An Optimization Model for Simultaneous Periodic Timetable Generation and Stability Analysis

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Abstract

We present an optimization model which is able to generate feasible periodic timetables for networks given the line structure and the requested line frequencies, taking into account infrastructure constraints and train overtake locations. As the model uses the minimum cycle time as the objective function, the stability of the timetable is also simultaneously expressed. Dimension reduction techniques are presented taking advantage of the symmetries of periodic timetables. The model is applied to a case study of a dense corridor with heterogeneous traffic.

Keywords

Timetable design, Timetable stability, Optimization

1 Introduction

Periodic, regular interval passenger timetables offer consistent service and high frequencies throughout the day, result in a high utilization of existing infrastructure, vehicles and staff, and simplify planning and operations due to the repeating patterns. Furthermore, in rail-ways, providing differentiated supply, following fluctuations in demand, is possible even within a regular interval timetable, by using different train lengths. For these reasons, many railway operators follow a regular, in most cases hourly timetable.

Liebchen [11] describes different ways to model periodic timetables. Borndorfer and Liebchen [1] provides a particular short-term example when a regular-interval timetable is suboptimal with respect to the number of vehicles required. In the following we consider a regular interval timetable.

On many railway networks passenger demand is so high that trains run at the highest frequency possible given the infrastructure. This means a realistic timetabling model has to explicitly take into account the infrastructure capacity. Assuming sufficient amounts of vehicles and staff, the possible train frequencies are still limited by the available station platforms, station area capacity determined by the layout of the switch areas, minimum headway time possible on the open track based on the signalling blocks, and the capacity at junctions. All these capacities are furthermore dependent on signalling and automatic train protection (ATP) systems, the characteristics of the rolling stock, driver behaviour etc.

In the macroscopic model in this paper the infrastructure is described as a graph of stations, junctions, bottlenecks such as moveable bridges and connecting open track segments, and limited infrastructure capacity is represented by required minimum headway times between two trains at given locations.

The *nominal cycle time* of a timetable is the period according to which all events repeat, typically the full hour. We also use the *minimum cycle time* which is the shortest time duration in which all the events scheduled in the nominal cycle time are feasible if all dependency constraints such as minimum running times and minimum headway times required by the infrastructure are respected.

The relationship between the nominal and the minimum cycle time describes the capacity utilization of the timetable [7]: the timetable is stable exactly if the minimum cycle time T is less than the nominal cycle time T_0 , i.e. $T < T_0$, and the larger $T_0 - T$ is, the more time reserve there is available. This ratio T/T_0 is defined in Goverde [4] as *network throughput*. Therefore there is a strong relationship between the capacity of the physical network and the stability of the timetable: infrastructure capacity determines the pace at which the timetable can be executed, therefore the minimum cycle time T, and the stability of the timetable can be described as the relationship between T and T_0 . See also Goverde [5] for a discussion on timetable stability.

In this paper, the optimization model uses the minimum cycle time as the objective function, which then represents the feasibility and the stability of the timetable.

The complete optimization of a railway timetable of realistic size is an extraordinarily large mathematical problem. Smaller instances or smaller subproblems of the timetable optimization have been successfully solved before. The Periodic Event Scheduling Problem (PESP) was defined by Serafini and Ukovich [16]. Kroon et al. [9] developed a set of optimization tools which can find feasible solutions to the railway network of the Netherlands if it exists under the given initial parameters, or points to the critical constraints if a feasible solution does not exist. Goverde [3] defines an optimization problem with buffer times as decision variables and exploiting the graph structure of the network to reduce the number of variables. Liebchen and Mohring [12] optimized a homogeneous, high-frequency metro network. Liebchen et al. [13] define an opimization problem focusing on delay resistance, with predefined running times.

This paper presents an optimization approach for the generation of periodic timetables with flexible running times with timetable stability represented directly in the objective function. Therefore the method can be used both to generate timetable variants with different frequencies and the stability evaluation of these timetables. The new contribution of this paper is that the minimum cycle time is used as the optimization objective, with flexible running times for faster trains to allow for flexibility in case of dense following of trains of different speeds. The model ensures that train overtakings only happen at allowed locations whether at stations or on the open track. Furthermore, dimension reduction techniques are presented to speed up the optimization.

