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ABSTRACT

Three theoretical models of plane flames burning on a cooled porous-plug type of flame-holder are reviewed and compared with experimentally observed relationships between stand-off distance, flame speed and temperature.

It is shown that for most practical burners their conductance is large and that for near adiabatic conditions, the order of the non-dimensional stand-off distance ceases to be $O(1)$, but is $O(\ln\Theta)$ where Θ is the non-dimensional activation energy.

CONTENTS

1. INTRODUCTION

The theoretical understanding of the behaviour of premixed flames with heat loss is a subject now receiving quite a lot of attention (Buckmaster and Ludford 1982) particularly in the light of recent advances in the use of large activation energy asymptotic theory (Sivashinsky 1983). One of the most practical ways of observing flat flames in a laboratory is to anchor such flames to a porous-plug flame-holder the characteristics of which were first described by Hirschfelder, Curtiss and Cambell (1953). Such a flame-holder was designed and used in experimental tests by Botha and Spalding (1954) and more recently by Ferguson and Keck (1979). The heat loss of such flames is due to conduction to the holder and can have a marked affect on flame speed and flame temperature.

Various theories have recently been put forward to model burner anchored flames under steady conditions, the implications of which are not always the same. Theoretical analyses differ in particular in the way heat losses have been modelled. Carrier, Bush and Fendell (1978) use a Dirac- δ function heat sink in the preheat zone whereas the model used by Clarke and McIntosh (1980) adopts the flame-holder description advocated by Hirschfelder. Essentially these two models are shown to be in close agreement except in the resolution of the so-called 'cold boundary difficulty' (Williams 1965; p. 109). However the modifications made to the Dirac- δ function model by Matkowsky and Olagunju (1981) are shown to produce results which are different in some important respects. Therefore it is the purpose of this review to briefly summarise all these theories and then compare them with the empirically derived relationships between stand-off distance, flame speed and flame temperature. In that theories of the behaviour of flames under unsteady conditions are being built upon these basic steady solutions, it is vital that a realistic model is chosen.

This paper is meant to serve as a review and consequently only the main results of the theories considered are shown. For the detailed derivation of these results the reader is referred to the original papers (Carrier, Bush and Fendell 1978, Ferguson and Keck 1979, Clarke and McIntosh 1980, Matkowsky and Olagunju 1981).

2. NOTATION AND BASIC ASSUMPTIONS

To avoid confusion, the following set of symbols is used throughout to define the major quantities involved in this review.

C_p'	overall coefficient of specific heat
ρ'	mixture density
λ'	thermal conductivity of mixture
$\mathcal{D}'$	diffusion coefficient
Le	Lewis Number $\equiv \dfrac{\rho'\mathcal{D}'C_p'}{\lambda'}$
V'	mixture velocity
M'	Mass flux $= \rho'V' = \rho_0'V_0'$ (Inlet Mass Flux)
M_a'	Adiabatic mass flux
T_b'	Non-adiabatic flame temperature
T_a'	Adiabatic flame temperature
x_f'	Flame stand-off distance
E_A'	Overall Activation Energy
R'	Universal Gas Constant
θ_1	Non dimensional activation energy (based on T_b') $\equiv \dfrac{E_A'}{R'T_b'}$
θ	Non dimensional activation energy (based on T_a') $\equiv \dfrac{E_A'}{R'T_a'}$
θ^*	Non dimensional activation energy (based on $T_b'-T_{ud}'$) $\equiv E_A'/R'(T_b'-T_{ud}')$
T_h'	Upstream holder temperature
T_D'	Downstream holder temperature

P_e Pedet number $\equiv M_o' C_p' \int_0^{x_f'} \frac{dx'}{\lambda'}$ (chap. 3)

y_f Non dimensional flame stand-off distance $\equiv \frac{M_o' C_p'}{(\rho'\lambda')Le} \int_0^{x_f'} \rho' dx'$ (chap. 4)

q_o' Heat loss to flame-holder per unit area per unit time (chap. 4)

K' Conductance of flame-holder (i.e. thermal conductivity of holder divided by the width)

K Non dimensional conductance $\equiv \frac{K'}{C_p' M_o'}$ (chap. 4)

ξ_f Non dimensional flame stand-off distance $\equiv \frac{M_o' C_p'}{Le} \int_0^{x_f'} \frac{dx'}{\lambda'}$ (chap. 5)

T_{ud}' Far upstream temperature (chap. 5)

T_{od}' Mixture temperature at holder (chap. 5)

T_{cd}' 'Characteristic' temperature of holder in heat loss term added to energy equation (chap. 5)

K_d' Heat transfer coefficient of holder in heat loss term added to energy equation (chap. 5)

K_d Non dimensional heat transfer number $\equiv \frac{K_d'}{C_p' M_o'}$ (chap. 5)

Dashed ($'$) symbols always represent dimensional quantities. The subscripts ('o') and ('b') denote that the relevant quantity is evaluated immediately downstream of the holder and in the burnt stream respectively.

In this brief review, the assumption is made that mixture strength is far from stoichiometric and constant throughout. Neither assumption is vital but they help to focus attention on the differences in the models used by current authors. A schematic of the flame/flame-holder system is given in Fig. 1.

3. EMPIRICAL RELATIONSHIPS FOR STAND-OFF DISTANCE, FLAME SPEED AND FLAME TEMPERATURE

In practical experiments, one specifies mixture strength and mass flux (M_o') at the flame-holder. The flame-holder will have certain characteristics which, given an upstream temperature (T_h') will determine the downstream face temperature (T_o') of the porous plug through which the gas mixture flows. Given these inlet conditions, there will be a stand-off distance (x_f') and flame temperature (T_b') . Ferguson and Keck (1979) use the empirical relationship of Kaskan (1957),

$$\frac{M_o'}{M_a'} = \exp\left[-\frac{E_A'}{2R'T_a'T_b'} (T_a' - T_b') \right] , \tag{3.1*}$$

to link flame temperature (T_b') and inlet mass flux (M_o'). Using the energy equation, they then derive a result for the Peclet number (P_e),

$$P_e \equiv M_o' C_p' \int_0^{x_f'} \frac{dx'}{\lambda'} = \ln\left(\frac{T_a' - T_o'}{T_a' - T_b'} \right) , \tag{3.2a,b}$$

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*The mass flux M_o' is given by Clarke and McIntosh (1980) in the form

$$M_o' = (\text{constant})\, T_b'^2 \exp(-E_A'/2R'T_b') ,$$

where T_b' is the burnt temperature under non-adiabatic conditions. It is not difficult to calculate a burnt, or final, temperature value under adiabatic conditions and quite independently of any flow/flame geometry or of the presence of a flame-holder. This value is T_a' . When T_a' is substituted into the above equation, M_o' is equal to M_a' , and this is what we mean here by the adiabatic mass flux M_a'.

However M_a' is a fiction in the case of the present flame/flame-holder configuration, albeit a useful one, since there is no <u>theoretical</u> limit to the input mass flux at the holder. It has been shown by Clarke (1983) that significant structural changes occur when M_o' approaches and exceeds M_a' .

which links stand-off distance (x_f') to flame temperature (T_b') assuming a constant value of T_o'. They found that in their particular experiments, variations in T_o' were small, and that these two relationships matched experimental results to a high degree of accuracy. They also define a modified Peclet number (P_e^*) based on adiabatic flame speed,

$$P_e^* \equiv M_a' C_p' \int_0^{x_f'} \frac{1}{\lambda'} dx' = \exp\left[\frac{E_A'}{2R'}\left(\frac{1}{T_b'} - \frac{1}{T_a'}\right)\right] \ln\left(\frac{T_a' - T_o'}{T_a' - T_b'}\right), \quad (3.3)$$

and, as will be seen below, this is used to compute actual stand-off distances. Differentiation of (3.3) with $\partial x_f'/\partial T_b'$ set to zero yields an approximation to the distance of closest approach of the flame to the holder for a given composition. For this particular condition, T_b' is very close to T_a'.

