DTNSRDC-78/084

APPROXIMATE EVALUATION OF ADDED MASS AND DAMPING COEFFICIENTS OF TWO-DIMENSIONAL SWATH SECTIONS

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

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by

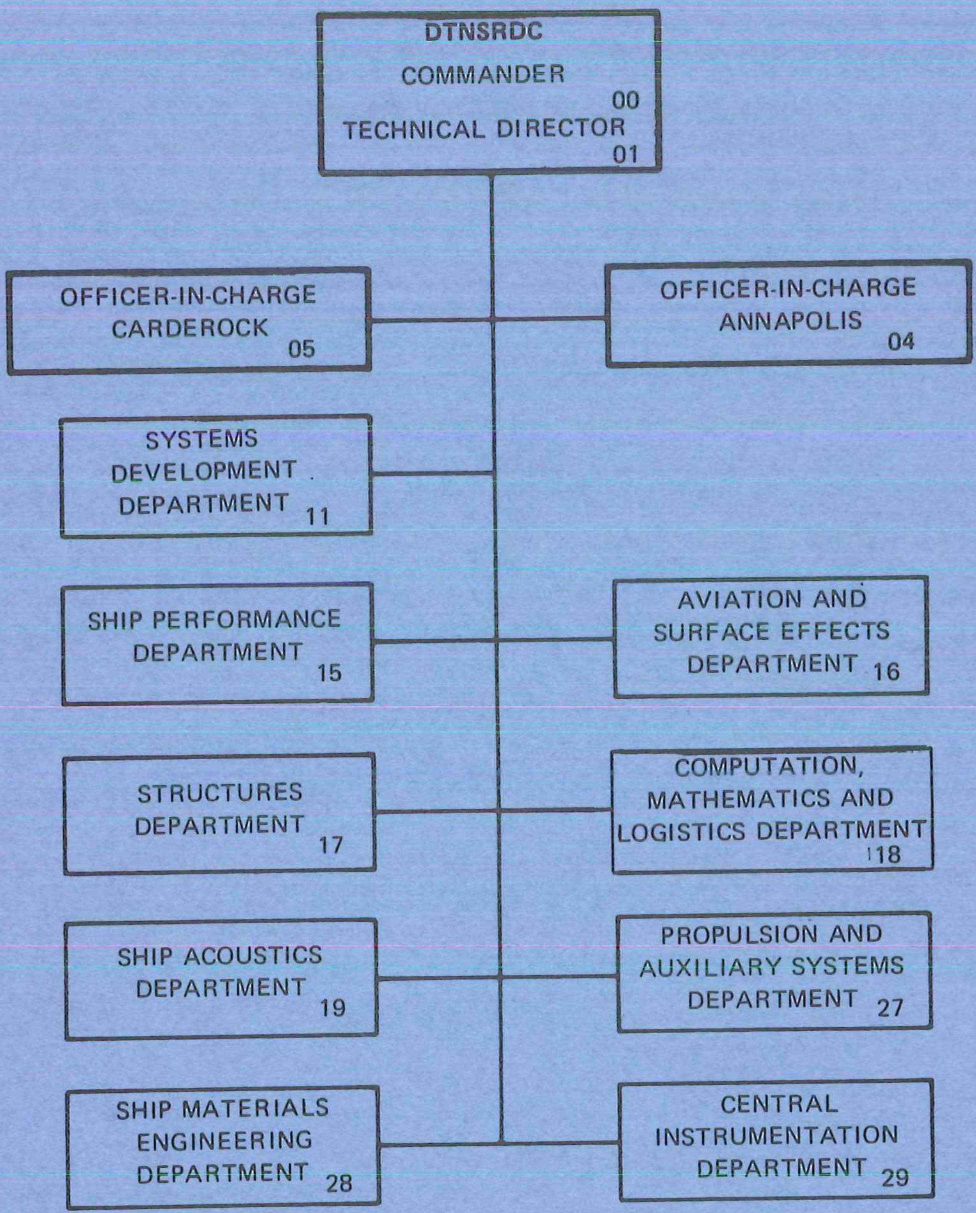
Choung M. Lee

SHIP PERFORMANCE DEPARTMENT
RESEARCH AND DEVELOPMENT REPORT

October 1978

DTNSRDC-78/084

MAJOR DTNSRDC ORGANIZATIONAL COMPONENTS



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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER DTNSRDC-78/084	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) APPROXIMATE EVALUATION OF ADDED MASS AND DAMPING COEFFICIENTS OF TWO-DIMENSIONAL SWATH SECTIONS	5. TYPE OF REPORT & PERIOD COVERED Final
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Choung M. Lee	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS David W. Taylor Naval Ship Research and Development Center Bethesda, Maryland 20084	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (See reverse side)
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE October 1978
	13. NUMBER OF PAGES 41
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Small-Waterplane-Area, Twin-Hull Ships
Ship Motion in Waves
Two-Dimensional Added Mass and Damping

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Derivation of approximate formulas for determining the added mass and damping coefficients of two-dimensional, small-waterplane-area, twin-hull (SWATH) sections is described. The added mass and damping coefficients of interest are those associated with a forced oscillation of SWATH sections in heave, sway, or roll mode in a free surface. The objective of deriving the

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S/N 0102-LF-014-6601

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(Block 10)

Program Element 62543N
Project ZF-43421
Task Area ZF-43421001
Work Unit 1-1500-102

(Block 20 continued)

approximate formulas for the hydrodynamic coefficients is to simplify the computation of motion of SWATH ships in waves without sacrificing its accuracy significantly.

