FINITE TIME LYAPUNOV EXPONENTS AND EXTREME CONCENTRATION FLUCTUATIONS IN 2D TURBULENCE

Hua Xia[1], Nicolas Francois[1], Horst Punzmann[1], Kamil Szewc[2] and Michael Shats[1]
[1] *Research School of Physics and Engineering, The Australian National University, Canberra, Australia*
[2] *Institute of Fluid-Flow Machinery, Polish Academy of Sciences, ul. Fiszera 14, 80952 Gdańsk, Poland*

Abstract The statistics of the Finite time Lyapunov exponent (FTLE) has been investigated in detail in laboratory 2D flows. The balance of the forward and backward FTLE suggests the incompressible nature of the turbulence in both the electromagnetic and Faraday wave driven experiments. The tail in the PDF of the FTLE field is correlated with the extreme concentration of the passive scale, the 'unmixing' events.

Passive scalar is known to be more intermittent than the velocity field. The question is where this intermittency come from. It is generally accepted that the stretching field is the best indicator of the passive scalar spreading [1]. However, in incompressible flow, the stretching is compensated by the un-stretching. The statistics of stretching fields can be investigated using the finite-time Lyapunov exponent (FTLE), which is the logarithm of the stretching divided by the finite integration time. Here we investigate the statistics of the forward and backward FTLE and their relaitonship to the extreme fluctuations in the passive scalar concentration.

Two-dimensional flows are generated in the stratified layers of fluid, electromagnetically driven turbulence (EMT) [2] and on the surface of Faraday wave (FWT) [3]. In the EMT turbulence, the range of the forcing-scale Reynolds numbers $Re = \langle\tilde{u}\rangle_{rms} L_f/\nu$ changes in the range from 10 to 120 reaching at higher currents regime of fully developed 2D turbulence. Here $\langle\tilde{u}\rangle_{rms}$ is the mean square root velocity fluctuations, $L_f = 9$ mm is the forcing scale, and $\nu \approx 10^{-6}$ m^2s^{-1} is the kinematic viscosity. In the FWT experiments, the Reynolds number can reach up to 200. The characteristic time scale of the particle trajectories, the Lagrangian integral time T_L is defined as $T_L = \int_0^\infty \rho(t)dt$, where $\rho(t)$ is the Lagrangian autocorrelation function, $\rho(t) = \langle u(t_0)u(t_0+t)\rangle/u^2$. For the experiments described here, the Lagrangian integral time ranges from $T_L = 1.9$ s in the spatially periodic flow and $T_L = 0.84$ s in turbulence.

FTLE [4] is the logarithm of the stretching divided by the finite integration time τ, which is determined at location x_0 and time t_0 as $\Lambda(\mathbf{x}(x_0, t_0), \tau) = \left(\frac{1}{\tau}\right)\log(|\delta\mathbf{x}(\tau)|/|\delta\mathbf{x}(0)|)$. Here $\delta\mathbf{x}(t)$ is the separation at time $t_0+\tau$ between two points which were close together and centred at location $\mathbf{x}_0$ at time t_0. In Fig.1, the contour plots of the FTLE fields obtained for two EMT experiments at low (Re=30) and high (Re=112) Reynolds number are shown.

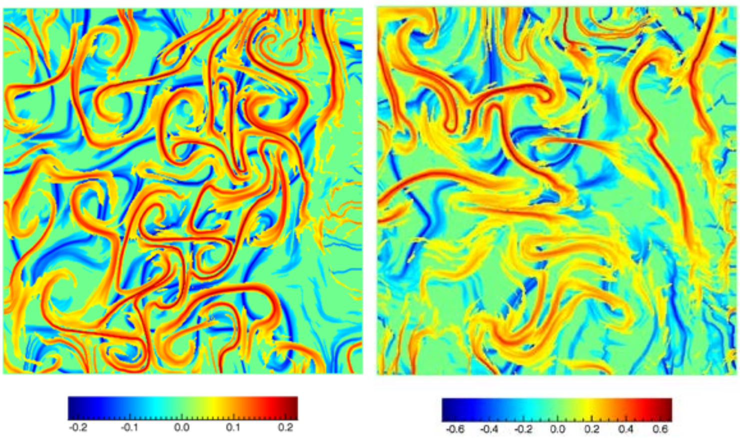

Figure 1. Contour plot of the FTLE for two cases at low and high Reynolds number.

