Estimation of the precipitation recycling ratio

by

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In partial fulfilment of the requirements for the degree of

Master of Science
in Civil Engineering, Water Management track
at the Delft University of Technology

and the degree of

Master of Science
in Hydraulic Engineering and Water Resources Management
at the National University of Singapore

To be publicly defended on 23th May 2016

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The struggle itself towards the heights is enough to fill a man's heart.

One must imagine Sisyphus happy.

Albert Camus

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Abstract

The recycling ratio R is said to be an indicator of the potential interaction between surface hydrologic processes and atmospheric hydrologic processes for a determined equilibrium state (Eltahir and Bras (1994)). Several precipitation recycling models that provide an estimate of R have been developed in literature, with each having various assumptions and limitations (Brubaker et al. (1993)) (Eltahir and Bras (1994)) (Dominguez et al. (2006)) (Van Der Ent (2010)).

The present thesis is focused on applying some of the most commonly used precipitation recycling models (models of (Brubaker et al. (1993)), (Eltahir and Bras (1994)) , (Dominguez et al. (2006)) or dynamic model and (Van Der Ent (2010)) or Water Accounting Model (WAM)) to the upper Mississippi region including U.S. Midwest under the hypothesis that it is possible to estimate the recycling ratio making use of observed relationships. The data used comes from satellite observations and reanalysis products from the ERA-Interim.

Estimates of precipitation recycling obtained by using the models of (Brubaker et al. (1993)) and (Eltahir and Bras (1994)), which are applicable at monthly timescale, are approximated with a linear equation of the form $R = a + bE * dt$, where $E * dt$ is the monthly evaporation depth [L] and a and b are empirical coefficients. The coefficient of determination (R^2) between the model estimates and the linear equation are 0.91 and 0.82, for the period 2004-2014, respectively for (Brubaker et al. (1993)) and (Eltahir and Bras (1994)).

Although several attempts of estimating the daily recycling ratio based on the dynamic model, the WAM, and the atmospheric fluxes were made, it was not possible to identify an equation that produces estimates within an acceptable range ($R^2 > 0.65$). However, when the definition of the recycling ratio was applied to the daily estimates in order to determine the monthly values, the correlation between the monthly evaporation and the monthly recycling ratio improved significantly.

Previous researches have correctly claimed that the recycling ratio is dependent on the size of the region studied (Eltahir and Bras (1996)), (Dominguez et al. (2006)), (Van Der Ent (2010)), some have even proposed equations that estimate the regional recycling ratio in seasonal or annual timescales as a function of the area (Dominguez et al. (2006)), (Dirmeyer and Brubaker (2007)). As a means to quantify the influence of the size of the region, the recycling ratio was estimated with the WAM without considering the influx of locally generated water vapour so that the transport of moisture within the region is null, and it was found that the maximum contribution of the evaporation to the recycling of moisture was approximately 15 %.

With the analysis of three regions with different sizes, it was demonstrated that the dependence of the regional recycling ratio on the size of the region is due to the transport of moisture generated in the upwind zones to the downwind zones, and that in small regions (region 1 in this study) there was not a clear difference between the effects of local coupling and moisture recycling, which agrees with the findings of (Goessling and Reick (2011)).

The estimates of the regional recycling ratio from the three regions were fitted to an equation which has a similar form of the one proposed by (Dominguez et al. (2006)) and (Van Der Ent and Savenije (2011))

Finally in order to test accuracy of the estimates obtained with the new proposed equation, the monthly recycling ratio of two new regions are determined with the WAM, both have different

geometry, sizes and are located in different zones. The estimates obtained with the proposed equation showed to have a good correlation, R^2=0.86 and 0.90 respectively, with the estimates from the WAM.

CHAPTER 1

1.1 Introduction

The role of precipitation within the hydrological cycle is evidently important. Precipitation provides the element that triggers surface hydrological processes like runoff generation, rapid sub-surface flow and ground water restoration. The phenomenon of precipitation is complex as it requires of four steps for its generation: 1) supply of moisture, 2) cooling below the point of condensation, 3) condensation 4) growth of particles (Tu Delft Faculteit Civiele Techniek En Geowetenschappen. Section Water Resources (2007)).

The precipitation in an area is said to be composed by water vapour advected and water vapour locally evaporated that reached the atmosphere; the percentage of contribution of the evaporated water to the total precipitation within a region is the recycling ratio. Precipitation recycling or moisture recycling refers to the contribution of evaporation within a region to the precipitation in that same region (Brubaker et al. (1993)).

Precipitation recycling has been linked to the potential degree of interaction between atmosphere and surface hydrology, it can also be used as an indicator for sensitivity of climate to land use changes (Brubaker et al. (1993)), (Savenije (1995)).

The link between precipitation and evaporation has been studied in several regions, the understanding of their interaction have changed throughout the years. (Benton et al. (1950)) showed that in previous works of Horton (1943): “little or no vapour of truly oceanic origin may ever reach small tributary areas at the headwaters of large rivers”, which suggest that the water present in the continents is mainly of local origin, and that more evaporation will lead automatically to more precipitation. (Benton et al. (1950)) argues otherwise. They proposed that the lifting mechanism acts as an important factor for precipitation occurrence, stating that the increase of water vapour in the atmosphere does not necessarily increase precipitation. (Mcdonald (1962)) pointed out that the contribution of evaporation to local precipitation was smaller than what was previously believed, he justified his claim arguing that there were misconceptions due to the incomplete understanding of the rainfall mechanisms, and to overlook of the scale of the phenomenon. Besides being an indicator of the potential interaction between surface hydrology and the atmosphere, the precipitation recycling ratio can be used as a tool to study the origin of water vapour that contributes to the precipitation in a determined area as in (Van Der Ent (2010)) and (Gimeno et al. (2012)).

The recycling ratio can be studied with a) analytical box models b) numerical models and c) isotopic models. Each have advantages and disadvantages when compared with the other. The analytical box models are simple and require few data, but usually make use of assumptions and don’t incorporate boundary processes. The numerical models allow more realistic representation in a Lagrangian or Eulerian frame of reference but also are highly dependent on the model used. The isotopic models are mainly used to validate the results of the models previously mentioned, however isotopic models can be sensitive to the isotopic signal and calculation time (Gimeno et al. (2012)).

1.2 Hypothesis and research questions

The recycling ratio has been historically estimated using analytical models, the present thesis hypothesises that:

“It is possible to estimate the regional recycling ratio or the recycled precipitation without a model, by making use of observed relationships of the atmospheric fluxes”

Among the research questions this thesis will answer are:

- Which model studied provides results of the recycling ratio with more accuracy.
- How much of the precipitation that occurs in Illinois is sustained by the local evaporation?
- How much does evaporation and moisture transport influence the calculation of the precipitation recycling ratio.

CHAPTER 2

2.1 Literature Review

Precipitation recycling is a concept that helps to better understand the water cycle, which is of special interest as this is the most basic of all geochemical cycles. It influences on other cycles and affects global atmospheric and oceanic circulation, shaping weather and climate (Eagleson (1986)). The precipitation recycling is characterized by the precipitation recycling ratio. The precipitation recycling ratio concept must be carefully interpreted and applied as it has been previously pointed by researchers:

- The estimates of the precipitation recycling ratio depend on the size of the region and are a measure of the degree of control exerted by land surface hydrology on the climatic regime. As the climatic regime is the driving force for land-surface hydrology the recycling ratio is a measure of the land surface-atmosphere feedback (Brubaker et al. (1993)).
- The precipitation recycling ratio is a diagnostic measure of the predominant climatic regime, and it cannot be used for predictions due to the complex interrelated factors that control the precipitation (Brubaker et al. (1993)).
- The precipitation recycling ratio depends on the geographical location, temporal scale, spatial scale and may vary seasonally (Eltahir and Bras (1996))
- The precipitation recycling ratio is characteristic of equilibrium status and cannot be taken as constant, hence can't be used for forecasting, but it can be used as diagnostic of potential interactions in the current climate (Eltahir and Bras (1996))
- The precipitation recycling ratio should be considered as an index rather than a physical quantity (Bosilovich and Schubert (2001)).

Past studies on precipitation recycling have been mainly focused in the use and development of analytical and numerical models that are able to show the possible feedbacks and influence of the land surface hydrology with the atmosphere. These type of models have evolved through the years and become more realistic, at the same time their spectrum of application has also widened from investigating the potential influence of the local evaporation in the local precipitation (Brubaker et al. (1993)), (Eltahir and Bras (1994)), (Bisselink and Dolman (2008)), (Bisselink and Dolman (2009)), to investigate the sources of moisture during floods or droughts (Dirmeyer and Brubaker (1999)),(Dominguez et al. (2006)) and the impacts of irrigation in the regional water cycle (Harding and Snyder (2012)).

Analytical box models are most used for studying the recycling of moisture, the motivation resides in the desire of understanding of how changes in the surface hydrology of a region, due to anthropogenic influences or natural variability, which are likely to modify the climate through changes in the water cycle (Gimeno et al. (2012)).

According to (Burde and Zangvil (2001)) the first analytical box model to study precipitation recycling was developed by Budyko and Drozdov in 1953. This model is the simplest of all and laid the foundation for the study of precipitation recycling with analytical models.

The result obtained with the analytical models proposed by (Brubaker et al. (1993)), (Eltahir and Bras (1994)) and (Dominguez et al. (2006)), are scale dependent. (Savenije (1995)) was the first and only to propose an analytical model that is not dependent on the size of the region, but rather in the distance that an air pocket travels from the ocean across a continent.

The most utilized models are (Brubaker et al. (1993)), (Eltahir and Bras (1994)),(Dominguez et al. (2006)), and (Van Der Ent (2010)), which have been used in several regions and periods of time. For a summarized table with results of several researches please go to **Appendix 1**.

2.2 Basic Definitions

2.2.1 Moisture or precipitation recycling ratio

Precipitation recycling is defined as the contribution of precipitation falling in a region which originates from evaporation within the same region (Brubaker et al. (1993)), (Eltahir and Bras (1994)) (Savenije (1995)), (Van Der Ent (2010)). Precipitation in an area is assumed to compose of two parts (Brubaker et al. (1993)):

$$P = P_m + P_a \tag{1}$$

Where P is the total precipitation in an area, P_m is the precipitation of water vapour evaporated in the same area, P_a is precipitation of water vapour that is advected to that area.

The percentage of precipitation recycling is represented by the moisture recycling ratio or precipitation recycling ratio, from now on mentioned as recycling ratio, which can be local (ρ)(Eltahir and Bras (1994)),(Burde and Zangvil (2001)), regional (R) (Eltahir and Bras (1994; Eltahir and Bras (1996)) and continental (ρ_c) (Savenije (1995; Van Der Ent (2010)), (Gimeno et al. (2012; Yoshimura et al. (2004)).

The *local recycling ratio ρ* is said to be specific for a point (x, y) in a region (Burde and Zangvil (2001)), but in reality is the recycling ratio of a cell of a given resolution which contains that specific point (x, y). The value of ρ varies with position (x, y), time (t) and size of the cells or resolution of the area studied. It is defined as:

$$\rho\ (t, x, y) = \frac{P_m\ (t, x, y)}{P\ (t, x, y)} \tag{2}$$

where P_m is the precipitation originated from evaporation of the same cell [L], P_m is the total precipitation [L].

The *regional recycling ratio R* is defined for a specific region, it is representative for the whole area studied. This ratio is dependent of time t, the size of the region A, and its shape. It can be defined as:

$$R = \frac{\int P_m\ (t, x, y) dA}{\int P(t, x, y) dA} = \frac{\sum_{i=1}^{n} \rho_i P_i\, A_i}{\sum_{i=1}^{n} P_i\, A_i} \tag{3}$$

The *continental recycling ratio ρ_c* makes the distinction between precipitation of water vapour from continental origin P_C and precipitation of water vapour from oceanic origin P_O which in this case would be from locally evaporated water vapour and of advected water vapour respectively (Savenije (1995)).

$$P = P_C + P_O \tag{4}$$

The continental recycling ratio can be calculated for a Lagrangian or Eulerian work frame. (Savenije (1995)) calculates the recycling ratio along a streamline that crosses a continent as a function of the distance of a pocket of air to the ocean. While the definitions proposed by (Yoshimura et al. (2004)), (Van Der Ent (2010)) are a modification of the local recycling ratio in which the limiting factor of shape and size of the region are overcome by using the continental surface as the region of study.

The continental recycling ratio in a 2D Eulerian work frame is usually defined as in Eq.(5)(Van Der Ent (2010)):

$$\rho_c\,(t,x,y) = \frac{P_C\,(t,x,y)}{P\,(t,x,y)} \tag{5}$$

2.2.2 Equations

The starting point for studying the precipitation recycling is the *atmospheric water balance* (AWB) of water vapour, also known as mass balance equation.

$$\frac{\partial w}{\partial t} + \frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} = E - P \; or \; \frac{\partial w}{\partial t} + \nabla\overrightarrow{Q_o} = E - P \tag{6}$$

where:

w= precipitable water [L^3]
u = precipitable water weighted vertically integrated water vapour velocity in the x direction [LT^{-1}]
v= precipitable water weighted vertically integrated water vapour velocity in the y direction [LT^{-1}]
P = precipitation [L^3T^{-1}]
E= evaporation [L^3T^{-1}]
$\overrightarrow{Q_o} = Q_{ox}\,\vec{\imath} + Q_{oy}\,\vec{\jmath}$ vector of water vapour flux [L^4T^{-1}]

The *weighted velocities also called effective velocities* are defined as shown in eq. (7)

$$u = \frac{Q_{ox}}{w};\; v = \frac{Q_{oy}}{w} \tag{7}$$

The first term in the left of eq. (6) represents the change of the precipitable water or atmospheric storage in time for a specific particle of water vapour; the second and third terms respectively represent the change of the vertically integrated moisture flux in the x and y directions or the divergence of moisture flux.

The *divergence of the water vapour flux* from an area can be calculated using the Gauss theorem:

$$\overrightarrow{\nabla Q_o} = \frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} = \frac{1}{A}\oint_s \overrightarrow{Q_o}\cdot\vec{n}ds = O - I \tag{8}$$

where:
A = area of the region [L^2]
$\vec{n}$ = unit vector perpendicular to the border of the area
ds = differential of the length of the border [L]
O = outflux [L^3T^{-1}]
I = influx [L^3T^{-1}]

Physically the divergence of a vector flux is interpreted as the amount of flux entering or leaving a closed surface of a volume, a positive divergence means that more flux is leaving the volume, hence acting as a source while a negative divergence means that the volume is acting as a sink of the flux. The *influx of water vapour flux* towards an area and *outflux from an area* is calculated as

$$I = -\frac{1}{A} \oint_{\lambda_{in}} \overrightarrow{Q_o} \cdot \overrightarrow{n_\lambda}\, ds \tag{9}$$

$$O = -\frac{1}{A} \oint_{\lambda_{out}} \overrightarrow{Q_o} \cdot \overrightarrow{n_\lambda}\, ds \tag{10}$$

where:
$\overrightarrow{n_\lambda}$ = unitary vector perpendicular to the direction of the flow
λ = direction for integration. It may be inwards or outwards.

Finally the *atmospheric water vapour* and the components of the *moisture vapour flux vector* are calculated as follow:

$$w = \frac{A}{\delta g} \int_{pu}^{ps} q dp \tag{11}$$

$$Q_{ox} = \frac{A}{\delta g} \int_{pu}^{ps} q \hat{u} dp \tag{12}$$

$$Q_{oy} = \frac{A}{\delta g} \int_{pu}^{ps} q \hat{v} dp \tag{13}$$

where:
δ = density of liquid water [ML^{-3}]
g = gravity [LT^{-2}]
q = specific humidity [M/M]
dp = differential of the atmospheric pressure [$MT^{-2}L^{-1}$]
$\hat{u}$ = horizontal velocity of the air in the x direction [LT^{-1}]
$\hat{v}$ = horizontal velocity of the air in the y direction [LT^{-1}]
pu = pressure when water vapour content is negligible [$MT^{-2}L^{-1}$]
ps = pressure at surface [$MT^{-2}L^{-1}$]
Q_{ox} = x component of the vertically integrated moisture flux vector [$M\ L^{-1}\ T^{-1}$]
Q_{oy} = y component of the vertically integrated moisture flux vector [$M\ L^{-1}\ T^{-1}$]

It is important to note that the influx I and outflux O are scalars, while $\overrightarrow{Q_o}$ is a vector.

2.3 Analytical Box Models of the recycling ratio

The present thesis will only consider the most widely used analytical box models for the determination of the recycling ratio; as said before these models have the advantage of being computationally efficient and require a small number of parameters and data (Gimeno et al. (2012)).

The box analytical models sacrifice complexity in order to gain speed of calculation. To do this the models make use of several assumptions that simplify the calculation of the recycling ratio.

2.3.1 Assumptions

The analytical models have used several assumptions since the first model was developed; as new models have emerged these assumptions have been suppressed. Here a brief description of all the assumptions ever used are provided/ listed:

1. *Use of time averaged data*

   This assumption is especially important for the determination of the moisture influx in monthly time scale.

   Most of the data used for the calculation of the influx is measured by satellite. The measurements are taken 4 times per day and then processed into daily and monthly values. (Fitzmaurice (2007)) identified two possible ways of processing the data, called the time-averaged method and accumulated method.

   The first one refers to this assumption, here the influx is calculated from a unique value of $\overrightarrow{Q_o}$ which is the result of averaging all the instantaneous $\overrightarrow{Q_o}$.

   The accumulated method calculates the monthly influx as the average of instantaneous influx. The use of the accumulated method is more realistic and overrides the necessity of this assumption.

2. *Storage of atmospheric water vapour is small compared to the atmospheric water vapour fluxes*

   This assumption refers to the time scale used for the analysis, (Eltahir and Bras (1994)) demonstrated that for a monthly time-scale the variation of precipitable water is small when compared with the outflux and evaporation from an area.

   The pitfall of this assumption is that the use of large time-scales i.e. monthly neglects the daily variation of moisture storage changes, which can influence the results for recycling ratio as has been demonstrated by (Fitzmaurice (2007)) and (Dominguez et al. (2006)).

3. *The atmosphere is assumed to be mixed*

   The assumption of a well-mixed atmosphere implies that the particles of advected and evaporated moisture have the same probability of precipitating. This means that the ratios of advected to evaporated particles are the same in the water that is being precipitated in a rainfall event and in the precipitable water present in the atmosphere.

   In the early years of precipitation recycling research this assumption was accepted and widely used, however, the increase of computational power allowed the formulation of more detailed transport models that led to a more detailed representation of the vertical distribution of the water vapour. The use of the well mixed atmosphere assumption is justified by arguing that most of the water vapour in the atmosphere is contained in the planetary boundary layer (PBL), where the moisture is well mixed due to convective processes (for more reference data and explanation (see (Eltahir and Bras (1994))). (Bosilovich (2002)) demonstrated that the mixing in the atmosphere doesn't occur quickly after evaporation as Budyko in 1974 and others proposed. (Bosilovich (2002)) concluded that the recycling ratio can substantially vary depending on the vertical distribution of moisture transport and the presence of convective processes, and that similar recycling ratios don't imply the same vertical distribution of water vapour.

   In places like the Amazon where most of the evaporated vapour stays in the lower troposphere and falls right back to the surface, the use of the well mixed assumption may lead to underestimation of the recycling ratio values as claimed by (Lettau et al. (1979)), who argued that in the Amazon region fast recycling can account for at least 30% of the total precipitation.

The well mixed assumption sets the limitation for the timescale of the analysis. It has to be longer than the time of boundary layer mixing (Dominguez et al. (2006)).
(Fitzmaurice (2007)) proposed a method to relax this assumption, but it introduces coefficients that are difficult to determine.

$$\frac{\bar{P_a}}{\bar{P}_m} = \frac{\overline{w_a}}{\overline{w}_m} \; or \; \frac{P_m}{P} = \frac{w_m}{w} \tag{14}$$

where a denotes advected and m of local origin.

2.3.2 *Description of the models*

The analytical box models can be classified into stationary and non-stationary models according to the treatment given to the time derivative of the precipitable water.

The stationary models neglect the change in time of the precipitable water, thus the estimates of the recycling ratio are valid only for monthly, seasonal or yearly timescales.

The non-stationary models consider the time derivative of the precipitable water, which enables the estimation of the recycling ratio at daily or smaller timescale.

2.3.2.1 *Model of Budyko and Drozdov.*

This is the first model developed for estimating the precipitation recycling. This model considers rectangular regions traversed by parallel uniform atmospheric flow. The variables for precipitation and evaporation are assumed to be constant and equal to their average values. The 1D models solve a modified version of the AWB, where all the processes are considered along a single straight streamline. The base equation of this model in the x direction is:

$$\frac{d(wu)}{dx} = E - P \tag{15}$$

In addition to the assumption of negligible change of the precipitable water and time averaged data, Budyko and Drozdov also used the assumption of a well-mixed atmosphere for average values, and defined the parameter β which is calculated as:

$$\beta = \frac{\overline{w}}{\overline{w_a}} = \frac{\bar{P}}{\bar{P_a}} \tag{16}$$

The parameter β mathematically shows the proportion of precipitation that the evaporation adds to the advected water vapour. With the application of boundary conditions to eq. (15) and the assumptions previously mentioned, an equation for the estimation of the regional recycling ratio is obtained, which is defined as:

$$R = 1 - \beta^{-1} = \frac{eL}{eL + 2Q_{ox}} \tag{17}$$

where L represents the length of the region along the stream line and e is equal to $E\Delta t/A$.