The approximate formulas are derived based on the potential-flow theory. The damping coefficients are obtained in terms of the outgoing wave amplitudes, and the added mass coefficients are obtained by using the damping coefficients through the so-called Kramers-Kronig relations. Within the frequency range of practical interest, the approximate formulas provide satisfactory results.

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NOTATION

$\bar{A} = A_1/h_o$	Ratio of wave amplitude to oscillation amplitude
A_1	Amplitude of outgoing wave
a_{ij}	Two-dimensional added mass in the ith mode due to the motion in the jth mode
$(\bar{a}_{22},\bar{a}_{33}) = \dfrac{(a_{22},a_{33})}{\rho S_A}$	Nondimensional (sway, heave) added mass coefficient
$\bar{a}_{24} = \dfrac{a_{24}}{(\rho b S_A)}$	Nondimensional sway-roll coupling added mass
$\bar{a}_{44} = \dfrac{a_{44}}{(\rho b^2 S_A)}$	Nondimensional roll added inertia
b	One-half of the distance between the centerline of each hull
b_{ij}	Two-dimensional wavemaking damping in the ith mode due to the motion in the jth mode
b_o	Strut thickness at the mean waterline
$(\bar{b}_{22},\bar{b}_{33}) = \dfrac{(b_{22},b_{33})}{\rho\omega S_A}$	Nondimensional (sway, heave) damping coefficient
$\bar{b}_{24} = \dfrac{b_{24}}{(\rho\omega b S_A)}$	Nondimensional sway-roll coupling damping coefficient
$\bar{b}_{44} = \dfrac{b_{44}}{(\rho\omega b^2 S_A)}$	Nondimensional roll damping coefficient
C_i	Correction factor (i = 1,2,3)
℄	Centerline of the twin bodies

D	Draft of cross section
d_c	Depth to the center of circular hull
d_s	Depth of strut
g	Gravitational acceleration
h_o	Amplitude of oscillation of body
$i = \sqrt{-1}$	Imaginary unit
$K = \omega^2/g$	Wave number
(n_y, n_z)	y- and z-component of unit normal vector pointing into body
R	Radius of circular portion of hull
(r, θ)	Cylindrical coordinates as defined on page 5
S_A	Submerged cross sectional area of one hull
U	Sway velocity
V	Heave velocity
(y, z)	Right-handed Cartesian coordinate system; Oz is directed vertically upward and Oy coincides with calm waterline
μ_{22}	One-half of sway added mass of twin circles submerged under a rigid surface
μ_{33}	Heave added mass of SWATH demihull cross section at zero frequency
ρ	Water density
(ϕ_H, ϕ_S, ϕ_R)	Velocity potential representing flow field disturbed by (heave, sway, roll) oscillation