4. HIRSCHFELDER MODEL OF FLAME-HOLDER: SUMMARY OF MAIN RELATIONSHIPS OBTAINED

The reader is referred to the earlier paper by Clarke and McIntosh (1980) for a full derivation of the main relationships obtained for flame-speed, flame temperature and stand-off distance. The theory uses the tool of matched asymptotic expansions based on large activation energy $(\theta_1 \equiv E_A'/R'T_b')$ to derive the connection between mass flux (constant , M_o') and flame temperature. Under far from stoichiometric conditions, one obtains

$$\frac{M_o'}{M_a'} = \left(\frac{T_b'}{T_a'}\right)^2 \exp\left[\frac{-E_A'}{2R'T_a'T_b'}(T_a'-T_b')\right] . \qquad (4.1)$$

One immediately sees the similarity between (4.1) and (3.1). The experimental measurements can easily miss the comparitively mild algebraic factor in the face of the strong exponential dependence on T_b' (particularly for $E_A'/R'T_b' >> 1$ as is generally the case).

A non dimensional stand-off distance is defined as,

$$y_f \equiv \frac{M_o' C_p'}{(\rho'\lambda')L_e}\int_0^{x_f'} \rho' dx' , \qquad (4.2)$$

and it is assumed in this theory that,

$$(\rho'\lambda') = \text{constant} . \qquad (4.3)$$

This assumption is in fact close to reality. Density ρ' is inversely proportional to temperature T' for these essentially isobaric flames and it has been observed that thermal conductivity λ' is proportional to T'^h with $0.75 \lesssim h \lesssim 0.94$ (Hirschfelder, Curtiss and Bird (1954), Kanury (1975)). Use of this further assumption shows that $L_e y_f$ defined in (4.2) is identical to the Peclet number defined in (3.2a).

At the holder it is assumed no product species diffuse back upstream (Hirschfelder condition), and the heat loss is linked directly to the temperature gradient at the holder. Thus

$$q_o' \equiv \lambda_o' \frac{dT'}{dx'}\bigg|_{x'=0} = k'(T_o' - T_h') \quad , \qquad (4.4a,b)$$

where k' is the conductance of the holder, and the theory allows for the temperature T_o' on the downstream side of the holder to vary whilst keeping the temperature T_h' on the upstream side fixed. It is found that

$$Le\, y_f = \ln\left[K^{-1}\left(\frac{T_a' - T_h'}{T_o' - T_h'}\right)\right] = \ln\left[(1-K^{-1})\left(\frac{T_a' - T_h'}{T_a' - T_b'}\right)\right] , \qquad (4.5a,b)$$

with

$$K \equiv \frac{k'}{C_p' M_o'} = \frac{(T_o' - T_h') + (T_a' - T_b')}{(T_o' - T_h')} . \qquad (4.6a,b)$$

Another form of equation (4.5a) eliminates K to give,

$$Le\, y_f = \ln\left[\frac{(T_a' - T_h')}{(T_a' - T_b') + (T_o' - T_h')}\right] . \qquad (4.7)$$

This general result for stand-off distance becomes the result (3.2b) for the case when $T_o' = T_h'$ (i.e. $k' = \infty$, q_o' finite; see (4.4b)).

The dimensional stand-off distance can be obtained by reversing the definition (4.2). Thus,

$$x_f' = \frac{\lambda_o' Le}{T_o' M_o' C_p'} \int_0^{y_f} T' dy \quad , \qquad (4.8)$$

where T' in the pre-heat zone is given by,

$$T' = T_b' - \left(\frac{T_b' - T_0'}{1 - e^{-Le\,y_f}}\right)\left(1 - e^{Le(y - y_f)}\right) \quad . \tag{4.9}$$

Equation (4.8) then yields,

$$x_f' = \frac{\lambda_0'}{T_0' M_0' C_p'}\left[(T_b' - T_0') + \left(\frac{T_0' - T_b' e^{-Le\,y_f}}{1 - e^{-Le\,y_f}}\right) Le\, y_f\right] \quad , \tag{4.10}$$

which with (4.1), (4.7) and for $T_0' = T_h'$ yields:

$$x_f' = \frac{\lambda_0'}{T_0' M_a' C_p'}\left(\frac{T_a'}{T_b'}\right)^2 \exp\left[\frac{E_a'}{2R'}\left(\frac{1}{T_b'} - \frac{1}{T_a'}\right)\right]\left\{(T_b' - T_0') + \left[\frac{T_0'(T_a' - T_0') - T_b'(T_a' - T_b')}{(T_b' - T_0')}\right] \ln\left(\frac{T_a' - T_0'}{T_a' - T_b'}\right)\right\} \quad . \tag{4.11}$$

The results obtained from (4.11) are very similar in form to those obtained by using (3.3). The essential functional form comes from the exponential and logarithmic term. The other algebraic terms only slightly alter the curves. The minimum stand-off distance is predicted to occur for quite small heat losses, and with T_b' close to T_a' . This is in agreement with the findings of Ferguson and Keck in their experiments. Note however that though x_f' is at a minimum, y_f need not be small. (cf. P_e^* and P_e respectively in Chap. 3). The reader is referred to Fig. 1 in Ferguson and Keck (1979), Figs. 7a,b in Clarke and McIntosh (1980), and Fig. 2 (P.30) of Buckmaster and Ludford (1982).

The two basic results are (4.1) and (4.7), which except for the algebraic dependence in flame speed are identical to the empirical relationships (3.1) and (3.2b) for T_0' assumed constant. Such close agreement underscores the essential correctness of the Hirschfelder model of the flame/flame-holder system and the valuable insight a proper application of this model can give to the understanding of flame behaviour.

5. DIRAC- δ MODEL OF FLAME-HOLDER: SUMMARY OF MAIN RELATIONSHIPS OBTAINED

In this section, the reader is referred to Fig. 2 which illustrates the approach used by Carrier, Bush and Fendell (1978) to model the flame-holder. The holder is represented by a δ -function heat sink situated within the inert pre-heat domain, with stand-off distance defined as the distance between the flame sheet and the heat sink.

For an order 2 reaction and far from stoichiometric conditions, using matched asymptotic expansions based on the largeness of $\Theta^* \equiv E_A'/R'(T_b'-T_{ud}')$, the following relationship for mass flux is derived:

$$\frac{M_o'}{M_a'} = \left(\frac{T_b'-T_{ud}'}{T_a'-T_{ud}'}\right) \exp\left[\frac{-E_A'(T_a'-T_b')}{2R'(T_a'-T_{ud}')(T_b'-T_{ud}')}\right] . \qquad (5.1)$$

This relationship resembles (3.1) and (4.1) except that all temperatures are lowered by T_{ud}', the upstream temperature. This *ad hoc* addition to strict Arrhenius kinetics is necessary in this model in order to overcome the cold boundary difficulty at the far upstream boundary. The altered Arrhenius term is found in Equation (2.6) of Carrier et al (1978), and carries all the way through their analysis. Nevertheless as the authors point out, since $T_b', T_a' >> T_{ud}'$, (5.1) approximates well to (3.1) (Kaskan's observed relationship).

