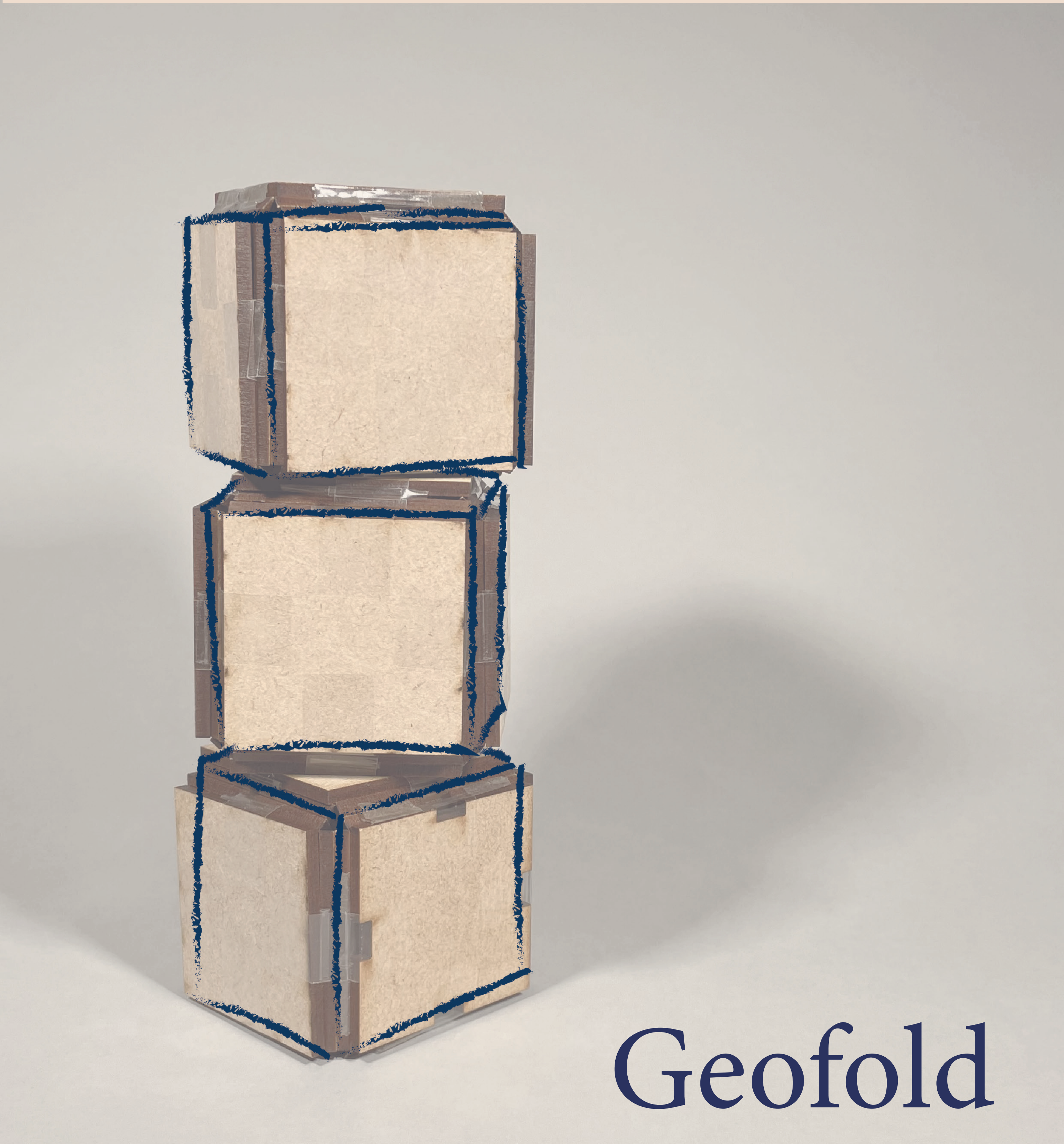

Geofold

Emma Heilig

Master Thesis
Msc. Design For Interaction

June 2025

Where geometry unfolds. A toolkit that helps high school students develop spatial skills through tangible play and structured exploration.

Master Thesis
June 2025

Written by **Emma Heilig**

MSc. Design For Interaction
Delft University of Technology
Faculty of Industrial Design Engineering

TU Delft supervisory team
Chair: Prof.dr. P.J. Stappers
Department of Human Centered Design

Mentor: ir. A.J.M. van Leeuwen
Department of Human Centered Design

In collaboration with
Sciene Hub Tu Delft
Client supervision: L. Sonneveld MSc

Preface

Over the past few years, I have tutored over 30 different high school students for the course mathematics. Every student, despite coming from many different schools, brought the same material: A book, with explanations and exercises, a notebook to write down their answers and sometimes a calculator if they did not forget to bring it. Every exercise, every explanation, was a race against the clock, with an important test lurking in the background. At the beginning, I considered it normal; it reflected what I had done for the subject back when I was in high school. But as time went on, I started wondering, isn't there a more fun way to teach, a way where students actually want to learn?

This question has kept me thinking for the past few years, taking every opportunity during my studies to design for education and motivation. When the day finally came that I could start my own project, I decided to do exactly what I have been wanting to do for a long time; designing to make learning more fun.

I would like to thank my supervisors PJ and Arno for keeping me on the right track throughout the project and Leonie who got me in touch with all the right people. I would also like to give a special thanks to Tom from the Makersspace who helped me realise my crazy prototype ideas. And I would like to thank my friends and family who I could count on in the crucial moments; especially Laura, Agueda and Fien.

Abstract

Mathematics education in secondary schools is often experienced as abstract and disconnected from reality, leading to declining motivation among students. To address this, both the Dutch Ministry of Education and the SLO are working to integrate spatial skills more explicitly into the curriculum. Simultaneously, there is growing interest in inquiry- and design-based learning, which aligns well with the spatial, tangible changes that are coming.

This research investigates the question: How can 12 to 15 year old havo/vwo students develop their spatial skills during math classes through a tangible object? Sub-questions explore the definition of spatial skills and the structure of a typical math lesson.

The process was divided into a research cycle and a design cycle. The research cycle involved literature review, interviews, and observations, and used context mapping to identify similarities and differences between student needs and school systems. The design cycle followed a research through design approach, with iterative prototyping and user testing.

The literature review revealed that spatial skills are not just about mental rotation or perspective-taking, but depend on the object, its surroundings, and how we engage with it; mentally, physically, or through movement in the environment. The context mapping analysis shows that schools prioritize structure, efficiency, and test preparation, while students expressed a desire for interactive and collaborative experiences. This led to a design space that balances both needs: structured lessons that still allow for active exploration.

The resulting concept is a magnetic cube toolkit designed to enhance spatial reasoning through geometry tasks that align with the curriculum. Instead of adding extra work, the tool replaces optional exercises with open-ended, spatial challenges. Students select an assignment card and build with modular cubes, progressing from 2D to 3D thinking and from closed to open questions.

User testing showed that the haptic experience of the magnetic cube increased engagement. However, improvements were needed in the clarity and length of task descriptions, since the connection between the questions and the cube was not always clear. The physical size and production cost of the prototype were also mentioned as limitations, making large-scale classroom implementation unrealistic in its current form.

As a recommendation, a more affordable, compact version was developed, even if this means losing the tactile quality of the magnets. Future steps include refining the design for production and conducting full-classroom trials to test feasibility and educational impact.

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Glossary

The Dutch school system works different from other countries, this glossary will shortly explain how high schools work and several words will be defined.

In the Netherlands, after primary school (which ends around age 12), students enter secondary education, which is better known as high school. High school is divided into different tracks based on academic ability. Two of the main tracks are HAVO and VWO.

High school: High school in the Netherlands can be compared to grades 7 through 12.

Second and third year high school students: Students aged 13-15. Comparable to grade 8 and 9 in other countries. When referring to high school students in the report, this group is meant.

HAVO: A five year secondary education track that focuses on preparing students for higher professional education, which is comparable to universities of applied sciences. HAVO is considered mid to high level of secondary education.

VWO: A six year secondary education track that focuses on preparing students for research universities. The curriculum is more theoretical and research orientated.

OOL: Research and Design Learning course taught in high schools (Onderzoekend en Ontwerpend Leren).

STEM: An umbrella term for education being focussed on Science, Technology, Engineering and Mathematics.

1. Introduction

This chapter will introduce the project brief and research question that formed the start of this project. It will elaborate on the reasons that led to the development of the project, such as the importance of spatial skills in mathematics education and describe how it is relevant. Lastly, it will introduce the client for the project: Science Hub TU Delft.

Nowadays, school feels like a chore; you sit, you listen, you do homework and you take a test. Learning, which comes so naturally to people, suddenly does not feel so fun anymore. Learning is a process, where mistakes and exploration take you further than staying within the lines. However, schools and their grading system have taken away this freedom that comes with learning, resulting in a declining motivation for many subjects.

For many subjects, the textual way of learning works just fine, but for a subject like mathematics, which also covers spatial concepts, this textual way of learning can cause problems. Mathematics requires abstract thinking and strong spatial skills, especially in topics such as geometry, Pythagoras theorem and volumes and areas. Many high school students struggle with this these topics, which hinders their understanding and motivation in mathematics (Atit et al., 2020).

1.1 Maths education and spatial skills

In the early years of high school, the main goal of mathematics education is to develop students' mathematical literacy. This includes learning to use the 'mathematical language'. Being mathematically literate and numerate involves a coherent understanding of numbers, measurement concepts, and spatial reasoning (SLO, 2023). It also includes the ability to interpret graphs and tables, as well as to recognize and work with shapes in both two and three dimensions.

Although not always explicitly emphasized, the role of spatial reasoning within mathematical literacy is supported by research. Studies show that students with strong spatial abilities often perform better in math classes. They can easily understand mathematical concepts and recognise patterns, because spatial skills help the development of critical thinking, problem-solving and logical thinking (Harris, 2020). This is caused by the close relation between mathematics and spatial skills. Literature from psychology, education and neuroscience give four explanations to why the two are related (Hawes, 2020):

1. The mental number line: every person thinks of numbers as being arranged in space, either in a straight line from left to right, a wave or ladder.

2. Shared brain processing: The parts of the brain that processes math problems, is the same area that light up of CT- scans when faced to spatial challenges. A theory called 'neuro-recylcing' states that this is caused by the evolution of the brain, where it was originally used to think about space, this part of the brain is now adapted to solving math, which contains problems that have evolved from spatial problems.

3. Spatial modelling: Some theories suggest that math is connected to physical and visual experiences, where spatial skills are needed to imagine and manipulate numbers to understand them, just like we do with real life objects.

4. Working memory: A few theories think the link is due to the ability to hold and process information in the mind (working memory). Although some research has shown that spatial skills and the working memory work together, but operate separately.

1.2 Relevance

The SLO, an organization that develops the curriculum for the Dutch high schools, has acknowledged the importance of spatial skill development as part of high school education. For the first time in 15 years, they are changing their learning objectives and exam program for the course mathematics. This change puts more emphasis on spatial skills within existing topics, such as geometry (SLO, 2024). One of the reason for this change according to Rijksoverheid, is the ongoing changes in society like digitalisation and the increasing importance of STEM subjects. They state that education should adapt to fit these changes. (Ministerie van Onderwijs, Cultuur en Wetenschap, 2025)

Another reason for the change was the declining motivation for the course mathematics (Scherrer & Preckel, 2018; Wijsman et all., 2015). While mathematics is considered an important subject in early education, its perceived importance diminishes in high school, leading to a decline in students' intrinsic motivation. This can be explained by the lack of challenging material; In mainstream education, more remedial lessons are offered than enrichment lessons, resulting in limited opportunities for students who show potential to excel. (SLO, 2021)

1.3 Science Hub TU Delft

People who also acknowledge the importance of spatial skill development, are the researchers at the science hub at TU Delft, who are the client for this project. At the hub, employees, researchers and designers work together with primary and secondary school teachers on projects in which the design process is central. Their aim is to stimulate both students and teachers in creative thinking, designing, and researching (wetenschapsknooppunt, z.d.). At this moment, this approach to teaching is mainly used in specially designed courses like OOL. However, the Science Hub aims to integrate this method into a broader range of subjects.

They are currently exploring how students of different age groups can improve their spatial abilities through the use of tangible and material based learning tools. This focus on spatial skills aligns with the principles of research and design learning. Since the development of spatial abilities often benefits from hands-on, visual, and iterative activities. By integrating spatial skill development into the mathematics curriculum in a way that aligns with the design process and creative thinking, education can become more engaging and meaningful for students. This brings us to the didactic foundation of the project: research and design learning.

1.4 Research and design learning

Research and design learning is a teaching method that stimulates students to approach a problem in creative and hands-on ways. This didactic approach is similar to the way professional designers work, where experimentation, iteration and solution-based thinking is emphasised. The main goal is to help students develop crucial skills, like creativity and collaboration that are needed in the 21st century (Klapwijk & Holla, 2018).

Research and design learning follows a structured, iterative process, often visualised as a 'design and research cycle', consisting of several steps that guide students from problem definition to tangible solutions (figure 1). The design cycle is slowly finding its' way into the Dutch curriculum, where it is now implemented in high schools in courses like OOL (Onderzoekend en Ontwerpend leren).

What makes this course interesting is that it allows for multiple solutions to the same question. Unlike many traditional courses that focus on finding the one 'correct' answer, this course puts the emphasis on broader learning objectives (figure 2). Students are encouraged to think in different directions, empathize with others, experiment and share their thinking.

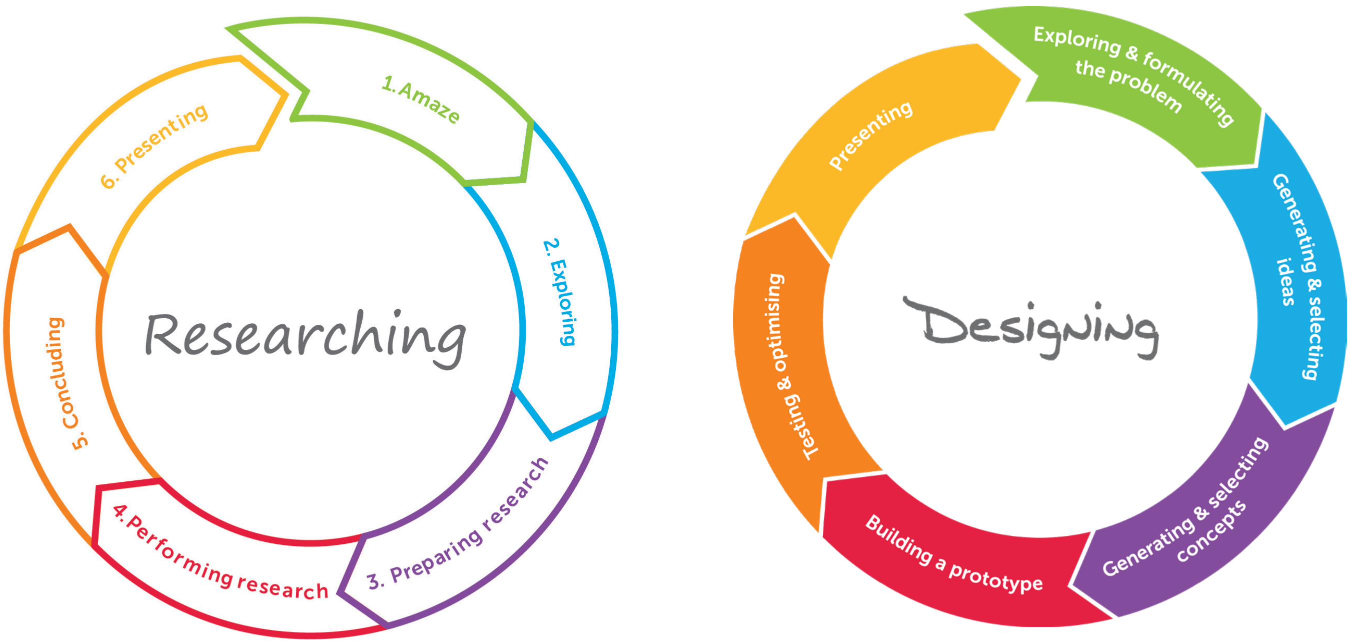


Figure 1 – The research and design cycle (Klapwijk & Holla, 2018)

Science Hub has been actively working to integrate design learning not only into OOL, but across other subjects as well. For mathematics, this would mean to incorporate reflective and inquisitive learning, referred to as the 'mathematical attitude'. When math education is limited to mindlessly applying rules and procedures, important opportunities are missed to make meaningful connections between skills and their practical applications. By encouraging reflection and discussing solution spaces, students will learn to recognise underlying mathematics in various (new) situations. (Jonkers & Wijers, 2016)

Figure 2 – Learning objectives for design learning (Klapwijk & Holla, 2018)

1.5 Research question

Collaborating with the Science Hub at TU Delft allows for a material-based learning experience that encourages students to explore and problem-solve while working on their spatial skills.

Together, this leads to the research question of this project:

How can 12 to 15 year old havo/vwo students develop their spatial skills during math classes through a tangible object?

Opportunities in this domain lie in the introduction of interactive tools that allow students to move beyond textbooks and actively engage with three-dimensional figures, allowing students to develop a deeper understanding of spatial relationships. However, the challenge lies in practical implementation: schools have limited time and resources, making it difficult to integrate new teaching methods into an already packed curriculum. Additionally, teachers are often not familiar with alternative teaching methods, which can hinder effective adoption.

1.6 Approach

To address the research question, the project was structured in two phases, each following the structure of the previously discussed research and design cycles (Figure 3).

In the first phase, the research cycle was carried out to gain an understanding of the context and the definition of spatial skills. This involved qualitative methods such as conducting interviews with relevant stakeholders and performing classroom observations. These activities provided in-depth insights into the educational environment and the current challenges related to spatial skill development. In addition, desk research was conducted to analyse existing literature and definitions, which helped to frame spatial skills within an academic context.

The second phase focused on the design cycle, in which the obtained insights from the research phase were translated into concrete design opportunities. This phase involved ideation and the development and testing of prototypes. These prototypes were created and refined through multiple iterative testing rounds, in which feedback was collected and used to improve the design.

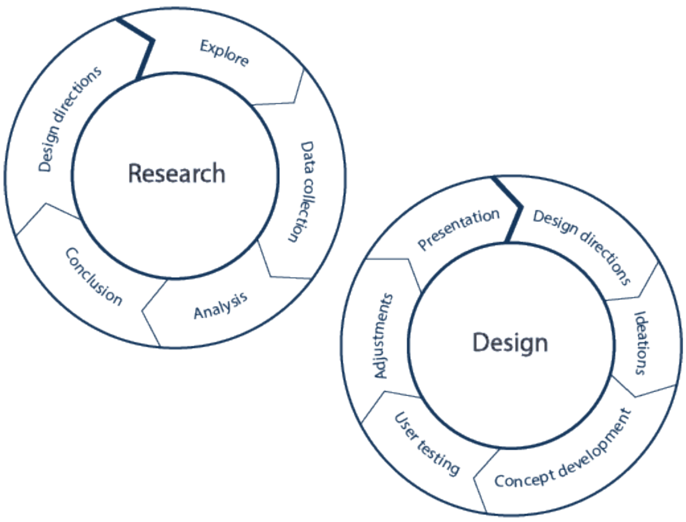
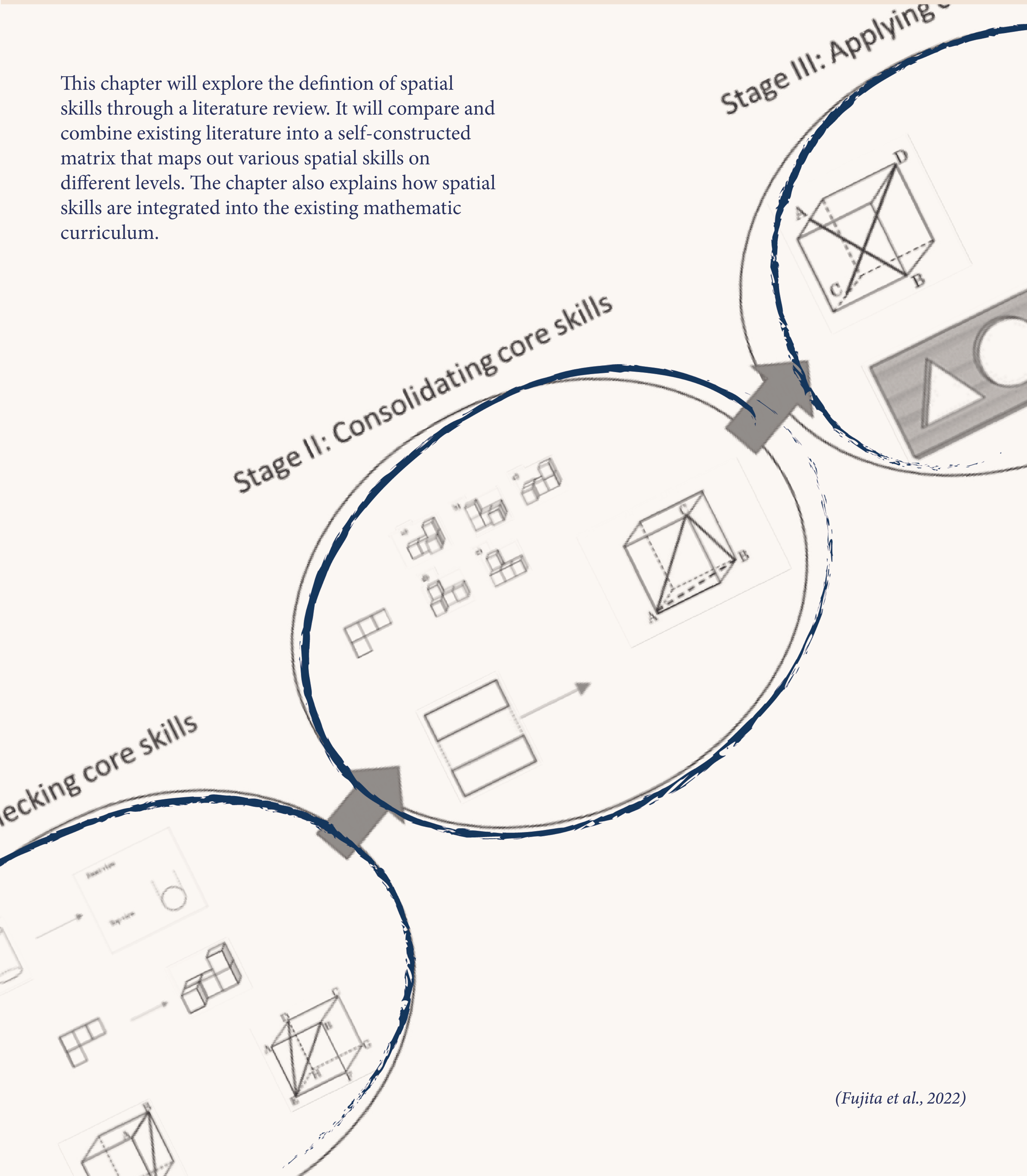


Figure 3 - My approach based on the research and design cycles

2. Exploration

This chapter will explore the defintion of spatial skills through a literature review. It will compare and combine existing literature into a self-constructed matrix that maps out various spatial skills on different levels. The chapter also explains how spatial skills are integrated into the existing mathematic curriculum.



(Fujita et al., 2022)

2.1 The definition of spatial skills

In order to understand how spatial skills can be developed, it is important to define what spatial skills are. Spatial skills is an umbrella term for different spatial abilities. For most people, it means something in the sense of: the ability to generate and manipulate an images in the mind. Battista (2007) looked deeper into the definition and describe spatial skills as 'the ability to see, inspect and reflect on spatial objects, relationships and transformations'.

Over the years, many researchers have tried to define the term 'spatial skills' by comparing the different types of spatial problems and placing those into groups. Ramful et all. (2016) for example, organised these spatial problems into a three tier framework, categorising the different types of abilities as: mental rotation, spatial orientation and spatial visualization. Skills like the rotation of objects, map reading and recognising symmetry should fit the categories respectively.

A similar distinction was made by Fujita et all. (2022), who came to the same categories, but added the relationship between the object and the observer. It expands the existing groups by comparing them in a three dimensional space, showing how important it is if the observer or the object is moving.

The fact that movement is important for defining spatial skills also comes forward in a paper by Uttal et all. (2013). They define spatial skills by crossing two dimensions: intrinsic versus extrinsic information and static versus dynamic tasks. Intrinsic information refers to distinguishing specific properties and therefor recognising an object, while extrinsic information is the relation between this object and another. Static tasks are about a non-moving object, where ratios are consistent and dynamic tasks include movement and thus a changing ratio.

Based on the different definitions, a self-constructed matrix was created to map the various skills involved in spatial reasoning (figure 4). The axis create four quarters: object recognition, mental transformation, spatial orientation and spatial visualisation.

What all these definitions have in common is that they focus primarily on categorizing the specific skills needed for solving spatial puzzles. However, they often overlook the real-life contexts in which

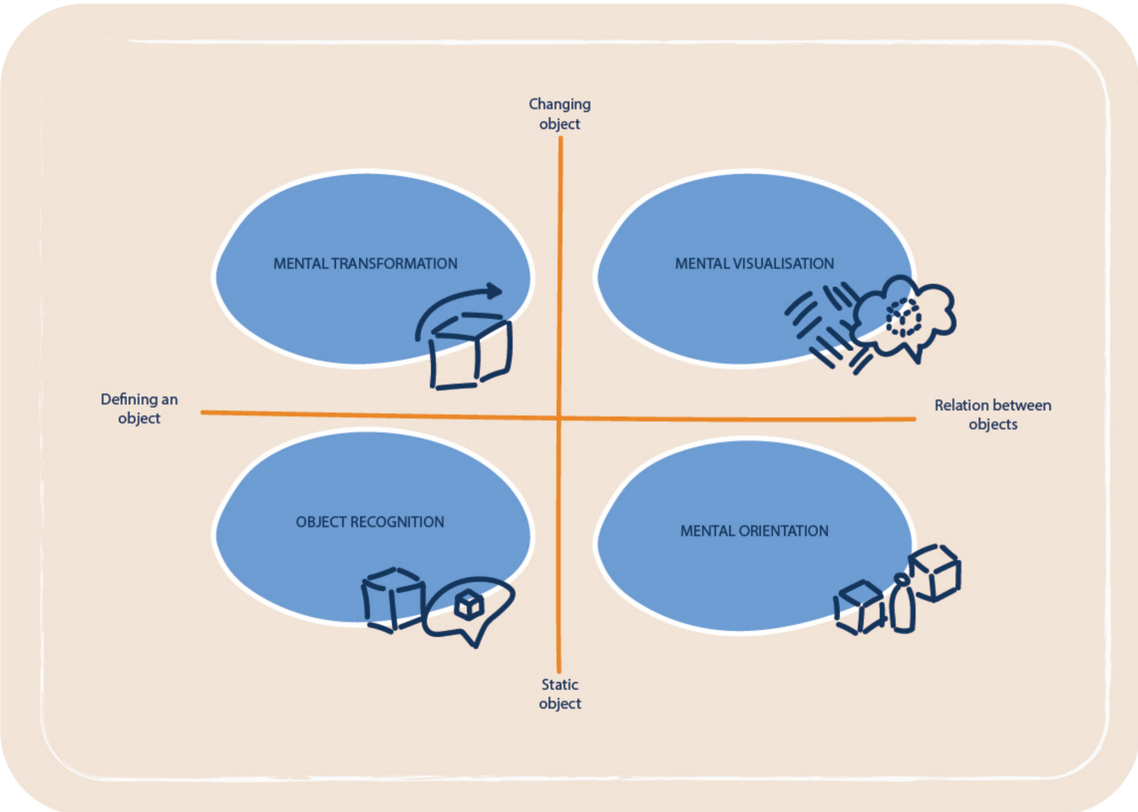


Figure 4 – A matrix of the combined spatial skill definitions

these spatial skills are actually applied. This means that while the definitions are useful for breaking down the different components of spatial reasoning, they rarely explain how these abilities manifest in everyday tasks and situations.

Object recognition can be explained as recognising the key that opens your front door based on its' shape and colour. A good example of mental transformation is trying to fit all your luggage in the back of your car the night before you go on holiday. By looking at how the different sized bags or suitcases can be placed in the very compact trunk, you have to be able to mentally rotate the objects in front of you. Spatial orientation becomes important when you walk out of a store after buying something and figuring out what way you should go now that you are looking at the street from a different perspective. Setting up a tent is an example of spatial visualisation. You scan the grassy area, notice nearby trees or uneven ground, and picture how the tent will stand once the poles are in place, before even touching anything.

These examples suggest that spatial skills are tools that help to solve a problem within a specific environment by having the body and mind work together. A spatial puzzle or problem can thus be solved in the mind by thinking about it, with the body by actively moving things or by analysing and including the elements of the environment. When adding this dimension to the previous matrix, you get a matrix that looks like figure 5.

To clearly illustrate the distinction between the three layers, it can be compared to implementing tactics in a team sport. The first layer, the mind, is like the team receiving an explanation of the tactic during a tactical meeting. Players discuss positioning and movements, drawing runs and placements on a whiteboard to understand the concept mentally. The second layer, the body, comes into play during training sessions, where those same tactical movements are physically practiced. Through repetition, players train their bodies to instinctively move into the right positions on the field. The final layer, the environment, can be compared to the actual match. The team attempts to follow the trained tactic, but must now adapt it to a dynamic game situation, with unexpected plays, an unpredictable opponent, and changing conditions on the field.

2.2 Spatial skills in mathematics education

When comparing this matrix with the current mathematics curriculum of the second and third year of high school (figure 6), it becomes clear that most of the mathematical concepts which include spatial skills that are taught in school (Havo 2 & Vwo 2 - Wiskunde Academie, 2019; Havo 3 & VWO 3 - Wiskunde Academie, 2019) can be placed on the defining side of the matrix, in the first layer. This side of the matrix shows geometry topics like calculating the area and volume of different shapes. This suggest that the curriculum mostly focusses on teaching mental transformation and object recognition by engaging only the mind.

This might be caused by the subject's theoretical nature. Unlike subjects such as physics or geography, which often require students to apply knowledge in hands-on experiments or real-world environments, mathematics is traditionally taught in a more abstract way. Where physics might bring a formula to life through a moving object or an experiment, mathematics tends to stop at the conceptual understanding. As a result, there is less natural integration to the second (physical engagement) and third layer (adaptation to dynamic contexts). This might unintentionally limit the development of spatial skills that rely on embodied experience or environmental interaction, such as spatial orientation or advanced spatial visualisation.

2.3 Development of spatial skills

Research shows that one of the best ways to improve spatial skills, is by practising them. Making a lot of puzzles on rotated or mirrored objects, will most likely improve your mental transformation, just like practising your public speaking will make you better at giving presentations. What is special about spatial skills though, is that practising specific type of spatial exercise, such as mental transformations, can lead to an increased understanding of another spatial category, like spatial orientation (Uttal et all., 2013).

