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Measuring Spill-over Effects of Disruptions in Public Transport Networks

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Abstract— **Transit is vital to the functioning of most major cities in the world today and therefore evaluation of the robustness – capacity to withstand disruptions with minimal impact to the system – of transit networks is essential. In this paper, we study the spatial extent of link disruption impacts in urban public transport networks (PTNs) due to spill-over effects that occur as affected passengers choose alternative routes. Quantifying link criticality in terms of its spatial propagation effects is important for prioritizing robustness measures as well as devising methods to encapsulate disruption spill-over effects. To this end, a new local criticality indicator – spatial criticality – that is based on: (i) the magnitude of relative change in load on other links due to closure of a link and (ii) the topological distance between these other links and the disrupted link is introduced. Further, a stochastic user equilibrium, PTN assignment model is developed. The model explicitly considers the service layer of the PTNs and accounts for on-board crowding and denied boarding at stops to represent passenger spill-over effects. Finally, to demonstrate the proposed indicator and investigate its relationship with conventional, local topological indicators, the urban rail-bound network of Amsterdam is used as a case study.**

Keywords*— *public transport; criticality; assignment model; crowding;

I. INTRODUCTION

Public transport networks (PTNs) handle a significant proportion of human mobility in many cities and disruptions in these systems can reduce the efficiency or even completely halt the functioning of these cities. Moreover, public transportation is much more complex and rigid than personal (road) transportation due to pre-determined service routes, fixed schedules and interdependency caused by service capacity limits. Because of these factors, even planned disruptions in PTNs, such as maintenance works, can lead to large reductions in efficiency that spread throughout the system. There is thus a pressing need to understand and assess the robustness of PTNs, in particular the criticality of different components to support the prioritization of investments and design of contingency plans.

Robustness of a system is defined as the capacity of the system to absorb disturbances and retain its functionality [1]. Since PTNs consist of stops and ordered connections between them, disturbances are either on the stops or on the connections. The criticality of these components is then defined as the combination of the probability of disruption of that component and its impact on the system [2]. By representing stops as nodes and the connections as edges of a graph, complex network analysis is often used to study PTN robustness. Such analysis is based on the hypothesis that the design of the network, in terms of its topology, has a strong impact on its performance in normal and disturbed situations. Component criticality has been studied through hypothetical attacks on links or nodes of PTNs followed by evaluation of network performance indicators and comparison with local topological indicators [2, 3]. In this way, critical links or nodes are found and key topological indicators are identified.

The effect of component disruptions is evaluated through network performance indicators. Mattsson and Jenelius [4] report that most studies on PTN robustness are of a generic topological nature. Many of them, such as [5-7] have used purely structural performance indicators such as network fragmentation and diameter. Other indicators considering shortest path lengths include the average distance between nodes and the efficiency indicator by [8]. While these studies are able to evaluate the topology of the infrastructure of PTNs they do not take into consideration the service layer [9] of the system and thus do not give any information about the flow or distribution of passengers within the network.

The explicit representation of transit lines is essential for the analysis of the travel time components: waiting for a vehicle, actual movement on-board the vehicle and transferring between different lines. Moreover, representing the flow of passengers after disruptions requires modelling their distribution over the network and only a few studies have done this. An all-or-nothing assignment of passengers to shortest paths was performed in [10, 11] when measuring the increase in average travel time as a result of link disruptions. Although a considerable improvement on the purely topological studies, this study assumes that disruption of link has no effect on the travel times on other links. A comprehensive agent-based model of public transportation that is able to assign passengers in a dynamic and stochastic manner was used for analyzing network vulnerability in [3]. By also considering variable traffic conditions and on-board congestion effects it takes into consideration that the effect of link disruptions is not limited to the primary effect on disrupted links alone. Link criticality was assessed in terms of the changes in passenger welfare or travel costs.