In the following, Section 2 describes how to model the railway network as an eventactivity graph. Section 3 introduces the optimization model including the proposed dimension reduction techniques. Section 4 applies the optimization model on a case study based on a real-life railway corridor with dense and heterogeneous traffic. Finally, Section 5 concludes the paper.

2 Modelling the railway network

2.1 Events and activities

The railway network is modelled on a macroscopic level including stations and junctions but not directly modelling signalling blocks. The line structure and the minimum headway times caused by limited infrastructure capacity can be modelled by a event-activity graph [4], also called an event-activity network [2]. Lef this network be $\mathcal{N} = (\mathcal{E}, \mathcal{A})$, where events can be arrivals and departures at stations and junctions. If the train does not stop at a node in the physical network, the arrival and departure events are classified *arr.thr* and *dep.thr*:

$$\mathcal{E} = \mathcal{E}_{dep} \cup \mathcal{E}_{arr} \cup \mathcal{E}_{dep.thr} \cup \mathcal{E}_{arr.thr}$$

The activities can be *dwell* activities \mathcal{A}_{dwell} connecting an event in \mathcal{E}_{arr} to an event in \mathcal{E}_{dep} , *run* activities \mathcal{A}_{run} connecting an event in \mathcal{E}_{dep} or $\mathcal{E}_{dep,thr}$ to an event in \mathcal{E}_{arr} or $\mathcal{E}_{arr,thr}$, *through* activities \mathcal{A}_{thr} connecting an event in $\mathcal{E}_{arr,thr}$ to an event in $\mathcal{E}_{dep,thr}$, *infrastructure* activities \mathcal{A}_{infra} between any two activities of different vehicle journeys symbolizing the minimum headway time between those events using the same physical infrastructure and *regularity activities* \mathcal{A}_{reg} taking into account different train lines offering a regular-interval service on their shared sections:

$$\mathcal{A} = \mathcal{A}_{\mathrm{dwell}} \cup \mathcal{A}_{\mathrm{run}} \cup \mathcal{A}_{\mathrm{thr}} \cup \mathcal{A}_{\mathrm{infra}} \cup \mathcal{A}_{\mathrm{reg}}.$$

We only consider a subnetwork for which the border nodes of this subnetwork can be considered to have sufficient capacity for turning. We assume therefore that the existing timetable is operated outside this subnetwork, and any train of the existing timetable which is cancelled inside the subnetwork, and any new train within the subnetwork, can turn at these border nodes. This means that there is no need to define turn activities.

2.2 Graph definition

We define T_0 as the nominal period length of the regular interval timetable to be designed, in most cases the full hour. The generated timetables can have a cycle time different from T_0 , denoted by T.

The train lines to be included are defined by their stop patterns as one chain of events in \mathcal{E} per direction connected by activites in $\mathcal{A}_{dwell} \cup \mathcal{A}_{run} \cup \mathcal{A}_{thr}$. For every line, a required frequency F is defined. If F > 1 then the event chain is duplicated into further runs with time offsets $\frac{1}{F}T$, $\frac{2}{F}T$... $\frac{F-1}{F}T$ and between every identical event of consecutive runs, a regularity activity $a \in \mathcal{A}_{reg}$ is added with process time $\frac{1}{F}T$. Furthermore, if a common infrastructure resource means a required minimum headway time between certain events, then an infrastructure activity $a \in \mathcal{A}_{infra}$ is added between all these event pairs.

3 The optimization model

This section presents a Mixed-Integer Linear Programming (MILP) model which takes the event-activity graph of a train network with given line structures and frequencies and calculates the event times that minimizes the cycle time of the timetable.

3.1 Variables

Unlike most timetabling models, in this optimization model the period length of the regular interval timetable, the cycle time, is variable, denoted as T. This means that when event timestamps and process durations are constrained within the interval [0, T), the upper bound of this interval becomes an additional constraint in the MILP formulation including two variables.

The variable x_i represents the planned event time for every event $i \in \mathcal{E}$ and $0 \leq x_i < T \quad \forall i \in \mathcal{E}$.

The variable w_{ij} denotes the planned process duration, for every process $(i, j) \in \mathcal{A} \subset \mathcal{E} \times \mathcal{E}$ and therefore

$$w_{ij} = x_j - x_i \pmod{T}.$$
 (1)

As w_{ij} represents a process duration, it is necessary that for every process $0 \le w_{ij}$. Furthermore, from (1) it follows that $w_{ij} < T$. In case there is a process for which the duration can be larger than the cycle time, this process can be divided into multiple shorter processes inserting additional events to ensure that the new process durations are less than the cycle time.

