Imaging the mode shapes of microdrum resonators

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Abstract

A setup was built to measure the local frequency response of (graphene) microdrums in order to image mode shapes. To this end, a laser interferometer that measures the local deflection of a microdrum was equipped with a very precisely movable sample stage. The setup was used to image the mode shapes of a single few layer graphene microdrum, consisting of a graphene flake suspended over a cavity.

The local frequency response and accompanying mode shapes of this drum were successfully imaged under three different actuation mechanisms: 1) electrostatic actuation by using a gate electrode at the bottom of the cavity; 2) optothermal actuation by periodic heating from an actuation laser, inducing periodic thermal contractions and expansions; and 3) by thermally induced Brownian motion.

The mode shapes of the graphene drum under electrical actuation and Brownian motion were consistent and resembled those predicted by membrane or plate theory, but their eigenfrequencies did not match up with either of these regimes. This might be an indication that the drum is in a cross-over regime. It might also be explained by impurities in the drum surface, supported by an atomic force microscopy (AFM) image of the drum surface. The mode shapes of the graphene drum under optothermal actuation had different features, but this might be caused by the fact that the actuation laser was scanned along with the measurement laser in this scheme.

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Chapter 1 Introduction

Since the experimental discovery of graphene in 2004 [11], two-dimensional crystals have been the focus of extensive research. Hoped to be the next big thing in applications such as microelectronics, sensor technology and nano-electromechanical systems, many studies are done on its interesting mechanical, electronic and optical properties. This research has focused on the vibrational mechanical properties of a few layer graphene microdrum.

Two-dimensional crystals are materials that are atomically thin or only a few atomic layers thick in one direction. The properties of such materials can differ appreciably from those of the bulk material. One of such two-dimensional materials is graphene, which consists of a single layer of carbon atoms ordered into a hexagonal grid. If it consists of a few layers of carbon atoms, such a crystal is called a few layer graphene (FLG) flake. Graphene fabricated by mechanically exfoliating (tearing off) flakes off a graphite crystal, which is a bulk crystal of carbon atoms. The thickness of the flake can be reduced by repeating the exfoliation process.

When we allow graphene flakes to move freely over some area, for example by suspending them over a hole etched into some substrate material, they can be turned into microscopic mechanical resonators. Resonators are systems that exhibit some recurring motion over time, such as pendula or spring-mass systems. This oscillatory behaviour is characterized by a few different parameters, such as the natural (resonance) frequencies of the system, the amplitude of the motions, the dampening of the motion over time and the amplitude and phase response to external oscillatory forces at frequencies around the natural frequency of the system. Many systems also show multiple distinct vibrational modes, each with their own natural frequency and particular amplitude distribution. Resonance systems can be studied in the time-domain, meaning that the motion of the system is followed over time, or in the frequency-domain, meaning that the response of the system to driving forces of different frequencies is measured.

Two-dimensional crystals stretched over a hole exhibit resonance behaviour similar to that of the skin of a drum or a circular plate. Devices such as these are called microdrums and the motion of such microdrums can be measured using laser interferometry. Light waves reflected from the drum surface interfere with light that has been transmitted through the drum and reflected off the bottom of the drum cavity. From the interference intensity information about the deflection of the drum can be deduced. When the drum is driven by an electrical or thermomechanical force, multiple resonance modes are observed [2, 3], which seem to correspond with different vibrational modes of the drum surface.

To support the characterization of these vibrational modes, information of the resonance be-

haviour in the spatial domain would be useful in addition to time domain and frequency domain measurements. This entails measuring resonance parameters such as amplitude and phase over the surface of the drum, in an attempt to reveal the shape of the vibrational modes. For silicon nitride microdisks with dimensions on the order of 200 µm this has been done by Wang, Lee, and Feng [15]. Studies on graphene microdrums of a few micrometer in diameter is more challenging due to a low Q factor, small displacement and high frequency and are as of yet still lacking.

The aim of this project is the development of a spatial scanning setup and measuring the mode shapes of a graphene microdrum.

Chapter 2

Theory

Vibration of (quasi) two-dimensional crystals such as graphene or molybdenum disulfide drums is described well by classical physics. In this chapter the necessary theoretical foundation to characterize such vibrations is derived from first principles. The first section deals with vibrations in a general sense, starting from the basic harmonic oscillator and including the wave equation for membranes. Then the governing equations for vibration of plates, derived from classical or Kirchhoff plate theory are discussed. Finally, actuation of such drums by thermally induced Brownian motion is treated.

2.1 The harmonic oscillator

One of the simplest classes of oscillatory systems is that of the harmonic oscillators. These systems all share two main features: firstly there is some mechanism that generates a restoring force if the system is perturbed from equilibrium and secondly the system exhibits some form of inertia. In the case of harmonic oscillators this restoring force is linearly proportional to the size of the perturbation. Common examples of harmonic oscillators include mechanical systems such as spring-mass systems (including stretched strings and membranes) and pendula (in good approximation) but also electromagnetic systems such as LC-circuits.

To describe the behaviour of harmonic oscillators, we start by using the spring-mass system illustrated in figure 2.1 as an example. In such a system, the necessary inertia is provided by motional inertia of the mass m. The linear restoring force is generated by the spring k.

Taking the origin of the x-axis to coincide with the equilibrium position of the mass, Hooke's law gives us the following expression for the restoring force F_r in the system if the mass is at position x.

$$F_r = -kx \tag{2.1}$$

This force acts on the mass m together with the external force F and the damping force F_d . This damping force is often assumed to be proportional to the velocity of the mass. We will call the damping proportionality factor c. Combining this net force F_{net} with Newton's second law leaves us with the following equation of motion for the system.

$$m\frac{d^2x}{dt^2} = F_{net} = F_{res} + F + F_d = -kx - c\frac{dx}{dt} + F$$
(2.2)



Figure 2.1: Schematic representation of a spring-mass system. A massless spring with spring constant k is fixed at one end and connected to a mass m at the other end. The oscillation is damped by damper B and driven by an external force F.

This equation is mostly written in the following form, in which the external force is assumed to be time-dependent.

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$
(2.3)

This linear second order differential equation is the most general equation describing the behaviour of a harmonic oscillator. It can be applied to all harmonic oscillators if the appropriate physical quantities are taken into account and for the physically interesting case of constant coefficients and a sinusoidal driving force it can be solved exactly.

2.1.1 Free harmonic oscillation

In the simplest case for a spring-mass system with no external driving force and no damping, and defining $\omega = \sqrt{k/m}$, equation (2.3) reduces to

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \tag{2.4}$$

The solutions to this differential equation are given by

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$
(2.5)

The values A and B will have to be fixed by applying appropriate initial conditions to the system. Regardless of the initial state of the system, we see that the angular frequency of the oscillation is given by $\omega = \sqrt{k/m}$. Stiffening the spring increases the oscillation frequency, while increasing the mass lowers the frequency.

2.1.2 Driven and damped harmonic oscillation

We return to equation (2.3) to describe the more general case of driven and damped harmonic oscillation. We want to study the response of the system to sinusoidal driving forces with an

angular frequency ω_d that does not necessarily equal the eigenfrequency of the free system. For this, we plug in the following expression for the driving force.

$$F(t) = F_0 \cos(\omega_d t) = \Re\{F_0 e^{i\omega_d t}\}$$
(2.6)

This leaves us with the following equation of motion.

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0\cos(\omega_d t)$$
(2.7)

Instead of m, k and c to characterize the system, often the undamped oscillation frequency $\omega_0 = \sqrt{k/m}$ and the dampening coefficient $\gamma = c/2m$ are employed. Both of these parameters have units of time⁻¹. The dampening time γ^{-1} indicates what the time scale is over which the system decays. Similarly, the oscillation time ω_0^{-1} expresses the time scale of oscillations in the system.

For simplicity, we switch to a complex position variable \overline{x} , satisfying $x = \Re{\{\overline{x}\}}$. Because (2.7) is linear, its solutions are given by the real part of the solutions to the following complex differential equation.

$$m\frac{d^2\overline{x}}{dt^2} + c\frac{d\overline{x}}{dt} + k\overline{x} = F_0 e^{i\omega_d t}$$
(2.8)

All solutions to (2.8) can be written as the sum of a particular solution of the differential equation and the solutions to the associated homogeneous problem.

Homogeneous problem

We first turn to the homogeneous problem. Upon plugging in the ansatz $\overline{x}(t) = e^{i\omega t}$, we find its characteristic equation

$$-m\omega^2 + ic\omega + k = 0 \tag{2.9}$$

Systems for which the time scale of decay is larger than the time scale of oscillation are called *underdamped* systems. For these systems we have $\gamma < \omega$ or likewise $c^2 < 4km$. In this case the roots $\omega_{1,2}$ to the characteristic equation are given by

$$\omega_{1,2} = \frac{ic \pm \sqrt{4km - c^2}}{2m} = i\gamma \pm \sqrt{\omega_0^2 - \gamma^2}$$
(2.10)

If $c^2 > 4km$ or equivalently $\gamma > \omega$ the system is said to be *overdamped*. We find the roots $\omega_{1,2}$ to be purely imaginary, as the following expression indicates.

$$\omega_{1,2} = \frac{ic \pm \sqrt{4km - c^2}}{2m} = i\gamma \pm i\sqrt{\gamma^2 - \omega_0^2}$$
(2.11)

A system for which $c^2 = 4km$ or likewise $\omega_0 = \gamma$ is said to be *critically damped*. In that case we find a double root for ω , leaving us with only one solution for the homogeneous problem to (2.8). We can find another linearly independent solution by assuming the other solution to be of the form $\overline{x}_2(t) = u(t)\overline{x}_1(t) = u(t)e^{-\gamma t}$. This is called the method of reduction of order. It turns out another linearly independent solution is given by

$$\overline{x}_2(t) = t e^{-\gamma t} \tag{2.12}$$

We are now ready to write down the solutions to the homogeneous problem associated with (2.8) for all classical harmonic oscillator systems. They are given by

$$\overline{x}(t) = e^{-\gamma t} \left[A e^{i\sqrt{\omega_0^2 - \gamma^2}} + B e^{-i\sqrt{\omega_0^2 - \gamma^2}} \right] \qquad (underdamped) \qquad (2.13)$$

$$\overline{x}(t) = e^{-\gamma t} \left[A + Bt \right] \qquad (critically damped) \qquad (2.14)$$

$$\overline{x}(t) = e^{-\gamma t} \left[A e^{\sqrt{\gamma^2 - \omega_0^2}} + B e^{-\sqrt{\gamma^2 - \omega_0^2}} \right] \qquad (overdamped) \qquad (2.15)$$

Particular solution

Finally, we try to find a particular solution to (2.8). We plug in the ansatz $\overline{x}_p(t) = A(\omega_d)e^{i\omega_d t}$ and after dividing by $me^{i\omega_d t} \neq 0$ we find

$$A(\omega_d)\left(-\omega_d^2 + i\frac{c}{m}\omega_d + \frac{k}{m}\right) = A(\omega_d)\left(-\omega_d^2 + i\gamma\omega_d + \omega_0\right) = \frac{F_0}{m}$$
(2.16)

After rearranging this equation, the amplitude of the driven oscillation can be written as a function of the driving frequency.

$$A(\omega_d) = \frac{F_0/m}{\omega_0^2 - \omega_d^2 + 2i\gamma\omega_d}$$
(2.17)

This leaves us with the following particular solution

$$\overline{x}_p(t) = A(\omega_d)e^{i\omega_d t} \tag{2.18}$$

General solution

In all cases the homogeneous solutions contribute a transient that decays on the time scale of γ^{-1} to the general solution of (2.8). After this short transient the solution is dominated by the particular solution found in (2.18). We see that the system oscillates at the frequency of the driving force with a (complex) amplitude given by (2.17). The system is then said to resonate with the driving force.

