

Agent-based Modelling and Simulation of TCAS Operations under Uncertainty

Volume I: Thesis Body

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Acknowledgements

I dedicate this Thesis to my mother, Beatrix Anna Josefine Lenner-Gellert (18 March 1955 – 13 January 2000), who would be proud to see me defending it.

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Summary

In order to accommodate increasing traffic demand, SESAR and NextGEN pose higher requirements on collision avoidance. Also the increasing use of unmanned aerial vehicles (UAV) poses new collision avoidance issues. Therefore, the FAA is currently developing a new airborne collision avoidance system (ACAS) called ACAS X. It is supposed to be successor of the current collision avoidance system TCAS II (Traffic Collision Avoidance System). One of the main novelties is that ACAS X shall have different version which are each tailored for a specific type of aircraft, e.g. ACAS X_A for commercial passenger aircraft, ACAS X_U for unmanned vehicles. Next, each version of ACAS X shall function both with other ACAS X versions and with TCAS II.

Since TCAS II was not designed to cooperate with other systems, it is of high interest to investigate the risk of collision between TCAS II equipped aircraft and aircraft with a different ACAS, e.g. an UAV.

Studies do exist which model TCAS II operations, but it became clear that none of them can both be extended with a different ACAS model and is modelling TCAS in sufficient detail, including the effect of various uncertainties.

Therefore, it was decided to develop a new agent-based model for TCAS II operations, which offers flexibility for extension, including the details of the minimum operational performance standards (MOPS) of TCAS II and various uncertainties.

The research was carried out in three parts. In Part I a TCAS II model was developed based on the MOPS and ICAO specification and additional agents were specified. The TCAS II model followed a modular structure.

- A Slant Range and Vertical Range Filters module estimates the states between the own and an intruder aircraft (relative position and velocity).
- A Traffic Advisory module notifies the pilot in command about traffic in the vicinity.
- A Threat Detection module determines whether the intruder is a threat and a Resolution Advisory (RA) should be issued.
- A Horizontal Miss Distance Filter module supports the Threat Detection module by suppressing unnecessary Resolution Advisories in cases where the horizontal miss distance is sufficiently great.
- A Sense Selection module selects whether an upwards or downwards manoeuvre should be performed.
- A Strength Selection module determines the altitude velocity limits issued in an RA.
- A Threat Evolution Monitoring module monitors the evolution of the situation, and decides when RA adjustments are needed.

In addition to the TCAS II agent, agent “Aircraft i” models the aircraft dynamics, agent “Cockpit i” models the behaviour of the pilot, and agent “Communication i” models the communication between two aircraft.

In this agent-based model the following eight types of uncertainties have been captured:

- jitter in range measurements,

- bias in range measurements,
- jitter in own altitude estimates,
- bias in own altitude estimates,
- jitter in own altitude velocity estimates,
- small variation in pilot delay,
- probability of reception of an interrogation response, and
- variation in starting time of TCAS computation cycle.

In Part II a computer implementation of the new TCAS model was developed. From the mathematical specification of the agents a Petri Net model was developed, which after completion was implemented in a MATLAB program. The MATLAB program was successfully verified in two phases. In phase 1 the MATLAB program was verified with disabled uncertainties. In phase two a second verification was carried out with enabled uncertainties.

In Part III the new Petri Net model was validated versus the EURCOCONTROL TCAS II simulator "InCAS". For validation nine cases were simulated. However, since InCAS does not model uncertainties, for this comparison the uncertainties in the new TCAS II model were disabled.

The effect of various uncertainties was evaluated through conducting Monte Carlo simulations for the same nine cases. In total three Monte Carlo simulation set ups were addressed. In the first set up all uncertainty models were enabled. In the second set up the random pilot delay variation was disabled. In the third set up for each case eight Monte Carlo simulations were performed where in each Monte Carlo simulation only one of the eight uncertainties was enabled.

It was identified that uncertainties have a large and unpredictable impact on TCAS operations. For the nine cases, the findings are as follows:

- Although the mean time of issue of RAs is not significantly affected by uncertainties, the variation in time of issue of RAs may be significantly affected by uncertainties.
- Uncertainties have a significant effect on whether an RA is issued or not.
- Uncertainties have a significant effect on the type of RA issued.
- Uncertainties have a significant effect on the number of consecutive RA adjustments.
- TCAS II Version 7.1 may cause a loop of conflict detection and clear of conflict declaration, which may continue until both aircraft have passed the CPA.
- The variation in pilot delay may affect both the number of RA adjustments and the vertical miss distance at CPA.
- Uncertainties clearly have an effect on safety improvement by TCAS II.
- The dynamic and static range errors may only have an impact on the vertical miss distance, if the horizontal closing speeds are low.
- Different uncertainties play main role in different encounters.

It remains to be studied if these findings also apply to a much larger set of aircraft and encounters.

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Glossary of Symbols

a_{lim}	Vertical miss distance threshold value
$\mathbf{a}_t^i$	Acceleration vector of aircraft 'i' at time 't' in Cartesian coordinates
$a_{t,A}^i$	Altitude acceleration of aircraft 'i' at time 't'
$\mathbf{a}_{t,H}^i$	Horizontal acceleration vector of aircraft 'i' at time 't' in x and y coordinates
c_t^*	Indicator whether altitudes will cross before closest point of approach
$B_{t,AT}$	Boolean of altitude test for RA
$B_{AT,CA}$	Boolean of time to co-altitude is small test
$B_{t,AT,TA}$	Boolean of TA altitude test
$B_{t,CROSS}$	Boolean of altitude crossing intent
$B_{t,D}$	Boolean of sense selection process
$B_{t,firm}$	Boolean of firmness state of parabolic range tracker
$B_{t,HMDF}$	Boolean of horizontal miss distance filter status
$B_{t,HMDF,ACC}$	Boolean of acceleration test of horizontal miss distance filter
$B_{t,HMDF,HMD}$	Boolean of horizontal miss distance test of horizontal miss distance filter
$B_{t,HMDF,MAN}$	Boolean of manoeuvre test of horizontal miss distance filter
$B_{t,RA,EVO}$	Boolean of evolution monitoring process
$B_{t,RT}$	Boolean of range test for RA
$B_{t,RT,TA}$	Boolean of TA range test
$B_{t,TA}$	Boolean of TA test
$C_{t,BBT}$	Covariance of range and cross range measurements of bearing based tracker at time 't'
$C_{t t-T,BBT}$	Prediction of covariance of range and cross range measurements of bearing based tracker at time 't' subject to estimated covariance at time 't-T'
$C_{t t-T,BBT}^\mu$	Residual of covariance estimate at time t subject to estimated covariance at time 't-T'
D_t	Selected sense at time 't'
D_t^A	Virtual selected sense
$d_{t,HMD,BBT}$	Estimated horizontal miss distance by bearing based tracker at time 't'
$d_{t,HMD,PRT}$	Estimated horizontal miss distance by parabolic range tracker at time 't'
d_{mod}	RA distance threshold value
$d_{mod,TA}$	TA distance threshold value
H_m	Horizontal miss distance threshold value

HMD	Horizontal miss distance
m_t^A	Virtual sent coordination message
$m_{t,sent}^{ik}$	Coordination message sent from aircraft 'i' to aircraft 'k'
$m_{t,received}^{ki}$	Coordination message received by aircraft 'i' from aircraft 'k'
$P_{t,firm}$	Firmness parameter of parabolic range tracker
$P_{t,firm,BBT}$	Firmness parameter of bearing based tracker
$\dot{P}_m$	Parameter to determine whether divergence is slow
$\begin{pmatrix} q_{t,X,BBT} \\ q_{t,Y,BBT} \end{pmatrix}$	Relative position estimates in Cartesian coordinates of bearing based tracker at time 't'
$\begin{pmatrix} \dot{q}_{t,X,BBT} \\ \dot{q}_{t,Y,BBT} \end{pmatrix}$	Relative velocity estimates in Cartesian coordinates of bearing based tracker at time 't'
$\begin{pmatrix} q_{t t-T,X,BBT} \\ q_{t t-T,Y,BBT} \end{pmatrix}$	Prediction of relative position in Cartesian coordinates of bearing based tracker at time 't' subject to relative position, relative velocity estimates at time 't-T'
$\begin{pmatrix} \dot{q}_{t t-T,X,BBT} \\ \dot{q}_{t t-T,Y,BBT} \end{pmatrix}$	Prediction of relative velocity in Cartesian coordinates of bearing based tracker at time 't' subject to relative velocity estimates at time 't-T'
r_t	Estimated slant range at time 't'
$\dot{r}_t$	Estimated slant range rate
$r_{t t-T}$	Prediction of slant range at time 't' subject to range and range rate estimates at time 't-T'
$r_{t,A}$	Estimated relative altitude at time 't'
$\dot{r}_{t,A}$	Estimated relative altitude velocity at time 't'
$r_{t t-T,A}$	Prediction of relative altitude at time 't' subject to relative altitude and relative altitude velocity estimates at time 't-T'
$r_{t,CT}$	Estimated slant range of Cartesian tracker at time 't'
$\dot{r}_{t,CT}$	Estimated relative velocity of Cartesian tracker at time 't'
$\ddot{r}_{t,CT}$	Estimated relative acceleration of Cartesian tracker at time 't'
$r_{t t-T,CT}$	Prediction of slant range of Cartesian tracker at time 't' subject to estimates at time 't-T'
$\dot{r}_{t t-T,CT}$	Prediction of relative velocity of Cartesian tracker at time 't' subject to estimates at time 't-T'
$\ddot{r}_{t t-T,CT}$	Prediction of relative acceleration of Cartesian tracker at time 't' subject to estimates at time 't-T'
$r_{t,XS,CT}$	Cartesian extrapolation of relative range in x direction at time 't'
$\dot{r}_{t,XS,CT}$	Cartesian extrapolation of relative velocity in x direction at time 't'

$r_{t,YS,CT}$	Cartesian extrapolation of relative range in y direction at time 't'
$\dot{r}_{t,YS,CT}$	Cartesian extrapolation of relative velocity in y direction at time 't'
$r_{t t-T,XS,CT}$	Prediction of relative range in x direction of Cartesian tracker at time 't' subject to relative range in x direction estimate at time 't-T'
$r_{t t-T,YS,CT}$	Prediction of relative range in y direction of Cartesian tracker at time 't' subject to relative range and velocity in y direction estimates at time 't-T'
$\dot{r}_{t t-T,YS,CT}$	Prediction of relative velocity in y direction of Cartesian tracker at time 't' subject to relative velocity in y direction estimate at time 't-T'
$r_{CPA,A t}$	Predicted vertical miss distance at closed point of approach
$r_{t,CPA,A t}^{adjust}$	Adjusted predicted relative altitude at closest point of approach
$r_{CPA,A t}^{reac+}$	Predicted vertical miss distance at closed point of approach if pilot would pull up after assumed reaction time
$r_{CPA,A t}^{reac-}$	Predicted vertical miss distance at closed point of approach if pilot would push down after assumed reaction time
$r_{CPA,A t}^{+}$	Adjusted predicted vertical miss distance at closest point of approach
$r_{CPA,A t}^{-}$	Adjusted predicted vertical miss distance at closest point of approach
$r_{CPA,A t}^{*}$	Predicted relative altitude at closest point of approach if own aircraft levels off after an assumed reaction time
$r_{CPA,A,F t}^{*}$	Predicted relative altitude at closest point of approach if own aircraft levels off after an assumed reaction time
$r_{t,PRT}$	Estimated slant range of parabolic range tracker at time 't'
$\dot{r}_{t,PRT}$	Estimated relative velocity of parabolic range tracker at time 't'
$\ddot{r}_{t,PRT}$	Estimated relative acceleration of parabolic range tracker at time 't'
$\dddot{r}_{t,PRT}$	Estimated jerk of parabolic range tracker at time 't'
$r_{t t-T,PRT}$	Prediction of slant range of parabolic range tracker at time 't' subject to slant range, relative velocity and relative accelerations estimates at time 't-T'
$\dot{r}_{t t-T,PRT}$	Prediction of relative velocity of parabolic range tracker at time 't' subject to relative velocity and relative accelerations estimates at time 't-T'
$\mathbf{s}_t^i$	Position state vector of aircraft 'i' at time 't' in Cartesian coordinates
$\mathbf{s}_t^{ik}$	Relative position vector between aircraft 'i' and aircraft 'k' with reference to aircraft 'i'
$s_{t,A}^i$	Altitude of aircraft 'i' at time 't'
$\mathbf{s}_{t,H}^i$	Horizontal position state vector of aircraft 'i' at time 't' in x and y coordinates
T_{A1}	Acceleration threshold value
$t_{t,RA}$	Elapsed time since RA
t_{reac}	Time after an assumed reaction time Δ_{reac}

T_{RV}	Threshold value of manoeuvre test of horizontal miss distance filter
$t_{t, timer}$	Timer to count down to estimated time of closest point of approach at initial RA
T_v	Time to co-altitude is small test threshold value
$\mathbf{v}_t^i$	Velocity vector of aircraft 'i' at time 't' in Cartesian coordinates
$\mathbf{v}_t^{ik}$	Relative velocity vector between aircraft 'i' and aircraft 'k' with reference to aircraft 'i'
$v_{t,A}^i$	Altitude velocity of aircraft 'i' at time 't'
$v_{t,A,min}^i$	Lower bound of target altitude velocity of aircraft 'i' at time 't'
$v_{t,A,max}^i$	Upper bound of target altitude velocity of aircraft 'i' at time 't'
$v_{t,A,RA,min}^0$	Minimum altitude velocity limit indicated by RA at time 't'
$v_{t,A,RA,max}^0$	Minimum altitude velocity limit indicated by RA at time 't'
$v_{\Delta A,max}$	Upper altitude velocity bound for RA selection
$v_{\Delta A,min}$	Lower altitude velocity bound for RA selection
$\mathbf{v}_{t,H}^i$	Horizontal velocity vector of aircraft 'i' at time 't' in x and y coordinates
$y_{t,A}^i$	Measured pressure altitude of aircraft 'i' at time 't'
$y_{t,A}^{ik}$	Measured relative altitude difference between aircraft 'i' and aircraft 'k' with reference to aircraft 'i' at time 't'
$y_{t,r}^{ik}$	Measured distance of aircraft 'k' to aircraft 'i' by aircraft 'i' (slant range)
$y_{t,\theta}^{ik}$	Relative bearing measurement by aircraft 'i' of aircraft 'k' at time 't'
z_{thr}	RA altitude separation threshold value
$z_{thr,TA}$	TA altitude separation threshold value
α, β	Smoothing parameters of range $\alpha - \beta$ filter
α_A^i	Smoothing factor of first order autoregressive model of the pressure altitude jitter of aircraft 'i'
$\alpha_{CT}, \beta_{CT}, \gamma_{CT}$	Smoothing parameters of Cartesian tracker
α_{MAN}	Smoothing parameter for $\ddot{r}_{t,PRT}$ estimation
$\alpha_{PRT}, \beta_{PRT}, \gamma_{PRT}$	Smoothing parameters of parabolic range tracker
$\alpha_{\rho,BBT}, \beta_{\rho,BBT}$	Smoothing parameter of range estimates of bearing based tracker
$\alpha_{\rho x,BBT}, \beta_{\rho x,BBT}$	Smoothing parameter of cross range estimates of bearing based tracker
α_v, β_v	Smoothing parameters of altitude $\alpha - \beta$ filter
Δ	Time step of aircraft evolution
Δ_{RA}	RA time threshold value
Δ_{reac}	Assumed reaction time of pilot

$\Delta_{\text{reac},F}$	Assumed fast reaction time of pilot
Δ_{TA}	TA time threshold value
$\mathcal{E}_{A,bias}^i$	Bias of measured pressure altitude of aircraft 'i'
$\mathcal{E}_{t,A,jitter}^i$	Jitter of measured pressure altitude of aircraft 'i' at time 't'
$\mathcal{E}_{t,r,jitter}^i$	Slant range measurement error of aircraft 'i' at time 't' induced by jitter
$\mathcal{E}_{r,bias}^i$	Bias of slant range measurement error of aircraft 'i'
$\mathcal{E}_{t,\theta}^{ik}$	Bearing measurement error of measurement by aircraft 'i' of aircraft 'k' at time 't'
θ_t^i	Magnetic heading of aircraft 'i' at time 't'
$\dot{\theta}_t^i$	Turn rate of aircraft 'i' at time 't'
$\mu_{t,A}$	Relative altitude residual at time 't'
$\mu_{t,r}$	Range measurement residual at time 't'
$\mu_{t,r,CT}$	Range measurement residual of Cartesian tracker at time 't'
$\mu_{t,r,PRT}$	Range residual of parabolic range tracker at time 't'
$\begin{pmatrix} \mu_{t,\rho,BBT} \\ \mu_{t,\rho x,BBT} \end{pmatrix}$	Residuals of range and cross range measurements at time 't'
σ_A^i	Standard deviation pressure altitude measurement by aircraft 'i'
$\sigma_{A,noise}^i$	Standard deviation of white noise of first order autoregressive model of the pressure altitude jitter of aircraft 'i'
$\sigma_{t,m}$	Estimate of standard error of prediction at time 't' of parabolic range tracker
$\sigma_{r,jitter}^i$	Standard deviation of jitter of slant range measurement by aircraft 'i' at time 't'
$\tau_{A t}$	Time to co-altitude
$\tau_{A t}^{adjust}$	Adjusted time to co-altitude
$\tau_{CPA t}$	Estimated time until closes point of approach at time 't'
$\tau_{\text{mod} t}$	Modified time until closest point of approach for RA
$\tau_{\text{mod},TA t}$	Modified time until closest point of approach for TA
$\mathcal{X}_{\text{ModeS}}^i$	Aircraft ID of aircraft 'i'

Glossary of Terms

ABM	Agent-Based Modelling
ABMS	Agent-Based Modelling and Simulation
ACAS	Airborne Collision Avoidance System
ADS-B	Automatic dependent surveillance – broadcast
ALIM	Altitude separation limit
AT	Altitude test
BBT	Bearing based tracker
CA	Co-altitude
CDTI	Cockpit display of traffic information
CL	Climb
COC	Clear of conflict
CPA	Closest point of approach
CT	Cartesian tracker
DAA	Detect and Avoid
DCL	Do Not Climb
DDE	Do Not Descend
DE	Descend
InCas	Interactive Collision Avoidance Simulator
IPN	Interconnecting Petri Net
LO	Level-Off
LPN	Local Petri Net
MAC	Mid-air collision
NMAC	Near mid-air collision
PRT	Parabolic range tracker
QNH	Pressure setting of pilot
RA	Resolution advisory

RT	Range test
SMM	System Monitor Module
STM	Surveillance Tracking Module
TA	Traffic Advisory
TCAS	Traffic Collision Avoidance System
TRM	Threat Resolution Module
UAS	Unmanned Aerial Systems
UAV	Unmanned Aerial Vehicle

1 Introduction

Unmanned Aerial Systems (UAS) are the flavour of the time. They are available in all sizes and may be used for a variety of purposes, both commercially and civil. In particular, the low cost and high availability of toy UAS lead to high sale numbers. Smaller UAS are typically allowed, if flown within visual range, and operated away from people and foreign property. So far, UAS operation are only permitted in non-segregated airspace, when a safety case has proven a sufficient level of safety, and then only with operational limitations. However, recent incident data shows that UAS operators are often not complying with the current air space restrictions on UAS. To assess the safety impact of those operators, studies are currently carried out to assess the severity impact between small UAS and manned aircraft. Furthermore, it was also identified that the biggest issue is the lack of appropriate airborne collision avoidance system in order to allow UAS free operation in non-segregated airspace. FAA is leading in this regard with their ACAS X concept, which aims at introducing both vertical and horizontal collision avoidance manoeuvres for UAS.

In order to estimate the risk of collision of between UAS and conventional aircraft the aim is to develop an agent-based model which simulates operations of conventional TCAS equipped aircraft with UAS which is equipped with a DAA system according to the ACAS X principles.

The initial idea was to reuse an available agent-based model about TCAS and extend it with an agent-based model about an UAS. However, in the beginning of the research it became apparent that the available TCAS model lacks details to such an extent that one could argue that the agent-based model is not modelling TCAS but a generic collision avoidance system with some features similar to TCAS. Especially, the assumption that all aircraft know the exact location, velocity and heading of aircraft in the vicinity raised questions about the validity of the model.

Therefore, it was decided that in order to produce meaningful results about the risk of collision between a TCAS equipped aircraft and a UAS, first an agent-based model for TCAS operation considering uncertainties needs to be created.

1.1 Research question

The initial research question was the following:

To develop an agent-based model and simulation for risk analysis of controlled en-route airspace between an UAV, which is not TCAS II compliant due to limited climb performances, with a large commercial TCAS II, equipped aircraft?

The question is to be answered by developing a detect and avoid concept based on the ACAS XU concept, developing an agent-based model, simulating the number of collisions between two commercial aircraft and between an UAV and a commercial aircraft, and analysing the risk ratio by comparing the simulated collision probability.

Due to the lack of valid agent-based models for TCAS operations, it was decided that this research will focus on supporting the research questions by:

To develop an agent-based model and simulation for risk analysis of TCAS II Ver. 7.1 operation including the effects of various uncertainties.

1.2 Methodology

The Thesis follows the steps presented in [Figure 1-1](#). After successful completion of the literature study prior to the Thesis, the first step was to specify the agent TCAS II Ver. 7.1.

This is followed by the specification of additional agents.

The third step is to develop a Petri Net model capturing the specified agents. The Petri Net specifications are according to (Everdij et al., 2003).

In order to enable simulations, the Petri Net model is programmed in MATLAB.

Next, the MATLAB program is verified and validated versus the EUROCONTROL TCAS simulator InCAS V.3.3. For validation 9 cases are simulated in both InCAS and in the Petri Net model. As InCAS is not capable of simulating with uncertainties, uncertainties are set to zero, and all systems are set to be working throughout the validation simulations.

Last, Monte Carlo simulations are carried out for the 9 cases used for validation to assess the effects of uncertainties. In one Monte Carlo simulation all uncertainties are enabled. In the second Monte Carlo simulation pilot delay uncertainty is disabled. In the third Monte Carlo simulation only one uncertainty is simulated at a time.

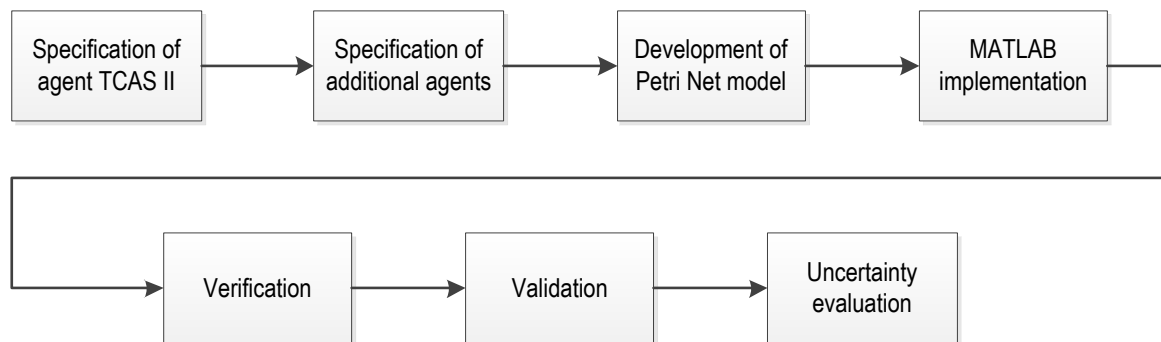


Figure 1-1: Methodology followed during the Thesis.

1.3 Organisation of the Thesis

The Thesis is organised in two volumes: Volume I - Thesis Body, and Volume II - Appendices. This volume includes the main body of the Thesis, which is organised as follows. First, Section 2 contains a short literature study to give the reader an introduction to current and future airborne collision avoidance systems, current collision avoidance models, and the concept of agent-based modelling and Simulation, which is applied in this Thesis.

Next, Volume I – Thesis Body consists of three parts.

- Parts 1 - Specification of the agents:
 - In Sections 3-11 TCAS II Ver. 7.1 is specified.
 - In Section 12 the additional agents are specified, which are

- agent “Aircraft i” in Section 12.1,
 - agent “Cockpit i” in Section 12.2, and
 - agent “Communication i” in Section 12.3.
- In Section 13 the adopted assumption are summarised.
- Part 2 - Description of the Petri Net model and the validation:
 - In Section 14 the Petri Net model is described in the following order
 - agent “TCAS i” in Section 14.1,
 - agent “Aircraft i” in Section 14.2,
 - agent “Cockpit i” in Section 14.3, and
 - agent “Communication i” in Section 14.4.
 - In Section 15 the implementation of the Petri Net model in a MATLAB program is described, where
 - in Section 15.1 the implementation strategy is presented, and
 - in Section 15.2 the verification of the Petri Net model is given.
- Part 3 - Validation of Petri Net model, Monte Carlo simulation of TCAS operations, and conclusions drawn from the simulation results:
 - In Section 16 validation of the Petri Net model is presented, where
 - in Section 16.1 notable issues of the results of the EUROCONTROL TCAS simulator, InCAS, are explained,
 - in Section 16.2 the similarities of the Petri Net model results and the InCAS results are presented,
 - in Section 16.3 the differences of the Petri Net model results and the InCAS results are presented, and
 - in Section 16.4 conclusions are drawn on the validation of the Petri Net model.
 - In Section 17 the Monte Carlo simulations of TCAS operations are presented, where three different Monte Carlo simulation set ups were applied:
 - in Section 17.1 all uncertainties were enabled,
 - in Section 17.2 the random variation of pilot delay was removed, and
 - in Section 17.3 several Monte Carlo simulations were carried out with only one random variable being activated at a time.
 - Last, in Section 18 conclusions are drawn on the simulation results of the simulated TCAS operations.
- Additionally, Volume 2 – Appendices presents the following appendices:
 - In Appendix A the Petri Net model specification of the new TCAS model is presented.
 - In Appendix B a preliminary detect and avoid system for UAS is developed.

- In Appendix C the Petri Net model of the detect and avoid system for UAS is developed.
- In Appendix D the Petri Net model specification of the detect and avoid system for UAS is presented.

2 Literature study

Prior to this research a literature study was carried out. Next, parts of the literature study are presented.

Section 2.1 elaborates on the current airborne collision avoidance system by describing the general system, how range detection is carried out, and how the systems of different aircraft cooperate with each other.

Section 2.2 presents a new airborne collision avoidance system, called ACAS X. The basic concept and its components are elaborated upon, following by a description of the UAS tailored, and the commercial aircraft tailored ACAS systems.

Section 2.3 gives a short overview on current Collision avoidance models.

Section 0 elaborates on the agent-based modelling concept, which is the intended modelling technique of this research.

2.1 Current ACAS system (TCAS)

In order to prevent mid-air collisions the airborne collision avoidance system (ACAS) was developed. The result is the current traffic collision avoidance system (TCAS). The main tasks of TCAS are to detect near mid-air collision conflicts and give advisories to the pilot. A near mid-air collision is considered to occur when two aircraft are closer than 500ft horizontally and 100ft vertically (Jeannin et al., 2015). There are two types of advisories: First, the traffic advisory (TA), which informs the pilot about traffic in the vicinity and the associated threat; second, resolution advisories (RA), which recommends necessary actions to the pilot to avoid a collision. Both TA and RA are communicated aurally and visually on the flight instruments. In RA TCAS will indicate the pilot to (Chryssanthacopoulos, 2012)

- Climb or descend at 1,500ft/min,
- Level off,
- Maintain current climb or descend rate,
- Limit climb or descend rate to 500, 1,000, or 2,000ft/min, or
- Strengthen climb or descend rate to 2,500ft/min.

Hence, separation is assured by vertical separation only. The TCAS system can be seen in [Figure 2-1](#).

So far there have been different versions of TCAS: namely TCAS I and TCAS II. TCAS II can further be distinguished in Version 7.0 and Version 7.1 (Haessig, 2016). TCAS I only provides TA while TCAS II provides TA

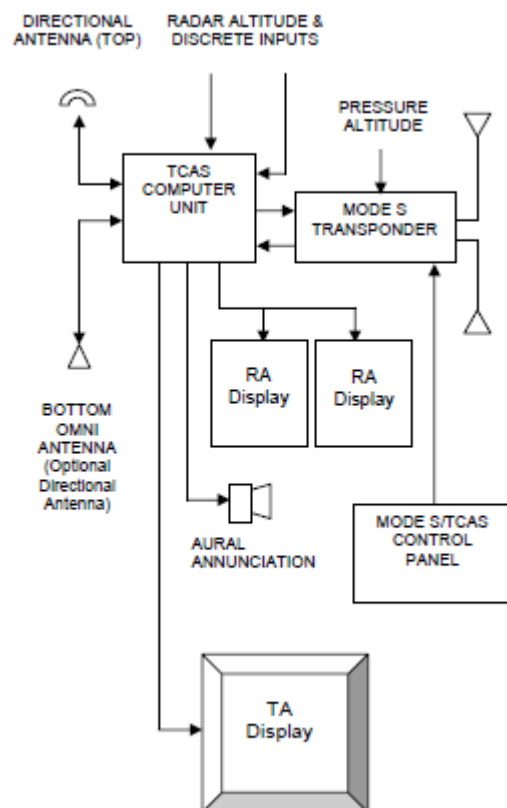


Figure 2-1: TCAS II technical system block diagram (FAA, 2011).

and RA. In TCAS Version 7.1 the ability of reversing a RA was added, e.g. from climb to descent to prevent scenarios such as the Überlingen accident (Haessig, 2016). TCAS I was never mandatory but as of January 2000 all civil fixed-wing aircraft with a maximum take-off weight of 15,000kg or more, or being able to carry 30 passengers or more had to be TCAS II Ver. 7.0 equipped (Haessig, 2016). In January 2005 the rule was extended by including all aircraft with a maximum take-off weight of 5,700kg or more, or being able to carry at least 19 passengers (Haessig, 2016).

TCAS Range Detection

TCAS is equipped with a Mode S transponder. The Mode S transponder sends out the aircraft ID on a regular basis. Once an aircraft notices other aircraft in the vicinity, each second it sends an addressed interrogation to each aircraft at a frequency of 1,090MHz (ECTL, 2010). The addressed aircraft will reply with its aircraft ID and altitude at a frequency of 1,30MHz. From the response, TCAS computes the slant range through the measured time delay, and the slant rate through the Doppler shift (Haessig, 2016).

Also TCAS sends out a “Mode C all-call” at a rate of once per second. All aircraft equipped with a Mode C transponder will reply to the message but do not send their altitude. The frequencies of Mode C are the same as for Mode S and, therefore, TCAS can compute the slant range and slant rate as well. However, due to lack of altitude in the response, the altitude is not known.

TCAS Coordination

To avoid two aircraft from performing the same avoiding manoeuvre, TCAS coordinates the actions (ECTL, 2010). When TCAS detects a near mid-air collision, it first checks whether the other aircraft has sent a resolution advisory complement message, such as “don’t pass above” or “don’t pass below”. If not, TCAS will make its decision freely and sends out the complementary message. In case both aircraft select the same RA, the TCAS with the higher Mode S address will revert its RA.

2.2 Future ACAS systems

TCAS has certain limitations. On the one hand it is bound to large commercial aircraft, as a redesign is expected to be very costly (Chryssanthacopoulos, 2012). On the other hand, TCAS has drawbacks, such as a high number of false alerts and high maintenance costs (Chryssanthacopoulos, 2012).

That is why the FAA is currently developing a new ACAS system, called ACAS X (FAA, 2015b). ACAS X is expected to have a higher flexibility, be more robust and safer, while having lower implementation and maintenance costs compared to a TCAS redesign (Chryssanthacopoulos, 2012; Haessig, 2016).

A key word which describes ACAS X best is interoperability. ACAS X will work with a variety of sensors in a plug and play fashion, so different aircraft types, such as small general aviation aircraft but also unmanned aircraft can use it. Nevertheless, no matter which configuration is chosen, all ACAS X and old TCAS II are supposed to work together to solve conflicts between all possible aircraft types (Haessig, 2016).

ACAS X Concept

ACAS X will contain several varieties (FAA, 2015b):

- ACAS X_A: intended for large aircraft (current TCAS II users),
- ACAS X_O: intended for specific operations,
- ACAS X_P: intended for general aviation, and
- ACAS X_U: intended for unmanned aircraft.

All varieties will take advantage of the more precise ADS-B surveillance technology in addition to active interrogation reply functionality, except for general aviation which is planned to rely solely on ADS-B. Since, it can be expected that unmanned aircraft may not have the same performance features, the special category ACAS X_U was created. ACAS X_U will also be able to take advantage of additional non-cooperative sensors and more flexible avoidance procedures. While ACAS X_A will still be handling conflicts with vertical avoidance manoeuvres, ACAS X_U will also have the capability of horizontal manoeuvring when vertical manoeuvres are not possible or undesirable.

However, it is not very clear when vertical and when horizontal manoeuvring should be used. On the one hand, the ACAS X concept of use (FAA, 2015b) describes that vertical manoeuvring will be used in encounters with cooperative intruders, such as TCAS II and ACAS A, while horizontal manoeuvres are reserved to non-cooperative traffic. On the other hand, when due to performance issues vertical manoeuvres cannot be carried out, the UAS should switch into horizontal avoidance mode. Furthermore, when the UAS identifies that it will not be able to perform vertical collision avoidance, it should switch from automatic collision avoidance to also automatic self-separation. This means it will perform avoidance manoeuvres earlier.

In this logic it can already be seen that there are rules which contradict each other. Imagine the scenarios where the UAS identifies while performing a vertical avoidance manoeuvre that it cannot carry it out properly due to performance issues. Should the UAS stay in the vertical avoidance mode which might lead to a collision, because it is dealing with a cooperative threat, or should it violate the rule and switch to horizontal manoeuvring?

ACAS X_U Components

According to (FAA, 2015b) ACAS X_U will have the following components which can also be seen in [Figure 2-2](#):

- Surveillance Tracking Module (STM): ACAS X_U will have an input from a variety of surveillance sources. The minimum requirement will be a Mode S transponder which will be capable of ADS-B, active interrogation of cooperative aircraft. Additionally, it is required to carry sensors to detect non-cooperative aircraft. However, what the minimum required non-cooperative sensors are has still to be determined. The STM will use the multiple sensor input to estimate the tracks and correlate tracks of same intruders measured with different sensors. As such, the STM will be able to validate the measurements against each other and detect outliers. This feature has already been successfully tested in successfully in flight test in 2014 (Kotegawa, 2016).

- **Threat Resolution Module (TRM):** The TRM will use the estimated tracks by the STM as input and determines whether any aircraft will pose a threat. If so, it will determine a collision avoidance manoeuvre and forward the RA to the aircraft control system. The difference to other ACAS X systems is that the TRM will be able use vertical and horizontal collision avoidance logic based on pre calculated logic tables similar to TCAS (Chryssanthacopoulos, 2012). This is to also support UAS with low vertical performance manoeuvres down to 500ft/min. Therefore, a nucleus will decide when to use which collision avoidance manoeuvres. Nevertheless, vertical manoeuvring is the desired feature for cooperative encounters, while horizontal collision avoidance will be used with non-cooperative intruders or special cases only.
- **Automatic Control System:** For UAS it is expected that they follow RA automatically to reduce the risk of delayed due to communication latency or loss link with the pilot. Nevertheless, the pilot will be warned about RA and is able to intervene.
- **System Monitor Module (SMM):** The SMM will monitor the health of all systems of the UAS and provides integrity knowledge.

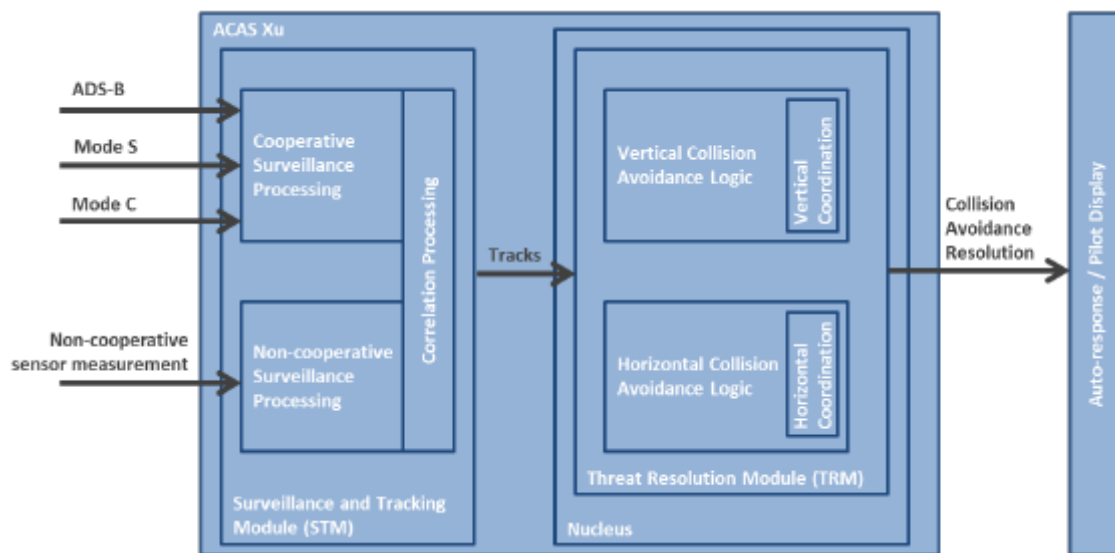


Figure 2-2: ACAS Xu system concept (FAA, 2015b).

ACAS X_A Model

The Lincoln Laboratory, in collaboration with other organizations has developed the current TCAS II technology and is pioneering the development of ACAS X (Chryssanthacopoulos, 2012). It can be noted that main contributors are M.J. Kochenderfer and J.P. Chryssanthacopoulos (Kochenderfer, 2010, 2011; Chryssanthacopoulos, 2012).

Similar to TCAS II, ACAS X_A is supposed take advantage of pre-calculated look up tables. The on-board system on the aircraft will estimate the intruder's position and looks up the resolution advisory from the look up tables. The look up tables were created using methods described in (Kochenderfer, 2010, 2011).

The problem was formulated as a Markov decision process. The optimization function may be solved using different methods. The method used is (stochastic) dynamic programming, as it is an efficient tool (Chryssanthacopoulos, 2012). According to (Chryssanthacopoulos, 2012) the computation time to compute one set of look up tables was only 10 minutes.

The robustness and validity was then tested using different models and data: real operational radar data, airspace encounter models, procedure specific models, and stress-testing models. The simulation resulted in 38% more advisories in parallel approach encounters, but risk collision decrease by 47% and 40% fewer alerts in total compared to TCAS II (Chryssanthacopoulos, 2012).

2.3 Collision avoidance models

TCAS II operations have been modelled in many instances. For example (Billingsley et al., 2009) made a multiple threat encounter analysis of TCAS II operations and (Kuchar, 2005) analysed the performance of UAVs, which are equipped with TCAS II. However, it seems that in both studies only one uncertainty, a variation in pilot delay, was included in the assessments (Billingsley et al., 2009, p. 5; Kuchar, 2005, p. 7). Another example is an agent-based model of TCAS II by (Netjasov, 2010). But in this model no uncertainties were modelled, either.

Next to TCAS II, there are also a variety of other collision avoidance algorithms available of which many are reviewed and classified (Jenie et al., 2016).

One method is the Selective Velocity Obstacle method (Jenie et al., 2013, 2016). The method works by adopting the rules of the air and applies horizontal collision avoidance manoeuvres. Necessary knowledge is the velocity and position of own and intruder's aircraft. These three vectors are sufficient to determine which avoidance manoeuvre is required according to the rules of the air. To do so, a protecting sphere is modelled around the intruder. Then a cone is created using own's aircraft position and the tangential points of the sphere, which is projected using the intruder's velocity. This creates the projected dangerous zone and if the own's aircraft velocity vectors ends up in this cone, a collision avoidance manoeuvre is issued. Avoidance resolutions are in the form of turns limited by maximal rate of turns. The Velocity Obstacle method has also been applied in another similar study by NLR which has focused on agent-based modelling (Blom et al., 2016). The study mentions model parameter which may be useful as reference for further research. It should also be noted that the efficiency of these new methods relative to TCAS are not yet known.

Another horizontal avoidance algorithm was established using the same method as used for ACAS X_A. The model focuses on multirotor UAS taking horizontal manoeuvring and velocity changes into account. The velocity changes do not have a minimum velocity limit as hovering is within the capabilities. Nevertheless, tuning of the cost function required effort to avoid the trivial solution to stop every time in order to postpone the collision.

Including velocity variation in collision avoidance algorithms has also been considered in other research. (Galatolo et al., 2016) developed an algorithm which allows both horizontal avoidance manoeuvres according to the rules of the air, but also allows velocity variations. Furthermore, the algorithm also tries to keep deviations to intended flight path as low as possible. The protected zone of an intruder is modelled as an ellipsoid and the relative position and speed of both aircraft is known.

There are also models specified for UAS which implement vertical manoeuvring. In the study (Yu et al., 2016) a model was designed specific for the landing phase. Opposed to the other mentioned models, this model is aircraft specific and uses an advance longitudinal aircraft description, which uses elevator deflection and thrust to resemble desired vertical rates. In short, the model knows the relative position and velocity of intruders. Then it estimates the optimal solution by looking for a trajectory as close as possible to the optimal trajectory. After passing the intruder, it takes advantage of an energy function integrated in the aircraft controller to get back to the intended flight path. This model is very complex, as it is not trivial to derive the optimal elevator deflections and thrust settings, and uses a biogeography-based optimization method. Biogeography-based optimization is categorized as an artificial heuristic. Last but not least, due to the aircraft specific modelling this method cannot be easily used on other aircraft if the aircraft characteristics are not known.

2.4 Agent-Based Modelling and Simulation (ABMS)

This chapter elaborates on agent-based modelling and simulation by giving an overview of the modelling method, its benefits, disadvantages and typical areas of application, and how it can be simulated with the Monte Carlo method. Note that the research required the use of an agent-based model and therefore only a brief overview is given.

What is Agent-Based Modelling?

According to Gilbert (Gilbert, 2008) “Formally, agent-based modelling is a computational method that enables a researcher to create, analyse, and experiment with models composed of agents that interact within an environment.” The first part of the definition means that it enables experiments of models with a computational method. Basically instead of doing live trials, it is possible to describe the to be researched systems in models and achieve results virtually. Furthermore, these models are composed of agents. An agent is a distinct entity in the model representing social actors or bodies (Gilbert, 2008). Importantly, the agents can be interconnected with each other and the environment. This may be due to communication and/or observation.

Opposed to Gilbert (Gilbert, 2008), Bonabeau (Bonabeau, 2002) defines agent-based modelling as follows: “ABM is a mind-set more than a technology. The ABM mind-set consists of describing a system from the perspective of its constituent units.” His reasoning is that many people think describing a model with differential equation is the alternative to ABM. However, an ABM may consist of differential equations but each describing one of the systems units. For example, instead of describing the traffic jam behaviour as a group of cars, in ABM each traffic participant is modelled separately. An argument supporting Bonabeau’s vision on ABM could be explained by the fact that in an ABM several different modelling techniques may be including. For example learning behaviour of humans might be modelled with heuristic evolution algorithms while vehicles might be modelled with simple dynamic or even linear equations. Hence, given the variety of agents and modelling techniques it may makes sense to define agents in categories. According to Blom¹ there are two types of agents:

¹ H. Blom. AE4448 - Agent Based Safety Risk Analysis. TU Delft, Aerospace, 2015. Lecture slides.

- Proactive agent: a proactive agent is able to exhibit goal-directed and adaptive behaviour by taking own initiative,
- Reactive agent: a reactive agent is able to perceive their environment, and respond in a timely fashion to changes that occur in it (stimulus-response behaviour and delayed response behaviour).

In regard to the environment one can usually distinguish between the following types¹:

- Accessible vs inaccessible,
- Static vs dynamic,
- Deterministic vs non-deterministic, and
- Discrete vs continuous.

Monte Carlo Simulation

Monte Carlo simulation is a type of simulation to evaluate theoretical models. The methodology is rather simple by approximating a solution of a given model through simulating many times with different random parameters (Johnson, 2013).

Furthermore, it is a common tool in many fields and often used to find emergent behaviour in ABM (Johnson, 2013). For example (Jenie et al., 2013; Blom et al., 2016; Zou et al., 2016) use Monte Carlo simulation to approximate results.

One of the application fields mentioned in (Johnson, 2013) is socio sciences. As this is the nature of the research to be carried out, it is advised to also take advantage of Monte Carlo simulation

Benefits and Disadvantages

According to Bonabeau (Bonabeau, 2002) there are three main advantages of ABMS.

First, ABMS offers a natural description of a system. Instead of describing processes, ABMS focuses on activities. Therefore, one does not need to know the whole process in advance but can focus on rules and behaviour of single agents. Especially in a known environment this is advantages, as it may be difficult to be unbiased through experience when describing processes.

Second, ABMS offers a high degree of flexibility. Since each agent and the environment are modelled separately, it is easy to adopt changes, e.g. add agents, change rules, or change the environment. This also makes it possible to change the level of observation, e.g. by reducing the simulation to a subset.

Third, ABMS captures emergent behaviour. Seeing in literature (Bonabeau, 2002; Gilbert, 2008; Blom et al., 2016), this is the most important benefit. For complex systems it may be difficult to forecast the resulting behaviour resulting from the interactions of different agents. However, this holds true for models with simple rules as well. With ABM it is possible to capture this behaviour and identify potential unexpected effects of small rule changes.

¹ H. Blom. AE4448 - Agent Based Safety Risk Analysis. TU Delft, Aerospace, 2015. Lecture slides.

As being described so far, ABM seems to be the ultimate tool to model everything. However, the bottom up modelling approach has its disadvantages, too. For example, ABM requires a lot of time, understanding of the system, and resources. The system needs to be known extensively, as all the agents, the environment, and all interactions have to be known. In general, it can be said that a model should have just the right amount of details to serve its purpose (Bazghandi, 2012; Bonabeau, 2002). Then with increasing number of agents and interactions, the computation time to solve such models increases as well. Therefore, given the available knowledge, resources, time and desired detail of results, other simpler methods may be superior.

Areas of application

Due to the capability to capture emergent behaviour, ABMS is used to model socio-technical systems¹. To be more precise, ABMS is commonly applied in the following areas (Bonabeau, 2002):

- Flows: evacuation, traffic, and customer flow management.
- Markets: stock market, shopbots and software agents, and strategic simulation.
- Organizations: operational risk and organizational design.
- Diffusion: diffusion of innovation and adoption dynamics.

Part I

3 TCAS II Ver. 7.1

This section elaborates on the general principals of the airborne collision avoidance system TCAS II Ver. 7.1.

3.1 Introduction to TCAS II

The Traffic Collision Avoidance System (TCAS) is a system to prevent mid-air collisions. The current version is TCAS II Ver. 7.1 and is mandatory for all aircraft with a maximum take-off mass of 5,700kg or more, or being able to carry at least 19 passengers (Haessig, 2016).

TCAS is specified in (EUROCAE, 2008) and explained in (FAA, 2011). It is based on the ACAS specification of ICAO which is elaborated in (ICAO, 2006). However, often the explanations are in text form only. Based on this, a mathematical model will be developed. When available and within the scope of this Thesis, equations have been taken from the specifications and are cited accordingly.

TCAS uses Mode S communication to detect threats and coordinate avoidance manoeuvres. If a threat has been detected, TCAS warns the pilot with an aural and visual traffic advisory (TA), which is supposed to inform the pilot about the threat and help him to locate the intruder. If the conflict remains unsolved and a certain threshold has been reached, TCAS issues a resolution advisory (RA), which recommends necessary actions to the pilot to avoid a collision. The RA is both aural and visual, as well and is coordinated between TCAS of involved aircraft. The RA may recommend to

- climb or descend at 1,500ft/min;
- level off;
- maintain current climb or descend rate;
- limit climb or descend rate to 500, 1,000, or 2,000ft/min; or
- strengthen climb or descend rate to 2,500ft/min.

After the conflict has been resolved, TCAS notifies the pilot with a clear of conflict message.

3.2 TCAS II Ver. 7.1

In this study TCAS Ver. 7.1 has been modelled to the specification gathered from (FAA, 2011), (EUROCAE, 2008) and (ICAO, 2006). The way the model is working is visualised in a simplified manner in [Figure 3-1](#). It may be divided into four phases, threat detection, sense selection, strength selection and threat evolution assessment.

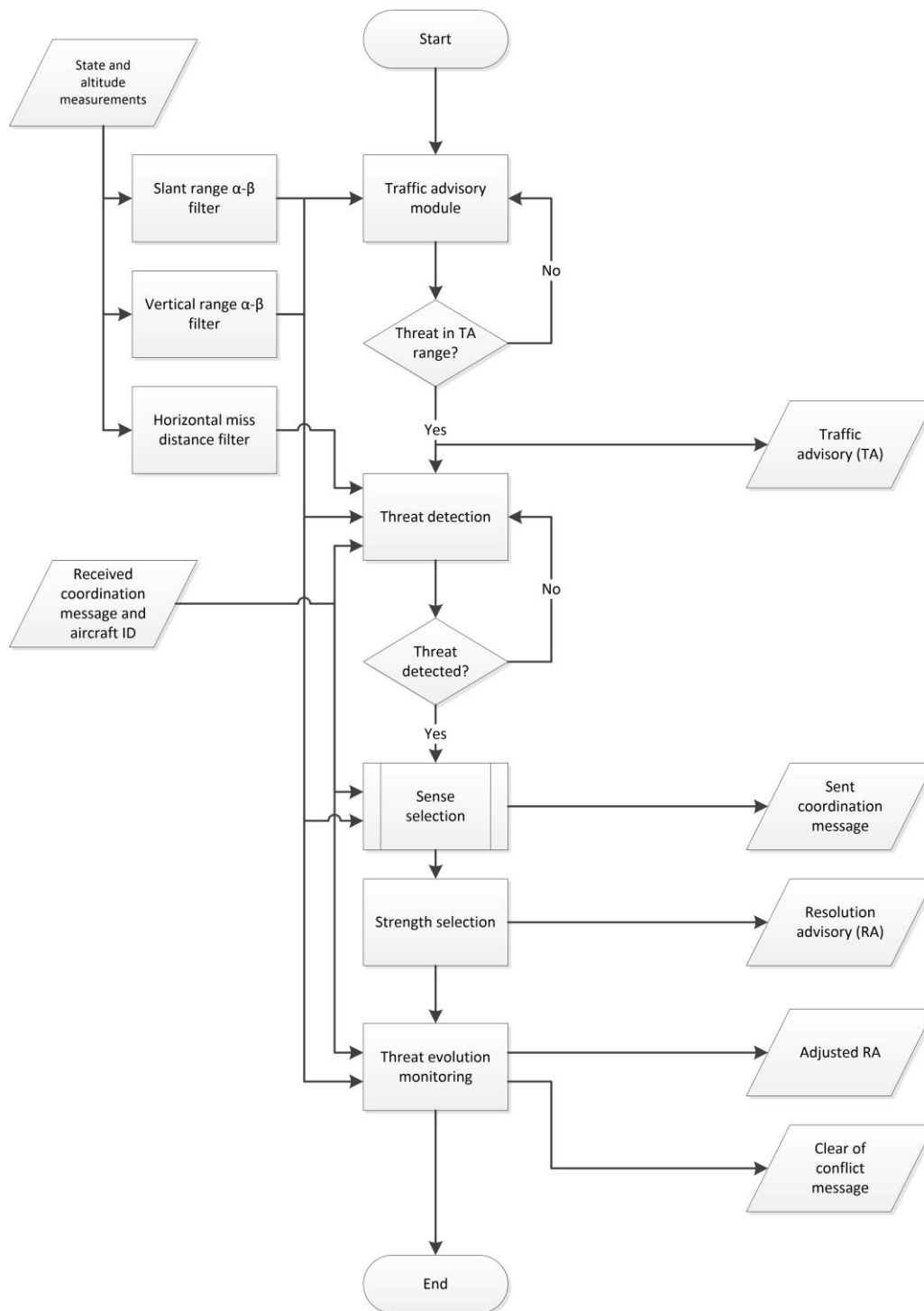


Figure 3-1: Functional flow diagram of TCAS II Ver. 7.1.

First, the aircraft states, own and foreign aircraft, are used to determine whether the foreign aircraft is a threat and, hence, is an intruder. TCAS uses two tests to determine threats. The range test checks the measured slant range versus the measured slant range rate and the altitude test checks the vertical displacement versus the relative vertical rate. Only if both tests are positive, an RA will be issued.

Second, after a threat has been detected a sense is selected. The sense defines the nature of RA and may be positive or negative. A positive sense means that a change in vertical velocity is required for a safe pass. This velocity change may be either in upwards or downwards direction. For example, if the own aircraft is descending and the sense is positive upwards, the possible RA's may be to level off or to climb. In contrast, a negative sense means that no vertical velocity change is required, but certain vertical speed limitations exist. For example, if the own aircraft is flying level, possible RA may be to not climb.

The geometrical rules to determine the sense are as followed:

- Select sense which results in greatest miss distance;
- Except, avoid altitude crossings if miss distance suffice

Furthermore, the sense selection also takes advantage of Mode S coordination with the intruder. TCAS checks whether a coordination message has been received. In a coordination message the TCAS of the own aircraft tells the TCAS of the intruder, and vice versa, which its intended resolution is. Depending on the resolution the coordination message may demand the other aircraft to either not pass above, or to not pass below. This is necessary to avoid both aircraft initiating an avoidance manoeuvre in the same direction. Therefore, if no coordination message has been received by one TCAS, it will determine the sense depending on the encounter geometry. However, if a coordination message has been received, then the selection space of the sense is limited accordingly.

In rare cases both aircraft select the same sense at the same time. In this case the aircraft with the higher aircraft ID will go back to the sense selection and changes its sense. Hence, this check is carried out until a message has been received and the selected senses of both aircraft harmonise or the aircraft has the lower aircraft ID.

Third, the strength of the RA is determined and the RA is issued to the pilot.

Fourth, the evolution of the threat is being monitored until the aircraft is clear of conflict. If the current RA is identified to be insufficient, e.g. if the pilot reacts too late or the intruder is not following its selected sense, then the RA may be strengthened or the sense reversed.

Following, the structure of the mathematical model is following the structure of [Figure 3-1](#).

- Section 4 elaborates on information received by TCAS; state and altitude measurements, and received coordination message and aircraft ID,
- Section 5 elaborates on the slant range and vertical range filtering modules,
- Section 6 describes the Traffic Advisory module,
- Section 7 explains the horizontal miss distance filter which is used as an additional input in the threat detection module,
- Section 8 elaborates on the threat detection module, in which another aircraft is declared a threat,

- Section 9 describes the sense selection module, which results in a coordination message sent to the intruder,
- Section 9.5 elaborates on the strength selection module, which results in the RA for the own aircraft, and
- Section 10 describes the threat evolution monitoring module which determines strength/sense adjustments.

4 State and altitude measurements

The own aircraft receives information about the location of other aircraft via measurements through interrogation messages. Also through coordination messages the intent of other aircraft is communicated. In this section first the communication between an aircraft 'i' and 'k' are explained. Then the state and velocity variables are defined, followed up by the measurements of aircraft 'i'. Last, the received coordination message and aircraft ID of aircraft 'k' by aircraft 'i' are described.

4.1 State and velocity variables

The position state of aircraft 'i' at time 't' is specified in Cartesian coordinates as $\mathbf{s}_t^i$. The change of position, $\dot{\mathbf{s}}_t^i$, is defined as velocity vector $\mathbf{v}_t^i$. The change of velocity, $\dot{\mathbf{v}}_t^i$, is defined as $\mathbf{a}_t^i$.

$$\mathbf{s}_t^i = \begin{pmatrix} s_{t,x}^i \\ s_{t,y}^i \\ s_{t,z}^i \end{pmatrix} \in \mathbb{R}^3 \quad (1)$$

$$\mathbf{v}_t^i = \begin{pmatrix} v_{t,x}^i \\ v_{t,y}^i \\ v_{t,z}^i \end{pmatrix} \in \mathbb{R}^3 \quad (2)$$

$$\dot{\mathbf{s}}_t^i = \mathbf{v}_t^i \quad (3)$$

$$\dot{\mathbf{v}}_t^i = \mathbf{a}_t^i \quad (4)$$

Furthermore, the position, velocity and acceleration states are decoupled in a horizontal, subscript 'H', and altitude, subscript 'A', components. The horizontal components at time 't' of aircraft 'i' are the two dimensional position, $\mathbf{s}_{t,H}^i$, the two dimensional velocity, $\mathbf{v}_{t,H}^i$, and the two dimensional acceleration, $\mathbf{a}_{t,H}^i$, all in x and y coordinates.

$$\mathbf{s}_{t,H}^i = \begin{pmatrix} s_{t,x}^i \\ s_{t,y}^i \end{pmatrix} \in \mathbb{R}^2 \quad (5)$$

$$\mathbf{v}_{t,H}^i = \begin{pmatrix} v_{t,x}^i \\ v_{t,y}^i \end{pmatrix} \in \mathbb{R}^2 \quad (6)$$

$$\mathbf{a}_{t,H}^i = \begin{pmatrix} a_{t,x}^i \\ a_{t,y}^i \end{pmatrix} \in \mathbb{R}^2 \quad (7)$$

The altitude components at time 't' of aircraft 'i' are one dimensional. $s_{t,A}^i$ is the altitude of aircraft 'i'. $v_{t,A}^i$ is the rate of climb of aircraft 'i'. If $v_{t,A}^i$ is negative, then we also call $|v_{t,A}^i|$ the rate of descend. $a_{t,A}^i$ is the altitude acceleration of aircraft 'i'.

$$s_{t,A}^i \in \mathbb{R}^1 \quad (8)$$

$$v_{t,A}^i \in \mathbb{R}^1 \quad (9)$$

$$a_{t,A}^i \in \mathbb{R}^1 \quad (10)$$

It is assumed that each aircraft 'i' has a system to know its own rate of climb, $v_{t,A}^i$, and that this information is made available to TCAS of aircraft 'i'. According to (NASA, 2017, p. 12) the altitude velocity has a jitter of 1.707m/s with 95% confidence.

Besides, the relative position, s_t^{ik} , between aircraft 'i' and aircraft 'k' is defined as the position vector between aircraft 'i' and aircraft 'k' with reference to aircraft 'i'. v_t^{ik} is the relative velocity vector between aircraft 'i' and aircraft 'k' with reference to aircraft 'i'.

$$s_t^{ik} \triangleq s_t^k - s_t^i \in \mathbb{R}^3 \quad (11)$$

$$v_t^{ik} \triangleq v_t^k - v_t^i \in \mathbb{R}^3 \quad (12)$$

If $i=0$ and $k=1$ then it is shortly written:

$$s_t \triangleq s_t^1 - s_t^0 \in \mathbb{R}^3 \quad (13)$$

$$v_t \triangleq v_t^1 - v_t^0 \in \mathbb{R}^3 \quad (14)$$

4.2 Measurements by aircraft 'i'

The exact position and speed vectors of an intruder aircraft are not known to own aircraft but are estimated by TCAS with alpha-beta filters (ICAO, 2006) from physical range and bearing tests using the Mode S signal and received altitude reports.

Every 8 to 10 seconds each TCAS equipped aircraft sends out a Mode S message including its aircraft ID (EUROCAE, 2008, p. B1). This message is being picked up by other aircraft which start interrogating each other. An interrogation message is always addressed to one specific aircraft which responds accordingly. The response also includes the pressure altitude, aircraft ID and a coordination message. From the response delay the slant range can be measured. Interrogations are nominally carried out once per second (EUROCAE, 2008, p. B3).

$$y_{t,r}^{ik} = \|s_t^{ik}\| + \varepsilon_{t,r,jitter}^i + \varepsilon_{r,bias}^i \quad (15)$$

$y_{t,r}^{ik}$ is the aircraft 'i' measured distance of aircraft 'k' to aircraft 'i'. According to (EUROCAE, 2008, p. 34) the slant range measurement should not exceed a bias of up to 125ft and a jitter, $\sigma_{r,jitter}^i$, of 50ft root mean square, but independently of transponder effects, the range measurement error will not exceed 35ft root mean square jitter. Also according to (Thompson, 2000, p.7) the bias remains "essentially constant". In (Hammer, 1996a) the jitter of the slant range measurement error is stated to be white Gaussian noise of 30ft root mean square. Here, the same assumption is made. Hence,

$\mathcal{E}_{t,r,jitter}^i$ is the slant range measurement error of aircraft 'i' at time 't' induced by jitter, and $\mathcal{E}_{r,bias}^i$ is the constant error of the slant range measurement of aircraft 'i', also called bias.

$$y_{t,\theta}^{ik} = \text{angle}(\mathbf{v}_t^i) - \text{angle}(\mathbf{s}_t^{ik}) + \mathcal{E}_{t,\theta}^{ik} \quad (16)$$

$y_{t,\theta}^{ik}$ is the relative bearing measurement by aircraft 'i' of aircraft 'k' measured in clockwise direction. Because the relative bearing is measured in the aircraft body axis system, Equation (16) applies under the assumption that the velocity vector points in the direction of the aircraft body (no wind or sideslip present). Hence, the relative bearing measurement is equal to the angle of velocity vector of aircraft 'i', $\text{angle}(\mathbf{v}_t^i)$, minus the angle of the relative position vector between aircraft 'i' and aircraft 'k' with reference to aircraft 'i', $\text{angle}(\mathbf{s}_t^{ik})$, and added measurement error, $\mathcal{E}_{t,\theta}^{ik}$. Note that $\text{angle}(\mathbf{v}_t^i)$ and $\text{angle}(\mathbf{s}_t^{ik})$ are measured counter clockwise in the mathematical reference system, while the relative bearing is measured clockwise from the direction of flight of aircraft 'i'. Also, in case the relative bearing measurement exceeds the range $[-180^\circ, 180^\circ]$, then the relative bearing measurement is corrected in quantisation of 360° . According to (EUROCAE, 2008, p.117) the bearing measurement error should not exceed 9 degrees root mean square with a maximum of 27 degrees.

Pressure altitude is reported either in 25 or 100ft quantization. This model assumes 25ft quantization reports.

$$y_{t,A}^i = s_{t,A}^i + \mathcal{E}_{A,bias}^i + \mathcal{E}_{t,A,jitter}^i \quad (17)$$

$y_{t,A}^i$, as defined in Equation (17), is the measured pressure altitude of aircraft 'i'. It is assumed that all measurements are carried out with QNH setting equal to 0. $\mathcal{E}_{A,bias}^i$ is the constant altitude measurement error. According to (ICAO, 2006, pp. 444-445) the total measurement error may be modelled as a Gaussian variable with a 99.7% error bound as stated in Table 4-1. $\mathcal{E}_{t,A,jitter}^i$ is the variable measurement error. From available flight data, it was identified that the measured altitude of a steady aircraft varies with a standard deviation of about 5ft with an autocorrelation of about 8s. Hence, the jitter of the altitude measurement is modelled in a first order autoregressive as given in Equation (18) with $\alpha_A^i = 0.8$. $\mathcal{E}_{t,A,noise}^i$ is white Gaussian noise with zero mean and a standard deviation of $\sigma_{A,noise}^i$, which is assumed to be 3ft in this case. α_A^i has been deducted from the autocorrelation of the flight data. The standard deviation of the white noise, $\sigma_{A,noise}^i$, in the autoregressive model has been deducted from the standard deviation of the flight data, σ_A^i , and is given in Equation (19).

$$\mathcal{E}_{t,A,jitter}^i = \alpha_A^i \mathcal{E}_{t-\Delta,A,jitter}^i + \mathcal{E}_{t,A,noise}^i \quad (18)$$

$$\sigma_{A,noise}^i = \sigma_A^i \sqrt{(1 - \alpha_A^{i^2})} \quad (19)$$

Table 4-1: TCAS specified altitude measurement 99.7% error bound (ICAO, 2006, Table 3-1).

Own altitude [ft]	99.7% error bound [ft]
Mean sea level	135
5,000	144
10,000	156
15,000	174
20,000	195
25,000	213
30,000	234
35,000	258
40,000	285

Hence, the relative measured altitude difference, $y_{t,A}^{ik}$, between aircraft 'i' and aircraft 'k' is the difference between measured altitudes of aircraft 'i' and aircraft 'k':

$$y_{t,A}^{ik} = 25\text{ft} \left\lfloor \frac{y_{t,A}^k}{25\text{ft}} \right\rfloor - y_{t,A}^i \quad (20)$$

where the measured altitude of aircraft 'k' is rounded to the closest 25ft increment (in line with the 25ft quantization assumption). $\left\lfloor \cdot \right\rfloor$ is defined as rounding to the nearest integer.

Similar as in (13)-(14), if $i=0$ and $k=1$, then it is shortly written:

$$y_{t,r} \triangleq y_{t,r}^{01} \quad (21)$$

$$y_{t,A} \triangleq y_{t,A}^{01} \quad (22)$$

$$y_{t,\theta} \triangleq y_{t,\theta}^{01} \quad (23)$$

4.3 Received coordination message and aircraft ID

The coordination message received by aircraft 'i' from aircraft 'k' depends on the coordination message sent by the aircraft 'k'. A coordination message sent and a coordination message received from aircraft 'i' to aircraft 'k' are defined as, respectively:

$$m_{t,\text{sent}}^{ik} \triangleq \begin{cases} 0, & \text{no message sent by a/c 'i' to a/c 'k'} \\ 1, & \text{a/c 'i' should pass above a/c 'k'} \\ -1, & \text{a/c 'i' should pass below a/c 'k'} \end{cases} \quad (24)$$

$$m_{t,\text{received}}^{ki} \triangleq \begin{cases} 0, & \text{no message received by a/c 'i' from a/c 'k'} \\ 1, & \text{a/c 'k' should pass above a/c 'i'} \\ -1, & \text{a/c 'k' should pass below a/c 'i'} \end{cases} \quad (25)$$

Hence, if there are no communication errors, then the coordination message received by aircraft 'i' from aircraft 'k' satisfies:

$$m_{t, received}^{ki} = m_{t, sent}^{ki} \quad (26)$$

According to (EUROCAE, 2008, p.133) both aircraft also acknowledge the reception of a coordination message in a reply. Therefore, it is assumed that the coordination message is only being transmitted until the other aircraft has acknowledged the reception of the message.

The aircraft ID, χ_{ModeS}^i , of aircraft 'i' is defined as a positive or zero integer:

$$\chi_{ModeS}^i \triangleq \{0, 1, 2, 3, \dots\} \quad (27)$$

Similar as in (13)-(14), if $i=0$ (own aircraft) and $k=1$ (intruder), then the message sent by own aircraft is shortly written as:

$$m_{t, sent} \triangleq m_{t, sent}^{01} \quad (28)$$

Similarly the message received by own aircraft is shortly written as:

$$m_{t, received} \triangleq m_{t, received}^{10} \quad (29)$$

In the following it is always considered that own aircraft is number '0' and the other aircraft is number '1', which allows to use short notations from Equations (13)-(14), (21)-(23), and (28)-(29).

5 Slant range and vertical range filters

As the measurements contain errors, state estimators are used to reduce these errors. This section describes the applied state estimation α - β filters.

5.1 Slant range α - β filter

With the range measurements and an α - β filter the slant range and slant range rate are estimated. These are defined in Equations (30)-(35) based on (ICAO, 2006, pp. 3-9 – 3-10).

$$r_t = r_{t|t-T} + \alpha \mu_{t,r} \quad (30)$$

$$\dot{r}_t = \dot{r}_{t-T} + \frac{\beta}{T} \mu_{t,r} \quad (31)$$

$$r_{t|t-T} = r_{t-T} + \dot{r}_{t-T} T \quad (32)$$

r_t is the estimated slant range, $\dot{r}_t$ is the estimated slant range rate, and $r_{t|t-T}$ is the prediction of slant range at time 't' subject to range and range rate estimates at time 't-T'.

$$\mu_{t,r} = \begin{cases} y_{t,r} - r_{t|t-T}, & \text{if } \exists \text{ measurement } y_{t,r} \\ 0, & \text{if no measurement} \end{cases} \quad (33)$$

$\mu_{t,r}$ is the range measurement residual and T is the elapsed time since the previous interrogation message.

The presented values of the smoothing parameters α and β are (ICAO, 2006, p. 3-9):

$$\alpha = 0.67 \quad (34)$$

$$\beta = 0.25 \quad (35)$$

5.2 Vertical range α - β filter

The estimates $r_{t,A}$ and $\dot{r}_{t,A}$ of the relative altitude, $s_{t,A}$, and relative altitude velocity, $v_{t,A}$, satisfy Equations (36)-(41) according to (ICAO, 2006, p. 3-50).

$$r_{t,A} = r_{t|t-T,A} + \alpha_A \mu_{t,A} \quad (36)$$

$$\dot{r}_{t,A} = \dot{r}_{t-T,A} + \frac{\beta_A}{T} \mu_{t,A} \quad (37)$$

$$r_{t|t-T,A} = r_{t-T,A} + \dot{r}_{t-T,A} T \quad (38)$$

$$\mu_{t,A} = \begin{cases} y_{t,A} - r_{t|t-T,A}, & \text{if } \exists \text{ measurement } y_{t,A} \\ 0 & \text{, if no measurement} \end{cases} \quad (39)$$

$r_{t,A}$, $\dot{r}_{t,A}$, $r_{t|t-T,A}$, $\mu_{t,A}$ are the relative altitude estimate, relative altitude velocity estimate, the relative altitude prediction based on the previous measurements and the relative altitude residual, respectively, all at time 't'.

The presented values of the smoothing parameters α_V and β_V depend on the vertical range rate estimate and the vertical range measurement residual as defined in Equations (40) and (41) (ICAO, 2006, p. 3-50).

$$\alpha_A = \begin{cases} 0.4, & \text{if } \left\{ \begin{aligned} & \left[(\dot{r}_{t-T,A} < 7\text{ft/s}) \wedge (r_{t-T,A} \geq 0) \right] \vee \\ & \left[(\dot{r}_{t-T,A} > -7\text{ft/s}) \wedge (r_{t-T,A} \leq 0) \right] \end{aligned} \right\} \\ 0.5, & \text{if } \left\{ \begin{aligned} & \left[(\mu_{t-T,A} < 22.5\text{ft}) \wedge (\dot{r}_{t-T,A} \geq 7\text{ft/s}) \wedge (r_{t-T,A} \geq 0) \right] \\ & \left[(\mu_{t-T,A} > -22.5\text{ft}) \wedge (\dot{r}_{t-T,A} \leq -7\text{ft/s}) \wedge (r_{t-T,A} \leq 0) \right] \end{aligned} \right\} \\ 0.6, & \text{otherwise} \end{cases} \quad (40)$$

$$\beta_A = \begin{cases} 0.100, & \text{if } \left\{ \begin{aligned} & \left[(\dot{r}_{t-T,A} < 7\text{ft/s}) \wedge (r_{t-T,A} \geq 0) \right] \vee \\ & \left[(\dot{r}_{t-T,A} > -7\text{ft/s}) \wedge (r_{t-T,A} \leq 0) \right] \end{aligned} \right\} \\ 0.167, & \text{if } \left\{ \begin{aligned} & \left[(\mu_{t-T,A} < 22.5\text{ft}) \wedge (\dot{r}_{t-T,A} \geq 7\text{ft/s}) \wedge (r_{t-T,A} \geq 0) \right] \\ & \left[(\mu_{t-T,A} > -22.5\text{ft}) \wedge (\dot{r}_{t-T,A} \leq -7\text{ft/s}) \wedge (r_{t-T,A} \leq 0) \right] \end{aligned} \right\} \\ 0.257, & \text{otherwise} \end{cases} \quad (41)$$

6 Traffic Advisory

With the range estimates from the α - β filters TCAS tracks the distance to aircraft in the vicinity. If the range estimates satisfy certain threshold values, then TCAS warns the pilot with a Traffic Advisory (TA) that an aircraft is close (EUROCAE, 2008, p. 140). This is purely informative and is intended to make the pilot aware of other aircraft as they may become a threat to the own aircraft. Whether the other aircraft is sufficiently close to issue a TA is determined by a range and an altitude test (ICAO, 2006, p. 3-57).

6.1 Range test of TA

The range test (RT), as described in the ACAS specification (ICAO, 2006, pp.3-58 - 3-59), determines whether the intruder is a Range threat. As defined in Equation (115), the range test for a TA, defined as Boolean $B_{t,RT,TA}$, is true if the modified time until closest point of approach (CPA), $\tau_{\text{mod},TA|t}$, is within a certain threshold, Δ_{TA} , and the aircraft are converging, or the aircraft are within a specified distance, $d_{\text{mod},TA}$, diverging in range, and the divergence is “slow”. The modified time until CPA, $\tau_{\text{mod},TA|t}$, accounts for horizontal position inaccuracies and is defined in Equation (116) (ICAO, 2006, p. 3-59). Aircraft are considered to divert slowly when the multiplication of estimated range and estimated velocity is less than the value of a parameter $\dot{P}_m$.

$$B_{t,RT,TA} = \text{TRUE} \quad , \text{iff} \quad \left\{ \begin{aligned} & \left[\left(0 \leq \tau_{\text{mod},TA|t} < \Delta_{TA} \right) \wedge \left(\dot{r}_t \leq 0 \right) \right] \vee \\ & \left[\left(r_t < d_{\text{mod},TA} \right) \wedge \left(\dot{r}_t > 0 \right) \wedge \left(r_t \dot{r}_t < \dot{P}_m \right) \right] \end{aligned} \right. \quad (42)$$

$$\tau_{\text{mod},TA|t} = \begin{cases} \frac{r_t^2 - d_{\text{mod},TA}^2}{r_t |\dot{r}_t|} & , \text{ if } (r_t \geq d_{\text{mod},TA}) \\ 0 & , \text{ otherwise} \end{cases} \quad (43)$$

$$\dot{r}_t' = \min(\dot{r}_t, -3\text{m/s}) \quad (44)$$

The alarm threshold values of the range test are a function of the own altitude and defined in [Table 6-1](#). However, in (EUROCAE, 2008, p. 125) some exceptions are stated. One is that in cases where the threshold values of the intruder aircraft ‘k’ are larger than the threshold values of the own aircraft ‘i’ then the larger threshold values are used by own aircraft ‘i’ as well. Exceptions exist when the own altitude is below 1,000ft or TCAS has been manually set to a mode which only issues TAs. Here these exceptions are not considered and the threshold value is determined by the maximum altitude of aircraft ‘i’ and ‘k’.

Table 6-1: Alarm thresholds of the range test for a Traffic Advisory (FAA, 2011, Table 2).

Own altitude [ft]	$\dot{P}_m$ [NM ² /s]	Δ_{TA} [s]	$d_{mod,TA}$ [NM]
< 1,000 (AGL)	N/A	20	0.30
(1,000 – 2,350] (AGL)	0.0020	25	0.33
(2,350 – 5,000]	0.0028	30	0.48
(5,000 – 10,000]	0.0028	40	0.75
(10,000 – 20,000]	0.0028	45	1.00
(20,000 – 42,000]	0.0040	48	1.30
≥ 42,000	0.0040	48	1.30

Furthermore, the specification also describes that the range test is positive if a miss distance could not be calculated on the current or the miss distance is violating the horizontal miss distance threshold. However, from the description it is not clear whether this specification is only considered for RAs or also for TAs. Since, TAs are purely informative and the description is not clear, the horizontal miss distance test has been omitted from the range test regarding TAs.

6.2 Altitude test of TA

The altitude test (AT) for TAs, as described in the ACAS specification (ICAO, 2006, p.3-57), determines whether the intruder is an altitude threat. The altitude test, as defined as Boolean $B_{t,AT,TA}$, is true if the current aircraft altitude separation is small, or the aircraft are converging in altitude and the time to co-altitude (both aircraft at same altitude) is small. If $B_{AT,CA}$ is defined as the outcome of the latter “co-altitude is small” test, then the altitude test for TAs becomes:

$$B_{t,AT,TA} = TRUE, \text{ iff } \left\{ \begin{array}{l} \left[|r_{t,A}| < z_{thr,TA} \right] \vee \\ \left[(\dot{r}_{t,A} < 0) \wedge (r_{t,A} \geq 0) \wedge B_{AT,CA} \right] \vee \\ \left[(\dot{r}_{t,A} > 0) \wedge (r_{t,A} < 0) \wedge B_{AT,CA} \right] \end{array} \right. \quad (45)$$

where $z_{thr,TA}$ is an altitude separation threshold value. According to (ICAO, 2006, p.3-61) the time to co-altitude is small if:

“ τ_v is declared “small” if $\tau_v < T_v$ for encounters in which the magnitude of own aircraft’s vertical rate is not more than 600 ft/min or own aircraft’s vertical rate has the same sign as but smaller magnitude than that of the intruder. For all other encounters τ_v is declared “small” if $\tau_v < T_v$.”

In our setting this means that the time to co-altitude, τ_{Alt} , is small, i.e. $B_{AT,CA}$ is true,

- if the time to co-altitude, τ_{Alt} , is below a threshold value T_v and the own aircraft’s altitude velocity is below 600ft/min, or

- if the time to co-altitude, $\tau_{A|t}$, is below a threshold value T_v and the own aircraft's altitude velocity has the same sign but is smaller in magnitude than the altitude velocity of the intruder, and
- for all other encounters if the time to co-altitude, $\tau_{A|t}$, is below threshold value Δ_{RA} .

