_{k_{\parallel}} \frac{1}{\bar{k}_{\perp}} \right). \quad (6)$$

$k_{\perp}^A = [k_F^2 - (2m/\hbar^2)U_A - k_{\parallel}^2]^{1/2}$ ,  $k_{\perp}^B = [k_F^2 - (2m/\hbar^2)U_B - k_{\parallel}^2]^{1/2}$ , and  $\bar{k}_{\perp} = (k_{\perp}^A + k_{\perp}^B)/2$ . Note that here  $\langle \text{tt}^\dagger \rangle = \text{Re}\langle \text{t} \rangle \neq \langle \text{t} \rangle \langle \text{t}^\dagger \rangle$ , which means that vertex corrections are very important. The expressions for the transmission through two or more disordered interfaces can be calculated analogously. However, since  $E_F - U_{A,B} \gg 0$  a semiclassical approximation is even better justified for disordered than for ideal multilayers. By neglecting interferences due to multiple reflection at the interfaces [17] we find for the transmission of a mode  $k_{\parallel}$  through two interfaces, i.e., a single barrier,

$$T_{k_{\parallel}}^{(2)} = \frac{k_{\perp}^A}{k_{\perp}^A + 2M_{\text{IR}}} \quad (7)$$

when  $k_{\perp}^A, k_{\perp}^B > 0$  and zero otherwise. The bulk impurity scattering (BI) within the layers can be calculated analogously by semiclassically concatenating the transmission through infinitesimal slices of impure material. The transmission through a slab of  $A$  with thickness  $L_A$  becomes

$$T_{k_{\parallel}}^A = \frac{k_{\perp}^A}{k_{\perp}^A + L_A M_{\text{BI}}^A}, \quad (8)$$

defining

$$M_{\text{BI}}^A = n_{\text{BI}}^A L_A \left( \frac{m \gamma_{\text{BI}}^A}{\hbar} \right)^2 \left( \frac{1}{S} \sum_{k_{\perp}^A} \frac{1}{k_{\perp}^A} \right), \quad (9)$$

where  $n_{\text{BI}}^A, \gamma_{\text{BI}}^A$  are the concentration and scattering potentials of point impurities in  $A$ . The conductance of  $N$  nonmagnetic bilayers is now easily calculated:

$$G^{(N)} = \frac{e^2}{h} \sum_{k_{\parallel}}^{k_{\perp}^A, k_{\perp}^B > 0} \frac{k_{\perp}^A}{k_{\perp}^A + N \{ 2M_{\text{IR}} + L_A M_{\text{BI}}^A + (k_{\perp}^A/k_{\perp}^B) L_B M_{\text{BI}}^B \}}, \quad (10)$$

which is the main result of the present paper. In order to obtain a simple analytic formula, evanescent states, the differences of the bulk impurity scatterings in  $A$  and  $B$ , and the effects of  $\Delta U$  in the intermediate wave-vector summations are disregarded:

$$\frac{G^{(N)}}{G_0} = \frac{G_{\text{con}}}{G_0} - \frac{2N}{\bar{N}} \sqrt{1 - \frac{\Delta U}{E_F}} + 2 \left( \frac{N}{\bar{N}} \right)^2 \ln \left( 1 + \frac{\bar{N}}{N} \sqrt{1 - \frac{\Delta U}{E_F}} \right), \quad (11)$$

where  $\bar{N} = k_F / [2M_{\text{IR}} + L_A M_{\text{BI}}^A + L_B M_{\text{BI}}^B (1 - \Delta U / E_F)^{1/2}]$  is the mean free number of traversed interfaces, i.e.,  $\bar{N}(L_A + L_B) \equiv \bar{N}L$  is the (elastic) mean free path normal to the interfaces. For large  $N$  and vanishing  $\Delta U$  a Drude-like (Ohm's law) expression is obtained for the conductivity of a thick multilayer:

$$\sigma_{\infty} = \lim_{N \rightarrow \infty} NLG^{(N)} / S = \frac{2e^2}{h} \frac{k_F^2}{3\pi} \bar{N}L, \quad (12)$$

which agrees with the results by Zhang and Levy [6].

The conductance of a nonmagnetic multilayer (Fig. 2) is determined by  $\bar{N}$  and  $\Delta U / E_F$ . These quantities are not completely independent, but physically meaningful parameters. Note that the Drude result is approached rather slowly and only when the contact term can be disregarded. Figures 3 and 4 represent the relative magnetoconductance of an antiferromagnetically

coupled multilayer  $\Delta G / \bar{G}$ , where  $\Delta G = G^F - G^{\text{AF}}$  and  $\bar{G} = (G^F + G^{\text{AF}}) / 2$  and  $G^F, G^{\text{AF}}$  are straightforward generalizations of Eq. (11). In Fig. 3 the contact conductance is assumed to be independent of spin such that the spin-valve effect is exclusively due to the spin-dependent scattering from imperfections. In such a (rather unphysical) limit the effect increases with the number of layers and approaches the Drude result for samples which are much thicker than the mean free path. In Fig. 4 only the contact potential is assumed spin dependent. As anticipated