Apart from practising, there is also other ways to develop your spatial skills. Research has shown for example that playing videogames and training motor skills are also ways to improve spatial abilities. (Moreau et all., 2012). A way to train motor skills, is through practising sports. Sports, and especially ball- and team sports make use of spatial visualisation (Peterson et all., 2020); as a player you have to

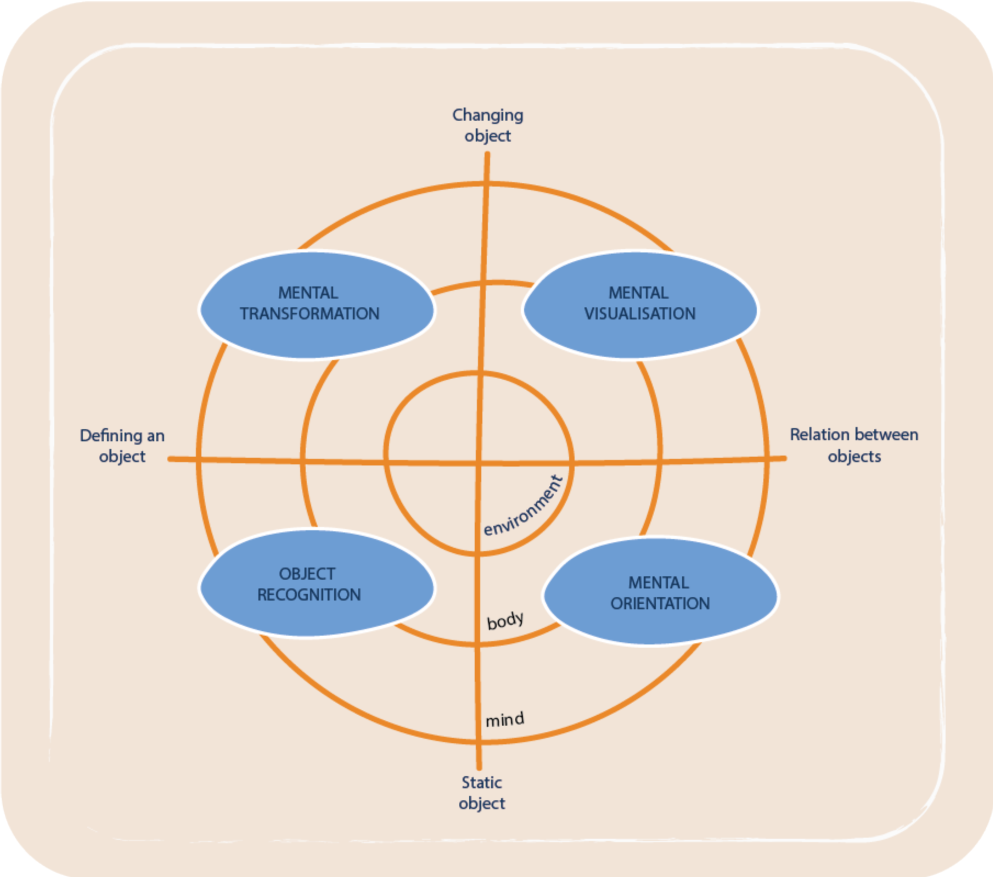


Figure 5 – Constructed matrix to organise all spatial skills

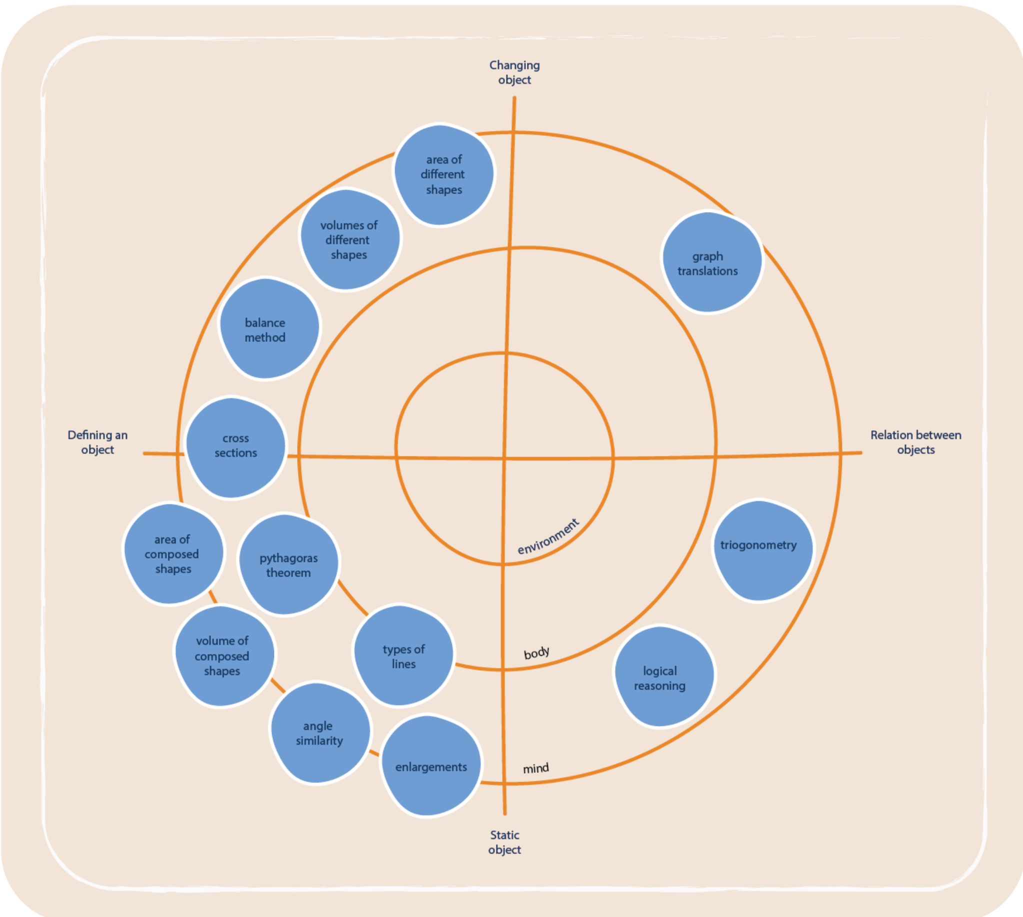


Figure 6 – Matrix filled with different curriculum subjects

estimate how your movements and the movements of your team member will eventually lead to the desired outcome. The interesting thing about this, is that this estimation is something you do not consciously do; it is the result of years of practise and experience.

The fact that spatial skills are developed by practise and experience suggest that active learning or learning while doing, is the best way to develop the skill. The research of Ha & Fang (2017) suggest a similar thing; Their experiment with a tangible object in combination with a simulation program concluded that a tangible tool increases the understanding of the object and the way it can change.

2.4 Active learning with origami

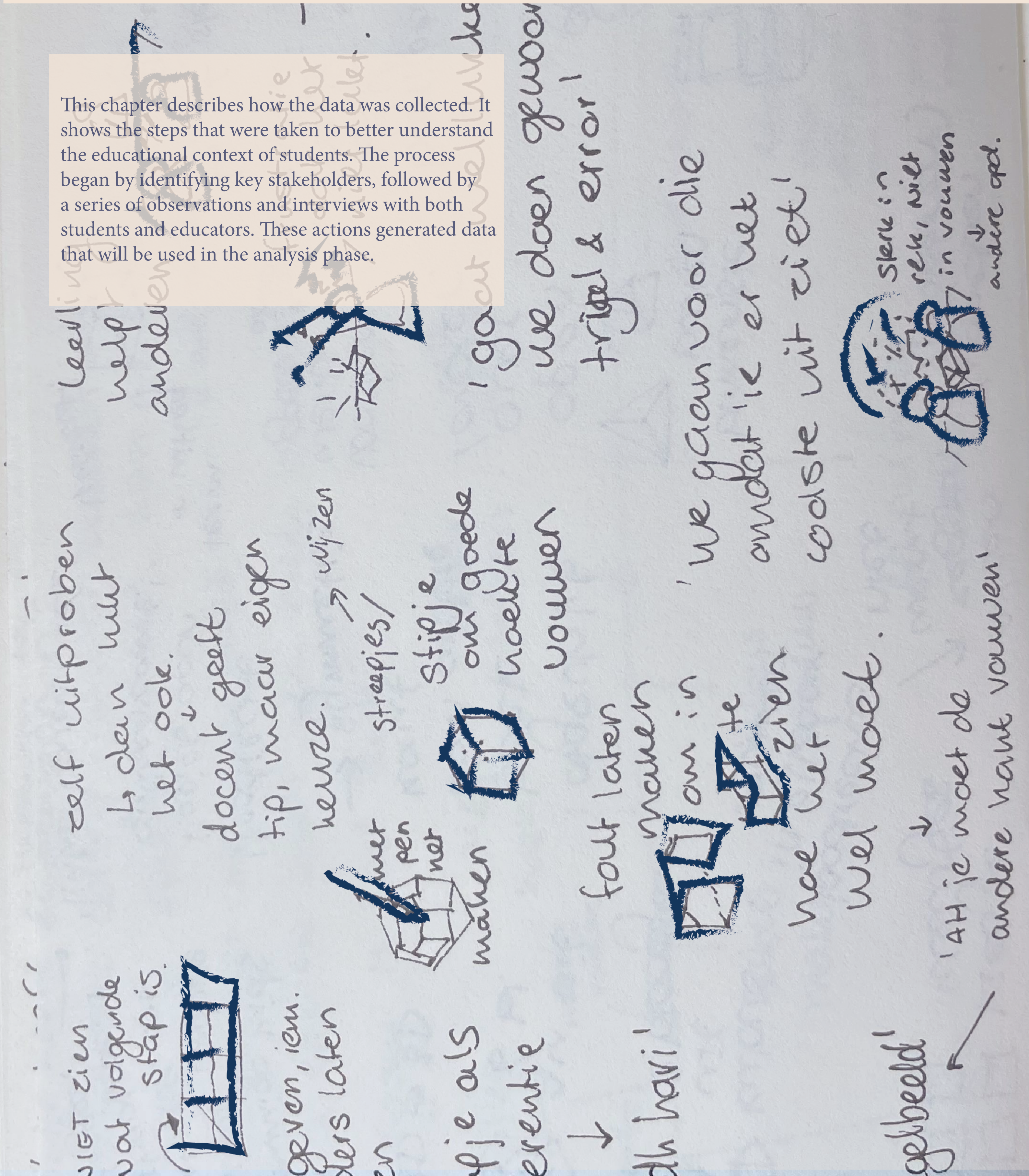
Origami is the art of folding papers. It is a great example of how a tangible tool can enhance the understanding of a spatial and changing object. The idea of using origami, is to explain abstract concepts by making them concrete. Mathematics is an abstract representation of the real world, but this connection is often overlooked because the mathematical language has become increasingly more abstract throughout the years, making it decreasingly tangible. (Visser, 2024)

Research shows that abstract concepts are better understood when they are made concrete; Experiencing these concepts in the real world is necessary in order to gain a deeper understanding, whereas knowledge based on solely verbal instructions tend to remain superficial. (Visser, 2024)

In Japanese education, origami is added into the curriculum as a learning tool. Influenced by Friedrich Fröbel, who believed that play was essential for learning, origami has shown to support the development of spatial skills, as well as increase students' interest in the subject of mathematics (Marji et al., 2023). By folding paper and geometrically transforming two-dimensional shapes to form figures, geometrical concepts can be developed in an engaging way. (Hanada, 2022)

3. Data collection

This chapter describes how the data was collected. It shows the steps that were taken to better understand the educational context of students. The process began by identifying key stakeholders, followed by a series of observations and interviews with both students and educators. These actions generated data that will be used in the analysis phase.



3.1 Stakeholders

Schools do not operate in isolation, they are part of a larger educational system and must adapt to policies and regulations made higher up. In any school system, the students are the primary focus of education, with various stakeholders influencing their learning experience and development. These stakeholders can be separated into a direct, indirect and system level.

On the direct level are those who interact with students on a daily or regular basis. Teachers are the most visible stakeholders on this level, delivering the material and motivating the students. Alongside them, mentors and counsellors support the students on a more personal basis, helping them with their study planning or emotional challenges. School leaders, such as the school principle or the department heads also influence the experience of the students. Although not being in direct contact with the students, school leaders decide over school policy, teacher selection and the creation of the learning environment. The last stakeholder on this level is peers and classmates, who form a significant part of a student's daily interaction.

External stakeholders do not always engage with students directly, but play a supporting role. Parents or guardians can be found on this level, providing emotional and practical support while learning. Examination boards can also effect the learning experience indirectly through checking the quality of the education and the objectivity of the grading.

At the system level, stakeholders often operate on a larger, national scale, making guidelines by setting learning objectives in order to minimise the differentiation between schools. Curriculum developers, such as the SLO, determine the learning goals and content for students of all ages and levels in the Netherlands. They work closely together with Ministry of Education (OCW) in order to make learning objectives that are to be achieved throughout the whole country. Schools are then checked by the education inspection, our final stakeholder on this level, to see if these goals and learning objectives are followed.

Operating on all levels are the educational suppliers. Publishers translate the set goals and objectives into books or online learning materials that can be chosen by students and teachers to study the necessary subjects.

In order to gather data from the stakeholders, interviews and observations were set up with the most relevant stakeholders: students, teachers and curriculum developers. Over the course of six weeks, seven math classes were observed and three teachers, three students and two curriculum developers were interviewed.

The classes that were observed were taught by three different teachers, at three different schools. The students that were interviewed attended schools

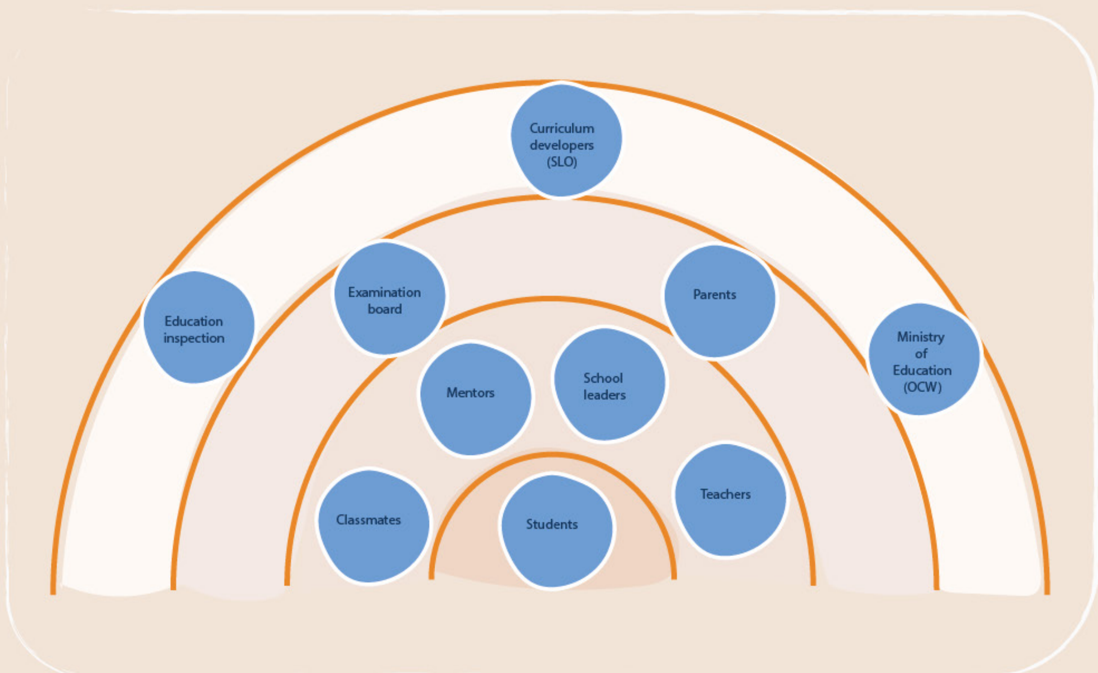


Figure 7 – stakeholder map

outside from those.

3.2 Observation set-up

The aim of observations was to map out what the average math class looks like and to gain insights into what tools are provided to help them learn.

During observations I took a seat at the back of the classroom, having an overview of the whole class, while experiencing the lesson like one of the students. My points to pay attention to included: the approach teachers took to explain new material, how students reacted to the explanation, how a lesson proceeds and what kind of questions or obstacles students found. This information was collected in a notebook in the form of drawings and notes (Appendix B).

Observations were done in three different schools in a number of different classes. At each school, the observation was done in a different class, taught by the same teacher. In school 1, the same class was observed at two different moments of the week. In total, the following classes were attended:

School 1
- 2GymnasiumA (geometry)
- 3HavoTechnasium (algebra)
- 2GymnasiumA (geometry)

School 2
- 2Vwo (Pythagoras theorem)
- 2Havo (Pythagoras theorem)

School 3
- 6GymnasiumA (extra class: origami)
- 6GymnasiumB (extra class: origami)

The origami classes at school 3 were attended as a form of inspiration. Origami is a prime example of how a two dimensional object can transform into a three dimensional object. This transition is similar to the transition that is needed to move from books to tangible learning tools and therefor relevant to the project. Origami has since this point been an recurring source of inspiration throughout the project.

3.3 Interviews set-up

The aim of the interviews with students was to further map out the average math class, but also gather information on how students learn best from their perspective and what challenges they face while being in the classroom. This was done by asking them a set of questions about their preferred environment and way of teaching. The interview ended with a spatial puzzle that they had to solve. This gave insight into how they would approach a spatial problem and what tools they would reach to first. The students I interviewed were in their second and third year of high school, either in Havo or Vwo.

The interviews with teachers had a more casual setting, where it had more resemblance to a conversation about their teaching methods and the challenges they face by having many different students in one classroom. It also elaborated on observation in their classroom prior to the interview, how they tried helping students and how they notice if their students exceed or struggle.

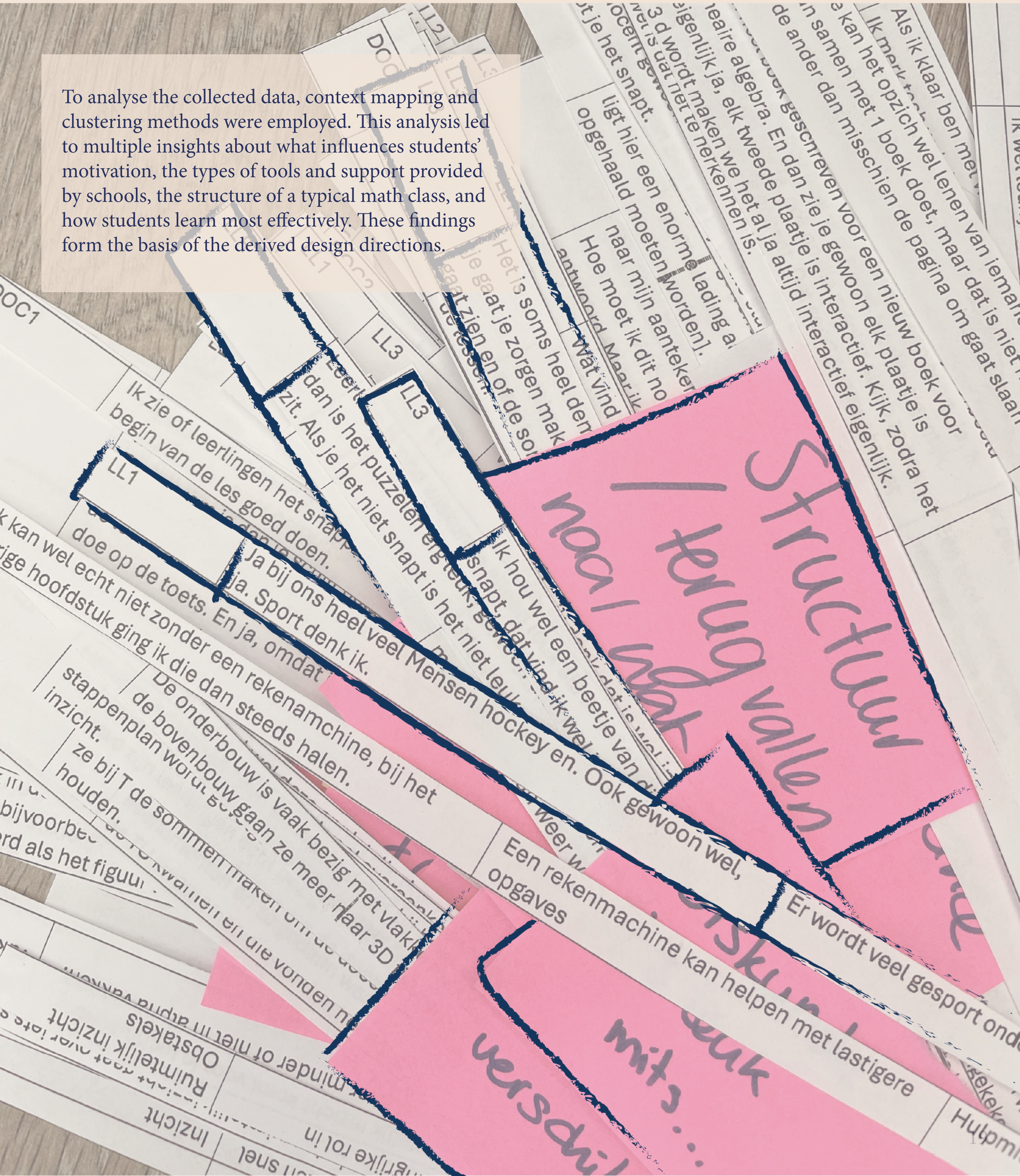
The interviews with curriculum developers again followed a set of questions and aimed to gain insights into how decision are made higher up and what the process of making tools looks like. Two participants were found that can be associated with this group. The first one is the Head of the Dutch association of mathematic teachers (Nederlandse Vereniging van Wiskundeleraren). The association offers a platform for sharing knowledge about mathematics education through blogs, forums and exchanging teaching material, contributing to the development of the profession. The board of the association also maintains close relations with parties like the preciously named SLO and the Ministry of Education.

The second participant is an employee from PRIME (The PRogramme of Innovation in Mathematics Education). This initiative aims to redesign existing mathematics courses at the TU Delft to strengthen the connection between mathematics and engineering. PRIME supports this goal by providing teaching materials for all the mathematical courses at the university. They do so in the form PowerPoint presentations and interactive online simulations. Even though PRIME is not directly linked to high school education, their approach to developing teaching materials can offer interesting insights.

All interview questions can be found in appendix C, as well as the Informed Consent Form (Appendix D).

4. Analysis

To analyse the collected data, context mapping and clustering methods were employed. This analysis led to multiple insights about what influences students' motivation, the types of tools and support provided by schools, the structure of a typical math class, and how students learn most effectively. These findings form the basis of the derived design directions.



The data that was generated from the interviews with students and the classroom observations was used for context mapping. By clustering the data, insights were gained into students' experience of math classes, their motivators and the classroom structure. The same data was also used to identify patterns that define a typical math class.

4.1 Context mapping

To analyse the dataset, the DIKW model was applied (Figure 8). DIKW stands for Data, Information, Knowledge, and Wisdom. This approach makes data more accessible for discussion, making it a suitable method to promote collaboration and shared understanding within a research team. Often, the analysis leads to both insights and inspiration. (Stappers et al, 2023)

Data	- Raw data and unorganised facts.
Information	- Organised data with a pattern or meaning.
Knowledge	- Interpreted information to make connections with the context.
Wisdom	- Created understanding to make further decisions.

To move from one level to another, the data was prepared, organized, and analysed. To move from Data to Information, quotes are extracted from the interview transcripts and paraphrased by translating them into one's own words and insights in the form of statement cards. To move from Information to Knowledge, the relationships between themes and insights must be explored within the context by clustering similar statement together. Wisdom can be achieved when the gained insights are applied, for example, in the form of design directions and prototypes.

4.2 Application DIKW-model

4.2.1 Data to information

To jump from data to information, the transcripts were carefully reviewed. While reading, interesting passages or quotes were highlighted, with brief notes added to indicate their relevance to the research. After reviewing all transcripts, the highlighted sections were critically examined once more. The most meaningful fragments were then converted into so-called statement cards, an analytical tool used to transform raw data into interpreted insights.

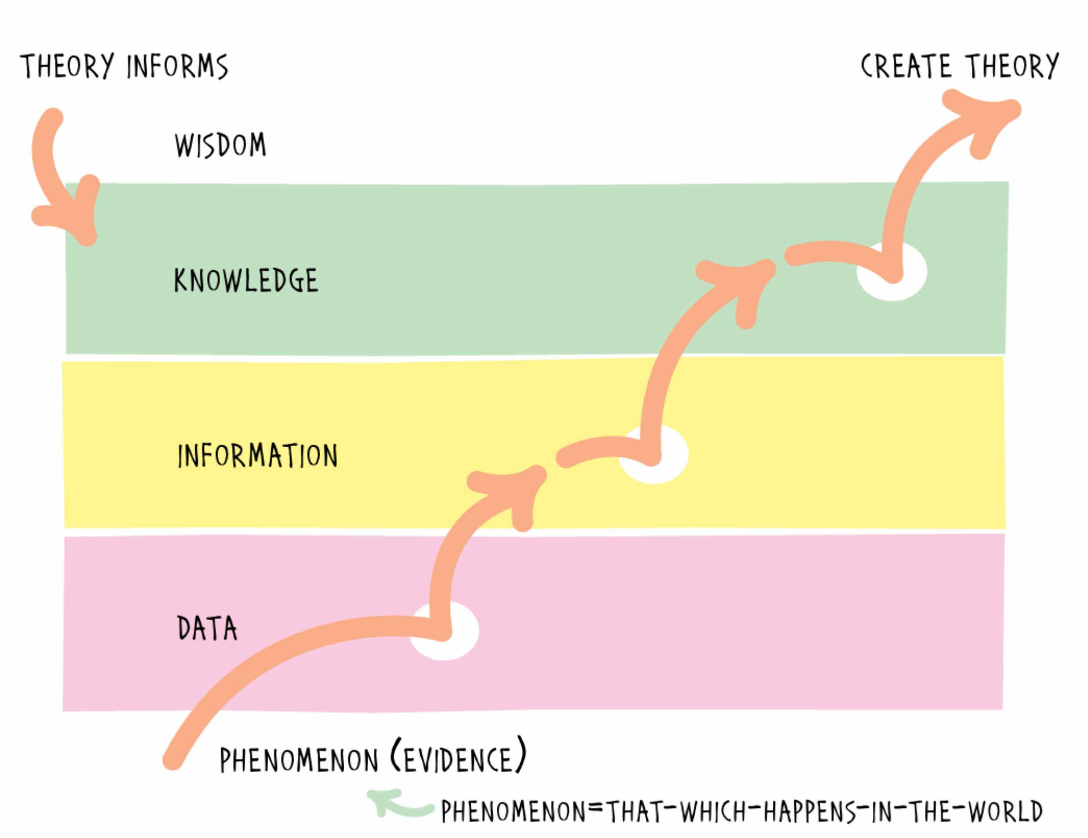


Figure 8 - The DIKW-model (Sanders & Stappers, 2012)

Each statement card contained four components (Figure 10):

- the original quote or fragment
- a marker indicating which transcript the quote comes from
- an interpretation of the fragment, i.e., the statement
- tags that summarised the statements in words.

A statement card usually only contains the first three components, but by adding the modification of a tag, it was easier to compare similar statements.

The interpretation is the most important part of the card and is therefore visually the most prominent. It is a rephrasing of the quote, making the main idea quickly and clearly recognizable.

4.2.2 Information to knowledge

To convert the obtained information into knowledge, the statement cards and the observation drawings were used as the basis of the analysis. In preparation of the clustering session, all statement cards and drawings were printed and cut. In total 115 statement cards and around 100 observation drawings were used. A preselection of 40 cards and drawings was made, deliberately choosing a wide variety of topics to provide a clear overview of the diverse information. The result of the clustering session can be seen in figure 11.

Supplies:

- Statements cards
- Observation drawings
- Coloured post-its
- Pens and markers
- A wall or floor
- (Tape)

Approach

1. Initial clustering

- Starting with one statement card from the preselection, the statement is read and placed somewhere on the designated area.
- While placing the rest of the cards and drawings, similarities with previously placed cards are identified, and related cards are grouped together.
- Once all 40 cards are placed, clusters begin to emerge. The researchers assess what the cards within each cluster have in common. Each cluster is given a label consisting of a short sentence with a verb. This prevents the clusters from being seen as simple groupings (e.g., instead of labeling a cluster “lesson structure,” one might choose: “lessons follow the structure of the book”)

2. Deepening and restructuring

- After a short break, the remaining statement cards and drawings are reviewed.
- The remaining cards are now processed: cards are either added to existing clusters, placed in a logical new spot, or used to form a new cluster.
- Finally, after placing all the cards and drawings, the relationships between the clusters are addressed. This helps to clarify contrasts and similarities and potential connections to existing literature.

3. Wrap up

After the analysis, the results are carefully documented.

- Photos are taken of the complete board and of the individual clusters.
- An audio or video is recorded to discuss the clusters and their connections
- All the cards are removed from the board are archived per cluster (e.g. in an envelope per category)

4.3 Insights and connections

Analysing the clustered data shows several insights and connections that reveal how students engage with mathematics, what supports their learning and how schools structure their lessons (figure 12).