Since the integration time of the FTLE calculation affect the probability distribution of the computed FTLE, to compare the stretching field statistics of different flows, we determine the integration time needed with respect to the Lagrangian integral time T_L. For longer integration time (larger than T_L), the PDF of FTLE is truncated at higher values. The PDF converges for shorter integration time of $0.4T_L$ and $0.6T_L$. For the results shown here, an integration time of $\tau = 0.6T_L$ is used. The PDFs are obtained with an ensemble average over $10T_L$.

PDFs of Λ normalized by their mean values <Λ> are illustrated in Fig. 2(a). The figure shows spatially and ensemble averaged PDFs of the forward and backward FTLEs for two EMT experiments. Statistically averaged PDFs of the forward and backward FTLE are effectively the same. This confirms that the studied flows are incompressible 2D flows since the sum of positive and negative Lyapunov exponents vanishes, as it should [5]. This is also true for the FWT experiments at higher Reynolds number.

Probabilities of large Λ are noticeably higher for higher Reynolds number experiments, as seen from the PDF tails. PDFs of the FTLE obtained in these experiments are well described by the Weibull distributions, $P(\Lambda) = (b/a)(\Lambda/a)^{b-1}exp(-\Lambda^b/a^b)$. The shape parameter has values of b=1.95 and b=1.7 for EMT Re=30 and Re=112 respectively. The shape parameter for the FWT higher Reynolds experiments is b=1.3. The observed shape parameter is in the same range as those derived from the oceanographic data [6]. Our data confirm that the lower values of b correspond to more developed turbulence.

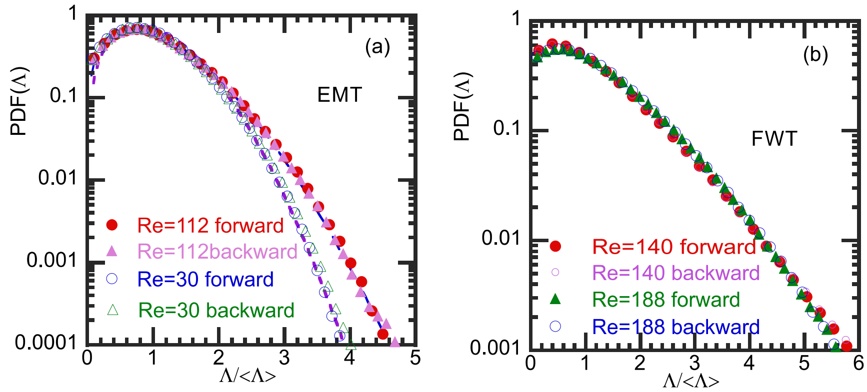

Figure 2. PDF of the forward and backward FTLE (a) EMT turbulence at Re=30 and 112 respectively. (b) FWT turbulence at Re=140 and Re=188 respectively

Strong bursts in the local dye concentration in turbulence are observed as shown in Fig.3 (b). The recurrence of bright concentration blobs, appearing randomly in time and in space, represent local 'unmixing' events.

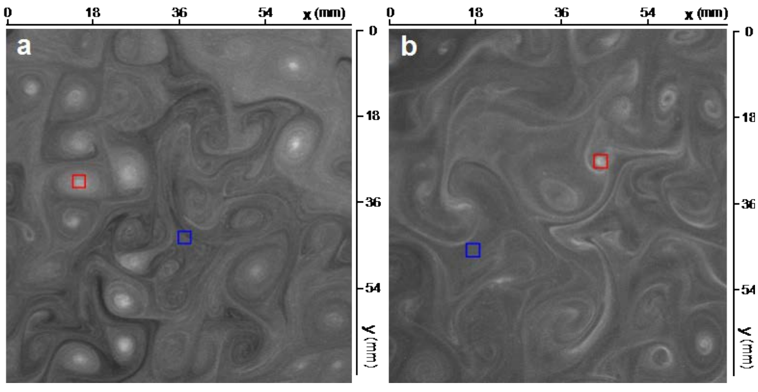

Figure 3. Dye concentration at EMT turbulence at (a) Re=30 and (b) 112 respectively.

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