Estimating uncertainties in co-operative networks

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Abstract

The cooperative output regulation problem has attracted considerable attention due to its wide applications in several real life problems; in the past decade, the cooperative control of multi-agent systems has become a trendy topic in the control community. In the cooperative output regulation problem the main purpose consists in achieving reference tracking and disturbance rejection.
State-of-the-art methodologies can solve cooperative output regulation only by assuming some critical a priori knowledge: either the exosystem dynamics (e.g. their harmonic frequencies) can be globally shared; or some structural parameters of the communication graph (e.g. structural eigenvalues) are known; or initial stabilizing controllers are available for each system.
However, from a practical point of view, it is crucial to develop adaptive methodologies to effectively handle uncertainty in cooperative control of network systems, exploiting as little a priori information as possible.
This work addresses and solves the cooperative output regulation problem without exploiting any of the aforementioned critical knowledge.
In fact, the distinguishing feature of the proposed solution is to assume an uncertain cooperative scenario where neither follower nor leader dynamics are globally known. In particular, the exosystem dynamics are assumed to correspond to harmonic oscillators with unknown frequencies.
Cooperative output regulation is achieved by designing, for each system in the network, fully distributed adaptive controllers, i.e. requiring no knowledge of the structural eigenvalues nor initial stabilizing control law.

Table of Contents

Acknowledgements **vii**

1 Introduction **1**
1-1 Open problems and proposed solutions 2

2 Output Regulation Problem **3**
2-1 Linear Output Regulation 3
2-2 Robust Linear Output Regulation 9

3 Cooperative Output Regulation **13**
3-1 Basic Concepts of Graph theory 13
3-2 Kronecker Product 14
3-3 Notation 15
3-4 Decentralized Control 16
3-5 Distributed Observer 17
3-6 Distributed Internal Model 19

4 Adaptive Distributed Observer **23**
4-1 The distributed observer approach 23
4-2 The Adaptive Distributed Observer 24

5 Exosystem Estimator **29**
5-1 Problem formulation 29
5-2 Distributed exosystem estimator 31

6 Adaptive observer **35**
6-1 The adaptive solution to the Regulator Equations 35
6-2 Main result 37

7 Simulations Results **41**
7-1 Persistently Exciting Case . . . 41
7-1-1 Experiment 1 . . . 43
7-1-2 Experiment 2 . . . 45
7-1-3 Experiment 3 . . . 47
7-2 Future work: no persistency of excitation . . . 49

8 Conclusions **53**

Bibliography **55**

List of Figures

2-1 Unity feedback control 4
2-2 Decentralized Feedforward Control. 9

3-1 Example of communication graph. 13
3-2 Decentralized Control Scheme 16
3-3 Distributed Internal Model Approach. 17
3-4 Distributed Internal Model Approach. 19

5-1 To reconstruct the exosystem state, non-target nodes exchange auxiliary variables according to the communication graph. 31

7-1 Communication graph $\mathscr{G}$ 41
7-2 Regulated outputs 43
7-3 Estimates of the exosystem 44
7-4 Estimates of the solution pair (X_i, p_i) to the regulator equations for every agent. 45
7-5 Regulated outputs 46
7-6 Estimates of the the system matrices A_i and b_i for every agent. 47
7-7 Regulated outputs with unknown agents and leader 48
7-8 Estimates of the exosystem 48
7-9 Estimates of the solution pair (X_i, p_i) to the regulator equations for every agent. 49
7-10 Regulated Outputs 50
7-11 Parameters estimation of the first agent 51
7-12 Estimates of the solution pair (X_i, p_i) to the regulator equations for every agent. 52

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Chapter 1

Introduction

The cooperative output regulation problem refers to the problem of making a network of systems (sometimes referred to as follower agents) to follow the behavior of a leader exosystem (sometimes referred to as leader agent).
Cooperation arises from solving the problem in a distributed way when not all systems in the network can access the signals of the leader. The main idea is that the systems not directly connected to the exosystem reconstruct the exosystem signals through communication with neighbors [1, 2].
In the traditional formulations of cooperative output regulation, the system dynamics are considered to be known or belonging to a sufficiently small uncertainty set, and in addition, the exosystem dynamics are assumed to be globally known in the network [3, 4, 5, 6].
Many are the applications of the cooperative control in real scenarios, such as cooperative platooning [7, 8], smart energy [9, 10] and smart traffic [11, 12, 13, 14].
However, in real-life networked environments, information is partial or not available: therefore, it is desirable to consider cooperative output regulation problems in which:

- The system dynamics involve parametric uncertainties.
- The exosystem dynamics are not globally known.

In the first case, a distributed observer with adaptive coupling gains can be used to reconstruct the state of the exosystem. Variants of this technique have been adopted for homogeneous uncertain dynamics [15, 16], heterogeneous known dynamics [17], or special classes of heterogeneous unknown dynamics (e.g. Euler-Lagrange [18, 19], model reference [20, 21, 22], or passifiable [23, 24] dynamics).
In the second case, when the leader dynamics are not globally known, it has been assumed that the systems connected to the leader know them and share this information across the network through consensus dynamics [25].
However, this requires an extra communication channel. To avoid the use of this extra communication, estimation techniques for the leader dynamics have been combined with a robust design [26] or with a learning-based design [27].

1-1 Open problems and proposed solutions

Distributed adaptive techniques offer strong potential to deal with large parametric uncertainties in networks [28]. However, despite the recent advances presented above, uncertain cooperative problems are still solved assuming some critical a priori knowledge:

- Globally known leader dynamics [24]
- Globally shared leader dynamics through consensus [25]
- Sufficiently small uncertainty for fixed-gain robust control [26]
- Initial stabilizing feedback available for each system [27].

The main contribution of this thesis is to solve the cooperative regulation problem without exploiting any of the aforementioned critical knowledge. In particular, in the presence of uncertain dynamics and large parametric uncertainties that cannot be handled by state-of-the-art methodologies, our approach implements three steps of adaptation.

- The first step involves an on-line fully distributed estimation of the exosystem dynamics, which are assumed to be completely unknown (the exosystem is assumed to be a multi-dimensional harmonic oscillator with unknown frequencies).
- The second step involves the on-line estimation of the unknown parameters of a minimum state-space realization of the systems, and the use of adaptive observers to estimate the corresponding states.
- Finally, in the third step, the estimated exosystem and adaptively generated parameters and states of the systems are used to solve the regulator equations and update the parameters of the output regulators. It is shown analytically that, overall, the three adaptation steps lead to asymptotic cooperative regulation.

The thesis is organized as follows: the second chapter concisely presents the output regulation problem, the third chapter introduces the the cooperative output regulation problem for linear systems; the fourth chapter gives a review of different methodologies found in literature to solve the cooperative output regulation problem.
Then, the fifth chapter introduces the distributed exosystem estimator whereas the sixth covers both the adaptive observer for the system dynamics, the adaptive solution to the regulator equations and presents the main result.
Several simulations are carried out in the seventh chapter, while in the eighth and last chapter,possible future development and conclusions are presented.

Chapter 2

Output Regulation Problem

In this chapter a concise but self-contained analysis of the output regulation problem for LTI systems is given, in order to have a clear understanding of this topic when it will be extended to the cooperative case. In the first section, the linear output regulation problem is defined whereas in the second section the robust approach to parametric uncerainties in the plant is presented.

2-1 Linear Output Regulation

Many practical control problems such as car's cruise control, trajectory planning of a robot, attitude control of an airplane, and so on, fall into the classical scenario represented in Fig. 2-1.
We have a plant, subject to a disturbance $d(t)$ and a controller that guarantees disturbance rejection and reference tracking, meaning that not only the closed loop system is not affected by the disturbance but also that the output $y(t)$ asymptotically tracks the reference r(t), leading the error $e(t)$ to 0 as follows

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}(y(t) - r(t)) = 0 \tag{2-1}$$

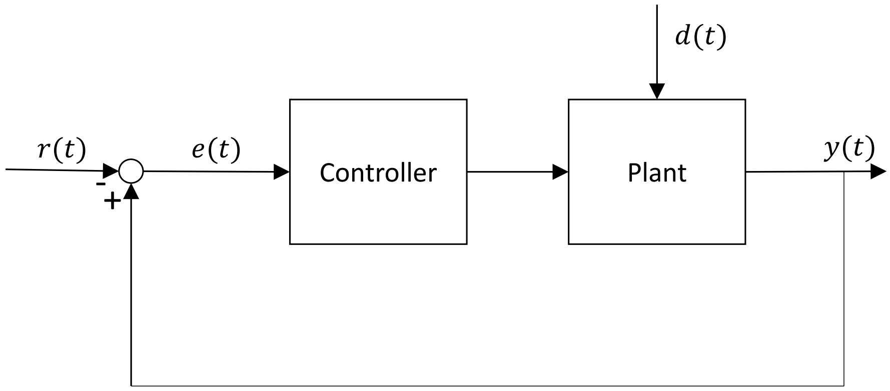


Figure 2-1: Unity feedback control

From control theory, it is known that a linear system with with m inputs, p outputs and n state variables can be represented in its state space form as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{2-2}$$

where $x(\cdot)$ is the state vector, with $x(t) \in \mathbb{R}^n$; $y(\cdot)$ is the output vector, with $y(t) \in \mathbb{R}^p$; $u(\cdot)$ is the input (or control) vector, with $u(t) \in \mathbb{R}^m$; A is the state (or system) matrix of dimension $n \times n$, B is the input matrix, of dimension $n \times m$; C is the output matrix, of dimensions $p \times n$ and, finally, D is the feedthrough (or feedforward) matrix, of dimensions $p \times m$.
When the system doesn't have a direct feedthrough, the matrix D is equal to 0.
However, it is possible to incorporate in this representation also the disturbance acting on the plant as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu + E_d d \\ y &= Cx + Du + F_d d \end{aligned} \tag{2-3}$$

Therefore, the tracking error e is given by

$$e = Cx + Du + F_d d - r \tag{2-4}$$

In order to give a systematic representation of the output regulation problem, we should be able to design a single controller capable to simultaneously handle class of references and disturbances; this objective is achieved by generating both references and disturbances through a autonomous linear differential equation defined as:

$$\dot{r} = S_{1r} r, \;\; r(0) = r_0, \;\; \dot{d} = S_{1d} d, \;\; d(0) = d_0 \tag{2-5}$$

where r_0 and d_0 are the initial states. Stacking together the reference and the disturbance in a vector v, it is possible two consider the two differential equation as a single autonomous dynamic system, defined as follows:

$$\dot{v} = Sv, \;\; v(0) = v_0 \tag{2-6}$$

where

$$v = \begin{bmatrix} r \\ d \end{bmatrix}, \;\; S = \begin{bmatrix} S_{1r} & 0 \\ 0 & S_{1d} \end{bmatrix}, \;\; v_0 = \begin{bmatrix} r_0 \\ d_0 \end{bmatrix} \tag{2-7}$$

Thus, equations (2-3) can be put in the following form

$$\begin{aligned} \dot{x} &= Ax + Bu + Ev \\ e &= Cx + Du + Fv \end{aligned} \tag{2-8}$$

where

$$\begin{bmatrix} E \\ F \end{bmatrix} = \begin{bmatrix} 0 & E_d \\ -I & F_d \end{bmatrix} \tag{2-9}$$

In the continuation of the thesis, the autonomous dynamic system in (2-6) will be referred to as exosystem (or leader in the multi-agent case).
With this new formulation, the asymptotic reference tracking and the disturbance rejection can be treated as the regulation of the error e to the origin and this is the reason why the problem of asymptotic tracking and disturbance rejection can be referred to as *output regulation*. Two are the classes of feedback control that solves the linear output regulation:

- *Static State Feedback*

  $$u = K_x x + K_v v \tag{2-10}$$

  where K_x and K_v are constant matrices named feedback gain and feedforward gain, respectively

- *Dynamic Measurement Output Feedback*

  $$u = Kz \qquad \dot{z} = G_1 z + G_2 y_m \tag{2-11}$$

  where y_m represent the output available for measurement and it is defined as

  $$y_m = C_m x_m + D_m u_m + F_m v. \tag{2-12}$$

  However, in many cases the available output is the error e itself, and, as a consequence, we have that $C_m = C$, $D_m = D$, $F_m = F$ and the obtained controller is defined as *Dynamic Error Feedback*.

The static case is also referred to as "full information case", since it is based on the assumption that all the states x and v can be measured and, therefore, used by the controller.

Definition 1. [Linear Output Regulation Problem] Given the system (2-8), a controller (2-10) or (2-11) must be found such that:

- The closed loop system is Hurwitz
- For any initial condition $x(0)$, $v(0)$, the tracking error is asymptotically steered to 0

  $$\lim_{t \to \infty} e(t) = 0 \tag{2-13}$$

By closing the loop using either (2-10) or (2-11), the following results are obtained

$$\begin{aligned} \dot{x}_{cl} &= A_{cl} x_{cl} + B_{cl} v, \;\; x_{cl}(0) = x_{cl0} \\ \dot{v} &= Sv, \\ e &= C_{cl} x_{cl} + D_{cl} v \end{aligned} \tag{2-14}$$

where, for the static case $x_{cl} = x$ and

$$\begin{aligned} A_{cl} &= A + BK_x, \quad & B_{cl} &= E + BK_v \\ C_{cl} &= C + DK_x \quad & D_{cl} &= F + DK_v \end{aligned} \tag{2-15}$$

while for the dynamic case we have that $x_{cl} = \begin{bmatrix} x \\ z \end{bmatrix}$ and

$$\begin{aligned} A_{cl} &= \begin{bmatrix} A & BK \\ G_2C_m & G_1 + G_2D_mK \end{bmatrix}, \quad B_{cl} = \begin{bmatrix} E \\ G_2F_m \end{bmatrix} \\ C_{cl} &= \begin{bmatrix} C & DK \end{bmatrix}, \quad D_{cl} = F \end{aligned} \tag{2-16}$$

In this scenario, controllers (2-10), (2-11) manage to keep the closed loop exponentially stable.
In [29], the closed loop system (2-14) is said to be exponentially stable if the following conditions hold

- A_{cl} is Hurwitz
- for every initial state x_{c0} and v_0, the dynamics described by (2-20) satisfy

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (C_{cl}x_c(t) + D_{cl}v(t)) = 0 \tag{2-17}$$

Furthermore, the following assumptions are needed in order the solve the linear output regulation problem

Assumption 1. S has no negative eigenvalues

Assumption 2. the pair (A, B) is stabilizable

Assumption 3. The pair

$$\left(\begin{bmatrix} C_m & F_m \end{bmatrix}, \begin{bmatrix} A & E \\ 0 & S \end{bmatrix} \right) \tag{2-18}$$

is detectable

The first asssumption guarantees that the modes associated with the eigenvalues of the exosystem S will not decay to asymptotically while the second one guarantees that the problem is solvable through state feedback, meaning that it is always possible to determine a feedback matrix K_x such that $A + BK_x$ is Hurwitz.
Finally , the third assumption together with the second one guarantees that the problem is solvable through output feedback.