In deriving relationships for stand-off distance, the authors of this δ -function model, use a non dimensional stand-off distance variable defined (in our notation, and with the distance origin at the heat sink) as:

$$\xi_f \equiv \frac{M_o' C_p'}{Le} \int_0^{x_f'} \frac{dx'}{\lambda'} . \qquad (5.2)$$

In this theory, $(\rho'\lambda')$ is not assumed constant. One can in fact integrate the species and energy equations as long as $Le \equiv \rho'\mathcal{D}'c_p'/\lambda'$ is treated as a constant. However, in order to make comparisons with the Hirschfelder model one can assume $(\rho'\lambda')$ is constant with little loss of generality (see note in Chap. 4) which in (5.2) implies (see (4.2)),

$$\xi_f = y_f \qquad . \tag{5.3}$$

The energy loss at the holder is modelled by putting an additional term into the energy equation such that, in volumetric units,

Energy loss at holder per unit area per unit time $= K_d' (T_{od}' - T_{cd}')$ (5.4)

where K_d' is a heat transfer coefficient; the ratio

$$K_d \equiv \frac{K_d'}{c_p' M_o'} \quad , \tag{5.5}$$

is termed a heat-transfer number for the holder, and T_{cd}' is a characteristic temperature for the holder. For any apparatus, this model regards K_d and T_{cd}' as given values. Here we have purposely given a subscript 'd' to K', K, T_o' and T_c' in order to make it clear that it is the δ-function that is being referred to.

It is found that the non dimensional stand off distance ξ_f is given by, (see equation (4.7a,b) of Carrier et al (1978)),

$$Le\,\xi_f \; (= Le\,y_f) = \ln\left[\frac{(T_a' - T_{ud}')}{(T_a' - T_b') + (T_{od}' - T_{ud}')}\right] \quad , \tag{5.6}$$

and that (equation (2.20) in the same reference), K_d can be related to the temperatures as follows;

$$K_d = \frac{T_a' - T_b'}{T_{od}' - T_{cd}'} \quad . \tag{5.7}$$

Integrating the original energy equation yields a link between the jump in gradients across the heat sink and the temperature at the heat sink. One obtains,

$$\lambda_o' \frac{dT'}{dx'}\bigg|_{o+} - \lambda_o' \frac{dT'}{dx'}\bigg|_{o-} = K_d'(T_{od}' - T_{cd}') \quad , \tag{5.8}$$

where '$o+$', '$o-$' refer to just downstream and just upstream of the heat sink respectively.

If one now compares results (5.6 - 5.8) with (4.7), (4.6b) and (4.4b) of the Hirschfelder model, it becomes clear that to make proper comparisons, one should regard the following temperatures as equivalent:

$$T_{od}' = T_o' \quad , \tag{5.9a}$$

$$T_{ud}' = T_h' \quad , \tag{5.9b}$$

$$T_{cd}' = T_h' \quad . \tag{5.9c}$$

Thus the theory of Carrier et al (1978) allows some extra flexibility (in general) by having $T_{ud}' \neq T_{cd}'$. Using (5.9a-c), it becomes clear that the K (conductance) of the previous theory is linked to the present K_d (heat transfer number) (see equations (4.6b) and (5.7) above) by,

$$K_d = K - 1 \quad . \tag{5.10}$$

If one now assumes that $\lambda_o'(dT'/dx')_{x'=o}$ in (4.4b) is the same as $\lambda_o'(dT'/dx')_{x'=o+}$ in (5.8), then

$$\frac{dT'}{dx'}\bigg|_{o-} = \frac{c_p' M_o'}{\lambda_o'}(T_o' - T_h') \quad . \tag{5.11}$$

Equations (5.1) and (5.6) are now consistent with the empirical formulae referred to in chapter 3. When $K_d' \to \infty, T_{od}' \to T_{cd}'$ for a finite rate of heat loss (see (5.8)). In particular $(dT'/dx')_{0+}$ remains finite, but $(dT'/dx')_{0-}$ is zero (see (5.11)). The non dimensional stand-off distance from (5.6) then takes on the simple form (as in (3.2b)),

$$Le\,y_f = \ln\left(\frac{T_a' - T_o'}{T_a' - T_b'}\right) . \tag{5.12}$$

One can invert the definition (5.2), as done in Chapter 4 to obtain the dimensional stand-off distance. A similar relationship to (4.11) materialises and as stated previously, the essential functional form for dimensional stand-off distance is given by (3.3).

6. NEAR ADIABATIC CONDITIONS

In the previous chapters we have shown that the two significant relationships linking flame speed, stand-off distance and flame temperature are, in non dimensional terms, (see (4.1) and (4.7)),

$$M_o = T_b^2 \exp\left[-\frac{\Theta}{2T_b}(1-T_b) \right] , \qquad (6.1)$$

$$Le\, y_f = \ln\left[\frac{(1-T_h)}{(1-T_b) + (T_o - T_h)} \right] , \qquad (6.2)$$

where

$$T_b \equiv \frac{T_b'}{T_a'} \; ; \quad T_o \equiv \frac{T_o'}{T_a'} \; ; \quad T_h \equiv \frac{T_h'}{T_a'} , \qquad (6.3a,b,c)$$

$$M_o \equiv \frac{M_o'}{M_a'} \; ; \quad \Theta \equiv \frac{E_A'}{R'T_a'} . \qquad (6.4a,b)$$

Note also that, as in (4.6), (5.5) and (5.7), K and K_d can be expressed as,

$$K \equiv \frac{K'}{C_p' M_o'} = \frac{K_a}{M_o} = 1 + \left(\frac{1-T_b}{T_o - T_h} \right) , \qquad (6.5)$$

$$K_d \equiv \frac{K_d'}{C_p' M_o'} = \frac{K_{da}}{M_o} = \left(\frac{1-T_b}{T_o - T_h} \right) = K - 1 , \qquad (6.6)$$

where we have introduced the definitions,

$$K_a \equiv \frac{K'}{C_p' M_a'} \quad ; \quad K_{da} \equiv \frac{K_d'}{C_p' M_a'} \quad . \tag{6.7a,b}$$

Using either (6.5) or (6.6), the term $(T_o - T_h)$ in (6.2) can be eliminated so one can write (using (6.5) here),

$$\boxed{\begin{aligned} M_o &= T_b^2 \exp\left[-\frac{\theta}{2T_b}(1-T_b)\right] && (6.8) \\ Le\, y_f &= \ln\left[\left(\frac{1-T_h}{1-T_b}\right)\left(\frac{K_a/M_o - 1}{K_a/M_o}\right)\right] \quad . && (6.9) \end{aligned}}$$

These two equations are boxed since they represent in summary the complete description of a real low speed flame/ flame-holder system. In practice M_o, K_a and T_h will be specified in an experiment (with fixed mixture strength). Then, using (6.8) and (6.9), one can predict the temperature T_b and stand-off distance y_f (non dimensional). As shown in chapters 3 to 5, these are a close model of reality. The essential behaviour of the dimensional stand-off distance can be described by the approximation

$$x_f \equiv \frac{x_f' M_a' C_p'}{\lambda'} \approx \frac{Le\, y_f}{M_o} \quad ; \tag{6.10a}$$

i.e.

$$x_f \approx \frac{1}{T_b^2} \ln\left[\left(\frac{1-T_h}{1-T_b}\right)\left(\frac{K_a/M_o - 1}{K_a/M_o}\right)\right] \exp\left[\frac{\theta}{2T_b}(1-T_b)\right] . \tag{6.10b}$$

We have used non dimensional quantities in (6.8), (6.9) since one can more readily understand the salient features of the model as we consider near adiabatic conditions. The results

(6.8), (6.9) have been shown to be justified by using large activation energy asymptotic theory where $\Theta_1 \equiv E_A'/R'T_b'$ is considered to be much larger than unity.

