CHAPTER TWO

Pedestrian flow and crowd operation variables

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Abstract

Traffic flow variables are essential for understanding, analysing, and optimising how pedestrians move in urban areas. In this chapter, we introduce the variables that can be used to describe a pedestrian flow. These variables can be microscopic (individual pedestrians), macroscopic (aggregate level), or mesoscopic (distributions of microscopic quantities). We start with the most detailed level of description, a trajectory. The trajectory describes the dynamics of an individual pedestrian as a function of time by its (two-dimensional) position in space. Based on the trajectory, we derive the most relevant microscopic variables, velocity and acceleration. Then, we give different definitions of density, one of the key macroscopic flow variables, by taking a snapshot of a pedestrian traffic situation at a time instant. Density is used to express crowdedness and level-of-service. Next, we look at the space-mean velocity of a pedestrian flow, followed by the third key macroscopic variable, the flow rate. The flow rate is the number of pedestrians passing a cross-section during a certain time period. We end this chapter with the generalised definitions of flow, density, and velocity for a time-space region, and show how they are related.

Advances in Transport Policy and Planning, Volume 15
ISSN 2543-0009, https://doi.org/10.1016/bs.atpp.2025.04.005

1. Introduction

Traffic flow variables are essential for understanding, analysing, and optimising how pedestrians and pedestrian flows move in urban areas. They help engineers, city planners, and researchers improve traffic management, reduce walking and (over) crowdedness, enhance safety, and design better transportation systems and sustainable and liveable cities.

Pedestrian operations occur – in contrast to car traffic – in two-dimensional space. This makes these flows more complex to describe than car traffic, since the generalisation of well-known concepts (e.g., distance headway, flow rate) is not trivial. Moreover, pedestrian traffic is more sensitive to the environment and the need for dedicated infrastructure and heterogeneity (the fact that each pedestrian has different characteristics and different behaviour) plays a more important role as well.

In this chapter, we will introduce the different variables that can be used to describe a pedestrian flow. These variables can be *microscopic* (describe a flow on the level of the individual pedestrian), *macroscopic* (describe a flow at the aggregate level, in terms of aggregate variables such as density, flow, velocity), or *mesoscopic* (in terms of *distributions* of microscopic quantities).

Note that unless specified differently, we will use SI units for all quantities (meter, seconds, meter/second, etc.).

In providing the different definitions, we aim to provide the material that will allow the reader to define, compute, and assess pedestrian flow traffic variables. In doing so, we focus on those variables that are needed describe and understand pedestrian flows, to model and simulate pedestrian flows, to analyse and illustrate relations between variables and the pedestrian traffic flow state, and to evaluate and manage functioning of pedestrian facilities.

The chapter is organised as follows: in the next section, we will discuss the basis microscopic quantity, the trajectory, and the different characteristics that can be derived directly from the trajectory of a pedestrian. Next, we will look at the key macroscopic quantities, namely density (Section 3), velocity (Section 4) and flow rates (Section 5). In Section 6, we introduced generalised definitions of density, velocity and flow according to Edie. We end with a short summary in Section 7.

2. Trajectories and related quantities

From the perspective of a traffic flow, the *trajectory* is the most detailed level of description. The trajectory describes the dynamics of an individual

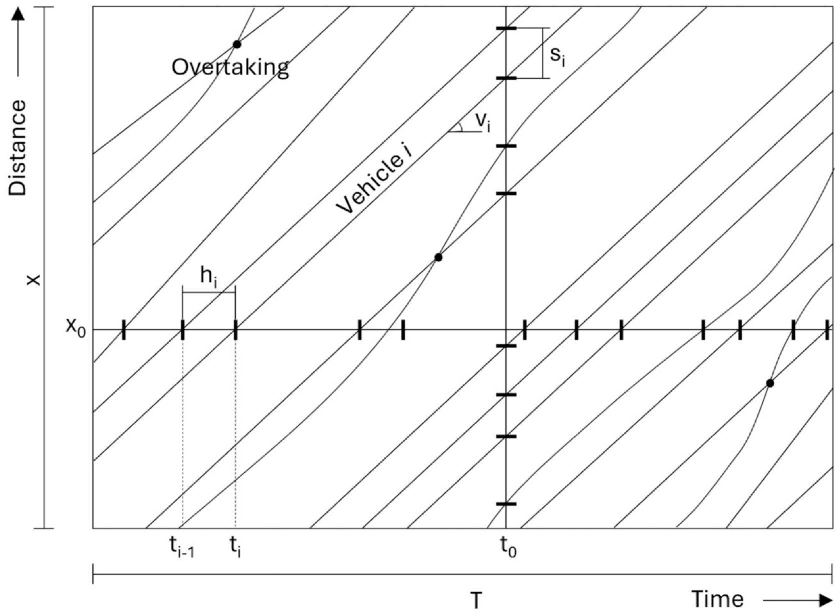

Fig. 1 Vehicle trajectories on a two-lane road. Vehicles move from bottom to top.

traffic participant as a function of time *t* by its position in space. As car traffic is simpler, as it is moving in only one direction, we start by introducing trajectories of vehicles on a two-lane road. A graphical representation of these trajectories is given in Fig. 1. The trajectories are drawn in a space time plot, with time on the horizontal axis and space on the vertical axis. By definition, trajectories do not get back in time, so the trajectories always move from left to right. In Fig. 1 we show one-directional traffic, meaning that the trajectories also following increasing distance. In case of opposing traffic, trajectories move in decreasing distance.