A fundamental result for the output regulation theory is given by the following Lemma , whose demonstration can be found in [29, Chap. 1]

Lemma 1. *Under Assumptions 1-3, given the controller* (2-10) *or* (2-11) *that makes the closed loop matrix A_{cl} Hurwitz, the following statements are equivalent:*

1. *The closed loop system satisfies* (2-17).
2. *The given controller solves the output regulation problem*
3. *Exists a unique matrix X_{cl} that solves the following algebraic equations*

$$\begin{aligned} X_{cl}S &= A_{cl}X_c + B_{cl} \\ 0 &= C_{cl}X_{cl} + D_{cl} \end{aligned} \tag{2-19}$$

This result is extremely important and represents the cornerstone for the output regulation theory: by linking the solvability of the output regulation problem to the one of a set of linear matrix equations, mathematical tools of linear algebra can be exploited to solve the problem.
Since during the continuation of this thesis work, a modified version of the static state feedback will be adopted as controller, let us focus our attention on this case.
Let us rewrite the closed loop system equations (2-14) and the algebraic equations (2-19) for the static case in order to underline some fundamental concepts:

$$\begin{aligned} \dot{x} &= (A + BK_x)x + (E + BK_v)v, \\ \dot{v} &= Sv, \\ e &= (C + DK_x)x + (F + DK_v)v \end{aligned} \tag{2-20}$$

$$\begin{aligned} X_{cl}S &= (A + BK_x)X_{cl} + E + BK_v \\ 0 &= (C + DK_x)X_{cl} + F + DK_v \end{aligned} \tag{2-21}$$

The main idea behind the static state feedback controller consists in using the feedback term K_x to make the closed loop matrix A_{cl} Hurwitz while the feedforward term K_v annihilate the steady state error, achieving, simultaneously, reference tracking and disturbance rejection.
More specifically, the feedforward term drives the closed loop system towards a subspace define by the hyperplane $(C + DK_x)x + (F + DK_v)v = 0$, where the trajectories remain since A_{cl} is Hurwitz.
Consequently, there is a straightforward and schematic way to solve the linear output regulation:

1. Choose a feedback term K_x such that A_{cl} is Hurwitz
2. Find a solution pair (X_{cl}, K_x) to the set of equations (2-21)
3. Define the control input as $u(t) = K_x x(t) + K_v v(t)$

The clear drawback of this approach lies in the dependency of X_{cl} and K_v on the feedback term K_x, leading to a new computation of X_{cl} and K_v, for every new design of the feedback term K_x.
This problem can be overcome by defining two matrices X and U such that:

$$\begin{bmatrix} X \\ U \end{bmatrix} = \begin{bmatrix} I_n & 0_{n\times m} \\ K_x & I_m \end{bmatrix} \begin{bmatrix} X_{cl} \\ K_v \end{bmatrix} \tag{2-22}$$

leading to the the well known *Regulator Equations*

$$\begin{aligned} XS &= AX + BU + E \\ 0 &= CX + DU + F \end{aligned} \tag{2-23}$$

The obtained results can be summarized in the following lemmma

Lemma 2. *Under Assumptions 1 and 2, the linear output regulation problem is solvable through static state feedback*

$$u = K_x x + K_v v \tag{2-24}$$

by choosing K_x such that A_{cl} is Hurwitz and K_v such that

$$K_v = U - K_x X \tag{2-25}$$

where (X,U) is a solution pair to the Regulator Equations (2-23)

By analyzing more accurately the Regulator Equations, it is possible to infer some useful concepts: by considering v to be constant, the exosystem matrix S becomes, obviously, equal to 0.
As a consequence, equations (2-21) and (2-23) become respectively:

$$\begin{aligned} 0 &= (A+BK_x)X_{cl} + E + BK_v \\ 0 &= (C+DK_x)X_{cl} + F + DK_v \end{aligned} \tag{2-26}$$

and

$$\begin{aligned} 0 &= AX + BU + E \\ 0 &= CX + DU + F \end{aligned} \tag{2-27}$$

From (2-26), it can be inferred that $X_{cl}v$ represents the equilibrium of the closed loop system at which the output is zero; furthermore, $X_{cl}v$ represents also the steady state value of the closed loop system. Instead, by analyzing (2-27), it can be deduced that Uv represents the input that drives the open loop system towards the equilibrium point Xv, where the output is 0.
Since from the linear transformation (2-22) it has been established that $X = X_{cl}$, it can be seen that the steady state value of the input $u(t)$ will converge to Uv:

$$\lim_{t\to\infty} u(t) = (K_x X + K_v)v = Uv \tag{2-28}$$

These considerations can be extended to the general case, where v is not constant, leading to the following results, achieved under any controller able to solve the output regulation problem:

$$\lim_{t\to\infty} x_{cl}(t) - Xv(t) = 0 \tag{2-29}$$

and

$$\lim_{t\to\infty} u(t) - Uv(t) = 0 \tag{2-30}$$

What emerges from these results is that, when the output regulation problem is solvable, the steady state behavior of the closed loop depends on the solution of the regulator equations.

Assumption 4. For every $\lambda \in \sigma(S)$, where $\sigma(S)$ is the spectrum of the matrix S

$$rank \begin{bmatrix} A-\lambda I & B \\ C & D \end{bmatrix} = n + p \tag{2-31}$$

where n is the state space dimension, while p the output space dimension.

This assumption is fundamental to an important result, presented in the following lemma:

Lemma 3. *For any matrices E and F, the regulator equations* (2-23) *are solvable if and only Assumption 4 holds*

Without going into excessive details (the proof can be found in [29]), in this lemma it is demonstrated that it is possible to transform (2-23) into a standard linear algebraic equation of the form:

$$Qx = b \tag{2-32}$$

where

$$\begin{aligned} Q &= S^T \otimes \begin{bmatrix} I_n & 0_{n\times m} \\ 0_{p\times n} & 0_{p\times m} \end{bmatrix} - I_q \otimes \begin{bmatrix} A & B \\ C & D \end{bmatrix} \\ x &= \text{vec}\left(\begin{bmatrix} X \\ U \end{bmatrix}\right) \quad b = \text{vec}\left(\begin{bmatrix} E \\ F \end{bmatrix}\right) \end{aligned} \tag{2-33}$$

where $\otimes$ denotes the Kronecker product, whose properties can be found in the next chapter, while the notation vec indicate a vector-valued function of a matrix defined such as, given a matrix $H \in R^{m\times n}$ we obtain:

$$\text{vec}(H) = \begin{bmatrix} H_1 \\ \vdots \\ H_n \end{bmatrix} \tag{2-34}$$

where for $i = 1,\dots,m$, H_i is the i_{th} column of H.

2-2 Robust Linear Output Regulation

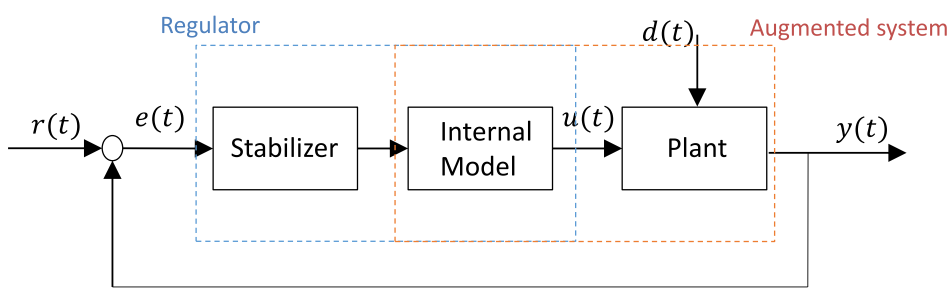


Figure 2-2: Decentralized Feedforward Control.

In the previous section, the proposed approaches only works in a scenario where the plant dynamics are completely known.
The situation completely changes when we try to keep into account plant uncertainties, leading to the following class of systems:

$$\begin{aligned} \dot{x} &= (A+\Delta A)x + (B+\Delta B)u + (E+\Delta E)v \\ e &= (C+\Delta C)x + (D+\Delta D)u + (F+\Delta F)v \end{aligned} \tag{2-35}$$

Here A, B, C, D, E, F,C, denote the nominal part of the plant while ΔA, ΔB, ΔC, ΔD, ΔF, denote the uncertain part. For ease of notation, it is convenient to collect together the uncertainties in the vector $w = vec(\begin{bmatrix} \Delta A & \Delta B & \Delta C \\ \Delta D & \Delta E & \Delta F \end{bmatrix})$,
Then, by defining

$$\begin{aligned} A_w &= A + \Delta A \quad & B_w &= B + \Delta B \quad & E_w &= E + \Delta E \\ C_w &= C + \Delta C \quad & D_w &= D + \Delta D \quad & F_w &= F + \Delta F \end{aligned} \tag{2-36}$$

it is possible to rewrite (2-35) in a compact way, together with the exosystem (2-6):

$$\begin{aligned} \dot{x} &= A_w x + B_w u + E_w v \\ \dot{v} &= Sv \\ e &= C_w x + D_w u + F_w v \end{aligned} \tag{2-37}$$

In the previous section a static state feedback and a dynamic output feedback were shown to solve the linear outptut regulation problem.
Because of the presence of the uncertain parameter w, no static state feedback can solve the robust output regulation problem and, for this reason, a dynamic state or output feedback is needed, such as:

- Dynamic State Feedback

$$\begin{aligned} u &= K_1 x + K_2 z \\ \dot{z} &= G_1 z + G_2 y_m \end{aligned} \tag{2-38}$$

or

- Dynamic Output Feedback

$$\begin{aligned} u &= Kz \\ \dot{z} &= G_1 z + G_2 y_m \end{aligned} \tag{2-39}$$

where y_m is the measurement output.

Definition 2. [Linear Robust Output Regulation Problem] Given the system (2-35), a controller (2-38) or (2-39) must be found such that:

- The closed loop system is Hurwitz
- For any initial condition $x(0)$, $v(0)$, the tracking error is asymptotically steered to 0

$$\lim_{t \to \infty} e(t) = 0 \tag{2-40}$$

It is worth delving deeper into the reasons why a static feedback no longer represents a valid option to solve the output regulation problem for the robust case.
Let us first consider the closed loop matrices, obtained for the two classes of controllers:

$$\begin{aligned} A_{wcl} &= \begin{bmatrix} A_w + B_w K_1 & B_w K_2 \\ G_2(C_w + D_w K_1) & G_1 + G_2 D_w K_2 \end{bmatrix}, \quad B_{wcl} = \begin{bmatrix} E_w \\ G_2 F_w \end{bmatrix} \\ C_{wcl} &= \begin{bmatrix} C_w + D_w K_1 & D_w K_2 \end{bmatrix}, \quad D_{wcl} = F_w \end{aligned} \tag{2-41}$$

for the state feedback case and

$$A_{wcl} = \begin{bmatrix} A_w & B_w K \\ G_2 C_w & G_1 + G_2 D_w K \end{bmatrix}, \quad B_{wcl} = \begin{bmatrix} E_w \\ G_2 F_w \end{bmatrix}$$
$$C_{wcl} = \begin{bmatrix} C_w & D_w K \end{bmatrix}, \quad D_{wcl} = F_w \tag{2-42}$$

for the output feedback case. In this way, (2-37) becomes:

$$\begin{aligned} \dot{x}_{cl} &= A_{wcl} x_{cl} + B_{wcl} v \\ \dot{v} &= S v \\ e_{cl} &= C_{wcl} x_{cl} D_{wcl} v \end{aligned} \tag{2-43}$$

where $x_{cl} = col(x, z)$.
Similarly to the previous section, the closed loop system (2-43) is said to be exponentially stable if the following conditions hold

- A_{wcl} is Hurwitz
- for every initial state x_{c0} and v_0, the dynamics described by (2-20) satisfy

$$\lim_{t \to \infty} e(t) = \lim_{t \to \infty} (C_{wcl} x_{cl}(t) + D_{wcl} v(t)) = 0 \tag{2-44}$$

Then, a result, whose demonstration can be found in [29], is given as follows

Lemma 4. *Under Assumptions 1-3, given a controller such as* (2-38) *or* (2-39) *that renders the closed loop matrix A_{wcl} Hurwitz, the following statements are equivalent:*

1. *There exists a neighbourhood W of $w = 0$ such that the closed loop system* (2-43) *satisfies* (2-44)
2. *The controller solves the linear robust output regulation problem*
3. *For each $w \in W$ such that A_{wcl} is exponentially stable, exists a unique matrix X_{cw} that solves the following matrix equations*

$$\begin{aligned} X_{wcl} S &= A_{wcl} X_c + B_{wcl} \\ 0 &= C_{wcl} X_{wcl} + D_{wcl} \end{aligned} \tag{2-45}$$

Differently from the previous section, these equations depend on the uncertain parameter w and this makes the robust output regulation problem more difficult to solve.
In fact, whilst for the previous, non-robust, case it was possible to find a solution pair (X, U) to the regulator equations (2-23), this cannot apply to the current case since the uncertain regulator equations (2-45) depend on variable parameter w.
As a consequence, no fixed-gain static feedback controller is able to solve the robust output regulation.
Since using the solutions of the regulator equations is no longer a valid option, another methodology is presented: the well known *Internal Model Principle*, first introduced in [30].
This methodology works as follows:

1. A dynamic compensator of the form

$$\dot{z} = G_1 z + G_2 y_m \tag{2-46}$$

is chosen such that it incorporates a p-copy internal model of the system (2-37)

2. An augmented system

$$\begin{aligned} \dot{x} &= Ax + Bu + Ev \\ \dot{z} &= G_1 z + G_2 y_m \\ e &= Cx + Du + Fv \end{aligned} \tag{2-47}$$

is defined, where the uncertainties no longer appear since they are keep into account by the internal model

3. Finally, either a dynamic state feedback or a dynamic output feedback that stabilizes the augmented system (2-47) is designed.
In this way, the robust output regulation problem is solved in an open neighborhood W of $w = 0$.

The mathematical details of how the internal model is able to satisfy the third statement of Lemma 5 lie outside the purpose of this thesis work and can be found in [29, Theorem 1.30]
What is clear from this brief summary about the output regulation theory, is that

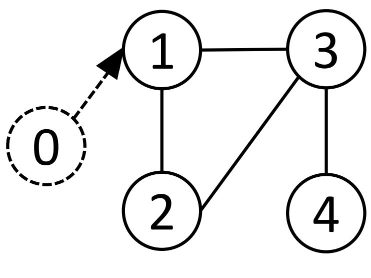
- Parameters uncertainties are usually dealt with exploiting a robust approach.
- The robust approach only works with small uncertainties while the adaptive control can overcome this problem, otherwise unsolved.
- It is still necessary to explain how to solve solve the problem in the cooperative case.

Chapter 3

Cooperative Output Regulation

Since the core of this project is the development of an adaptive, fully distributed solution to the Cooperative output regulation problem, all the concepts presented in Chapter 2 will be now extended to the cooperative case. In the first section a small overview of the graph theory is given, then the Kronecker product, together with its properties, is given; in the third section the notation that will be used throughout the thesis work is given, while in the fourth section the reason why the decentralized solution is impossible in our case are shortly explained.
Then, the distributed observer is introduced in the fifth section while in the last section a short explanation of how is usually solved the robust output regulation for the cooperative case is given.

3-1 Basic Concepts of Graph theory



$$\mathcal{G} = \{\mathcal{V}, \mathcal{E}\} \qquad \bar{\mathcal{G}} = \{\mathcal{V}, \mathcal{E}, \mathcal{T}\}$$
$$\mathcal{V} = \{1, 2, 3, 4\} \qquad \mathcal{T} = \{1\}$$
$$\mathcal{E} = \{(1,2), (2,1), (1,3), (3,1), (3,2), (2,3), (3,4), (4,3)\}$$

Figure 3-1: Example of communication graph.

In the study of coordination and synchronization problems of Multiagent Systems (MAS), communication between agents plays a fundamental role.
Suppose, therefore, that multiple systems interact with each other according to a precise communication topology: appears natural modeling these interactions between systems by graphs.
We consider networks of dynamical systems (also referred to as nodes), which are linked to each other via a *communication graph*, that describes the allowed information flow.
In other words, we say that system i has a *directed* connection to system j if the second can receive

information from the first.
When the information can flow in both directions, the connection is said to be *undirected*.
In a communication graph, a special role is played by the *leader* node, which is a system (typically indicated as system 0) that does not receive information from any other system in the network.
The communication graph describing the allowed information flow between all the systems, *leader excluded*, is completely defined by the pair $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, \ldots, N\}$ is a finite nonempty set of nodes, and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is a set of pairs of nodes, called edges.
To include the presence of the leader in the network we define $\bar{\mathscr{G}} = \{\mathscr{V}, \mathscr{E}, \mathscr{T}\}$, where $\mathscr{T} \subseteq \mathscr{V}$ is the set of those nodes, called *target nodes*, which receive information from the leader. Fig. 3-1 provides a simple exle of how $\mathscr{V}$, $\mathscr{E}$, and $\mathscr{T}$ can be defined. Two square matrices are instrumental to find many useful properties of a communication graph: the *adjacency matrix* $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ and the *Laplacian matrix* $\mathscr{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$.
Specifically, the adjacency matrix of an undirected communication graph is defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathscr{E}$, where $i \neq j$; the Laplacian matrix is defined as $l_{ii} = \sum_j a_{ij}$ and $l_{ij} = -a_{ij}$, if $i \neq j$.
The adjacency and the Laplacian matrices corresponding to the exle in Fig. 3-1 are

$$\mathscr{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathscr{L} = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

In addition, we use a square diagonal matrix, the *target matrix* $\mathscr{M} = [m_{ij}] \in \mathbb{R}^{N \times N}$, to describe the directed communication of the leader with the target nodes.
Specifically, the target matrix is defined as $m_{ii} = 1$ if $i \in \mathscr{T}$ and $m_{ii} = 0$ otherwise. In the exle of Fig. 3-1, we have $\mathscr{M} = \text{diag}(1,0,0,0)$.
An undirected graph $\mathscr{G}$ is said to be *connected* if, taken any arbitrary pair of nodes (i, j) where $i, j \in \mathscr{V}$, there is a path that leads from i to j. Note that the graph $\mathscr{G}$ in Fig. 3-1 is undirected and connected.
Finally, let us define the *leader-follower topology matrix* as $\mathscr{B} = \mathscr{L} + \mathscr{M}$. When $\mathscr{L}$ is the Laplacian matrix of an undirected and connected graph, $\mathscr{B}$ is positive definite by construction.

