

MECHANICS OF SEDIMENT TRANSPORT
BY THE COMBINATION OF WAVES AND CURRENT.
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1. Introduction.

The motivation to develop a transport formulation for the combination of waves and current has been the necessity to model longshore transport passing tidal in- and outlets. The generally applied C.E.R.C. formula for the longshore transport is not applicable here because that formula relates the longshore transport to the longshore current caused by the breaking waves, without using this current explicitly.

For this reason a formulation was looked for, which related the sediment transport to the current taking the wave influence into consideration. It is explicitly assumed that the waves itself do not transport material. As long as the direction of the wave propagation is not close to the current direction this assumption gives no problems. The problems for almost equal directions will be discussed in chapter 7.

The basic concept is that waves stir up the sediment and the current transports it. Therefore any transport formulation which allows the introduction of this effect adequately, will do.

In the following discussion a formula will be presented which has been developed between '66 and '71. (Bijker 1971)

At the Delft University of Technology and at Delft Hydraulics work is in good progress to replace this formula by a more accurate and physically better justified one. The present formula gives, however, still a good insight in the physical process and since the new formula is not yet ready this one will be discussed.

2. Basics of bed load transport formulation.

For the transport formulation the formula of Kalinske - Frijlink is used because this formula enables an easy distinction between the "transporting" part and a possible "stirring up" part.

This formula was originally written as (Frijlink, 1952) :

$$S_b = b D_{50} \frac{v}{C} \sqrt{\mu g} \exp \left[-0.17 \frac{\Delta D_{50} C^2}{\mu v^2} \right] \quad (1)$$

where

- S_b = bed load transport
 b = experimentally derived coefficient ≈ 5
 D_{50} = Mean sediment grain size
 v = average velocity
 C = Chezy friction coefficient
 μ = ripple factor
 g = acceleration of gravity
 -0.27 = experimental coefficient
 Δ = relative density $= (\rho_s - \rho) / \rho$
 ρ = fluid density
 ρ_s = sediment density

This formula is nothing more nor less than a good curve fitting of all at that moment existing data. The form has been chosen in order to distinguish a transporting and stirring-up part. The stirring-up part is represented by the exponential function. Although the formula is presented as a bed load transport formula, it is in principle a total load transport formula since in the circumstances from where data were collected (rivers and medium sized flumes) a great part of the sediment was transported in suspension.

Also the double place of the ripple coefficient μ , in the transporting part and in the stirring-up part, is somewhat inconsequent. This ripple factor indicates the part of the bed shear stress which is related to the grain shear stress and can be "used" to lift the grains out of the bed.

$$\mu = \tau_{\text{grains}} / \tau_c \quad (2a)$$

where

$$\tau_{\text{grains}} = \rho_s v^2 / C_{90}^2$$

$$\tau_c = \text{bed shear stress} = \rho_s v^2 / C^2$$

According to this definition the ripple factor should be proportional to $(C / C_{90})^2$

where C = Chezy coefficient of the bed = $18 \log(12 h/r)$

C_{90} = Chezy coefficient for a flat bed with grain size

of D_{90}

This, however, underestimates the role of the ripples on the stirring-up process.

A better formula for μ is therefore:

$$\mu = (C/C_{90})^{3/2} = (\tau_{\text{grains}}/\tau_c)^{3/4} \quad (2b)$$

So the term μ , which is regarded as that part of the shear stress responsible for the stirring-up of the grains reads now:

$$\mu \tau_c = (\tau_{\text{grains}})^{3/4} \tau_c^{1/4} \quad (2c)$$

This is apparently a "mixture" of both the grain- and the total shear stress.

According to this reasoning the term μ should appear therefore only in the stirring-up part of the formula, the exponential term. This influences also the value of the factor b . This factor should, as compared to that in the original Kalinske-Frijlink formula, be multiplied with a factor $1/\mu$. But also because a great part of the transport calculated by this formula is transported as suspended load, "b" should be smaller when the formula is used to describe exclusively bed load. Tests by Bijker (1967) and recomputation of some data where bed and suspended load were distinguished lead to a value $b = 2$

The formula to describe the bed load transport will be written now as:

$$S_b = \underbrace{2D_{50} \frac{v}{C} \sqrt{g}}_{\text{transport}} \exp \left[\underbrace{-0.27 \frac{\Delta D_{50} C^2}{\mu v^2}}_{\text{stirring-up}} \right] \quad (3)$$

3. Basics of suspended load transport formulation.

Under the assumption of a uniform flow, the relation between average velocity (v), waterlevel slope (i), waterdepth (h) and bed shear friction coefficient (C) is given by the well known Chezy formula:

$$v = C \sqrt{hi} \rightarrow l = \frac{v^2}{C^2 h} \quad (4)$$

In that case the bed shear stress can be written as

$$\tau_c = \rho g v^2 / C^2 \quad (5)$$

For such a flow the vertical velocity gradient ($dv(z)/dz$) can be written as

$$\frac{dv(z)}{dz} = \frac{\tau(z)}{\rho \epsilon_f} \quad (6)$$

where ϵ_f = fluid diffusion coefficient

$\tau(z)$ = shear stress at height z from the bed.

The mixing length theory of Prandtle (1926) resulted in the following equation for

$$\epsilon_f = l^2 \frac{dv(z)}{dz} \quad (7)$$

where l = mixing length.

Von Karman (1930) developed an equation for the mixing length near the bed.

$$l = \kappa z,$$

where κ = Von Karman coefficient = 0.4

For the entire depth the equation can be written as:

$$l = \kappa z \sqrt{(1-z/h)} \quad (8)$$

so

$$\tau(z) = \rho (\kappa z)^2 (1-z/h) \left(\frac{dv(z)}{dz} \right)^2 \quad (9a)$$

So the shear stress is found to vary linearly with the height above the bed.

$$\tau(z) = \tau_c (1-z/h) \quad (9b)$$

The vertical velocity gradient can be written as

$$\frac{dv(z)}{dz} = \frac{\sqrt{\tau_c}}{\kappa z \sqrt{\rho}} \quad (10)$$

Solving this differential equation leads to the well known logarithmic velocity distribution

$$v(z) = \sqrt{\frac{\tau_c}{\rho}} \frac{1}{\kappa} \ln \frac{z}{z_0} \quad (11a)$$

where z_0 = distance above the bed at which the velocity is zero.

Sometimes $\sqrt{\tau_c/\rho}$ is defined as the shear stress velocity v_* . This results in the well known Prandtle - Von Karman logarithmic velocity profile

$$v(z) = \frac{v_*}{\kappa} \ln \frac{z}{z_0} \quad (11b)$$

The shear stress velocity is difficult to interpret physically. It is the velocity occurring at an elevation z^1 above the bed, assuming a logarithmic velocity profile.

In that case

$$z^1 = z_0 e^{\kappa} \quad (12)$$

A physically more important velocity is the velocity which marks the change of the turbulent flow of the logarithmic velocity profile to a much less turbulent or even laminar sublayer close to the bed.

This assumed velocity distribution is shown in Figure 1. In the lower part close to the bed this distribution is assumed linear and is tangent with the logarithmic velocity distribution at a height z_t above the bed.