2.3.2.2 *Model of Brubaker et al (1993).:*

The model suggested by (Brubaker et al. (1993)) was a modification to the one proposed by Budyko and Drozdov. This model still treats the values of E, P_a, and P_m as constants equal to their average values, and assumes that the magnitude of the total fluxes in the region $\|\overrightarrow{Q_o}\|$, are equal to the arithmetic mean of the incoming I and outgoing moisture fluxes O, which implies the assumption of a linear variation of $\|\overrightarrow{Q_o}\|$ within the region.

(Brubaker et al. (1993)) claim that this model is 2D, because it considers the influx to a region I along 2 directions, however the process followed to arrive to an expression for the regional recycling ratio starts from the eq. (15), which only considers one direction.

The method used to calculate the influx eq. (9) allows for non-uniform moisture distribution of the moisture entering the region and can provide a correct extension of the model into 2D if the region is traversed by parallel inflow, but the boundaries are not parallel to the streamlines (Burde and Zangvil (2001)).

Moreover is incorrect to try to obtain the magnitude of a vector $\|\overrightarrow{Q_o}\|$ from the sum of scalar variables as are the influx and outflux I, O respectively.

The formula for regional recycling ratio proposed by (Brubaker et al. (1993)) is as follows:

$$R = \frac{EA}{EA + 2I} = \frac{1}{1 + \frac{2I}{EA}} \tag{18}$$

The process of estimation of the recycling ratio with the model of Brubaker et al. (1993) is the simplest, as it only requires the values of influx I, evaporation E and area of the region A.

The eq. (18) shows that in the case when $EA \gg 2I$ the regional recycling ratio tends to 1; while if $2I \gg E$ it tends to 0, and their relationship can be described with a power function.

2.3.2.3 *Model of Eltahir and Bras (1994):*

(Eltahir and Bras (1994)) introduced a model that estimates the local recycling ratio in a spatially distributed grid, for timescales of months and longer. This model no longer makes use of spatially averaged data of evaporation E and precipitation P.

Similar to the model of Brubaker et al., the model of Eltahir and Bras uses as input the influx of water vapour I and evaporation E, but performs the calculations for the local recycling ratio in each of the grid cells.

The equations of conservation of mass are derived for two different kinds of moisture according to their origin as shown in the eq. (19) and (20),

$$\frac{\partial w_i}{\partial t} = I_i + E - O_i - P_i \tag{19}$$

$$\frac{\partial w_o}{\partial t} = I_o - O_o - P_o \tag{20}$$

where I represents the flux coming into the cell and O the flux going out of the cell, the subscripts i indicate from within the region and o from outside the region. Here the definition of region includes all the area of study. Each of the fluxes I and O are the summation of the components of the flux in the two horizontal directions.

With the application of the assumptions of negligible change of precipitable water and well mixed atmosphere together with spatially distributed data, the estimation of the local recycling ratio ρ is possible. The formula for ρ is defined as:

$$\rho = \frac{I_i + EA}{I_i + EA + I_0} = \frac{I_i + EA}{I + EA} \tag{21}$$

The advantage of this model is the possibility of using values that are spatially distributed, which results in a spatial distribution of ρ, this is a more realistic approach than the model of Brubaker et al. However, it should be kept in mind that the resolution of the data should be small enough to resolve the spatial variability of the fluxes and evaporation (Eltahir and Bras (1994)).

The regional recycling ratio R can be obtained applying the eq. (3) to the results of eq. (21).

The relationship observed for the variables of eq. (21) is similar for the variables of the eq. (18), because $I_i \ll I$; but eq. (21) estimates the local recycling ratio, and the resolution of the cells can be taken only until certain extent without affecting the spatial variability.

The estimation procedure for the local recycling ratio consist of an iteration process. The area of study is divided with a grid where the values of evaporation E and precipitation P are interpolated into the nodes. The values of influx I are determined with the eq. (9), the conservation of mass is a critical component for this model, it should be noted that the outflux from one cell is the influx to the one adjacent. The local recycling ratio ρ is estimated in the middle of the nodes.

With an arbitrary, but sufficiently low, initial value of the local recycling ratio assigned for all the cells the partition of fluxes into advected and of local origin is done; with the recently estimated values of the fluxes a new value for the recycling ratio is calculated. This process is repeated for all the cells in the region until the difference between last value obtained for the local recycling ratio and the value obtained in the previous iteration is within an acceptable range.

Although this model allows the estimation of the local recycling ratio for each cell, the numerical implementation is done only for the dominant direction of the moisture flux, because as stated before, the influx is scalar and is the result of the sum of the influx in both horizontal directions, which is equivalent to say that the model of (Eltahir and Bras (1994)) is an spatially distributed model in 1D.

2.3.2.4 *Model of (Dominguez et al. (2006))*

This model is derived from the AWB in 2D, it allows the estimation of the local recycling ratio ρ for timescales of days or longer and only uses the assumption of the well mixed atmosphere.

The starting point of this model is the equation for mass conservation of water vapour that has its origin within the region as shown in eq. (22). With the use of the well mixed assumption, later eq. (22) can be written in terms of (1-ρ), as show in eq. (23)

$$\frac{\partial w_i}{\partial t} + \frac{\partial u w_i}{\partial x} + \frac{\partial v w_i}{\partial y} = E - P_i \tag{22}$$

$$w\frac{\partial(1-\rho)}{\partial t} + wu\frac{\partial(1-\rho)}{\partial x} + wv\frac{\partial(1-\rho)}{\partial y} = -E(1-\rho) \tag{23}$$

Then to facilitate the arrival to a solution for the differential equation (23) a new coordinate system is introduced:

$$\begin{gathered} \chi = x - ut \\ \xi = y - vt \\ \tau = t \end{gathered} \tag{24}$$

After substitution of the new system and the application of chain rule to eq. (23) the final expression for a modified recycling ratio is obtained:

$$\Re(\chi, \xi, \tau) = 1 - exp\left[-\int_{o}^{\tau} \frac{\varepsilon(\chi, \xi, \tau)}{\omega(\chi, \xi, \tau)} \partial\tau'\right] \tag{25}$$

where: $\Re(\chi, \xi, \tau)$, $\varepsilon(\chi, \xi, \tau)$, $\omega(\chi, \xi, \tau)$, represent $\rho(x, y, t)$, $E(x, y, t)$, $w(x, y, t)$ respectively, the value of $\Re$ can be transformed into the original ρ with the use of eqs.(24).

In order to overcome the issue of the new coordinate system, a backward trajectory scheme is used for the numerical implementation of the model. This model follows a control volume of water particles through its path until it leaves the region.

The scheme implemented is simple and computational efficient, it describes the displacement of particle in a forward trajectory between the points x^n and x^{n+1} as being the equivalent to the backward distance between from x^{n+1} to x^n, as shown in eq. (26) and (27). For more details see (Merrill et al. (1986)).

$$x^{n-1} = x^n + \left(\frac{u^n + u^{n-1}}{2}\right)\Delta t \tag{26}$$

$$y^{n-1} = y^n + \left(\frac{v^n + v^{n-1}}{2}\right)\Delta t \tag{27}$$

where Δt is negative.

The process of calculation starts when precipitation occurs and the initial position and velocity of the control volume is known. With the help of eq. (26) and (27) the position of the control volume for the previous time step is calculated, for this position in space and time the values of evaporation E, precipitable water w are stored while the corresponding velocity (u, v) is used for the determination of the previous position. These steps are repeated until the control volume leaves the region.

The local recycling ratio ρ is determined by adding the values stored E/w and applying the eq. (25), to obtain the regional recycling ratio R the eq. (3) has to be applied.

The dynamic model only picks up data of evaporation and precipitable water for each cell within the domain without considering the internal circulation (due to the influx and outflux), this means that there is no flux of moisture in between cells and each one is treated like a closed region.

The recycling ratio estimated with the dynamic model is sensitive to the size and shape of the area of study, it may be the case of a control volume that moves out of the region at a certain time step but in the following returns to the region and continues within it; if the limits of integration are considered only until the particle leaves the region, it's like that the recycling ratio is underestimated.

2.3.3 Numerical model

2.3.3.1 *Water Accounting Model (WAM)*

(Van Der Ent (2010)) proposed a model based in the 2D AWB. This model is capable of estimating the local recycling ratio ρ, regional recycling ratio R and the continental recycling ratio ρ_c for daily timescales. This model tags the water molecules so they can be tracked down to a particular origin. The AWB for the tracing of sources is defined as follows:

$$\frac{\partial w_\Omega}{\partial t} + \frac{\partial u w_\Omega}{\partial x} + \frac{\partial v w_\Omega}{\partial y} = E_\Omega - P_\Omega \tag{28a}$$

or

$$\frac{d w_\Omega}{dt} + O_{\Omega_{x,y,t}} - I_{\Omega_{x,y,t}} = E_\Omega - P_\Omega \tag{28b}$$

where the subscript Ω denotes the area of origin of the water vapour and $O_{\Omega_{x,y,t}}$ is the out flux with origin Ω for a specific coordinate *(x, y)* and time *t.*

The numerical implementation of the WAM may be done with a Forward in Time Central in Space (FTCS) scheme if the eq. (28a) is discretized, or with explicit Euler if the Green's theorem is applied which is the case of the eq. (28b). In any case the unknown variable is the precipitable water in time step t+1. The WAM uses the assumption of a well-mixed atmosphere.

The regional recycling ratio is estimated for a specific area and period of time with the following equation:

$$R = \frac{\sum_{t=1}^{n}\left(\sum_{i=1}^{m} P_i \frac{w_{\mathrm{i}\Omega}}{w_i}\right)}{\sum_{t=1}^{n}\left(\sum_{i=1}^{m} P_i\right)} \tag{29}$$

where:

$n =$ is the superior limit of the integration period,

$m =$ is the number of cells in the domain.

CHAPTER 3

3.1 Data and region of study

The data used for the calculation of the recycling ratios was downloaded from the ERA-Interim daily data server of the European Centre for Medium-Range Weather Forecasts (ECMWF), for a region that lays between 47°N - 34.5°N and 99°W – 87°W, as shown in Figure 1, for the period of 2004 to 2014.

The variables used are presented in the Table 1:

Table 1. Variables used in the models

Variable	Units
Vertical integral of eastward water vapour flux	kg/(ms)
Vertical integral of northward water vapour flux	kg/(ms)
Vertical integral of water vapour	kg/m^2
3 hourly evaporation depth	m
3 hourly precipitation	m

The vertical integral of the eastward, northward water vapour flux and the water vapour are measured every six hours daily, while the evaporation and precipitation are forecasted and accumulated values every three hours two times a day.

The use of the regional recycling ratio has raised some criticism from (Van Der Ent (2010)), (Dirmeyer and Brubaker (2007)), mostly due to the issues listed in section 3.3. However, it should not be discarded as a useful index that is able to provide information about the land atmosphere interaction.

The regional recycling ratio gains importance when it comes to studies of regions that have special economical interest (Usda (2012)) or ecological interest (Dominguez and Kumar (2008)).

The area of the present study covers the upper Mississippi river sub basin, and a portion of the Great Plains (size $1.363*10^6$ Km^2)

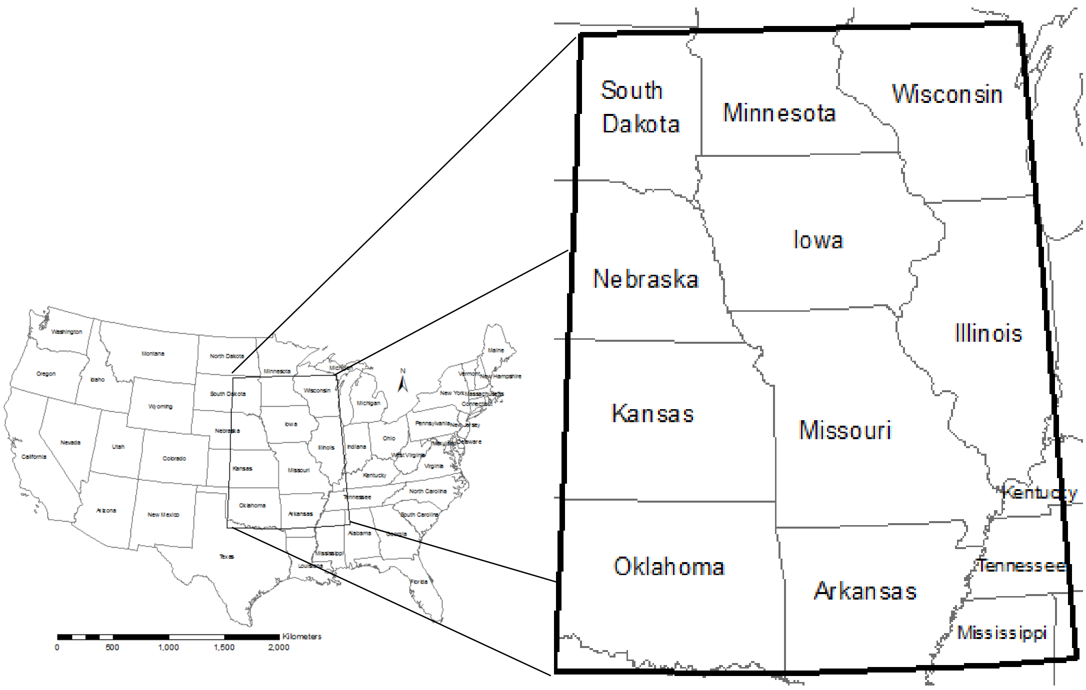


Figure 1. Location of the area of study

About 52% of the area is used as cropland, forestland accounts for 20%, permanent pasture and hayland represent 9%, urban areas make up about 8% and the rest (11%) is classified as rangeland, wetlands, and barren land. The upper Mississippi River Basin is an important economic and agricultural region, its sales in 2007 were reported to be about $44 billion of which 57% came from crops, being corn and soybeans the principal ones, according to the 2007 Census of Agriculture. Irrigation is not common in the region and only about 2 percent of the harvested area was irrigated in 2007.

The region of study has a continental climate, with a dominant westerly moisture flux during summer, and in northeast direction during spring and autumn as shown in Figure 2.

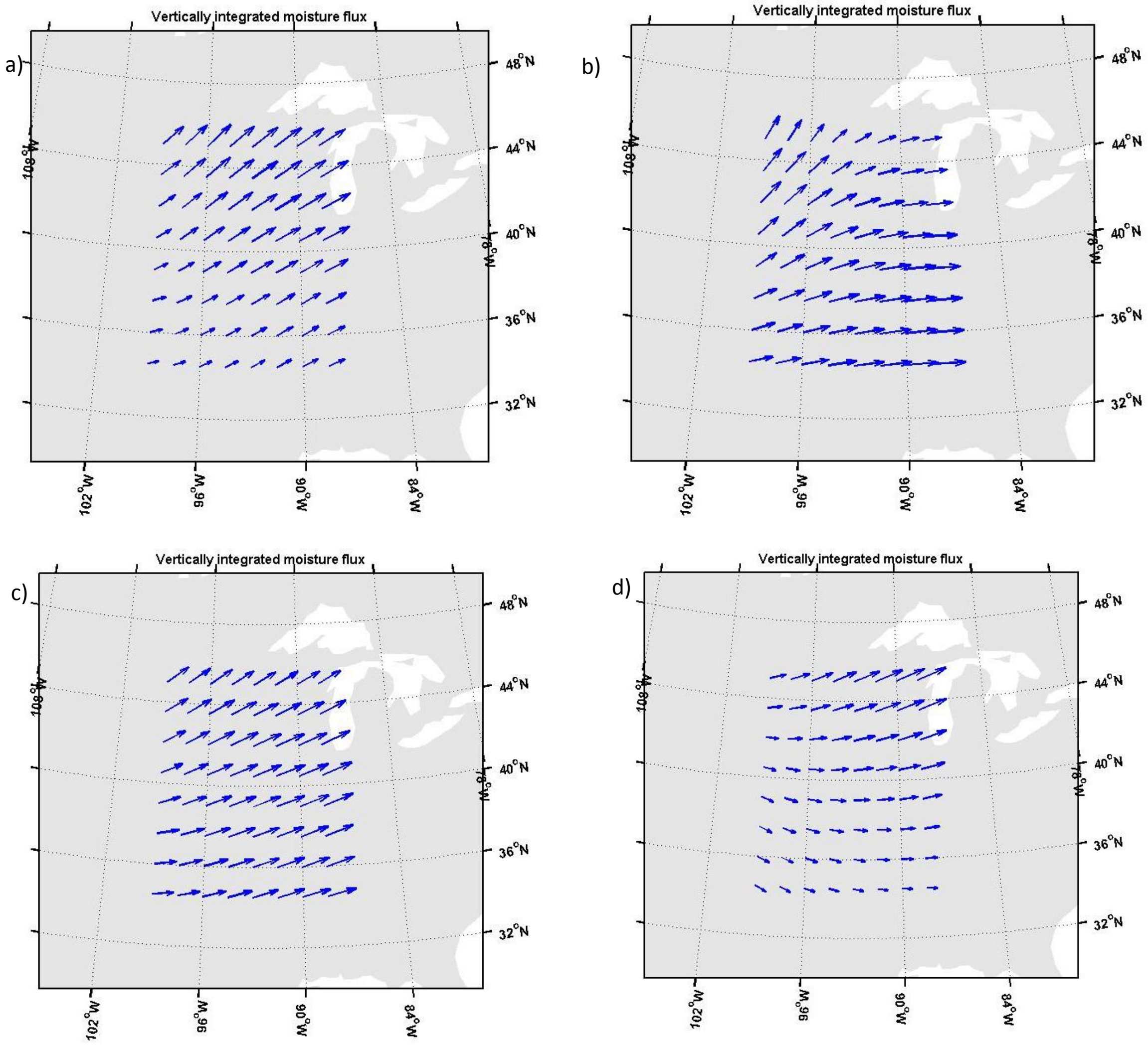


Figure 2.Vertically integrated moisture flux for a) spring b) summer c) autumn d) winter

The seasonal variations of fluxes are presented in the Figure 3, it can be seen that the influx and outflux have a very similar trend with maximums in June (summer) and minimums during February (winter). The monthly mean influx is larger than the outflux from January to May and from October to December, this is reflected in the divergence of the vertically integrated moisture flux, divergence (D) from now on, which shows a positive net influx of moisture for these periods.

The divergence shows a peak in July (summer) and minimum in December (winter). When the lowest influx and outflux occur in February the difference between both values is not large enough to create a minimum D. The mean monthly evaporation depth (E) describes the shape of a bell curve with the maximum occurring in July (summer), the minimum in December (winter). The mean monthly precipitation (P) in the area describes two peaks, the first one in April and the second one on October, which are months when the moisture flux is converging to the region, hence the rainfall events with the highest amount of precipitation occur when the atmosphere has a convergent character.

Precipitation increases when the moisture flux has a convergent character (January to April and August to October), while it decreases when the moisture flux is divergent (April to August). The average annual evaporation for the period of study was 803.3[mm], while the mean annual precipitation was 764.7[mm].The curve of E –P shows that from May to September the region is a source of moisture, while from October to April it's a sink. Comparing E, P and D the pattern/behaviour described by both is similar, the only difference occurs for May because according to the divergence the moisture is converging while according to E-P is diverging.

The divergence is maximum in July when the out flux is maximum and the difference with the influx is the largest; Figure 3 c) and d) suggest that the surplus of moisture diverged comes from the evaporation which is maximum for that month.

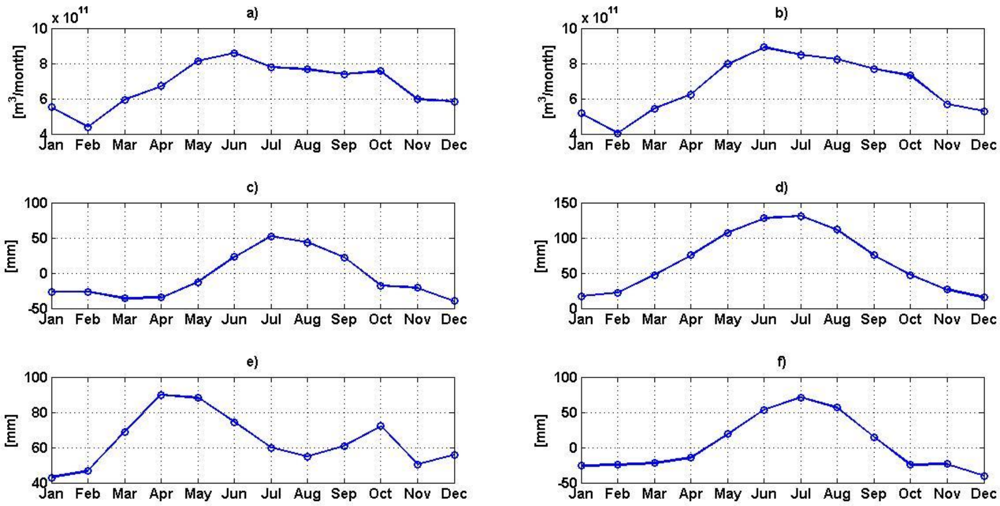


Figure 3. Seasonal variation of a)Influx b)Out flux c)Divergence d)Monthly evaporation depth e)Monthly precipitation depth f)Monthly evaporation depth – monthly precipitation depth

Every satellite data have uncertainties associated to their measurements or forecasts, and the data from ERA-Interim is not an exception. A good way to assess the uncertainty of the data is by calculating the atmospheric water balance for the region, the results and data of such calculation are shown in the Table 2. The term referred as alpha (α) in the last column is the closure or residual term.