The fundamental microscopic variables describing car traffic are speed, space headway, and time headway (see Fig. 1). These variables or characteristics of the flow are easy to derive from trajectories. Speed represents the distance a vehicle travels per unit of time. This corresponds to the slope of a trajectory. Steeper trajectories correspond to a higher vehicle speed. This also implies that the trajectories cannot be vertical, as that reflects an infinite speed. In some cases, the inverse of speed is a useful measure; this represents the time a vehicle takes to cover a unit of distance and is referred to as the pace. Another key variable is space headway or spacing, which measures the distance between a vehicle and its leader. Here, we distinguish between the net space headway (the gap between the front bumper of a vehicle and the rear bumper of the vehicle ahead) and the gross space headway (the distance from the rear bumper of the leading vehicle to the rear bumper of the following vehicle, so including the length of the following vehicle itself. Similarly, we can consider the time it takes for the front bumper of a following vehicle to reach the position of the rear bumper of its leader. This is known as the net time headway. If we additionally account for the time required to travel the length of the vehicle itself, we obtain the gross time headway.

Contrary to cars moving in uni-directional (along the road), pedestrians move in a two-dimensional plane, where they can follow multiple directions. Another difference between car traffic and pedestrian traffic is that the concept of distance headway cannot be generalized. Cars travel in designated lanes and maintain relatively structured linear movements, while pedestrians move freely in open space, taking flexible paths.

Given the two-dimensional character of pedestrian movements, in this chapter, we will use the vector notation and use the subscript i to refer to the traffic participant. That is, $\vec{x}_i(t)$ denotes the position of pedestrian i in a 2-dimensional plane at time instant t.[1]

In some cases, a three-dimensional representation may also be useful. This would mean that also the z-direction is considered (i.e., height). This may be useful if considering multi-level facilities, or if we want to explicitly use incline or decline in the modelling which in turn could have an affect on the walking speeds of the pedestrians. For the sake of simplicity, we will not consider this generalisation in this chapter; generalisations are left to the reader.

Fig. 2 shows an example of a set of trajectories collected during a crossing flow experiment. In this experiment, two groups of pedestrians were instructed to traverse an eight by eight meter square. One group walked from top to bottom; one from right to left. Using image analysis techniques, the trajectories of all pedestrians were collected. We refer to Daamen and Hoogendoorn (2003) for a complete description of the experiment and the data collection set-up. Contrary to car trajectories, pedestrian trajectories cannot be fully visualised in a two-dimensional figure, as they vary in three dimensions: two spatial dimensions and one temporal dimension. Therefore, three different projections are shown. In the top-left projection the red trajectories are fully shown, as those

[1] Note that by *position*, we mean the position of some (fixed) reference point of the pedestrian (e.g., the head of a pedestrian).

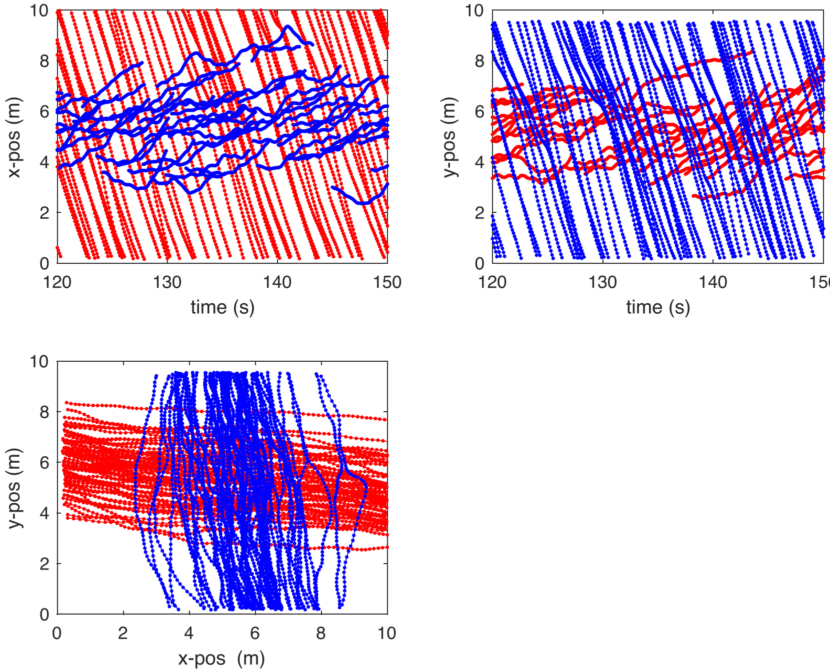

Fig. 2 Different projections of trajectories from the crossing experiment (top-left shows x-position versus time, top-right figure shows y-position over time, and the figure below shows x-position versus y-position. The two colours (blue and red) indicate the walking directions (blue: from top to bottom; red: from right to left; see Daamen and Hoogendoorn (2003)).

pedestrians move in the x-direction, so their trajectories are shown from the moment they enter the experiment until the moment they leave. This projection also directly shows their moving speed in x-direction. The projection at the top-right gives the same information for the blue trajectories. The bottom picture represents a top-down view of the experiment area, and does not include the time dimension. Speeds can therefore not be derived directly from the figure, but the shape of the trajectory suggests the corresponding speed because of the swaying behaviour of pedestrians. Swaying behaviour refers to the natural lateral oscillations or side-to-side movements that pedestrians exhibit while walking. This behavior is influenced by biomechanics, that is, the fact that pedestrians shift their body weight between steps. The degree of sway depends on walking speed, stride length, and individual gait patterns (Wang et al., 2018). When a pedestrian is walking slowly, the lateral distance of the swaying remains similar, but the forward movement is small. This implies that the trajectory of a slow moving pedestrian shows much more swaying than the trajectory of a fast moving pedestrian.