From $dv_t/dz = v_t/z_t$ follows

$$z_t = ez_0 \quad (13)$$

The velocity at this height is found to be

$$v_t = \frac{v_*}{\kappa} = \frac{\sqrt{g}}{\kappa C} \quad (14)$$

With equation (10) a new formulation for the bed shear τ_c is found

$$\tau_c = \rho \kappa^2 v_t^2 \quad (15)$$

This relation is developed in order to relate the bed shear stress to the velocity near the bed for the combination of the two fundamentally different velocity profiles of a uniform flow and the orbital velocity due to waves.

The value of z_0 is related to the apparent bottom roughness r . Experimentally Nikuradse found

$$z_0 = r/33 \quad (16)$$

For a flat bed r is related to the grain size. There is a wide range of estimates for r , for instance

| | | |
|----------------------------|-------------------|-----|
| Ackers and White (1973) | $r = 1.25 D_{35}$ | 17a |
| Einstein (1950) | $r = D_{65}$ | 17b |
| Engclund and Hansen (1967) | $r = 2 D_{65}$ | 17c |
| Kamphuis (1975) | $r = 2.5 D_{90}$ | 17d |
| van Rijn (1984) | $r = 3 D_{90}$ | 17e |

D_x is defined as the diameter of the material which is exceeded by (1 - x) %.

In formula 2b for the ripple coefficient, C is determined with $r = D_{90}$. This value is also in reasonable agreement with the so called sand grain roughness of Nikuradse.

For a rippled bed the apparent roughness is determined by the height and the length of the ripples, for instance:

$$\text{Swart (1976)} \quad r/\eta = 25 \eta/\lambda \quad 18a$$

$$\text{van Rijn (1982)} \quad r/\eta = 1.1 (1 - e^{-25 \eta/\lambda}) \quad 18b$$

where η = ripple height
 λ = ripple length.

A reasonable estimate for ripples of normal shape is $r = \eta$.

The above given information is required to calculate the concentration of the material in suspension. This material is kept in suspension by exchange of upward and downward transport as result of the turbulent diffusion (See Figure 2) This upward diffusion coefficient for the sediment is related to the turbulent fluid diffusion coefficient, but not necessarily equal to it (van de Graaff and Roelvink 1984 and van de Graaff 1988). The upward transport due to the turbulent diffusion is in the equilibrium situation equal to the downward motion of the sediment due to the fall velocity.

$$\text{So} \quad wc(z) + \epsilon_s(z) \frac{dc(z)}{dz} \quad (19)$$

where w = fall velocity of the sediment particles in still water
 $c(z)$ = average concentration at height z above the bed

$\epsilon_s(z)$ = diffusion coefficient for the sediment at height z
 z = height above the bed.

Rouse and Einstein suggested a parabolic changing diffusion coefficient (See Figure 3)

$$\epsilon_s(z) = 4 \epsilon_{s \max} \frac{z}{h} \left[\frac{h-z}{h} \right] \quad (20)$$

This results in a concentration distribution denoted by

$$c(z) = c_a \left[\frac{h-z}{z} \frac{a}{h-a} \right]^{z/a} \quad (21)$$

$$\text{where } z_w = \frac{w}{\kappa v_*} = \frac{hw}{4 \epsilon_s \max} \quad (22)$$

c_i = reference concentration at level $z=a$ above the bed.

In his formulation of the suspended load transport, Einstein calculated c_i at a height "a" of only some grain diameters from the bed (Einstein 1950). For a rippled or undulated bed this assumption is not so realistic. Bijker assumed therefore that "a" would be equal to the bed roughness "r".

The concentration c_i at the top of this layer is calculated under the assumption that the bedload is transported in this layer with the average velocity over this layer and a, over this layer constant, concentration.

In formula

$$S_b = r v_{\text{bed layer}} c_a \quad (23)$$

Over the height ez_0 the velocity distribution is linear, from ez_0 to r the velocity distribution is logarithmic (See Figure 4)

This results in the following formula for the average velocity v_r in the bottom layer:

$$v_{\text{bed layer}} = \frac{1}{r} \left[\frac{1}{2} \frac{v_*}{\kappa} ez_0 + \int_{ez_0}^r \frac{v_*}{\kappa} \ln \frac{z}{z_0} dz \right] \quad (24)$$

$$\text{or } v_{\text{bed layer}} = 6.34 v_* \quad (24b)$$

The average concentration in this layer which is equal to the reference concentration

c_i is

$$c_i = \frac{S_b}{6.34 v_* r} \quad (25)$$

So, resumming, this approach is as follows:

$$S_b = b D_{50} \frac{v}{C} \sqrt{g} \exp \left[-0.27 \frac{\Delta D_{50} C^2}{\mu v^2} \right] \quad (3)$$

or with $\tau_c = \rho g (v^2/C^2)$

$$S_b = b D_{50} \frac{v}{C} \sqrt{g} \exp \left[-0.27 \frac{\Delta D_{50} \rho g}{\mu \tau_c} \right] \quad (3a)$$

$$c_z = \frac{S_b}{6.34 v_* r} \left[\frac{(h-z)}{z} \frac{r}{(h-r)} \right]^{z_*} \quad (21a)$$

$$\begin{aligned} z_* &= w / (\kappa v_*) \\ \tau_c &= \rho v_*^2 \end{aligned} \quad (22) \quad (26)$$

Einstein calculated the suspended load S_s as

$$S_s = 11.6 \sqrt{\frac{E}{\rho}} r c_a [I_1 \ln(33h/r) + I_2] \quad (27)$$

where I_1 and I_2 are the so called Einstein integrals.

These integrals can be written as

$$I_1 = 0.216 \frac{A^{(z_*-1)}}{(1-A)^{z_*}} \int_A^1 \left[\frac{(1-\zeta)}{\zeta} \right]^{z_*} d\zeta \quad (28)$$

$$I_2 = 0.216 \frac{A^{(z_*-1)}}{(1-A)^{z_*}} \int_A^1 \left[\frac{(1-\zeta)}{\zeta} \right]^{z_*} \ln \zeta d\zeta \quad (29)$$

where $A =$ the dimensionless roughness $= r/h$

$\zeta =$ dimensionless elevation above the bed $= z/h$

Einstein provided in the pre-computer era tables for I_1 and I_2 . Although the integrals can be calculated numerically now relatively simple, Bogaard and Bakker (1977) calculated

$$[I_1 \ln(33h/r) + I_2] = Q \text{ for various values of } z \text{ and } r/h.$$

Their results are summarized in Table 1.

4. Introduction of wave influence.

The typical form of the vertical distribution of the orbital velocity of short waves is given in Figure 5.

The greater part is exponentially but near the bottom the distribution is determined by friction.

Jonsson (1966) carried out experiments to determine the bed shear stress under waves for a rough bed and turbulent current. He found that this bed shear stress could be described in terms of the near bed velocity amplitude and the wave friction factor f_w .

$$\tau_w = \frac{1}{2} \rho f_w \hat{u}_o^2 \sin \omega t \quad (30a)$$

$$\hat{\tau}_w = \frac{1}{2} \rho f_w \hat{u}_o^2 \quad (30b)$$

where

- τ_w = bed shear stress for waves as function of time
 \hat{t}_w = maximum bottom shear stress
 f_w = friction factor
 \hat{u}_o = maximum horizontal velocity component just outside the boundary layer
 ω = angular velocity = $2\pi/T$
 T = wave period

Wave friction factors depend on the Reynolds number and the relative roughness. For small Reynolds numbers the wave friction factor depends only on this Reynolds number, for high Reynolds numbers the influence of the Reynolds number is very small. The best solution for Reynolds number depended friction factors is given by Kamphuis (1975). Since prototype conditions can be expected to be fully turbulent and thus have a high Reynolds number, the Reynolds number dependence can be ignored. Therefore the original relation of f_w of Jonsson is used, See Figure 6.