Given this description $B_{AT,CA}$ is captured in:

$$B_{AT,CA} = TRUE, \text{ iff } \left\{ \begin{array}{l} \left[(0 < \tau_{A|t} < T_v) \wedge (|v_{t,A}^0| < 600 \text{ft/min}) \right] \vee \\ \left[(0 < \tau_{A|t} < T_v) \wedge \left(\frac{(\dot{r}_{t,A} + v_{t,A}^0)}{v_{t,A}^0} > 1 \right) \right] \vee \\ \left[(0 < \tau_{A|t} < \Delta_{RA}) \wedge (|v_{t,A}^0| \geq 600 \text{ft/min}) \wedge \left(\frac{(\dot{r}_{t,A} + v_{t,A}^0)}{v_{t,A}^0} \leq 1 \right) \right] \end{array} \right. \quad (46)$$

The predicted time to co-altitude, $\tau_{A|t}$, is given in Equation (47):

$$\tau_{A|t} = \begin{cases} -\frac{r_{t,A}}{\dot{r}_{t,A}}, & \text{if } (|\dot{r}_{t,A}| > 0) \\ -1, & \text{otherwise} \end{cases} \quad (47)$$

The threshold values $z_{thr,TA}$, Δ_{TA} , Δ_{RA} and T_v are a function of the current altitude as given in Table 6-2.

Table 6-2: Alarm thresholds for the altitude test for a Traffic Advisory (FAA, 2011, Table 2).

Own altitude [ft]	Δ_{TA} [s]	Δ_{RA} [s]	T_v [s]	$z_{thr,TA}$ [ft]
< 1000 (AGL)	20	N/A	N/A	580
(1,000 – 2,350] (AGL)	25	15	15	850
(2,350 – 5,000]	30	20	18	850
(5,000 – 10,000]	40	25	20	850
(10,000 – 20,000]	45	30	22	850
(20,000 – 42,000]	48	35	25	850
$\geq 42,000$	48	35	25	1,200

6.3 Traffic Advisory test

Now a TA is issued if both the range and altitude test have been passed:

$$B_{t,TA} = TRUE, \text{ iff } [B_{t,RT,TA} \wedge B_{t,AT,TA}] \quad (48)$$

7 Horizontal miss distance filter

The horizontal miss distance filter, as described in report (Hammer, 1996a) and in US patent 5,566,074 (Hammer, 1996b), is supposed to filter nuisance resolution advisories. The summarized concept is given in Figure 7-1. The filter is activated if it recognises that the estimated relative acceleration is below a defined threshold, the estimated horizontal miss distance is above a defined threshold, and if it detects that none of the involved aircraft is performing a manoeuvre. The calculations are performed in a parabolic range tracker (PRT) and a Cartesian tracker (CT), which just use the range measurements, and a bearing based tracker (BBT), which uses both the range and the bearing measurements.

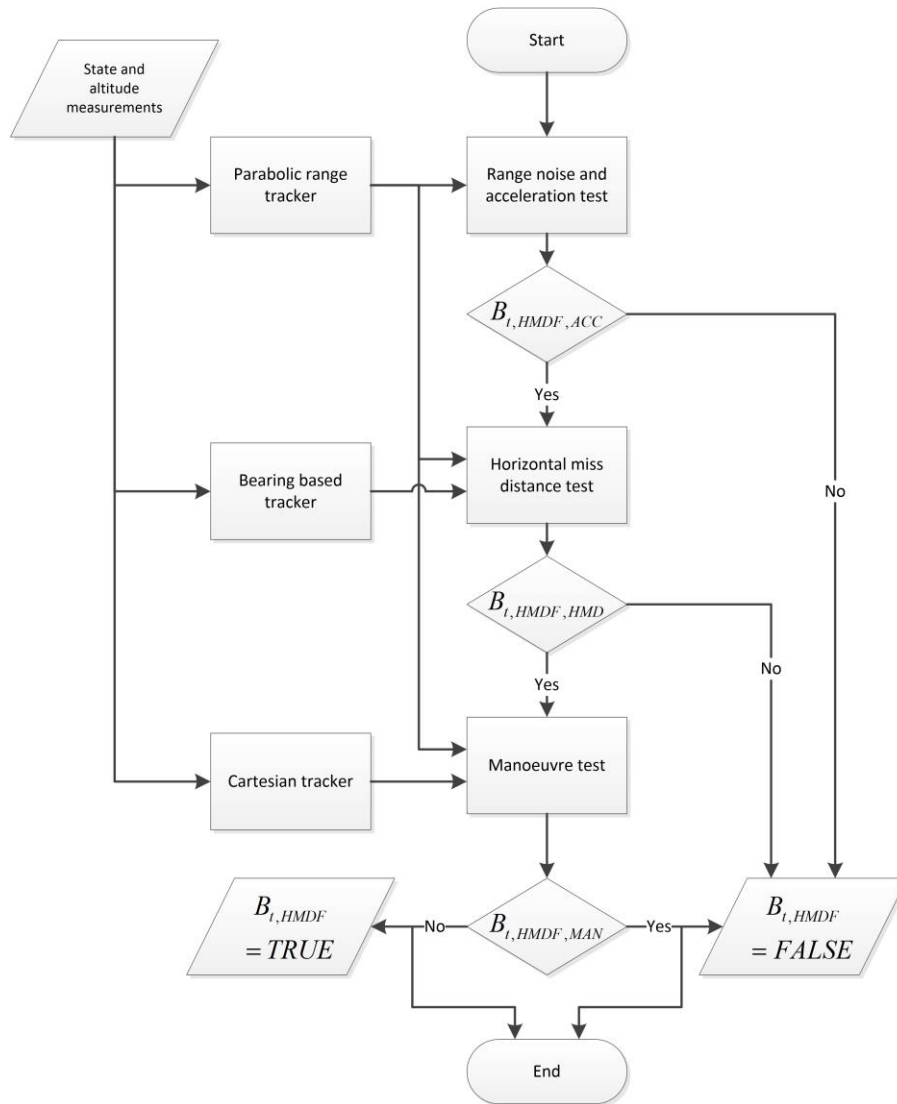


Figure 7-1: Concept flow diagram of horizontal miss distance filter.

7.1 Parabolic range tracker

The parabolic range tracker is an α - β - γ filter (Hammer, 1996a). It uses the same range measurement, $y_{t,r}$, as used in the α - β filter in equation (30) (ICAO, 2006, pp. 3-9 – 3-10).

$$r_{t,PRT} = r_{t|t-T,PRT} + \alpha_{PRT} \mu_{t,r,PRT} \quad (49)$$

$$\dot{r}_{t,PRT} = \dot{r}_{t|t-T,PRT} + \frac{\beta_{PRT}}{T} \mu_{t,r,PRT} \quad (50)$$

$$\ddot{r}_{t,PRT} = \ddot{r}_{t|t-T,PRT} + \frac{\gamma_{PRT}}{T^2} \mu_{t,r,PRT} \quad (51)$$

$r_{t,PRT}$, $\dot{r}_{t,PRT}$, $\ddot{r}_{t,PRT}$ are the estimated slant range, relative velocity, and relative acceleration at time 't' respectively.

$$r_{t|t-T,PRT} = r_{t-T,PRT} + \dot{r}_{t-T,PRT} T + \ddot{r}_{t-T,PRT} \frac{T^2}{2} \quad (52)$$

$$\dot{r}_{t|t-T,PRT} = \dot{r}_{t-T,PRT} + \ddot{r}_{t-T,PRT} T \quad (53)$$

$r_{t|t-T,PRT}$, $\dot{r}_{t|t-T,PRT}$ are the predictions at time 't' of the just mentioned slant range, velocity and acceleration estimates at time 't' subject to estimates at time 't-T'.

$$\mu_{t,r,PRT} = \begin{cases} y_{t,r} - r_{t|t-T,PRT} & , \text{ if } \exists \text{ measurement } y_{t,r} \\ 0 & , \text{ if no measurement} \end{cases} \quad (54)$$

$\mu_{t,r,PRT}$ is the predicted range error, and T , as previously defined, is the elapsed time since the previous interrogation.

$$r_{t-T,PRT} = 0, \text{ initial condition} \quad (55)$$

$$\dot{r}_{t-T,PRT} = 0, \text{ initial condition} \quad (56)$$

$$\ddot{r}_{t-T,PRT} = 0, \text{ initial condition} \quad (57)$$

The initial range, velocity and acceleration estimates are set equal to zero, i.e. when for the first time a measurement is established.

$$P_{t,firm} \in \{0,1,2,...,8\} \\ \triangleq \begin{cases} 0, \text{ initial value} \\ \min(P_{t-T,firm} + 1, 8), \text{ if } \exists \text{ measurement } y_{t,r} \\ \max(P_{t-T,firm} - 1, 0), \text{ if no measurement} \end{cases} \quad (58)$$

The smoothing parameters, α_{PRT} , β_{PRT} , γ_{PRT} , and the estimated slant range, $r_{t,PRT}$, depend on the firmness parameter, $P_{t,firm}$, which is a function of successful measurements. The evolution of the firmness parameter is defined in Equation (58) and visualised in [Figure 7-2](#). The smoothing

parameters are given in Table 7-1. If the firmness parameter is equal to 0 or 1, then the current slant range estimate is set equal to the current slant range measurement (Hammer, 1996b). Furthermore, the firmness parameter is used for the components of the horizontal miss distance filter to identify whether a stable track has been established (Hammer, 1996a, p. 8). This is captured in Boolean $B_{t,firm}$. If the firmness parameter reaches value 8, then Boolean $B_{t,firm}$ is set to TRUE, meaning a reliable track has been established. If the firmness parameter drops below value 3, then $B_{t,firm}$ is set to FALSE, which means no reliable track has been established.

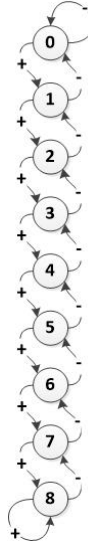


Figure 7-2: Evolution of firmness parameter $P_{t,firm}$.

Table 7-1: Smoothing parameters of Parabolic Range Tracker (Hammer, 1996b).

$P_{t,firm}$	0	1	2	3	4	5	6	7	8
α_{PRT}	1.0	1.0	0.83	0.70	0.60	0.46	0.40	0.40	0.40
β_{PRT}	0.0	1.0	0.50	0.30	0.20	0.11	0.10	0.10	0.10
γ_{PRT}	0.0	0.0	0.16	0.07	0.035	0.013	0.01	0.01	0.01

7.2 Range noise estimator and acceleration test

The range noise estimator, as defined in (Hammer, 1996b), uses the prediction error to estimate the standard error of prediction, $\sigma_{t,m}$. The estimate standard error will be used in an acceleration test to define the measurement validity region.

$$\sigma_{t,m} \simeq \left\{ E \left[\mu_{t,r,PRT}^2 \right] \right\}^{1/2} \quad (59)$$

$$\sigma_{t,m}^2 = \alpha_{RNE} \sigma_{t-T,m}^2 + (1 - \alpha_{RNE}) \mu_{t,r,PRT}^2 \quad (60)$$

$$\alpha_{RNE} = 0.1 \quad (61)$$

The acceleration test, as defined in (Hammer, 1996b), checks whether the estimated relative acceleration is higher than a threshold T_{A1} . The minimal threshold value is 1.5ft/s^2 . It is increased if the estimated standard deviation of prediction error is higher than 35ft/s^2 . This test is intended to ensure sufficient confidence of horizontal miss distance estimates.

$$T_{A1} = \max\left(1, \frac{\sigma_{t,m}}{35\text{ft}}\right) 1.5\text{ft/s}^2 \quad (62)$$

$$B_{t,HMDF,ACC} = \text{TRUE}, \text{ iff } \ddot{r}_{t,r,PRT} > T_{A1} \quad (63)$$

7.3 Bearing based tracker

The bearing based tracker is an α - β filter and uses the range measurement and the bearing measurement for the estimation of relative position and relative velocity. The relative position and velocity estimates are calculated as follows:

$$\begin{pmatrix} q_{t,X,BBT} \\ q_{t,Y,BBT} \end{pmatrix} = \begin{pmatrix} q_{t|t-T,X,BBT} \\ q_{t|t-T,Y,BBT} \end{pmatrix} + \begin{bmatrix} \sin(y_{t,\theta}) & \cos(y_{t,\theta}) \\ \cos(y_{t,\theta}) & -\sin(y_{t,\theta}) \end{bmatrix} \begin{bmatrix} \alpha_{\rho,BBT} & 0 \\ 0 & \alpha_{\rho x,BBT} \end{bmatrix} \begin{pmatrix} \mu_{t,\rho,BBT} \\ \mu_{t,\rho x,BBT} \end{pmatrix} \quad (64)$$

$$\begin{pmatrix} \dot{q}_{t,X,BBT} \\ \dot{q}_{t,Y,BBT} \end{pmatrix} = \begin{pmatrix} \dot{q}_{t|t-T,X,BBT} \\ \dot{q}_{t|t-T,Y,BBT} \end{pmatrix} + \begin{bmatrix} \sin(y_{t,\theta}) & \cos(y_{t,\theta}) \\ \cos(y_{t,\theta}) & -\sin(y_{t,\theta}) \end{bmatrix} \begin{bmatrix} \frac{\beta_{\rho,BBT}}{T} & 0 \\ 0 & \frac{\beta_{\rho x,BBT}}{T} \end{bmatrix} \begin{pmatrix} \mu_{t,\rho,BBT} \\ \mu_{t,\rho x,BBT} \end{pmatrix} \quad (65)$$

$$\begin{pmatrix} q_{t|t-T,X,BBT} \\ q_{t|t-T,Y,BBT} \end{pmatrix} = \begin{pmatrix} q_{t-T,X,BBT} \\ q_{t-T,Y,BBT} \end{pmatrix} + \begin{pmatrix} \dot{q}_{t-T,X,BBT} \\ \dot{q}_{t-T,Y,BBT} \end{pmatrix} T \quad (66)$$

$$\begin{pmatrix} \dot{q}_{t|t-T,X,BBT} \\ \dot{q}_{t|t-T,Y,BBT} \end{pmatrix} = \begin{pmatrix} \dot{q}_{t-T,X,BBT} \\ \dot{q}_{t-T,Y,BBT} \end{pmatrix} \quad (67)$$

$$\begin{pmatrix} \mu_{t,\rho,BBT} \\ \mu_{t,\rho x,BBT} \end{pmatrix} = \begin{cases} \begin{pmatrix} y_{t,r} - (\sin(y_{t,\theta})q_{t|t-T,X,BBT} + \cos(y_{t,\theta})q_{t|t-T,Y,BBT}) \\ -\cos(y_{t,\theta})q_{t|t-T,X,BBT} + \sin(y_{t,\theta})q_{t|t-T,Y,BBT} \end{pmatrix}, & \text{if } \exists \text{ measurement } y_{t,r} \wedge y_{t,\theta} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, & \text{else} \end{cases} \quad (68)$$

$q_{t,X,BBT}$, $q_{t,Y,BBT}$ are the relative position estimates in Cartesian coordinates, and, $\dot{q}_{t,X,BBT}$, $\dot{q}_{t,Y,BBT}$ are the relative velocity estimates in Cartesian coordinates, all relative to own aircraft's position. $q_{t|t-T,X,BBT}$, $q_{t|t-T,Y,BBT}$, $\dot{q}_{t|t-T,X,BBT}$ and $\dot{q}_{t|t-T,Y,BBT}$ are the predictions of just mentioned relative position and velocity estimates at time 't' subject to estimates at time 't-T'. $\alpha_{\rho,BBT}$, $\alpha_{\rho x,BBT}$, $\beta_{\rho,BBT}$

and $\beta_{\rho x, BBT}$ are the smoothing parameters. $\mu_{t, \rho, BBT}$ and $\mu_{t, \rho x, BBT}$ are the slant and cross range measurements residuals respectively, as given in Equation (68) .

The values of the smoothing parameters $\alpha_{\rho, BBT}$ and $\beta_{\rho, BBT}$ are given in Table 7-2 and correspond to the firmness parameter of the bearing based tracker, $P_{t, firm, BBT}$, which increases by one when both bearing and range measurements are successful and decreases by one when not both measurements were successful. The minimum and maximum values of the firmness parameter are zero and eight. Once the firmness parameter of bearing based tracker reaches the value 8, the horizontal miss distance estimates of the bearing based tracker may be used by the horizontal miss distance filter.

Table 7-2: Smoothing parameters of Bearing Based Tracker (Hammer, 1996a).

$P_{t, firm, BBT}$	0	1	2	3	4	5	6	7	8
$\alpha_{\rho, BBT}$	1.0	1.0	0.83	0.70	0.60	0.46	0.39	0.31	0.278
$\beta_{\rho, BBT}$	0.0	1.0	0.50	0.30	0.20	0.11	0.07	0.05	0.0453

The values for $\alpha_{\rho x, BBT}$ and $\beta_{\rho x, BBT}$ are calculated as follows.

The initial range and velocity estimates are set equal to zero, i.e. when for the first time a measurement is established.

$$\begin{pmatrix} q_{t-T, X, BBT} \\ q_{t-T, Y, BBT} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (69)$$

$$\begin{pmatrix} \dot{q}_{t-T, X, BBT} \\ \dot{q}_{t-T, Y, BBT} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (70)$$

Initially $\alpha_{\rho x, BBT} = \beta_{\rho x, BBT} = 0$. At the second successive measurement $\alpha_{\rho x, BBT} = \beta_{\rho x, BBT} = 1$ and the initial covariance is estimated as given in Equation (71), where σ_θ equals 0.0087 radians.

$$C_{t, BBT} = \begin{bmatrix} y_{t,r}^2 \sigma_\theta^2 & \frac{y_{t,r}^2 \sigma_\theta^2}{T} \\ \frac{y_{t,r}^2 \sigma_\theta^2}{T} & \frac{2y_{t,r}^2 \sigma_\theta^2}{T^2} \end{bmatrix} \quad (71)$$

After the second successive measurement on at each measurement moment the predicted covariance is estimated as follows:

$$C_{t|t-T, BBT}(1,1) = C_{t-T, BBT}(1,1) + 2TC_{t-T, BBT}(1,2) + T^2C_{t-T, BBT}(2,2) + \frac{T^4Q}{4} \quad (72)$$

$$C_{t|t-T, BBT}(1,2) = C_{t-T, BBT}(1,2) + TC_{t-T, BBT}(2,2) + \frac{T^3Q}{2} \quad (73)$$

$$C_{t|t-T, BBT}(2,1) = C_{t-T, BBT}(2,1) + TC_{t-T, BBT}(2,2) + \frac{T^3Q}{2} \quad (74)$$

$$C_{t|t-T, BBT}(2,2) = C_{t-T, BBT}(2,2) + T^2Q \quad (75)$$

Then the smoothing parameters are computed as given in Equations (77)-(78).

$$C_{t|t-T, BBT}^\mu = C_{t|t-T, BBT}(1,1) + y_{t,r}^2 \sigma_\theta^2 \quad (76)$$

$$\alpha_{\rho x, BBT} = \begin{cases} \frac{C_{t|t-T, BBT}(1,1)}{C_{t|t-T, BBT}^\mu}, & \text{if } \exists \text{ measurement } y_{t,r} \wedge y_{t,\theta} \\ 0 & , \text{ else} \end{cases} \quad (77)$$

$$\beta_{\rho x, BBT} = \begin{cases} \frac{C_{t|t-T, BBT}(2,1)}{C_{t|t-T, BBT}^\mu}, & \text{if } \exists \text{ measurement } y_{t,r} \wedge y_{t,\theta} \\ 0 & , \text{ else} \end{cases} \quad (78)$$

From the third successive measurement on, at each measurement moment the covariance is updated as follows:

$$C_{t, BBT}(1,1) = (1 - \alpha_{\rho x, BBT})C_{t|t-T, BBT}(1,1) \quad (79)$$

$$C_{t, BBT}(1,2) = (1 - \alpha_{\rho x, BBT})C_{t|t-T, BBT}(1,2) \quad (80)$$

$$C_{t, BBT}(2,1) = (1 - \alpha_{\rho x, BBT})C_{t|t-T, BBT}(2,1) \quad (81)$$

$$C_{t, BBT}(2,2) = C_{t|t-T, BBT}(2,2) - \beta_{\rho x, BBT}C_{t|t-T, BBT}(1,2) \quad (82)$$

Additionally, there are four inconsistency tests. The first test checks whether the cross range residual is greater than three times the square root of expected residual covariance, $C_{t|t-T, BBT}^\mu$.

The second test checks whether the sign of the cross range residual has not changed in the past 10 cycles and the cross range residual is greater than 0.7 times the square root of expected residual covariance, $C_{t|t-T, BBT}^\mu$. If either of the first or the second test is passed, then the bearing based tracker is reset.

The third inconsistency test checks whether the range residual is above 150ft.

The fourth inconsistency test checks whether a manoeuvre has been detected. If either of the third or fourth test is passed, then the smoothing parameter values of the bearing based tracker are increased. In (Hammer, 1996a) it is not stated by how much the smoothing parameters should be increased¹.

7.4 Horizontal miss distance test

Following (Hammer, 1996a), if the acceleration test has been passed ($B_{t,HMDF,ACC}=TRUE$), then the horizontal miss distance estimates from the parabolic range and bearing based tracker are compared with a reference miss distance, H_m . If both miss distances are greater than the reference miss distance, the horizontal miss distance test is passed:

$$B_{t,HMDF,HMD} = TRUE, \text{ iff } \left\{ \left[\left(d_{t,HMD,PRT} \wedge d_{t,HMD,BBT} \right) > H_m \right] \right\} \quad (83)$$

$$d_{t,HMD,PRT} = \sqrt{\frac{r_{t,PRT}^2 - \left(r_{t,PRT} \dot{r}_{t,PRT} \right)^2}{r_{t,PRT} \ddot{r}_{t,PRT} + \dot{r}_{t,PRT}^2}} \quad (84)$$

$$d_{t,HMD,BBT} = \frac{\left| r_{t,X,BBT} \dot{r}_{t,X,BBT} - r_{t,Y,BBT} \dot{r}_{t,Y,BBT} \right|}{\sqrt{\dot{r}_{t,X,BBT}^2 + \dot{r}_{t,Y,BBT}^2}} \quad (85)$$

$d_{t,HMD,PRT}$ and $d_{t,HMD,BBT}$ are the predicted horizontal miss distances estimated with the parabolic range tracker and bearing based tracker respectively. Equations (84)-(85) are not calculated if the acceleration test has been failed ($B_{t,HMDF,ACC}=False$), i.e. when aircraft are really close. The horizontal miss distance threshold is a function of the own altitude and defined in Table 7-3.

Table 7-3: Horizontal miss distance threshold (ICAO, 2006, p. 3-59).

Own altitude [ft] ²	H_m [ft]
< 1,000 (AGL)	N/A
(1,000 – 2,350] (AGL)	1,251
(2,350 – 5,000]	2,126
(5,000 – 10,000]	3,342
(10,000 – 20,000]	4,861
(20,000 – 42,000]	6,683
≥ 42,000	6,683

¹Due to this lack of information, at this moment only the second inconsistency test will be implemented.

² As discussed in Section 6 the maximum altitude of aircraft 'i' and 'k' is used to determine the threshold values.

7.5 Cartesian tracker

The Cartesian tracker is an α - β - γ filter specified by (Hammer, 1996b). Opposed to the parabolic range tracker, it is based on linear trajectories through Cartesian extrapolation. This works well under the restrictive assumption that no course changes are carried out by any of the aircraft considered. If this restrictive assumption is violated, then a high range prediction error may result. Hence, a dedicated manoeuvre test is specified later on. The range estimation and smoothing is identical to the parabolic range tracker and is computed throughout Equations (86) to (106) and based on (Hammer, 1996b). The range estimate, $r_{t,CT}$, the range rate estimate, $\dot{r}_{t,CT}$, and the relative acceleration estimate, $\ddot{r}_{t,CT}$, are:

$$r_{t,CT} = r_{t|t-T,CT} + \alpha_{CT} \mu_{t,r,CT} \quad (86)$$

$$\dot{r}_{t,CT} = \dot{r}_{t|t-T,CT} + \frac{\beta_{CT}}{T} \mu_{t,r,CT} \quad (87)$$

$$\ddot{r}_{t,CT} = \ddot{r}_{t|t-T,CT} + \frac{\gamma_{CT}}{T^2} \mu_{t,r,CT} \quad (88)$$

$r_{t|t-T,CT}$, $\dot{r}_{t|t-T,CT}$ and $\ddot{r}_{t|t-T,CT}$ are the predictions of slant range, velocity and acceleration made at the previous measurement. $\mu_{t,r,CT}$ is the measurement residual:

$$\mu_{t,r,CT} = \begin{cases} y_{t,r} - r_{t|t-T,CT} & , \text{ if } \exists \text{ measurement } y_{t,r} \\ 0 & , \text{ if no measurement} \end{cases} \quad (89)$$

The smoothing parameters α_{CT} , β_{CT} , γ_{CT} are not defined in (Hammer, 1996b). Hence smoothing is assumed to be identical to the parabolic range tracker with equal smoothing parameters:

$$\alpha_{CT} = \alpha_{PRT} \quad (90)$$

$$\beta_{CT} = \beta_{PRT} \quad (91)$$

$$\gamma_{CT} = \gamma_{PRT} \quad (92)$$

The Cartesian extrapolation is defined in Equations (99)-(100) and Equations (104)-(106) based on the following five implicitly adopted assumptions (Hammer, 1996b):

$$\dot{r}_{t,YS,CT}^2 = \dot{r}_{t,CT}^2 + r_{t,CT} \ddot{r}_{t,CT} \quad (93)$$

$$r_{t,YS,CT} \dot{r}_{t,YS,CT} = r_{t,CT} \dot{r}_{t,CT} \quad (94)$$

$$r_{t,XS,CT}^2 + r_{t,YS,CT}^2 = r_{t,CT}^2 \quad (95)$$

$$\dot{r}_{t,XS,CT} = 0 \quad (96)$$

$$\ddot{r}_{t,CT} r_{t,CT}^3 = r_{t,XS,CT}^2 \dot{r}_{t,YS,CT}^2 \quad (97)$$

Then the Cartesian extrapolation is defined in Equations (99)-(100).

$$\dot{r}_{t,YS,CT} = \begin{cases} \sqrt{r_{t,CT} \ddot{r}_{t,CT} + \dot{r}_{t,CT}^2}, & \text{if } r_{t,CT} \ddot{r}_{t,CT} + \dot{r}_{t,CT}^2 \geq 1\text{ft/s} \\ 1\text{ft/s} & , \text{otherwise} \end{cases} \quad (98)$$

$$r_{t,YS,CT} = \frac{r_{t,CT} \dot{r}_{t,CT}}{\dot{r}_{t,YS,CT}} \quad (99)$$

$$r_{t,XS,CT} = \begin{cases} \sqrt{r_{t,CT}^2 - r_{t,YS,CT}^2}, & \text{if } r_{t,CT}^2 - r_{t,YS,CT}^2 \geq 1\text{ft}^2 \\ 1\text{ft} & , \text{otherwise} \end{cases} \quad (100)$$

This is followed by the estimate prediction defined in Equations (101)-(106).

$$r_{t|t-T,XS,CT} = r_{t-T,XS,CT} \quad (101)$$

$$r_{t|t-T,YS,CT} = r_{t-T,YS,CT} + \dot{r}_{t-T,YS,CT} T \quad (102)$$

$$\dot{r}_{t|t-T,YS,CT} = \dot{r}_{t-T,YS,CT} \quad (103)$$

$r_{t|t-T,XS,CT}$ and $r_{t|t-T,YS,CT}$ are the predictions at time 't' subject to estimates at time 't-T'. $\dot{r}_{t|t-T,YS,CT}$ is the prediction at time 't' subject to the corresponding relative velocity estimate at time 't-T'. With these two dimensional predictions the final slant range, velocity and acceleration predictions are calculated as followed:

$$r_{t|t-T,CT} = \sqrt{r_{t|t-T,XS,CT}^2 + r_{t|t-T,YS,CT}^2} \quad (104)$$

$$\dot{r}_{t|t-T,CT} = \frac{r_{t|t-T,YS,CT} \dot{r}_{t|t-T,YS,CT}}{\max(r_{t|t-T,CT}, 1\text{ft})} \quad (105)$$

$$\ddot{r}_{t|t-T,CT} = \frac{r_{t|t-T,XS,CT}^2 \dot{r}_{t|t-T,YS,CT}^2}{\max(r_{t|t-T,CT}^3, 1\text{ft}^3)} \quad (106)$$

The initial range, velocity and acceleration estimates are set equal to zero, i.e. when for the first time a measurement is established.

$$r_{t-T,CT} = 0, \text{ initial condition} \quad (107)$$

$$\dot{r}_{t-T,CT} = 0, \text{ initial condition} \quad (108)$$

$$\ddot{r}_{t-T,CT} = 0, \text{ initial condition} \quad (109)$$

7.6 Manoeuvre test

The manoeuvre test consists of four sub tests as defined in Equation (110) (Hammer, 1996b). It is only carried out if both the acceleration test and the horizontal miss distance test have been passed ($B_{t,HMDF,ACC}=TRUE$ and $B_{t,HMDF,HMD}=TRUE$).

$$B_{t,HMDF,MAN} = TRUE, \text{ iff } \left\{ \begin{array}{l} \left[\ddot{r}_{t,PRT} < 0 \right] \vee \\ \left[\mu_{t,r,CT} < T_{RV} \right] \vee \\ \left[d_{t,HMD,BBT} < 0.5d_{t,HMD,PRT} \right] \vee \\ \left[\left(\ddot{r}_{t,CT} \vee \ddot{r}_{t,PRT} \right) < -T_{A1} \right] \end{array} \right. \quad (110)$$

First the acceleration determined by the parabolic range tracker, Equation (51), is used to estimate the relative jerk, $\ddot{r}_{t,PRT}$, of the relative position vector, $\|\mathbf{s}_t^i\|$, through an alpha filter as defined in Equation (111) (Hammer, 1996b). If the jerk is negative, meaning that the acceleration of the slant range is decreasing, then a manoeuvre is declared and no RA is suppressed. Manoeuvres are also declared, if the range measurement residual of the Cartesian tracker is sufficiently negative, or if the estimated horizontal miss distance of the bearing based tracker is smaller than half the estimated horizontal miss distance of the parabolic range tracker, or if the estimated range acceleration of the Cartesian or parabolic range tracker are smaller than the acceleration threshold defined in Equation (62). According to (Hammer, 1996a, p.30), if the manoeuvre test has been passed, then the following 10 seconds no RA will be filtered, as the state estimation are expected to be erroneous.

$$\ddot{r}_{t,PRT} = \begin{cases} (1 - \alpha_{MAN}) \ddot{r}_{t-T,PRT} + \alpha_{MAN} (\ddot{r}_{t|t-T,PRT} - \ddot{r}_{t,PRT}), & \text{if } B_{t,HMDF,ACC} = TRUE \\ 0, & \text{otherwise} \end{cases} \quad (111)$$

According to (Hammer, 1996b) the smoothing parameter of the acceleration estimate is:

$$\alpha_{MAN} = 0.1 \quad (112)$$

The parameter T_{RV} is not specified in (Hammer, 1996b). But in (Hammer, 1996a, p. 35) it is stated that the manoeuvre is declared if the range residual of the Cartesian tracker is larger than 3 times the range measurement error as calculated in the range noise estimator and acceleration test, $\sigma_{t,m}$ in Equations (59)-(61). However, in (Hammer, 1996a, p. 45) the manoeuvre detection threshold value is indicated to be approximately constant at about 105ft. From this it is concluded that the manoeuvre detection threshold for the range residual of the Cartesian tracker is calculated similar to the acceleration threshold value as calculated in Equation (62), and uses the maximum between $\sigma_{t,m}$ and 35ft. Hence, the threshold parameter T_{RV} is set to:

$$T_{RV} = -3\max(\sigma_{t,m}, 35\text{ft}) \quad (113)$$

7.7 Horizontal miss distance filter activation

The state of the horizontal miss distance filter is activated if both the acceleration and the horizontal miss distance test, $B_{t,HMDF,ACC}$ and $B_{t,HMDF,HMD}$, have been passed, the manoeuvre test, $B_{t,HMDF,MAN}$, is negative and a reliable track of the aircraft, $B_{t,firm}$, has been established:

$$B_{t,HMDF} = TRUE, \text{iff } \{ [B_{t,HMDF,ACC}] \wedge [B_{t,HMDF,HMD}] \wedge [\neg B_{t,HMDF,MAN}] \wedge [B_{t,firm}] \} \quad (114)$$

8 Threat detection

In the threat detection process TCAS determines whether another aircraft is a threat towards its own aircraft. To do so, a range and an altitude test are carried out. Furthermore, information from a received coordination message, if received, and the status of the horizontal miss distance filter are considered as well.

8.1 Range test

The range test (RT) in the threat detection test is similar to the range test in the traffic advisory module described in section 6.

The range test, as described in the ACAS specification (ICAO, 2006, pp.3-58 - 3-59), determines whether the intruder is a Range threat. As defined in Equation (115), the range test, defined as Boolean $B_{t,RT}$, is positive if the modified time until closest point of approach (CPA), $\tau_{\text{mod}|t}$, is within a certain threshold, Δ_{RA} , and the aircraft are converging, or the aircraft are within a specified distance, d_{mod} , diverging in range, and the divergence is “slow”. The modified time until CPA, $\tau_{\text{mod}|t}$, accounts for horizontal position inaccuracies and is defined in Equation (116) (ICAO, 2006, p. 3-59). Aircraft are considered to diverge slowly when the multiplication of estimated range and estimated velocity is less than the value of a parameter $\dot{P}_m$.

$$B_{t,RT} = \text{TRUE} \quad , \text{iff} \quad \left\{ \begin{array}{l} \left[\left(0 < \tau_{\text{mod}|t} \leq \Delta_{RA} \right) \wedge \left(\dot{r}_t \leq 0 \right) \right] \vee \\ \left[\left(r_t < d_{\text{mod}} \right) \wedge \left(\dot{r}_t > 0 \right) \wedge \left(r_t \dot{r}_t < \dot{P}_m \right) \right] \end{array} \right. \quad (115)$$

$$\tau_{\text{mod}|t} = \begin{cases} \frac{r_t^2 - d_{\text{mod}}^2}{r_t |\dot{r}_t'|} & , \text{ if } (r_t \geq d_{\text{mod}}) \\ 0 & , \text{ otherwise} \end{cases} \quad (116)$$

$$\dot{r}_t' = \min(\dot{r}_t, -3\text{m/s}) \quad (117)$$

The alarm threshold values of the range test for RAs are a function of the own altitude and defined in [Table 8-1](#).

Table 8-1: Alarm thresholds of the range test for RAs (ICAO, 2006, p. 3-58) (FAA, 2011, Table 2).

Own altitude [ft] ¹	$\dot{P}_m$ [NM ² /s]	Δ_{RA} [s]	d_{mod} [NM]	z_{thr} [ft]
< 1,000 (AGL)	N/A	N/A	N/A	N/A
(1,000 – 2,350] (AGL)	0.0020	15	0.20	600
(2,350 – 5,000]	0.0028	20	0.35	600
(5,000 – 10,000]	0.0028	25	0.55	600
(10,000 – 20,000]	0.0028	30	0.80	600
(20,000 – 42,000]	0.0040	35	1.10	700
≥ 42,000	0.0040	35	1.10	800

Furthermore, the specification also describe that the range test is positive if a miss distance could not be calculated on the current or the miss distance is violating the horizontal miss distance threshold, which also has been used in the horizontal miss distance filter. Since TCAS already uses the horizontal miss distance filter this aspect has been excluded from this model.

8.2 Altitude test

The altitude test (AT), as described in the ACAS specification, determines whether the intruder is a vertical threat (ICAO, 2006, pp.3-59 - 3-61):

- “a) the aircraft are converging in range, the current altitude separation is “small” and the vertical miss distance is “small”;
- b) the aircraft are converging in range and altitude, the time to co-altitude is “small” and either the vertical miss distance is “small”, or co-altitude is predicted to occur before CPA ($\tau_v < \tau_u$); or
- c) the aircraft are diverging in range and the current altitude separation is “small”;

In our setting this means that the altitude test is true:

- if the aircraft are converging in range, the current relative altitude, $r_{t,A}$, and an adjusted predicted relative altitude at CPA, $r_{t,CPA,A|t}^{adjust}$, are violating the vertical separation threshold, z_{thr} , or
- if the aircraft are converging in range and altitude, the time to co-altitude, $\tau_{A|t}$, is small, and either the adjusted predicted relative altitude is violating the vertical separation threshold, or the aircraft cross altitudes before CPA, or
- if the aircraft are diverging in range and the current altitudes separation is violating the vertical separation threshold.

The above is captured as Boolean $B_{t,AT}$ in Equation (118):

¹ As discussed in Section 6 the maximum altitude of aircraft ‘i’ and ‘k’ is used to determine the threshold values.

$$B_{t,AT} = TRUE, \text{ iff } \begin{cases} \left[\left(\dot{r}_t \leq 0 \right) \wedge \left(|r_{t,A}| < z_{thr} \right) \wedge \left(|r_{t,CPA,A|t}^{adjust} < z_{thr} \right) \right] \vee \\ \left[\left(\dot{r}_t \leq 0 \right) \wedge \left(\dot{r}_{t,A} < 0 \right) \wedge \left(r_{t,A} > 0 \right) \wedge B_{AT,CA} \wedge \left(|r_{t,CPA,A|t}^{adjust} < z_{thr} \right) \right] \vee \\ \left[\left(\dot{r}_t \leq 0 \right) \wedge \left(\dot{r}_{t,A} > 0 \right) \wedge \left(r_{t,A} \leq 0 \right) \wedge B_{AT,CA} \wedge \left(|r_{t,CPA,A|t}^{adjust} < z_{thr} \right) \right] \vee \\ \left[\left(\dot{r}_t \leq 0 \right) \wedge \left(\dot{r}_{t,A} < 0 \right) \wedge \left(r_{t,A} > 0 \right) \wedge B_{AT,CA} \wedge \left(0 < \tau_{A|t} < \tau_{A|t}^{adjust} \right) \right] \vee \\ \left[\left(\dot{r}_t \leq 0 \right) \wedge \left(\dot{r}_{t,A} > 0 \right) \wedge \left(r_{t,A} \leq 0 \right) \wedge B_{AT,CA} \wedge \left(0 < \tau_{A|t} < \tau_{A|t}^{adjust} \right) \right] \vee \\ \left[\left(\dot{r}_t > 0 \right) \wedge \left(|r_{t,A}| < z_{thr} \right) \right] \end{cases} \quad (118)$$

where $\tau_{A|t}$ satisfies Equation (47) and $B_{AT,CA}$ satisfies Equation (46). $r_{t,CPA,A|t}^{adjust}$ is the adjusted predicted relative altitude dependent on the modified time to CPA, $\tau_{mod|t}$, and an adjusted time to co-altitude, $\tau_{A|t}^{adjust}$, as defined in (ICAO, 2006, p.3-60) and captured in Equations (119) and (120).

$$r_{t,CPA,A|t}^{adjust} = \begin{cases} \min \left(\left(r_{t,A} + \dot{r}_{t,A} \tau_{A|t}^{adjust} \right), \left(r_{t,A} + \dot{r}_{t,A} \tau_{mod|t} \right) \right) & , \text{ if } r_{t,A} + \dot{r}_{t,A} \tau_{A|t}^{adjust} > 0 \\ \max \left(\left(r_{t,A} + \dot{r}_{t,A} \tau_{A|t}^{adjust} \right), \left(r_{t,A} + \dot{r}_{t,A} \tau_{mod|t} \right), 0 \right) & , \text{ otherwise} \end{cases} \quad (119)$$

$$\tau_{A|t}^{adjust} = \min \left(\left| \frac{r_t}{\dot{r}_t} \right|, \Delta_{RA} \right) \quad (120)$$

The alarm thresholds for the altitude test are a function of the own altitude and defined in Table 8-2.

Table 8-2: Alarm thresholds for the altitude test (FAA, 2011, Table 2).

Own altitude [ft] ¹	Δ_{RA} [s]	z_{thr} [ft]
< 1000 (AGL)	N/A	N/A
(1,000 – 2,350] (AGL)	15	600
(2,350 – 5,000]	20	600
(5,000 – 10,000]	25	600
(10,000 – 20,000]	30	600
(20,000 – 42,000]	35	700
≥42,000	35	800

8.3 Threat detection test

As described in the TCAS specification (EUROCAE, 2008, pp. 127-128), TCAS issues an RA if the intruder is declared a range threat and a vertical threat and the state of the horizontal miss distance

¹ As discussed in Section 6 the maximum altitude of aircraft ‘i’ and ‘k’ is used to determine the threshold values.

filter is deactivated, i.e. $B_{t,HMDF} = FALSE$. Furthermore, TCAS also issues an RA if the intruder has sent a crossing intent via a coordination message and the range test has been passed, or if the intruder has sent any coordination message and both the range and altitude test have been passed. Whether TCAS issues an RA is defined as the Boolean $B_{t,RA}$ which satisfies Equation (121).

$$B_{t,RA} = TRUE, \text{ iff } \begin{cases} [B_{t,RT} \wedge B_{t,AT} \wedge \neg B_{t,HMDF}] \vee \\ [B_{t,RT} \wedge (m_{t,received} \neq 0) \wedge B_{t,CROSS}] \vee \\ [B_{t,RT} \wedge B_{t,AT} \wedge (m_{t,received} \neq 0)] \end{cases} \quad (121)$$

Whether the own aircraft has received from the intruder a coordination message and that coordination message indicates a crossing intent, e.g. the intruder is higher/lower in altitude than own aircraft but the received coordination message indicates that the intruder wants to pass below/above own aircraft, is determined by Boolean $B_{t,CROSS}$:

$$B_{t,CROSS} = TRUE, \text{ iff } \begin{cases} [(m_{t,received} = 1) \wedge (r_{t,A} < 0)] \vee \\ [(m_{t,received} = -1) \wedge (r_{t,A} > 0)] \end{cases} \quad (122)$$

9 Sense selection

After another aircraft has been declared a threat, TCAS selects the sense of the RA. The sense selection process is explained in words in (FAA, 2011) and (EUROCAE, 2008):

“Based on the range and altitude tracks of the intruder, the CAS logic models the intruder’s flight path from its present position to CPA. The CAS logic then models upward and downward sense RAs for own aircraft, [...], to determine which sense provides the most vertical separation at CPA. [...] In encounters where either of the senses results in the TCAS aircraft crossing through the intruder’s altitude, TCAS is designed to select the non-altitude crossing sense if the non-crossing sense provides the desired vertical separation (ALIM) at CPA. If the non-altitude crossing sense provides at least ALIM feet of separation at CPA, this sense will be selected even if the altitude crossing sense provides greater separation. If ALIM cannot be obtained in the non-altitude crossing sense, an altitude crossing RA will be issued” (FAA, 2011, p. 29);

“The basic rule for sense selection in a TCAS/TCAS encounter is that each TCAS must check to see if it has received an intent message from the other aircraft before selecting an RA sense. If an intent message has been received, TCAS selects the opposite sense from that selected by the other aircraft and communicated via the coordination interrogation.

[...] If TCAS has not received an intent message, the sense is selected based on the encounter geometry in the same manner as would be done if the intruder were not TCAS equipped.

[...]Occasionally, the two aircraft declare each other as threats simultaneously, and therefore both aircraft will select their RA sense based on the encounter geometry. In these encounters, there is a chance that both aircraft will select the same sense. When this happens, the aircraft with the higher Mode S address will detect the selection of the same sense and will reverse its sense.” (FAA, 2011, p. 34).

The sense selection model has been developed from these descriptions plus the specification of possible RAs given in [Table 9-1](#). A graphical representation of the sense selection process is given in [Figure 9-1](#).

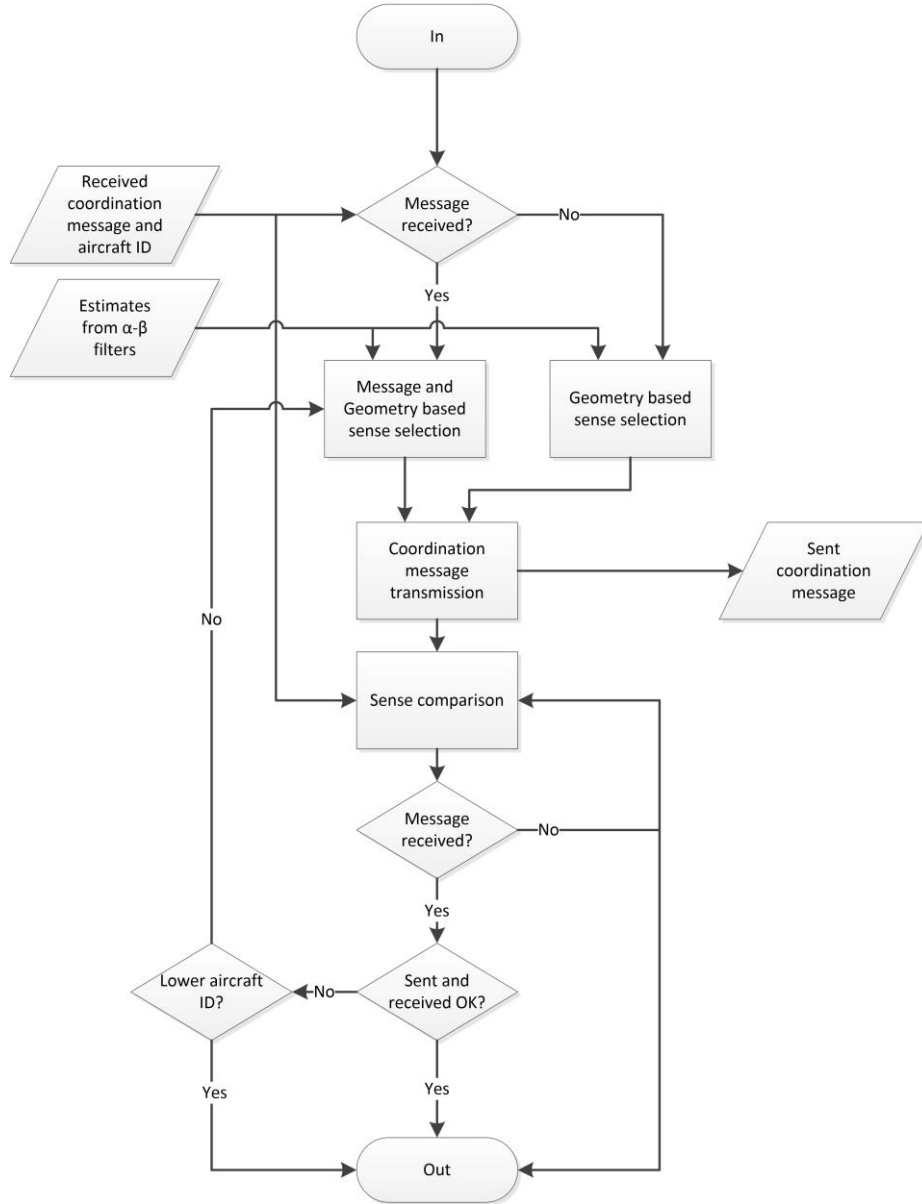


Figure 9-1: Functional flow diagram of sense selection module.

The sense is selected only once. The exception is, if both aircraft have selected the same sense, which is described later. The Boolean $B_{t,D}$ is defined to determine whether a sense has been selected. So if no sense has been selected, $\neg B_{t,D}$, and if an RA is present, $B_{t,RA}$, then the sense is selected. Subsequently a coordination message is sent and $B_{t,D}$ is set to true, as defined in Equation (123). Furthermore, a timer, $t_{t, timer}$, is set equal to the time until CPA, $\tau_{CPA|t}$, in order to determine later in section 10 whether the aircraft have passed the initially predicted CPA.

$$\begin{aligned}
& \text{if } [\neg B_{t,D} \wedge B_{t,RA}] \\
& \quad \text{then } \begin{cases} \text{compute } D_t \text{ using eq. (126-136)} \\ \text{compute } m_{t,sent} \text{ using eq. (137)} \\ B_{t,D} = TRUE \\ t_{t,timer} = \tau_{CPA|t} \end{cases} \quad (123)
\end{aligned}$$

$$\tau_{CPA|t} = \min\left(\Delta_{RA}, \max\left(\tau_{t,A|t}, \tau_{mod|t}, 0\right)\right) \quad (124)$$

For a time step $\Delta > 0$:

$$t_{t,timer} = t_{t-\Delta,timer} - \Delta \quad (125)$$

As described above, according to (FAA, 2011) the sense selection depends on the encounter geometry and chooses the sense which leads to the greatest miss distance as long as altitudes do not cross. The sense resulting in non-altitude crossings while sustaining a minimal required vertical miss distance is favoured over the greater miss distance. However, the sense selection algorithm takes the geometric position and, if available, the coordination message received from the intruder into account.

According to (FAA, 2011, p. 30) TCAS II Ver.7.1 May choose between a positive and a negative sense. The possible initial RAs based on sense selection have been summarised in [Table 9-1](#). A positive sense indicates that the own aircraft should either start or increase climbing or descending. Hence it is also labelled as a “Positive (Corrective)” sense with either upwards or downwards direction. A negative sense indicates that the own aircraft should limit its climb or descend. However there are two cases. One is the “Negative (Preventive)” sense, which requires no action from the pilot. The other is the “Negative (Corrective)” sense, which requires an action from the pilot, e.g. reduce climb or descent to 0ft/min.

Table 9-1: Possible initial RA of TCAS Ver.7.1 (FAA, 2011, Table 3) with the three clusters used in the model.

RA Type	Upwards Sense:		Downwards Sense:	
	RA	Required Vertical Rate [fpm]	RA	Required Vertical Rate [fpm]
Positive (Corrective)	Climb	1,500 – 2,000	Descend	-1,500 – -2,000
Positive (Corrective)	Crossing Climb	1,500 – 2,000	Crossing Descend	-1,500 – -2,000
Positive (Corrective)	Crossing Maintain Climb	1,500 – 4,400	Crossing Maintain Descend	-1,500 – -4,400
Positive (Corrective)	Maintain Climb	1,500 – 4,400	Maintain Descend	-1,500 – -4,400
Negative (Corrective)	Reduce Descent	0	Reduce Climb	0
Negative (Preventive)	Do Not Descend	>0	Do Not Climb	<0
Negative (Preventive)	Do Not Descend >500fpm	>-500	Do Not Climb >500fpm	<500
Negative (Preventive)	Do Not Descend >1,000fpm	>-1,000	Do Not Climb >1,000fpm	<1,000
Negative (Preventive)	Do Not Descend >2,000fpm	>-2,000	Do Not Climb >2,000fpm	<2,000

Based on the possible initial RAs the senses are clustered in three:

- cluster corrective upwards sense, as indicated by top left box in Table 9-1;
- cluster corrective downwards sense, as indicated by top right box in Table 9-1;
- cluster preventive sense, as indicated by bottom located box in Table 9-1.

Hence, the sense in this model is defined as D_t , as defined in Equation (126) where $D_t = 1$ represents a corrective upwards sense, $D_t = -1$ represents a corrective downwards sense, and $D_t = 0$ represents a preventive sense. So the clustering is based on whether the pilot needs to adjust the current vertical velocity, and if yes, whether the pilot needs to increase or decrease vertical velocity

$$D_t \in \{-1, 0, 1\} \quad (126)$$

To determine D_t eight decision variables have been identified. Seven of them are binary and the eighth has three parameters. Hence, there are 384 combinations possible ($2^7 \cdot 3 = 384$). Two third of these combinations, 256, are possible if the own aircraft has received a coordination message from the intruder, and one third, 128, if the intruder has not received any coordination message.

9.1 Message and geometry based sense selection

If a coordination message from the intruder has been received, then the to be selected sense, D_t , is based on the received coordination message and the encounter geometry.

Obviously, the received coordination message simplifies the sense selection process. For example, if the received coordination message advises to pass above the intruder aircraft, then a descent manoeuvre is not considered. Hence, it only needs to be checked whether the prediction of the current flight paths would result in a predicted relative altitude greater than the minimal required altitude miss distance, a_{lim} . a_{lim} is specified in (FAA, 2011, Table 2) as a function of the own altitude and is given in Table 9-2.

Table 9-2: Definition of minimal required altitude miss distance (FAA, 2011, Table 2).

Own altitude [ft] ¹	a_{lim} [ft]
< 1000 (AGL)	N/A
(1,000 – 2,350] (AGL)	300
(2,350 – 5,000]	300
(5,000 – 10,000]	350
(10,000 – 20,000]	400
(20,000 – 42,000]	600
≥42,000	700

So if a coordination message has been received, i.e. $m_{t,received} \in \{-1,1\}$, then the own sense, D_t , is selected as defined in Equation (127).

$$D_t = \begin{cases} 1, & \text{if } \left[\left[(m_{t,received} = -1) \wedge (r_{CPA,A|t} \geq 0) \right] \vee \right. \\ & \left. \left[(m_{t,received} = -1) \wedge (r_{CPA,A|t} < 0) \wedge (-r_{CPA,A|t}^- < a_{lim}) \right] \right] \\ -1, & \text{if } \left[\left[(m_{t,received} = 1) \wedge (r_{CPA,A|t} < 0) \right] \vee \right. \\ & \left. \left[(m_{t,received} = 1) \wedge (r_{CPA,A|t} \geq 0) \wedge (r_{CPA,A|t}^+ < a_{lim}) \right] \right] \\ 0, & \text{if } \left[\left[(m_{t,received} = -1) \wedge (r_{CPA,A|t} < 0) \wedge (-r_{CPA,A|t}^- \geq a_{lim}) \right] \vee \right. \\ & \left. \left[(m_{t,received} = 1) \wedge (r_{CPA,A|t} \geq 0) \wedge (r_{CPA,A|t}^+ \geq a_{lim}) \right] \right] \end{cases} \quad (127)$$

with $r_{CPA,A|t}$ the predicted relative altitude of the intruder above own aircraft at CPA:

$$r_{CPA,A|t} = r_{t,A} + \tau_{CPA|t} \dot{r}_{t,A} \quad (128)$$

¹ As discussed in Section 6 the maximum between the altitudes of aircraft 'i' and 'k' is used to determine the threshold values.

As defined in Equation (124), the time until CPA is determined as the minimum of Δ_{RA} and the maximum of the time to co-altitude from the altitude test, the modified time to CPA from the range test, and zero. The Δ_{RA} is necessary to exclude large times until CPA, e.g. when the distance to the intruder is below the threshold distance and closing rates are very small or negative.

Now the additional tests against a_{lim} in Equation (127) will be explained. These tests take into account that the relative altitude velocity may be adjusted, because TCAS only gives preventive RAs in quantization of 500ft/s (see Table 9-1). It might be possible that the current flight paths result in a sufficient vertical miss distance, but there is no valid negative RA available. For example imagine that all altitude velocities above 350ft/s of own aircraft would result in insufficient vertical miss distance. So if the own aircraft is flying at 250ft/s it could continue, but TCAS can only indicate to not climb with more than 500ft/s, which would be insufficient, or to not climb with more than 0ft/s, which would not be a preventive RA, as the pilot would need to adjust the vertical speed. Hence, if the intruder would pass above at current relative altitude velocity, then the own aircraft altitude velocity is set to zero if the own altitude velocity is below zero, or rounded up to the next 500ft/s increment and the adjusted predicted relative altitude, $r_{CPA,A|t}^+$, is defined in Equation (129).

$$r_{CPA,A|t}^+ = r_{CPA,A|t} + \tau_{CPA|t} \left(v_{t,A}^0 - \left\lceil \frac{\max(v_{t,A}^0, 0)}{500\text{ft/min}} \right\rceil 500\text{ft/min} \right) \quad (129)$$

If the intruder would pass below at current relative altitude velocity, then the own aircraft altitude velocity is set to zero if the own altitude velocity is above zero, or rounded down to the next 500ft/s increment and the adjusted predicted relative altitude, $r_{CPA,A|t}^-$, is defined in Equation (130).

$$r_{CPA,A|t}^- = r_{CPA,A|t} + \tau_{CPA|t} \left(v_{t,A}^0 - \left\lfloor \frac{\min(v_{t,A}^0, 0)}{500\text{ft/min}} \right\rfloor 500\text{ft/min} \right) \quad (130)$$

Because, the adjusted predicted relative altitude may have changed signs compared to the predicted relative altitude. The direction of the adjusted predicted relative altitude is considered in Equations (127) and (131) by comparing the negative of $r_{CPA,A|t}^-$ with the required altitude miss distance.

9.2 Geometry only based sense selection

If the own aircraft has not received a coordination message from the intruder, then the sense is selected as defined in Equation (131). A graphical representation of example encounter geometries for the seven conditions of Equation (131) in which a corrective upwards sense is selected is given in Figure 9-2.

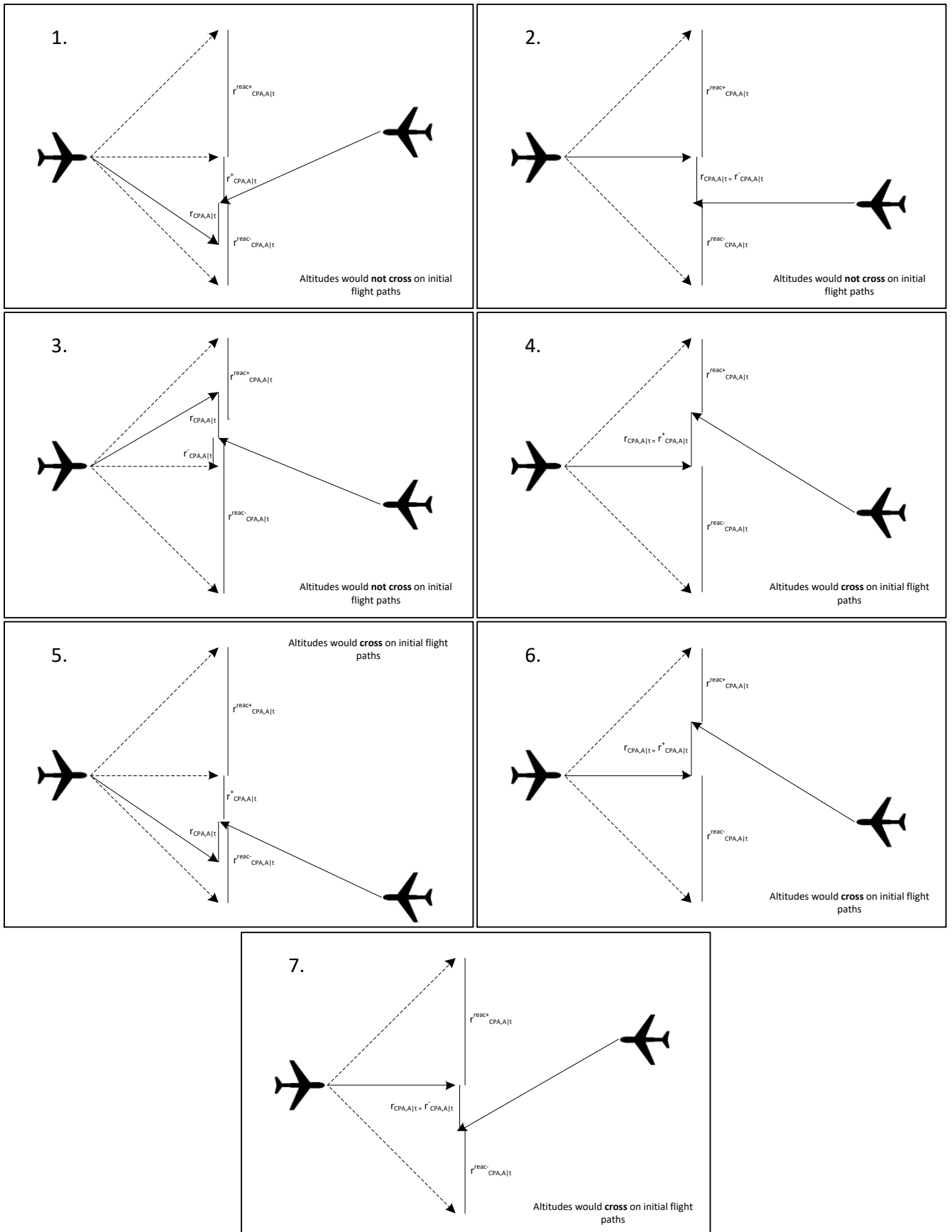


Figure 9-2: Example encounter geometries where TCAS selects corrective upwards sense based on Equation (131), i.e. when $m_{t, received} = 0$. Note: numbering of example encounter geometry is based on order of appearance in Equation (131).

$$D_t = \begin{cases} 1, \text{ if } \left\{ \begin{aligned} & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| > |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac-}| < a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| > |r_{CPA, A|t}^{reac-}|) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| \leq |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac+}| \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ \geq a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| > |r_{CPA, A|t}^{reac-}|) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| \leq |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac+}| \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| > |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac-}| < a_{lim}) \right] \end{aligned} \right\} \\ \\ -1, \text{ if } \left\{ \begin{aligned} & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| > |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac-}| \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| \leq |r_{CPA, A|t}^{reac-}|) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| \leq |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac+}| < a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| \leq |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac+}| < a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- \geq a_{lim}) \wedge (|r_{CPA, A|t}^{reac-}| \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| > |r_{CPA, A|t}^{reac-}|) \wedge (|r_{CPA, A|t}^{reac-}| \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- < a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| \leq |r_{CPA, A|t}^{reac-}|) \right] \end{aligned} \right\} \\ \\ 0, \text{ if } \left\{ \begin{aligned} & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* > 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- \geq a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ \geq a_{lim}) \wedge (|r_{CPA, A|t}^{reac+}| < a_{lim}) \right] \vee \\ & \left[(m_{t, received} = 0) \wedge (c_t^* \leq 0) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- \geq a_{lim}) \wedge (|r_{CPA, A|t}^{reac-}| < a_{lim}) \right] \end{aligned} \right\}
\end{cases}
\quad (131)$$

Here c_t^* indicates whether the aircraft are predicted to cross each other's altitudes during the time period $[t_{reac}, t_{CPA|t}]$, as defined in Equation (132). t_{reac} is the time after an assumed reaction time Δ_{reac} as defined in Equation (133). If c_t^* is smaller than zero, then the aircraft predictions have crossed in altitude, and if c_t^* is greater than zero, then the aircraft predictions have not crossed in altitude.

$$c_t^* = r_{CPA, A|t} r_{t_{reac}, A|t} \quad (132)$$

$$t_{reac} = t + \Delta_{reac} \quad (133)$$

$$t_{CPA|t} = t + \tau_{CPA|t} \quad (134)$$

$r_{CPA,A|t}^{+}$ and $r_{CPA,A|t}^{-}$ are conditionally predicted relative altitude at CPA as defined in Equations (135) and (136).

$$r_{CPA,A|t}^{+} = r_{CPA,A|t} + (\tau_{t,CPA} - \Delta_{reac}) (v_{t,A}^0 - v_{\Delta A}) \quad (135)$$

$r_{CPA,A|t}^{+}$ is the conditionally predicted relative altitude at CPA if own aircraft would fly upwards with an altitude velocity of $v_{\Delta A}$ after assumed reaction time t_{reac} .

$$r_{CPA,A|t}^{-} = r_{CPA,A|t} + (\tau_{t,CPA} - \Delta_{reac}) (v_{t,A}^0 + v_{\Delta A}) \quad (136)$$

$r_{CPA,A|t}^{-}$ is the conditionally predicted relative altitude at CPA if own aircraft would fly downwards with an altitude velocity of $v_{\Delta A}$ after an assumed reaction time Δ_{reac} . The reaction time considered for initial RA's is 5 seconds and the vertical velocity, $v_{\Delta A}$, is 1,500ft/min (FAA, 2011, p.29).

9.3 Coordination message transmission

As mentioned above, just when the sense has been selected, a coordination message is sent to the intruder. The structure of the own coordination message is identical to the coordination message from the intruder and tells the intruder where to pass. As defined in Equation (137), if a positive sense has been selected, then the message is the opposite, and if a negative sense has been selected, then the message depends on the predicted vertical miss distance vector.

$$m_{t,sent} = \begin{cases} D_t & , \text{if } [D_t \neq 0] \\ 1 & , \text{if } [(D_t = 0) \wedge (r_{CPA,A|t} < 0)] \\ -1 & , \text{if } [(D_t = 0) \wedge (r_{CPA,A|t} \geq 0)] \end{cases} \quad (137)$$

9.4 Sense comparison

In the unlikely event that both aircraft select the same sense at the same time and therefore send conflicting coordination messages, the aircraft with the higher Mode S address will reverse its sense. This is achieved by setting the Boolean $B_{t,D}$ false, in order to repeat computations from Equation (123) on. Hence, Equation (127) is now used instead of Equation (131), which results in a new sense selection.

$$\begin{aligned} & \text{if } [(m_{t,sent} = m_{t,received}) \wedge (\chi_{ModeS}^0 > \chi_{ModeS}^1)] \\ & \text{then } B_{t,D} = FALSE \end{aligned} \quad (138)$$

9.5 Strength selection

After the sense has been selected the TCAS algorithm continues with the strength selection. Strength selection refers to the determination of the minimum and maximum target rate of climb or

descend to be applied by the own aircraft. The process is explained in words in (FAA, 2011, pp.29-32) and (EUROCAE, 2008, pp.131-132). The strength selection model has been developed from these descriptions.

TCAS II has three main options to choose from. Either to issue a level off command, to issue a climb or descend, or to inform the pilot about prohibited vertical velocities (FAA, 2011, Table 3) and previously presented in Table 9-1. The latter is being issued when a negative sense has been selected upon. When a positive sense has been selected another geometrical test is carried out. This has been incorporated in Equation (139).

$$[v_{t,A,RA,min}^0, v_{t,A,RA,max}^0] = \begin{cases} \left[\left\lceil \frac{\max(v_{t,A}^0, 0)}{500\text{ft/min}} \right\rceil 500\text{ft/min}, v_{A,max}^0 \right] & , \text{if } [(D_t = 0) \wedge (m_{t,sent} = 1)] \\ \left[v_{A,min}^0, \left\lfloor \frac{\min(v_{t,A}^0, 0)}{500\text{ft/min}} \right\rfloor 500\text{ft/min} \right] & , \text{if } [(D_t = 0) \wedge (m_{t,sent} = -1)] \\ [v_{\Delta A}, v_{\Delta A,max}] & , \text{if } [(D_t = 1) \wedge (r_{CPA,A|t}^* > -a_{lim})] \\ [v_{\Delta A,min}, -v_{\Delta A}] & , \text{if } [(D_t = -1) \wedge (r_{CPA,A|t}^* < a_{lim})] \\ [0, 0] & , \text{if } \left\{ \begin{aligned} & [(D_t = 1) \wedge (r_{CPA,A|t}^* \leq -a_{lim})] \\ & [(D_t = -1) \wedge (r_{CPA,A|t}^* \geq a_{lim})] \end{aligned} \right. \end{cases} \quad (139)$$

$$v_{\Delta A,max} = \begin{cases} 4,400\text{ft/min} & , \text{if } v_{t,A}^0 \geq 1,500\text{ft/min} \\ 2,000\text{ft/min} & , \text{otherwise} \end{cases} \quad (140)$$

$$v_{\Delta A,min} = \begin{cases} -4,400\text{ft/min} & , \text{if } v_{t,A}^0 \leq -1,500\text{ft/min} \\ -2,000\text{ft/min} & , \text{otherwise} \end{cases} \quad (141)$$

$v_{t,A,RA,min}^0$ is the lower bound and $v_{t,A,RA,max}^0$ is the upper bound of the in the RA advised altitude velocity range for the own aircraft. Note, when a climb or descend command is supposed to be issued, TCAS may choose from two options (FAA, 2011, pp.29-32): if the own aircraft is not climbing or descending, then it may issue an RA to climb or descend with an altitude velocity between 1,500 to 2,000ft/min, or if the own aircraft is already climbing or descending, then it may issue an RA to climb or descend with an altitude velocity between $\pm 1,500\text{ft/min}$ to $\pm 4,400\text{ft/min}$. Which maximum altitude velocity upper bound, $v_{\Delta A,max}$, or minimum altitude velocity lower bound, $v_{\Delta A,min}$, is applicable, is determined by Equations (140)-(141).

Furthermore, initial preventive RAs of TCAS II Ver.7.1 are limited to eight RA solutions (see Table 9-1). TCAS II Ver.7.1 may issue the pilot to not climb with an altitude velocity greater than 0, 500, 1,000, or 2,000ft/min, or to not descend with an altitude velocity smaller than 0, -500, -1,000, or -2,000ft/min. A difference may be observed in this model. As now action from the pilot is required for

these RAs, the strength selection for preventive RAs in this model has been designed to allow altitude velocity limitation advisories in multiples of 500ft/min. Hence, the RA in this model may be in form of: “Do Not Climb with more than 0, 500, 1,000, 1,500, 2,000, 2,500, etc. ft/min”; or “Do Not Descend with less than 0, -500, -1,000, -1,500, -2,000, -2,500, etc. ft/min”.

$$r_{CPA,A|t}^* = r_{CPA,A|t} + \left(\tau_{CPA|t} - \Delta_{\text{reac}} \right) v_{t,A}^0 \quad (142)$$

Equation (142) establishes the predicted relative altitude at CPA, $r_{CPA,A|t}^*$, between own aircraft and intruder, if the own aircraft levels off after the reaction has passed. Moreover, the sign of the miss distance indicates the predicted relative vertical location of both aircraft. This is used to determine whether a level off command would conflict with the selected sense. So if a positive upwards sense has been selected, level-off is only chosen if the own aircraft is currently descending and a level-off would result in sufficient separation.

10 Threat evolution monitoring

In the threat evolution monitoring module it is assessed whether the given RA needs to be adjusted or may be removed in form of a clear of conflict message.

To support the computations, once an initial RA is issued the values of the sent coordination message, $m_{t, \text{sent}}$, and the initial selected sense, D_t , are copied to variables m_t^A and D_t^A , respectively. Furthermore, a timer, $t_{t, RA}$, is set to 8s, as the pilot needs to be given sufficient time before adjusting the RA. The assumption is made that there must be 8 seconds between two consecutive RAs.

$$m_t^A = m_{t, \text{sent}} \quad (143)$$

$$D_t^A = D_t \quad (144)$$

$$t_{t, RA} = 8s \quad (145)$$

For a time step $\Delta > 0$, $t_{t, RA}$ evolves as:

$$t_{t, RA} = t_{t-\Delta, RA} - \Delta \quad (146)$$

10.1 RA adjustment

TCAS checks whether the other aircraft is not a threat anymore. In (FAA, 2011, pp. 33-34) it is stated:

“During an RA, if the CAS logic determines that the response to a Positive RA has provided ALIM feet of vertical separation prior to CPA (i.e. the aircraft have become safely separated in altitude while not yet safely separated in range) before CPA, the initial RA will be weakened to either a Do Not Descend RA (after an initial Climb RA) or a Do Not Climb RA (after an initial Descend RA). This is done to minimize the displacement from the TCAS aircraft’s original altitude.

In Version 7.0 and later, after ALIM feet of separation has been achieved, the resulting Do Not Descend or Do Not Climb RA is designated as corrective. In Version 7.0, the RA is annunciated as “Adjust Vertical Speed, Adjust.” In Version 7.1, the RA is annunciated as “Level Off, Level Off.” (Version 6.04a keeps the original preventive designation, meaning that the RA is annunciated as “Monitor Vertical Speed.”)

In Version 7.0 and later, negative RAs will not be weakened and the initial RA will be retained until CPA unless it is necessary to strengthen the RA or reverse the RA sense.

After CPA is passed and the range between the TCAS aircraft and threat aircraft begins to increase, or if the horizontal miss distance filter is able to determine prior to CPA that there will be sufficient horizontal miss distance, all RAs are cancelled” (FAA, 2011, pp. 33-34).

This means that a positive RA may be weakened to a “Do Not Climb” or “Do Not Descend” RA if vertical separation has been achieved. This is modelled as the following algorithm, which will be carried out after each measurement cycle:

$$\begin{aligned}
& \text{if } \left[\left(D_t^A = 1 \right) \wedge \left(r_{CPA,A,F|t}^* \leq -a_{\text{lim}} \right) \wedge \left(r_{t,A} \leq -a_{\text{lim}} \right) \right] \\
& \quad \text{then } \begin{cases} \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = \left[0, v_{\Delta A,\max} \right] \\ D_t^A = 0 \\ m_t^A = 1 \\ t_{t|RA} = 8s \end{cases} \\
& \text{else if } \left[\left(D_t^A = -1 \right) \wedge \left(r_{CPA,A,F|t}^* \geq a_{\text{lim}} \right) \wedge \left(r_{t,A} \geq a_{\text{lim}} \right) \right] \\
& \quad \text{then } \begin{cases} \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = \left[v_{\Delta A,\min}, 0 \right] \\ D_t^A = 0 \\ m_t^A = -1 \\ t_{t|RA} = 8s \end{cases} \\
& \qquad \qquad \qquad r_{CPA,A,F|t}^* = r_{CPA,A|t} + \tau_{CPA|t} v_{t,A}^0
\end{aligned} \tag{147}$$

$$r_{CPA,A,F|t}^* = r_{CPA,A|t} + \tau_{CPA|t} v_{t,A}^0 \tag{148}$$

The current relative altitude, $r_{t,A}$, and the prediction of the relative altitude at CPA, $r_{CPA,A,F|t}^*$, if the own aircraft would be in levelled-off flight, are compared to the required altitude miss distance, a_{lim} . Both comparisons are necessary as a level-off manoeuvre may result in both aircraft approaching each other. If the conditions are satisfied, then a new sense, D_t^A , and a virtual message, m_t^A , are set. Last but not least, an RA weakening is a secondary RA. Hence, the timer, $t_{t,RA}$, is reset to eight seconds.

Moreover, after an RA has been issued, TCAS also monitors whether the threat remains or the situation even worsens, i.e. if the intruder is not following its intended sense. The process is explained in words in (FAA, 2011, pp. 31-32) and (EUROCAE, 2008, pp.132-133):

“In some events, the intruder aircraft will maneuver vertically in a manner that thwarts the effectiveness of the issued RA. In these cases, the initial RA will be modified to either increase the strength or reverse the sense of the initial RA. Reversed sense RAs will be discussed separately. A VSL is strengthened by changing to a more restrictive VSL or to a positive Climb or Descend RA. A Climb or Descend RA is strengthened to an Increase Climb/Descend RA. An Increase Climb/Descend RA can only be issued after a Climb/Descend RA has been displayed either as an initial RA, a strengthening of a negative RA, or a sense reversal RA” (FAA, 2011, pp. 31-32).

This is modelled by another assessment of the geometrical situation. As the pilot needs to be given the sufficient time to react upon an RA, that this test is carried out after each new range and rate estimate, but when the timer $t_{t|RA}$ has elapsed to zero:

$$B_{t,RA,EVO} = TRUE, \text{ iff } \left\{ \begin{array}{l} \left[\left(m_t^A = 1 \right) \wedge \left(r_{CPA,A|t} > 0 \right) \wedge \left(t_{t|RA} \leq 0 \right) \wedge \left(0 \leq \tau_{CPA|t} < \Delta_{RA} \right) \right] \vee \\ \left[\left(m_t^A = -1 \right) \wedge \left(r_{CPA,A|t} < 0 \right) \wedge \left(t_{t|RA} \leq 0 \right) \wedge \left(0 \leq \tau_{CPA|t} < \Delta_{RA} \right) \right] \vee \\ \left[\left(r_{CPA,A|t}^+ < a_{lim} \right) \wedge \left(r_{CPA,A|t} > 0 \right) \wedge \left(t_{t|RA} \leq 0 \right) \wedge \left(0 \leq \tau_{CPA|t} < \Delta_{RA} \right) \right] \vee \\ \left[\left(-r_{CPA,A|t}^- < a_{lim} \right) \wedge \left(r_{CPA,A|t} < 0 \right) \wedge \left(t_{t|RA} \leq 0 \right) \wedge \left(0 \leq \tau_{CPA|t} < \Delta_{RA} \right) \right] \end{array} \right. \quad (149)$$

Boolean $B_{t,RA,EVO}$ is true if the intruder is not following its coordinated sense or if the current predicted vertical miss distance is not sufficient and if the pilot has been given sufficient time to react upon an RA. The former is checked by comparing the sent coordination message, $m_{t,sent}$, and the predicted relative altitude at CPA, $r_{CPA,A|t}$. The latter is checked by comparing the adjusted predicted relative altitude at CPA, $r_{CPA,A|t}^+$, $r_{CPA,A|t}^-$, versus the required altitude miss distance, a_{lim} . If one of these cases is satisfied, then a strength and/or sense adjustment is performed.

From (FAA, 2011, pp. 31-32) it is concluded that TCAS may change a “Climb/Descend” RA into a “Descend/Climb” RA or into a strengthened “climb/descend” RA, or change a “Level-Off” RA or “Do Not Climb/Descend” RA into a “Climb/Descend” RA. The new strengthened altitude speed limit, $v_{\Delta A,strengthened}$, is 2500ft/min (Kochenderfer et al, 2012).

From the text, TCAS should determine when to issue a secondary RA by comparing the virtual message, m_t^A , with the predicted relative altitude at CPA, $r_{CPA,A|t}$, to identify whether the intruder is following its intended sense. Next the own selected sense, D_t^A , needs to be compared against the conditionally predicted relative altitudes at CPA, $r_{CPA,A,F|t}^{reac+}$, $r_{CPA,A,F|t}^{reac-}$, in order to decide whether a climb/descend or strengthened climb/descend RA is required.