Resonance

The resonance amplitude $A(\omega_d)$ is a complex number, and as such it can be expressed by an absolute value |A| and a phase angle ϕ : $A(\omega_d) = |A(\omega_d)|e^{i\phi(\omega_d)}$. This yields the following expression for the complex position of the system as a function of time.

$$\overline{x}(t) = A(\omega_d)e^{i\omega_d t} = |A(\omega_d)|e^{i(\omega_d t + \phi(\omega_d))}$$
(2.19)

Now we return to the real problem in (2.7). Its solutions are given by

$$x(t) = \Re(\overline{x}(t)) = |A(\omega_d)| \cos(\omega_d t + \phi(\omega_d))$$
(2.20)

We see that the system oscillates at the driving frequency with an amplitude of $|A(\omega_d)|$ but out of phase with the driving force by a phase difference of $\phi(\omega_d)$. The precise behaviour of the resonance amplitude will be studied further in section 2.1.4.

2.1.3 Dissipation and the quality factor Q

The dampening force in the damped, driven harmonic oscillator dissipates energy at a rate

$$P_d(t) = F_d(t)x'(t) = -cx'(t)^2 = -cA^2\omega_d^2\sin(\omega_d t)^2$$
(2.21)

This amounts to an energy loss per oscillation cycle given by

$$E_{d} = \int_{0}^{T} P_{d}(t)dt = -cA^{2}\omega_{d}^{2}\int_{0}^{T} \sin(\omega_{d}t)^{2}dt = -cA^{2}\omega_{d}^{2}\int_{0}^{T} \frac{1}{2} + \frac{1}{2}\cos(2\omega_{d}t)dt$$

$$= \frac{cA^{2}\omega_{d}^{2}T}{2} = -2\pi \frac{cA^{2}\omega_{d}}{2}$$
(2.22)

The integration is understood to be taken over a single oscillation period $T = 2\pi/\omega_d$.

A widely used parameter to describe the dampening of a system is the quality factor or Q factor. It is defined as the ratio between the energy stored and the energy dissipated per radian of oscillation. For driving frequencies close to the resonance frequency, the energy stored in the oscillation is given by

$$E_{s} = K + V \approx \frac{1}{2}kA^{2}\sin(\omega_{d}t)^{2} + \frac{1}{2}mA^{2}\omega_{0}^{2}\cos(\omega_{d}t)^{2}$$

$$= \frac{1}{2}kA^{2}\left(\sin(\omega_{d}t)^{2} + \cos(\omega_{d}t)^{2}\right) = \frac{1}{2}kA^{2}$$
(2.23)

For the Q factor we find the expression

$$Q = \frac{E_s}{E_d/2\pi} \approx \frac{kA^2}{cA^2\omega_0} = \frac{\omega_0}{2\gamma}$$
(2.24)

The Q factor is actually a measure for the underdampedness of the system: a higher Q factor implies a less damped system. This is evident in the damped but undriven harmonic oscillator, where the Q factor equals the number of oscillations the system makes before its energy has dropped by a factor e.

2.1.4 Resonance peaks

Now we take a closer look at the expression for the complex resonance amplitude in (2.17) to describe the system's behaviour as it is driven at different frequencies.

The resonance oscillation amplitude $|A(\omega_d)|$ is given by

$$|A(\omega_d)| = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + 4\gamma^2 \omega_d^2}}$$
(2.25)

The phase difference $\phi(\omega_d)$ is given by

$$\phi(\omega_d) = -\arctan\left(\frac{2\gamma\omega_d}{\omega_0^2 - \omega_d^2}\right) \tag{2.26}$$

For lightly damped systems ($\gamma \ll \omega$), the maximum amplitude is approximately attained if the driving frequency matches the oscillation frequency of the undriven, undamped system. This maximum amplitude is equal to

$$|A_0| = \frac{F_0}{2m\gamma\omega_0} = \frac{1}{2\gamma} \frac{F_0\omega_0}{k} = Q \frac{F_0}{k}$$
(2.27)



Figure 2.2: Relative resonance amplitude and phase lag as a function of driving frequency. Each line represents the response of a system with a different Q factor, ranging from Q = 1 for the blue line to Q = 5 for the dark yellow line. The dashed line indicates the frequency and amplitude of the resonance maxima for different Q factors. It can be seen that the maximum resonance frequencies approach the natural frequency ω_0 as the Q factor increases.

At this driving frequency, the phase difference is $\phi_0 = -\pi/2$.

Figure 2.2 shows the resonance behaviour of systems with different Q factors. We see that systems with a higher Q factor driven by the same force will resonate with a higher amplitude. The width of the resonance peak is directly influenced by the dampening coefficient. It can be shown that the full width at half the maximum $\Delta \omega$ of the resonance peak equals 2γ . This enables us to determine Q factors from resonance spectra via

$$Q = \frac{\omega_0}{2\gamma} = \frac{\omega_0}{\Delta\omega} \tag{2.28}$$

2.2 Membrane vibrations

An essentially two-dimensional piece of material whose dynamic behaviour is dictated by in-plane tensional forces is called a membrane. An ideal membrane exhibits no rigidity against bending of the surface. Such approximations do indeed hold for some realistic systems such as the skin of a drum. It may also be applicable to some two-dimensional microstructures such as graphene monolayers. To describe the behaviour of membranes, we follow the derivation by Habermann [6] closely.

We introduce the displacement u(x, y, t) of the membrane as a function of position and time. If the displacements are sufficiently small, we can assume the membrane to move only perpendicularly to the plane of the membrane in rest. In that case, the equation that governs the motion of an ideal membrane is the two-dimensional wave equation, given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = c^2 \nabla^2 u \tag{2.29}$$

The wave velocity c depends on the tension T_0 in the membrane and its areal density ρ_0 through the relation $c^2 = T_0/\rho_0$. The Laplace operator is given by $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$. Given appropriate initial and boundary conditions, equation (2.29) can be solved analytically

Given appropriate initial and boundary conditions, equation (2.29) can be solved analytically in some cases. This applies in particular to circular geometries, which will also be of concern in this research. In the following section we will therefore explore the behaviour of a circular membrane with radius a with fixed boundaries.

For circular membranes it is useful to express positions in a polar coordinate system. Positions are then described by the radial distance r to the origin and the angle θ to a fixed axis. The wave equation now becomes:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u = c^2 \left(\frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial r^2} \right)$$
(2.30)

We define product solutions to solve this equation by separation of variables:

$$u(r,\theta,t) = \phi(r,\theta)h(t) \tag{2.31}$$

After separating variables and introducing the separation constant λ , we find a second order differential equation for the temporal part h(t), akin to the harmonic oscillator equation (2.4):

$$\frac{d^2h}{dt^2} = -\lambda c^2 h \tag{2.32}$$

We know its solutions to be given by a sinusoid that oscillates at an angular frequency $\omega = c\sqrt{\lambda}$, as illustrated in (2.5).

For the spatial part we are left with a two-dimensional eigenvalue problem:

$$\nabla^2 \phi = -\lambda \phi \tag{2.33}$$

Again we employ separation of variables by introducing:

$$\phi(r,\theta) = R(r)\Theta(\theta) \tag{2.34}$$

Plugging this in the eigenvalue equation (2.33) we get:

$$\frac{1}{r}\frac{\partial R}{\partial r}\Theta + \frac{\partial^2 R}{\partial r^2}\Theta + \frac{1}{r^2}\frac{\partial^2 \Theta}{\partial \theta^2}R = -\lambda R\Theta$$
(2.35)

We separate by dividing by $R\Theta$, multiplying by r^2 and introducing the separation constant m^2 :

$$\frac{1}{R}\left(r\frac{\partial R}{\partial r} + r^2\frac{\partial^2 R}{\partial r^2}\right) + \lambda r^2 = -\frac{1}{\Theta}\frac{\partial^2 \Theta}{\partial \theta^2} = m^2$$
(2.36)

Now we have an eigenvalue problem for each spatial component:

$$\frac{\partial^2 \Theta}{\partial \theta^2} + m^2 \Theta = 0 \tag{2.37}$$

$$r^{2}\frac{\partial^{2}R}{\partial r^{2}} + r\frac{\partial R}{\partial r} + (\lambda r^{2} - m^{2})R = 0$$
(2.38)

The eigenvalue problem for $\Theta(\theta)$ can be solved by taking the periodic nature of the θ -coordinate into account and applying appropriate boundary conditions. The eigenvalues are given by m^2 for $m = 0, 1, 2, \ldots$ For m > 0, two corresponding linearly independent eigenfunctions are given by

$$\Theta(\theta) = \sin(m\theta) \tag{2.39}$$

$$\Theta(\theta) = \cos(m\theta) \tag{2.40}$$

This reduces to one eigenfunction $\Theta(\theta) = 1$ for m = 0. The degeneracy of this eigenfunction expresses the different rotations of the possible eigenmodes.

The differential equation (2.38) is equivalent to the well-studied Bessel differential equation. Its solutions are given by linear combinations the Bessel functions of order $m: J_m(\sqrt{\lambda}r)$ and $Y_m(\sqrt{\lambda}r)$.

To find the solutions to the eigenvalue problem we now have to apply the appropriate boundary conditions. At r = 0 the solution should be well-behaved, i.e. it should not blow up. Therefore, we will require $|R(0)| < \infty$. This excludes the Bessel functions of second kind $Y_m(\sqrt{\lambda}r)$ as valid solutions to the eigenvalue problem. As the edge of the membrane is fixed, we will also require R(a) = 0. This implies that

$$J_m(\sqrt{\lambda a}) = 0 \tag{2.41}$$

Unfortunately, the roots of the Bessel equation cannot be expressed in closed form. We introduce z_{mn} as the *n*th root of $J_m(z)$, of which there are infinitely many for each m. This allows us to solve for the eigenvalues:

$$\lambda_{mn} = \left(\frac{z_{mn}}{a}\right)^2 \tag{2.42}$$

These can be related to the natural oscillation frequencies ω_{mn} of the eigenmodes of the membrane:

$$\omega_{mn} = c\sqrt{\lambda} = \frac{c}{a} z_{mn} \tag{2.43}$$

Table 2.1 lists the frequencies for the first few modes of a circular membrane.