Table 2.Average atmospheric water balance for the region of study and the period of 2004-2014

	dW/dt [mm/month]	E [mm/month]	P [mm/month]	D [mm/month]	alpha [mm/month]
Jan	**-0.91**	**16.84**	**42.97**	**-26.69**	**-1.46**
Feb	**-0.18**	**22.25**	**46.65**	**-24.13**	**0.09**
Mar	**3.34**	**46.89**	**68.81**	**-32.86**	**-7.60**
Apr	**2.85**	**75.26**	**89.90**	**-32.07**	**-14.58**
May	**6.43**	**107.13**	**88.23**	**-11.57**	**-24.05**
Jun	**6.56**	**127.72**	**74.32**	**20.91**	**-25.94**
Jul	**3.64**	**130.78**	**59.78**	**47.59**	**-19.77**
Aug	**-0.59**	**111.35**	**54.89**	**39.63**	**-17.42**
Sep	**-6.46**	**75.51**	**60.83**	**20.11**	**-1.04**
Oct	**-7.12**	**47.44**	**72.04**	**-16.05**	**1.43**
Nov	**-5.30**	**26.68**	**50.43**	**-18.98**	**-0.53**
Dec	**-2.30**	**15.49**	**55.86**	**-36.42**	**1.65**
Total	**-0.03**	**803.35**	**764.70**	**-70.53**	**-109.21**

The values of alpha (α) demonstrate that the data from ERA-Interim have errors (see Figure 4), when compared to the magnitude of the rest of the fluxes, α is larger than the divergence especially for the months of May and June. This may lead equivocally to the conclusion that the divergence is underestimated for the region. But if a comparison of all the data is made one can observe that the precipitation is the variable that is being underestimated in ERA-Interim for the months of May, April and September when the mean monthly precipitation exceeds 100[mm/month] according to (Sutton (1933)). (Dominguez et al. (2006)) and (Van Der Ent (2010)) attributed the errors in closure of the water balance to errors in the forecasting of the values of evaporation and precipitation.

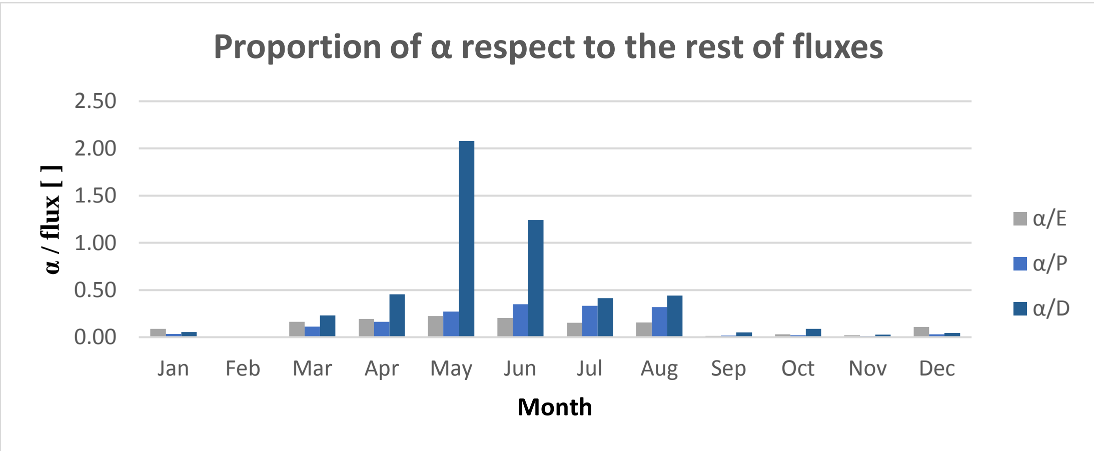


Figure 4. Proportion of closure term with respect to the other fluxes

The inter-annual variation of the fluxes is presented in the Figure 5; the blue line correspond to the annual value, the green one to the mean plus the standard deviation, and the red line to the mean minus the standard deviation.

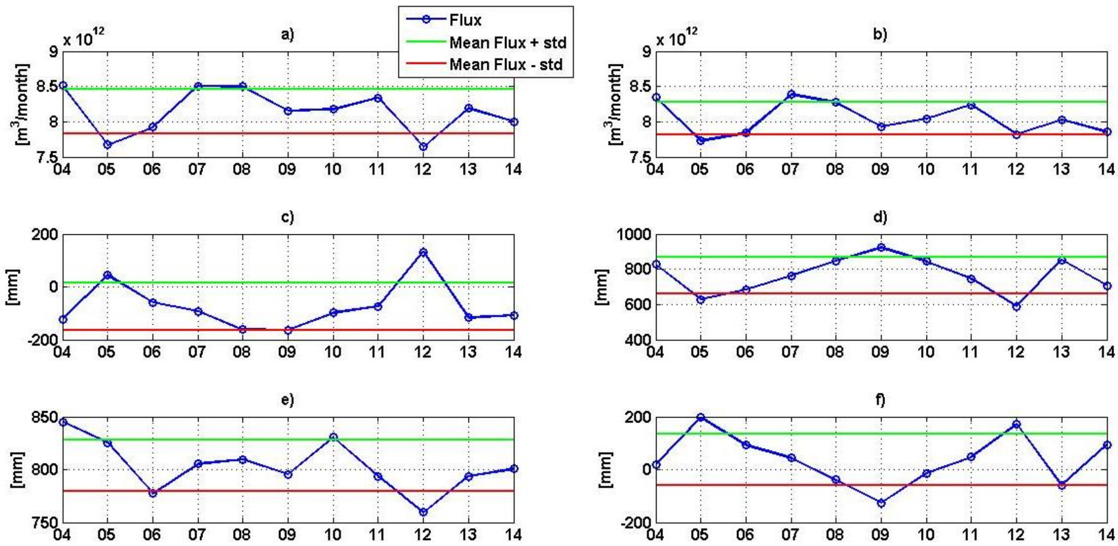


Figure 5. Inter-annual variability of the fluxes a)Influx b)Out flux c)Divergence d)annual evaporation depth e)annual precipitation depth f)Evaporation-Precipitation

The influx and outflux show a similar trend as expected, similar as they did in Figure 3 a) and b), the minimums occur in 2005 and in 2012 and a maximum in 2007. The mean annual influx is larger than the mean annual outflux, which explains that the majority (9 out of 11) of the years the net flux was positive. The divergence is positive in 2005 and 2012, when the moisture flux and the evaporation are minimum. The precipitation is minimum in 2006 and 2012, and maximum in 2010.

The observed relationships show that when a year has low evaporation the outflux is larger than the influx, which means that the moisture is diverging from the region, and the precipitation is low too.

The trend described by the E-P Figure 5f) is similar to the trend of the divergence with minimum in 2009 and maximum in 2005 and 2012, however according to the values the region was a net source of moisture (E>P) for seven years, this differs from the divergence Figure 5c) which shows that for nine years the moisture was converging.

The dynamic model and WAM require data with smaller temporal resolution, due to spatial constrains the daily data of 2011, for which the fluxes are within the range of +/- one standard deviation, are presented in the Figure 6.

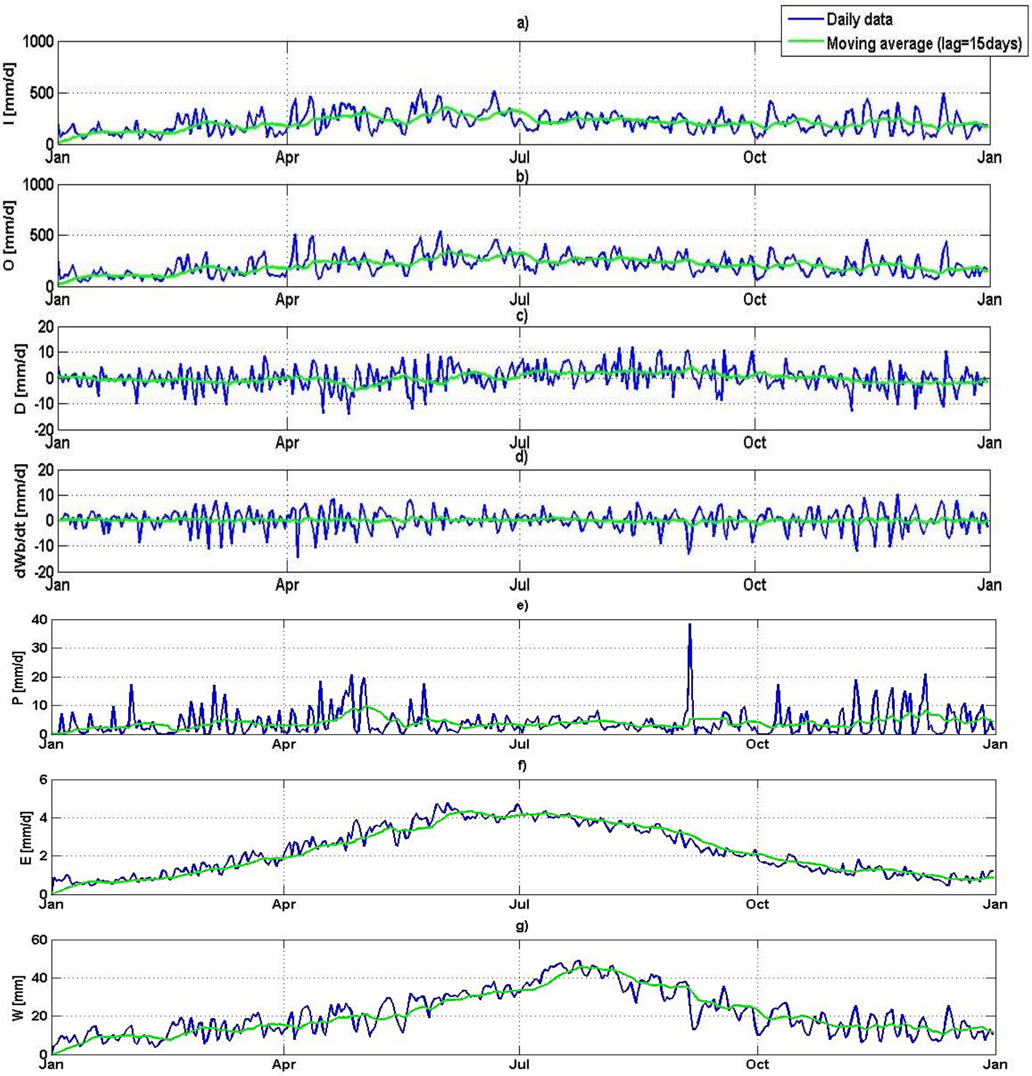


Figure 6. Daily regional fluxes of 2005 a) influx, b) outflux, c) divergence, d) daily variation of precipitable water dW/dt, e) precipitation depth, f) evaporation depth, g) precipitable water

In order to have all the fluxes in the same units to facilitate comparison, the influx and outflux were divided by the lateral area of the region, it was assumed that the mean altitude was 500 m.a.m.s.l. and the height of the atmosphere 24 Km. The influx and outflux have the same trend, they are two orders of magnitude larger than evaporation, and one order of magnitude larger than the precipitable water.

The divergence and the change of precipitable water seem to have their peaks occurring in the same time. From January to approximately April when the moisture transport is convergent the relative moisture storage decreases, from May to September when the moisture flux is diverging there is a relative increase of the precipitable water, from October to December the relationship is the same as the one observed from January to April. The months when these two kinds of relationships are observed coincide with the months when the moisture transport is convergent (January-April and October-December) and divergent (May-September) as showed in the Figure 3.

The data plotted in the Figure 6e) is the mean precipitation in the cells of the region where the precipitation is larger than zero. It shows that when a precipitation event occurs, it’s accompanied with a relative decrease of the precipitable water. The magnitude of the precipitation is larger than the decrease of the precipitable water especially during June, July and August.

The daily evaporation reaches its peak values (E ≈ 4mm/d) in July; the precipitable water has the shape of a negative skewed bell curve and is maximum in the end of July (W ≈ 40mm), when the precipitation is low. Two rainy seasons can be observed, the first one occurs during spring and the second one in November and December.

The values of the recycling ratios obtained after the application of the models described above will be presented in the following sections. Given the characteristics and limitations of each model, the results will be grouped into two sections, one for the models of Brubaker and Eltahir which are valid on a monthly timescale and another one for the models of Dominguez and WAM.

3.1.1 Results from the model of Brubaker et al. and Eltahir and Bras.

The results of the monthly recycling ratios are presented in the Figure 7, it can be seen that they follow closely the trend of the evaporation. The values for the months of September to February from the model of Eltahir are larger, while the contrary occurs for the rest of the months.

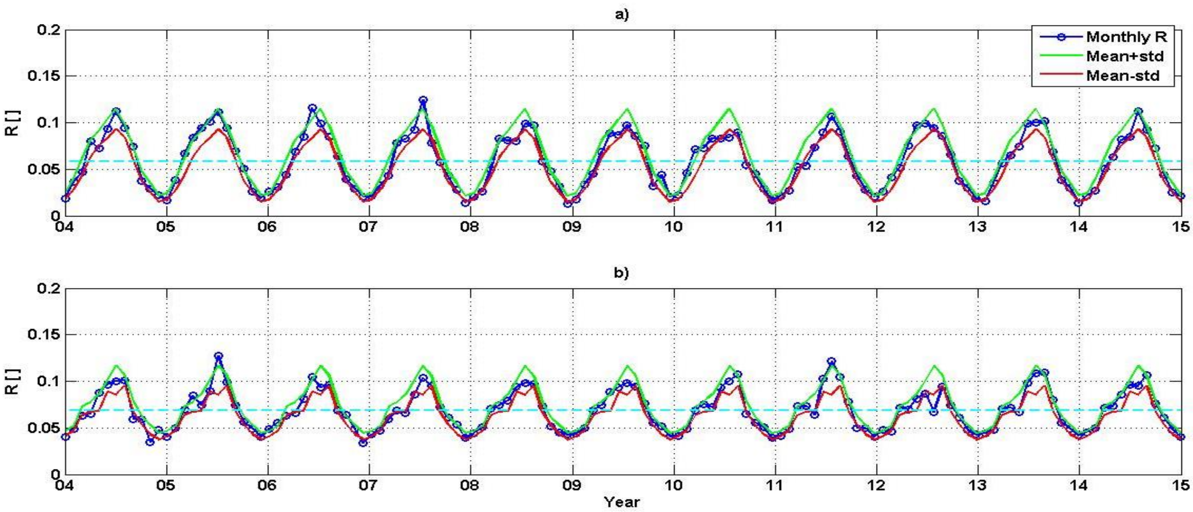


Figure 7. Monthly regional recycling ratio for the period 2004-2014 a)Brubaker et al. b) Eltahir and Bras

The long term average R for the case of Eltahir is estimated to be 6.8% and for the case of Brubaker 5.8%.

The Figure 7 show that for dry years i.e. 2005 and 2012 with ($E_{annual} < (E_{mean} - \sigma_E)$) the recycling ratio is larger than the average while the precipitation recycled is small (see Figure 8).

The largest precipitation recycled are estimated in 2010, see Figure 8, which has large evaporation ($E_{annual} > (E_{mean} + \sigma_E)$) and also large precipitation($P_{annual} > (P_{mean} + \sigma_P)$).

Although 2009 has the largest evaporation the results of the models are not the highest, because the precipitation is relatively low and the divergence is maximum.

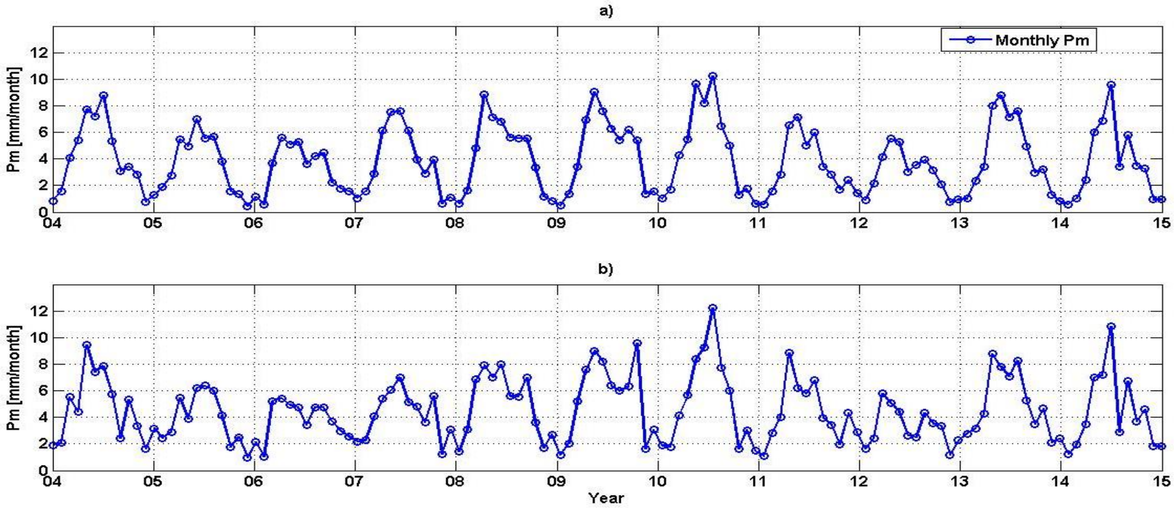


Figure 8. Monthly precipitation of local origin (Pm) from a) Brubaker et al. b) Eltahir and Bras

The precipitation of local origin (Pm) which is the result of the product between P and R, reflect a time series very similar to the R, hence to the E. As expected the minimums of Pm are larger when calculated with R from the model of Eltahir.

3.1.2 Results from the model of Dominguez et al. (Dynamic) and WAM

The dynamic model and WAM were applied to estimate the daily recycling ratio, the data for the dynamic model had 2.5°x2.5° spatial resolution and 6 hours temporal resolution, just as (Dominguez et al. (2006)) who used 6 hourly data from the R-II project to estimate the recycling ratio over the contiguous U.S. Due to the resolution of the data the region analysed with the dynamic model expands from 47°N - 34.5°S and 99°W – 87°W.

For the WAM the data used had 1.5°x1.5° spatial resolution and 6 hours temporal scale which was interpolated to 30 minutes, just as (Van Der Ent (2010)).

The accuracy of the results of any model are subjected to the accuracy of the data and the calculation process, and the science behind it. The dynamic model and WAM were developed with solid and widely accepted scientific concepts, which means that the concern of the accuracy of the results for the present project depends only in the accuracy of the data and calculation.

The accuracy of the calculation process is especially important in a complex model as is the WAM, it requires the estimation at a cell scale of the influx, outflux and water balance. Once the values of influx and outflux are obtained the divergence can be calculated. To evaluate the accuracy of the calculation process the calculated divergence is compared with the divergence downloaded from ERA interim after interpolating it to 30 minutes, as shown in Figure 9. It can be seen that there is a good agreement in the trend, until approximately the step $1.7x10^5$, when the downloaded data seem to have a systematic error that overestimates the values. Figure 9 demonstrates that the accuracy of the calculation method is acceptable, the difference between both variables may be attributed to inaccuracies in the distances used in the calculation (projection used). The discrepancy between the downloaded and the estimated values start in October 2013.

Another source of errors in the calculation process in the WAM is the method of interpolation of precipitation. If the precipitation data is continuously interpolated, the interpolated data may end up with a larger amount of precipitation than the original one and with a bigger number of days with rainfall. This happens due to the sharp decrease typical of the data i.e. in two contiguous days of which the first one has forecasted precipitation at 24:00h and the second doesn't have precipitation, if the precipitation is linearly interpolated the second day will have precipitation >0 during the first hours of the day. In order to preserve the trend of the data the precipitation interpolated has sharp changes.

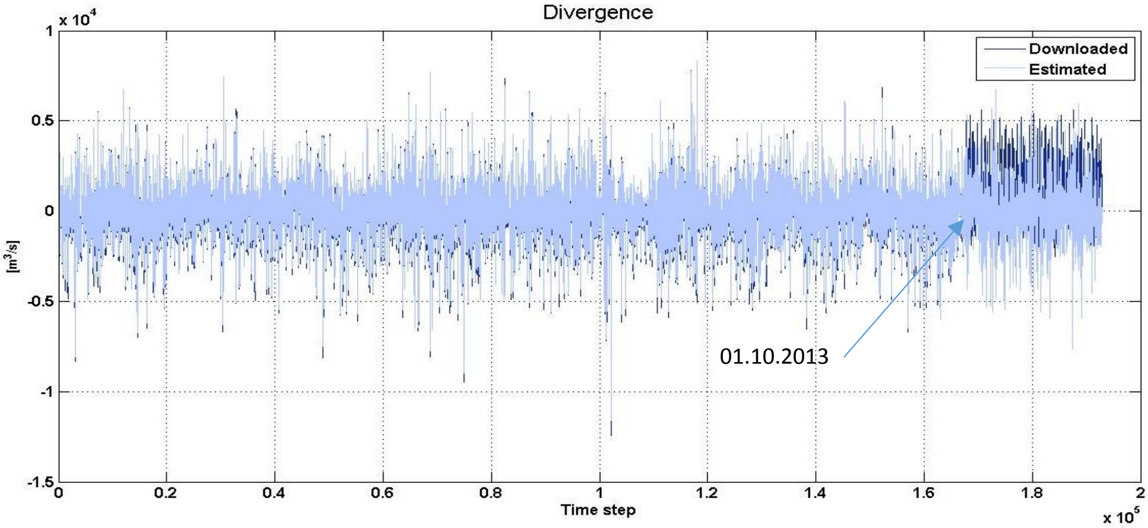


Figure 9. Estimated and downloaded divergence after interpolation

The accuracy of the data is evaluated with the closure term α of the water balance of the region, although this does not guarantee the accuracy of the data it may shed light in the size of the error. Figure 10 shows the variation and magnitude of the closure term for the time series. The pattern of α in the negative part has the same trend as the evaporation, and in the positive side has the trend of precipitation. According to (Dominguez et al. (2006)) the data of precipitation and evaporation from the ECMWF are variables type C that depend completely on the model physics. The term α was calculated with the precipitable water from ERA Interim and divergence calculated from the vertical integral of the moisture fluxes in north and east directions.