Swart (1976) has rewritten this relation in the following formula:

$$f_w = \exp\left[-5.977 + 5.213(a_o/r)^{-0.194}\right] \quad (31a)$$

for $a_o/r > 1.59$

$$f_w = 0.30 \quad \text{for } a_o/r < 1.59 \quad (31b)$$

where r = bed roughness
 a_o = maximum horizontal displacement of water particles just outside the boundary layer.

In order to be able to combine the effect of waves and current, Bijker (1966) defined the bed shear stress due to waves in the same way as the current shear stress.

So $\tau_w = \rho \kappa^2 u_t^2$, where u_t is the velocity at the place where the assumed linear velocity distribution very close to the bed is tangent with the velocity distribution in the viscous boundary layer of the orbital motion. This u_t can be compared with the velocity v_t for a velocity profile of a uniform current and will be related to the orbital velocity at the top of the viscous boundary by a factor β .

$$\text{So } u_t = \beta u_o$$

From this follows

$$\tau_w = \rho \kappa^2 u_t^2 = \rho \kappa^2 (p \hat{u}_0)^2 \sin^2 \omega t \quad (32a)$$

$$\hat{\tau}_w = \rho \kappa^2 (p \hat{u}_0)^2 \quad (32b)$$

From comparison with the expression derived by Jonsson (Eq.30) follows

$$p = \frac{1}{\kappa} \sqrt{\frac{\hat{\tau}_w}{2}} \quad (33)$$

In comparison with the shear stress velocity of a uniform flow also a shear stress velocity for waves can be defined as

$$u_{*w} = \sqrt{\frac{\hat{\tau}_w}{\rho}} = \sqrt{\frac{\hat{\tau}_w}{2}} \frac{\hat{u}_0}{\kappa} \quad (34)$$

The approach used to calculate the transport, bed load as well as suspended load, is to increase the shear stress in the stirring-up parameter (Eq.3a) by the influence of the waves. This modified and increased shear stress due to the combination of waves and current is denoted as τ_{cw} .

Since the shear stress due to current can be written as

$$\tau_c = \rho \kappa^2 v_t^2 \quad (15)$$

and the shear stress due to waves as

$$\tau_w = \rho \kappa^2 u_t^2 \quad (32)$$

the two velocities v_t and $u_t = p u_0$ can be combined. For this combination it is assumed that the wave crests make an arbitrary angle ϕ with the current, see Figure 7.

The resultant velocity at that specific height v_r equals

$$v_r = \sqrt{v_t^2 + (p \hat{u}_0 \sin \omega t)^2 + 2 v_t p \hat{u}_0 \sin \omega t \sin \phi} \quad (35)$$

$$\cos \theta = \frac{v_t + p \hat{u}_0 \sin \omega t \sin \phi}{v_r} \quad (36)$$

The shear stress for combined waves and current is denoted by

$$\tau_{cw} = \rho K^2 v_r^2 \quad (37a)$$

or

$$\tau_{cw} = \rho K^2 \left[v_t^2 + (p \hat{u}_o \sin \omega t)^2 + 2 v_t \hat{u}_o \sin \omega t \sin \varphi \right] \quad (37b)$$

The variation of this shear stress over the wave period is shown in Figure 8. The direction changes with the phase of the orbital velocity. This actual direction is, however, not important for the stirring-up of the material. Only the value itself is of importance since it is assumed that the material, when after been stirred-up in suspension, is moved by the main current. To take this into account the time averaged value of the total shear stress τ_{cw} is calculated.

It should be stated specifically that this approach is used for convenience. Basically the exponential function

$$\exp \left[-0.27 \frac{\Delta D_{50} \rho g}{\mu \tau_{cw}} \right] \text{ should be averaged.}$$

The separate components between brackets of Eq. 37b are shown in Figure 9. The time-averaged values of the two time dependent factors in Eq. 37b are:

$$\frac{1}{T} \int_0^T \sin \omega t \, dt = 0 \quad (38)$$

$$\frac{1}{T} \int_0^T \sin^2 \omega t \, dt = 1/2 \quad (39)$$

With these simplifications Eq 37b becomes:

$$\bar{\tau}_{cw} = \rho K^2 \left[v_t^2 + \frac{1}{2} p \hat{u}^2 \right] \quad (40)$$

with $p = \frac{1}{K} \sqrt{\frac{f_w}{2}}$, $\tau_c = \rho K^2 v_t^2$ (Eq 15)

and $v_t = \frac{v \sqrt{g}}{K}$ (Eq 14)

Eq. 40 can be written as:

$$\bar{\tau}_{cw} = \tau_c \left[1 + \frac{1}{2} \left(\xi \frac{\hat{u}_o}{v} \right)^2 \right] \quad (41)$$

where

$$\xi = \frac{p K C}{\sqrt{g}} = C \sqrt{\frac{f_w}{2g}} \quad (42)$$

The factor $1 + \frac{1}{2} \left(\frac{\hat{u}_o}{v} \right)^2$ can be seen as the increasing factor of the current shear stress due to the wave motion.

Since $\hat{\tau}_w = \rho \kappa^2 (\rho \hat{u}_o)^2$ Eq.(32 b),

Eq.40 can also be written as

$$\tau_{cw} = \tau_c + \frac{1}{2} \hat{\tau}_w \quad (43)$$

It should be noted that the same result would have been obtained if the shear stress velocities and not the velocities at elevation z_i had been combined.

5. Transport formula for current and waves.

The above considerations result in the following procedure to calculate the sediment transport under the combined action of waves and current.

$$S_b = b D_{50} \frac{v}{C} \sqrt{g} \exp \left[\frac{-0.27 \Delta D_{50} \rho g}{\mu \tau_{cw}} \right] \quad (43a)$$

or

$$S_b = b D_{50} \frac{v}{C} \sqrt{g} \exp \left[\frac{-0.27 \Delta D_{50} \rho g}{\mu \tau_c \left\{ 1 + (1/2) \left(\frac{\hat{u}_o}{v} \right)^2 \right\}} \right] \quad (43b)$$

or

$$S_b = b D_{50} \frac{v}{C} \sqrt{g} \exp \left[\frac{-0.27 \Delta D_{50} \rho g}{\mu (\tau_c + (1/2) \hat{\tau}_w)} \right] \quad (43c)$$

or

$$S_b = b D_{50} \sqrt{\frac{\tau_c}{\rho}} \exp \left[\frac{-0.27 \Delta D_{50} \rho g}{\mu (\tau_c + (1/2) \hat{\tau}_w)} \right] \quad (43d)$$

or

$$S_b = b D_{50} \frac{v}{C} \sqrt{g} \exp \left[\frac{-0.27 \Delta D_{50} C^2}{\mu v^2 \left\{ 1 + (1/2) \left(\frac{\hat{u}_o}{v} \right)^2 \right\}} \right] \quad (43e)$$

The formula is written in various forms because one specific form may be special convenient in a specific application.