ABSTRACT

Derivation of approximate formulas for determining the added mass and damping coefficients of two-dimensional, small-waterplane-area, twin-hull (SWATH) sections is described. The added mass and damping coefficients of interest are those associated with a forced oscillation of SWATH sections in heave, sway, or roll mode in a free surface. The objective of deriving the approximate formulas for the hydrodynamic coefficients is to simplify the computation of motion of SWATH ships in waves without sacrificing its accuracy significantly.

The approximate formulas are derived based on the potential-flow theory. The damping coefficients are obtained in terms of the outgoing wave amplitudes, and the added mass coefficients are obtained by using the damping coefficients through the so-called Kramers-Kronig relations. Within the frequency range of practical interest, the approximate formulas provide satisfactory results.

ADMINISTRATIVE INFORMATION

This work was sponsored by the Naval Material Command as part of the High Performance Vehicle Hydrodynamic Program of the Ship Performance Department, David W. Taylor Naval Ship R&D Center (DTNSRDC). Funding was provided under the Ships, Subs and Boats Program, Element 62543N, Task Area ZF-43421001, Project Number ZF-43421, Work Unit 1-1500-102.

INTRODUCTION

One of the advantages expected from a small-waterplane-area, twin-hull (SWATH) configuration is the improved seakeeping qualities compared to monohull ships in moderate sea conditions. To assist in the development of the SWATH concept, an analytical prediction of motion of SWATH ships in waves has been developed at the David W. Taylor Naval Ship R&D Center (DTNSRDC).[1*,2] The analytical method can predict the motion of a SWATH ship in five degrees-of-freedom in regular and irregular waves. The surge motion is excluded due to its minor practical significance.

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*A complete listing of references is given on page 31.

Although the existing analytical method has its own merits and serves important purposes, it has been felt that it is too cumbersome to be used in the conceptual design stage at which numerous candidate hull forms are examined. Dalzell[3] has addressed this problem by developing a simplified method of computing the hydrodynamic coefficients associated with heave-pitch coupled motion of SWATH ships at zero speed in head waves. Since the major portion of the computer program[2] for prediction of motion is comprised of computing the sectional added mass and damping, the accomplishment of Dalzell in reducing the size and computation time of the computer program is regarded to be very worthwhile.

In this report, an extension of the Dalzell effort to all hydrodynamic coefficients associated with the five degrees-of-freedom of motion of SWATH ships is described. The approach employed here is similar to the Dalzell approach. That is, the sectional wavemaking damping coefficients of SWATH cross sections are obtained using the far-field potential-flow theory. By application of the Kramers-Kronig relations[4] to the damping coefficients, the added-mass coefficient, which can be regarded as the conjugate pair of the damping, is determined.

The advantage of this procedure over the conventional multiple-expansion method[5] or source-distribution method[6] is that the wavemaking damping can be obtained by determining the outgoing wave amplitude at a far field rather than by determining the pressure distribution on the body. The outgoing wave amplitude can be closely approximated by representing the flow disturbances by a pulsating single source, a dipole, or a combination of both. Furthermore, the Kramers-Kronig relations can be expressed in terms of a half-range double Fourier transform. Because an efficient numerical procedure such as fast Fourier transform (FFT)[7] is a commonly available computer routine, the process of obtaining the added-mass coefficients can be quickly performed.

Once the sectional added mass and damping coefficients in heave, sway, roll, and roll-sway coupled modes are obtained, the rest of the motion prediction process follows identically to that described in Reference 1.