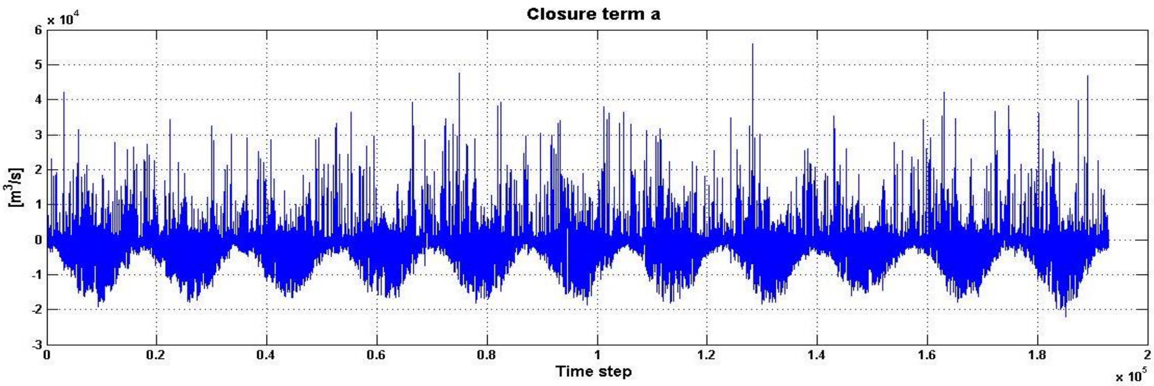


Figure 10. Closure term α of the water balance

Figure 11 shows, in light blue, the daily recycling ratio estimated with the dynamic model, and in blue its moving average for a time period of 15 days. The recycling ratio describes a behaviour similar to the evaporation and precipitable water, with minimums during the winter months and maximums during spring or summer. Although the results from Brubaker et al. and Eltahir's models are not in the same time scale of the results from the dynamic model it can be seen that the peaks reached by the latter are higher.

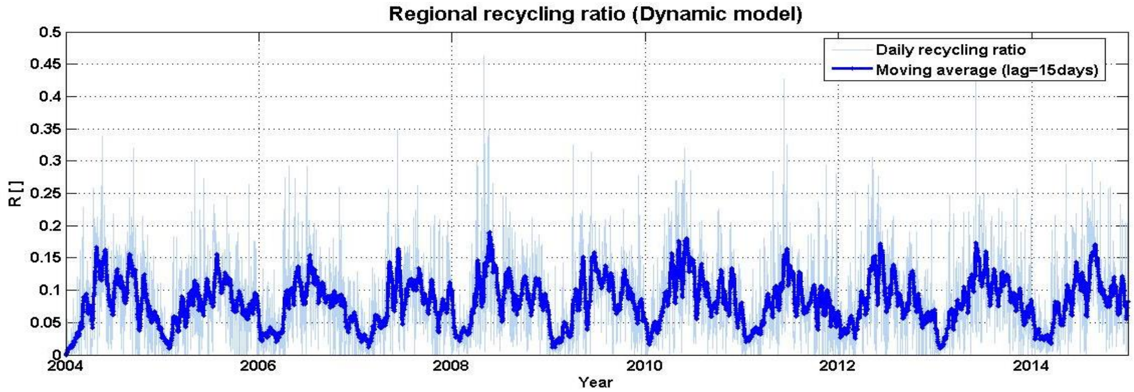


Figure 11. Daily regional recycling ratio obtained with the dynamic model

The estimation of the recycled precipitation is subjected to the averaging method of the regional precipitation. Here two possible cases are identified: the *actual* and *theoretical* recycled precipitation.

The *actual* and *theoretical* mean daily recycled precipitation are shown in the Figure 12, it's called actual because it's the recycled precipitation that is actually produced over the region where the rainfall happened.

For the actual recycled precipitation the regional precipitation is calculated as the average of precipitation of the cells with P>0 [mm], while the theoretical recycled precipitation uses the average of the precipitation in all cells of the region, even when precipitation has not occurred.

Because the concept of the estimate of the actual recycled precipitation is closer to the reality, this is only one that will be used in the hypothesis testing, and presented for the results obtained from the WAM.

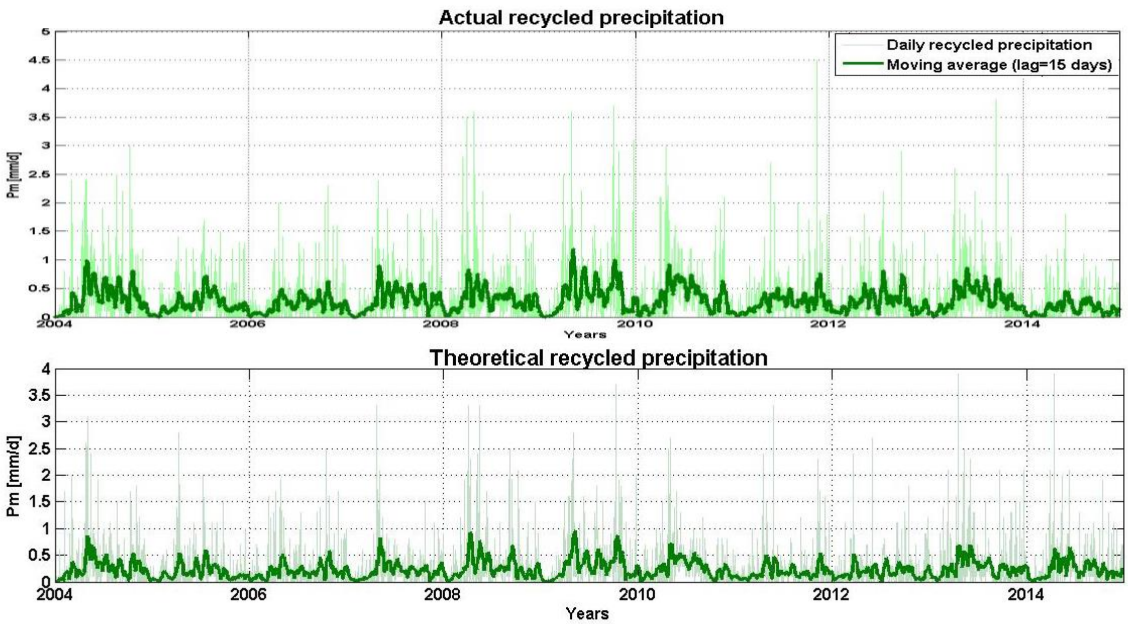


Figure 12. Mean actual and theoretical recycled precipitation

In order to be consistent with the calculation procedure of the dynamic model (each cell is a closed region) the WAM also calculates the recycling ratio for each as a closed region according to the eq. (29) and the origin of the precipitable water is calculated as shown in eq. (30)

$$w_{\Omega_{x,y}}^{n+1} = w_{\Omega_{x,y}}^{n} + E_{\Omega_{x,y}}^{n} - P_{\Omega_{x,y}}^{n} \frac{w_{\Omega_{x,y}}^{n}}{w_{x,y}} - O_{\Omega_{x,y}}^{n} \frac{w_{\Omega_{x,y}}^{n}}{w_{x,y}} + \alpha_{\Omega_{x,y}}^{n} \frac{w_{\Omega_{x,y}}^{n}}{w_{x,y}} \tag{30}$$

where the superscript n indicates the step in time and α is the closure term due to the errors associated with E and P.

The recycling ratio obtained with the WAM is presented in Figure 13, the trend described by the moving average is consistent with the one described by the estimates obtained with the dynamic model.

A comparison of the value of the estimates from these two models is not useful, as the data used in the WAM for the estimation of the local recycling ratio includes the precipitation, evaporation and alfa, while for the dynamic model it's required only the evaporation.

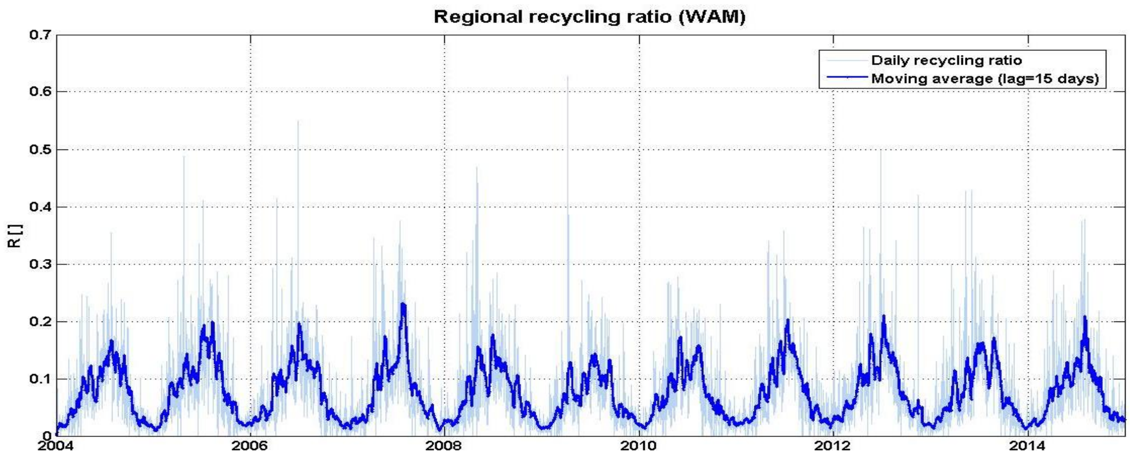


Figure 13. Daily regional recycling ratio obtained with the WAM

The actual recycled precipitation follows the trend of the recycling ratio, with maximum estimates occurring during spring or summer and the minimums during winter. The peaks reached with the estimates from the WAM are smaller when compared to the estimates from the dynamic model.

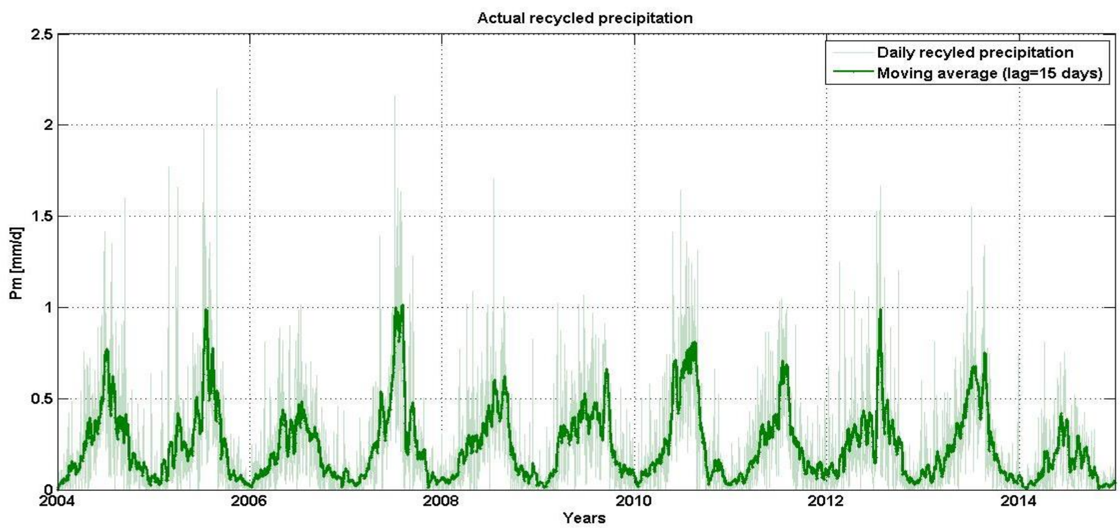


Figure 14. Mean actual recycled precipitation

3.2 Hypothesis testing

Similar to the previous section the hypothesis will be tested in two groups, one for the models that are applicable at a monthly timescale, and one for the models that can be applied at a daily timescale.

Some studies have correlated the recycling ratio to several parameters from fluxes to solar radiation and temperature (Zangvil et al. (2004)) or sensible heat and soil moisture (Dominguez and Kumar (2008)); the final product of those researches is the correlation factors between the recycling ratio and the variables. Even when a relatively good value for the correlation coefficient between sensible heat and the recycling ratio was found the present study will only make use of fluxes, because they all have similar or equivalent units, and according to (Goessling and Reick (2011)) the influence that evaporation exerts on precipitation over a sufficiently large region occurs over the atmosphere's moisture budget. Therefore one may think that the findings in (Dominguez and Kumar (2008)) may not only reflect the interaction caused by the recycling ratio but also due to local coupling.

3.2.1 Models of Brubaker et al. and Eltahir and Bras

The approach used for the estimation of the Pm, is based on the use of linear correlations between the variables of the models like I, E, P, O, other fluxes like D, E-P and the R, together with the basic definition of the recycling ratio as defined in the eq. (2) and eq.(3).

In Table 3 and Table 4 it can be seen that the recycling ratio has the largest coefficient of determination when is correlated to the evaporation, for the model of Brubaker, R^2=0.91 and for the model of Eltahir, R^2=0.82.

	R	I [m³/month]	O [m³/month]	D [mm/month]	E [mm/month]	P [mm/month]	Pm [mm/month]	E- P [mm/month]
R	1.00	0.24	0.42	0.49	0.91	0.03	0.62	0.71
I	0.24	1.00	0.92	0.05	0.50	0.38	0.46	0.11
O	0.42	0.92	1.00	0.24	0.68	0.18	0.45	0.32
D	0.49	0.05	0.24	1.00	0.42	0.19	0.05	0.85
E	0.91	0.50	0.68	0.42	1.00	0.10	0.71	0.66
P	0.03	0.38	0.18	0.19	0.10	1.00	0.49	0.09
Pm	0.62	0.46	0.45	0.05	0.71	0.49	1.00	0.17
E- P	0.71	0.11	0.32	0.85	0.66	0.09	0.17	1.00

Table 3. Coefficients of determination R^2 for linear correlations of the variables and the fluxes for the model of Brubaker

	R	I [m³/month]	O [m³/month]	D [mm/month]	E [mm/month]	P [mm/month]	Pm [mm/month]	E- P [mm/month]
R	1.00	0.22	0.39	0.49	0.82	0.01	0.27	0.72
I	0.22	1.00	0.92	0.05	0.50	0.38	0.52	0.11
O	0.39	0.92	1.00	0.24	0.68	0.18	0.39	0.32
D	0.49	0.05	0.24	1.00	0.42	0.19	0.00	0.85
E	0.82	0.50	0.68	0.42	1.00	0.10	0.44	0.66
P	0.01	0.38	0.18	0.19	0.10	1.00	0.78	0.09
Pm	0.27	0.52	0.39	0.00	0.44	0.78	1.00	0.02
E- P	0.72	0.11	0.32	0.85	0.66	0.09	0.02	1.00

Table 4.Coefficients of determination R^2 for linear correlations of the variables and fluxes for the model of Eltahir

Although the influx of moisture is used in both models and its order of magnitude is larger or equal to the order of magnitude of evaporation in the region, the correlation with the recycling ratio is small R^2=0.24 and R^2=0.22 respectively.

As the best correlation observed occurs with the evaporation, a curve is fitted between the evaporation and the recycling ratio. Figure 15a) shows the corresponding points as dots and the fitted curve in red, it can be seen that the spread of the points is relatively small with outliers in the range from 20 mm to 55 mm, and in the range from 115 mm to 140 mm. The first group of outliers have a corresponding recycling ratio lager than the one predicted with the fitted curve, they occur in February 2010 and March 2005, 2010, 2013, when the evaporation is larger than the mean, or in some cases the mean plus the standard deviation of that month among years and the precipitation is smaller than the mean or mean minus standard deviation.

The second group of the outliers are overestimated with the fitted equations, they occur in August 2004, June 2008, 2010, and 2014 and July 2010. These data are characterized for having the influx of moisture larger than the mean plus the standard deviation of that month among years. In some cases the evaporation is also larger than the mean but the magnitude of the influx is one order of magnitude larger than the mean values.

a)

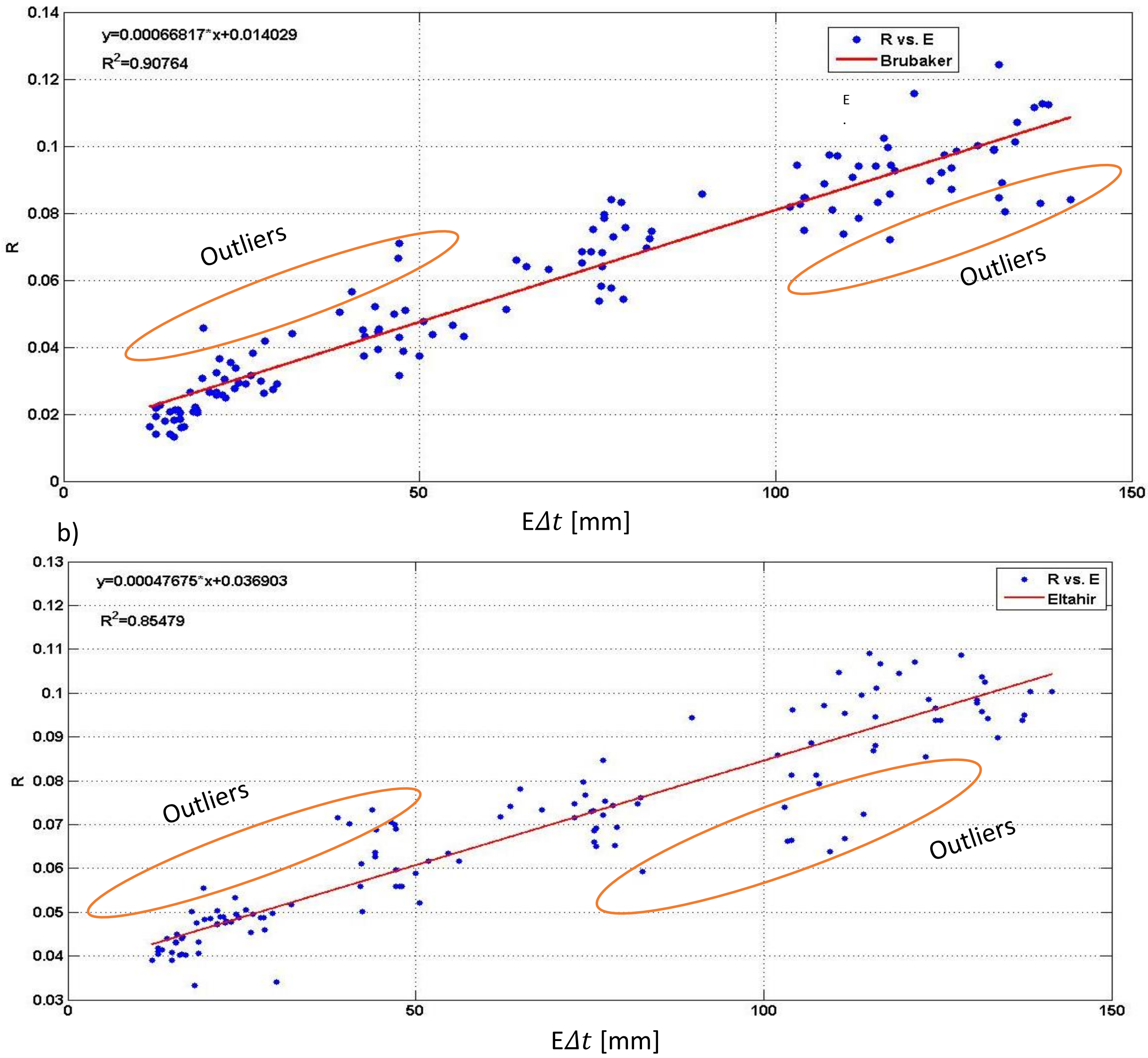


Figure 15. Curve fitting of the recycling ratio and evaporation for the model of a) Brubaker et al. and b) Eltahir and Bras

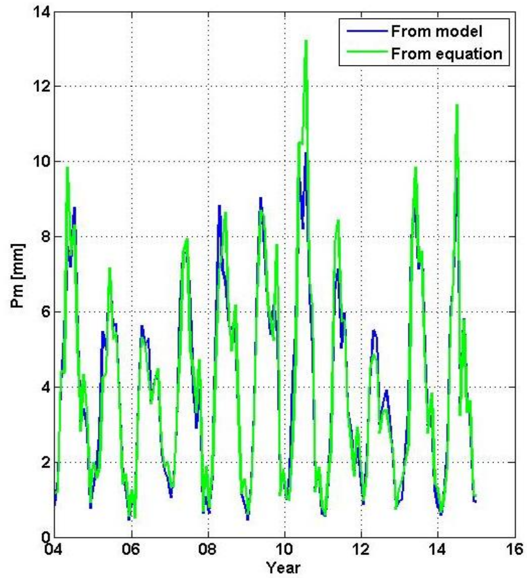
For the case of the model of Eltahir the outliers occur in the same months as with the model of Brubaker.