As explained in Chapter 3 the suspended transport S_s can be written as:

$$S_s = 11.6 \sqrt{\frac{\tau_c}{\rho}} r_{ca} \left[I_1 \ln(33h/r) + I_2 \right] \quad (27)$$

or
$$S_s = 11.6 \sqrt{\frac{\tau_c}{\rho}} r_{ca} Q \quad (27a)$$

In the Einstein integrals I_1 and I_2

the factor
$$z_* = \frac{w}{\kappa V_*} \quad \text{Eq. (22)}$$

is now changed into

$$z_* = \frac{w}{\kappa V_* c w} \tag{44}$$

where

$$V_* c w = \sqrt{\frac{\tau_c}{\rho}} = \sqrt{\frac{\tau_c}{\rho} \left[1 + \frac{1}{2} \left(\xi \frac{\hat{u}_o}{v} \right)^2 \right]} \tag{45}$$

So

$$z_* = \frac{w \sqrt{\rho}}{\kappa \sqrt{\tau_c} \left[1 + \frac{1}{2} \left(\xi \frac{\hat{u}_o}{v} \right)^2 \right]} \tag{46}$$

It should be stated that in the Einstein integrals of Eqs. 28 and 29, the by the wave motion increased value of z_* should be used.

In the term $\sqrt{\frac{\tau_c}{\rho}}$ the shear stress as result of the mere current should be used because $\sqrt{\frac{\tau_c}{\rho}}$ can be written also as $\frac{v}{C} \sqrt{\xi}$ and does indicate the rate at which the suspended material is transported. This velocity is not increased by the wave motion.

The suspended load follows from

$$S_s = \int_0^h c(z) v(z) dz \tag{47}$$

In this equation $c(z)$ is calculated by Eq. 21, with the increased z value of Eq. 46. $v(z)$ is the normal current distribution according to Eq. 11,b. When using the total Einstein integral term Q this leads to

$$S_s = 1.83 Q S_b$$

This indicates that the suspended load transport is directly and linearly proportional to the bed load. This is logical since the suspended load is directly related through c_s to S_b

The total transport can now be written as

$$S = S_b + S_s = S_b (1+1.83 Q) \tag{45}$$

In Table 2 and in Figure 10 the values of Q and S_s/S_b are given as function of r/h and z_* . In this case z_* has the by the wave motion increased value:

$$z_* = \frac{w}{\kappa V_* c w} \tag{44}$$

In order to summarize the procedure an example for a case with and without waves is given. In this case the value of b is assumed to be 2.

Example:

Given:

$$\begin{aligned} h &= 3 \text{ m} \\ V &= 1 \text{ m/s} \\ r &= 0.06 \text{ m} \end{aligned}$$

$$\begin{aligned} H &= 1.18 \text{ m} \\ T &= 8 \text{ s} \end{aligned} \quad] \quad \hat{u}_o = 1 \text{ m/s}$$

$$\begin{aligned} D_{50} &= 200 \text{ } \mu\text{m} \\ D_{90} &= 300 \text{ } \mu\text{m} \end{aligned}$$

Currents only:

$$S_b = 5D_{50} \frac{V\sqrt{g}}{C} \exp\left[\frac{-0.27\Delta D_{50}\rho g}{\mu r_c}\right]$$

To calculate this bottom transport we need the following parameters:

$$C = 18 \log(12h/r) = 50 \sqrt{\text{m/s}}$$

$$C_{90} = 18 \log(12h/D_{90}) = 91.4 \sqrt{\text{m/s}}$$

$$\mu = (C/C_{90})^{3/2} = 0.405$$

$$r_c = \rho g(V/C)^2 = 3.92 \text{ N/m}^2$$

This gives a bottom transport of:

$$S_b = 0.000036 \text{ m}^3/\text{sm}$$

The suspended transport, using Fig.3.5.4 or Table 3.5.1, is a function of A ($= r/h$) and z_* .

$$S_s = f(A, z_*)$$

$$A = r/h = 0.02$$

$$z_* = w/(\kappa V_*)$$

$$w = 0.025 \text{ m/s}$$

$$D_{50} = 200 \text{ } \mu\text{m}$$

$$V_* = \sqrt{r_c/\rho} = 0.063 \text{ m/s}$$

$$z_* \approx 1$$

Using Fig.3.5.4 or Table 3.5.1:

$$S_s \approx 6 \cdot S_b = 0.000216 \text{ m}^3/\text{sm}$$

For the total sediment transport:

$$S = S_s + S_b \approx 7 \cdot S_b = 0.000252 \text{ m}^3/\text{sm}$$

Currents and waves:

$$S_b = 5D_{50} \frac{V\sqrt{g}}{C} \exp\left[\frac{-0.27\Delta D_{50}\rho g}{\mu\tau_{cw}}\right]$$

The shear stress is found using:

$$\overline{\tau_{cw}} = \tau_c + 1/2 \hat{\tau}_w$$

$$\hat{\tau}_w = 1/2 \rho f_w \hat{u}_o^2$$

$$f_w = \exp\left[-5.977 + 5.213(a_o/r)^{-0.194}\right] = 0.045$$

$$\hat{\tau}_w = 22.5 \text{ N/m}^2$$

$$\overline{\tau_{cw}} = 3.92 + 11.25 = 15.17 \text{ N/m}^2$$

$$S_b = 0.000054 \text{ m}^3/\text{sm}$$

Again the suspended sediment is found by using Fig.3.5.4 or Table 3.5.1:

$$S_s = f(A, z^*)$$

$$A = r/h = 0.02$$

$$z^* = w/(\kappa V_{*cw})$$

$$V_{*cw} = \sqrt{\tau_{cw}/\rho} = \sqrt{15.17/1000} = 0.123 \text{ m/s}$$

$$z^* \approx 0.5$$

$$S_s \approx 25 S_b = 0.00135 \text{ m}^3/\text{sm}$$

$$S = S_b + S_s \approx 26 S_b = 0.001404 \text{ m}^3/\text{sm}$$

By superimposing waves on the current the suspended sediment increases a factor ≈ 6.3 , while the total transport increases by a factor ≈ 5.6 .

6. Comments on this approach.

As stated already in the introduction, the transport formula is only a very rough approximation of the reality.

1. It has been mentioned already that it is not correct to average over v_{cw} for the computation of the exponential stirring-up term. The averaging should have taken place over the total exponential function.
2. It is assumed that the turbulent diffusion coefficient for the waves is equal to that for the fluid (water). This is most likely not true (van de Graaff, 1988).
3. It is not certain whether the increased turbulence near the bed spreads indeed over the total height of the fluid so that for the concentration distribution the increased value of z_* should be used. Tests in the pulsating watertunnel of Delft Hydraulics indicate that this is possibly not entirely true.
4. The ripple factor is calculated as the ratio between the shear stress caused by only the grains and the total shear stress. The same value is used in the case for a combination of waves and currents. It should have been better to use for both values those increased by the waves. The influence will be, however, rather limited.
5. The formula is derived for a current as main transporting agency, with waves only as an increasing factor. As soon as the orbital motion at the bed, \hat{u}_1 , becomes almost equal to the current near the bed, v_1 , the results should be regarded with some suspicion.