Comparison of the results obtained by experiments, the method of source distribution,[8] and the present approximate method are presented in graphs. Agreements are, in general, fair. The approximate method yields savings in the computation time by an order of magnitude compared to the conventional method of Reference 8. The real merit of the approximate method will be evaluated in the future when an analytical method of prediction of SWATH motions in waves is developed.

ANALYTICAL METHODS

BACKGROUND

Within a linear analysis, the added mass and damping coefficients of oscillating twin-cylindrical bodies of arbitrary cross section have been determined by the method of pulsating source distribution.[8] This method has shown satisfactory agreement with the experimental results of various cross sectional shapes such as twin semicircles, rectangles, triangles,[8] and SWATH sections.[9]

To obtain reasonably accurate added mass and damping coefficients of a SWATH cross section, at least 15 to 20 points of source distribution on the submerged contour of one hull should be taken for each frequency of oscillation. To obtain all the hydrodynamic coefficients in the linear equations of motion covering all degrees-of-freedom, the sectional added mass and damping coefficients should be computed for the sway, heave, and roll modes for about 20 cross sections along the length of a ship. If the motion of a SWATH ship in irregular waves for different wave headings and ship speeds is to be computed, the foregoing number of calculations should be repeated for each wave heading, ship speed, and frequency of oscillation. Thus, one can easily expect that a significant amount of computer time will be required to evaluate the seakeeping qualities of a ship.

In spite of this large expense of computation time, what we are obtaining, at best, are motions based on the hydrodynamic coefficients which are obtained under the assumption of two-dimensional flow conditions at each cross section. Furthermore, unlike monohull ships, the motion of SWATH ships cannot be predicted accurately by using only the coefficients obtained from the potential-flow theory. There are viscous effects and fin effects to be included in the hydrodynamic coefficients. It seems reasonable, therefore, to seek a simplified method of obtaining the sectional hydrodynamic coefficients, if the economic savings in the labor and time can justify the loss of accuracy of the results within a certain margin.

ANALYSIS

From the principle of conservation of energy, the wavemaking damping of an oscillating two-dimensional body can be determined by the amplitude of the radiating waves at a far distance from the body. This implies that the damping coefficient can be determined by examining the far-field behavior of the disturbances generated by the oscillating body. Once the wavemaking damping is known, the so-called Kramers-Kronig relations can be invoked to obtain the added mass of the body. Thus, both coefficients can be obtained by examining the far-field behavior of the outgoing waves. The advantage of examining the far-field behavior of the fluid disturbances lies in the fact that the solution of the potential function can be obtained by examining the asymptotic behavior of the function as the horizontal coordinate, for example y, goes to infinity.

The analysis will be carried out in the following manner. First, the potential function representing the outgoing wave at the far field, which is generated by oscillating the demihull of a two-dimensional SWATH form, will be determined. Then, the interference and blocking effects of the other hull on the outgoing waves at the far field will be determined. The next step is to determine the damping coefficient in terms of the wave amplitude and then, the added-mass coefficient by the Kramers-Kronig relations.

Heave Damping Approximation

Assume that the circular portion of a cross section of the demihull of a SWATH ship can be represented by a vertical dipole, with the pulsating velocity V located at a certain point on the submerged area of the demihull cross section, which is shown in Figure 1. The notations denoting the dimensions of the cross section are also given in Figure 1.

The velocity potential representing the vertical dipole in an unbounded fluid is given by

$$\phi_d = -\frac{V}{2\pi}\left(S_A + \frac{\mu_{33}}{\rho}\right)\frac{\cos\theta}{r} \tag{1}$$

where ρ = water density

S_A = submerged cross sectional area of the demihull

μ_{33} = heave added mass of the demihull cross section in an unbounded fluid

(r,θ) = cylindrical coordinates such that $y = r\sin\theta$ and $z = r\cos\theta - d_o$, where d_o is the depth of the point where the dipole is located