The equations of the fitted curves for each of the models are presented in the Table 5. The coefficient of the evaporation has dimensions of length [mm], the second term is dimensionless. Both equations reflect what was previously stated, the minimum values of the recycled precipitation are larger with the equation of Eltahir; and the maximums are larger with equation of Brubaker.

Model	Equation
Brubaker	R=0.00066817*E+0.014029
Eltahir	R=0.00047675*E+0.036903

Table 5. Equations of the fitting curves for the recycling ratio and evaporation

Given the characteristics of the recycling ratio the applicability of the equations in Table 5 is dictated for the limits of the data for which it was obtained, and for months when the fluxes are within the range of the mean plus standard deviation or mean minus standard deviation



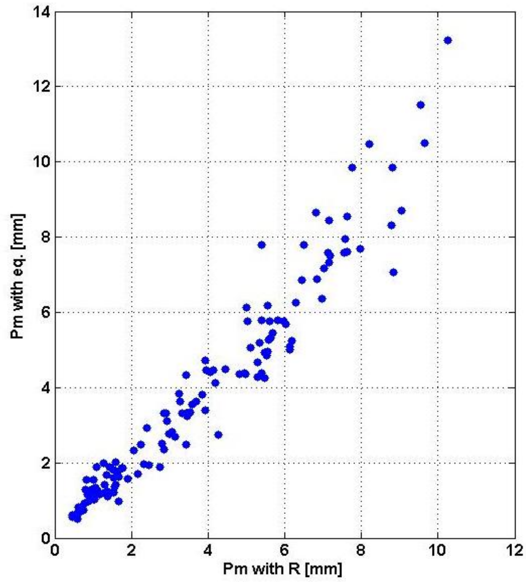
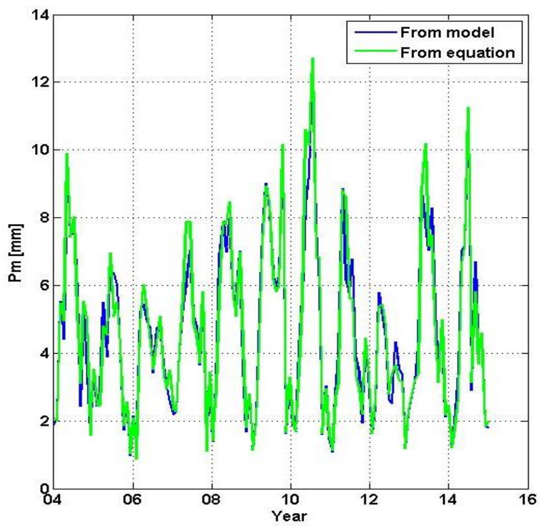


Figure 16. Recycled precipitation with the equation and with the model of Brubaker



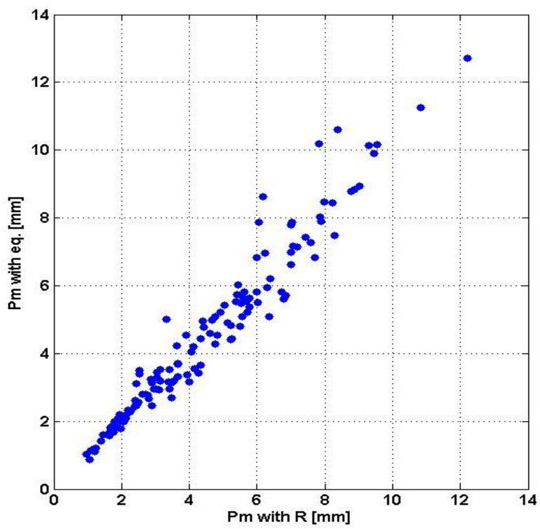


Figure 17. Recycled precipitation with the equation and with the model of Eltahir

Figure 16 and Figure 17 show the time series of the monthly recycled precipitation, the outliers are the same points that were mentioned before.

3.2.2 Dynamic model and WAM

The first attempt to test the hypothesis with the results from the dynamic model and the WAM is done by grouping data according to the amount of mean precipitation as in (Zangvil et al. (2004)).

Besides the fluxes used in the previous section, additional data from ERA-Interim, such as convective precipitation and large scale precipitation, may be used to test the hypothesis; for the present, these fluxes won't be taken into account because they are also type C variables and it's very likely that they also contain errors. Hence, the use of these dataset will make the results of the hypothesis tests even more dependent on the data used.

Because the precipitable water is used in the estimation of the recycling ratio in both of the models, this variable, will be used to test the hypothesis too.

The data is divided into 7 groups with the classification given in Table 6.

Group	Amount of precipitation [mm/day]
1	0 – 0.5
2	0.5 – 1
3	1 – 1.5
4	1.5 – 2.0
5	2.0 – 4.0
6	4.0 – 6.0
7	> 6

Table 6. Ranges for precipitation height

According to the Figure 18 none of the variables reflect direct relationship with the daily recycling ratio when there's no lag of the variables, this means that the recycling ratio of the 5th of August is compared with the estimated fluxes of the 5th of August.

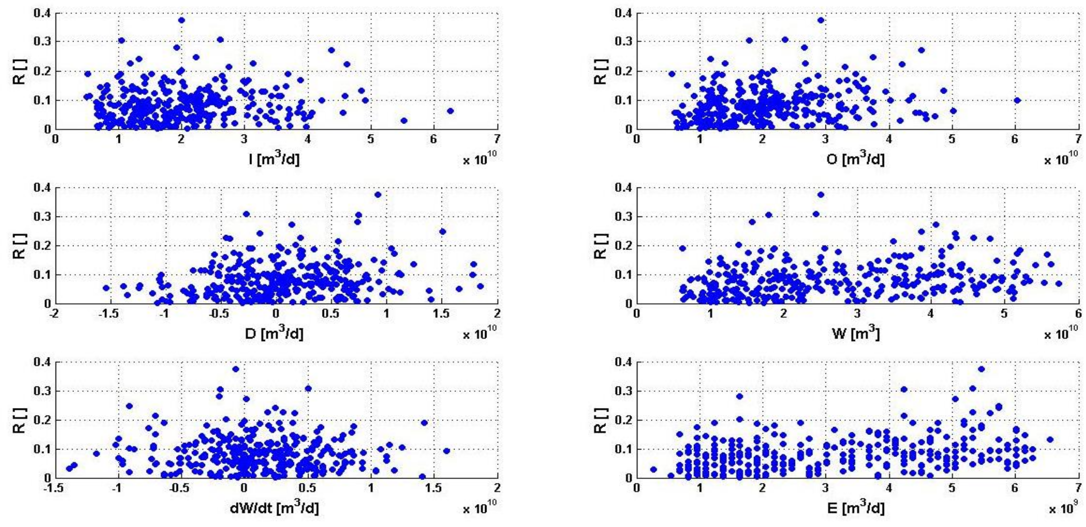


Figure 18. Scatter plot of the daily regional recycling ratio for the group 3

The hypothesis was also tested for combination of the variables like:

- $\frac{E}{W}$ because it resembles the term used in the dynamic mode
- $\frac{E}{I}$ because it's a proportion of the flux locally generated with respect to the advected

- $\frac{E}{I-dW/dt}$ because the evaporation is the driver of moisture recycling, the influx represents the advected moisture and the change in precipitable water accounts for moisture lost
- $\frac{E}{W}+\frac{E}{I-dW/dt}$ as the addition of previous combinations

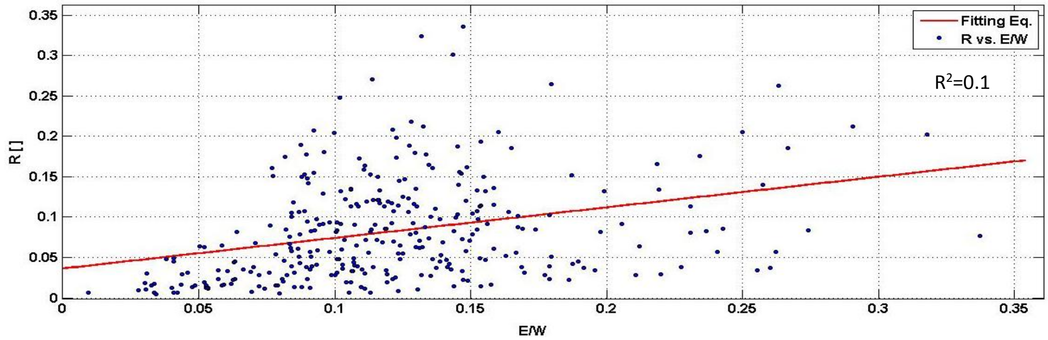


Figure 19. R vs. E/W for group 3

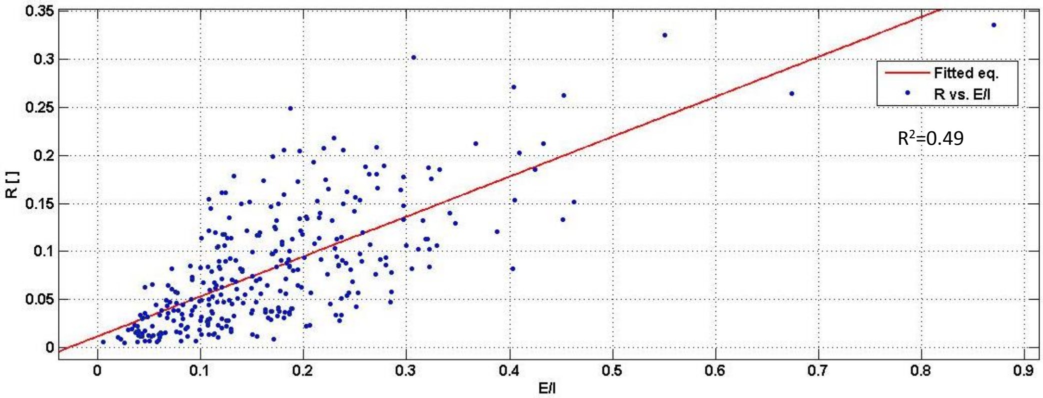


Figure 20. R vs. E/I for group 3

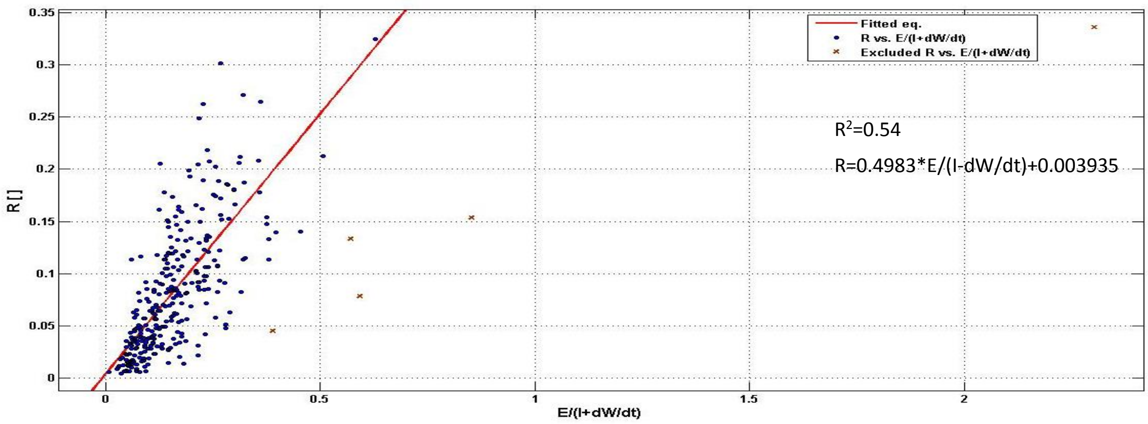


Figure 21.R vs. E/(I-dW/dt) for group 3

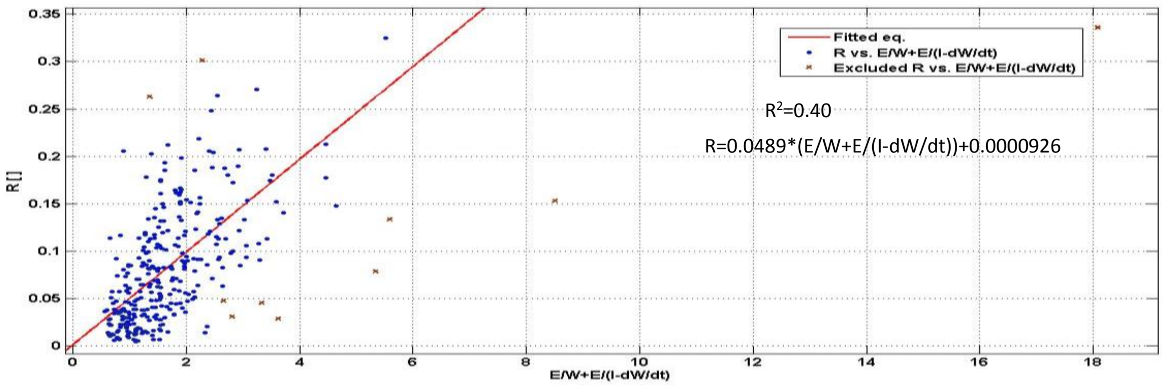


Figure 22. R vs. E/W +E/(I-dW/dt) for group 3

The best relationship between the variables and the recycling ratio was found to be for the one in Figure 21, because when a linear equation is fitted to the scattered points R^2 is 0.54.

As the dynamic model integrates the values of the evaporation and precipitable water with a backwards trajectory scheme, the recycling ratio will be cross correlated to the rest of the fluxes, in an attempt to find if the relationship between the recycling ratio and the fluxes improves when they are lagged an specific number of time steps or in this case days.

Variable	Lag (days)	Cross correlation coefficient
E	-2	0.82
I	-1	0.78
O	0	0.79
D	0	0.05
W	-1	0.81
dW/dt	0	0.08

Table 7. Cross correlation of the variables and the recycling ratio

The cross correlation of the entire series indicate that the pattern of recycling ratio is more similar to the pattern of the evaporation and precipitable water, if they are lagged two and one days respectively, however this doesn't increase the determination coefficient between the recycling ratio and the fluxes, as shown in Figure 23, because the daily recycling ratio varies a lot around the mean, and if the data used for the analysis is a large time series like in this case (11 years) the variation range increases.

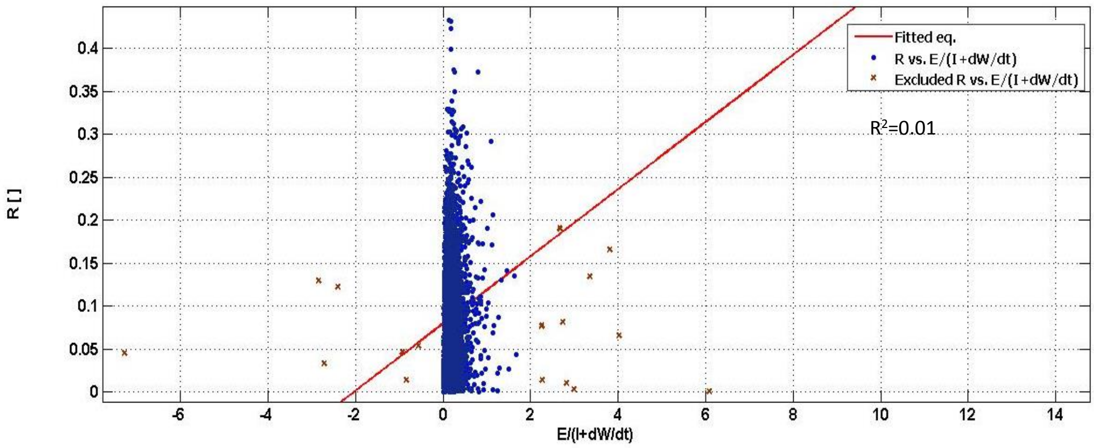


Figure 23. R vs. E/(I-dW/dt) for the entire time series

As the analysis of the entire time series has not provided better results than the analysis when the data is divided in groups depending on the amount of precipitation, the next attempt to test the hypothesis groups the data according to the months.

Figure 24 shows scatter plots for the precipitation vs. recycling ratio for groups of data of 2 months, it can be seen that there's not a clear and observable relationship to describe the behaviour of one variable as a function of the values of the other.

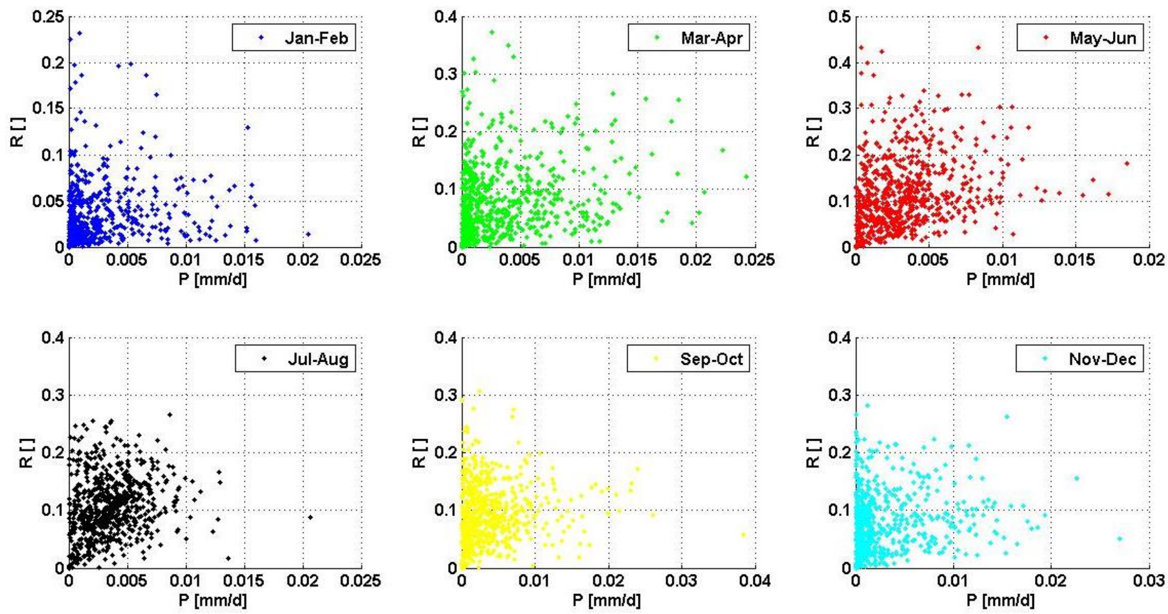


Figure 24. Precipitation vs. recycling ratio

For a change of approach, instead of relating the recycling ratio to the fluxes, now the recycled precipitation is related to the rest of the fluxes and the best relationship was found to occur with the precipitation from July to October as shown in the Figure 25.

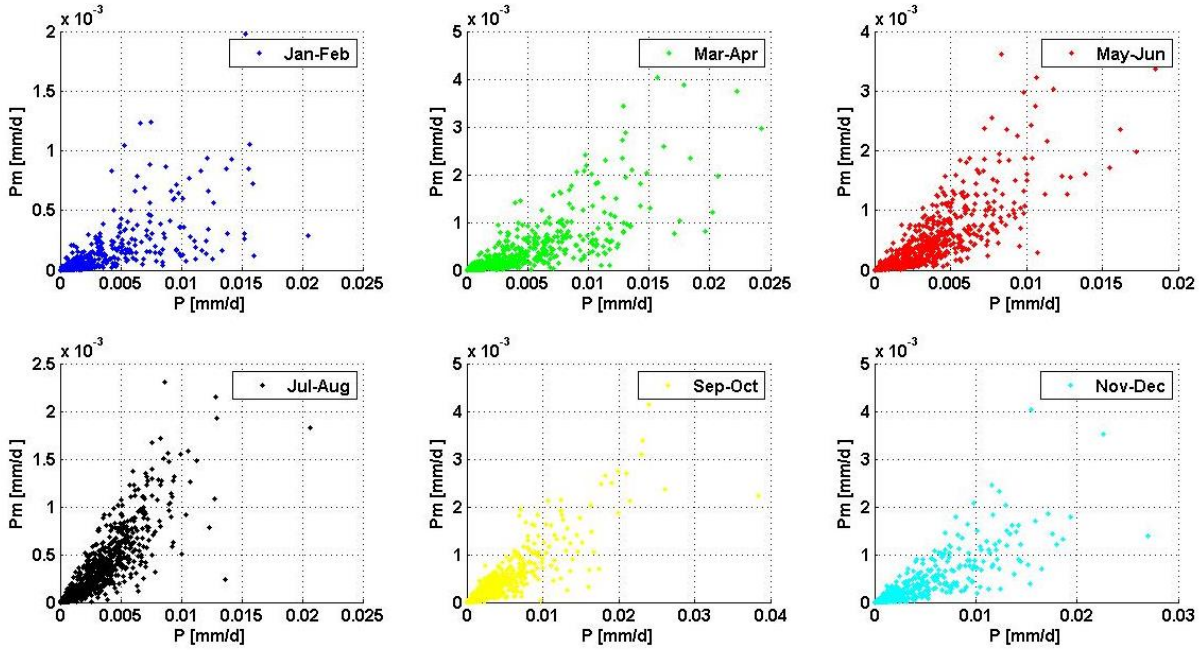


Figure 25. Precipitation vs. recycled precipitation estimated with the dynamic model

For the data of July and August the coefficient of determination for a fitted linear equation is R^2= 0.68, and for the data of September and October its R^2=0.75, the factor that relates both variables in Figure 25 is the average regional recycling ratio, and the reason to have a good fit is only due to the relatively small variation of the recycling ratio.