At last we are able to formulate the eigenmode solutions to the wave equation 2.29 for a circular membrane. They are given by

$$u_{mn}(r,\theta,t) = R_{mn}(r)\Theta_m(\theta)h_{mn}(t)$$

= $u_0 \sin\left(\frac{c}{a}z_{mn}t - \phi_0\right) J_m\left(z_{mn}\frac{r}{a}\right) \sin\left(m(\theta - \theta_0)\right),$ (2.44)

where u_0 is the amplitude, ϕ_0 the initial phase and θ_0 the rotation angle. The corresponding mode shapes are depicted in figure 2.3.



Figure 2.3: Mode shapes for a circular membrane of radius a. The separation constant m indicates the number of nodal lines while n indicates the number of nodal circles (including the outer circle).

Table 2.1: Natural frequencies of eigenmodes of a circular membrane. The modes are sorted by frequency in ascending order

m	n	ω
0	1	ω_0
1	1	$1.59\omega_0$
2	1	$2.14\omega_0$
0	2	$2.30\omega_0$
3	1	$2.65\omega_0$
1	2	$2.91\omega_0$

2.3 Plate vibrations

Two-dimensional continuous objects whose motion is governed by the bending rigidity of the material are called plates. One of the most basic theories to describe the behaviour of plates is the classical or Kirchoff plate theory [4, 13, 9]. Under the assumptions made in this theory, the motion of a plate can be completely described by the z-displacement w(x, y, t) of the plate mid-plane as a function of position and time. In the absence of external forces, this mid-plane displacement is governed by the following equation of motion:

$$\frac{\partial^2 w}{\partial t^2} = -\frac{D}{\rho h} \left(\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 y^2} + \frac{\partial^4 u}{\partial y^4} \right) = -\frac{D}{\rho h} \nabla^4 w \tag{2.45}$$

In this equation, ρ is the volume density of the material, h is the thickness of the plate and $\nabla^4 = \nabla^2 \nabla^2$ is the biharmonic operator. The bending stiffness D is a structural parameter of the plate derived from its height h and two material properties: the Young's modulus E and the Poisson's ratio ν . It is defined as

$$D = \frac{Dh^3}{12(1-\nu^2)} \tag{2.46}$$

The vibrations of the two-dimensional microdrums encountered in this research might be modeled as plate vibrations. We will therefore explore the behaviour of a plate with radius a. It is easiest now to express positions in polar coordinates.

In polar coordinates, the bending moment in the plate in the r-direction is given by [9, p. 2]:

$$M_r(r,\theta) = -D\left[\frac{\partial^2 w}{\partial r^2} + \nu \left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right]$$
(2.47)

Equation (2.45) is a fourth-order differential equation. As such, four boundary conditions are necessary to solve it. In a circular geometry, two of these boundary conditions are provided by requiring regularity at the origin r = 0:

$$|w(0,\theta,t)| < \infty \tag{2.48}$$

$$\left|\frac{\partial w}{\partial r}(0,\theta,t)\right| < \infty \tag{2.49}$$

The other two boundary conditions must be derived from the attachment at the edge of the plate. A few common cases are listed below.

1. *Fixed edges.* In this case the edges are clamped into the surrounding material, fixing its position as well as the slope of the plate at the edge. Mathematically this is expressed as

$$w(a,\theta,t) = 0 \tag{2.50}$$

$$\frac{\partial w}{\partial r}(a,\theta,t) = 0 \tag{2.51}$$

2. Simply supported edges. In this case the edges are attached pointwise to the surrounding material. No bending moment can be exerted on the plate by the environment, but its position is fixed. Mathematically this is expressed as

$$w(a,\theta,t) = 0 \tag{2.52}$$

$$M_r(a,\theta,t) = -D\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] = 0$$
(2.53)

Once again, separation of variables proves to be useful to solve (2.45). We start by defining the product solutions

$$w(r,\theta,t) = \phi(r,\theta)T(t) \tag{2.54}$$

Separating and introducing the separation constant λ^4 yields

$$-\frac{\rho h}{D}\frac{1}{T}\frac{d^2T}{dt^2} = \frac{1}{\phi}\nabla^4\phi = \lambda^4$$
(2.55)

$$\frac{d^2T}{dt^2} = \lambda^4 \frac{D}{\rho h} T \tag{2.56}$$

$$\nabla^4 \phi = \lambda^4 \phi \tag{2.57}$$

The temporal differential equation yields a sinusoidal oscillation at an angular frequency $\omega = \lambda^2 \sqrt{D/(\rho h)}$. The spatial eigenvalue problem can be factored into

$$(\nabla^2 + \lambda^2)(\nabla^2 - \lambda^2)\phi = 0 \tag{2.58}$$

To solve this equation, we must require at least one of the factors to be zero. This generates two second-order eigenvalue problems, of which the solutions are then also solutions to (2.58). These two eigenvalue problems are given by

$$(\nabla^2 + \lambda^2)\phi = 0 \tag{2.59}$$

$$(\nabla^2 - \lambda^2)\phi = 0 \tag{2.60}$$

The first eigenvalue problem is similar to the one we found in equation (2.33) for the membrane. We know its solutions to be given by

$$\phi(r,\theta) = J_m(\lambda r)\cos(m(\theta - \theta_0)) \tag{2.61}$$

with eigenvalues λ .

After separating the radial part R(r) and the angular part $\Theta(\theta)$, it can be shown that an differential equation is obtained for the angular part that is equivalent to the modified Bessel's differential equation. Its solution is given by a linear combination of the eigenfunctions

$$R(r) = I_m(\lambda r) \tag{2.62}$$

$$R(r) = K_m(\lambda r) \tag{2.63}$$

Table 2.2: Natural frequencies of eigenmodes of a circular plate with clamped boundary conditions. The modes are sorted by frequency in ascending order [9, 10]

m	n	λa	ω
0	1	3.20	ω_0
1	1	4.61	$2.08\omega_0$
2	1	5.91	$3.41\omega_0$
0	2	6.31	$3.89\omega_0$
3	1	7.14	$5.00\omega_0$
1	2	7.80	$5.95\omega_0$

The functions I_m and K_m are known as the modified Bessel functions of order m of the first and the second kind respectively. At r = 0, K_m diverges and therefore it does not satisfy the regularity boundary condition at the origin. Thus, for a circular plate only I_m is a valid solution.

The solution for the angular part of this eigenvalue problem is again given by $\Theta(\theta) = \cos(m(\theta - \theta_0))$ for $m = 0, 1, 2, \ldots$ and θ_0 an angle of rotation of the mode.

By superposition, the solutions to (2.58) are then given by

$$\phi_{\lambda}(r,\theta) = [a_{\lambda}J_m(\lambda r) + b_{\lambda}I_m(\lambda r)]\cos(m(\theta - \theta_0))$$
(2.64)

2.3.1 Fixed edges

To find the possible eigenvalues λ in a plate with fixed edges we apply the boundary conditions (2.50) and (2.51). The first boundary condition yields

$$a_{\lambda}J_m(\lambda a) + b_{\lambda}I_m(\lambda a) = 0 \tag{2.65}$$

Now we can express b_{λ} in terms of a_{λ} :

$$b_{\lambda} = -\frac{J_m(\lambda a)}{I_m(\lambda a)} \tag{2.66}$$

It can be shown [9] that applying the second boundary conditions yields the relation

$$J_m(\lambda a)I_{m+1}(\lambda_a) + I_m(\lambda a)J_{m+1}(\lambda a) = 0$$
(2.67)

The roots λ_{mn} of this relation dictate the eigenfrequencies of the plate. Values for the first few eigenfrequencies can be found in table 2.2 and the accompanying mode shapes in figure 2.4.



Figure 2.4: Mode shapes for a circular plate of radius a with fixed edges. Note that they are very similar to the membrane mode shape except at the edges. There they are flatter because the edges are clamped stiffly. 18

2.4 Thermal excitation and Brownian motion

At the microscopic level, thermal energy is made up of the motion and vibration of individual atoms and molecules. These microscopic particles bump in to each other and exchange energy in pattern that behaves randomly if viewed at the macroscopic scale. The microscopic motion of systems in this world of random collisions is called Brownian motion.

2.4.1 Spectral density

Random forces from thermal fluctuations originate from independent processes, implying that the forces that act on a system follow a Gaussian distribution. This leads to so-called Brownian motion [14]. The precise motion of the system is of course random, but a useful way to characterize the resultant motion is through its spectral density. The spectral density of the trace of a random process is defined as the expected value of the Fourier transform of its energy, defined as the square of the amplitude. For thermal noise, the power spectral density is constant: $S(\omega) = S_0$.

We would like to be able to determine the response of a system to a random input. Fortunately, this can be easily calculated for linear systems. If a stochastic signal with a certain power spectral density $S_0(\omega)$ is fed into a linear system, the power spectral density $S(\omega)$ of the output of the linear system is given by multiplying $S_0(\omega)$ and the frequency response $|H(\omega)|^2$ of the system:

$$S(\omega) = S_0(\omega) |H(\omega)|^2$$
(2.68)

2.4.2 Harmonic oscillation driven by thermal noise

Thermal noise can also be the driving force F(t) for a harmonic oscillator. To investigate this, we return to equation (2.3):

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$
(2.3 revisited)

Its frequency response can be calculated from (2.17) and is given by

$$|H(\omega)|^2 = \frac{\omega_0^4}{k^2} \frac{\omega_0^4}{(\omega_0^2 - \omega^2)^2 + 4\gamma^2 \omega^2} = \frac{1}{m^2} \frac{1}{(\omega_0^2 - \omega^2)^2 + \omega^2 \omega_0^2/Q^2}$$
(2.69)

As it is a linear system, the spectral density of the position of the oscillator can then be calculated using (2.68) and is given by $S_x(\omega) = S_F(\omega)|H(\omega)|^2$, where $S_F(\omega) = S_0$ is the constant noise force spectral density.

The equipartition theorem of thermodynamics states that the average energy in every degree of freedom in the system equals $(1/2)k_BT$ when the system is in thermal equilibrium with its environment. For the harmonic oscillator, which has a degree of freedom in its position and in its velocity, this means that the total average energy equals [5]

$$\langle E \rangle = k_B T \tag{2.70}$$

The average thermal energy can be calculated from the power spectral density of the position:

$$\langle E \rangle = 2 \langle V \rangle = m \omega_0^2 \langle x^2 \rangle = \frac{m \omega_0^2}{2\pi} \int_0^\infty S_x(\omega) \, d\omega = \frac{m \omega_0^2}{2\pi} \int_0^\infty |H(\omega)|^2 S_F(\omega) \, d\omega$$

$$= \frac{m \omega_0^2 S_0}{2\pi} \int_0^\infty |H(\omega)|^2 \, d\omega = \frac{m \omega_0^2 S_0}{2\pi} \cdot \frac{\pi Q}{2m^2 \omega_0^3} = S_0 \frac{Q}{4m \omega_0}$$

$$(2.71)$$

Equating (2.70) and (2.71) provides an expression for the noise force spectral density:

$$S_0 = \frac{4k_B T m \omega_0}{Q} \tag{2.72}$$

Ultimately an expression for the position spectral density is obtained:

$$S_x(\omega) = \frac{4\omega_0 k_B T}{Qm} \frac{1}{(\omega_0^2 - \omega^2)^2 + \omega^2 \omega_0^2/Q^2}$$
(2.73)

Chapter 3

Microdrum resonance and laser interferometry

The focus of this research was studying the resonance behaviour of an oscillating microdrum. This chapter will shine a light on the methods used to this end. The techniques and setup described in this chapter have been used as a starting point for the development of the spatial scanning setup built in this project, which will be elaborated in chapter 4.