Marker	Quote	Statement	Tag
LL1	Bij andere vakken zou ik denken wat saai, maar wiskunde is wel zo'n vak waarbij je niet heel veel opdrachten samen hebt. Ik vind het wel prima om gewoon te overleggen of een beetje uit te leggen aan elkaar.	**Samenwerken bij wiskunde is niet heel erg nodig, maar wel fijn voor korte uitleg of overleg.**	Samenwerken Les

Figure 10 – An example of a statement card

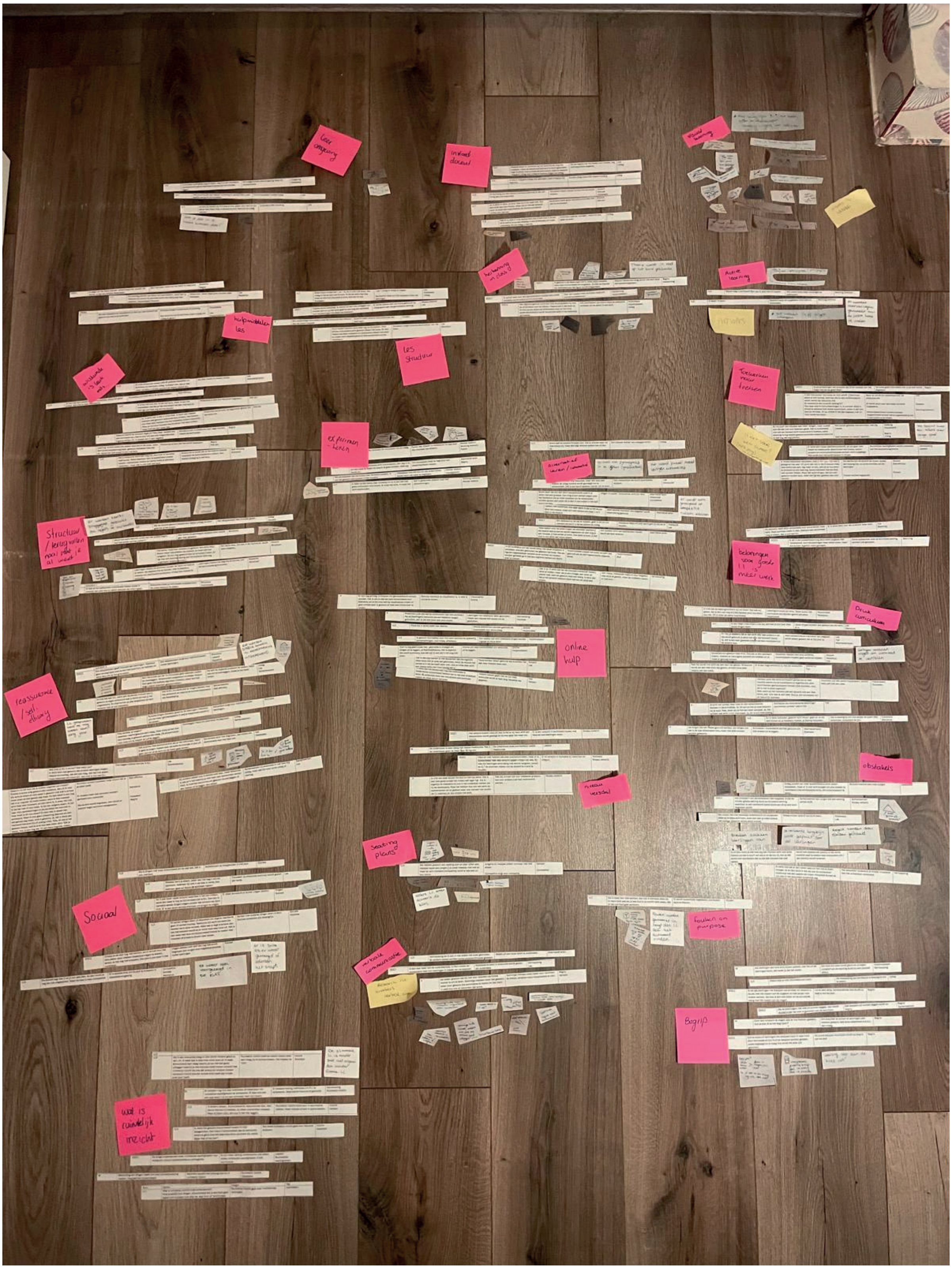


Figure 11 - Results of clustering session

4.3.1 Overall insights

Math is fun
The most interesting insight, was the fact that students actually think mathematics is fun, as long as the material is understood. When they understand the material, they see the exercises as puzzles and like to solve them. This links back to the self-efficacy that students need to have. Students like to have the feeling that they can do hard things themselves, which is why multiple students state that it is crucial to have enough personal time to try and solve a problem by themselves. However, once they think they have the right answer, they need reassurance. This is done by either checking the answers of a problem, or discussing it with peers.

Social pressure
This constant need of reassurance comes from the social aspect that school has. Students want to fit in and not want to be considered stupid by their peers. This anxiety can sometimes cause them to refrain from asking questions altogether. Students state that they would therefor prefer to sit next to someone they know and like, in order to feel at ease with asking for help. Schools do not see this necessity, because most classes have a fixed seating plan. Teachers state that this separation of friends is necessary in order to have a quiet classroom, so that all students get the opportunity to learn.

Peaceful learning environment
That the learning environment is important also comes forward when looking at what is on student's desks. The classrooms have small tables that are closely put together, with students needing to bring books, notebooks, calculators, pencil cases and sometimes even laptops. The mess that this creates contradicts the need for a peaceful environment. Students state that in order to learn best, they need a decluttered desk and a silent room, something schools do not offer.

Structured lessons
When looking at how lessons are structured, it becomes clear that it often follows the structure of the books. In a step-by-step explanation, teachers build up to the understanding of the material. They state that this is important to give students the confidence to solve a problem, because it gives them something to hold on to, also when they are stuck in a problem. This circles back to the important of self-efficacy in the course. The step-by-step approach can also be seen in the way students come to a solution to a math problem. They start at the beginning, follow the steps and come to the answer. This is a rigid approach, which limits the amounts of mistakes, but also leaves little room for creative thinking.

No room for mistakes
This need for no mistakes comes from the test mentality that most schools have. Mathematics is a busy subject with a lot of material and a test almost every month. This cancels out the opportunity to learn from mistakes and just experiment, while it is deemed very effective and even part of our natural way of learning (Visser, 2023).

Busy curriculum
The busy schedule also effects the time for alternative learning. Students state that the most fun things to do in math classes are things outside of the standard lessons: Quizzes, escape rooms or other puzzles. Students get really excited when they are allowed to do something different and in a more experimental and playful way. This comes forward in observations as well, where the mentioning of the word 'puzzle' brings great excitement to the classroom. Interviews with students show the same thing; when provided with a spatial puzzle, they would not stop until they figured out the answer. During this little test, students stated that they would want to make the puzzle in real life so they could figure it out more easily. This suggests that there is a need for active learning, or learning while doing.

Visual vs. verbal learning
The final big insight that was found during the analysis was the contrast between visual and verbal learning. Curriculum developers and teachers state that classes consist of different learners, not only do they vary in academic performance, they also vary in being a visual or a verbal learner. To tend to all the students in the classroom, they therefor try to explain new material in a verbal way, as well as a visual way. However, observations show that in every classroom where only verbal explanation was used, students did not understand the assignment, while visual explanation was always effective. This suggests that schools put too much emphasis of verbal learning, while it is not effective on its own.

The textual way of explaining is caused by two things: the fact that we have been learning from textbooks for the past decades and the fact that test

Figure 12 - The named clusters and its relations

are always given in a textual way. A similar thing was discovered by Koolstra (2018). He states that tests are so textual that it is unsure if students are examined on their mathematics skills or on their reading comprehension. This question applies to most subjects and can therefor cause friction in equal opportunities for students.

4.3.2 The average math class

As shown in figure 13, most lessons tend to follow a similar structure. A class always begins with a plenary introduction to the day's planning. It then proceeds into either making exercises about the previously discussed material or new material, based on the teacher's preference. These exercises are then checked or discussed after which a cycle is entered. This cycle includes: explaining new material, practising material individually and practicing material collaboratively with the whole class. Depending on the time that was set for the class, this cycle continues one or multiple times. The three activities may differ in duration, so in case a student finishes early with their individual practice, most teachers will offer some extra work while waiting for the rest of the class to finish. At the end of each class, a plenary closing statement is made to wrap up the work. A complete lesson is often 40 or 45 minutes, while there are also schools that do so called 'double hours' of 80 minutes.

Students also mentioned a few alternative ways of learning that less often used in classes, but do not belong to the average math class: watching videos, making quizzes or doing puzzles. These activities are considered most fun by students, but only occur if extra time can be found in the busy class schedule. The lesson that is mapped out, is part of a bigger picture. Every year, eight chapters need to be covered and tested. With an average of 40 weeks of school every year, this leaves a little more than 4 weeks per chapter. Schools make room for around three full hours of math classes a week, which are converted into 2-4 lessons depending on the length of classes set by the school. This means that students get around 12 hours to learn the necessary material. The first lesson is often taken to create a sense of wonder for the students, to spark enthusiasm for the content of the chapter. This is often the only time that teachers include alternative ways of teaching in their classrooms.

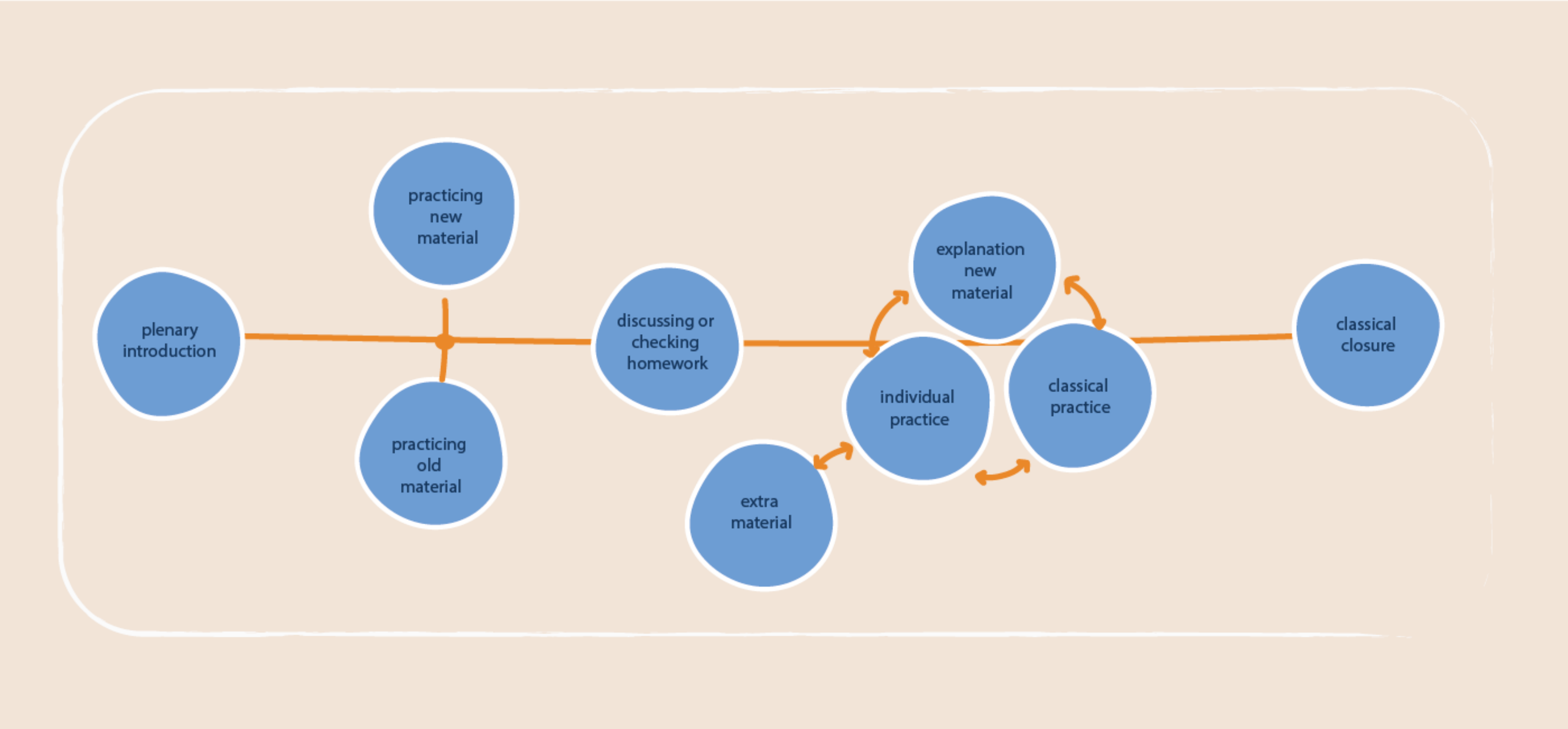


Figure 13 - The average math class

4.4 Conclusion

The findings show insights from two perspectives: the students and the schools.

For students, there is an unmet need for experimental and physical learning. The observed excitement around puzzles and spatial challenges show the potential of hands-on learning to improve the understanding and enjoyment of the subject. Alternative forms of learning, such as quizzes or games, spark enthusiasm and engagement, indicating that including interactive and physical elements into the curriculum can have a positive effect on student motivation. Additionally, students express a strong desire to feel competent. When they understand the material, they perceive math as a fun challenge. However, a significant part of the learning process is social. Students often seek reassurance from peers before feeling confident in their answers, as they fear being perceived as 'dumb'. This suggests that a math class is not only about content but also about social dynamics.

From the schools' perspective, the emphasis is on the need for structure. Lessons follow the textbook step-by-step to ensure clarity and reduce mistakes. The curriculum is heavily test-oriented, leaving little room for mistakes and exploratory learning. The pressure of frequent testing contributes to a packed schedule, making it difficult to implement alternative lesson formats in lessons or the curriculum, even if these would benefit student engagement.

4.5 Design space

For students, the perfect math class would be an interactive class where they can explore different concepts with their peers through activities like puzzles and games without feeling time-pressure or working towards a test. Schools on the other hand, want their classes to be structured, efficient and aligned with the curriculum. Lessons need to follow a clear, step-by-step format where mistakes are limited, to prepare students for frequent testing, a measurable outcome.

The design space lies in the middle of the two ideal scenarios: A structured math class, that allows room for active and collaborative learning while not going beyond the borders of lesson plans, with lessons that follow a consistent format to meet curriculum goals but including interactive elements like puzzles or group work to improve motivation. This middle ground balances the need for alternative learning approaches of students with the schools' need for structure.

4.6 Design directions

The design space leads to a few design directions, which are shown in figure 14.

1. Physical and experimental learning within the lesson structure

Create a way to integrate active learning that connect to the existing learning objectives. The activities need to be short and playful, to integrate it perfectly into the structure of the lesson plan.

2. Balancing individual work with collaborative learning

Stimulating structured peer-interaction that allows students to find their own solutions, but needing one another to find the right answers. The activity should have clear instructions and divisions of roles.

3. visual instead of verbal learning

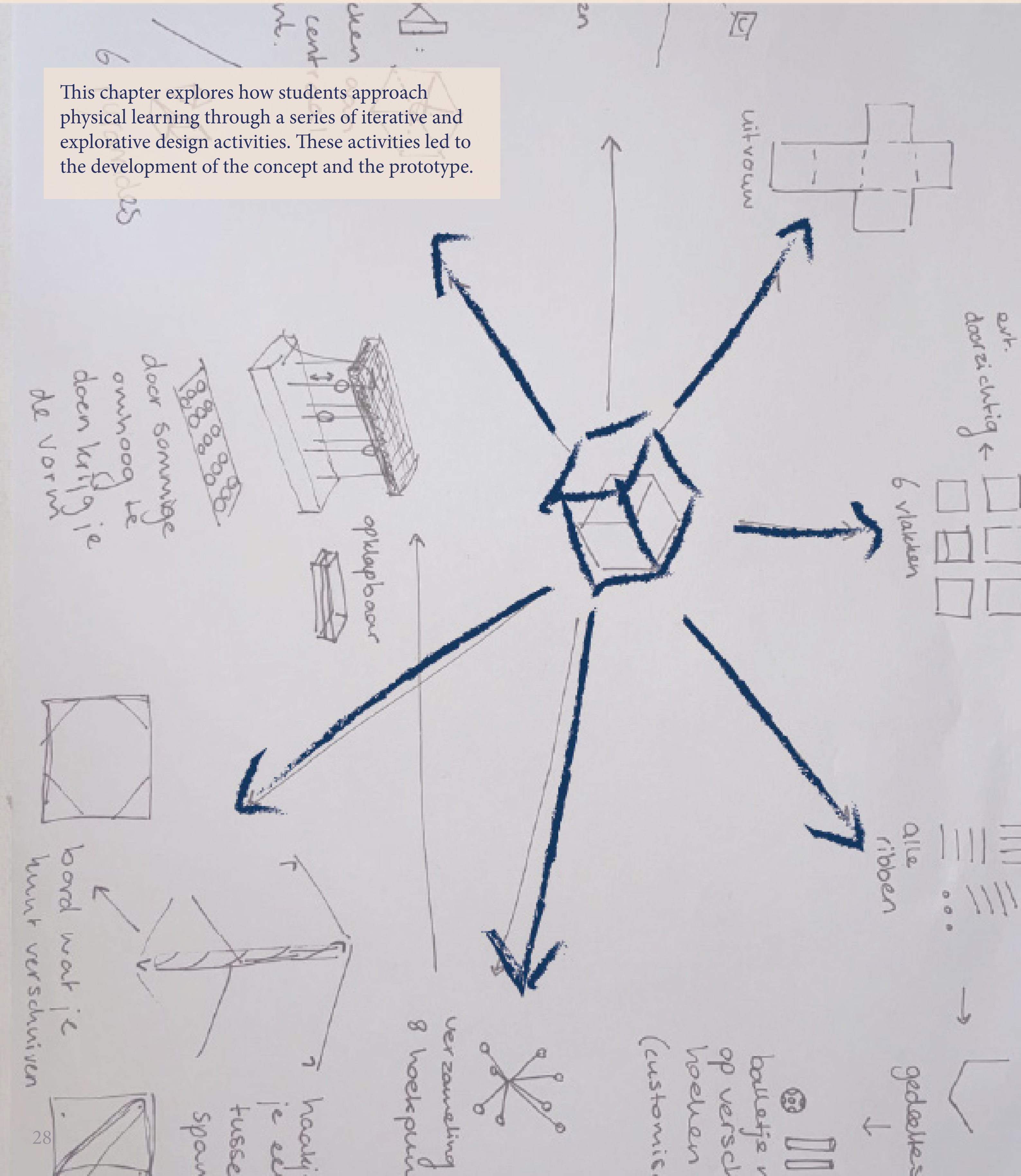
Only use visual explanations like visualisations, tangible objects or a symbolic game. The activity should use a limited amount of text, emphasise the use of colours and shapes to convey information.

Comparing these design directions to the design brief, the most fitting direction would be the first one: Bringing physical and experimental learning into the lesson structure. The others can still be considered in the final concept as sources of inspiration. Ideas that came forward from choosing this design direction can be found in Appendix E.

Figure 14 - Three design directions

5. Ideation

This chapter explores how students approach physical learning through a series of iterative and explorative design activities. These activities led to the development of the concept and the prototype.



5.1 Crafting workshop

One of the main things that comes forward in the analysis, is the need to do more fun learning activities. Something that is considered fun amongst students, is making puzzles. A mathematical exercise can also be considered as a puzzle, but only if the material is understood and the students has an idea on how to approach the puzzle.

In mathematics education, exercises and puzzles are usually made with pen and paper. But for this project, it was explored what physical tools are reached for when trying to solve a puzzle.
A workshop was set up with three students that fit the target group. They were asked to solve several spatial puzzles (Figure 15), with the crafting materials that was in front of them (Appendix F).

What comes forward in this workshop, is the need to mimic the form of the shapes in the spatial questions. When a cube is used in the question, participants mimic this by using a dice, or making the cube out of clay (figure 15: A, B, D) and when a triangle is used, they cut the shape out of paper (Figure 15: C, H). There is not a specific type of material that is favoured, but if a specific material like paper, is mentioned or if something flat is suggested, paper is often immediately used. They want to stay close to the 'real thing'.

Another interesting insight, is the need for similar colours. To show differently coloured sides of a cube, stickers, markers or other attributes are used. The same happened with the colour of the paper, (Figure 15: C,H), the question shows a red triangle, so the participant decided to use red paper. They also chose to use smaller paper, that was sticky and not non-sticking larger paper, only because 'they would have to cut less'. However, when icons are used (figure 15: E, G, I), the need to use colour disappears. This suggests that colour is only a helpful tool if the question contains specific colours.

From this workshop, it becomes evident that the shape is the most important thing that needs to be mimicked in order to solve and thus improve spatial questions.

5.2 Exploring shapes

Given the importance of shape and form, different

Figure 15 - The tools used to solve different spatial puzzles

shapes were explored. To be able to answer a variety of spatial questions, it would be beneficial if the tool could transform into different shapes. An exploration was done to take a closer look at one of the shape that is most commonly used in mathematics: the cube. (figure 16) (Appendix G)

This studies concluded that a cube can be seen in many different ways: as a collection of corner points, a group of edges or even as a central point that holds the faces together. This realisation led to multiple ideas. (Figure 17)

1. This idea is based on a cube being constructed from six pyramids that join in the centre of the cube. This makes it possible to switch between a cube, pyramid, triangle and square in one shape.

2. This idea uses rope with clickable beats that can form cornerpoints. This way every non-organic shape can be constructed.

3. This idea uses a connection point and rods. The connection point has pre-made holes at different angles, making it possible to construct different shapes by placing the rods in different holes.

4. This idea is an outside frame with hooks. By connecting string through different hooks, multiple shapes can be constructed.

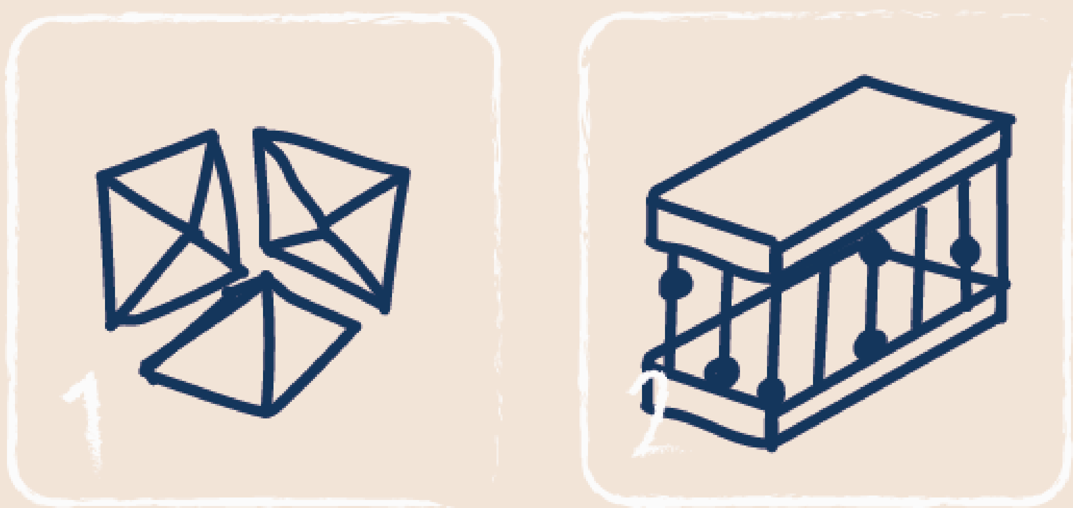


Figure 17 - Sketches of the four ideas

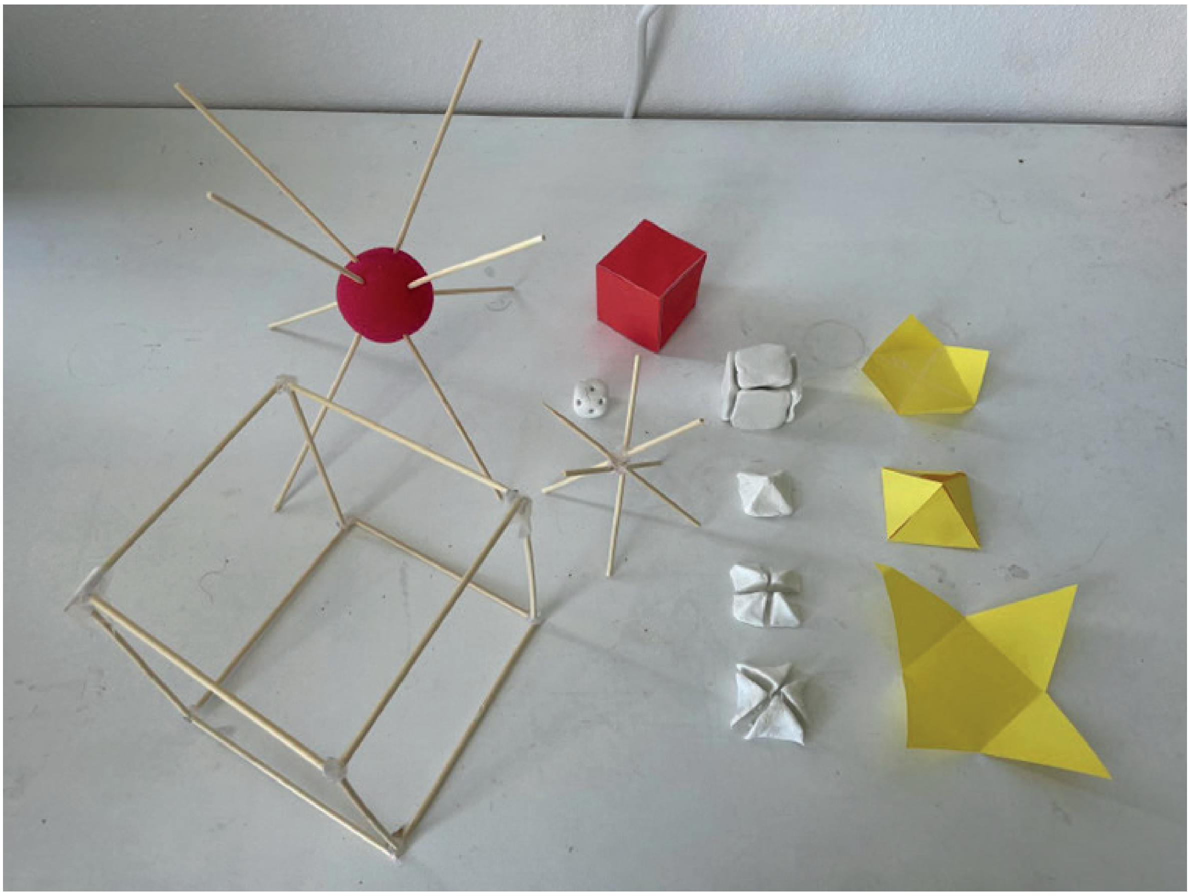
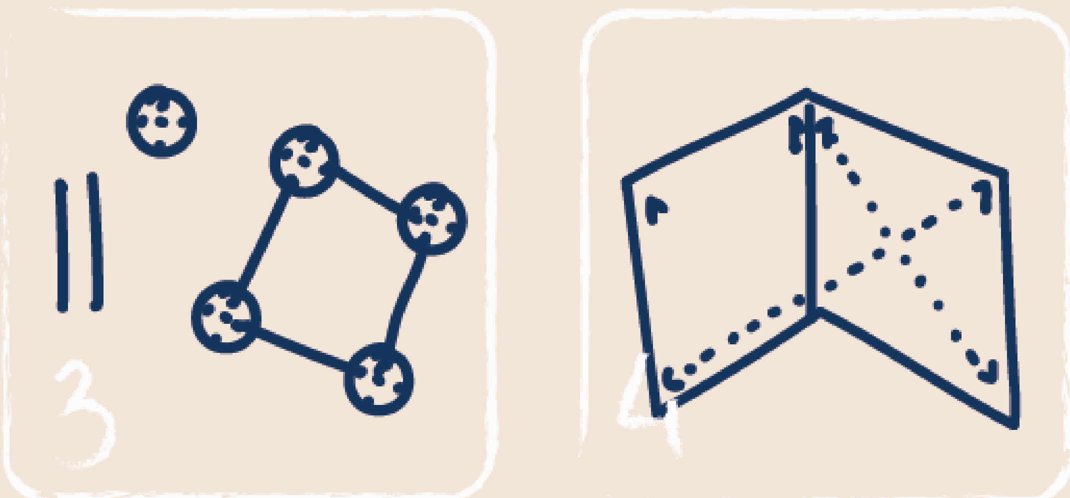


Figure 16 - Shape exploration



5.3 The shapeshifting cube

What makes idea one particularly interesting is that it contains multiple geometric shapes. When all the pyramids are assembled, the structure forms a cube. Once disassembled, it separates into six individual pyramids. When a pyramid is unfolded, it reveals both a square and a triangle (Figure 18). This variety of shapes allows the tool to be used in many different ways, making it suitable for exploring a wide range of spatial problems.

The easy transition between the two-dimensional and three-dimentional world aligns with the transition to hand-on learning and thus fits the research findings best.

5.4 Prototype development

5.4.1 Creating a pyramid

In order to realise this shapeshifting cube, various mechanisms were explored using origami techniques (Figure 19, A, B, C), add-ons, like rope, a lid, an elastic or slots (Figure 19, D-I) and magnets (Figure 19, J, K, L). The mechanism needed to meet several requirements: it had to fold in both directions to allow a pyramid to form on either side of the base and it needed to bring all the pyramid points together at the top.