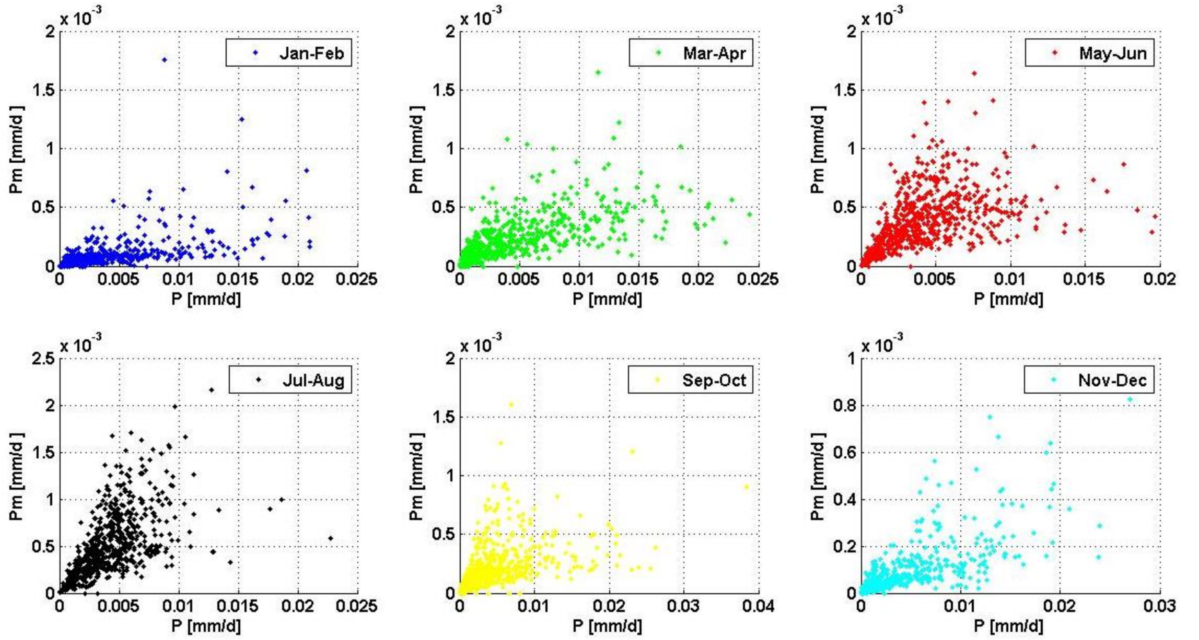


Figure 26. Precipitation vs. recycled precipitation estimated with the WAM

Figure 26 confirms the previous statement because when the data of the recycling ratio has a greater amplitude of variation (standard deviation) the goodness of the fit is reduced to 0.43 and 0.28 respectively.

If the definition of the recycling ratio is applied to the results of the daily recycling ratio from the WAM and dynamic model it's possible to estimate the monthly recycled precipitation and the monthly values of the recycling ratio as shown in:

$$R_{month\ j} = \sum_{i=1}^{n} \frac{R_{i,j} * P_{i,j}}{P_{i,j}} \qquad (31)$$

where:

$R_{month\ j}$= Regional recycling ratio for the month j
$R_{i,j}$ = Regional recycling ratio for the day i of the month j
$P_{i,j}$ = Mean precipitation depth for the day i of the month j [L^{-1}]
n = number of days in the month j

The use of eq. (31) allows the comparison of the results from the models of Brubaker et al., Eltahir and Bras, dynamic and WAM, which are shown in Figure 27. It shows that the recycling ratio estimated with the dynamic and the WAM is larger than when it's estimated with the model of Brubaker and Eltahir, as it was demonstrated by (Dominguez et al. (2006)).

The recycling ratio estimated with the WAM, Brubaker and Eltahir show a similar trend of highs and lows, while the one estimated with the dynamic model show several peaks during a year.

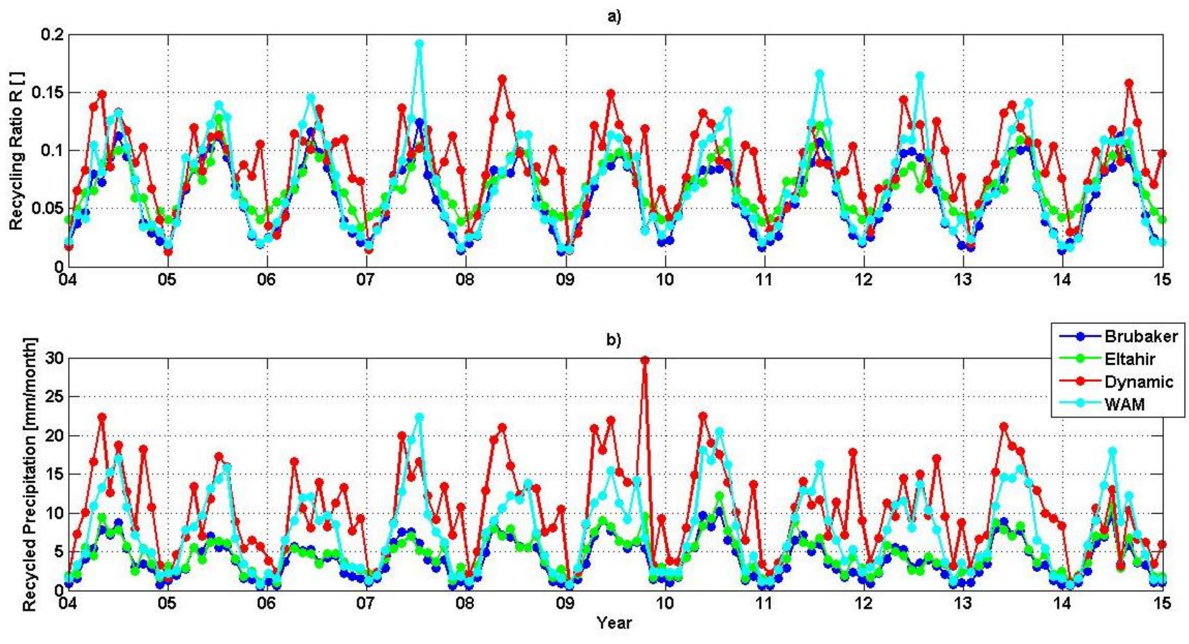


Figure 27. Estimated a) monthly recycling ratio and b) recycled precipitation

The recycled precipitation is larger with the dynamic model than with the WAM as showed the recycling ratio.

If the hypothesis is tested with the results from the dynamic model, the WAM and the monthly mean evaporation, the linear correlation coefficient R^2 found was 0.48 and 0.85 respectively. The estimates of the recycling ratio with the dynamic model show more spreading around the fitted curve, hence, most of the values obtained with the curve will be far from the estimates with the models.

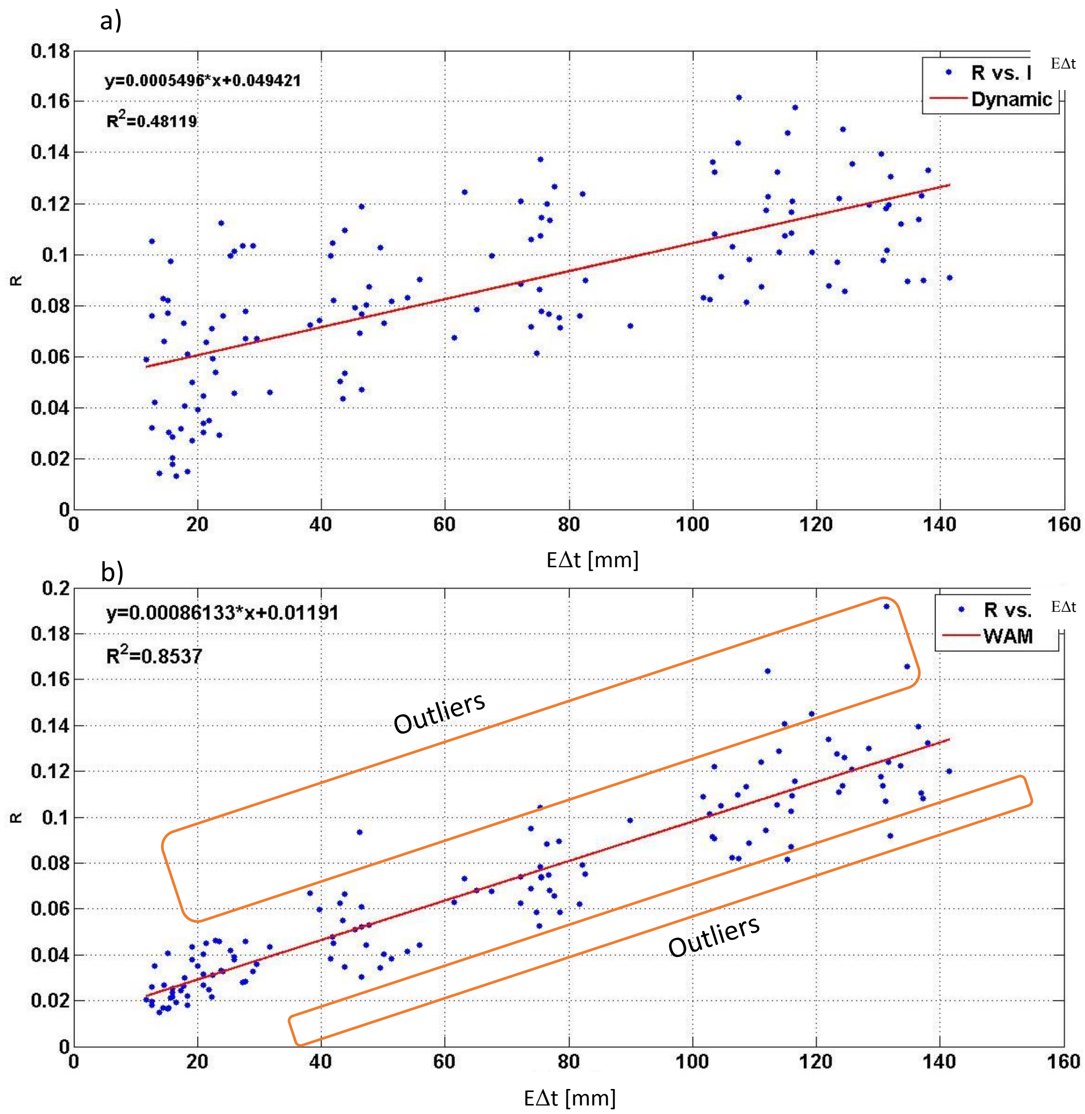


Figure 28. Fitted equations of the recycling ratio vs. evaporation a) dynamic, b) WAM

The fitted equation that estimates the regional recycling ratio as a function of the evaporation with the WAM is R=0.00086133*E+0.01191.

The recycling ratio is underestimated in March 2005, April 2004, July 2011, August 2012, July 2007, July 2010, and overestimated in May 2004, June 2007. The underestimated points have as common characteristic that their divergence is greater than the average plus the standard deviation, and for the overestimated points the divergence is smaller than the mean minus the standard deviation, the precipitation is abnormally high too.

3.3 Inherent issues of the recycling ratio concept:

As stated above, the recycling ratio has limitations with respect to its area, geometry (shape and orientation) and applicability.

The issue of area dependence is explained by (Eltahir and Bras (1996)) with a thought experiment (see Figure 29) where they consider two extremes cases of the recycling ratio, one where the considered area is the entire world (Figure 29a) and other where the area is reduced to a point (Figure 29b); if a climatic equilibrium is assumed the recycling ratio for the former is one while the recycling ratio for the latter is zero because the vapour evaporated from the surface won't precipitate in the same point due to the atmospheric transport. (Eltahir and Bras (1996)) calculated the regional recycling ratio for several areas of varying sizes in the Amazon region and found that the variation of the regional recycling ratio with respect to the size of the area can be explained by an exponential equation that relates the length (square root of the area) to the regional recycling ratio.

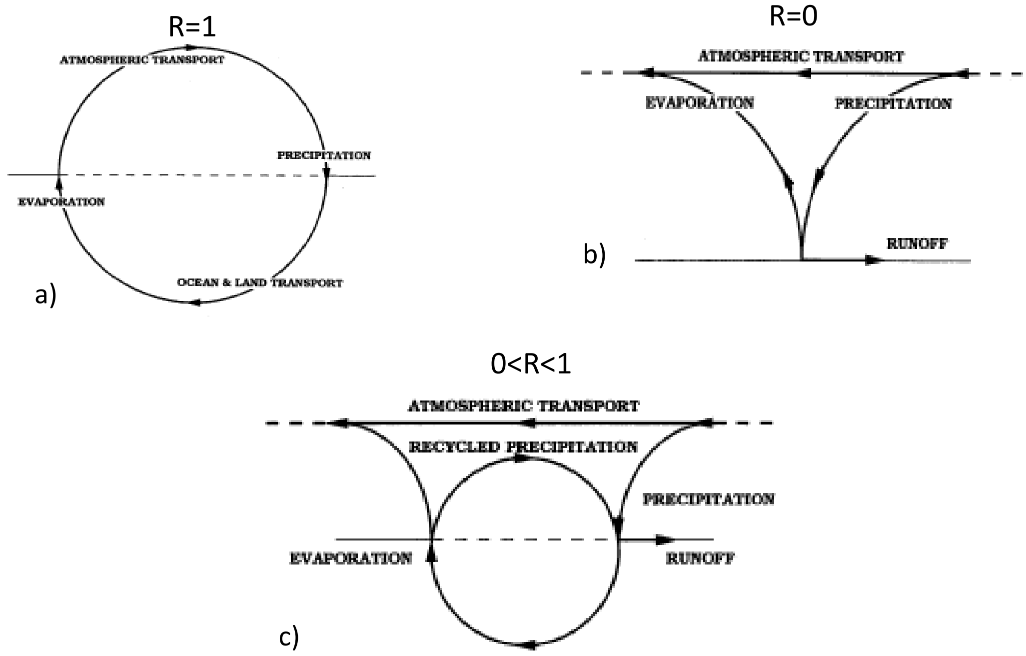


Figure 29. Regional recycling ratio. Source: Eltahir and Brass (1996)

(Brubaker et al. (2001)) explained the dependence of the regional recycling ratio on the area with a similar thought experiment as (Eltahir and Bras (1996)). The study of (Brubaker et al. (2001)) took place in the Mississippi basin and produced an exponential relationship between the square root of the area and the regional recycling ratio for the spring and summer months.

(Dominguez et al. (2006)) calculated the monthly recycling ratio for the summer months in the U.S. for several areas that started from $2*10^5$ Km^2 centred over southern Illinois and extended to $4*10^6$ Km^2 and found an exponential correlation between area and magnitude of the recycling ratio.

(Dirmeyer and Brubaker (2007)) justified the area dependence of the recycling ratio with a similar example as (Eltahir and Bras (1996)). They used a back-trajectory analysis on a global investigation and obtained a series of exponents applicable for specific areas that relate the annual regional recycling ratio and the size of the area.

(Van Der Ent and Savenije (2011)) proposed an approach to quantify the spatial and temporal scales of moisture recycling that doesn't depend on the size of the area or its shape. It consisted in obtaining the length scale of precipitation and evaporation recycling, which represent the distance in which the recycling ratio changes approximately 63%. The length scale is interpreted as the mean distance travelled by the water particles, which allows to draw the conclusion that small lengths indicate high feedback between land and atmosphere, hence, higher recycling ratio. Although this approach is indeed independent of the shape, size and allows the comparison between regions in different climates the length scale is a concept more abstract than the regional recycling ratio.

Although previous researches demonstrated that the recycling ratio increases as the size of the region increases (Dominguez et al. (2006)), (Brubaker et al. (2001)), (Dirmeyer and Brubaker (2007)), they don't identify the reasons for this relationship. In the present study this relationship (influence) will be scrutinized by studying the influence of the evaporation and the influence of the internal transport of moisture or the transport of moisture from the upwind zones to the downwind zones.

The influence of evaporation is studied by estimating the recycling ratio with the WAM according to eqs. (30) and (31) for three regions of different size as shown in the Figure 30. The largest region is the original one (1377665.74 Km^2) for which the hypothesis was tested, the medium (652356.17 Km^2) and smaller regions (262941.00 Km^2) are contained within it.

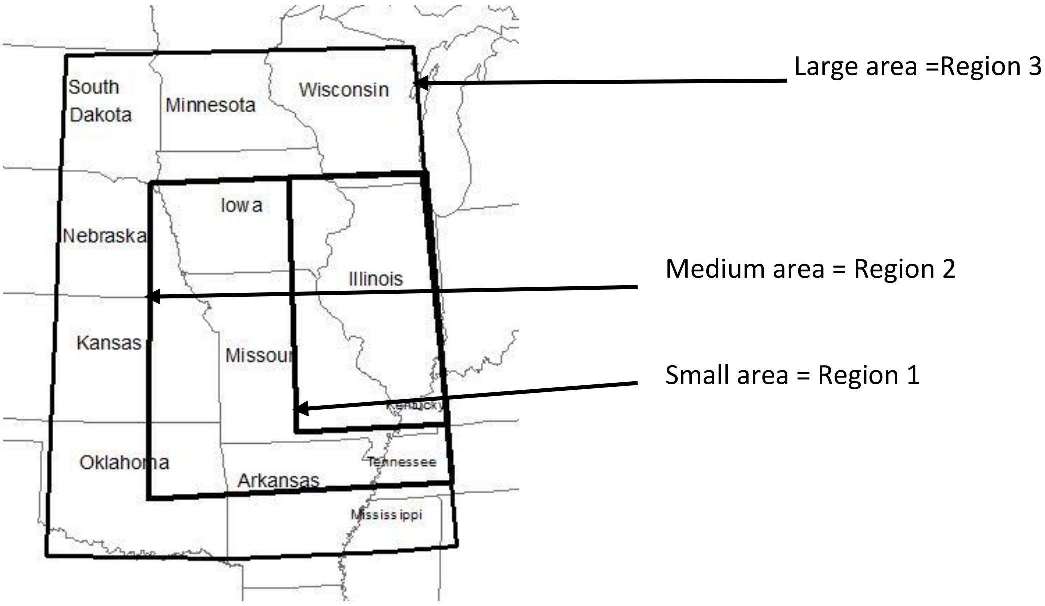


Figure 30. Regions of different sizes

The monthly estimates of the recycling ratio are presented in the Figure 31, they follow the same trend and show small differences, except in July 2007 where the recycling ratio of the largest and medium region are more than 50% bigger when compared to the one of the smallest region.

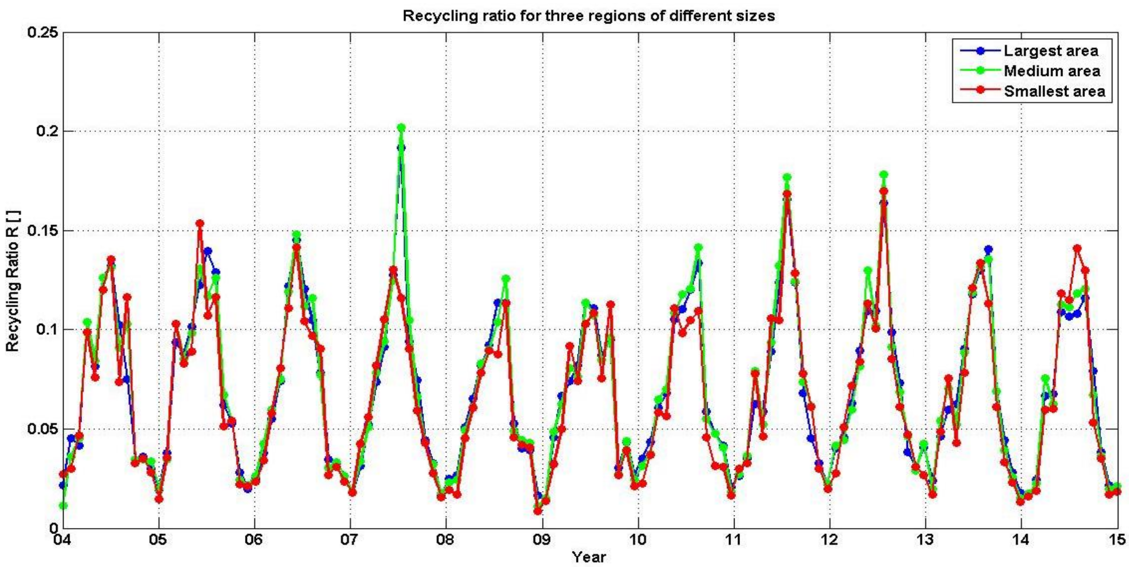


Figure 31. Recycling ratio for the three regions

Although the difference in the size of the regions is of one order of magnitude the estimated recycling ratios show that for regions of different sizes with similar spatial values of evaporation the influence of it doesn't depend on the size of the region, as it shown in the Figure 32 and Figure 33 where the spatial distribution of the monthly evaporation and local recycling ratio are depicted.