By introducing the binary variables z_{ij} , (1) can be rewritten as the following equation:

$$w_{ij} = x_j - x_i + z_{ij}T \quad \forall (i,j) \in \mathcal{A}, \quad z_{ij} \in \{0,1\}$$
 (2)

3.2 Process duration bounds

The parameters contained in the first instance of the optimization model are the process duration lower and upper bounds. These constants l_{ij} , u_{ij} for a process (i, j) can be defined as follows.

Dwell activities The minimum dwell time should be an estimate of the minimum duration required for the boarding and alighting of passengers, taking into account the estimated demand based on station type and train type. The maximum dwell time is the highest acceptable duration a train can stay at a platform on an intermediate stop.

Run activities The minimum running time of a train on a given segment between two geographical points, stations, junctions or other imporant locations, is the shortest running time possible, i.e. excluding time reserve, on the segment in moderate weather conditions taking into account the stop pattern of the line and the expected rolling stock dynamics. It can be an output of measurements or a simulation model. The maximum running time is the minimum running time plus the maximum acceptable time reserve on the segment.

Through activities Through activities connect the arrival and the departure events of a train at a location where the train does not stop, therefore these activities have a constant minimum and maximum process time $l_{ij} = u_{ij} = 0$.

Infrastructure activities Minimum headway times represent the constraints when events of two trains have to be separated in time because of a shared infrastructure resource. In a macroscopic model including stations and also junctions and other important infrastructure

locations, however, not modelling signalling blocks and station areas in detail, these infrastructure restrictions can be modelled by adding a headway process between all train event pairs using a shared resource. Because infrastructure constraints are defined in pairs such as (i, j) and (j, i), it is not necessary to define upper bounds.

Regularity activities If a train line is planned with a regular interval timetable with a headway time lower than the cycle time, then this headway time can be expressed by adding *regularity* constraints between the identical events of multiple runs of the same line. In a timetable with a period length T_0 and a line with a frequency F > 1, these regularity activities have a constant value T_0/F . However, as T is a variable, the duration of the regularity processes also becomes variable T/F.

3.3 Handling train overtakings

Because of the way infrastructure constraints are represented in the above model, with headway constraints between event times without fixed train order, in case of certain parameters, it is possible that a solution is generated including train overtakings at locations where this is not physically possible. The current model formulation, however, is able to implicitly respect also these constraints, if the following observations are followed.

The constraints of limited infrastructure on the railway network can be classified as follows:

Headway times when entering or exiting stations or junctions These can be natively modelled by two infrastructure activities $(i, j), (j, i) \in A_{infra}$ between departure or arrival events *i* and *j* at the same station or junction.

Headway times of following trains on the open track Kroon and Peeters [10] have shown that in case of flexible running times with lower and upper bounds, it is possible to ensure that no prohibited overtaking takes place by introducing further through events if necessary. We furthermore show that it is sufficient to split the segment into

$$n = \left[\frac{u_1 - l_2}{2\min\left(l_{12}^d, l_{21}^a\right)}\right] \tag{3}$$

shorter segments to prevent train 2 to overtake train 1, where u_1 is the upper bound of the running time of train 1, l_2 is the lower bound of the running time of train 2, l_{12}^d is the departure headway between train 1 and train 2 at the beginning of the segment and l_{21}^a is the arrival headway between train 2 and train 1 at the end of the segment. This is true because a forbidden overtake can take place if

$$u_1 \ge l_2 + l_{12}^d + l_{21}^a,\tag{4}$$

or in words, if the slower train is allowed to have such a long running time that allows for the minimum departure headway, the minimum run time of the overtaking train and the minimum arrival headway. If we split the segment into n shorter segments with each segment the updated runing times $l'_2 = l_2/n$, $u'_1 = u_1/n$ and headway times $l'_{12} = l'_{21} = \min(l_{12}^d, l_{21}^a)$, then if n is chosen according to (3) then (4) becomes false for all new segments. Note that in practice, if the running time upper bounds are not much larger than the lower bounds, then most often n = 1, i.e. there is no need for splitting the segment.

Headway times of following trains through stations Analogous to the previous paragraph, it can be shown that train 2 cannot overtake train 1 at a station if the dwell processes are split into n equal processes according to (3) where in this case u_1 and l_2 are the dwell time bounds. n is in most cases zero again if the dwell time buffers are not too large.