In contrast to car flows, where we often can put different types of restrictions on a trajectory, the nature of a pedestrian flow is such that quite often these restrictions are not necessary. For instance, for the trajectory of a car, we quite often assume that $\frac{dx_i}{dt} \geq 0$, implying that a car would not be (very likely to be) driving backward. In general, these types of restrictions will not apply to pedestrians, since they are more flexible in their dynamics and will – at least on occasion – step backward, sideways, etc.

Based on the trajectory of traffic participant i, we can derive various microscopic flow variables. Below, we describe the most relevant ones for the contents of this book. The velocity $\vec{v}_i(t)$ is defined by:

$$\vec{v}_i(t) = \frac{d}{dt}\vec{x}_i(t) \tag{1}$$

It is important to note that the velocity is a vector (i.e., it has a direction). This in contrast to the (absolute) speed $w_i(t)$ of participant i, which is defined by:

$$w_i(t) = ||\vec{v}_i(t)|| \tag{2}$$

where $||\vec{z}|| = \sqrt{z_1^2 + z_2^2}$. Note that in pedestrian flow theory, it is common to use the term speed for the directionless velocity.

The direction $\vec{e}_i(t)$ of traffic participant i can be determined from the velocity and the speed as follows:

$$\vec{e}_i(t) = \frac{\vec{v}_i(t)}{w_i(t)} \tag{3}$$

This means that we have $\vec{v}_i(t) = w_i(t) \cdot \vec{e}_i(t)$.

In a similar way, we can define the acceleration $\vec{a}_i(t)$:

$$\vec{a}_i(t) = \frac{d}{dt}\vec{v}_i(t) = \frac{d^2}{dt^2}\vec{x}_i(t) \tag{4}$$

and the jerk $\vec{\gamma}_i(t)$:

$$\vec{\gamma}_i(t) = \frac{d}{dt}\vec{a}_i(t) \tag{5}$$

These quantities describe the basic microscopic variables for an individual traffic participant i (in isolation, so not in relation to other pedestrians, obstacles, etc.). To describe a pedestrian traffic flow, other quantities describing how pedestrian i relates to the other pedestrians k in the flow may also be required (e.g., distances and time headways). As said in the introduction of this chapter, these are much less trivial than for an – essentially one-dimensional – car flow.

3. Definitions of density

In this section, we will start by considering a snapshot of a pedestrian traffic situation at time instant t. At this time instant, multiple pedestrians j are present in the considered facility (stadion, station, road, etc.). The positions of all these pedestrians are again denoted by $\vec{x}_j(t)$.

3.1 Density in a region

The density is one of the key macroscopic flow variables in characterizing a pedestrian flow. It is used to express crowdedness, and level-of-service, and it is one of the many variables needed to macroscopically model pedestrian dynamics (e.g., macroscopic or continuum modelling).

The density describes the average or mean number of pedestrians in a certain area. While this sounds simple, the definition is however not trivial, which is one of the reasons why various definitions have been presented in literature. Here, we provide a (non-complete) overview of definitions needed for the purpose of this book. For a comprehensive overview, we refer to Duives et al. (2015).

Let us begin with the most basic definition of the *density*. Consider a region Ω in the two-dimensional space. This region can have any shape (square, rectangle, triangle).

The density for this region $\rho_\Omega(t)$ is defined by the number of pedestrians j in the region, divided by the area of the region, i.e.:

$$\rho_\Omega(t) = \frac{\sum_j 1_{\overrightarrow{x_j(t)\in\Omega}}}{||\Omega||} = \frac{n_\Omega(t)}{||\Omega||} \tag{6}$$

where $1_{\vec{x}\in\Omega} = 1$ if $\vec{x} \in \Omega$ and 0 elsewhere. Eq. (6) counts the number of pedestrians $n_\Omega(t)$ in Ω at time instant t and divides this number by the area $||\Omega||$. As the density is defined at a time instant, we call this an *instantaneous* traffic flow variable. To provide some insight into how densities involve over time, Fig. 3 provides the density during the crossing flow experiment (Daamen and Hoogendoorn, 2003).

The figure shows how the (instantaneous) density changes from the one moment to the next due to the fact that pedestrians move in and out of the

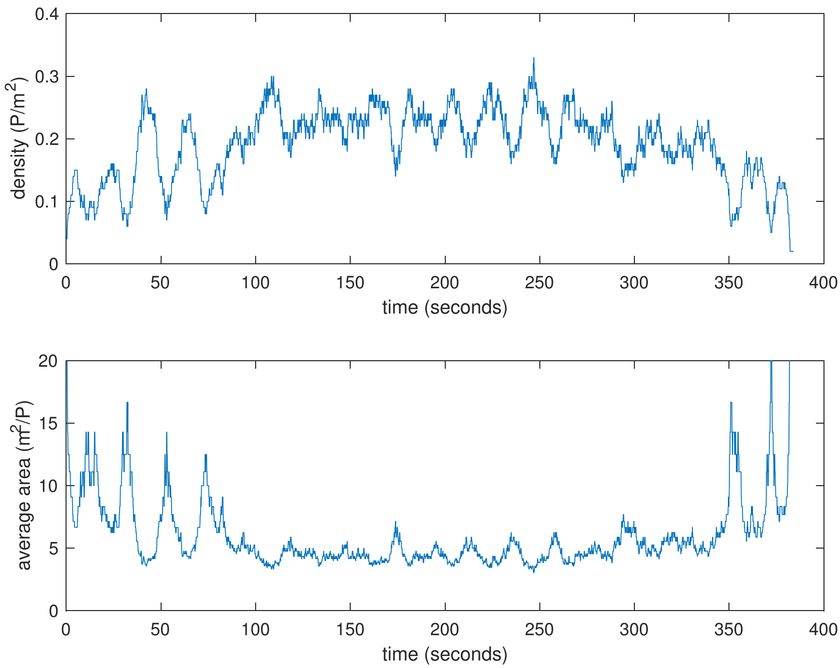

Fig. 3 Top: Example of instantaneous density over the area for crossing flow experiment as function of time; Bottom: example of the average area per pedestrian for same experiment as function of time.