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Similar to other networks, the performance of links and nodes in a PTN is interdependent; that is, the effect of a disruption in a PTN component, such as a link, does not remain confined to the immediate impact on the link itself. Rather, due to the dynamic nature of public transport supply, the impact may spread through the disrupted line and to other lines in the network. From the demand side, the existing saturation level of the link and the reaction of passengers, in the form of taking alternative lines, cause the effects of disruption to spill-over through the rest of the network. These spill-over effects are in the form of increase in loads of un-disrupted lines that are used as alternatives. On-board congestion and denied boarding at some stops caused thereby are detrimental to the passengers: the former in terms of discomfort and the latter by inducing greater waiting time. It should be noted that due to spill-over effect of disruptions even passengers that did not make use of the disrupted link suffer welfare losses [12].

Except for a handful of studies, works on the robustness of PTNs have not considered system characteristics such as transit lines, vehicle headways, or vehicle capacities. Furthermore, in spite of its importance, previous studies have rarely discussed and none of them has quantified and analyzed the spatial extent of disruptions in urban transit systems.

In this study, we develop a static, stochastic user equilibrium (SUE) transit network assignment model which can account for on-board crowding and denied boarding at stops. The model considers the service layer of PTNs by explicitly modelling the network as a combination of transit lines rather than solely in topological terms. This model can, thus, correctly estimate the spill-over effects of planned disruptions, such as maintenance works, of PTN links (of given transit lines) due to passenger demand under equilibrium conditions. The flow distribution model enables the analysis of the criticality of links in PTNs based on the extent of the spatial propagation of the effects of their closure. To this end, we formulate a local criticality indicator – spatial criticality – which is based on the magnitude of the effect of disruption on other links and the topological distance between them and the disrupted link. The urban rail network, consisting of metro and tram, of Amsterdam is used as a case study. We deploy a full scan approach to link closures to identify critical links which can be important not only for prioritizing measures for link protection but also for devising methods to contain the spill-over effects of disruptions.

Next, in section 2 the spatial criticality indicator is formulated. In section 3, the methodology of the PTN assignment model is presented. The Amsterdam urban rail network case study setup and results are given in section 4 followed by conclusions in section 5.

II. Spatial Criticality Indicator

Apart from transportation networks, cascade based vulnerability can also be found in several other networks such as the internet, electric power grids and economic systems [13]. Complex system theoretical studies regarding this have sought to understand the extent of these effects and the conditions under which they occur using simplified graph models [13, 14]. They compare network performance indicators such as efficiency or size of the largest network fragment in the post disruption equilibrium with different initial network conditions such as capacity tolerance [13, 14] or clustering coefficient [15].

The propagation of the impact of a link closure is difficult to predict because of the inherent complexity of PTNs and spatial variations in demand and link saturation levels. Therefore, in this study, as described in the next section, a more complex model that considers specific properties of PTNs is used to find passenger flow distribution over the network in user equilibrium states and to find the criticality of each link under a full scan of link failure disruptions. None of the abovementioned studies has proposed an indicator for link criticality based on the spill-over effects of its failure. Such an indicator would enable the ranking of links regarding their cascade based vulnerability, and the comparison of vulnerability with initial local conditions rather than network conditions.

This section introduces the spatial criticality indicator which is defined based on: (i) the magnitude of relative change in load on other links due to closure of a link and (ii) the topological distance between these other links and the disrupted link. The extent of spill-over effects of link closure can be described by the average topological distance from the disrupted link up to which the impact occurs. This average distance is calculated by weighting the topological distance of links with their relative change in load so that if the highest changes occur in the proximity of the disrupted link, with only a few changes farther away, the spatial criticality of the link is low.