7. Application for waves and current in almost equal directions.

In principle the formula can be used also in this case. However, due to the generation of the suspended sediment over the ripples (Bijker et al 1976) the applied time and bed averaging is probably not any more allowed. When the waves are small in comparison with the current it might be allowed, but not when they are of equal magnitude. In practice this means that the formula may be used for grid models as the sediment transport module but not to calculate on- and offshore transport in the breakerzone. In this case the actual transport in the direction of the waves is not neglectable in comparison with the transport by the current, even when increased by the wave action. The principle of this motion is explained in Figure 11.

The sand transport is caused by eddies formed behind the ripple crests (1). The high orbital motion causes erosion of the lee side of the ripple and of the ripple crest. Most of this sand is stored in the eddy. When the orbital velocity decreases and reverses, the eddy "explodes" and the sand is carried backward in suspension. When the distance over which the sand is transported by the backward orbital velocity is larger than the ripple length (which is often the case) this results in a resultant backward transport. During the backward orbital velocity the same phenomenon occurs. Since, however, normally the forward orbital motion (in the direction of the wave propagation) is stronger than the backward motion, the resultant sediment transport is in the opposite direction of the wave propagation.

8. Application of the formula in the breakerzone and the influence of turbulence.

Although, as mentioned in Chapter 6, the formula is developed for a situation with a current which is predominant over the waves, the procedure is also applied to calculate the longshore transport by the wave induced current in the breakerzone.

The fact that the phenomena described in the previous chapter does occur in this situation does not make the application of the formula in this case impossible. The orbital motion may cause some on- or offshore transport, but the transport of this material by the longshore current is still described sufficiently well by using the time and bed averaged concentration.

The real point of concern is the fact that for the greater part of this zone the waves are breaking and therefore cause an increased turbulence. This results in a diffusion coefficient which is considerably higher than that which would result from a current with normal, not breaking waves.

Basically the best procedure would be to introduce this effect in the exponential "stirring-up" term. Since, however, not yet sufficient research has been performed in this phenomenon, this effect has been accounted for by increasing the factor "b" to the value 5. This value has been obtained by calibration with data from nature and models.

At this moment tests are in progress at the Delft University of Technology in which sediment concentrations are measured under these conditions. This research will probably lead the way to a better understanding of this effect. This will then, however, most likely lead also to another and better transport formulation.

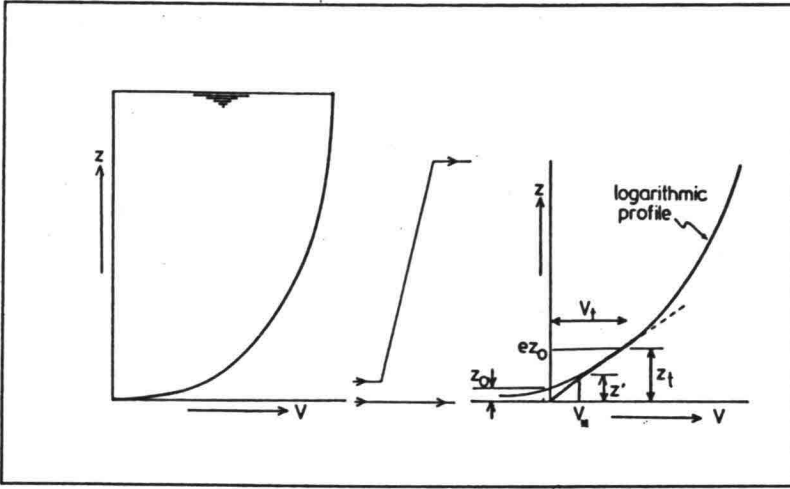


Fig. 1 Velocity distribution for a uniform stationary current.

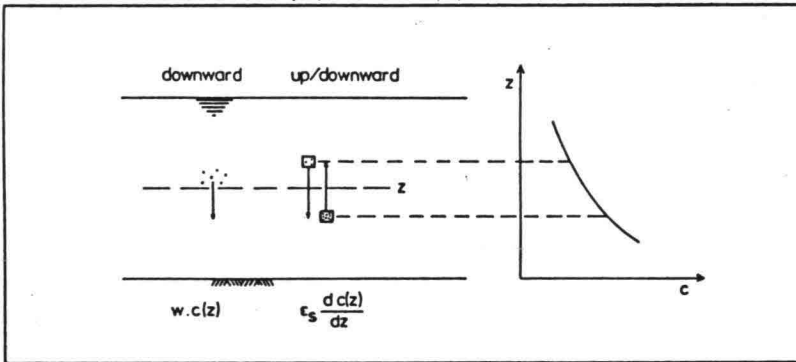


Fig. 2 Mass balance of sediment.

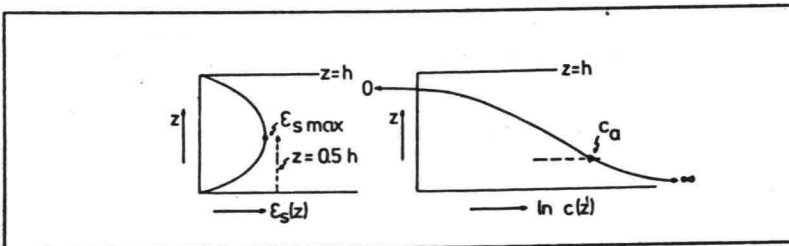


Fig. 3 Rouse/Einstein distribution of the sediment diffusion coefficient.

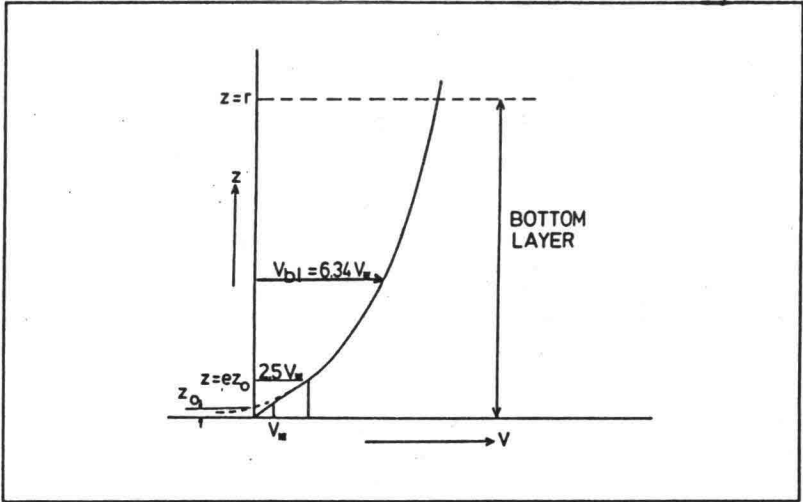


Fig. 4 Computation of the mean velocity in the bottom layer.