3-2 Kronecker Product

In mathematics, in the field of linear algebra, the Kronecker product is indicated with $\otimes$ and represents an operation between two arrays of arbitrary size, always applicable. If A Is a matrix $m \times n$ and B a matrix $p \times q$, then their Kronecker product is a matrix defined as follows:

$$A \otimes B = \begin{pmatrix} a_{11}B & \ldots & a_{1n}B \\ \ldots & \ldots & \ldots \\ a_{m1}B & \ldots & a_{mn}B \end{pmatrix} \tag{3-1}$$

or, if we expand every term

$$A \otimes B = \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & \cdots & a_{11}b_{1q} & \cdots & \cdots & a_{1n}b_{11} & a_{1n}b_{12} & \cdots & a_{1n}b_{1q} \\ a_{11}b_{21} & a_{11}b_{22} & \cdots & a_{11}b_{2q} & \cdots & \cdots & a_{1n}b_{21} & a_{1n}b_{22} & \cdots & a_{1n}b_{2q} \\ \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \ddots & \vdots \\ a_{11}b_{p1} & a_{11}b_{p2} & \cdots & a_{11}b_{pq} & \cdots & \cdots & a_{1n}b_{p1} & a_{1n}b_{p2} & \cdots & a_{1n}b_{pq} \\ \vdots & \vdots & & \vdots & \ddots & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & & \ddots & \vdots & \vdots & & \vdots \\ a_{m1}b_{11} & a_{m1}b_{12} & \cdots & a_{m1}b_{1q} & \cdots & \cdots & a_{mn}b_{11} & a_{mn}b_{12} & \cdots & a_{mn}b_{1q} \\ a_{m1}b_{21} & a_{m1}b_{22} & \cdots & a_{m1}b_{2q} & \cdots & \cdots & a_{mn}b_{21} & a_{mn}b_{22} & \cdots & a_{mn}b_{2q} \\ \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \ddots & \vdots \\ a_{m1}b_{p1} & a_{m1}b_{p2} & \cdots & a_{m1}b_{pq} & \cdots & \cdots & a_{mn}b_{p1} & a_{mn}b_{p2} & \cdots & a_{mn}b_{pq} \end{pmatrix} \quad (3\text{-}2)$$

The Kronecher product has the following properties:

- $A \otimes (B+C) = A \otimes B + A \otimes C$
- $(A+B) \otimes C) = A \otimes C + B \otimes C$
- $(kA) \otimes B) = A \otimes (kB) = k(A \otimes B)$
- $(A \otimes B) \otimes C = A \otimes (B \otimes C)$

However, the commutative property does not hold for this kind of product. It is worth mentioning how Kronecker product and regular matrix product work together:

- $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$
- $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$

In the rest of the thesis work the Kronecker product will play a fundamental role, making the notation for the demonstrations over the networks much more compact.

3-3 Notation

The transpose of a matrix or of a vector is indicated with X^T and x^T. The $n \times n$ identity matrix is denoted by I_n.
A diagonal matrix $\Delta = \text{diag}(\delta_1, \delta_2, \ldots, \delta_n)$ is denoted as $\text{diag}(\delta_k)_{\overleftarrow{n}}$; a block-diagonal matrix is denoted as $\Delta = \text{bdiag}(\Delta_k)_{\overleftarrow{n}}$.
A matrix $M \in \mathbb{R}^{n\times n}$ is said to be negative definite if, for every non-zero vector $x \in \mathbb{R}^n$, it results $x^T M x < 0$.
A vector signal $x(\cdot)$ is said to belong to $\mathscr{L}_2$ ($x \in \mathscr{L}_2$), if $\int_0^t \left\|x(\tau)\right\|^2 \mathrm{d}\tau < \infty$, $\forall t \geq 0$. A vector signal $x(\cdot)$ is said to belong to $\mathscr{L}_\infty$ ($x \in \mathscr{L}_\infty$), if $\max\limits_{t\geq 0} \left\|x(t)\right\| < \infty$, $\forall t \geq 0$.

3-4 Decentralized Control

Let us define a linear heterogeneous multi-agent system composed by N agents as follows:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v \\ e &= C_i x_i + D_i u_i + F_i v \end{aligned} \tag{3-3}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, $e_i \in R^{p_i}$, and $v \in R^q$.
As for the single agent case, let us also define the exosystem, that generates either the reference r and the disturbance d as follows:

$$\dot{v} = Sv \tag{3-4}$$

Then, systems Eq. (3-3) and Eq. (3-4) can be considered as a multi-agent system of $N+1$ agents, where the exosystem Eq. (3-4) is the leader and the agents Eq. (3-3) are the N followers. If every follower can access the state v of the leader, then the output regulation problem of system Eq. (3-3) can be handled by a so-called decentralized control scheme, shown in Fig. 3-2

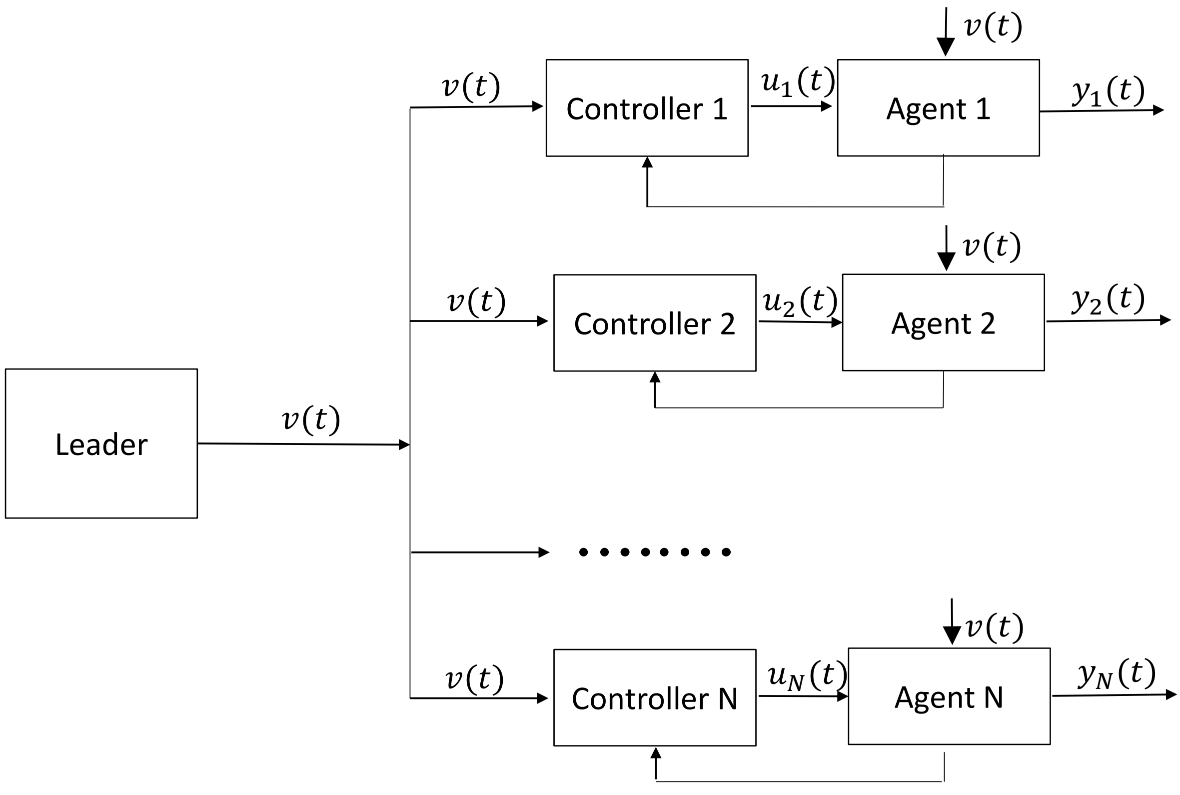


Figure 3-2: Decentralized Control Scheme

However, in the cooperative output regulation problem, the information flow must satisfy some constraints, such as not every agent can communicate with the exosystem and/or with every other agent. Consequently, two types of agents are considered:

- informed followers that can directly communicate with the leader
- uninformed followers that cannot communicate directly with the leader

For this the reason, it is necessary to find a way to solve the output regulation problem for a multi-agent system by using a distributed control strategy (i.e. exploiting only own measurements and measurements from neighbors).
Basically, our objective is to steer every i_{th} regulated output e_i to 0, as follows

$$\lim_{t \to \infty} e_i(t) = 0 \qquad\qquad i = 1, \dots, N \tag{3-5}$$

A possible approach used to solve the cooperative output regulation for a linear multi-agent system, when no uncertainties are taken into account, consists in exploiting the so called *Distributed Observer*.

3-5 Distributed Observer

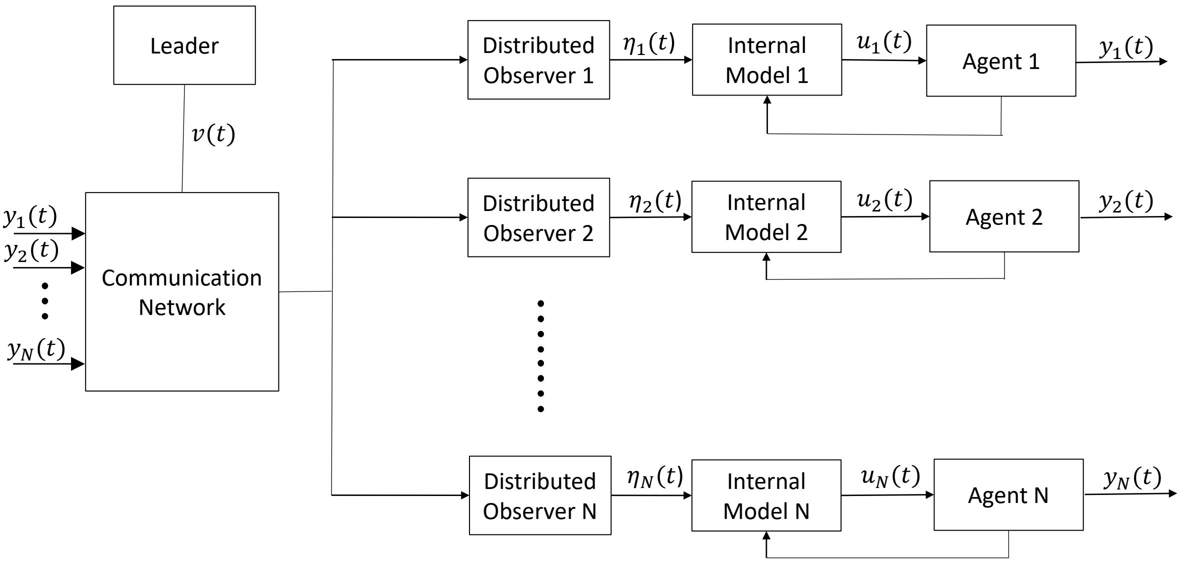


Figure 3-3: Distributed Internal Model Approach.

As mentioned in the previous section, this approach is based on a dynamic compensator, the distributed observer, that, by exploiting only neighboring information, is able to provide to every agents an estimation of the leader dynamics.
An example of distributed observer is given by:

$$\dot{\eta}_i = S\eta_i + \mu \left(\sum_{j \in N_i} a_{ij}(\eta_j - \eta_i) + m_{ii}(v - \eta_i) \right) \tag{3-6}$$

where μ is some positive number, $\eta_i \in \mathbb{R}^q$ is the estimate of v. The dynamics η_i are clearly influenced by η_j, meaning that the approximation of the exosystem state v is obtained by the exploitation of neighboring information.
Clearly, the i_{th} compensator depends on v if and only if $m_{ii} \neq 0$, meaning that the agent i is a neighbor of the leader.
Similarly to the single agent case, the following assumptions are needed for the solution of the cooperative output regulation problem:

Assumption 5. The exosystem matrix S has no eigenvalues with negative real part

Assumption 6. The pairs (A_i,B_i) are stabilizable, $i = 1 \dots N$

Assumption 7. The regulator equations

$$\begin{aligned} X_i S &= A_i X_i + B_i U_i + E_i \\ 0 &= C X_i + D U_i + F_i \end{aligned} \tag{3-7}$$

have solution pairs (X_i,U_i)

Then, in the full information case (i.e. the states of the agents are measurable), the distributed controller becomes

$$\begin{aligned} u_i &= K_{x_i} x_i + K_{\eta_i} \eta_i \\ \dot{\eta}_i &= S\eta_i + \mu \left(\sum_{j \in N_i} a_{ij}(\eta_j - \eta_i) + a_{i0}(v - \eta_i) \right) \end{aligned} \tag{3-8}$$

where K_{x_i} is chosen such that $A + BK_{x_i}$ is Hurwitz and K_{η_i} is obtained by exploiting the solution of the regulator equation (3-7)

$$K_{\eta i} = U_i - K_{xi} X_i \tag{3-9}$$

Moreover, also the output feedback solution, as for the single agent case, is possible by considering some extra assumptions.
Let us define a measurement output y_{m_i}, then the equations describing the dynamics of the N agents become:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v \\ y_{m_i} &= C_{m_i} x_i + D_{m_i} + F_{m_i} u_i \\ e &= C_i x_i + D_i u_i + F_i v \end{aligned} \tag{3-10}$$

Then by adding the following detectability assumptions

Assumption 8. The pairs (C_{m_i},A_i) are detectable, for every informed follower

Assumption 9. The pairs $\left(\begin{bmatrix} C_{m_i} & F_{m_i} \end{bmatrix}, \begin{bmatrix} A_i & E_i \\ 0 & S \end{bmatrix} \right)$ are detectable, for every uninformed follower

the problem can be solved by a dynamic output feedback: such as

$$\begin{aligned} u_i &= K_{xi} z_i + K_{\eta i} \eta_i \\ \dot{\eta}_i &= S\eta_i + \mu \left(\sum_{j \in N_i} a_{ij}(\eta_j - \eta_i) + a_{i0}(v - \eta_i) \right) \\ \dot{z}_i &= G_{1i} z_i + G_{2i} y_{mi} \end{aligned} \tag{3-11}$$

where G_{1i} and G_{2i} are constant matrices.
Assumption 8 establishes that for all the uninformed agents (i.e. the ones not directly linked to the leader) the state x_i is detectable from the measurement output y_{m_i}, while Assumption 9 establishes that, for all the informed agents (i.e. the ones directly linked to the leader) the state x_i and the exosystem state v is detectable from the measurement output y_{m_i}.

Definition 3. [Linear Cooperative Output Regulation Problem] Given the multi-agent system (3-3) and a graph $\mathcal{G}$, a controller (3-8) or (3-11) must be found such that:

- The closed loop system is Hurwitz
- For any initial condition $x_i(0)$, $\eta_i(0)$ for $i = 1 \dots N$ and $v(0)$, the tracking error is asymptotically steered to 0

$$\lim_{t \to \infty} e_i(t) = 0 \quad i = 1 \dots N \tag{3-12}$$

Under Assumptions 5-9, this is possible if and only if the graph $\mathcal{G}$ contains a directed spanning tree, namely there is a subgraph that connects every vertex of $\mathcal{G}$.

3-6 Distributed Internal Model

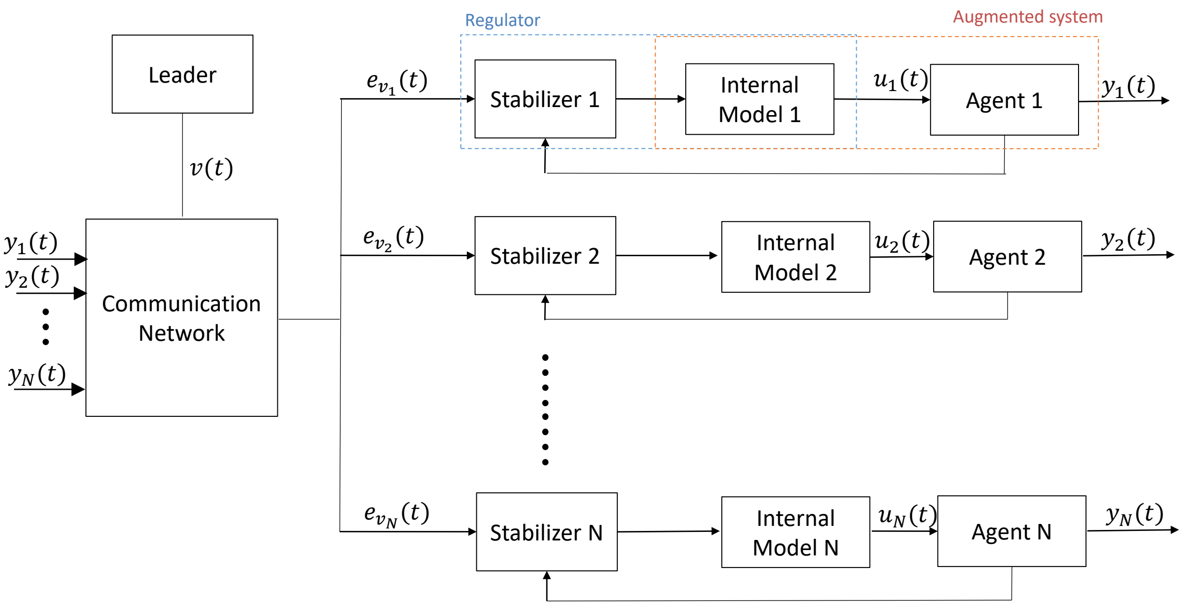


Figure 3-4: Distributed Internal Model Approach.