The PTN is described as a directed graph with stops as nodes and edges as links between stops. Let E denote the set of links (edges) each of which is defined by a beginning and end node. The load on link $j \in E$ in the base scenario with no disruptions is $q_j(0)$ and $q_j(i)$ is the corresponding load when link i is disrupted. The topological distance between links i and j, d_{ij}, is the number of links in the shortest path between the beginning nodes of the two links. Then, link spatial criticality is given by the following formula:

$$s_i = \sum_{j \in E, j \neq i} \frac{|\Delta q_j(i)| \times d_{ij}}{\sum |\Delta q_j(i)|} \quad (1)$$

Where $\Delta\, q_j(i)$ is the relative change in link load defined as:

$$\Delta\, q_j(i) = \frac{q_j(i) - q_j(0)}{q_j(0)} \times 100 \quad (2)$$

After disruption of a link, reductions in link loads (e.g. due to infeasible interchanges) occur alongside the expected rise in saturation levels. The criticality indicator is therefore defined so that these effects would not cancel each other out and thus return incorrect estimates of the extent of impact. Therefore, the absolute value of the link load change is used to calculate the spatial criticality indicator. This allows the measurement of the extent of the spill-over effect of a link regardless of its direction. This way the impact a certain link has on the rest of the network can be quantified.

III. PTN ASSIGNMENT METHODOLOGY

Fig. 1 depicts the methodology framework developed for this study. In the following sub-sections, each of the methodological steps is described, resulting with the link spatial criticality indicator defined in the previous section.

A. Input

The inputs required for the PTN assignment model are: (1) service layer representation of the PTN, (2) service properties of the PTN, (3) passenger demand matrix of origin-destination stops (O-D) and (4) passenger behavioural parameters. The service layer of the PTN is described by the sequence of stops in each transit line along with the average travel time between these stops. In addition, the frequency (vehicle departures per hour) of each transit line, and sitting and standing capacities for each vehicle are specified. Hourly passenger demand is given in the form of a matrix of origin-destination stops and it is assumed that demand for the PTN is inelastic.

B. PTN Representation

In order to apply complex network theory analysis to PTNs, several methods of representation have been used. This study uses the L'-space representation to explicitly represent the service layer – nodes are stops and links are transit service connections between stops. This representation is similar to the one displayed in transit maps. For computational purposes the PTN is also represented in P-space in which a node is directly connected to all other nodes that can be reached without changing transit lines, that is, without making a transfer.

The network representation is encoded through adjacency matrices where, if a connection between two nodes exists then the in-vehicle travel time is given as a link label. To describe the sequence of stops that passengers traverse through to reach a destination from an origin we use the following terminology. Routes are the sequence of stops passengers actually step onto – the origin stop, all the transfer stops and the destination stop. Routes choices are, thus, best described in the P-space which represents the service rather than the infrastructure. The spatial dimension of the latter is preserved in the notion of paths. Paths are the sequence of all the stops that passengers pass through during their travel, including their origin and destination and are therefore represented in the L'-space. Paths are used to assign passengers to the links between consecutive nodes which is necessary to emulate the on-board congestion and denied boarding effects. It should be noted that the same route can refer to different paths as two nodes may be connected by more than one line.

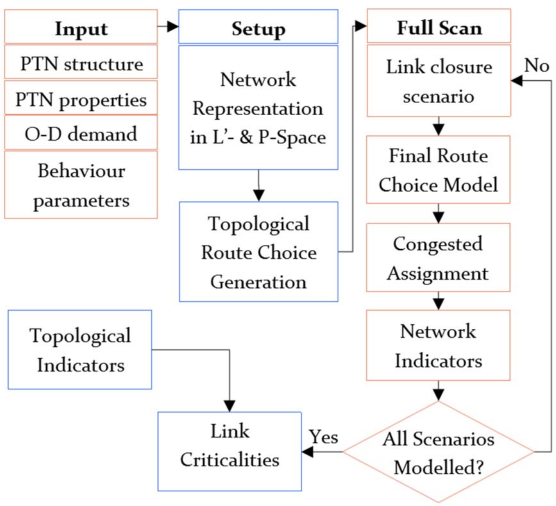

Fig. 1. Methodology framework

C. Topological Route Choice Generation

Route choice generation involves generating routes for all O-D pairs that are connected in the network. This phase refers to the non-iterative part of the route choice generation process since it does not take into consideration the link disruption scenario. The route choice generation follows a breadth-first search algorithm: each node in the network is taken separately as the origin which is the vertex root of a search tree; its immediate neighbours in P-space are stored; out of these neighbours, transfer nodes (nodes connected to more than one line) become origins for the next level of the tree and their respective immediate connections are stored retrospectively starting from the origin node, once again the transfer nodes become the origins and so on. Thus, routes between the origin node and different destinations are known.