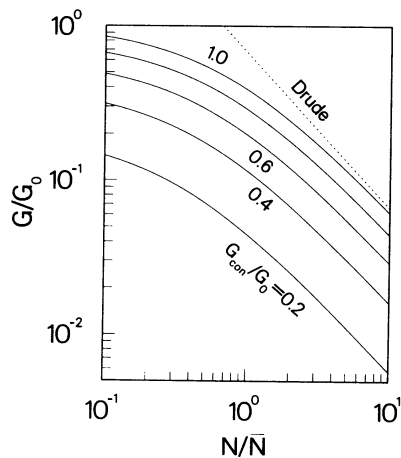


FIG. 2. Conductance of a spinless multilayer as a function of the contact potential  $\Delta U$  and the number of layers  $N$  relative to the mean free number  $\bar{N}$ . The dashed line is the Drude result [6].

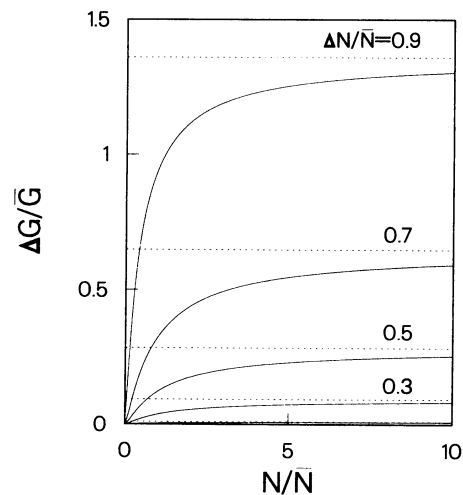


FIG. 3. Magnetoconductance of an antiferromagnetically coupled magnetic multilayer ( $N \geq 2$  and even,  $G_{\text{con}} = 0.8G_0$ ). The solid curves illustrate the effect of the spin-dependent, mean free number of layers, where the difference between both spin channels is  $\Delta \bar{N}$  and  $\bar{N}$  is the spin-averaged result. The dashed lines are the corresponding Drude results for a superlattice [6]. For  $\Delta \bar{N} = 0$  the magnetoconductance vanishes, and it becomes exactly 2 for  $\Delta \bar{N} = \bar{N}$ .

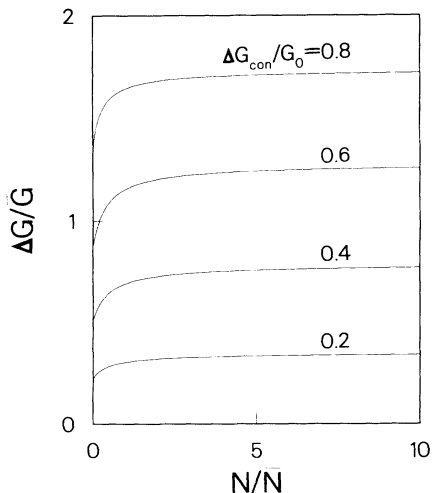


FIG. 4. Magnetoconductance of an antiferromagnetically coupled magnetic multilayer ( $N \geq 2$ ) with equal impurity roughness scattering parameter  $\bar{N}$  for both spin directions, but spin-dependent contact potentials  $\Delta U$  and a fixed  $G_{\text{con}}^{\text{F}} = G_0$ . For  $\Delta G_{\text{con}} = 0$  the magnetoconductance vanishes, and it becomes exactly 2 for  $\Delta G_{\text{con}}/G_0 = 1$ .

above, the ferromagnetically coupled spin-filter improves with increasing thickness by the scattering from imperfections, even if the latter is not spin selective. In this regime the Drude result Eq. (12) completely fails (it vanishes identically). An experimental test and parametrization of the present model should proceed in three steps. The contact conductances should first be determined by focusing on single interfaces and double bilayers of highly perfect samples grown by molecular beam epitaxy. The effects of the BI scattering can be obtained by thicker homogeneous samples of a single material. Only then will systematic transport measurements on multilayers give conclusive evidence of the importance of the interface roughness.

In conclusion, the LB scattering formalism is shown to be well suited to describe the spin-valve magnetoresistance effect in metallic multilayers. A simple closed formula is derived, which, in the absence of spin-flip relaxation processes, unites different published approaches [6, 8]. The parameters of the theory are as yet phenomenological, but the formalism can be extended to include

band structure effects and more realistic models of impurity and interface roughness scatterings which opens the way to first-principles calculations of the transport properties of metallic multilayers. In conjunction with experiments on microstructured samples [9] important new insights into the mesoscopic physics of magnetism are expected.

I would like to thank Martin Gijs for sharing his idea of perpendicular magnetoresistance. I benefited from discussions with Paul Kelly and Henk van Houten.

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