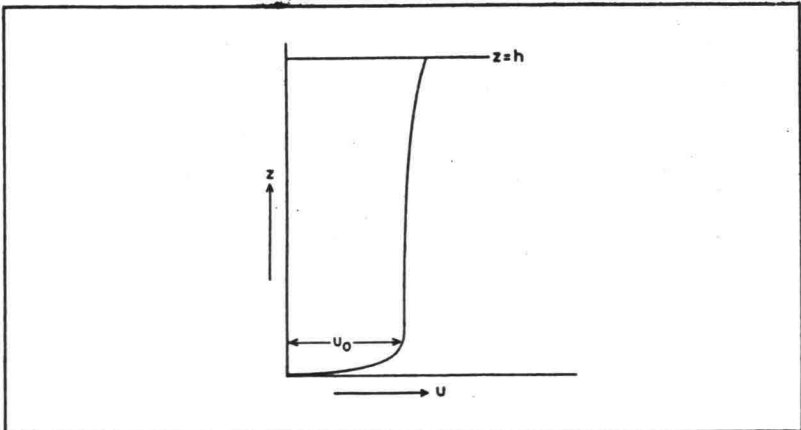


Fig. 5 Variation of velocity with height.

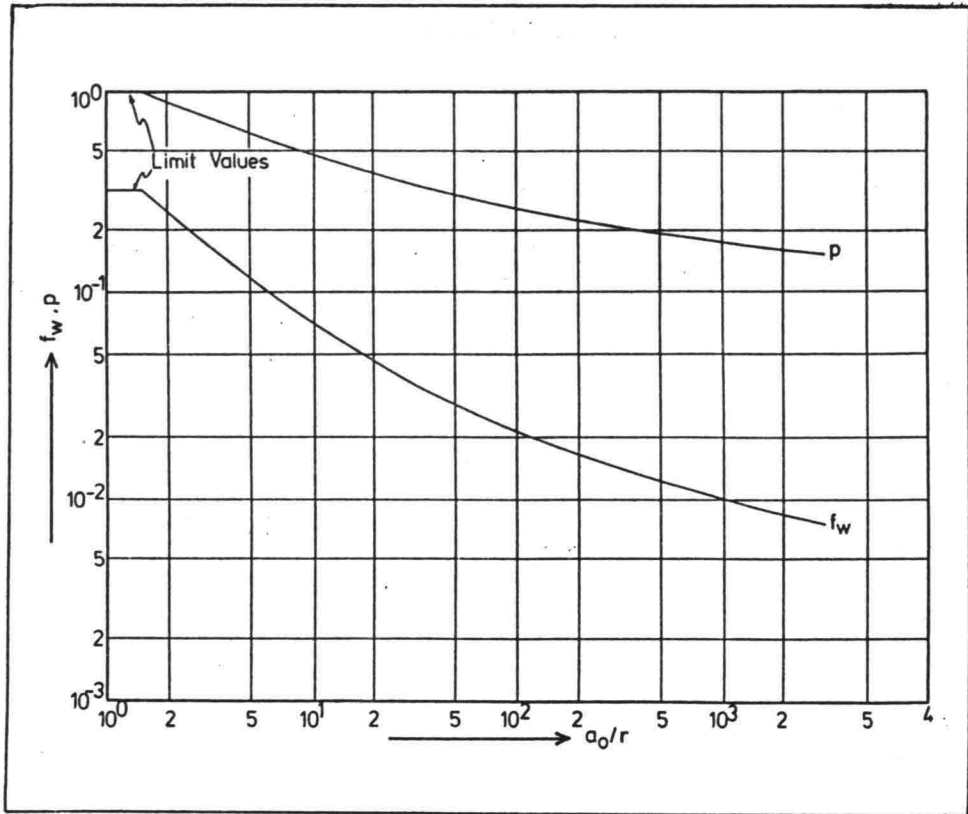


Fig. 6 Wave friction parameters (p , f_w).

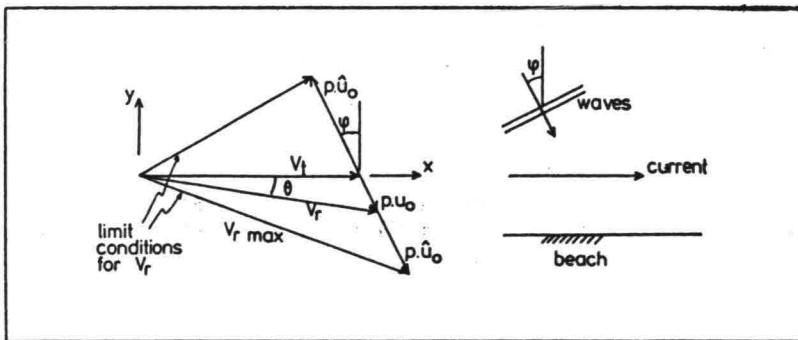


Fig. 7 Plan view and specific velocity components at an elevation z_c above the bottom.

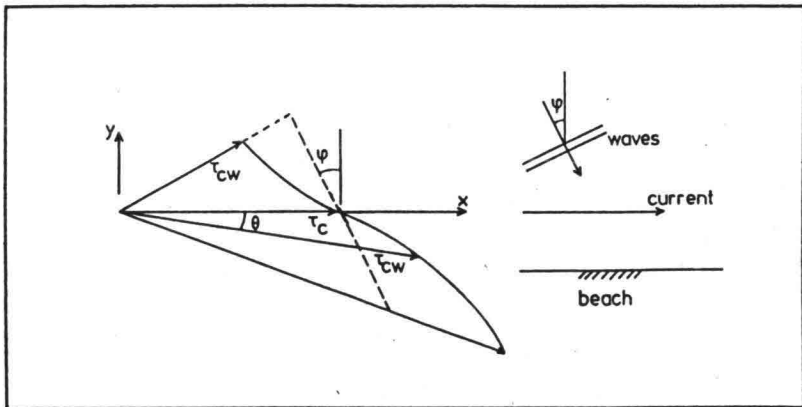


Fig. 8 Shear stress component at an elevation z_t above the bottom.

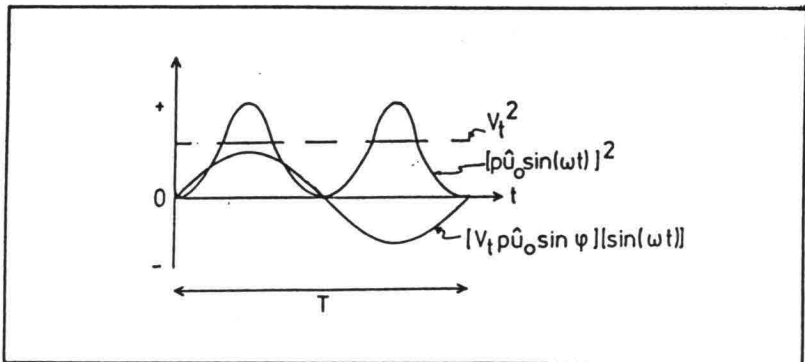


Fig. 9 Components of the mean shear stress.