Headway times of following trains on the open track Similarly for train overtakings through stations, it can be shown that the boundary values can reasonably be set in a way that prohibited overtakings on the open track do not happen, or else adding intermediate dummy through events to both trains with infrastructure activities between them. See Kroon and Peeters [10] for dealing with train overtakings using variable trip times in the PESP formulation.

3.4 The MILP formulation

In the following, we introduce a basic formulation of the optimization model that we transform into a true MILP model in a second step.

$$\min T$$
 (5)

such that

$$w_{ij} = x_j - x_i + z_{ij}T \quad \forall (i,j) \in \mathcal{A}, \tag{6}$$

$$x_i < T \quad \forall i \in \mathcal{E},\tag{7}$$

$$w_{ij} < T \quad \forall (i,j) \in \mathcal{A}, \tag{8}$$

$$l_{ij} \le w_{ij} \le u_{ij} \quad \forall (i,j) \in \mathcal{A}_{\text{run}} \cup \mathcal{A}_{\text{dwell}},\tag{9}$$

$$l_{ij} \le w_{ij} \quad \forall (i,j) \in \mathcal{A}_{infra}, \tag{10}$$

$$0 = w_{ij} \quad \forall (i,j) \in \mathcal{A}_{\text{thr}},\tag{11}$$

$$w_{ij} = \frac{1}{F_{ij}}T \quad \forall (i,j) \in \mathcal{A}_{\text{reg}},\tag{12}$$

$$z_{ij} \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}.$$

$$(13)$$

$$0 \le x_i \quad \forall i \in \mathcal{E},\tag{14}$$

$$0 \le w_{ij} \quad \forall (i,j) \in \mathcal{A},\tag{15}$$

The objective function (5) minimizes the cycle time of the timetable. Therefore if a solution is found, it represents a timetable that is feasible if and only if $T \le T_0$ where T_0 is the nominal cycle time. A measure of timetable stability is the $T_0 - T$ difference between the scheduled and the minimums cycle time: the larger, the more time reserve the timetable contains.

Constraints (6) and (13) define the relationships between process durations and event times identical to the earlier (2). The range of event times and process durations, dependent on the variable cycle time, are defined in (7), (8), (14) and (15). The lower and upper bounds of run, dwell and infrastructure processes, as well as the cycle time-dependent value of the regularity process durations, are given in equations (9)–(12).

In the following, modifications of the above model definition are explained in order to conform with the requirements of an MILP model.

Because the cycle time is a variable in this model, constraint (6) contains the product $z_{ij}T$ of two variables, which violates the MILP conditions. Let u be an upper bound for the objective value T, if this leads to infeasibility the upper bound can be increased. $u = T_0$ would be a suitable upper bound restricting the model to only return feasible timetables, but one might prefer to choose a larger value to retrieve solutions with $T_0 < T$ which, even though being infeasible, can describe the limits of the requested timetable plan, for example by showing the critical headways. A product of the binary variable z and a bounded continuous variable $0 \le T \le u$ can be reformulated as the following four linear constraints using the new variable y = zT [15, 17]:

$$y \le uz,\tag{16}$$

$$y \le T,\tag{17}$$

$$y \ge T - u\left(1 - z\right),\tag{18}$$

$$y \ge 0. \tag{19}$$

Constraints (7) and (15) include a strict inequality relating to cycle time T. This can be replaced by a non-strict inequality with $T - \delta$ where δ is a suitably small value, such as 1 second.

In order to write constraints (9)-(11) and (15) more concisely, we define

$$u_{ij} = u \quad \forall (i,j) \in \mathcal{A}_{infra} ,$$

 $u_{ij} = l_{ij} = 0 \quad \forall (i,j) \in \mathcal{A}_{thr}$

and ensure that $\ \ l_{ij} \geq 0 \quad \forall \, (i,j) \in \mathcal{A} \quad .$