The heave added mass μ_{33} will be approximated by

$$\mu_{33} = \rho\pi R^2\left(1 - \frac{b_o^2}{4}\right) \tag{2}$$

Due to the presence of the surface-piercing vertical strut, the displaced area of the cross section changes according to the vertical oscillation of the body. The change in the displaced area is equivalent to the flux through the width of the strut which is Vb_o. Thus, an equivalent source strength to generate this flux can be obtained by $Vb_o/(2\pi)$, and the corresponding source located at the point $(0,-d_s)$ is expressed by

$$\phi_s = \frac{Vb_o}{2\pi}\,\ell n\sqrt{y^2+(z+d_s)^2} \tag{3}$$

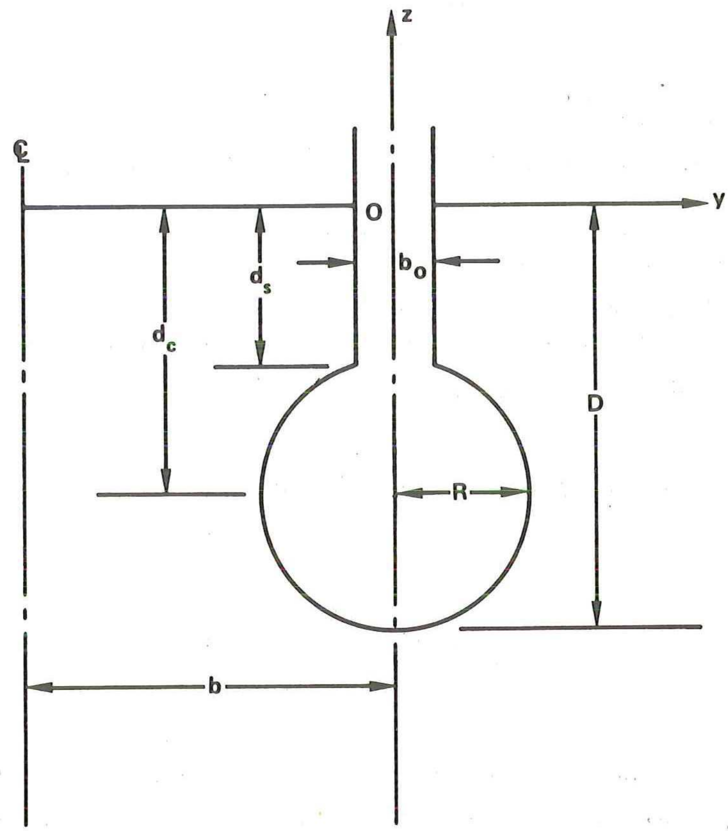

Figure 1 - Demihull Cross Section of a SWATH Ship

Thus, the velocity potential representing a heaving demihull SWATH cross section is obtained approximately by

$$\phi_H = \phi_d + \phi_s$$

$$= -\frac{V}{2\pi}\left(S_A + \frac{\mu_{33}}{\rho}\right)\frac{\cos\theta}{r} + \frac{Vb_o}{2\pi}\ \ell n\ \sqrt{y^2+(z+d_s)^2} \qquad (4)$$

The far-field behavior of the singularities represented by Equation (4), where the z = 0 is regarded as the calm-water line, can be obtained by[11]

$$\phi_H \sim \left[-\left(S_A + \frac{\mu_{33}}{\rho}\right)Ke^{K(z-d_o+iy)} + b_o e^{K(z-d_s+iy)}\right](-i\omega h_o e^{-i\omega t}) \qquad (5)$$

where only the real part of the right-hand side is to be realized, and

$V = -i\omega h_o e^{-i\omega t}$

$K = \omega^2/g$, the wave number

ω = circular frequency of oscillation

g = gravitational acceleration

$i = \sqrt{-1}$

Hereafter, when a real function is expressed in a complex form involving $e^{-i\omega t}$, it will be tacitly assumed that only the real part of the complex expression is meant.