3.1 Thin microdrums

A thin flake of a 2-dimensional material such as graphene or molybdenum disulfide is suspended on top of a hole of a few microns in diameter etched in a substrate material. The flake is held in place by van der Waals forces between the substrate and the flake in the area surrounding the hole. Over the hole itself the flake is free to move up and down like the skin of a drum.

3.2 Microdrum resonance measurement techniques

The resonance behaviour of microdrums has been studied extensively in both the frequency and time domain using laser interferometry setups. Frequency response measurements [3, 2] (figure 3.1) show the presence of multiple vibrational modes, which can be contributed to multiple plate or membrane modes depending on the thickness and the material properties of the microdrum. In addition to the resonance frequency of these modes, other resonance parameters that can be deduced from frequency response measurements include the line width of the resonance peak, the Q factor and the phase lag with respect to the driving signal.

More recently, the resonance behaviour of microdrums has also been studied in the time domain. The relaxation or 'ring-down' of microdrums to their equilibrium state was measured [8], illustrated in figure 3.2. This kind of measurement opens the possibility of measuring the ring-down Q-factor from the time domain dampening behaviour in addition to the spectral Q-factor derived from the frequency domain resonance behaviour.

The next interesting measurement domain to study the resonance behaviour of microdrums is the spatial domain. This would enable us to image the actual shape of the vibrational modes and to link these to the theoretically predicted membrane or plate modes. It would also enable us to



Figure 3.1: Resonance behaviour of circular MoS_2 mechanical oscillators in the frequency domain, as reported by Castellanos-Gomez et al. [3]. (a) Measured resonance frequency as a function of the number of layers for MoS_2 mechanical resonators 2 µm in diameter (red squares in the top panel) and 3 µm in diameter (blue circles in the lower panel). The lines indicate the calculated frequency vs. thickness relationship for different cases: circular membrane (dotted), circular plate (dashed) and the combination of these two cases (solid). A The inset in the lower panel in (a) includes data measured for seven thick MoS_2 resonators (more than 30 layers), which are in the plate limit. (b) Mechanical resonance spectra measured for a single-layer MoS_2 resonator (top panel) and a nine layers thick MoS_2 resonator (lower panel), showing higher order modes. While for the single-layer the higher modes appear at frequencies similar to those expected for a circular membrane, for the thicker resonator the resonance spectra agrees with a circular plate.

measure the variations of other resonance parameters such as the Q factor, resonance frequency or line width over the surface of the drum. Opening up the spatial domain for resonance measurements has been the focus of this research project, on which chapter 4 will elaborate.

3.3 Deflection measurement by interferometry

The local deflection of the drum is measured using laser interferometry, as shown in figure 3.3. The beam of the measurement laser is focused onto a particular spot where it is partially reflected off the drum and partially transmitted through. The transmitted light will hit the bottom of the hole over which the drum is suspended and be reflected from there. This bottom-reflected light is then transmitted through the drum again (neglecting higher-order reflections in the cavity), coinciding with the light that was initially reflected off the drum.



Figure 3.2: Time domain ring-down measurements on microdrums, as reported by Van Leeuwen et al. [8]. (a) Frequency response (magnitude |H| and phase ϕ) of a single layer MoS₂ resonator with a diameter of 2 µm, and a harmonic oscillator fit with $f_0 = 22.2$ MHz and the $Q_S = 41.5 \pm 0.3$. (b) Ring-down measurement of the same device, and exponential fit with $\tau = 0.6$ µs, which corresponds to $Q_R = 41.8 \pm 0.4$.



Figure 3.3: Simplified schematic cross-section of the few layer graphene (FLG) microdrum used in this research. A circular hole is etched into a silicon oxide substrate on which gold-paladium is deposited as a ground electrode. At the bottom of the hole another layer of gold-paladium is deposited as a gate electrode, enabling electrical actuation of the microdrum. On top of this hole an FLG flake is stamped as drum surface. To measure the local deflection u(x, y) of the microdrum, laser interferometry is employed with the drum surface acting as a moving mirror for this Fabry-Pérot cavity. The incoming laser beam I, which is depicted under an angle here for conceptual clarity, is partially reflected off the drum surface into beam R_1 and partially transmitted through into beam T_1 . Beam T_1 is then reflected off the cavity bottom as beam R_2 . This beam is in turn transmitted through the drum surface again as beam T_3 (neglecting higher order reflections). The optical path difference Δl between beams R_1 and T_3 will eventually cause interference in the reflected beam. This enables us to relate the interference intensity in the reflected beam to the local drum deflection u(x, y). The interference intensity response to the deflection is maximal when the undeflected interference intensity is exactly at the halfway point between destructive and constructive interference. This happens if the cavity depth is an odd multiple of one eighth of the laser wavelength λ : $\Delta l \approx \frac{2k+1}{8}\lambda$. For this setup $\lambda = 633 \text{ nm}$ and $\frac{\Delta l}{\lambda} = \frac{280 \text{ nm}}{633 \text{ nm}} = 0.44$, which is not ideal but near the $\frac{3}{8}\lambda$ responsivity maximum. Note that Van der Waals attraction near the edges of the drum and the electrical actuation force might pull the drum surface further downward, thus moving the drum closer to the responsivity maximum.

The extra distance traveled by the bottom-reflected light will cause it to pick up an extra phase relative to the drum-reflected light. This phase difference induces a variation in the intensity of the reflected beam through interference. A photodiode is used to measure this intensity, providing a measure for the optical path difference and the deflection.

When the setup is aligned and adjusted properly, the photodiode voltage responds approximately linearly to the deflection of the drum for deflections on the order of magnitude of interest. This is expressed by the responsivity \Re :

$$V_{ph}(t) = \Re u(t), \tag{3.1}$$

where $V_{ph}(t)$ is the photodiode voltage and u(t) the local deflection of the drum.

As \Re depends on the thickness of the drum, the refractive index of the drum material and the cavity dimensions, it has to be determined individually for every microdrum under investigation. If for some situation the deflection is known, this is straightforward. A suitable method for calibrating the responsivity is by using the expected value of the deflection power when the drum is excited by thermal Brownian motion, which is given by (2.73). Unfortunately, this involves a detailed calculation of the effective modal masses of the resonance modes, which is outside the scope of this research.

The setup that is present in the Room Temperature Nanomechanics Lab is described in detail in the supplementary material in reference [3]. A schematic representation is presented in figure 3.4.

3.4 Actuation of the microdrum

Two methods of external actuation of the microdrum were employed: optothermal actuation and electric actuation. In the first method, a blue laser ($\lambda = 405 \text{ nm}$), is focused on the drum. A varying driving signal is modulated on top of the laser power, heating up the drum periodically. Although the heat is applied locally, it is spread through the drum much faster than the oscillation period of the actuation. These temperature changes are then converted to mechanical vibrations by thermal expansion and contraction.

For the second method of actuation the driving signal is supplied to the electrode at the bottom of the cavity with a DC offset voltage. The varying electric potential at the bottom electrode induces periodic electrostatic attraction of the electrons in the drum material, resulting in mechanical vibrations.

The maximal drum deflection that can be achieved is greater under optothermal actuation than under electrical actuation, as the maximal gate voltage is limited by the breakdown voltage of the insulating SiO_2 layer between the gate electrode lead and the ground electrode of about 16 V.

3.5 Driving and readout of the setup

A vector network analyzer (VNA), Rohde & Schwarz model ZVB-4, is used to simultaneously drive the actuation (be it either electrical or optothermal) of the microdrum and read out the photodiode response. The VNA outputs sinusoidally varying voltages while sweeping the frequency. It monitors the response of the system at that same frequency, measuring the response amplitude as well as its phase lag with respect to the driving signal.

For the Brownian motion measurements, the photodiode is connected to a spectrum analyzer (*Rohde & Schwarz* model FSL). The spectrum analyzer gives the Fourier transform of the input signal over a given frequency range.



Figure 3.4: Schematic representation of the optical setup used to measure the deflection of microdrum resonators [3]. The sample with the suspended drum is mounted into a vacuum chamber. In front of the vacuum chamber a microscope objective (OBJ) is mounted to focus incoming light onto a single spot on the drum. The red HeNe laser acts as the interferometry laser in the cavity. The reflected light is projected by a beam splitter (PBS) onto a photodiode (PD) to measure the interference intensity. The blue diode laser is used for actuation and projected onto the drum by a dichroic mirror (DM), which only reflects blue light but lets red light pass through unhindered. Another beam splitter (M) is used to image the drum and the laser spots on a camera CCD.

Chapter 4

Development of a spatial scanning interferometry setup

4.1 Introduction

In this project we expanded the interferometry setup described in the previous chapter with spatial scanning capabilities, enabling us to measure resonance behaviour of microdrums in the spatial domain. This setup was then used to perform measurements on a few-layer graphene microdrum (figure 5.1). The results of these measurements will be discussed in detail in the next chapter.

The main idea of our spatial scanning setup is that the interferometry laser spot can be moved relative to the drum. The laser is scanned over the drum surface and at each position relevant resonance parameters are extracted from the local frequency response under actuation or from the Brownian motion power spectrum. These local parameters, such as resonance amplitude, frequency, phase with respect to the driving signal, Q factor and line width can then be plotted spatially. The spatial resolution is only physically limited by the diffraction limited finite laser spot size, which is discussed in section 4.8.

Spatial amplitude and phase information provide a way to actually image the vibrational mode shapes. The other parameters might also provide information about other interesting physical properties, such as for example thermal transport properties through the resonance frequency temperature response.

4.2 Main approach to spatial scanning

To perform a spatial scan of the local deflection of the microdrum, two approaches have been explored: fixing the sample while scanning the interferometer laser beam and fixing the laser beam while moving the sample. In both methods, the extremely small movements that are needed to obtain significant spatial resolution on a drum with lateral dimension of only a few micrometers would be produced by Picomotors, which are described in more detail in section 4.3.

Initially the first method seemed to be the easiest way to achieve spatial scanning, as we already had a very precisely rotatable picomotorized mirror stage at our disposal. However, as the laser beam covers the entire entrance pupil of the objective, steering it using only one mirror means that



Figure 4.1: Left: Picomotor model 8301, right: motorized optical mount model 8816-6. Both are manufactured by New Focus. These devices were used in this research project.

part of the beam falls outside of the entrance pupil. Furthermore, the laser beam would enter the objective under an angle, possibly deteriorating the reflected signal.

These considerations led to the development of the second method. We mounted the Picomotor screws onto the sample stage, replacing the manual micrometer screws. This enabled us to move the sample around while keeping the laser spot focused at the same spot. The rest of this chapter will be devoted to explaining in detail the implementation of this method. The experimental challenges, which include hysteresis in the picomotor behaviour, calibration of the sub-µm motion and locating the laser spot position are discussed.