On its own, this method will generate unreasonable routes and take a long time to run. Therefore, the search algorithm is constrained by the following logical rules: (i) no transfer to the same line; (ii) no loops, and; (iii) no reverse movement. However, even with these constraints the algorithm can take too long to run because of the unlimited number of transfers possible. Therefore, as a behavioural parameter that takes into consideration feasibility of routes, it is assumed that passengers accept a maximum of two transfers to prune the search tree.

The route choice generation phase is designed to generate routes that might be reasonable in case of disruptions. For this reason, previously used lines may be visited again – just not consecutively. Although it is not logical to visit the same line again under normal conditions, it is allowed so that a part of the route choice generation process can be made non-iterative. Since, a link closure is assumed to be managed by short-turning on the remaining operational parts of the line (i.e. dividing a line into two) using the same line again must be allowed now but will be corrected later for undisrupted cases.

D. Link Disruptions

A full scan approach is used to identify critical links in the network. When a link is disrupted, it is assumed that vehicles can no longer traverse it but the supply of vehicles in the rest of the line and network remains unaffected. Furthermore, it is assumed that the demand also does not change and passengers use the remaining, undisrupted, network to reach their destination. Finally, the disruptions are assumed to be planned and last sufficiently long so that a user equilibrium is obtained.

E. Route Choice Model

The final route choice model is a two-step process: first, the final sets of feasible and non-disrupted routes are determined and second, the travellers are assigned to routes using discrete choice modelling.

1) Scenario-specific route choice-set

To obtain the final route choice set, routes that are unavailable due to link closure or are behaviourally unacceptable are removed. First, paths are derived from routes and then, using this, the set of links connecting the nodes in the route are identified. Thereby, routes which contain the disrupted link and do not have any alternative are identified and removed. At this stage, nodes that are no longer connected due to the disruption are recorded for later analysis. Next, the nominal in-vehicle and waiting times of the remaining routes are calculated. Waiting time is calculated on the assumption that passenger arrivals are uniformly distributed through the hour and that they take the first vehicle they can board. Further, vehicle arrival schedules are assumed to be distributed evenly throughout the hour. Thus, waiting time at a stop is given by half of the headway of vehicles of common transit lines. Dominancy rules based on the number of transfers and the in-vehicle times are applied to remove strictly dominated routes.

2) Route choice modelling

To assign passengers to routes a multinomial logit model is used. The utility of a route is given by the sum of different travel components: in-vehicle time, waiting time and number of transfers weighted by their respective parameters. The effect of overlapping on route choice was neglected as the introduction of a path size logit model on this case study was found to have a very marginal effect on route choice probabilities while resulting in a considerably longer computational time.

F. User Eqilibrium Capacitated Assignment

The assignment is carried out in the following steps: first the network is loaded considering vehicle capacities and denied boarding at stations, then the (perceived) in-vehicle time and waiting time are calculated based on the on-board crowding and number of boarding denials, respectively. Route utility values are then recalculated and new loads for the routes are found using the method of successive averages. Finally, the duality gap difference between the last two iterations is calculated to check if another iteration is required.

1) Network loading

Using the route loads, first a load matrix is derived for route nodes which is then used to load each line sequentially (link-by-link) in L'-space so that the number of persons boarding a line from a station is constrained by the space available on-board. During this sequential loading, the number of passengers who cannot board a vehicle on a transit line out of all those who want to – denied boarding fraction – is calculated.