area. It shows these 'microtrends', but it also shows more global trends reflecting changes in demand.

The density of a region is the most commonly used density definition. It is often used as a measure for crowdedness in an area. It is also often applied for macroscopic flow modelling, where an area or network is divided into (small) cells $\mathbf{\Omega}_i$, often in a grid-like structure, for which the dynamics of the densities are described based on inflows and outflows from the cells.

The definition of the density using a fixed region does however have downsides, in particular if we want to characterise the aggregate behaviour of the flow. For instance, we know that the area a person needs to make a step depends on the walking speed of that person. Macroscopically, this leads to the so-called fundamental diagram describing the relation between the density and the average speed: the lower the speed, the higher the density (or vice versa). For more insights into this fundamental diagram we refer to Chapter 3.

Fig. 4 shows a situation where this average relation between density and speed will not hold, solely due to the definition of the area $\mathbf{\Omega}$. Because the

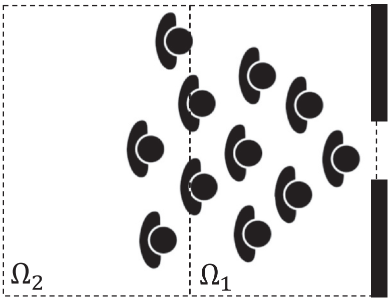

Fig. 4 Instantaneous density over the area of a queue of pedestrians waiting to go through a door opening.

pedestrians who are queuing are on the edge of the area Ω_2 (and not uniformly distributed over the available space), the average speed in the area is low, while the density is also low. This results in bias in the speed-density diagram, which can be remedied using another definition of density.

3.2 Average area per pedestrian

The density describes the average number of pedestrians per square meter. It is a *macroscopic* variable that describes the aggregate status of the pedestrian flow.

Using the density, we can clearly also provide insight into the (average) situation from a microscopic (individual) perspective by looking at the *average area* $A(t)$ that is available for each pedestrian. We have:

$$A(t) = \frac{||\Omega||}{n_\Omega(t)} = \frac{1}{\rho_\Omega(t)} \tag{7}$$

That is, the average area at the disposal of a pedestrian in the considered region is equal to one over the density in that region. Fig. 3 shows the dynamics of this quantity for the crossing flow experiment. The average density as well as the area (or space) have been used to define levels-of-service of a pedestrian flow; see Fruin (1971), Raad and Burke (2018), Highway Capacity Manual (2010). Examples will be provided later in this book.

3.3 Local density and area per pedestrian

In the previous subsection, we showed how the density can be used to compute the average area or space that is available to a pedestrian in the region considered.

Vice versa, the area per pedestrian j can also be used as a starting point for defining density. Let $A_j(t)$ be the part of the region Σ that pedestrian j is using. The (instantaneous) average area for pedestrians j in Ω can be determined easily via:

$$A(t) = \frac{1}{n_{\Omega(t)}} \sum_{\vec{x_j(t)} \in \Omega} A_j(t) \tag{8}$$

Using Eq. (7), the density can be computed from the average area per pedestrian.

The area $A_i(t)$ available per pedestrian is the microscopic equivalent of the density. Using trajectory information, several ways have been proposed in literature to partition the region Ω into subregions $\Omega_i(t)$ with area $A_i(t)$ reflecting the available areas for each pedestrian i. A well-known approach is the use of the so-called *Voronoi diagram*.[2] This diagram partitions a region into subregions $\Omega_i(t)$ based on the positions of the pedestrians $\vec{x}_i(t)$ (Steffen and Seyfried, 2010).

Fig. 5 shows an example of a Voronoi diagram applied to data from the crossing flow experiment. It shows how for each pedestrian i (shown in the figure by the '+'), an area is identified that signifies the individual area A_i of the pedestrian.

Based on the identified areas $A_i = A_i(t)$ at time instant t, we can determine a local density $\rho_i(t)$ value as follows:

$$\rho_i(t) = \frac{1}{A_i(t)} \tag{9}$$

We can easily show that the (global) density $\rho(t)$ then becomes:

$$\rho(t) = \frac{n_\Omega}{\sum_j \frac{1}{\rho_j(t)}} \tag{10}$$

Note that the micro definition of the density also allows us to determine other spatial statistics next to the mean density. For instance, we can compute the spatial variance $\sigma^2(t)$ in the density as a measure of how homogeneous the pedestrians are distributed over space at a particular moment of time t. Applications of this quantity can be found in Hoogendoorn et al. (2017).

The concept of *local densities* was also introduced without explicitly determining the available subregion per pedestrian. Johansson et al. (2008) propose using the following local density definition:

$$\rho_i(t, \vec{x}) = \frac{1}{\pi R^2} \sum_j \exp(-||\vec{x}_j(t) - \vec{x}||^2/R^2) \tag{11}$$

[2] We refer to https://en.wikipedia.org/wiki/Voronoi_diagramfor a more elaborate discussion.