In the previous section, the cooperative output regulation problem was tackled when no uncertainty is taken into account; in order to obtain a robust result, a distributed internal model approach must be chosen.
It consists in converting the cooperative output regulation problem of a multi-agent system into a stabilization problem of a multiple augmented system composed of the given multi-agent system and the distributed internal model.
Let us define the uncertain multi agent system as:

$$\begin{aligned} \dot{x}_i &= (A_i + \Delta A_i)x_i + (B_i + \Delta B_i)u_i + (E_i + \Delta E_i)v \\ e_i &= (C_i + \Delta C_i)x + (D_i + \Delta D_i)u_i + (F_i + \Delta F_i)v \end{aligned} \tag{3-13}$$

Here A_i, B_i, C_i, D_i, E_i, F_i, denote the nominal part of the agents while ΔA_i, ΔB_i, ΔC_i, ΔD_i, ΔE_i, ΔF_i, denote the uncertain part. For ease of notation, it is convenient to collect together the uncertainties in

the vector $w = vec(\begin{bmatrix} \Delta A_i & \Delta B_i & \Delta C_i \\ \Delta D_i & \Delta E_i & \Delta F_i \end{bmatrix})$.
Then, similar to the single agent case, by defining

$$\begin{aligned} A_{w_i} &= A_i + \Delta A_i \quad B_{w_i} = B_i + \Delta B_i \quad E_{w_i} = E_i + \Delta E_i \\ C_{w_i} &= C_i + \Delta C_i \quad D_{w_i} = D_i + \Delta D_i \quad F_{w_i} = F_i + \Delta F_i \end{aligned} \tag{3-14}$$

it is possible to write (2-35) in a compact way, that together with the leader (5-3) constitute the following system:

$$\begin{aligned} \dot{x}_i &= A_{w_i} x_i + B_{w_i} u_i + E_{w_i} v \\ \dot{v} &= Sv \\ e_i &= C_{w_i} x_i + D_{w_i} u_i + F_{w_i} v \end{aligned} \tag{3-15}$$

In order to overcome the communication constraints that prevent the uninformed agents from accessing the exosystem state, a different solution from the distributed observer can be used: a virtual regulated output for each agent.
Let us define e_{vi} to be equal to y_{m_i}, the measurement output, for the informed followers, while for the uninformed followers

$$e_{vi} = \sum_{j=1}^{N} \frac{1}{N_i} (y_{m_i} - y_{m_j}) \tag{3-16}$$

where N_i is the cardinality of the set $(i = 1 \dots N)$.
Clearly, e_{vi} is defined in such a way because the informed agents can directly access the leader's state, whereas the uninformed agents cannot and they estimate it by exchanging information with other agents.
In this way, the information generated by the leader is passed to every agent of the network, overcoming the communication constraints.
Finally, the standard methodology is similar to the single agent case; two kind of controllers can be used:

- Dynamic State Feedback

$$u_i = K_{1i} x_i + K_{2i} z_i \tag{3-17}$$

$$\dot{z}_i = G_{1i} z_i + G_{2i} e_{vi} \tag{3-18}$$

  and

- Dynamic Output Feedback

$$u_i = K z_i \tag{3-19}$$

$$\dot{z}_i = G_{1i} z_i + G_{2i} e_{vi} \tag{3-20}$$

where in both cases the dynamical compensator $\dot{z}_i = G_1 z + G_2 e_{vi}$ incorporates a p-copy internal model of the uncertain composite system Eq. (3-15). Then, the dynamical compensators (that keep into account plant uncertainties) together with the agent dynamics without uncertainties constitutes the following augmented system:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v \\ e_i &= C_i x_i + D_i u_i + F_i v \\ \dot{z}_i &= G_{1i} z_i + G_{2i} e_{vi} \end{aligned} \tag{3-21}$$

Definition 4. [Linear Cooperative Robust Output Regulation Problem]
Given the augmented system (3-21) and a graph $\mathcal{G}$, a controller such as (3-17) and (3-19) must be found such that:

- The closed loop system is Hurwitz
- For any initial condition $x_i(0)$, $\eta_i(0)$ for $i = 1 \dots N$ and $v(0)$, the tracking error is asymptotically steered to 0
$$\lim_{t \to \infty} e_i(t) = 0 \quad i = 1 \dots N \tag{3-22}$$

Under Assumptions 5-9, this is possible if and only if the graph $\mathcal{G}$ contains a directed spanning tree.

In this chapter, the following concepts has been presented

- The reason why the decentralized approach is not valid.
- The distributed observer approach and how to solve the linear cooperative output regulation problem.
- The cooperative robust approach in case of small parametric uncertainties.

In the following chapter, it will be given a brief overview about the different approaches found in literature to solve the cooperative output regulation problem with particular attention the the approach introduced in [25], that represented the starting point of this thesis work.

Chapter 4

Adaptive Distributed Observer

In the first section, a general overview about different methodologies, based on the distributed observer, to solve the cooperative output regulation problem is given.
Then, in the second section is presented the work [25], that represented the starting point of this thesis work.

4-1 The distributed observer approach

The distributed observer (3-6) has been first introduced in [2], where the cooperative output regulation for a network of heterogeneous linear systems is achieved via a dynamic full information.
The problem here faced includes the synchronized output regulation of networked systems studied in [1] as a special case by assuming that all the agents to be identical with the feedthrough matrix D equal to 0.
The solution proposed in [1] requires some extra assumption due to the different design of the distributed obersver:

$$\dot{\eta}_i = S\eta_i + \sum_{j=1}^{N} a_{ij} H(x_i - x_j) \tag{4-1}$$

where η_i represents the i_{th} estimation of the exosystem state v, a_{ij} represents the element of coordinate (i, j) of the adjacency matrix A and H is defined as the distributed synchronous protocol gain.
The term m_{ii} is here absent because the adjacency matrix is defined over the entire network (exosystem included) and the connection between the agents and the exosystem is represented by a_{i1}, since in this "extended" adjacency matrix, the leader is always considered as the first element of the network.
The drawbacks of this observer respect to the one presented in [2] are several:

- The stability analysis of the closed loop system is much more complicated since (4-1) exploits the states of the agents to reconstruct the exosystem state.
- The eigenvalues of the Laplacian matrix describing the network must be located in a certain, allowed region

- The feedforward gain K_f, according to Lemma 2 of [1], must be chosen such that (B,S) are detectable.

Then, as thoroughly explained in section 4-2, by exploiting some extra detectability assumption, an improvement to the result obtained in [2] is achieved in [31]: the cooperative output regulation is indeed obtained through distributed measurement output feedback control.
This result is important because in many practical situations neither the state of the plant nor the state of the exosystem is directly available for feedback control.

However, the control design presented in [2] explicitly depends on certain nonzero eigenvalues of the Laplacian matrix associated with the communication graph. This represents a problem because any nonzero eigenvalue of the Laplacian matrix is global information of the communication graph.
Using these global information prevents from designing fully distributed the controllers.
This issue has been overcome by [32], where the proposed adaptive distributed observer

$$\dot{\eta}_i = S\eta_i + L(F\eta_i - v) \tag{4-2}$$

where (4-2) works for the informed followers,

$$\begin{aligned} \dot{\eta}_i &= S\eta_i - d_i p_i \sum_{j=1}^{N} a_{ij}(\eta_i - \eta_j) \\ d_i &= \left[\sum_{j=1}^{N} a_{ij}(\eta_i - \eta_j)\right]^T \Gamma \left[\sum_{j=1}^{N} a_{ij}(\eta_i - \eta_j)\right] \end{aligned} \tag{4-3}$$

while (4-3) works for the uninformed followers. It is important to underlie that the proposed control scheme relies only on the agent dynamics and the local information of neighboring subsystems, independent of any global information of the communication graph, meaning that a fully distributed solution has been obtained.
Another major drawback of the distributed observer approach is that the matrix S is used by every follower agent. This means that every follower have information about the leader (e.g. in case of a sinusoidal reference, every follower would know the frequency of the sine). An improvement of the classical distributed observer (3-6) has been obtained in [25], with the definition of the adaptive distributed observer. Let us focus our attention on this work.

4-2 The Adaptive Distributed Observer

The current section is completely based on the work presented in [25]. Let us consider an exosystem

$$\dot{v} = Sv \tag{4-4}$$

and a network of N agents

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v \\ e_i &= C_i x_i + D_i u_i + F_i v \\ y_{mi} &= C_{mi} x_i + D_{mi} u_i + F_{mi} v \end{aligned} \tag{4-5}$$

As explained in the previous chapter, the system composed of (4-4) and (4-5) is treated as a multi-agent system of $(N+1)$ agents with 4-4 as the leader and the N subsystems of 4-5 as N followers. The adjacency matrix $\mathscr{A}$ is, for this work, defined over the entire network. Moreover, in order to solve the linear output regulation problem, the classical Assumptions 5-9 must hold and the graph $\mathscr{G}$ in this case considered directed, must contain a spanning tree with the node 0 as the root. The adaptive distributed observer is defined as follows:

$$\begin{aligned} \dot{S}_i &= \mu_1 \sum_{j=0}^{N} a_{ij}(S_j - S_i) \\ \dot{\eta}_i &= S_i \eta_i + \mu \sum_{j=0}^{N} a_{ij}(\eta_j - \eta_i) \end{aligned} \tag{4-6}$$

where $\eta_0 = v$, $\eta_i \in R^q$, $i = 1,\dots,N$,, $S_0 = S$, $S_i \in R^{q\times q}$,$i = 1,\dots,N$, $\mu_1,\mu_2 > 0$. for $i = 1,\dots,N$. The main strength of this new observer is that it incorporates a mechanism for estimating the matrix S, based on the fact that the informed followers know the matrix S.
Let us define $\tilde{\eta}_i = \eta_i - v$; then, for any initial condition $S_i(0)$ and $\eta_i(0)$, we have

- for any $\mu_1 > 0$, for $i = 1,\dots,N$,
$$\lim_{t\to\infty} \tilde{S}_i(t) = 0 \tag{4-7}$$
exponentially, and
- for any $\mu_2 > 0$, for$i = 1,\dots,N$,
$$\lim_{t\to\infty} \tilde{\eta}_i(t) = 0 \tag{4-8}$$

Because of the estimation mechanism, also the regulator equations, that relies on the exosystem matrix S, need to be adaptively calculated, by using the estimate S_i.
For this purpose, for $i = 1,\dots,N$, let

$$x_i = vec\left(\begin{bmatrix} X_i \\ U_i \end{bmatrix}\right) \qquad b_i = vec\left(\begin{bmatrix} E_i \\ F_i \end{bmatrix}\right) \tag{4-9}$$

and

$$Q_i(t) = S_i(t)^T \otimes \begin{bmatrix} I_{ni} & 0 \\ 0 & 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \tag{4-10}$$

where S_i is provided by (4-6).
Then the following results hold: for $i = 1,\dots,N$, for any initial condition $\zeta_i(0)$, each of the following equations

$$\zeta_i = -\mu_3 Q_i(t)(Q_i(t)\zeta_i - b_i) \tag{4-11}$$

where $\mu_3 > 0$, has a unique bounded solution over $t \geq 0$.
Moreover, let us define $\Xi_i(t) = M^q_{(n_i+m_i)}(\zeta_i(t))$, where $M^q_{(n_i+m_i)}(\zeta_i(t)) = \begin{bmatrix}\zeta_{i1} \cdots \zeta_{iq}\end{bmatrix}$, with $\zeta_{ij} \in \mathbb{R}^{(n_i+m_i)}$, for $j = 1,\dots,q$.
Then, for some solution $[X_i^*, U_i^*]$ of the regulator equations, we have

$$\lim_{t\to\infty}\left(\Xi_i(t) - \begin{bmatrix} X_i^* \\ U_i^* \end{bmatrix}\right) = 0 \tag{4-12}$$

exponentially.

At this point, since the adaptive solution to the regulator equation was achieved, it is possible to solve the cooperative output regulation problem by state feedback control.
Let $\Xi_i(t) = \left[X_i(t)^T, U_i(t)^T\right]^T$ where $X_i(t) \in R^{n_i \times q}$ and $U_i(t) \in R^{m_i \times q}$.
Since (A_i, B_i) is stabilizable, let $K_x i$ be such that $\tilde{A}_i = A_i + B_i K_{xi}$ is Hurwitz and $K_{\eta i}(t)$ be given as

$$K_{\eta i}(t) = U_i(t) - K_{xi} X_i(t) \tag{4-13}$$

For $i = 1, \dots, N$, we design the following state feedback controller

$$u_i = K_{xi} x_i + K_{\eta i}(t) \eta_i \tag{4-14}$$

Then the following result is obtained: given systems (4-5),(4-4) and a graph $\mathscr{G}$ that contains a spanning tree, under Assumptions 5-7, the cooperative output regulation problem is solvable by the control law composed of (4-6), (4-11), and (4-14).

In fact, by defining $\tilde{x}_i = x_i - X_i^* v, \tilde{X}_i(t) = X_i(t) - X_i^*$ and $\tilde{U}_i(t) = U_i(t) - U_i^*$ we obtain

$$\begin{aligned}\dot{\tilde{x}}_i &= A_i x_i + B_i u_i + E_i v - X_i^* S v \\ &= \tilde{A}_i \tilde{x}_i + B_i U_i^* \tilde{\eta}_i + B_i \tilde{U}(t)_i \eta_i - B_i K_{xi} X_i^* \tilde{\eta}_i - B_i K_{xi} \tilde{X}(t)_i \eta_i\end{aligned} \tag{4-15}$$

Under the aforementioned assumptions, it is known that $\|v\|$ is bounded above by a polynomial function.
Thus, $(B_i U_i^* \tilde{\eta}_i + B_i \tilde{U}(t)_i \eta_i - B_i K_{xi} X_i^* \tilde{\eta}_i - B_i K_{xi} \tilde{X}(t)_i \eta_i)$ tends to 0 exponentially as $t \to \infty$.
Since $\tilde{A}_i$ is Hurwitz, we have

$$\lim_{t \to \infty} \tilde{x}_i(t) = 0 \tag{4-16}$$

On the other hand, let $\tilde{C}_i = Ci + D_i K_{xi}$.
Then we have

$$\begin{aligned}e_i &= C_i x_i + D_i u_i + F_i v \\ &= \tilde{C}_i \tilde{x}_i + D_i U_i^* \tilde{\eta}_i + D_i \tilde{U}_i(t) \eta_i - D_i K_{xi} X_i^* \tilde{\eta}_i - D_i K_{xi} X_i(t) \tilde{\eta}_i\end{aligned} \tag{4-17}$$

Again, since $\tilde{\eta}_i(t)$, $Ui(t)$, $\tilde{X}_i(t)$,$\tilde{x}_i(t)$ all tend to 0 exponentially. Finally, as $t \to \infty$, we have

$$\lim_{t \to \infty} e_i(t) = 0 \tag{4-18}$$

In case the state is not available, the problem is still solvable by adopting a measurement output feedback control. Let K_{xi} and $K_i(t)$ be defined as in the state feedback control law (4-14). If $(C_m i, A_i)$ is detectable, then there exists L_i such that $(A_i + L_i C_m i)$ is Hurwitz.
For $i = 1, \dots, N$, let

$$\begin{aligned}u_i &= K_{xi} \xi_i + K_{\eta_i} \eta_i \\ \dot{\xi}_i &= A_i \xi_i + B_i u_i + E_i \eta_i + L_i (C_{mi} \xi_i + D_{mi} u_i + F_{mi} \eta_i - y_{mi})\end{aligned} \tag{4-19}$$

Then the following result is obtained: given systems (4-5),(4-4) and a graph $\mathscr{G}$ that contains a spanning tree, under Assumptions 5-9, the cooperative output regulation problem is solvable by the control law composed of (4-6), (4-11), and (4-19).

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This work constituted the starting point of this thesis work, since we tried and managed to remove the two main drawbacks of this approach:

- The estimation mechanism present in (4-6) is based on the assumption that every informed follower access the matrix S. In our approach the estimation is achieved by just passing the state v to the informed followers, in a fully distributed way
- Since no internal model is used to solve the output regulation (as explained in section 4-2 for the multi agent case and in section 2-1 for the single agent case), any small uncertainty in the agents' parameters would make the approach fail. This problem has been overcome through adaptive theory, using adaptive observers to estimate each agent parameters.

Chapter 5

Exosystem Estimator

In this chapter it is introduced the main contribution of this thesis work, the distributed exosystem estimator.
Let us start by defining, in the next section, what type of dynamical systems we are going to work with, which assumptions are needed and what is the the problem to be solved. Then, in the second section, the distributed exosystem estimator is derived.

5-1 Problem formulation

The following network of heterogeneous uncertain single-input single-output systems is considered

$$y_i = \frac{b_{i,1}s^{n_i-1} + \ldots + b_{i,n_i-1}s + b_{i,n_i}}{s^{n_i} + a_{i,1}s^{n_i-1} + \ldots + a_{i,n_i-1}s + a_{i,n_i}} u_i, \quad i \in \mathcal{V} \tag{5-1}$$

whose minimal state-space realization in the observable form is given as:

$$\begin{aligned}
\dot{x}_i &= \underbrace{\begin{bmatrix} -a_{i,1} & \\ \vdots & I_{n_i-1} \\ -a_{i,n_i} & 0\cdots 0 \end{bmatrix}}_{A_i} x_i + \underbrace{\begin{bmatrix} b_{i,1} \\ \vdots \\ b_{i,n_i} \end{bmatrix}}_{b_i} u_i \\
y_i &= \underbrace{[1\ 0\ \cdots\ 0]}_{c_i^T} x_i\ , \quad i \in \mathcal{V}
\end{aligned} \tag{5-2}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$, $y_i \in \mathbb{R}$ are the state, the control input, and the output of the i-th system, respectively. The coefficients of the numerator and denominator polynomials in (5-1) which appear in the $A_i \in \mathbb{R}^{n_i \times n_i}$ and $b_i \in \mathbb{R}^{n_i}$, are *unknown* constants.
In line with most adaptive designs [33, 34], we are focusing on uncertain single-input single-output systems, where c_i is known, in view of the observer form.
The control objective is to design, for every system (5-2), a distributed control strategy for u_i, that

makes each y_i track the output of an exosystem, or leader system.
The exosystem is taken in the form of a multi-dimensional harmonic oscillator

$$\begin{aligned} \dot{v} &= \underbrace{\text{bdiag}\left(\begin{bmatrix} 0 & \omega_k \\ -\omega_k & 0 \end{bmatrix}\right)_{\overleftarrow{q/2}}}_{S} v \\ r &= [\,\underbrace{\overbrace{0\;1}^{q/2\text{ times}}\;0\;1\;\cdots\;0\;1}_{c_0^T}\,]v \\ e_i &= y_i - r = c_i^T x_i - c_0^T v \end{aligned} \tag{5-3}$$

where $\omega_k > 0$, $k = 1,\ldots,q/2$, are the frequencies of the leader system, which are distinct and are assumed to be *unknown* to all systems in the network. In (5-3), $v \in \mathbb{R}^q$ is the leader state, $r \in \mathbb{R}$ is the reference signal to be tracked, and $e_i \in \mathbb{R}$ is the regulation error to be driven to zero.

