Addendum to the master's thesis

Instruments for seismic isolation

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It has come to the attention of the author that the Cramér-Rao Lower Bound (CRLB) calculation, see Section 2.2.1 at pages 23-27 in the master's thesis document, was improperly compared with Rasnik's measured Amplitude Spectral Density noise floor, $A_n(f)$, in Table 2.4 at page 38. The noise levels $A_n(f)$ of a measurement, with Gaussian distributed noise, compare to the RMS in time domain of a measurement as:

$$\text{RMS} = \sqrt{\int_{f_1}^{f_2} |A_n(f)|^2 df} \tag{1}$$

As the CRLB models a *variance* of a minimum observable shift in x or y, by means of a Gaussian distributed pixel noise $\hat{\sigma}$, the model forms a lower bound for the RMS value of a measurement, instead of for the frequency dependent $A_n(f)$:

$$\sigma_{\text{CRLB}} < \text{RMS} \tag{2}$$

Recall that, in simplified form, the stdev CRLB is given as (Equation 2.7, page 24):

$$\sigma_{\text{CRLB,x}} = \sqrt{\frac{\hat{\sigma}^2}{\sum_S I_x^2}} \tag{3}$$

with $\hat{\sigma}$ the pixel noise of the image sensor, and I_x the derivative of the image in the shift direction x, providing a model of what shifts could minimally be detected when only pixel noise limits Rasnik's spatial resolution (no environmental effects).

Essential to this model is the pixel noise $\hat{\sigma}$. For this, the original document considered the mean pixel value (the mean of all $\hat{\mu}(x, y)$) for each image in a set of $N = 150$ images, and calculated the standard deviation of this set of 150 means. This resulted in a pixel noise of $\hat{\sigma} = 0.08$.

The new interpretation of the pixel noise considers a Gaussian distributed noise over the white areas, as well as over the black areas, and adds them in quadrature. The interpretation of a noise normally distributed over the image with area S, is in accordance with Pham et al. [2005], who established the CRLB

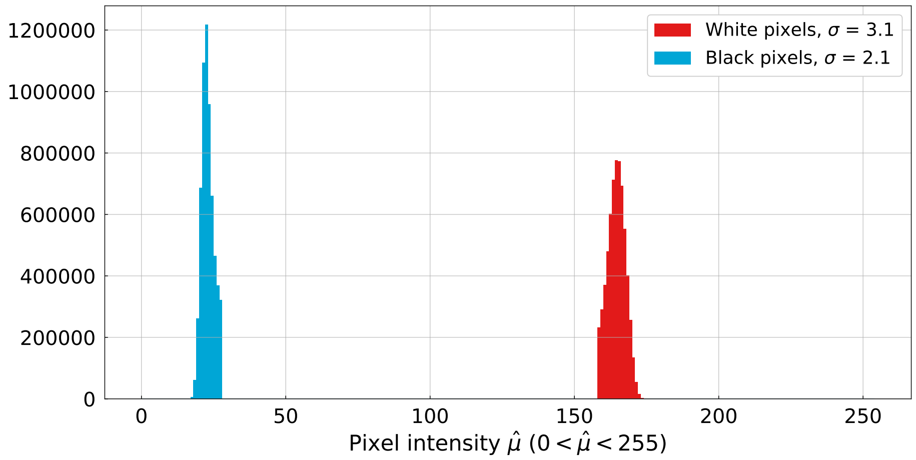

Figure 1: Estimation of the pixel noises σ_{black} and σ_{white}. This histogram shows pixel counts of $N = 150$ images (304×404) at $I = 80$ mA.

LED current I	CRLB σ_x	Measured RMS	Measured A_n
10 mA	16 nm	16 nm	1.9 $\text{nm}/\sqrt{\text{Hz}}$
20 mA	10 nm	10 nm	1.1 $\text{nm}/\sqrt{\text{Hz}}$
40 mA	7.5 nm	7.5 nm	0.68 $\text{nm}/\sqrt{\text{Hz}}$
80 mA	5.9 nm	6.7 nm	0.46 $\text{nm}/\sqrt{\text{Hz}}$

Table 1: Modelled Cramér-Rao limits, as well as measured RMSs of the covered setup with $f_s = 120$ Hz, for different LED intensities, back-illuminating the mask. The measured spectral noise floors are listed for comparison.

for image shifts. In Figure 1 the distribution of pixel counts in white and black areas is displayed, as derived from the pixels of $N = 150$ images with 304×404 pixels, with LED light current $I = 80$ mA (this image is created from the same data as is displayed in Figure 2.5.b. on page 26).

From Figure 1 the pixel count distributions are estimated as $\sigma_{\text{black}} = 2.1$ and $\sigma_{\text{white}} = 3.1$. Summing in quadrature yields $\hat{\sigma} = \sqrt{\sigma_{\text{black}} + \sigma_{\text{white}}} = 3.6$, for the $I = 80$ mA measurement. The resulting CRLB of each LED current measurement is displayed in Table 1.

This interpretation of the Cramér-Rao limit, where the shift variance is compared with the integrated RMS, concludes that the covered Rasnik setup (Section 2.3.3.), operates at its pixel noise limit. Only the measurement at $I = 80$ mA has a RMS slightly above the modelled lower bound, indicating that this system is not merely pixel noise limited. This is affirmed by the spectral density plot of $I = 80$ mA in Figure 2.19, on page 37, showing a declined resolution below 1 Hz, which was determined to be due to environmental effects.

References

Tuan Q. Pham, Marijn Bezuijen, Lucas J. van Vliet, Klamer Schutte, and Cris L. Luengo Hendriks. Performance of optimal registration estimators. In *Visual Information Processing*, 2005.