It is important to mention that, although the folded-out pyramids appear to form a perfect square, the shape needs to be slightly adjusted to become three-

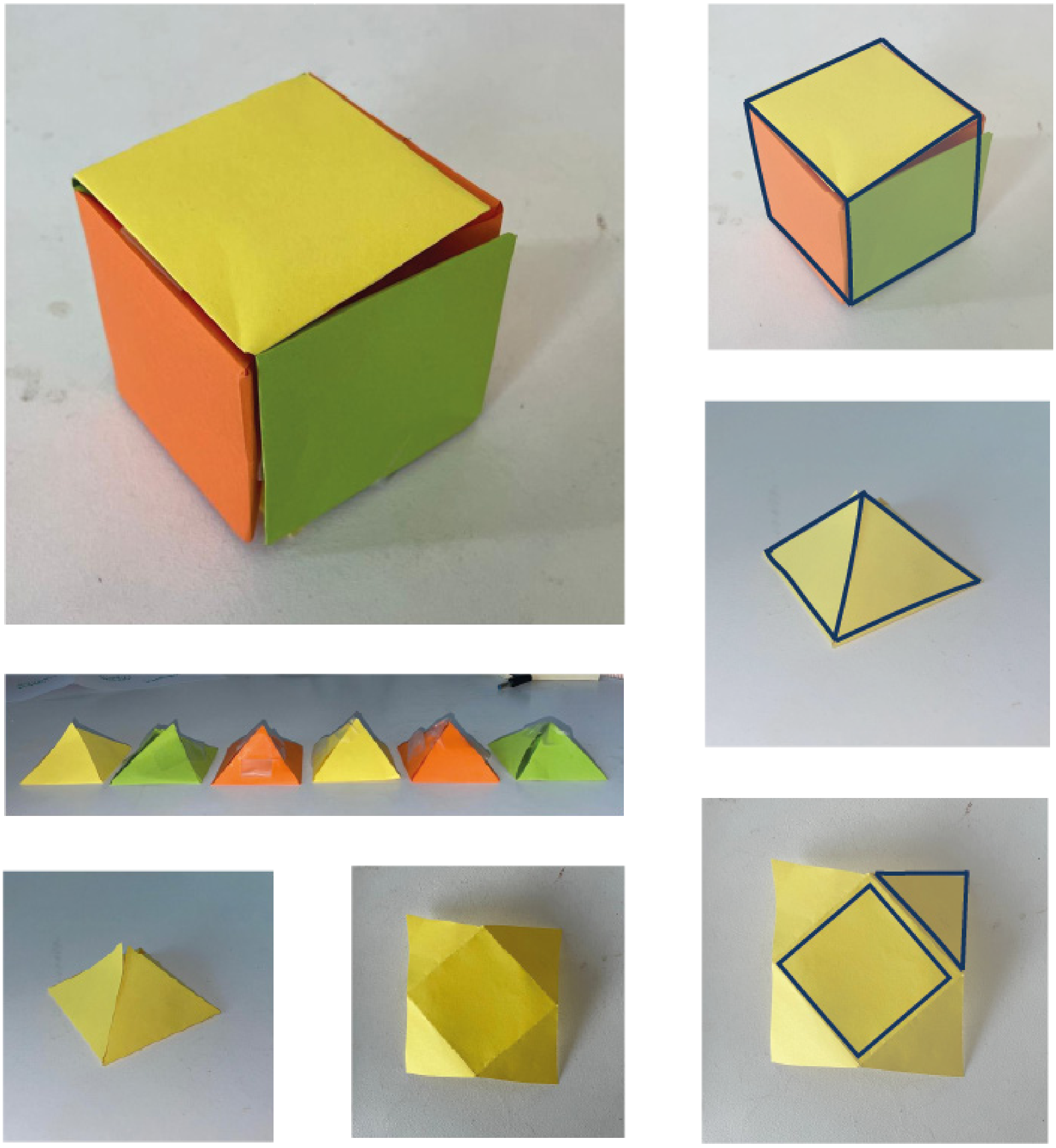


Figure 18 - The shapeshifting cube and it's multiple shapes

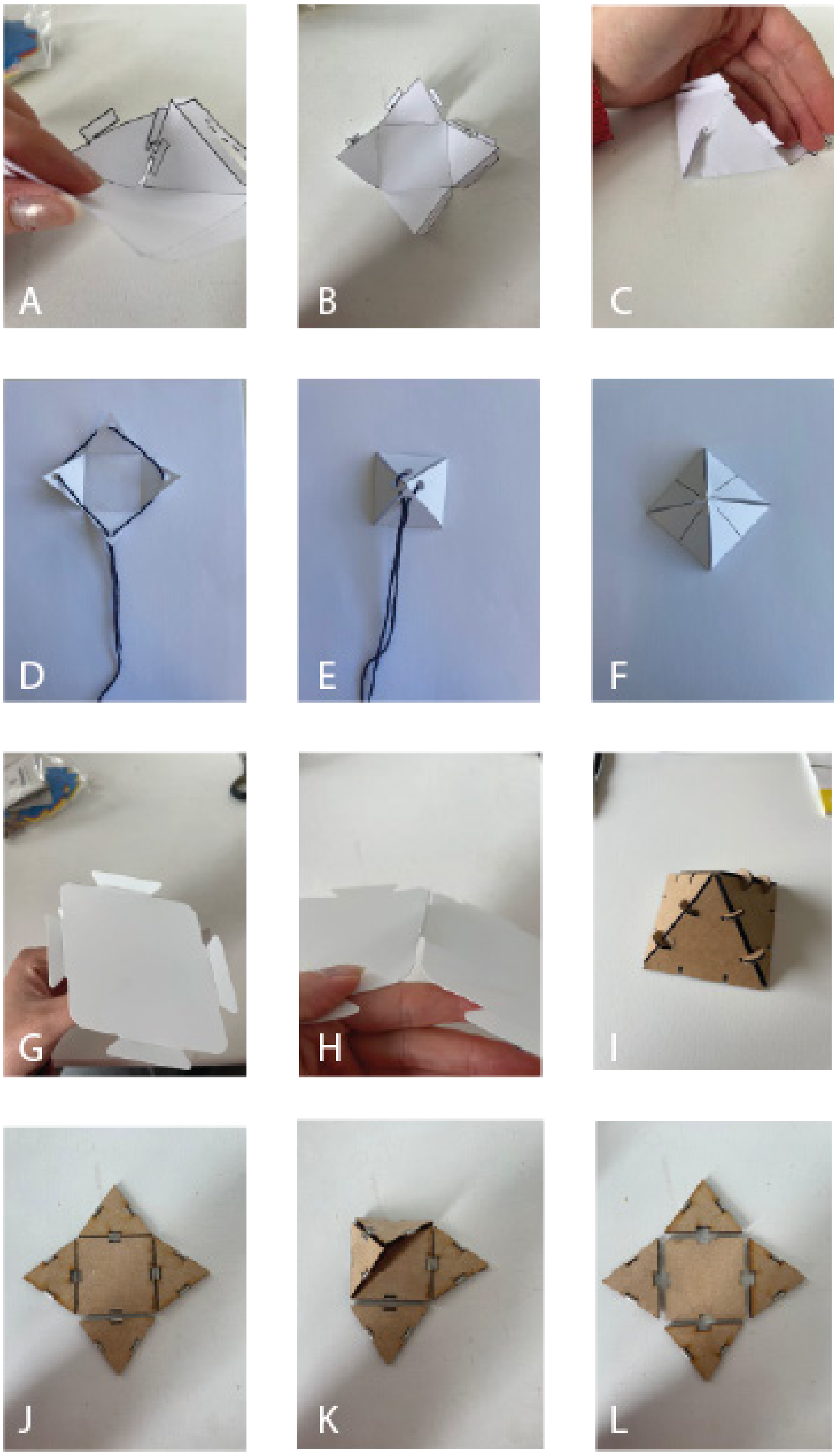


Figure 19 - Exploration of various mechanisms

dimensional. In order for the pyramid tops to meet in the centre when folded, the legs of the triangles have to be slightly longer than the geometric centre of the square. As a result, the base may look like a square, but its proportions are slightly skewed. This subtle modification is necessary to create volume, which creates the three-dimensional form.

5.4.2 Creating a cube

The final requirement was for the pyramids to lock into place in different orientations to create a stable cube. Although many techniques made it possible to create a pyramid, not every technique seemed viable for interlocking.

The technique that deemed most effective to meet this requirement was the one that used magnets as a form of connection (Figure 20). By dividing the fold-out into one square and four isosceles triangles and connecting those with magnets, it was possible to form a pyramid at both sides of the base. The magnets were also helpful to connect the six pyramids to one another as a cube, although polarisation should be considered.

5.4.3 The magnetic cube

The polarisation of the cube is an important aspect of the design (Figure 21). This has to do with how magnets attract: the north and south poles always need to face each other in order to connect. That means the magnets have to be placed very precisely into the small slots of each pyramid. Since every magnet has two sides, the orientation becomes even more important. If a fold-out is assembled the wrong way around, the polarisation ends up reversed, causing some pyramids to repel instead of click together. The way to make the polarisation match again is by folding the fold-out the other way, to create the pyramid on the opposite side.

The magnetic mechanism initially used 96 magnets for one cube. However, an issue that occurred with only using magnets was that the loose components caused the triangles to stick to the inside of the cube when disassembling it. To prevent this form happening, the fold-out should consist of 'one' piece. This was achieved by replacing the magnets between the square and triangle with clear tape. As a result, the number of magnets required per cube was reduced to 48. However, this solution introduced a

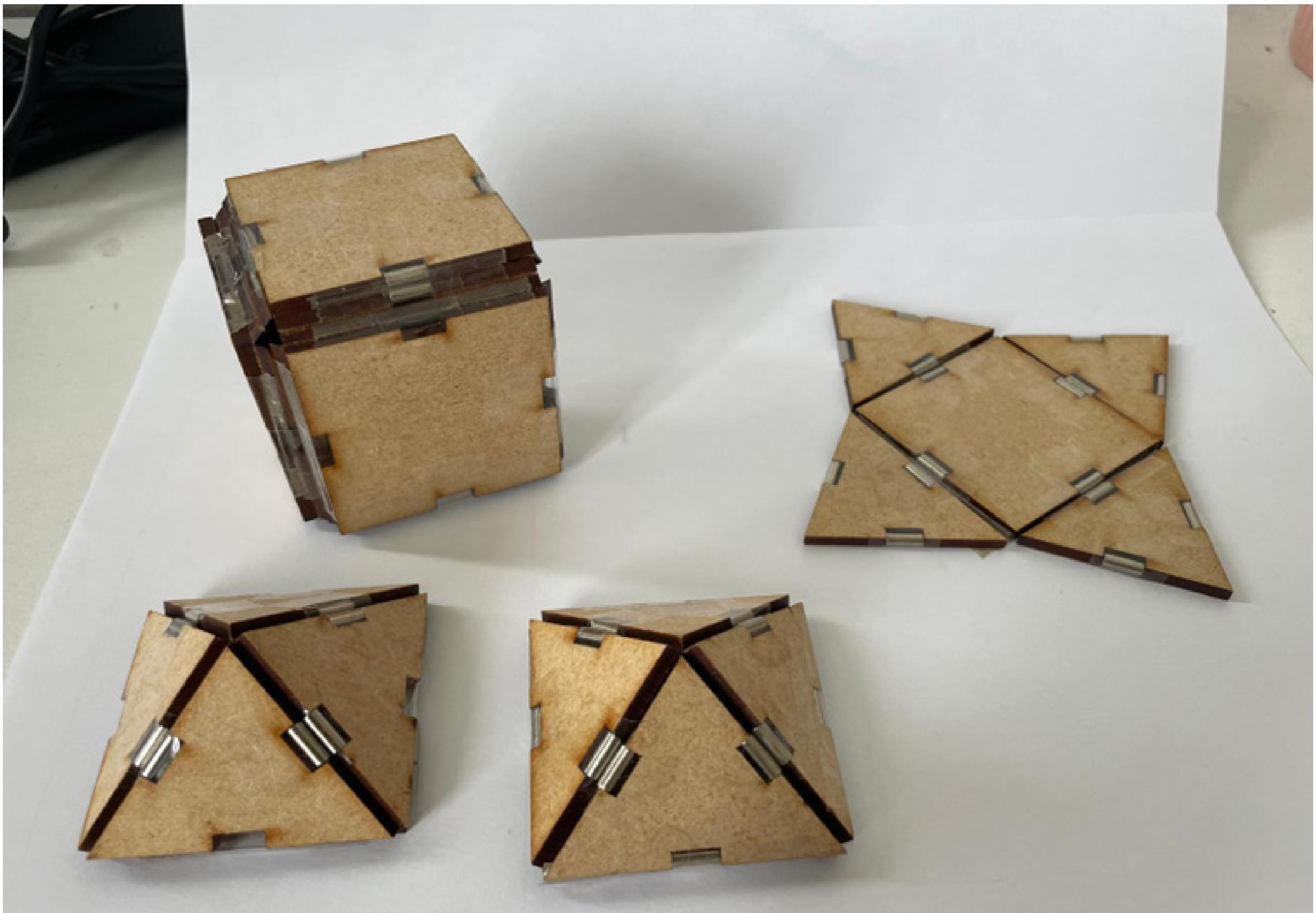


Figure 20 - The magnetic mechanism of the shape-shifting cube

new challenge: the taped connections require a small gap between the different shapes to allow for smooth folding. (Figure 22)

5.5 Game analysis

With the mechanism functioning properly, the next step was to design the types of questions that could be asked to students. The only requirement was that the questions should feel like puzzles; being engaging and challenging, rather than straightforward exercises. To gain inspiration, three puzzle-based games were analysed: an escape card game, a murder mystery game, and a clue box (appendix H).

The escape card game demonstrated the use of spatial skills in a very concrete way. It challenged players to extend patterns or navigate mazes, using recognition to match clues with the correct solutions.

The murder mystery game layered multiple storylines onto a single two-dimensional map, making it possible for players to answer various questions using just that one visual.

The clue cube incorporated puzzles on different sides of the cube; one side might present a puzzle, while another revealed the corresponding answer. The puzzles lacked clear instructions, encouraging players to experiment until the correct approach became apparent. Its many physical components allowed for surprises, hiding new challenges in unexpected places. Despite this complexity, the game still guided players by revealing only the pieces and information relevant at each stage.

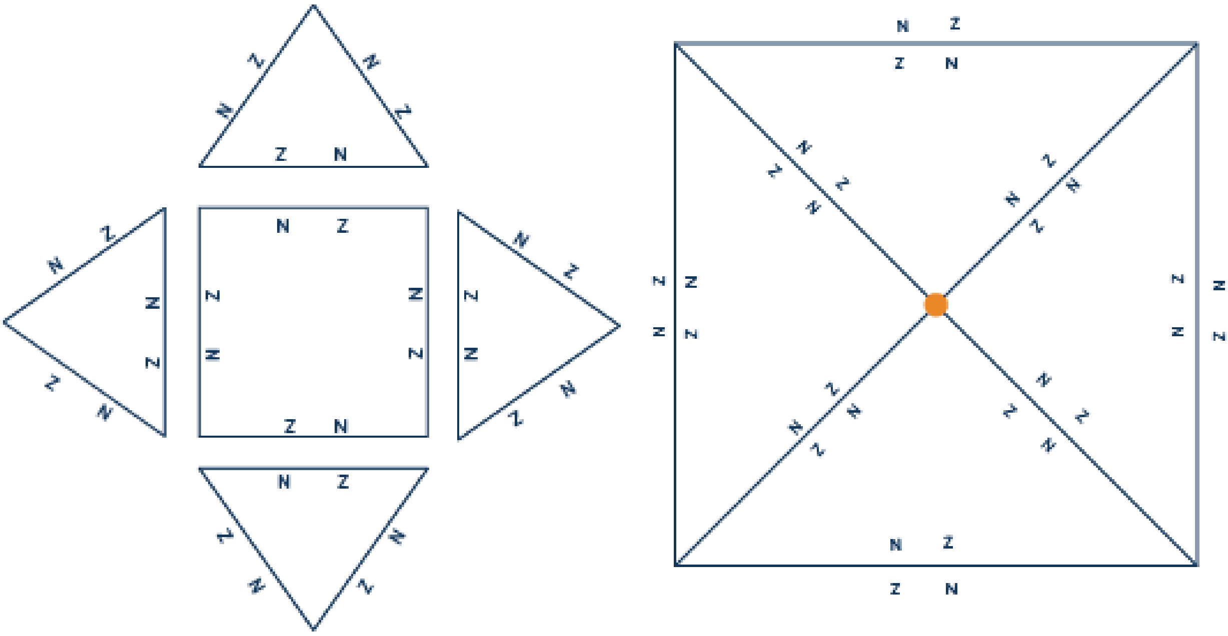


Figure 21 - Polarisation of the design

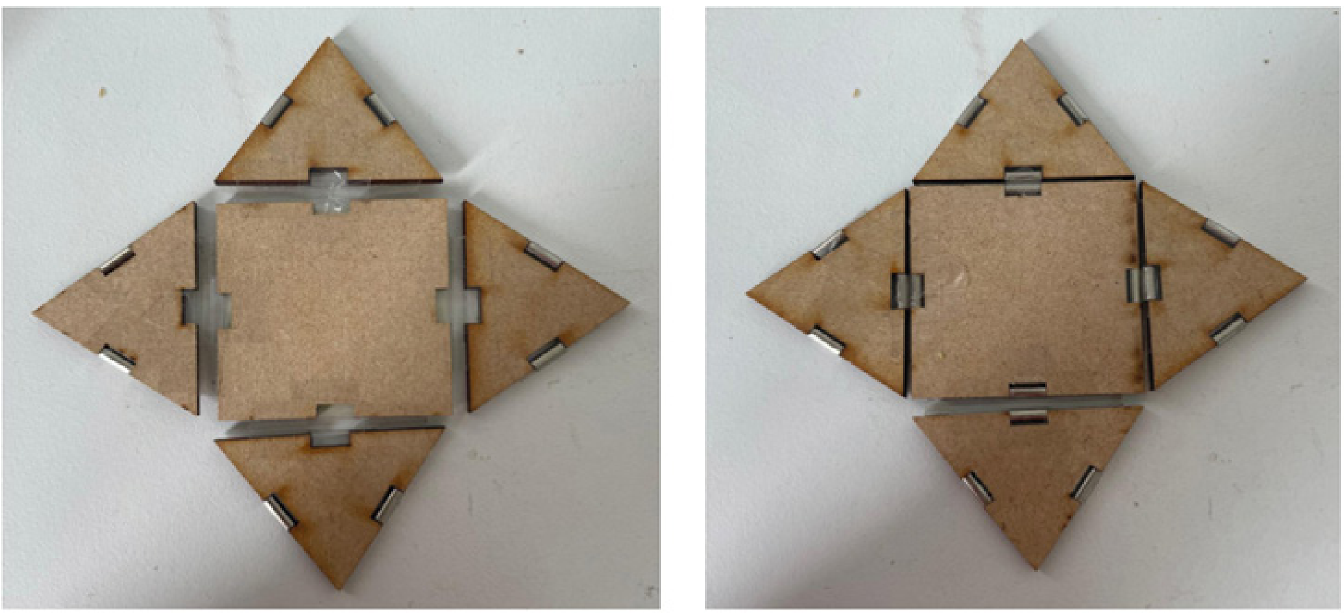


Figure 22 - A tape connection (left) and a magnetic connection (right)

Concept 1

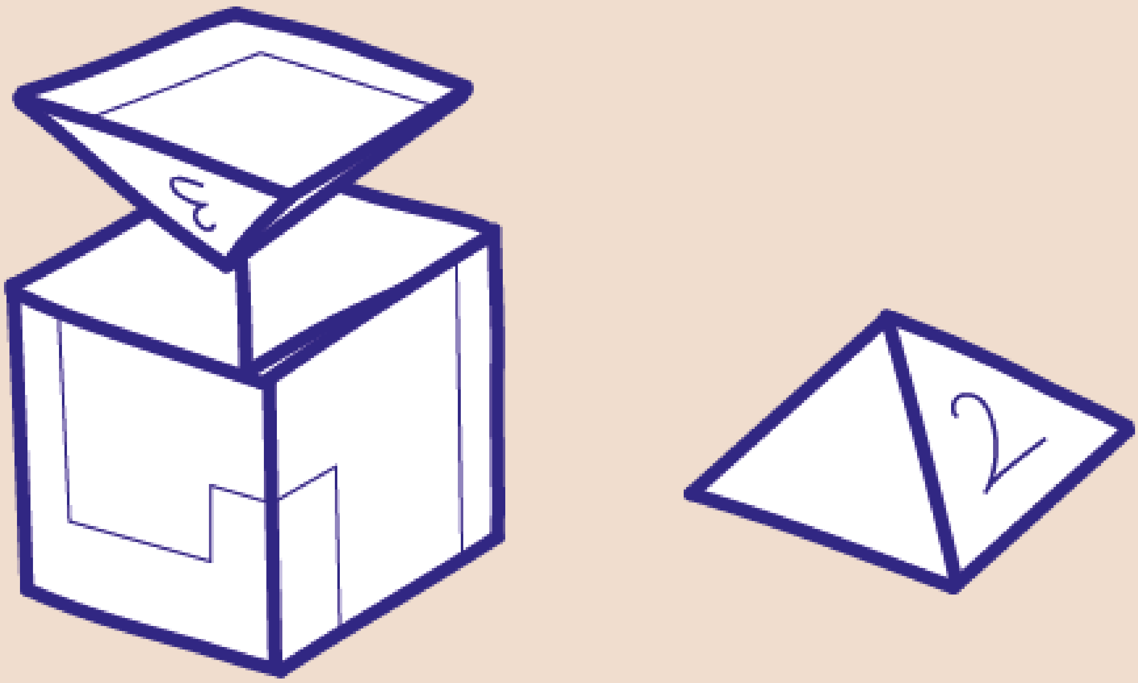


Figure 23 - Concept drawing for the escape game cube

The game analysis resulted in the escape game cube (Figure 23, 24). This prototype used the faces of the different shapes to create puzzles that used the orientation of the cubes to find the right answers. With the help of an exercise booklet, a student would be guided through the different questions. By transforming and playing around with the cube, students can indirectly improve their spatial skills.

Discussing this concept, it became clear that it was too focused on puzzles, losing the connection to the mathematic world. The guidance that was given with a booklet and only being able to continue when you have the right answer, is very similar to an escape game, but does not align with the design learning method, where the goal is to experiment. In order to implement it into the mathematic curriculum, some changes should be made that would allow the concept to fit into a math class and align with the learning objectives that are set for year 2 and 3 in high schools.

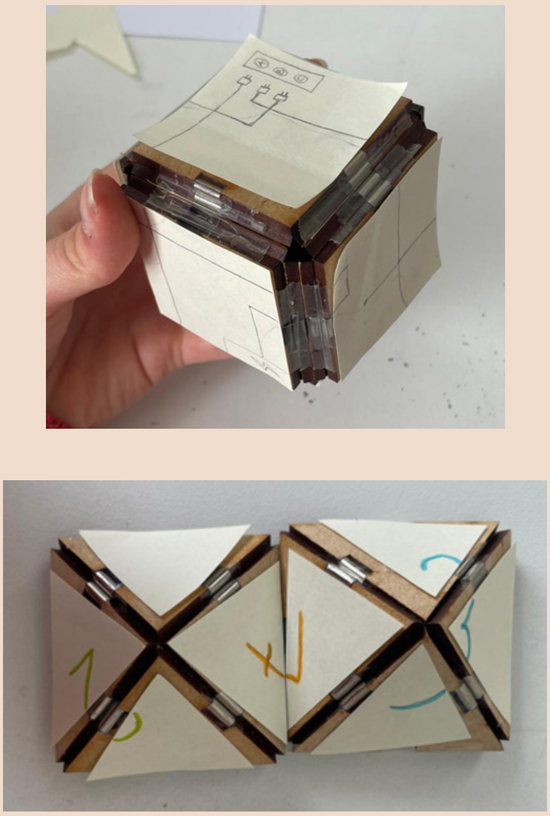


Figure 24 - Prototype of the escape game cube

Concept 2

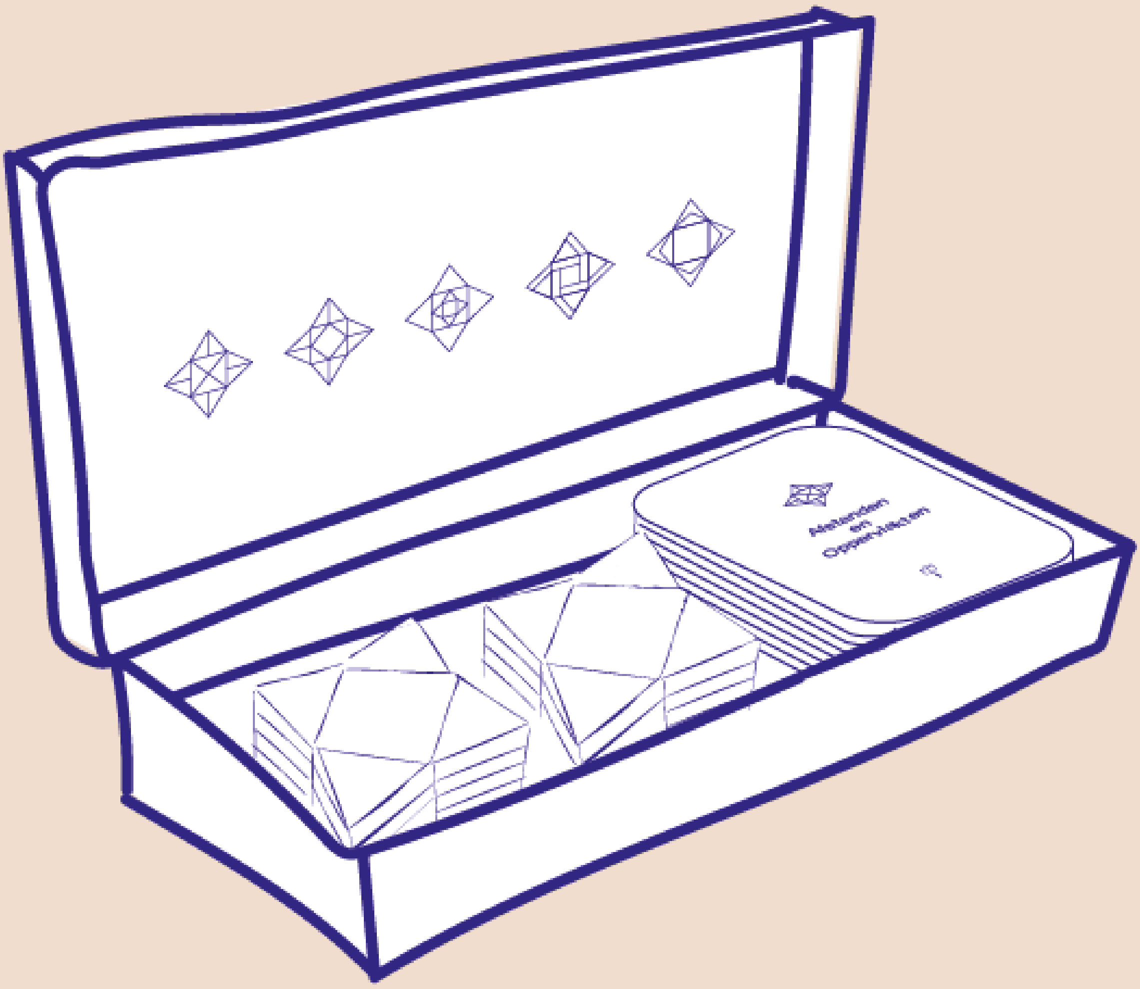


Figure 24 - Concept drawing of the cube toolkit

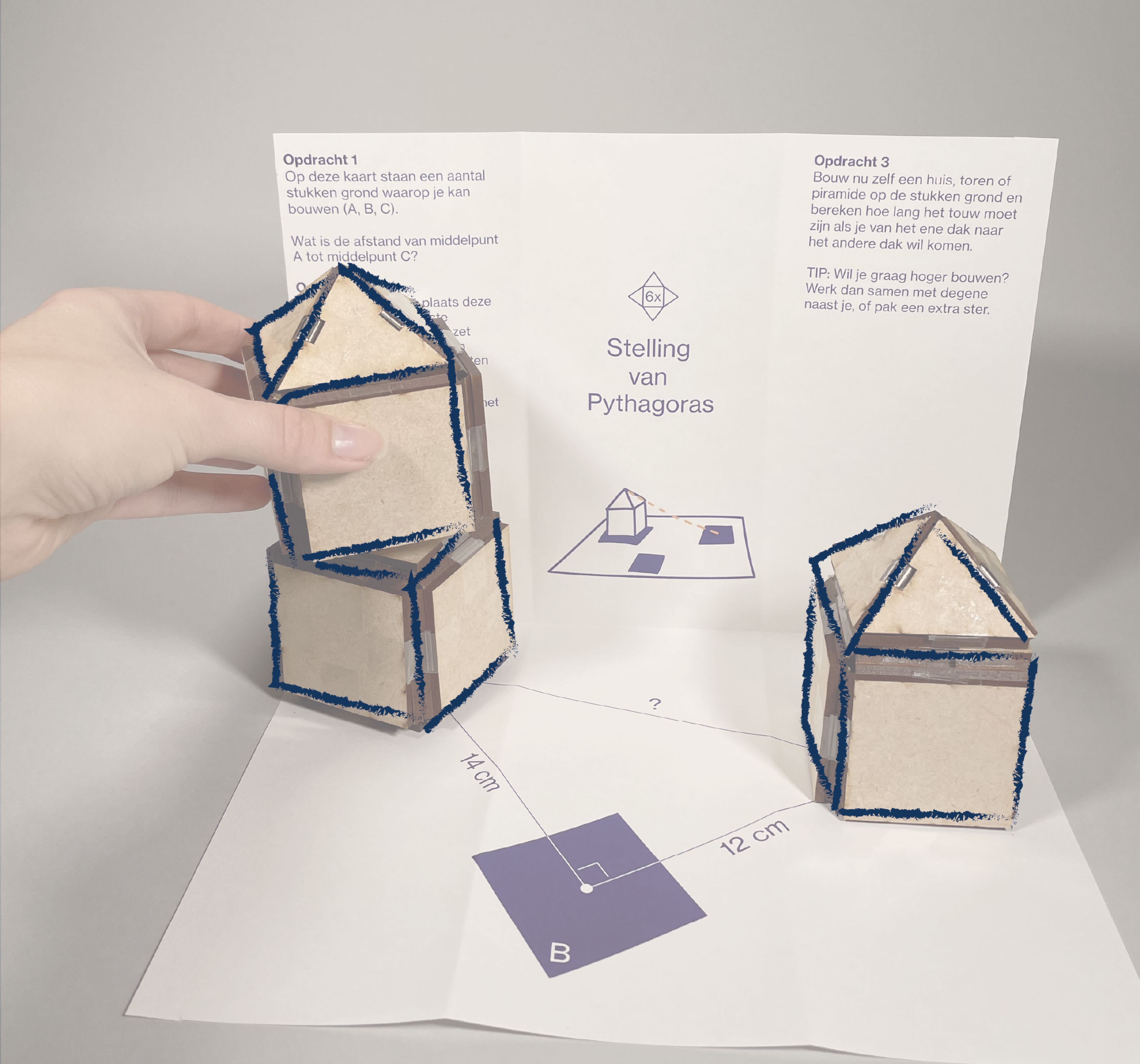
In this concept, the cubes are not used as questions themselves, but become part of the questions by physically placing them onto the assignment cards. Each card presents a spatial task that aligns with different topics within the math curriculum. The questions invite students to interact with the shapes, moving from 2D to 3D reasoning in a hands-on way.

The shape-shifting cubes, along with the corresponding sets of assignment cards, are delivered to schools as a complete teaching kit (Figure 24, 25). Each box contains enough fold-out templates to construct 30 cubes. Thanks to their stackable design, the entire set fits into a compact format, making it easy to store and distribute in a classroom setting.

Figure 25 - Prototype for the cube toolkit

6. Geofold toolkit

This chapter explains the final product that was designed; how it aligns with the curriculum, how it can be implemented into classrooms and it will show the types of interaction with the toolkit.



6.1 Aligning with the curriculum

To ensure the concept fits well within the existing school curriculum, an analysis was conducted of the chapters typically covered in the second and third years of Dutch high schools as set by educational publishers. These 16 chapters can be grouped into three main categories: algebra, statistics, and geometry. Spatial skills align most closely with the geometry category.

Within the curriculum, five geometry chapters are covered:

- Distances and area
- Pythagoras theorem
- Volumes and enlarging
- Similarity
- Trigonometry

Since each of these chapters has a different focus and teaching approach, it was decided to develop five unique sets of question sheets; each tailored to one specific geometry topic.

6.2 Implementation in the classroom

The box will be placed at the front of the classroom as a supplementary learning tool (Figure 26). When students finish their individual work, rather than doing extra textbook exercises (Figure 27), they can take a question card and the necessary fold-outs from the box.

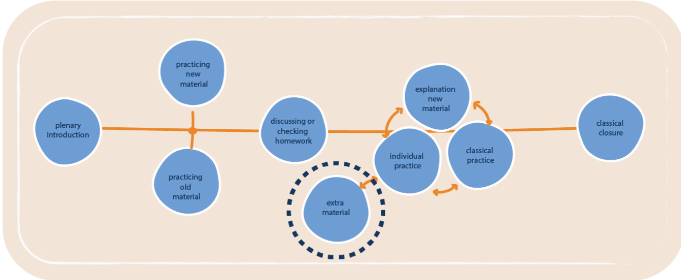


Figure 27 - How Geofold will fit into lesson plans

At their desks, students use the cards to guide their interaction with the fold-outs and cubes. Each card has two sides: one with the questions, and the other serving as the 'playing field'; a space for arranging the shapes.

The questions increase in complexity. They begin with simple tasks using just the playing field. Gradually, the tasks become more advanced, requiring students to interact with 3D shapes.

For example, students begin with a straightforward task related to the theory they've learned in class, like calculating the length of a side in a triangle using the Pythagorean theorem. Next, they are asked to place the cube or pyramid on the playing field in a specific way, allowing them to apply the same theory in a three-dimensional context. Finally, the students are challenged with an open-ended task in which they

Figure 26 - Storyboard of interaction with the Geofold

create their own configuration on the playing field and do calculations based on that setup. This final step encourages creativity and exploration, aligning with the principles of design-based learning, while still having the structure that many students need.

6.3 Haptics in the design

Haptics, or tactile feedback, play an important role in this design. In this toolkit, the haptic element comes from the magnetic cubes that click into place with a clear, physical response. This adds a sensory layer to the learning experience that goes beyond the visual and mental effort normally associated with math exercises.

The physical response of the magnets makes the product more intuitive to use. Instead of needing a long explanation on how to interact with the parts, clicking pieces together feels natural and guides their actions. This intuitive quality helps students focus on the content, rather than the tool itself.

The magnetic click also offers direct, tactile feedback. When a shape connects correctly, students feel it. This removes some of the uncertainty that often comes with abstract problems and replaces it with an intuitive way of working. The result is a learning activity that feels more like building or solving a puzzle, which is something students previously mentioned as a more enjoyable experience.