4.3 **Picomotor characterization**

Picomotors, shown in figure 4.1, are used to provide the precise motions needed for spatial scans of high resolution. The picomotor is a very high resolution linear actuator; it offers a step size on the order of tens of nanometers. This little actuator is used to align the measurement laser and the microdrums very precisely so that the oscillation of the microdrums can be measured as a function of the location.

Figure 4.2 shows a schematic of the inner working of the picomotor, which is a screw that is turned by two jaws in a way that is very similar to turning a screw by hand, using the thumb and the index finger. The jaws first turn the screw by moving slowly and clamping the screw by static friction. To get back to the initial position for the next move, the jaws move back relatively quickly. The screw then stays in place due to its inertia and the fact that dynamic friction is weaker than static friction. The thread on the screw is very fine (31.5 threads per cm), enabling a very small linear displacement per step typically in the order of 30 nm.

The jaws are put in motion by a piezoelectric element that expands or contracts under the influence of an applied voltage. By rising the voltage quickly or slowly the speed of the jaw movement can be controlled. A single step consists of a slow rise in voltage to move the jaws slowly and a quick fall to move the jaws back quickly. By repeating this voltage waveform multiple steps can be performed, turning the screw over larger angles. The waveform frequency of the driving signal directly corresponds to the stepping rate of the motor.



Figure 4.2: Schematic overview of the working of a picomotor actuator [1]. A: Inside the picomotor, a screw is clamped by two jaws that are connected to a piezo-electric element that can contract and expand under the influence of an applied voltage. B: The jaws first move slowly, turning the screw by static friction. C: The jaws return to their initial state in a quicker motion, leaving the screw at rest due to the screw's inertia and the weaker dynamic friction.

When no power is applied, the screw stays in its position: this makes it very suitable for setand-hold applications. This set-up takes advantage of that by using it to 'set' the measurement alignment and then 'holding' it during measurement.

Although this system is very suitable for precise motion control, it also has a profound downside: poor repeatability. The step size may depend on load, direction and wear of the picomotor.

As the load on the screw increases, the screw experiences more friction between its screw thread and the screw thread in the housing. This makes the jaw movement more inclined to slip, resulting in a smaller rotation. As a result, step size can vary considerably. Furthermore, the step size may depend on the direction of motion, because the piezo element might behave differently when contracting than when expanding. Even the wear that comes with using the picomotor may impact the step size as little scratches or particles of dust change the friction inside the picomotor. This makes it hard to determine the actual position of the screw, based on prior movements of the picomotor.

Operating the picomotor without any feedback on the position of the screw is called open loop operation, and is very inaccurate because of the aforementioned reasons. An important part of designing the set-up therefore consisted of devising a suitable feedback system for controlling the motion of the picomotor and its attachments. To get a first impression of the capabilities of the picomotors, we carried out preliminary experiments to characterise their motion and compare their performance to the specifications supplied by the manufacturer.

4.3.1 Specifications

The picomotor that was used in this set-up is Picomotor model 8301, manufactured by New Focus. In addition, optical mount model 8816 by the same company, that incorporates these picomotors as well, was used. The motors are driven by a waveform generator (*New Focus* type 8742) that has been designed specifically for these motors. This driver can be controlled by a computer over a GPIB-interface.

The motors can operate at a maximum rate of 2000 steps per second. This happens if the motor is supplied with a driving signal of 2000 Hz. However, the driver we used (model 8732 by New Focus) has a maximum frequency of 1500 Hz. Moreover, the driver can only generate signals at frequencies that are integer divisions of the maximum frequency of 1500 Hz, down to a minimum of 0.023 Hz (1500 Hz/65535).

Model 8301 and 8816

The specifications of the picomotors we used as provided by the manufacturer are listed in table 4.1. With respect to the repeatability of the step size, the manufacturer gives the following information: "We have no statistical data on the variation in step size since the step size depends on a number of factors such as load and age. The Picomotor may not be an appropriate solution to replace stepper motors in applications that require repeatable step sizes.". A graph of step size versus load as supplied by the manufacturer is presented in figure 4.3.

Table 4.1: Specifications of picomotor model 8301 and motorized optical mount 8816 as provided by the manufacturer.

Motorized screw 8301			
Travel range	12.7 mm (0.5 inch)		
Minimum Incremental Motion	<30 nm		
Angular resolution (screw rotation)	<0.6 mrad		
Motorized optical mount 8816			
Angular range	±4 °		
Angular resolution	$0.7 \ \mu rad$		
Motorized axes	2		
Load capacity	22 N		
Drive torque	$0.018 \ \mathrm{Nm}$		
Maximum driving frequency	2 kHz		
Operating temperature	10-40 °C		

4.3.2 Beam steering

The first experiments we conducted were intended to explore the capabilities of these picomotors and their potential shortcomings that needed to be dealt with. Also a lot of the techniques and computer software that were needed to operate the actual set-up were developed in this stage of the research, such as the camera read-out software and image processing techniques.



Figure 4.3: Average Picomotor step size versus load, as supplied by the manufacturer [12]. This data was taken using a interferometer as the picomotor lifted loads ranging from 1 to 6 pounds (0.45 to 2.72 kg).

First try-out: pointing at the wall

As a first try-out, the optical mount was fitted with a mirror, at which a laser pointer was aimed. The reflected beam was projected on a wall approximately 4 meters from the mirror. The mirror was then moved back and forth with the picomotors, causing the laser spot on the wall to move.

The effects of a non-constant step size became apparent immediately: moving the mirror for a certain number of steps in one direction and subsequently moving back by the same number of steps showed that the spot on the wall did not return to its initial position. Repeating the same experiment along the other motorized axis of the optical mount yielded similar results. From this we concluded that the average step size differed between directions and axes.

First steps towards automated quantitative step size measurements

To get some quantitative data on the step size, an automated set-up was designed to measure the movement of the laser spot in response to mirror rotations by the picomotors. The wall as an imaging device in the previous experiment was replaced by a webcam at a distance of about 37 cm from the center of the mirror. The mirror was positioned so that the beam would be reflected at an angle of 90° (i.e. the mirror surface is at an angle of 45° to the incoming beam). A photograph of this set-up can be seen in figure 4.4.

Figure 4.5 shows the geometry of this set-up in detail. The changes in distances and angles brought forward by actuating the picomotor are small enough to justify small-angle approximations in deriving their relations. Upon displacing the mirror at the picomotor screw mounting point by a length Δb , we find the following expression for the mirror rotation angle α .

$$\alpha \approx \sin(\alpha) = \frac{\Delta b}{a} \tag{4.1}$$



Figure 4.4: Preliminary set-up to measure quantitative data on the step size of the picomotors. A laser pointer (L) is directed at a picomotorized optical mount fitted with a mirror (M) at 45° to the incoming beam. The reflected beam is projected directly, without a lens, on the CCD of a webcam (C). As the mirror is rotated by the picomotors, the spot moves over the CCD. The position of the spot on the CCD can be calculated by processing the webcam image. The black stopper left of the mirror is used to block secondary reflections produced by the semi-transparent mirror used in this set-up.

If we neglect the shift in the point where the beam hits the mirror after rotation, we can derive the following relation for the displacement of the laser spot on the CCD.

$$\Delta x = l\sin(2\alpha) \approx 2l\alpha \approx \frac{2l}{a}\Delta b \tag{4.2}$$

The main load on the picomotor screw when rotating the mirror is produced by the attachment spring.

Equation 4.2 shows an approximate linear relation between Δx and Δb . Every picomotor motion is magnified by a factor $\frac{2l}{a} \approx 17.8$ on the CCD. Any initial rotation of the mirror before rotating the mirror in a step size measurement is of negligible influence on this magnification factor. As a fortunate consequence, the laser spot displacements on the CCD are independent of the initial screw length.

The next stage in designing this experiment was developing a method to calculate the position of the center of the laser spot on the CCD automatically. The first method we tried was using the image circle finding function supplied with Matlab. Internally, this function first uses an edge detection algorithm to extract the boundaries of objects in the images. It then tries to find circular edges in the image by using the Hough Circle Transform (HCT) [7]. This algorithm checks for every pixel in the image if it could potentially be the center of a circle by fitting circles of varying radius r around that pixel and calculating how many edge points in the image that fitted circle overlays. The pixels with the highest (local) scores are then marked as circle centers.



Figure 4.5: Geometry of the set-up used to measure the step size of the picomotors (figure 4.4). The picomotorized optical mount (on the right) is fitted with a mirror (thick line), attached to the mount by a spring. The mirror hinges on the fixed point H and on the picomotor screw. This figure shows one of the two motorized axes of the optical mount. The mirror is used to reflect the laser beam incoming from below towards the CCD of a webcam (on the left). In this configuration, a typical picomotor step of 30 nm corresponds to a displacement of 530 nm. This is approximately 0.05 px on the webcam CCD in live capture mode.



Figure 4.6: Typical image of the laser spot as recorded by the webcam (Philips type SPC900NC) CCD. The black spots in the image are dirt particles on the CCD.

Figure 4.6 shows an image of the laser spot as acquired by the webcam. Unfortunately, the laser spot image was not very circular but rather elliptical. Furthermore, the webcam image proved to flicker appreciably, especially around the edges of the spot. As the HCT algorithm is based on extrapolating circle centers from the shape of circular edges in the image, this made the detection of the center of the laser spot inaccurate. Without moving the laser spot, the detected center would typically fluctuate over a distance of about 10% of the spot diameter: far too inaccurate for actual step size measurements which are on the order of 100 nm or less. Adjusting HCT or edge detection parameters and even switching to Python/OpenCV for other implementations of the HCT and edge detection algorithms did not prove to be fruitful.

Spot detection algorithm

In an attempt to solve the image quality problems encountered in the webcam approach, the webcam was replaced by a digital single lens reflex camera (DSLR) - a Canon EOS 1200D. As the CCD and camera control options of this camera were far more advanced compared to the webcam, the image quality was much better. Furthermore, a camera of this type is already mounted in the interferometer setup, where it is used to image the drum and the laser spot. The rest of the design of the set-up was kept the same.

Reading images from the Canon camera did not prove to be trivial as a useful Python driver was not available, but eventually the camera image could be reliably transferred into a Python environment for further processing. To this end, a custom Python wrapper for the Canon EDSDK camera driver was written.

In addition, another method for determining the spot center in the recorded image was developed, based on calculating the virtual center of gravity (centroid) of the spot. First, all pixels corresponding to the laser spot were selected by picking all pixels whose intensity fell above a visually selected threshold value. Then the center of gravity of the spot was established by calculating the average x- and y-position of all pixels deemed to belong to the laser spot. The main advantage of this method over the HCT circle detection algorithm is that the precise shape of the spot doesn't matter as long as its (thresholded) image is roughly symmetric (which it seems to be in our case). Figure 4.7 shows an example result of this algorithm.



(a) Direct image of the laser spot.



Figure 4.7: Laser spot as recorded by the Canon EOS 1200D camera sensor and subsequent spot center calculation using the centroid method. The blue diamond corresponds to the calculated spot center. In the thresholded image, all pixels with a blue intensity value was above 40% of the maximum were selected. From these selected pixels the centroid was calculated as the spot center.