Because the wave amplitude, which is denoted by A_1, is obtained by

$$A_1 = \left|\frac{\omega\phi_H}{g}\right|_{\substack{z\ =\ 0\\ |y|\ =\ \infty}}$$

then

$$\bar{A} = \frac{A_1}{h_o} = \left| -K^2 \left(S_A + \frac{\mu_{33}}{\rho} \right) e^{-Kd_o} + b_o K e^{-Kd_s} \right| \tag{6}$$

From the conservation of energy, the rate of work imparted on the fluid by the body during one cycle of oscillation should be equal to the rate of energy carried by the radiating surface waves during the same period. This means that

$$\int_0^T F_j \, V \, dt = -2\rho \int_0^T dt \int_{-\infty}^0 \phi_t \phi_y dz \tag{7}$$

where

T	= period of oscillation
F_j	= $[-\omega^2(M+a_{jj}) \cos \omega t - \omega b_{jj} \sin \omega t + c_{jj} \cos \omega t] h_o$
V	= $-\omega h_o \sin \omega t$
ϕ	= $\dfrac{gA_1 \cos (Ky-\omega t+\alpha) e^{Kz}}{\omega}$
α	= phase angle with respect to the vertical motion of the body
(a_{jj}, b_{jj}, c_{jj})	= (added mass, damping, restoring) in the jth mode due to the motion in the jth mode
M	= mass of the body

From Equation (7)

$$b_{jj} = \rho\omega \left(\frac{A_1}{Kh_o} \right)^2 \tag{8}$$

Substituting Equation (6) into (8)

$$b_{33}^{(1)} = \rho\omega \left(\frac{\bar{A}}{K} \right)^2 \tag{9}$$

where the superscript 1 is used to indicate the quantities corresponding to the demihull only.

If $b_{33}^{(1)}$ is nondimensionalized by $\rho\omega S_A$, then

$$\bar{b}_{33}^{(1)} \equiv \frac{b_{33}^{(1)}}{\rho\omega S_A}$$

$$= \frac{\left[-K^2\left(S_A+\frac{\mu_{33}}{\rho}\right)e^{-Kd_o}+b_oKe^{-Kd_s}\right]^2}{K^2S_A} \qquad (10)$$

To take into account the interference or blockage effects from the other hull, the reflected images at y = -2b of the dipole and source introduced in the foregoing analysis should be considered. Further, assume that the demihull is replaced by a vertical barrier having the depth D; hence, the waves generated by one hull cannot be transmitted completely beyond the other hull.

If we let ε denote the ratio of the wave amplitude of the incident wave before the barrier to the amplitude of the transmitted wave behind the barrier, the waves generated by the singularities can be expressed in the form

$$\zeta = \cos(Ky-\omega t) + \varepsilon \cos(Ky+2b-\omega t+\alpha_T) \qquad (11)$$

where the incident wave amplitude is assumed to be unity, and α_T is the change of the phase angle of the transmitted wave.

Equation (11) can be expressed as

$$\zeta = \beta_o \cos(Ky-\omega t+\delta) \qquad (12)$$

where

$$\beta_o = [1+\varepsilon^2+2\varepsilon \cos (2Kb+\alpha_T)]^{1/2}$$

$$\delta = \tan^{-1} \frac{-\varepsilon \sin (2Kb+\alpha_T)}{1+\varepsilon \cos (2Kb+\alpha_T)} \tag{13}$$

The transmission coefficient ε and the phase angle α_T are given by[12]

$$\varepsilon = \frac{K_1(KD)}{\sqrt{\pi^2 I_1^{\,2}(KD)+K_1^{\,2}(KD)}} \tag{14}$$

$$\alpha_T = \tan^{-1}\left(\frac{\pi I_1(KD)}{K_1(KD)}\right) \tag{15}$$

where I_1 and K_1 are the modified Bessel functions. For $\chi < 1$, it can be shown that

$$I_1(\chi) = \frac{\chi}{2} + \frac{\chi^3}{16} + \frac{\chi^5}{384} + \frac{\chi^7}{18432} + \ldots \tag{16}$$