The haptic element turns spatial reasoning into a tangible experience, where students can test their ideas and reflect on their actions. By making the tasks physical and responsive, the tool increases engagement and motivation. Students are more likely to stay involved because the product rewards interaction in a direct and satisfying way.

6.4 The exercises

The exercises cover five different geometry chapters. Every chapter has their own assignment card and its corresponding use of the toolkit. Each assignment card consists of a 'playing field', where the cube plays a role and a set of questions that guide you through the different exercises. Each assignment card gradually builds up through three exercises; It starts with a question that is similar to exercises in the book, often two-dimensional. It then progresses to a three-dimensional question where the cube or pyramids are involved. The final exercise uses the design thinking method, encouraging creative exploration through an open-ended question.

To realise the design, three chapters were prototyped:

- Distances and area
- Pythagoras theorem
- Volumes and enlarging

Pictures of the questions in the assignment card can be found in appendix I. The intended interactions between the assignment cards and the cube can be found on the following sections.

6.4.1 Distances and area

The spatial skills that are covered in this chapter are object recognition and perspectives. The questions reflect this by having to recreate figures out of different shapes and taking different perspectives to match them to the figures on the assignment cards.

The follow-up questions dive into calculating the area of the shapes with the help of the constructed figures. The final question let's students experiment with making new figures. (Figure 28)

6.4.2 Pythagoras Theorem

Pythagoras theorem is about calculating distances with a right angle. The chapter gradually moves from simple two-dimensional problems to more difficult three-dimensional problems. This assignment card follows the same structure, but physically demonstrated the three-dimensional problems. The final questions offers space to explore different configurations to make personal Pythagoras problems. (Figure 29)

6.4.3 Volumes and enlarging

This chapter focuses on how volumes change when certain measurements change. The toolbox visualises these changes by showing dents or bulges when the cube is placed on the assignment card. (figure 30)

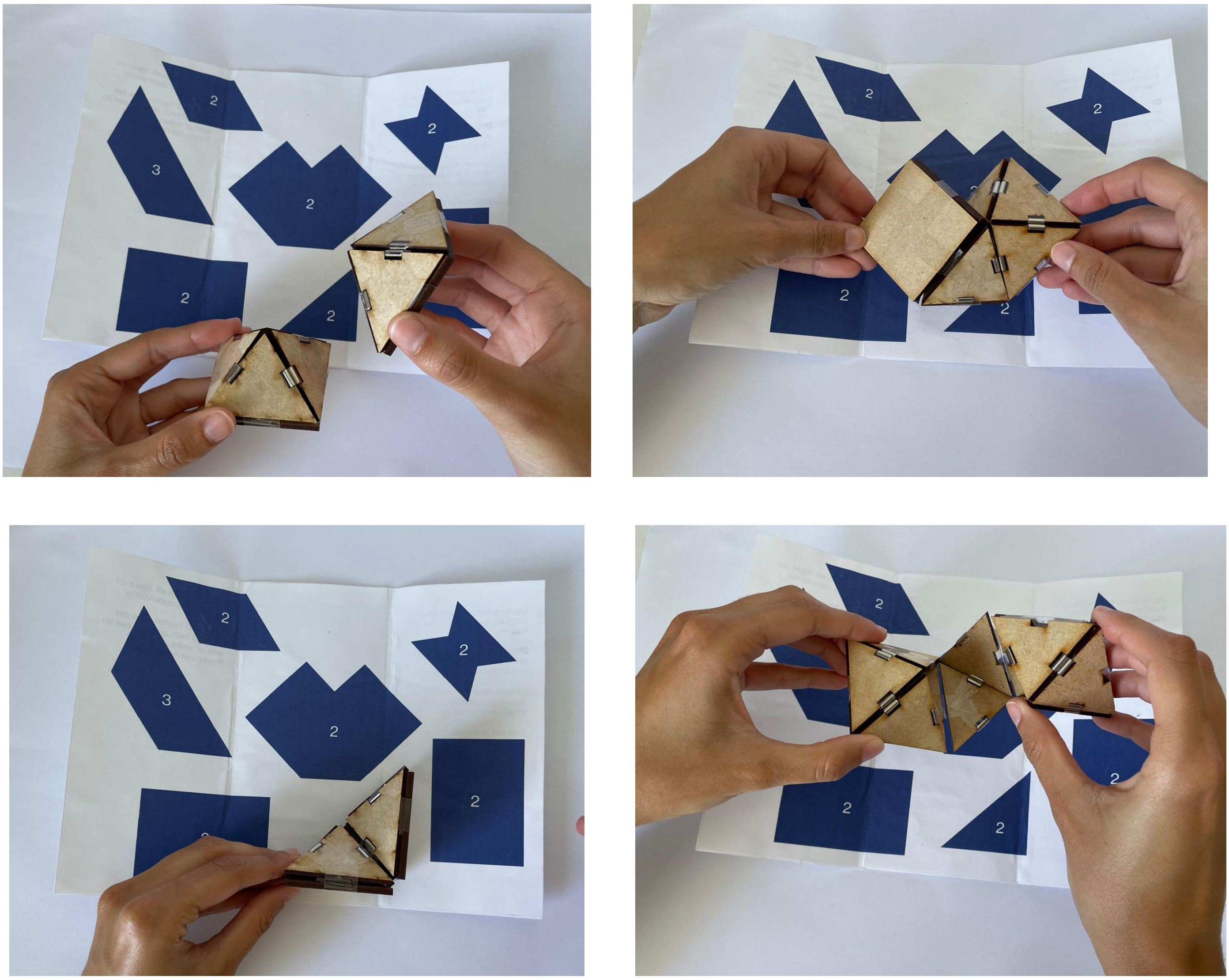


Figure 28 - Intended interaction with distances and area assignment card

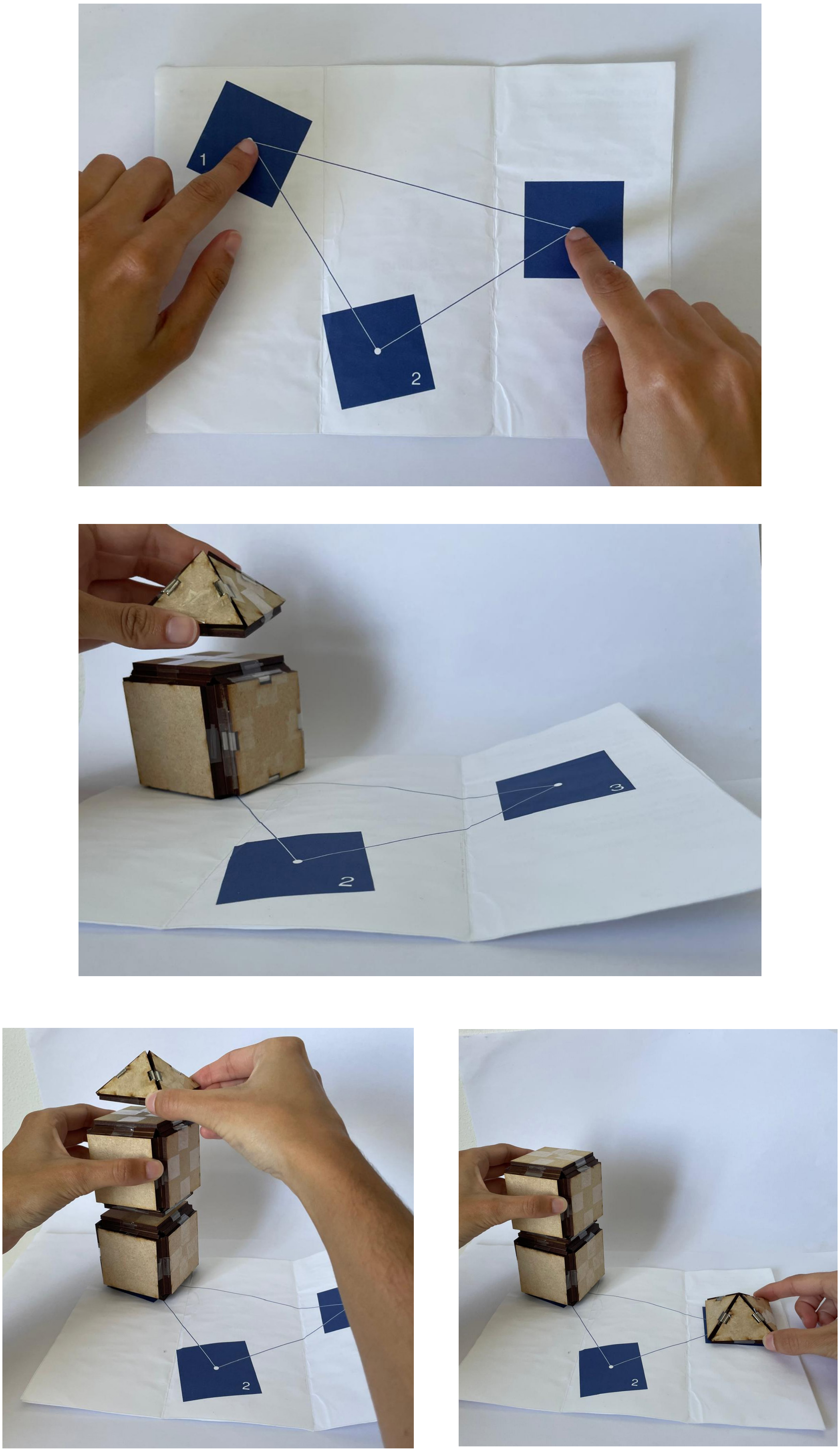


Figure 29 - Intended interaction with Pythagoras Theorem assignment card

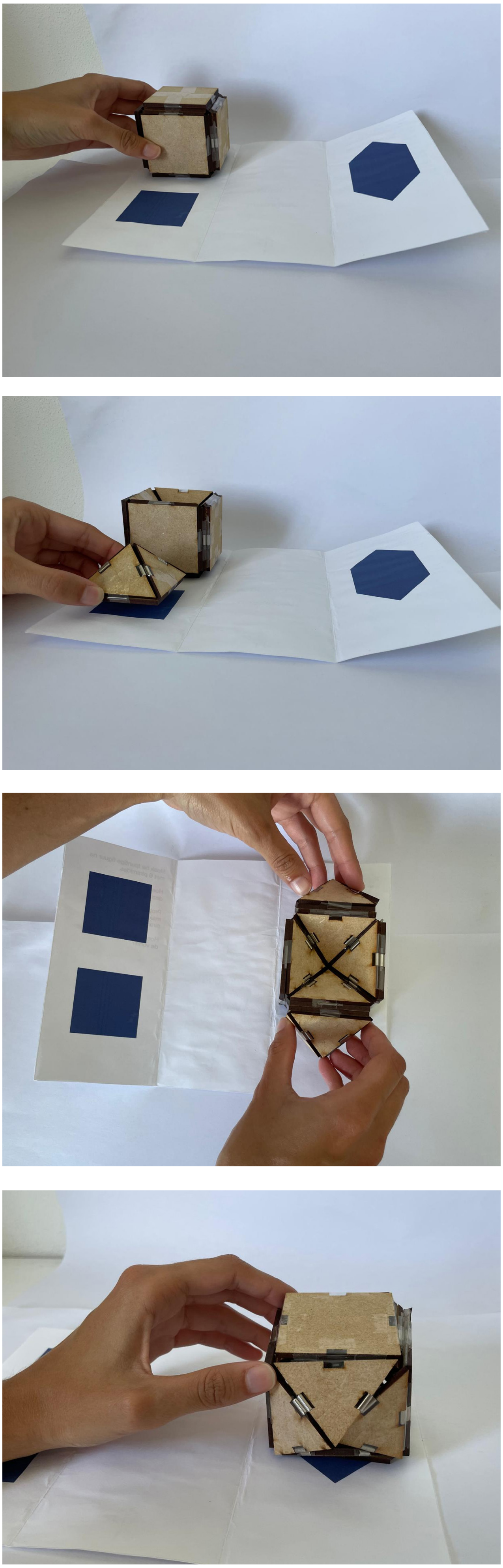


Figure 30 - Intended interaction with volumes and enlargement assignment card

6.5 How Geofold improves spatial skills

This toolkit improves spatial skills by incorporating spatial reasoning into regular math lessons through physical interaction. Instead of approaching spatial problems only on a mental level, students are invited to engage with foldable shapes in tasks that activate both the body and environment (Figure 31). The assignment cards visualise abstract mathematical concepts, making them easier to understand.

The questions are designed to cover a variety of spatial tasks that align with the developed spatial skills matrix: they involve both single objects (object recognition) and relationships between multiple elements (relations between objects), tasks that are static (placing a cube on an allocated area or orienting connected pieces) and dynamic (transforming two pyramids into a new shape), and creates engagement through moving between mind, body and environment.

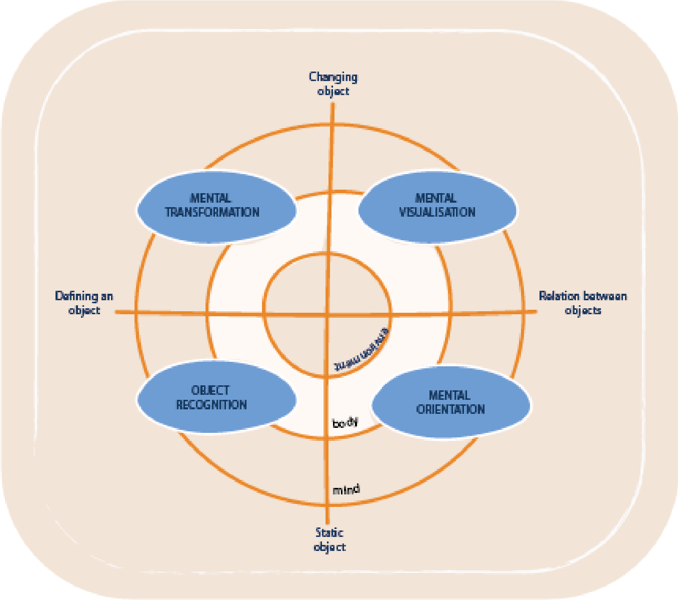


Figure 31 - The spatial skills that are covered by Geofold

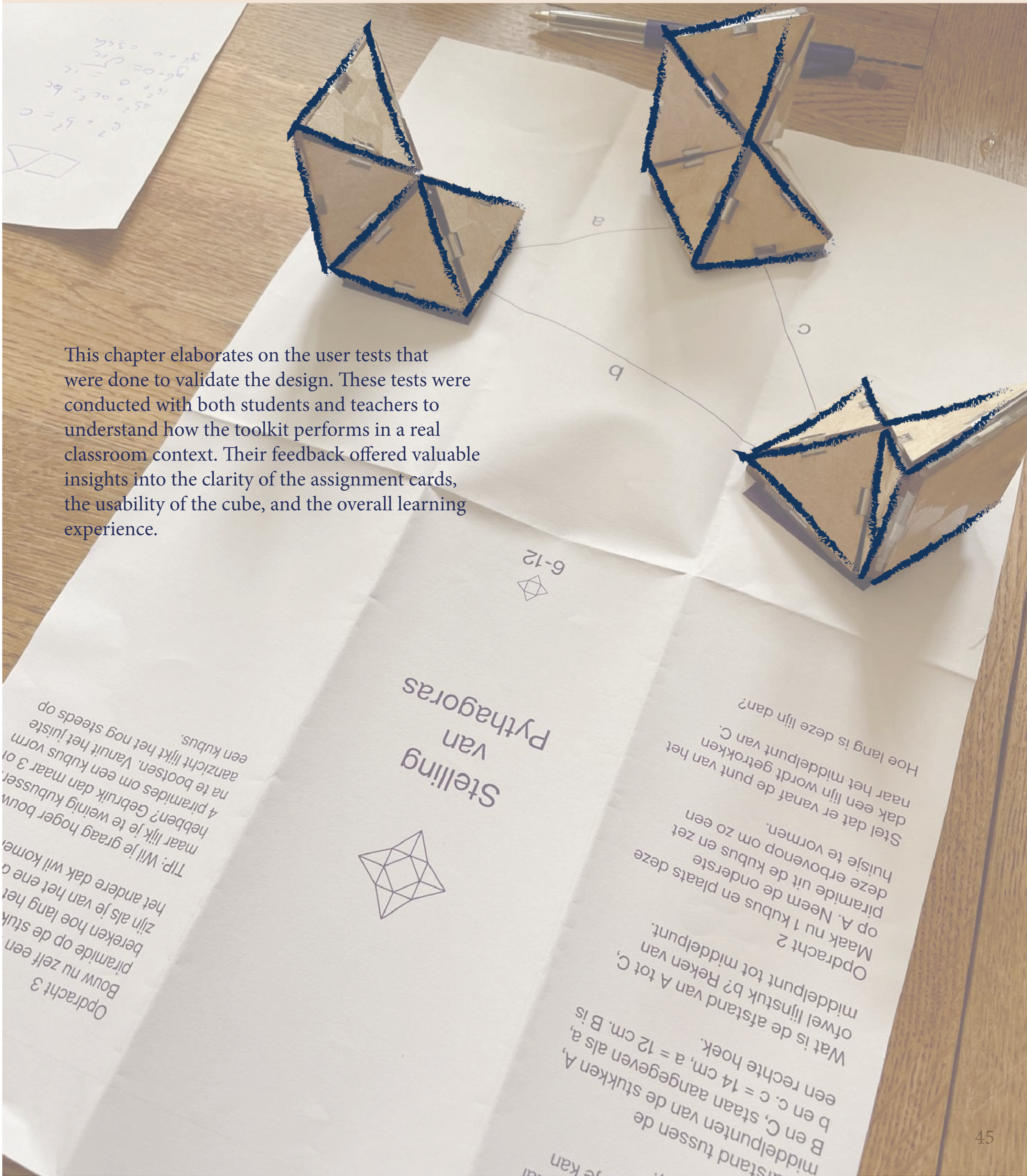
6.6 Costs

The costs for the cubes depend on the material costs and production time. However, because of the tape connection, assembly time plays a big role in the realisation of the product. According to the table in figure 32, it would cost €28.15 to make a cube.

Category	Laser cutting
Production time	8 min x €25/h = €3.25
Material	€4.00 per cube
Assembly time	6 x 5m = 30 min
Assembly costs (€13/h)	0.5 x €13 = €6.50
Magnets (48 x €0.30)	€14.40
Total costs per cube	**€28.15**

Figure 32 - Production costs of one Geofold cube

7. Validation



This chapter elaborates on the user tests that were done to validate the design. These tests were conducted with both students and teachers to understand how the toolkit performs in a real classroom context. Their feedback offered valuable insights into the clarity of the assignment cards, the usability of the cube, and the overall learning experience.

To ensure that the product would fit the needs and wants of students and teachers, a test was set up. During the test, participants engaged with the cube and the assignment cards of the cube toolkit. The material consisted of a set of folded out pyramids and assignment cards, each linked to specific chapters from the curriculum. 10 teachers and 4 students participated. (Figure 33)

7.1 Validation with teachers

Set-up
For the user tests with teachers, I went to a lecture with 10 mathematical teachers in training. The group was divided into three, where each group got one assignment card and the cube was passed on to the next group after 8 minutes.

Procedure
At the beginning of the test, the concept was briefly explained and a demonstration was shown on how to assemble the cube.

After the introduction, the group was divided into three and each group was given their own assignment card. The aim of the test was to see if the questions were at the right level and if students would be able to comprehend the instructions. I moved from group to group with the cube to see how they would approach the questions. By the end the groups wrote down their overall feedback. (Appendix J)

7.2 Validation with students

The user test with students was done with two different versions of the assignment cards. The first participant used the same version as the teachers, while the remaining three students got an improved and iterated version.

Set-up
For the test I was set across the table from two students. A stack of stars and assignment cards were laid out in front of them, similar to how they would be presented in a classroom setting.

Procedure
At the beginning of the session, participants were introduced to the concept through a brief explanation provided by the instructor. This included a video demonstration showing how the star elements could be assembled into pyramids and a

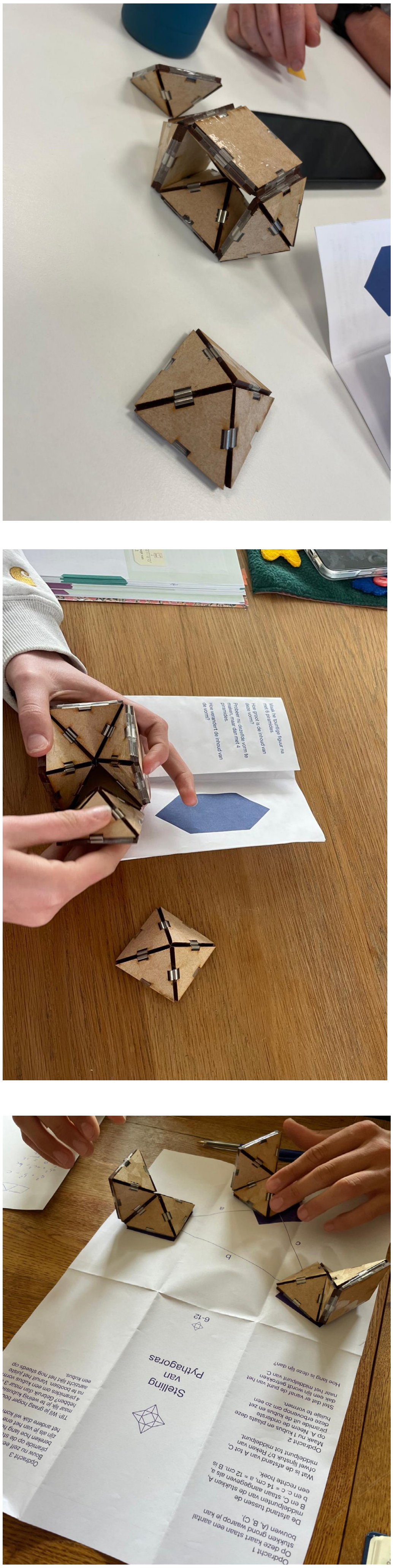


Figure 33 - User tests with teachers and students

cube structure.

Following the introduction, participants were invited to select a assignment card and begin the tasks. The emphasis during the test was not on solving every question correctly, but rather on observing the participants' handling of the cube, their interpretation of the instructions, and the reasoning behind their choices.

Upon completion of the tasks, participants answered a series of follow-up questions. These were aimed at gaining deeper insight into the comprehension of the questions and their overall experience with the tool.

7.3 Feedback

The user tests showed that both the cube and the assignment cards required adjustments in four areas: The instructions, the visualisation and design, the teacher support, practical matters. The overall comments on the assignment cards are summarised in figure 34 to 36.

Instructions
At first sight, the instructions appear as a full body of text. According to some of the students, this made the questions overwhelming and therefor they were considered more difficult than they were. This contradicts with the feedback teachers gave, who emphasized that instructions should be as clear and explicit as possible. A mistake both parties made, was starting with the wrong question after opening and flipping the assignment cards. They stated that this was due to the lack of hierarchy and visual structure in the instructions.

Visualisation & design
Teachers and students interacting with the toolkit had some confusion about how the cube related to the shapes on the assignment cards. When placing the cube on the designated squares, the cube felt too large. Additionally, participants were often unsure whether the shapes on the assignment card were views, cross sections or just dimensions. Some shapes were also too similar, where the changes were so small that they were disregarded or overlooked.

Teacher support
Multiple participants in the user tests asked the same question: How high are the cube and the pyramids? This information was not stated explicitly in the questions, but was supposed to be derived from the cube's dimensions (5x5x5cm). However, this was unclear to the users. Teachers suggested including a dedicated support section for teachers, where important background information, hints, possible solutions and troubleshooting guidance can be provided.

Practical matters
There were some practical matters that stood out during the user tests. The haptic feedback that participants got when clicking two magnets together was considered satisfying, and interacting with the stars, pyramids and cubes was therefore intuitive and logical. However, due to the different types of questions, the magnets did not always have to physically be connected in order to find the right answer. In some tasks, the pieces had to click together, while in others the pieces were just stacked together, without a connection. This inconsistency resulted in a reduction of the importance of the haptic element. As a result, participants began to ignore the need to connect the pieces properly, with led to imprecise answers.

This diversity in questions and the multi-usage of the cube also caused confusion about whether the pyramids should be folded out (to remain 2D) or folding up (to form a pyramid in 3D). Since the assignment cards are two dimensional, while the pyramids can be both two- or three dimensional, participants were sometimes unsure on how to use them correctly. Although this does encourage creative thinking, it reduces the effectiveness of the toolkit. When users avoid engaging with the spatial aspect of the material, they miss opportunities to practice and develop their spatial skills. This might cause the intended learning goals related to spatial skill development to not be achieved.

The final practical matter that needs adjustment is the flipping of the assignment cards. While folding the assignment cards helps to save space and keep the environment tidy, most participants mentioned that it was inconvenient not being able to see both the questions and the playing field at the same time. This separation can interrupt the workflow, making it harder to stay focused during the assignment.

Positive User Experience
Despite the practical and design-related challenges, students responded enthusiastically to the toolkit. Many indicated that they enjoyed working with the material, comparing the activity to the practical

experiments in science or physics classes. Several students mentioned that they would rather work on the assignment cards than to do their regular homework. One students even mentioned that they would use the tool to solve three-dimensional problems in the book, since the shape of the cube or pyramids resambles the two-dimensional pictures. These reactions show that the toolkit has the potential to increase engagement and motivation.