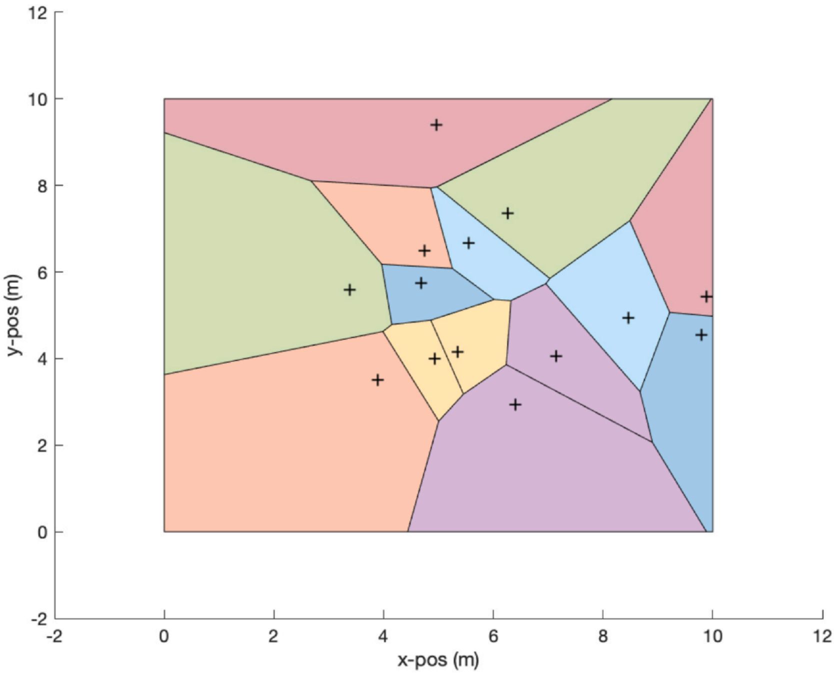

Fig. 5 Example of the Voronoi diagram.

Here, $\vec{x}$ denotes the location for which we are assessing the local density, $\vec{x}_j(t)$ denote the positions of pedestrians j, and $R > 0$ denotes a parameter (related to the radius of influence). From Eq. (11), we can see that the farther a pedestrian j is from the location $\vec{x}$, the less its contribution to the local density will be. In other words, the local density is high when many pedestrians j are near the assessment location $\vec{x}$. The greater the parameter R, the larger the smoothing area that is considered.

4. Space-mean velocity and speed

In the previous section, we showed the relation between the trajectory of a pedestrian j and the velocity $\vec{v}_j(t)$. Having determined the velocities of each active pedestrian j allows us to determine the *space-mean* velocity $\vec{u}_\Omega(t)$ at time instant t:

$$\vec{u}_\Omega(t) = \frac{1}{n_\Omega} \sum_{\overrightarrow{x_j(t)\in\Omega}} \vec{v}_j(t) \tag{12}$$

This instantaneous average velocity is a vector: it has an absolute value $U_\Omega(t)$ (the space-mean speed) and a direction $\vec{f}_\Omega\,(t)$.

Fig. 6 shows the space-mean velocities for the crossing flow experiment. The figure clearly shows that the interpretation of the space-mean velocity is not trivial due to its two-dimensional nature. Furthermore, the figure combines the data of two groups of pedestrians: those moving from top to bottom, and those moving from right to left. Fig. 7 shows the results when separating the two groups of pedestrians walking in different directions.

Note that in general, the space-mean speed $U_\Omega\,(t) = ||\vec{u}_\Omega\,(t)||$ is not the same as the space-mean of the individual speeds $w_j(t)$. Consider for instance a bi-directional flow in which 50 % of the pedestrians walk from left to right $\vec{v}_j = (1,\ 0)$, and the other 50 % of the pedestrians walk from right to left $\vec{v}_j = (-1,\ 0)$. If we compute the instantaneous average velocity, we would find $\vec{u}_\Omega = (0,\ 0)$ and thus $U_\Omega = 0$ m/s. Taking the average of the absolute speeds $w_j = ||\vec{v}_j||$ would give a value of 1 m/s. Note that if the

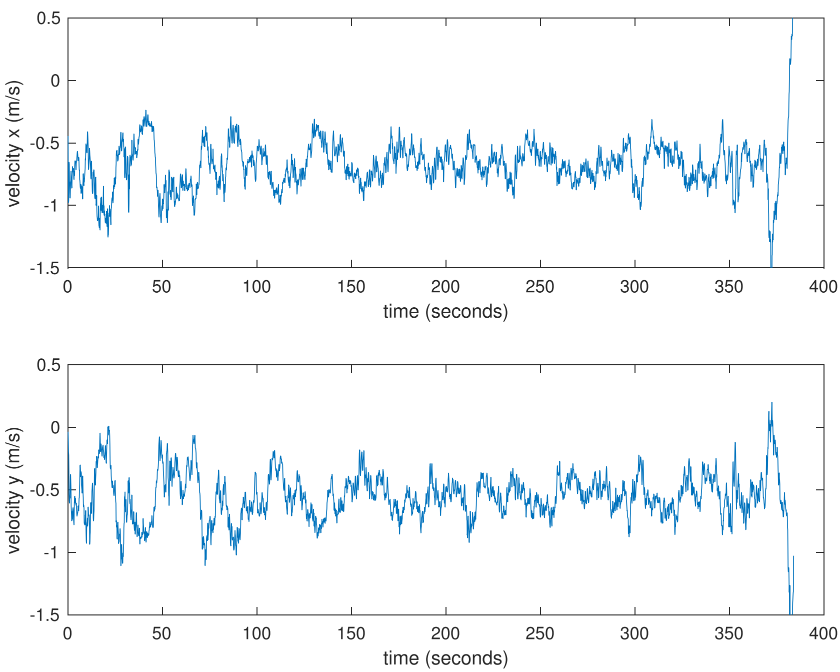

Fig. 6 Average velocity for crossing experiment.