Equation (6.10b) can be used to estimate the variation of stand-off distance x_f with final temperature T_b for a given composition, upstream face holder temperature T_h and conductance K_a. Plots for different values of K_a are shown in Fig. 3. Although K_a is generally large, we include for completeness plots of x_f near $T_b = 1$ for a wide range of K_a. We note there is always a minimum stand-off distance for any $K_a > 1$. If $K_a = 1$ exactly, one finds x_f has a limiting value at $T_b = 1$ given by,

$$x_f(K_a = 1, T_b = 1) = \ln\left[(1 - T_h)\left(2 + \tfrac{1}{2}\Theta\right)\right] , \qquad (6.11)$$

but if $K_a < 1$ there is theoretically a value of T_b where x_f diminishes to zero. However these facts are only of passing academic interest since generally K_a is large for most practical burners. As pointed out by Carrier et al (1978;p.45), K_d $\left(= K_{da}/M_0 = K - 1 = K_a/M_0 - 1\right.$; see Equations (6.5,6)$\left.\right)$ is in fact large. $K_d = 10$ is quite within the bounds of possibility. In the above reference, Fig. 2, $K_d = 1$ is termed "an implausibly small value". Thus K_d approaching zero $\left(K \to 1, K_a \to 1\right.$ with M_0 near 1 see (6.5,6)$\left.\right)$ should be discounted as impractical. (Although in Clarke and McIntosh (1980) the case $K_a = 1$ (corresponding to $K' = K'_{crit}$ in that reference) was allowed for, it was acknowledged that generally K_a is large (see caption to Fig. 2 of that paper)). Thus one can conclude that in practical experiments the special case K_a near 1 (i.e. K_{da} near zero) is <u>not</u> typical. Certainly K_a greater than 5 would be typical for most experiments and we observe in Fig. 3 that the curves for this range of K_a are all very similar to the $K_a = \infty$ curve. Note that at $K_a = \infty$ equation (6.10b) is exactly that of Buckmaster and Ludford (1982;p.29).

An alternative approach to the investigation of near adiabatic conditions is to approximate the closeness of T_b to unity by the expansion

$$T_b = 1 - \Theta^{-1}\eta(\Theta) \quad , \tag{6.12}$$

where $\Theta \equiv E_A'/R'T_a'$ is now the relevant large parameter and the exact order of η is not yet known other than for the restriction

$$\mathrm{Ord}(\eta) \leq 1 \quad . \tag{6.13}$$

Using such an approach one can approximate (6.8)-(6.10) by the following

$$M_o \approx e^{-\eta/2} \quad , \tag{6.14}$$

$$Le\, y_f \approx \ln\left[\frac{\Theta(1-T_h)}{\eta}\left(\frac{K_a e^{\eta/2}-1}{K_a e^{\eta/2}}\right)\right] \tag{6.15}$$

$$x_f \approx \ln\left[\frac{\Theta(1-T_h)}{\eta}\left(\frac{K_a e^{\eta/2}-1}{K_a e^{\eta/2}}\right)\right] e^{\eta/2} \quad , \tag{6.16}$$

with, from (6.6);

$$T_o \approx T_h + \frac{\eta}{\Theta(K_a e^{\eta/2}-1)} \tag{6.17}$$

which shows that near adiabatic conditions with $K_a \gg 1$, the temperature of the downstream face of the holder (T_o) is very nearly the same as that of the upstream face (T_h).[†]

[†]Equation (6.15) shows that y_f is $O(\ln\Theta)$ for practical K_a values, and that one has therefore moved out of the order classes of temperature and distance implicit in the derivation of the present, leading order, results. However, this is a technical point and careful investigation reveals what one might intuitively assume, namely that results like (6.14) - (6.16) are in fact quite correct.

The result (6.16) can be used in a similar way as (6.10b) to yield plots of stand-off distance x_f for a given T_b (through η). These plots are shown in Figs. 4 and 5. The essential features of Fig. 3 are preserved except that all the distances for a given T_b are noticeably reduced due to the approximate nature of (6.16).

Fig. 6 shows the effect of varying θ on the plots of x_f versus T_b from both equation (6.10b) and equation (6.16). These plots are for $K_a = \infty$, but a similar trend will be observed for all large K_a values. The results from the two approximations for x_f only take on similar values when T_b is very close to 1 $(T_b > 0{\cdot}98)$. The main reason for this is that the algebraic term T_b^{-2} is missing in (6.16) as compared with (6.10b). Consider for example $T_b = 0{\cdot}96$, $\theta = 20$. Equation (6.10b) gives $x_f = 5{\cdot}031$ whereas equation (6.16) yields $x_f = 4{\cdot}560$. The factor $T_b^{-2} = 1{\cdot}085$ multiplied by this latter value brings x_f back up to $4{\cdot}948$ and therefore accounts for a large part of the discrepancy. This indicates that one can only make qualitative predictions using these relations. Accurate quantitative estimates depend much upon a correct value of θ and T_b. These simple examples do expose a limitation of large activation energy asymptotics, where the numerical values of the large parameter are in reality <u>only</u> as large as ten or twenty.

We now consider the order of the quantities involved in equations (6.14)-(6.17) around the minimum stand-off distance. From the above discussion K_a $(\equiv K'/C_p'M_a'$ (see (6.5)) is a property of the flame-holder and is well above unity in value. Consequently one must come to the conclusion from (6.15) that $Le\, y_f$ is in fact of order $\ln(\theta)$ to leading order for near adiabatic conditions, and is no longer of order unity*. In that the non dimensional heat loss at the holder

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*This result is closely linked with the matters referred to in the footnote to equation (3.1).

is given by,

$$q_o \equiv \frac{Le\, q_o'}{C_p' T_b' M_o'} = \frac{Le\, K_a}{M_o}(T_o - T_h) = \frac{Le\, K_a}{M_o}\left(\frac{1 - T_b}{K_a/M_o - 1}\right), \tag{6.18}$$

in near adiabatic conditions one obtains,

$$q_o \approx \frac{Le\, K_a\, e^{\eta/2}\, \eta}{\Theta\,(K_a e^{\eta/2} - 1)} \quad . \tag{6.19}$$

Thus as $Le\, y_f$ becomes of order $\ln(\Theta)$, q_o becomes of order (Θ^{-1}).

Lastly we consider the order of η near the minimum stand-off point. Since in Figs. 4 and 5 the $K_a = 5$ and over curves are so similar to the $K_a = \infty$ curve, we approximate (6.16) with $K_a = \infty$ to highlight the main arguments involved and ease the algebraic complication. Thus we have,

$$x_f \approx \ln\left[\frac{\Theta(1 - T_h)}{\eta}\right] e^{\eta/2} \quad , \tag{6.20}$$

which yields for stationary points, the condition,

$$\eta \ln\left(\frac{\eta}{N}\right) = -2 \quad , \tag{6.21}$$

where,

$$N \equiv \Theta(1 - T_h) \quad . \tag{6.22}$$

There are two solutions to (6.21). One solution is for η large like Θ ; by virtue of the restrictions (6.12) and (6.13) to near-adiabatic conditions this solution is invalid. The other solution yields the minimum point corresponding to that illustrated in Fig. 5 (where for $\Theta = 10$, $T_h = 0{\cdot}15$, $\eta_{min} \approx 0{\cdot}91$). Equation (6.21) can be rewritten as,

$$\eta \ln \eta - \eta \ln\left(\Theta(1-T_h)\right) = -2 \qquad , \tag{6.23}$$

so that to preserve correct ordering η cannot be greater in order than $(\ln\Theta)^{-1}$. If we write

$$\eta_{min} = \frac{a_{min}}{\ln\Theta} \qquad , \tag{6.24}$$

then (6.23) yields to leading order,

$$a_{min} \approx 2 \qquad ; \qquad \eta_{min} \approx \frac{2}{\ln\Theta} \qquad . \tag{6.25}$$