The following assumptions are made.

Assumption 10. The pairs (A_i, b_i) are controllable, the pairs (c_i, A_i) are observable, and n_i is known, $\forall\, i \in \mathscr{V}$.

Assumption 11. The zeros of (5-1) do not coincide with the eigenvalues of S.

Assumption 12. The order q of the exosystem satisfies $\frac{q}{2} \geq \bar{n} = \max_i n_i$.

Assumption 13. The graph $\mathscr{G}$ of the leaderless network is undirected and connected, and the leader interacts with at least one system ($\mathscr{T} \neq \emptyset$).

Remark 1. *Assumptions 10 and 11 guarantee the solvability of the output regulation problem, which, as previously mentioned, is equivalent to the existence of solution pairs (X_i, p_i), $\forall\, i \in \mathscr{V}$ to the linear regulator equations*

$$\begin{aligned} X_i S &= A_i X_i + b_i p_i^T \\ 0 &= c_i^T X_i - c_0^T \end{aligned} \tag{5-4}$$

which can be expressed in the compact form

$$Q_i \xi_i = \beta_i \tag{5-5}$$

where

$$\xi_i = \text{vec}\left(\begin{bmatrix} X_i \\ p_i^T \end{bmatrix}\right), \; \beta_i = \text{vec}\left(\begin{bmatrix} 0 \\ -c_0^T \end{bmatrix}\right), \tag{5-6}$$

$$Q_i = S^T \otimes \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} A_i & b_i \\ c_i^T & 0 \end{bmatrix}. \tag{5-7}$$

Finally, Assumption 12 guarantees the correct identification of all parameters in (5-1)*, while Assumption 13 is a standard network connectivity assumption that allows the distributed estimation of the frequencies in* (5-3).

We are now ready to give the problem formulation.

Problem 1. Under Assumptions 10-13, given the network of uncertain systems (5-2) with uncertain exosystem (5-3), design distributed adaptive control laws u_i such that the signals of the closed-loop network system are bounded, and the regulation errors e_i satisfy

$$\lim_{t\to\infty} e_i(t) = 0, \quad \forall\, i \in \mathscr{V}.$$

Remark 2. *The distributed nature of u_i refers to the fact that the adaptive control strategy must respect the allowed information flow. Solving Problem 1 in a distributed way presents at least three challenges: (a) the exosystem dynamics are unknown to all systems, which make it necessary to design, for each system $i \in \mathscr{V}$, an estimator of* (5-3)*; (b) the coefficients in* (5-2) *are unknown, which require the design of adaptive observers; (c) the solution to the regulator equations* (5-4) *must be obtained in a stable adaptive way from the above-mentioned estimates.*

5-2 Distributed exosystem estimator

The first step for solving Problem 1 is the design of a distributed exosystem estimator. The task of such estimator, as sketched in Fig. 5-1, is twofold: estimating S for all systems, and reconstructing the state v for the non-target nodes. In the following, all variables and estimates are intended to depend on time, and time dependence is omitted whenever obvious.

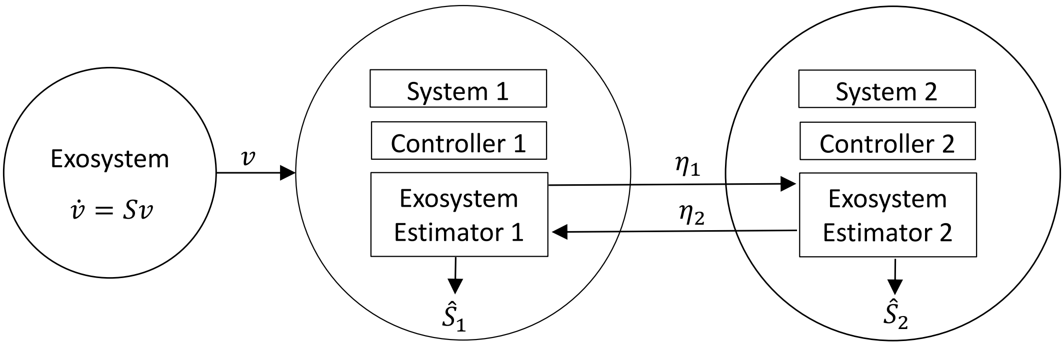


Figure 5-1: To reconstruct the exosystem state, non-target nodes exchange auxiliary variables according to the communication graph.

Let us start by defining the local observation error ε_i

$$\varepsilon_i = \sum_{j=1}^{N} a_{ij}(\eta_i - \eta_j) + m_{ii}(\eta_i - v), \tag{5-8}$$

where a_{ij} and m_{ii} come from the adjacency and the target matrices.
In (5-8), the variable v is available only to the target nodes (through m_{ii}). For each system i, the variable η_i in (5-8) represents the locally reconstructed exosystem state to be exchanged according to the communication graph (through a_{ij}), c.f. Fig. 5-1. Note that the error in (5-8) represents a *consensus* error over the variable of interest v; in other words, $\varepsilon_i \to 0$ $\forall i \in \mathscr{V}$ implies the local reconstruction of the exosystem state, $\eta_i \to v$ $\forall i \in \mathscr{V}$.

To represent in a compact form variables for the overall network, it is now convenient to adopt Kronecker product notation (denoted by $\otimes$). For example, after defining $\eta = [\eta_1^T, \eta_2^T, \ldots, \eta_N^T]^T \in \mathbb{R}^{Nq}$ and $v_m = [v^T, v^T, \ldots, v^T]^T \in \mathbb{R}^{Nq}$, it is easy to write the overall observation error $\varepsilon = [\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_N^T]^T$, stemming from (5-8), as

$$\varepsilon = (\mathscr{L} \otimes I_q)\eta + (\mathscr{M} \otimes I_q)(\eta - v_m).$$

Exploiting the fact that $(\mathscr{L} \otimes I_q)v_m = 0$ [35], we can write

$$\varepsilon = (\mathscr{B} \otimes I_q)(\eta - v_m).$$

The design of the distributed exosystem estimator is provided by the following theorem.

Theorem 1. *Under Assumption 13, consider the following distributed dynamics for* η_i

$$\dot{\eta}_i = \hat{S}_i \eta_i + (A_m - \hat{S}_i)\varepsilon_i \tag{5-9}$$

with the following Hurwitz diagonal matrix $A_m \in \mathbb{R}^{q \times q}$

$$A_m = -\ \text{bdiag}\,(a_k \cdot I_2)_{\overleftarrow{q/2}}, \quad a_k > 0,\ k = 1, \ldots, q/2. \tag{5-10}$$

Furthermore, let us write η_i *and* ε_i *component-wise as*

$$\eta_i = \begin{bmatrix} \eta_{i,1} \\ \vdots \\ \eta_{i,q} \end{bmatrix}, \quad \varepsilon_i = \begin{bmatrix} \varepsilon_{i,1} \\ \vdots \\ \varepsilon_{i,q} \end{bmatrix}, \tag{5-11}$$

and let $\hat{S}_i$ *in* (5-9) *be*

$$\hat{S}_i = \text{bdiag}\left(\begin{bmatrix} 0 & (\hat{\omega}_k)_i \\ -(\hat{\omega}_k)_i & 0 \end{bmatrix} \right)_{\overleftarrow{q/2}} \tag{5-12}$$

with $(\hat{\omega}_k)_i$ *being the estimate of* ω_k *for system i, generated by*

$$(\dot{\hat{\omega}}_k)_i = \kappa_k \Big[(\eta_{i,(2k-1)} - \varepsilon_{i,(2k-1)})\varepsilon_{i,(2k)} - (\eta_{i,(2k)} - \varepsilon_{i,(2k)})\varepsilon_{i,(2k-1)} \Big], \tag{5-13}$$

with initial conditions $(\hat{\omega}_k)_i(0)$ *and where* $\kappa_k > 0$ *is a constant gain.*
Then, the adaptation laws (5-13) *guarantee that* $\eta_i \to v$ *and* $\hat{S}_i \to S$ *as* $t \to \infty$, $\forall i \in \mathscr{V}$.

Proof. The dynamics (5-9) can be equivalently written as a function of the local error (5-8) and of the estimation error $\tilde{S}_i = \hat{S}_i - S$

$$\dot{\eta}_i = S\eta_i + (A_m - S)\varepsilon_i + \tilde{S}_i(\eta_i - \varepsilon_i). \tag{5-14}$$

Moreover, by defining

$$\tilde{S}_d(t) = \text{diag}(\tilde{S}_1(t), \tilde{S}_2(t), \ldots, \tilde{S}_N(t))$$

we can write (5-14) for the overall network as

$$\dot{\eta} = (I_N \otimes S)\eta + [I_N \otimes (A_m - S)]\varepsilon + \tilde{S}_d(\eta - \varepsilon). \tag{5-15}$$

Let us write the overall error dynamics, using (5-8) and (5-15) as

$$\begin{aligned}\dot{\varepsilon} =&(\mathscr{B}\otimes I_q)(I_N\otimes S)(\eta - v_m) + (\mathscr{B}\otimes I_q)[I_N\otimes(A_m - S)]\varepsilon + (\mathscr{B}\otimes I_q)\tilde{S}_d(\eta-\varepsilon)\\ =&\Big[(I_N\otimes S)+\big(\mathscr{B}\otimes(A_m-S)\big)\Big]\varepsilon + (\mathscr{B}\otimes I_q)\tilde{S}_d(\eta-\varepsilon).\end{aligned} \tag{5-16}$$

Positive-definiteness of $\mathscr{B}$ leads to the existence of a unitary matrix $\mathscr{U}\in\mathbb{R}^{N\times N}$ such that $\mathscr{U}^T\mathscr{B}^{-1}\mathscr{U} = \mathrm{diag}(\delta_1,\delta_2,\dots,\delta_N)\triangleq\Delta$, where δ_i, $i=1,\dots,N$, are the eigenvalues of the topology matrix $\mathscr{B}$. This can be used to define the transformation $\varepsilon = (\mathscr{U}\otimes I_n)\bar{\varepsilon}$ with $\bar{\varepsilon} = [\bar{\varepsilon}_1^T,\bar{\varepsilon}_2^T,\dots,\bar{\varepsilon}_N^T]^T$ [36].
Consider the positive definite Lyapunov function candidate

$$V = \frac{1}{2}\varepsilon^T(\mathscr{B}^{-1}\otimes I_q)\varepsilon + \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{q/2}\frac{(\tilde{\omega}_k)_i^2}{\kappa_k}. \tag{5-17}$$

Using (5-13), we have

$$\begin{aligned}\dot{V} =&\varepsilon^T(\mathscr{B}^{-1}\otimes I_q)\Big[(I_N\otimes S)+\big(\mathscr{B}\otimes(A_m-S)\big)\Big]\varepsilon + \varepsilon^T(\mathscr{B}^{-1}\otimes I_q)(\mathscr{B}\otimes I_q)\tilde{S}_d(\eta-\varepsilon)\\ &+\sum_{i=1}^{N}\sum_{k=1}^{q/2}\frac{(\tilde{\omega}_k)_i}{\kappa_k}(\dot{\tilde{\omega}}_k)_i\\ =&\varepsilon^T\Big[(\mathscr{B}^{-1}\otimes S)+\big(I_N\otimes(A_m-S)\big)\Big]\varepsilon+\varepsilon^T\tilde{S}_d(\eta-\varepsilon)+\sum_{i=1}^{N}\sum_{k=1}^{q/2}\frac{(\tilde{\omega}_k)_i}{\kappa_k}(\dot{\tilde{\omega}}_k)_i\\ =&\sum_{i=1}^{N}\bar{\varepsilon}_i^T(\delta_i S + A_m - S)\bar{\varepsilon}_i + \sum_{i=1}^{N}\varepsilon_i^T\tilde{S}_i(\eta_i-\varepsilon_i)+\sum_{i=1}^{N}\sum_{k=1}^{q/2}(\tilde{\omega}_k)_i\Big[(\eta_{i,(2k-1)}-\varepsilon_{i,(2k-1)})\varepsilon_{i,(2k)}\\ &-(\eta_{i,(2k)}-\varepsilon_{i,(2k)})\varepsilon_{i,(2k-1)}\Big].\end{aligned} \tag{5-18}$$

Considering the first summation in the last equation of (5-18), we have that each matrix

$$\delta_i S - S + A_m = \mathrm{bdiag}\left(\begin{bmatrix}-a_k & \omega_k(\delta_i-1)\\ -\omega_k(\delta_i-1) & -a_k\end{bmatrix}\right)_{\overleftarrow{q/2}} \tag{5-19}$$

is always negative definite for each i, since, taken a non-zero vector $s\in\mathbb{R}^q$ it results

$$\begin{aligned}&\begin{bmatrix}s_1 & \dots & s_q\end{bmatrix}\mathrm{bdiag}\left(\begin{bmatrix}-a_k & \omega_k(\delta_i-1)\\ -\omega_k(\delta_i-1) & -a_k\end{bmatrix}\right)_{\overleftarrow{q/2}}\begin{bmatrix}s_1\\ \vdots\\ s_q\end{bmatrix}\\ &=-\sum_{j=0}^{(q/2)-1}\sum_{k=1}^{q/2}a_k\left(s_{2j+1}^2+s_{2j+2}^2\right)\end{aligned} \tag{5-20}$$

Considering now the second summation in the last equation of (5-18), we can write for system i

$$\begin{aligned}&\begin{bmatrix}\varepsilon_{i,1} & \dots & \varepsilon_{i,q}\end{bmatrix}\mathrm{bdiag}\left(\begin{bmatrix}0 & (\tilde{\omega}_k)_i\\ -(\tilde{\omega}_k)_i & 0\end{bmatrix}\right)_{\overleftarrow{q/2}}\begin{bmatrix}\eta_{i,1}-\varepsilon_{i,1}\\ \vdots\\ \eta_{i,q}-\varepsilon_{i,q}\end{bmatrix}\\ &=\sum_{i=1}^{N}\sum_{k=1}^{q/2}(\tilde{\omega}_k)_i\Big[-\varepsilon_{i,(2k)}(\eta_{i,(2k-1)}-\varepsilon_{i,(2k-1)})+\varepsilon_{i,(2k-1)}(\eta_{i,(2k)}-\varepsilon_{i,(2k)})\Big].\end{aligned}$$

Therefore, we have

$$\dot{V} = \sum_{i=1}^{N} \bar{\varepsilon}_i^T (\delta_i S + A_m - S)\bar{\varepsilon}_i \tag{5-21}$$

which is negative semi-definite in view of (5-20). Since $V > 0$ and $\dot{V} \leq 0$, $V(t)$ is non-increasing and bounded from below by zero, which implies the existence of a limit

$$\lim_{t\to\infty} V(\varepsilon(t), \tilde{\Omega}(t)) = V_\infty < \infty \tag{5-22}$$

where $\tilde{\Omega} = [(\tilde{\omega}_1)_1 ... (\tilde{\omega}_{q/2})_1 ... (\tilde{\omega}_1)_N ... (\tilde{\omega}_{q/2})_N]$ collects all the parametric errors.
Boundedness of $V(t)$ implies that the error ε and the estimates $\tilde{\Omega}_k$ are bounded functions of time. Furthermore, we derive that $\dot{V}(t)$ is a uniformly continuous function of time because $\ddot{V}(t)$ is a uniformly bounded function of time.
In fact

$$\ddot{V} = 2\sum_{i=1}^{N} \bar{\varepsilon}_i^T (\delta_i S + A_m - S)\dot{\bar{\varepsilon}}_i \tag{5-23}$$

where boundedness of $\dot{\bar{\varepsilon}}_i$ can be derived by looking at (5-16), after noticing that: the homogeneous part of (5-16) leads to an exponentially stable system; the input term in (5-16) is a bounded function of time because ε and $\tilde{\Omega}_k$ are bounded. According to Barbalat's lemma [33, Lemma 3.2.6], lower boundedness of $V(t)$, negative semi-definiteness and uniformly continuity of $\dot{V}(t)$, imply that $\dot{V} \to 0$ as $t \to \infty$ and hence $\varepsilon \to 0$. Therefore $\eta_i \to v$ is derived, $\forall i \in \mathscr{V}$.

The fact that $\hat{S}_i$ converges to S is derived as follows. Since v is the state of a harmonic oscillator with distinct frequencies, according to [37, Definition 14.1], it is persistently exciting, i.e. it satisfies the property

$$\int_t^{t+T_0} v(\tau)v^T(\tau)\mathrm{d}\tau \geq \alpha_0 T_0 I$$

for some T_0, $\alpha_0 > 0$ and for all $t > 0$. This implies, using [33, Thm. 4.3.2] that $(\hat{\omega}_k)_i \to \omega_k$ (i.e. $\hat{S}_i \to S$).
This concludes the proof. Finally, it is possible to show that the result of Theorem 1 holds also in case of parameter projection of the estimates (5-13) in $\mathbb{R}^+$, i.e. exploiting the a priori knowledge that $\omega_k > 0$ [33, Sect. 4.4]. □

The following remark applies to (5-9)-(5-13).