Consequently, the rewritten MILP formulation is as follows:

$$\min T \tag{20}$$

such that

$$x_i \le T - \delta \quad \forall i \in \mathcal{E},\tag{21}$$

$$w_{ij} \le T - \delta \quad \forall (i,j) \in \mathcal{A}, \tag{22}$$

$$w_{ij} = x_j - x_i + y_{ij} \quad \forall (i,j) \in \mathcal{A},$$
(23)

$$y_{ij} \le u z_{ij} \quad \forall (i,j) \in \mathcal{A}, \tag{24}$$

$$y_{ij} \le T \quad \forall \, (i,j) \in \mathcal{A},\tag{25}$$

$$y_{ij} \ge T - u \left(1 - z_{ij}\right) \quad \forall (i, j) \in \mathcal{A}, \tag{26}$$

$$0 \le T \le u,\tag{27}$$

$$w_{ij} = \frac{1}{F_{ij}}T \quad \forall (i,j) \in \mathcal{A}_{\text{reg}},\tag{28}$$

$$l_{ij} \le w_{ij} \le u_{ij} \quad \forall (i,j) \in \mathcal{A}_{\rm run} \cup \mathcal{A}_{\rm dwell} \cup \mathcal{A}_{\rm infra} \cup \mathcal{A}_{\rm thr},\tag{29}$$

$$0 \le x_i \quad \forall i \in \mathcal{E}, \tag{30}$$

$$z_{ij} \in \{0,1\} \quad \forall (i,j) \in \mathcal{A},\tag{31}$$

$$0 \le y_{ij} \quad \forall (i,j) \in \mathcal{A}. \tag{32}$$

The differences in (21)-(32) compared to (5)-(13) are as follows. In (21)-(22) the strict inequalities of (7)-(8) are replaced by non-strict ones. In (23), the new variables y_{ij} replace the product of variables $z_{ij}T$ in (6). $y_{ij} = z_{ij}T$ is ensured by (24)-(27) and (31)-(32). Constraint (29) sums up previous constraints (9)-(11).

3.5 Interpreting the optimization model results

The goal of the above optimization model is to calculate the minimum cycle time of all timetables following a predefined line structure, nominal cycle time T_0 , stop patterns, frequencies and constraints regarding running times, dwell times and infrastructure capacity.

If a solver finds an optimal solution for the model with calculated cycle time T, for which $T \leq T_0$, that means that feasible timetables exist given the parameters and one such timetable, with optimal minimum cycle time, is $\mathbf{x} = [x_1 \dots x_{|\mathcal{E}|}]$. If, however $T > T_0$, then no such feasible timetable exists.

If a solver results in infeasibility, then no feasible timetable exists with a cycle time $T \leq u$.

If within a given time frame only a feasible, possibly non-optimal solution is found, then this solution represents a feasible timetable within cycle time t and the calculated cycle time serves as an upper bound to the minimum cycle time of all timetables given the parameters.

Finally, if a solver does not return either a solution or a message of infeasibility within a time frame, no such conclusions can be made. In this case, the following dimension reduction techniques are helpful.

3.6 Dimension reduction techniques

The following additional steps aim ad reducing the search space of the MILP model.

Connected components

In the graph representation, different train lines are connected to each other because of the infrastucture constraints. However, it can still be that the graph is separable to multiple connected components, for example if a train line is operated independently of other lines or if the two directions of a double-track railway line have no shared resources. As one connected component has no effect on the others, the optimization model could be executed separately for each connected component.

Greatest common divisor of frequencies

In case of regular-interval timetables, often many train services have a headway time of less than the full hour, namely 30, 20, 15 or sometimes even 10 minutes. If the greatest common divisor g of all line frequencies is larger than 1, the timetable can be calculated with updated frequencies F' = F/g. This reduces the number of events by a factor of 1/g and reduces the number of processes even to a larger extent, because of the structure of the infrastructure

and

processes connecting all events related to a given resource. The new calculated timetable with cycle time t can then be simply scaled back to the original frequencies by duplicating the events at times x_i to new times $x_i + \frac{1}{g}T$, $x_i + \frac{2}{g}T$... $x_i + \frac{g-1}{g}T$ and the re-scaled timetable has a minimum cycle time $g \cdot T$.

Identical timetables

Two constraints are added to avoid multiple identical solutions. Firstly, if two solutions are identical except a uniform shift in time for all event times, they also represent timetables with identical characteristics and minimum cycle time. Therefore, it is possible without loss of generality to choose a single event and fix its event time. Therefore if $I = \{e_{init}\}$ denotes the set containing this one initial event,

$$x_{\text{init}} = 0. \tag{33}$$

The other constraint considers train lines with frequencies larger than one. In this case, a new timetable created by shifting the event times of this line with a multiple of the headway time T/F is identical to the original timetable. Therefore if \mathcal{F} is a set containing one event for each train line and F_i is the frequency of the line of event i,

$$x_i < \frac{T}{F_i} \quad \forall i \in \mathcal{F}.$$
(34)

Similarly to constraint (21), the strict inequality can be replaced by a non-strict one by replacing T with $T - \delta$. Note that there is no need to restrict constraint (34) to the lines with frequencies larger than one: if $F_i = 1$ then this constraint degenerates to the existing (21).