For $\Theta = 10$, (6.25) yields $\eta_{min} \approx 0{\cdot}87$ which is a fair approximation to that obtained numerically (Fig. 5 : $\eta_{min} \approx 0{\cdot}91$). Thus from (6.12), the correct orderings for T_b, M_o, Ley_f and x_f in this region are in fact,

$$T_b \approx 1 - \frac{a}{\Theta \ln\Theta} \qquad , \tag{6.26}$$

$$M_o \approx 1 - \frac{a}{2\ln\Theta} \qquad , \tag{6.27}$$

$$Ley_f \approx \ln\left[\frac{(1-T_h)}{a}\Theta\ln\Theta\right] = \ln\Theta + \ln(\ln\Theta) + \ln\left(\frac{1-T_h}{a}\right) + \cdots ; \tag{6.28}$$

$$x_f \approx \ln\left[\frac{(1-T_h)}{a}\Theta\ln\Theta\right] \cdot \left[1 + \frac{a}{2\ln\Theta}\right] \qquad , \tag{6.29a}$$

i.e.

$$x_f \approx \ln\Theta + \ln(\ln\Theta) + \frac{a}{2} + \ln\left(\frac{1-T_h}{a}\right) + \cdots \qquad . \tag{6.29b}$$

The latter expansion includes up to $O(1)$ terms and highlights the fact that x_f and $Le\, y_f$ are in the same order class i.e. $\ln\theta + \ln(\ln\theta)$. However x_f has the additional $a/2$ term on the $O(1)$ scale so that the $O(1)$ terms

$$X \equiv \frac{a}{2} + \ln\left(\frac{1-T_h}{a}\right) , \tag{6.30}$$

have a minimum when $a = 2$, as derived in (6.25).

So in summary, the fact that the <u>non dimensionalised</u> stand-off distance $Le\, y_f$ must be of order $\ln(\theta)$ in this region does not preclude one from still finding the <u>dimensional</u> distance x_f' of closest approach. The order of the difference of T_b from unity is in fact $(\theta \ln\theta)^{-1}$ in this region. Note that the units of x_f' are $\lambda_o'/M_a' C_p'$ and x_f' is then typically between 0.01cm and 0.1 cm.

7. MODIFIED DIRAC-δ MODEL OF FLAME-HOLDER

In the paper by Matkowsky and Olagunju (1981), a further model of the flame/flame-holder system is proposed. It is based on the δ -function model used by Carrier et al (1978) but modified such that some results are altered significantly.

It is assumed in this model that density is constant which further simplifies the analysis of chapter 5 by discounting the thermal expansion of the mixture. However the main features of the analysis are not affected by such an assumption. The only significant effect is to rationalise the approximation (6.10a) for dimensional stand-off distance x_f' , so that in this model,

$$x_f \equiv \frac{M_a' c_p' x_f'}{\lambda_o'} = \frac{Le\, y_f}{M_o} \quad . \qquad (7.1a,b)$$

Matkowsky and Olagunju (1981) further assume near adiabatic conditions, so that (as (6.12))

$$T_b = 1 - \eta \theta^{-1} \quad , \qquad (7.2)$$

but here η is assumed to be $O(1)$. The implications of this assumption have been considered in chapter 6. One obtains relations (6.14)-(6.16) which rewritten in terms of $K_{da}(= K_a - M_o)$ are:

$$M_o = e^{-\eta/2} \quad , \qquad (7.3)$$

$$Le\, y_f = \ln\left[\frac{\theta(1-T_h)}{\eta} \cdot \frac{K_{da}/M_o}{(1+K_{da}/M_o)}\right] \quad , \qquad (7.4)$$

$$x_f = \frac{1}{M_o} \cdot \ln\left[\frac{\theta(1-T_h)}{\eta} \cdot \frac{K_{da}/M_o}{(1+K_{da}/M_o)}\right] \quad . \qquad (7.5)$$

As pointed out in chapter 6, equations (7.3)-(7.5) agree with the empirical results of Kaskan (1957) and Ferguson and Keck (1979) since in practical experiments K_{da} is large and the temperature of the holder does not vary a great deal. Equation (7.5) thus becomes,

$$x_f \approx \ln\left[\frac{\theta(1-T_h)}{\eta}\right] = \ln\left[\frac{T_a'-T_h'}{T_a'-T_b'}\right], \qquad (7.6)$$

which is in agreement with (3.2b) under the constant density assumption.

However, Matkowsky and Olgunju (1981) do not follow this reasoning. Instead they make a further assumption that K_{da} is small; specifically they write (in our notation)

$$K_{da} = \frac{H}{\theta(1-T_h)} . \qquad (7.7)$$

This reduces (7.3)-(7.5) to,

$$M_o = e^{-\eta/2} , \qquad (7.8)$$

$$M_o x_f = Le\, y_f \approx \ln\left[\frac{H}{\eta M_o}\right] = \ln\left[\frac{K_d'}{C_p' M_o'}\left(\frac{T_a'-T_h'}{T_a'-T_b'}\right)\right], \qquad (7.9)$$

where the last result here restores dimensional quantities through the various definitions of H etc. In order to get agreement with (3.2b) they then require

$$\frac{K_d'}{C_p' M_o'} \left(= \frac{K_{da}}{M_o}\right) = 1 . \qquad (7.10)$$

This has a serious implication in (7.7) for, if (7.10) holds true, then (7.7) implies,

$$M_o = \frac{H}{\theta(1-T_h)} . \qquad (7.11)$$

Since M_o must be $O(1)$ one is then forced to conclude that H is not $O(1)$, which contradicts (7.7).

The correct approach to burner flame modelling is not to force an impractical ordering of the adiabatic heat transfer number (K_{da}) or conductance (K_a) onto the problem but to keep these as $O(1)$ quantities. As shown in chapter 6, when this is done and adiabatic conditions are approached, the flame stand-off distance then ceases to be $O(1)$ but becomes $O(\ln \theta)$, whilst the heat loss to the holder becomes $O(\theta^{-1})$.

Stability analyses are now being made (Margolis and Kerstein 1983) on the basis of the modified Dirac-δ model of the flame-holder described at the beginning of this section. But the above analysis shows that the basic steady model is not true to the real situation, and doubts must be raised as to the validity of the stability predictions. Some further work on these matters is necessary.

8. CONCLUSIONS

A review has been presented of three theoretical models of plane flames burning on a cooled porous-plug type flame-holder. It has been shown that the Dirac delta function type of holder gives satisfactory agreement with observation provided one is prepared to modify the Arrhenius kinetics of the burning reaction. Care must be exercised with the asymptotic orderings of the several quantities of physical significance. In particular small $O(\theta^{-1})$ heat loss rates give rise to $O(\ln\theta)$ flame stand-off distances. It is important to note that the minimum stand-off distance of a near-adiabatic flame occurs within this order class of quantities. The Hirschfelder type of flame-holder model gives excellent agreement with observation without the need for modification of Arrhenius kinetics.