Remark 3. *Let us refer to Fig. 5-1 to explain the contribution of Theorem 1. In [25], the matrix S is known to the target nodes, which share this information to all other nodes using extra communication. The proposed exosystem estimator* (5-13) *solves the case in which the exosystem dynamics are unknown to all systems. In addition, because no estimate of S is shared along the graph, it provides a much simpler communication architecture. Another contribution worth mentioning is that the matrices in* (5-19) *are always negative definite, regardless of the values of δ_i and ω_k. This makes the estimator in Theorem 1 fully distributed, i.e. convergence does not require the knowledge of any structural parameter of the communication graph (e.g. structural eigenvalues). This provides an improvement with respect to the scheme in [27], where sufficiently high gains (whose values depend on some structural eigenvalue of the communication graph) are required to achieve convergence.*

Chapter 6

Adaptive observer

The adaptive observer and the adaptive solution to the regulator equation are presented in the first section whereas, in the second section, the main result is derived.

6-1 The adaptive solution to the Regulator Equations

Since the states x_i are not measurable and the parameters of A_i and b_i are unknown, it is necessary to estimate them on-line simultaneously using an adaptive observer. We adopt a Luenberger observer where A_i, b_i are replaced with their estimates $\hat{A}_i$ and $\hat{b}_i$, that is

$$\begin{aligned} \dot{\hat{x}}_i &= \underbrace{\begin{bmatrix} -\hat{a}_{i,1} & \\ \vdots & I_{n_i-1} \\ -\hat{a}_{i,n_i} & 0\cdots 0 \end{bmatrix}}_{\hat{A}_i} \hat{x}_i + \underbrace{\begin{bmatrix} \hat{b}_{i,1} \\ \vdots \\ \hat{b}_{i,n_i} \end{bmatrix}}_{\hat{b}_i} u_i + l_i(y_i - \hat{y}_i) \\ \hat{y}_i &= \underbrace{[1\ 0\ \cdots\ 0]}_{c_i^T} \hat{x}_i \end{aligned} \tag{6-1}$$

where $\hat{x}_i$ is the observed state, the time-varying observer gain $l_i(t)$ is

$$l_i(t) = \begin{bmatrix} a^*_{i,1} - \hat{a}_{i,1}(t) & \\ \vdots & 0_{n_i-1} \\ a^*_{i,n_i} - \hat{a}_{i,n_i}(t) & 0\cdots 0 \end{bmatrix}$$

and $a^*_{i,1}, \ldots, a^*_{i,n_i}$ are chosen as the coefficients of a stable polynomial. Several methods can be used to generate the parameter estimates $\hat{a}_{i,1}, \ldots, \hat{a}_{i,n_i}$ and $\hat{b}_{i,1}, \ldots, \hat{b}_{i,n_i}$ at each time t.
The methods rely on expressing the system equation (5-1) in the form of a linear-in-the-parameter model [33, Sect. 2.4]

$$z_i = \theta_i^{*T} \phi_i \tag{6-2}$$

where

$$z_i = \frac{s^{n_i}}{\Lambda_i(s)} y_i = y_i + \lambda_i^T \phi_{2_i}$$

$$\phi_i = \left[\frac{\alpha_{n_i-1}^T(s)}{\Lambda_i(s)} u_i, -\frac{\alpha_{n_i-1}^T(s)}{\Lambda_i(s)} y_i \right]^T = \left[\phi_{1_i}^T, \phi_{2_i}^T \right]^T$$

$$\Lambda_i(s) = s^{n_i} + \lambda_i^T \alpha_{n_i-1}(s)$$

$$\lambda_i = [\lambda_{n_i-1} \ \lambda_{n_i-2} \ \dots \ \lambda_1 \ \lambda_0]^T$$

$$\alpha_{n_i-1}(s) = [s^{n_i-1} \ s^{n_i-2} \ \dots \ s \ 1]^T$$

and the unknown coefficients of (5-1) are included in the unknown vector

$$\theta_i^* = [b_{i,1} \ \dots \ b_{i,n_i} \ a_{i,1} \ \dots \ a_{i,n_i}]^T$$

with $\Lambda_i(s)$ a Hurwitz polynomial of degree n_i chosen by the designer.
In view of (6-2), a possible adaptive law to estimate on-line the unknown vector θ^* (i.e. the coefficients $a_{i,1}, \dots, a_{i,n_i}$ and $b_{i,1}, \dots, b_{i,n_i}$) is a gradient algorithm based on integral cost [33, Chap. 4], which takes the form

$$\begin{aligned} \dot{\theta}_i &= -\Gamma_i(\Psi_i \theta_i + \rho_i) \\ \dot{\Psi}_i &= -\gamma_i \Psi_i + \frac{\phi_i \phi_i^T}{m_i^2}, \quad \Psi_i(0) = 0 \\ \dot{\rho}_i &= -\gamma_i \rho_i - \frac{z_i \phi_i^T}{m_i^2}, \quad \rho_i(0) = 0 \end{aligned} \tag{6-3}$$

with the following choices for the design parameters: $m_i^2 = 1 + n_{s_i}^2$, with n_{s_i} chosen so that $\phi_i / m_i \in \mathscr{L}_\infty$ (e.g., $n_{s_i}^2 = \alpha_i \phi_i^T \phi_i, \alpha_i > 0$); $\gamma_i > 0$; $\Gamma_i = \Gamma_i^T > 0$.
The following convergence properties, whose proof can be found in [33, Chap. 4], apply to (6-3).

Lemma 5. *The adaptive observer formed by combining the observer equation* (6-1) *and the adaptive law* (6-3) *based on the parametric model* (6-2) *guarantees that:*

(i) all signals are uniformly bounded;

(ii) $\dot{\theta}_i \in \mathscr{L}_2 \cap \mathscr{L}_\infty$ *and* $\theta_i \in \mathscr{L}_\infty$*;*

(iii) the output observation error $\tilde{y}_i = y_i - \hat{y}_i$ *converges to zero as* $t \to \infty$*;*

(iv) if u_i *is sufficiently rich of order* $2n_i$*, then the state observation error* $\tilde{x}_i = x_i - \hat{x}_i$ *and the parameter error* $\tilde{\theta}_i = \theta_i - \theta_i^*$ *converge to zero with exponential rate of convergence.*

Next, we show how to solve the regulator equations (5-4) adaptively. To this purpose, let us replace ξ_i and Q_i in (5-6) and (5-7) with

$$\hat{\xi}_i(t) = \text{vec}\left(\begin{bmatrix} \hat{X}_i(t) \\ \hat{p}_i^T(t) \end{bmatrix} \right),$$

$$\hat{Q}_i(t) = \hat{S}_i^T(t) \otimes \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} \hat{A}_i(t) & \hat{b}_i(t) \\ c_i^T & 0 \end{bmatrix}$$

where $\hat{S}_i$ is provided by (5-13), $\hat{A}_i$, $\hat{b}_i$ are provided by (6-3), and $\hat{\xi}_i(t)$ collects the estimates $(\hat{X}_i, \hat{p}_i)$ of the solution to the regulator equations.
Based on these estimates, the regulator equations can be solved adaptively via the systems of linear equations

$$\hat{Q}_i\hat{\xi}_i = \beta_i, \qquad \forall i \in \mathscr{V}. \tag{6-4}$$

From the properties of systems of linear equations we have that if $\hat{A}_i$, $\hat{b}_i$ converge to A_i, b_i, then $\hat{X}_i$, $\hat{p}_i$ converge to the actual solutions X_i, p_i.

6-2 Main result

The solution to Problem 1 arises from combining the distributed exosystem estimators of Theorem 1 with the adaptive observers and the adaptive solutions to the regulator equations.

Theorem 2. *The control law composed of: the distributed exosystem estimators* (5-9) *with adaptive laws* (5-13)*, the adaptive observers* (6-3)*, the adaptive solutions of the regulator equations* (6-4)*, and the output-feedback control input*

$$u_i(t) = -k_i^T(t)\hat{x}_i(t) + f_i^T(t)\eta_i(t) \tag{6-5}$$

where

$$f_i^T(t) = \hat{p}_i^T(t) + k_i^T(t)\hat{X}_i(t) \tag{6-6}$$

solves Problem 1 provided that the gains k_i, l_i are chosen such that

$$\hat{A}_i(t) - \hat{b}_i(t)k_i^T(t) \qquad \hat{A}_i(t) - l_i(t)c_i^T(t)$$

are Hurwitz at every time instant t.

Proof. First, we will prove persistency of excitation of the control input (6-5), that can be rewritten as

$$u_i(t) = -k_i^T(t)(\hat{x}_i(t) - \hat{X}_i\eta_i) + \hat{p}_i^T\eta_i. \tag{6-7}$$

Let us write the dynamics of $\hat{x}_i - \hat{X}_i\eta_i$

$$\begin{aligned}\dot{\hat{x}}_i - \dot{\hat{X}}_i\eta_i - \hat{X}_i\dot{\eta}_i =&(\hat{A}_i - \hat{b}_ik_i^T)(\hat{x}_i - \hat{X}_i\eta_i) + l_ic_i^T(x_i - \hat{x}_i)\\ &- \dot{\hat{X}}_i\eta_i - \hat{X}_i(A_m - \hat{S}_i)\varepsilon_i,\end{aligned} \tag{6-8}$$

where we have substituted the regulator equation $\hat{X}_i\hat{S}_i = \hat{A}_i\hat{X}_i + \hat{b}_i\hat{p}_i^T$.
By observing the terms on the right-hand side in (6-8), we have that $c_i^T(x_i - \hat{x}_i) \to 0$, $\varepsilon_i \to 0$ (from Lemma 5 and Theorem 1, respectively).
In addition, since $\dot{\hat{A}}_i, \dot{\hat{b}}_i \in \mathscr{L}_2 \cap \mathscr{L}_\infty$ from the properties of the estimator, the system of linear equations (6-4) allows us to conclude that $\dot{\hat{X}}_i \in \mathscr{L}_2 \cap \mathscr{L}_\infty$. Consequently, with $\hat{A}_i - \hat{b}_ik_i^T$ Hurwitz, using notions of input/output stability [33, Lemma 3.3.3], we obtain that $\hat{x}_i - \hat{X}_i\eta_i \in \mathscr{L}_2 \cap \mathscr{L}_\infty$.
Therefore, using standard properties of persistently exciting signals [33, Lemma 4.8.3], we have that u_i is sufficiently rich of order $2\bar{n}$.
Now, define $\tilde{A}_i = \hat{A}_i - A_i$, $\tilde{b} = \hat{b}_i - b_i$ $\tilde{x}_i = \hat{x}_i - x_i$, $\tilde{\chi}_i = x_i - X_i v$, $\tilde{u}_i = u_i - p_i^T v$, $\tilde{\eta}_i = \eta_i - v$, $f_i^{*T} = p_i^T + k_i^T X_i$, $\tilde{f}_i = f_i - f_i^{*T}$, and $\tilde{\zeta}_i = \hat{x}_i - X_i v$.

By making use of the regulator equations, it is possible to obtain the dynamics of the errors just defined.
Let us start with $\tilde{\chi}_i$:

$$
\begin{aligned}
\tilde{\chi}_i &= A_i x_i + b_i u_i - X_i S v \\
&= A_i(\tilde{\chi}_i + X_i v) + b_i(\tilde{u}_i + p_i^T) - X_i S v \\
&= A_i \tilde{\chi}_i + b_i \tilde{u}_i
\end{aligned}
\tag{6-9}
$$

In order to determine the error on the control input $\tilde{u}_i$, let us first rewrite $\tilde{x}_i$ as follows:

$$
\tilde{x}_i = \hat{x}_i - x_i = (\tilde{z}_i + X_i v) - (\tilde{\chi}_i + X_i v) = \tilde{\zeta}_i - \tilde{\chi}_i. \tag{6-10}
$$

Then $\tilde{u}_i$ becomes

$$
\begin{aligned}
\tilde{u}_i &= -k_i^T \hat{x}_i + f_i^T \eta_i - p_i^T v \\
&= -k_i^T (\tilde{x}_i + x_i) + f_i^T \eta_i - p_i^T v \\
&= -k_i^T x_i + k_i^T (\tilde{\chi}_i - \tilde{\zeta}_i) + f_i^T \eta_i - p_i^T v \\
&= k_i^T (\tilde{\chi}_i - \tilde{\zeta}_i) - k_i^T \tilde{\chi}_i + \tilde{f}_i^T v + f_i^T \tilde{\eta}_i
\end{aligned}
\tag{6-11}
$$

It is possible to express the regulated output as:

$$
\begin{aligned}
e_i &= c_i^T x_i - c_0^T v \\
&= c_i^T \hat{x}_i + c_i^T X_i v - c_0^T v \\
&= c_i^T \hat{x}_i
\end{aligned}
\tag{6-12}
$$

The dynamics of the error $\tilde{\zeta}_i = \hat{x}_i - X_i v$ are:

$$
\begin{aligned}
\dot{\tilde{\zeta}}_i &= \hat{A}\hat{x}_i + \hat{b}_i u_i + l_i c_i (x_i - \hat{x}_i) - X_i S v \\
&= \hat{A}_i(\tilde{\zeta}_i + X_i v) + \hat{b}_i(\tilde{u}_i + p_i^T v) + l_i c_i(\tilde{\chi}_i - \tilde{\zeta}_i) - X_i S v \\
&= \hat{A}_i \tilde{\zeta}_i + \hat{b}_i \tilde{u}_i + l_i c_i^T (\tilde{\chi}_i - \tilde{\zeta}_i) + \tilde{A}_i X_i v + \tilde{b}_i p_i^T v
\end{aligned}
\tag{6-13}
$$

If we collect together the obtained results, we get:

$$
\begin{aligned}
\dot{\tilde{\chi}}_i &= A_i \tilde{\chi}_i + b_i \tilde{u}_i \\
\dot{\tilde{\zeta}}_i &= \hat{A}_i \tilde{\zeta}_i + \hat{b}_i \tilde{u}_i + l_i c_i^T (\tilde{\chi}_i - \tilde{\zeta}_i) + \tilde{A}_i X_i v + \tilde{b}_i p_i^T v \\
e_i &= c_i^T \tilde{\chi}_i \\
\tilde{u}_i &= k_i^T (\tilde{\chi}_i - \tilde{\zeta}_i) - k_i^T \tilde{\chi}_i + \tilde{f}_i^T v + f_i^T \tilde{\eta}_i
\end{aligned}
\tag{6-14}
$$

Finally, by closing the loop we obtain:

$$
\begin{aligned}
\begin{bmatrix} \dot{\tilde{\chi}}_i \\ \dot{\tilde{\chi}}_i - \dot{\tilde{\zeta}}_i \end{bmatrix} &= \begin{bmatrix} \hat{A}_i - \hat{b}_i k_i^T & b_i k_i^T \\ 0 & \hat{A}_i - l_i c_i^T \end{bmatrix} \begin{bmatrix} \tilde{\chi}_i \\ \tilde{\chi}_i - \tilde{\zeta}_i \end{bmatrix} \\
&+ \begin{bmatrix} -(\tilde{A}_i - \tilde{b}_i k_i^T)\tilde{\chi}_i + b_i \tilde{f}_i^T v + b_i f_i^T \tilde{\eta}_i \\ -\tilde{A}_i x_i - \tilde{b}_i u_i \end{bmatrix}.
\end{aligned}
\tag{6-15}
$$

Since the control input u_i is sufficiently rich of order $2\bar{n}$, we can conclude that the terms $\tilde{A}_i$ and $\tilde{b}_i$ in (6-15) converge to zero exponentially fast.
Moreover, from Theorem 1 we know that $\tilde{\eta}_i \to 0$ and $\hat{S}_i \to S$ also exponentially fast. Now, we can conclude $\hat{X}_i \to X_i$ and $\hat{p}_i \to p_i$, that means also $\tilde{f}_i \to 0$. Then, the Hurwitz property of $\hat{A}_i - \hat{b}_i k_i^T$ and $\hat{A}_i - l_i c_i^T$ (that converge to $A_i - b_i k_i^T$ and $A_i - l_i c_i^T$, respectively) guarantees that $\tilde{\zeta}_i \to \tilde{\chi}_i \to 0$ exponentially, from which we obtain convergence of e_i to zero. This concludes the proof. □

The following remarks apply to Theorem 2.