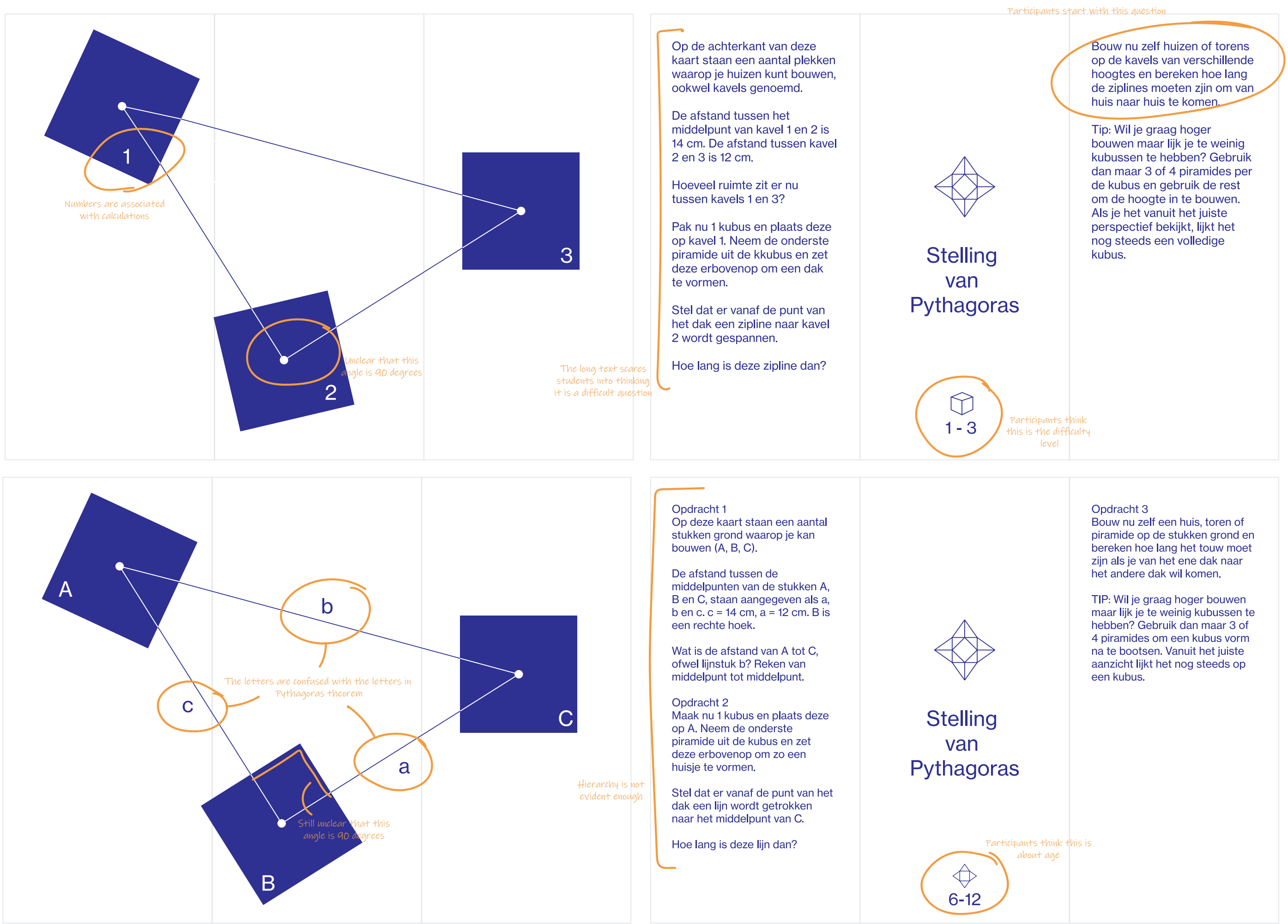


Figure 34 - Summary of comments on Pytahgoras Theorem assignment card

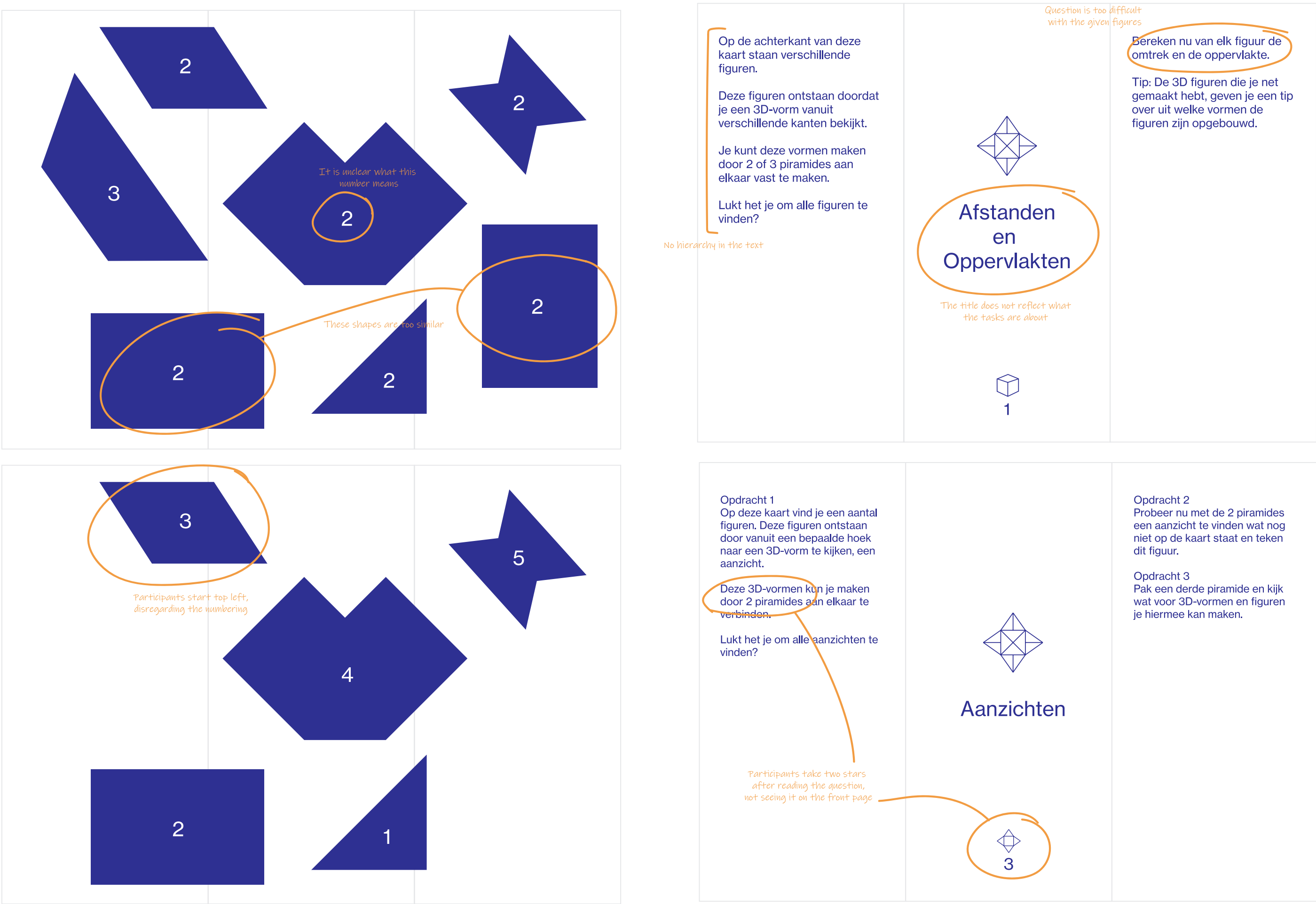


Figure 35 - Summary of comments on distances and area assignment card

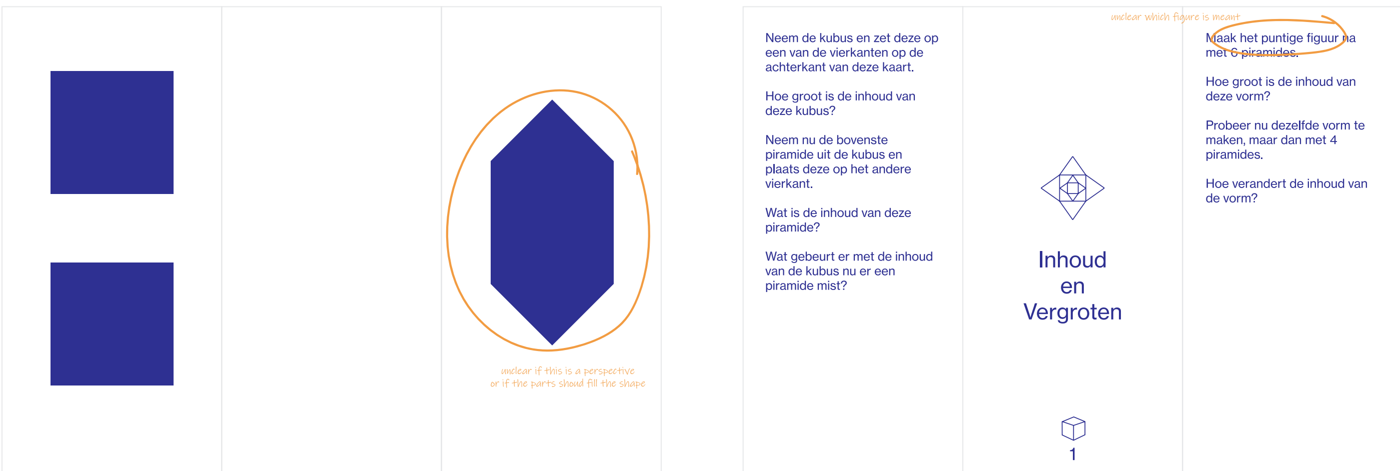
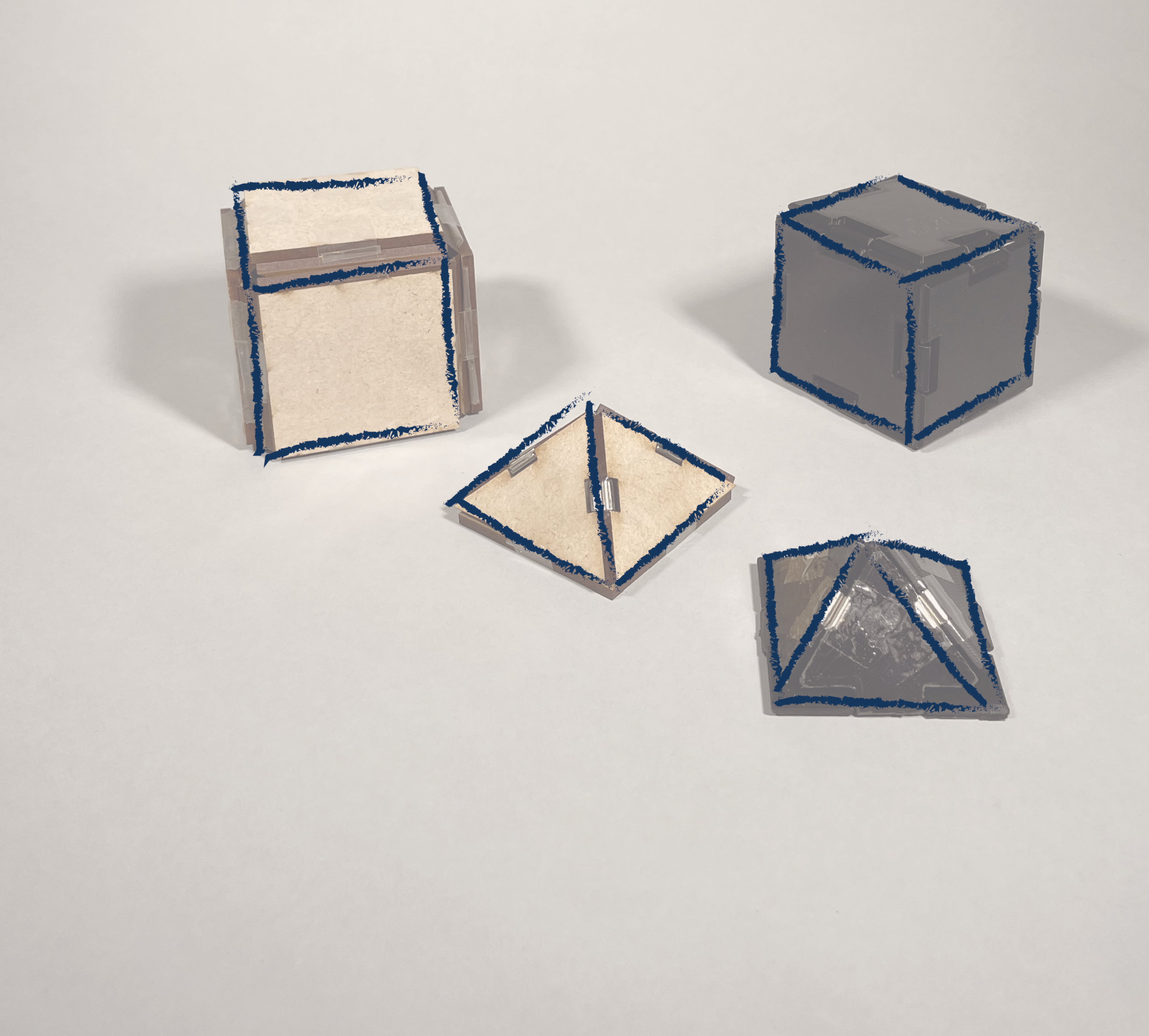


Figure 36 - Summary of comments on volumes an enlargements assignment card

8. Adjustments

Based on the feedback of the user-tests, adaptations were made for the assignment cards and the prototype. This chapter will highlight all the changes and presents the finalisesd concept.



8.1 Adjustments assignment card

Two iterations were made for the assignment cards. The first iteration filtered out small errors, so the focus for the next test could be more on the interaction with the toolkit, instead of on the misinterpretation of the questions. The instructions were simplified and made more explicit and the cards were designed to show both the questions and the playing field at once. Small adjustments were made to the playing fields to avoid confusion between similar-looking figures or line segments.

In the second iteration, a clearer text hierarchy was introduced. Elements that could be visualised were integrated into the playing fields to decrease the amount of text used in the questions. To have a clearer connection between the cube and the assignment cards, a hint was given in the form of a visualisation on the front of the card that reflects the interaction of the cube or pyramids with the assignment card. The final changes can be found in figure 37.

8.2 Adjustments prototype

One point of feedback on the prototype was that it appeared too large and bulky, lacking the proportions of a proper cube. To address this, a more refined version was developed (Figure 38). The triangles of the stars were made smaller and integrated into a hinge mechanism, allowing the sides of the cube to fit together more seamlessly. This adjustment ensures a tighter closure and gives the object a more accurate cube-like appearance.

Another important aspect of the prototype is the connection between the squares and the triangles. In the laser-cut version, this connection was achieved by applying tape to both sides of the joint area, creating a double-acting hinge. While this method functions well in terms of flexibility and stability, it is labour-intensive and would not be practical if the toolkit were to be produced on a larger scale.

To address this issue, the 3D printed version used a different connection method: a click system (Appendix K). This was implemented by adding two small dots to the connection area of the square and a corresponding hollowed-out dots on the triangle.

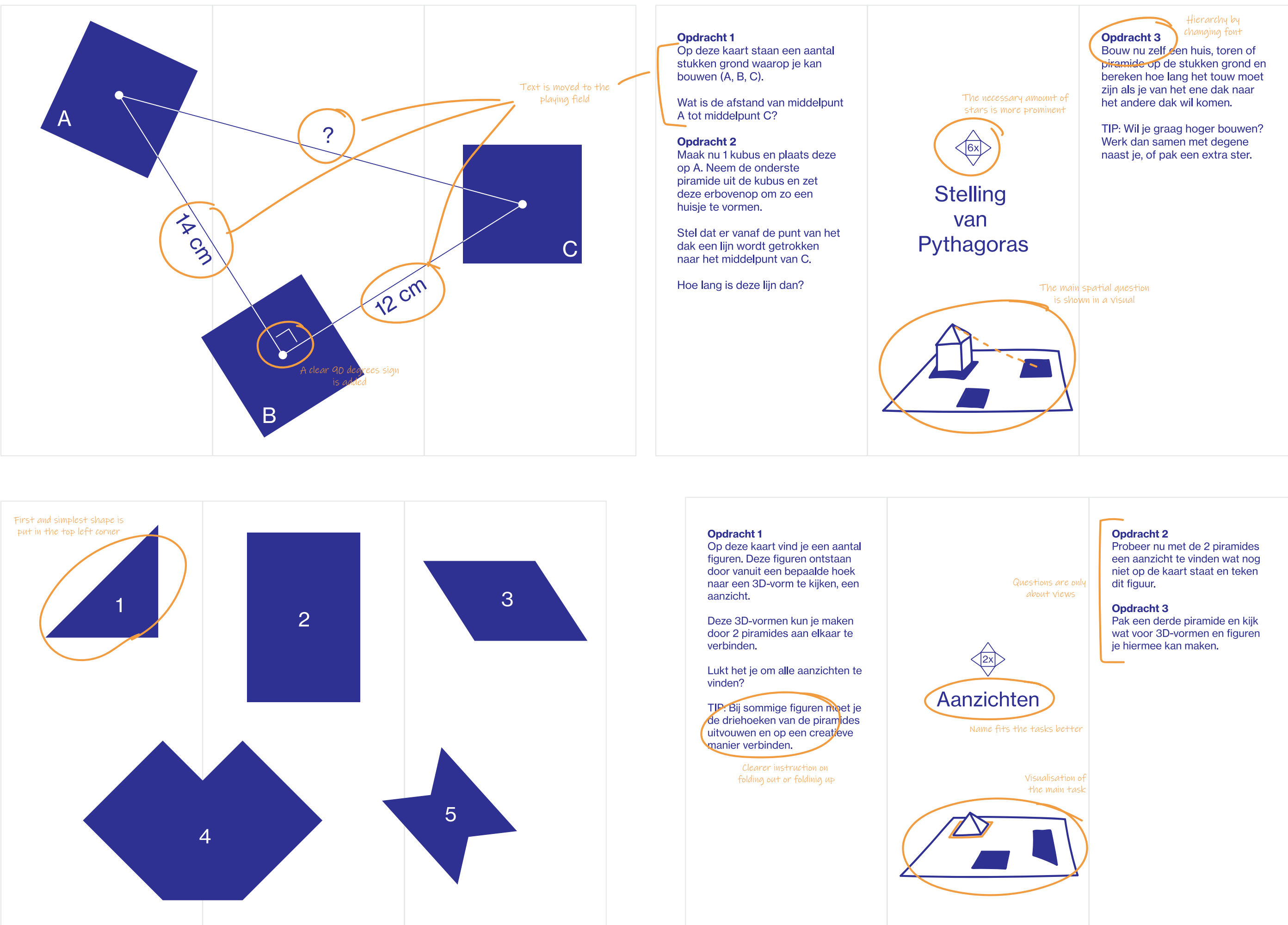


Figure 37 - Final changes to the assignment cards

These parts could then be snapped together, forming a mechanical hinge without the need for additional materials, reducing both the material costs and assembly time, making it more suitable for mass production.
However, the inaccuracies of standard 3D printers introduced a new challenge: instability in the connection. Because the printed parts are not printed exact, the click system can become loose, causing the connections to detach unintentionally. This compromises the durability and functionality of the prototype.

8.3 Cost comparison

This cost comparison table indicates that 3D printing the model is slightly less expensive than laser cutting. The biggest difference lies in the assembly time: taping all the components of the laser-cut version takes approximately 30 minutes per model, whereas the click system used in the 3D printed version reduces assembly time to just 5 minutes. This makes the 3D printed version significantly more efficient in terms of manual labour during assembly. (Figure 39)

However, when looking at the total production time per unit, 3D printing proves to be much more time-consuming. Producing a complete cube via 3D printing takes around 6 hours per unit, while a laser-cut version can be completed in approximately 40 minutes. This means that despite the more efficient assembly process, 3D printing is less time-efficient overall due to the long printing time.

8.4 Final design

The final design covered assignment cards of two of the geometry chapters and used the laser cut prototype for stabilty. The pictures can be found on the following pages (Figure 40-43).

Figure 38 - The refined 3D printed version

Category	Laser cutting	3D printing
Production time	8 min x €25/h = €3.25	6h x €1.50/h = €9.00
Material	€4.00 per cube	Included
Assembly time	6 x 5m = 30 min	6 x 20s = 2 min
Assembly costs (€13/h)	0.5 x €13 = €6.50	0.033 x €13 = €0.43
Magnets (48 x €0.30)	€14.40	€14.40
Total costs per cube	**€28.15**	**€23.83**

Figure 39 - Cost comparison between laser cutting and 3D printing

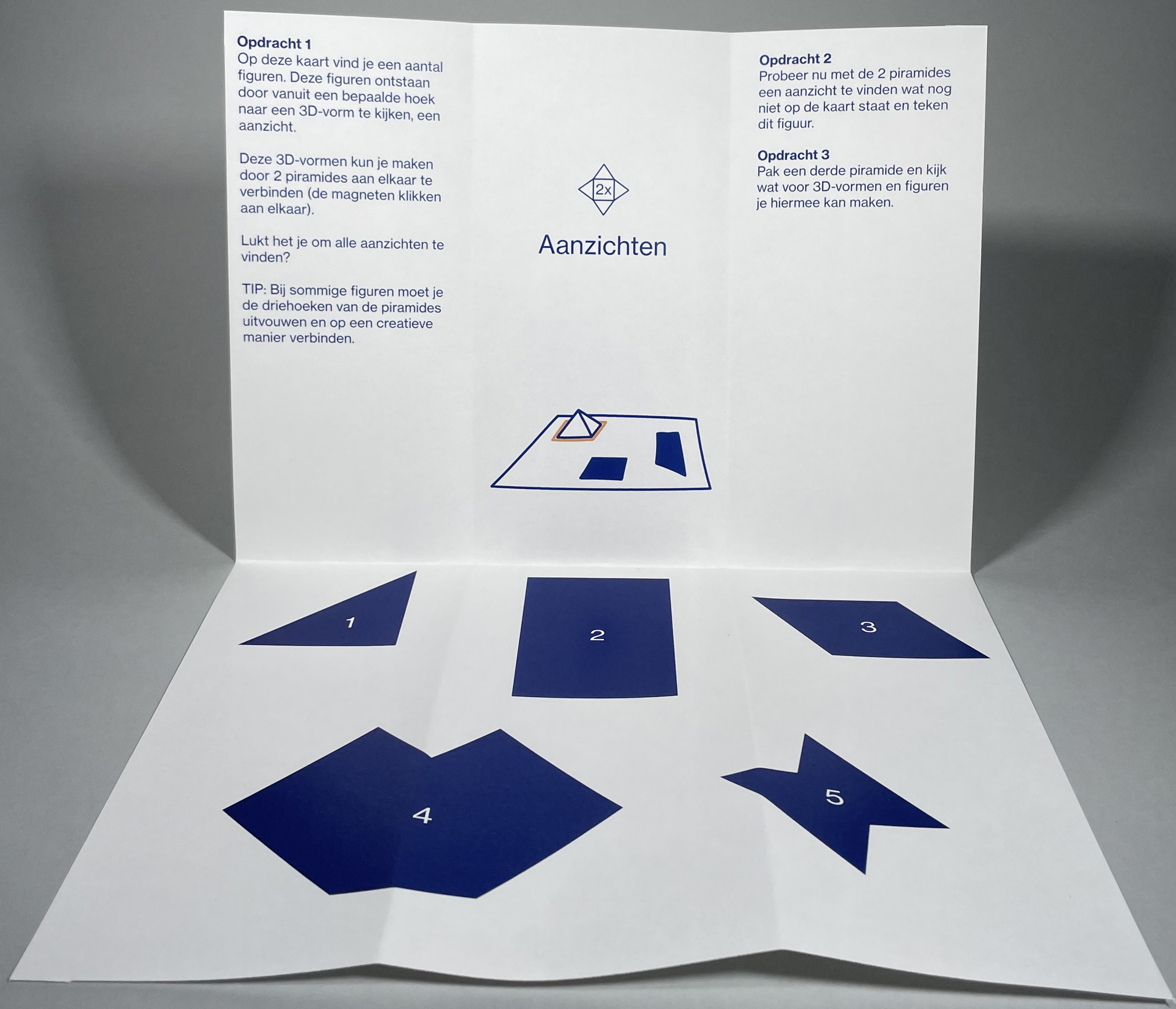


Figure 40 - Final design photo perspective assignment card

Figure 41 - Final design photo perspective asignment card with Geofold

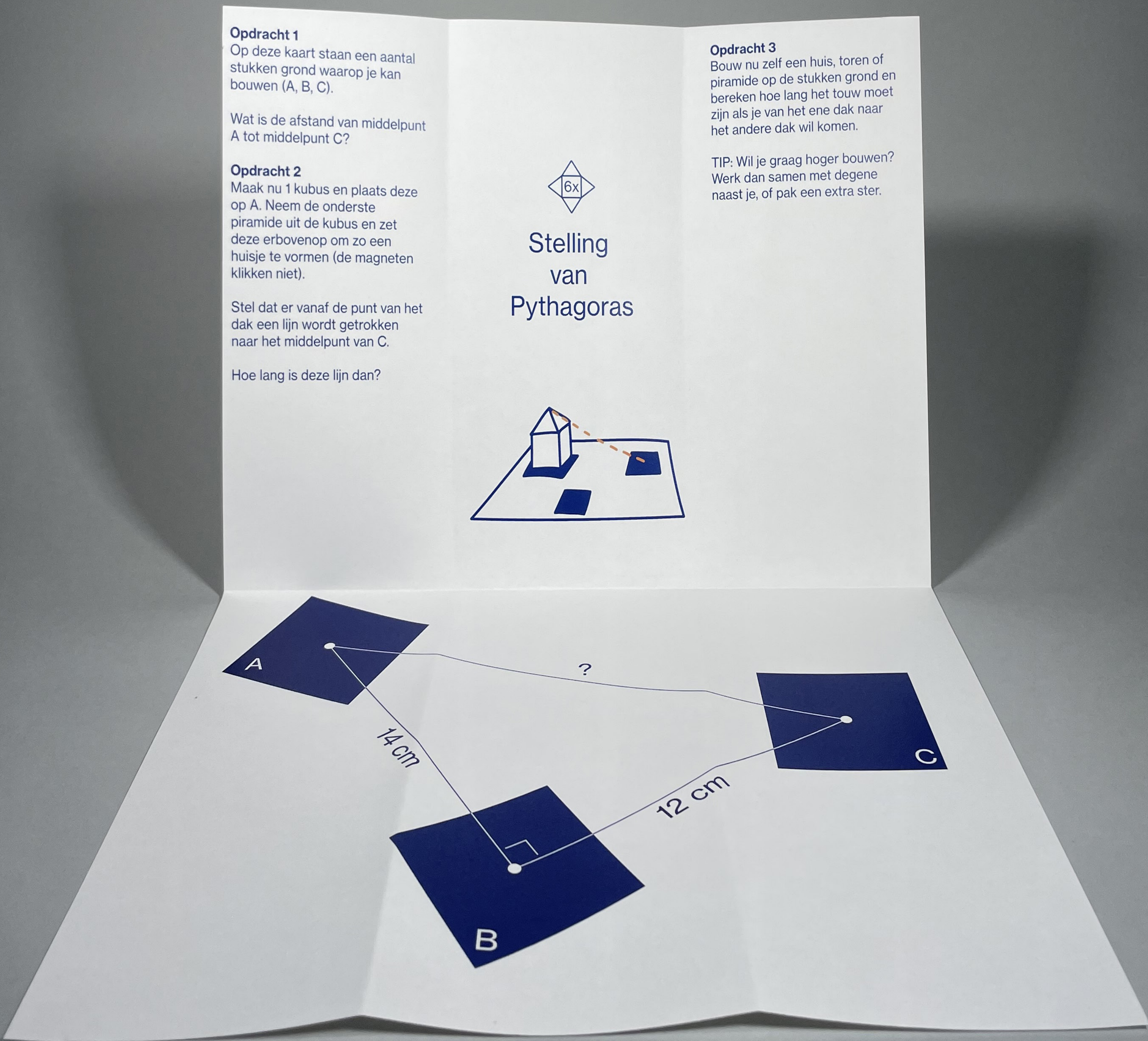


Figure 42 - Final design photo Pythagoras Theorem assignment card

Figure 43 - Final design photo Pythagoras Theorem asignment card with Geofold

9. Limitations

When bringing the prototype into an educational context, practical constraints become visible. Important factors such as school budget and student interaction become crucial. In case of the geofold toolkit, several of these limitations became apparent during development.

9.1 Production and stabiltiy

Laser cutting is relatively expensive and labour-intensive, as each part requires manual handling and assembly. While 3D printing offers a slightly cheaper alternative with less manual labour, the current connection method lacks stability and reliability. To improve this, several directions can be explored. Switching to a higher-resolution print method could lead to a more secure click. Another option is to test materials that are slightly more flexible, so they grip better when pressed together.

For larger-scale production, moving from 3D printing to injection moulding could be a logical next step. Although the upfront costs for a mould are high, the production time per part drops significantly, making it faster and cheaper over time. Injection moulding also brings higher precision and consistency, which would help make the click mechanism more reliable.

In short, 3D printing is useful for testing and prototyping thanks to its low labour demand and easy production. Laser cutting currently works best for small batches. But if the toolkit is to be scaled up, injection moulding offers the most realistic way forward due to its speed, precision and cost-efficiency once tooling is in place.

9.2 Implementation

As outlined in the research section of this report, schools often work with a tight budget. The development of the current cube toolkit prototype has proven to be relatively expensive. With an estimated production cost of around €25 per cube, supplying a full classroom of approximately 30 students would require a budget of over €750. This makes the current version of the toolkit financially unfeasible for implementation in most educational institutions.

To address this issue, a simplified, cost-effective version of the toolkit was developed using cardboard. While this version sacrifices the tactile, interactive quality of the original design, it still allows students to engage with the core educational content.

The cardboard version of the toolkit relies on the friction of the material rather than using an additional connection method. Instead of having moving parts, the individual squares and triangles are provided with a zig-zag edge to interlink the pieces in different positions. Although this version lacks the haptic feedback of the magnets, it enables students to engage with the same spatial questions and learning process in a more affordable way.

The cardboard version can be made by lasercutting the pieces of cardboard, reducing the costs to just the material and production time. This makes a cube well under 5 euro. While some experiential quality is lost, the educational value of the assignments remains intact.

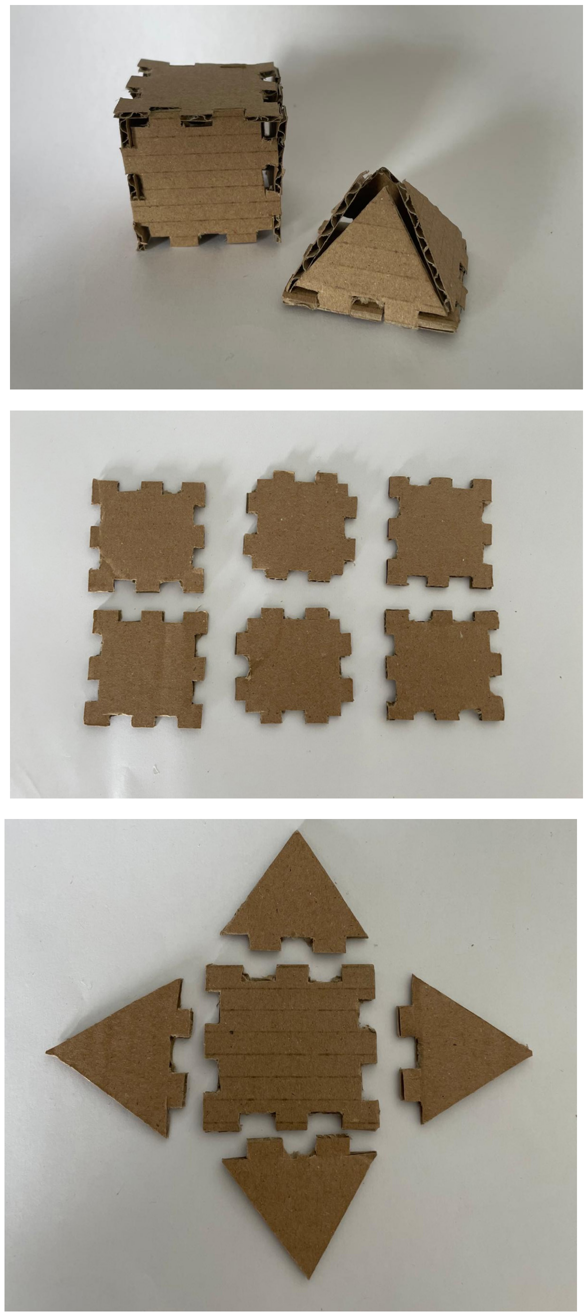


Figure 44 - Cost-efficient version of Geofold

Conclusion

The main question in the research was: How can 12 to 15 year old havo/vwo students develop their spatial skills during math classes through a tangible object? Mathematics is seen as an abstract course and is considered to be losing its touch with reality. Yet, it is exactly this connection to real-life experience that can help students better comprehend the material and counteract declining motivation for the course mathematics.

The research cycle shows that spatial skills have been difficult to define. Researchers often describe them as groups of spatial questions, missing the connection to everyday life. To create a clearer understanding of what spatial skills actually are, a matrix was developed that categorises different types of spatial skills. This matrix combines three axes: whether the task involves a single object or multiple objects, whether it is static or dynamic, and how the task is approached; mentally (mind), physically (body), or through movement in the environment (environment). By combining these dimensions, spatial tasks can be more easily explained and compared, offering a more structured way to understand and train spatial skills.

Research also shows that spatial skills can be improved through practice, particularly through hand-on activities like origami.

To explore how this could be applied in the context of mathematics education, three main stakeholders were identified: Students, teachers and curriculum developers. Interviews and observations were analysed by using context mapping, leading to a two-sided perspective.

Students express a strong desire for experimental and hand-on learning activities. They also value a sense of competence and often turn to peers for help, showing that a math class is not only about content, but also contains social dynamics. Schools however, prefer structure. Step-by-step instructions are the norm and mistakes are minimised, to focus heavily on tests. Combined with a busy curriculum, this leaves little room for alternative ways of teaching.

Entering the design cycle, these insights were translated into a design space and design directions. The focus was on finding a middle ground: a structured math class that allows room for active and collaborative learning while remaining within curriculum constraints.

The proposed solution includes lessons that follow a consistent format to meet curriculum goals but include interactive elements like puzzles and group work to improve motivation.

The chosen design direction explored how physical and experimental learning could be included within the lesson structure. The resulting design was Geofold, a hands-on learning tool, that aligns with the geometry chapters of the curriculum.

Rather than offering fixed exercises, Geofold introduces open-ended, tactile tasks that invite creative problem-solving. These spatial challenges allow students to explore, which is an important part of design learning. The design replaces students' time for extra exercises, minimizing the impact it has on the overall structure of math lessons, while still offering space for creative engagement.

Validation with students and teachers revealed mixed findings. The haptic experience that the magnets offer, was considered a positive experience; it encouraged exploration, engagement, and collaboration. However, the prototype was considered too bulky and impractical for typical classroom settings. Furthermore, the connection between the material and the assignment cards was not always intuitive. In particular, the assignment cards required clearer instructions and less textual complexity.

One significant limitation was the high costs of the prototype, which makes the implementation of the design difficult due to strict budgets. To address this, more cost-efficient design alternatives were proposed. Further development of the prototype is recommended, as well as tests with a complete set in a full classroom setting.

Resources

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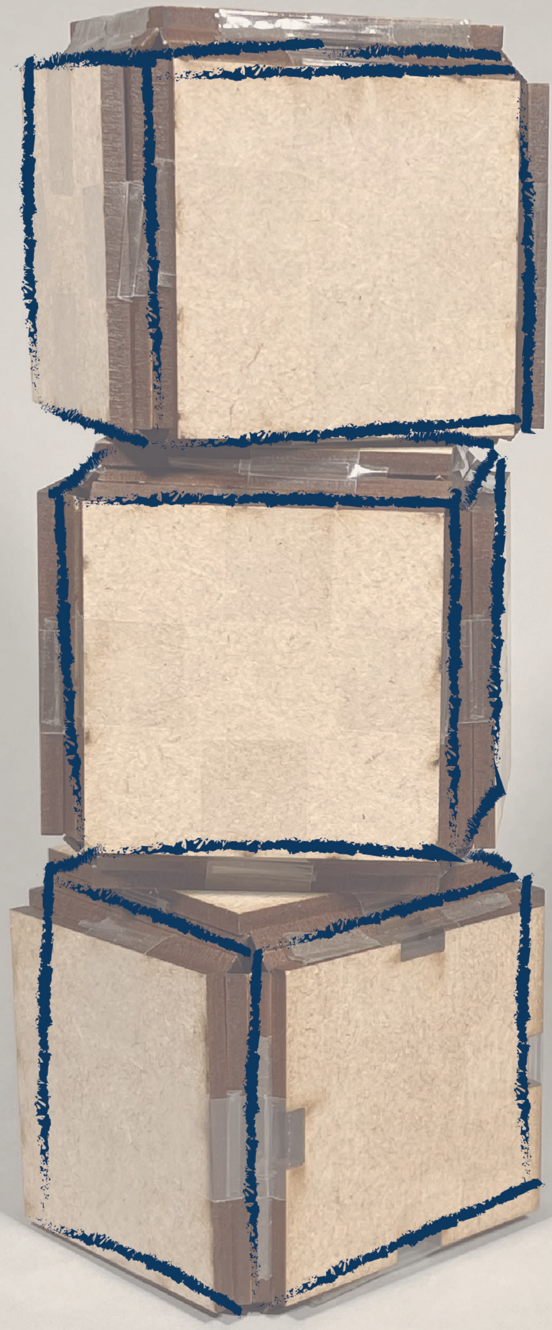
Heilig



Geofold

Emma Heilig

Master Thesis
Msc. Design For Interaction

June 2025

Where geometry unfolds. A toolkit that helps high school students develop spatial skills through tangible play and structured exploration.