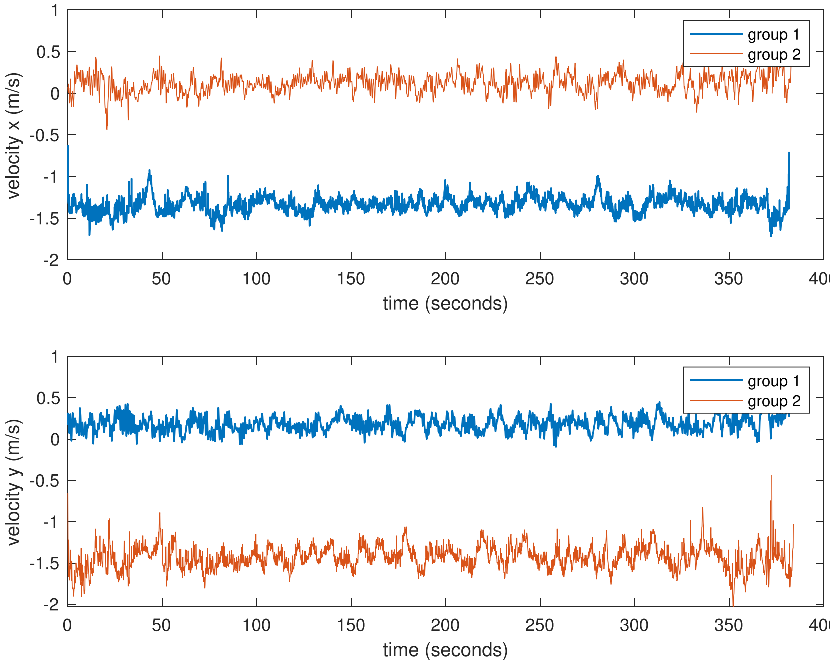

Fig. 7 Average velocity for crossing experiment per walking direction. Group 1 walks from right to left; group 2 walks from top to bottom.

pedestrians all walk into the same direction, the two values are equal. The choice of the most appropriate value depends on the application at hand.

The top part of Fig. 8 shows the difference between the space-mean (absolute) speed (where we take the mean of the absolute velocities at a time instant) and the absolute mean speed (where we take the absolute value of the space-mean velocity). The bottom part of Fig. 8 shows that the differences between the definitions vanish in case we consider the walking directions separately. It is left to the reader to interpret the differences between the figures.

A typical mean pedestrian speed lies around 1.34 m/s, with a standard deviation of 0.26 m/s (Bosina and Weidmann, 2017). However, many personal (e.g., age and gender) as well as contextual factors (e.g., weather and inclination) influence the speeds of individuals as well as the total pedestrian flow. We refer to Chapter 5 for more insights into these human factors.

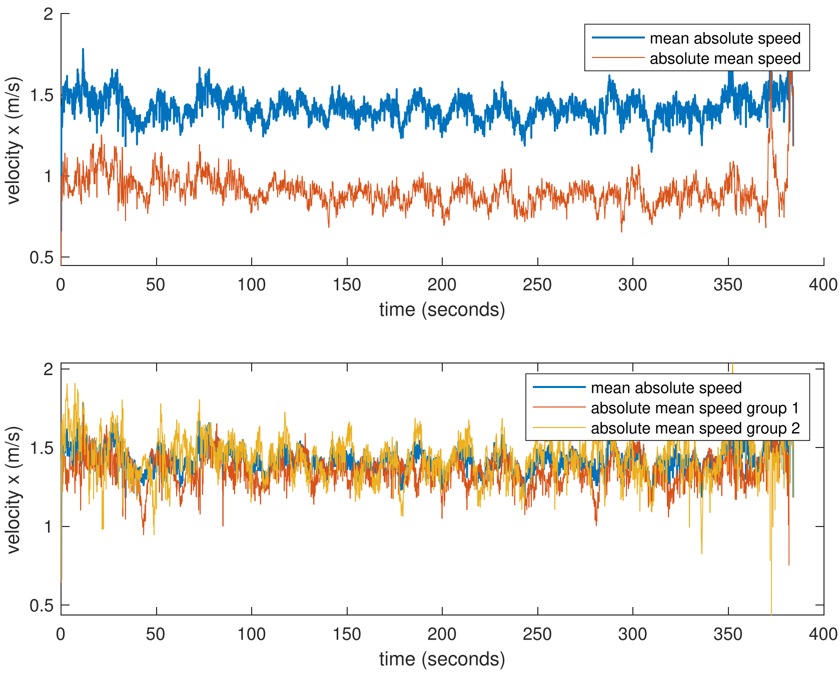

Fig. 8 Difference between different definitions of the speed: Top: difference between mean absolute speed and absolute mean speed; Bottom: differences between mean absolute speed and absolute mean speed per group.