This is captured in Equations (150)-(153), which determine the adjusted lower and upper bounds of the secondary RA.

$$\begin{aligned}
& \text{if } \left\{ \begin{aligned} & \left[B_{t,RA,EVO} \wedge (m_t^A = 1) \wedge (r_{CPA,A|t} \geq 0) \wedge (D_t^A = 0) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| > \left| r_{CPA,A,F|t}^{reac-} \right| \right) \wedge \left(\left| r_{CPA,A,F|t}^{reac-} \right| \geq a_{\text{lim}} \right) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = 1) \wedge (r_{CPA,A|t} \geq 0) \wedge (D_t^A = 0) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| \leq \left| r_{CPA,A,F|t}^{reac-} \right| \right) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = 1) \wedge (r_{CPA,A|t}^A \geq 0) \wedge (D_t^A = 1) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = -1) \wedge (r_{CPA,A|t} < 0) \wedge (D_t^A = 0) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| \leq \left| r_{CPA,A,F|t}^{reac-} \right| \right) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| < a_{\text{lim}} \right) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = -1) \wedge (r_{CPA,A|t} \geq 0) \wedge (D_t^A = 0) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = -1) \wedge (r_{CPA,A|t} \geq 0) \wedge (D_t^A = -1) \wedge (v_{t-\Delta_t,A,RA,\min}^0 \geq v_{t-\Delta_t,A,RA,\max}^0) \right] \\ & \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = \left[-v_{\Delta A,\max}, -v_{\Delta A} \right] \\ & D_t^A = -1 \\ & m_t^A = -1 \\ & t_{t|RA} = 8s \end{aligned} \right. \\
& \text{then } \left\{ \begin{aligned} & D_t^A = -1 \\ & m_t^A = -1 \\ & t_{t|RA} = 8s \end{aligned} \right.
\end{aligned} \tag{150}$$

$$\begin{aligned}
& \text{if } \left\{ \begin{aligned} & \left[B_{t,RA,EVO} \wedge (m_t^A = 1) \wedge (r_{CPA,A|t} \geq 0) \wedge (D_t^A = 0) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| > \left| r_{CPA,A,F|t}^{reac-} \right| \right) \wedge \left(\left| r_{CPA,A,F|t}^{reac-} \right| < a_{\text{lim}} \right) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = 1) \wedge (r_{CPA,A|t} < 0) \wedge (D_t^A = 0) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = 1) \wedge (r_{CPA,A|t} < 0) \wedge (D_t^A = 1) \wedge (v_{t-\Delta_t,A,RA,\min}^0 \geq v_{t-\Delta_t,A,RA,\max}^0) \right] \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = -1) \wedge (r_{CPA,A|t} < 0) \wedge (D_t^A = 0) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| \leq \left| r_{CPA,A,F|t}^{reac-} \right| \right) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| \geq a_{\text{lim}} \right) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = -1) \wedge (r_{CPA,A|t} < 0) \wedge (D_t^A = 0) \wedge \left(\left| r_{CPA,A,F|t}^{reac+} \right| > \left| r_{CPA,A,F|t}^{reac-} \right| \right) \right] \vee \\ & \left[B_{t,RA,EVO} \wedge (m_t^A = -1) \wedge (r_{CPA,A|t} < 0) \wedge (D_t^A = -1) \right] \\ & \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = \left[v_{\Delta A}, v_{\Delta A,\max} \right] \\ & D_t^A = 1 \\ & m_t^A = 1 \\ & t_{t|RA} = 8s \end{aligned} \right. \\
& \text{then } \left\{ \begin{aligned} & D_t^A = 1 \\ & m_t^A = 1 \\ & t_{t|RA} = 8s \end{aligned} \right.
\end{aligned} \tag{151}$$

$$\begin{aligned}
& \text{if } \left[B_{t,RA,EVO} \wedge (m_t^A = 1) \wedge (r_{CPA,A|t} < 0) \wedge (D_t^A = 1) \wedge (v_{t-\Delta_t,A,RA,\min}^0 < v_{t-\Delta_t,A,RA,\max}^0) \right] \\
& \text{then } \begin{cases} \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = \left[v_{\Delta A,strengthened}, 4400\text{ft/min} \right] \\ D_t^A = 1 \\ m_t^A = 1 \\ t_{t|RA} = 8s \end{cases} \quad (152)
\end{aligned}$$

$$\begin{aligned}
& \text{if } \left[B_{t,RA,EVO} \wedge (m_t^A = -1) \wedge (r_{CPA,A|t} \geq 0) \wedge (D_t^A = -1) \wedge (v_{t-\Delta_t,A,RA,\min}^0 < v_{t-\Delta_t,A,RA,\max}^0) \right] \\
& \text{then } \begin{cases} \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = \left[-4400\text{ft/min}, -v_{\Delta A,strengthened} \right] \\ D_t^A = -1 \\ m_t^A = -1 \\ t_{t|RA} = 8s \end{cases} \quad (153)
\end{aligned}$$

$r_{CPA,A,F|t}^{+}$, $r_{CPA,A,F|t}^{-}$ are defined in Equations (154)-(155) and are similar to the previously defined $r_{CPA,A|t}^{+}$, $r_{CPA,A|t}^{-}$ in Equations (135)-(136). However, the previously assumed reaction time, Δ_{reac} , of 5s is replaced by the faster assumed reaction time, $\Delta_{reac,F}$, of 2.5s (ICAO, 2006, p. 4-2).

$$r_{CPA,A,F|t}^{+} = r_{CPA,A|t} + \left(\tau_{CPA|t} - \Delta_{reac,F} \right) \left(v_{t,A}^0 - v_{\Delta A} \right) \quad (154)$$

$$r_{CPA,A,F|t}^{-} = r_{CPA,A|t} + \left(\tau_{CPA|t} - \Delta_{reac,F} \right) \left(v_{t,A}^0 + v_{\Delta A} \right) \quad (155)$$

10.2 Clear of conflict message

In the threat evolution monitoring module it is also tested each step whether the threat situation has been solved. This is announced with a clear of conflict message. When the clear of conflict message is supposed to be issued is modelled by checking whether the time to CPA, $t_{t,timer}$, at initial RA has been elapsed to zero and whether the Range Test, $B_{t,RT}$, or Altitude Test, $B_{t,AT}$, have failed, or if the horizontal miss distance filter has been activated:

$$\begin{aligned}
& \text{if } \begin{cases} \left[(\neg B_{t,RT} \vee \neg B_{t,AT}) \wedge (t_{t,timer} \leq 0) \right] \vee \\ B_{t,HMDF} \end{cases} \\
& \text{then } \begin{cases} \text{clear of conflict message} \\ \text{reset system} \end{cases} \quad (156) \\
& \text{else repeat the monitoring cycle}
\end{aligned}$$

Furthermore, together with the clear of conflict message all variables and parameters are set to initial values.

1.1 Model verification

In order to verify whether all possible combinations in Equations (127) and (131) have been covered, these equations are now analysed as follows. First the possible values of the key time-dependent variables are identified:

$$\begin{aligned}
 m_{t, received} &\in \{-1, 0, 1\} \\
 c_t^* &\in \{\leq 0, > 0\} \\
 r_{CPA, A|t} &\in \{< 0, \geq 0\} \\
 -r_{CPA, A|t}^- &\in \{< a_{lim}, \geq a_{lim}\} \\
 r_{CPA, A|t}^+ &\in \{< a_{lim}, \geq a_{lim}\} \\
 |r_{CPA, A|t}^{reac+}| &\in \left\{ \leq |r_{CPA, A|t}^{reac-}|, > |r_{CPA, A|t}^{reac-}| \right\} \\
 |r_{CPA, A|t}^{reac+}| &\in \{< a_{lim}, \geq a_{lim}\} \\
 |r_{CPA, A|t}^{reac-}| &\in \{< a_{lim}, \geq a_{lim}\}
 \end{aligned}$$

This comprises 7 binary values and 1 variable with 3 possible values. From this there are 384 combinations possible ($2^7 \cdot 3 = 384$). For Equation (127) $m_{t, received} \in \{-1, 1\}$, hence, 256 ($=2^8$) combinations should be covered by Equation (127). The verification of this is done by evaluating for each row in Equation (127) how many combinations are merged. Next it is verified that these numbers add up to 256 combinations.

Number of combinations:

$$D_t = \begin{cases} 1, & \text{if } \left\{ \begin{aligned} &\left[(m_{t, received} = -1) \wedge (r_{CPA, A|t} \geq 0) \right] \vee \\ &\left[(m_{t, received} = -1) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- < a_{lim}) \right] \end{aligned} \right\} & \begin{aligned} &2^6 \\ &2^5 \end{aligned} \\ -1, & \text{if } \left\{ \begin{aligned} &\left[(m_{t, received} = 1) \wedge (r_{CPA, A|t} < 0) \right] \vee \\ &\left[(m_{t, received} = 1) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ < a_{lim}) \right] \end{aligned} \right\} & \begin{aligned} &2^6 \\ &2^5 \end{aligned} \\ 0, & \text{if } \left\{ \begin{aligned} &\left[(m_{t, received} = -1) \wedge (r_{CPA, A|t} < 0) \wedge (-r_{CPA, A|t}^- \geq a_{lim}) \right] \vee \\ &\left[(m_{t, received} = 1) \wedge (r_{CPA, A|t} \geq 0) \wedge (r_{CPA, A|t}^+ \geq a_{lim}) \right] \end{aligned} \right\} & \begin{aligned} &2^5 \\ &2^5 \end{aligned} \end{cases}$$

Hence, in total $2 \cdot 2^6 + 4 \cdot 2^5 = 2^8 = 256$ combinations are covered in Equation (127).

For Equation (131) $m_{t, received} \in \{0\}$, hence, 128 ($=2^7$) combinations should be covered by Equation (131). Similar as above, the verification of this is done by evaluating for each row in Equation (131) how many combinations are merged. Next it is verified that these numbers add up to 128 combinations.

Number of combinations:

[illegible]

Hence, in total $8*2^2+8*2^3+2*2^4=32+64+32=128$ combinations are covered in Equation (131).

12 Additional Agents

Next to TCAS, three additional agents are modelled:

- Aircraft 'i',
- Cockpit 'i', and
- Communication 'i'.

It should be noted that the agents are developed in a generic manner to allow interchangeability with other systems, i.e. an UAV with a different collision avoidance system.

12.1 Agent Aircraft i

The evolution of the position coordinates of aircraft 'i', $s_{t,x}^i, s_{t,y}^i, s_{t,z}^i$, are based on the ground and altitude velocities, $v_{t,H}^i, v_{t,A}^i$, the heading, θ_t^i , and the time step, Δ , as given in Equations (157)-(159), where θ_t^i is the magnetic heading of aircraft 'i' at time 't' and defined to be zero in direction of North (positive y-axis) and increasing in a clockwise direction. Function $f_M(\theta_t^i)$ converts the magnetic heading, $\theta_t^i \in \{0, \dots, 360\}$, into the equivalent angle of the standard mathematical reference system (zero in direction of the positive x-axis and increasing counter clockwise). In this model the discrete time step, Δ , is set to 0.1s and differs to the operating cycle of TCAS.

$$s_{t,x}^i = s_{t-\Delta,x}^i + \|\mathbf{v}_{t-\Delta,H}^i\| \cos(f_M(\theta_{t-\Delta}^i)) \Delta \quad (157)$$

$$s_{t,y}^i = s_{t-\Delta,y}^i + \|\mathbf{v}_{t-\Delta,H}^i\| \sin(f_M(\theta_{t-\Delta}^i)) \Delta \quad (158)$$

$$s_{t,z}^i = s_{t-\Delta,z}^i + v_{t-\Delta,A}^i \Delta \quad (159)$$

In order to deviate from the current trajectory, the evolution model of aircraft 'i' may process the following input parameters of aircraft 'i' at time 't', which resemble actions by the pilot or autopilot of aircraft 'i' (see Section 12.2):

- Target altitude velocity set point limits, $v_{t,A,\min}^i, v_{t,A,\max}^i$,
- altitude acceleration set point, $a_{t,A}^i$, and
- turn rate set point, $\dot{\theta}_t^i$.

Note, as described in Section 12.2, the target altitude velocity set point limits, $v_{t,A,\min}^i, v_{t,A,\max}^i$, in the agent "Aircraft i" are set through input from agent "Cockpit i" according to the issued velocity boundaries in an RA, see Equations (139), (147)-(148), and (150)-(153). The altitude acceleration set point, $a_{t,A}^i$, in the agent "Aircraft i" is set through input from agent "Cockpit i" equal to 0.25g for initial RAs and equal to 0.35g for RA adjustments. The turn rate set point is set through input from agent "Cockpit i" to zero, if the pilot receives an RA. The latter is not a set point directly issued by

TCAS but comes from the assumption that the pilot will stop the aircraft from turning, if he or she reacts upon an RA.

Furthermore, it is assumed that the magnitude of the ground velocity, $\|\mathbf{v}_{t,H}^i\|$, remains constant.

Now, based on the turn rate of aircraft 'i', $\dot{\theta}_t^i \in \mathbb{R}$, the new heading is given in Equation (160).

$$\theta_t^i = \theta_{t-\Delta}^i + \dot{\theta}_{t-\Delta}^i \Delta \quad (160)$$

In case the heading, θ_t^i , exceeds the limits of $[0^\circ, 360^\circ)$ degrees, it is corrected in quantisation of 360° until it is within the limits.

Based on the target altitude velocity set point limits, $v_{t,A,\min}^i, v_{t,A,\max}^i$, and the altitude acceleration set point, $a_{t,A}^i$, the new altitude velocity is given in Equation (161), where the altitude velocity is adjusted if it is not bounded by the altitude velocities bounds, $v_{t,A,\min}^i, v_{t,A,\max}^i$.

$$v_{t,A}^i = \begin{cases} \min\left((v_{t-\Delta,A}^i + a_{t-\Delta,A}^i \Delta), v_{t-\Delta,A,\max}^i\right) & , \text{ if } v_{t-\Delta,A}^i < v_{t-\Delta,A,\min}^i \\ \max\left((v_{t-\Delta,A}^i - a_{t-\Delta,A}^i \Delta), v_{t-\Delta,A,\min}^i\right) & , \text{ if } v_{t-\Delta,A}^i > v_{t-\Delta,A,\max}^i \\ v_{t-\Delta,A}^i = v_{t-\Delta,A}^i & , \text{ otherwise} \end{cases} \quad (161)$$

Implicitly two assumptions are adopted. First, there is no wind, such that the heading is also the course. Second, the aircraft body direction equals the heading.

12.2 Agent Cockpit i

In this model, the pilot may be human or an autopilot and reacts only upon RAs.

In TCAS equipped aircraft, RAs are communicated acoustically through an aural annunciation and visually through target vertical velocity indications on the cockpit display. Also, only the human pilot may adjust velocities based on RAs. As human pilots cannot react instantly, reaction time is considered. According to ICAO, the nominal reaction time for initial RAs is 5 seconds and for secondary RAs it is 2.5 seconds (ICAO, 2006, p. 4-2).

Also according to (ICAO, 2006, p. 4-2) the pilot climbs or descends with an altitude acceleration of 0.25g for initial RAs, and with 0.35g for adjusted RAs. In this model, this leads to the setting of the set points of the agent "Aircraft i", as described in Section 12.1.

In order to use a non-constant reaction time, it is assumed that the human pilot reaction time is Gaussian with mean equal to the nominal reaction time assumed by ICAO and a standard deviation of 0.5s.

Furthermore, it is assumed that the human pilot only picks up an RA, if he/she was given both an aural and a visual signal. The failure rate of the cockpit display the aural annunciation system is

assumed to be 10^{-7} per flight hour, which is in line with system requirements for critical aircraft systems where a system failure would be categorized as hazardous (JARUS, 2014, p. 30). In rare cases where the pilot has not reacted upon an RA and a new RA is issued, then the pilot will only react upon the latest RA.

Regarding the behaviour of the pilot upon preventive RAs and clear of conflict messages, two assumptions are made. First, if the pilot receives a do not climb/descend RA then he/she continues flying with an altitude velocity equal to 10% of the allowable altitude velocity range. E.g. the pilot continues flying with 250ft/min if the indicated velocity range of the RA is [0,2500] ft/min.

Second, the pilot levels off when he/she receives a clear of conflict message. This is achieved through issuing a level off adjusted RA.

The target altitude velocity set point limits, $v_{t,A,\min}^i, v_{t,A,\max}^i$, in the agent “Aircraft i” are set according to the issued velocity boundaries in an RA, see Equations (139), (147)-(148), and (150)-(153). The altitude acceleration set point, $a_{t,A}^i$, in the agent “Aircraft i” is set equal to 0.25g for initial RAs and equal to 0.35g for RA adjustments.

Finally, in case the human pilot is performing a manoeuvre and TCAS issues an RA, then the pilot stops the aircraft from turning and only climbs, descends, or levels off. In such cases, the turn rate set point, $\dot{\theta}_t^i$, in the agent “Aircraft i” is set equal to zero at any RA.

In regard to model flexibility, there is also an autopilot which may react immediately upon RAs. However, in the current model the autopilot is not activated for TCAS operations.

12.3 Agent Communication ‘i’

In TCAS the exact position and speed vectors of an intruder aircraft are not known to own aircraft but are estimated by TCAS with alpha-beta filters (ICAO, 2006) from physical range and bearing tests using the Mode S signal and received altitude reports.

The Mode S connection is established by aircraft ‘i’ using its transponder. Every 8 to 10 seconds the transponder of a TCAS equipped aircraft ‘i’ sends out a Mode S message including its aircraft ID (EUROCAE, 2008, p. B1). This message is being picked up by the transponders of other aircraft in the vicinity. Then when an aircraft ‘i’ knows the aircraft ID of aircraft ‘k’ in the vicinity, it sends out an interrogation message to aircraft ‘k’. Interrogations are nominally carried out once per second (EUROCAE, 2008, p. B3). The interrogated aircraft ‘k’ should respond to this interrogation by sending a message to aircraft ‘i’ including its aircraft ID, altitude and maybe a coordination message. From the response delay and the direction of the message from aircraft ‘k’ to ‘i’, aircraft ‘i’ is able to calculate the slant range and relative bearing between aircraft ‘k’ and ‘i’.

Besides, according to (RTCA, 2017, p. Q-6), the probability of reception of a response is 0.95.

13 Assumptions adopted

Throughout the model specification of TCAS and the agents Aircraft 'i', Cockpit 'i' and Communication 'i' several assumptions have been made. First, the assumption affecting TCAS operations are summarized:

- TCAS 'i':
 1. Only single encounters are considered.
 2. The jitter of the slant range measurement error may be modelled as white Gaussian noise of 30ft root mean square.
 3. The jitter of altitude measurements may be modelled as a first order regression model with alpha equal to 0.8, mean equal to zero, and standard deviation of 3ft.
 4. The reported altitude of aircraft 'i' is in 25ft quantisation.
 5. The jitter of the own altitude velocity measurements may be modelled as white Gaussian noise of 0.8535m/s root mean square (1.707m/s 95% confidence).
 6. Only the third inconsistency test of the bearing based tracker of the horizontal miss distance filter is applied in the current model.
 7. The smoothing parameters of the Cartesian tracker are equal to the smoothing parameters of the parabolic range tracker.
 8. "Do Not Climb/Descend" RAs may be issued in all possible 500ft steps. E.g. do not climb with more than 0, 500, 1000, 1500, etc. feet per minute.
 9. An RA adjustment may only be issued at least 8 seconds after the previous RA. The exception is an RA weakening, which may be issued at any time.
- Aircraft 'i':
 1. The human pilot or autopilot inputs are altitude velocity bound set points, altitude acceleration set point, and turn rate set point.
 2. The aircraft evolutions are discretized in time step Δ .
 3. There is no wind, such that the heading is also the course.
 4. The aircraft body axis direction equals the heading.
- Cockpit 'i':
 1. The reaction time of the human pilot is Gaussian with mean equal to the nominal reaction time assumed by ICAO (5s for initial RA and 2.5s for an adjusted RA) and a standard deviation of 0.5s.
 2. The climb or descend of a pilot upon an RA has an altitude acceleration of 0.25g for initial RAs and 0.35g for adjusted RAs.

3. The pilot does only react upon an RA if he was informed both visually and aurally.
 4. If the pilot has not reacted yet upon a previous RA, then the pilot will only react upon the latest RA.
 5. The pilot should stop the aircraft from turning if he reacts upon an RA.
 6. If the pilot receives a do not climb/descend RA then he/she continues flying with an altitude velocity equal to 10% of the allowable altitude velocity range. E.g. the pilot continues flying with 250ft/min if the velocity range of the RA is [0,2500] ft/min.
 7. The pilot levels off when he/she receives a clear of conflict message. This is achieved through a level off adjusted RA.
 8. In case aural and visual RAs are different, then the pilot follows the visual RA.
- Communication 'i':
 1. Aircraft 'i' knows that aircraft 'k' is in the vicinity.
 2. Aircraft 'i' knows the aircraft ID of aircraft 'k'.
 3. Interrogations are carried out at a frequency of 1Hz.
 4. Probability of reception of a response message is 0.95.

Part II

14 Petri Net Model of TCAS

A Petri Net model according to (Everdij et al., 2003) has been created from the previously presented TCAS model. A graphical representation of the Petri Net model is given in Figure [Figure 14-1](#). In total there are the following 4 agents:

- TCAS i,
- Aircraft i,
- Cockpit i, and
- Communication i.

The agents have the following local Petri Nets (LPN) and interconnecting Petri Nets (IPN):

- Agent “TCAS i” consists of 7 LPNs and 6 IPN.
 - LPNs:
 - State Estimation,
 - Received Coordination Message,
 - TA Module,
 - Threat Detection,
 - Sense Selection,
 - Strength Selection, and
 - Evolution Monitoring.
 - IPNs:
 - TA,
 - Timer,
 - Sense Coordination,
 - RA,
 - Adjusted RA, and
 - COC.
- Agent “Aircraft i” consists of 1 LPN and no IPN.
 - LPN:
 - State.
- Agent “Cockpit i” consists of 3 LPN and 2 IPN.
 - LPNs:
 - Pilot Flying,
 - CDTI, and
 - Aural Annunciation.
 - IPNs:
 - Pilot Input
 - Auto Pilot
- Agent “Communication i” consists of 4 LPNs and 1 IPN.
 - LPNs:
 - Transponder,
 - Mode S Reply,
 - Measurements, and
 - Interrogation.

- IPN:
 - Int. mes. from aircraft i to k.

Next, each agent is described. The Petri Net model specifications are given in Appendix A.

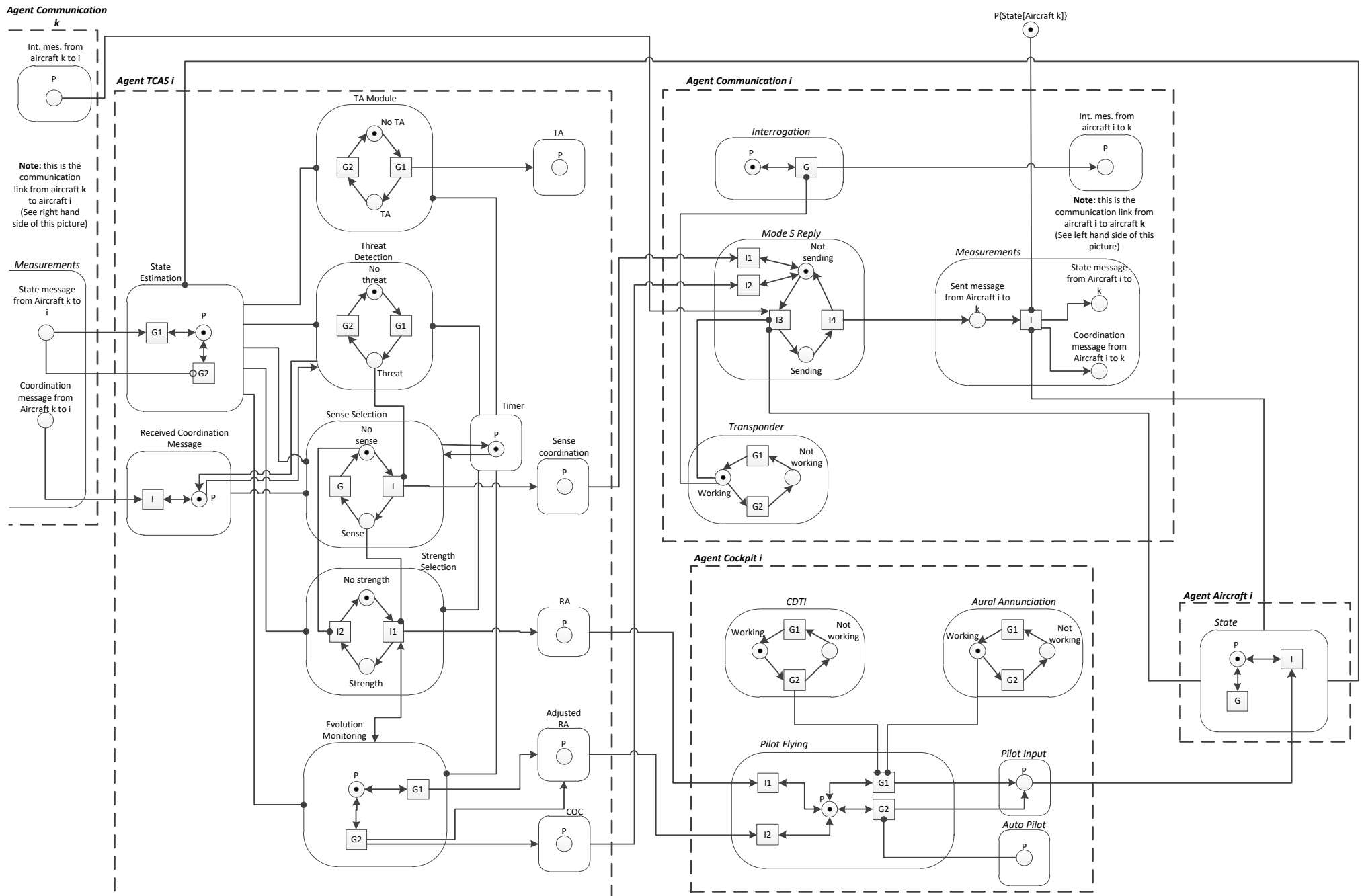


Figure 14-1: Petri Net model of agents TCAS 'i', Communication 'i', Cockpit 'i' and Aircraft 'i'.

14.1 Agent “TCAS i”

Agent “TCAS i” consists of 7 LPNs and 6 IPN. The LPNs are the following:

- State Estimation,
- Received Coordination Message,
- TA Module,
- Threat Detection,
- Sense Selection,
- Strength Selection, and
- Evolution Monitoring.

The IPNs are the following:

- TA,
- Timer,
- Sense Coordination,
- RA,
- Adjusted RA, and
- COC.

The above mentioned LPNs and IPNs are addressed shortly below.

LPN “State Estimation”

This LPN has one place, “P” and two transitions, G1 and G2. There is always a token present. The LPN resembles the slant range and vertical range filters, as described in Section 5, and the horizontal miss distance filter, as described in Section 7. As such the token in place “P” has saved as colours the slant range, slant rate, vertical range and vertical rate, status of the horizontal miss distance filter, own altitude, own vertical velocity, and additional information necessary to update the just listed variables.

The update of the token is carried out through transitions G1 and G2 at a frequency of 1Hz. In case a message from aircraft ‘k’ has been sent, then a token is present in place “State message from Aircraft k to i”. The enabling arc from that place in addition to the enabling arc from place “P” of agent “Aircraft i” will enable transition G1 and the state estimation and horizontal miss distance filter status are updated using the measurement data and information about the own aircraft from agent “Aircraft i”. Else transition G2, which has also one enabling arc from agent “Aircraft i”, is used and the update is carried out without measurements about aircraft ‘k’.

In order to prevent transitions G1 and G2 from becoming active at the same time, an inhibitor arc from place “State message from Aircraft k to i” to transition G2 ensures that G2 only becomes active when no message has been sent by aircraft ‘k’.

LPN “Received Coordination Message”

This LPN has one place, “P”, and one transition I. There is always one token at the place (except when the “Threat Detection” LPN is removing and returning it) and its colour represents the coordination message received from the other aircraft. The colour is updated through transition I which also has an incoming arc from place “Coordination message Aircraft k to i”.

Additionally, place “P” has outgoing and incoming arcs to and from transitions G1 and G2 of LPN “Threat Detection”, where transition G1 keeps the token’s colour unchanged and transition G2 changes the token’s colour to zero, meaning no message has been received, which is part of a TCAS reset.

LPN “TA Module” and IPN “TA”

The LPN has two places, “TA” and “No TA”, and two transitions, G1 and G2. The IPN has only one place “P”. Both resemble the module to issue a TA, as described in Section 6, where the IPN receives a token if a TA is issued.

In the LPN, there is initially a token at place “No TA” which is being transferred to place “TA” via transition G1. Transition G1 uses the information about the slant range, slant rate, vertical range, vertical rate, own aircraft’s altitude and own aircraft’s vertical rate from the token in place “P” of LPN “State Estimation” through an enabling arc, and it has an enabling arc from place “P” of IPN “Timer”, which has no effect on the time of enabling of the transition as there is always a token present. Additionally, transition G1 moves one token without colour to place “P” of IPN “TA” to signal that a TA is issued.

When the dangerous situation has passed, TCAS needs to be reset, as described in Equation (156) in Section 10. In the “TA Module” LPN this is done by transition G2 which transfers the token from place “TA” to “No TA”. To check whether the transition should take place, input is taken through an enabling arc from place “P” of LPN “State Estimation” and an enabling arc from place “P” of IPN “Timer”.

LPN “Threat Detection”

This LPN has two places, “Threat” and “No threat”, and two transitions, G1 and G2. This LPN resembles the threat detection module which declares the other aircraft as a threat, as described in Section 8.

Initially there is a token at place “No threat” which is being transferred to place “Threat” via transition G1. Transition G1 uses the information about the slant range, slant rate, vertical range, vertical rate, own aircraft’s altitude and own aircraft’s vertical rate from the token in place “P” of LPN “State Estimation” through an enabling arc, and the information about a received coordination message from the other aircraft through an incoming arc from place “P” of LPN “Received Coordination Message” to determine whether the other aircraft is supposed to be declared a threat. There is also an enabling arc from place “P” of IPN “Timer” to transition G1, which has no effect on the time of enabling the transition as there is always a token present. Additionally, transition G1 also returns the token coming from place “P” of LPN “Received Coordination Message” back to its initial place with unchanged colour.

From place “Threat” there is also an enabling arc to transition I of LPN “Sense Selection” which is meant to enable the sense selection process after the other aircraft has been declared a threat.

When the dangerous situation has passed, TCAS needs to be reset, as described in Equation (156) in Section 10. In the “Threat Detection” LPN this is done by transition G2 which transfers the token from place “Threat” to “No threat”. To check whether the transition should take place, input is taken

through an enabling arc from place “P” of LPN “State Estimation” and an enabling arc from place “P” of IPN “Timer”. Additionally, there is an incoming and outgoing arc from and to place “P” of LPN “Received Coordination Message”, which returns that token with a colour value meaning “no message received”. This is necessary, as else the previously received coordination message may trigger transition G1 again and the other aircraft would be declared faulty a new threat.

LPN “Sense Selection” and IPNs “Timer” and “Sense Coordination”

The LPN “Sense Selection” has two places, “Sense” and “No sense”, and two transitions, I and G. IPNs “Timer” and “Sense Coordination” have both one place “P”. Together, the LPN and IPNs resemble the sense selection process, as described in Section 9.

Initially, there is a token in place “No sense” of LPN “Sense Selection” and a token in place “P” of IPN “Timer”

IPN “Sense Coordination” has one incoming arc from Transition I of LPN “Sense Selection” and one outgoing arc to transition I1 of LPN “Mode S Reply” of agent “Communication i”. It transfers the in the sense selection selected intent, see Equation (137), to the agent “Communication i” from where the intent is sent to aircraft ‘k’.

IPN “Timer” has outgoing enabling arcs to LPNs “TA Module”, “Threat Detection”, “Strength Selection” and “Evolution Monitoring”. Furthermore, there are outgoing and incoming arcs between the IPN “Timer” and LPN “Sense Selection”. Transition I of LPN “Sense Selection” removes the token at place “P” and returns a token with a colour value equalling the time until CPA, see Equation (124). The saved time is discounted and indicates whether the initial CPA should have passed.

In LPN “Sense Selection” all the calculations as described in Section 9 are carried out. The necessary information about the aircraft states and the intruder intent comes from enabling arcs from LPN “State Estimation” and “Received Coordination Message” and goes to all transitions in LPN “Sense Selection”. Whether the intruder has been declared a threat is determined through an enabling arc from place “Threat” of LPN “Threat Detection” to transition I. When there is a token in place “No sense”, meaning no sense is selected and a token in place “Threat” of LPN “Threat Detection”, meaning the intruder has been declared a threat, then transition I selects the sense and fires three tokens. One token is removed from place “No sense” and fired to place “Sense” with a colour value equalling the selected sense. The second token is removed from place “P” of IPN “Timer” and returned with a colour value equalling the time until CPA, as described above. The third token is additionally produced by transition I and fired to place “P” of IPN “Sense Coordination”, as described above.

Transition G has the same incoming and enabling arcs from outside the LPN. Within the LPN it transfers a token from place “Sense” to place “No sense”. It is used to reset the sense selection process. Opposed to the “TA Module” and “Threat Detection” LPNs, the sense selection process may be reset in two cases. First, when the dangerous situation has passed, as described in Equation (156) in Section 10. Second, when both aircraft have selected the same sense, then the aircraft with the higher aircraft ID selects the sense again using the new information, as described in Equation (138). Besides, opposed to transition I, the outgoing arc to IPN “Timer” just returns the incoming token from that IPN with unchanged colours.

From place “Sense” there is also an enabling arc to transition I1 of LPN “Strength Selection” which is meant to enable the strength selection process after the sense has been selected.

From place “No sense” there is also an enabling arc to transition I2 of LPN “Strength Selection” which is meant to reset that LPN, i.e. a new strength may be selected when a new sense has been selected.

LPN “Strength Selection” and IPN “RA”

The LPN has two places, “No strength” and “Strength”, and two transitions, I1 and I2. The IPN has one place “P”. Together they resemble the module which selects the strength and as such the RA, as described in Section 9.5.

Initially there is only a token at place “No strength” of LPN “Strength Selection”, which is being transferred to place “Strength” via transition I1 after TCAS has selected its sense. As such, transition I1 has three enabling arcs and two incoming arcs.

Two enabling arcs are from place “P” of LPN “State Estimation” and place “P” of IPN “Timer” which always have a token present. The third enabling arc comes from place “Sense” of LPN “Sense Selection” which only has a token present if a sense has been selected. The incoming arcs come from place “No Strength” and from place “P” of LPN “Evolution Monitoring”.

In transition I1 the RA is created by selecting the strength. To do so, information about the aircraft states are taken from the colours of the token in place “P” of LPN “State Estimation”, about the selected sense and to be sent coordination message from the colours of the token in place “Sense” of LPN “Sense Selection”, and about the estimated time until CPA from the colour of the token in place “P” of IPN “Timer”.

Transition I1 has also three outgoing arcs. One outgoing arc goes to place “Strength”, which fires a token without colour. One outgoing arc goes to place “P” of LPN “Evolution Monitoring”, which fires a token with colours containing the target minimum and maximum vertical velocity, the to be sent coordination message, and parameters needed for the monitoring cycle to be enabled and its initial cycle delay times. And one outgoing arc to place “RA” of IPN, which fires a token with colours containing the target minimum and maximum vertical velocity, target turn rate, which is set to zero as no horizontal manoeuvre is desired during a TCAS RA, and a delay time to model the time until the pilot reacts upon the RA.

Transition I2 has also three enabling arcs, one incoming arc, but only one outgoing arc. The enabling arcs are coming from LPNs “State Estimation”, “Timer” and place “No sense” of LPN “Sense Selection”. The incoming arc comes from place “Strength” and the outgoing arc goes to place “No Strength”.

In case the “Sense Selection” LPN is reset by having a token in place “No sense” and a token is available in place “Strength”, then transition I2 causes an immediate transition of the token from place “Strength” to “No strength”. The additional enabling arcs from place “P” of LPN “State Estimation” and place “P” of IPN “Timer” to transition I2 have no effect on the time of enabling, as at each place always a token is present.

LPN “Evolution Monitoring” and IPNs “Adjusted RA” and “COC”

The LPN has only one place, “Evolution monitoring” and two transitions, G1 and G2. Both IPNs also have one place each called “P”. Together they resemble the module to monitor the encounter, as described in Section 10, where the IPNs resemble the outputs as an adjusted RA or a clear of conflict message.

Once an RA has been issued, which is captured in the colour of the token in place “P” of LPN “Evolution Monitoring” through the colours being altered by transition I1 of LPN “Strength Selection”, the guards G1 and G2 become active.

Transition G1 includes two checks: first, whether the current RA may be weakened; second whether the current RA needs to be strengthened or reversed.

For these checks transition G1 has two enabling arcs and one incoming arc. One enabling arc comes from place “P” of LPN “State Estimation” to gather information about the aircraft states. The second enabling arc comes from place “P” of IPN “Timer” and has no effect on the transition, as there is always a token present. The remaining information about the current RA comes from the incoming token from place “P”. Additionally, there is a time delay of 8 seconds after an initial RA and 5 seconds after a secondary RA to give the pilot sufficient reaction time to react upon the current RA.

In case a secondary RA is deemed necessary, guard G1 is activated and fires two tokens. One token back to place “P” with updated colours, i.e. target minimum and maximum vertical velocities, virtual sense, virtual coordination message, and the time delay until another RA change may be allowed is set to 5s. The second token is fired to place “P” of IPN “Adjusted RA” with colours containing the new RA information being target minimum and maximum vertical velocity, target turn rate, a delay time to model the time until the pilot reacts upon the RA, and a time stamp of the RA. The target turn rate is always zero as no horizontal manoeuvre is desired during a TCAS RA.

Transition G2 has no additional time delay and checks whether a clear of conflict message may be issued or the current RA may be weakened in a secondary RA. However, transition G2 has an implicit cycle rate of 1Hz due to the update rate of the incoming information from place “P” of LPN “State Estimation”. Similar to transition G1, it has two enabling arcs and one incoming arc. One enabling arc comes from place “P” of LPN “State Estimation” to gather information about the aircraft states. The second enabling arc comes from place “P” of IPN “Timer” and its colour is used to determine whether the initial time until CPA has been elapsed. The information about the current RA the own aircraft’s intention comes from colour of the incoming token in place “P”.

In case transition G2 determines that a clear of conflict message may be issued, three tokens are fired. One token without colour is fired to place “COC”. A second token is fired to place “P” of IPN “Adjusted RA” which is equivalent to a level off secondary RA. This is based on the assumption that the pilot would level off after the aircraft is clear of conflict and would consult ATC for further instructions. The third token is fired back to place “P” and resets its colours to initial values, such that evolution monitoring is deactivated.

14.2 Agent “Aircraft i”

Agent “Aircraft i” consists of 1 LPN and no IPN. The LPN is the following:

- State.

The above mentioned LPNs are addressed shortly below.

LPN “State”

LPN has one place, “P”, and two transitions, I and G. Place “P” contains one coloured token, where the colours of the token resemble the physical properties of one aircraft ‘i’, i.e. location, velocities, accelerations, heading, and target velocities and target accelerations.

At each time step of the simulation the location and velocity parameters change. This is done through transition G.

Additionally, at any point in time the pilot may change the settings of the aircraft, such as target horizontal velocity and acceleration, target vertical velocity and acceleration, and target heading and turn rate. These parameters are updated through transition I using input from place “P” in IPN “Pilot Input” which resembles the behaviour of the pilot (or autopilot in case the aircraft is an UAV).

14.3 Agent “Cockpit i”

Agent “Cockpit i” consists of 3 LPN and 2 IPN. The LPNs are the following:

- Pilot Flying,
- CDTI, and
- Aural Annunciation.

The IPN is the following one:

- Pilot Input
- Auto Pilot

The above mentioned LPNs and the IPNs are addressed shortly below.

LPN “Pilot Flying” and IPN “Pilot Input” and “Auto Pilot”

The LPN “Pilot Flying” and IPN “Pilot Input” and “Auto Pilot” resemble the pilot of aircraft ‘i’. LPN “Pilot Flying” has one place “P” and four transitions, I1, I2, G1 and G2. Initially there is one token in place “P”.

Transition I1 has two incoming arcs, one from LPN “RA” of agent “TCAS i” or “ACAS UAV i” and one from place “P”. If an RA is present, then the tokens from the incoming places are eaten and a token with the colours of the token from place “P” of LPN “RA” of agent “TCAS i” or “ACAS UAV i” is fired to place “P” through an outgoing arc.

Transition I2 has also two incoming arcs, one from LPN “Adjusted RA” of agent “TCAS i” or “ACAS UAV i” and one from place “P”. If a secondary RA is present, then the tokens from the incoming places are eaten and a token with the colours of the token from place “P” of LPN “Adjusted RA” of agent “TCAS i” or “ACAS UAV i” is fired to place “P” through an outgoing arc.

Transition G1 has one incoming, two outgoing and two incoming enabling arcs. The incoming arc comes from place “P”. The enabling arcs are coming from place “Working” of LPN “CDTI” and place “Working” of LPN “Aural Annunciation”. A token in each of the places mean that the cockpit display instruments and the aural annunciation system are working fine. If those systems are working and the colour of the token in place “P” indicates that an RA is present and the pilot reaction time has elapsed, then a token is fired back to place “P” with colours indicating that no RA is present and a token to place “P” of IPN “Pilot Input” with colours describing the pilot input to the aircraft.

Transition G2 has one incoming, two outgoing, and one enabling arc. The incoming arc comes from place “P”. The enabling arc is coming from place “P” of IPN “Auto Pilot”. If the aircraft is flying with the autopilot, then there is a token in place “P” of IPN “Auto Pilot”, which has then no effect on the time of enabling transition G2. In this case, immediately when the colour of the token in place “P” indicates that an RA is present, then a token is fired back to place “P” with colours indicating that no RA is present and a token to place “P” of IPN “Pilot Input” with colours describing the pilot input to the aircraft.

IPN “Auto Pilot” has one place “P” which has initially a token if an autopilot is reacting upon RAs, e.g. in an UAV, else there is no token. In this Petri net model it is assumed that the pilot, including the autopilot, does only react upon RAs. The colour of the token in place “P” determines whether an RA is present or not.

Place “P” of IPN “Pilot Input” is the only place in the IPN and a present token would mean that the pilot or autopilot is changing the current aircraft settings, i.e. velocity or heading. Initially there is no token in place “P”.

LPN “CDTI”

The LPN “CDTI” consists of one LPN with two places and two transitions. The places are “Working” and “Not working” and there is always one token in one of the two. A token in place “Working” means that the cockpit display system is working properly and a token in place “Not working” means that the system is not working properly. Therefore, one enabling arc is going from place “Working” to transition G in LPN “Pilot Flying” to let the pilot react upon an RA.

Tokens in places “Working” and “Not working” have one colour, a delay time, and may move places through transitions G1 and G2. Transition G1 transfers a token from place “Not working” to “Working” and transition G2 transfers a token from place “Working” to “Not working”. Both transitions require the delay time to have elapsed and set a new delay time for the new place.

The delay time to prevent a transition from “Working” to “Not working” through transition G2 will be based on technical requirements for aircraft systems, which typically require system failures to be less than $1 \cdot 10^{-9}$ per flight hour.

Additionally, it is assumed that a system failure may only be repaired on the ground. Therefore the time delay for a transition from “Not working” to “Working” is set equal to the length of one simulation run.

LPN “Aural Annunciation”

The LPN “Aural Annunciation” consists of one LPN with two places and two transitions. The places are “Working” and “Not working” and there is always one token in one of the two. A token in place “Working” means that the aural annunciation system is working properly and a token in place “Not working” means that the system is not working properly. Therefore, one enabling arc is going from place “Working” to transition G in LPN “Pilot Flying” to let the pilot react upon an RA.

Tokens in places “Working” and “Not working” have one colour, a delay time, and may move places through transitions G1 and G2. Transition G1 transfers a token from place “Not working” to “Working” and transition G2 transfers a token from place “Working” to “Not working”. Both transitions require the delay time to have elapsed and set a new delay time for the new place.

The delay time to prevent a transition from “Working” to “Not working” through transition G2 will be based on technical requirements for aircraft systems, which typically require system failures to be less than $1 \cdot 10^{-9}$ per flight hour.

Additionally, it is assumed that a system failure may only be repaired on the ground. Therefore the time delay for a transition from “Not working” to “Working” is set equal to the length of one simulation run.

14.4 Agent “Communication i”

Agent “Communication i” consists of 3 LPNs and 2 IPNs. The LPNs are the following:

- Transponder,
- Mode S Reply, and
- Interrogation.

The IPN is the following:

- Measurements, and
- Int. mes. from aircraft i to k.

The above mentioned LPNs and IPNs are addressed shortly below. However it should be already noted that in this model two assumptions are made. First, aircraft ‘i’ knows that aircraft ‘k’ is in the vicinity and no identification message is being sent every 8-10 seconds. Second, the measurements about slant range and relative bearing and the estimate about vertical range are included in the interrogation response of any aircraft ‘i’, see Equations (21)-(23).

LPN “Transponder”

This LPN has two places, “Working” and “Not working”, and two transitions, G1 and G2. It is similar to the LPNs “CDTI” and “Aural Annunciation” of agent “Cockpit i”.

Initially there is only one token in place “Working” with a colour describing a time delay determining when the next transition takes place. Transition G2 moves the token from place “Working” to “Not working” and sets a new time delay. Transition G1 moves the token from place “Not working” to “Working” and sets a new time delay.

Additionally there is one enabling arc from place “Working” to transition I3 of LPN “Mode S Reply” and one enabling arc from place “Working” to transition G of LPN “Interrogation”. As such this agent models whether the transponder of aircraft ‘i’ is working.

LPN “Mode S Reply”

The LPN has two places, “Not sending” and “Sending”, and four transitions, I1, I2, I3 and I4. This LPN creates the reply message to be sent to aircraft “k” for an interrogation message from aircraft “k”.

Initially there is a token in place “Not sending” meaning that no message is being sent. This token also has a colour containing information about the sense coordination.

From place “Not sending” there are in total three outgoing arcs, to transitions I1, I2 and I3, and three incoming arcs, from transitions I1, I2 and I4.

Transition I1 is activated when there is a token both in place “Not sending” and in place “P” of LPN “Sense coordination” of agent “TCAS i”. In this case the tokens from the input places are eaten and one token is fired back to place “Not sending” with colour values describing the coordination intent and whether a coordination message should be sent

Transition I2 is similar to I1, but it is activated when there is a token both in place “Not sending” and in place “P” of LPN “COC” of agent “TCAS i”. In this case the tokens from the input places are eaten

and one token is fired back to place “Not sending” with colour values describing that no coordination is available.

Transition I3 has two incoming arcs, two enabling arcs and one outgoing arc. The incoming arcs are coming from place “Not sending” and from place “P” of LPN “Int. mes. from aircraft k to i” of agent “Communication k”. The enabling arcs are coming from place “P” of LPN “State” of agent “Aircraft i” and from place “Working” of LPN “Transponder”. The outgoing arc goes to place “Sending”. If an interrogation message of aircraft ‘k’ is available, no message is being sent and the transponder is working, then transition I3 is activated and a token is fired to place “Sending”. The colours of the token contain the information about the sense coordination and own measured altitude. Transition I4 has only one incoming arc from place “Sending” and two outgoing arcs to place “Not sending” and place “Sent message from Aircraft i to k” of LPN “Measurements”. Hence, if a token is available in place “Sending”, then immediately one token is fired to place “Not sending” with colours containing that no sense coordination needs to be carried out. Sense coordination is only done once as it is assumed that the other aircraft receives the coordination message once it has been sent. The second token is fired to place “Sent message from Aircraft i to k” of LPN “Measurements” with the colours of the incoming token.

IPN “Measurements”

The IPN has three places and one transition. Place “Sent message from Aircraft i to k” has one incoming arc from transition I4 of LPN “Mode S Reply” and one outgoing arc to transition I. A token is present there if a message has been sent by aircraft ‘i’.

In that case, the token is immediately eaten by transition I and two tokens are fired. One token is fired to place “Coordination message from Aircraft i to k” with the colours containing information about the sense coordination from the colours of the incoming token.

The second token is fired to place “State message from Aircraft i to k” with colours containing the state information from the colours of the incoming token and the measurements about the slant range, vertical range, and relative bearing. The additional measurements, see Equations (21)-(23), are computed using the state information coming from the enabling arcs from agents “Aircraft i” and “Aircraft k”.

The tokens in places “State message from Aircraft i to k” and “Coordination message from Aircraft i to k” may then be picked up by the agent “TCAS k” through outgoing arcs.

IPN “Interrogation” and IPN “Int. mes. from aircraft i to k”

The IPN has one place “P” and one transition G. Initially at place “P” a token is present with a colour value equalling the delay until the next interrogation message should be sent.

Transition G has one incoming arc from place “P”, one enabling arc from place “Working” of LPN “Transponder” and two outgoing arcs. If the time delay in place “P” has elapsed and the transponder is working then and one token is fired back to place “P” and its colour value is set to 1s delay, and one token without colour is fired to place “P” of IPN “Int. mes. from aircraft i to k”.

The token in place “P” of IPN “Int. mes. from aircraft i to k” may then be picked up by the intruder through an outgoing arc to the LPN “Mode S Reply” of agent “Communication k”.

15 Petri Net Model MATLAB

The Petri Net model for TCAS operations is simulated in MATLAB. First, the implementation strategy is presented. Second, the verification process of the Petri Net model of TCAS in MATLAB is given.

15.1 Implementation Strategy

The specified Petri Net model has been implemented in MATLAB using the following strategy:

- Inside out method has been applied. I.e. first the places and transitions have been programmed, followed by the petri net process of agents with multiple LPNs, followed by the main Petri Net process running the complete simulation.
- Places are captured variables and transitions are captured as functions.
- Variables are in Matrix shape (n x m).
- Each column of any variable contains the state of a place of aircraft 'i', whether a token is present, and its colours, if colours available. Since only single threat encounters are assumed, each variable has two columns. The first for aircraft 'i' and the second for aircraft 'k'.
- LPNs which have two places and there is always a token present in one of both, are combined as one variable, where the first row of a variable determines at which location the token is. E.g. for LPN "CDTI": if the first entry is 1, then the token is in place "Working", else the first entry is 0 and the token is in place "Not working".
- For places where always a token is present, the first entry is skipped as the value would never change.
- The Petri Net process of all agents is programmed as the script file called *main.m*.
- The Petri Net processes of agents with multiple LPNs are programmed as separate script files. E.g. script *main.m* is calling agent "TCAS Aircraft i" by calling script *TCAS_cycle.m* which then calls the functions of the LPNs.
- Guards, which check for the same condition, are programmed as a single function with multiple inputs and outputs for the different places. E.g. guards checking for Equation (156) are modelled in the function *Reset_TCAS_States.m*.
- Transitions, which may change the colours of the same token, may be programmed in a single function to save computation power. E.g. transitions I and G of agent "State Aircraft i" are captured in function *Update_AC_States.m* where the functions first checks for transition I and afterwards for transition G.
- Random variables, i.e. to model measurement errors, are created using inbuilt MATLAB functions, e.g. *normrnd(X,Y)* to draw a sample from a normal distribution with mean 'X' and standard deviation 'Y', using the setting *rng('shuffle')* for different samples in different simulations.

Furthermore, there are two special cases in the program:

- In order to simplify the program and save computation time, transition G of LPN “Sense Selection” of agent “TCAS Aircraft i” is split into two MATLAB functions.
 - One function checks for the condition of Equation (156) such that a TCAS reset with a clear of conflict message is performed globally to save computation power.
 - Equation (138) is accounted for in the script of agent “TCAS Aircraft i”, as it only consists of one if statement.
- In order to simplify the program and save computation time, the Petri Net process from places “Sense coordination” and “COC” of IPN until places “Messages to aircraft k with slant range measurement” and “Message to Aircraft k (2)” of IPN through LPN “Mode S Reply” is programmed in a separate script, *Mode_S_Link.m*, calling the function *Send_Message.m*.

15.2 Verification of Petri Net Model of TCAS

The Petri Net model implementation in MATLAB has been successfully verified in two phases. This section presents the verification strategy, the results of phase 1, and the results of phase 2.

Verification Strategy

The MATLAB model has been verified successfully in two phases:

- Phase 1: Without measurement errors and pilot delay.
- Phase 2: With measurement errors and pilot delay.

Both phases were carried out using the following strategy:

0. If an error is found and corrected, the verification process will have to start from point 1 again.
1. Verification of variables:
 - a. Are all places captured in variables?
 - b. Have the variables the correct dimensions?
2. Verification of functions:
 - a. Assessing the performance of a function by inputting different input parameters and checking whether the output is as expected.
 - b. Checking each step of a function by using Debug Mode of MATLAB and assessing each calculation step separately.
3. Verification of agent processes (scripts *TCAS_cycle.m* and *Mode_S_Link.m*)
 - a. Assessing the overall performance by inputting different input parameters and checking whether the output is as expected.
 - b. Checking each step of the script by using Debug Mode of MATLAB and assessing each calculation step separately. In cases a called function does not give the expected output, each calculation step within the function is assessed using the "Step In" debug option.
4. Verification of complete Petri Net process, script *main.m*,
 - a. Assessing the overall performance by inputting different input parameters and checking whether the output is as expected. The output is assessed in two ways:
 - i. Assessing the values of places whether they are as expected.
 - ii. Visually assessing the plots of aircraft positions and altitudes.
 - b. Checking each step of the script by using Debug Mode of MATLAB and assessing each calculation step separately. In cases a called function does not give the expected output, each calculation step within the function is assessed using the "Step In" debug option.

Verification Results of Phase 1

Verification of Phase 1 was carried out successfully for almost two weeks. During the verification process the following corrections and model improvements have been made:

- Several small programming errors, i.e. typos, wrong or missing unit conversion, have been corrected.
- The complete sequence of calling transitions has been optimized. I.e. first agent "TCAS i" is carried and afterwards agents "Communication i" and "Cockpit i" are carried out. Also within agent "TCAS i" first the transitions are carried out which check whether the system should be reset, e.g. transition G2 of LPN "TA Module" or transition G2 of LPN "Evolution monitoring". Only afterwards the remaining transitions are carried out in the order how TCAS operates, e.g. first LPNs "State Estimation" and "Received Coordination Message", then LPN "TA Module" and so on.
- Functioning of the Cartesian Tracker of the horizontal miss distance filter has been identified to be erroneous when the prediction error is large. It was identified that this behaviour is according to the horizontal miss distance specification. Further research resulted that when the horizontal miss distance filter declares a manoeuvre, then the horizontal miss distance filter should be paused for 10 seconds (Hammer, 1996a, p.30).
- Erroneous determination of smoothing parameters of vertical range filter. It was identified that the specification (ICAO, 2006, pp.3-58 - 3-59) assumes only positive altitude measurements. Equations (40) and (41) have been updated accordingly.
- Erroneous sense coordination between aircraft. This problem could be traced back to a programming error which updated the places of the wrong aircraft.

Verification Results of Phase 2

After the successful completion of phase 1 verification, the verification of phase 2 was carried out in less than a week. Equations unaffected by measurement errors and pilot delay have not been verified, as these have been verified in Phase 1 already.

In Phase 2 no errors could be found, however unexpected behaviour was identified. In some cases RAs resulting in an altitude crossing of both aircraft were issued although both aircraft were in levelled flight. I.e. Aircraft 1 is located higher in altitude than Aircraft 2 and both aircraft are neither climbing nor descending, but Aircraft 1 decides to pass below Aircraft 2, although a "Climb" RA would lead the greater vertical miss distance and avoids an altitude crossing.

This behaviour could be traced back to the measurement errors and the vertical range filter. In some cases the vertical range filter indicates that both aircraft are approaching in altitude in such way, that only an altitude crossing would lead to sufficient vertical separation, although the real altitude difference was constant.

Therefore, it is concluded that the MATLAB implementation has been verified successfully.

Part III

16 Validation of New Model of TCAS

In the study (Netjasov, 2010), nine cases were simulated in InCAS using TCAS Version 7.0. Using the initial conditions stated in the study (Netjasov, 2010), the nine cases have been simulated in the Petri Net model and in InCAS 3.3 using TCAS Version 7.1. As InCAS requires radar plots as input, radar plots have been created by extrapolating the initial conditions for 300s. It should be noted that in the format used, swisscontrol, the x and y coordinates are quantised with unit 1/64NM and the altitude is quantised with unit 1/100ft (Flight Level).

The simulation results have then been used to validate the new TCAS model versus InCAS. The results are given in [Table 16-1](#) and discussed in this section.

First in Section 16.1 the InCAS results are checked for their credibility.

Next, in Section 16.2 similarities between InCAS and the new TCAS model are identified in cases 1, 7, 8 and 9.

Then in Section 16.3 differences between InCAS and the new TCAS model identified in cases 2, 3, 4, 5 and 6.

Last, a conclusion is drawn on the validity of the new TCAS model.

Table 16-1: Validation Results of new TCAS model and InCAS.

Case	Model type	A/C	Initial condition					TA			Initial RA					Secondary RA					COC			Min. separation			
			X coord. [NM]	Y coord. [NM]	Altitude [ft]	vertical velocity [ft/min]	Ground speed [knots]	Magnetic heading [°]	time [s]	vert. sep. [ft]	hor. sep. [NM]	time [s]	sense	strength (min,max) [ft/min]	vert. sep. [ft]	hor. sep. [NM]	time [s]	sense	strength (min,max) [ft/min]	vert. sep. [ft]	hor. sep. [NM]	time [s]	vert. sep. [ft]	hor. sep. [NM]	time [s]	vert. sep. [ft]	hor. sep. [NM]
1	Own model	1	0	5	26000	0	540	360	N/A			N/A					N/A					N/A			156.7	2410	1.28
		2	-10.6	23.5	16800	2600	240	65	N/A			N/A					N/A					N/A					
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						N/A			N/A					N/A					N/A			157	2399	1.3
		2							N/A			N/A					N/A					N/A					
2	Own model	1	0	4.3	40500	-3600	480	360	144.1	2854	1.08	168.1	LO	0,0	1414	0.23	N/A					192.1	995	0.95	170.9	1246	0.23
		2	-6.5	7	29000	0	440	18	167.1	1474	0.25	168.1	DE	-2000,-1500	1414	0.23	174.1	DCL	-2000,0	1059	0.29	192.1	995	0.95			
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						144	2860	1.1	169	LO	0,0	1360	0.2	N/A					177	976	0.4	171	1240	0.2
		2							144	2860	1.1	169	DCL	-4400,0	1360	0.2	170	DES	-2000,-1500	1300	0.2	177	976	0.4			
3	Own model	1	0	4	36700	-1600	430	360	166.1	1271	0.95	N/A					N/A					N/A			175.7	1015	0.01
		2	15.7	11.8	31000	0	420	310	N/A			N/A					N/A					N/A					
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						165	1300	1.1	N/A					N/A					N/A			176	1009	0
		2							158	1487	1.8	N/A					N/A					N/A					
4	Own model	1	0	13.8	39900	-2500	410	360	46.1	1979	10.19	59.1	LO	0,0	1438	7.12	N/A					94.1	1111	1.19	89.3	1111	0.36
		2	1.5	34.8	36000	0	440	186	N/A			N/A					N/A					N/A					
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						46	1987	10.2	59	LO	0,0	1445	7.1	N/A					95	1153	1.4	89	1153	0.4
		2							45	2029	10.4	N/A			1196	4.8	N/A				N/A						
5	Own model	1	0	4.2	18200	-500	460	360	129.1	848	5.85	144.1	CL	1500, 2000	598	3.89	154.1	DDE	0, 4400	611	2.59	174.1	1029	0.07	174	1026	0.07
		2	-18	18	15200	500	410	65	129.1	848	5.85	144.1	DE	-2000, -1500	598	3.89	154.1	DCL	-4400, 0	611	2.59	174.1	1029	0.07			
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						128	860	6	145	LO	0,0	587	3.8	146	CL	1500, 2000	573	3.6	180	1075	0.8	174	1075	0.1
		2							128	860	6	144	DE	-2000, -1500	600	3.9	158	LO	0,0	972	2.1	180	1075	0.8			
6	Own model	1	0	3.3	26100	-2800	360	360	125.1	4790	1.26	144.1	LO	0,0	2890	1.02	N/A					174.1	2053	0.64	224.2	2053	0.05
		2	2.8	2.8	8800	3200	370	353	125.1	4490	1.22	144.1	LO	0,0	2890	1.02	N/A					174.1	2053	0.64			
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						125	4800	1.3	138	LO	0,0	3500	1.1	N/A					251	2684	0.3	192	2684	0.4
		2							126	4700	1.2	139	LO	0,0	3400	1.1	N/A					251	2684	0.3			
7	Own model	1	0	1.3	4200	-750	150	360	135.1	850	0.65	N/A					N/A					N/A			177.8	707	0.2
		2	-2.3	2.5	5500	-950	130	20	135.1	850	0.65	N/A					N/A					N/A					
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						133	849	0.7	N/A					N/A					N/A			177	706	0.2
		2							132	852	0.7	N/A					N/A					N/A					
8	Own model	1	0	4.3	29000	-2300	460	360	131.1	2997	6.71	144.1	LO	0,0	2174	4.52	N/A					179.1	1711	1.61	170.5	1711	0.68
		2	21.6	23.5	17700	1500	470	278	131.1	2997	6.71	144.1	LO	0,0	2174	4.52	N/A					179.1	1711	1.61			
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						130	3054	6.7	143	LO	0,0	2231	4.4	N/A					175	1798	1.1	169	1798	0.4
		2							131	2990	6.5	144	LO	0,0	2169	4.3	N/A					175	1798	1.1			
9	Own model	1	0	-33.2	38900	-2300	520	360	192.1	464	11.67	N/A					N/A					N/A			233	2032	4.63
		2	-11.8	26.8	32000	0	430	165	192.1	464	11.67	N/A					N/A					N/A					
	InCAS V3.3	1	300 measurement points in swiss radar plot format gained through extraloplation of above condition for t=0,....,299s						192	461	11.7	N/A					N/A					N/A			233	2031	4.6
		2							192	461	11.7	N/A					N/A					N/A					

Legend of RAs: LO - Level Off; CL - Climb; DE - Descend; DCL - Do not climb; DDE - Do not descend

16.1 Notable issues with InCAS V3.3 results

From the results given in given in [Table 16-1](#), two issues regarding TA generation and one issue regarding RA generation could be identified.

In cases 2, 3 and 4 InCAS generates TAs for both aircraft at approximately the same time. However, the “co-altitude is small” test, as described in Section 6, dictates different TA threshold values for aircraft climbing or descending with less than 600ft/min. The different threshold values are given in [Table 6-2](#) and vary between 10s and 23s. In cases 2, 3 and 4 the first aircraft is descending with more than 600ft/min and the second aircraft are in levelled flight. Hence, one would expect that issuing of a TA for the second aircraft to be at least 10s later.

In cases 2 and 5 it can be observed that InCAS issues an RA for an aircraft and one second later it issues a secondary RA for that aircraft.

In case 5 the initial RA for aircraft 1 is a corrective “Level Off” RA and one second later a corrective “Climb” RA is issued. It is not clear why InCAS does not give the pilot sufficient reaction time before strengthening the RA. In case 2 the initial RA for aircraft 2 is a preventive “Do Not Climb” RA and one second later a corrective “Descend” RA is issued. Opposed to case 5 this looks okay in case 2 as the pilot does not need to take actions upon the initial RA.

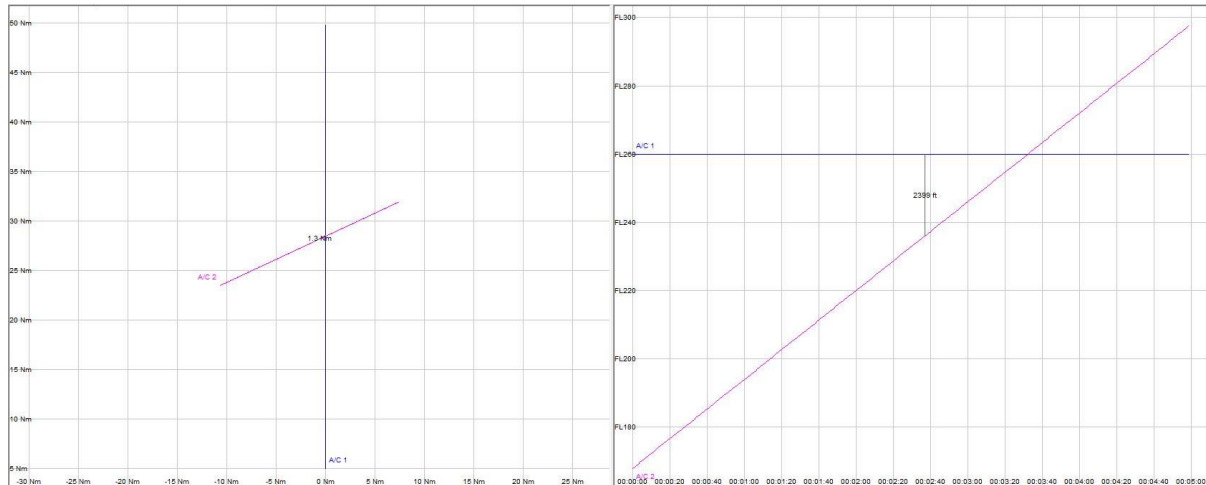
However, in both cases the pilots have not reacted upon the initial RAs. If the velocities are constant between two points in time, then the projected vertical miss distance should be constant as well. Therefore, it does not make sense why equal projected vertical miss distance estimates result in different RAs.

16.2 Similarities of InCAS vs. New TCAS Model Results

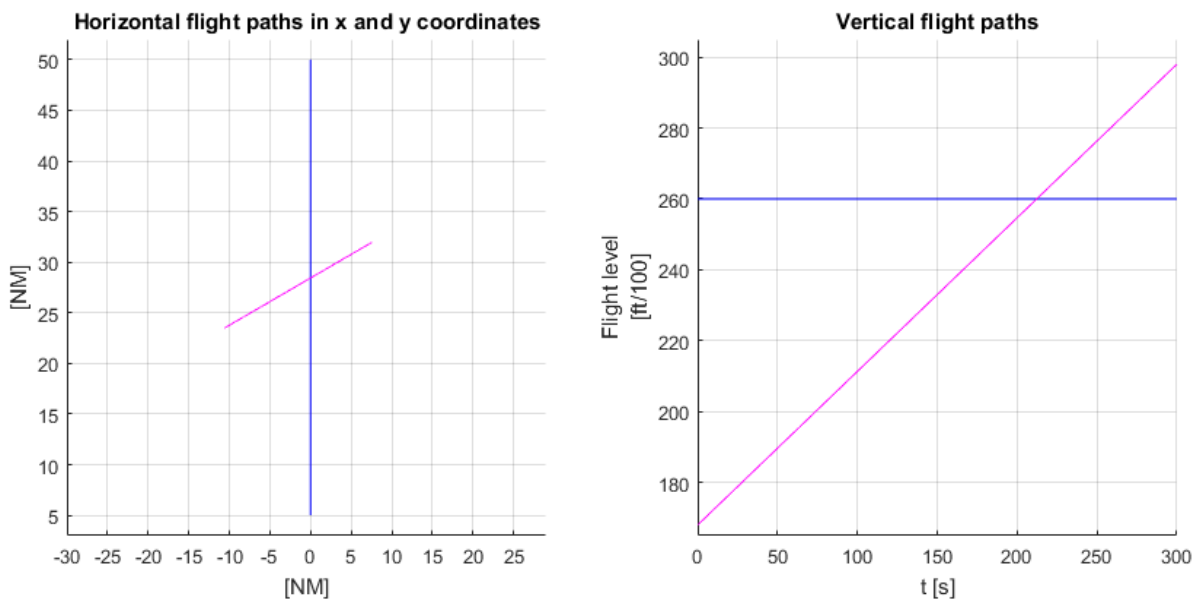
For cases 1, 7, 8 and 9 the results are the same. Only very small differences can be spotted which result from the quantization of the InCAS input. Next the results for cases 1, 7, 8 and 9 are presented.

Case 1:

As can be seen in Figure 16-1, both models do not issue TAs or RAs. Therefore the results are exactly the same.



a) Simulation results of InCAS V3.3.

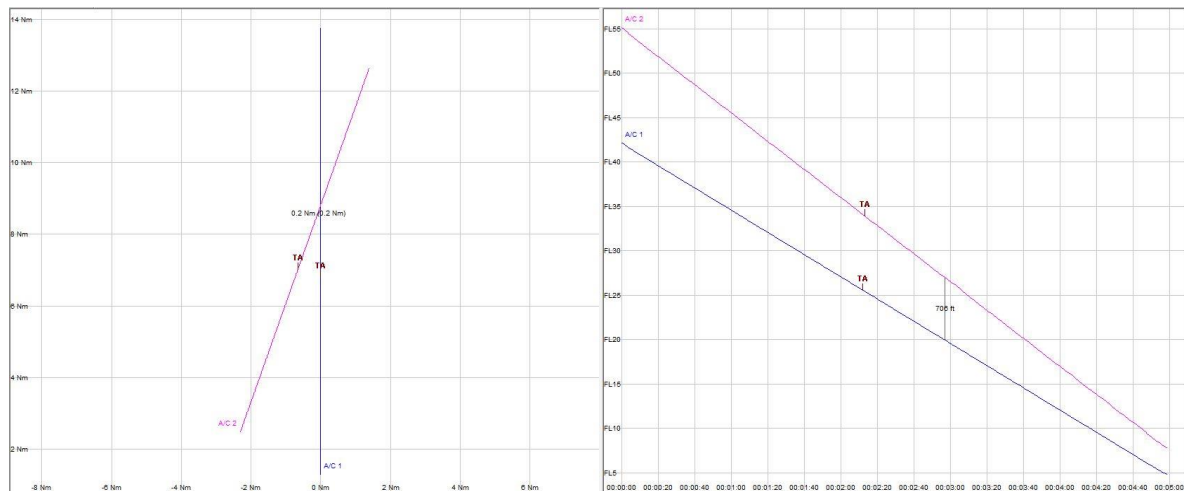


b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2.

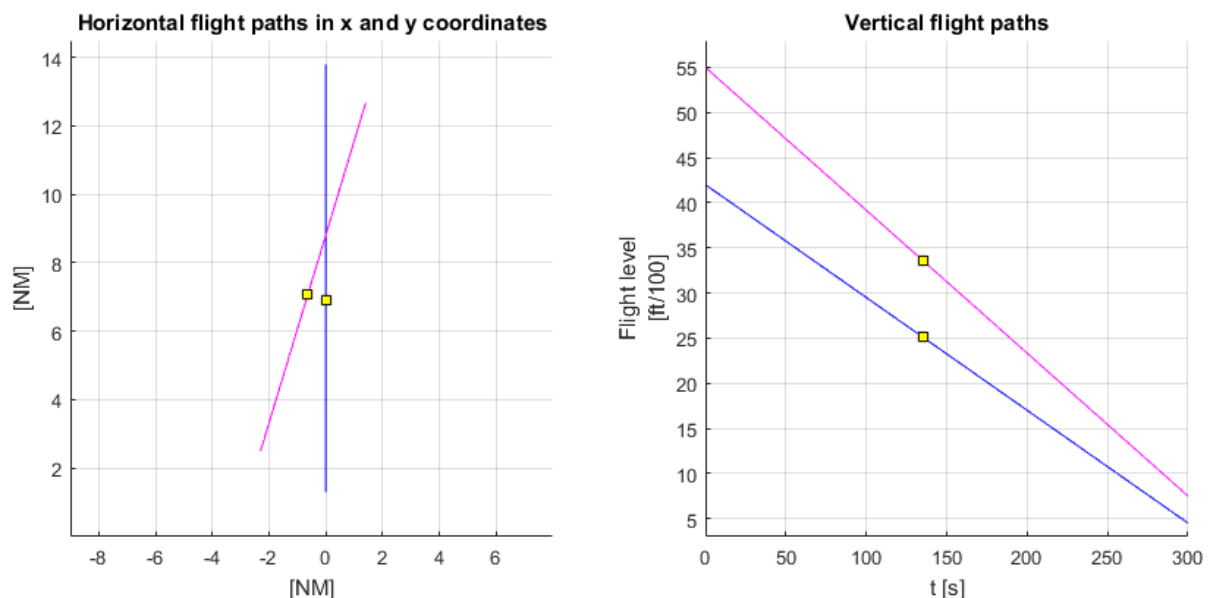
Figure 16-1: Graphical simulation results of case 1. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

Case 7:

In [Figure -16-2](#) it can be seen that both models issue TAs for both aircraft. Also, in [Table 16-1](#) it can be seen that the new TCAS model issues the TAs after 135.1s when the separation is 850ft vertically and 0.65NM horizontally. InCAS on the other hand issues a TA for aircraft 1 after 133s when the separation is 549ft vertically and 0.7NM horizontally, and for aircraft 2 after 132s when the separation is 852ft vertically and 0.7NM horizontally. Hence, the vertical separation distance is almost the same in both models when TAs are issued. So the small timing difference in issue of TAs between both models can be explained through the quantization of the input data of InCAS.



a) Simulation results of InCAS V3.3.

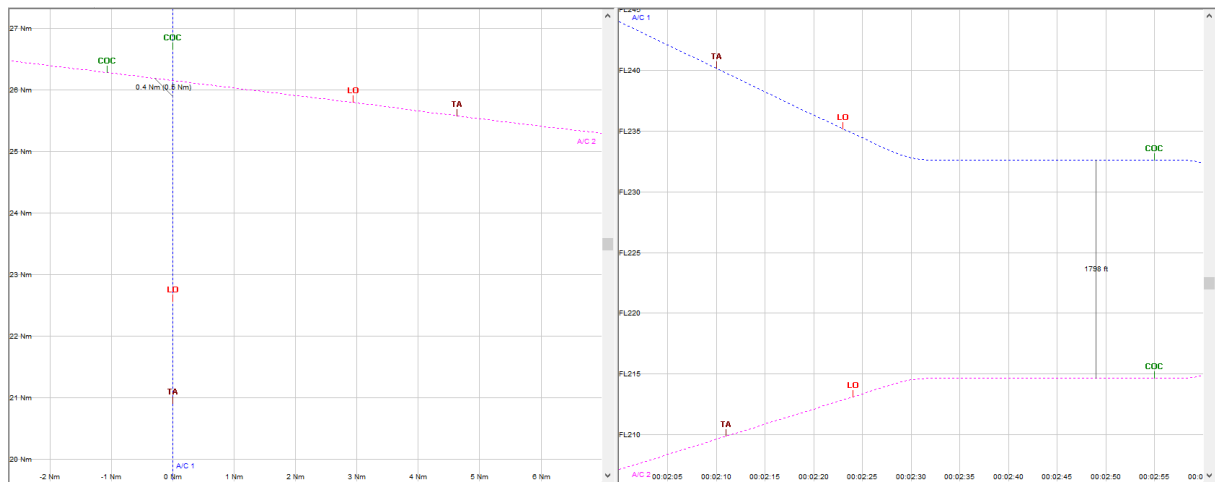


b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA.

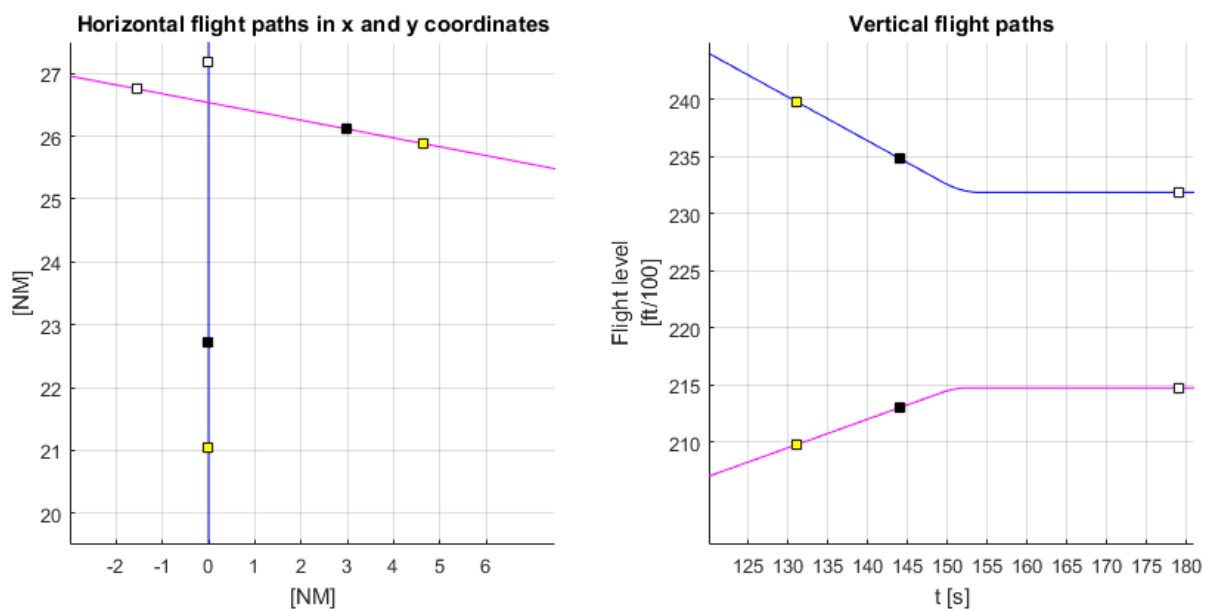
Figure -16-2: Graphical simulation results of case 7. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

Case 8:

In [Figure 16-3](#) it can be seen that both models respond the same. In [Table 16-1](#) small differences can be seen in the vertical separation achieved through the RAs which is 1711ft in the new TCAS model and 1798ft in InCAS. The difference can be traced back that InCAS issues the “Level-Off” RA of aircraft 1 one second before the RA of aircraft 2. This is likely due to the quantization of the input of InCAS.



a) Simulation results of InCAS V3.3.