Figure 32 corresponds to March 2006, a month for which the regional recycling ratio in each of the three areas is 0.055, 0.059 and 0.057 (relatively small difference) for the largest, medium and small region respectively. The mean local recycling ratio is not proportional to the area either, because the largest one is calculated for the medium size region. These values support the previous claim that with respect to evaporation a larger area does not necessarily lead to a larger regional recycling ratio.

In a larger area the possibility of having large gradients of evaporation is bigger, this is demonstrated by the coefficient of variation in Table 8, a large gradient will result in a smaller mean evaporation and a smaller regional recycling ratio due to evaporation; whereas smaller regions with lower coefficient of variations and larger means will result in larger recycling ratios as is the case for the regions 2 and 3.

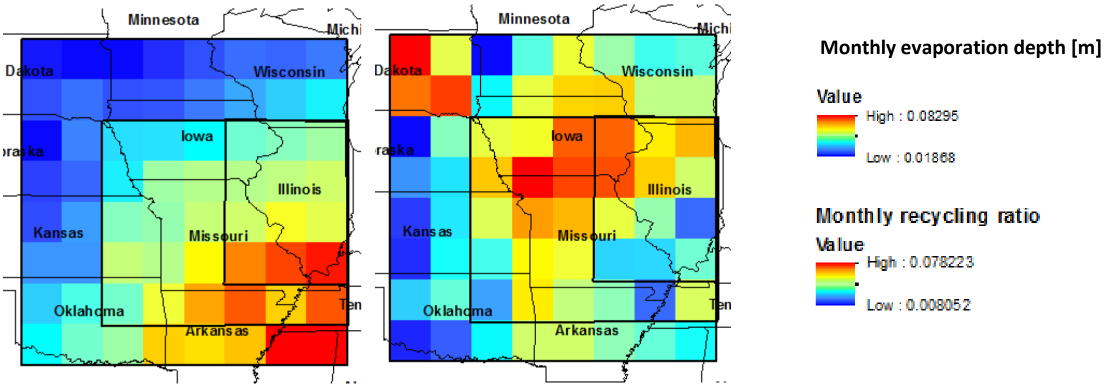


Figure 32. Monthly evaporation (left) in m and monthly local recycling ratio (right) of March 2006

	Area	Monthly evaporation depth			Recycling ratio		
	[Km^2]	Mean [m]	E*A [Hm^3]	Coeff. of Var	ρ mean	Coeff. of Var	R
Large	1377665.74	0.038	51.756	0.601	0.053	0.337	0.055
Medium	652356.17	0.047	30.421	0.492	0.057	0.279	0.059
Small	262941	0.046	12.143	0.487	0.055	0.275	0.057

Table 8. Statistical indexes of the evaporation and local recycling ratio of March 2006

The second case analysed is for July 2007 because it shows a large difference between the estimates of the recycling ratio of the region 3 and 2 compared to the region 1. Figure 33 shows the spatial distribution of monthly evaporation and the spatial distribution of the local recycling ratio; in the latter the mean is the smallest for the region 1 which coincidently results in the smallest regional recycling ratio due to evaporation. If the size of the region is increased towards areas with larger local recycling ratios its mean and the regional recycling ratio will also increase, as show the values in Table 9.

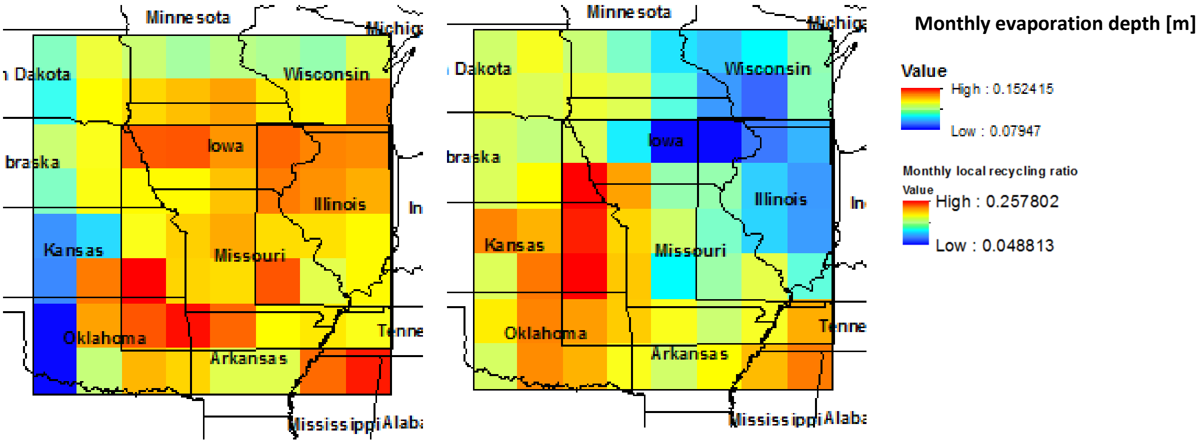


Figure 33. Monthly evaporation (left) in m and monthly local recycling ratio (right) of July 2007

	Area	Monthly evaporation depth			Recycling ratio		
	[Km^2]	Mean [mm]	E*A [Hm^3]	Coeff. of Var	ρ mean	Coeff. of Var	R
Large	1377665.74	0.131	180.892	0.601	0.164	0.080	0.19
Medium	652356.17	0.138	90.085	0.492	0.156	0.087	0.20
Small	262941.23	0.138	36.206	0.487	0.114	0.125	0.11

Table 9. Statistical indexes of the evaporation and local recycling ratio of July 2007

Due to the similarity of the data the Figure 31 suggest that it's possible to fit an equation that estimates the monthly regional recycling ratio due to evaporation as a function of the mean monthly evaporation. Two approaches will be used one that uses the evaporation depth and another one that uses the volume of evaporation, which is calculated as the sum of the product of evaporation times the area within region.

Figure 34 shows the regional recycling ratio due to evaporation for the three regions in the y axis and the regional mean monthly evaporation depth in the x axis; it can be seen that the first order curve was fitted with a relative good agreement. The coefficient of correlation is 0.815 for values of evaporation depth between 10 and 150 mm.

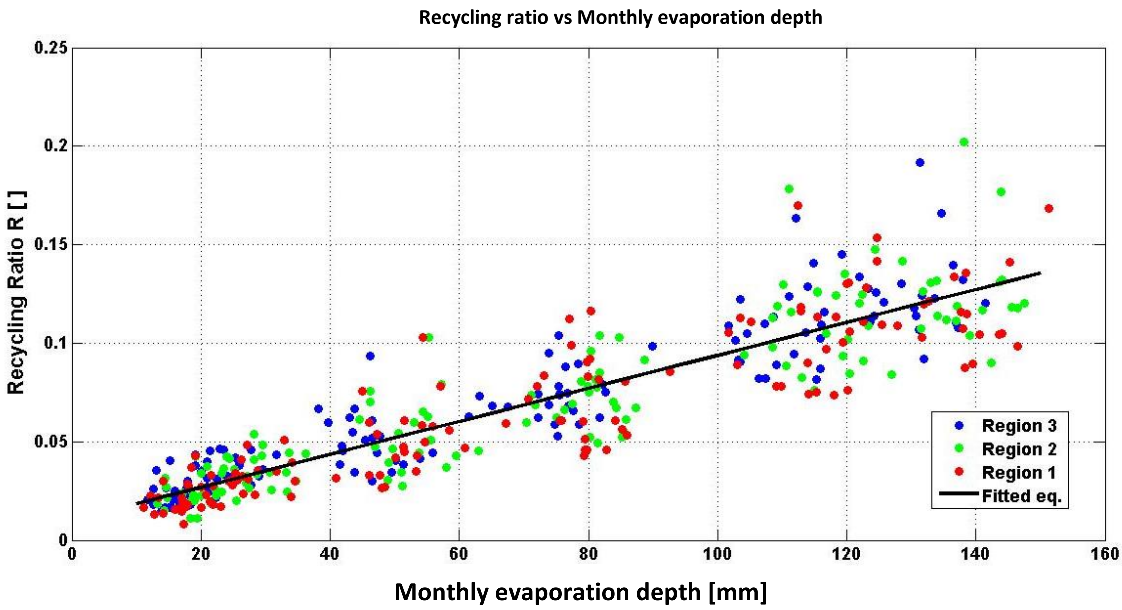


Figure 34.Regional recycling ratio of the regions vs. mean monthly regional evaporation depth in mm

When the volume of evaporation is plotted against the regional recycling ratio due to evaporation, three groups of results can be distinguished, one for each of the regions as is depicted in the Figure 35a), which shows that the regional recycling ratio due to evaporation does not necessarily increase when the area (in this case presented as evaporation times area) increases. The slope of the curves decrease as the area increases.

The data in Figure 35a) suggest that there is an upper limit of the recycling ratio due to evaporation. In order to verify this the recycling ratio due to evaporation for the entire continental U.S. was calculated and plotted in Figure 35b), it shows that the recycling ratio is bounded between 0 and 0.15. The data of the recycling ratio was fitted to equations of the form $R = 1 - e^{-x/b}$, where the term b obtained after the fitting is similar to the area hence the final equation is only function of the monthly evaporation depth value in mm.

The fitted curves from Figure 34 and Figure 35 are given in Table 10.

	Equation	R^2	Units of EΔt and EAΔt
All regions vs evaporation depth	R=0.00083709*EΔt+0.010018	0.815	[mm]
Region 1	R=1-exp(-EΔt*A/(0.2697*100000))	0.853	[Hm^3]
Region 2	R=1-exp(-EΔt*A/(0.6521*100000))	0.809	[Hm^3]
Region 3	R=1-exp(-EΔt*A/(13.07*100000))	0.835	[Hm^3]
Continent	R=1-exp(-EΔt*A/(9.761*1000000000))	0.901	[Hm^3]

Table 10. Equations of the fitted curves for the two cases (evaporation as depth and as volume)

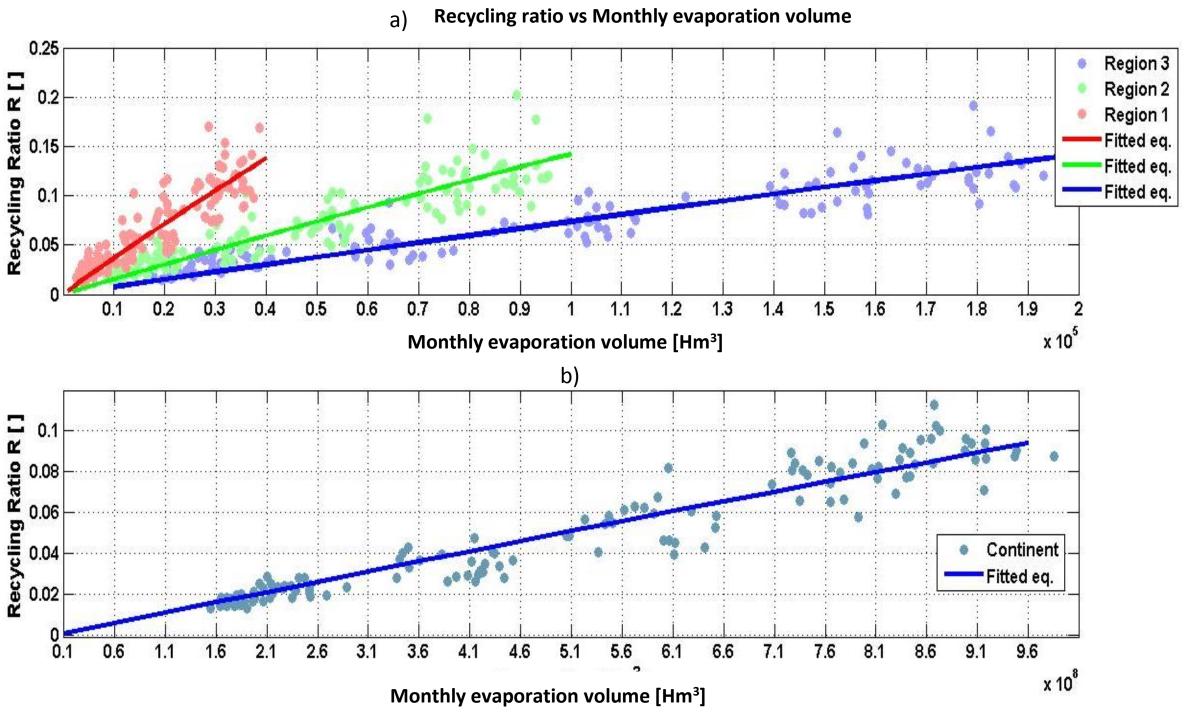


Figure 35. Regional recycling ratio due to evaporation vs. evaporation volume for the three regions

Previous researches successfully demonstrated that the regional recycling ratio depends on the size of the region, and even obtained generic equations for the recycling ratio for the summer season as a function of the size of the region. However these equations are only applicable for the area where they were obtained as the recycling ratio is depends on meteorological, and topographical conditions.

The influence of the size of the region will be analysed for the same three regions with the use of eq. (29) that includes the flux of moisture from cell to cell.

$$w_{\Omega_{x,y}}^{n+1} = w_{\Omega_{x,y}}^{n} + E_{\Omega_{x,y}}^{n} - P_{\Omega_{x,y}}^{n} \frac{w_{\Omega_{x,y}}^{n}}{w_{x,y}} - \sum O_{\Omega_{x,y}}^{n} \frac{w_{\Omega_{x,y}}^{n}}{w_{x,y}} + \alpha_{\Omega_{x,y}}^{n} \frac{w_{\Omega_{x,y}}^{n}}{w_{x,y}} + \sum I_{\Omega_{x,y}}^{n} \frac{w_{\Omega_{x,y}}^{n}}{w_{x,y}} \quad (32)$$

where I is the influx of moisture to the cell, the rest of the terms have been previously explained.

Eq. (32) not only considers the effect of the inner circulation, it is also the effect of evaporation, and as the latter was previously isolated the equations obtained before will be used to decompose the results of the recycling ratio as a function of the influence of evaporation and area.

When the influx of moisture is considered, the water vapour of local origin increases, and consequently the recycling ratio. If the size of the region studied increases, the path that the moisture travels increases and the possibility of picking up and dropping a larger amount of local moisture also does.

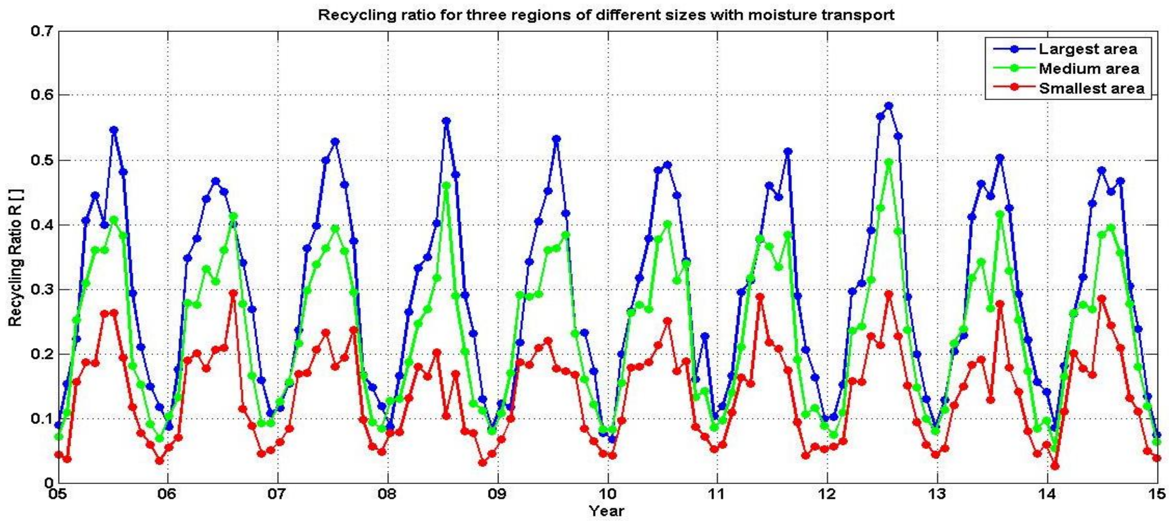


Figure 36. Regional recycling ratio due to evaporation and moisture transport for the three regions

Figure 36 shows the monthly recycling ratio for all three regions, for the period 2005 – 2014. The results of 2004 had to be excluded because the model takes approx. 1 month to overcome the influence of the initial values.

If Figure 36 and Figure 31 are compared the influence of the size of the region becomes clear, for the entire period the recycling ratio of the smallest region is the lowest, and is the largest for the biggest region, with the difference being more noticeable from April to July.

Because for the same time period the recycling ratio is different among the regions it can be concluded that there won't be a linear or unique relationship between it and the evaporation depth as the one in Figure 34, and only the evaporation volume will be analysed. Figure 37 shows the scatter plots and fitted equations of the three regions (Figure 37a) and the continental U.S. (Figure 37b).

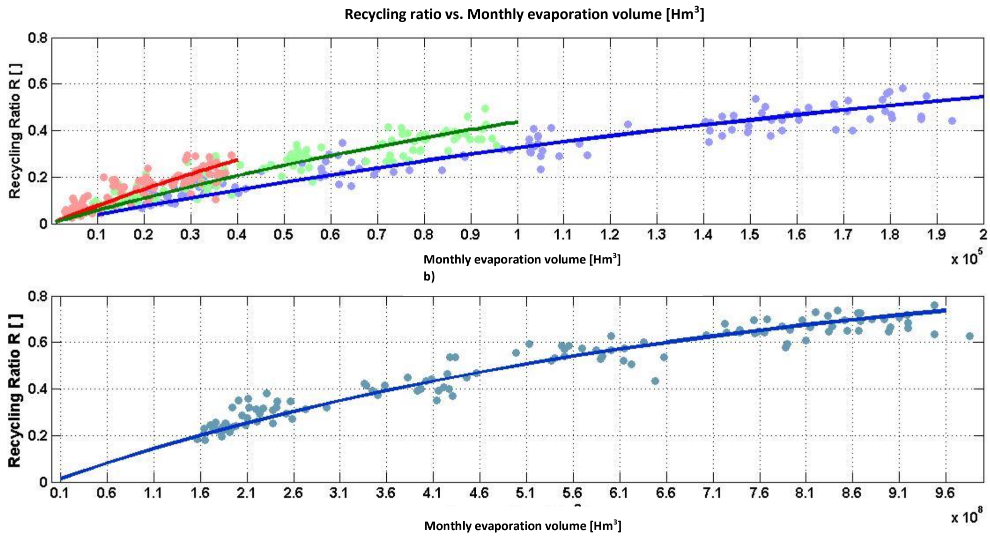


Figure 37. Recycling ratio due to moisture transport and evaporation vs. monthly evaporated volume for a) the three regions, b) continental U.S.

The data was fitted to equations of the form $y = 1 - e^{-x/b}$, they are presented in the Table 11, the correlation coefficient R^2 for every region is above 0.65 (relatively good fit). The coefficient b is unique for each size is plotted against the area in the Figure 38.

	Equation	R^2	Units of E and EA
Region 1	R=1-exp(-EΔt*A/(0.1258*1000000))	0.9027	[Hm^3]
Region 2	R=1-exp(-EΔt*A/(0.1740*1000000))	0.728	[Hm^3]
Region 3	R=1-exp(-EΔt*A/(0.2545*1000000))	0.9027	[Hm^3]
Continent	R=1-exp(-EA/(0.7203*1000000000))	0.9328	[Hm^3]

Table 11. Fitted equations of the regional recycling ratio

Figure 38 shows a fitted equation that makes it possible to obtain an estimated value for the coefficient b as a function of the area in Km^2. In order to use the equations of the Table 11 the coefficient b must be multiplied by 10^n where n is the order of magnitude of EA plus one.