Infrastructure constraint pairs

As train orders are flexible during optimization, infrastructure constraints are defined between all pairs of events sharing the same resource. This means that infrastructure constraints exist in pairs, such as

$$\begin{cases} l_{ij} \leq w_{ij} \\ l_{ji} \leq w_{ji} \end{cases} \} \quad \forall (i,j) \mid \{(i,j), (j,i)\} \subseteq \mathcal{A}_{infra} \end{cases}$$

where $w_{ij} + w_{ji} = T$. Therefore for each pair, it is possible to eliminate variables w_{ji} and y_{ji} to turn the lower bound of w_{ji} into an upper bound of w_{ij} :

$$l_{ij} \le w_{ij} \le T - l_{ji}.$$

Note that this is very similar to the classical PESP constraint, except that in our case T is a variable. More formally, aligning variables on the left side and parameters on the right side:

$$\begin{cases} w_{ij} \ge l_{ij} \\ T - w_{ij} \ge l_{ji} \end{cases} \quad \forall (i,j) \mid (i,j) \in \mathcal{A}_{infra} \cap i < j ,$$

where i < j ensures that exactly one of each infrastructure event pair is taken into account.

4 Case Study

An example of the above model is shown for an approximately 60 km rail corridor between Den Haag CS and Utrecht railway stations in the Netherlands. This line is a double-track

	train	hourly frequency								
	nr.	orig.	S 1	S 2	S 3	S4	S5			
Den Haag CS–Utrecht	150	4	4		1	C	4			
Intercity	150 4		4	-	1	2	4			
Den Haag CS–Utrecht	640/	2	4	6	6	4+2	2			
Local train	6400									
(Rotterdam–)Gouda–Utrecht	127	4					4			
Intercity	137	4	-	-	-	-	4			
(Leiden-)Woerden-Utrecht	310	2	-	-	-	-	2			
Intercity										
(R'dam-)Gouda-Woerden(-Amsterdam)	632	2	-	-	-	-	2			
Local train										
(Rotterdam–)Gouda–Gouda Goverwelle	635	2	-	-	-	-	2			
Local train										
Den Haag CS–Gouda Goverwelle	645	2	-	-	-	-	2			
Local train										

Table 1: Train line frequencies in scenarios S1-S5

line with a dedicated track in each direction with overtaking possibilities at the four track sections between stations Gouda and Gouda Goverwelle and between stations Woerden and Vleuten. The schematic track layout in one direction is shown in Figures 1–3. In this case, diverging and merging railway tracks are not further modelled, but in the last scenario, relevant sections of merging and diverging train lines are included.

4.1 Line structure

Based on the actual 2007 timetable of the line, the lines used in the defined scenarios are listed in Table 1. The notation "4+2" means that 6 trains per hour are defined as two regular interval services with frequencies 4 and 2 with no restrictions for regularity times between the two groups.

Scenario S1 is an example for a "balanced" timetable with 4 intercity and 4 local trains per hour. Scenarios S2-S4 all include a local train frequency increased to 6 per hour, with 0, 1 and 2 intercity trains per hour respectively, and for S4, relaxing the regularity for the local trains. Finally, Scenario S5 has the same train lines and frequencies as the original timetable.

4.2 Defining activity duration bounds

Timetable data including minimum process times is used from the timetable stability analysis tool PETER (see Goverde and Odijk [6]), including pre-processed data based on the timetable planning system DONS (see Hooghiemstra et al. [8]) and the simulation tool SI-MONE (see Middelkoop [14]). As DONS models the railway network not only as stations and track segments connecting stations, but also including junctions, bridges and other locations that can be infrastructure bottlenecks, this makes it possible to model the infrastructure constraints more realistically than station-only models, without the need for microscopic simulation.

Dwell activities The lower and upper bound of dwell activity durations are defined as the minimum and the scheduled dwell times, respectively, from the DONS model. Note that in many cases then $l_{ij} = u_{ij}$.