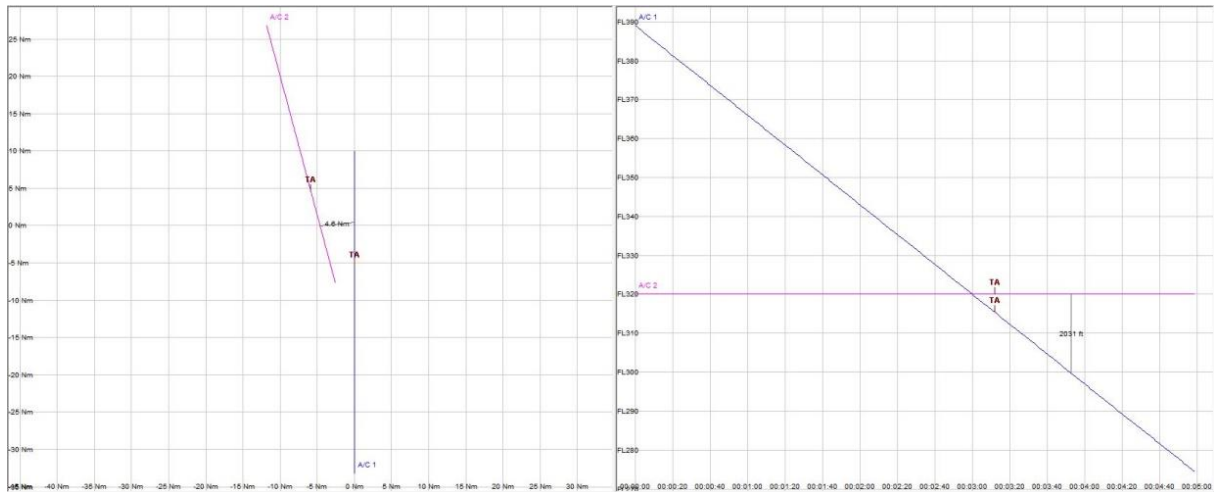
b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA; black box: RA; white box: COC message.

Figure 16-3: Graphical simulation results of case 8. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

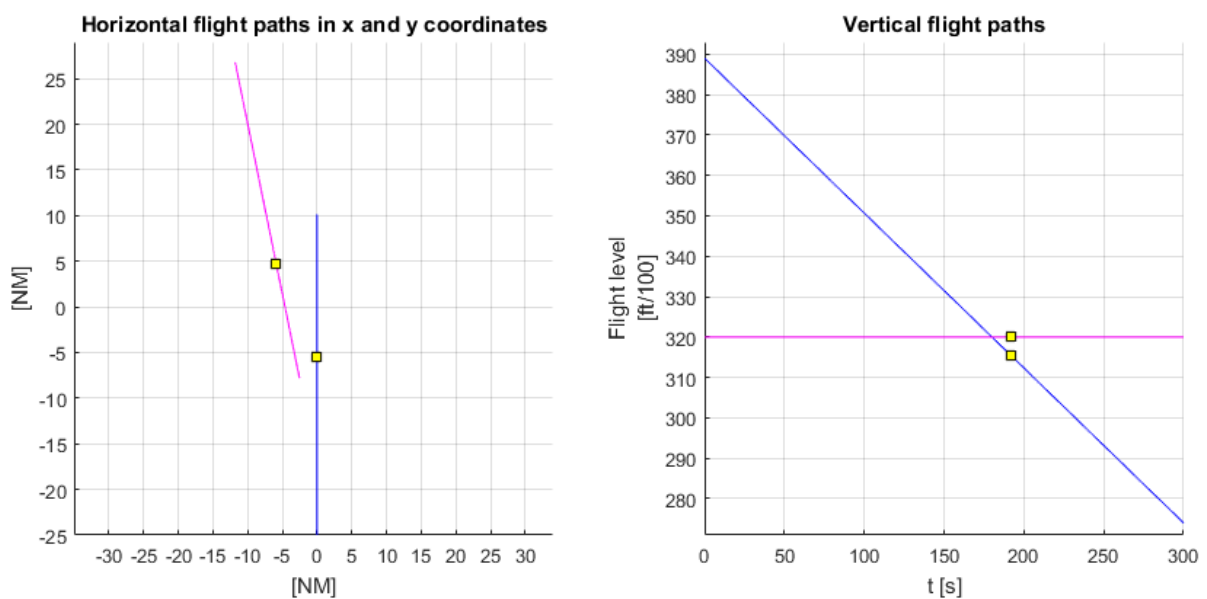
Also in [Table 16-1](#) it can be seen that the clear of conflict message in InCAS is issued after 175s and in the new TCAS model after 179.1s. The difference is small and it is likely that InCAS estimated a slightly shorter time until CPA due to the quantization of the input.

Case 9:

In [Figure 16-4](#) it can be seen that the results of the new TCAS model and InCAS are exactly the same. Also, in [Table 16-1](#) it can be seen that the issue of TAs in both models happens at the same time and separation, and the minimum separation is also exactly the same.



a) Simulation results of InCAS V3.3.



b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA.

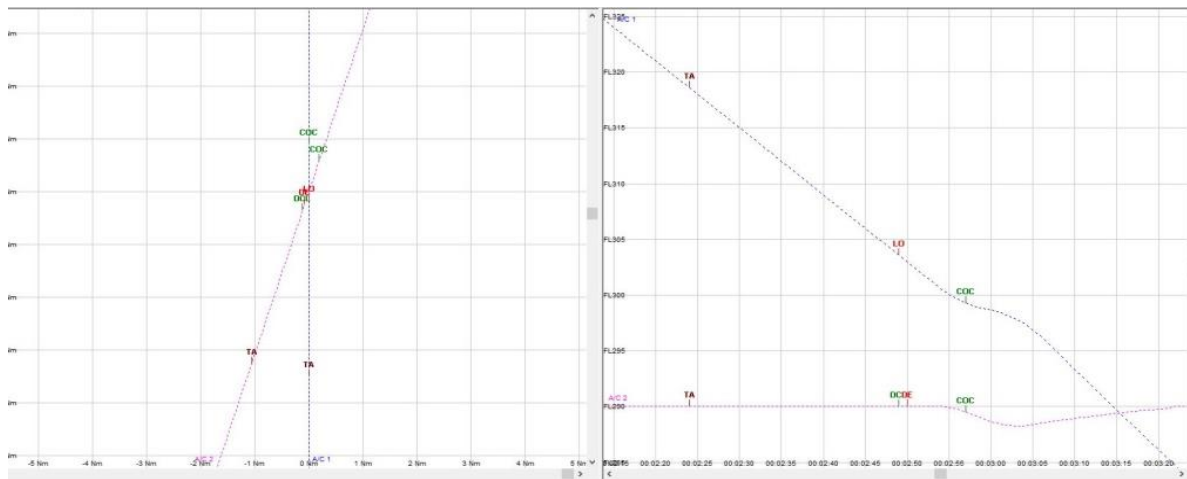
Figure 16-4: Graphical simulation results of case 9. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

16.3 Differences of InCAS vs. New TCAS Model Results

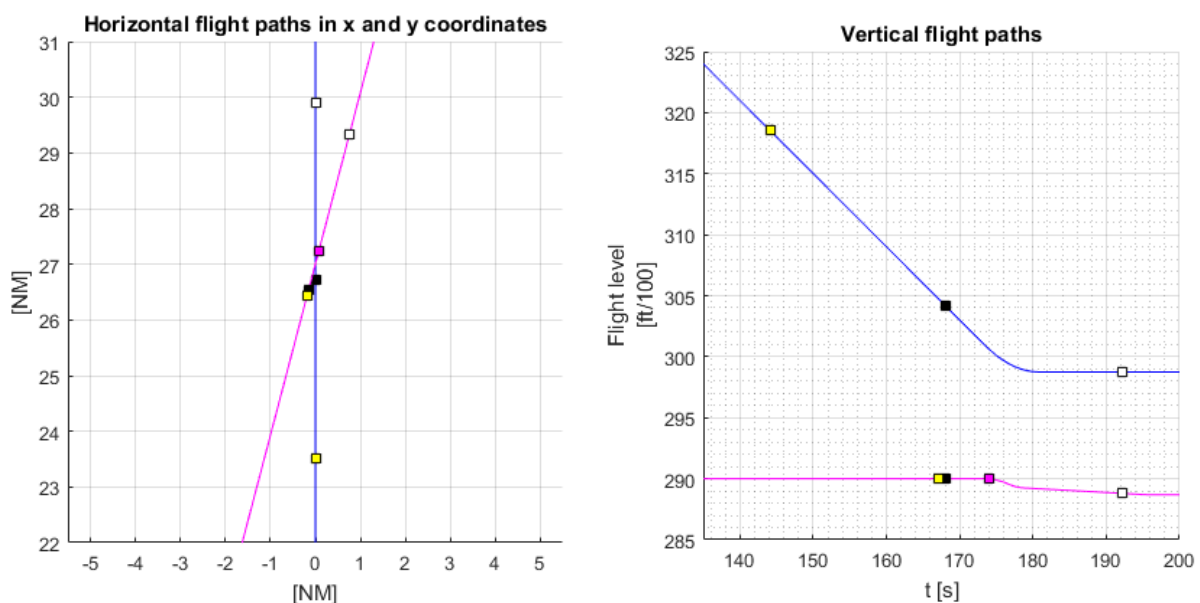
Differences in results can be found in cases 2, 3, 4, 5 and 6. Next the differences per case are discussed.

Case 2:

The graphical results of case 2 are given in the figure below. The numerical results are given in [Table 16-1](#).



a) Simulation results of InCAS V3.3.



b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA; black box: RA; magenta box: RA adjustment; white box: COC message.

Figure 16-5: Graphical simulation results of case 2. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

In [Figure 16-5](#) it can be seen that InCAS issues TAs for both aircraft at the same time, while the new TCAS model issues the TA for the second aircraft 23s later. As already discussed previously, this behaviour of TCAS does not make sense while the new TCAS model shows the expected behaviour,

as the TA threshold difference is exactly 23s, see [Table 6-1](#), for aircraft between flight level 100 and 200 and for aircraft above flight level 200.

Further differences can be identified in the issue of RAs. The new TCAS model issues a “Level-Off” RA for aircraft 1 and a “Descend” RA for aircraft 2. Then 6s later the “Descend” RA of aircraft 2 is weakened by a “Do Not Climb” RA. InCAS also issues a “Level-Off” RA for aircraft 1 but a “Do Not Climb” RA for aircraft 2. Then 1s later InCAS updates the RA of aircraft 2 with a “Descend” RA.

As already discussed before, it does not make sense that InCAS corrects an RA only 1s after it has been issued. First, InCAS does not consider measurement errors. Second, the pilot reacts 5s after an initial RA. From this it can be deduced that the velocities of both aircraft are constant until $t=174s$, which also leads to the fact that predicted vertical miss distance is constant until $t=174s$. Hence, if a “Do Not Climb” RA is valid at $t=169s$ it should also be valid at any point in time until the velocity of one aircraft changes, which is $t=174s$.

Nevertheless, it should be noted that InCAS corrects the RA to the same RA issued by the new TCAS model from the beginning. Therefore the trajectories flown by both aircraft in both models are very similar.

Another difference can be identified at the time the clear of conflict messages are issued. InCAS issues the clear of conflict messages at $t=177s$. The new TCAS model issues the clear of conflict messages significantly later at $t=192.1s$.

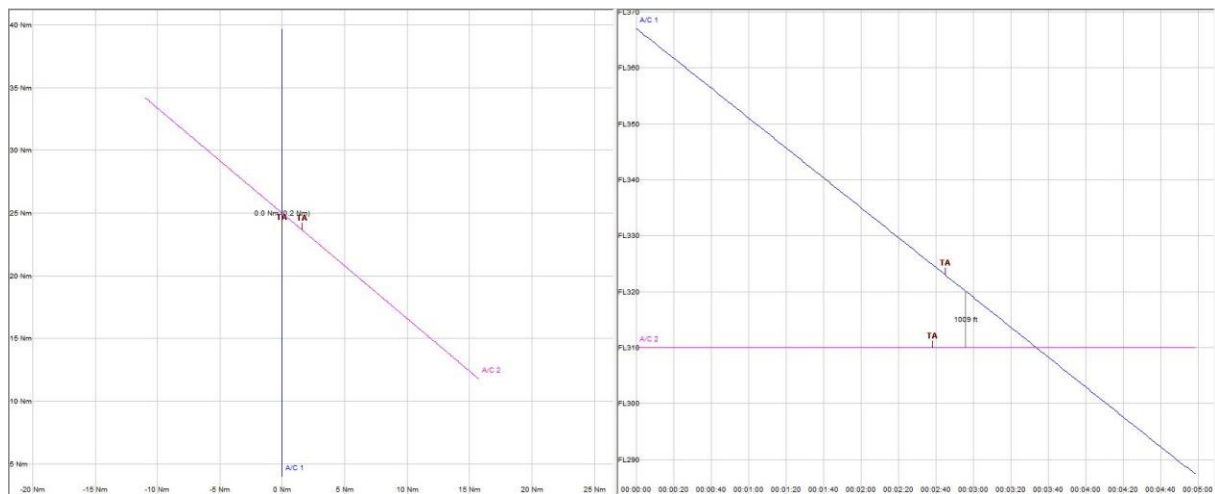
As described in Section 10, a conflict message may only be issued if the range and altitude test are negative and the time until initially estimated CPA has been elapsed or the horizontal miss distance filter is activated.

In the new TCAS model the vertical separation at the initial RAs was 1414ft at time $t=168.1s$. Given the relative vertical velocity of 3600ft/min of both aircraft, the estimated time to co-altitude at initial RAs is 23.57s. Hence, one would expect the clear of conflict messages to be issued earliest at $t=191.67s$, which is what happened in the Petri Net simulation.

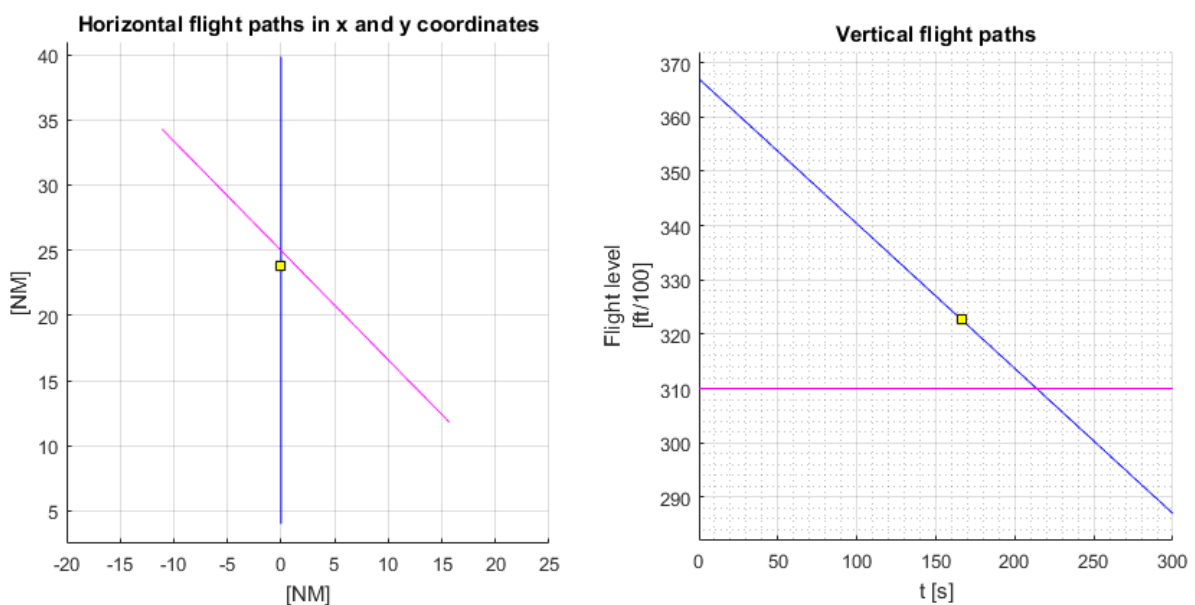
However, for InCAS, the RA was issued at $t=169s$ where the time to co-altitude was 22.67s. Hence, one would expect the clear of conflict messages earliest at $t=192s$ like it is in the new TCAS model. Therefore, it cannot be explained why InCAS issued the clear of conflict messages significantly earlier at $t=177s$.

Case 3:

The graphical results of case 3 are given in the figure below. The numerical results are given in [Table 16-1](#).



a) Simulation results of InCAS V3.3.



b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA.

Figure 16-6: Graphical simulation results of case 3. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

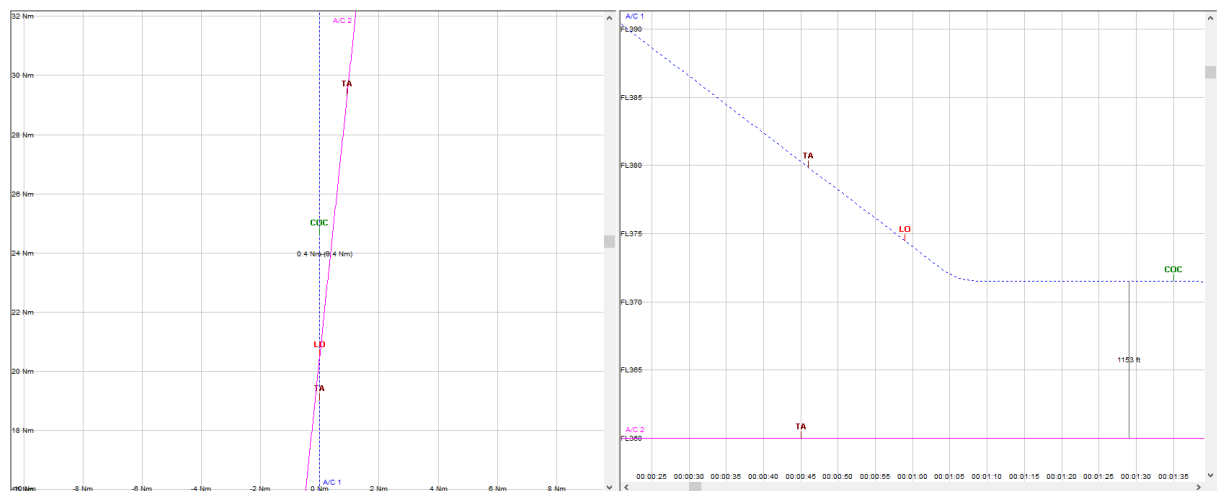
As can be seen in [Figure 16-6](#) both models issue a TA for aircraft 1 at the same time. However, for aircraft 2, the new TCAS model does not issue a TA at all, while InCAS issues a TA significantly earlier.

According to the “time to co-altitude is small” test, as described in Section 6, and the TA threshold values given in [Table 6-2](#), one would expect a TA for aircraft 1, which is descending with 1,600ft/min, to be issued at 48s before CPA, and for aircraft 2, which is in levelled flight, to be issued at 25s before CPA.

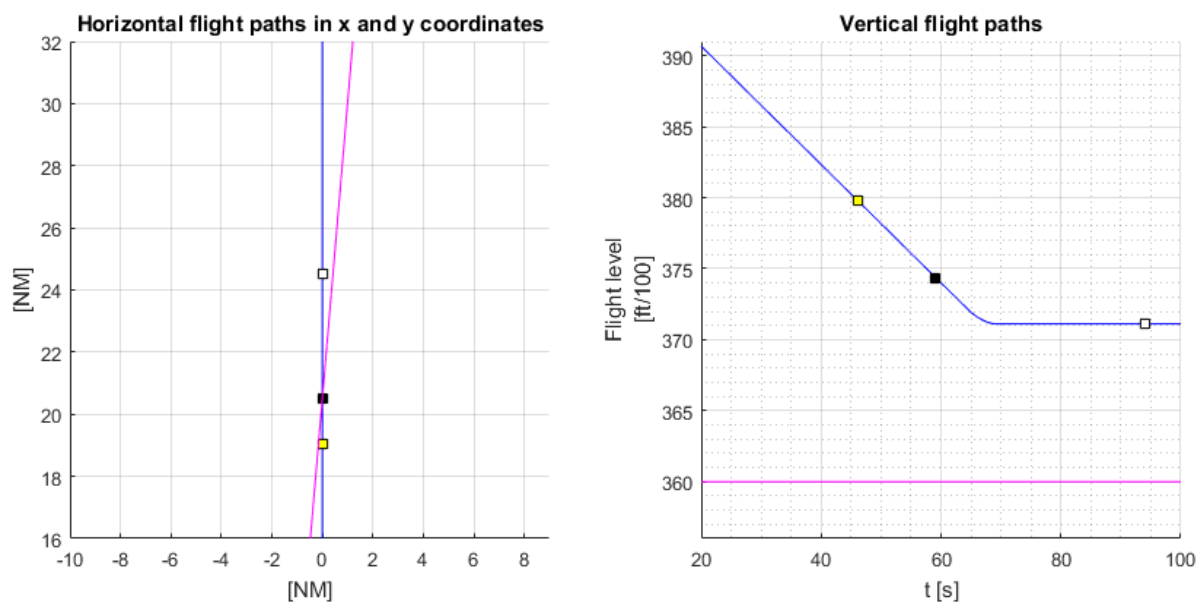
Additionally, from the vertical separation at CPA and the vertical velocities given in [Table 16-1](#) it can be calculated by hand that at CPA the time to co-altitude is 37.8s which is above the threshold value of aircraft 2 but below the threshold value of aircraft 1. Hence, it makes sense that the new TCAS model does only issue a TA for aircraft 1 and it cannot be explained why this is not the case in InCAS.

Case 4:

The graphical results of case 4 are given in the figure below. The numerical results are given in [Table 16-1](#).



a) Simulation results of InCAS V3.3.



b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA; black box: RA; white box: COC message.

Figure 16-7: Graphical simulation results of case 4. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

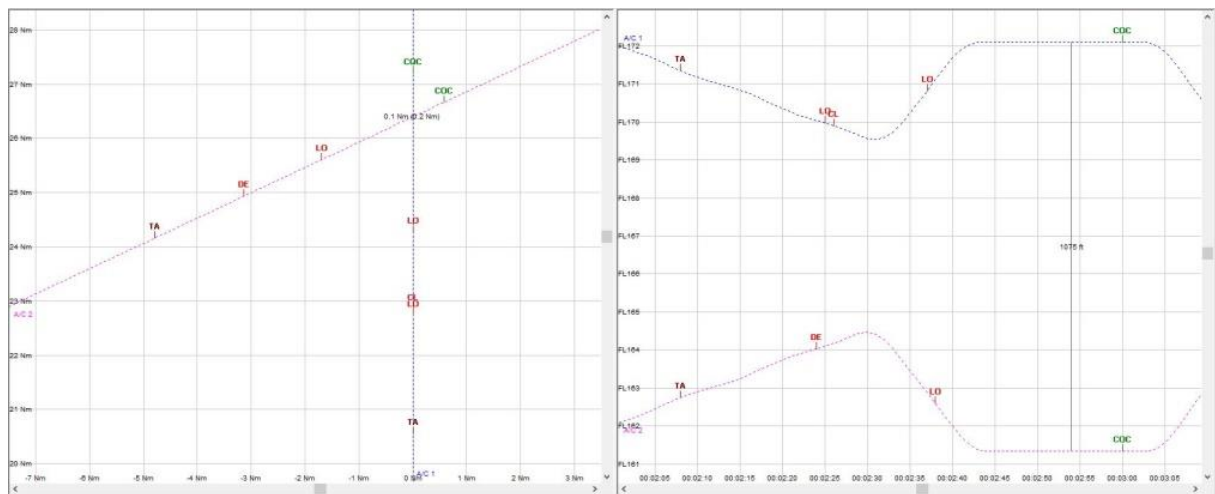
As can be seen in [Figure 16-7](#) the results of both models for case 4 are exactly the same with one exception. InCAS issues a TA for aircraft 2 and the new TCAS model does not issue a TA for aircraft 2.

According to the “time to co-altitude is small” test, as described in Section 6, and the TA threshold values given in [Table 6-2](#), one would expect a TA for aircraft 1, which is descending with 2,500ft/min, to be issued at 48s before CPA, and for aircraft 2, which is in levelled flight, to be issued at 25s before CPA.

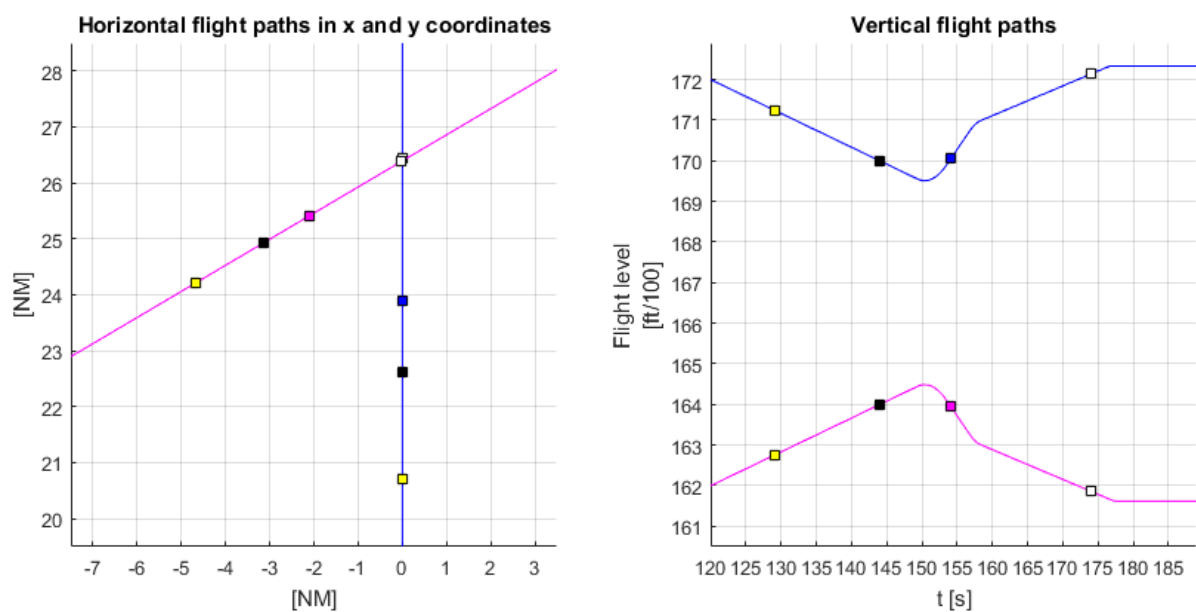
Therefore, one would expect the TA for aircraft 2 to be issued 23s after the TA has been issued in for aircraft 1, so at $t=69s$. However, in [Figure 16-7](#) it can be seen that at $t=69s$ (or 1:09min) the first aircraft is already in levelled flight and the aircraft are not approaching each other vertically. Thus, no TA is required after aircraft 1 has levelled off and it makes sense that the new TCAS model is not issuing a TA for aircraft 2. Why this is not the case in InCAS cannot be explained.

Case 5:

The graphical results of case 5 are given in the figure below. The numerical results are given in [Table 16-1](#).



a) Simulation results of InCAS V3.3.



b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA; black box: RA; blue/magenta box: RA adjustment; white box: COC message.

Figure 16-8: Graphical simulation results of case 5. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

In [Figure 16-8](#) differences in the issue of RAs can be identified. Both models issue the initial RAs at almost the time. The new TCAS model issues a “Climb” RA for aircraft 1 and a “Descend” RA for aircraft 2.

InCAS also issues a “Descend” RA for aircraft 2 but a “Level-Off” RA for aircraft 1. However, one second later InCAS issues a new RA for aircraft 2, which is a “Climb” RA.

It cannot be explained why InCAS issues first a “Level-Off” RA and then updates it one second later for two reasons.

First, the velocities of both aircraft are constant until about $t=150s$. Therefore, the projected vertical miss distance is constant until $t=150s$. So an RA update should only be necessary once the projected vertical miss distance is changing.

Second, the first “Level-Off” RA is a corrective RA which requires action by the pilot. So the pilot should be given sufficient time to react before updating the RA. As 5seconds is assumed as standard reaction time, as described in Section 10, it does not make sense that the RA is updated only after one second.

A second difference can be identified in the issue of secondary RA. The new TCAS model issues preventive “Do Not Climb” and “Do Not Descend” RAs at $t=154.1s$, but InCAS issues corrective “Level-Off” RAs at $t=157s$ and $t=158s$. The latter is inconsistent with Section 10.1 in (FAA, 2011, pp. 33-34) where it is stated that TCAS issues a “Do Not Descend” or “Do Not Climb” RA adjustment if sufficient altitude separation has been achieved:

“In Version 7.0 and later, after ALIM feet of separation has been achieved, the resulting Do Not Descend or Do Not Climb RA is designated as corrective. In Version 7.0, the RA is annunciated as “Adjust Vertical Speed, Adjust.” In Version 7.1, the RA is annunciated as “Level Off, Level Off.” (Version 6.04a keeps the original preventive designation, meaning that the RA is annunciated as “Monitor Vertical Speed.”)”

The above means that in version 7.1 the visual RA is “Do Not Climb/Descend” and the aural RA is “Level-Off, Level-Off”. Therefore, it is likely that InCAS made the assumption that the pilot follows the aural RA annunciation and not the visual.

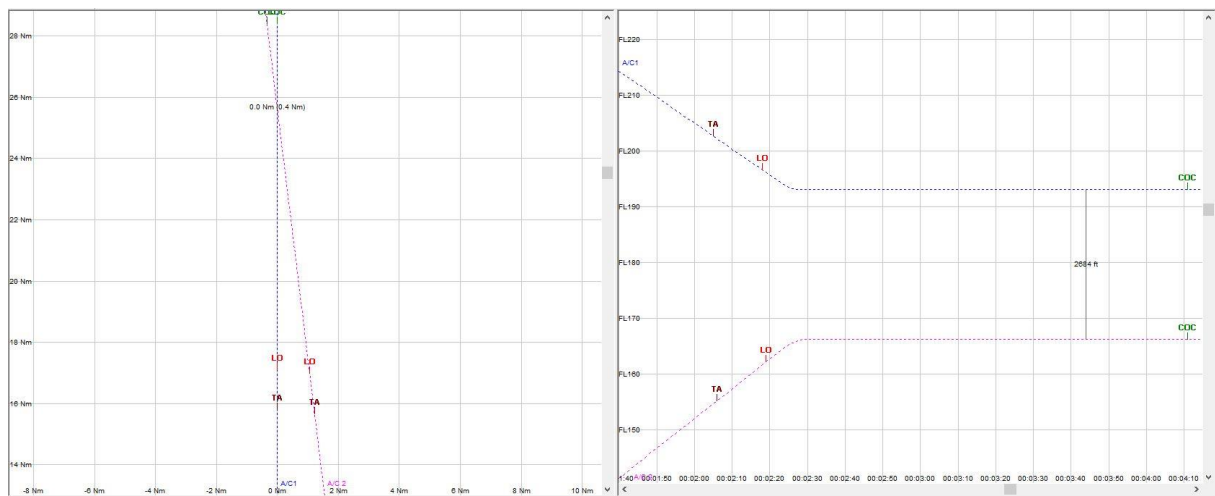
Hence, there are the following two options available:

- Option 1: Pilot follows aurally announced level off command.
- Option 2: Pilot follows indicated altitude velocity limits on cockpit display.

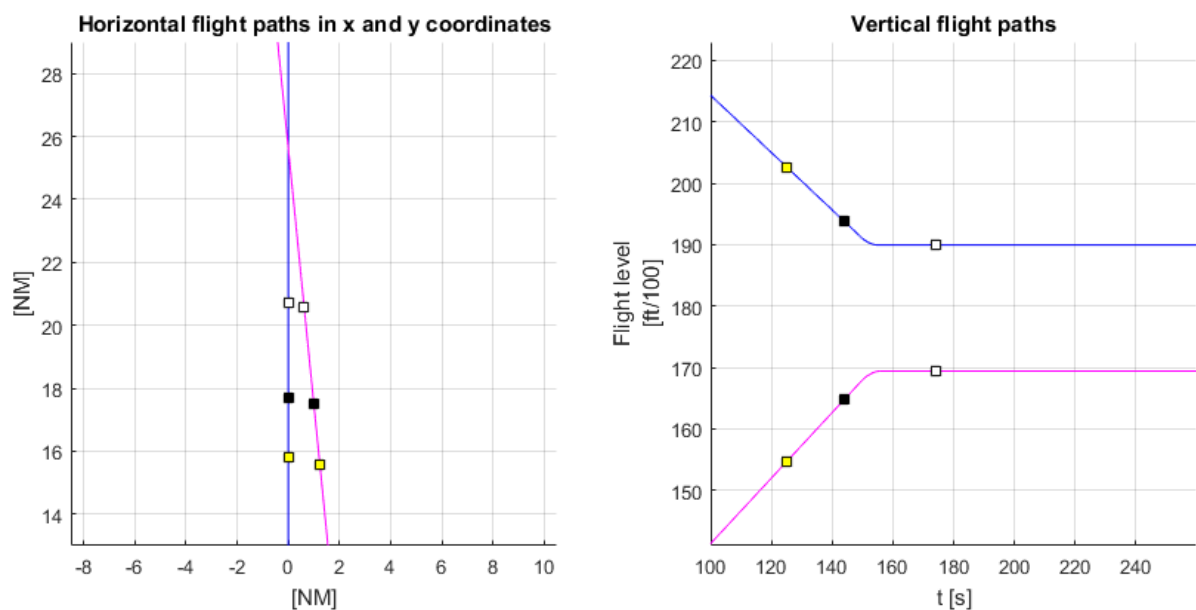
At this moment it is not clear which of the two options is the better one. If option 1 or a mixture of the two turns out to be the better one, then this needs to be modified in the new TCAS model.

Case 6:

The graphical results of case 6 are given in the figure below. The numerical results are given in [Table 16-1](#).



a) Simulation results of InCAS V3.3.



b) Simulation results of new TCAS model. Blue line: aircraft 1; magenta line: aircraft 2; yellow box: TA; black box: RA; white box: COC message.

Figure 16-9: Graphical simulation results of case 6. a) Simulation results of InCAS V3.3; b) simulation results of new TCAS model.

As can be seen in [Figure 16-9](#) the results of both models look similar. However, in the numerical results in [Table 16-1](#) three significant differences can be seen.

First, InCAS issues “Level-Off” RAs significantly earlier, 5-6s earlier, compared to the new TCAS model. In [Figure 16-9](#) it can be seen that both aircraft are between flight level 100 and 200 after $t=130s$. According to [Table 6-2](#), the RA threshold value in this altitude region is 30s. Given the

relative altitude velocity of 6,000ft/min (=100ft/s), the altitude test described in Section 8 should only be passed when the vertical separation is less than 3,000ft.

In InCAS the vertical separation at initial RA of aircraft 1 and 2 are 3,500ft and 3,400ft respectively. There is no explanation for InCAS passing the altitude test at these vertical separations.

In the new TCAS model the vertical separation at initial RAs is 2,890ft which is clearly below the 3,000ft boundary. However, it should be noted that also the range test has to be passed in order to issue an RA. So it is not surprising that the new TCAS model does not issue RAs exactly at 3,000ft vertical separation.

Second, a difference in the issue of clear of conflict messages can be identified. In [Table 16-1](#) it can be seen that the new TCAS model issues clear of conflict messages 30s after the RAs. In InCAS the clear of conflict messages are issued 112s and 113s after the RAs.

As described in Section 10, a conflict message may only be issued if the range and altitude test are negative and the time until initially estimated CPA has been elapsed or the horizontal miss distance filter is activated.

As noted above, the time to CPA threshold for RAs for the aircraft is 30s, see [Table 6-2](#). Also in [Figure 16-9](#) it can be seen that the aircraft are not approaching each other vertically after about $t=148s$ in InCAS and after $t=155s$ in the new TCAS model. Hence, issuing the clear of conflict messages 30s after the RA is the expected behaviour. Why this is not the case in InCAS cannot be explained.

16.4 Conclusions

In general the results of both models are very similar or lead to similar avoidance manoeuvres. No cases could be identified where InCAS chooses an upward sense and the Petri Net chooses a downward sense or vice versa.

However, differences between the models were still found. As described in the previous subsection, for all differences the behaviour of the new TCAS model was according to the expected behaviour deducted from the specifications (FAA, 2011; ICAO, 2006; EUROCAE, 2008) while the behaviour of InCAS had some minor deviations. Also an issue was identified regarding the use of aural or visual RAs by the pilot in case of differences.

Therefore, it can be concluded that the new TCAS model resembles TCAS V7.1 and is therefore validated successfully.

17 Monte Carlo simulations of TCAS Operations

For each of the nine cases used in the validation of the new TCAS model Monte Carlo simulations of N_{MC} runs will be carried out. A Monte Carlo simulation may consider random measurement errors, probability of reception of interrogation response, the possibility that both aircraft choose the same sense at the same time, random pilot reaction time, and a shift in TCAS operating cycle time between both aircraft. Note that the systems are assumed to working continuously.

In total there were Monte Carlo simulations carried out for three different set ups.

In the first set up for each case $N_{MC} = 10^6$ simulation runs under uncertainty were carried out. The information captured in each run is the following and the results are presented in Section 17.1:

- Time until issue of TA,
- Time until issue of initial RA and up to five adjusted RAs,
- Velocity boundaries of initial RA and adjusted RAs,
- The miss distance at CPA, and
- Whether a mid-air collision (MAC) or near mid-air collision (NMAC) was encountered.

To determine whether a MAC has occurred, the aircraft size of a Boeing 737-800 is taken as reference, which is about 40m in both length and wingspan, and about 12.3m in height¹. Hence, a collision is detected if at CPA the horizontal separation is less than 40m and the vertical separation is less than 12.3m. To determine whether a NMAC has happened, the separation distance stated in (EUROCAE, 2008, p. 15) is used, which is 500ft horizontally and 100ft vertically, and the sizes of the aircraft are neglected.

In the second set up, for each case $N_{MC} = 10^6$ simulation runs under uncertainty were carried out. However, this time the variation of the pilot delay was switched off to analyse the effect of pilot behaviour. The information captured in each run is the same as the information captured in the first simulation set up. In Section 17.2 the difference of the results of the first and second simulation set up are presented.

Third, as the simulation results of the first and second set up have not shown large differences, for the nine cases several Monte Carlo simulations were carried. This time the number of runs was set to only $N_{MC} = 10^4$, but for each Monte Carlo simulation a different combination of enabled uncertainties was selected and the results are presented in Section 17.3.

¹ Source Wikipedia: https://en.wikipedia.org/wiki/Boeing_737; last accessed at 09.08.2017

17.1 Simulation Results I

From the results of the first simulation set up the following information were computed:

- mean, variance, minimum, maximum, median of time of issue of TA,
- mean, variance, minimum, maximum, median of time of issue of initial RA,
- mean, variance, minimum, maximum, median of time of issue of adjusted RAs,
- 50%, 95% and 99% confidence interval of time of issue of TA,
- 50%, 95% and 99% confidence interval of time of issue of RA,
- 50%, 95% and 99% confidence interval of time of issue of adjusted RAs,
- type of initial RA and adjusted RAs, and
- number of MAC and NMAC.

These results are also compared to the reference results obtained in Section 16 for the new TCAS model, but without random effects.

Next, the results are discussed case by case. Please note that the following notations are used in this section:

- CL: "Climb" RA,
- LO: "Level-Off" RA,
- DE: "Descend" RA,
- DCL: "Do Not Climb" RA, and
- DDE: "Do Not Descend" RA.

Case 1:

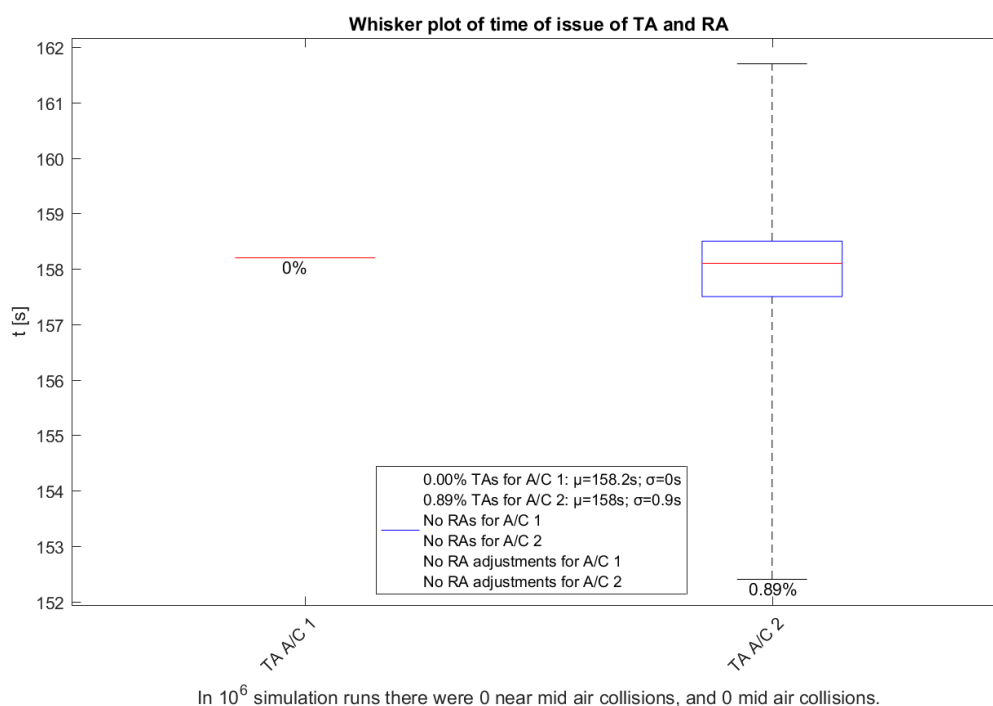


Figure 17-1: Graphical visualization of Monte Carlo simulation results for case 1. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on.

Table 17-1: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 1.

		Time of Issue [s]								
Type	A/C	Reference case						Confidence interval		
			Mean	σ	Min	Max	Median	50%	95%	99%
TA	1	-	158.2	0.0	158.2	158.2	158.2	[158.2,158.2]	[158.2,158.2]	[158.2,158.2]
	2	-	158	0.9	152.4	161.7	158.1	[157.5,158.5]	[155.9,159.5]	[155.0,160.1]
RA	1	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-

Table 17-2: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 1.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	1	0.00	-	-	-	-	-	-	-	-
	2	8909	0.89	-	-	-	-	-	-	-	-
RA	1	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-

As described in Section 16, in the reference case neither TAs nor RAs were issued. Also no near mid-air collision or mid-air collision was encountered.

The results of the Monte Carlo simulation are presented graphically in [Figure 17-1](#) and numerically in [Table 17-1](#) and [Table 17-2](#). In the Monte Carlo simulation TAs were issued for both aircraft. For

aircraft 1 only in one simulation a TA was issued. For aircraft 2 in 0.89% of the cases a TA was issued. The mean time of issue is 158s and the standard deviation is 0.9s, which is very small. In rare cases, less than 1% of the simulation, a TA is issued between 2.1 and 3.7s later than the mean (between 160.1 and 161.7s) or between 3.0 and 5.6s earlier than the mean (between 155.0 and 152.4s). So for rare cases a tendency for an earlier time of issue can be identified.

Also, no mid-air or near mid-air collisions were encountered in the Monte Carlo simulation.

The difference between the reference case and the Monte Carlo simulation can be traced back to the bias of the altitude measurement error. In order to pass the altitude test for a TA aircraft 1 needs to measure an altitude difference of less than 1,083ft if the relative altitude velocity is 2,600ft/min. For aircraft 2 the measured altitude separation would need to be less than 2,080ft. In the reference case the vertical separation at CPA was 2,4010ft. Given the 99.7% error bounds of the altitude measurements, 174-195ft for aircraft 1 and 195-213ft for aircraft 2, it is reasonable that almost never the altitude separation estimate just before CPA by aircraft 1 is 1,317ft smaller than actual separation and in 0.89% of the simulation the altitude separation estimate just before CPA by aircraft 2 is 330ft smaller than the actual separation.

Case 2:

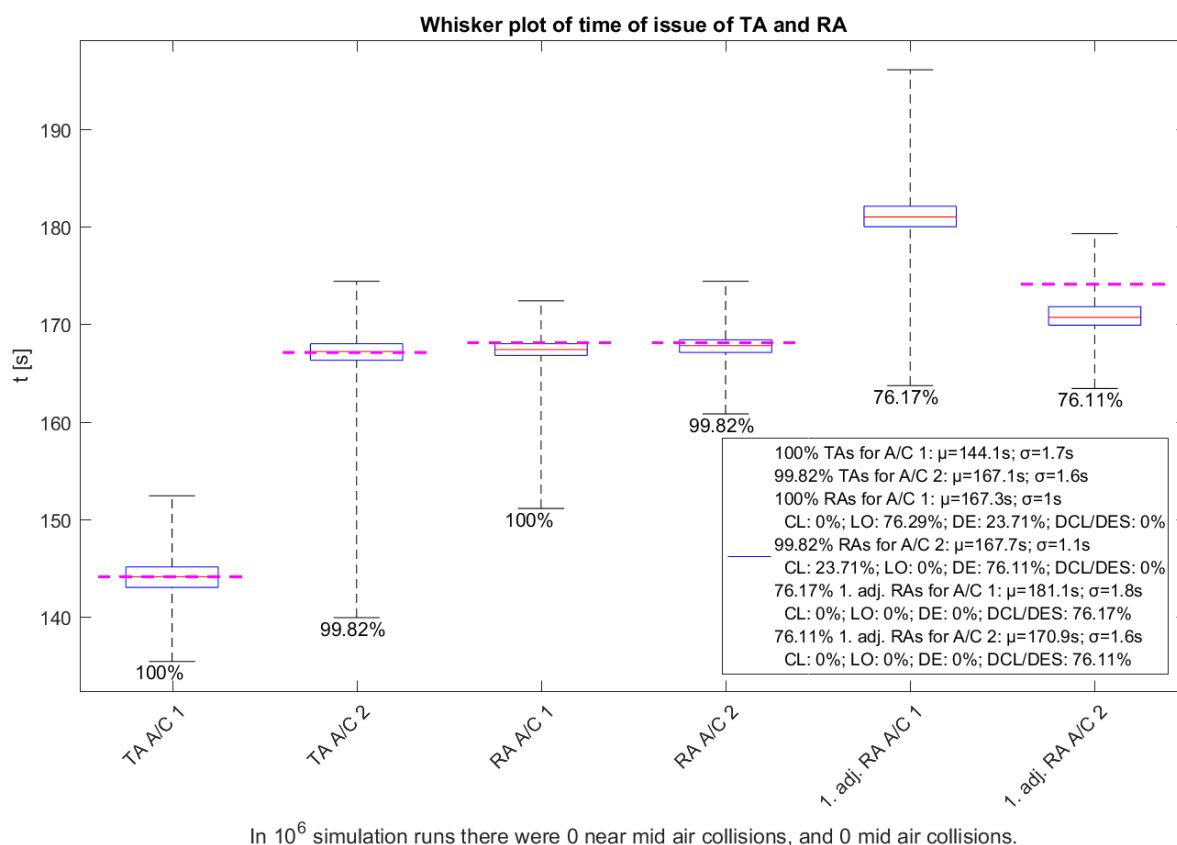


Figure 17-2: Graphical visualization of Monte Carlo simulation results for case 2. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-3: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 2.

Type	A/C	Time of Issue [s]								
		Reference case	Mean	σ	Min	Max	Median	Confidence interval		
TA	1	144.1	144.1	1.7	135.4	152.4	144.1	[143,145.1]	[140.6,147.3]	[139.4,148.4]
	2	167.1	167.1	1.6	139.9	174.4	167.2	[166.3,168]	[164.6,169.6]	[163.3,170.4]
RA	1	168.1	167.3	1	151.1	172.4	167.4	[166.8,168]	[165.1,169.1]	[163.9,169.6]
	2	168.1	167.7	1.1	160.8	174.4	167.8	[167.1,168.4]	[165.5,169.8]	[164.5,170.5]
1 st RA adj.	1	-	181.1	1.8	163.7	196.1	181	[180.0,182.1]	[178.0,185.4]	[176.7,187.5]
	2	174.1	170.9	1.6	163.4	179.3	170.7	[169.9,171.8]	[168.1,174.4]	[167.1,175.4]

Table 17-4: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 2.

Note: Bold font indicates a TA and the type of RA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	1000000	100	-	-	-	-	-	-	-	-
	2	998242	99.82	-	-	-	-	-	-	-	-
RA	1	1000000	100	-	-	762914	76.29	237086	23.71	-	-
	2	998187	99.82	237097	23.71	-	-	761090	76.11	-	-
1 st RA adj.	1	761661	76.17	-	-	-	-	-	-	761661	76.17
	2	761090	76.11	-	-	-	-	-	-	761090	76.11

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in [Figure 17-2](#) and numerically in [Table 17-3](#) and [Table 17-4](#).

In the reference case a TA was issued by aircraft 1 at 144.1s and by aircraft 2 at 167.1s

In the Monte Carlo simulation a similar behaviour can be found. In 100% of the simulation a TA is issued by aircraft 1 at a mean time of 144.1s and a standard deviation of 1.7s. Aircraft 2 issued in 99.82% of the simulation a TA at a mean time of 167.1s and a standard deviation of 1.6s. Although the standard deviation is smaller compared to aircraft 1, the minimum time of issue deviated up to 27.2s. So in rare cases aircraft 2 issued significantly earlier a TA or no TA at all.

In the reference case a RA was issued by both aircraft at 168.1s. Aircraft 1 issued a “Level-Off” RA and aircraft 2 issue a “Descend” RA.

In the Monte Carlo simulation the time of issue by both aircraft was similar with a low standard deviation. Aircraft 1 always issued an RA at a mean time of 167.3s with a standard deviation of 1.0s. In 99.82% of the simulation Aircraft 2 issued an RA at a mean time of 167.3s with a standard deviation of 1.1s. However, differences can be observed in the type of RA issued. In 23.71% of the simulation aircraft 1 issued a “Descend” RA instead of a “Level-Off” RA and aircraft 2 a “Climb” RA instead of a “Descend” RA. Hence, in 23.71% TCAS chose an altitude crossing intent, although the reference case has shown that the aircraft could safely pass each other without an altitude crossing.

Regarding adjusted RAs, a difference between the reference case and the Monte Carlo simulation can be seen as well. In the reference case only aircraft 2 issued a “Do Not Climb” RA adjustment at 174.1s. In the Monte Carlo simulation both aircraft issue a do not climb/descend adjusted RA in about 76% of the simulation. By looking at the exact number of adjusted RAs and initial RAs, it can also be seen that aircraft 2 always issued a “Do Not Descend” RA, which is similar to the reference case. But also aircraft 2 issued a “Do Not Climb” RA in almost all simulations where the aircraft chose the same initial RA type as in the reference case. This is not the case in the reference case.

Also it can be seen that usually no adjusted RA is issued if the aircraft are crossing altitudes.

Neither the reference case, nor the Monte Carlo simulation show mid-air or near mid-air collisions. Nevertheless, it should be noted that without any flight path adjustments, no mid-air collisions or near mid-air collision would have occurred either.

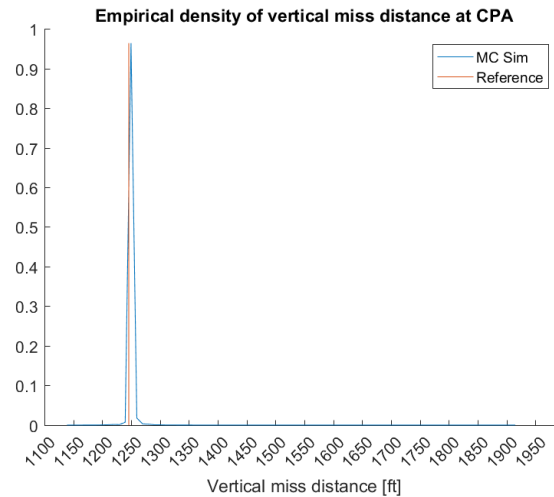


Figure 17-3: Empirical density of vertical miss distance at CPA of Monte Carlo simulation (blue line) and the reference case (orange line) for case 2.

Finally, in order to better understand the effects of the uncertainties, we take a look at the empirical density of the vertical miss distance at CPA in given in [Figure 17-3](#).

From [Figure 17-3](#) two observations can be made. First, almost all vertical miss distances are within a range of 10ft to the reference case. Second, in rare cases the vertical miss distance reduces to 1,140ft or increases to 1910ft.

The positive outliers are due to the initial RAs of aircraft 1 presented in [Figure 17-2](#). There it can be seen that in rare cases aircraft 1 issued an initial RA 16.2s earlier than the mean. This resulted in an earlier manoeuvring by the pilot, which is likely to have caused the greater miss distance. However, for the negative outliers the cause is not clear.

To be certain about the exact causes, additional Monte Carlo simulations are needed.

Case 3:

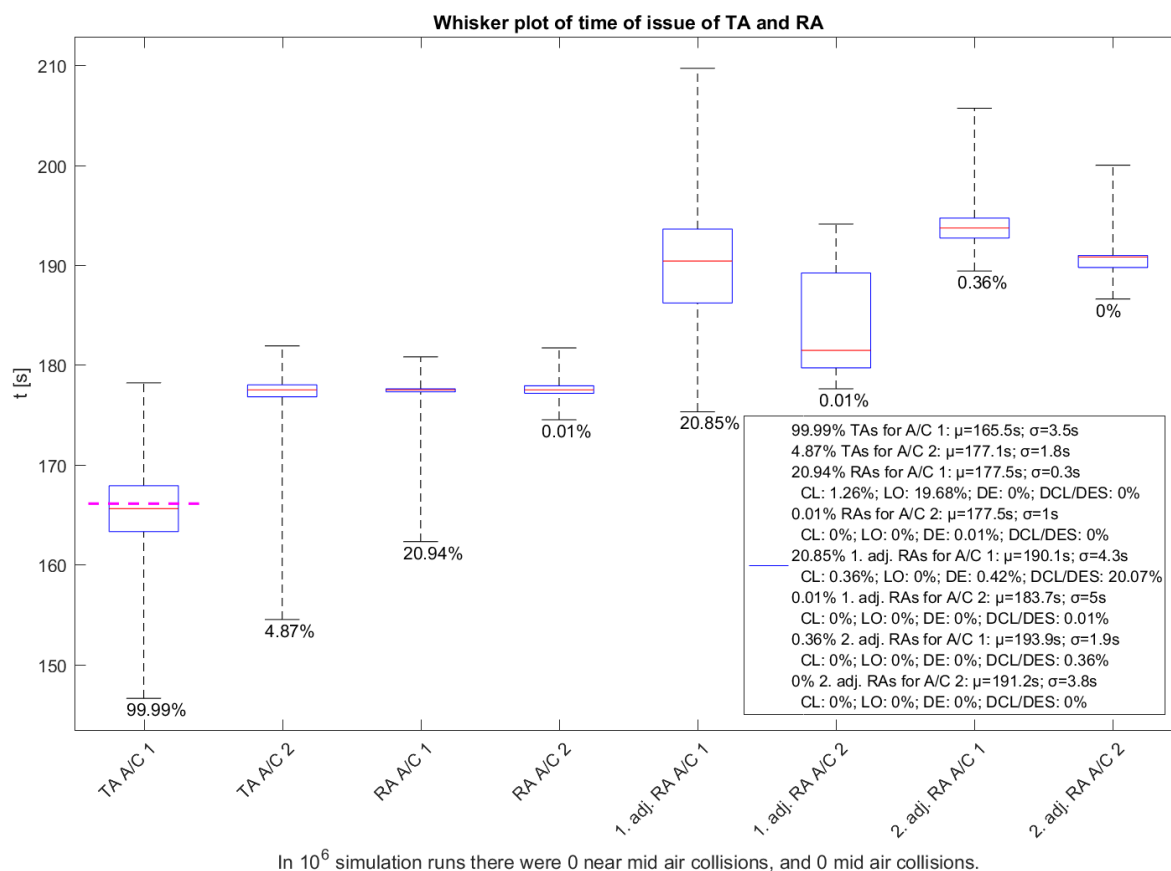


Figure 17-4: Graphical visualization of Monte Carlo simulation results for case 3. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-5: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 3.

		Time of Issue [s]								
Type	A/C	Reference case						Confidence interval		
			Mean	σ	Min	Max	Median	50%	95%	99%
TA	1	166.1	165.5	3.5	146.6	178.2	165.6	[163.3,167.9]	[158.0,172.0]	[155.4,173.9]
	2	-	177.1	1.8	154.5	181.9	177.5	[176.8,178.0]	[172.4,178.9]	[165.3,179.3]
RA	1	-	177.5	0.3	162.3	180.8	177.5	[177.3,177.6]	[177.0,177.8]	[176.8,178.6]
	2	-	177.5	1.0	174.5	181.7	177.5	[177.2,177.9]	[175.4,179.7]	[174.5,181.6]
1 st RA adj.	1	-	190.1	4.3	175.3	209.7	190.4	[186.2,193.6]	[185.2,198.5]	[184.5,201.6]
	2	-	183.7	5.0	177.6	194.1	181.5	[179.7,189.2]	[177.6,193.2]	[177.6,194.1]
2 nd RA adj.	1	-	193.9	1.9	189.4	205.7	193.7	[192.7,194.7]	[190.6,197.8]	[189.7,200.7]
	2	-	191.2	3.8	186.6	200	190.8	[189.8,191.0]	[186.6,200.0]	[186.6,200.0]

Table 17-6: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 3.
Note: Bold font indicates a TA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	999943	99.99	-	-	-	-	-	-	-	-
	2	48682	4.87	-	-	-	-	-	-	-	-
RA	1	209396	20.94	12588	1.26	196798	19.68	10	0.00	-	-
	2	108	0.01	32	-	-	-	76	0.01	-	-
1 st RA adj.	1	208492	20.85	3593	0.36	-	-	4235	0.42	200664	20.07
	2	102	0.01	7	0.00	-	-	-	-	95	0.01
2 nd RA adj.	1	3575	0.36	-	-	-	-	-	-	3575	0.36
	2	8	0.00	-	-	-	-	1	0.00	7	0.00

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in [Figure 17-4](#) and numerically in [Table 17-5](#) and [Table 17-6](#).

In the reference case only a TA was issued by aircraft 1 after 166.1s.

In the Monte Carlo Simulations TAs, RAs and up to 2 adjusted RAs could be observed for both aircraft.

By aircraft 1 in 99.99% of the Monte Carlo simulation a TA was issued on average at 165.5s and with a standard deviation of 3.5s. So on average the TA is issued slightly earlier due to the measurement errors. Also it can be observed that in rare cases the TA is issued up to 18.9s earlier but only up to 12.7s later compared to the mean; and in 0.01% of the simulation no TA was issued.

By aircraft 2 in 4.87% of the Monte Carlo simulation a TA was issued, where the mean time of issue is 177.1s and the standard deviation is 1.8s. Also from the confidence intervals it can be seen that normally the time of issue spreads equally about the mean time but in rare cases a TA is issued up to 22.6s earlier but only 4.8s later than the mean time. It should be noted, that the time of issue of the TA is coinciding with the time of CPA which is 175.7s and the aircraft 2 estimated relative altitude separation needs to be below 667ft in order to trigger a TA. So the uncertainties lead in 4.87% of the simulation to a slightly delayed CPA, by 1.4s, and a relative altitude estimate of less than 348ft compared to the actual situation.

In regard to RA, aircraft 1 issued in 20.94% of the simulation an RA, in 20.85% an adjusted RA, and in 0.36% a second adjusted RA. In the type of the RAs it can be seen that the majority of initial RAs are level-off, 19.68%, and the remaining initial RAs are “Climb” RAs, 1.26%. As aircraft 1 is descending and aircraft 2 is flying levelled, all initial RAs are avoiding altitude crossings. The majority of the first RA adjustments are “Do Not Climb/Descend” RAs, 20.07%, and only a very few percentage are climb or “Descend” RAs, 0.36% and 0.42% respectively. All second RA adjustments are “Do Not Climb/Descend” RAs, 0.36%.

The difference to the reference case can be explained through the altitude measurement errors. First, it can be observed that the time of issue of initial RA, 177.5, is coinciding with the time of CPA, 175.7s. Second, the required estimated relative altitude needs to be less than 933ft just before CPA in order to trigger an RA. Given the altitude measurement error bounds, it is reasonable that the relative estimated altitude is 82ft lower than the actual relative altitude in about 21% of the time.

Aircraft 2 issued only in 0.01% initial and an adjusted RA, and only in 8 simulations, 0.00%, a second adjusted RA. As all initial RAs are “Descend” RAs it can be concluded that also aircraft 2 always tried to avoid altitude crossings.

Neither the reference case, nor the Monte Carlo simulation show mid-air or near mid-air collisions. Nevertheless, it should be noted that without any flight path adjustments, no mid-air collisions or near mid-air collision would have occurred either.

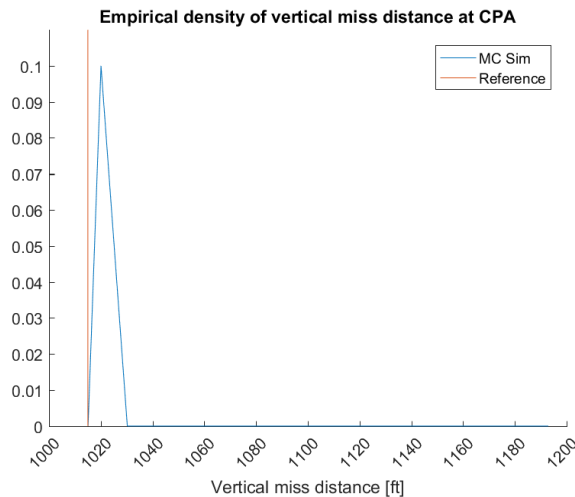


Figure 17-5: Empirical density of vertical miss distance at CPA of Monte Carlo simulation (blue line) and the reference case (orange line) for case 3.

Finally, in order to better understand the effects of the uncertainties, we take a look at the empirical density of the vertical miss distance at CPA in given in Figure 17-5.

From Figure 17-5 two observations can be made. First, almost all vertical miss distances are within 1,015 and 1,025ft and hence marginally higher than the reference miss distance of 1,015ft. The small increase may be explained by the fact that in 80% of the simulations the vertical miss distance was equal to the reference case, as no RAs were issued, and that in the remaining 20% of the simulation the issued RA always lead to a higher vertical miss distance.

Second, in rare cases the vertical miss distance increases to 1,190ft. Similar to case 2, these positive outliers in the vertical miss distance are likely correlated to the rare cases in which the initial RAs of aircraft 1 were issued up to 15.2s earlier than the mean time of issue. However, to be sure additional Monte Carlo simulations would be needed.

Case 4:

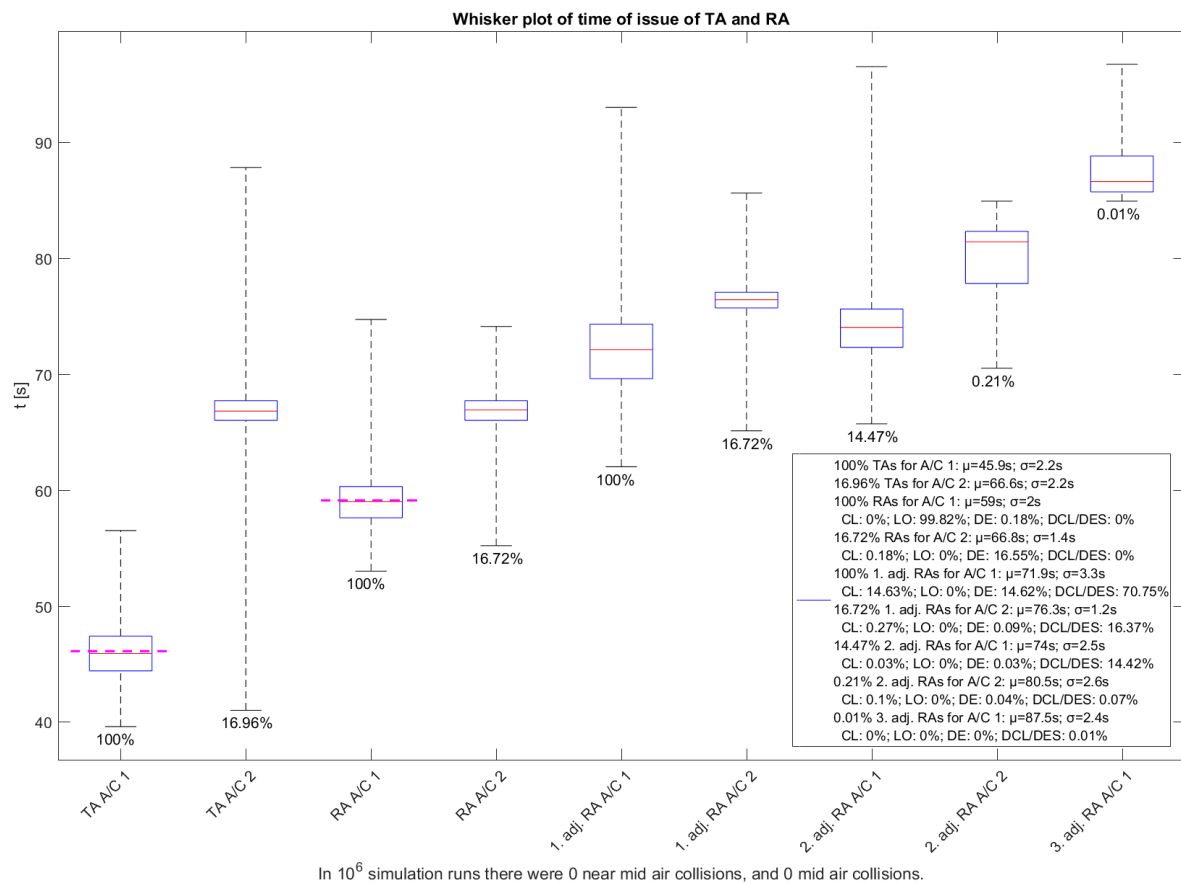


Figure 17-6: Graphical visualization of Monte Carlo simulation results for case 4. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-7: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 4.

Type	A/C	Reference case	Time of Issue [s]					Confidence interval		
			Mean	σ	Min	Max	Median	50%	95%	99%
TA	1	166.1	45.9	2.2	39.6	56.5	45.9	[44.4,47.4]	[41.4,50.2]	[40.8,51.7]
	2	-	66.6	2.2	41	87.8	66.8	[66.0,67.7]	[63.5,69.3]	[50.4,70.3]
RA	1	59.1	59	2	53	74.7	59	[57.6,60.3]	[54.9,62.9]	[54.1,64.2]
	2	-	66.8	1.4	55.2	74.1	66.9	[66.0,67.7]	[64.1,69.3]	[62.5,70.2]
1 st RA adj.	1	-	71.9	3.3	62	93	72.1	[69.6,74.3]	[65.7,78.0]	[64.2,80.3]
	2	-	76.3	1.2	65.1	85.6	76.4	[75.7,77.1]	[73.7,78.5]	[72.0,79.2]
2 nd RA adj.	1	-	74	2.5	65.7	96.5	74	[72.3,75.6]	[69.3,78.8]	[68.0,82.7]
	2	-	80.5	2.6	70.5	84.9	81.4	[77.8,82.3]	[75.8,84.4]	[73.1,84.8]
3 rd RA adj.	1	-	87.5	2.4	84.9	96.7	86.6	[85.7,88.8]	[85.2,93.7]	[84.9,96.5]
	2	-	-	-	-	-	-	-	-	-

Table 17-8: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 4.

Note: Bold font indicates a TA and the type of RA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	1000000	100	-	-	-	-	-	-	-	-
	2	169560	16.96	-	-	-	-	-	-	-	-
RA	1	999997	100	-	-	998219	99.82	1778	0.18	-	-
	2	167244	16.72	1778	0.18	-	-	165466	16.55	-	-
1 st RA adj.	1	999988	100	146318	14.63	-	-	146162	14.62	707508	70.75
	2	167244	16.72	2653	0.27	-	-	914	0.09	163677	16.37
2 nd RA adj.	1	144749	14.47	261	0.03	-	-	280	0.03	144208	14.42
	2	2106	0.21	1037	0.1	-	-	390	0.04	679	0.07
3 rd RA adj.	1	130	0.01	-	-	-	-	-	-	130	0.01
	2	-	-	-	-	-	-	-	-	-	-

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in [Figure 17-6](#) and numerically in [Table 17-7](#) and [Table 17-8](#).

In the reference case only a TA and an RA were issued by aircraft 1. Nothing was issued by aircraft 2.

In the Monte Carlo simulation in 100% of the simulation a TA was issued by aircraft 1. On average this happened at 45.9s which is very close to the reference case where it was at 46.1s.

Opposed to the reference case, in 16.96% of the simulation a TA was issued by aircraft 2. The mean time of issue is 66.6 with a variance of 2.2s. From the confidence interval, it can also be seen that in rare cases the time of issue is 25.6s earlier or 21.2s later than the mean time of issue. Reflecting on the amount of TAs issued by aircraft 2 and looking at the separation at CPA, one would expect more often TAs by aircraft 2. At CPA the altitude separation is 1,111ft but already an estimate of 1,042 just before CPA would result in a TA. So given the altitude measurement error bounds for both aircraft between 258-285ft, one would expect the threshold to be undercut in 40-50% of the simulation. However, as being described next, aircraft 1 issues on average an RA 7.6s before aircraft 2 issues a TA. Therefore, in many cases aircraft 1 has reacted upon an RA which results in a greater vertical separation at CPA, such that aircraft 2 estimates only for 16.96% of the simulation a vertical separation of less than 1,042ft any time before CPA.

Regarding the RA, the RAs issued by aircraft are similar to the RA of the reference case. In 100% of the time an RA was issued by aircraft 1 at a mean time of 59.0s. In the reference case this was at 59.1s. Also in most simulations, 99.82%, a “Level-Off” RA was issued, such as in the reference case. However, in the remaining 0.18% a “Descend” RA was issued, which results in altitude crossings. Hence, in rare cases the measurement errors, and probably also missed messages, lead to faulty relative altitude velocities, which identify that a level off or “Climb” RA would not result in sufficient vertical miss distance, but only an altitude crossing RA.

Also, it can be noticed that in 100% of the simulation aircraft 1 issued an adjusted RA, in 14.47% a second adjusted RA, and in 0.21% a third adjusted RA. It can also be seen that most adjusted RAs are “Do Not Climb/Descend” RAs.

For aircraft 2 the number of simulations in which RAs were issued are significantly lower compared to aircraft 1. In 16.72% an initial RA, in 16.72% an adjusted, and in 0.21% a second adjusted RA was

issued by aircraft 1. Initially, the types of RAs are mostly “Descent”, followed by “Do not Climb/Descend” adjusted RAs. It can also be seen that in 0.18% of the simulation the initial RA by aircraft 2 is a “Climb” RA and by aircraft 1 a “Descend” RA. So it can be concluded that in all cases where one aircraft issues an altitude crossing RA, the other aircraft issues the complementary RA.

Neither the reference case, nor the Monte Carlo simulation show mid-air or near mid-air collisions. Nevertheless, it should be noted that without any flight path adjustments, no mid-air collisions or near mid-air collision would have occurred either.

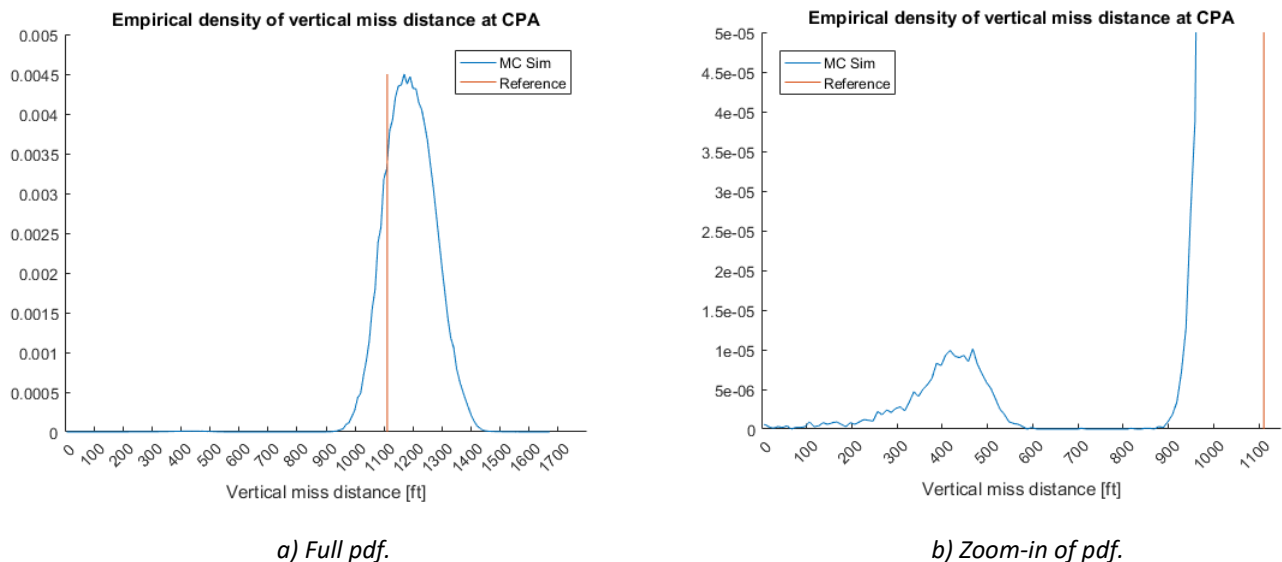


Figure 17-7: Empirical density of vertical miss distance at CPA of Monte Carlo simulation (blue line) and the reference case (orange line) for case 4. a) full pdf graph; b) zoom-in left of pdf.

Finally, in order to better understand the effects of the uncertainties, we also take a look at the empirical density of the vertical miss distance at CPA in given in [Figure 17-7](#).

In the graph on the left hand side of [Figure 17-5](#) two observations can be made. First, almost all vertical miss distances are within 1,100 and 1,300ft and are on average about 100ft higher than the reference miss distance of 1,111ft. The increase is probably caused by the simulations in which aircraft 2 issued a “Descend” RA. [Table 17-8](#) shows this was in 16.55% of the simulation while in the reference case no RA was issued for aircraft 2.

A closer look on the graph on the right hand side of [Figure 17-5](#) reveals that there is also a small peak at about 450ft. The peak may be explained by the simulation runs in which aircraft 1 selected a “Descend” RA and aircraft 2 issued a “Climb” RA. These RAs resulted in an altitude crossing, occurred in 0.18% of the simulation runs, and therefore could explain the separate concentration of vertical miss distance different.

In rare cases the vertical miss distance decreases to values below 100ft. It is likely that these lower vertical miss distances are due to rare cases in which aircraft 1 issued the RA up to 15.7s later than the mean time.

To be certain about the exact causes, additional Monte Carlo simulations are needed.

Case 5:

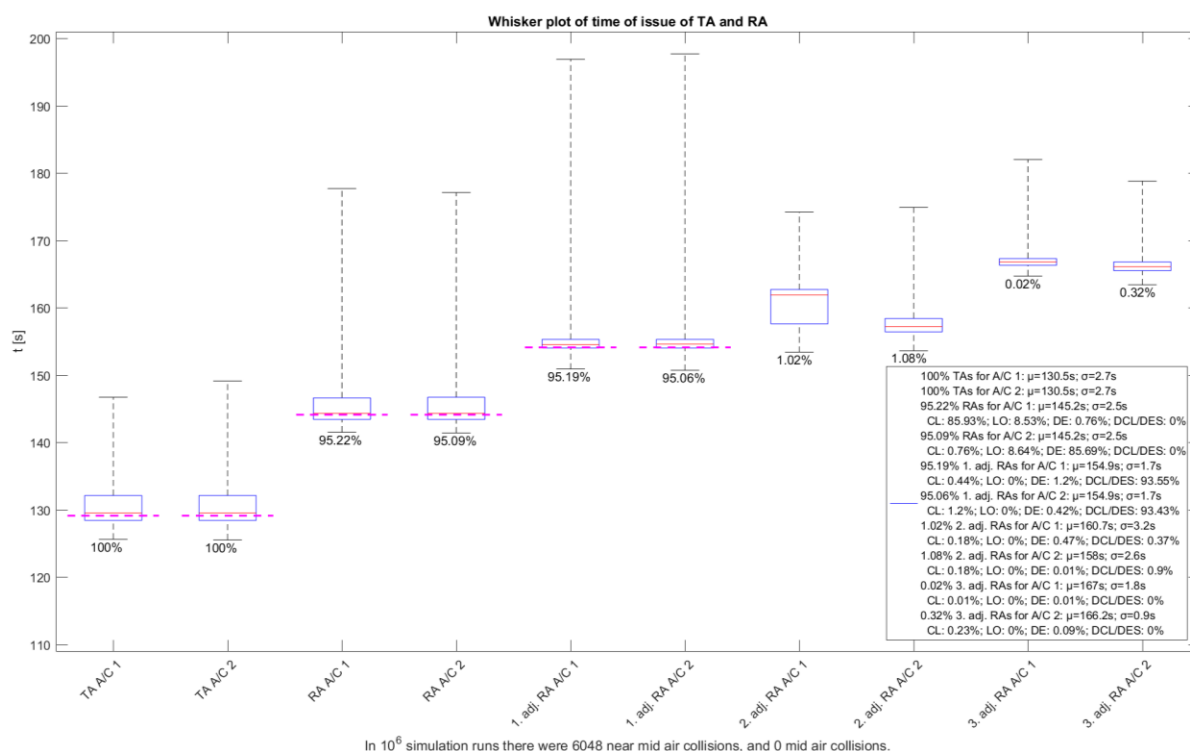


Figure 17-8: Graphical visualization of Monte Carlo simulation results for case 5. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-9: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 5.