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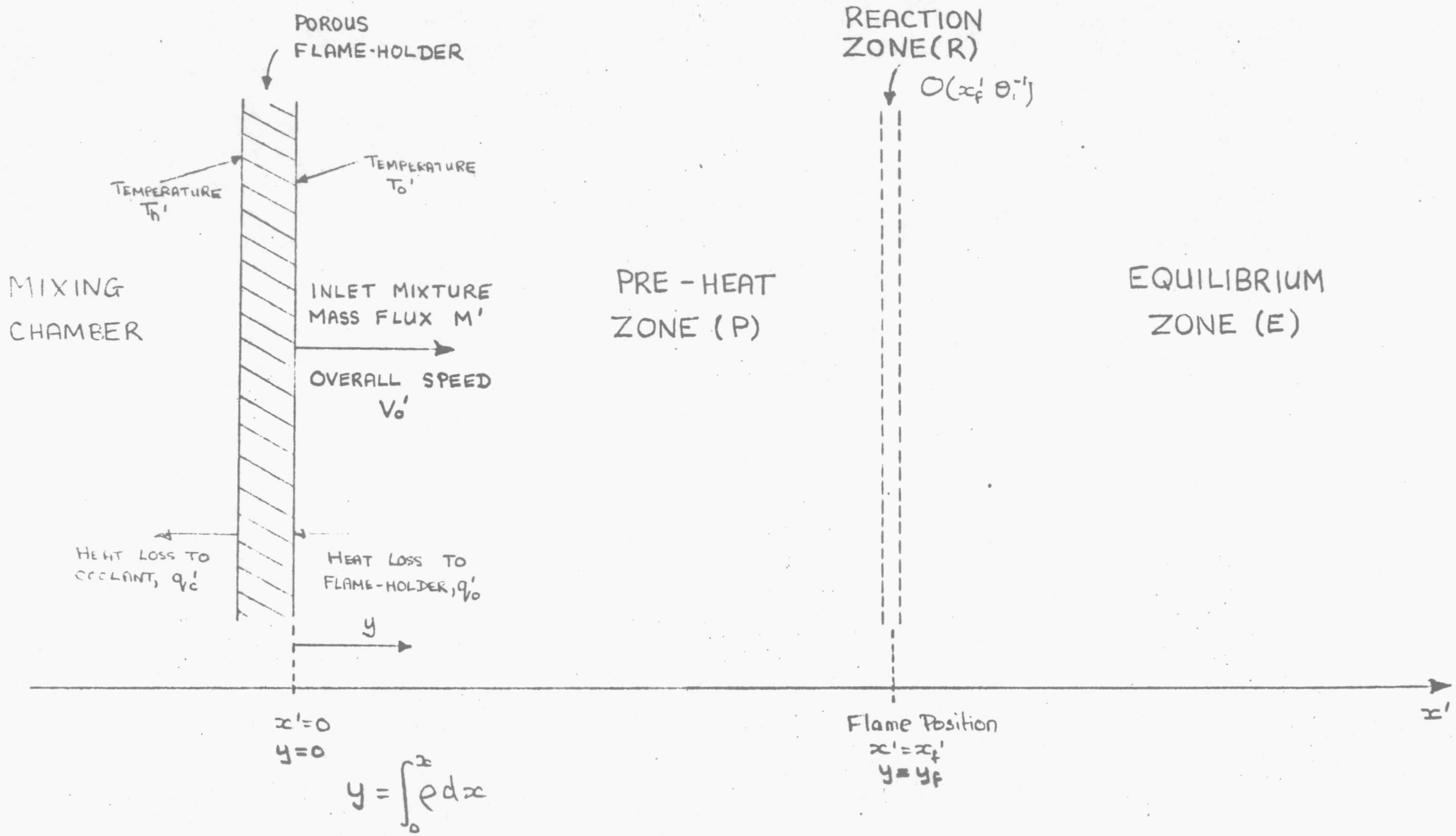

FIG. 1. SCHEMATIC OF ONE-DIMENSIONAL PRE-MIXED FLAME WITH FLAME-HOLDER.

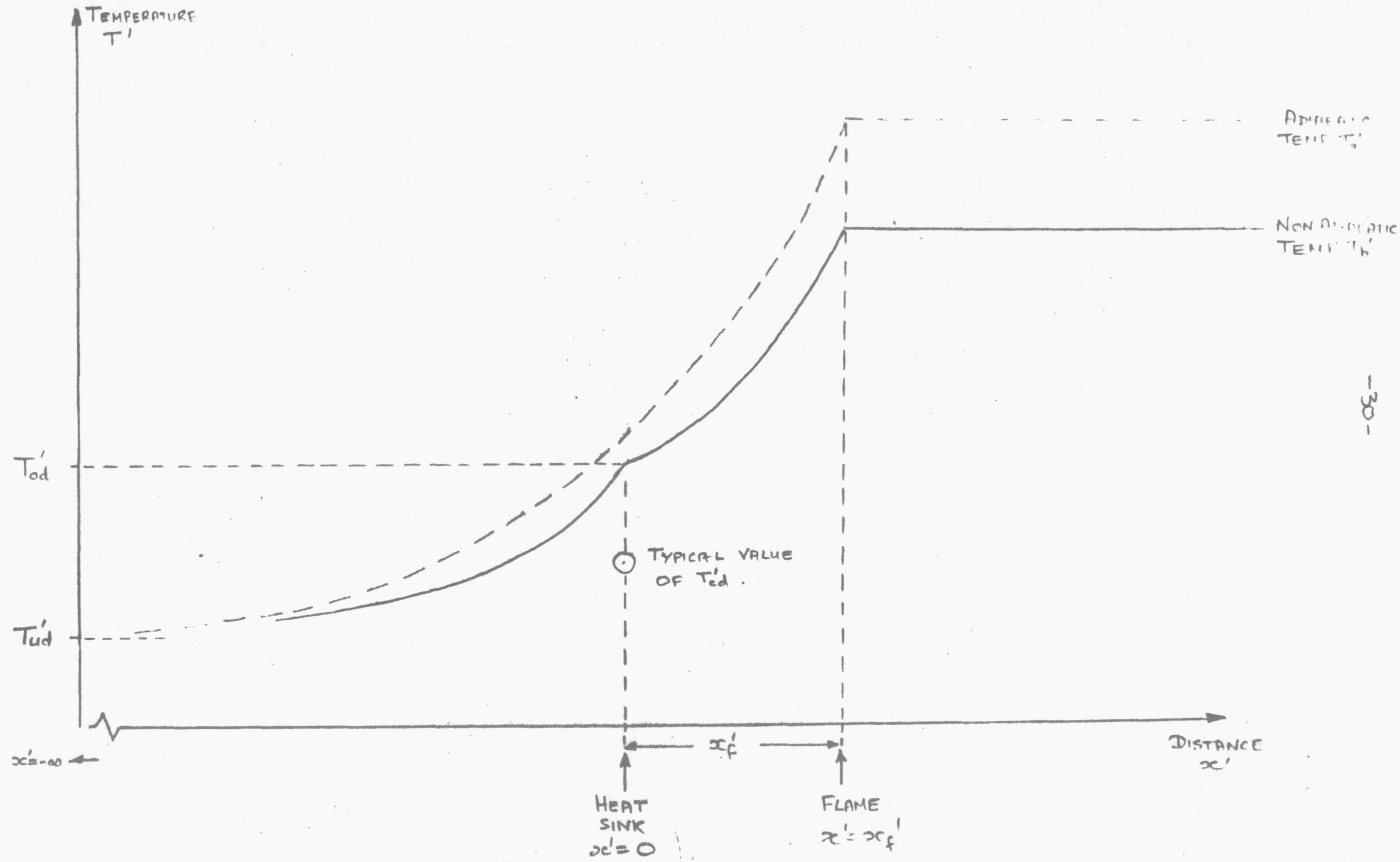

FIG. 2. DIRAC δ - FUNCTION MODEL OF FLAME - HOLDER.