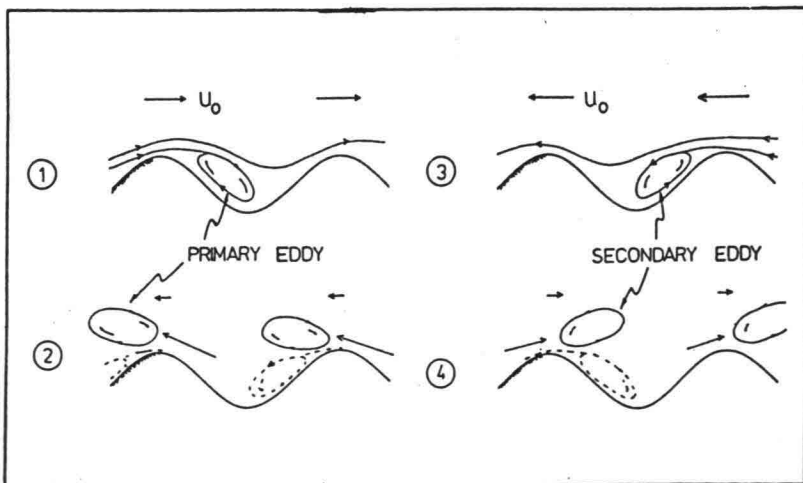


Fig. 11 Eddy formation near ripples.

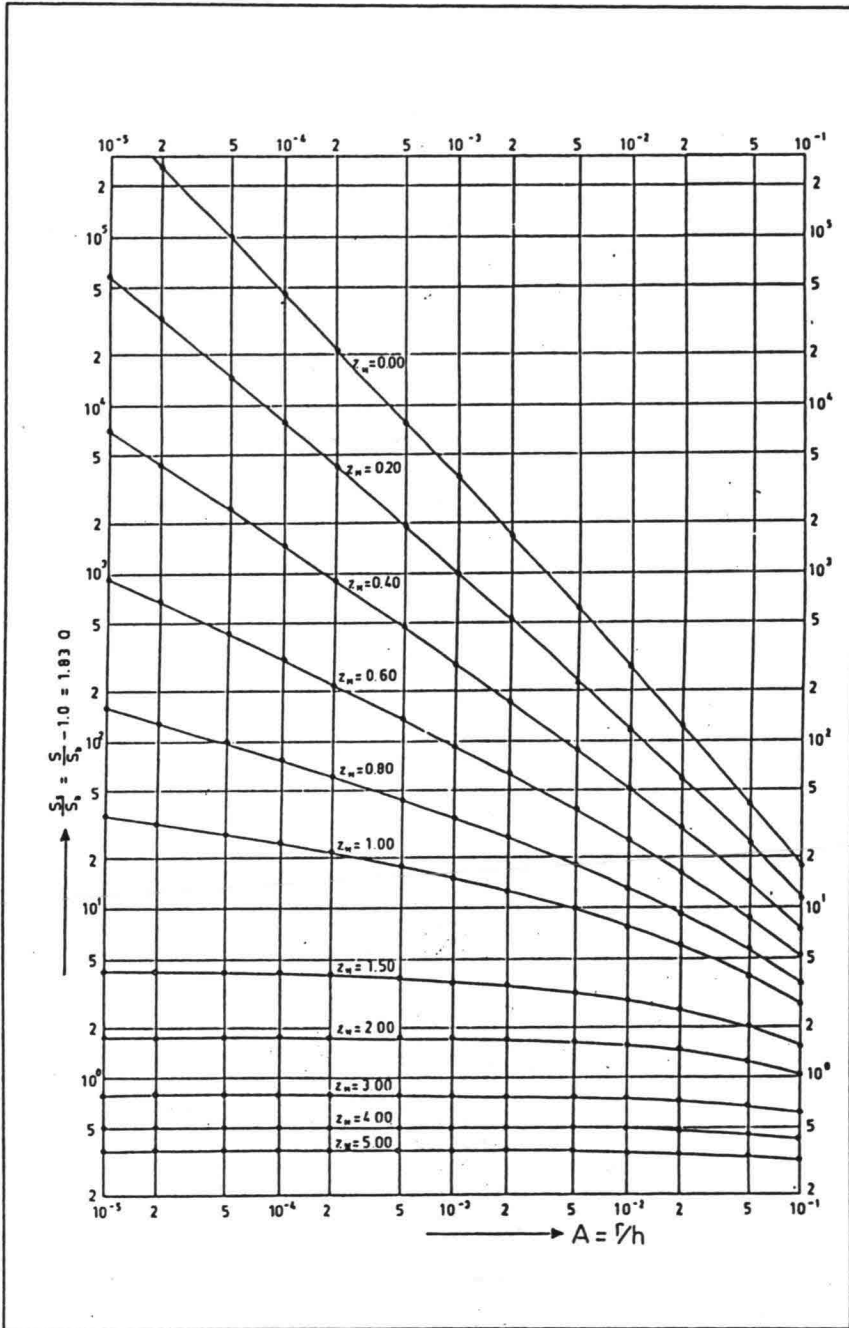


Fig. 10 Suspended sediment transport parameters.

| r/h (-) | z* (-) | | | | | | | | |
|--------------------|--------|-------|------|------|------|------|------|-------|-------|
| | 0.00 | 0.20 | 0.40 | 0.60 | 0.80 | 1.00 | 1.50 | 3.00 | 4.00 |
| 1 10 ⁻⁶ | 303000 | 32800 | 3880 | 527 | 88.0 | 20.0 | 2.33 | 0.432 | 0.276 |
| 2 10 ⁻⁶ | 144000 | 17900 | 2430 | 377 | 71.6 | 17.9 | 2.31 | 0.432 | 0.276 |
| 5 10 ⁻⁶ | 53600 | 7980 | 1300 | 239 | 53.6 | 14.4 | 2.28 | 0.432 | 0.276 |
| 1 10 ⁻⁴ | 25300 | 4320 | 803 | 169 | 42.7 | 13.6 | 2.25 | 0.432 | 0.276 |
| 2 10 ⁻⁴ | 11900 | 2330 | 496 | 119 | 33.9 | 11.9 | 2.21 | 0.431 | 0.275 |
| 5 10 ⁻⁴ | 4360 | 1020 | 260 | 74.3 | 24.6 | 9.8 | 2.13 | 0.431 | 0.275 |
| 1 10 ⁻³ | 2030 | 545 | 158 | 51.2 | 19.1 | 8.4 | 2.05 | 0.430 | 0.275 |
| 2 10 ⁻³ | 940 | 289 | 95.6 | 35.1 | 14.6 | 7.0 | 1.96 | 0.428 | 0.274 |
| 5 10 ⁻³ | 336 | 123 | 48.5 | 20.8 | 10.0 | 5.4 | 1.78 | 0.424 | 0.273 |
| 1 10 ⁻² | 153 | 63.9 | 28.6 | 13.8 | 7.3 | 4.3 | 1.62 | 0.417 | 0.279 |
| 2 10 ⁻² | 68.9 | 32.8 | 16.5 | 8.9 | 5.2 | 3.3 | 1.42 | 0.404 | 0.264 |
| 5 10 ⁻² | 23.2 | 13.1 | 7.7 | 4.8 | 3.1 | 2.2 | 1.10 | 0.374 | 0.249 |
| 1 10 ⁻¹ | 9.8 | 6.3 | 4.1 | 2.8 | 2.0 | 1.5 | 0.84 | 0.339 | 0.236 |
| 2 10 ⁻¹ | 3.9 | 2.8 | 2.0 | 1.5 | 1.2 | 0.9 | 0.55 | 0.317 | |
| 5 10 ⁻¹ | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.17 | | |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | |

(all items are dimensionless)

Table 1 Values of Einstein integral factor Q.