Type	A/C	Reference case	Time of Issue [s]					Confidence interval		
			Mean	σ	Min	Max	Median	50%	95%	99%
TA	1	129.1	130.5	2.7	125.6	146.7	129.5	[128.4,132.1]	[127.4,137.0]	[127,139.4]
	2	129.1	130.5	2.7	125.5	149.1	129.5	[128.4,132.1]	[127.4,137.0]	[127,139.5]
RA	1	144.1	145.2	2.5	141.5	177.7	144.3	[143.4,146.6]	[142.6,150.4]	[142.3,152]
	2	144.1	145.2	2.5	141.4	177.1	144.3	[143.4,146.7]	[142.6,150.4]	[142.3,152]
1 st RA adj.	1	154.1	154.9	1.7	150.9	196.9	154.5	[154.0,155.3]	[152.9,158.5]	[152.3,162]
	2	154.1	154.9	1.7	150.7	197.7	154.6	[154.0,155.3]	[152.9,158.5]	[152.2,162]
2 nd RA adj.	1	-	160.7	3.2	153.4	174.2	161.9	[157.6,162.7]	[154.9,166.4]	[154.5,167.8]
	2	-	158	2.6	153.6	174.9	157.2	[156.4,158.4]	[154.9,163.9]	[154.5,167.7]
3 rd RA adj.	1	-	167	1.8	164.7	182	166.8	[166.3,167.3]	[165.6,167.9]	[164.8,180.7]
	2	-	166.2	0.9	163.4	178.8	166.1	[165.5,166.8]	[164.6,167.7]	[164.2,167.9]

Table 17-10: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 5.
Note: Bold font indicates a TA and the type of RA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	1000000	100	-	-	-	-	-	-	-	-
	2	1000000	100	-	-	-	-	-	-	-	-
RA	1	952188	95.22	859314	85.93	85263	8.53	7565	0.76	46	-
	2	950890	95.09	7565	0.76	86419	8.64	856867	85.69	39	-
1 st RA adj.	1	951862	95.19	4358	0.44	-	-	12010	1.20	935494	93.55
	2	950577	95.06	12029	1.20	-	-	4204	0.42	934344	93.43
2 nd RA adj.	1	10247	1.02	1837	0.18	13	0.00	4709	0.47	3688	0.37
	2	10829	1.08	1761	0.18	12	0.00	102	0.01	8954	0.9
3 rd RA adj.	1	189	0.02	86	0.01	-	-	99	0.01	4	0.00
	2	3182	0.32	2289	0.23	-	-	891	0.09	2	0.00

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in [Figure 17-8](#) and numerically in [Table 17-9](#) and [Table 17-10](#).

In the reference case a TA, an initial RA and one RA adjustment were issued by both aircraft.

The results of the Monte Carlo simulation show a similar outcome. For example the mean time of issue for a TA, an initial RA and the first RA adjustment do not deviate more than 1.4s to the time of issue in the reference case.

The standard deviations of the time of issue of TAs and RAs are not large either. For both aircraft they range between 0.9s and 3.2s. But it can be observed that for both aircraft in rare cases the time of issue is largely later than the mean time of issue, while it is not largely earlier. E.g. for initial RAs, the difference between the mean time and earliest time of issue is only 3.7s for aircraft 1 and only 3.8s for aircraft 2 but the difference between the mean time and latest time of issue is 32.5s for aircraft 1 and 31.9s for aircraft 2. As given in [Table 16-1](#), in the reference case the closest point of approach is at 174.0s. Hence, in these cases the initial RAs were issued after the closest point of approach.

However, three small differences can be identified, too. First, in only about 95% of the simulation runs an initial RA and an RA adjustment were issued. Hence, the measurement errors lead both aircraft to not take any action.

Second, the majority of the initial RAs, in about 86% of the simulation runs, are similar to the reference case. But in about 9% of the simulation runs both aircraft issued a “Level-Off” RA, and in 0.76% of the simulation runs aircraft 1 was issued to descend and aircraft 2 was issued to climb.

By keeping in mind that aircraft 1 was initially descending, and aircraft 2 was initially climbing, it can be concluded that the measurement errors lead in about 9% of the simulation runs to a greater vertical miss distance, such that a less strong RA is issued (level off), but in 0.76% of the simulation runs the vertical miss distance estimate is such smaller, that RAs with an altitude crossing intent were issued.

Third, a very low number of 2nd and 3rd RA adjustments were encountered in the Monte Carlo simulation. For aircraft 1 in 1.02% and 0.02% of the simulation runs a second RA adjustment and third RA adjustment were issued respectively. For aircraft 2 in 1.08% and 0.32% of the simulation

runs a second RA adjustment and third RA adjustment were issued respectively. Remarkable is that in 25 simulation runs “Level-Off” RA adjustments were issued, as indicated in red font in [Table 17-10](#). As described in Sections 10 and 12.2, a “Level-Off” RA adjustment is just a mean to model a clear of conflict message. Also, it should be noted that in the result evaluation only the last RA adjustment was counted as a clear of conflict message. Therefore, the “Level-Off” RA adjustments indicate that in these rare cases several clear of conflict messages were issued, which means that in these cases a new conflict was detected after the aircraft were declared clear of conflict. This is likely caused due to the assumption that the pilot levels off when he or she receives a clear of conflict message, which puts the aircraft back to a collision course.

Next, in the reference case no near mid-air collision or mid-air collision was encountered. In the Monte Carlo simulation 6048, 0.6%, near mid-air collisions but no mid-air collisions were encountered. Since without any flight path adjustments a near mid-air collision would have occurred, it is likely that the number of near mid-air collisions is caused by the rare cases in which the initial RAs were issued by both aircraft after the closest point of approach had been passed. To better understand what is going on, we also take a look at the empirical density of the vertical miss distance at CPA which is given in [Figure 17-9](#).

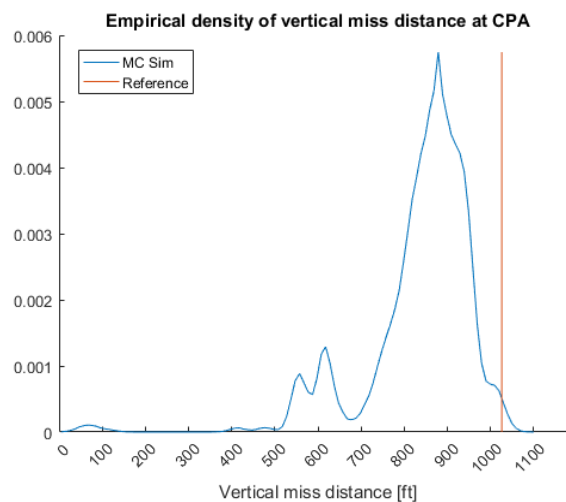


Figure 17-9: Empirical density of vertical miss distance at CPA of Monte Carlo simulation (blue line) and the reference case (orange line) for case 5.

Remarkably, almost all vertical miss distances in the Monte Carlo simulation are smaller than the vertical miss distance of the reference case, and there are six local maxima visible.

The highest peak covers most of the data and, therefore, is likely to represent the 86% of the simulations runs in which the RAs of the reference case were issued. Though, it remains to be explained why this did not lead to a similar vertical miss distances as in the reference case.

The second and third highest peaks are located to the left of the highest peak, and probably represent 10-15% of the data. It is likely that one peak is from the simulation runs where one of the aircraft issued a “Level-Off” RA instead of climb or “Descend” RA (8-9% of the simulation runs) and the other peak is from the runs when no aircraft issued an RA at all (about 5% of the simulation runs).

The fourth and fifth peaks are very small and located in the range of 400-500ft. It is not clear how these two peaks relate to the results in [Figure 17-8](#).

The sixth peak is very low and centred at about 50ft. This explains the low number of near mid-air collisions encountered. Since, the horizontal miss distance at CPA is about 480ft, it can be concluded, that in this case mid-air collisions could have happened, if the horizontal miss distance would have been lower.

To be certain about the exact causes, additional Monte Carlo simulations are needed.

Case 6:

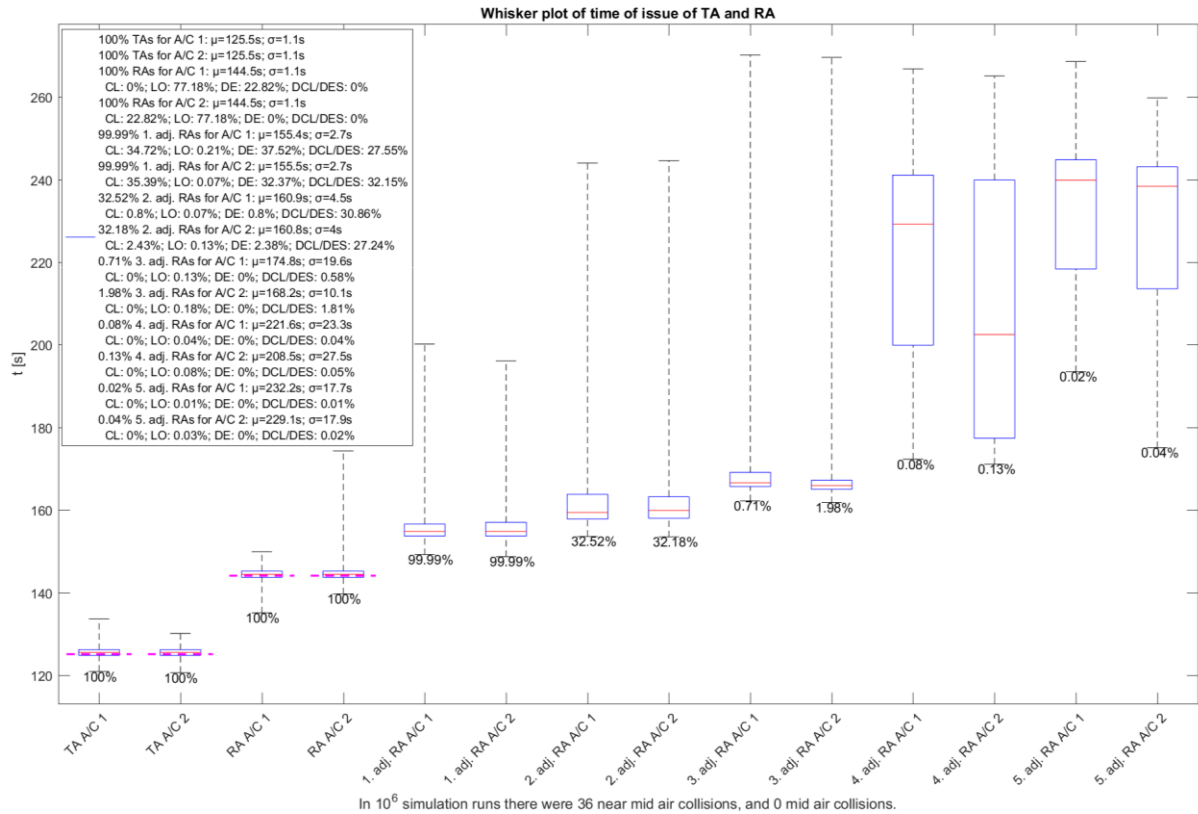


Figure 17-10: Graphical visualization of Monte Carlo simulation results for case 6. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-11: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 6.

Type	A/C	Reference case	Time of Issue [s]							Confidence interval		
			Mean	σ	Min	Max	Median			50%	95%	99%
TA	1	125.1	125.5	1.1	120.9	133.6	125.5			[124.8,126.2]	[123.5,127.6]	[122.9,128.3]
	2	125.1	125.5	1.1	120.6	130.1	125.5			[124.8,126.2]	[123.4,127.5]	[122.7,128.1]
RA	1	144.1	144.5	1.1	135.1	149.9	144.4			[143.7,145.2]	[142.5,146.7]	[142.0,147.4]
	2	144.1	144.5	1.1	139.7	174.3	144.4			[143.7,145.2]	[142.5,146.7]	[142.0,147.4]
1 st RA adj.	1	-	155.4	2.7	149.2	200.2	154.8			[153.7,156.6]	[152.1,160.7]	[151.4,164.0]
	2	-	155.5	2.7	148.7	196.1	154.8			[153.7,157.0]	[152.0,161.0]	[151.3,163.8]
2 nd RA adj.	1	-	160.9	4.5	153.6	244	159.4			[157.8,163.8]	[156.2,167.9]	[155.5,184.5]
	2	-	160.8	4.0	153.5	244.6	159.9			[158.0,163.2]	[156.3,167]	[155.7,173.4]
3 rd RA adj.	1	-	174.8	19.6	162.2	270.2	166.6			[165.7,169.1]	[164.3,239.9]	[163.7,243.7]
	2	-	168.2	10.1	161.8	269.6	165.9			[165.0,167.2]	[163.7,192.1]	[163.2,241.9]
4 th RA adj.	1	-	221.6	23.3	172.3	266.8	229.2			[199.9,241.1]	[173.6,248.5]	[172.8,255.2]
	2	-	208.5	27.5	171.1	265.1	202.5			[177.4,239.9]	[172.4,246.6]	[171.7,257.1]
5 th RA adj.	1	-	232.2	17.7	193.5	268.6	239.9			[218.4,244.8]	[197.1,255.5]	[194.6,266.8]
	2	-	229.1	17.9	175.1	259.8	238.4			[213.6,243.1]	[195.0,250.0]	[176.6,256.5]

Table 17-12: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 6.
Note: Bold font indicates a TA and the type of RA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	1000000	100	-	-	-	-	-	-	-	-
	2	1000000	100	-	-	-	-	-	-	-	-
RA	1	1000000	100	1	0.00	771787	77.18	228212	22.82	-	-
	2	1000000	100	228222	22.82	771778	77.18	-	-	-	-
1 st RA adj.	1	999931	99.99	347160	34.72	2081	0.21	375181	37.52	275509	27.55
	2	999855	99.99	353900	35.39	700	0.07	323721	32.37	321534	32.15
2 nd RA adj.	1	325154	32.52	7950	0.8	664	0.07	7989	0.8	308551	30.86
	2	321825	32.18	24347	2.43	1274	0.13	23839	2.38	272365	27.24
3 rd RA adj.	1	7059	0.71	-	-	1250	0.13	-	-	5809	0.58
	2	19837	1.98	-	-	1756	0.18	-	-	18081	1.81
4 th RA adj.	1	790	0.08	-	-	356	0.04	-	-	434	0.04
	2	1259	0.13	-	-	774	0.08	-	-	485	0.05
5 th RA adj.	1	176	0.02	-	-	86	0.01	-	-	90	0.01
	2	439	0.04	-	-	250	0.03	-	-	189	0.02

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in Figure 17-10 and numerically in Table 17-11 and Table 17-12.

In the reference case both aircraft issued a TA and a “Level-Off” RA.

In the Monte Carlo simulation both aircraft issued a TA in all simulation runs and the mean time of issue is similar to the reference case and the standard deviation of time of issue is also small with 1.1s.

Also in all simulation runs, an RA was issued by each aircraft. However, the type of RA differs to the reference case. In only 77% of the simulation runs also a level RA was issued but in 23% of the simulations a “Descend” RA was issued by aircraft 1 and a “Climb” RA was issued by aircraft 2. Hence, in 23% of the simulation runs RAs are selected with the intent to let the aircraft cross altitudes.

Moreover, also many RA adjustments were issued, which is opposed to the reference case. The distribution of the type of first RA adjustments is similar for both aircraft. About 35% were “Climb” RAs, about 32-38% were “Descend” RAs, and about 28-32% were “Do Not Climb/Descend” RAs.

Then two drops in RA issues can be seen in the results. First, the number of second RA adjustments of both aircraft reduces to 32%, of which most were “Do Not Climb/Descend” RAs. Second, the numbers of third to fifth RA adjustments drop to 0-2%.

Next, all “Level-Off” RA adjustments in Table 17-12 are in red font. This is to express that these RA adjustments actually represent clear of conflict messages in simulation runs where more than one clear of conflict message was issued, i.e. TCAS declared the other aircraft again as a threat after it has determined to be clear of conflict. Because it was not anticipated that TCAS would cause loops of conflict detection and conflict resolution until the CPA has been passed, only five RA adjustments have been stored.

In order to better understand the dynamics of TCAS which could cause such loops in real operations, it is recommended to modify the way the output is saved, such that correlated RAs and clear of conflict messages can be made visible.

It is likely that also in real TCAS operations the pilots would go into a loop of conflict detection and clear of conflict declaration until the aircraft arrive at the CPA. In the reference case the time of CPA was at 224.2s, see [Table 16-1](#), which is about 100s later than the mean time of initial RA issue. So it is likely that once a loop was entered, many cycles appeared. Probably, this is also the reason why the standard deviation of the time of issue of third to fifth RA adjustments is very large, 10.1-27.5s.

In the reference case no near mid-air collision or mid-air collision was encountered, but in the Monte Carlo simulation 36 near mid-air collisions were encountered. Because no near mid-air collision would have happened on the initial flight paths of the aircraft, these are TCAS induced near mid-air collisions.

In order to better understand what is going on, we take a look at the empirical density of the vertical miss distance in [Figure 17-11](#).

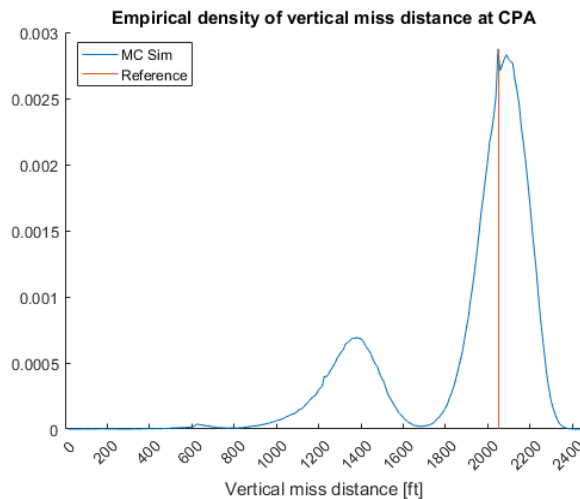


Figure 17-11: Empirical density of vertical miss distance at CPA of Monte Carlo simulation (blue line) and the reference case (orange line) for case 6.

In the graph three peaks can be found. The highest peak is at about 2,100ft in the range of 1,700-2,400ft, which is slightly higher than the vertical miss distance of the reference case, 2053ft. It covers approximately 70-80% of the data. Therefore it is likely that these vertical miss distances occurred when both aircraft issue the same RA like in the reference case, about 77% of the simulation runs.

The second peak is at about 1,400ft in the range of 1,000-1,700ft. This region covers approximately 20% of the data, and probably relates to the simulation runs in which aircraft 1 issued a “Descend” RA and aircraft 2 issued a “Climb” RA, about 23% of the simulation runs.

The third peak is a very low and in the range of 500-700ft; it covers less than 1% of the data. It is likely that these vertical miss distances appear when the aircraft issue a third or higher RA adjustment.

Case 7:

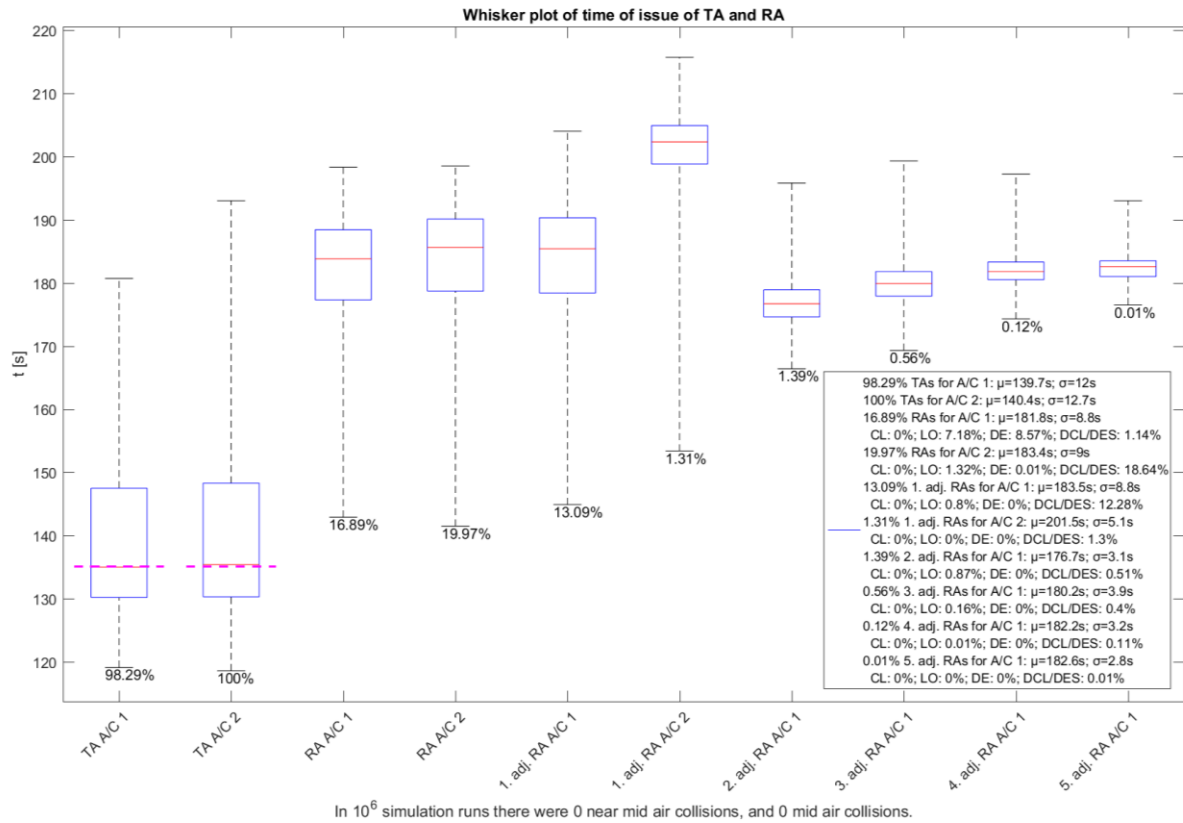


Figure 17-12: Graphical visualization of Monte Carlo simulation results for case 7. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-13: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 7.

Type	A/C	Reference case	Time of Issue [s]					Confidence interval		
			Mean	σ	Min	Max	Median	50%	95%	99%
TA	1	135.1	139.7	12	119.1	180.7	135	[130.2,147.5]	[126.6,168.6]	[125.0,174.1]
	2	135.1	140.4	12.7	118.6	193	135.4	[130.3,148.3]	[126.6,171.9]	[125.0,177.0]
RA	1	-	181.8	8.8	142.9	198.3	183.8	[177.3,188.4]	[159.6,193.3]	[150.0,194.8]
	2	-	183.4	9	141.5	198.5	185.6	[178.7,190.1]	[160.8,194.8]	[151.2,196.1]
1 st RA adj.	1	-	183.5	8.8	144.9	204	185.4	[178.4,190.3]	[161.8,195.5]	[154.5,197.5]
	2	-	201.5	5.1	153.4	215.7	202.3	[198.8,204.9]	[191.9,209.3]	[178.1,211.3]
2 nd RA adj.	1	-	176.7	3.1	166.4	195.8	176.7	[174.6,178.9]	[170.8,182.7]	[169.0,184.9]
	2	-	-	-	-	-	-	-	-	-
3 rd RA adj.	1	-	180.2	3.9	169.3	199.3	179.9	[177.9,181.8]	[174.2,192.6]	[172.3,196.6]
	2	-	-	-	-	-	-	-	-	-
4 th RA adj.	1	-	182.2	3.2	174.3	197.2	181.8	[180.5,183.3]	[177.1,192.7]	[175.9,195.2]
	2	-	-	-	-	-	-	-	-	-
5 th RA adj.	1	-	182.6	2.8	176.5	193	182.6	[181.0,183.5]	[177.9,193.0]	[176.5,193.0]
	2	-	-	-	-	-	-	-	-	-

Table 17-14: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 7.
Note: Bold font indicates a TA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	982888	98.29	-	-	-	-	-	-	-	-
	2	1000000	100	-	-	-	-	-	-	-	-
RA	1	168880	16.89	-	-	71765	7.18	85710	8.57	11405	1.14
	2	199673	19.97	2	0.00	13150	1.32	80	0.01	186441	18.64
1 st RA adj.	1	130884	13.09	-	-	8048	0.80	-	-	122836	12.28
	2	13060	1.31	2	0.00	1	0.00	23	0.00	13034	1.3
2 nd RA adj.	1	13859	1.39	-	-	8719	0.87	-	-	5140	0.51
	2	-	-	-	-	-	-	-	-	-	-
3 rd RA adj.	1	5582	0.56	-	-	1572	0.16	-	-	4010	0.40
	2	-	-	-	-	-	-	-	-	-	-
4 th RA adj.	1	1238	0.12	-	-	114	0.01	-	-	1124	0.11
	2	-	-	-	-	-	-	-	-	-	-
5 th RA adj.	1	78	0.01	-	-	6	0.00	-	-	72	0.01
	2	-	-	-	-	-	-	-	-	-	-

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in Figure 17-12 and numerically in Table 17-13 and Table 17-14.

In the reference case both aircraft issued a TA at 135.1s. No RAs were issued.

In the Monte Carlo simulation also TAs by both aircraft were encountered. The median of the time of issue is coinciding with the time of issue of the reference case. For aircraft 1 the median is 135.0s and for aircraft 2 135.4s. However, a difference between the mean and the median can be seen. The mean time of issue of a TA is 139.7s for aircraft 1 and 140.4s for aircraft. So the measurement errors lead to a larger deviation of the TAs issued later than in the reference case.

Another effect of the measurement errors can also be seen in the number of TAs issued. In the Monte Carlo simulation in 98.29% of the simulation a TA was issued for aircraft 1 and in 100% of the simulation for aircraft 2. This is remarkably, as both aircraft should have the same thresholds for the altitude range test for a TA, and one would expect that a TA is issued by both aircraft at the same time. The difference can be explained through the “time to co-altitude is small” test, see Section 6.2. If the altitude velocity of both aircraft has the same sign, then the aircraft with the lower altitude has should use difference threshold values, see threshold value T_v in Table 6-2. As aircraft 1 has the lower magnitude of altitude velocity, it is reasonable that in some cases the threshold values of aircraft 1 are lower than the threshold values of aircraft 2. This could lead to aircraft 2 reacting upon an RA before aircraft 1 reaches the necessary conditions to issue a TA.

Regarding RAs, in 16.89% of the simulation an RA was issued by aircraft 1 with a mean time of issue of 181.8s, and in 19.97% of the simulation an RA was issued by aircraft 2 with a mean time of issue of 183.4s. The lower number of RAs for aircraft 1 is likely due to the reason causing the lower number of TAs issued by aircraft 1 as explained above. However, also a large standard deviation of the time of issue for TAs and RAs is present. The standard deviations of the time of issue of TAs are 12.0 and 12.7s, and of RAs 8.8 and 9.0s. This is probably due to the low closing speed of 200ft/min, because this is the only case with a low closing speed and the only case in which such large standard deviations occurred.

In the type of RA a difference can be spotted between the two aircraft. Aircraft 1, which is at a lower altitude but descends with a lower altitude velocity compared to aircraft 2, most often selected a “Level-Off” RA, 7.18%, or a “Descend” RA, 8.57%, and only sometimes a “Do Not Climb/Descend” RA, 1.14%. The high number of “Level-Off” RAs is remarkable, as this is an altitude crossing RA and the reference case showed that no RA is necessary.

For aircraft 2, the observed RAs in the Monte Carlo simulation are different to aircraft 1. In 1.32% of the simulation a “Level-Off” RA was issued, in 0.01% of the simulation a “Descend” RA was issued, in 18.64% of the simulation a “Do Not Climb/Descend” RA was issued, and almost never, in only two simulation runs, a “Climb” RA was issued. Hence, opposed to aircraft 1, aircraft 2 issued in most simulations a preventive RA and in only 0.01% of the simulation an altitude crossing RA.

Regarding RA adjustments, it could be observed that for aircraft 1 an RA adjustment occurred regularly. In 13.09% of the simulation a first RA adjustment was issued of which most were “Do Not Climb/Descend” RAs: 12.28% do not climb or descend, and 0.8% level off. Additionally, a low number of 2nd, 3rd, 4th and 5th RA adjustments were issued by aircraft 1 as well: 1.39%, 0.56%, 0.12% and 0.01% respectively. It should be noted that the simulation output only captured up to five RA adjustments per aircraft, so it is possible that more adjustments were issued.

Also, all “Level-Off” RA adjustments in [Table 17-14](#) are in red font. Similar to case 6, this is to express that these RA adjustments actually represent clear of conflict messages in simulation runs where more than one clear of conflict message was issued, i.e. TCAS declared the other aircraft again as a threat after it has determined to be clear of conflict. Because it was not anticipated that TCAS would cause loops of conflict detection and conflict resolution until the CPA has been passed only five RA adjustments have been stored.

In order to better understand the dynamics of TCAS which could cause such loops in real operations, it is recommended to modify the way the output is saved, such that correlated RAs and clear of conflict messages can be made visible.

Next, we take a closer look into the simulation results for the cases where “Level-Off” RA adjustments were issued consecutively. The likely cause is the assumption that the pilot levels off when he or she receives a clear of conflict message (see assumption 7 of agent “Cockpit i” in Section 13). Since both aircraft are descending and aircraft 1 is below aircraft 2, a level off manoeuvre by aircraft 1 would result that the aircraft approach each other in altitude. Hence, it can be concluded that for aircraft 1 the measurement errors and/or the pilot behaviour cause in rare cases a loop where an RA is issued, followed by a clear of conflict message at sufficient altitude separation, followed by another RA issue, and so on until the closest point of approach had been reached. However, the validity of underlying assumption 7 needs to be verified. If it turns out that pilot reaction upon clear of conflict messages is differently, then this needs to be adopted in the new TCAS model.

Next, in order to get further insight into the results, we take a look into the empirical density of the vertical miss distance in [Figure 17-13](#). In the graph it can be seen that the vertical miss distances of the simulation runs are centred at the vertical miss distance of the reference case. Furthermore, it can be seen that rare cases spread over the range of 410 to 1180ft. Hence, it can be concluded that with very few exceptions the uncertainties do not have an impact on the vertical miss distance in

this case. Hence, in neither the reference case, nor in the Monte Carlo simulation mid-air or near mid-air collisions were encountered.

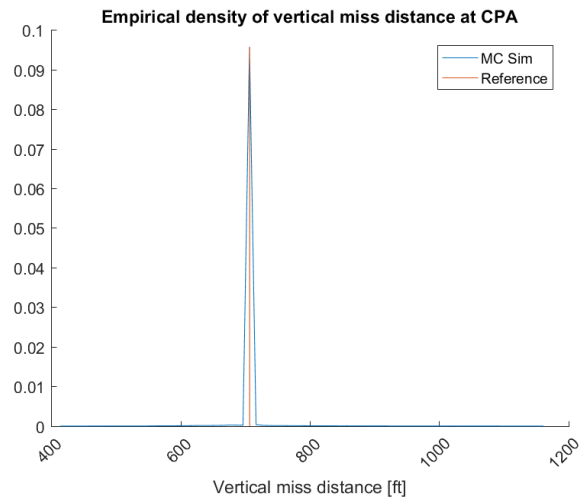


Figure 17-13: Empirical density of vertical miss distance at CPA of Monte Carlo simulation (blue line) and the reference case (orange line) for case 7.

Case 8:

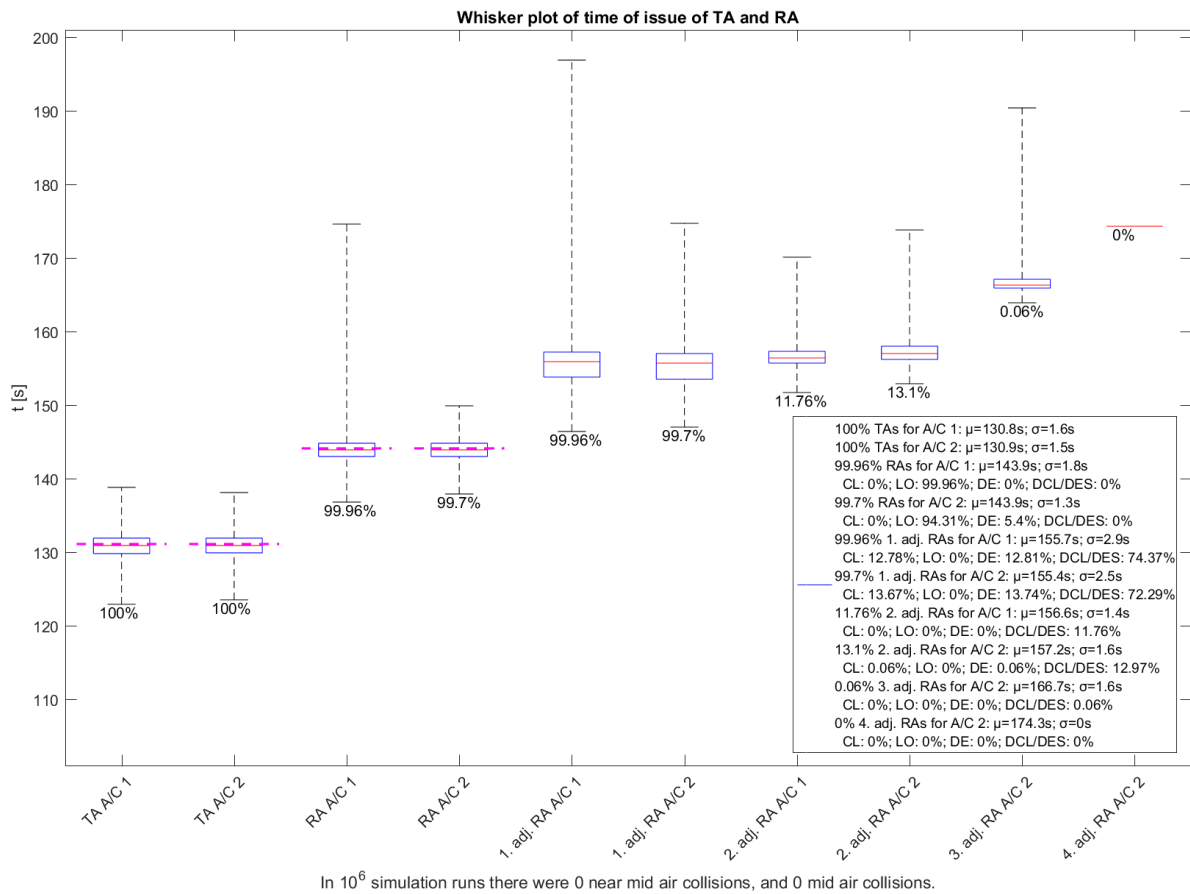


Figure 17-14: Graphical visualization of Monte Carlo simulation results for case 7. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in Figure 17-14 and numerically in Table 17-15 and Table 17-16.

Table 17-15: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 8.

Type	A/C	Time of Issue [s]									
		Reference case	Mean	σ	Min	Max	Median	Confidence interval			
TA	1	131.1	130.8	1.6	122.9	138.8	130.9	[129.8,131.9]	[127.6,133.9]	[126.4,135.0]	
	2	131.1	130.9	1.5	123.5	138.1	130.9	[129.9,131.9]	[127.9,133.8]	[127.0,134.7]	
RA	1	144.1	143.9	1.8	136.8	174.6	143.9	[143.0,144.8]	[141.1,146.6]	[140.2,147.7]	
	2	144.1	143.9	1.3	137.9	149.9	143.9	[143.0,144.8]	[141.3,146.5]	[140.5,147.2]	
1 st RA adj.	1	-	155.7	2.9	146.4	196.9	155.9	[153.8,157.2]	[150.8,160.7]	[149.7,163.7]	
	2	-	155.4	2.5	147	174.7	155.7	[153.5,157.0]	[150.9,160.3]	[149.9,162.7]	
2 nd RA adj.	1	-	156.6	1.4	151.7	170.1	156.4	[155.7,157.3]	[154.3,159.9]	[153.6,161.9]	
	2	-	157.2	1.6	152.9	173.8	157.0	[156.2,158.0]	[155.0,161.0]	[154.4,164.2]	
3 rd RA adj.	1	-	-	-	-	-	-	-	-	-	
	2	-	166.7	1.6	163.9	190.4	166.3	[165.9,167.1]	[165.5,169.3]	[165.4,175.1]	
4 th RA adj.	1	-	-	-	-	-	-	-	-	-	
	2	-	174.3	0.0	174.3	174.3	174.3	[174.3,174.3]	[174.3,174.3]	[174.3,174.3]	

Table 17-16: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 8.
Note: Bold font indicates a TA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	1000000	100	-	-	-	-	-	-	-	-
	2	1000000	100	-	-	-	-	-	-	-	-
RA	1	999598	99.96	-	-	999598	99.96	-	-	-	-
	2	997042	99.7	-	-	943068	94.31	53974	5.40	-	-
1 st RA adj.	1	999592	99.96	127759	12.78	-	-	128139	12.81	743694	74.37
	2	997040	99.7	136730	13.67	-	-	137379	13.74	722931	72.29
2 nd RA adj.	1	117589	11.76	-	-	-	-	-	-	117589	11.76
	2	130976	13.1	638	0.06	2	0.00	632	0.06	129704	12.97
3 rd RA adj.	1	-	-	-	-	-	-	-	-	-	-
	2	556	0.06	2	0.00	1	0.00	-	-	553	0.06
4 th RA adj.	1	-	-	-	-	-	-	-	-	-	-
	2	1	0.00	-	-	1	0.00	-	-	-	-

In the reference case both aircraft issued a TA at 131.1s and a “Level-Off” RA at 144.1s.

In the Monte Carlo simulation a similar behaviour can be identified. Both aircraft issued a TA at a mean time of issue of 130.8 and 130.9s. In regard to RAs, small differences can be observed. Aircraft 1 did not issue an RA in all simulation runs, 99.96%, but if it did, it was always a level RA similar to the reference case. Aircraft 2 did not issue an RA in all simulations either, 99.7%, and when it did it, it issued sometimes a “Descend” RA instead of a level RA, 5.4% of the simulation runs.

Moreover, in Table 17-16 it can be seen, that if an RA was issued, in almost all cases one RA adjustment was issued, too. Also, most of the adjustments were of preventive nature, in 72-75% of the simulation runs, and the remaining were climb or “Descend” RA adjustments. Then in about 12% of the simulation aircraft 1 issued a second RA adjustment of which all are “Do Not Climb/Descend” RAs, and no third to fifth RA adjustments. Aircraft 2 also issued second RA adjustments, 13.1% of the simulation runs, but also 556 third and 1 fourth RA adjustment. Most of these were “Do Not Climb/Descend” RAs, but in 4 cases a “Level-Off” RA adjustment was issued.

The few “Level-Off” RA adjustments are remarkable, because a “Level-Off” RA adjustment is just a mean to model a clear of conflict message, as described in Sections 10 and 12.2. Also, it should be noted that in the result evaluation only the last RA adjustment was counted as a clear of conflict message. Therefore, the “Level-Off” RA adjustments indicate that in these rare cases a new threat was declared after TCAS issued a clear of conflict message.

Neither in the reference case, nor in the Monte Carlo simulation were mid-air or near mid-air collisions encountered. This can be also seen in the empirical density in the graphs in Figure 17-15. As can be seen in the graph on the left hand side of the figure, in general a higher vertical miss distance was achieved compared to the reference case. A likely reason is that in the model it is assumed that a pilot reacts upon a do not climb or “Do Not Descend” RA adjustment by descending or climbing with low altitude velocity, such that he or she is not flying at the edge of the issued altitude velocity limits, see Section 12.2. Hence, this reaction could have caused an increase in vertical velocity in 72-74% of the simulation where a “Do Not Climb/Descend” RA adjustment was issued.

Finally, an observation can be made in the graph on the right hand side of the figure. At about 1,100ft and 1,400ft there are two small peaks. An educated guess is that these vertical miss distances are related to the simulation runs where a third or fourth RA adjustment was issued.

Last but not least, again no clear correlation between the uncertainties and the results could be found.

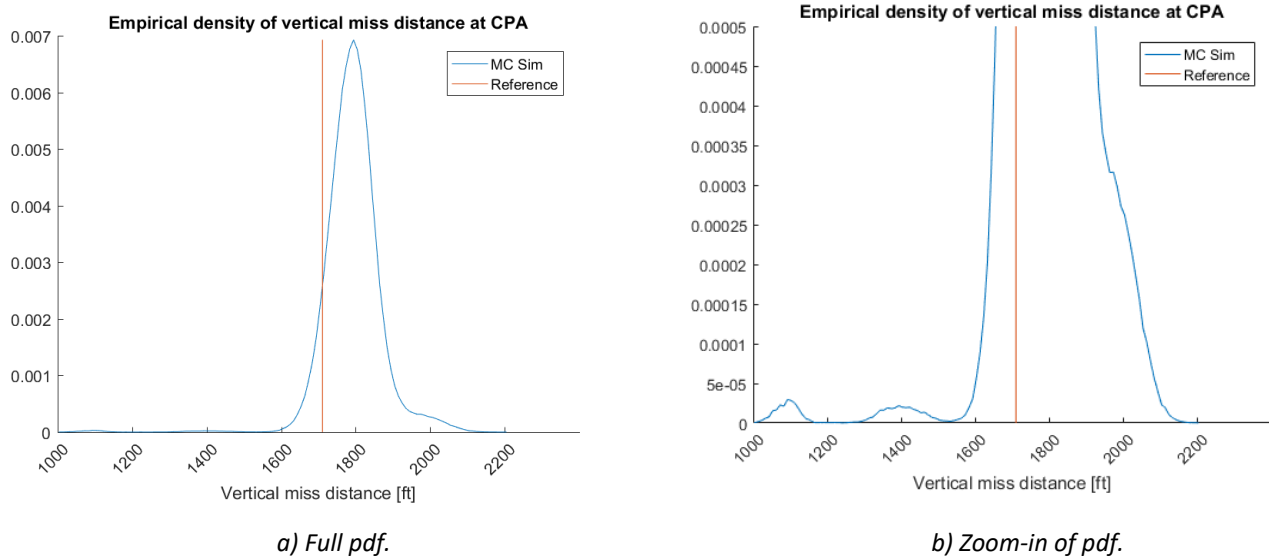


Figure 17-15: Empirical density of vertical miss distance at CPA of Monte Carlo simulation (blue line) and the reference case (orange line) for case 8. a) full pdf graph; b) zoom-in left of pdf.

Case 9:

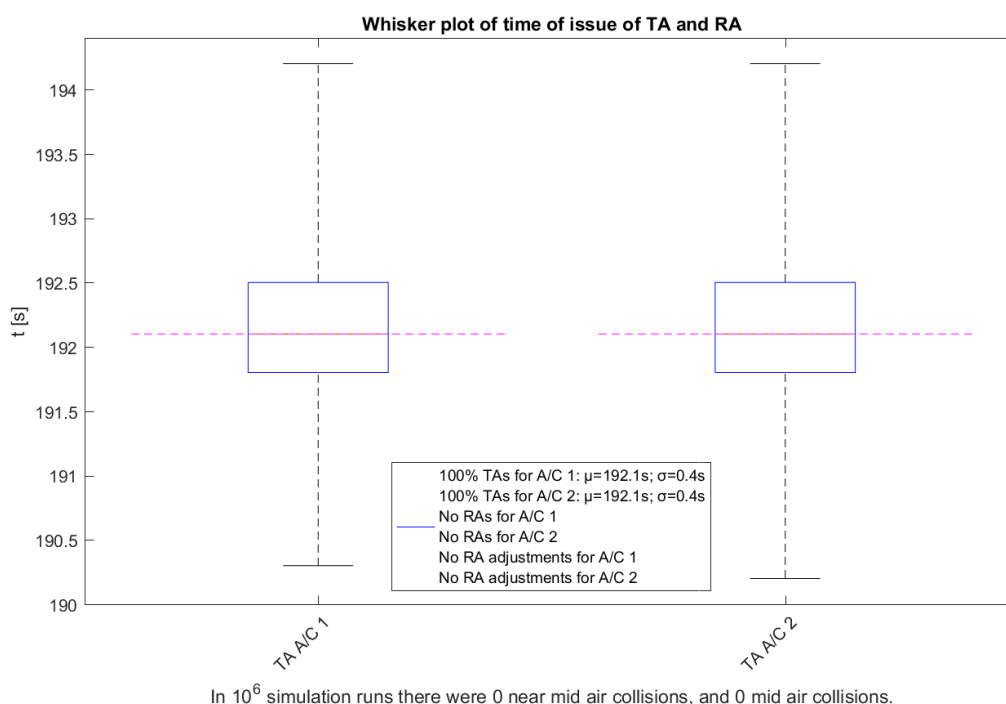


Figure 17-16: Graphical visualization of Monte Carlo simulation results for case 9. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-17: Monte Carlo simulation results of time of issue of TAs, RAs and adjusted RAs for case 9.

Type	A/C	Time of Issue [s]								
		Reference case	Mean					Confidence interval		
			Mean	σ	Min	Max	Median	50%	95%	99%
TA	1	192.1	192.1	0.4	190.3	194.2	192.1	[191.8,192.5]	[191.3,193.0]	[191.0,193.2]
	2	192.1	192.1	0.4	190.2	194.2	192.1	[191.0,193.2]	[191.3,193.0]	[191.0,193.2]
RA	1	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-

Table 17-18: Monte Carlo simulation results of number and type of TAs, RAs and adjusted RAs for case 9.

Note: Bold font indicates a TA in the reference case.

Type	A/C	Number of RA/TA		Number of CL		Number LO		Number of DE		Number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	1000000	100	-	-	-	-	-	-	-	-
	2	1000000	100	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-
RA	1	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-

The results of the reference case, as described in Section 16, and of the Monte Carlo simulation are presented graphically in Figure 17-16 and numerically in Table 17-17 and Table 17-18.

Case 9 reflects the reference case. In 100% of the Monte Carlo simulations TAs were issued by both aircraft. The mean time of issue is for both aircraft equal to the time of issue of the reference case, 192.1s. The standard deviation is both aircraft also equal and low at 0.4s.

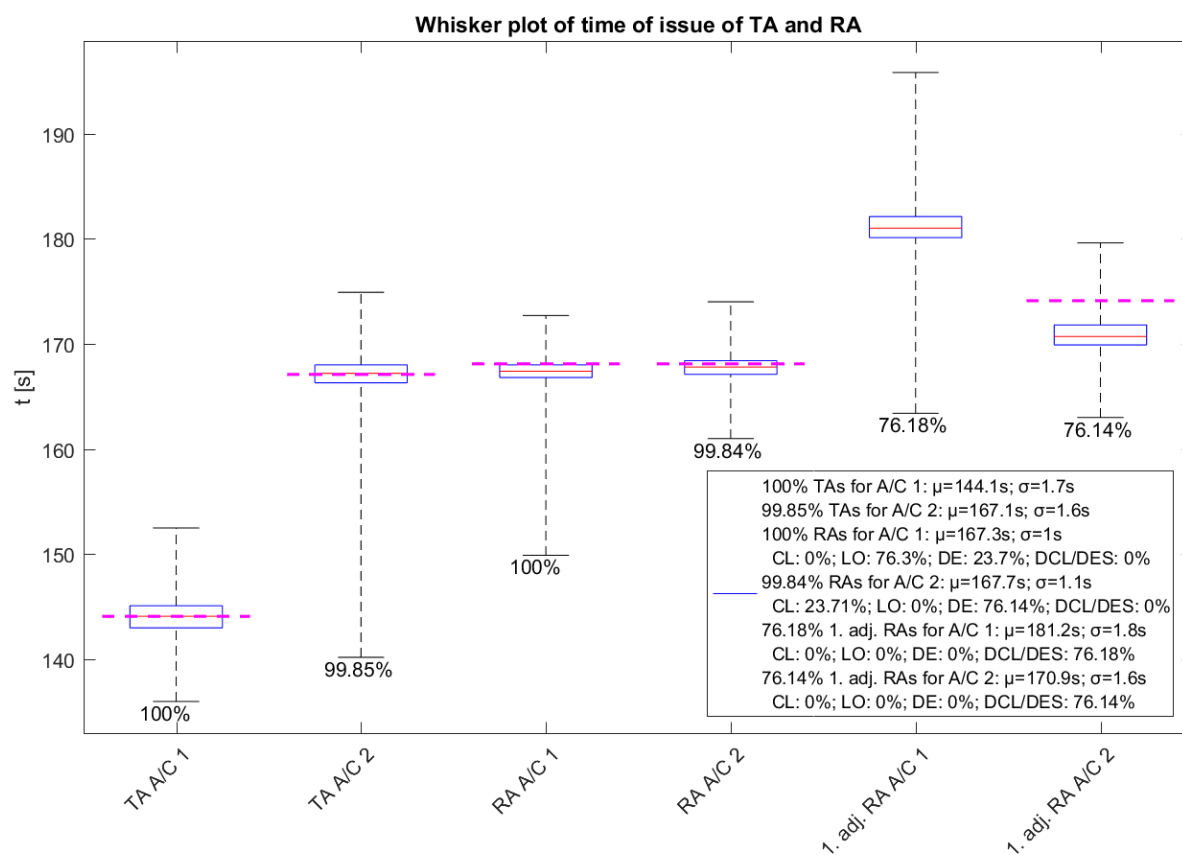
Neither in the reference case, nor in the Monte Carlo simulation a near mid-air collision or mid-air collision was encountered. But it should be noted that without any flight path adjustments no mid-air collisions or near mid-air collision would have occurred either.

17.2 Simulation Results II (Effect of Pilot Delay Variations)

In the second Monte Carlo set up, the random variation in pilot delay was omitted. Since in cases 1 and 9 no RA was issued, the pilot delay variation does not affect the Monte Carlo simulation.

Next, cases 2-8 are evaluated on the effect of variation in pilot delay.

Case 2:



In 10^6 simulation runs there were 0 near mid air collisions, and 0 mid air collisions.

Figure 17-17: Graphical visualization of Monte Carlo simulation results for case 2 with pilot delay variation modified to zero. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

Table 17-19: Difference in time of issue of TA/RA of case 2 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in time of issue [s]				
		Mean	σ	Min	Max	Median
TA	1	-	-	0.6	0.1	-
	2	-	-	0.3	0.5	-
RA	1	-	-	-1.2	0.3	-
	2	-	-	0.2	-0.4	-
1 st RA adj.	1	0.1	-	-0.3	-0.3	-
	2	-	-	-0.4	0.3	-

Table 17-20: Difference in number and type of TA/RA of case 2 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in number of RA/TA		Difference in number of CL		Difference in number LO		Difference in number of DE		Difference in number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	-	-	-	-	-	-	-	-	-	-
	2	244	0.03	-	-	-	-	-	-	-	-
RA	1	-	-	-	-	41	0.01	-41	-0.01	-	-
	2	243	0.02	-40	-0.00	-	-	283	0.03	-	-
1 st RA adj.	1	117	0.01	-	-	-	-	-	-	117	0.01
	2	283	0.03	-	-	-	-	-	-	283	0.03

In [Figure 17-17](#) the results of the Monte Carlo simulation, which did not account for pilot delay variation are given. In [Table 17-19](#) and [Table 17-20](#) the differences between the Monte Carlo simulation with pilot delay variation and without pilot delay variation are presented. It can be seen that the differences are very small. For aircraft 2, the increased number of “Do Not Climb/Descend” RA adjustments could be caused by the difference in initial “Descend” RAs, which are unaffected by the pilot delay variation. Also for both aircraft the difference in RA adjustments is too small for meaningful feedback.

In [Figure 17-18](#) the empirical densities of vertical miss distance are given for both Monte Carlo simulations. The new results are centred at the reference vertical miss distance. Hence, it can be concluded that a pilot delay variation causes a small increase in vertical miss distance.

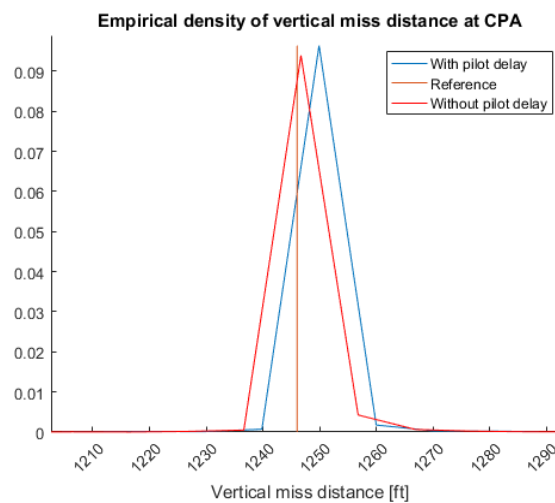


Figure 17-18: Empirical densities of vertical miss distance of case 2 of Monte Carlo simulations I and II.

Case 3:

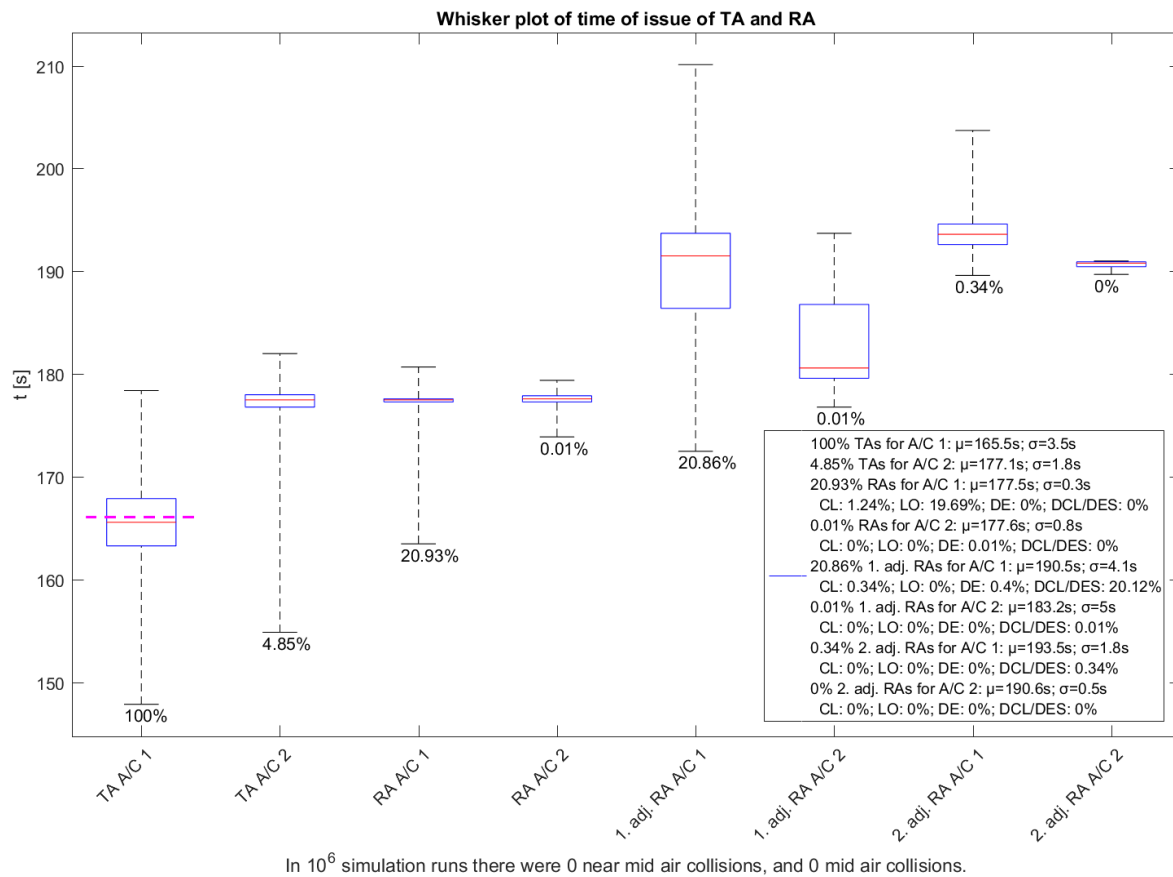


Figure 17-19: Graphical visualization of Monte Carlo simulation results for case 3 with pilot delay variation modified to zero. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

In Figure 17-19 the results of the Monte Carlo simulation, which did not account for pilot delay variation, are given. In Table 17-21 and Table 17-22 the differences between the Monte Carlo simulation with pilot delay variation and without pilot delay variation are presented. It can be seen that the differences are very small and are too small for meaningful feedback.

Table 17-21: Difference in time of issue of TA/RA of case 3 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in time of issue [s]				
		Mean	σ	Min	Max	Median
TA	1	-	-	1.3	0.2	-
	2	-	-	0.4	0.1	-
RA	1	-	-	1.2	-0.1	-
	2	0.1	-0.2	-0.6	-2.3	0.1
1 st RA adj.	1	0.4	-0.2	-2.8	0.4	1.1
	2	-0.5	-	-0.8	-0.4	-0.9
2 nd RA adj.	1	-0.4	-0.1	0.2	-2	-0.1
	2	-0.6	-3.3	3.1	-9	-

Table 17-22: Difference in number and type of TA/RA of case 3 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in number of RA/TA		Difference in number of CL		Difference in number LO		Difference in number of DE		Difference in number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	9	0.01	-	-	-	-	-	-	-	-
	2	-175	-0.02	-	-	-	-	-	-	-	-
RA	1	-115	-0.01	-209	-0.02	89	0.01	5	-	-	-
	2	12	-	1	-	-	-	11	-	-	-
1 st RA adj.	1	76	0.01	-160	-0.02	-	-	-266	-0.02	502	0.05
	2	9	-	-1	-	-	-	-	-	10	-
2 nd RA adj.	1	-141	-0.02	-	-	-	-	-	-	-141	-0.02
	2	-3	-	-	-	-	-	-1	-	-2	-

In [Figure 17-20](#) the empirical densities of vertical miss distances are given for both Monte Carlo simulations. No difference can be seen, which means that pilot delay variation has no effect for case 3.

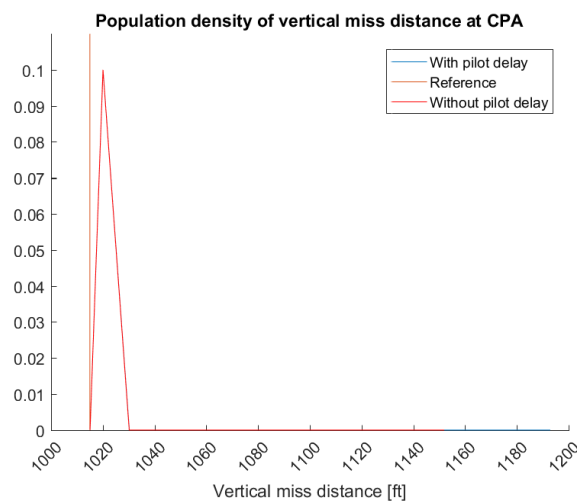


Figure 17-20: Empirical densities of vertical miss distance of case 3 of Monte Carlo simulations I and II.

Case 4:

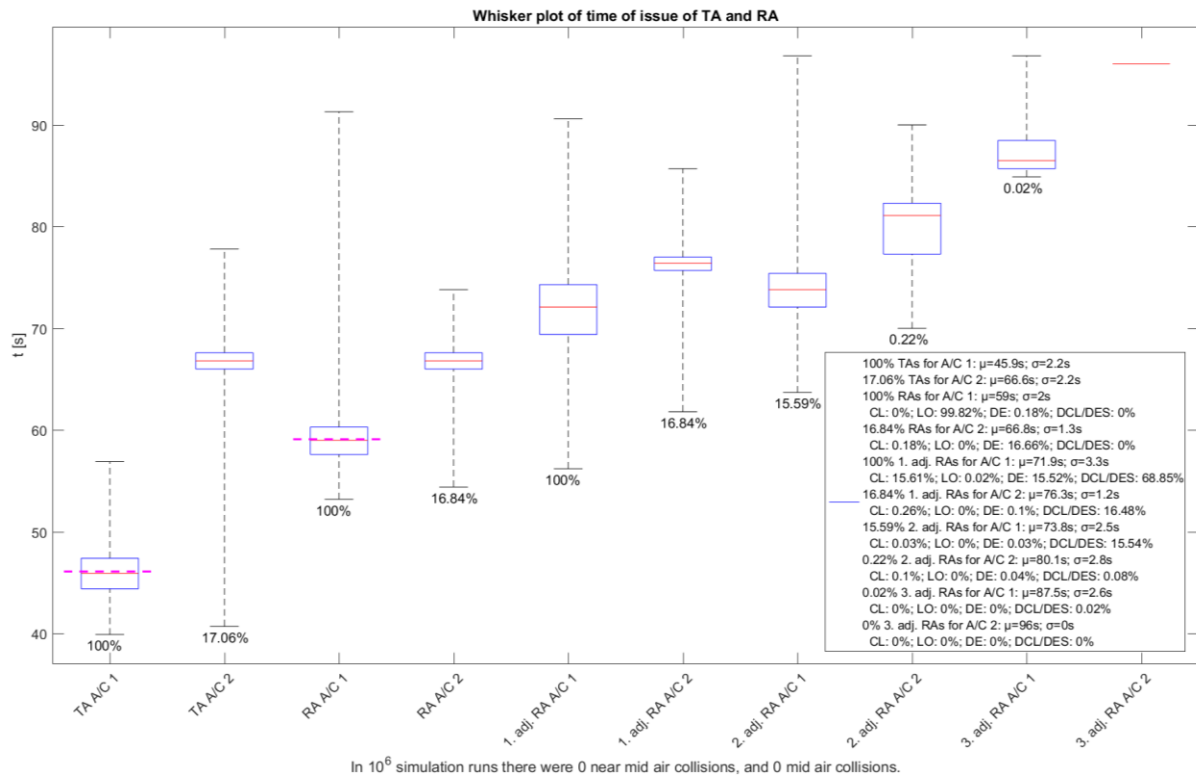


Figure 17-21: Graphical visualization of Monte Carlo simulation results for case 4 with pilot delay variation modified to zero. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

In Figure 17-21 the results of the Monte Carlo simulation, which did not account for pilot delay variation are presented. In Table 17-23 and Table 17-24 the differences between the Monte Carlo simulation with pilot delay variation and without pilot delay variation are given.

In Table 17-23 differences can be found in the time of issue of 3rd RA adjustments of aircraft 2. However, the big difference in the time of issue is caused due to the fact that no third RA adjustments by aircraft 2 were issued in the Monte Carlo simulation with pilot delay variation.

Table 17-23: Difference in time of issue of TA/RA of case 4 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in time of issue [s]				
		Mean	σ	Min	Max	Median
TA	1	-	-	0.3	0.4	-
	2	-	-	-0.3	-10	-
RA	1	-	-	0.2	16.6	-
	2	-	-0.1	-0.8	-0.3	-0.1
1 st RA adj.	1	-	-	-5.8	-2.4	-
	2	-	-	-3.3	0.1	-
2 nd RA adj.	1	-0.2	-	-2	0.3	-0.2
	2	-0.4	0.2	-0.5	5.1	-0.3
3 rd RA adj.	1	-	0.2	-	0.1	-0.1
	2	96	-	96	96	96

In Table 17-24 a large difference can be seen for second RA adjustments of aircraft 2. Without pilot delay variation in 11186 simulation runs more a “Do Not Climb/Descend” RA adjustments was issued. This is remarkable since the number of initial RAs and first RA adjustments were almost identical. Hence, without pilot delay variation in 1.12% more of the simulation runs the aircraft reached a position in which it was safe for aircraft 1 to weaken the RA. For aircraft 2 a difference could be observed as well, but since the differences of the initial RAs and first RA adjustments are equal, this is not due to the removed pilot delay variation, but due to measurement errors.

Table 17-24: Difference in number and type of TA/RA of case 4 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in number of RA/TA		Difference in number of CL		Difference in number LO		Difference in number of DE		Difference in number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	-	-	-	-	-	-	-	-	-	-
	2	1042	0.1	-	-	-	-	-	-	-	-
RA	1	-1	-	-	-	-1	-	-	-	-	-
	2	1117	0.12	-1	-	-	-	1118	0.11	-	-
1 st RA adj.	1	-9	-	9808	0.98	182	0.02	9049	0.9	-19048	-1.9
	2	1117	0.12	-45	-0.01	1	-	40	0.01	1121	0.11
2 nd RA adj.	1	11186	1.12	13	-	-	-	9	-	11164	1.12
	2	142	0.01	-29	-	1	-	27	-	143	0.01
3 rd RA adj.	1	29	0.01	-	-	-	-	-	-	29	0.01
	2	1	-	-	-	-	-	-	-	1	-

Although a difference in issue of RA adjustments was observed, no large differences can be found in the empirical densities of the vertical miss distance, as given in Figure 17-22. It can only be noted that the pilot delay variation is smoothing the outcome.

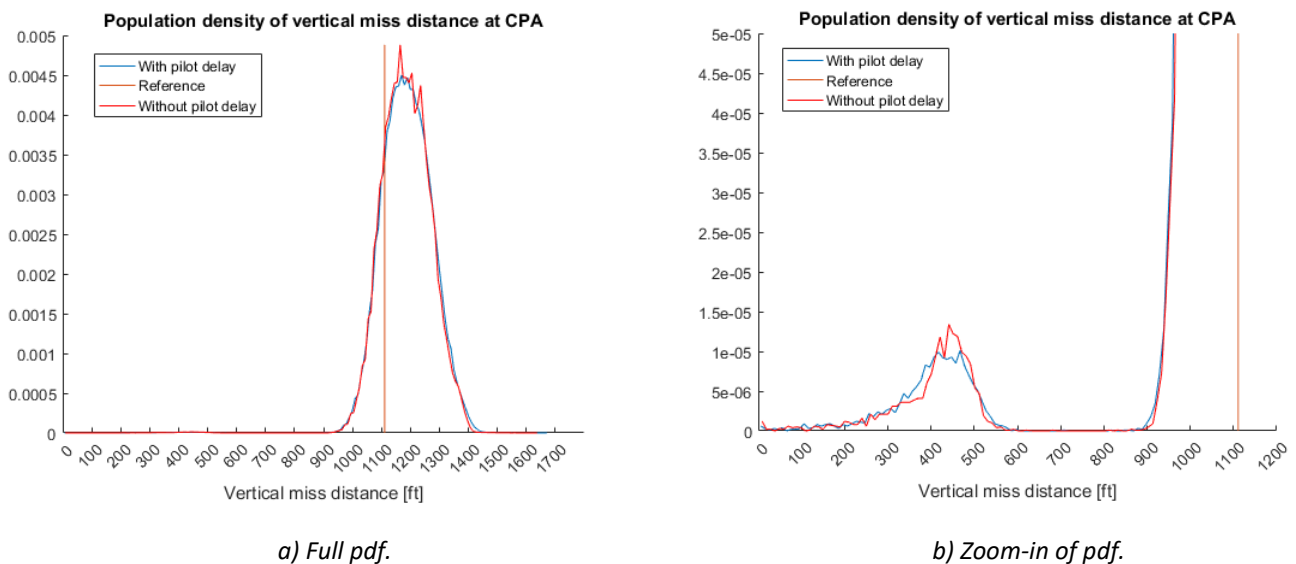


Figure 17-22: Empirical density of vertical miss distance at CPA of Monte Carlo simulations I and II for case 4. a) full pdf graph; b) zoom-in left of pdf.

Case 5:

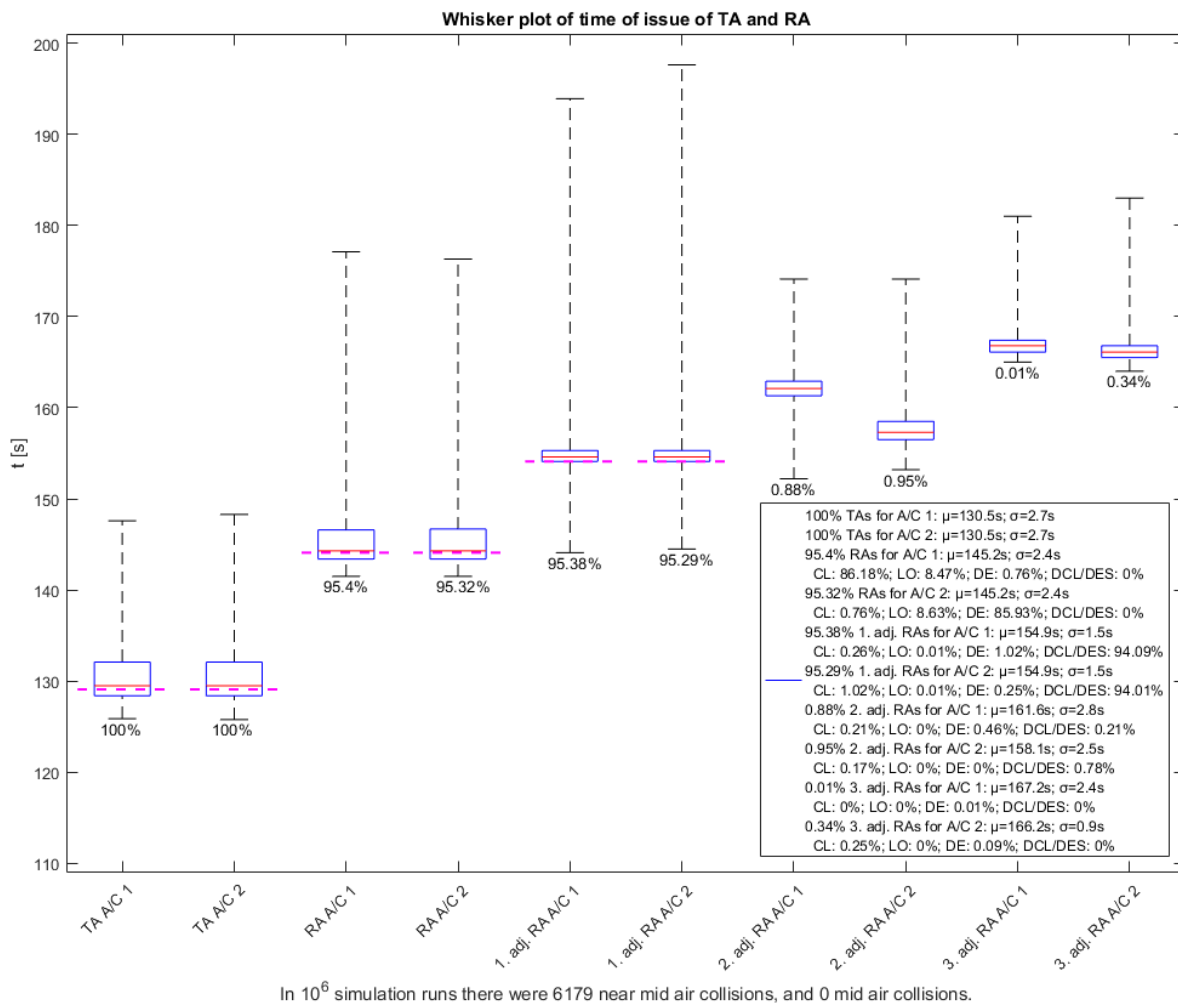


Figure 17-23: Graphical visualization of Monte Carlo simulation results for case 5 with pilot delay variation modified to zero. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

In Figure 17-23 the results of the Monte Carlo simulation, which did not account for pilot delay variation are presented. In Table 17-23 and Table 17-26 the differences between the Monte Carlo simulation with pilot delay variation and without pilot delay variation are given.

In Table 17-26 it can be seen that there is a large increase in “Do Not Climb/Descend” RAs for first RA adjustments increased twice as much as the number of initial RA, but the number of “Do Not Climb/Descend” RAs for second RA adjustments decreased. Hence, the pilot delay variation is causing in a small number of runs that TCAS may not issue a weakening of RA prior to a clear of conflict message.

Nevertheless, from the empirical densities in Figure 17-24 this effect cannot be observed, but only that the variation in pilot delay is smoothing the results.

Table 17-25: Difference in time of issue of TA/RA of case 5 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in time of issue [s]				
		Mean	σ	Min	Max	Median
TA	1	-	-	0.3	0.9	-
	2	-	-	0.3	-0.8	-
RA	1	-	-0.1	-	-0.6	-
	2	-	-0.1	0.1	-0.8	-
1 st RA adj.	1	-	-0.2	-6.8	-3	0.1
	2	-	-0.2	-6.2	-0.1	-
2 nd RA adj.	1	0.9	-0.4	-1.2	-0.1	0.2
	2	0.1	-0.1	-0.4	-0.8	0.1
3 rd RA adj.	1	0.2	0.6	0.3	-1	-
	2	-	-	0.6	4.2	-

Table 17-26: Difference in number and type of TA/RA of case 5 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in number of RA/TA		Difference in number of CL		Difference in number LO		Difference in number of DE		Difference in number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-
RA	1	1822	0.18	2442	0.25	-591	-0.06	7	-	-36	-
	2	2273	0.23	2	-	-158	-0.01	2454	0.24	-25	-
1 st RA adj.	1	1891	0.19	-1781	-0.18	64	0.01	-1780	-0.18	5388	0.54
	2	2340	0.23	-1798	-0.18	69	0.01	-1660	-0.17	5729	0.58
2 nd RA adj.	1	-1398	-0.14	294	0.03	-	-	-90	-0.01	-1602	-0.16
	2	-1288	-0.13	-43	-0.01	-2	-	-60	-0.01	-1183	-0.12
3 rd RA adj.	1	-88	-0.01	-41	-0.01	-	-	-47	-	-	-
	2	239	0.02	231	0.02	-	-	6	0.00	2	0.00

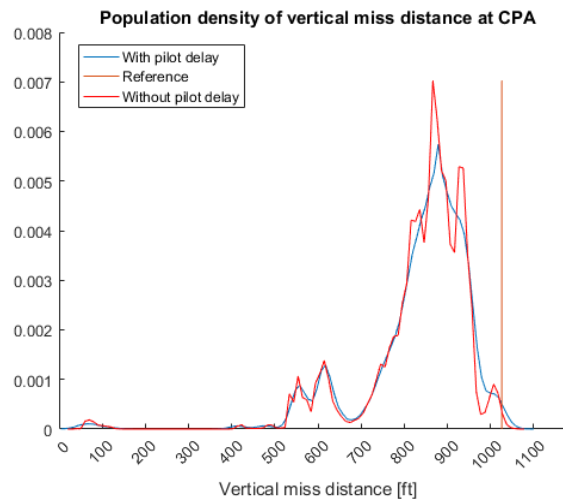


Figure 17-24: Empirical densities of vertical miss distance of case 5 of Monte Carlo simulations I and II.

Case 6:

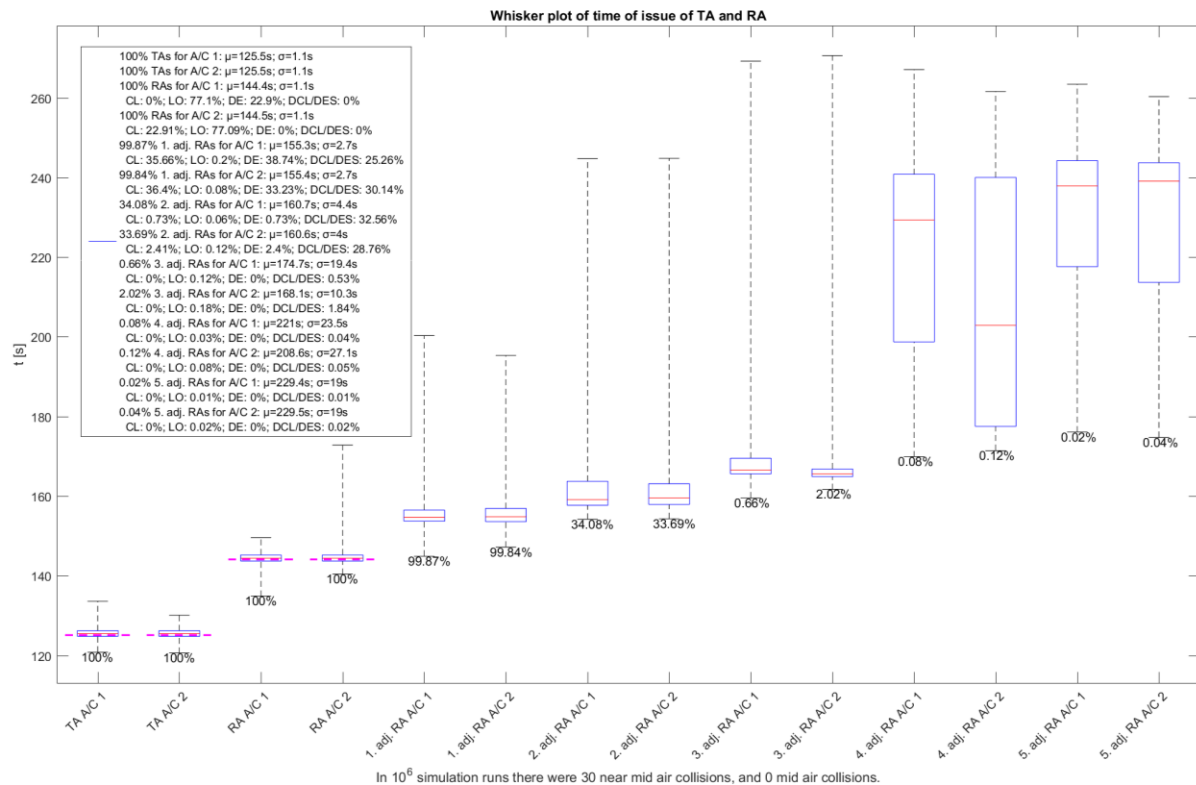


Figure 17-25: Graphical visualization of Monte Carlo simulation results for case 6 with pilot delay variation modified to zero. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

In Figure 17-25 the results of the Monte Carlo simulation, which did not account for pilot delay variation are presented. In Table 17-27 and Table 17-28 the differences between the Monte Carlo simulation with pilot delay variation and without pilot delay variation are given.