2) Iterative loading process

Perceived increase in in-vehicle travel time due to on-board crowding is represented by multiplying in-vehicle travel times by crowding factors that are based on the volume-capacity ratio of each link. The fraction boarding (a line) at a stop is used to calculate the number of boarding denials faced by the hourly demand before it is satisfied and using this the average waiting time, including excessive waiting time due to denied boarding, is determined. New route utilities are calculated and then multinomial logit is used to find route loads according to the new route utilities. The method of successive averages uses the iteration number to adjust the old route loads with the route loads based on the new utilities to find the new route loads that will be used in the next iteration.

3) Stopping criterion

The stopping criterion used here is the absolute difference between relative duality gaps of successive iterations. The relative duality gap is the ratio of the product-sum of route utilities and loads, and the product-sum of the highest route utility and load for each O-D pair. A threshold for the absolute difference between relative duality gaps of successive iterations is used as the stopping criterion.

G. Network Indicators

After reaching the stopping criterion the link criticalities are calculated. Apart from the spatial criticality, the generalized travel cost (GTC) criticality which indicates the total impact of closing a link is also calculated. GTC is the total utility of all the passengers travelling in the PTN and the GTC criticality is calculated as the change in GTC due to the disruption as a percentage of the no-disruption scenario.

Direct comparison of the no-disruption and disruption scenarios is not possible when nodes are disconnected since the passengers between these nodes no longer travel on the PTN. The reduction in the number of passengers is accounted for by subtracting the disconnected passengers' impacts in the no-disruption scenario. For example, the total utility of the disconnected passengers is subtracted from the no-disruption GTC before calculating the GTC criticality. Similarly, before calculating spatial criticality, the link loads due to disconnected passengers are deducted from the no-disruption link loads. While these direct impacts are removed, it should be noted that the indirect influences of disconnected passengers on the PTN are not. With fewer passengers in the PTN, congestion costs will decrease and consequently, some links may become more attractive than others leading to re-routing of passengers. Thus, as an indirect influence of the disconnected passengers, the GTC may decrease due to lower congestion on links and the link loads may change due to rerouting. However, the goal here is to quantify the spatial spillover consequences of link disruption for which subtracting the disconnected demand from the no disruption case provides a benchmark.

IV. Case Study: Amsterdam Urban Rail Network

The Amsterdam urban rail network is used as a case study to demonstrate the above methodology and derive and analyse the indicators discussed above.

A. Setup

The Amsterdam urban rail network consists of 4 metro and 15 tram bidirectional lines that serve a total of 233 stops. The demand matrix used represents the situation in morning peak hours. As the demand data is derived from multiple sources, the frequency of some lines is adjusted so that the no-disruption link saturation levels are somewhat representative of ground reality. The PTN considered consists of 780 unidirectional links in L' space, each of which is disrupted in the full scan. However, due to the assumption of continuous demand and the level of saturation, three links in the periphery of the city center could not be disrupted without continuous denied boarding at some stations leading to extremely long waiting times. These links have been excluded from the full scan to prevent them from heavily skewing summary statistics. Finally, the values for passenger behavioural parameters such as crowding factors have been derived from literature.

B. *Analysis*

The analysis is split into three sections wherein first the effect of disconnected passengers is described, followed by a short evaluation of the impact of disruptions on the net network performance, and finally the spill-over effects of link disruptions is scrutinized.

1) *Disconnected passengers*

Fig. 2 shows the link spatial criticalities (color) and the number of disconnected passengers (thickness) for the Amsterdam urban rail network. The links at the beginning of line branches disconnect most passengers as alternatives are not available. Although the direct impacts of disconnected passengers have been removed, the indirect influences on disruption of end links have a strong effect on their criticality values. This is because rerouting due to the disruption itself is not present. It is observed that these links have relatively low GTC criticality (not shown) and high spatial criticality.