This center-detection algorithm turned out to be very stable: next to no scatter in the calculated center position was observed for repeated measurements. As for accuracy, the calculated center seemed to correspond to the center of the spot nicely upon visual inspection, as long as the center of the spot was still on the CCD.

Calibrated scanning of laser

Now it is possible to perform an actual experiment to measure step size as a function of direction and of initial screw length. At the start of each measurement run an initial screw length was set. Next the rotation of the entire optical mount adjusted so that the laser spot hit the CCD in a certain initial location (in the lower left corner as seen from the direction of the laser beam).

From this initial location the mirror was first rotated right for 20k steps and then rotated left for the same number of steps. The final locations of the laser spot on the CCD after each track were calculated and saved. The spot is then guided back to its initial position for the next measurement track. These measurement tracks were repeated twice for each different initial screw length.

Figure 4.8a presents the results of this experiment. The step size decreases as the screw length and thus the attachment spring force increases, which was expected. Furthermore a difference between step size and rotation direction was found. This was also observed by just measuring the change in length of the screw when it rotated unloaded in both directions. A possible explanation for this effect is that the piezoelectric element behaves differently when operating in different directions.

Figure 4.8b shows the results of a similar measurement for different driving frequencies. The driving frequency seems not to influence the step size significantly, except for the maximum frequency.



Figure 4.8: Step size variation. The laser spot displacement Δx for 20k steps and the individual step size (secondary y-axis) is plotted versus initial screw length b (a) and driving frequency f (b) for two measurement runs per measurement point. Crosses represent the left/"screw in" direction against the spring force, circles represent the right/"screw out" direction for the set-up geometry as outlined in figure 4.5. The effect of the increasing pre-load by the spring for increasing screw lengths is visible as a decrease in step size in figure (a). The driving frequency seems not to be of significant influence on the step size, except for the maximum frequency.



Figure 4.9: Partial interferometry set-up used to developing suitable methods for drum positioning using a camera (C) for feedback. A white light source (L) is used to illuminate the sample via a semi-transparent mirror (M) and a beam splitting prism (B). The laser light in the actual interferometry set-up would follow the red dotted line. Both the laser light and the white light are focused on to the sample stage (S) by objective lens (O). Part of the reflected light is passed through beam splitter (B) towards the rest of the interferometry set-up, and part of the reflected light is deflected towards the camera (C), passing through the semi-transparent mirror (M) along the way.

4.4 Drum positioning using camera feedback

For the actual mode shape measurements, feedback on the current position of the drum relative to the interferometry laser spot is desired. One method of acquiring such feedback is by including a camera in the set-up to watch and track the position of the drum as the motorized sample stage moves the drum around.

In the interferometry set-up a camera can be included by adding a beam splitter just before the final objective lens and the sample, as is shown in figure 3.4. The beam splitter is used to illuminate the sample with white light from an LED and allows the camera to receive part of the reflected light. To develop suitable software algorithms for drum positioning, this part of the interferometry set-up was built separately as an experimental sandbox. Figure 4.9 illustrates this partial set-up. To recreate the circumstances of the actual interferometry setup, a chip with circular cavities etched in the substrate, on which 2D-crystal flakes are to be stamped in a next step, was mounted on the sample stage.

The focus of the image on the camera could be adjusted by changing the distance between the objective and the sample. When in focus, a sample image of a drum cavity on the camera is shown in figure 4.10.



Figure 4.10: Image of the try-out chip mounted on the sample stage, as projected on the camera CCD in the set-up illustrated in figure 4.9. The drum cavity finding algorithm we developed was instructed to find the center of gravity of detected shapes in the red box. The red diamond shows the detected center, up to an accuracy of about 5% of the cavity diameter.

4.4.1 Cavity detection

To find the center of the cavities, an algorithm similar to the spot detection algorithm in the beam steering set-up was developed using the same center-of-gravity technique. The first endeavour in developing this algorithm was finding a suitable image processing method to determine which pixels belong to the cavity and which pixels don't.

A simple thresholding of the luminosity (grayscale values) of the image proved to be ineffective. The differences in luminosity were very small and an overall luminosity gradient was observed over the image, rendering a global threshold value useless, as illustrated in figure 4.11.

We found out that this could be countered by subtracting different colour layers of the image. Because the luminosity gradient affects all colour layers, subtracting two of them should cancel out its effect. Especially the red and green layers proved to be useful in this respect, as the cavity is greener and less red than its surroundings.

Thresholding the red minus green image turned out to be an adequately accurate way of identifying cavity pixels over a reasonably large area of the camera image, as illustrated in figure 4.12. To actually detect the center of a single cavity, we applied the center-of-gravity technique to a selected portion of the image. This is presented in figure 4.13. The detected center seems to accord nicely with the center of the cavity on visual inspection.

4.4.2 Cavity steering

The cavity can be moved around by the picomotorized stage. Now that we have an algorithm to detect the center of a cavity, a method to guide this center to a specified location can be developed.

During motion, the selection square should move along with the cavity. This is achieved by continuously monitoring the position of the cavity and updating the position of the selection square







age at 34.1% lumi-

nosity.





(c) Thresholded im-(d) Thresholded image at 33.7% lumiage at 32.9% luminosity.

Figure 4.11: Thresholded images of a cavity as recorded by the camera in the sandbox set-up, in an attempt to apply the center-of-gravity method to find the cavity center. Unfortunately, average luminosity was found to vary over the image as a whole (increasing in the south-west direction). This makes applying a global luminosity threshold infeasible.

nosity.



Figure 4.12: Different colour layers of the image of the cavity as captured by the camera in the sandbox set-up. Observe that the cavities are less red and more green than the surroundings. Subtracting the green layer from the red layer cancels out the luminosity variation over the image. This yields an image that can be fairly accurately thresholded to find which pixels belong to the drum and which don't.



Figure 4.13: Detection of a single cavity center (red diamond), obtained by applying the centerof-gravity technique to a selected portion of the image (red square).

accordingly.

Vector calculus is employed to determine how large movements in every direction should be to end up at the specified location. Firstly, direction vectors that relate the number of picomotor steps in each direction to the center displacement in pixels need to be determined. This can be done manually beforehand or at the start of each use of the motion script.

The displacement vector from the initial to the final position can then be decomposed into multiples of the appropriate direction vectors. The multiplication factors indicate the number of picomotor steps that are needed to end up with the cavity at the desired position.

As the picomotor step size may change over the course of the motion, the cavity doesn't actually end up at the specified location most of the time. This is solved by calculating the motion vectors and moving the cavity iteratively, while redetermining the direction vectors from the displacements by the latest picomotor motions. The results of this method were pretty satisfactory - most of the time the cavity could be moved to the desired location within pixel resolution in just three iterations.

4.5 Building the scanning interferometry setup

The next step towards a spatial scanning setup lies in combining the picomotorized sample stage and the interferometry setup. To this end, the picomotorized sample stage sandbox setup was moved into the interferometer optical path. Unfortunately, the desired focal distance between the objective and the sample for the laser beam did not align with the focal distance needed for the camera to produce a sharp image, due to poorly collimated light from the white lamp. Tuning the camera focal distance by adding lenses to the camera optical path proved to be unfruitful. This meant that no visual feedback could be provided by the camera on the drum position with respect to the laser spot.

In the sample stage we loaded a few layer graphene (FLG) circular microdrum as the object of study. This microdrum is depicted schematically in figure 3.3 and an optical microscope image is shown in figure 5.1. The interferometer setup was used to measure the frequency response of this microdrum.

4.6 Data analysis

4.6.1 Analysis of VNA data for actuated measurements

To extract the the resonance frequency, amplitude and line width from the VNA voltage response data, we fitted the real and imaginary parts of the response voltage simultaneously to the theoretically derived expression for the resonance response given by equation (2.17) (multiplied by a local phase factor) using a least squares method. This gave excellent results for most frequency sweeps, as illustrated by figure 4.14.

4.6.2 Analysis of spectrum analyzer data for Brownian motion measurements

We fitted a Lorentzian function superposed onto a linear noise floor against the spectral amplitude as measured by the spectrum analyzer to extract the resonance amplitude and line width. The



Figure 4.14: Harmonic oscillation resonance response (red) fitted to a VNA voltage response frequency sweep (blue). The left graph shows the real part of both functions while the right graph shows the imaginary part. This shows that the VNA output can be reliably fitted to a harmonic oscillator model if the resonance peak amplitude is above the noise level.

precise fitting function is given by the following expression:

$$L(\omega; A_0, \omega_0, \gamma, N_0, n) = A_0 \frac{\gamma^2}{(\omega - \omega_0)^2 + \gamma^2} + (n\omega + N_0),$$
(4.3)

where ω is the frequency, A_0 is the resonance amplitude maximum, γ is half width at half the maximum, n is the slope of the linear noise floor and N_0 is the *y*-intercept of the noise floor. This function is a widely used and fairly accurate approximation of the actual resonance amplitude as given by equation (2.25).

4.6.3 Data cleanup

If a resonance peak is too small in comparison to the noise or even completely absent (when the laser is focused on a nodal point of the mode shape or outside of the drum), the results of the fitting algorithm are aberrant. Sometimes the algorithm fits a single or a few noise peaks (which are quite sharp in general - much sharper than a valid resonance peak), or sometimes the algorithm would fit a very wide subtle overall trend in the voltage response. These misfits, sometimes with very extreme amplitudes or line widths, contaminated the data appreciably, especially for the higher modes that have lower resonance amplitudes anyway.

The amount of data generated is far too large to remove these misfits manually. Therefore, we set out to devise validity criteria for each data set. Particularly line width proved to be a useful validity criterion, excluding misfits at noise peaks. Determining a range of valid line widths was done manually for each resonance mode.

In addition to the line width criterion, fits were validated or rejected based on resonance frequency (which doesn't deviate abruptly from spot to spot) and amplitude (which shouldn't be too extreme in comparison to the neighbouring amplitudes).



Figure 4.15: Exemplary results of the first spatial scans of fundamental mode resonance of the microdrum depicted in 3.3. (a) Schematic of the scan motion. Crosses indicate the scan positions. The sample was scanned from left to right and back to left in one movement for each scan line. Lines were scanned from top to bottom. Especially the scan step sizes s_r and s_l need to be balanced for a successful scan. (b & c) The color axis shows the fundamental mode vibration amplitude while the x and y-axes indicate the position in terms of scan steps. The scan grid spanned approximately $15 \,\mu\text{m} \times 15 \,\mu\text{m}$ and every for every pixel the interferometry measurement took about 10 seconds. This amounts to a total measurement time of 1 to 1.5 hours. (b) Unbalanced step sizes in the left and right scan direction cause the drum to drift out of view as the scan progresses further downward in the uncalibrated scan. (c) After calibration of the scan step sizes the drift is minimal, showing the circular shape of the fundamental mode shape clearly.

4.7 Calibrating scan step sizes

At first, we tried scanning the drum without any positional feedback. This highlighted the poor repeatability of the picomotor step size once more, especially in different directions. This is clearly visible in figure 4.15b, where the drum drifts out of view after a few scan lines.