Large differences can be found for first and second RA adjustments in Table 17-28. In regard to first RA adjustments, in about 2% of the simulation runs a climb or “Descend” RA is issued instead of a “Do Not Climb/Descend” RA. In regard to second RA adjustments, an increase in the total number of RA adjustment can be seen, of which most were “Do Not Climb/Descend” RAs, about 1.5% of the simulation runs.

Table 17-27: Difference in time of issue of TA/RA of case 6 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in time of issue [s]				
		Mean	σ	Min	Max	Median
TA	1	-	-	-0.1	-	-
	2	-	-	-	-	-
RA	1	-0.1	-	-0.2	-0.3	-
	2	-	-	0.7	-1.5	-
1 st RA adj.	1	-0.1	-	-4.3	0.1	-0.1
	2	-0.1	-	-1.5	-0.8	-
2 nd RA adj.	1	-0.2	-0.1	0.6	0.7	-0.3
	2	-0.2	-	0.8	0.2	-0.4
3 rd RA adj.	1	-0.1	-0.2	-2.7	-1	-0.1
	2	-0.1	0.2	-0.1	1	-0.3
4 th RA adj.	1	-0.6	0.2	-2.4	0.3	0.2
	2	0.1	-0.4	0.3	-3.5	0.4
5 th RA adj.	1	-2.8	1.3	-17.4	-5.2	-2
	2	0.4	1.1	-0.4	0.5	0.7

Table 17-28: Difference in number and type of TA/RA of case 6 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in number of RA/TA		Difference in number of CL		Difference in number LO		Difference in number of DE		Difference in number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-
RA	1	-	-	-1	-	-830	-0.08	831	0.08	-	-
	2	-	-	836	0.09	-836	-0.09	-	-	-	-
1 st RA adj.	1	-1265	-0.12	9483	0.94	-64	-0.01	12189	1.22	-22873	-2.29
	2	-1431	-0.15	10090	1.01	78	0.01	8574	0.86	-20173	-2.01
2 nd RA adj.	1	15623	1.56	-698	-0.07	-38	-0.01	-675	-0.07	17034	1.7
	2	15045	1.51	-202	-0.02	-74	-0.01	120	0.02	15201	1.52
3 rd RA adj.	1	-505	-0.05	-	-	-22	-0.01	-	-	-483	-0.05
	2	394	0.04	-	-	31	-	-	-	363	0.03
4 th RA adj.	1	-28	-	-	-	-20	-0.01	-	-	-8	-
	2	-31	-0.01	-	-	-23	-	-	-	-8	-
5 th RA adj.	1	-11	-	-	-	-1	-	-	-	-10	-
	2	-39	-	-	-	-23	-0.01	-	-	-16	-

Finally, a small effect can be identified in the empirical density of the vertical miss distance. In [Figure 17-26](#) and increase in concentration of vertical miss distance at reference miss distance is present. This means that pilot delay variation takes some weight away from the reference point and yields in return a slightly higher vertical miss distance.

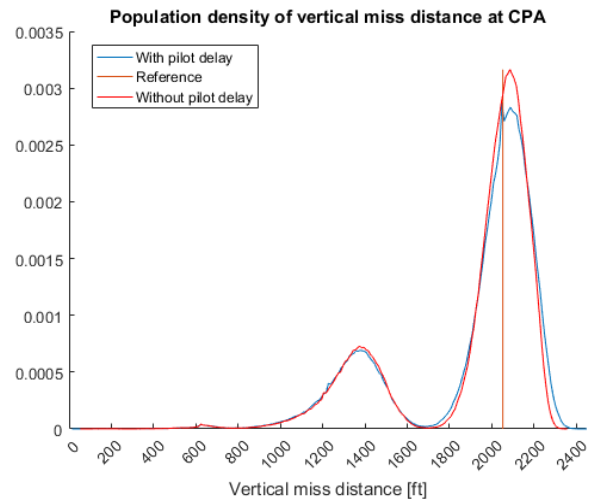


Figure 17-26: Empirical densities of vertical miss distance of case 6 of Monte Carlo simulations I and II.

Case 7:

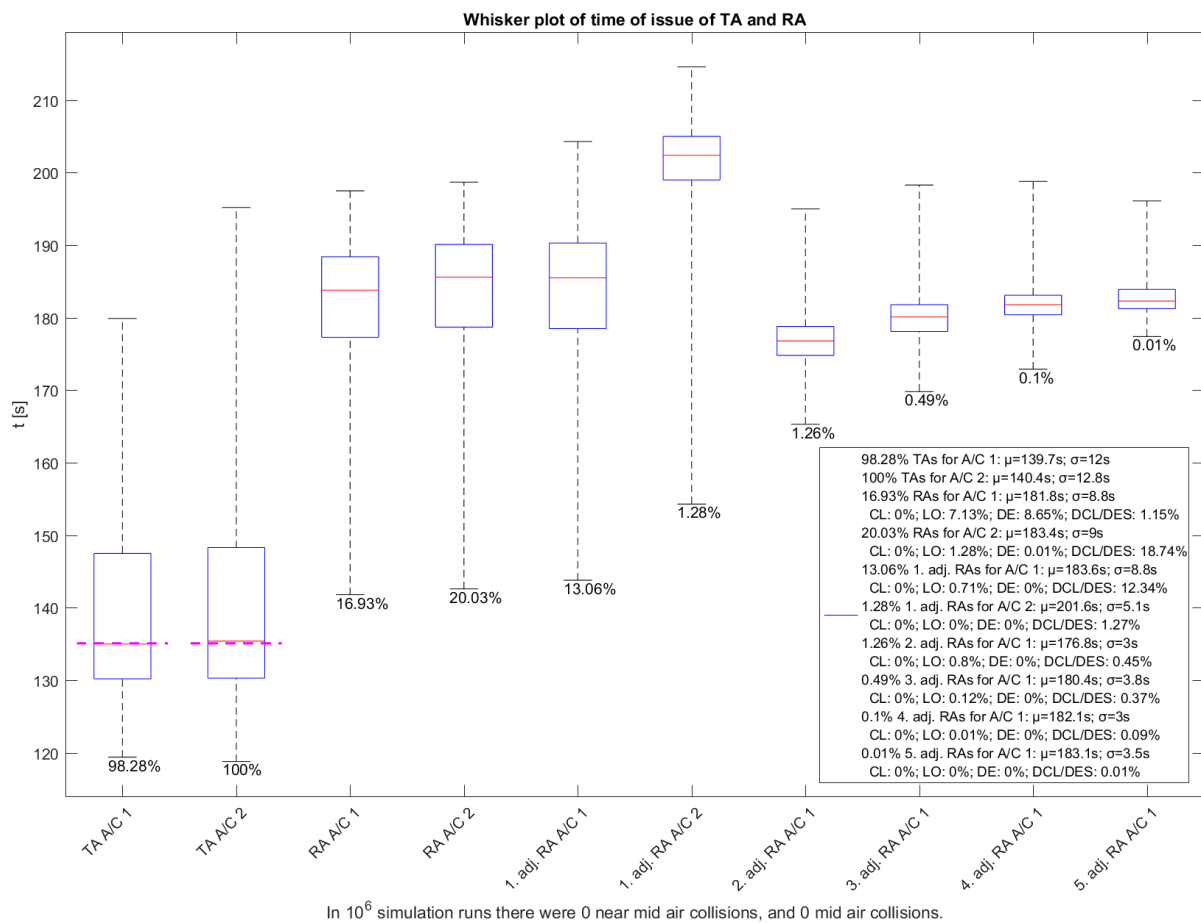


Figure 17-27: Graphical visualization of Monte Carlo simulation results for case 7 with pilot delay variation modified to zero. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

In Figure 17-27 the results of the Monte Carlo simulation, which did not account for pilot delay variation are presented. In Table 17-29 and Table 17-30 the differences between the Monte Carlo simulation with pilot delay variation and without pilot delay variation are given.

Neither in time of issue and number and type of TA and RA, nor in the empirical density of the vertical miss distance, see Figure 17-28, large differences can be found. Hence, for case 7, the pilot delay variation does not have a significant influence.

Table 17-29: Difference in time of issue of TA/RA of case 7 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in time of issue [s]				
		Mean	σ	Min	Max	Median
TA	1	-	-	0.3	-0.8	-
	2	-	0.1	0.2	2.2	-
RA	1	-	-	-1.1	-0.8	-
	2	-	-	1.1	0.2	-
1 st RA adj.	1	0.1	-	-1.1	0.3	0.1
	2	0.1	-	0.9	-1.1	0.1
2 nd RA adj.	1	0.1	-0.1	-1.1	-0.8	0.1
	2	-	-	-	-	-
3 rd RA adj.	1	0.2	-0.1	0.5	-1	0.2
	2	-	-	-	-	-
4 th RA adj.	1	-0.1	-0.2	-1.4	1.6	-
	2	-	-	-	-	-
5 th RA adj.	1	0.5	0.7	0.9	3.1	-0.3
	2	-	-	-	-	-

Table 17-30: Difference in number and type of TA/RA of case 7 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in number of RA/TA		Difference in number of CL		Difference in number LO		Difference in number of DE		Difference in number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	-56	-0.01	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-
RA	1	383	0.04	-	-	-418	-0.05	744	0.08	57	0.01
	2	601	0.06	-1	-	-333	-0.04	-13	-	948	0.1
1 st RA adj.	1	-334	-0.03	-	-	-931	-0.09	-	-	597	0.06
	2	-303	-0.03	3	-	-1	-	-3	-	-302	-0.03
2 nd RA adj.	1	-1292	-0.13	-	-	-685	-0.07	-	-	-607	-0.06
	2	-	-	-	-	-	-	-	-	-	-
3 rd RA adj.	1	-659	-0.07	-	-	-394	-0.04	-	-	-265	-0.03
	2	-	-	-	-	-	-	-	-	-	-
4 th RA adj.	1	-282	-0.02	-	-	-27	-	-	-	-255	-0.02
	2	-	-	-	-	-	-	-	-	-	-
5 th RA adj.	1	-15	-	-	-	-2	-	-	-	-13	-
	2	-	-	-	-	-	-	-	-	-	-

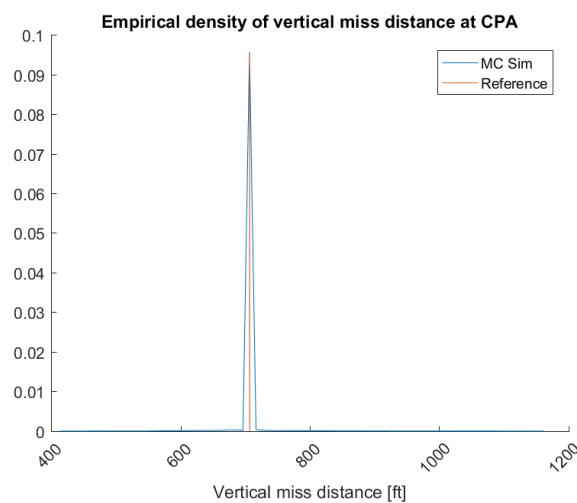


Figure 17-28: Empirical densities of vertical miss distance of case 7 of Monte Carlo simulations I and II.

Case 8:

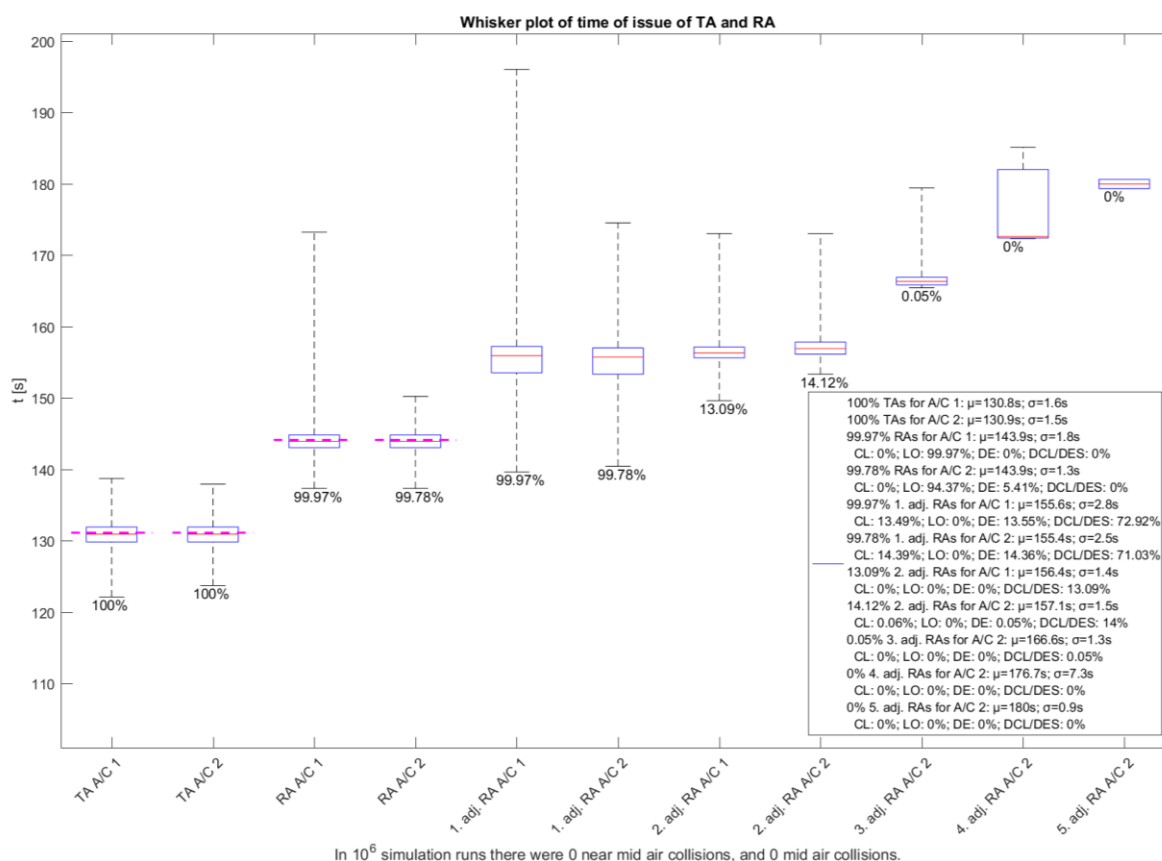


Figure 17-29: Graphical visualization of Monte Carlo simulation results for case 8 with pilot delay variation modified to zero. Red line: median; blue box: range of 50% confidence interval; dashed black line with T shaped ends: minimum and maximum values; percentages: what part of the Monte Carlo runs the results are based on; magenta dashed line: time of issue in the reference case.

In Figure 17-29 the results of the Monte Carlo simulation, which did not account for pilot delay variation are presented. In Table 17-31 and Table 17-32 the differences between the Monte Carlo simulation with pilot delay variation and without pilot delay variation are given.

Table 17-31: Difference in time of issue of TA/RA of case 8 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in time of issue [s]				
		Mean	σ	Min	Max	Median
TA	1	-	-	-0.8	-0.1	-
	2	-	-	0.2	-0.2	-
RA	1	-	-	0.5	-1.4	-
	2	-	-	-0.6	0.3	-
1 st RA adj.	1	-0.1	-0.1	-6.8	-0.9	-
	2	-	-	-6.6	-0.2	-
2 nd RA adj.	1	-0.2	-	-2.1	2.9	-0.1
	2	-0.1	-0.1	0.4	-0.8	-0.1
3 rd RA adj.	1	-	-	-	-	-
	2	-0.1	-0.3	1.5	-11	-
4 th RA adj.	1	-	-	-	-	-
	2	2.4	7.3	-2	10.8	-1.7
5 th RA adj.	1	-	-	-	-	-
	2	180	0.9	179.3	180.6	180

Similar to case 6, large differences can be found for first and second RA adjustments in Table 17-32. In regard to first RA adjustments, in about 1-1.5% of the simulation runs a climb or “Descend” RA is issued instead of a “Do Not Climb/Descend” RA. In regard to second RA adjustments, an increase in the total number of RA adjustment can be seen, of which most were “Do Not Climb/Descend” RAs, about 1-1.3% of the simulation runs.

Table 17-32: Difference in number and type of TA/RA of case 8 between MC simulation I and II (effect of pilot delay variation).

Type	A/C	Difference in number of RA/TA		Difference in number of CL		Difference in number LO		Difference in number of DE		Difference in number of DCL/DDE	
		#	%	#	%	#	%	#	%	#	%
TA	1	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-
RA	1	100	0.01	-	-	100	0.01	-	-	-	-
	2	740	0.08	-	-	613	0.06	127	0.01	-	-
1 st RA adj.	1	104	0.01	7165	0.71	10	-	7389	0.74	-14460	-1.45
	2	741	0.08	7185	0.72	12	-	6177	0.62	-12633	-1.26
2 nd RA adj.	1	13307	1.33	-	-	-	-	-	-	13307	1.33
	2	10185	1.02	-61	-	-2	-	-95	-0.01	10343	1.03
3 rd RA adj.	1	-	-	-	-	-	-	-	-	-	-
	2	-83	-0.01	-2	-	3	-	-	-	-84	-0.01
4 th RA adj.	1	-	-	-	-	-	-	-	-	-	-
	2	2	-	-	-	1	-	-	-	1	-
5 th RA adj.	1	-	-	-	-	-	-	-	-	-	-
	2	2	-	-	-	-	-	-	-	2	-

Finally, an effect can also be identified in the empirical density of the vertical miss distance. In Figure 17-30 there is an increase in concentration of vertical miss distance of the simulations without pilot delay variation at the mean of the vertical miss distance of the Monte Carlo simulation with pilot delay variation.

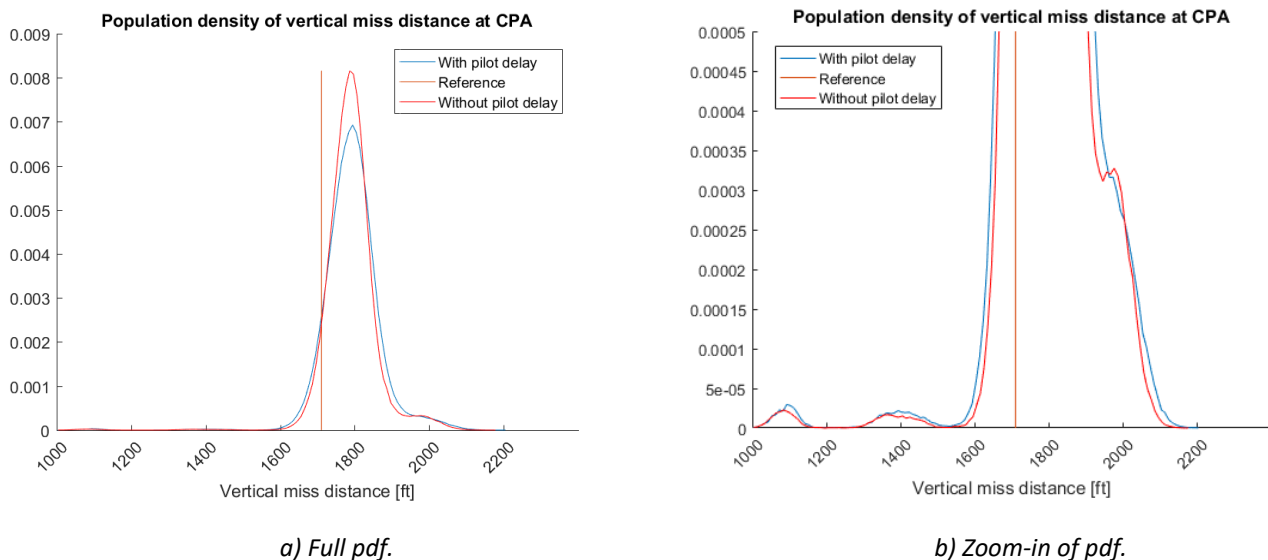


Figure 17-30: Empirical density of vertical miss distance at CPA of Monte Carlo simulations I and II for case 8. a) full pdf graph; b) zoom-in left of pdf.

17.3 Simulation Results III (Effect of various Uncertainties)

In the third simulation set up several Monte Carlo simulations were carried for cases 2-8. In each Monte Carlo simulation only one uncertainty was enabled to investigate the effect of each uncertainty on the vertical miss distance. Since, no RAs were issued in cases 1 and 9, the vertical miss distance remains unaffected. Hence, cases 1 and 9 are left out from the evaluation.

To recap, there are the following eight uncertainties:

- a) Dynamic range error, due to jitter in range measurements,
- b) Static range error, due to bias in range measurements,
- c) Dynamic altitude error, due to jitter in own altitude estimates,
- d) Static altitude error, due to bias in own altitude estimates,
- e) Dynamic error in own rate of climb/descend, due to jitter in own altitude velocity estimates,
- f) variation in pilot delay,
- g) probability of missed response (non-reception of an interrogation response), and
- h) variation in starting time of TCAS cycle.

Note, that the same order is applied in the figures of the empirical vertical miss distances.

Case 2:

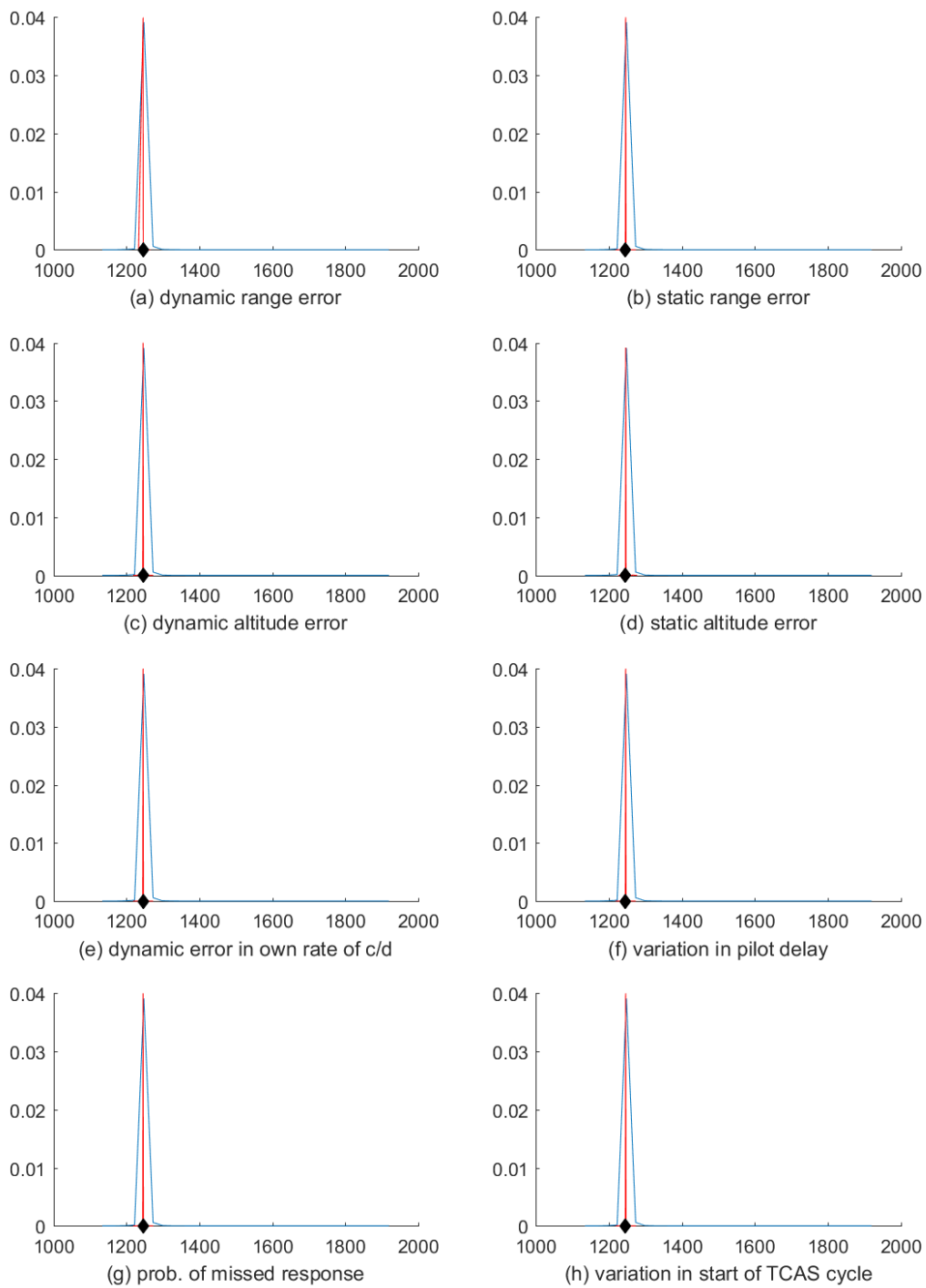


Figure 17-31: Empirical densities of vertical miss distance at CPA of case 2 of Monte Carlo simulations I and III. Black marker: reference. Blue: MC Sim I. Red: MC Sim III.

From the results in [Figure 17-31](#) it can be seen that none of the single uncertainties has a significant impact on the vertical miss distance at CPA.

Case 3:

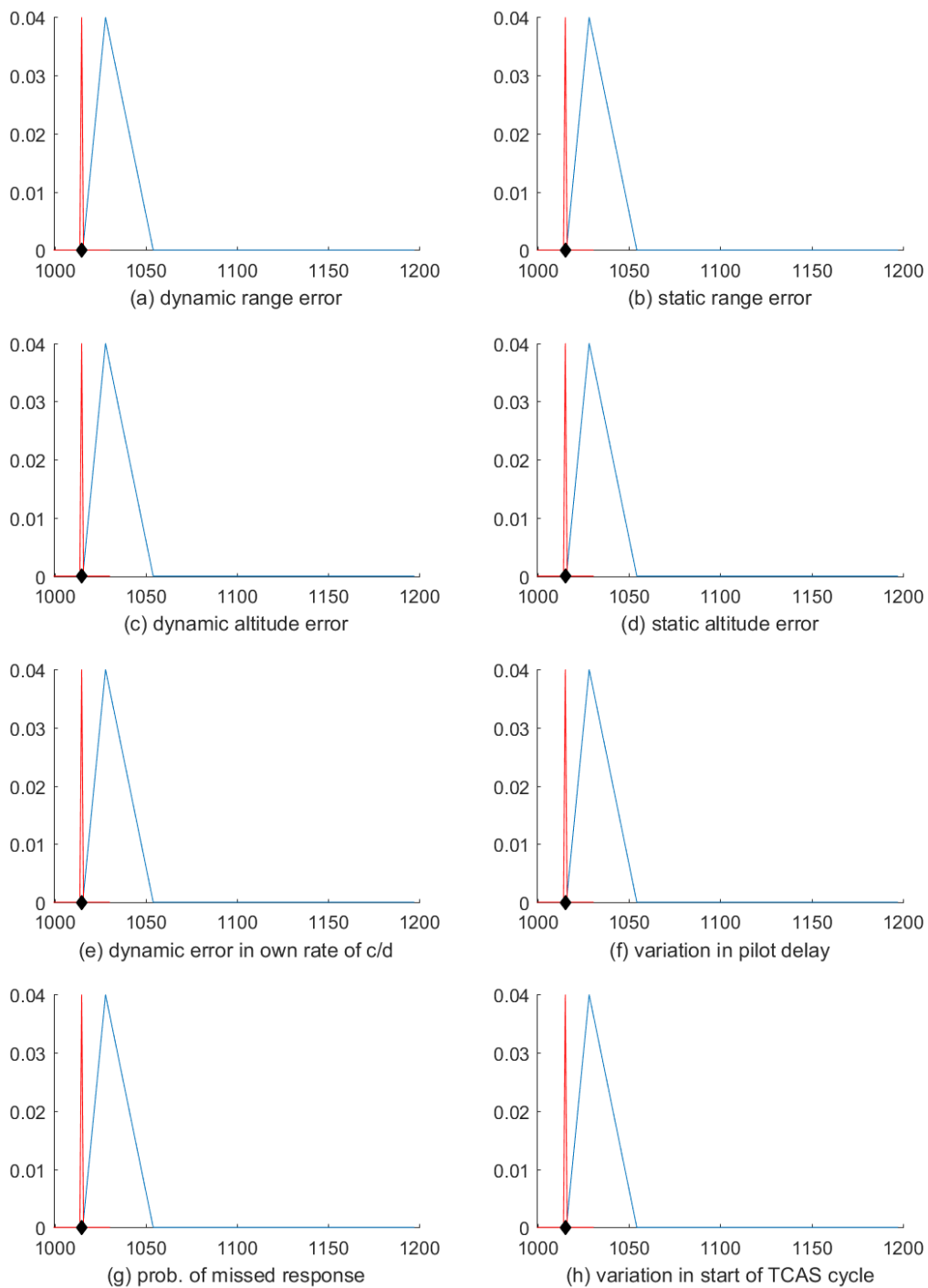


Figure 17-32: Empirical densities of vertical miss distance at CPA of case 3 of Monte Carlo simulations I and III. Black marker: reference. Blue: MC Sim I. Red: MC Sim III.

From the results in [Figure 17-32](#) it can be seen that none of the single uncertainties has a significant impact on the vertical miss distance at CPA. Hence, there must be a combination of uncertainties that lead together to the increase of the vertical miss distance.

Case 4:

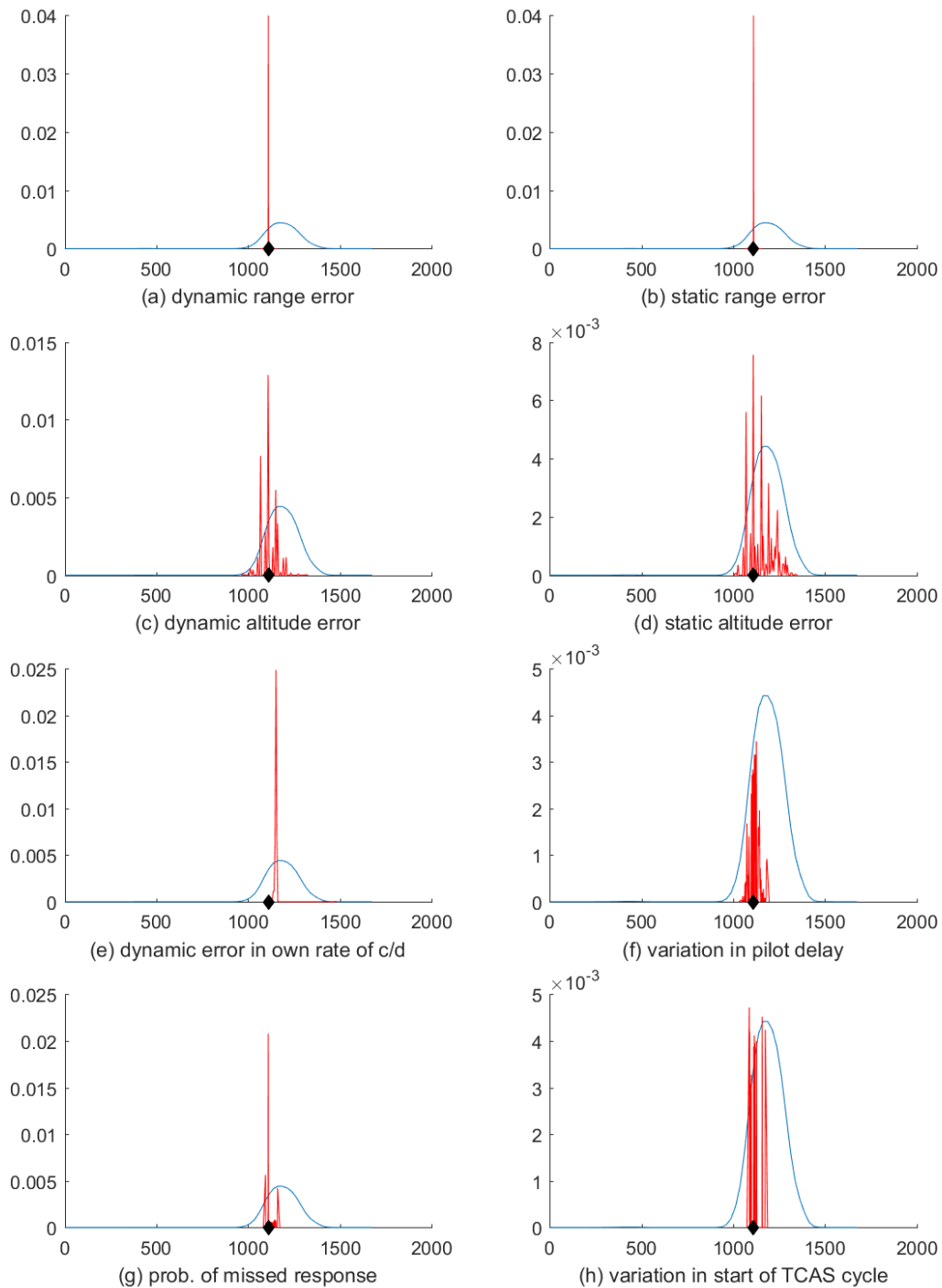


Figure 17-33: Empirical densities of vertical miss distance at CPA of case 4 of Monte Carlo simulations I and III. Black marker: reference. Blue: MC Sim I. Red: MC Sim III. Unit of x-axis: [ft].

From the results in Figure 17-33 it can be seen that different uncertainties have different effects.

The dynamic and static range measurement errors do not have any effect.

The dynamic and static altitude measurement errors have the largest effect and cause a spread of the data with many peaks. Also it can be seen that in between many peaks the graph goes almost

down to zero. Because the relative altitude velocity between the aircraft is relatively low compared to the relative range velocity, only a small deviation in altitude error may have significant impact.

The dynamic error in the measurement of own rate of climb or descend has no significant influence. In rare cases it causes an increase of 500ft in vertical miss distance (up to 1,500ft). A likely explanation is that this error has mostly an effect on the rare issue of preventive RA adjustments.

For the variation in pilot delay it can be seen that it causes a normal shaped distribution of the vertical distance around the reference miss distance. Hence, the pilot delay variation causes a smoothing of the empirical density.

The probability of missed response has in about 10% of the cases a new concentration at about 1,100ft.

The variance in the start time of the TCAS also leads to a concentration of vertical miss distances at about 1,100ft; even more significant than the probability of a missed response. Probably there is a tipping point for the starting time which leads to different RAs.

Case 5:

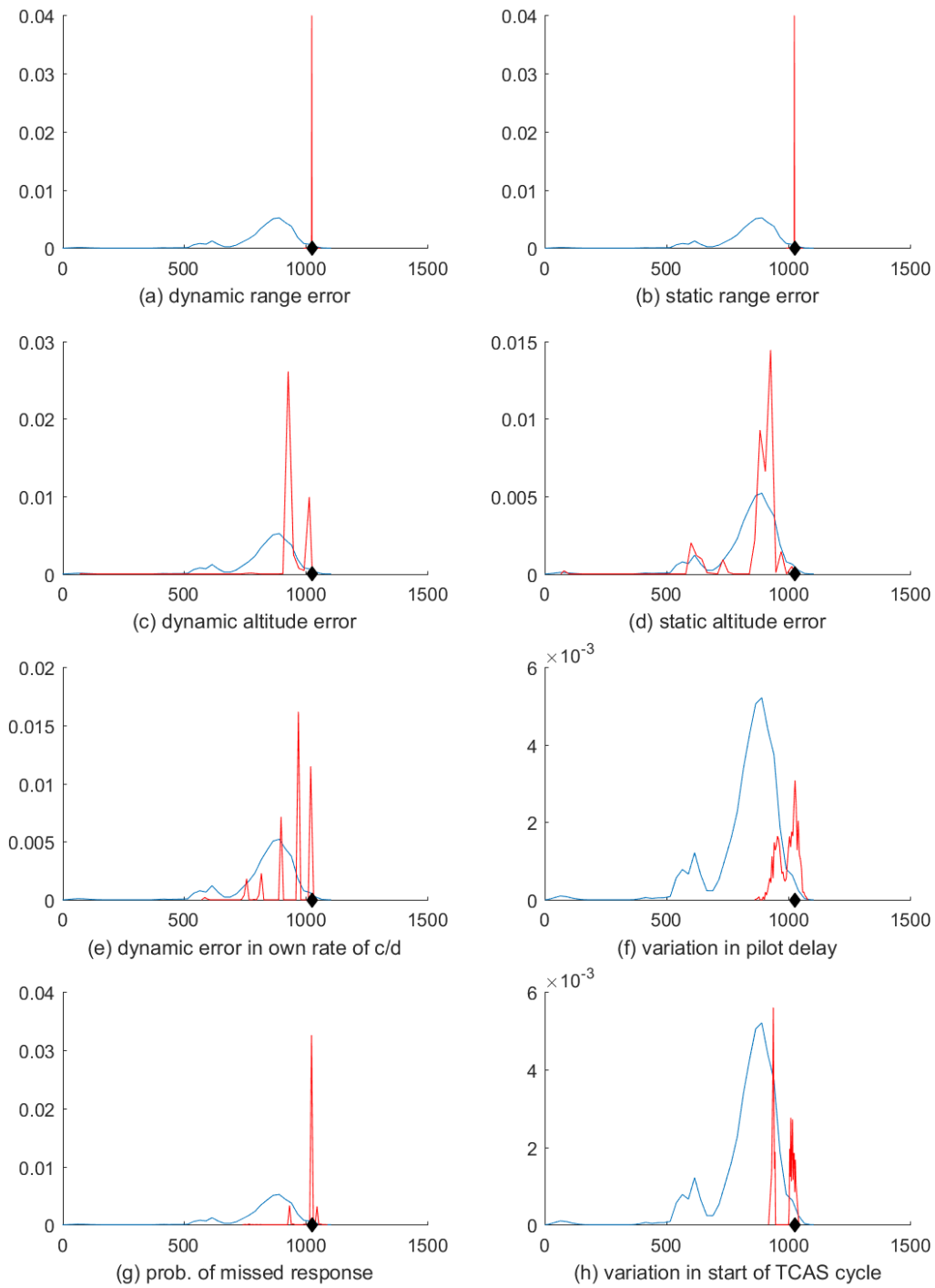


Figure 17-34: Empirical densities of vertical miss distance at CPA of case 5 of Monte Carlo simulations I and III. Black marker: reference. Blue: MC Sim I. Red: MC Sim III.

From the results in [Figure 17-34](#) it can be seen that that different uncertainties have different effects.

The dynamic and static range measurement errors do not have any effect.

The dynamic and static altitude measurement errors have the most significant effect. Both lead to vertical miss distances below 100ft and therefore could cause a mid-air collision. Besides, it can also be seen that the static altitude measurement error alone largely causes the type of shape of the empirical density function with all uncertainties enabled.

The dynamic error in the measurement of own rate of climb or descend causes vertical miss distances below 600ft. It can also be seen that six clearly separated peaks were created, which means that it causes 6 different behaviors of the RA generation.

For the variation in pilot delay it can be seen that it spreads out the vertical miss distance around the reference point. Hence, it is likely that the pilot delay variation causes a smoothing in the overall pdf.

The probability of missed responses has no large impact. Only in rare cases it causes a decrease in miss distance down to 750ft.

Similar to case 4, the variance in the start time of the TCAS leads to two large concentrations of vertical miss distance, where one is concentrated at the reference point and the other is concentrated at 900ft. So probably there is again a tipping point for the starting time which leads to different RAs.

Case 6:

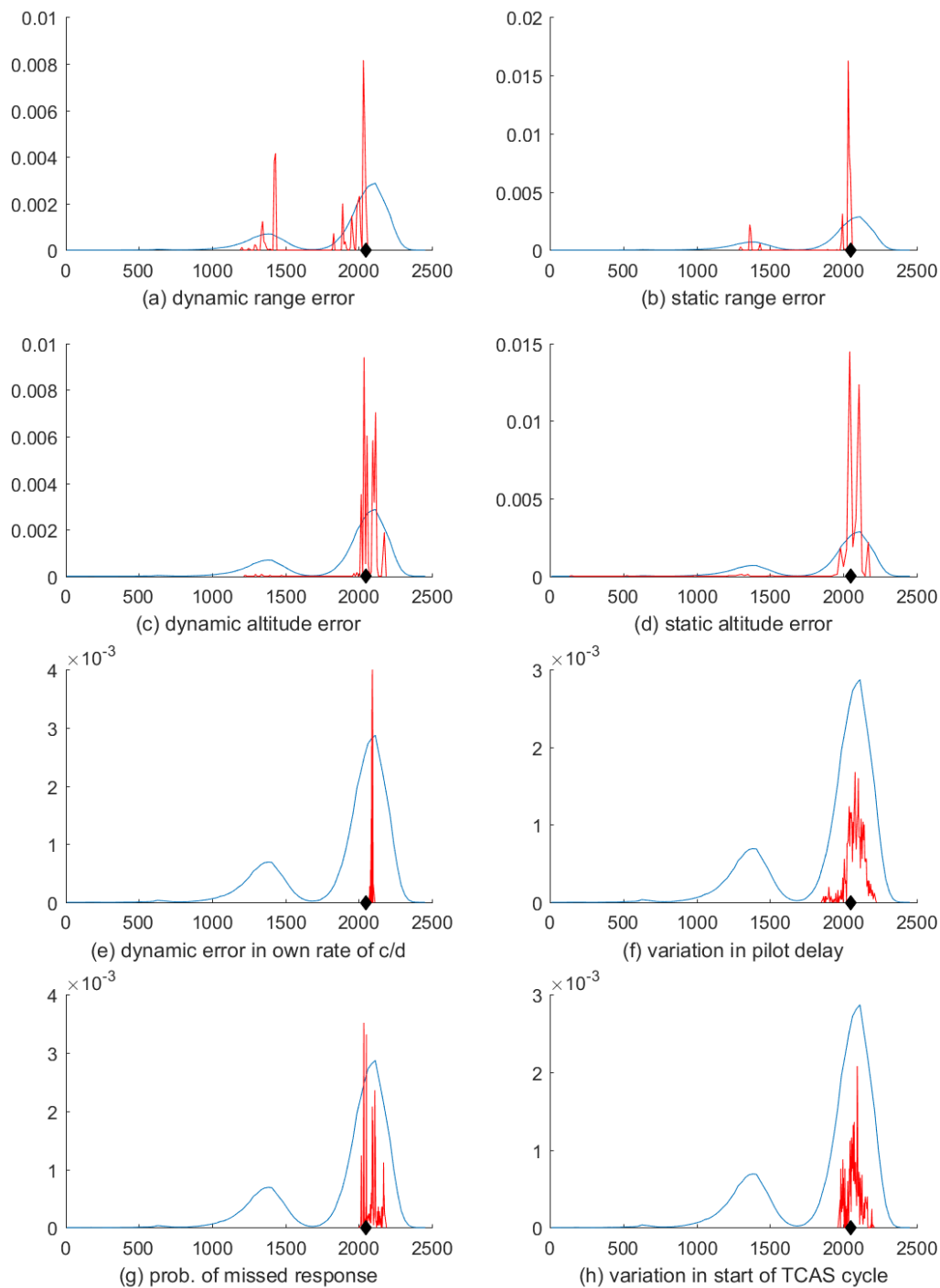


Figure 17-35: Empirical densities of vertical miss distance at CPA of case 6 of Monte Carlo simulations I and III. Black marker: reference. Blue: MC Sim I. Red: MC Sim III.

From the results in [Figure 17-35](#) it can be seen that that different uncertainties have different effects.

The dynamic and static range measurement errors do have the largest effect. The dynamic range measurement error is causing two large concentrations of vertical miss distance at the same peak locations of the simulations where all uncertainties were enabled. The static range error also causes

a small peak around 1,400ft. The significant influence of the static and dynamic range errors for case 6 is likely due to the low relative range velocity, causing small deviations to have a relative high effect.

The dynamic and static altitude measurement errors have less effect than range errors. They cause some peaks but all are very close to the reference miss distance.

The dynamic error in the measurement of own rate of climb or descend has a small effect; there is only one very large peak which is at a marginally higher vertical miss compared to the reference.

The variation in pilot delay, the probability of missed responses, and the start time of the TCAS cause each a small spread of the vertical miss distance where the peaks are at a marginally higher vertical miss compared to the reference.

Case 7:

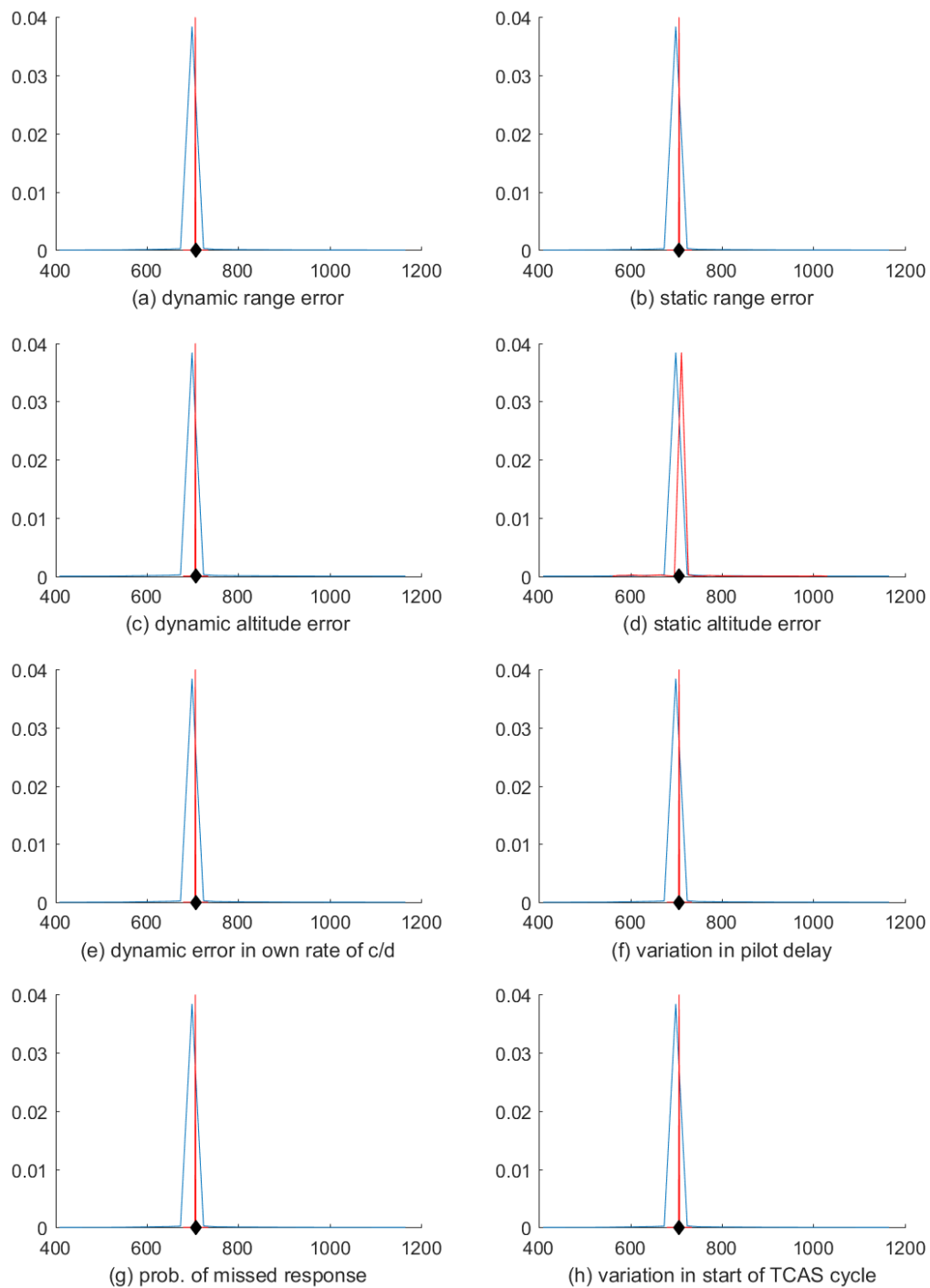


Figure 17-36: Empirical densities of vertical miss distance at CPA of case 7 of Monte Carlo simulations I and III. Black marker: reference. Blue: MC Sim I. Red: MC Sim III.

From the results in [Figure 17-36](#) it can be seen that none of the single uncertainties has a significant impact on the vertical miss distance at CPA. Only for the static altitude measurement error it can be seen that in rare cases the vertical miss distance decreases below 600ft and increases above 1,000ft.

Case 8:

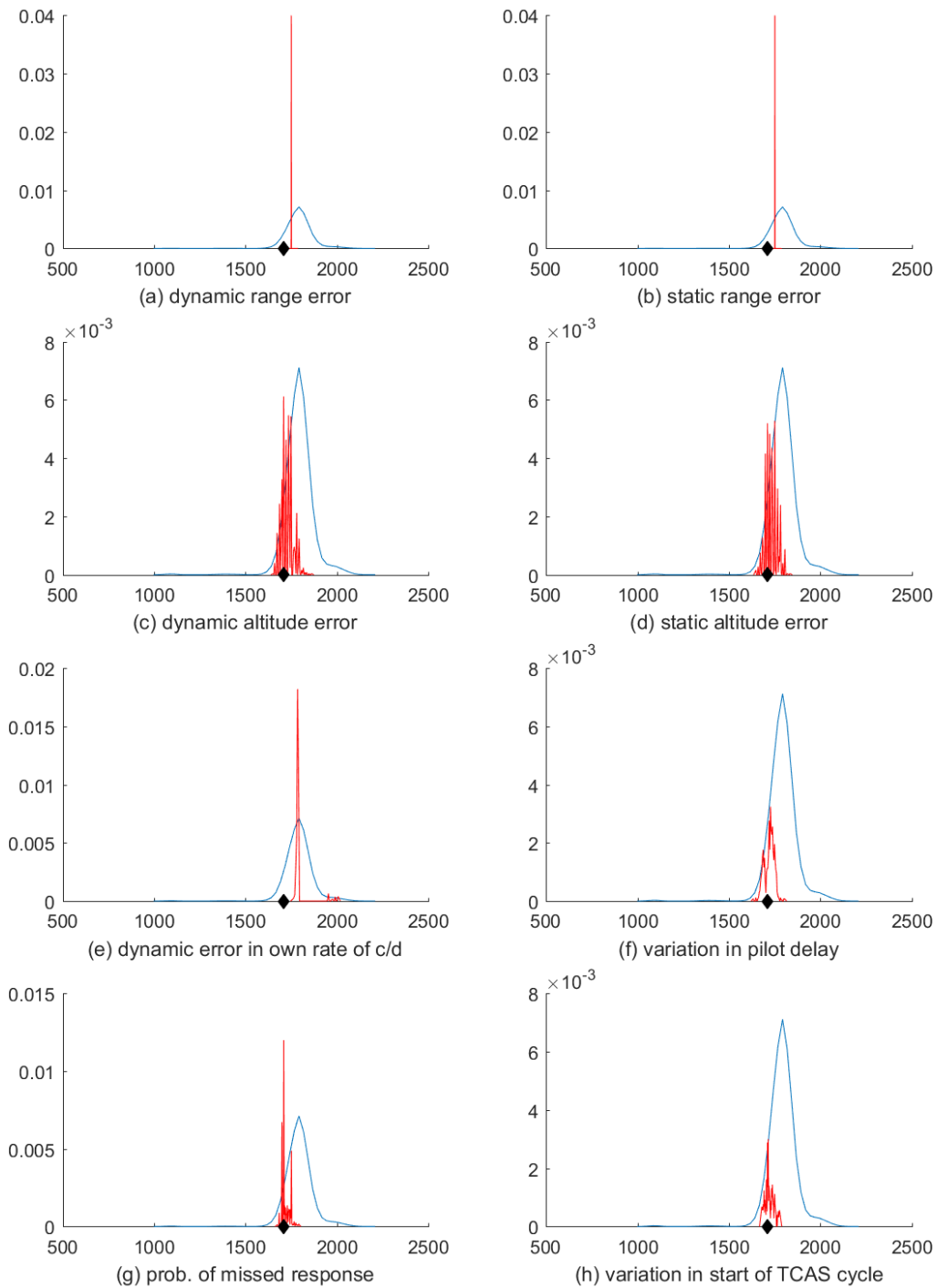


Figure 17-37: Empirical densities of vertical miss distance at CPA of case 8 of Monte Carlo simulations I and III. Black marker: reference. Blue: MC Sim I. Red: MC Sim III.

From the results in [Figure 17-37](#) it can be seen that the effects are small.

The dynamic and static range measurement errors do not have a large effect, but only increase the location of the peak by about 50ft.

The dynamic and static altitude measurement errors cause both a relatively spread of the vertical miss distance around the reference point.

The dynamic error in the measurement of own rate of climb or descend has one large peak. It is remarkably that this peak is located exactly at the location where the peak of the simulation with all uncertainties enabled.

For the variation in pilot delay, the probability of missed responses, the variance in the start time of the TCAS have a similar effect and cause a small spread of the vertical miss distance around the reference point.

Hence, it can be concluded that in reference to the simulation where all uncertainties were enabled, the dynamic error in the measurement of own rate of climb or descend causes location of the peak, and the dynamic and static error in altitude cause the shape of the peak.

18 Conclusions

From the simulation results for the nine cases studied, the following is concluded.

First, the mean time of issuing an RA is often not significantly affected by uncertainties. This could be observed in the results of the Monte Carlo simulation results in which all uncertainties were enabled, i.e. the highest difference between the mean time of issue of an RA and the reference case was 1.1s for case 5.

Second, the variation in time of issuing an RA is significantly affected by uncertainties in cases where the vertical closing speeds are low. E.g. in case 7 the relative altitude velocity is 200ft/min and the maximum standard deviation in time of issue of initial RAs is 9s (aircraft 2). In case 5 the relative altitude velocity is 1,000ft/min and the standard deviation in time of issue of initial RAs is 2.5s for both aircraft. In the other cases the relative altitude velocities are equal or above 1,600ft/min and the standard deviations in time of issue of initial RAs are equal or below 2.0s.

Third, uncertainties have a significant effect on whether an RA is issued or not, i.e. in the following cases an RA was issued although there was none in the reference case: about 21% in case 3 for aircraft 1; in about 21% in case 4 for aircraft 2; about 17% aircraft 1, and about 20% for aircraft 2 in case 4. In contrast, in case 5 in about 5% of the simulations each aircraft issued no RA, although there were initial RAs for both aircraft in the reference case.

Fourth, uncertainties have a significant effect on the type of RA issued. For example, the type of RA issued deviated in about 24% of the simulation runs in case 2 and in about 23% of the simulation runs in case 6. In both cases, the different selected RAs resulted in an altitude crossing, although the reference case has shown that both aircraft could have ensured sufficient separation without crossing altitudes (Note: as described in Section 9 TCAS II prefers to select RAs which do not lead to altitude crossings).

Fifth, uncertainties have a significant effect on the number of consecutive RA adjustments. In all cases where an RA was issued (cases 2-8), more RA adjustments were issued compared to the reference case. In cases 6 and 7, up to five RA adjustments were issued. Since only up to five RA adjustments were stored in the data, it is possible that even more RA adjustments were issued.

Sixth, TCAS II Version 7.1 may cause loops of conflict detection and clear of conflict declaration until both aircraft have passed the CPA. However, it needs to be determined whether this is due to assumption 7 of agent "Cockpit i" in Section 13, which assumes that the pilot levels off after he/she has received a clear of conflict message, or due to RAs issued by TCAS, or a mixture of the two.

Seventh, the effect on the variation in pilot delay has either an effect on both the number of RA adjustments and the vertical miss distance at CPA, or none of the two. For example, in cases 2 and 3 no large differences could be found in the empirical density of vertical miss distance and in the number of RA adjustments issued. In cases 4 to 8, a difference could be observed for both the empirical density of vertical miss distance and the number of RA adjustments. Moreover, it was identified that the variation in pilot delay has a smoothing effect on the empirical density of vertical miss distance.

Eighth, uncertainties clearly have an effect on the safety of TCAS II operations. In case 6, near mid-air collisions were encountered, although this would not have occurred if no RAs were issued. Moreover, the root cause is the dynamic and static error in altitude measurements. In the results of the third Monte Carlo simulation, it was observed that only the dynamic or static altitude errors cause the vertical miss distance to decrease below 100ft.

Ninth, the dynamic and static range errors only have an impact on the vertical miss distance, if the horizontal closing speeds are low, i.e. in case 6 the closing speed was relatively low due to similar headings of the aircraft (353 and 360°), and both the static and dynamic range error influenced the vertical miss distance. For all the other cases, no impact by the both the dynamic or static range error could be found and the horizontal closing speeds were relatively large, since the headings of the aircraft differed by at least 82°.

Tenth, only through conducting rare MC simulation it can become clear which uncertainties play a significant role under which encounter conditions. It is expected that more can be learned by conducting rare event MC simulations for other encounter types and under other conditions.

Finally, from the conclusions drawn, it can be seen that uncertainties have a large and unpredictable impact on TCAS operations. Although only nine specific cases were evaluated, it was identified that uncertainties can induce mid-air collisions, and therefore, it is recommended to include uncertainties in future risk assessments of TCAS II operations.

A risk assessment that is currently of interest in aviation is the estimation of risk of collision between a TCAS II equipped aircraft and an UAV. For example, due to the large increase in use of UAVs, the FAA is currently developing a collision avoidance system especially for UAVs, ACAS XU. In order to assess the performance of such new ACAS, the new TCAS model may be used.

Hence, next to the development of the new TCAS model, an ACAS for an UAV was developed, too. This model is based on the ACAS XU concept of the FAA. The surveillance is done through automatic dependent surveillance-broadcast (ADS-B) and can select between vertical and horizontal avoidance manoeuvring. From the developed model, a Petri Net model was created, which was also programmed in MATLAB. Furthermore, the model has been already successfully verified. The description of the model and the Petri Net model are included in Appendix B, C and D. Currently, Monte Carlo simulations are carried out which simulate one TCAS II equipped aircraft and one UAV to assess the performance of the developed ACAS for UAV.

References

- (Bazghandi, 2012) A. Bazghandi. Techniques, advantages and problems of agent based modelling for traffic simulation. *International Journal of Computer Science Issues*, 9(3):115-119, 2012.
- (Billingsley et al., 2009) T.B. Billingsley, L.P. Espindle, J. D. Griffith. TCAS Multiple Threat Encounter Analysis, 22 October 2009. Report number: ATC-359, Affiliation: Massachusetts Institute of Technology, Lincoln Laboratory.
- (Bonabeau, 2002) E. Bonabeau. Agent-based modelling: Methods and techniques for simulating human systems. *PNAS*, 99(3):7280-7287, 2002.
- (Blom et al., 2016) H.A.P. Blom, G.J. Bakker. Agent-Based Modelling and Simulation of Trajectory Based Operations under Very High Trajectory Demand, 8-10 November 2016. Schaefer, Dirk (Editor) *Proceedings of the SESAR Innovation Days (2016) EUROCONTROL*. ISSN 0770-1268.
- (Chryssanthacopoulos, 2012) J.P. Chryssanthacopoulos, M.J. Kochenderfer, J.E. Holland. Next-generation airborne collision avoidance system. *Lincoln Laboratory Journal*, 19(1):17-33, 2012.
- (ECTL, 2010) EUROCONTROL. Unmanned Aircraft Systems - ATM Collision Avoidance Requirements, May 2010. Report CND/CoE/CNS/09-156, Edition Number 1.3, Brussels, Belgium.
- (EUROCAE, 2008) EUROCAE. Minimum Operational Performance Standards for Traffic Alert and Collision Avoidance System II (TCAS II) - Volume I, September 2008. Report ED-143; Malakoff, France.
- (Everdij et al., 2003) M.H.C. Everdij, H.A.P. Blom. Petri-Nets and Hybrid-State Markov Processes in a Power-Hierarchy of Dependability Models, 16-18 June 2003. In *Proceedings of IFAC Conference on Analysis and Design of Hybrid Systems 2003*, St Malo, Brittany, France.
- (FAA, 2010) FAA. Automatic Dependent Surveillance-Broadcast (ADS-B) Out equipment performance requirements, Amdt. 91-316, 75 FR 37712, June 30, 2010. Code of Federal Regulations: 14 CFR 91.227.
- (FAA, 2011) FAA. Introduction to TCAS II Version 7.1, 2011. Report HQ-111358.
- (FAA, 2015a) FAA. Advisory Circular Airworthiness Approval of Automatic Dependent Surveillance – Broadcast (ADS-B) Out Systems, July 2015. Advisory Circular AC 20-165B.
- (FAA, 2015b) FAA. Concept of Use for the Airborne Collision Avoidance System (ACAS) Xu, September 2015. Report ACAS CON 15 001 V2R0, Version 2.
- (Galatolo et al., 2016) R. Galatolo, G. Migliaccio, G. Mengali. A solution to detect and avoid conflicts for civil remotely piloted aircraft systems into on-segregated airspaces. *J Aerospace Engineering*, 230:1655-1667, 2016.
- (Gilbert, 2008) N. Gilbert. Agent-based models. SAGE Publications, 2008. ISBN 978-1412949644.
- (Hammer, 1996a) Jonathan B. Hammer. A Horizontal Miss Distance Filter Design for TCASII, January 1996. Report MTR 96W0000134.

(Hammer, 1996b) Jonathan B. Hammer. Horizontal miss distance filter system for suppressing false resolution alerts, 15th October 1996. United States Patent 5,566,074.

(Haessig, 2016) D.A. Haessig, R.T. Ogan and M.Olive. Sense and Avoid - What's required for aircraft safety. IEEE, 30 March-3 April 2016. Proceedings of SoutheastCon 2016; Norfolk, VA; United States. ISSN 1558-058X.

(ICAO, 2006) ICAO. Airborne Collision Avoidance System (ACAS) Manual, 2006. Doc 9863 AN/461.

(JARUS, 2014) JARUS. Remotely Piloted Aircraft Systems, Systems Safety, January 2014. JARUS Scoping Paper to AMC RPAS.1309, Issue 1.

(Jeannin et al., 2015) Jean-Baptiste Jeannin, Khalil Ghorbal, Yanni Kouskoulas, Ryan Gardner, Aurora Schmidt, Erik Zawadzki, and Andre Platzer. A Formally Verified Hybrid System for the Next-Generation Airborne Collision Avoidance System, pages 21-36. Springer Berlin Heidelberg, Berlin, Heidelberg, 2015. ISBN 978-3-662-46681-0. Proceedings of 21st International Conference, TACAS 2015, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2015, London, UK, April 11-18, 2015.

(Jenie et al., 2013) Y. I. Jenie, E.J. van Kampen, C. C. de Visser, Q.P. Chu. Selective Velocity Obstacle Method for Cooperative Autonomous Collision Avoidance System for UAVs. ARC, 19-22 August 2013. Proceedings of the AIAA Guidance, Navigation, and Control (GNC) Conference; Boston, MA; United States; AIAA 2013-4627.

(Jenie et al., 2016) Y.I. Jenie, E.J. van Kampen, J. Ellerbroek, J.M. Hoekstra. Safety Assessment of Unmanned Aerial Vehicle Operations in an Integrated Airspace. AIAA, January 2016. Proceedings of AIAA Infotech @ Aerospace, AIAA SciTech Forum, San Diego, California, USA. AIAA 2016-1000.

(Johnson, 2013) P. E. Johnson. Monte Carlo Analysis in Academic Research, pages 454-479. Oxford University Press, New York, 2013. ISBN 978-0199934874.

(Kochenderfer, 2010) M.J. Kochenderfer, J.P. Chryssanthacopoulos, L.P. Kaelbling, T. Lozano-Perez. Model-Based Optimization of Airborne Collision Avoidance Logic, 26 January 2010. Project Report number: ATC-360, Affiliation: Massachusetts Institute of Technology, Lincoln Laboratory.

(Kochenderfer, 2011) M.J. Kochenderfer, J.P. Chryssanthacopoulos. Robust Airborne Collision Avoidance through Dynamic Programming, January 2011. Report number: ATC-371, Affiliation: Massachusetts Institute of Technology, Lincoln Laboratory.

(Kochenderfer et al, 2012) M.J. Kochenderfer, J.E. Holland and J.P. Chryssanthacopoulos. Next-generation airborne collision avoidance system. Lincoln Laboratory Journal , 19(1):17–33, 2012.

(Kotegawa, 2016) T. Kotegawa. Proof-of-concept airborne sense and avoid system with ACAS-Xu fight test. IEAA A&E Systems Magazine, 1(10.1109):53-62, 2016.

(Kuchar, 2005) J.K. Kuchar. Safety Analysis Methodology for Unmanned Aerial Vehicle (UAV) Collision Avoidance Systems, 27-30 June 2005. Proceedings of the 6th USA/Europe Air Traffic Management Research and Development Seminar; Baltimore, United States

(NASA, 2017) NASA. Mitigating the Impact of Sensor Uncertainty on Unmanned Aircraft Operations. Langley, United States, June 2017. Report NASA/CR-2017-219630.

(Netjasov, 2010) F. Netjasov. Risk Analysis and Safety Assessment of Air Traffic Control System. University of Belgrade, Serbia, July 2010. Doctoral Thesis.

(RTCA, 2017) RTCA. Minimum Operational Performance Standards (MOPS) for Detect and Avoid (DAA) Systems, May 2017. Report RTCA DO-365.

(Thompson, 2000) S.D. Thompson. Comparison of Active TCAS Slant Range Measurements with Interpolated Passive Position Reports for Use in Hybrid Surveillance Applications: Measurements from the June 1999 Los Angeles Basin Flight Tests, October 2000. Report ATC-289.

(Yu et al., 2016) X. Yu, X. Fu, Y. Zhang. An advance sense and collision avoidance strategy for unmanned aerial vehicles in landing phase. IEAA A&E Systems Magazine, 1(10.1109):40-52, 2016.

(Zou et al., 2016) M. X. Zou, R. Alexander, J. McDermid. On the Validation of a UAV Collision Avoidance System Developed by Model-Based Optimization: Challenges and a Tentative Partial Solution. IEEE, 28 June-1 July 2016 2016. Proceedings of 46th Annual IEEE/IFIP International Conference on Dependable Systems and Networks Workshop (DSN-W). ISBN 978-1-5090-3688-2.

Agent-based Modelling and Simulation of TCAS Operations under Uncertainty

Volume II: Appendices

Master of Science Thesis

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	IPN “Adjusted RA”	18
	IPN “COC”	19
	IPN “Reset”	20
	IPN “Hor. Man Message”	21
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Glossary of Symbols

a_{lim}	Vertical miss distance threshold value
$a_{t,A}^i$	Altitude acceleration of aircraft 'i' at time 't'
$a_{t,A,RA}^i$	Altitude acceleration set point of aircraft 'i' at time 't' due to RA
$B_{t,AT}$	Boolean of altitude test for RA
$B_{t,AT,UAV}$	Boolean of altitude test for RA for ACAS UAV
$B_{t,D,UAV}$	Boolean of sense selection process for ACAS UAV
$B_{t,\hat{f}irm}$	Boolean of firmness state of parabolic range tracker
$B_{t,HMDF}$	Boolean of horizontal miss distance filter status
$B_{t,HMDF,UAV}$	Boolean of horizontal miss distance filter status for ACAS UAV
$B_{t,RA,EVO,UAV}$	Boolean of evolution monitoring process
$B_{t,RA,UAV}$	Boolean of threat detection test of ACAS UAV
$B_{t,RT}$	Boolean of range test for RA
$B_{t,RT,UAV}$	Boolean of range test for RA for ACAS UAV
$B_{t,TA}$	Boolean of TA test
D_t	Selected sense at time 't'
$D_{t,UAV}$	Selected sense at time 't'
$d_{t,HMD,ADSB}$	Estimated horizontal miss distance by HMDF of ACAS UAV at time 't'
$d_{t,HMD,ADSB i}$	Estimated horizontal miss distance by HMDF of ACAS UAV at time 't+i' if aircraft turns right
$d_{t,HMD,ADSB k}$	Estimated horizontal miss distance by HMDF of ACAS UAV at time 't+k' if aircraft turns left
d_{mod}	RA distance threshold value

H_m	Horizontal miss distance threshold value
$L_{t,UAV}$	Avoidance logic selection
$m_{t,sent}^{ik}$	Coordination message sent from aircraft 'i' to aircraft 'k'
$m_{t,received}^{ki}$	Coordination message received by aircraft 'i' from aircraft 'k'
$P_{t,firm}$	Firmness parameter of parabolic range tracker
p_{mes}	
$\begin{pmatrix} q_{t,X,BBT} \\ q_{t,Y,BBT} \end{pmatrix}$	Relative position estimates in Cartesian coordinates of bearing based tracker at time 't'
$\begin{pmatrix} \dot{q}_{t,X,BBT} \\ \dot{q}_{t,Y,BBT} \end{pmatrix}$	Relative velocity estimates in Cartesian coordinates of bearing based tracker at time 't'
r_t	Estimated slant range at time 't'
$\dot{r}_t$	Estimated slant range rate
$r_{t,A}$	Estimated relative altitude at time 't'
$\dot{r}_{t,A}$	Estimated relative altitude velocity at time 't'
$r_{t,CT}$	Estimated slant range of Cartesian tracker at time 't'
$\dot{r}_{t,CT}$	Estimated relative velocity of Cartesian tracker at time 't'
$\ddot{r}_{t,CT}$	Estimated relative acceleration of Cartesian tracker at time 't'
$r_{t,PRT}$	Estimated slant range of parabolic range tracker at time 't'
$\dot{r}_{t,PRT}$	Estimated relative velocity of parabolic range tracker at time 't'
$\ddot{r}_{t,PRT}$	Estimated relative acceleration of parabolic range tracker at time 't'
$\dddot{r}_{t,PRT}$	Estimated jerk of parabolic range tracker at time 't'
$\hat{S}_{CPA,A t}$	Predicted vertical miss distance at CPA

$\hat{S}_{CPA,A t}^+$	Predicted vertical miss distance at CPA if the own aircraft would be flying with maximum vertical speed
$\hat{S}_{CPA,A t}^-$	Predicted vertical miss distance at CPA if the own aircraft would be flying with minimum vertical speed
$S_{t,A}^i$	Altitude of aircraft 'i' at time 't'
$\hat{S}_{t,A}^{ik}$	Estimate of relative altitude between aircraft 'i' and aircraft 'k' at time 't'
$S_{t,H}^i$	Horizontal position state of aircraft 'i' at time 't' in x and y coordinates
$\hat{S}_{t,H,ADSB}^{ik}$	Relative horizontal range estimate between aircraft 'i' and aircraft 'k' at time 't'
$\hat{S}_{t+i,H,ADSB t,i}$	Estimated relative horizontal position vector between own aircraft and intruder at time 't+i' given own aircraft turns right
$\hat{S}_{t+k,H,ADSB t,k}$	Estimated relative horizontal position vector between own aircraft and intruder at time 't+k' given own aircraft turns left
$t_{t,RA,UAV}$	Elapsed time since RA
T_{ADSB}	Length of one ADS-B reporting cycle
t^G	Time delay until next update/measurement
t_{HMDF}	Timer to pause HMDF
$t_{t,timer}$	Timer to count down to estimated time of closest point of approach at initial RA
$\hat{V}_{t,H,ADSB}^{ik}$	Estimated relative horizontal velocity between aircraft 'i' and aircraft 'k' with reference to aircraft 'i'
$\hat{V}_{t+i-\Delta,H,ADSB t,i}$	Estimated relative horizontal velocity vector between own aircraft and intruder at time 't+i' given own aircraft turns right
$\hat{V}_{t+k-\Delta_k,H,ADSB t,k}$	Estimated relative horizontal velocity vector between own aircraft and intruder at time 't+k' given own aircraft turns left
$V_{t,A}^i$	Altitude velocity of aircraft 'i' at time 't'
$\hat{V}_{t,A}^{ik}$	relative altitude velocity estimate between aircraft 'i' and aircraft 'k' at time 't'
$V_{t,A,\min}^i$	Lower bound of target altitude velocity of aircraft 'i' at time 't'

$v_{t,A,\max}^i$	Upper bound of target altitude velocity of aircraft 'i' at time 't'
$v_{t,A,RA,\min}^i$	Minimum altitude velocity limit indicated by RA at time 't'
$v_{t,A,RA,\max}^i$	Minimum altitude velocity limit indicated by RA at time 't'
$y_{t,A}^i$	Measured pressure altitude of aircraft 'i' at time 't'
$y_{t,A}^{ik}$	Measured relative altitude difference between aircraft 'i' and aircraft 'k' with reference to aircraft 'i' at time 't'
$y_{t,A,ADSB}^i$	Reported altitude of aircraft 'i' at time 't'
$y_{t,A,v,ADSB}^i$	Reported measurement of the altitude velocity of aircraft 'i' at time 't'
$y_{t,H,ADSB}^i$	Reported measurement of horizontal position of aircraft 'i' at time 't'
$y_{t,H,v,ADSB}^i$	Reported measurement of ground speed of aircraft 'i' at time 't'
$y_{t,r}^{ik}$	Measured distance of aircraft 'k' to aircraft 'i' by aircraft 'i' (slant range)
$y_{t,\theta}^i$	Measured heading of aircraft 'i' at time 't' in reference to true North
$y_{t,\theta}^{ik}$	Relative bearing measurement by aircraft 'i' of aircraft 'k' at time 't'
z_{thr}	RA altitude separation threshold value
Δ	Time step of aircraft evolution
Δ_i	Time step of iteration
Δ_k	Time step of iteration
Δ_{RA}	RA time threshold value
$\mathcal{E}_{t,v,A}^i$	Altitude velocity measurement error of aircraft 'i' at time 't'
$\mathcal{E}_{t,x,ADSB}^i$	Measurement error in x direction of aircraft 'i' at time 't'
$\mathcal{E}_{t,y,ADSB}^i$	Measurement error in y direction of aircraft 'i' at time 't'
$\mathcal{E}_{t,v,H}^i$	Measurement error of groundspeed of aircraft 'i' at time 't'

$\varepsilon_{t,\theta}^i$	Heading measurement error of aircraft 'i' at time 't'
$\varepsilon_{t,\theta}^{ik}$	Bearing measurement error of measurement by aircraft 'i' of aircraft 'k' at time 't'
θ_t^i	Magnetic heading of aircraft 'i' at time 't'
$\dot{\theta}_t^i$	Turn rate of aircraft 'i' at time 't'
$\dot{\theta}_{\max}^i$	Maximum turn rate of aircraft 'i'
$\dot{\theta}_{t,RA}^i$	Indicated turn rate in RA for aircraft 'i' at time 't'
$\mu_{t,A}$	Relative altitude residual at time 't'
$\sigma_{t,m}$	Estimate of standard error of prediction at time 't' of parabolic range tracker
$\tau_{t,A,ADSB}$	Time to co-altitude for ACAS UAV
$\tau_{t,CPA,ADSB}$	Estimated time until closes point of approach at time 't' for ACAS UAV
$\tau_{t,H,\lim}^i$	Threshold value determining the minimum required time until CPA which allows for an effective horizontal avoidance manoeuvre for aircraft 'i'
$\tau_{t,H,\text{mod},ADSB}$	Modified time until closest point of approach for RA for ACAS UAV
χ_{ModeS}^i	Aircraft ID of aircraft 'i'

Note: symbols regarding the colour of a token in the Petri Net specification are only listed at that location.

Glossary of Terms

ACAS	Airborne Collision Avoidance System
ADS-B	Automatic dependent surveillance – broadcast
AT	Altitude test
BBT	Bearing based tracker
CDTI	Cockpit display of traffic information
COC	Clear of conflict
CPA	Closest point of approach
CT	Cartesian tracker
DAA	Detect and Avoid
HMD	Horizontal miss distance
HMDF	Horizontal miss distance filter
IPN	Interconnecting Petri Net
LPN	Local Petri Net
PRT	Parabolic range tracker
RA	Resolution Advisory
RT	Range test
TA	Traffic Advisory
TCAS	Traffic Collision Avoidance System
UAV	Unmanned Aerial Vehicle

Introduction

The Thesis is structured in two volumes: Main Body, and Appendices. This volume includes the following appendices belonging to Agent-based Modelling and Simulation of TCAS Operations under Uncertainty - Volume I: Main Body.

- In Appendix A the Petri Net model specification of the new TCAS model is presented.
- In Appendix B a preliminary detect and avoid system for UAS is developed.
- In Appendix C the Petri Net model of the detect and avoid system for UAS is developed.
- In Appendix D the Petri Net model specification of the detect and avoid system for UAS is presented.

Note, numbering of sections and equations are continuous throughout Volume I and Volume II.

Appendix

A Petri Net Model Specification

In total there are the following 4 agents:

- TCAS i,
- Communication i,
- Cockpit i, and
- Aircraft i.