Run activities Similar to dwell times, the lower bounds of run processes are defined as the minimum run times given from the DONS model. The upper bounds of run processes are equal to the lower bound for local trains and are 25% larger than the lower bound for Intercity trains to allow for more dense following of local and Intercity trains. Therefore by allowing a flexibility in some run times, this model is an extension to DONS.

Through activities Through activities have a constant minimum and maximum process time $l_{ij} = u_{ij} = 0$.

Infrastructure activities Minimum headway times are defined between all activity pairs which use the same resource on the train paths. No upper bound is defined for headway times.

Regularity activities For lines with frequencies larger than one, regularity activities are defined as described in Section 2.2.

4.3 Results

The MILP models were solved by CPLEX version 12.4 on an IBM PC with 12 GB RAM and a six-core 3.47 GHz CPU. The time needed to find the optimal solution, the time needed to prove its optimality, and the calculated minimum cycle time with nominal cycle time $T_0 =$ 1h is shown in Table 2. To give an example of the effectiveness of the dimension reduction methods, computational times are also given *without* processing the infrastructure constraint pairs as described in Section 3.6. The time-distance diagram of scenarios S1, S4 and S5 in one direction are shown in Figures 1–3: local trains are shown with continuous lines and intercity trains with dashed lines with one color per line number, displayed along the lines. The time axis shows one cycle of the length of the minimum cycle time. displayed at the maximum of the time axis. Finally, where the trains are only separated by the minimum infrastructure constraints, this is shown with horizontal black dotted lines.

For these case study instances, the solver was able to find the optimal solutions typically in seconds and in less than 10 minutes for the most complex case. Proving optimality, however, can take much more time than finding the optimal solution. We note that for larger instances, even if the solver cannot prove optimality or infeasibility, the intermediate solver state with a solution with a cycle time less than nominal cycle time, or with an LP bound larger than the nominal cycle time, already proves that a feasible timetable exists or does not exist, respectively.

While optimality proves that no timetable with a lower cycle time exists for the given parameters, it is the relation of the cycle time to the nominal cycle time which describes the feasibility and the stability of the timetable. In our case study, all scenarios but S3 have a

Table 2: Computational results (h:mm:ss)										
	S 1	S2	S3	S4	S5					
Cycle time	45:12	48:00	1:07:48	57:12	57:12					
Time to solution	00:01	<00:01	00:03	00:06	09:55					
Time to prove opt.	00:01	<00:01	00:03	00:48	39:29					
Without preprocessing infrasctructure constraint pairs:										
Time to solution	00:03	<00:01	00:10	00:45	22:41					
Time to prove opt.	00:03	<00:01	00:10	01:39	>60:00					

cycle time less than one hour, this means that feasible timetables exist for all scenarios but S3, and one with minimal cycle time is calculated by the solver.

The minimum cycle time can be seen as a measure of timetable stability: the lower the cycle time, the faster the timetable can recover from disturbances. This means that S1 is substantially more stable than the rest, at the cost of not using full capacity. On the other hand, as S4 and S5 have the same minimum cycle time, this implies that although S5 contains many more train services per hour, its stability is still comparable to that of S4.

From these examples we can conclude that our approach can decide whether a feasible timetable exists for given line patterns, and calculate a timetable optimized for stability, with favorable computational times. Furthermore, we suggest that the approach could help decide the timetable feasibility also for much larger instances, even when finding or proving an optimal timetable might not be possible in acceptable time.



Figure 1: Time-distance diagram of scenario S1



Figure 2: Time-distance diagram of scenario S4



Figure 3: Time-distance diagram of scenario S5

5 Conclusion

In this paper we presented an optimization model for the design of a regular interval railway timetable for given line patterns and frequencies and with the minimum cycle time as objective function. This makes it possible to simultaneously decide if there exists a feasible timetable under current conditions and evaluate the stability of the timetable using the minimum cycle time.

A few techniques are presented to reduce the size of the problem. The performance of the model is presented in a case study of a heavily utilized railway corridor with heterogeneous, merging and diverging traffic. The solver was unable to prove optimality for all defined scenarios but a feasible timetable and therefore also an upper bound for the minimum cycle time was calculated for the cases without an optimal solution.

Future research shall focus on further dimension reduction techniques to enable the approach to perform successfully on larger instances.

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