| r/h | $z_* = 0.00$ | | $z_* = 0.20$ | | $z_* = 0.40$ | | $z_* = 0.60$ | | $z_* = 0.80$ | |
|---------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|
| | Q | S_s/S_b | Q | S_s/S_b | Q | S_s/S_b | Q | S_s/S_b | Q | S_s/S_b |
| 0.00001 | 303000 | 554000 | 32800 | 60000 | 3880 | 7100 | 527 | 964 | 88 | 161 |
| 0.00002 | 144000 | 263000 | 17900 | 32700 | 2430 | 4440 | 377 | 689 | 71.6 | 131 |
| 0.00005 | 53600 | 98000 | 7980 | 14600 | 1300 | 2370 | 239 | 438 | 53.6 | 98.0 |
| 0.0001 | 25300 | 46300 | 4320 | 7900 | 803 | 1470 | 169 | 310 | 42.7 | 78.2 |
| 0.0002 | 11900 | 21800 | 2330 | 4260 | 496 | 907 | 119 | 218 | 33.9 | 62.0 |
| 0.0005 | 4360 | 7980 | 1020 | 1870 | 260 | 475 | 74.3 | 136 | 24.6 | 45.0 |
| 0.001 | 2030 | 3720 | 545 | 998 | 158 | 290 | 51.2 | 93.7 | 19.1 | 34.9 |
| 0.002 | 940 | 1720 | 289 | 529 | 95.6 | 175 | 35.1 | 64.2 | 14.6 | 26.7 |
| 0.005 | 336 | 615 | 123 | 226 | 48.5 | 88.7 | 20.8 | 38.1 | 10.0 | 18.3 |
| 0.01 | 153 | 280 | 63.9 | 117 | 28.6 | 52.3 | 13.8 | 25.2 | 7.3 | 13.4 |
| 0.02 | 68.9 | 126 | 32.8 | 60.0 | 16.5 | 30.2 | 8.9 | 16.3 | 5.2 | 9.5 |
| 0.05 | 23.2 | 42.4 | 13.1 | 24.0 | 7.7 | 14.1 | 4.8 | 8.7 | 3.1 | 5.7 |
| 0.1 | 9.8 | 18.0 | 6.3 | 11.5 | 4.1 | 7.5 | 2.8 | 5.1 | 2.0 | 3.6 |
| 0.2 | 3.9 | 7.1 | 2.8 | 5.1 | 2.0 | 3.7 | 1.5 | 2.8 | 1.2 | 2.1 |
| 0.5 | 0.8 | 1.5 | 0.7 | 1.3 | 0.6 | 1.1 | 0.5 | 0.9 | 0.4 | 0.7 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | |
| r/h | $z_* = 1.00$ | | $z_* = 1.50$ | | $z_* = 2.00$ | | $z_* = 3.00$ | | $z_* = 4.00$ | |
| | Q | S_s/S_b | Q | S_s/S_b | Q | S_s/S_b | Q | S_s/S_b | Q | S_s/S_b |
| 0.00001 | 20.0 | 36.6 | 2.33 | 4.26 | 0.973 | 1.78 | 0.432 | 0.790 | 0.276 | 0.505 |
| 0.00002 | 17.9 | 32.8 | 2.31 | 4.23 | 0.973 | 1.78 | 0.432 | 0.790 | 0.276 | 0.505 |
| 0.00005 | 14.4 | 28.2 | 2.28 | 4.17 | 0.967 | 1.77 | 0.432 | 0.790 | 0.276 | 0.505 |
| 0.0001 | 13.6 | 24.9 | 2.25 | 4.11 | 0.967 | 1.77 | 0.432 | 0.790 | 0.276 | 0.505 |
| 0.0002 | 11.9 | 21.8 | 2.21 | 4.04 | 0.967 | 1.77 | 0.431 | 0.789 | 0.275 | 0.504 |
| 0.0005 | 9.8 | 17.9 | 2.13 | 3.90 | 0.962 | 1.76 | 0.431 | 0.788 | 0.275 | 0.504 |
| 0.001 | 8.4 | 15.3 | 2.05 | 3.76 | 0.951 | 1.74 | 0.430 | 0.787 | 0.275 | 0.503 |
| 0.002 | 7.0 | 12.8 | 1.96 | 3.58 | 0.940 | 1.72 | 0.428 | 0.784 | 0.274 | 0.502 |
| 0.005 | 5.4 | 9.8 | 1.78 | 3.26 | 0.907 | 1.66 | 0.424 | 0.776 | 0.273 | 0.499 |
| 0.01 | 4.3 | 7.8 | 1.62 | 2.96 | 0.869 | 1.59 | 0.417 | 0.763 | 0.270 | 0.494 |
| 0.02 | 3.3 | 6.0 | 1.42 | 2.59 | 0.809 | 1.48 | 0.404 | 0.740 | 0.264 | 0.483 |
| 0.05 | 2.2 | 4.0 | 1.10 | 2.02 | 0.694 | 1.27 | 0.374 | 0.684 | 0.249 | 0.456 |
| 0.1 | 1.5 | 2.7 | 0.84 | 1.53 | 0.568 | 1.04 | 0.339 | 0.620 | 0.236 | 0.432 |
| 0.2 | 0.9 | 1.6 | 0.55 | 1.01 | 0.414 | 0.76 | 0.317 | 0.580 | | |
| 0.5 | 0.3 | 0.6 | 0.17 | 0.32 | | | | | | |
| 1 | 0 | 0 | 0 | 0 | | | | | | |

Table 2 Values of Einstein integral factor, Q, and values of the ratio suspended load to bed load, S_s/S_b , according to Bijkers transport formula.

SYMBOLS.

| | | |
|------------------|---|-----------------------|
| a | height above bed of reference concentration | [1] |
| b | coefficient | - |
| c | concentration | [m/l ³] |
| f _w | wave friction factor | - |
| g | acceleration of gravity | [1/t ²] |
| h | depth | [1] |
| i | energy slope | - |
| p | velocity factor | - |
| r | bed roughness | [1] |
| t | time | [t] |
| u | orbital velocity | [1/t] |
| v | current velocity | [1/t] |
| v _* | shear stress velocity | [1/t] |
| v _t | v _* /κ | [1/t] |
| w | fall velocity of sediment | [1/t] |
| ζ | height above bed | [1] |
| r _* | Rouse number | - |
| A | dimensionless roughness = r/h | - |
| C | Chezy friction coefficient = 18 log (12h/r) | [1 ^{1/2} /t] |
| D | grain diameter | [1] |
| I _{1,2} | Einstein integrals | - |
| Q | Einstein integral factor: I ₁ ln(h/r)+I ₂ | - |
| S | Sediment Transport | [1 ³ /t] |
| T | wave period | [t] |
| e | diffusion coefficient | [1 ² /t] |
| ζ | dimensionless distance above bed: z/h | - |
| η | ripple height | [1] |
| κ | von Karman constant | - |
| μ | ripple factor | - |
| ξ | combined wave and current friction factor | - |
| ρ | density | [m/l ³] |
| τ | bed shear stress | [m/lt ²] |
| φ | angle between wave crest and current | [o] |
| ω | angular velocity: 2π/T | [1/t] |
| Δ | relative density of grains | - |
| θ | angle between current and resultant shear stress | [o] |

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