Remark 4. *In contrast with approaches based on fixed-gain robust control [26], in Theorem 2 no assumption is made on the size of the parameter uncertainty set. Also, differently from the learning-based approach of [27], no initially stabilizing feedback is required. Finally, learning-based solutions require injecting an external probing signal in the input to induce persistency of excitation (an input sufficiently rich of order* $\frac{\bar{n}(\bar{n}+1)}{2} + \bar{n} + 1$ *is required in [27] to estimate both the Lyapunov function and the control gains). In our case, the adaptive closed loop* (6-15) *reduces the requirements on the sufficiently richness of the input, which is only of order* $2\bar{n}$. *Therefore, Assumption 12 makes any additional external probing signal unnecessary.*

Remark 5. *Theorem 2 requires the estimated pairs* $(\hat{A}_i, \hat{b}_i)$, $(c_i, \hat{A}_i)$ *to be controllable and observable at every time instant (which is necessary and sufficient to have* $\hat{A}_i - \hat{b}_i f_i^T$ *and* $\hat{A}_i - l_i c_i^T$ *Hurwitz). This assumption is in line with the well-known 'loss-of-controllability/observability' situation of indirect pole-placement adaptive control [33, Chap. 7], where the calculation of the controller parameters is performed based on estimated dynamics that must be controllable/observable at every time instant. In our case, a sufficiently rich input of order* $2\bar{n}$ *guarantees exponential convergence of the estimated parameters to their true values. Since the true parameters correspond to a controllable and observable system, it is implied by continuity that the estimated pairs* $(\hat{A}_i, \hat{b}_i)$, $(c_i, \hat{A}_i)$ *will enter in finite time a set corresponding to a controllable and observable system [33, 38, 39]. This leads to the following estimation strategy: if the estimated pairs are not controllable/observable during the initial estimation transient, it suffices to freeze the controller parameters to their previous values; then, exponential convergence guarantees that any loss-of-controllability/observability issue is removed in finite time.*

Chapter 7

Simulations Results

In this chapter the simulation results are presented: however, in the first section 3 experiments are run, in order to test the performances of our new methodology compared to the work of [25]. Instead, in the second section, a surprising result discovered during the simulations is presented.

7-1 Persistently Exciting Case

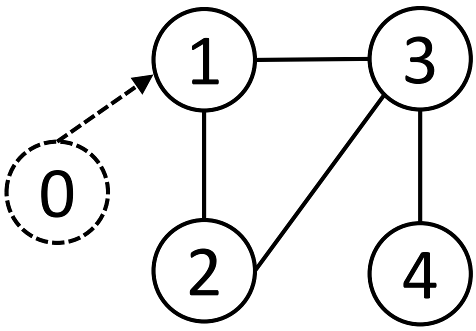


Figure 7-1: Communication graph $\mathscr{G}$

The four systems connected as in the communication graph of Fig. 3-1 are used as numerical validation of the proposed approach. As already stated in Chapter 3, the communication graph $\mathscr{G}$ is defined by the pair $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, \ldots, N\}$ is a finite nonempty set of nodes, and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is a set of pairs of nodes, called edges.
In this case, we have

- $\mathscr{V}$={1,2,3,4}
- $\varepsilon = \{(1,2),(2,1)(1,3)(3,1)(3,2)(2,3)(3,4)(4,2)\}$

It is, then, necessary to calculate the 4 matrices that completely describe the communication structure:

- *Adjacency matrix $\mathscr{A}$*

$$\mathscr{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{7-1}$$

- *Laplacian Matrix $\mathscr{L}$*

$$\mathscr{L} = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \tag{7-2}$$

- *Target Matrix $\mathscr{M}$*

$$\mathscr{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7-3}$$

- *Leader-Topology Matrix $\mathscr{B}$*

$$\mathscr{B} = \mathscr{L} + \mathscr{M} = \begin{bmatrix} 3 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \tag{7-4}$$

The exosystem, represented as system 0 in Fig. 3-1, is given by

$$\dot{v} = \begin{bmatrix} 0 & 3 & 0 & 0 \\ -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix} v \qquad r = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix} v \tag{7-5}$$

where the frequencies of the generated sinusoids are $\omega_1 = 3$ and $\omega_2 = 2$.

The heterogeneous followers are given by:

$$\dot{x}_1 = \begin{bmatrix} -10 & 1 \\ -24 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u \qquad y_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$\dot{x}_2 = \begin{bmatrix} -12 & 1 \\ -11 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u \qquad y_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$\dot{x}_3 = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \qquad y_3 = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$\dot{x}_4 = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u \qquad y_4 = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

.

For the simulations we will consider three settings, that represent different levels of uncertainty:

- *Experiment 1*. The systems are known and the exosystem is known only to system 1. This corresponds to the setting in [25];
- *Experiment 2*. The systems are unknown and the exosystem is known only to system 1. This requires to augment the method in [25] with a system estimator;
- *Experiment 3*. The systems are unknown and the exosystem is unknown to all systems. This can be handled by the proposed fully distributed method.

In all experiments, the initial state of the exosystem is $v(0) = [1, 0.2, 0.5, 1]^T$ and all the states of the systems are initialized to zero. Whenever used, the initial state of the estimator is $\theta(0) = [1, 1, 1, 1]^T$, and all the other estimators are initialized to zero.
The other design parameters are described in the following.

7-1-1 Experiment 1

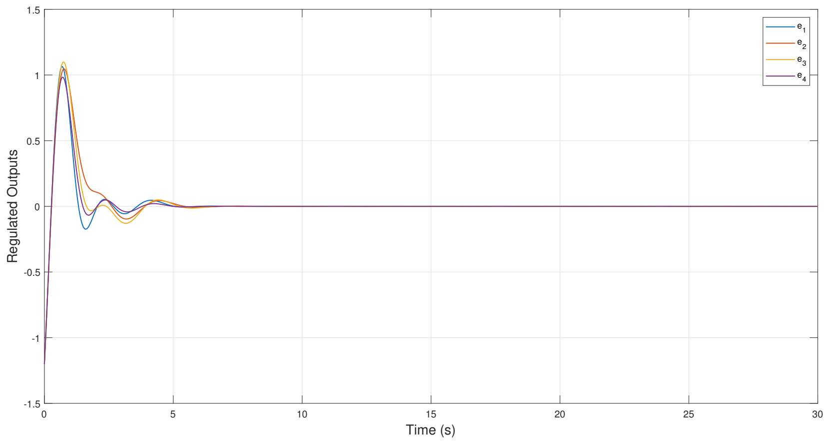


Figure 7-2: Regulated outputs

For experiment 1, we use the algorithm in [25] with gains $\mu_1 = \mu_2 = 10$ (gains for consensus over S and v) and $\mu_3 = 40$ (gain for adaptive solution to regulator equations). The regulated outputs, shown in Fig. 7-2 go to 0 smoothly, without violent oscillations, in approximately 15 seconds.

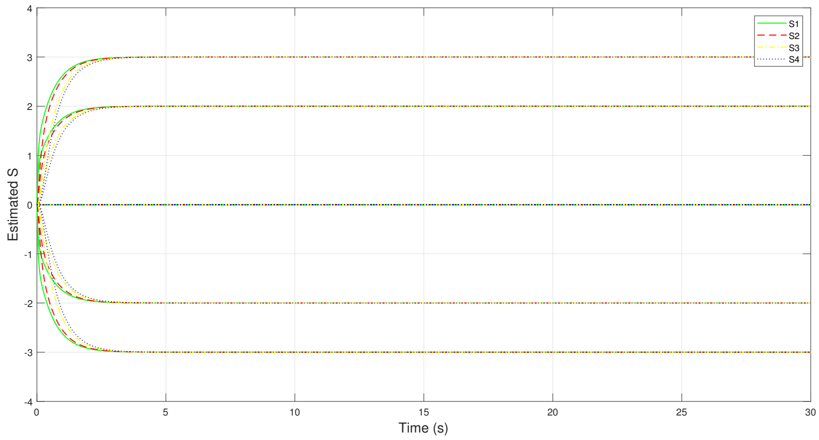
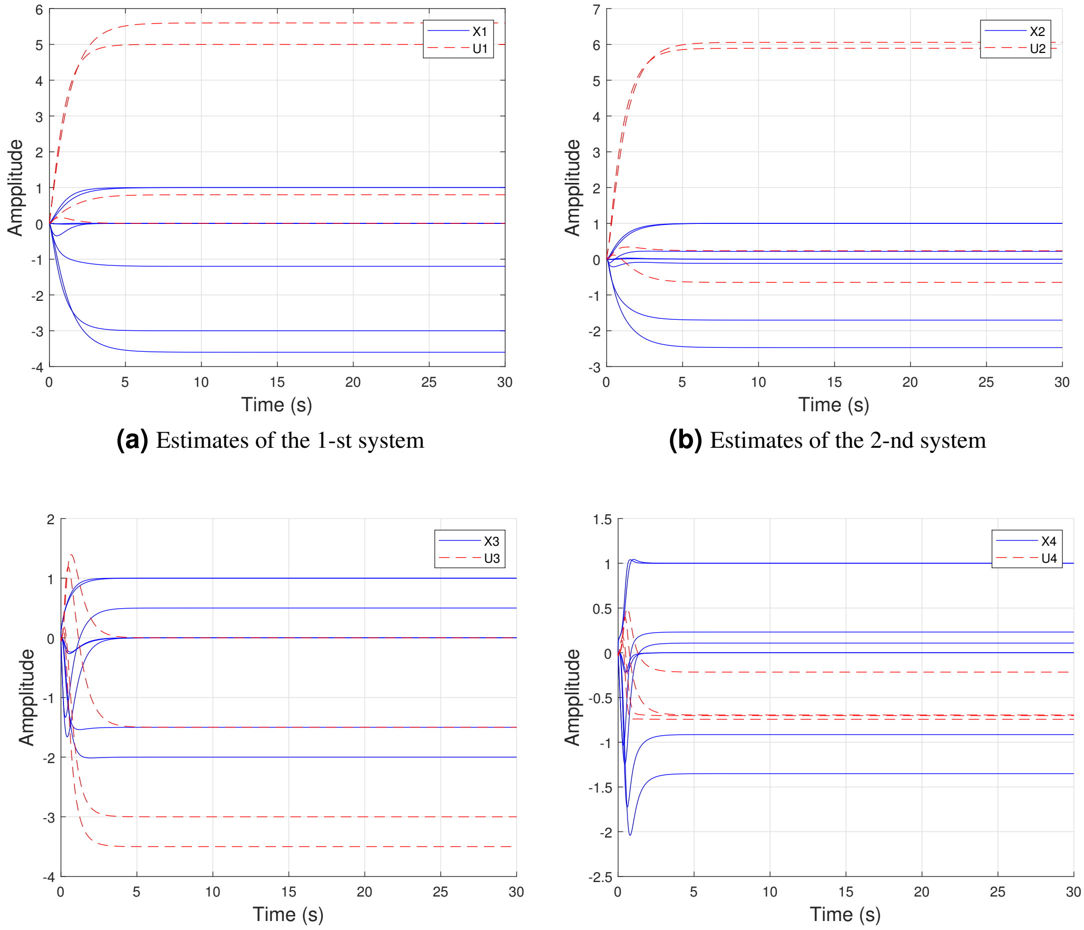


Figure 7-3: Estimates of the exosystem

In Fig. 7-3 it is shown how rapidly the matrix S is estimated by the agents: this result was expected, since all the informed followers (in this case agent 1) have the real S, leading to an estimation time of approximately 3 seconds.
Worth mentioning are also the adaptive solution to the regulator equations: the true result are here listed

- $U_1 = \begin{bmatrix} 0 & 5 & 0.8 & 5.6 \end{bmatrix}$ $X_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -3 & 0 & -3.6 & -1.2 \end{bmatrix}$
- $U_2 = \begin{bmatrix} -0.6 & 5.9 & 0.23 & 6 \end{bmatrix}$ $X_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -1.7 & 0.2 & -2.5 & -0.1 \end{bmatrix}$
- $U_3 = \begin{bmatrix} -1.5 & -3.5 & 0 & -3 \end{bmatrix}$ $X_3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -1.5 & 0.5 & -2 & 0 \end{bmatrix}$
- $U_4 = \begin{bmatrix} 0 & 1 & 0.8 & 5.6 \end{bmatrix}$ $X_4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -0.9 & 0.2 & -1.3 & 0.1 \end{bmatrix}$

Instead in Fig. 7-4 are shown how the adaptive solutions converge to the true values for all the agents.



(a) Estimates of the 1-st system

(b) Estimates of the 2-nd system

(c) Estimates of the 3-rd system

(d) Estimates of the 4-th system

Figure 7-4: Estimates of the solution pair (X_i, p_i) to the regulator equations for every agent.

7-1-2 Experiment 2

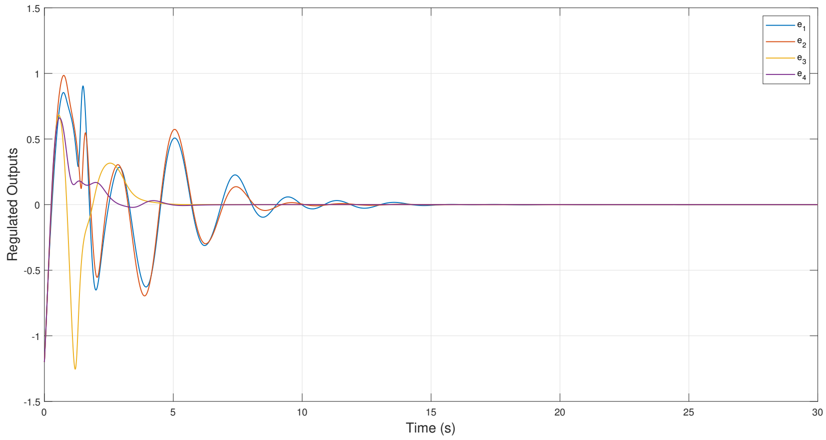
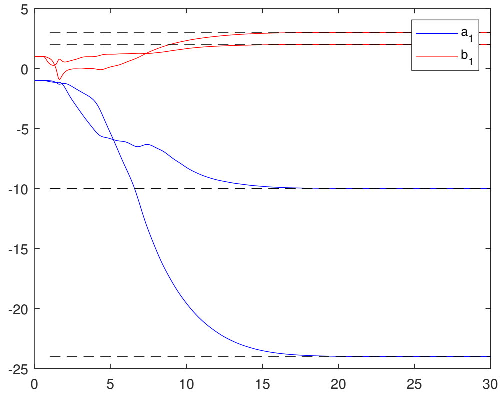


Figure 7-5: Regulated outputs

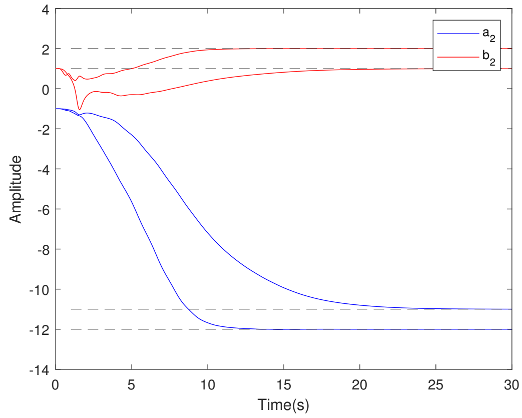
For experiment 2, we add a system estimator as in (6-1) and (6-3), with the parameters:

- $\alpha_i = 0.01$
- $\gamma_i = 0.01$
- $\Gamma_i = \begin{bmatrix} 180 & 0 & 0 & 0 \\ 0 & 180 & 0 & 0 \\ 0 & 0 & 180 & 0 \\ 0 & 0 & 0 & 180 \end{bmatrix}$
- $a^* = \begin{bmatrix} 15 & 56 \end{bmatrix}^T$
- $\Lambda_i(s) = s^2 + 2s + 1$

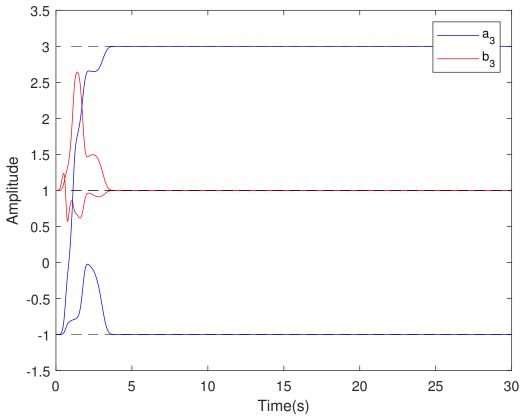
The resulting regulated outputs in Fig. 7-5 show a larger transient than Fig. 7-2 due to the estimation of A_i and b_i. The consensus over S is identical to Fig. 7-3, and therefore not shown.



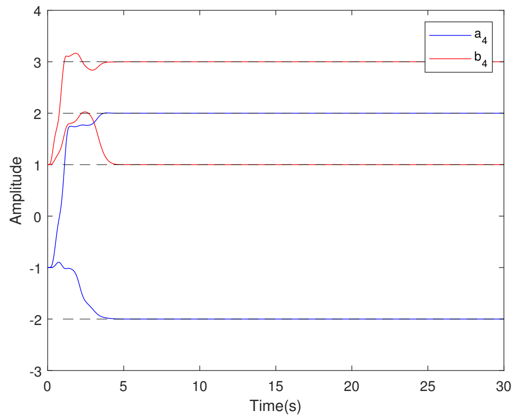
(a) Estimates of the 1-st system



(b) Estimates of the 2-nd system



(c) Estimates of the 3-rd system



(d) Estimates of the 4-th system

Figure 7-6: Estimates of the the system matrices A_i and b_i for every agent.

Rather, in Fig. 7-6 we show, the convergence of the estimated parameters $\hat{A}$, $\hat{b}$ to the actual A, b for every agent. In the second experiment, the behaviour of the adaptive solution to the regulator equation is basically equal to the first experiment, and for this reason is not shown.