Each aircraft is modelled by four agents. If the aircraft is TCAS equipped, then these agents are: TCAS i, Communication i, Cockpit i, and Aircraft i. If the aircraft is an UAV, then these agents are: ACAS UAV i, Communication i, Cockpit i, and Aircraft i. Next the specifications of the agents are given.

A.1 Agent “TCAS i”

Since TCAS is a complex system consisting of several modules, each module is modelled as a LPN. An additional IPN is added to model the timer which models the elapsed time until CPA, as this timer is required for several modules in order to determine whether these should be reset. The outputs, i.e. TA, RA, sense coordination, secondary RA, COC message, are also modelled as IPNs. Hence, agent “TCAS i” has in total 7 LPNs and 6 IPNs which are described below.

LPN “State Estimation”

The state estimation local Petri net represents the slant range and vertical range filtering, and the horizontal miss distance filter modules as described in Section 5.

Incoming arcs within same agent

- None

Outgoing arcs within same agent

- Two enabling arcs from place “P” to LPN “TA Module” transitions G1 and G2
- Two enabling arcs from place “P” to LPN “Threat Detection” transitions G1 and G2
- Two enabling arcs from place “P” to LPN “Sense Selection” transitions I and G
- Two enabling arcs from place “P” to LPN “Strength Selection” transitions I1 and I2
- Two enabling arcs from place “P” to LPN “Evolution Monitoring” transitions G1 and G2

Incoming arcs from other agent

- Incoming arc from place “State message from aircraft k to i” of LPN “Measurements” of agent “Communication k” to transition G1
- Inhibitor arc from place “State message from aircraft k to i” of LPN “Measurements” of agent “Communication k” to transition G2
- Two enabling arcs from place “P” of LPN “State” of agent “Aircraft i” to transitions G1 and G2

Outgoing arcs to other agent

- None

Places

The following colours equal the variables given in Equations (30)-(41) and (49)-(114) in Sections 5 and 7. Exception is colour t^G which is added to control the TCAS cycle of 1Hz.

Table A-1: Places of LPN State Estimation of agent TCAS i.

Places	Colour type	Explanation	Colour function
P	$r_t \in \mathbb{R}$	Slant range estimate	None
	$\dot{r}_t \in \mathbb{R}$	Slant rate estimate	None
	$r_{t,A} \in \mathbb{R}$	Relative altitude estimate	None
	$\dot{r}_{t,A} \in \mathbb{R}$	Relative altitude rate estimate	None
	$\mu_{t,A} \in \mathbb{R}$	Altitude residual	None
	$v_{t,A}^i \in \mathbb{R}$	Measured altitude velocity of aircraft 'i'	None
	$y_{t,A}^i \in \mathbb{R}$	Measured altitude of aircraft 'i'	None
	$t^G \in \mathbb{R}$	Time delay until next update/measurement	$\dot{t}^G = -1$
	$\chi_{ModeS}^i \in \mathbb{R}$	Aircraft ID of aircraft 'i'	None
	$\chi_{ModeS}^k \in \mathbb{R}$	Aircraft ID of aircraft 'k'	None
	$B_{t,HMDF} \in \{0,1\}$	State of horizontal miss distance filter. Equals 1 if filter active	None
	$r_{t,PRT} \in \mathbb{R}$	Slant range estimate of parabolic range tracker of HMDF	None
	$\dot{r}_{t,PRT} \in \mathbb{R}$	Relative velocity estimate of parabolic range tracker of HMDF	None
	$\ddot{r}_{t,PRT} \in \mathbb{R}$	Relative acceleration estimate of parabolic range tracker of HMDF	None
	$\ddot{r}_{t,PRT} \in \mathbb{R}$	Relative jerk estimate of parabolic range tracker of HMDF	None
	$P_{t,firm} \in \{0,...,8\}$	Firmness parameter of HMDF	None
	$\sigma_{t,m} \in \mathbb{R}$	Range noise estimator of HMDF	None
	$q_{t,X,BBT} \in \mathbb{R}$	X coordinate of range estimate of bearing based tracker	None
	$q_{t,Y,BBT} \in \mathbb{R}$	Y coordinate of range estimate of bearing based tracker	None
	$\dot{q}_{t,X,BBT} \in \mathbb{R}$	X coordinate of velocity estimate of bearing based tracker	None
	$\dot{q}_{t,Y,BBT} \in \mathbb{R}$	Y coordinate of velocity estimate of bearing based tracker	None
	$r_{t,CT} \in \mathbb{R}$	Slant range estimate of Cartesian tracker of HMDF	None
	$\dot{r}_{t,CT} \in \mathbb{R}$	Relative velocity estimate of Cartesian tracker of HMDF	None
	$\ddot{r}_{t,CT} \in \mathbb{R}$	Relative acceleration estimate of Cartesian tracker of HMDF	None

$t_{HMDF} \in \mathbb{R}$	Timer to pause HMDF	$\dot{t}_{HMDF} = -1$
$B_{t,firm} \in \{0,1\}$	Indicator whether reliable aircraft tracks have been established	None

Initial markings

Initially, there is a token in place “P”. The initial colours of this token are defined in Sections 5 and 7 (see Equations (30)-(41) and (49)-(114)). Except, the initial colour of t^G is determined initially by drawing a random sample from a uniform distribution between 0 and 1.

Transitions

Table A-2: Transitions of LPN State Estimation of agent TCAS i.

ID	Transition	Condition
G1	<i>State message from aircraft k to i</i> $\{\text{Measurements}[\text{Communication } k]\} \wedge P\{\text{State}[\text{Aircraft } i]\} \wedge P \rightarrow P$	$t^G \leq 0$
G2	<i>NOT State message from aircraft k to i</i> $\{\text{Measurements}[\text{Communication } k]\} \wedge P\{\text{State}[\text{Aircraft } i]\} \wedge P \rightarrow P$	$t^G \leq 0$

Table A-3: Firing functions of transitions of LPN State Estimation of agent TCAS i.

ID	Firing function
G1	One token is fired to place P with colours. The colours are set according to Equations (30)-(41) and (49)-(114) in Sections 5 and 7. Some of the colour components are updated as described below: $y_{t,r} = y_{t,r} \{ \text{State message from aircraft } k \text{ to } i \{ \text{Measurements}[\text{Communication } k] \} \}$ $y_{t,A} = y_{t,A} \{ \text{State message from aircraft } k \text{ to } i \{ \text{Measurements}[\text{Communication } k] \} \}$ $v_{t,A}^i = v_{t,A}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \}$ $y_{t,A}^i, \text{ where } s_{t,z}^i = s_{t,z}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \}, \text{ see Equation (17);}$ $y_{t,\theta} = y_{t,\theta} \{ \text{State message from aircraft } k \text{ to } i \{ \text{Measurements}[\text{Communication } k] \} \}$ And according to: $t^G = 1s$ Note, a measurement about the intruder is present.
G2	One token is fired to place P with colours. The colours are set according to Equations (30)-(41) and (49)-(114) in Sections 5 and 7. Some of the colour components are updated as described below:

$$v_{t,A}^i = v_{t,A}^i \left\{ P \left\{ \text{State}[\text{Aircraft } i] \right\} \right\}$$

$$y_{t,A}^i, \text{ where } s_{t,z}^i = s_{t,z}^i \left\{ P \left\{ \text{State}[\text{Aircraft } i] \right\} \right\} \text{ see Equation (17);}$$

And according to:

$$t^G = 1s$$

Note, no measurements about the intruder are present.

LPN “Received Coordination Message”

The received coordination message, as defined in Equation (26), is saved in LPN “Received Coordination Message”.

Incoming arcs within same agent

- Two incoming arcs from transitions G1 and G2 of LPN “Threat Detection” to place “P”

Outgoing arcs within same agent

- Two outgoing arcs from place “P” to LPN “Threat Detection” transitions G1 and G2
- Two enabling arcs from place “P” to LPN “Sense Selection” transitions I and G

Incoming arcs from other agent

- Incoming arc from place “Coordination message from aircraft k to i” of LPN “Measurements” of agent “Communication k” to transition I

Outgoing arcs to other agent

- None

Places

Place “P” stores the value of the received coordination message as described in Section 4 (see Equation (26)).

Table A-4: Place of LPN Received Coordination Message of agent TCAS i.

Places	Colour type	Explanation	Colour function
P	$m_{t, received} \in \{-1, 0, 1\}$	Received coordination message	None

Initial markings

Initially there is a token present in place “P”. The initial colour is described below:

Table A-5: Initial marking of place of LPN Received Coordination Message of agent TCAS i.

Places	Colour	Explanation
P	$m_{t, received} = 0$	No message received

Transitions

Table A-6: Transition of LPN Received Coordination Message of agent TCAS i.

ID	Transition	Condition
I	Coordination message from aircraft k to i $\{\text{Measurements}[\text{Communication k}]\} \wedge P \rightarrow P$	None

Whenever a new coordination message is present, then the stored coordination message is updated according to Equation (26). As described in Section 4, a coordination message will be transmitted until the recipient has acknowledged the reception. The Petri Net model models the behaviour such that it continuously transmits a message but indicates through the additional parameter p_{mes} whether the intruder is sending a new coordination message.

Table A-7: Firing function of transition of LPN Received Coordination Message of agent TCAS i.

ID	Firing function
I	One token is fired with the colour: if $p_{mes} \{ \text{Coordination message from aircraft } k \text{ to } i$ $\{ \text{Measurements}[\text{Communication } k] \} = 1$ $m_{t, received} = m_{t, sent} \{ \text{Coordination message from aircraft } k \text{ to } i$ $\{ \text{Measurements}[\text{Communication } k] \}$,according to Equation(26) else $m_{t, received} = m_{t, received} \{ P \}$,stored received coordination message remains unchanged

LPN “TA Module”

The Traffic Advisory LPN represents the Traffic Advisory module as described in Section 6.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions G1 and G2
- Two enabling arcs from place “P” in IPN “Timer” to transitions G1 and G2

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc from transition G1 to place “P” in IPN “TA”

Places

There are two places without colours: “TA” and “No TA”.

Initial markings

Initially there is a token in place “No TA” without colour. The places represent $B_{t,TA}$ from Equation (48). A token in place “TA” represents the condition $B_{t,TA} = TRUE$. A token in place “No TA” represents the condition $B_{t,TA} = FALSE$.

Transitions

Table A-8: Transitions of LPN TA Module of agent TCAS i.

ID	Transition	Condition
G1	$No\ TA \wedge P\{State\ Estimation\} \wedge P\{Timer\} \rightarrow TA \wedge P\{TA\}$	If $B_{t,TA} = TRUE$ from Equation (48), where $r_t = r_t \{P\{State\ Estimation\}\}$ $\dot{r}_t = \dot{r}_t \{P\{State\ Estimation\}\}$ $r_{t,A} = r_{t,A} \{P\{State\ Estimation\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{State\ Estimation\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{State\ Estimation\}\}$ $y_{t,A}^0 = y_{t,A}^0 \{P\{State\ Estimation\}\}$
G2	$TA \wedge P\{State\ Estimation\} \wedge P\{Timer\} \rightarrow No\ TA$	If $\left\{ \left[\left(\neg B_{t,RT} \vee \neg B_{t,AT} \right) \wedge \left(t_{t,timer} \leq 0 \right) \right] \vee B_{t,HMDF} \right\}$ from Equation (156) is satisfied, where $r_t = r_t \{P\{State\ Estimation\}\}$ $\dot{r}_t = \dot{r}_t \{P\{State\ Estimation\}\}$

	$r_{t,A} = r_{t,A} \{P\{\text{State Estimation}\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{\text{State Estimation}\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{\text{State Estimation}\}\}$ $y_{t,A}^0 = y_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $t_{t,timer} = t_{t,timer} \{P\{\text{Timer}\}\}$ $B_{t,HMDF} = B_{t,HMDF} \{P\{\text{State Estimation}\}\}$ Note, this is the same test as to issue a clear of conflict message and reset the system
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Table A-9: Firing functions of transitions of LPN TA Module of agent TCAS i.

ID	Firing function
G1	Two tokens without colours are fired. One token to $P\{TA\}$, and one token to TA
G2	One token without colour is fired to $No\ TA$.

IPN “TA”

IPN to indicate issue of TA.

Incoming arcs within same agent

- Incoming arc from transition G1 of LPN “TA Module” of agent “TCAS Aircraft i” to place “TA”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There is one place “P” without colour.

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

LPN “Threat Detection”

The Threat Detection LPN represents the Threat Detection module as described in Section 8.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions G1 and G2
- Two incoming arcs from place “P” in LPN “Received Coordination Message” to transitions G1 and G2
- Two enabling arcs from place “P” in LPN “Timer” to transitions G1 and G2

Outgoing arcs within same agent

- Enabling arc from place “Threat” to transition I of LPN “Sense Selection”
- Two outgoing arcs from transitions G1 and G2 to place “P” in LPN “Received Coordination Message”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There are two places without colour: “Threat” and “No threat”. The places represent $B_{t,RA}$ from Equation (121). A token in place “Threat” represents the condition $B_{t,RA} = TRUE$. A token in place “No threat” represents the condition $B_{t,RA} = FALSE$.

Initial markings

Initially there is one token without colour in place “No Threat”.

Transitions

Table A-10: Transitions of LPN Threat Detection of agent TCAS i.

ID	Transition	Condition
G1	$No\ threat \wedge P\{State\ Estimation\} \wedge P\{Received\ Coordination\ Message\} \wedge P\{Timer\} \rightarrow Threat \wedge P\{Received\ Coordination\ Message\}$	If $B_{t,RA} = TRUE$ from Equation (121), where $r_t = r_t \{P\{State\ Estimation\}\}$ $\dot{r}_t = \dot{r}_t \{P\{State\ Estimation\}\}$ $r_{t,A} = r_{t,A} \{P\{State\ Estimation\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{State\ Estimation\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{State\ Estimation\}\}$ $y_{t,A}^i = y_{t,A}^i \{P\{State\ Estimation\}\}$ $m_{t,received} = m_{t,received} \{P\{Received\ Coordination\ Message\}\}$
G2	$Threat \wedge P\{State\ Estimation\} \wedge P\{Received\ Coordination\ Message\} \wedge P\{Timer\} \rightarrow No\ threat \wedge P\{Received\ Coordination\ Message\}$	If $\left[\left(\neg B_{t,RT} \vee \neg B_{t,AT} \right) \wedge (t_{timer} \leq 0) \right] \vee B_{t,HMDF}$ from Equation (156) is satisfied, where $r_t = r_t \{P\{State\ Estimation\}\}$ $\dot{r}_t = \dot{r}_t \{P\{State\ Estimation\}\}$ $r_{t,A} = r_{t,A} \{P\{State\ Estimation\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{State\ Estimation\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{State\ Estimation\}\}$ $y_{t,A}^0 = y_{t,A}^0 \{P\{State\ Estimation\}\}$ $t_{timer} = t_{timer} \{P\{Timer\}\}$ $B_{t,HMDF} = B_{t,HMDF} \{P\{State\ Estimation\}\}$ Note, this is the same test as to issue a clear of conflict message and reset the system

Table A-11: Firing functions of transitions of LPN Threat Detection of agent TCAS i.

ID	Firing function
G1	In total two tokens are fired: One token without colour is fired to place <i>Threat</i>. One token with unchanged colour from $P\{\text{Received Coordination Message}\}$ is fired to $P\{\text{Received Coordination Message}\}$
G2	In total two tokens are fired: One token without colour is fired to place <i>No threat</i>. One token with the following colour is fired to $P\{\text{Received Coordination Message}\}$: $m_{t, received} \{P\{\text{Received Coordination Message}\}\} = 0$

LPN “Sense Selection”

The Sense Selection LPN represents the Sense Selection module as described in Section 9.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions I and G
- Two enabling arcs place “P” in from LPN “Received Coordination Message” to transitions I and G
- Two incoming arcs place “P” in from LPN agent “Timer” to transitions I and G
- Enabling arc from place “Threat” of LPN “Threat Detection” to transition I

Outgoing arcs within same agent

- Enabling arc from place “No sense” to transition I2 of LPN “Strength Selection”
- Enabling arc from place “Sense” to transition I1 of LPN “Strength Selection”
- Two outgoing arcs from transitions I and G to place “P” of IPN “Timer”
- Outgoing arc from transition I to place “P” of IPN “Sense coordination”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

The places represent $B_{t,D}$ from Equations (123) and (138). A token in place “Sense” represents the condition $B_{t,D} = TRUE$. A token in place “No sense” represents the condition $B_{t,D} = FALSE$.

Table A-12: Places of LPN Sense Selection of agent TCAS i.

Places	Colour type	Explanation	Colour function
No sense	None	None	None
Sense	$D_t \in \{-1, 0, 1\}$	Selected sense, see Equations (127) and (131)	None
	$m_{t,sent} \in \{-1, 0, 1\}$	Sent coordination message, see Equation (137)	None

Initial markings

Initially there is one token without colour in place “No sense”

Transitions

Table A-13: Transitions of LPN Sense Selection of agent TCAS i.

ID	Transition	Condition
I	$No\ sense \wedge P\{State\ Estimation\} \wedge P\{Received\ Coordination\ Message\} \wedge P\{Timer\} \wedge Threat\{Threat\ Detection\} \rightarrow Sense \wedge P\{Sense\ coordination\} \wedge P\{Timer\}$	N/A
G	$Sense \wedge P\{State\ Estimation\} \wedge P\{Received\ Coordination\ Message\} \wedge P\{Timer\} \rightarrow No\ sense \wedge P\{Timer\}$	If $\left[\left(\neg B_{t,RT} \vee \neg B_{t,AT} \right) \wedge (t_{t,timer} \leq 0) \right] \vee B_{t,HMDF}$ from Equation (156), or if $\left[(m_{t,sent} = m_{t,received}) \wedge (\chi_{ModeS}^0 > \chi_{ModeS}^1) \right]$ from Equation (138) is satisfied, where $r_t = r_t \{P\{State\ Estimation\}\}$ $\dot{r}_t = \dot{r}_t \{P\{State\ Estimation\}\}$ $r_{t,A} = r_{t,A} \{P\{State\ Estimation\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{State\ Estimation\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{State\ Estimation\}\}$ $y_{t,A}^i = y_{t,A}^i \{P\{State\ Estimation\}\}$ $t_{t,timer} = t_{t,timer} \{P\{Timer\}\}$ $B_{t,HMDF} = B_{t,HMDF} \{P\{State\ Estimation\}\}$ $m_{t,received} = m_{t,received} \{P\{Received\ Coordination\ Message\}\}$ $\chi_{ModeS}^i = \chi_{ModeS}^i \{P\{State\ Estimation\}\}$ $\chi_{ModeS}^k = \chi_{ModeS}^k \{P\{State\ Estimation\}\}$

Table A-14: Firing functions of transitions of LPN Sense Selection of agent TCAS i.

ID	Firing function
I	In total three tokens are fired. The colours are determined according to Equations (123)-(138), where $r_t = r_t \{P\{State\ Estimation\}\}$ $\dot{r}_t = \dot{r}_t \{P\{State\ Estimation\}\}$ $r_{t,A} = r_{t,A} \{P\{State\ Estimation\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{State\ Estimation\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{State\ Estimation\}\}$ $y_{t,A}^i = y_{t,A}^i \{P\{State\ Estimation\}\}$ $t_{t,timer} = t_{t,timer} \{P\{Timer\}\}$ $B_{t,HMDF} = B_{t,HMDF} \{P\{State\ Estimation\}\}$ $m_{t,received} = m_{t,received} \{P\{Received\ Coordination\ Message\}\}$ One token is fired to <i>Sense</i> with the following colours:

	D_t and $m_{t,sent} \{Sense\}$, according to Equations (123)-(137) One token is fired to $P\{Timer\}$ with colour: $t_{t,timer}$, according to Equations (123)-(125) One token is fired to $P\{Sense\ Coordination\}$ with the following colour: $S_{Sense_coordination} = m_{t,sent} \{Sense\}$, according to Equations (123)-(137)
G	Two tokens are fired. One token is fired to <i>No sense</i> without colour. One token is fired to $P\{Timer\}$ with colour: $t_{t,timer} = 0$

IPN “Timer”

The timer measures whether the time until CPA at initial RA has elapsed.

Incoming arcs within same agent

- Incoming arcs from LPN “Sense Selection” transitions I and G

Outgoing arcs within same agent

- Two enabling arcs from place “P” to LPN “TA Module” transitions G1 and G2
- Two enabling arcs from place “P” to LPN “Threat Detection” transitions G1 and G2
- Two outgoing arcs from place “P” to LPN “Sense Selection” transitions I and G
- Two enabling arcs from place “P” to LPN “Strength Selection” transitions I1 and I2
- Two enabling arcs from place “P” to LPN “Evolution Monitoring” transitions G1 and G2

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

Table A-15: Place of IPN Timer of agent TCAS i.

Places	Colour type	Explanation	Colour function
P	$t_{t, timer} \in \mathbb{R}$	Time until CPA based on initial RA, see Equation (125)	$\dot{t}_{t, timer} = -1$

Initial markings

Initially there is one token present in place “P”. The colour is described below:

Table A-16: Initial marking of place of IPN Timer of agent TCAS i.

Places	Colour	Explanation
P	$t_{t, timer} = 0$	Initial value not relevant

Transitions

There are no transitions in the IPN

IPN “Sense coordination”

Incoming arcs within same agent

- Incoming arc from transition I of LPN “Sense Selection”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc from place “P” to transition I1 of LPN “Mode S Reply” of agent “Communication i”

Places

Table A-17: Place of IPN Sense Coordination of agent TCAS i.

Places	Colour type	Explanation	Colour function
P	$m_{t, sent} \in \{-1, 0, 1\}$	Coordination message from aircraft i to k	None

Initial markings

Initially there is no token in place “P”.

Transitions

There are no transitions in the IPN.

LPN “Strength Selection”

The Strength Selection LPN represents the Strength Selection module as described in Section 9.5.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions I1 and I2
- Two enabling arcs from place “P” in LPN “Timer” to transitions I1 and I2
- Enabling arc from place “Sense” of LPN “Sense Selection” to transition I1
- Enabling arc from place “No sense” LPN “Sense Selection” to transition I2
- Incoming arc from place “P” in LPN “Evolution Monitoring” to transition I1

Outgoing arcs within same agent

- Outgoing arc from transition I1 to place “P” in LPN “Evolution Monitoring”
- Outgoing arc from transition I1 to place “P” in IPN “RA”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There are two places without colour: “Strength” and “No strength”.

Initial markings

Initially there is one token without colour in place “No strength”

Transitions

Table A-18: Transitions of LPN Strength Selection of agent TCAS i.

ID	Transition	Condition
I1	$No\ strength \wedge Sense\{Sense\ Selection\} \wedge P\{State\ Estimation\} \wedge P\{Timer\} \wedge P\{Evolution\ Monitoring\} \rightarrow Strength \wedge P\{Evolution\ Monitoring\} \wedge P\{RA\}$	N/A
I2	$Strength \wedge No\ sense\{Sense\ Selection\} \wedge P\{State\ Estimation\} \wedge P\{Timer\} \rightarrow No\ strength$	N/A

Table A-19: Firing functions of transitions of LPN Strength Selection of agent TCAS i.

ID	Firing function
I1	In total three tokens are fired. The colours are determined according to Equations (42)-(48) and (139)-(142) where $r_t = r_t \{P\{State\ Estimation\}\}$

	$\dot{r}_t = \dot{r}_t \{P\{\text{State Estimation}\}\}$ $r_{t,A} = r_{t,A} \{P\{\text{State Estimation}\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{\text{State Estimation}\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{\text{State Estimation}\}\}$ $y_{t,A}^i = y_{t,A}^i \{P\{\text{State Estimation}\}\}$ $D_t = D_t \{Sense\{\text{Sense Selection}\}\}$ $m_{t,sent} = m_{t,sent} \{Sense\{\text{Sense Selection}\}\}$ One token without colour is fired to place “Strength”. One token with colours is fired to $P\{RA\}$. The colours are determined as follows: $v_{t,A,RA,min}^i = v_{t,A,RA,min}^i$ $v_{t,A,RA,max}^i = v_{t,A,RA,max}^i$ $a_{t,A,RA}^i = 0.25g, \text{ assumed acceleration by the pilot flying (ICAO, 2006)}$ $\dot{\theta}_{t,RA}^i = 0, \text{ TCAS does not include horizontal avoidance logic}$ $t_{RA} = N(5, 0.5^2)s, \text{ the reaction time of the pilot flying is modelled by drawing a sample from a normal distribution with the mean equal to the assumed reaction time by (ICAO, 2006) and a standard deviation of 0.5s.}$ One token with colours is fired to $P\{\text{Evolution Monitoring}\}$. The colours are determined as follows: $D_t^A = D_t \{Sense\{\text{Sense Selection}\}\}$ $m_{t,sent}^A = m_{t,sent} \{Sense\{\text{Sense Selection}\}\}$ $v_{t,A,RA,min}^i = v_{t,A,RA,min}^i$ $v_{t,A,RA,max}^i = v_{t,A,RA,max}^i$ $t_{t RA} = 8s$
12	One token without colour is fired to <i>No strength</i>

IPN “RA”

IPN to indicate the issue of an RA.

Incoming arcs within same agent

- Incoming arc from transition I1 of LPN “Strength Selection”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc to transition I1 of LPN “Pilot Flying” of agent “Cockpit i”

Places

There is one place “P” with the following colours.

Table A-20: Place of IPN RA of agent TCAS i.

Places	Colour type	Explanation	Colour function
P	$v_{t,A,RA,\min}^i \in \mathbb{R}$	Target lower altitude velocity bound of RA	None
	$v_{t,A,RA,\max}^i \in \mathbb{R}$	Target upper altitude velocity bound of RA	None
	$a_{t,A,RA}^i \in \mathbb{R}$	Target altitude rate of RA	None
	$\dot{\theta}_{t,RA}^i \in \mathbb{R}$	Target turn rate of RA	None
	$t_{RA} \in \mathbb{R}$	Assumed reaction time of the pilot	None

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

LPN “Evolution Monitoring”

The Evolution Monitoring LPN represents the evolution monitoring module as described in Section 10.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions G1 and G2
- Incoming arc from transition I1 of LPN “Strength Selection” to place “P”
- Two enabling arcs from place “P” of IPN “Timer” to transitions G1 and G2

Outgoing arcs within same agent

- Outgoing arc from place “P” to transition I1 of LPN “Strength Selection”
- Outgoing arc from transition G1 to place “P” of IPN “Adjusted RA”
- Outgoing arc from transition G2 to place “P” of IPN “Adjusted RA”
- Outgoing arc from transition G2 to place “P” of agent IPN “COC”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

Next to the variables used in Section 10, the Petri net model has an additional colour, $S_{Evolution_monitoring}$, to determine when the evolution monitoring algorithm (Equations (143)-(156)) should be carried out.

Table A-21: Place of LPN Evolution Monitoring of agent TCAS i.

Places	Colour type	Explanation	Colour function
P	$S_{Evolution_monitoring} \in \{0,1\}$	Equals 1 if monitoring active	None
	$D_t^A \in \{-1,0,1\}$	Updated selected sense	None
	$m_{t,sent}^A \in \{-1,0,1\}$	Virtual sent coordination message	None
	$v_{t,A,RA,min}^i \in \mathbb{R}$	Minimum target altitude velocity of current RA	None
	$v_{t,A,RA,max}^i \in \mathbb{R}$	Maximum target altitude velocity of current RA	None
	$t_{t RA} \in \mathbb{R}$	Interval time of monitoring cycle for transition G1	$\dot{t}_{t RA} = -1$

Initial markings

Initially there is one token present in place “P”. The colours are defined as described below:

Table A-22: Initial marking of place of LPN Evolution Monitoring of agent TCAS i.

Places	Colour	Explanation
P	$S_{Evolution_monitoring} = 0$	Initially no monitoring
	$D_t^A = 0$	Initially no monitoring, so value not of interest
	$m_{t,sent}^A = 0$	Initially no monitoring, so value not of interest
	$v_{t,A,RA,min}^i = 0$	Initially no monitoring, so value not of interest
	$v_{t,A,RA,max}^i = 0$	Initially no monitoring, so value not of interest
	$t_{t RA} = 0$	Initially no monitoring, so value not of interest

Transitions

Table A-23: Transitions of LPN Evolution Monitoring of agent TCAS i.

ID	Transition	Condition
G1	$P \wedge P\{\text{State Estimation}\} \wedge P\{\text{Timer}\} \rightarrow P \wedge P\{\text{Adjusted RA}\}$	If either Equation (147) or Equation (149) are satisfied where $r_t = r_t \{P\{\text{State Estimation}\}\}$ $\dot{r}_t = \dot{r}_t \{P\{\text{State Estimation}\}\}$ $r_{t,A} = r_{t,A} \{P\{\text{State Estimation}\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{\text{State Estimation}\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{\text{State Estimation}\}\}$ $y_{t,A}^i = y_{t,A}^i \{P\{\text{State Estimation}\}\}$
G2	$P \wedge P\{\text{State Estimation}\} \wedge P\{\text{Timer}\} \rightarrow P \wedge P\{\text{Adjusted RA}\} \wedge P\{\text{COC}\}$	If $\left[\left(\neg B_{t,RT} \vee \neg B_{t,AT} \right) \wedge (t_{t,timer} \leq 0) \right] \vee B_{t,HMDF}$ from Equation (156) is satisfied, where $r_t = r_t \{P\{\text{State Estimation}\}\}$ $\dot{r}_t = \dot{r}_t \{P\{\text{State Estimation}\}\}$ $r_{t,A} = r_{t,A} \{P\{\text{State Estimation}\}\}$ $\dot{r}_{t,A} = \dot{r}_{t,A} \{P\{\text{State Estimation}\}\}$ $v_{t,A}^i = v_{t,A}^i \{P\{\text{State Estimation}\}\}$ $y_{t,A}^i = y_{t,A}^i \{P\{\text{State Estimation}\}\}$ $t_{t,timer} = t_{t,timer} \{P\{\text{Timer}\}\}$ $B_{t,HMDF} = B_{t,HMDF} \{P\{\text{State Estimation}\}\}$ Note, this is the same test as to issue a clear of conflict message and reset the system

Table A-24: Firing function of transitions of LPN Evolution Monitoring of agent TCAS i.

ID	Firing function
G1	In total two tokens with colours are fired. One token is fired to place $P\{\text{Adjusted RA}\}$ with the following colours: $v_{t,A,RA,\min}^i$, according to Equations (42)-(48) and (143)-(155) $v_{t,A,RA,\max}^i$, according to Equations (42)-(48) and (143)-(155) $a_{t,A,RA}^i = 0.35g$, assumed acceleration by the pilot flying (ICAO, 2006) $\dot{\theta}_{Adjusted_RA}^i = 0$,TCAS does not include horizontal avoidance logic $t_{Adjusted_RA} = N(2.5, 0.5^2)s$,the reaction time of the pilot flying is modelled by drawing a sample from a normal distribution with the mean equal to the assumed reaction time by (ICAO, 2006) and a standard deviation of 0.5s. One token is fired to place “P” with the following colours: $S_{Evolution_monitoring} = 1$ and according to Equations (42)-(48) and (143)-(155): D_t^A $m_{t,sent}^A$ $v_{t,A,RA,\min}^i$ $v_{t,A,RA,\max}^i$ $t_{t RA}$
G2	In total three tokens are fired. One token without colour to place $P\{COC\}$. One token with colours to place P. The colours are as follows: $S_{Evolution_monitoring} = 0$ and according to Equations (42)-(48) and (143)-(155): D_t^A $m_{t,sent}^A$ $v_{t,A,RA,\min}^i$ $v_{t,A,RA,\max}^i$ $t_{t RA}$ One token with colours to place $P\{\text{Adjusted RA}\}$. The colours are as follows: $v_{t,A,RA,\min}^i$, according to Equations (42)-(48) and (143)-(155) $v_{t,A,RA,\max}^i$, according to Equations (42)-(48) and (143)-(155) $a_{t,A,RA}^i = 0.35g$, assumed acceleration by the pilot flying (ICAO, 2006) $\dot{\theta}_{Adjusted_RA}^i = 0$,TCAS does not include horizontal avoidance logic

$t_{Adjusted_RA} = N(2.5, 0.5^2)$ s, the reaction time of the pilot flying is modelled by drawing a sample from a normal distribution with the mean equal to the assumed reaction time by (ICAO, 2006) and a standard deviation of 0.5s.

Note that this latter token to place $P\{\text{Adjusted RA}\}$ is fired to reset the colour properties of a level off RA, as it is assumed that the pilot will level off the aircraft after a clear of conflict message and will ask the air traffic controller for further instructions.

IPN “Adjusted RA”

IPN to indicate the issue of a secondary RA.

Incoming arcs within same agent

- Incoming arc from transition G1 of LPN “Evolution Monitoring” to place “P”
- Incoming arc from transition G2 of LPN “Evolution Monitoring” to place “P”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc to transition I2 of LPN “Pilot Flying” of agent “Cockpit i”

Places

There is one place “P” with the following colours.

Table A-25: Place of IPN Adjusted RA of agent TCAS i.

Places	Colour type	Explanation	Colour function
P	$v_{t,A,RA,min}^i \in \mathbb{R}$	Lower velocity bound of RA	None
	$v_{t,A,RA,max}^i \in \mathbb{R}$	Upper velocity bound of RA	None
	$a_{t,A,RA}^i \in \mathbb{R}$	Assigned acceleration of RA	None
	$\dot{\theta}_{Adjusted_RA}^i \in \mathbb{R}$	Turn rate of RA	None
	$t_{Adjusted_RA} \in \mathbb{R}$	Time stamp of message	None

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

IPN “COC”

IPN to indicate the issue of a clear of conflict message.

Incoming arcs within same agent

- Incoming arc from transition G2 of LPN “Evolution Monitoring” to place “P”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc from place “P” to transition I2 of LPN “Mode S Reply” of agent “Communication i”

Places

There is one place “P” without colours.

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

A.2 Agent “Communication i”

This agent has 3 LPNs and 2 IPNs. The LPNs are the following:

- Transponder,
- Mode S Reply, and
- Interrogation.

The IPN is the following one:

- Measurements, and
- Int. mes. from aircraft i to k.

LPN “Transponder”

In case the transponder is not working or switched off at one of the two aircraft, then that aircraft is not able to communicate. Whether the transponder is working is modelled in this LPN.

Incoming arcs within same agent

- None

Outgoing arcs within same agent

- Enabling arc from place “Working” to LPN “Mode S Reply” transition I3
- Enabling arc from place “Working” to LPN “Interrogation” transition G

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

Table A-26: Places of LPN Transponder of agent Communication i.

Places	Colour type	Explanation	Colour function
Working	$t_{Transp}^{Working} \in \mathbb{R}$	Delay until next firing	$\dot{t}_{Transp}^{Working} = -1$
Not working	$t_{Transp}^{Not_working} \in \mathbb{R}$	Delay until next firing	$\dot{t}_{Transp}^{Not_working} = -1$

Initial markings

Initially there is one token in place “Working”. Currently, there are no assumptions made on the technical reliability of the transponder. Therefore, the time delay to enable the transition from place “Working” to “Not working” is set higher than the duration of the simulation, e.g. 999 hours. This ensures that the token remains in the place “Working”.

Table A-27: Initial marking of places of LPN Transponder of agent Communication i.

Places	Colour	Explanation
Working	$t_{Transp}^{Working} = 999h$	Delay until the transponder stops working

Transitions

Table A-28: Transitions of LPN Transponder of agent Communication i.

ID	Transition	Condition
G1	<i>Not working</i> → <i>Working</i>	$t_{Transp}^{Not_working} \leq 0$
G2	<i>Working</i> → <i>Not working</i>	$t_{Transp}^{Working} \leq 0$

Currently the initially condition of the token in place “Working” are set, such that the token remains in that place. Hence, the firing functions do not become active and don’t need to be defined yet.

Table A-29: Firing functions of transitions of LPN Transponder of agent Communication i.

ID	Firing function
G1	One token to <i>Working</i> with the following colour: $t_{Transp}^{Working} = TBD$
G2	One token to <i>Not working</i> with the following colour: $t_{Transp}^{Not_working} = TBD$

LPN “Mode S Reply”

It transmits the state information and a coordination message, if applicable, of aircraft ‘i’ to the IPN, such that aircraft ‘k’ may receive slant range, relative altitude, relative heading, and the coordination message, if applicable, as input for the TCAS system.

Incoming arcs within same agent

- Enabling arc from place “Working” of LPN “Transponder” to transition I3

Outgoing arcs within same agent

- Outgoing arc from transition “I4” to place “Sent message from Aircraft i to k” of LPN “Measurements”

Incoming arcs from other agent

- Incoming arc from place “P” of IPN “Sense coordination” of agent “TCAS i” to transition I1
- Incoming arc from place “P” of IPN “COC” agent “TCAS i” to transition I2
- Enabling arc from place “P” of LPN “State” of agent “Aircraft i” to transition I3
- Incoming arc from place “P” of IPN “Int. mes. from aircraft k to i” of agent “Communication k” to transition I3

Outgoing arcs to other agent

- None

Places

As described in the Section 4, the aircraft ‘i’ transmits information about the own altitude and a coordination message, if applicable. These information are created as colours in place “Sending”. After the message has been sent, only the coordination message is stored. Additionally, as described in Section 4, it is assumed that the coordination message is only sent repeatedly until the other aircraft has received the message. Therefore, an extra variable, $p_{Mode_S}^{Sending}$, is introduced to indicate whether TCAS has updated the coordination message.

Table A-30: Initial places of LPN Mode S Reply of agent Communication i.

Places	Colour type	Explanation	Colour function
Sending	$m_{t, sent} \in \{-1, 0, 1\}$	Sense coordination message	None
	$y_{t, A}^i \in \mathbb{R}$	Measured altitude of own aircraft i	None
	$p_{Mode_S}^{Sending} \in \{0, 1\}$	Indication whether the coordination message was updated	None
Not sending	$m_{t, sent} = \{-1, 0, 1\}$	Sense coordination message	None
	$p_{Mode_S}^{Not_sending} \in \{0, 1\}$	Indication whether the coordination message was updated	None

Initial markings

One token is in place “Not Sending” with the following colours:

Table A-31: Initial markings of places of LPN Mode S Reply of agent Communication i.

Places	Colour	Explanation
Not sending	$m_{t,sent} = 0$	No coordination message available
	$p_{Mode_S}^{Not_sending} = 0$	Coordination message was not updated

Transitions

There are three transitions as specified below. Transitions I1 and I2 transfer the coordination message generated by TCAS to the LPN “Mode S Reply”, such that the coordination message can be sent to the intruder. Transition I3 generates the altitude measurement about aircraft ‘i’. Transition I4 transmits the gathered information further to the IPN from where the information can be picked up by the intruder aircraft ‘k’.

Table A-32: Transitions of LPN Mode S Reply of agent Communication i.

ID	Transition	Condition
I1	$Not\ sending \wedge P\{Sense\ coordination[TCAS\ i]\} \rightarrow Not\ sending$	N/A
I2	$Not\ sending \wedge P\{COC[TCAS\ i]\} \rightarrow Not\ sending$	N/A
I3	$Not\ sending \wedge Working\{Transponder\} \wedge Int.\ mes.\ from\ aircraft\ k\ to\ i\{Measurements\} \wedge P\{State[Aircraft\ i]\} \rightarrow Sending$	N/A
I4	$Sending \rightarrow Not\ sending \wedge Sent\ message\ from\ aircraft\ i\ to\ k\{Measurements\}$	N/A

Table A-33: Firing functions of transitions of LPN Mode S Reply of agent Communication i.

ID	Firing function
I1	One token is fired with the following colours: $m_{t,sent} = m_{t,sent} \{P\{Sense\ coordination[TCAS\ i]\}\}$ $p_{Mode_S}^{Not_sending} = 1$
I2	One token is fired with the following colours: $m_{t,sent} = 0$ $p_{Mode_S}^{Not_sending} = 1$
I3	One token is fired to <i>Sending</i> with the following colours according to Equations (17) and (162)-(166): $m_{t,sent} = m_{t,sent} \{Not\ sending\}$ $p_{Mode_S}^{Sending} = p_{Mode_S}^{Not_sending} \{Not\ sending\}$

	$y_{t,A}^i$, where $s_{t,z}^i = s_{t,z}^i \{P\{\text{State}[\text{Aircraft } i]\}\}$
I4	The number of tokens fired depend on a sample drawing from a uniform distribution with range [0,1]. If the sample is smaller or equal than 0.95 (probability of reception), then following two tokens are fired: One token is fired to $\{\text{Sent message from Aircraft } i \text{ to } k\{\text{Measurements}\}\}$ with the following colours: $y_{t,A}^i = y_{t,A}^i \{\text{Sending}\}$ $m_{t,\text{sent}} = m_{t,\text{sent}} \{\text{Sending}\}$ $p_{\text{Coordination}} = p_{\text{Mode}_S}^{\text{Sending}} \{\text{Sending}\}$ One token is fired to <i>Not Sending</i> with the following colours: $m_{t,\text{sent}} = m_{t,\text{sent}} \{\text{Sending}\}$ $p_{\text{Mode}_S}^{\text{Not_sending}} = 0$ Else, only one token is fired to place <i>Not Sending</i> with the following colours: $m_{t,\text{sent}} = m_{t,\text{sent}} \{\text{Sending}\}$ $p_{\text{Mode}_S}^{\text{Not_sending}} = p_{\text{Mode}_S}^{\text{Sending}} \{\text{Sending}\}$

IPN “Measurements”

This IPN adds the measurements about slant range, relative bearing and relative altitude.

Incoming arcs within same agent

- Incoming arc from transition “I4” of LPN “Mode S Reply” to place “Sent message from Aircraft i to k”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- Enabling arc from place “P” in LPN “State” in agent “Aircraft i” to transition I
- Enabling arc from place “P” in LPN “State” in agent “Aircraft k” to transition I

Outgoing arcs to other agent

- Outgoing arc from place “State message from Aircraft i to k” to transition G1 of LPN “State Estimation” of agent “TCAS k”
- Inhibitor arc from place “State message from Aircraft i to k” to transition G2 of LPN “State Estimation” of agent “TCAS k”
- Outgoing arc from place “Coordination message from Aircraft i to k” to transition I of LPN “Received coordination message” of agent “TCAS k”

Places

Table A-34: Places of IPN Measurements of agent Communication i.

Places	Colour type	Explanation	Colour function
Sent message from Aircraft i to k	$y_{t,A}^i \in \mathbb{R}$	Measured altitude of aircraft ‘i’	None
	$m_{t, sent} \in \{-1, 0, 1\}$	Coordination message sent to own aircraft ‘i’	None
	$p_{mes} \in \{0, 1\}$	Indication whether the coordination message was updated	None
State message from Aircraft i to k	$y_{t,r} \in \mathbb{R}$	Slant range measurement between aircraft ‘i’ and ‘k’	None
	$y_{t,A} \in \mathbb{R}$	Relative altitude measurement between aircraft ‘i’ and ‘k’	None
	$y_{t,\theta} \in \mathbb{R}$	Relative heading measurement between aircraft ‘i’ and ‘k’	None
Coordination message from Aircraft i to k	$m_{t, sent} \in \{-1, 0, 1\}$	Coordination message sent to own aircraft ‘i’	None
	$p_{mes} \in \{0, 1\}$	Indication whether the coordination message was updated	None

Initial markings

Initially none of the places have a token.

Transitions

There is one transition I which generates the measurements about the slant range, relative heading and relative altitude. Besides, transition I also splits the sent message into two. The token sent to place “State message from Aircraft i to k” includes the state information, and the token sent to place “Coordination message from Aircraft i to k” includes the coordination intent.

Table A-35: Transition of IPN Measurements of agent Communication i.

ID	Transition	Condition
I	<i>Sent message from Aircraft i to k</i> $\wedge$ $P\{\text{State}[\text{Aircraft } i]\} \wedge P\{\text{State}[\text{Aircraft } k]\} \rightarrow$ <i>State message from Aircraft i to k</i> $\wedge$ <i>Coordination message from Aircraft i to k</i>	N/A

Table A-36: Firing function of transition of IPN Measurements of agent Communication i.

ID	Firing function
I	One token to <i>State message from Aircraft i to k</i> is fired with the following colours: $y_{t,r}$ according to Equation (15), where $s_{t,x}^i = s_{t,x}^i \{P\{\text{State}[\text{Aircraft } i]\}\}$ $s_{t,y}^i = s_{t,y}^i \{P\{\text{State}[\text{Aircraft } i]\}\}$ $s_{t,z}^i = s_{t,z}^i \{P\{\text{State}[\text{Aircraft } i]\}\}$ $s_{t,x}^k = s_{t,x}^k \{P\{\text{State}[\text{Aircraft } k]\}\}$ $s_{t,y}^k = s_{t,y}^k \{P\{\text{State}[\text{Aircraft } k]\}\}$ $s_{t,z}^k = s_{t,z}^k \{P\{\text{State}[\text{Aircraft } k]\}\}$ $y_{t,A}$ according to Equation (20), where $y_{t,A}^i = y_{t,A}^i \{\text{Sent message from aircraft } i \text{ to } k\}$ $s_{t,z}^k = s_{t,z}^k \{P\{\text{State}[\text{Aircraft } k]\}\}$ $y_{t,\theta}$ according to Equation (16), where $s_{t,x}^i = s_{t,x}^i \{P\{\text{State}[\text{Aircraft } i]\}\}$ $s_{t,y}^i = s_{t,y}^i \{P\{\text{State}[\text{Aircraft } i]\}\}$ $s_{t,x}^k = s_{t,x}^k \{P\{\text{State}[\text{Aircraft } k]\}\}$ $s_{t,y}^k = s_{t,y}^k \{P\{\text{State}[\text{Aircraft } k]\}\}$

$$\|\mathbf{v}_{t,H}^i\| = \|\mathbf{v}_{t,H}^i\| \{P\{\text{State}[\text{Aircraft } i]\}\}$$

$$\theta_t^i = \theta_t^i \{P\{\text{State}[\text{Aircraft } i]\}\}$$

One token to *Coordination message from Aircraft i to k* is fired with the following colours:

$$m_{t,sent} = m_{t,sent} \{Sent\ message\ from\ Aircraft\ i\ to\ k\}$$

$$p_{mes} = p_{mes} \{Sent\ message\ from\ Aircraft\ i\ to\ k\}$$

LPN “Interrogation”

This LPN sends out the interrogation message to aircraft ‘k’.

Incoming arcs within same agent

- Enabling arc from place “Working” of LPN “Transponder” to transition G

Outgoing arcs within same agent

- Outgoing arc to LPN “Int. mes. from aircraft i to k”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

Table A-37: Place of LPN Interrogation of agent Communication i.

Places	Colour type	Explanation	Colour function
P	$t_{\text{int}}^i \in \mathbb{R}$	Time delay until next interrogation message	$t_{\text{int}}^i = -1$

Initial markings

One token is in place “P” with the following colours:

Table A-38: Initial marking of place of LPN Interrogation of agent Communication i.

Places	Colour	Explanation
P	t_{int}^i is determined by a drawing of sample from a uniform distribution with a range of [0,1]	Time delay until next interrogation message

Transitions

There is one transition G which is activated if the colour value of the token in place “P” is below zero and if the transponder is working.

Table A-39: Transition of LPN Interrogation of agent Communication i.

ID	Transition	Condition
G	$P \wedge \text{Working}\{\text{Transponder}\} \rightarrow P \wedge P\{\text{Int. mes. from aircraft } i \text{ to } k\}$	$t_{\text{int}}^i \leq 0$

Table A-40: Firing function of transition of LPN Interrogation of agent Communication i.

ID	Firing function
G	In total two tokens are fired. One token is fired with the following colours to place P: $t_{\text{int}}^i = 1s$ One token is fired without colour to $P\{\text{Int. mes. from aircraft } i \text{ to } k\}$.

IPN “Int. mes. from aircraft i to k”

This IPN links agents “Communication i” and “Communication k” to transfer the interrogation message sent from aircraft ‘i’ to ‘k’.

Incoming arcs within same agent

- Incoming arc from transition G of LPN “Interrogation” to place “P”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc to transition I3 of LPN “Mode S Reply” of agent “Communication k”

Places

There is one place “P” without colours.

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

A.3 Agent “Cockpit i”

Agent “Cockpit i” has three LPN and two IPNs. The LPNs are the following:

- Pilot Flying,
- CDTI, and
- Aural Annunciation.

The IPNs are the following:

- Pilot Input, and
- Auto Pilot.

Next the LPNs and the IPNs are described.

LPN “Pilot Flying”

The pilot becomes active if he/she receives an RA and the reaction time has elapsed. Then the new target altitude velocity boundaries and the target turn rate are updated in the agent “State Aircraft i”. Please note that the reaction time of the pilot is modelled in place “P” of IPN “RA” and place “P” of IPN “Adjusted RA”.

Incoming arcs within same agent

- Enabling arc from LPN “CDTI” place “Working” to transition G1
- Enabling arc from LPN “Aural Annunciation” place “Working” to transition G1
- Enabling arc from IPN “Auto Pilot” place “P” to transition G2

Outgoing arcs within same agent

- Outgoing arc from transition G1 to place “P” of IPN “Pilot Input”
- Outgoing arc from transition G2 to place “P” of IPN “Pilot Input”

Incoming arcs from other agent

- Incoming arc from agent “TCAS i” IPN “RA” place “P” to transition I1
- Incoming arc from agent “TCAS i” IPN “Adjusted RA” place “P” to transition I2

Outgoing arcs to other agent

- None

Places

There is one place “P” which has colours as described below.

Table A-41: Place of LPN Pilot Flying of agent Cockpit i.

Places	Colour type	Explanation	Colour function
P	$S_{PF_active} \in \{0,1\}$	Indication whether an RA is present	None
	$v_{PF_active}^{A,min} \in \mathbb{R}$	Minimum target altitude velocity	None
	$v_{PF_active}^{A,max} \in \mathbb{R}$	Maximum target altitude velocity	None
	$a_{PF_active}^A \in \mathbb{R}$	Target altitude rate	None
	$\dot{\theta}_{PF_active} \in \mathbb{R}$	Target turn rate	None
	$t_{PF_active} \in \mathbb{R}$	Reaction time of pilot	$\dot{t}_{PF_active} = -1$

Initial markings

Initially there is a token present at place “P” with the colours described below.

Table A-42: Initial marking of place of LPN Pilot Flying of agent Cockpit i.

Places	Colour	Explanation
P	$S_{PF_active} = 0$	No RA present
	$v_{PF_active}^{A,min} = 0$	No RA present
	$v_{PF_active}^{A,max} = 0$	No RA present
	$a_{PF_active}^A = 0$	No RA present
	$\dot{\theta}_{PF_active} = 0$	No RA present
	$t_{PF_active} = 0$	No RA present

Transitions

Table A-43: Transitions of LPN Pilot Flying of agent Cockpit i.

ID	Transition	Condition
I1	$P\{RA[TCAS\ i]\} \wedge P \rightarrow P$	Immediate transition
I2	$P\{Adjusted\ RA[TCAS\ i]\} \wedge P \rightarrow P$	Immediate transition
G1	$Working\{CDTI\} \wedge Working\{Aural\ Annunciation\} \wedge P \rightarrow P\{Pilot\ Input\}$	$S_{PF_active}\{P1\} = 1 \wedge t_{PF_active} \leq 0$
G2	$P \wedge P\{Auto\ Pilot\} \rightarrow P\{Pilot\ Input\}$	$S_{PF_active}\{P1\} = 1$

Table A-44: Firing functions of transitions of LPN Pilot Flying of agent Cockpit i.

ID	Firing function
I1	One token with the following colours: $v_{PF_active}^{A,min} = v_{t,A,RA,min}^i \{P\{RA[TCAS\ i]\}\}$ $v_{PF_active}^{A,max} = v_{t,A,RA,max}^i \{P\{RA[TCAS\ i]\}\}$

	$a_{PF_active}^A = a_{RA}^A \{P\{RA[TCAS\ i]\}\}$ $\dot{\theta}_{PF_active} = \dot{\theta}_{RA} \{P\{RA[TCAS\ i]\}\}$ $t_{PF_active} = t_{RA} \{P\{RA[TCAS\ i]\}\}$
I2	One token with the following colours: $v_{PF_active}^{A,min} = v_{t,A,RA,min}^0 \{P\{Adjusted\ RA[TCAS\ i]\}\}$ $v_{PF_active}^{A,max} = v_{t,A,RA,max}^0 \{P\{Adjusted\ RA[TCAS\ i]\}\}$ $a_{PF_active}^A = a_{Adjusted_RA}^A \{P\{Adjusted\ RA[TCAS\ i]\}\}$ $\dot{\theta}_{PF_active} = \dot{\theta}_{Adjusted_RA} \{P\{Adjusted\ RA[TCAS\ i]\}\}$ $t_{PF_active} = t_{Adjusted_RA} \{P\{Adjusted\ RA[TCAS\ i]\}\}$
G1	One token with the following colours: $v_{PF_active}^{A,min} = v_{PF_active}^{A,min} \{P\}$ $v_{PF_active}^{A,max} = v_{PF_active}^{A,max} \{P\}$ $a_{PF_active}^A = a_{PF_active}^A \{P\}$ $\dot{\theta}_{PF_active} = \dot{\theta}_{PF_active} \{P\}$
G2	One token with the following colours: $v_{PF_active}^{A,min} = v_{PF_active}^{A,min} \{P\}$ $v_{PF_active}^{A,max} = v_{PF_active}^{A,max} \{P\}$ $a_{PF_active}^A = a_{PF_active}^A \{P\}$ $\dot{\theta}_{PF_active} = \dot{\theta}_{PF_active} \{P\}$

IPN “Auto Pilot”

IPN to indicate whether an autopilot is reacting upon RAs, i.e. in UAVs.

Incoming arcs within same agent

- None

Outgoing arcs within same agent

- Enabling arc from place “P” to transition G2 of LPN “Pilot Flying”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There is one place “P” without colours

Initial markings

If, the aircraft is an UAV then there is a token without colour present at place “P”, else no token is present.

Transitions

There are no transitions in the IPN.

IPN “Pilot Input”

IPN to transfer the pilot or autopilot input to agent “Aircraft i”.

Incoming arcs within same agent

- Incoming arc from transition G1 of LPN “Pilot Flying” to place “P”
- Incoming arc from transition G2 of LPN “Pilot Flying” to place “P”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc to transition I of LPN “State” of agent “Aircraft i”

Places

There is one place “P” with colours as described below.

Table A-45: Place of IPN Pilot Input of agent Cockpit i.

Places	Colour type	Explanation	Colour function
P	$v_{PF_active}^{A,min} \in \mathbb{R}$	Minimum target altitude velocity	None
	$v_{PF_active}^{A,max} \in \mathbb{R}$	Maximum target altitude velocity	None
	$a_{PF_active}^A \in \mathbb{R}$	Target altitude rate	None
	$\dot{\theta}_{PF_active} \in \mathbb{R}$	Target turn rate	None

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

LPN “CDTI”

The CDTI informs the crew visually about an RA.

Incoming arcs within same agent

- None

Outgoing arcs within same agent

- Enabling arc from place “Working” to LPN “Pilot Flying” transition G1

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

Table A-46: Places of LPN CDTI of agent Cockpit i.

Places	Colour type	Explanation	Colour function
Working	$t_{CDTI}^{Working} \in \mathbb{R}$	Delay until next firing	$\dot{t}_{CDTI}^{Working} = -1$
Not working	$t_{CDTI}^{Not_working} \in \mathbb{R}$	Delay until next firing	$\dot{t}_{CDTI}^{Not_working} = -1$

Initial markings

Initially there is one token in place “Working”. Its colour is specified according to the system reliability, p_{CDTI} . The system reliability for the CDTI system is assumed to be 10^{-7} per flight hour.

Table A-47: Initial markings of places of LPN CDTI of agent Cockpit i.

Places	Colour type	Explanation
Working	If a sample from a drawing of a uniform distribution of range [0,1] is larger than p_{CDTI} multiplied by the length of the simulation, then $t_{CDTI}^{Working} = 999h$. Else, $t_{CDTI}^{Working}$ is determined by a sample of uniform distribution of the range zero to length of simulation.	Delay time until CDTI stops working.

Transitions

Table A-48: Transitions of LPN CDTI of agent Cockpit i.

ID	Transition	Condition
G1	<i>Not working</i> $\rightarrow$ <i>Working</i>	$t_{CDTI}^{Not_working} \leq 0$
G2	<i>Working</i> $\rightarrow$ <i>Not working</i>	$t_{CDTI}^{Working} \leq 0$

It is assumed that once the system stops working, it remains until the end of the simulation. This is ensured due to the very large delay time of 999 hours.

Table A-49: Firing functions of transitions of LPN CDTI of agent Cockpit i.

ID	Firing function
G1	One token to <i>Working</i> with the following colour: $t_{CDTI}^{Working} = TBD$
G2	One token to <i>Not working</i> with the following colour: $t_{CDTI}^{Not_working} = 999h$

LPN “Aural Annunciation”

The Aural Annunciation system informs the crew aurally about an RA.

Incoming arcs within same agent

- None

Outgoing arcs within same agent

- Enabling arc from place “Working” to LPN “Pilot Flying” transitions G1

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

Table A-50: Places of LPN Aural Annunciation of agent Cockpit i.

Places	Colour	Explanation	Colour function
Working	$t_{Aural}^{Working} \in \mathbb{R}$	Delay until next firing	$\dot{t}_{Aural}^{Working} = -1$
Not working	$t_{Aural}^{Not_working} \in \mathbb{R}$	Delay until next firing	$\dot{t}_{Aural}^{Not_working} = -1$

Initial markings

Initially there is one token in place “Working”. Its colour is specified according to the system reliability, p_{Aural} . The system reliability for the Aural Annunciation system is assumed to be 10^{-7} per flight hour.

Table A-51: Initial markings of places of LPN Aural Annunciation of agent Cockpit i.

Places	Colour	Explanation
Working	If a sample from a drawing of a uniform distribution of range [0,1] is larger than p_{Aural} multiplied by the length of the simulation, then $t_{Aural}^{Working} = 999h$. Else, $t_{Aural}^{Working}$ is determined by a sample of uniform distribution of the range zero to length of simulation.	Delay until Aural Annunciation system stops working

Transitions

Table A-52: Transitions of LPN Aural Annunciation of agent Cockpit i.

ID	Transition	Condition
G1	<i>Not working</i> → <i>Working</i>	$t_{Aural}^{Not_working} \leq 0$
G2	<i>Working</i> → <i>Not working</i>	$t_{Aural}^{Working} \leq 0$

It is assumed that once the system stops working, it remains until the end of the simulation. This is ensured due to the very large delay time of 999 hours.

Table A-53: Firing functions of transitions of LPN Aural Annunciation of agent Cockpit i.

ID	Firing function
G1	One token to <i>Working</i> with the following colour: $t_{Aural}^{Working} = TBD$
G2	One token to <i>Not working</i> with the following colour: $t_{Aural}^{Not_working} = 999h$

A.4 Agent “Aircraft i”

This agent has one LPN “State”. Next the LPN is described.

LPN “State”

The agent computes the evolution of each aircraft ‘i’ according to parameters such as, position, velocity, and target velocity.

Incoming arcs within same agent

- None

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- Incoming arc from place “P” of IPN “Pilot Flying” of agent “Cockpit i” to transition I

Outgoing arcs to other agent

- Enabling arcs from place “P” to LPN “State Estimation” of agent “TCAS i” transitions G1 and G2
- Enabling arc from place “P” to LPN “Mode S Reply” of agent “Communication i” transition I3
- Enabling arc from place “P” to LPN “Measurements” of agent “Communication” transition I

Places

Place “P” has colours according to the aircraft states described in Section 4. A change in altitude velocity and heading are modelled through a turn rate input from the pilot, or a minimum and maximum altitude velocity, $v_{t,A,\min}^i$ and $v_{t,A,\max}^i$ (not described in Section 4), and altitude acceleration setting from the pilot, i.e. see transition I. The change of position and velocity is modelled through transition G after a predefined time step, Δ_{A/C_state} .

Table A-54: Place of LPN State of agent Aircraft i.

Places	Colour type	Explanation	Colour function
P	$s_{t,x}^i \in \mathbb{R}$	x position	None
	$s_{t,y}^i \in \mathbb{R}$	y position	None
	$s_{t,z}^i \in \mathbb{R}$	Altitude	None
	$\ \mathbf{v}_{t,H}^i\ \in \{0, \dots, \infty\}$	Ground speed	None
	$v_{t,A}^i \in \mathbb{R}$	Altitude velocity	None
	$\theta_t^i \in \mathbb{R}$	Heading	None
	$v_{t,A,\min}^i \in \mathbb{R}$	Target minimum altitude velocity	None
	$v_{t,A,\max}^i \in \mathbb{R}$	Target maximum altitude velocity	None

	$a_{t,A}^i \in \mathbb{R}$	Altitude rate	None
	$\dot{\theta}_t^i \in \mathbb{R}$	Turn rate	None
	$\mathcal{E}_{r,bias}^i \in \mathbb{R}$	Bias of the slant range measurements	None
	$\mathcal{E}_{A,bias}^i \in \mathbb{R}$	Bias of the altitude measurements	None
	$\Delta \in \mathbb{R}$	Length of discrete time step	None
	$t_{State}^G \in \mathbb{R}$	Delay until next update	$\dot{t}_{A/C_state}^G = -1$

Initial markings

One token is present in place “P”. The discrete time step of the simulation is set to 0.1s ($\Delta = 0.1s$) and the initial delay until the next time step is zero ($t^G = 0$). The initial parameters of aircraft ‘i’ depend on the to be modelled scenario, e.g. see scenarios in Section 16.

Transitions

Table A-55: Transitions of LPN State of agent Aircraft i.

ID	Transition	Condition
G	$P \rightarrow P$	$t_{A/C_state}^G \leq 0$
I	$P \wedge P\{\text{Pilot Input[Cockpit i]}\} \rightarrow P$	N/A

The horizontal position components, $s_{t,x}^i$ and $s_{t,y}^i$, are updated through the position change resulting from the ground velocity, $\mathbf{v}_{t,H}^i$, and heading, θ_t^i . The altitude, $s_{t,z}^i$, is updated using the current altitude velocity, $v_{t,A}^i$.

The heading, θ_t^i , is updated using the turn rate, $\dot{\theta}_t^i$. In case the new heading is out of the magnetic range (0-360 degrees) then the heading is corrected accordingly.

In case the vertical velocity, $v_{t,A}^i$, is not between the minimum target altitude rate, $v_{t,A,\min}^i$, and the maximum target altitude rate, $v_{t,A,\max}^i$, then the velocity is altered based on the current altitude acceleration setting, $a_{t,A}^i$.

Table A-56: Firing functions of transitions of LPN State of agent Aircraft i.

ID	Firing function
G	One token to P. The colours $s_{t,x}^i, s_{t,y}^i, s_{t,z}^i, v_{t,A}^i, \theta_t^i$, change according to Equations (157)-(161). The colour determining the guard transition delay is reset to $t_{State}^G = \Delta$ The remaining colours remain unchanged.
I	One token to P and the colours change according to: $v_{t,A,\min}^i = v_{PF_active}^{A,\min} \{P\{\text{Pilot Input}\{\text{Cockpit i}\}\}\}$ $v_{t,A,\max}^i = v_{PF_active}^{A,\max} \{P\{\text{Pilot Input}\{\text{Cockpit i}\}\}\}$ $a_{t,A}^i = a_{PF_active}^A \{P\{\text{Pilot Input}\{\text{Cockpit i}\}\}\}$ $\dot{\theta}_t^i = \dot{\theta}_{PF_active} \{P\{\text{Pilot Input}\{\text{Cockpit i}\}\}\}$ And the remaining colours remain unchanged.

B ACAS UAV

The ACAS X_U concept of (FAA, 2015b) describes that UAS should be capable of vertical and horizontal avoidance manoeuvring. Additionally the system should be capable of cooperating with aircraft equipped with different ACAS, i.e. ACAS X_A and TCAS II. Last, ACAS X_U equipped aircraft may have multiple surveillance systems, but ADS-B is a mandatory.

Therefore, a DAA system was developed which behaves similar to TCAS II for vertical collision avoidance manoeuvres but is also capable of horizontal avoidance manoeuvring. The decision which type of manoeuvring is selected is based on the predicted vertical and horizontal miss distances.

It should be noted, that although ACAS X the RA selection is based on look up tables which are computed through dynamic programming (Kochenderfer, 2010, 2011), in the model no look up tables are used, but the RA decision are determined by rule based calculations.

B.1 State and velocity variables

The state and velocity variables are the same as identified for the TCAS II Ver.7.1 model as defined in Equations (1)-(14).

B.2 State and altitude measurements by aircraft 'i'

According to (FAA, 2015b) ACAS X_U may process multiple measurement inputs, where ADS-B is a mandatory surveillance technology. In this model only ADS-B surveillance is considered. The horizontal position is reported in quantization which results in a precision of at least 1.1m¹. Additionally, a measurement error applies. According to (FAA, 2010, 2015a) the FAA requires an ADS-B position accuracy of 0.05NM with a 95% confidence interval. This leads to an error bound of 0.075NM with 99.7% confidence. As the error induced by quantization of reports is very small, in position measurements only the measurement error is considered. $y_{t,H,ADSB}^i$ is the measured horizontal position of aircraft 'i' at time 't'. $\varepsilon_{t,x,ADSB}^i$ and $\varepsilon_{t,y,ADSB}^i$ are the measurement errors in x and y direction respectively.

$$y_{t,H,ADSB}^i \triangleq s_{t,H}^i + \begin{bmatrix} \varepsilon_{t,x,ADSB}^i \\ \varepsilon_{t,y,ADSB}^i \end{bmatrix} \quad (162)$$

$y_{t,H,v,ADSB}^i$ is the measured ground speed of aircraft 'i' at time 't', where $\varepsilon_{t,v,H}^i$ is the measurement error. This model assumes perfect velocity measurements ($\varepsilon_{t,v,H}^i = 0$).

$$y_{t,H,v,ADSB}^i \triangleq \|\mathbf{v}_{t,H}^i\| + \varepsilon_{t,v,H}^i \quad (163)$$

¹ Source: <http://adsb-decode-guide.readthedocs.io/en/latest/velocity.html>; last accessed 20.02.2017.

The measured heading of aircraft 'i' in reference to true North, $y_{t,\theta}^i$, is a function of the heading of aircraft 'i' with added measurement error $\varepsilon_{t,\theta}^i$.

$$y_{t,\theta}^i \triangleq \theta_t^i + \varepsilon_{t,\theta}^i \quad (164)$$

The measurement of the altitude of aircraft 'i', $y_{t,A,ADSB}^i$, is reported in each ADS-B message and is similar to $y_{t,A}^i$ as previously defined in the TCAS II Ver.7.1 model in Equation (20).

$$y_{t,A,ADSB}^i = 25\text{ft} \left\lceil \frac{y_{t,A}^i}{25\text{ft}} \right\rceil \quad (165)$$

$\lceil \cdot \rceil$ is defined as rounding to the nearest integer. The altitude velocity reported by ADS-B is in 64ft/min increments¹. Hence, the vertical speed, $y_{t,A,v,ADSB}^i$, of aircraft 'i' at time 't' is rounded to the next 64ft/min increment. $\varepsilon_{t,v,A}^i$ is the altitude velocity measurement error. This model assumes perfect velocity measurements ($\varepsilon_{t,v,A}^i = 0$).

$$y_{t,A,v,ADSB}^i \triangleq 64\text{ft/min} \left\lceil \frac{v_{t,A}^i + \varepsilon_{t,v,A}^i}{64\text{ft/min}} \right\rceil \quad (166)$$

B.3 Received coordination message and aircraft ID

The coordination message received by aircraft 'i' from aircraft 'k', $m_{t,received}^{ik}$, has been previously defined in Equation (25). As ACAS Xu is supposed to cooperate with TCAS II, the coordination message is assumed to be of the same structure and content.

The aircraft ID, χ_{ModeS}^i , has been previously defined in Equation (27).

Similar as in (29), if i=0 and k=1, then it is shortly written:

$$m_{t,received} \triangleq m_{t,received}^{01} \quad (167)$$

In the following it is always considered from aircraft '0' perspective.

B.4 State estimates

The aircraft estimates the relative position and relative velocity in both the horizontal and vertical plane.

¹ Source: <http://adsb-decode-guide.readthedocs.io/en/latest/content/airborne-velocity.html>; last accessed 16.03.2017

Horizontal range and velocity estimates

Relative horizontal range estimate, $\hat{\mathbf{s}}_{t,H,ADSB}^{ik}$, is given in vector notation in Equation (168). If no relative horizontal range measurement could be established, then the estimate is computed using the relative range and velocity estimates at time 't-T_{ADSB}' in Equation (169), where $\hat{\mathbf{s}}_{t|t-T_{ADSB},H,ADSB}^{ik}$ is the previous relative horizontal position estimate, $\hat{\mathbf{v}}_{t-T_{ADSB},H,ADSB}^{ik}$ is the previous relative horizontal velocity estimate and T_{ADSB} is the length of one ADS-B reporting cycle.

$$\hat{\mathbf{s}}_{t,H,ADSB}^{ik} = \begin{cases} \mathbf{y}_{t,H,ADSB}^i - \mathbf{y}_{t,H,ADSB}^k & , \text{ if } \exists \text{ measurement } \mathbf{y}_{t,H,ADSB}^k \\ \hat{\mathbf{s}}_{t|t-T_{ADSB},H,ADSB}^{ik} & , \text{ if no measurement} \end{cases} \quad (168)$$

$$\hat{\mathbf{s}}_{t|t-T_{ADSB},H,ADSB}^{ik} = \hat{\mathbf{s}}_{t-T_{ADSB},H,ADSB}^{ik} + \hat{\mathbf{v}}_{t-T_{ADSB},H,ADSB}^{ik} T_{ADSB} \quad (169)$$

The estimate of the relative horizontal velocity between aircraft 'i' and aircraft 'k', $\hat{\mathbf{v}}_{t,H,ADSB}^{ik}$, is given in vector notation in Equation (170). If not both a horizontal velocity and a heading measurement of the intruder aircraft was received, then the estimate is equal to the previous horizontal velocity estimate.

$$\hat{\mathbf{v}}_{t,H,ADSB}^{ik} = \begin{cases} \mathbf{y}_{t,H,v,ADSB}^k \begin{pmatrix} \cos(y_{t,\theta}^k) \\ \sin(y_{t,\theta}^k) \end{pmatrix} - \mathbf{y}_{t,H,v}^i \begin{pmatrix} \cos(y_{t,\theta}^i) \\ \sin(y_{t,\theta}^i) \end{pmatrix} & , \text{ if } \exists \text{ measurements } y_{t,H,v,ADSB}^k \wedge y_{t,\theta}^k \\ \hat{\mathbf{v}}_{t-T_{ADSB},H,ADSB}^{ik} & , \text{ else} \end{cases} \quad (170)$$

Similar as in (13)-(14), if i=0 and k=1, then it is shortly written:

$$\hat{\mathbf{s}}_{t,H,ADSB} \triangleq \hat{\mathbf{s}}_{t,H,ADSB}^{01} \quad (171)$$

$$\hat{\mathbf{v}}_{t,H,ADSB} \triangleq \hat{\mathbf{v}}_{t,H,ADSB}^{01} \quad (172)$$

Altitude and altitude velocity estimates

Relative altitude estimate, $\hat{s}_{t,A}^{ik}$, between aircraft 'i' and aircraft 'k' is given in Equation (173). If no altitude measurement of the intruder aircraft was received, then the estimate is computed, as given in Equation (174), using the relative altitude and relative altitude velocity estimates at time 't-T_{ADSB}', where $\hat{s}_{t|t-T_{ADSB},A}^{ik}$ is the previous relative altitude estimate, $\hat{v}_{t-T_{ADSB},A}^{ik}$ is the previous relative vertical velocity estimate and T_{ADSB} is the length of one ADS-B reporting cycle.

$$\hat{s}_{t,A}^{ik} = \begin{cases} y_{t,A,ADSB}^k - y_{t,A}^i & , \text{ if } \exists \text{ measurement } y_{t,A,ADSB}^k \\ \hat{s}_{t|t-T_{ADSB},A}^{ik} & , \text{ if no measurement} \end{cases} \quad (173)$$

$$\hat{S}_{t|t-T_{ADSB},A}^{ik} = \hat{S}_{t-T_{ADSB},A}^{ik} + \hat{v}_{t-T_{ADSB},A}^{ik} T_{ADSB} \quad (174)$$

Next, the relative altitude velocity estimate, $\hat{v}_{t,A}^{ik}$, is computed as described in Equation (175). If no altitude velocity measurement of the intruder aircraft was received, then the estimate is equal to the previous estimate.

$$\hat{v}_{t,A}^{ik} = \begin{cases} y_{t,A,v,ADSB}^k - y_{t,A,v,ADSB}^i & , \text{ if } \exists \text{ measurement } y_{t,A,v,ADSB}^k \\ \hat{v}_{t-T_{ADSB},A}^{ik} & , \text{ if no measurement} \end{cases} \quad (175)$$

Similar as in (13)-(14), if $i=0$ and $k=1$, then it is shortly written:

$$\hat{S}_{t,A} \triangleq \hat{S}_{t,A}^{01} \quad (176)$$

$$\hat{v}_{t,A} \triangleq \hat{v}_{t,A}^{01} \quad (177)$$

Horizontal miss distance filter

The horizontal miss distance filter, modelled as Boolean $B_{t,HMDF,UAV}$ in Equations (178)-(179), determines whether the predicted horizontal miss distance at horizontal CPA, $d_{t,HMD,ADSB}$, is within threshold H_m . If the predicted horizontal miss distance is greater than the by TCAS II Ver.7.1 defined required horizontal miss distance, as given in [Table 7-3](#), then the filter is activated.

$$B_{t,HMDF,UAV} = TRUE \quad , \text{ iff } \{ d_{t,HMD,ADSB} > H_m \} \quad (178)$$

$d_{t,HMD,ADSB}$ consists of the current relative horizontal distance, the current relative horizontal velocity and the time until horizontal CPA.

$$d_{t,HMD,ADSB} = \left\| \hat{S}_{t,H,ADSB} - \hat{v}_{t,H,ADSB} \frac{\hat{S}_{t,H,ADSB} \cdot \hat{v}_{t,H,ADSB}}{\hat{v}_{t,H,ADSB}^2} \right\| \quad (179)$$

B.5 Threat detection

In the threat detection process it is determined whether another aircraft is a threat towards its own aircraft. To do so, a range and an altitude test are carried out. Furthermore, information from a received coordination message, if received, and a horizontal miss distance filter are considered as well.

Range test

The range test (RT), modelled as Boolean $B_{t,RT,UAV}$ in Equations (180)-(181), determines whether the intruder is a range threat. The test is positive if the relative horizontal distance to the intruder is within distance d_{mod} or if the modified time until horizontal CPA is within threshold $\tau_{t,RA}$. Distance

and time Δ_{RA} are taken from the TCAS II Ver.7.1 specification as defined in [Table 8-1](#). Also the time until CPA has been modified in a similar manner as performed in TCAS II Ver.7.1 (ICAO, 2006, p. 3-59).

$$B_{t,RT,UAV} = TRUE \quad , \text{iff} \begin{cases} \left[\left\| \hat{\mathbf{s}}_{t,H,ADSB} \right\| < d_{\text{mod}} \right] \vee \\ \left[0 \leq \tau_{t,H,\text{mod},ADSB} \leq \Delta_{RA} \right] \end{cases} \quad (180)$$

$$\tau_{t,H,\text{mod},ADSB} = \begin{cases} \frac{d_{t,\text{mod}}^2 - \hat{\mathbf{s}}_{t,H,ADSB}^2}{\hat{\mathbf{s}}_{t,H,ADSB} \cdot \hat{\mathbf{v}}_{t,H,ADSB}} & , \text{if } \left(-\hat{\mathbf{s}}_{t,H,ADSB} \cdot \hat{\mathbf{v}}_{t,H,ADSB} \leq 0 \right) \\ -1 & , \text{otherwise} \end{cases} \quad (181)$$

Altitude test

The altitude test (AT), modelled as Boolean $B_{t,AT,UAV}$ in Equations (182)-(183), determines whether the intruder is a vertical threat. The test is positive if the relative vertical distance to the intruder is within distance z_{thr} or if the modified time until horizontal CPA is within threshold Δ_{RA} . The distance and time thresholds are taken from the TCAS II Ver.7.1 specification as defined in [Table 8-2](#).

$$B_{t,AT,UAV} = TRUE \quad , \text{iff} \begin{cases} \left[\left| \hat{s}_{t,A} \right| < z_{thr} \right] \vee \\ \left[0 \leq \tau_{t,A,ADSB} \leq \Delta_{RA} \right] \end{cases} \quad (182)$$

$$\tau_{t,A,ADSB} = \begin{cases} -\frac{\hat{s}_{t,A}}{\hat{v}_{t,A}} & , \text{if } \left(\hat{v}_{t,A} \neq 0 \right) \\ -1 & , \text{otherwise} \end{cases} \quad (183)$$

Threat detection test

The threat detection logic defines a threat if both the range test and the altitude test are passed and if the horizontal miss distance filter is not activated. Furthermore, if a coordination message from another aircraft has been received, than the other aircraft is declared a threat as well. The test is defined as Boolean $B_{t,RA,UAV}$:

$$B_{t,RA,UAV} = TRUE, \text{iff} \begin{cases} \left[B_{t,RT,UAV} \wedge B_{t,AT,UAV} \wedge \neg B_{t,HMDF,UAV} \right] \vee \\ \left[m_{t,\text{received}} \neq 0 \right] \end{cases} \quad (184)$$

B.6 Avoidance logic selection

After a threat has been declared, it determines which avoidance logic is to be followed. A decision is made once and not changed afterwards. For horizontal avoidance manoeuvres the predicted horizontal miss distance is increasing exponentially as a function of own ground speed and turning

rate. Hence, horizontal avoidance has a minimal effect in short terms and should therefore be carried out from the beginning.