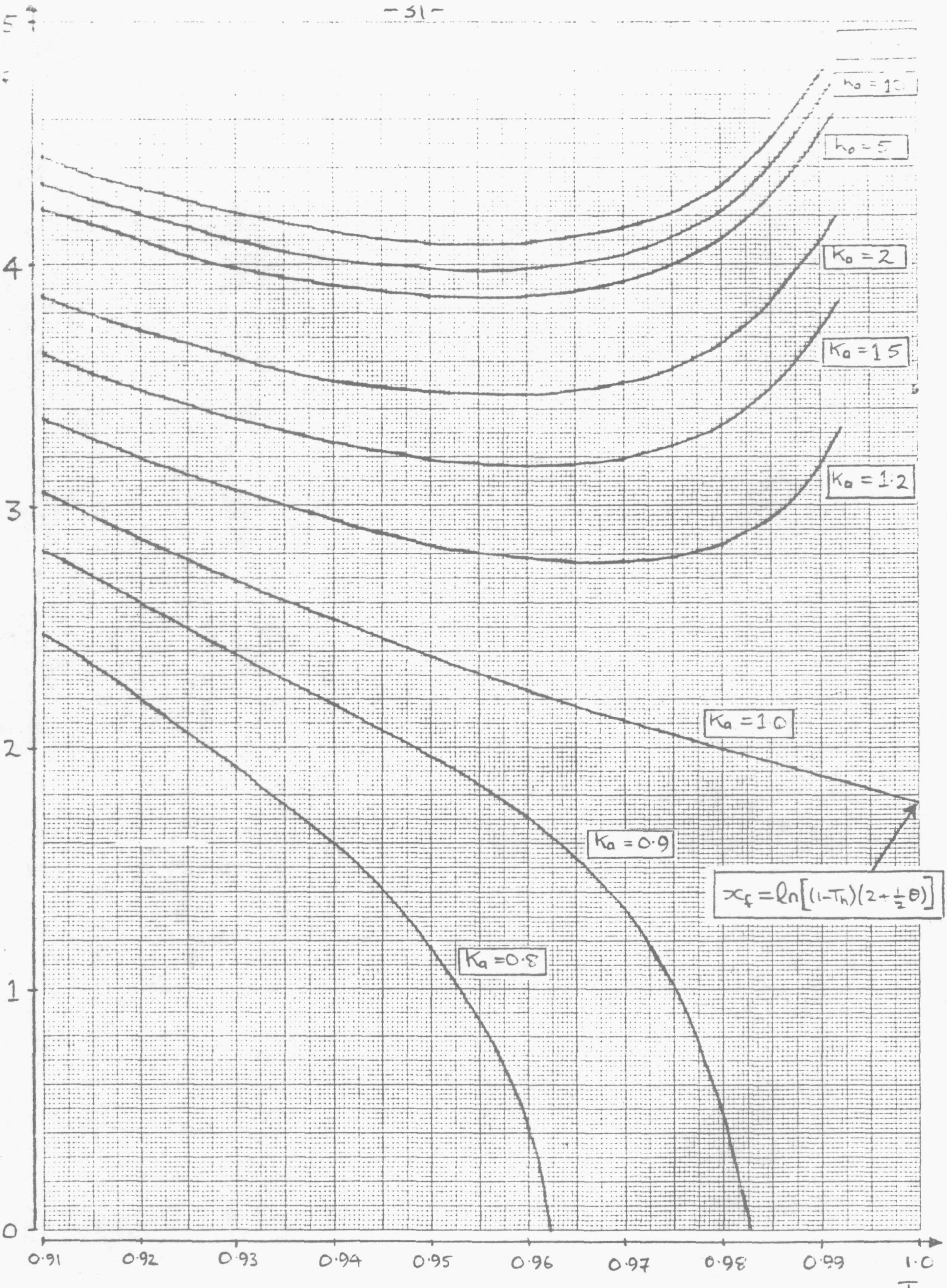

FIG. 3 STAND-OFF DISTANCE VERSUS TEMPERATURE USING EQUATION (6.10b). $\theta = 10$, $T_h = 0.15$. VARYING CONDUCTANCE K_A. NOTE USUALLY $K_a > 5$.

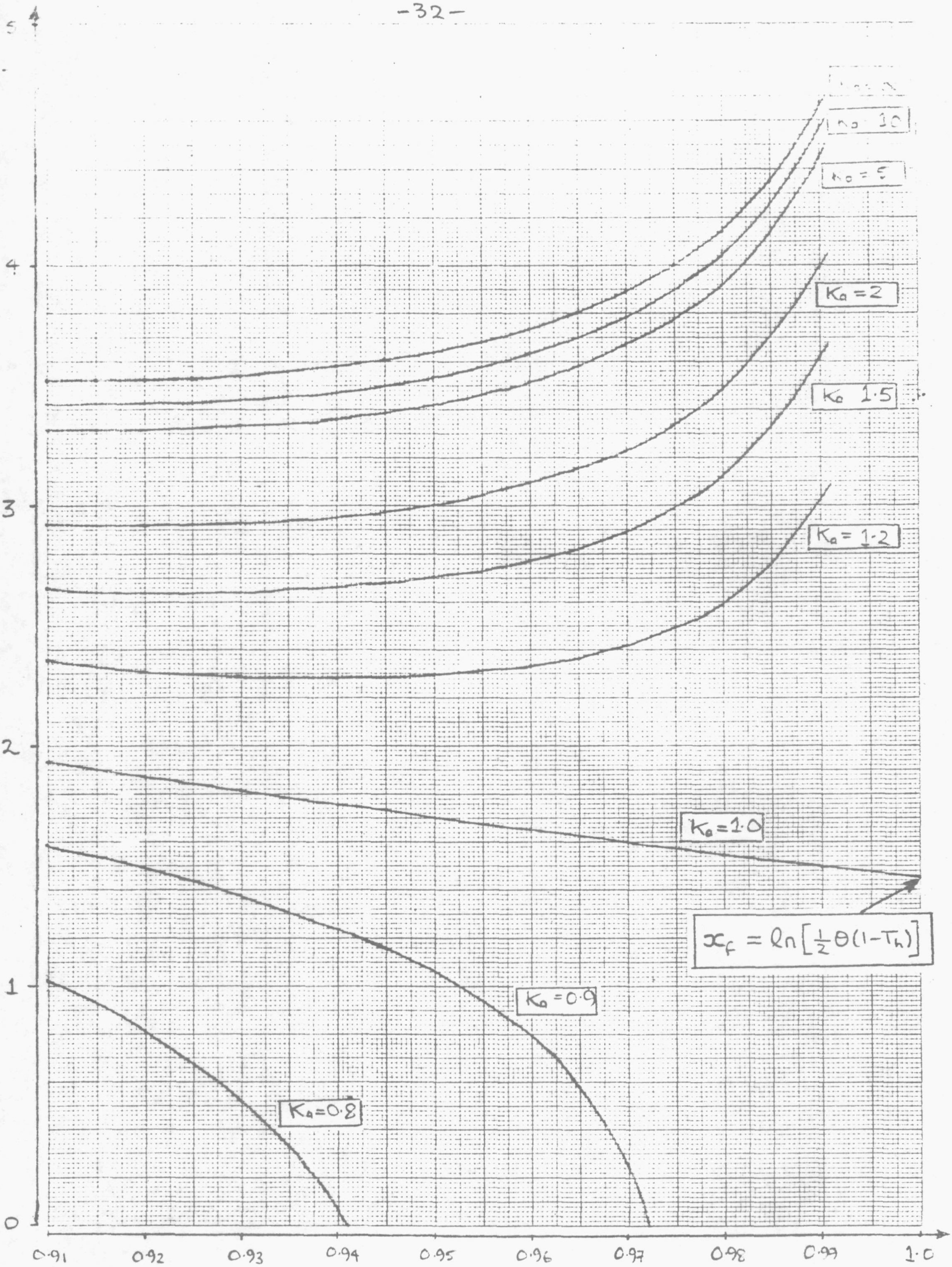

FIG. 4 STAND-OFF DISTANCE VERSUS TEMPERATURE USING EQUATION (6.16). $\theta = 10$, $T_h = 0.15$. VARYING CONDUCTANCE K_a. NOTE USUALLY $K_a > 5$.