The equation that approximates b is:

$$b = 5.927 * 10^{-8} * A + 0.1382 \qquad (33)$$

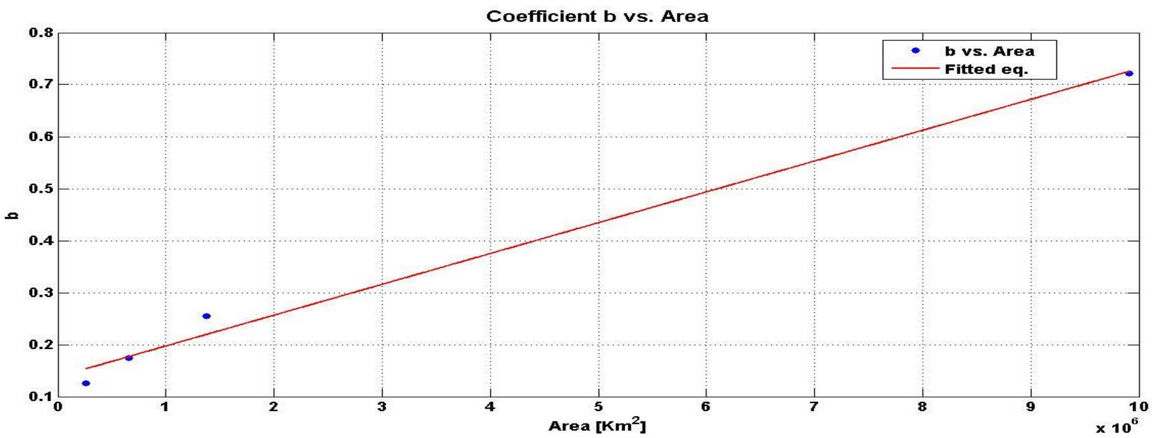


Figure 38. Coefficient b vs. area

Finally it is possible to arrive to an expression that approximates the monthly recycling ratio of a region as a function of the monthly evaporation and its size as shown in eq. (34)

$$R = 1 - \exp\left(-\frac{EA}{(5.927 * 10^{-8} * A + 0.1382) * 10^{n}}\right) \tag{34}$$

Because the regional recycling ratio is calculated from the values of the local recycling ratio and the latter is always linked to an origin, the regional recycling ratio for the same region can be different if we consider two different origins for the water vapour (Figure 39 and Figure 40) which show the local recycling ratio of continental origin and local recycling ratio with origin from the region 3 respectively.

The values of the continental recycling ratio are larger than those of the regional because the moisture is traveling from west to east and the further from the west coast a cell is, the larger the amount of moisture originated in the continent that the atmosphere will contain. This is a direct consequence of the distance of a cell, or cells in a group, to the ocean and the direction of the moisture flux.

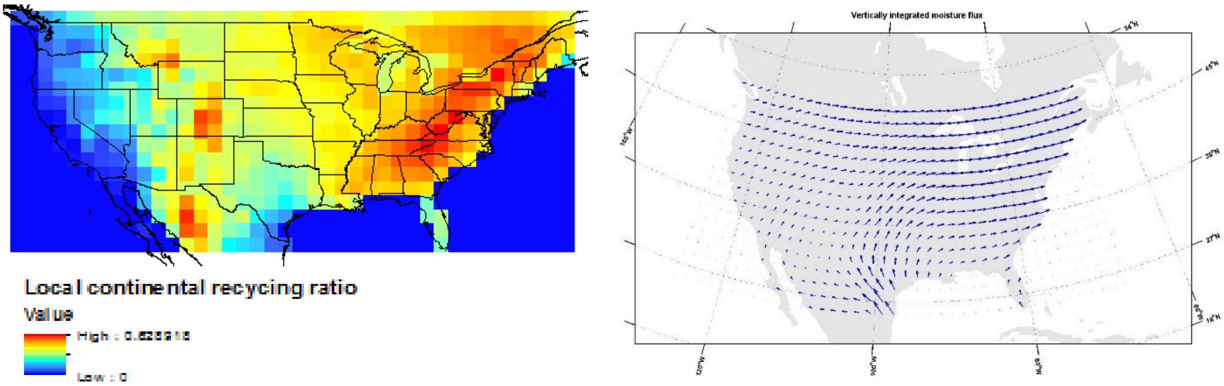


Figure 39. Mean local continental recycling ratio for summer 2005-2014

The local continental recycling ratio in Figure 39 is low in the Pacific coast and increases as it travels inland, its variation is in accordance with the moisture flux direction; however high values, which are larger than the surrounding ones, occur in the Rocky Mountains, which suggest that moisture recycling is favoured by orographic precipitation.

Comparing the values of the local recycling ratio when the origin of the moisture is the continent to the values when the origin is the region itself the difference becomes more apparent, the means are 0.40 and 0.18, and the maximums are 0.48 and 0.32 and the minimums 0.26 and 0.05 respectively.

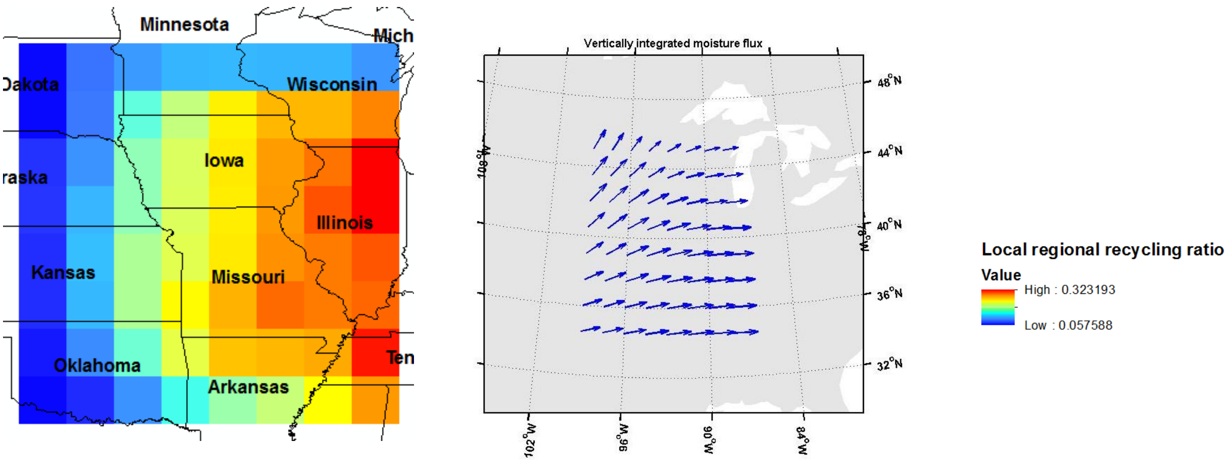


Figure 40. Mean local regional recycling ratio for summer 2005-2014

The regional recycling ratio is also sensitive to the geometry of the area chosen. This issue was addressed by (Van Der Ent (2010)) where he states the example of a long rectangular area which is affected by a dominant perpendicular moisture flux which may cause the regional recycling ratio to be zero, which from a theoretical point of view is plausible, as the probability that the water vapour to precipitate in the same area is practically zero.

In order to test eq. (34) the recycling ratio for two regions in different zones, with different sizes and geometry will be estimated using the WAM and these results will be compared with the estimates obtained with the equation. These regions are shown in Figure 41 and the data in Table 12**¡Error! No se encuentra el origen de la referencia.**. Region 4 extends over a portion of the Rocky Mountains, which have hotspot zones of higher recycling ratios, while the region 5 extends over the south of the conterminous U.S. where according to Figure 39 the recycling of moisture is mainly due to water vapour transport. Besides having a different size than all the previous regions the geometry is also different, for this case the ratio of the sides is 2.14, in the previous regions was 1.45, 1.17 and 1.89 in the regions 3, 2, 1 respectively.

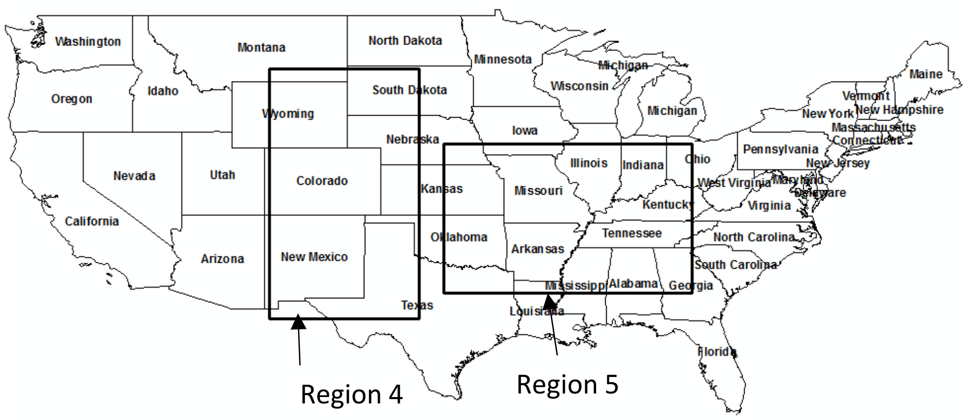


Figure 41. Rectangles showing the region to test eq. (34)

	Region 4	Region 5	
	Value	**Value**	**Unit**
Area	1297086.53	1330935.07	Km^2
Longitudinal side	776.9458	1334.0318	Km
Latitudinal side	1668.24581	998.76268	Km

Table 12. Data of region 4 and 5

The values of the volumetric evaporation are displayed in the Figure 42, they are of five orders of magnitude when expressed in [Hm^3], which means that the value of n that will be used in eq. (34) is 6.

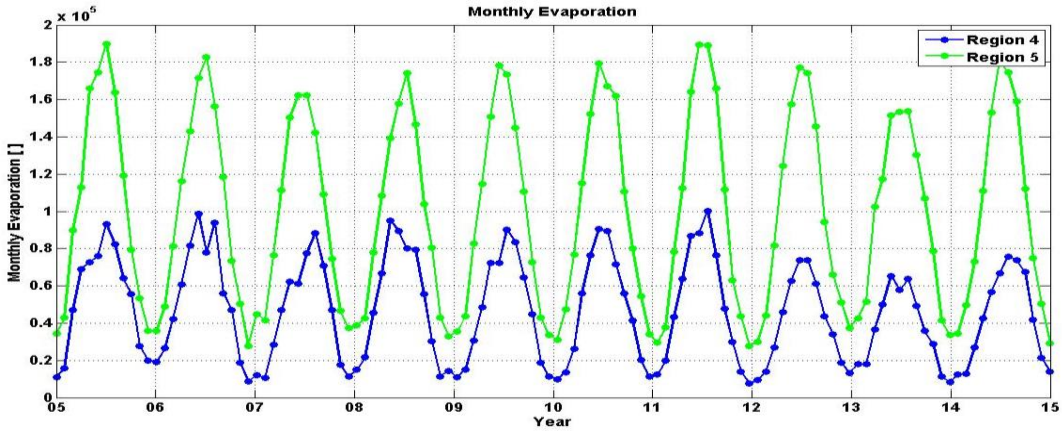


Figure 42. Monthly volumetric evaporation in Hm^3

The monthly variation of evaporation for the region 4 shows that for 2012, 2013 and 2014 the values of the peaks are smaller than the previous years. On average the peak evaporation from 2005 to 2011 was $9.1*10^4$ [Hm^3] while from 2012 to 2014 it was $7.5*10^4$ [Hm^3]. Region 5's evaporation is consistently larger than region 4's, the lowest peaks in the time series occur in 2007 and 2013.

The time series of the monthly recycling ratio is shown in Figure 43, as in previous cases the trend is well captured. The monthly recycling ratio for the region 4 shows good agreement in the minimums, while the peaks are underestimated by eq. (34); as previously said the region 4 contains hotspots of high moisture recycling which were not taken into account while eq. (34) was deduced, hence it cannot reproduce the effects of these localized zones with high recycling rates and the application of it results in underestimation of R. The correlation coefficient R^2 for the region 4 is 0.86.

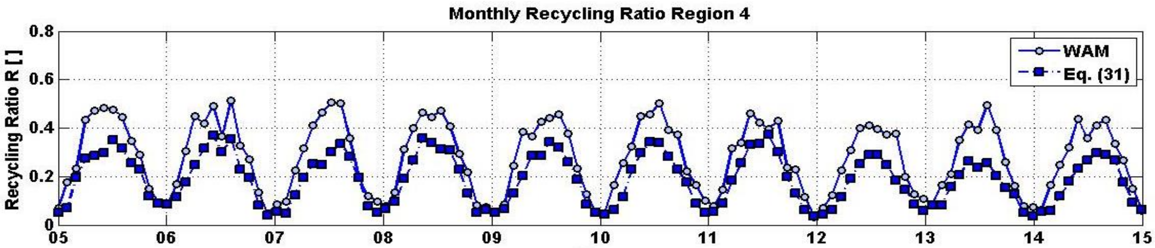


Figure 43. Monthly recycling ratio of the region 4 estimated with the WAM and the eq. (34)

The monthly recycling ratio estimated with eq. (34) for the region 5, see Figure 44, shows a better agreement with the results obtained from the WAM, and is reflected in the value of R^2= 0.90, demonstrating the usefulness of eq. (34) for a fast estimation in areas without hotspots.

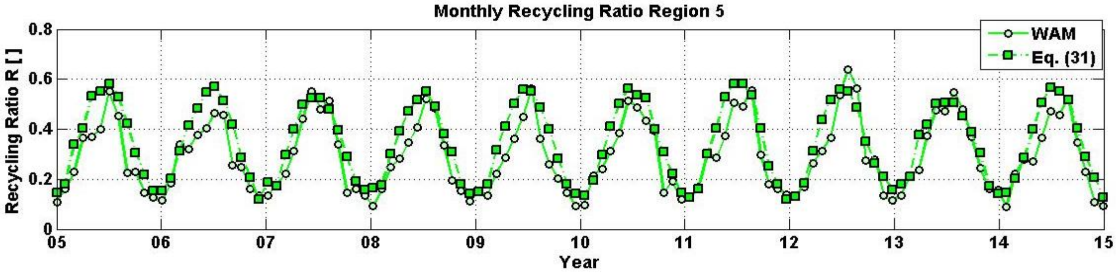


Figure 44. Monthly recycling ratio of the region 4 estimated with the WAM and the eq. (34)

CHAPTER 4

4.1 Discussion

The hypothesis testing produced successful results for all the models in a monthly timescale. Although one could argue that the use of evaporation is not practical because it is difficult to measure or estimate in field, and satellite data is likely to have errors. Evaporation is the term that links the terrestrial and atmospheric water balance and the main control of moisture recycling on the mentioned time scale.

For the models of Brubaker et al. and Eltahir and Bras the hypothesis was tested by fitting equations that relate the fluxes to the recycling ratio, it was found that the recycling ratio can be expressed as a function of the monthly evaporation depth. The monthly recycling ratio behaves in a similar fashion to the monthly mean evaporation. The equations obtained for the monthly models have the form $R = a + bE$; they reflect the proportion of evaporation that directly affects value of the recycling ratio around a minimum value, the term b has inverse units of the evaporation [1/mm], while the independent term is dimensionless, this term is smaller than the minimum recycling ratio because the monthly evaporation, according to the data, is always larger than zero.

It was not possible to determine an equation that estimates the recycling ratio at a daily time scale. In this timescale the interaction of the fluxes is intricate and a dominant relationship of the variables during the precipitation recycling process could not be found, moreover it was observed that the relationship of the daily recycling ratio with the fluxes is not unique, these means that for the same amount of precipitation or evaporation or any flux, several values of the daily recycling ratio are possible.

The dynamic model calculates the local recycling ratio at a grid scale, it depends on the ratio of E/W and the effective wind velocity at different steps in time, this means that the estimate of a daily recycling ratio is affected by the magnitude and direction of the wind during that day and in some cases the previous one, the averaging of data from 6 hours to 24 hours smoothens the drastic variations that the recycling ratio may have. The WAM also calculates the local recycling ratio at a grid scale, it's based on the discretization of the water balance equation which is dependent on the origin of the water particles, and the interaction of the fluxes becomes non-identifiable in such a small scale as is 30 minutes.

The results of the dynamic model and WAM when estimated at monthly timescale show strong correlation with the monthly evaporation, this may be caused by the use of accumulated fluxes in the process of calculation, which compensates for irregular values and tends to homogenize the final estimates.

Equation (34) may provide a fast estimated value of the monthly regional recycling ratio, even when it takes the influence of the size of the region into account it should not be used in other regions that are not the conterminous U.S. and in the case it is used for a region that includes the Rocky Mountains (zone with hotspots of high recycling due to orographic precipitation) it must be kept in mind that the summer estimates will be much smaller than the ones obtained from the model.

The proposed equation to estimate the recycling ratio eq. (34) has the form $R = 1 - e^{-EA/b}$, which is similar to equation (25) put forward by (Dominguez et al. (2006)). The term b has the same volumetric units as EA, it is estimated with a lineal equation function of the area in Km^2, the bigger the area the larger the b, mathematically a small b produces a curve with a steep slope.

The estimates of the recycling ratio from the models still suffer of the well mixed assumption, which implies immediate mixing of local ad advected water vapour. (Fitzmaurice (2007)) points out that this assumption is more representative of the reality when it is applied in the tropics where convection is rigorous and active, however for the central U.S. a larger influence of local water vapour is observed due to convective rainfall, approximately 60% to 70% larger than with the well mixed atmosphere.

Future research should focus more on the relationship of the recycled precipitation and the intensity of the rainfall events, with relaxation of the well mixed atmosphere. It is also suggested to investigate the coupling of land and atmosphere through the atmospheric water balance, the energy balance and terrestrial water balance

4.2 Conclusions

The results of a model depend on the data, the theory behind it and the implementation. The theory behind the dynamic model and the WAM are equally solid, one is a conceptual model with a formal derivation and the other one is a numerical approximation. If we assume that no error are done during the implementation the unique source of error is the data. As shown before the data of ERA contains errors in the values of evaporation and precipitation, this implies that the estimates from the dynamic model will have a larger error than the estimates from the WAM, because the term α is included in the latter.

Because the recycling ratio is characteristic for an equilibrium state, the estimates obtained with the equation (34) may be compared with estimates obtained from the models and indicate whether the equilibrium state has changed or is maintained.

The daily recycling ratio or the precipitation recycled cannot be estimated directly from the variables involved, due to spatial scale and time scale issues, however it is possible to estimate the monthly recycling ratio from the mean monthly evaporation.

The dependence of the recycling ratio on the size of the region is mostly due to the transport of moisture from the upwind zones, it was demonstrated that the value of the recycling ratio due to evaporation varies very little when the area changes, it rather depends on the presence of areas with high local values of the recycling ratio (see Figure 31 and Figure 33). Equations in Table 10 also suggest that the influence of evaporation alone in the recycling ratio has an upper limit which is approximately 15%.

Equation (34) is able to predict the monthly recycling ratio as a function of the size of the region and the monthly evaporation as volume. Its application is restricted to a minimum size, as shown in Figure 36 where the values or R for the region 1 (Illinois), are similar to those of the recycling ratio due to evaporation. Another limitation of equation (34) is the region for which it was obtained and tested, the area covered in this study is (45.75N – 30.75N, 108.75W- 83.25W).

The evaporation is one order of magnitude smaller than the rest of the fluxes, however it exerts great influence on the estimates of the recycling ratio. The evaporation and moisture transport have great influence over continental precipitation, but the recycling ratio should only be taken as an index of the degree of interaction between land and atmospheric hydrological processes.

Evaporation in a region is constrained by availability of water and energy. In the region 3 the evaporation is maximum during summer, the water evaporated comes from the soil moisture available that was accumulated from the previous rainy season (spring). This means that for summer (maximum radiation) the only constrain is the soil moisture. The recycling ratio is larger during summer because of the scarce precipitation (the less the rainfall, less the water vapour needed for a greater contribution). The months when the recycling ratio increases are characterized by high evaporation, low precipitation, divergence of moisture flux and small content of precipitable water; drastic changes in evaporation, moisture transport within the region and moisture advected to the region may lead to significant changes in precipitation.

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Appendix 1. Results of previous researches

Reference	Model	Assumptions	Period	Location	Area [Km^2]	Data Sources	Recycling ratio [%]	Observations
Budyko (1953)	Budyko et al.	1, 2, 3	N/A	Soviet Europe	N/A	Aerial observations	10	1D Model
Brubaker et al. (1993)	Brubaker et al.	1, 2, 3	1963 - 1973	Soviet Europe	2'230.000	GFDL	M=15, J=31, J=26	External sources dominate the supply of moisture
Brubaker et al. (1993)	Brubaker et al.	1, 2, 3	1963 – 1973	Mississippi Basin	1'948.000	GFDL	M=23 J=22, J=34	The area is net divergence of water vapour so it's a source
Brubaker et al. (1993)	Brubaker et al.	1, 2, 3	1963 – 1973	Amazon Region	5'307.000	GFDL	N= 31 ;D=32, J=27	The model is not a real 2D generalization The moisture flux is assumed to vary linearly along the area.
Brubaker (1993)	Brubaker et al.	1, 2, 3	1963 - 1973	African (south of Sahara)	2'535.890	GFDL	J=0.47, A= 0.48, S= 0.39	
Eltahir and Bras (1994)	Eltahir et al.	2, 3	1985 - 1990	Amazon Region	5'080.000	ECMWF	N=26 D=23 J=22,	Spatially distributed data
Dominguez et al. (2006)	Dynamic	3	1970 - 2000	USA	1'000.000	R-II	summer average= 18	The influence of atmospheric storage is shown in an increase of the recycling ratio
Bisselink et al. (2008)	Dynamic	3	1978 – 2008	Spain and Portugal	506.200	ERA 40	summer average= 15	Negative feedback
Bisselink et al. (2008)	Dynamic	3	1978 – 2008	Balkans	458.200	ERA 40	summer average = 19	Positive feedback
Harding and Snyder (2012)	Dynamic	3	1990	Great Plains	1'248.000	NARR	May to Sept average = 26	The increase of irrigation is depleting the Ogallala aquifer
van der Ent et al. (2010)	WAM	3	1998 - 2008	Amazon Region	6'724.200	ECWMF	annual average = 28	Numerical model, based on water balance

Assumptions: 1) Time averaged data 2) Change of moisture storage, 3) Atmosphere is well mixed.

GFDL=Geophysical Fluid Dynamics Laboratory; ECMWF=European Centre for Medium-Range Weather Forecasts

; R-II=Reanalysis II;