7-1-3 Experiment 3

Finally, for experiment 3, we used the proposed fully distributed exosystem estimator with

- $A_m = \begin{bmatrix} -15 & 0 & 0 & 0 \\ 0 & -15 & 0 & 0 \\ 0 & 0 & -15 & 0 \\ 0 & 0 & 0 & -15 \end{bmatrix}$
- $\kappa_1 = \kappa_2 = 60.$

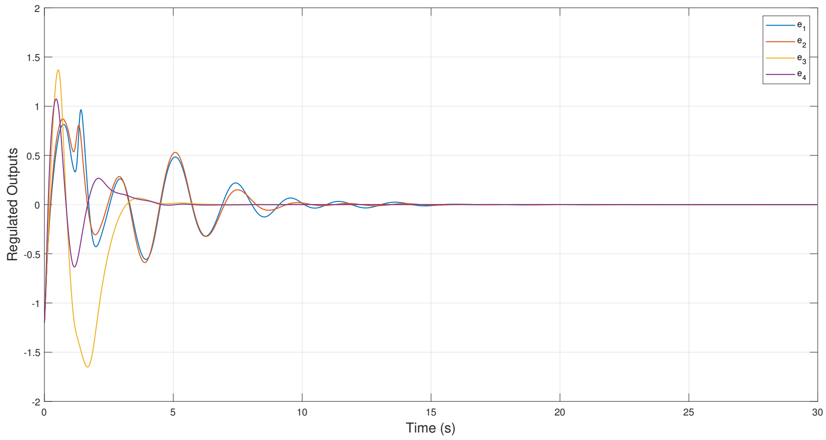


Figure 7-7: Regulated outputs with unknown agents and leader

The resulting regulated outputs, shown in Fig. 7-7, have a comparable transient with the one in Fig. 7-3, meaning that the triple level of adaptation of the proposed method doesn't affect the performances.

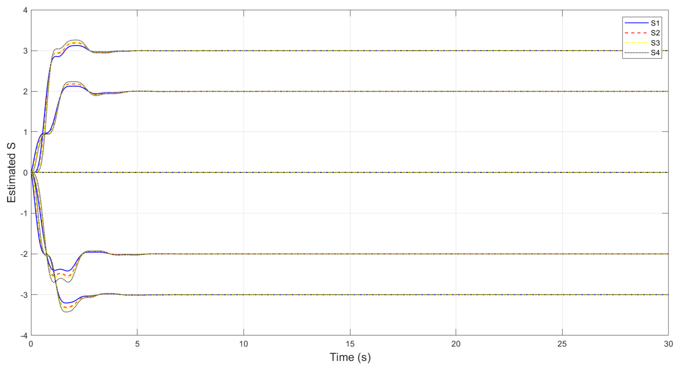
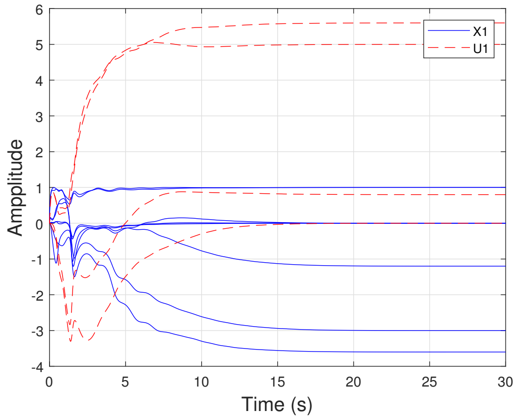
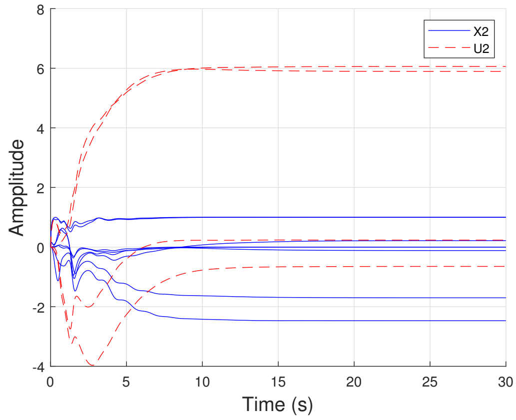


Figure 7-8: Estimates of the exosystem

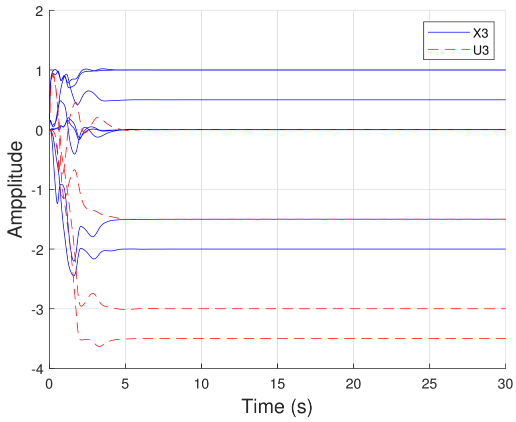
Even more interesting are the estimates of the exosystem in Fig. 7-6d, which are very different with the estimates in Fig. 7-3: this is because the systems are not allowed to do consensus over their estimates of S, and can perform the estimation only by communicating η_i.
Finally, the behaviour of the adatpive solutions to the regulator equations is shown in Fig. 7-9 for every agent



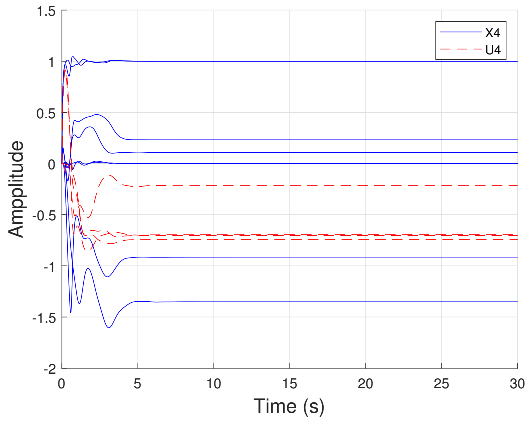
(a) Estimates of the 1-st system



(b) Estimates of the 2-nd system



(c) Estimates of the 3-rd system



(d) Estimates of the 4-th system

Figure 7-9: Estimates of the solution pair $(X_i,\ p_i)$ to the regulator equations for every agent.

7-2 Future work: no persistency of excitation

During the simulations, a very interesting result was discovered: even if one choose an exosystem matrix $S \in \mathbb{R}^{2\times 2}$ and agents of the fourth order, the cooperative output regulation problem is still solved by the new proposed method.
The adopted exosystem is:

$$\dot{v} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} v \qquad r = \begin{bmatrix} 0 & 1 \end{bmatrix} v \tag{7-6}$$

while the heterogeneous followers are given by:

$$\dot{x}_1 = \begin{bmatrix} -4 & 1 & 0 & 0 \\ -6 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 3 \\ 0.5 \\ 1 \end{bmatrix} u \qquad y_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

$$\dot{x}_2 = \begin{bmatrix} -8 & 1 & 0 & 0 \\ -24 & 0 & 1 & 0 \\ -32 & 0 & 0 & 1 \\ -16 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix} u \qquad y_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

$$\dot{x}_3 = \begin{bmatrix} 3 & 1 & 0 & 0 \\ -3.25 & 0 & 1 & 0 \\ 1.5 & 0 & 0 & 1 \\ -0.25 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 1.4 \\ 0.2 \end{bmatrix} u \qquad y_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

$$\dot{x}_4 = \begin{bmatrix} 0.5 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0.5 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 3 \\ 1 \\ 1 \\ 2 \end{bmatrix} u \qquad y_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

.

Then the chosen tuning parameters are:

- $A_m = \begin{bmatrix} -50 & 0 & 0 & 0 \\ 0 & -50 & 0 & 0 \\ 0 & 0 & -50 & 0 \\ 0 & 0 & 0 & -50 \end{bmatrix}$
- $\kappa_1 = \kappa_2 = 800$.

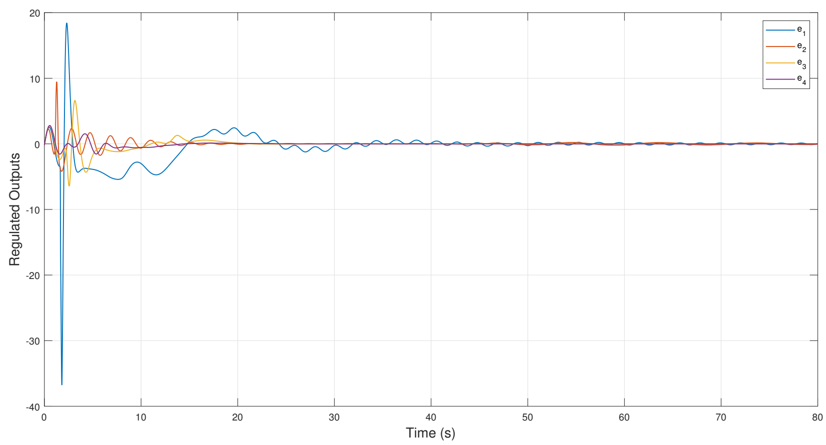
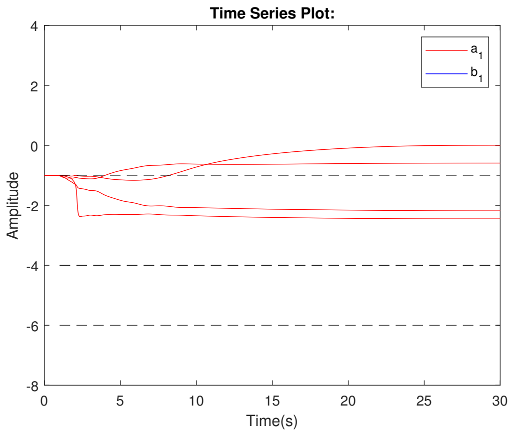
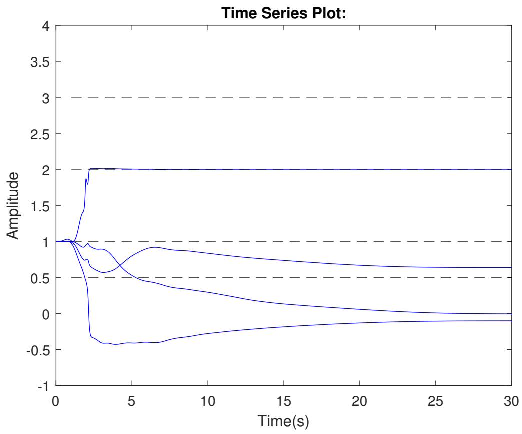


Figure 7-10: Regulated Outputs

As it is shown in Fig. 7-10, the cooperative output regulation problem is successfully solved by the proposed method, at cost of higher parameters and faster pole location for the matrix A_m. However, the errors converge to 0 less smoothly, with high peaks, and slower than the persistently exciting case.



(a) Estimates of A_1

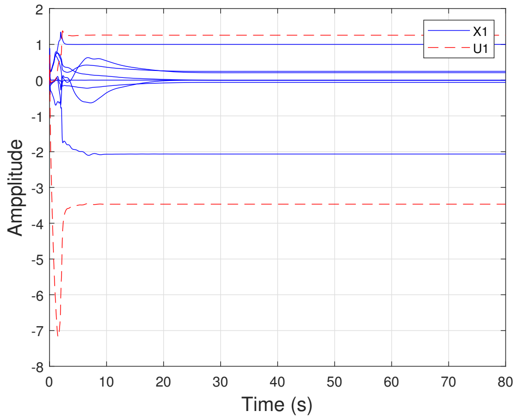


(b) Estimates of the b_1

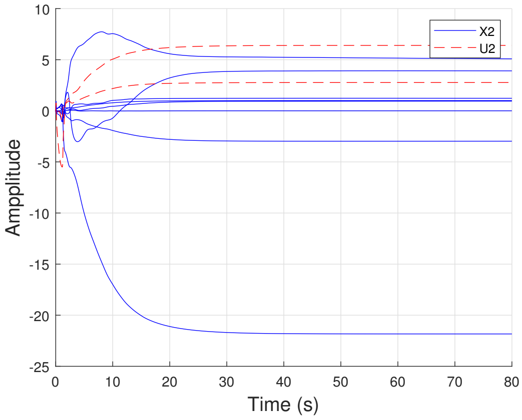
Figure 7-11: Parameters estimation of the first agent

As expected, the proposed method is no longer able to identify the true parameters of the followers, as shown in 7-11.
The most interesting result, that also represents the reason why the methodology works also without persistency of excitation, is that the adaptive solution to the regulator equations converge to the real parameters even without persistency.
The true values are here listed:

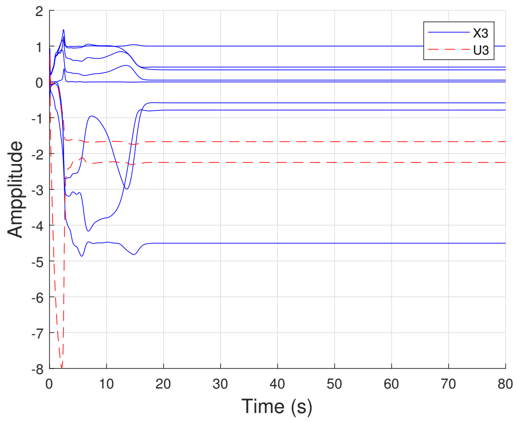
- $U_1 = \begin{bmatrix} -3.5 & 1.25 \end{bmatrix} \quad X_1 = \begin{bmatrix} 0 & 1 \\ -2.06 & 1.5 \\ -3 & 0.16 \\ 0.25 & 0.4 \end{bmatrix}$
- $U_2 = \begin{bmatrix} 6.4 & 2.76 \end{bmatrix} \quad X_2 = \begin{bmatrix} 0 & 1 \\ -21.8 & 2.46 \\ -21.6 & -0.56 \\ -7.7 & -2.13 \end{bmatrix}$
- $U_3 = \begin{bmatrix} -2.24 & -1.66 \end{bmatrix} \quad X_3 = \begin{bmatrix} 0 & 1 \\ -4.5 & 0.33 \\ -0.78 & 6.41 \\ -0.58 & 0.05 \end{bmatrix}$
- $U_4 = \begin{bmatrix} -4.11 & -0.27 \end{bmatrix} \quad X_4 = \begin{bmatrix} 0 & 1 \\ 3.35 & 0.33 \\ 1.13 & 0.63 \\ -1.55 & 0.91 \end{bmatrix}$



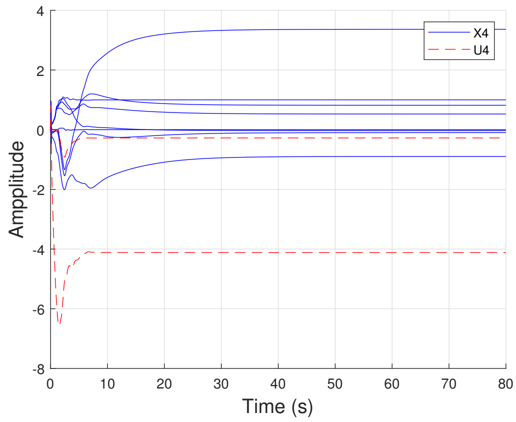
(a) Estimates of the 1-st system



(b) Estimates of the 2-nd system



(c) Estimates of the 3-rd system



(d) Estimates of the 4-th system

Figure 7-12: Estimates of the solution pair (X_i, p_i) to the regulator equations for every agent.

Chapter 8

Conclusions

This thesis work considered the problem of cooperative output regulation in the presence of both follower and leader uncertain dynamics.
A novel algorithm embodied with three levels of adaptation has been proposed: distributed adaptation was designed for estimating the leader dynamics, the follower dynamics, and the solution to the regulator equations.
Distinguishing features of the proposed approach are:

- No globally known information are used by the proposed controllers: in fact, the proposed exosystem estimator (5-13) solves the case in which the exosystem dynamics are unknown to every agent and no estimate of S is shared along the graph.
  Moreover, the matrices in (5-19) are always negative definite, regardless of the values of δ_i and ω_k, making the proposed estimator fully distributed.
- Due to the structure of the proposed methodology, no initial stabilizing controller is required because, as already mentioned, the matrices in (5-19) are always negative definite.
- Adaptive control easily handles big parametric uncertainties: in fact, in contrast with approaches based on fixed-gain robust control no assumption is made on the size of the parameter uncertainty set.

Therefore, similar features of traditional adaptive output regulation of individual systems has been recovered in a networked setting.
This allows handling situations in which minimum a priori information might make it impossible to have an initial stabilizing solution.

The surprising final result showed in the previous chapter suggest that possible future work should focus on how to mathematically demonstrate that even in absence of persistency of excitation, the cooperative output regulation problem is still solved by the proposed method.
Furthermore, in this work the graph have been always considered undirected, but in a real scenario it is difficult to ensure that every agent can communicate with each agent and, for this reason, a possible extension could be considering a directed communication graph.

Moreover, a possible future development could be considering cooperative control of systems with switching topology. While some work has been done for systems without uncertainties [40, 41], the joint presence of uncertainty and switching topologies makes the problems hard. Possibly a solution to this problem might come from recent tools of adaptive switched control [42, 43, 44, 45, 46].
Finally, the most challenging extension would be trying to develop a similar methodology also for agents characterized by non linear dynamics.

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