This prompted the need for another way to balance the step sizes in the left and right direction. For this purpose, a calibration algorithm was designed based on matching the resonance signal amplitudes for multiple scans of the same scan line. The algorithm starts out with certain preset values for the left and right step size (in picomotor steps). It will then perform iterative corrections of one of these values (keeping the other value fixed) until the actual displacement per scan step in the left and right direction matches up. In our case, the scan step size in the left direction was kept fixed while the size in the right direction was calibrated. The algorithm consists of two stages; a coarse calibration stage and a finer calibration stage

Firstly, the drum is moved outside of the laser spot. Then a single line is scanned and subsequently the drum is moved back. In this single line the reference edge of the drum is detected as the first scan step in which the resonance signal is above a certain amplitude. After the drum has been moved back the same scan line is scanned again and in this line scan the reference edge is also detected. If the scan step in which the reference edge is detected is not the same in both scans, which means that the drum has drifted, the step size in the calibration direction is updated to remedy this. This process is repeated until the reference edge of the drum matches up in subsequent scans of the same scan line.

After this coarse calibration stage, the algorithm switches to a finer calibration stage. In this

stage the algorithm tries to match up the value of the resonance amplitude at the reference edge. Again, the same line is scanned repeatedly while the scan step size in the calibration direction is updated in every run. It calculates this step size correction based on the derivative of the resonance amplitude at the reference edge:

$$\Delta s = \frac{\Delta A}{dA/ds} \tag{4.4}$$

The step size correction Δs is measured in picomotor steps, ΔA is the difference in amplitude between subsequent scans at the reference edge and the derivative dA/ds, indicating the amplitude change per picomotor step, is estimated numerically at the reference edge from the last line scan. This scheme is repeated until the difference in amplitude at the reference edge falls within the desired precision of 1%.

Typically, the algorithm finished in 3-5 coarse calibration runs (depending on the initial estimates) and 2-3 fine calibration runs. The final scan step sizes calculated by the algorithm can then be used to minimize drift in the actual scanning process. Unfortunately, the balanced values for the scan step size would not remain valid for multiple scans on different days, possibly because of wear in the picomotors or changes in the resistant force brought about by the vacuum tubes connected to the sample stage as they might have been moved slightly. This means that the calibration has to be repeated before every scan. Figure 4.15c shows the first scan with calibrated step sizes, with very small drift.

4.8 Effects of the finite laser spot size

The spatial resolution of the interferometer is ultimately limited by the finite laser spot size. The Rayleigh criterion states that the minimum achievable diffraction limited laser spot size is given by

$$d = \frac{1.22\lambda}{NA} = 1.3\,\mu\mathrm{m},$$
 (4.5)

where d is the laser spot diameter, $\lambda = 633 \,\mathrm{nm}$ is the laser wave length and NA = 0.6 is the numerical aperture of the microscope objective in the setup.

This means that the measured deflection is not in fact the local deflection, but the convolution of the local deflection and the Gaussian laser spot intensity profile over an area on the order of the laser spot area of about $1 \,\mu\text{m}^2$. Deconvolution might improve the spatial resolution, but was not done in this research, as the laser spot size was not exactly known and because of time constraints.

Chapter 5

Spatial scans of the resonance behaviour of an actuated FLG microdrum

A suspended microdrum made of a flake of few-layer graphene was the sample of interest in this study, as presented in figure 5.1. The flake was stamped onto a hole etched in a gold substrate. Using the interferometry setup, we studied the resonance behaviour of this drum under electrical and optothermal actuation as well as the thermal Brownian motion.

The resonance behaviour shows several interesting features. As figure 5.2 indicates, there are a few different resonance frequencies for this drum. This indicates that the drum can vibrate in one of several vibrational modes, as is expected from theory. Apart from the resonance frequency, important physical quantities that can be extracted from the resonance peak height, width and location include the vibration amplitude, line width, Q factor and phase lag of the local vibration of all the different modes.

Using the spatial scanning set-up we developed in this research project, we performed spatial scans of the local resonance behaviour under the three different actuation circumstances.



Figure 5.1: Optical microscope image of the drum, consisting of a flake of few-layer graphene stamped over a circular hole of 5 µm diameter etched in a gold substrate. On the bottom of this cavity there is another gold electrode to enable electrostatic actuation.



Figure 5.2: An example of the voltage transfer function of the microdrum interferometry measurement system as a function of driving frequency. This voltage transfer function is directly related to the (local) vibration amplitude and phase. In this measurement the drum was electrically actuated and the measurement laser was pointed just off the center. In this frequency response seven different resonance peaks could be fitted, with frequencies listed in table 5.1.

Table 5.1: Resonance frequencies of the vibrational modes of the drum. These values have been extracted from the frequency response just above the center of the drum in the high-resolution scan using electrical actuation.

f (MHz)	f/f_0
24.27	1.00
35.08	1.44
42.74	1.76
49.11	2.02
64.89	2.67
71.50	2.95
76.61	3.16



Figure 5.3: Classification of mode shapes of an FLG microdrum. (b) Local frequency response of the microdrum at the position indicated in the top left panel under electrical actuation as measured by the laser interferometer. The frequency response displays several resonance peaks. (a) The local frequency response has been scanned over the drum surface using the spatial scanning laser interferometer setup. The amplitude of the first resonance peak found in these frequency responses has been plotted spatially in this panel, revealing this peak to correspond to the fundamental vibrational mode. (d) The local amplitude of the other resonance peaks has been plotted spatially as well, revealing their vibrational mode shapes. (c) Optical microscope image of the microdrum. (e) Spatial plots of the local phase lag. In these phase plots we see different bulges oscillating in or out of phase. In all spatial plots grey areas represent measurement points outside of the drum, while dark blue or white pixels indicate no resonance peak could be found for fitting.

5.1 Classification of vibrational modes under electrical actuation

Figure 5.3 shows the results of our attempt to identify the vibrational modes of the graphene drum under electrical actuation. The resonance response of the drum under actuation was scanned in a grid of approximately $10 \,\mu\text{m} \times 10 \,\mu\text{m}$. In these resonance responses, seven different resonance peaks could be discerned (cf. table 5.1). Of these resonance peaks, the first four had a high enough signal-to-noise ratio over the area of the drum for a decent spatial map of their resonance parameters.

The first peak, with the highest amplitude on average, corresponds quite clearly to the fundamental mode of vibration of the drum. We see a single bulge, predicted from both plate and membrane theory, resonating in phase. It is hard to determine from this map alone whether the membrane resonates as a membrane, a plate or as some intermediate structure, since the mode shapes for the fundamental mode are very similar in both the membrane and plate case (cf. figures 2.3a and 2.4a).

For modes A and B we see two bulges oscillating half a cycle out of phase. This suggests a (1,1) membrane or plate mode, which might be degenerate along two different axes. Interestingly, their resonance frequencies lie quite a bit apart.

The amplitude shape of mode C resembles mode A's amplitude shape quite a bit. Again we have two major bulges, but this time they seem to be oscillating in phase with a small pocket in between and some pockets outside that oscillate out of phase. This phase difference seems to be less than half a cycle however.



Figure 5.4: Spatial plots of the resonance frequency of the first four resonance peaks found in the frequency response of the microdrum. For all modes a clear variation in the resonance frequency is observed, with the highest frequencies occuring when the interferometry laser is focused on the center of the drum. White pixels indicate no resonance peak could be found in the signal

5.2 Difficulties in scaling the mode shape maps

As the picomotors provide no reliable method of feedback on their position and step size, the scale of all spatial maps is not exactly known and may differ from scan to scan. An attempt to estimate the scan scales was made by measuring the motion of the sample stage using the visual feedback camera in the setup, but these measurements may not apply exactly to every scan. Furthermore, the edges of the drum were estimated from the signal-to-noise ratio of the first mode.

5.3 Resonance frequencies and theoretical predictions

The ratios of the resonance frequencies of the different vibrational modes, shown in table 5.1, do not match up with those predicted by theory for either the membrane case, presented in table 2.1. As the graphene flake stretches out significantly beyond the drum cavity, clamped boundary conditions seem most appropriate if the drum is assumed to be in the plate regime, but the theoretical eigenfrequencies of vibrations in this regime, presented in table 2.2, also do not match up with the measured frequencies. This might be an indication that the drum is actually in a cross-over regime or that surface wrinkles or other geometric defects may influence the local material parameters and consequently the mode shapes and their eigenfrequencies.

5.4 Variation in resonance frequencies induced by thermal strains

For all the modes we see a clear variation in the resonance frequency that ranges over approximately 1 MHz as the measurement laser is scanned over the drum (figure 5.4). This is most likely an effect of the (local) heating of the drum and the substrate by the measurement laser itself, causing thermal expansion of the substrate and contraction of the drum through its negative thermal expansion coefficient [16]. The resonance frequency is in turn affected by the pretension induced by these thermal strains.

This also means that the resonance frequency might differ between measurements as the ambient temperature varies.



Figure 5.5: Spatial plot of the fundamental mode resonance peak line width in the low resolution and the high resolution electrical actuation scan. In both scans we see an approximately constant line width except for one (low resolution) or two (high resolution) concentric rings. In the high resolution scan, the inner ring has a smaller line width while the outer ring has a larger line width.

5.5 Radial variations in resonance line width and Q factor

Spatial plots of the resonance peak line width reveal some radial variation, as seen in figure 5.5. Figure 5.5b, which was taken from a later, higher resolution scan shows a more or less constant line width over the entire drum area, except for two concentric rings. The inner ring exhibits a lower line width while the outer ring exhibits a higher line width. This is also reflected in the Q factor (figure 5.6).

Interestingly, these radial variations are absent under Brownian motion or optothermal actuation. This might suggest that it is in fact an effect of this specific actuation mechanism. For example, it could be an effect of the particular charge distribution in the gate electrode and the graphene flake, causing an annular actuation that might induce the phase behaviour we see in figure 5.7a as well as the radial variance in line width.



Figure 5.6: Spatial plot of the fundamental mode Q factor in the high resolution electrical actuation scan. An approximately constant Q factor of around 130 is observed, except for two concentric rings. The inner ring has a higher Q factor of about 160 while the outer ring has a lower Q factor of about 100.

5.6 Higher resolution scans under electrical actuation

After the scan shown in figure 5.3, we also performed a higher resolution scan. Figures 5.7, 5.8, 5.9 and 5.10 show the spatial map of all resonance parameters for the first three modes.

The fundamental mode now shows an appreciable dip in the amplitude near the center, which was not present in the earlier scan. This might be caused by a contamination of some kind that got attached to the drum or some other change in the drum structure from wear.

In the center of mode B we also now see two smaller bulges in between the larger bulges that oscillate in phase and anti-phase alternatively, reminiscent of a (2, 1) membrane or plate mode. This might also be caused by this anomaly. It seems unlikely that mode B is actually a (2, 1) mode, as we would expect the resonance frequency to be higher in that case, around $2.91f_0$ (membrane) to $5.95f_0$ (fixed plate).



Figure 5.7: High resolution spatial scan of the vibrational mode at ~ 25 MHz (fundamental mode) when driven by electrostatic force.