The avoidance logic selection is modelled as variable $L_{t,UAV}$ as defined in Equations (185)-(188). $\hat{S}_{CPA,A|t}^+$ is the predicted vertical miss distance at CPA if the own aircraft would be flying with maximum vertical speed. $\hat{S}_{CPA,A|t}^-$ is the predicted vertical miss distance at CPA if the own aircraft would be flying with minimum vertical speed. $\tau_{t,CPA,ADSB}$ is the time until CPA and $\tau_{t,H,lim}^i$ is a threshold value determining the minimum required time until CPA which allows for an effective horizontal avoidance manoeuvre.

$$L_{t,UAV} \triangleq \begin{cases} 0, \text{ no logic selected} \\ 1, \text{ vertical avoidance logic} \\ 2, \text{ horizontal avoidance logic} \end{cases} \in \{0,1,2\} \quad (185)$$

$$= \begin{cases} 0, \text{ initial selection} \\ 1, \text{ if } \left[\left(\hat{S}_{CPA,A|t}^+ \geq a_{lim} \right) \vee \left(\hat{S}_{CPA,A|t}^- \geq a_{lim} \right) \vee \left(\tau_{t,CPA,ADSB} < \tau_{t,H,lim}^0 \right) \right] \\ 2, \text{ if } \left[\left(\hat{S}_{CPA,A|t}^+ < a_{lim} \right) \wedge \left(\hat{S}_{CPA,A|t}^- < a_{lim} \right) \wedge \left(\tau_{t,CPA,ADSB} \geq \tau_{t,H,lim}^0 \right) \right] \end{cases} \quad (186)$$

$$\hat{S}_{CPA,A|t}^+ = \left| \hat{S}_{t,A} + \tau_{t,CPA,ADSB} \left(\hat{v}_{t,A}^1 - v_{A,max}^0 \right) \right| \quad (186)$$

$$\hat{S}_{CPA,A|t}^- = \left| \hat{S}_{t,A} + \tau_{t,CPA,ADSB} \left(\hat{v}_{t,A}^1 - v_{A,min}^0 \right) \right| \quad (187)$$

$$\tau_{t,CPA,ADSB} = \min \left(\tau_{RA}, \max \left(\tau_{t,V,ADSB}, \tau_{t,H,mod,ADSB}, 0 \right) \right) \quad (188)$$

The decision for a vertical avoidance manoeuvre is made, if the prediction of a vertical avoidance manoeuvre would result in sufficient separation, or if the time until CPA is so short that a vertical avoidance manoeuvre would achieve a greater separation compared to a horizontal turn. This threshold is computed using the following algorithm in Equation (190). The time threshold value, $\tau_{t,H,lim}^0$, is increased as long as the predicted horizontal miss distance at time 't+ $\tau_{t,H,lim}^0$ ' is smaller than the predicted vertical miss distance at time 't+ $\tau_{t,H,lim}^0$ ', and $\tau_{t,H,lim}^0$ is smaller than RA threshold time, Δ_{RA} . To account for measurement errors the 99.7% error measurement error bounds are deducted from the predicted vertical and horizontal miss distances.

The predicted vertical miss distance at time 't+ Δ_{RA} ' is given in Equation (189).

$$\hat{S}_{CPA,A|t} = \hat{S}_{t,A} + \tau_{t,CPA,ADSB} \hat{v}_{t,A} \quad (189)$$

$$\begin{aligned}
& \tau_{t,H,\text{lim}}^0 = 1; \\
& \text{while } \left\{ \begin{aligned} & \left(\left| \frac{\hat{\mathbf{v}}_{t,H,ADSB}^0}{\dot{\theta}_{\text{max}}^0} (1 - \cos(\dot{\theta}_{\text{max}}^0 \tau_{t,H,\text{lim}}^0)) \right| + d_{t,HMD,ADSB} - 3\sigma_H^0 \right) < \\ & \max \left(\left| \hat{S}_{CPA,A|t} + (\hat{v}_{t,A}^1 - v_{A,\text{max}}^0) \tau_{t,H,\text{lim}}^0 \right|, \left| \hat{S}_{CPA,A|t} + (\hat{v}_{t,A}^1 - v_{A,\text{min}}^0) \tau_{t,H,\text{lim}}^0 \right| \right) - 3\sigma_A^0 \\ & \wedge (\tau_{t,H,\text{lim}}^0 < \Delta_{RA}) \end{aligned} \right. \\
& \tau_{t,H,\text{lim}}^0 = \tau_{t,H,\text{lim}}^0 + 1 \\
& \text{end}
\end{aligned} \tag{190}$$

B.7 Vertical avoidance logic

If $L_{t,UAV} = 1$, then vertical avoidance logic is applied. First the sense, whether the own aircraft should fly upwards or downwards. Second, the strength, target rate of climb or descent, is determined. Third, the evolution of the situation is monitored.

Sense Selection

The sense selection process determines whether the UAV should fly upwards or downwards. The sense selection process is typically only performed once after a threat has been declared. This is defined in Equation (191). Boolean $B_{t,D,UAV}$ is set positive if a sense, $D_{t,UAV}$, has been selected. Also when the sense has been selected, a coordination message, $m_{t,\text{sent}}$, is sent accordingly.

$$\begin{aligned}
& \text{if } [\neg B_{t,D,UAV} \wedge B_{t,RA,UAV}] \\
& \text{then } [\text{compute } D_{t,UAV} \wedge \text{compute } m_{t,\text{sent}} \wedge B_{t,D,UAV} = \text{TRUE}]
\end{aligned} \tag{191}$$

Sense $D_{t,UAV}$ may either be equal to 1 or equal to -1. $D_{t,UAV} = 1$ is defined as an upward sense and $D_{t,UAV} = -1$ is defined as a downward sense. The rules are similar to the specification of TCAS II Ver.7.1 (FAA, 2011). If a coordination message has been received, $m_{t,\text{received}} \neq 0$, then the sense is selected as the opposite of the intruders sense. Else the sense is selected based on the encounter geometry. The preferred solution should avoid altitude crossings. However, if the non-altitude crossing solution does not yield a sufficient vertical separation distance, then the sense which would yield the greater vertical separation distance is selected. This is modelled in Equation (192) by comparing the current relative altitude against the predicted relative altitudes. In case an altitude crossing yields the larger miss distance, then it is checked whether the non-altitude crossing prediction also leads sufficient miss distance. This is sufficient to determine whether the aircraft would cross altitudes, as opposed to TCAS II Ver.7.1 no negative sense is allowed.

$$D_{t,UAV} = \begin{cases} -m_{t,received} & , \text{if } \left\{ \left[|m_{t,received}| > 0 \right] \right. \\ \\ 1 & , \text{if } \left\{ \left[\left(m_{t,received} = 0 \right) \wedge \left(\hat{s}_{t,A} \leq 0 \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| > \left| \hat{s}_{CPA,A|t}^- \right| \right) \right] \vee \right. \\ & \left[\left(m_{t,received} = 0 \right) \wedge \left(\hat{s}_{t,A} \leq 0 \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| \leq \left| \hat{s}_{CPA,A|t}^- \right| \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| \geq a_{lim} \right) \right] \vee \\ & \left[\left(m_{t,received} = 0 \right) \wedge \left(\hat{s}_{t,A} > 0 \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| > \left| \hat{s}_{CPA,A|t}^- \right| \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^- \right| < a_{lim} \right) \right] \vee \\ \\ -1 & , \text{if } \left\{ \left[\left(m_{t,received} = 0 \right) \wedge \left(\hat{s}_{t,A} > 0 \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| \leq \left| \hat{s}_{CPA,A|t}^- \right| \right) \right] \vee \right. \\ & \left[\left(m_{t,received} = 0 \right) \wedge \left(\hat{s}_{t,A} \leq 0 \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| \leq \left| \hat{s}_{CPA,A|t}^- \right| \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| < a_{lim} \right) \right] \vee \\ & \left[\left(m_{t,received} = 0 \right) \wedge \left(\hat{s}_{t,A} > 0 \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^+ \right| > \left| \hat{s}_{CPA,A|t}^- \right| \right) \wedge \left(\left| \hat{s}_{CPA,A|t}^- \right| \geq a_{lim} \right) \right] \vee \end{cases} \quad (192)$$

As mentioned above, just when the sense has been selected, a coordination message is sent to the intruder. The structure of the own coordination message is identical to the coordination message from the intruder and tells the intruder where to pass. As defined in Equation (193), the coordination message is equal to selected sense:

$$m_{t,sent} = D_{t,UAV} \quad (193)$$

As it could happen that both aircraft select the same sense at the exact same time, it is also checked whether the sent and received coordination message are conflicting. As given in Equation (194), if this is the case and the own aircraft has the higher ModeS address, then $B_{t,D,UAV}$ is set negative, such that the sense selection process in Equation (192) may be initiated again. This is done to comply with TCAS II Ver.7.1 (FAA, 2011).

$$\begin{aligned} & \text{if } \left[\left(m_{t,sent} = m_{t,received} \right) \wedge \left(\chi_{ModeS}^0 > \chi_{ModeS}^1 \right) \right] \\ & \text{then } \left[\neg B_{t,D,UAV} \right] \end{aligned} \quad (194)$$

Strength selection

After the sense has been selected, the strength, target rate of climb/descend, is selected:

$$\left[v_{t,A,RA,min}^0, v_{t,A,RA,max}^0 \right] = \begin{cases} \left[v_{A,max}^0, v_{A,max}^0 \right] & , \text{if } \{ D_{t,UAV} = 1 \\ \left[v_{A,min}^0, v_{A,min}^0 \right] & , \text{if } \{ D_{t,UAV} = -1 \end{cases} \quad (195)$$

The target rate of climb/descend is assumed to be the maximum climb/descend rate of own aircraft, because passenger and pilot comfort do not have to be taken into account.

Threat evolution monitoring and RA adjustment

After an RA has been issued, the evolution of the situation is monitored. The threat evolution monitoring detects whether situation is being resolved with the current RA. If not, then an RA adjustment is carried out, in which a new sense and RA is computed. Furthermore, it is also tested whether the threat has been passed. In such a case the own aircraft may be issued to level off. Also if the time until CPA has been passed and the threat has been resolved, then a clear of conflict message may be issued.

For the latter a timer, $t_{t,RA,UAV}$, is being set equal to the time until CPA, $\tau_{t,CPA,ADSB}$, such that the system knows when the CPA should have been passed:

$$t_{t,RA,UAV} = \tau_{t,CPA,ADSB} \quad (196)$$

For a time step $\Delta > 0$, $t_{t,RA,UAV}$ evolves as:

$$t_{t,RA,UAV} = t_{t-\Delta,RA,UAV} - \Delta \quad (197)$$

Threat evolution monitoring

Every second, but only after 5 seconds an RA has been issued, it is checked whether the intruder is not following its coordinated sense, or if current relative velocities do not yield a sufficient separation distance, which is captured in Boolean $B_{t,RA,EVO,UAV}$ as given in Equation (198). The delay is meant to give the intruder pilot sufficient time to react its potential own RA.

$$B_{t,RA,EVO,UAV} = TRUE, \text{ iff } \left\{ \left[\left(\frac{\hat{S}_{CPA,A|t}}{-D_{t,UAV}} \leq 0 \right) \vee \left(\left| \hat{S}_{CPA,A|t} \right| < a_{lim} \right) \right] \right\} \quad (198)$$

RA adjustment

If the threat evolution test positive, $B_{t,RA,EVO,UAV} = TRUE$, then the RA is adjusted. If the intruder is not following its coordinated sense, then the own sense selection is reversed. Else, the optimal sense is selected as given in Equation (199).

$$D_{t,UAV} = \begin{cases} 1 & , \text{if } \left\{ \left(\left| \hat{S}_{CPA,A|t}^+ \right| > \left| \hat{S}_{CPA,A|t}^- \right| \right) \wedge \left(\frac{\hat{S}_{CPA,A|t}}{-D_{t,UAV}} > 0 \right) \right\} \\ -1 & , \text{if } \left\{ \left(\left| \hat{S}_{CPA,A|t}^+ \right| \leq \left| \hat{S}_{CPA,A|t}^- \right| \right) \wedge \left(\frac{\hat{S}_{CPA,A|t}}{-D_{t,UAV}} > 0 \right) \right\} \\ -D_{t,UAV} & , \text{if } \left\{ \left(\frac{\hat{S}_{CPA,A|t}}{-D_{t,UAV}} \leq 0 \right) \right\} \end{cases} \quad (199)$$

The strength of the adjusted RA is then either maximum climb or maximum descend depended on the selected sense. The new altitude velocity setting is determined according to previously used Equation (195).

RA weakening and clear of conflict message

Furthermore, it is checked whether the other aircraft is not a threat anymore. If the range and altitude test are false and estimated time until CPA estimated at initial RA has elapsed, or if the horizontal miss distance filter is activated, then a clear of conflict message is issued and the own aircraft is set to level-off. Else, if the predicted vertical miss distance at CPA assuming own aircraft is in levelled off flight is larger than the required vertical miss distance would not result in a sense reversal, then the own aircraft is set to level-off. The algorithm is given in Equation (200) and the predicted vertical miss distance if own aircraft would fly level is given in Equation (201).

$$\begin{aligned}
 &\text{if } \left[\neg B_{t,RT,UAV} \wedge \neg B_{t,AT,UAV} \wedge (t_{t,RA,UAV} \leq 0) \right] \vee B_{t,HMDF,UAV} \\
 &\quad \text{then } \left\{ \begin{array}{l} \text{clear of conflict message} \\ \text{reset system} \\ \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = [0,0] \end{array} \right. \\
 &\quad \text{else if } \left[\left(\left| \hat{S}_{CPA,A|t}^{level} \right| \geq a_{\lim} \right) \wedge \left(\frac{\hat{S}_{CPA,A|t}^{level}}{-D_{t,UAV}} > 0 \right) \right] \\
 &\quad \quad \text{then } \left\{ \left[v_{t,A,RA,\min}^0, v_{t,A,RA,\max}^0 \right] = [0,0] \right. \\
 &\quad \text{else repeat the monitoring cycle}
 \end{aligned} \tag{200}$$

$$\hat{S}_{CPA,A|t}^{level} = \left| \hat{S}_{t,A} + \tau_{t,CPA,ADSB} \hat{V}_{t,A}^1 \right| \tag{201}$$

B.8 Horizontal avoidance logic

If $L_{t,UAV} = 2$, then horizontal avoidance logic is applied. This is done by first assessing the geometrical situation. To do so, first the iteration variables i and k are set to zero:

$$i = 0 \tag{202}$$

$$k = 0 \tag{203}$$

Then a left and a right turn are simulated. From the results a horizontal RA is selected and a coordination message is sent to the intruder.

Last but not least, evolution of the situation is monitored resulting in a clear of conflict message once the predicted horizontal miss distance is sufficiently large.

Assessing geometrical situation

A right turn indicated by iteration variable i . The horizontal miss distance, $d_{t,HMD,ADSB|i}$, dependent on own aircraft turns right is defined as follows:

$$d_{t,HMD,ADSB|i} = \left\| \hat{\mathbf{s}}_{t+i,H,ADSB|t,i} - \hat{\mathbf{v}}_{t+i,H,ADSB|t,i} \frac{\hat{\mathbf{s}}_{t+i,H,ADSB|t,i} \cdot \hat{\mathbf{v}}_{t+i,H,ADSB|t,i}}{\hat{\mathbf{v}}_{t+i,H,ADSB|t,i}^2} \right\| \quad (204)$$

Where $\hat{\mathbf{s}}_{t+i,H,ADSB|t,i}$ is the relative horizontal position vector between own aircraft and intruder at time 't+i' given own aircraft turns right, $\hat{\mathbf{v}}_{t+i-\Delta,H,ADSB|t,i}$ is the relative horizontal velocity vector between own aircraft and intruder at time 't+i' given own aircraft turns right, Δ_i is one time step in the iteration where own aircraft is predicted to turn right, and $\dot{\theta}_{\max}^0$ is the maximum turn rate of own aircraft.

$$\hat{\mathbf{s}}_{t+i,H,ADSB|t,i} = \hat{\mathbf{s}}_{t+i-\Delta,H,ADSB|t,i} + \hat{\mathbf{v}}_{t+i-\Delta,H,ADSB|t,i} \Delta_i \quad (205)$$

$$\hat{\mathbf{v}}_{t+i-\Delta,H,ADSB|t,i} = \hat{\mathbf{v}}_{t,H,ADSB}^1 - \hat{\mathbf{v}}_{t+i-\Delta,H,ADSB|t,i}^0 \quad (206)$$

$$\hat{\mathbf{v}}_{t+i-\Delta,H,ADSB|t,i}^0 = \begin{bmatrix} \cos(\dot{\theta}_{\max}^0 i) & -\sin(\dot{\theta}_{\max}^0 i) \\ \sin(\dot{\theta}_{\max}^0 i) & \cos(\dot{\theta}_{\max}^0 i) \end{bmatrix} \hat{\mathbf{v}}_{t,H,ADSB}^0 \quad (207)$$

A right turn indicated by iteration variable k . The horizontal miss distance, $d_{t,HMD,ADSB|k}$, dependent on own aircraft turns left is defined as follows:

$$d_{t,HMD,ADSB|k} = \left\| \hat{\mathbf{s}}_{t+k,H,ADSB|t,k} - \hat{\mathbf{v}}_{t+k,H,ADSB|t,k} \frac{\hat{\mathbf{s}}_{t+k,H,ADSB|t,k} \cdot \hat{\mathbf{v}}_{t+k,H,ADSB|t,k}}{\hat{\mathbf{v}}_{t+k,H,ADSB|t,k}^2} \right\| \quad (208)$$

$$\hat{\mathbf{s}}_{t+k,H,ADSB|t,k} = \hat{\mathbf{s}}_{t+k-\Delta,H,ADSB|t,k} + \hat{\mathbf{v}}_{t+k-\Delta,H,ADSB|t,k} \Delta_k \quad (209)$$

$$\hat{\mathbf{v}}_{t+k-\Delta,H,ADSB|t,k} = \hat{\mathbf{v}}_{t,H,ADSB}^1 - \hat{\mathbf{v}}_{t+k-\Delta,H,ADSB|t,k}^0 \quad (210)$$

$$\hat{\mathbf{v}}_{t+k-\Delta,H,ADSB|t,k}^0 = \begin{bmatrix} \cos(-\dot{\theta}_{\max}^0 k) & -\sin(-\dot{\theta}_{\max}^0 k) \\ \sin(-\dot{\theta}_{\max}^0 k) & \cos(-\dot{\theta}_{\max}^0 k) \end{bmatrix} \hat{\mathbf{v}}_{t,H,ADSB}^0 \quad (211)$$

where $\hat{\mathbf{s}}_{t+k,H,ADSB|t,k}$ is the relative horizontal position vector between own aircraft and intruder at time 't+k' given own aircraft turns left, $\hat{\mathbf{v}}_{t+k-\Delta,H,ADSB|t,k}$ is the relative horizontal velocity vector between own aircraft and intruder at time 't+k' given own aircraft turns left, and Δ_k is one time step in the iteration where own aircraft is predicted to turn left.

Then the iteration for both scenarios (turn right/left) are carried out until the predicted horizontal miss distances are greater than a_{lim} :

while $\left[\left(d_{t,HMD,ADSB|i} < a_{lim} \right) \vee \left(i < \tau_{t,H,mod,ADSB} \right) \right]$
 $i = i + \Delta_i$
 compute new $d_{t,HMD,ADSB|i}$ through Equations (183)-(186) (212)

end while

while $\left[\left(d_{t,HMD,ADSB|k} < a_{lim} \right) \vee \left(k < \tau_{t,H,mod,ADSB} \right) \right]$
 $k = k + \Delta_k$
 compute new $d_{t,HMD,ADSB|k}$ through Equations (183)-(186) (213)

end while

Horizontal Resolution Advisory selection

After the geometrical situation has been assessed, the direction of turn is determined. That turn direction is chosen, which is predicted to lead faster to the desired horizontal separation of a_{lim} . However, in cases where both a left and a right turn are predicted to not leading to a horizontal separation of a_{lim} within the time of horizontal CPA, then that direction is chosen, which would lead the greater horizontal separation:

$$\dot{\theta}_t^0 = \begin{cases} \dot{\theta}_{max}^0, & \text{if } \left\{ \begin{aligned} &(i \leq k) \vee \\ &\left[(i = k) \wedge \left(d_{t,HMD,ADSB|i} \geq d_{t,HMD,ADSB|k} \right) \right] \end{aligned} \right\} \\ -\dot{\theta}_{max}^0, & \text{otherwise} \end{cases} \quad (214)$$

Coordination message

Once a horizontal RA has been selected, the information needs to be coordinated with the other aircraft. It is assumed that ACAS X equipped aircraft will communicate that they are ACAS X equipped. Therefore, if no information regarding ACAS X has been received by the UAV, the UAV assumed that the other aircraft is TCAS II equipped and sends a coordination message understandable for TCAS II. This is also done when horizontal avoidance logic has been selected, as this message may trigger the threat detection test of the intruder.

If the own aircraft has received a coordination message from the intruder, then it selects the opposite sense, else it predicts the relative altitude after assumed reaction time of TCAS II Ver.7.1 and selects the coordination message according to a non-altitude crossing sense.

$$m_{t,sent} = \begin{cases} -m_{t,received} & , \text{ if } \left\{ \left| m_{t,received} \right| > 0 \right. \\ 1 & , \text{ if } \left\{ \left(\hat{s}_{t,A} < 0 \right) \wedge \left(m_{t,received} = 0 \right) \right. \\ -1 & , \text{ if } \left\{ \left(\hat{s}_{t,A} \geq 0 \right) \wedge \left(m_{t,received} = 0 \right) \right. \end{cases} \quad (215)$$

Furthermore, in order to comply with TCAS II Ver.7.1 specification (FAA, 2011), the sense communicated in the own coordination message is reversed, if both aircraft have selected the same sense and the own aircraft's ModeS address is higher than the intruder's ModeS address.

$$\begin{aligned} &\text{if } \left[(m_{t,sent} = m_{t,received}) \wedge (\chi_{ModeS}^0 > \chi_{ModeS}^1) \right] \\ &\quad \text{then } [m_{t,sent} = -m_{t,received}] \end{aligned} \quad (216)$$

Evolution monitoring

Evolution monitoring consists of 3 steps. First, it is tested whether the predicted horizontal miss distance is sufficiently large. If yes, then the target turn rate, $\dot{\theta}_t^0$, is set to zero. Second, if the predicted horizontal miss distance, $d_{t,HMD,ADSB|\theta_{t|RA}^0}$, is sufficiently large assuming the own aircraft is flying with its initial heading, heading at the time when the RA was issued, $\theta_{t|RA}^0$, then the target turn rate, $\dot{\theta}_t^0$, is set such that the own aircraft returns to its initial heading, $\dot{\theta}_t^0 = -\dot{\theta}_{t|RA}^0$. Third, when the initial heading has been reached, $y_{t,\theta}^0 \approx \theta_{t|RA}^0$, then a clear of conflict message is issued, the target turn rate, $\dot{\theta}_t^0$, is set to zero, and the ACAS is reset. Next the algorithm is given:

1. If $d_{t,HMD,ADSB} \geq d_{mod}$ then $\dot{\theta}_t^0 = 0$
2. If $d_{t,HMD,ADSB|\theta_{t|RA}^0} \geq d_{mod}$ then $\dot{\theta}_t^0 = -\dot{\theta}_{t|RA}^0$
3. If $|y_{t,\theta}^0 - \theta_{t|RA}^0| \leq \dot{\theta}_{max}^0$ then
 - a. Clear of Conflict message
 - b. System reset
 - c. $\dot{\theta}_t^0 = 0$

The predicted horizontal miss distance, $d_{t,HMD,ADSB|\theta_{t|RA}^0}$, assuming the own aircraft is flying with its initial heading is given in Equations (217)-(218).

$$d_{t,HMD,ADSB|\theta_{t|RA}^0} = \left\| \hat{\mathbf{s}}_{t,H,ADSB} - \hat{\mathbf{v}}_{adjust} \frac{\hat{\mathbf{s}}_{t,H,ADSB} \cdot \hat{\mathbf{v}}_{adjust}}{\hat{\mathbf{v}}_{adjust}^2} \right\| \quad (217)$$

$$\hat{\mathbf{v}}_{adjust} = \hat{\mathbf{v}}_{t,H,ADSB}^1 - \left\| \hat{\mathbf{v}}_{t,H,ADSB}^0 \right\| \begin{pmatrix} \cos(\theta_{t|RA}^0) \\ \sin(\theta_{t|RA}^0) \end{pmatrix} \quad (218)$$

C Petri Net Model ACAS UAV

A Petri Net model has been created from the ACAS UAV model given in Section B. In total there are the following 4 agents:

- ACAS UAV i,
- Communication i,
- Cockpit i, and
- Aircraft i.

Next, the agents are described below.

C.1 Agent “ACAS UAV i”

Agent “ACAS UAV i” consists of 3 LPNs and 10 IPN. The LPNs are the following:

- State Estimation,
- Received Coordination Message, and
- Threat Detection.

The IPNs are the following:

- Avoidance Logic,
- Sense/Strength Selection,
- Evolution Monitoring,
- Hor. Man. Message
- Hor. Man Selection,
- Reset,
- Sense Coordination,
- RA,
- Adjusted RA, and
- COC.

The Petri Net model of agent “ACAS UAV i” is given in [Figure C-1](#). Afterwards the above mentioned LPNs and IPNs are addressed shortly.

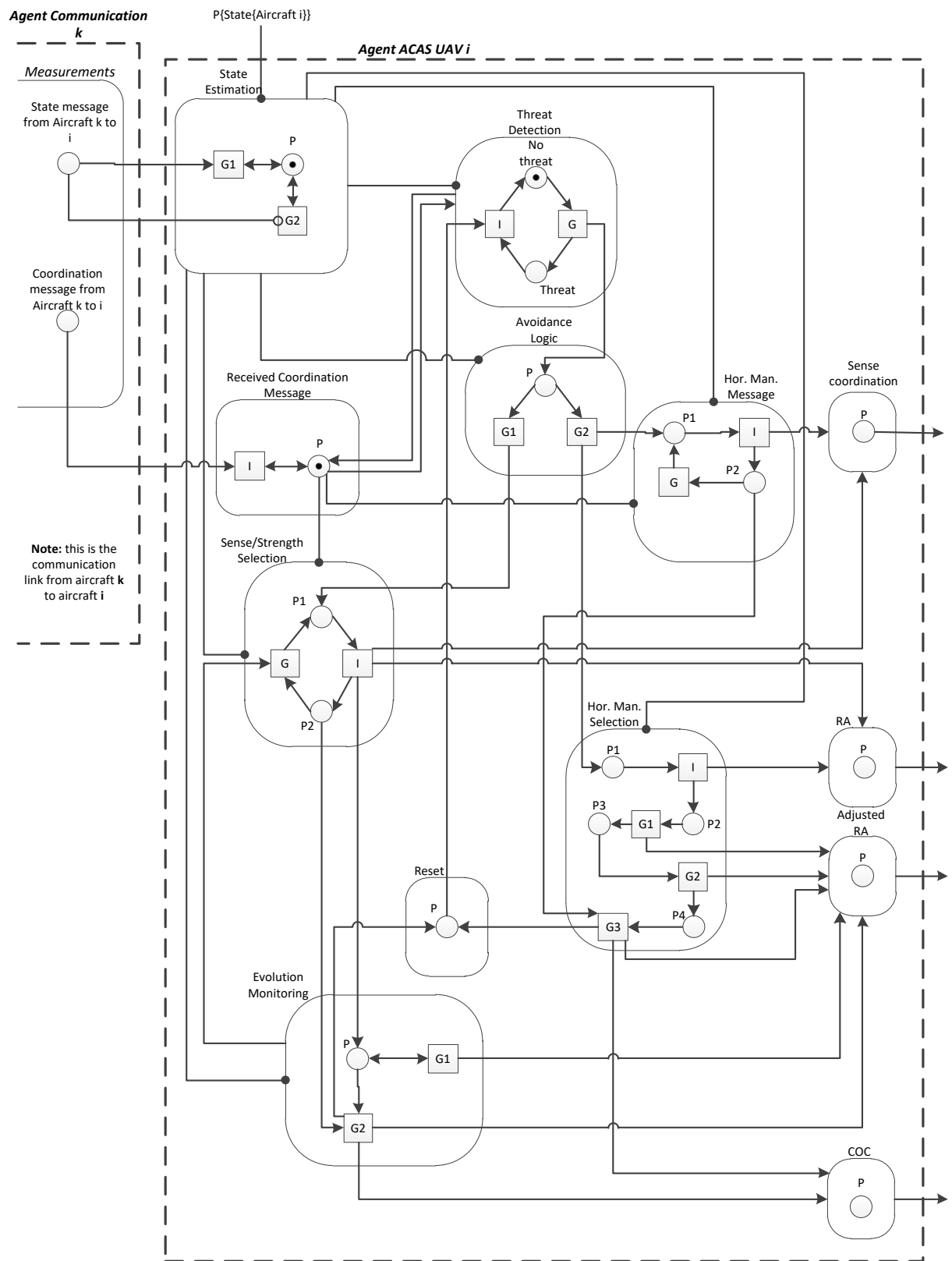


Figure C-1: Petri Net model drawing of agent "ACAS UAV i".

LPN “State Estimation”

This LPN has one place, “P” and two transitions, G1 and G2. There is always a token present. The LPN resembles state estimation and the horizontal miss distance filter, as described in Section B.4. As such the token in place “P” has the following colours: the relative horizontal position, the relative, own and intruder horizontal velocities as vectors, relative and own altitude positions, the relative and intruder altitude velocity, the heading of own aircraft, status of the horizontal miss distance filter, the ModeS addresses of both aircraft, and the performance limits of own aircraft.

The update of the token is carried out through transitions G1 and G2 at a frequency of 1Hz. In case a message from aircraft ‘k’ has been sent, then a token is present in place “State message from Aircraft k to i”. The enabling arc from that place in addition to the enabling arc from place “P” of agent “Aircraft i” will enable transition G1 and the state estimation and horizontal miss distance filter status are updated using the measurement data and information about the own aircraft from agent “Aircraft i”. Else transition G2, which has also one enabling arc from agent “Aircraft i”, is used and the update is carried out without measurements about aircraft ‘k’.

In order to prevent transitions G1 and G2 from becoming active at the same time, an inhibitor arc from place “State message from Aircraft k to i” to transition G2 ensures that G2 only becomes active when no message has been sent by aircraft ‘k’.

LPN “Received Coordination Message”

This LPN has one place, “P”, and one transition I. There is always one token at the place (except when the “Threat Detection” LPN is removing and returning it) and its colour represents the coordination message received from the other aircraft. The colour is updated through transition I which also has an incoming arc from place “Coordination message Aircraft k to i”.

Additionally, place “P” has outgoing and incoming arcs to and from transitions G and I of LPN “Threat Detection”, where transition G keeps the token’s colour unchanged and transition I changes the token’s colour to zero, meaning no message has been received, which is part of a TCAS reset.

The remaining outgoing enabling arcs go from place “P” to places “P1” and “P2” of both IPN “Sense/Strength Selection” and “Hor. Man. Message”, such that these IPNs may check whether both aircraft have coordinated the same resolution intent.

LPN “Threat Detection”

This LPN has two places, “Threat” and “No threat”, and two transitions, G and I. This LPN resembles the threat detection module which declares the other aircraft as a threat, as described in Section B.5.

Initially there is a token at place “No threat” which is being transferred to place “Threat” via transition G. Transition G uses the information about the aircraft states of aircraft ‘i’ and ‘k’ from the token in place “P” of LPN “State Estimation” through an enabling arc, and the information about a received coordination message from the other aircraft through an incoming arc from place “P” of

LPN “Received Coordination Message” to determine whether the other aircraft is supposed to be declared a threat.

Transition G has in total three outgoing arcs. One, as mentioned above, goes to place “Threat”. A second goes to place “P” of LPN “Received Coordination Message” which returns the token coming from there with unchanged colour, such that the information about a received coordination message remains saved in place “P” of LPN “Received Coordination Message”. A third goes to place “P” in IPN “Avoidance Logic”, which initiates the collision avoidance algorithm.

When the collision avoidance algorithm has finished, then a token is present in place “P” of IPN “Reset”. From there an incoming arc goes to transition I of the LPN “Threat Detection” which activates the transition. The transition removes the tokens in place “Threat” and place “P” of IPN “Reset” and fires one token to place “No threat”. Additionally, there is an incoming and outgoing arc from and to place “P” of LPN “Received Coordination Message”, which returns that token with a colour value meaning “no message received”. This is necessary, as else the previously received coordination message may trigger transition G1 again and the other aircraft would be declared faulty a new threat.

IPN “Sense Coordination”

IPN “Sense Coordination” has two incoming arcs and one outgoing arc. The outgoing arc goes to transition I1 of LPN “Mode S Reply” of agent “Communication i”. It transfers the in the sense selection selected intent to the agent “Communication i” from where the intent is sent to aircraft ‘k’.

The sense is selected either in IPN “Sense/Strength Selection”, see Equation (193), and comes from an incoming arc from transition I of that IPN, or in IPN “Hor. Man Message”, see Equations (215)-(216), and comes from an incoming arc from transition I of that IPN.

IPN “RA”

This IPN has two incoming arcs and one outgoing arc. The outgoing arc goes to transition I1 of LPN “Pilot Flying” of agent “Cockpit i”. It transfers initial RAs to agent “Cockpit i” such that the pilot may react upon it.

The RA is selected either in IPN “Sense/Strength Selection”, see Equation (195), and comes from an incoming arc from transition I of that IPN, or in IPN “Hor. Man Selection”, see Equations (214), and comes from an incoming arc from transition I of that IPN.

IPN “Adjusted RA”

This IPN has five incoming arcs and one outgoing arc. The outgoing arc goes to transition I2 of LPN “Pilot Flying” of agent “Cockpit i”. It transfers secondary RAs to agent “Cockpit i” such that the pilot may react upon it.

The adjusted RA is selected either in IPN “Evolution Monitoring”, see Equations (195) and (200), and comes from incoming arcs from transitions G1 and G2 of that IPN, or in IPN “Hor. Man Selection”,

see section about evolution monitoring for horizontal avoidance manoeuvres on page 13, and comes from incoming arcs from transitions G1, G2 and G3 of that IPN.

IPN “COC”

This IPN has two incoming arcs and one outgoing arc. The outgoing arc goes to transition I2 of LPN “Mode S Reply” of agent “Communication i”. It transfers the clear of conflict message to agent “Communication i” from where the clear of conflict message is sent to aircraft ‘k’.

The COC is created either in IPN “Evolution Monitoring”, see Equation (200), and comes from an incoming arc from transition G2 of that IPN, or in IPN “Hor. Man Selection”, see section about evolution monitoring for horizontal avoidance manoeuvres on page 13, and comes from an incoming arc from transition G3 of that IPN.

IPN “Avoidance Logic”

This IPN has one place “P” and two transitions, “G1” and “G2”. Initially there are no tokens present in this IPN. In case a threat has been detected, then IPN “Avoidance Logic” receives a token through an incoming arc from transition G of LPN “Threat Detection”. Based on information about the states of aircraft ‘i’ and ‘k’, which comes through enabling arcs to both transition, G1 and G2, from place “P” in LPN “State Estimation” it is decided whether a vertical or a horizontal manoeuvre should be carried out. If the logic determines that a vertical manoeuvre should be carried out then transition G1 is activated and transfers the token to place “P1” of IPN “Sense/Strength Selection”. If the logic determines that a horizontal manoeuvre should be carried out then transition G2 is activated and fires two tokens. One token to place “P1” of IPN “Hor. Man. Message” and one token to place “P1” in IPN “Hor. Man. Selection”.

IPN “Sense/Strength Selection”

This IPN has two places, “P1” and “P2”, and two transitions, “I and G”. Initially there are no tokens present in this IPN. In case the IPN “Avoidance Logic” has fired a token to place “P1” then the vertical avoidance algorithms are activated.

Transition I has enabling arcs from places “P” in LPNs “State Estimation” and “Received Coordination Message”, and one incoming arc from place “P1”. Based on the state estimates and a received coordination message, if any received, transition I determines whether the own aircraft should climb or descend.

There are four outgoing arcs which go to place “P2”, and places “P” of IPNs “Sense coordination”, “RA” and “Evolution Monitoring”. The tokens fired to places “P2” and “P” of IPNs “Sense coordination” has a colour value equal to the selected coordination message. The token fired to place “P” of IPN “RA” has colour values equal to the selected RA. The token to place “P” in IPN “Evolution Monitoring” has colour values equal to the time when the initial RA was issued and the selected sense of that RA, such that the encounter geometry may be monitored in IPN “Evolution Monitoring”.

Transition G has enabling arcs from places “P” in LPNs “State Estimation” and “Received Coordination Message”, one incoming arc from place “P2”, and one incoming arc from place “P” of IPN “Evolution Monitoring”. Transition G only compares the sent coordination message to aircraft “k” which is saved in the colour of the token in place “P2” against the received coordination message from aircraft ‘k’ in place “P” of IPN “Received Coordination Message”, and the saved ModeS addresses in place “P” in IPN “State Estimation”. Only if both aircraft have sent the same coordination message and aircraft ‘i’ has the higher ModeS address, then one colourless token is fired to place “P1”. Note that this also removes the token in place “P” in IPN “Evolution Monitoring” which stops the evolution monitoring process until a new RA has been selected by transition I.

IPN “Evolution Monitoring”

This IPN has one place “P” and two transitions, G1 and G2. Initially there is no token in place “P”. When a token has arrived from the incoming arc from transition I in IPN “Sense/Strength Selection”, then the guards perform the evolution monitoring process, as described in Section B.7.

Transition G1 checks for an RA strengthening, sense reversal or RA weakening, and if activated returns a token to place “P” with the updated sense and the time when the RA has been issued, and a token with colour values of the new RA to the IPN “Adjusted RA”.

Transition G2 checks whether a clear of conflict message may be issued, and if activated fires colourless tokens to IPNs “Reset”, “COC” and a token with colour values equal to a “Level-Off” RA, as the UAV should level off after a clear of conflict message. Note, that transition G2 does not only remove the token from place “P” but also from place “P2” in IPN “Sense/Strength Selection”. As such, when there is a token in place “P” in IPN “Reset” then there cannot be a token in IPNs “Sense/Strength Selection” and “Evolution monitoring”.

IPN “Hor. Man. Message”

This IPN has two places, “P1” and “P2”, and two transitions, I and G. Initially there is no token in this IPN. A token may only arrive in this IPN through an incoming arc to place “P1” from transition G2 in IPN “Avoidance Logic”.

A token may be transferred from place “P1” to “P2” through transition I. Transition I has also enabling arcs from places “P” in LPNs “State Estimation” and “Received Coordination Message” and another outgoing arc to place “P” in IPN “Sense coordination”. Transition I selects the coordination message which should be sent to aircraft ‘k’ which is saved as a colour value in both tokens that are fired.

A token in place “P2” may be transferred to place “P1” through transition G. Transition G has enabling arcs from places “P” in LPNs “State Estimation” and “Received Coordination Message” and compares the sent coordination message saved as colour in place “P2” against the received coordination message. Only if the sent and received coordination messages are equal and aircraft ‘i’ has the higher ModeS address, then transition G is activated. The information about a received coordination message comes from place “P” in IPN “Received Coordination Message” and

information about the ModeS addresses of both aircraft 'i' and aircraft 'k' comes from place "P" in IPN "State Estimation".

IPN "Hor. Man. Selection"

This IPN has 4 places, "P1", "P2", "P3" and "P4", and 4 transitions, I, G1, G2, and G3 and resembles the horizontal avoidance manoeuvre algorithm as described in Section B.8. Initially there is no token in this IPN. A token may only arrive in this IPN through in incoming arc to place "P1" from transition G2 in IPN "Avoidance Logic".

All transitions have enabling arcs from place "P" in IPN "State Estimation" and connect the places in the order as indicated by the numbers of the places. Transition I transfers a token from place "P1" to "P2" and also fires a token to place "P" in IPN "RA". As such transition I selects in which direction the aircraft should turn. If the aircraft 'i' has turned sufficiently such that the estimated horizontal miss distance is sufficiently large, transition G1 transfers the token from "P2" to "P3" and fires a token to place "P" in IPN "Adjusted RA" indicating an RA to stop aircraft 'i' from turning. Once it is safe for aircraft 'i' to return to its initial heading, then transition G2 transfers the token from place "P3" to "P4" and fires a token to place "P" in IPN "Adjusted RA" indicating an RA to turn back into the direction of its initial heading. Once the initial heading has been reached, then transition G3 is activated which removes the token in place "P4" and the token in place "P2" in IPN "Hor. Man. Message", and fires three tokens. Tokens without colour are fired to places "P" in IPNs "COC" and "Reset". The third token is fired to place "P" in IPN "Adjusted RA" indicating an RA to stop turning.

As such, transition G3 ensures that when there is a token in place "P" in IPN "Reset" then there cannot be tokens in IPNs "Hor. Man. Message" and "Hor. Man. Selection".

C.2 Agent "Aircraft i"

Agent "Aircraft i" is identical to the agent "Aircraft i" as designed in the TCAS Petri Net model, as described in Section 14.2.

C.3 Agent "Cockpit i"

The petri net model of the agent "Cockpit i" is identical to the agent "Cockpit i" of the TCAS Petri Net model, as described in Section 14.3. However, this time the autopilot is activated and will react immediately upon RAs (no reaction time) with the maximum possible altitude acceleration. Also the autopilot does not require visual or aural annunciation of the RA and the remote pilot does not intervene during collision avoidance manoeuvring.

It should be noted, that this time the RA and adjusted RA incomes comes from the agent "ACAS UAV i" and not from the agent "TCAS i".

C.4 Agent "Communication i"

The agent "Communication i" is similar to the agent "Communication i" of the TCAS Petri Net model, as described in Section 14.4. The following two extensions have been added to allow operability of agent "Communication i" of the TCAS Petri Net model between TCAS and UAV operations:

First, in order to cooperate with TCAS equipped aircraft, the UAV performs active surveillance through interrogation and response messages in the same manner as TCAS equipped aircraft do.

Second, it is assumed that the ADS- B data is transmitted in interrogation responses.

D Petri Net Model Specification Extension for Agent “ACAS UAV i”

In total there are the following 4 agents:

- ACAS UAV i,
- Aircraft i,
- Cockpit i, and
- Communication i.

D.1 Agent “ACAS UAV i”

The agent “ACAS UAV i” is interchangeable with agent “TCAS i” from the previously specified Petri Net model. Hence, the input and output to and from the agent are the same. Agent “ACAS UAV i” has in total 7 LPNs and 6 IPNs which are described below.

LPN “State Estimation”

The state estimation local Petri net represents the slant range and vertical range filtering, and the horizontal miss distance filter modules as described in Section 5.

Incoming arcs within same agent

- None

Outgoing arcs within same agent

- Two enabling arcs from place “P” to LPN “Threat Detection” transitions I and G
- Two enabling arcs from place “P” to IPN “Avoidance Logic” transitions G1 and G2
- Two enabling arcs from place “P” to IPN “Sense/Strength Selection” transitions I and G
- Two enabling arcs from place “P” to LPN “Evolution Monitoring” transitions G1 and G2
- Two enabling arcs from place “P” to IPN “Hor. Man. Message” transitions I and G
- Four enabling arcs from place “P” to IPN “Hor. Man. Selection” transitions I, G1, G2 and G3

Incoming arcs from other agent

- Incoming arc from place “State message from aircraft k to i” of LPN “Measurements” of agent “Communication k” to transition G1
- Inhibitor arc from place “State message from aircraft k to i” of LPN “Measurements” of agent “Communication k” to transition G2
- Two enabling arcs from place “P” of LPN “State” of agent “Aircraft i” to transitions G1 and G2

Outgoing arcs to other agent

- None

Places

The following colours equal the variables given in Equation (27) in Section 4 and in Equations (162)-(179) in Sections B.2 and B.4. Exception is colour t^G which is added to control the ACAS cycle of 1Hz.

Table D-1: Place of LPN State Estimation of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P	$\hat{\mathbf{s}}_{t,H,ADSB} \in \mathbb{R}$	Estimated relative horizontal position vector	None
	$\hat{\mathbf{v}}_{t,H,ADSB} \in \mathbb{R}$	Estimated relative horizontal velocity vector	None
	$\hat{\mathbf{v}}_{t,H,ADSB}^0 \in \mathbb{R}$	Estimated horizontal velocity vector of own aircraft	None
	$\hat{\mathbf{v}}_{t,H,ADSB}^1 \in \mathbb{R}$	Estimated horizontal velocity vector of intruder aircraft	None
	$\hat{s}_{t,A} \in \mathbb{R}$	Estimated relative altitude position	None
	$\hat{s}_{t,A}^0 \in \mathbb{R}$	Estimated altitude position of own aircraft	None
	$\hat{v}_{t,A} \in \mathbb{R}$	Estimated relative altitude velocity vector	None
	$\hat{v}_{t,A}^1 \in \mathbb{R}$	Estimated vertical altitude velocity of intruder aircraft	None
	$y_{t,\theta}^0 \in \mathbb{R}$	Heading of own aircraft	None
	$B_{t,HMDF} \in \{0,1\}$	State of horizontal miss distance filter. Equals 1 if filter active	None
	$t^G \in \mathbb{R}$	Time delay until next update/measurement	$t^G = -1$
	$\chi_{ModeS}^0 \in \mathbb{R}$	Aircraft ID of own aircraft	None
	$\chi_{ModeS}^1 \in \mathbb{R}$	Aircraft ID of intruder aircraft	None
	$\dot{\theta}_{\max}^0 \in \mathbb{R}$	Maximum turn rate of own aircraft	None
	$v_{A,\min}^0 \in \mathbb{R}$	Minimum altitude velocity of own aircraft	None
	$v_{A,\max}^0 \in \mathbb{R}$	Maximum altitude velocity of own aircraft	None

Initial markings

Initially, there is a token in place “P”. The initial colours depend on the scenario to be simulated. Typically all colours are set to zero, except, the initial colour of t^G is determined initially by drawing a random sample from a uniform distribution, and the fixed aircraft parameters, χ_{ModeS}^0 , χ_{ModeS}^1 , $\dot{\theta}_{\max}^0$, $v_{A,\min}^0$, $v_{A,\max}^0$, are set according to the properties of the aircraft to be simulated.

Transitions

Table D-2: Transitions of LPN State Estimation of agent ACAS UAV i.

ID	Transition	Condition
G1	State message from aircraft k to i $\{\text{Measurements}[\text{Communication } k]\} \wedge P\{\text{State}[\text{Aircraft } i]\} \wedge P \rightarrow P$	$t^G \leq 0$

G2	NOT <i>State message from aircraft k to i</i> $\{\text{Measurements}[\text{Communication } k]\} \wedge$ $P\{\text{State}[\text{Aircraft } i]\} \wedge P \rightarrow P$	$t^G \leq 0$
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Table D-3: Firing functions of transitions of LPN State Estimation of agent ACAS UAV i.

ID	Firing function
G1	One token is fired to place P with colours. The colours are set according to Equations (164)-(166) and (168)-(179) in Sections B.2 and B.4, where $y_{t,H,ADSB}^k = y_{t,H,ADSB}^k \{ \text{State message from aircraft } k \text{ to } i$ $\{ \text{Measurements}[\text{Communication } k] \} \}$ $y_{t,H,v,ADSB}^k = y_{t,H,v,ADSB}^k \{ \text{State message from aircraft } k \text{ to } i$ $\{ \text{Measurements}[\text{Communication } k] \} \}$ $y_{t,A}^k = y_{t,A}^k \{ \text{State message from aircraft } k \text{ to } i \{ \text{Measurements}[\text{Communication } k] \} \}$ $y_{t,A,v,ADSB}^k = y_{t,A,v,ADSB}^k \{ \text{State message from aircraft } k \text{ to } i$ $\{ \text{Measurements}[\text{Communication } k] \} \}$ $y_{t,A,v,ADSB}^i, \text{ where } v_{t,A}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \}$ $y_{t,A}^i, \text{ where } s_{t,z}^i = s_{t,z}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \}, \text{ see Equation (17);}$ $y_{t,\theta}^k = y_{t,\theta}^k \{ \text{State message from aircraft } k \text{ to } i \{ \text{Measurements}[\text{Communication } k] \} \}$ $y_{t,\theta}^i, \text{ where } v_{t,x}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \} \text{ and } v_{t,y}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \}$ And according to: $t^G = 1s$ Note, a measurement about the intruder is present.
G2	One token is fired to place P with colours. The colours are set according to Equations (164)-(166) and (168)-(179) in Sections B.2 and B.4, where $y_{t,A}^i, \text{ where } s_{t,z}^i = s_{t,z}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \}, \text{ see Equation (17);}$ $y_{t,\theta}^k = y_{t,\theta}^k \{ \text{State message from aircraft } k \text{ to } i \{ \text{Measurements}[\text{Communication } k] \} \}$ $y_{t,\theta}^i, \text{ where } v_{t,x}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \} \text{ and } v_{t,y}^i \{ P \{ \text{State}[\text{Aircraft } i] \} \}$ And according to: $t^G = 1s$ Note, no measurements about the intruder are present.

LPN “Received Coordination Message”

The received coordination message, as defined in Equation (26), is saved in LPN “Received Coordination Message”. It works the same as specified for TCAS.

Incoming arcs within same agent

- Two incoming arcs from transitions I and G of LPN “Threat Detection” to place “P”

Outgoing arcs within same agent

- Two outgoing arcs from place “P” to LPN “Threat Detection” transitions I and G
- Two enabling arcs from place “P” to IPN “Sense/Strength Selection” transitions I and G
- Two enabling arcs from place “P” to IPN “Hor. Man. Message” transitions I and G

Incoming arcs from other agent

- Incoming arc from place “Coordination message from aircraft k to i” of LPN “Measurements” of agent “Communication k” to transition I

Outgoing arcs to other agent

- None

Places

Place “P” stores the value of the received coordination message as described in Section 4 (see Equation (26)).

Table D-4: Place of LPN Received Coordination Message of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P	$m_{t, received} \in \{-1, 0, 1\}$	Received coordination message	None

Initial markings

Initially there is a token present in place “P”. The initial colour is described below:

Table D-5: Initial marking of place of LPN Received Coordination Message of agent ACAS UAV i.

Places	Colour	Explanation
P	$m_{t, received} = 0$	No message received

Transitions

Table D-6: Transition of LPN Received Coordination Message of agent ACAS UAV i.

ID	Transition	Condition
I	$\text{Coordination message from aircraft } k \text{ to } i$ $\{\text{Measurements}[\text{Communication } k]\} \wedge P \rightarrow P$	None

Whenever a new coordination message is present, then the stored coordination message is updated according to Equation (26). As described in Section 4, a coordination message will be transmitted until the recipient has acknowledged the reception. The Petri net mode models the behaviour such that it continuously transmits a message but indicates through the additional parameter p_{mes} whether the intruder is sending a new coordination message.

Table D-7: Firing function of transition of LPN Received Coordination Message of agent ACAS UAV i.

ID	Firing function
I	One token is fired with the colour: if $p_{Coordination} \{ \text{Coordination message from aircraft } k \text{ to } i$ $\{ \text{Measurements[Communication } k] \} \} = 1$ $m_{t, received} = m_{t, sent} \{ \text{Coordination message from aircraft } k \text{ to } i$ $\{ \text{Measurements[Communication } k] \} \}$,according to Equation(26) else $m_{t, received} = m_{t, received} \{ P \}$,stored received coordination message remains unchanged

LPN “Threat Detection”

The Threat Detection LPN represents the Threat Detection module as described in Section 8.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions I and G
- Two incoming arcs from place “P” in LPN “Received Coordination Message” to transitions I and G
- One incoming arc from place “P” of IPN “Reset” to transition I

Outgoing arcs within same agent

- One outgoing arc from transition G to place “P” of IPN “Avoidance Logic”
- Two outgoing arcs from transitions I and G to place “P” in LPN “Received Coordination Message”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There are two places without colour: “Threat” and “No threat”. The places represent $B_{t,RA,UAV}$ from Equation (184). A token in place “Threat” represents the condition $B_{t,RA,UAV} = TRUE$. A token in place “No threat” represents the condition $B_{t,RA,UAV} = FALSE$.

Initial markings

Initially there is one token without colour in place “No Threat”.

Transitions

Table D-8: Transitions of LPN Threat Detection of agent ACAS UAV i.

ID	Transition	Condition
G	$No\ threat \wedge P\{State\ Estimation\} \wedge P\{Received\ Coordination\ Message\} \rightarrow Threat \wedge P\{Received\ Coordination\ Message\} \wedge P\{Avoidance\ Logic\}$	If $B_{t,RA,UAV} = TRUE$ from Equation (184), where $\hat{s}_{t,H,ADSB} = \hat{s}_{t,H,ADSB} \{P\{State\ Estimation\}\}$ $\hat{v}_{t,H,ADSB} = \hat{v}_{t,H,ADSB} \{P\{State\ Estimation\}\}$ $\hat{s}_{t,A} = \hat{s}_{t,A} \{P\{State\ Estimation\}\}$ $\hat{v}_{t,A} = \hat{v}_{t,A} \{P\{State\ Estimation\}\}$ $m_{t,received} = m_{t,received} \{P\{Received\ Coordination\ Message\}\}$
I	$Threat \wedge P\{State\ Estimation\} \wedge P\{Received\ Coordination\}$	None

	$\text{Message} \wedge P\{\text{Reset}\} \rightarrow \text{No threat} \wedge P\{\text{Received Coordination Message}\}$	
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Table D-9: Firing functions of transitions of LPN Threat Detection of agent ACAS UAV i.

ID	Firing function
G	In total three tokens are fired: One token without colour is fired to place <i>Threat</i>. One token with unchanged colour from $P\{\text{Received Coordination Message}\}$ is fired to $P\{\text{Received Coordination Message}\}$ One token without colour is fired to place $P\{\text{Avoidance Logic}\}$
I	In total two tokens are fired: One token without colour is fired to place <i>No threat</i>. One token with the following colour is fired to $P\{\text{Received Coordination Message}\}$: $m_{t, \text{received}} \{P\{\text{Received Coordination Message}\}\} = 0$

IPN “Avoidance Logic”

The Avoidance Logic IPN represents the Avoidance Logic module as described in Section B.6.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions G1 and G2
- One incoming arc from transition G of LPN “Threat Detection” to place “P”

Outgoing arcs within same agent

- One outgoing arc from transition G1 to place “P1” of IPN “Sense/Strength Selection”
- Two outgoing arcs from transition G2 to places “P1” of IPNs “Hor. Man. Message” and “Hor. Man. Selection”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There is one place “P” without colour.

Initial markings

Initially there is no token in place “P”.

Transitions

Table D-10: Transitions of IPN Avoidance Logic of agent ACAS UAV i.

ID	Transition	Condition
G1	$P \wedge P\{\text{State Estimation}\} \rightarrow P1\{\text{Sense/Strength Selection}\}$	If $L_{t,UAV} = 1$ from Equation (185), where $\hat{\mathbf{s}}_{t,H,ADSB} = \hat{\mathbf{s}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB} = \hat{\mathbf{v}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A} = \hat{\mathbf{s}}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A} = \hat{\mathbf{v}}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A}^0 = \hat{\mathbf{s}}_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB}^0 = \hat{\mathbf{v}}_{t,H,ADSB}^0 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A}^0 = \hat{\mathbf{v}}_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A}^1 = \hat{\mathbf{v}}_{t,A}^1 \{P\{\text{State Estimation}\}\}$ $\dot{\theta}_{\max}^0 = \dot{\theta}_{\max}^0 \{P\{\text{State Estimation}\}\}$ $v_{A,\min}^0 = v_{A,\min}^0 \{P\{\text{State Estimation}\}\}$ $v_{A,\max}^0 = v_{A,\max}^0 \{P\{\text{State Estimation}\}\}$

G2	$P \wedge P\{\text{State Estimation}\} \rightarrow P1\{\text{Hor. Man. Message}\} \wedge P1\{\text{Hor. Man. Selection}\}$	If $L_{t,UAV} = 2$ from Equation (185), where $\hat{\mathbf{s}}_{t,H,ADSB} = \hat{\mathbf{s}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB} = \hat{\mathbf{v}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A} = \hat{\mathbf{s}}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A} = \hat{\mathbf{v}}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A}^0 = \hat{\mathbf{s}}_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB}^0 = \hat{\mathbf{v}}_{t,H,ADSB}^0 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A}^0 = \hat{\mathbf{v}}_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A}^1 = \hat{\mathbf{v}}_{t,A}^1 \{P\{\text{State Estimation}\}\}$ $\dot{\theta}_{\max}^0 = \dot{\theta}_{\max}^0 \{P\{\text{State Estimation}\}\}$ $\mathbf{v}_{A,\min}^0 = \mathbf{v}_{A,\min}^0 \{P\{\text{State Estimation}\}\}$ $\mathbf{v}_{A,\max}^0 = \mathbf{v}_{A,\max}^0 \{P\{\text{State Estimation}\}\}$
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Table D-11: Firing functions of transitions of IPN Avoidance Logic of agent ACAS UAV i.

ID	Firing function
G1	One token is fired with the following colour: $\tau_{t,CPA,ADSB}$, according to Equation (188) where $\hat{\mathbf{s}}_{t,H,ADSB} = \hat{\mathbf{s}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB} = \hat{\mathbf{v}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A} = \hat{\mathbf{s}}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A} = \hat{\mathbf{v}}_{t,A} \{P\{\text{State Estimation}\}\}$
G2	Two tokens are fired with each the following colour: $\tau_{t,CPA,ADSB}$, according to Equation (188) where $\hat{\mathbf{s}}_{t,H,ADSB} = \hat{\mathbf{s}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB} = \hat{\mathbf{v}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A} = \hat{\mathbf{s}}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,A} = \hat{\mathbf{v}}_{t,A} \{P\{\text{State Estimation}\}\}$

IPN “Sense coordination”

Incoming arcs within same agent

- Incoming arc from transition I of IPN “Sense/Strength Selection” to place “P”
- Incoming arc from transition I of IPN “Hor. Man. Message” to place “P”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc from place “P” to transition I1 of LPN “Mode S Reply” of agent “Communication i”

Places

Table D-12: Place of IPN Sense Coordination of agent UAV i.

Places	Colour type	Explanation	Colour function
P	$m_{t,sent} \in \{-1, 0, 1\}$	Coordination message from aircraft i to k	None

Initial markings

Initially there is no token in place “P”.

Transitions

There are no transitions in the IPN.

IPN “Sense/Strength Selection”

The Sense/Strength Selection IPN represents the Sense/Strength Selection module as described in Section B.79.5.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions I and G
- Two enabling arcs from place “P” in LPN “Received Coordination Message” to transitions I and G
- Incoming arc from transition G1 in IPN “Avoidance Logic” to place “P1”
- Incoming arc from place “P” in IPN “Evolution Monitoring” to transition G

Outgoing arcs within same agent

- Outgoing arc from transition I to place “P” of IPN “Sense Coordination”
- Outgoing arc from transition I to place “P” of IPN “RA”
- Outgoing arc from transition I to place “P” of IPN “Evolution Monitoring”
- Outgoing arc from place “P2” to transition G2 in IPN “Evolution Monitoring”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There are two places with the following colours.

Table D-13: Places of IPN Sense/Strength Selection of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P1	$\tau_{t,CPA,ADSB} \in \mathbb{R}$	Estimated time to CPA	$\dot{\tau}_{t,CPA,ADSB} = -1$
P2	$m_{t,sent} \in \mathbb{R}$	Coordination message from aircraft i to k	None

Initial markings

Initially there is no token in this IPN.

Transitions

Table D-14: Transitions of IPN Sense/Strength Selection of agent ACAS UAV i.

ID	Transition	Condition
I	$P1 \wedge P\{\text{State Estimation}\} \rightarrow P2 \wedge P\{\text{Evolution Monitoring}\} \wedge P\{\text{RA}\} \wedge P\{\text{Sense coordination}\}$	N/A

G	$P2 \wedge P\{\text{State Estimation}\} \wedge P\{\text{Evolution Monitoring}\} \rightarrow P1$	N/A
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Table D-15: Firing functions of transitions of IPN Sense/Strength Selection of agent ACAS UAV i.

ID	Firing function
I	In total four tokens are fired. The colours are determined according to Equations (192)-(195) where $\hat{s}_{t,A} = \hat{s}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{v}_{t,A} = \hat{v}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{s}_{t,A}^0 = \hat{s}_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $\hat{v}_{t,A}^0 = \hat{v}_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $\hat{v}_{t,A}^1 = \hat{v}_{t,A}^1 \{P\{\text{State Estimation}\}\}$ $\dot{\theta}_{\max}^0 = \dot{\theta}_{\max}^0 \{P\{\text{State Estimation}\}\}$ $v_{A,\min}^0 = v_{A,\min}^0 \{P\{\text{State Estimation}\}\}$ $v_{A,\max}^0 = v_{A,\max}^0 \{P\{\text{State Estimation}\}\}$ One token is fired to place $P2$ with the following colours: $m_{t,\text{sent}}$, according to Equations (192)-(195) $\tau_{t,CPA,ADSB} = \tau_{t,CPA,ADSB} \{P1\}$ One token with colours is fired to $P\{RA\}$. The colours are determined as follows: $v_{t,A,RA,\min}^0$, according to Equations (192)-(195) $v_{t,A,RA,\max}^0$, according to Equations (192)-(195) $a_{t,A,RA}^i = 0.25g$ $\dot{\theta}_{RA} = 0$ $t_{RA} = 0$ One token with colours is fired to $P\{\text{Evolution Monitoring}\}$. The colours are determined as follows: $D_{t,UAV} = D_{t,UAV}$ $t_{t RA} = 5s$ One token with the following colour is fired to place $P\{\text{Sense Coordination}\}$: $m_{t,\text{sent}}$, according to Equations (192)-(195)
G	One token with colour $\tau_{t,CPA,ADSB} = \tau_{t,CPA,ADSB} \{P2\}$ is fired to place $P1$

IPN “RA”

IPN to indicate the issue of an RA.

Incoming arcs within same agent

- Incoming arc from transition I of IPN “Sense/Strength Selection”
- Incoming arc from transition I in IPN “Hor. Man. Selection”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc to transition I1 of LPN “Pilot Flying” of agent “Cockpit i”

Places

There is one place “P” with the following colours.

Table D-16: Place of IPN RA of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P	$v_{t,A,RA,\min}^i \in \mathbb{R}$	Target lower altitude velocity bound of RA	None
	$v_{t,A,RA,\max}^i \in \mathbb{R}$	Target upper altitude velocity bound of RA	None
	$a_{t,A,RA}^i \in \mathbb{R}$	Target altitude rate of RA	None
	$\dot{\theta}_{RA} \in \mathbb{R}$	Target turn rate of RA	None
	$t_{RA} \in \mathbb{R}$	Assumed reaction time of the pilot	None

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

IPN “Evolution Monitoring”

The Evolution Monitoring LPN represents the evolution monitoring module as described in Section B.7.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions G1 and G2
- Incoming arc from transition I of IPN “Sense/Strength Selection” to place “P”
- Incoming arc from place “P2” of IPN “Sense/Strength Selection” to transition G2

Outgoing arcs within same agent

- Outgoing arc from place “P” to transition G of IPN “Sense/Strength Selection”
- Outgoing arc from transition G1 to place “P” of IPN “Adjusted RA”
- Outgoing arc from transition G2 to place “P” of IPN “Adjusted RA”
- Outgoing arc from transition G2 to place “P” of IPN “COC”
- Outgoing arc from transition G2 to place “P” in IPN “Reset”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There is one place “P” with the following colours:

Table D-17: Place of IPN Evolution Monitoring of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P	$D_{t,UAV} \in \{-1,0,1\}$	Currently selected sense	None
	$t_{t RA} \in \mathbb{R}$	Elapsed time since previous RA	$\dot{t}_{t RA} = -1$

Initial markings

Initially there is no token in this IPN.

Transitions

Table D-18: Transitions of IPN Evolution Monitoring of agent ACAS UAV i.

ID	Transition	Condition
G1	$P \wedge P\{\text{State Estimation}\} \rightarrow P \wedge P\{\text{Adjusted RA}\}$	If $B_{t,RA,EVO,UAV} = TRUE$ in Equation (198) and $t_{t RA} \leq 0$ or if $\left[\left(\left \hat{S}_{CPA,A t}^{level} \right \geq a_{lim} \right) \wedge \left(\frac{\hat{S}_{CPA,A t}^{level}}{-D_{t,UAV}} > 0 \right) \right] \text{ from Equation (200) is}$

		satisfied where $\hat{s}_{t,A} = \hat{s}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{v}_{t,A} = \hat{v}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{s}_{t,A}^0 = \hat{s}_{t,A}^0 \{P\{\text{State Estimation}\}\}$
G2	$P \wedge P\{\text{State Estimation}\} \wedge$ $P2\{\text{Sense/Strength Selection}\} \rightarrow$ $P\{\text{Reset}\} \wedge P\{\text{Adjusted RA}\} \wedge$ $P\{\text{COC}\}$	If $\left[\neg B_{t,RT,UAV} \wedge \neg B_{t,VT,UAV} \wedge (t_{t,RA,UAV} \leq 0) \right] \vee B_{t,HMDF,UAV}$ from Equation (200) is satisfied, where $\hat{s}_{t,A} = \hat{s}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{v}_{t,A} = \hat{v}_{t,A} \{P\{\text{State Estimation}\}\}$ $\hat{s}_{t,A}^0 = \hat{s}_{t,A}^0 \{P\{\text{State Estimation}\}\}$ $B_{t,HMDF} = B_{t,HMDF} \{P\{\text{State Estimation}\}\}$ Note, this is the same test as to issue a clear of conflict message and reset the system

Table D-19: Firing functions of transitions of IPN Evolution Monitoring of agent ACAS UAV i.

ID	Firing function
G1	In total two tokens with colours are fired. The colours are determined according to Equations (198)-(200). One token is fired to place $P\{\text{Adjusted RA}\}$ with the following colours: $v_{t,A,RA,\min}^0 = v_{t,A,RA,\min}^0$ $v_{t,A,RA,\max}^0 = v_{t,A,RA,\max}^0$ $a_{t,A,RA}^i = 0.35g$ $\dot{\theta}_{\text{Adjusted_RA}} = 0$ $t_{\text{Adjusted_RA}} = 0$ One token is fired to place “P” with the following colours: D_t^A $t_{t RA}$
G2	In total three tokens are fired. One token without colour to place $P\{\text{COC}\}$. One token is fired to place $P\{\text{Adjusted RA}\}$ with the following colours: $v_{t,A,RA,\min}^0 = v_{t,A,RA,\min}^0$ $v_{t,A,RA,\max}^0 = v_{t,A,RA,\max}^0$ $a_{t,A,RA}^i = 0.35g$ $\dot{\theta}_{\text{Adjusted_RA}} = 0$ $t_{\text{Adjusted_RA}} = 0$

	One token without colour to place $P\{\text{Reset}\}$.
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IPN “Adjusted RA”

IPN to indicate the issue of a secondary RA.

Incoming arcs within same agent

- Incoming arc from transition G1 in IPN “Evolution Monitoring”
- Incoming arc from transition G2 in IPN “Evolution Monitoring”
- Incoming arc from transition G1 in IPN “Hor. Man. Selection”
- Incoming arc from transition G2 in IPN “Hor. Man. Selection”
- Incoming arc from transition G3 in IPN “Hor. Man. Selection”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc to transition I2 of LPN “Pilot Flying” of agent “Cockpit i”

Places

There is one place “P” with the following colours.

Table D-20: Place of IPN Adjusted RA of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P	$v_{t,A,RA,\min}^i \in \mathbb{R}$	Lower velocity bound of RA	None
	$v_{t,A,RA,\max}^i \in \mathbb{R}$	Upper velocity bound of RA	None
	$a_{t,A,RA}^i \in \mathbb{R}$	Assigned acceleration of RA	None
	$\dot{\theta}_{Adjusted_RA} \in \mathbb{R}$	Turn rate of RA	None
	$t_{Adjusted_RA} \in \mathbb{R}$	Time stamp of message	None

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

IPN “COC”

IPN to indicate the issue of a clear of conflict message.

Incoming arcs within same agent

- Incoming arc from transition G2 of IPN “Evolution Monitoring”
- Incoming arc from transition G3 of IPN “Hor. Man. Selection”

Outgoing arcs within same agent

- None

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- Outgoing arc to transition I2 of LPN “Mode S Reply” of agent “Communication i”

Places

There is one place “P” without colours.

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

IPN “Reset”

IPN to reset the avoidance system algorithm.

Incoming arcs within same agent

- Incoming arc from transition G2 of IPN “Evolution Monitoring” to place “P”
- Incoming arc from transition G3 of IPN “Hor. Man. Selection” to place “P”

Outgoing arcs within same agent

- Outgoing arc from place “P” to transition I of LPN “Threat Detection”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There is one place “P” without colours.

Initial markings

Initially there is no token at place “P”.

Transitions

There are no transitions in the IPN.

IPN “Hor. Man Message”

This IPN selects the coordination message which is sent to aircraft ‘k’ as described in Section B.8.

Incoming arcs within same agent

- Two enabling arcs from place “P” in LPN “State Estimation” to transitions I and G
- Two enabling arcs from place “P” in LPN “Received Coordination Message” to transitions I and G
- Incoming arc from transition G2 of IPN “Avoidance Logic” to place “P1”

Outgoing arcs within same agent

- Outgoing arc from transition I to place “P” of IPN “Sense coordination”
- Outgoing arc from place “P2” to transition G3 of IPN “Hor. Man. Selection”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There are two places. Place “P1” does not have colours. Place “P2” has the following colour.

Table D-21: Places of IPN Hor. Man. Message of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P2	$m_{t,sent} \in \mathbb{R}$	Sent coordination message	None

Initial markings

Initially there is no token in this IPN.

Transitions

Table D-22: Transitions of IPN Hor. Man. Message of agent ACAS UAV i.

ID	Transition	Condition
I	$P1 \wedge P\{\text{State Estimation}\} \wedge P\{\text{Received Coordination Message}\} \rightarrow P2 \wedge P\{\text{Sense coordination}\}$	None
G	$P2 \wedge P\{\text{State Estimation}\} \wedge P\{\text{Received Coordination Message}\} \rightarrow P1$	If $\left[(m_{t,sent} = m_{t,received}) \wedge (\chi_{ModeS}^0 > \chi_{ModeS}^1) \right]$ from Equation (216), where $m_{t,sent} = m_{t,received} \{P\{\text{Received Coordination Message}\}\}$ $\chi_{ModeS}^0 = \chi_{ModeS}^0 \{P\{\text{State Estimation}\}\}$

		$\chi_{ModeS}^1 = \chi_{ModeS}^1 \{P\{\text{State Estimation}\}\}$
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Table D-23: Firing functions of transitions of IPN Hor. Man. Message of agent ACAS UAV i.

ID	Firing function
I	One token is fired with the colour $m_{t,sent}$ according to Equation (215)
G	One token without colours is fired

IPN “Hor. Man. Selection”

This IPN selects and controls the horizontal avoidance manoeuvre as described in Section B.8.

Incoming arcs within same agent

- Four enabling arcs from LPN “State Estimation” to transitions I, G1, G2 and G3
- Incoming arc from transition G2 of IPN “Avoidance Logic” to place “P1”
- Incoming arc from place “P2” of IPN “Hor. Man. Message” to transition G3

Outgoing arcs within same agent

- Outgoing arc from transition I to place “P” of IPN “RA”
- Outgoing arc from transition G1 to place “P” of IPN “Adjusted RA”
- Outgoing arc from transition G2 to place “P” of IPN “Adjusted RA”
- Outgoing arc from transition G3 to place “P” of IPN “Adjusted RA”
- Outgoing arc from transition G3 to place “P” of agent IPN “COC”
- Outgoing arc from transition G3 to place “P” in IPN “Reset”

Incoming arcs from other agent

- None

Outgoing arcs to other agent

- None

Places

There are four places. Place “P1” has no colours. Places “P2”, “P3” and “P4” have colours. Next the colours are described.

Table D-24: Places of IPN Hor. Man. Selection of agent ACAS UAV i.

Places	Colour type	Explanation	Colour function
P2	$\theta_{t RA}^0 \in \mathbb{R}$	Heading of own aircraft when initial RA was issued.	None
	$\dot{\theta}_{t RA}^0 \in \mathbb{R}$	Turn rate issued in initial RA	None
P3	$\theta_{t RA}^0 \in \mathbb{R}$	Heading of own aircraft when initial RA was issued.	None
	$\dot{\theta}_{t RA}^0 \in \mathbb{R}$	Turn rate issued in initial RA	None
P4	$\theta_{t RA}^0 \in \mathbb{R}$	Heading of own aircraft when initial RA was issued.	None

Initial markings

Initially there is no token in this IPN.

Transitions

Table D-25: Transitions of IPN Hor. Man. Selection of agent ACAS UAV i.

ID	Transition	Condition
I	$P1 \wedge P\{\text{State Estimation}\} \rightarrow P2 \wedge P\{\text{RA}\}$	None
G1	$P2 \wedge P\{\text{State Estimation}\} \rightarrow P3 \wedge P\{\text{Adjusted RA}\}$	If $d_{t,HMD,ADSB} \geq d_{\text{mod}}$ as described in Section B.8, where $\hat{\mathbf{s}}_{t,H,ADSB} = \hat{\mathbf{s}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB} = \hat{\mathbf{v}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A}^0 = \hat{\mathbf{s}}_{t,A}^0 \{P\{\text{State Estimation}\}\}$
G2	$P3 \wedge P\{\text{State Estimation}\} \rightarrow P4 \wedge P\{\text{Adjusted RA}\}$	If $d_{t,HMD,ADSB} \big _{\theta_{t RA}^0} \geq d_{\text{mod}}$ as described in Section B.8, where $\hat{\mathbf{s}}_{t,H,ADSB} = \hat{\mathbf{s}}_{t,H,ADSB} \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB}^1 = \hat{\mathbf{v}}_{t,H,ADSB}^1 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{v}}_{t,H,ADSB}^0 = \hat{\mathbf{v}}_{t,H,ADSB}^0 \{P\{\text{State Estimation}\}\}$ $\hat{\mathbf{s}}_{t,A}^0 = \hat{\mathbf{s}}_{t,A}^0 \{P\{\text{State Estimation}\}\}$
G3	$P4 \wedge P\{\text{State Estimation}\} \wedge P2\{\text{Hor. Man. Message}\} \rightarrow P\{\text{Adjusted RA}\} \wedge P\{\text{COC}\} \wedge P\{\text{Reset}\}$	If $\left y_{t,\theta}^0 - \theta_{t RA}^0 \right \leq \dot{\theta}_{\text{max}}^0$ as described in Section B.8, where $\hat{\theta}_t^0 = \hat{\theta}_t^0 \{P\{\text{State Estimation}\}\}$

Table D-26: Firing functions of transitions of IPN Hor. Man. Selection of agent ACAS UAV i.

ID	Firing function
I	In total two tokens with colours are fired. The colours are determined according to Equations (204)-(214). One token is fired to place $P\{\text{RA}\}$ with the following colours: $v_{t,A,RA,\min}^0 = 0$ $v_{t,A,RA,\max}^0 = 0$ $a_{t,A,RA}^i = 0.25g$ $\dot{\theta}_{RA} = \dot{\theta}_{RA}$ $t_{RA} = 0$ One token is fired to place “P2” with the following colours: $\theta_{t RA}^0$ $\dot{\theta}_{t RA}^0$
G1	In total two tokens are fired.

	One token is fired to place $P\{\text{Adjusted RA}\}$ with the following colours as described in Section B.8: $v_{t,A,RA,\min}^0 = 0$ $v_{t,A,RA,\max}^0 = 0$ $a_{t,A,RA}^i = 0.25g$ $\dot{\theta}_{RA} = 0$ $t_{RA} = 0$ One token is fired to place “P3” with the following colours: $\theta_{t RA}^0 = \theta_{t RA}^0 \{P2\}$ $\dot{\theta}_{t RA}^0 = \dot{\theta}_{t RA}^0 \{P2\}$
G2	In total two tokens are fired. One token is fired to place $P\{\text{Adjusted RA}\}$ with the following colours as described in Section B.8: $v_{t,A,RA,\min}^0 = 0$ $v_{t,A,RA,\max}^0 = 0$ $a_{t,A,RA}^i = 0.25g$ $\dot{\theta}_{RA} = -\dot{\theta}_{t RA}^0 \{P2\}$ $t_{RA} = 0$ One token is fired to place “P4” with the following colours: $\theta_{t RA}^0 = \theta_{t RA}^0 \{P3\}$
G3	In total three tokens are fired. One token is fired to place $P\{\text{Adjusted RA}\}$ with the following colours as described in Section B.8: $v_{t,A,RA,\min}^0 = 0$ $v_{t,A,RA,\max}^0 = 0$ $a_{t,A,RA}^i = 0.25g$ $\dot{\theta}_{RA} = 0$ $t_{RA} = 0$ One token without colour to place $P\{\text{COC}\}$. One token without colour to place $P\{\text{Reset}\}$.