TU Delft

DESIGN FOR our future

IDE Master Graduation Project

Project team, procedural checks and Personal Project Brief

In this document the agreements made between student and supervisory team about the student's IDE Master Graduation Project are set out. This document may also include involvement of an external client, however does not cover any legal matters student and client (might) agree upon. Next to that, this document facilitates the required procedural checks:

- Student defines the team, what the student is going to do/deliver and how that will come about
- Chair of the supervisory team signs, to formally approve the project's setup / Project brief
- SSC E&SA (Shared Service Centre, Education & Student Affairs) report on the student's registration and study progress
- IDE's Board of Examiners confirms the proposed supervisory team on their eligibility, and whether the student is allowed to start the Graduation Project

STUDENT DATA & MASTER PROGRAMME

Complete all fields and indicate which master(s) you are in

Family name

Initials

Given name

Student number

IDE master(s) IPD DfI SPD

2nd non-IDE master

Individual programme *(date of approval)*

Medisign

HPM

SUPERVISORY TEAM

Fill in he required information of supervisory team members. If applicable, company mentor is added as 2nd mentor

Chair dept./section

mentor dept./section

2nd mentor

client:

city: country:

optional comments

! Ensure a heterogeneous team. In case you wish to include team members from the same section, explain why.

! Chair should request the IDE Board of Examiners for approval when a non-IDE mentor is proposed. Include CV and motivation letter.

! 2nd mentor only applies when a client is involved.

APPROVAL OF CHAIR on PROJECT PROPOSAL / PROJECT BRIEF -> to be filled in by the Chair of the supervisory team

Sign for approval (Chair)

Name Date Signature

CHECK ON STUDY PROGRESS

To be filled in by SSC E&SA (Shared Service Centre, Education & Student Affairs), after approval of the project brief by the chair.
The study progress will be checked for a 2nd time just before the green light meeting.

Master electives no. of EC accumulated in total EC

Of which, taking conditional requirements into account, can be part of the exam programme EC

YES all 1st year master courses passed

NO missing 1st year courses

Comments:

Sign for approval (SSC E&SA)

Name Date Signature

APPROVAL OF BOARD OF EXAMINERS IDE on SUPERVISORY TEAM -> to be checked and filled in by IDE's Board of Examiners

Does the composition of the Supervisory Team comply with regulations?

YES Supervisory Team approved

NO Supervisory Team not approved

Comments:

Based on study progress, students is ...

ALLOWED to start the graduation project

NOT allowed to start the graduation project

Comments:

Sign for approval (BoEx)

Name Date Signature

DESIGN FOR our future

TUDelft

Personal Project Brief – IDE Master Graduation Project

Name student Emma Heilig **Student number**

PROJECT TITLE, INTRODUCTION, PROBLEM DEFINITION and ASSIGNMENT

Complete all fields, keep information clear, specific and concise

Project title Analysing and improving spatial skills of high school students through a tangible tool

Please state the title of your graduation project (above). Keep the title compact and simple. Do not use abbreviations. The remainder of this document allows you to define and clarify your graduation project.

Introduction

Describe the context of your project here; What is the domain in which your project takes place? Who are the main stakeholders and what interests are at stake? Describe the opportunities (and limitations) in this domain to better serve the stakeholder interests. (max 250 words)

Education has not changed much over the past decades. Children are put in a classroom with a book and expected to understand. For many subjects, a book works just fine, but for mathematics, other things are at play.

Mathematics requires not only abstract thinking, but also strong spatial skills, especially in topics such as geometry, graphs, and three-dimensional structures. Many high school students struggle with these skills, which hinders their understanding and performance in mathematics. (Atit et al., 2020)

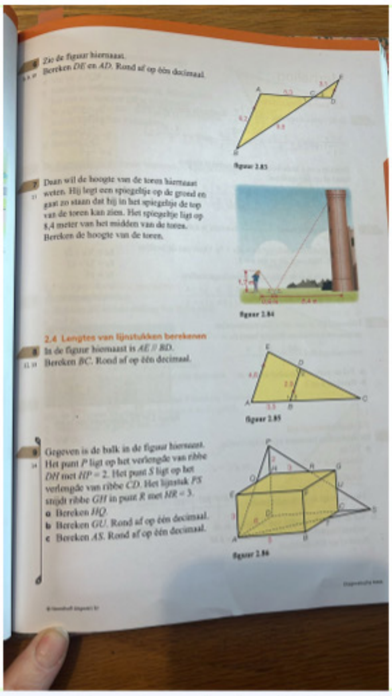
This project focuses on improving students' spatial ability in high school mathematics education. The main stakeholders are students, teachers, and educational institutions. Students benifit from improved skills and improved mathematical performance. Teachers gain effective tools to explain complex concepts more clearly. Educational institutions can strengthen their curriculum with innovative and effective teaching methods.

The focus of this project is based on the client involved: the 'Wetenschapsknooppunt' (Science Hub), a part of TU Delft. In the Science Hub TU Delft, employees, researchers, and designers of TU Delft work together with primary and secondary school teachers on projects in which the design process is central. The aim is to stimulate both students and teachers in creative thinking, designing, and researching. Currently, they are investigating how students' spatial abilities can be improved through the use of tanglible and material based learing tools

Opportunities in this domain lie in the introduction of interactive tools that allow students to move beyond textbooks and actively engage with three-dimensional figures, making abstract mathematical concepts more tangible and intuitive. However, the challenge lies in practical implementation: schools have limited time and resources, and not all teachers are trained in these techniques.

→ *space available for images / figures on next page*

introduction (continued): space for images


6 Zie de figuur hiernaast.
Bereken DE en AD. Rond af op één decimaal.

figuur 2.83

7 Daan wil de hoogte van de toren hiernaast weten. Hij legt een spiegeltje op de grond en gaat zo staan dat hij in het spiegeltje de top van de toren kan zien. Het spiegeltje ligt op 8,4 meter van het midden van de toren.
Bereken de hoogte van de toren.

figuur 2.84

2.4 Lengtes van lijnstukken berekenen

8 In de figuur hiernaast is $AE // BD$.
Bereken BC. Rond af op één decimaal.

figuur 2.85

9 Gegeven is de balk in de figuur hiernaast. Het punt P ligt op het verlengde van ribbe DH met $HP = 2$. Het punt S ligt op het verlengde van ribbe CD. Het lijnstuk PS snijdt ribbe GH in punt R met $HR = 3$.
a Bereken HQ.
b Bereken GU. Rond af op één decimaal.
c Bereken AS. Rond af op één decimaal.

figuur 2.86

image / figure 1 **A page of the most common mathematics textbook explaining spatial figures in 2D**

image / figure 2

Personal Project Brief – IDE Master Graduation Project

Problem Definition

What problem do you want to solve in the context described in the introduction, and within the available time frame of 100 working days? (= Master Graduation Project of 30 EC). What opportunities do you see to create added value for the described stakeholders? Substantiate your choice.
(max 200 words)

In traditional mathematics education, students often struggle with spatial reasoning, especially when working with three-dimensional figures. This might be caused by teaching methods heavily relying on textbooks and two-dimensional representations, limiting students' ability to fully understand essential mathematical concepts.

The key opportunity lies in connecting abstract mathematical theory with experiential learning. By providing interactive or tangible materials, this project can enhance student engagement and comprehension, leading to better academic performance and increased interest in a subject most students despise. The added value for stakeholders include innovative teaching tools for educators and improvement of the curriculum for institutions.

Assignment

This is the most important part of the project brief because it will give a clear direction of what you are heading for. Formulate an assignment to yourself regarding what you expect to deliver as result at the end of your project. (1 sentence) As you graduate as an industrial design engineer, your assignment will start with a verb (Design/Investigate/Validate/Create), and you may use the green text format:

Create a tangible object/tool to improve spetial skills for high school students in havo/vwo mathematics classes.

Then explain your project approach to carrying out your graduation project and what research and design methods you plan to use to generate your design solution (max 150 words)

The goals of the first phase will include the understanding of spatial skill development in children, as well as mapping out what the average maths class will look like and defining needs and requirements of the different stakeholders. These goals will be obtained by a combination of desk research, observations in classrooms and interviews with education experts, students and teachers. To sketch the average math class a journey map can be made next to the observations and interviews, and to define requirements benchmarking can be incorporated. The qualitative data derived from these activities can then be used to map out what influences the teaching of spatial skills (in math classes) by context mapping.

The second phase will focus on exploring different forms and functions the tool can take. Questions like 'what fits into the classroom and curriculum' will be central. Methods such as prototyping and co-creation sessions can be used to translate the gathered insights of phase 1 into potential designs for the tool.

Project planning and key moments

To make visible how you plan to spend your time, you must make a planning for the full project. You are advised to use a Gantt chart format to show the different phases of your project, deliverables you have in mind, meetings and in-between deadlines. Keep in mind that all activities should fit within the given run time of 100 working days. Your planning should include a ***kick-off meeting****,* ***mid-term evaluation meeting****,* ***green light meeting*** *and* ***graduation ceremony****. Please indicate periods of part-time activities and/or periods of not spending time on your graduation project, if any (for instance because of holidays or parallel course activities).*

Make sure to attach the full plan to this project brief.
The four key moment dates must be filled in below

Kick off meeting	11 Feb 2025
Mid-term evaluation	8 Apr 2025
Green light meeting	3 Jun 2025
Graduation ceremony	3 Jul 2025

In exceptional cases (part of) the Graduation Project may need to be scheduled part-time. Indicate here if such applies to your project

Part of project scheduled part-time	
For how many project weeks	
Number of project days per week	

Comments:

Motivation and personal ambitions

Explain why you wish to start this project, what competencies you want to prove or develop (e.g. competencies acquired in your MSc programme, electives, extra-curricular activities or other).

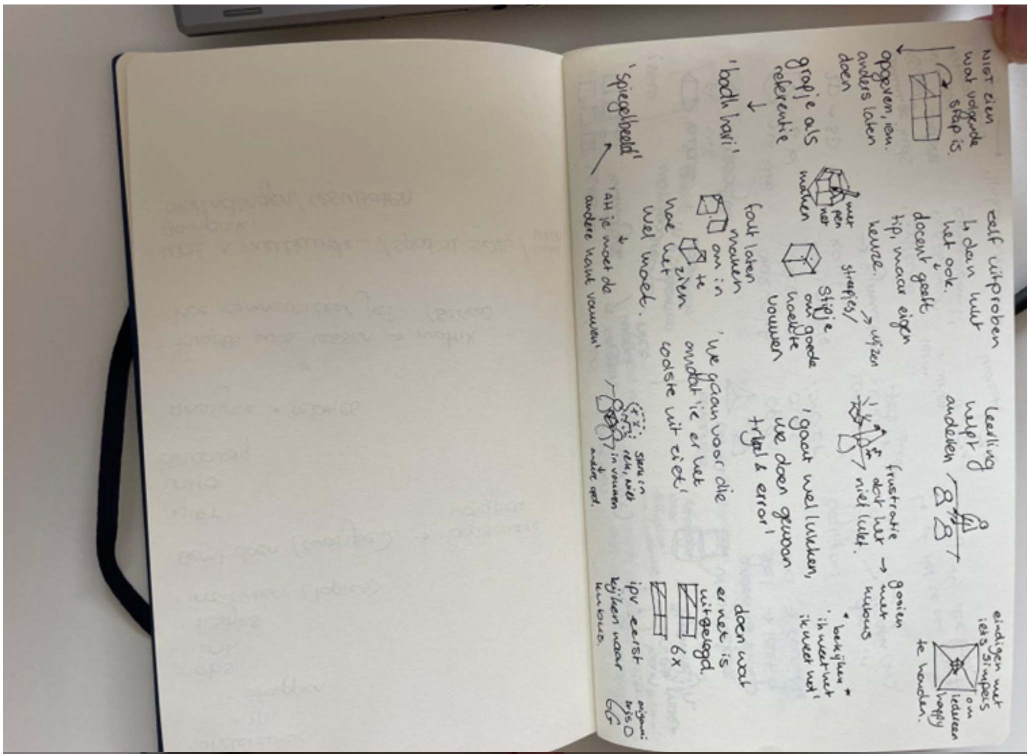
Optionally, describe whether you have some personal learning ambitions which you explicitly want to address in this project, on top of the learning objectives of the Graduation Project itself. You might think of e.g. acquiring in depth knowledge on a specific subject, broadening your competencies or experimenting with a specific tool or methodology. Personal learning ambitions are limited to a maximum number of five.
(200 words max)

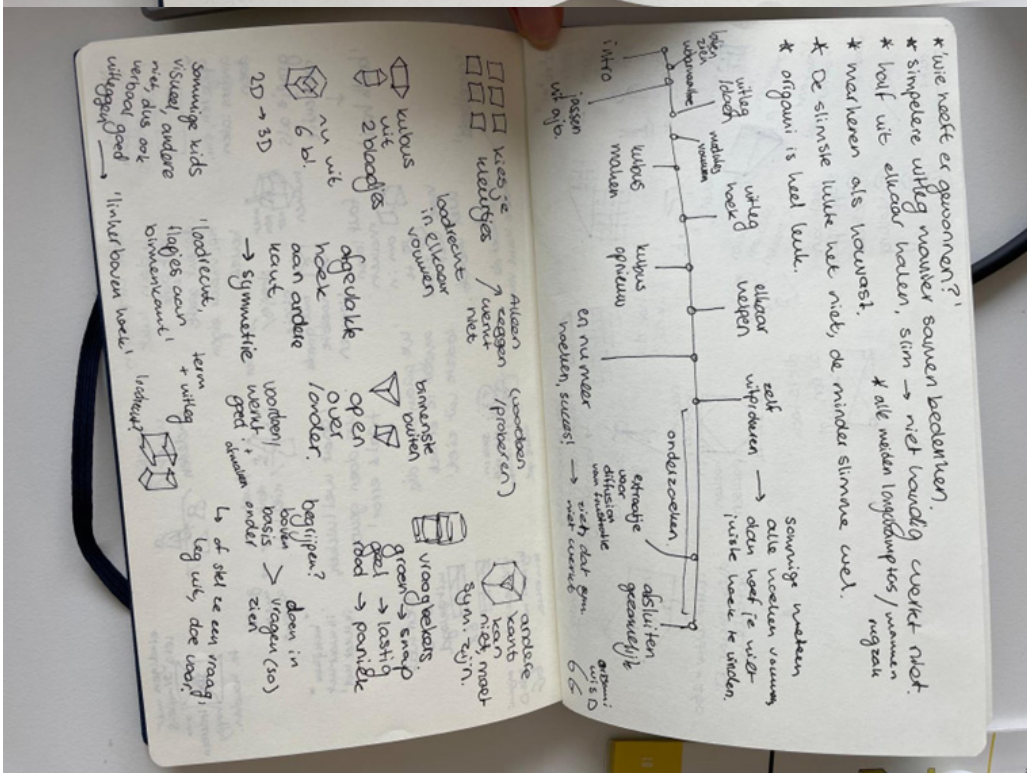
Over the past years, I have found that I am most interested in the following design aspects: behavioural change, education and gamification. In this project, I would therefor like to dive deeper into:

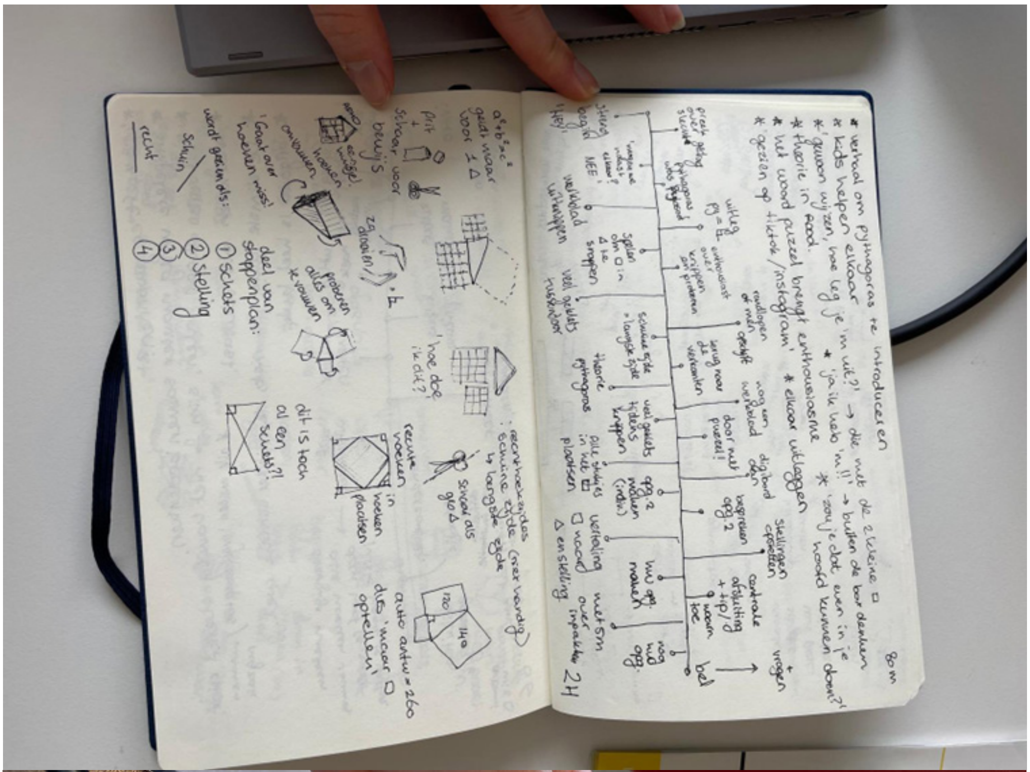
1. Designing effective educational tools, where the effectiveness is proved by theories that can be found in behavioural design theories. It should integrate seamlessly into the existing curriculum and target group

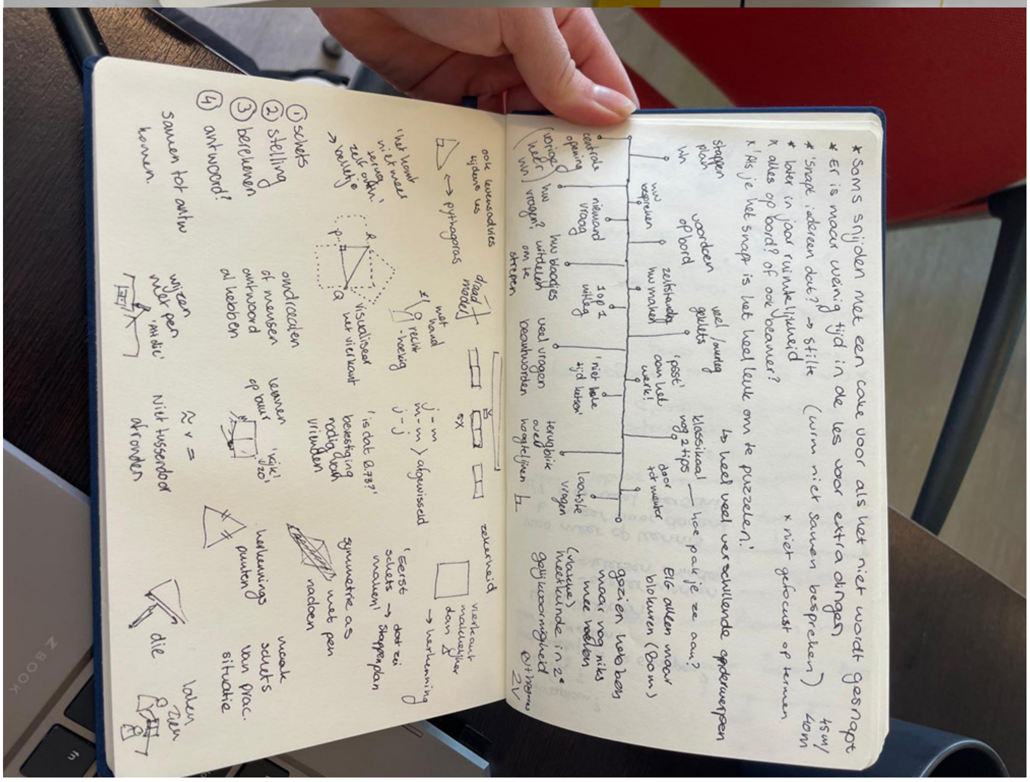
2. Strengthen my user-centered design skills, by setting up and conducting interviews or co-creating sessions and translating the data into insights, with the help of techniques like contextmapping.

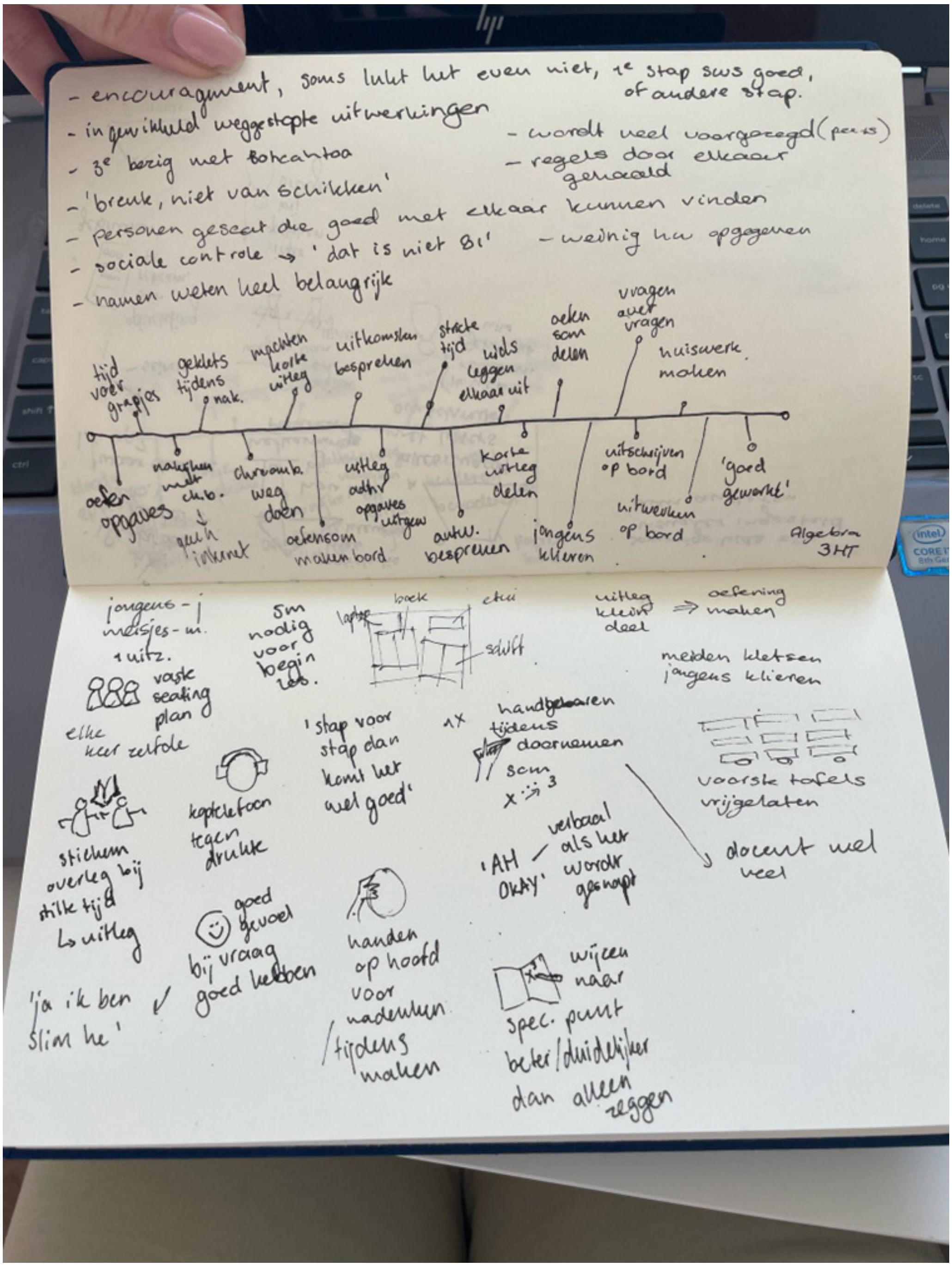
3. Improve my skills in (quick) prototyping, by making prototyping part of the process instead of the final product.

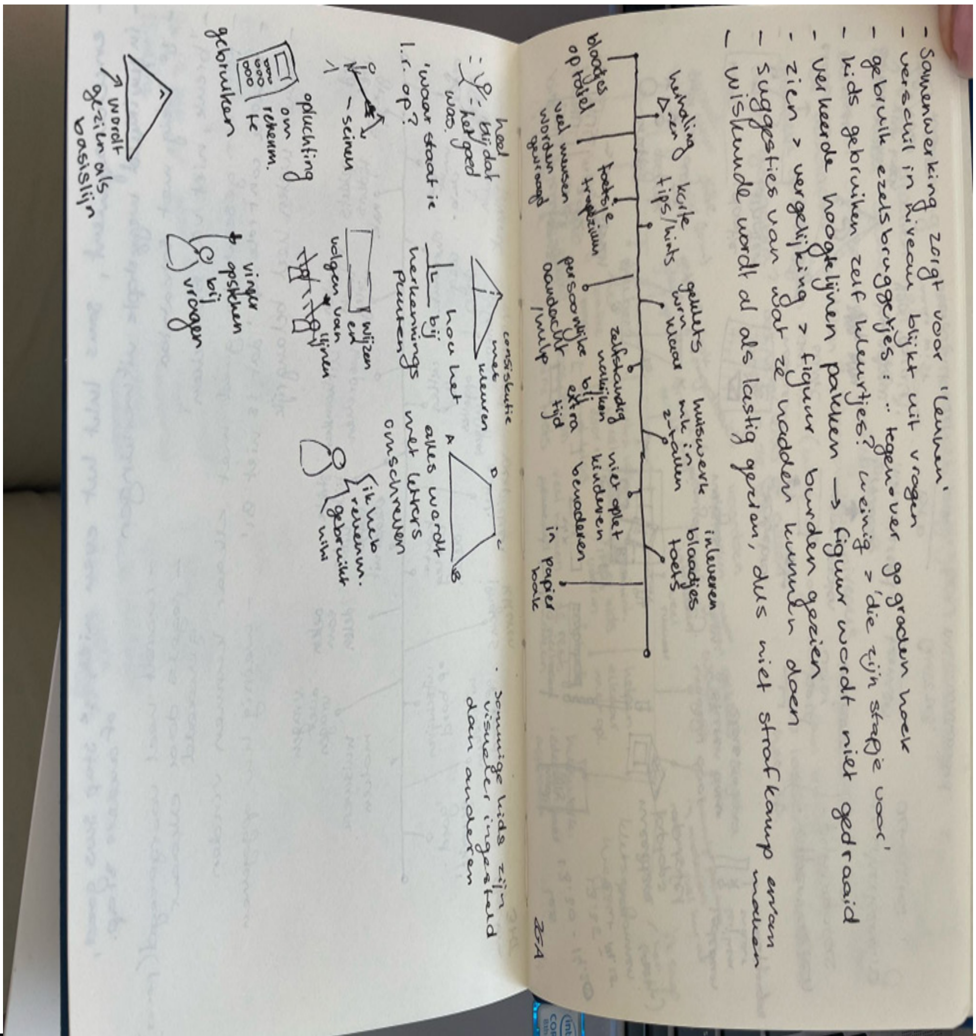


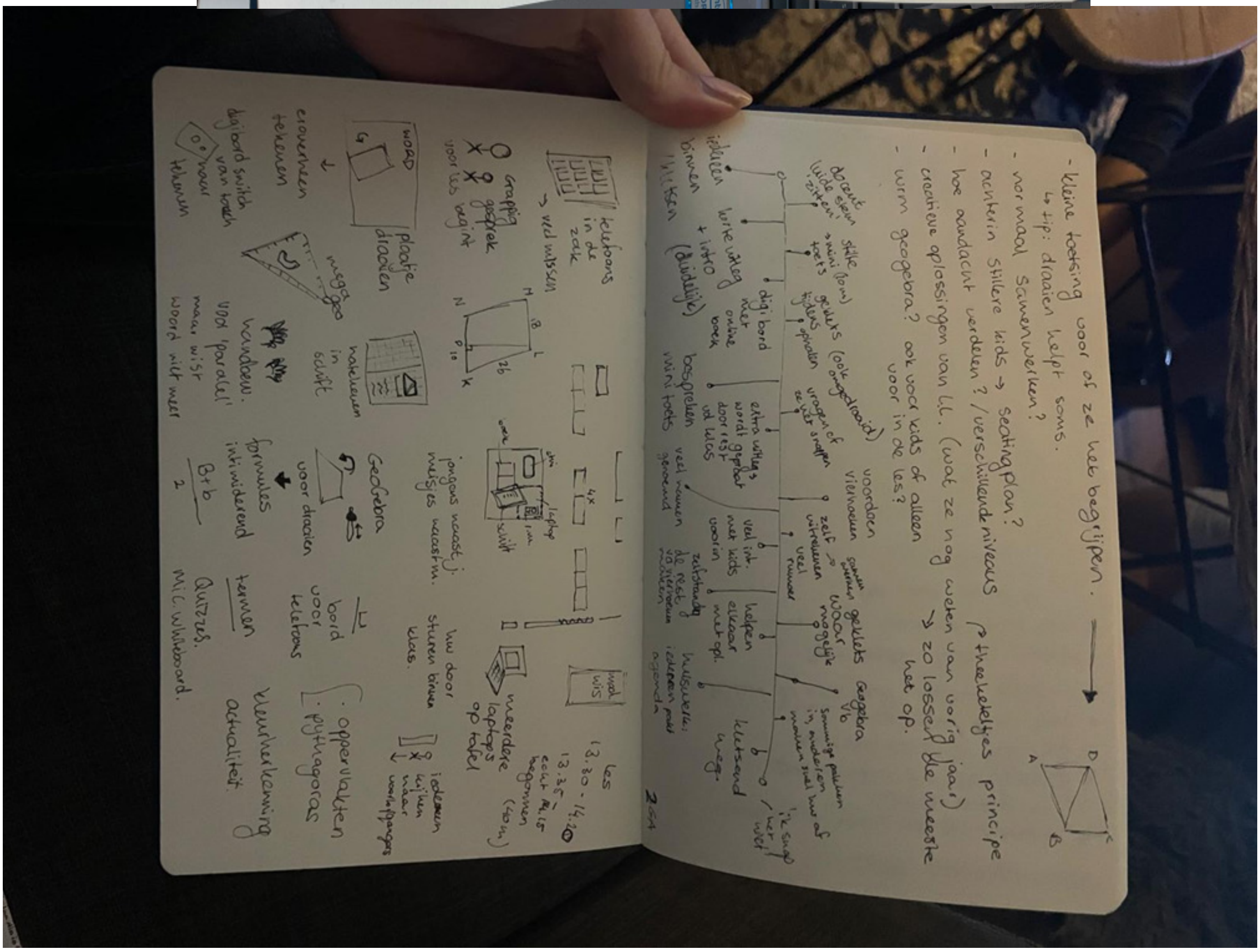







- encouragement, soms lukt het even niet, 1e stap sws goed, of andere stap.
- ingewikkeld weggestopte uitwerkingen
- 3e bezig met Sohcahtoa
- 'breuk, niet van schrikken'
- personen gezet die goed met elkaar kunnen vinden
- sociale controle → 'dat is niet 81'
- namen weten heel belangrijk
- wordt veel voorgezegd (peers)
- regels door elkaar genoemd
- weinig hw opgegeven
Algebra 3HT
jongens - j
meisjes - m.
vaste seating plan
elke keer zelfde
koptelefoon tegen drukte
goed gevoel bij vraag goed hebben
'ja ik ben slim he'
'stap voor stap dan komt het wel goed'
handen op hoofd voor nadenken / tijdens maken
'AH OKAY' - verbaal als het wordt gesnapt
wijzen naar spec. punt beter/duidelijker dan alleen zeggen
uitleg klein deel ⇒ oefening maken
meiden kletsen jongens klieren
voorste tafels vrijgelaten
docent veel


- Samenwerking zorgt voor 'leunen'
- verschil in niveau blijkt uit vragen
- gebruik ezelsbruggetjes: .. tegenover 90 graden hoek
- kids gebruiken zelf kleurtjes? weinig > die zijn stapje voor'
- verkeerde hoogtelijnen pakken → figuur wordt niet gedraaid
- wiskunde wordt al als lastig gezien, dus niet strafkamp ervan maken
sommige kids zijn visueler ingesteld dan anderen


- kleine toetsing voor of ze het begrijpen.
↳ tip: draaien helpt soms.
- normaal samenwerken?
- achterin stillere kids → seatingplan?
- hoe aandacht verdelen? / verschillende niveaus
- creatieve oplossingen van l.l.
- wim geogebra? ook voor kids of alleen voor in de les?