Figure 5.8: High resolution spatial scan of the vibrational mode at \sim 35 MHz (mode A) when driven by electrostatic force.



Figure 5.9: High resolution spatial scan of the vibrational mode at \sim 42 MHz (mode B) when driven by electrostatic force.



Figure 5.10: High resolution spatial scan of the vibrational mode at ${\sim}49\,\rm MHz$ (mode C) when driven by electrostatic force.

5.7 Resonance behaviour under optothermal actuation

After the electrical actuation measurements, we actuated the drum by optothermal excitation from a blue laser. Figures 5.11, 5.12, 5.13 and 5.14 show spatial scan results under these circumstances. We find that the resonance frequencies of the different modes are consistent with the measurements under electrical actuation, but that the measured mode shapes differ significantly.

5.7.1 Distinct differences in mode shapes in comparison to electrical actuation

Under optothermal actuation, the mode shapes as recorded by the spatial scanning setup seem to differ appreciably from the mode shapes found under electrical actuation. We see many more small bulges in the mode amplitude with distinct phases that do not seem to correspond nicely to membrane or plate mode shapes and that were not observed under electrical actuation. This is especially clear in the first mode, where we now see very distinct variations in the resonance phase lag over the drum surface. It is also interesting to note that the line width, and thus the Q factor, of the resonance peaks varies significantly between bulges under optothermal actuation for every mode.

This might be a consequence of the fact that not only the measurement laser is scanned over the drum surface but also the actuation laser. This means that the center of gravity of actuation moves over the drum, possibly affecting the preferred phase lag and mode shape orientation as the scan progresses.

5.7.2 Higher thermal strain induced resonance frequency variation

Another effect of scanning both lasers over the surface is that the thermal variations are greater than in the previous case. This is expressed in the fact that the resonance frequency variations now range over approximately 3 MHz.



Figure 5.11: Optothermal actuation spatial scans of the vibrational mode at ${\sim}25\,\rm MHz$ (fundamental mode)



Figure 5.12: Optothermal actuation spatial scan of the vibrational mode at ~ 35 MHz (mode A).



Figure 5.13: Optothermal actuation spatial scan of the vibrational mode at \sim 42 MHz (mode B).



Figure 5.14: Optothermal actuation spatial scan of the vibrational mode at ~ 49 MHz (mode C).



Figure 5.15: Atomic force microscopy (AFM) image of the sample. The triangular structure is the graphene flake that has been stamped over the drum cavity. The red ring indicates the edges of the drum cavity and has been determined by overlaying an optical microscope image of the drum. The white linear bulge coming in from below is raised by the electrical lead to the cavity bottom electrode. This image reveals that the flake and drum surface are actually far from smooth and exhibit wrinkles and a significant depression of about 50 nm in the drum area.

5.8 Physical properties of the drum

To establish a link between the resonance behaviour and the physical morphology of the drum, an atomic force microscopy (AFM) image was taken of the sample (figure 5.15). The AFM image shows that the drum surface is in fact quite rough: we see a few line-like wrinkles, as well as a distinct dip in the drum surface. Figure 5.15 shows the drum in the orientation corresponding to the orientation of the spatial scans.

5.8.1 Mode shapes and physical morphology

In figure 5.17 a contour map of the AFM scan has been superimposed on the mode shapes as measured in the high-resolution electrical actuation scan in attempt to find parallels between the mode shapes and the physical morphology of the drum. Unfortunately, the scaling and the position of the center of the drum in the mode shape plots are not exactly known, complicating the matching process.

A one-on-one correspondence between the drum morphology and the mode shapes is hard to establish but there are some corresponding features. The dip in the first mode amplitude might be related to the depression found in the drum surface. However, they don't overlap perfectly, but that might be because the drum center is actually not in the center of the mode shape plot.

On the other hand, the AFM overlay as it is scaled and positioned now does encompass all features of all mode shapes, such as the leftmost bulge of mode B, which might be an indication that it is positioned correctly.



Figure 5.16: Cut-outs of the AFM image displaying only the drum area. In the colormapped image a contour map of the drum surface has been overlayed as well, with the most prevalent linear structures added as black lines.



Figure 5.17: Spatial scans of the vibrational mode shapes under electrical actuation (cf. figures 5.7 - 5.10) overlayed with an AFM height contour map of the drum surface (cf. figure 5.16). A clear-cut correspondence between the mode shapes and the drum surface morphology seems to be hard to find.

Chapter 6

Spatial scans of the Brownian motion resonance of an FLG microdrum

In addition to measurements on an actuated microdrum, we also performed spatial scans of the thermally excited Brownian motion of the drum with no actuation. In the measurement range from 15 MHz to 80 MHz three resonance peaks could be identified, with frequencies corresponding to the resonance peaks found under driven oscillation for the fundamental mode and modes A and B. Figure 6.1 shows three spectra, measured in one spot, which display these three resonance peaks.

6.1 Mode shapes under Brownian motion

Figures 6.2, 6.3 and 6.4 show spatial scans of the microdrum resonance. The mode shapes seem to be consistent with the fundamental mode and modes A and B under actuation.

6.2 Temperature dependence of resonance frequency

Again we see a clear variation in resonance frequency for all the modes as the laser is scanned over the drum surface, caused by temperature differences and the accompanying differences in thermal strains as the heating by the measurement laser is moved over the drum surface. For the first mode this variation is on the order of 1 MHz, consistent with the variations found under electrical actuation.

6.3 Absence of radial variations in the first mode spectral Q factor

The spectral Q factor values for the first mode, which are on the order of 160 are similar to the values found under electrical actuation. Interestingly, we don't see the same radial variation in the



Q factor values, but those variations might be a result of the electrical actuation mechanism as explained in section 5.5.

Figure 6.1: Spectral power density of the signal of the photodiode in the interferometer setup with the measurement laser focused just outside of the center of the drum. Three resonance peaks could be identified in the signal (blue), superimposed on to a linear noise floor, which are plotted separately here. Lorentzian fits of resonance peaks in the spectral power density have been plotted in red.



Figure 6.2: Spatial scans of the Brownian motion resonance behaviour of the vibrational mode at $\sim 25 \text{ MHz}$ (fundamental mode).



Figure 6.3: Spatial scans of the Brownian motion resonance behaviour of the vibrational mode at ${\sim}35\,\rm MHz$ (mode A).



Figure 6.4: Spatial scans of the Brownian motion resonance behaviour of the vibrational mode at ${\sim}42\,\rm MHz$ (mode B).

Chapter 7 Summary and conclusion

The primary goal of this research project was to gain insight into the vibrational behaviour of a few layer graphene microdrum by mapping its mode shapes spatially. Theory predicted the graphene drums to vibrate like a membrane if the motion was dominated by in-plane tensile forces and like a plate if the motion was dominated by the bending rigidity of the material. The shapes of the vibrational modes in the membrane or plate regime proved to be very similar, but the ratios of the vibrational mode resonance frequencies differed greatly between these two regimes.

A laser interferometry setup to measure the deflection of the graphene drum surface was expanded with spatial scanning capabilities by motorizing the sample stage with very precise picomotor linear actuators. Careful characterization of the picomotor behaviour revealed that the picomotor actuators exhibited a strong asymmetry in their motion in different directions. This prompted the need for a calibration method, which was succesfully implemented.

Using this setup, the mode shapes of the microdrum under three different types of actuation were studied: 1) periodic electrostatic attraction to a gate electrode at the bottom of the drum cavity; 2) optothermal actuation by laser heating induced periodic thermal expansions and contractions; and 3) actuation by thermal Brownian vibrations.

Actuating the drum and measuring the frequency response of the drum motion was done using a vector network analyzer (VNA). Results were fitted using a driven-damped harmonic oscillator model. From these fits the resonance frequency, resonance amplitude, line width, Q factor and local phase lag with respect to the driving signal could be obtained for all measurable vibrational modes at every point of the drum surface. For the Brownian motion measurements, a spectrum analyzer was employed to measure the power spectral density of the drum resonance. Lorentzian fits were applied to these spectral densities, from which the resonance frequency, line width and spectral Q factor could be deduced. Data cleanup methods were necessary to remove misfits from the data.

In our microdrum, seven different resonance modes could be detected under electrical actuation, eight under optothermal actuation and three under Brownian motion. Of these resonance modes, the first four had a high enough signal-to-noise ratio for a useful spatial map under actuation and the first three were dominant enough for a spatial map of their Brownian motion mode shapes.

The mode shapes under electrical actuation and under Brownian motion proved to be consistent. Spatial maps of the microdrum resonance under optothermal actuation seemed to be different, but this might have been caused by the fact that the actuation laser was scanned along with the measurement laser in this measurement scheme, influencing the motion considerably. The mode shape of the first resonant mode, which was found around 25 MHz turned out to correspond nicely to a membrane or plate fundamental mode. Distinguishing between these two regimes proved to be infeasible based on mode shape alone. The second (around 35 MHz) and third (around 42 MHz) resonant mode seemed to correspond to a (2, 1) membrane or plate mode, degenerate along two different rotated axes. Interestingly, their resonance frequencies varied greatly, hinting at imperfections in the drum surface structure. The fourth mode (around 49 MHz) was hard to identify, but its amplitude map showed significant similarities to the second mode. This might be an indication that the impurities in the drum surface restrict the motion of the drum to a smaller area than the entire surface for some modes.

The mode resonance frequency ratios did not match with the values predicted by either membrane or plate theory. This was an indication that we are dealing with a cross-over regime or that the frequency ratios are severely altered by surface impurities.

In an attempt to uncover the influence of surface impurities, an atomic force microscopy (AFM) image of the drum surface was taken. Overlaying this image onto the mode shape maps was hard to do reliably as the scaling of the spatial maps was not exactly known. Matching AFM and mode shape images showed some corresponding features in the mode shape and the drum morphology. Most notably, a dip in amplitude in the center of the first mode shape seemed to correspond to a depression in the drum surface.

In conclusion, we have succeeded in building a setup to map the mode shapes of microdrums spatially, adding the spatial domain to the measurement toolbox of the NEMS experimentalist. Future improvements to this setup could involve taking into account the finite laser spot size and improving spatial resolution by deconvoluting the deflection amplitudes and the Gaussian laser spot intensity profile, improving the methods of determining the scaling of the mode shape scans by adding some extra reference method, by fixing the camera visual feedback system or by using closed loop linear actuators.

Interesting measurements that could be done with this setup entail mode shape measurements on drums with non-circular geometries such as dumbells or squares with engineered mode shapes, or on microdrums with intentionally altered surface morphologies in order to establish a link between the physical drum surface and the resulting mode shapes. Another possibility would be to alter the electrical actuation target by modifying the shape of the gate electrode at the bottom of the drum cavity. This might be used to force the drum to vibrate with a certain mode shape or provide further insight in the observed radial variances in the line width and phase lag of the modes under electrical actuation.

Spatial maps of the resonance behaviour could also be used to measure different physical properties of graphene, such as thermal transport properties through the strong temperature dependence of the vibrational mode resonance frequency. The setup could even be used to measure the static deflection of the drum under various circumstances. Finally, numerical simulations of the drum resonance behaviour could be used to expand the results that were obtained in this research.

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