2) *Network performance*

The performance of the network under disruptions is assessed by the GTC criticality. The GTC criticality values range from -0.08%, indicating an improvement in performance, to 2.69% with the mean value of 0.30%. In terms of in-vehicle minutes, link disruption effects vary from network wide savings of 1,922 min. to a maximum increase of 61,963 min. On average, a link closure leads to 6,942 min. increase in travel time in terms of in-vehicle time. The reason for such a small mean criticality may be that the indirect effects of disruption are being compensated by the advantages of having fewer passengers in the network due to disconnections. It is observed that GTC criticality increases modestly with an increase in link load (R^2=0.3195) and betweenness centrality (R^2=0.2857) (Fig. 3). Interestingly, for the Amsterdam PTN, links along all of the metro lines are found to be highly critical to network performance but only links on two of the tram lines are as important. This is explained by the higher density of the tram network which allows for short detours in case of disruptions while the metro lines play a more indispensable role.

3) *Spill-over effects*

The spatial criticality indicator measures the spill-over effects of link closures in PTNs. Spatial criticality varies between 2.39 and 26.44 with an average of 7.19. This means that, on average, effects of a link disruption propagate up to ~7 links from the closed link. Some distinctive patterns can be observed in the maps in Fig. 2: the spatial criticality of links in the dense section of the network in Amsterdam city center is the lowest; the values tend to increase in the more spread out network sections outside the city center; and, as discussed