Appendix C - Interview Questions

Vragen leerlingen:
- Hoe ziet jullie wiskundeles eruit? Hoe begint het, wat doen jullie gedurende de les? Hoe lang duurt de les?
- Werk je vaak in groepjes of alleen? Werk je liever samen of alleen?
- Werk je online, met werkbladen, voorwerpen of met pen en papier?
- Wat heb je op je bureau als je les hebt? Is dat hetzelfde als je thuis werkt?
- Wat is het aller stomste om te vergeten als je wiskunde les hebt? Dit had ik echt nodig gehad.
- Hoe zorg jij dat je wiskunde snapt? Wat gebruik je daarvoor?
- Hoe zou je de volgende vraag benaderen: zie foto
- Wat zijn dingen tijdens de les die je fijn/leuk vind?
- Wat zijn dingen in de les die je vervelend/stom vind?
- Wordt wiskunde gezien als een leuk vak? Hoe zou je wiskunde leuker kunnen maken?
- Wat vind je nou leuk om te doen op school? En buiten school?

Vragen docenten:
- Hoe toets je of leerlingen een onderwerp echt snappen?
- Wanneer ben je als docent tevreden met het resultaat van de klas? Is dat anders dan het gemiddelde cijfer?
- Welke onderwerpen hebben kinderen ruimtelijke vaardigheden voor nodig?
- Laat je kinderen wel eens samen werken en waarom?
- Is er een vaste seating plan?
- Gebruik je een vaste structuur in de les?
- Hoe ga je om met niveau verschil in de klas?
- Hoe kijk je naar ruimtelijke vaardigheden?
- Welke alternatieve manieren van lesgeven gebruik je?

Vragen PRIME:
- In het kort wat ze doen.
- Jullie maken vaak applets, wat is de keuze voor een online tool, in plaats van iets tastbaars? Hebben jullie onderzocht of dat ook de beste manier is? papers verschillen van mening.
- Hoe gaan jullie van abstract naar visualisaties? Hoe ziet dat proces eruit? Naar welke aspecten van de theorie kijk je?
- Als je simpelere wiskundige begrippen uit zou moeten leggen, hoe zou je dat aanpakken? Zou je wat jullie nu doen kunnen implementeren in middelbaar onderwijs?
- Wat zien jullie als spatial skills, als we breder denken dan wiskunde?
- Hoe denk je dat je spatial skills kunt ontwikkelen?

U, of uw kind wordt uitgenodigd om deel te nemen aan een onderzoek met de titel *'Het analyseren en verbeteren van ruimtelijk inzicht bij middelbare scholieren met behulp van een tastbare tool'*. Dit onderzoek wordt uitgevoerd door Emma Heilig van de TU Delft.

Het doel van deze studie is om vanuit verschillende perspectieven inzicht te krijgen in wiskundelessen en de manier waarop daarin aandacht wordt besteed aan de ontwikkeling van ruimtelijk inzicht. Het interview zal ongeveer 30 minuten van uw tijd in beslag nemen.

Tijdens het interview wordt het gesprek auditief opgenomen met een dictafoon en zullen er aantekeningen worden gemaakt. De verzamelde gegevens worden gebruikt voor kwalitatieve analyse en het schrijven van een scriptie. Tijdens het interview zullen we u een aantal vragen stellen en zal u worden gevraagd om een ruimtelijke puzzel op te lossen. Mogelijk wordt er ook gevraagd naar waargenomen interacties.

Zoals bij elke online activiteit bestaat er altijd een klein risico op een inbreuk op de privacy; zo is het mogelijk dat de antwoorden die u geeft indirect uw identiteit onthullen. Wij doen ons uiterste best om uw antwoorden vertrouwelijk te behandelen. Dit doen we door de verzamelde gegevens te pseudonimiseren en veilig digitaal op te slaan. Ook worden er geen directe (anonieme) antwoorden gebruikt in de onderzoeks-output zonder toestemming. De scriptie zal worden opgeslagen in de thesis repository van de TU Delft, maar bevat geen herkenbare persoonsgegevens.

Uw deelname aan dit onderzoek is volledig vrijwillig en **u kunt op elk moment besluiten om te stoppen**. U bent ook vrij om vragen over te slaan als u zich daar niet prettig bij voelt.

Mocht u vragen of zorgen hebben over de ethische aspecten van dit onderzoek die u liever niet bespreekt met de betrokken onderzoeker, dan kunt u contact opnemen met de verantwoordelijke begeleider:

Ik ga ermee akkoord dat mijn antwoorden of andere input **anoniem** kan worden aangehaald in

Handtekeningen

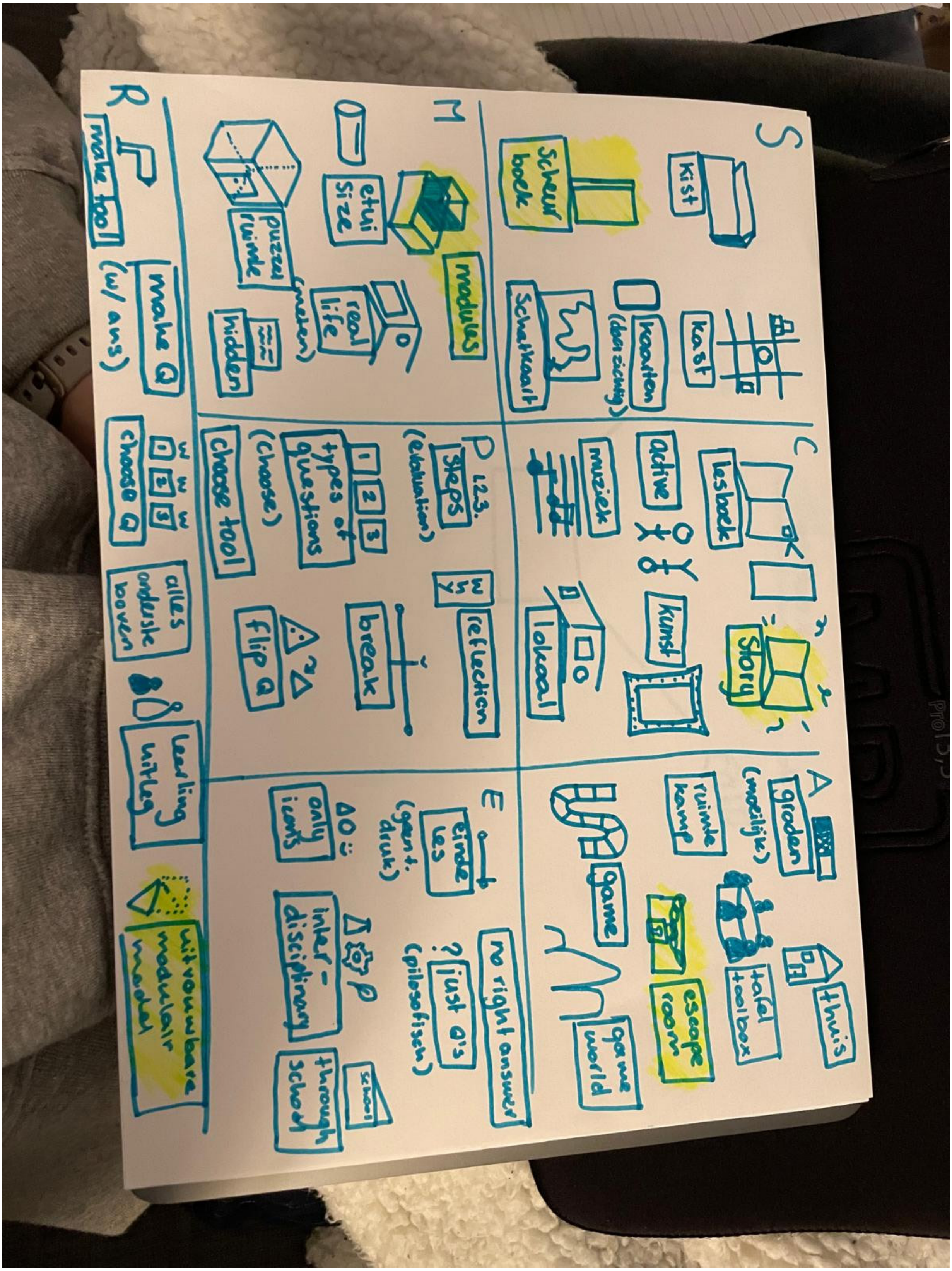
Naam deelnemer | Handtekening | Datum

Ik, als wettelijk vertegenwoordiger, bevestig dat ik getuige ben geweest van het correct voorlezen van het toestemmingsformulier aan de potentiële deelnemer en dat deze persoon de gelegenheid heeft gehad om vragen te stellen. Ik bevestig dat de persoon vrijwillig toestemming heeft gegeven

Naam ouder/verzorger | Handtekening | Datum

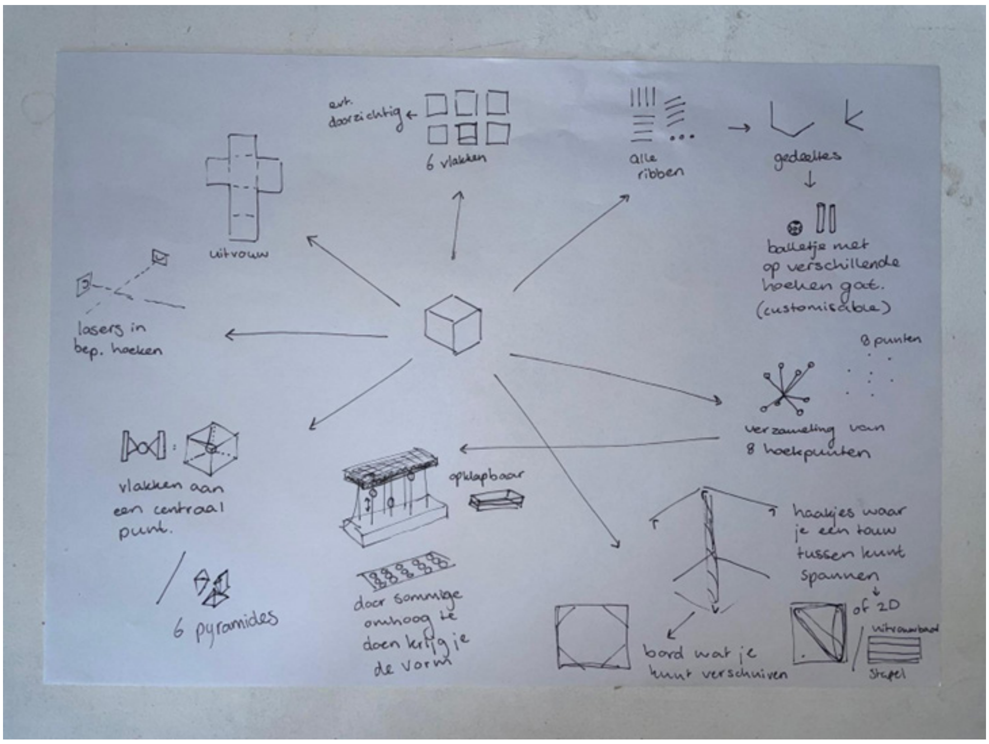
Ik, als onderzoeker, bevestig dat ik de informatiebrief correct heb voorgelezen aan de potentiële deelnemer en dat ik er, naar beste vermogen, voor heb gezorgd dat de deelnemer begrijpt waarvoor hij/zij vrijwillig toestemming geeft.

Naam onderzoeker | Handtekening | Datum


S
kist
scheur boek
kast
schatkaart
C
lesboek
active
muziek
kunst
story
lokaal
A
graden
thuis
tafel toolbox
escape room
game
game world
M
etui size
modulus
real life
puzzel ruimte
hidden
P
steps
(evaluation)
types of questions
(choose)
choose tool
reflection
break
flip Q
E
einde les
no right answer
just Q's
only icons
inter-disciplinary
through school
school
R
make tool (w/ ans)
make Q
choose Q
alles onderste boven
uitvouwbare modulair model

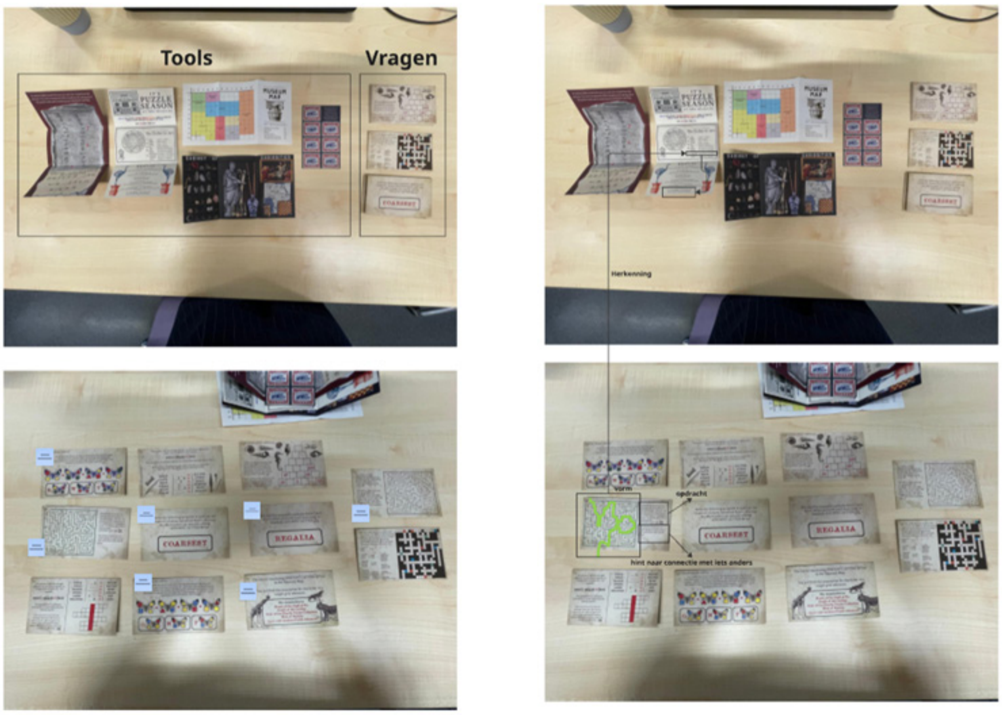
Appendix F - Crafting workshop materials

Appendix G - Cube studies brainstorm

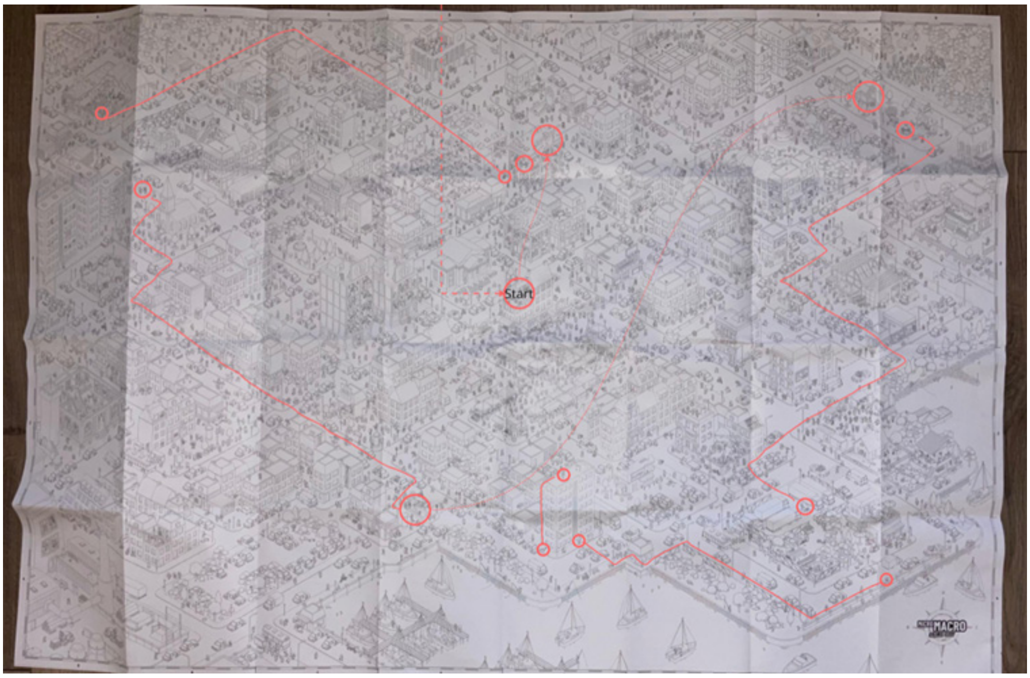


Appendix H - Game analysis

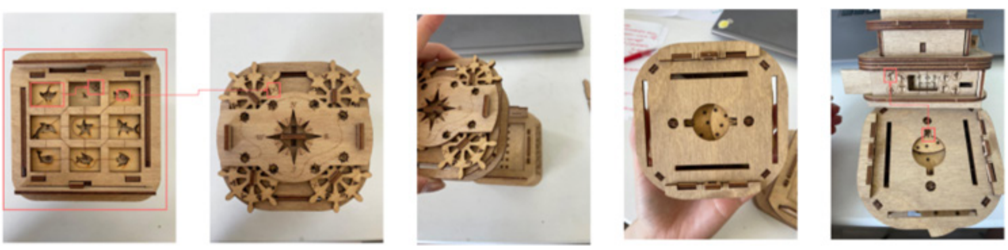
H.1 Cardgame

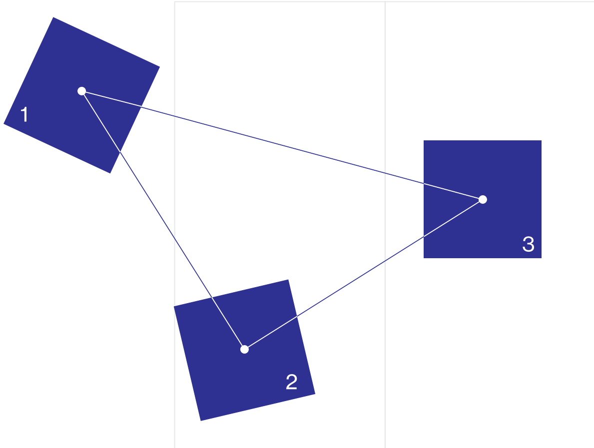


H.2 Micro Macro crime city



H.3 Clue box





Op de achterkant van deze kaart staan een aantal plekken waarop je huizen kunt bouwen, ookwel kavels genoemd.

De afstand tussen het middelpunt van kavel 1 en 2 is 14 cm. De afstand tussen kavel 2 en 3 is 12 cm.

Hoeveel ruimte zit er nu tussen kavels 1 en 3?

Pak nu 1 kubus en plaats deze op kavel 1. Neem de onderste piramide uit de kkubus en zet deze erbovenop om een dak te vormen.

Stel dat er vanaf de punt van het dak een zipline naar kavel 2 wordt gespannen.

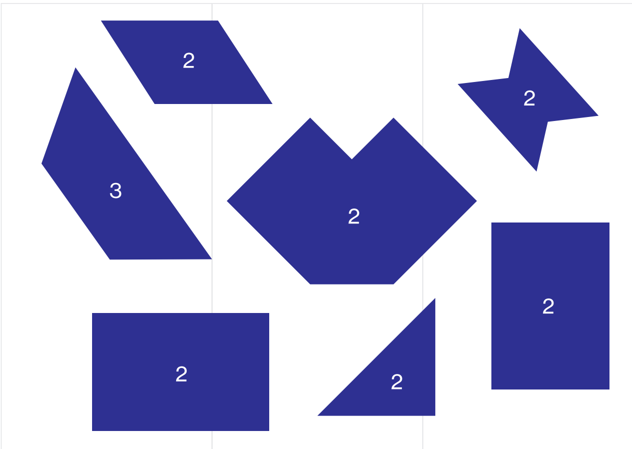
Hoe lang is deze zipline dan?

Stelling
van
Pythagoras

1 - 3

Bouw nu zelf huizen of torens op de kavels van verschillende hoogtes en bereken hoe lang de ziplines moeten zijn om van huis naar huis te komen.

Tip: Wil je graag hoger bouwen maar lijk je te weinig kubussen te hebben? Gebruik dan maar 3 of 4 piramides per de kubus en gebruik de rest om de hoogte in te bouwen. Als je het vanuit het juiste perspectief bekijkt, lijkt het nog steeds een volledige kubus.



Op de achterkant van deze kaart staan verschillende figuren.

Deze figuren ontstaan doordat je een 3D-vorm vanuit verschillende kanten bekijkt.

Je kunt deze vormen maken door 2 of 3 piramides aan elkaar vast te maken.

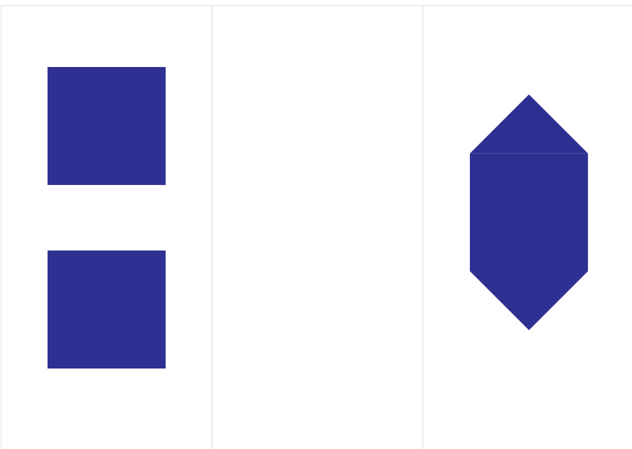
Lukt het je om alle figuren te vinden?

Afstanden
en
Oppervlakten

1

Bereken nu van elk figuur de omtrek en de oppervlakte.

Tip: De 3D figuren die je net gemaakt hebt, geven je een tip over uit welke vormen de figuren zijn opgebouwd.



Neem de kubus en zet deze op een van de vierkanten op de achterkant van deze kaart.

Hoe groot is de inhoud van deze kubus?

Neem nu de bovenste piramide uit de kubus en plaats deze op het andere vierkant.

Wat is de inhoud van deze piramide?

Wat gebeurt er met de inhoud van de kubus nu er een piramide mist?

Inhoud
en
Vergroten

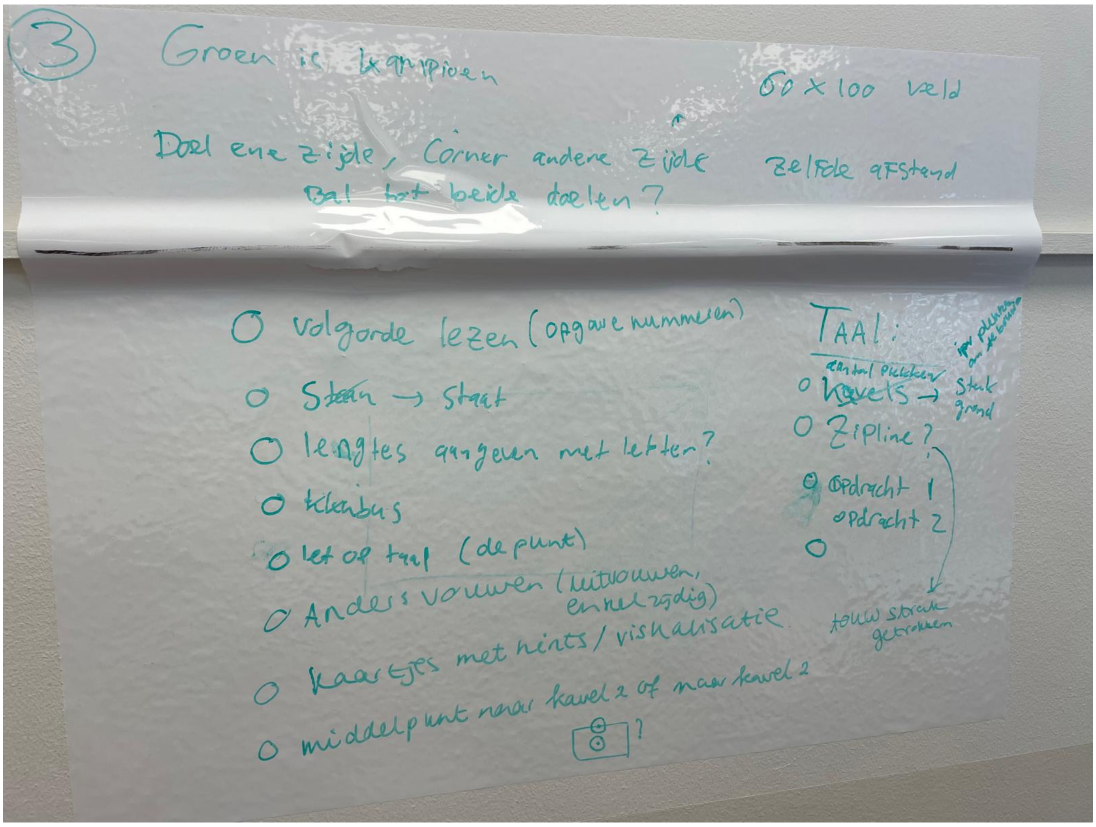
1

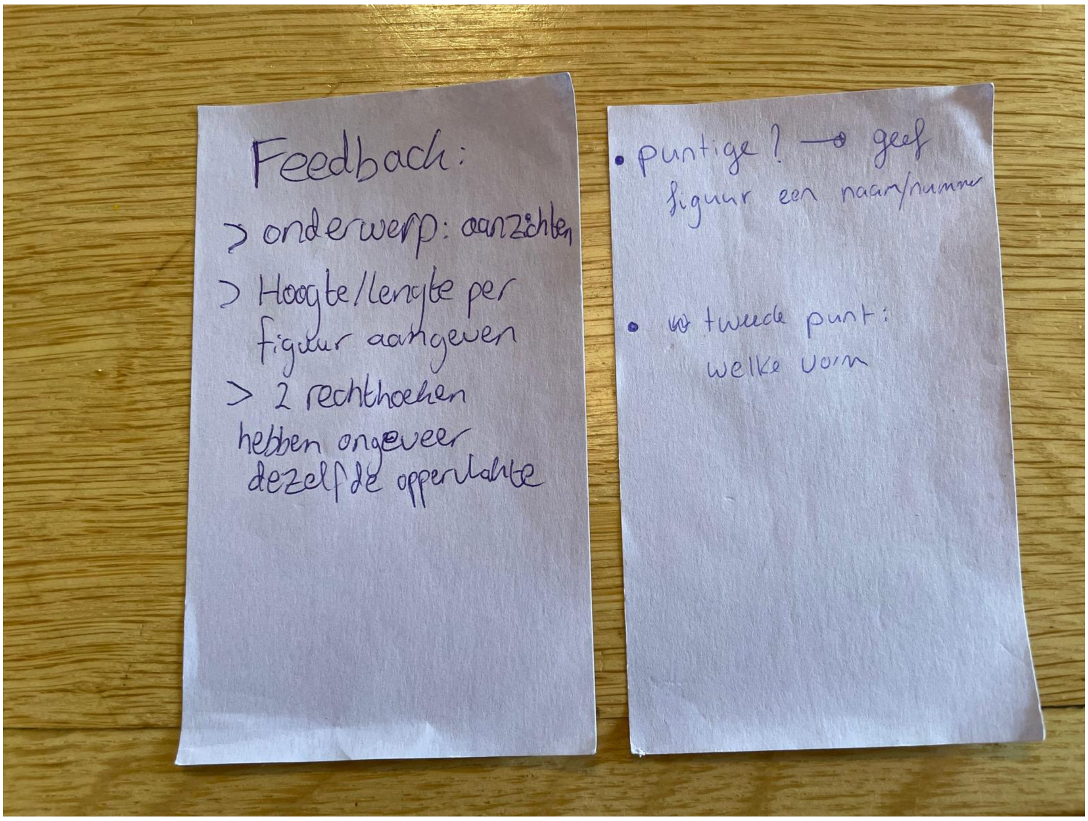
Maak he tpuntige figuur na met 6 piramides.

Hoe groot is de inhoud van deze vorm?

Probeer nu dezelfde vorm te maken, maar dan met 4 piramides.

Hoe verandert de inhoud van de vorm?


3
Groen is kampioen
60 x 100 veld
Doel ene zijde, Corner andere zijde
Bal tot beide doelen?
zelfde afstand
volgorde lezen (opgave nummeren)
Staat
lengtes aangeven met letter?
let op taal (de punt)
Anders vouwen (uitvouwen, enkelzijdig)
kaartjes met hints / visualisatie
TAAL:
kavels → Stuk grond
Zipline?
opdracht 1
opdracht 2
touw strak getrokken


Feedback:
> onderwerp: aanzichten
> Hoogte/lengte per figuur aangeven
> 2 rechthoeken hebben ongeveer dezelfde oppervlakte
• puntige? → geef figuur een naam/nummer
• tweede punt: welke vorm

Appendix K - The 3D printed mechanism