5. Flow rates and time headways

In car traffic, flow rates are generally defined by looking at the average number of vehicles that pass a cross-section for a certain time period. In this book, cross-sections are defined by (user-defined) lines in the plane in which the pedestrian flow moves. Below, we will explain these lines and how they can be used to determine flow rates as well as time headways.

5.1 Flow rate at a cross-section

We will define a cross-section as a *line* in two-dimensional space. Often, this line will be perpendicular to one of the (x or y) axes in two-dimensional space, but this is not necessarily the case: there are several conceivable applications for which other definitions are more appropriate.

We will use the following representation of a line l:

$$a \cdot x + b \cdot y + c = 0 \tag{13}$$

The line will be bound by taking minimum and maximum values of x and/or y. This means that generally, the line $||l||$ has a certain length L which will be used to normalise the flow rate (e.g., in P/m/s).

We will denote $N(t_0, t_1|l)$ as the (cumulative) number of pedestrians that have passed the cross-section l in the period $[t_0, t_1)$. The flow rate then becomes:

$$q(t_0, t_1|l) = \frac{N(t_0, t_1|l)}{(t_1 - t_0) \cdot L} \tag{14}$$

where L is the length of the considered line (cross-section). The unit of the flow rate is thus $\frac{P}{ms}$. Using this definition, we can compare the flow in two corridors of different widths. Note that in some cases, we will consider the flow rate over the entire line; in those cases, the unit of the flow rate is $\frac{P}{s}$. The latter definition could be relevant to indicate the total inflow or outflow of an area. As indicated before, the definition to use is therefore dependend on the application at hand.

One should also consider of the movement direction of the pedestrians, like for pedestrian speed. Consider again a bi-directional flow in which 50 % of the pedestrians walk from left to right, and the other 50 % of the pedestrians walk from right to left. We can then calculate three flow rates, namely the total flow rate, the flow rate from right to left and the flow rate from left to right. Note that negative flows typically do not occur, to avoid confusion with the movement directions.

In illustration, we plotted the flow rate of pedestrians (over a cross-section of 10 m wide) moving from right to left for both the lines $x = 5$ and $y = 5$ in Fig. 9. The figure shows that the flow across the line $x = 5$ is substantially higher than across $y = 5$, but the latter is not equal to zero since pedestrians still pass this line (e.g. due to lateral movement due to side stepping, and evading other pedestrians).

Flow rates are (local) time-averaged quantities. This is in contrast to densities, space-mean velocities and speeds considered in the previous sections. Without going into detail now, we can show that if the flow is both stationary and homogeneous, the continuity equation holds:

$$\vec{q} = \rho \cdot \vec{u} \tag{15}$$

where ρ again is the density, $\vec{u}$ is the space mean velocity (both for the considered region Ω) and $\vec{q}$ is the flow rate vector (which elements denote the flow-rate in the x and y direction respectively). We will leave the proof of this for the next section where Edie's generalised definitions are introduced.

In many of the (macroscopic) models that will be discussed in Chapters 7 and 8 of this book, the flow vector $\vec{q}(t, \vec{x})$ is used.

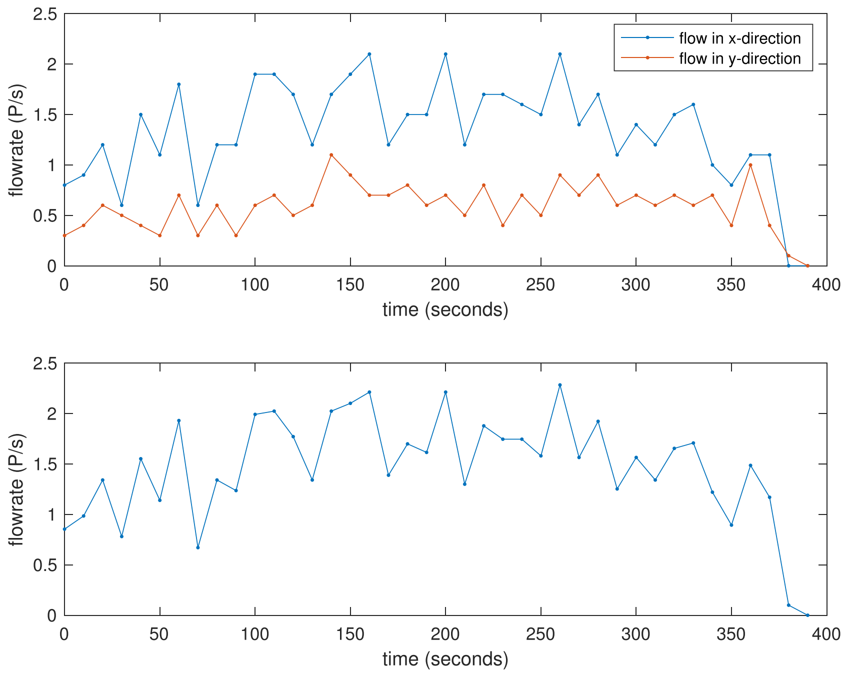

Fig. 9 Flow rates in the *x*-direction and the *y*-direction for pedestrians moving from right to left in crossing flow experiment. The bottom figure shows the absolute flow. We used a 10 s time period.

5.2 Time headways

Time headways h_i are defined by the difference in passage time at a cross-section l between two successive pedestrians $i-1$ and i. A time headway is a *microscopic quantity*, since it describes the property of an individual pedestrian (in relation to its predecessor). The flow rate and the time-averaged headway are related as follows:

$$\bar{h}(t_0, t_1|l) = \frac{t_1 - t_0}{N(t_0, t_1|l)} = \frac{1}{q(t_0, t_1|l)} \tag{16}$$

6. Edie's definitions of densities, flows, and velocities

In this section, we present the generalized definitions of flow rate, density and velocity for a time-space region. In proposing this generalisation, we relax the requirement of stationarity and homogeneity needed for the continuity equation $\vec{q} = \rho \cdot \vec{u}$.