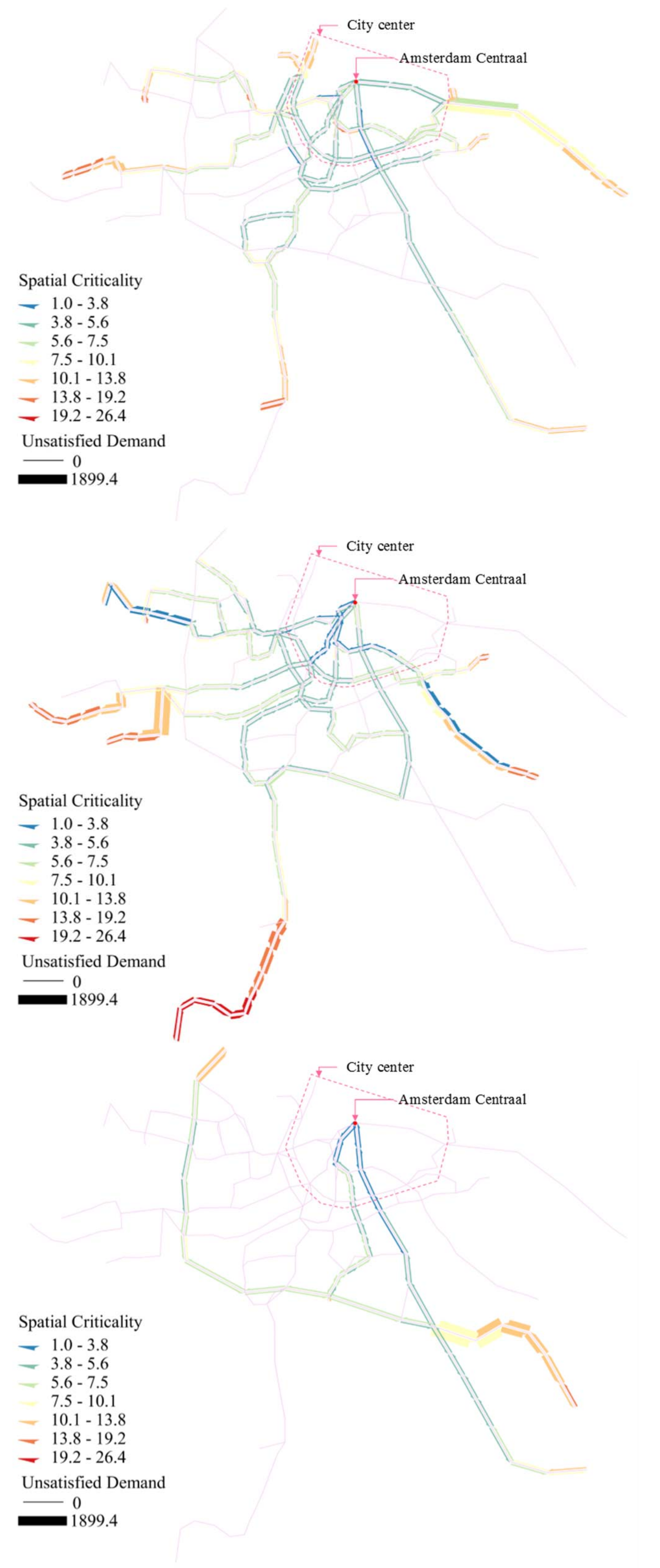

Fig. 2. Spatial criticality of links in the Amsterdam PTN. Map split to represent overlapping lines (top: metro lines: 54, tram lines: 3, 5, 10, 14, 17, 24, 26; middle: metro lines: 51, tram lines: 1, 2, 7, 9, 12, 13, 16; bottom: metro lines: 50, 53, tram lines: 4). Both directions of each line shown separately.

before, spatial criticality is the highest for links at the end of the lines on the outskirts of the network. The dense section consists of lines branching out from Amsterdam Centraal Station and 3 radial tramways which are served by several tram lines. Therefore, passengers are able to take a short detour around the disruption unlike the situation outside the city center where alternative routes take them far from their preferred route.

Spatial criticality does not correlate well with betweenness centrality (R^2=0.045) and only a slight decrease with increase in no-disruption link load (R^2=0.1105) is observed. This indicates that it is difficult to predict the spill-over effects of disruptions on the basis of static indicators making the described methodology necessary for analysis.

V. Conclusions

The contribution of this research is two-fold: first a methodology for a stochastic user equilibrium, PTN assignment model is developed. This model explicitly models the service layer of PTNs and takes into consideration boarding denials at stations and perceived increase in travel time due to on-board crowding. The modelling framework is especially suitable for executing full-scan network analysis. The model is then used for the second contribution: measuring spill-over effects of link disruptions. For this purpose, the spatial criticality indicator, defined as the average topological distance up to which the effects of a link closure propagate, is proposed.

Applying the above to the Amsterdam urban rail network, it is observed that the denser section of the network is found to have relatively low spatial criticality values as compared to more spatially sparse sections. Further, unlike criticality based on overall network performance, spatial criticality has a poor correlation with betweenness centrality and no-disruption link load. Therefore, to contain effects of link disruptions alternative services are more important in low network densities. Moreover, the described methodology may be used to predict spill-over effects and protect links with high volume-capacity ratio by guiding affected passengers through information [3] on preferred alternatives.

Further research should focus on examining topological indicators that are able to predict spatial criticality better and on analyzing more real-world PTNs in order to check whether the patterns observed in this study can be generalized.

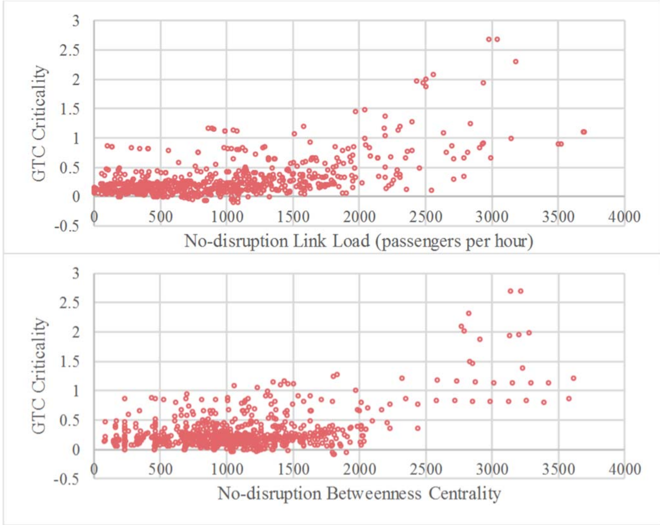

Fig. 3. GTC criticality versus no-disruption link load (top) and betweenness centrality (bottom)

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