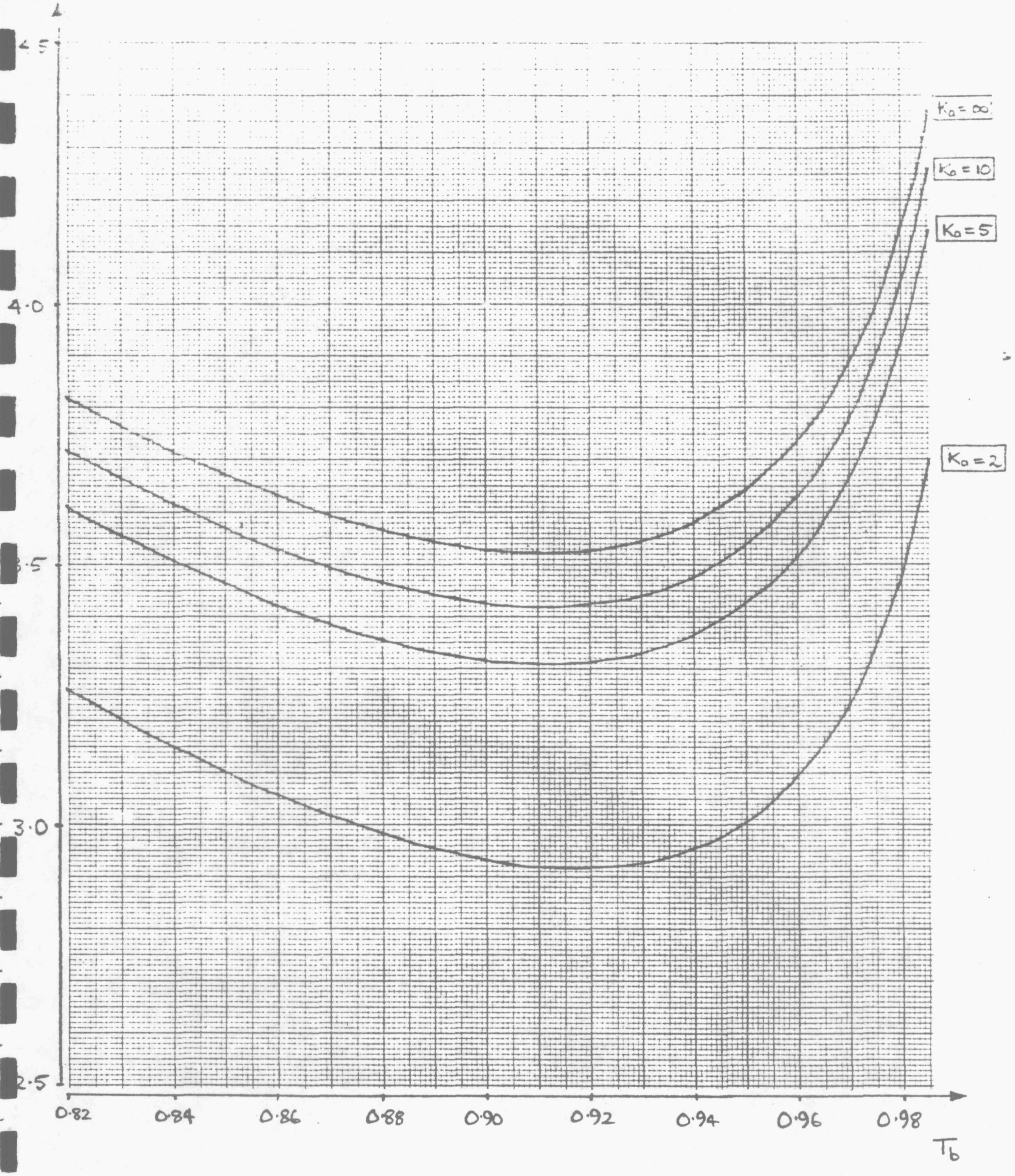

FIG. 5. DETAIL OF FIG 4. SHOWING MINIMUM POINTS FOR $K_a \geqslant 2$.

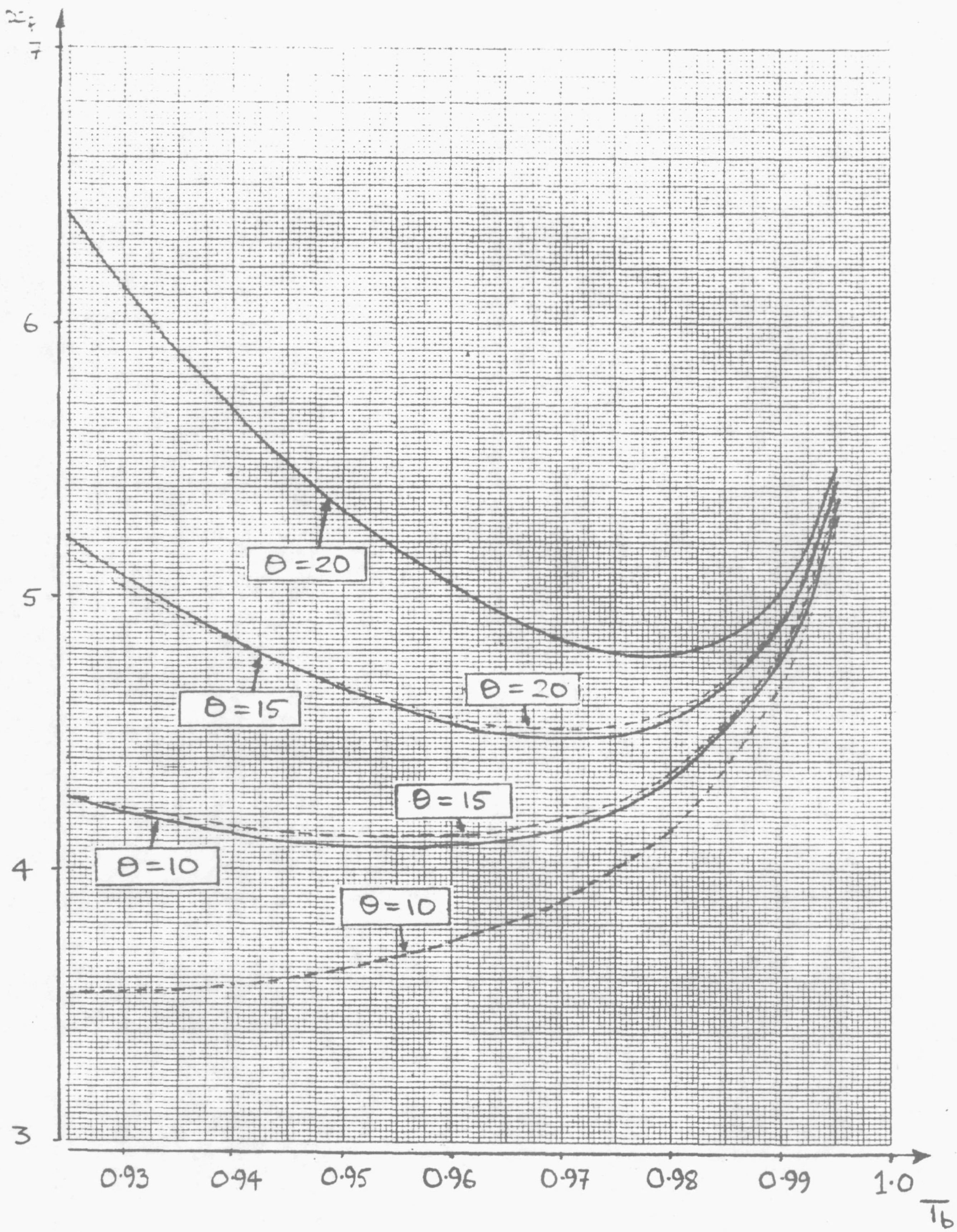

FIG. 6. STAND-OFF DISTANCE VERSUS TEMPERATURE FOR $K_a = \infty$, $T_h = 0{\cdot}15$, AND THREE VALUES OF θ AS MARKED. — USING EQUATION (6·10b) --- USING EQUATION (6·16)