We will show how these definitions are generalisations of the definitions proposed earlier. This derivation was first proposed by van Wageningen-Kessels et al. (2014) as a generalisation of Edie (1963). For the sake of simplicity, we consider a box in time and space defined by $T \times X \times Y$ where $T = [t_0, t_1]$, $X = [x_0, x_1]$ and $Y = [y_0, y_1]$. We let Δt, Δx and Δy denote respectively $t_1 - t_0$, $x_1 - x_0$ and $y_1 - y_0$.

We will consider trajectories of pedestrians moving in two-dimensional space. In the considered time-space region $T \times X \times Y$, partial trajectories will be observed; see Fig. 10. For each trajectory $\vec{x}_j$ of pedestrian j that moves through this region, we can determine the following quantities:

1. The time $T_j \leq \Delta t$ that j has spent in the region
2. The distance $X_j \leq \Delta x$ in the x-direction that j has traversed[3].
3. The distance $Y_j \leq \Delta y$ in the y-direction that j has traversed[4]

Intuitively, the average velocities are most easy to define. To this end, we consider the total distance travelled by all the pedestrians and divide it by the total time spent in the region. Let $\vec{u} = (u_x, u_y)$, then:

$$u_x = \frac{\sum_j X_j}{\sum_j T_j} \tag{17}$$

and

$$u_y = \frac{\sum_j Y_j}{\sum_j T_j} \tag{18}$$

The density is more involved, and defined by the following equation:

$$\rho = \frac{\sum_j T_j}{\Delta t \Delta x \Delta y} \tag{19}$$

Studying this equation shows the similarity to the instantaneous definition proposed before. $\Delta x \Delta y$ is the area A of the considered region. $\frac{\sum_j T_j}{\Delta t} = \sum_j (T_j / \Delta t)$ in the time-mean density over the period $[t_0, t_1]$, where we can interpret $T_j / \Delta t \leq 1$ as the share of the period $[t_0, t_1]$ where the pedestrian was in the region.

[3] Note that the distance can be negative: X_j is defined by the x exit point minus the x entry point

[4] Note that also this distance can be negative: similar to X_j, Y_j is defined by the y exit point minus the y entry point

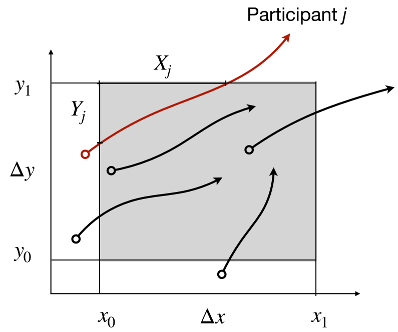

Fig. 10 Projections of trajectories in the region $T \times X \times Y$, and the definitions of the variables needed for the Edie definitions.

Note that if we let $\Delta t \downarrow 0$ (considered period is very short) that $T_j = \Delta t$ and Edie's definition reduces to the definition of instantaneous density we showed before.

For the flow rate (in the x-direction) we use:

$$q_x = \frac{\sum_j X_j}{\Delta t \Delta x \Delta y} \tag{20}$$

The flow rate is thus defined by the averaged distance travelled in the x direction (which can be negative) over the length of the considered region, per unit time and unit width. This can be seen from the definition by noticing that $\frac{X_j}{\Delta x} \leq 1$ is the share of the region that is traversed by j during the considered period.

Note that if we let $\Delta x \downarrow 0$, then the region $T \times X \times Y$ becomes equivalent to a cross-section at $x = x_0$. We have $X_j = \Delta x$ for all pedestrians j that enter/exit the infinitesimal region. We get:

$$q_x = \frac{\sum_j 1}{\Delta t \Delta y} \tag{21}$$

which is exactly the definition of the flow rate presented before.

For the flow-rate (in the y-direction) we use:

$$q_y = \frac{\sum_j Y_j}{\Delta t \Delta x \Delta y} \tag{22}$$

As pointed out by van Wageningen-Kessels et al. (2014), the definitions of flow and velocity may not result in a useful outcome if the pedestrians do not all move in the same direction. For example, if about half of the pedestrians walks from left to right, and the rest walks in the other direction, this causes the flows and velocities in x direction to (almost) cancel out. This can be dealt with by using a multi-class approach (see previous examples).

As a final, but important note, the continuity equation $\vec{q} = \rho \cdot \vec{u}$ is trivial when using Edie's generalised definitions. For instance, we can show that the flow q_x satisfies:

$$q_x = \frac{\sum_j X_j}{\Delta t \Delta x \Delta y} = \frac{\sum_j T_j}{\Delta t \Delta x \Delta y} \cdot \frac{\sum_j X_j}{\sum_j T_j} = \rho \cdot u_x \tag{23}$$

7. Summary and closing

In this chapter, we have presented formal definitions for the key flow variables characterising a pedestrian flow. We have not aimed to be complete here, nor have we tried to motivate or justify which definitions are most useful for which applications. This is not to say that there is no debate on this. Quite the contrary, as can be concluded from Duives et al. (2015). We will enter this debate – if appropriate – in the chapters of the book where it is most relevant. For instance, discussion on density definitions are closely related to which definition limits the amount of scatter in and the shape of the fundamental diagram (to be discussed in Chapter 5).

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