$$K_1(\chi) = \frac{1}{\chi} + \ln\left(\frac{\chi}{2}\right) I_1(\chi) - \frac{(1-2\gamma)}{4}\chi$$

$$- \frac{\left(2\frac{1}{2}\right)-2\gamma}{32}\chi^3 - \frac{\left(\frac{10}{3}-2\gamma\right)}{768}\chi^5 - \frac{\left(\frac{47}{12}-2\gamma\right)}{36864}\chi^7 \ldots \tag{17}$$

where

$$\gamma = \lim_{m \to \infty}\left(\sum_{n=1}^{m} \frac{1}{n} - \ln m\right)$$

$$= 0.57721566\ldots$$

is known as the Euler constant. For $KD \geq 1$, it can be approximated that $\varepsilon \doteq 0$ and $\alpha_T \doteq 0$.

Because β_o represents the interference and blockage effects on the wave amplitude at the far field, the twin-hull damping coefficient can be determined by

$$\bar{b}_{33} = C_1 \, \bar{b}_{33}^{(1)} \beta_o^{\,2} \tag{18}$$

where C_1 is introduced here as a correction factor if the need arises.

When the strut thickness is zero, the source potential given by Equation (3) can be disregarded because $b_o = 0$.

Also, due to the absence of the vertical barrier, it is assumed that $\varepsilon = 1$ and $\alpha_T = 0$. Thus, the heave damping of completely submerged twin circles can be obtained from Equations (9) and (18) as

$$b_{33} = 2C_1 \, \rho\omega K^2 e^{-2Kd_c} \left(S_A + \frac{\mu_{33}}{\rho} \right)^2 (1+ \cos 2Kb)$$

$$= 2C_1 \, \rho\omega\pi^2 R^4 K^2 e^{-2Kd_c} \left(2+ \frac{R^2}{2b^2} + \frac{R^2}{2d_c^{\,2}} \right)^2 (1+ \cos 2Kb)$$

or

$$\bar{b}_{33} = 2C_1 \, \pi R^2 K^2 e^{-2Kd_c} \left(2+ \frac{R^2}{2b^2} + \frac{R^2}{2d_c^{\,2}} \right)^2 (1+ \cos 2Kb) \tag{19}$$

where

$S_A = \pi R^2$

$\mu_{33} = \pi R^2 \left(1+ \dfrac{R^2}{2b^2} + \dfrac{R^2}{2d_c^{\,2}} \right)$

d_c = the depth to the center of the circle

The last two terms in the bracket of the expression of μ_{33} represent, respectively, the twin-hull effect and the free-surface effect which is assumed to be a rigid wall.[13]

Sway Damping Approximation

From a far-field view, the demihull of a SWATH section in the sway motion can be regarded as a vertical plate of the same draft as the body draft. For small frequencies of oscillation which are of practical interest to SWATH motion, it can be further assumed that, near the body, the fluid disturbances would be close to those made by a vertical plate of twice the length of the body draft in sway motion in an unbounded fluid.

If the sway velocity of the body motion is denoted by U, the flow field can be represented by the velocity potential which can be expressed in terms of the dipole distribution as

$$\phi_S = \frac{U}{\pi} \int_{-D}^{0} \sqrt{D^2-\zeta^2}\; y \left(\frac{1}{y^2+(z-\zeta)^2} + \frac{1}{y^2+(z+\zeta)^2} \right) d\zeta \tag{20}$$

where the dipole density $\sqrt{D^2-\zeta^2}$ can be obtained from the well-known solution of a flat plate subject to normal uniform flow by taking the potential jump across the plate.

The corresponding far-field expression which represents the radiating free-surface waves is obtained from Equation (13.29) of Reference 11 as

$$\phi_S \sim i2KUe^{iKy} \int_{-D}^{0} \sqrt{D^2-\zeta^2}\; e^{K(z+\zeta)} d\zeta$$

$$= i2KUe^{K(z+iy)} B(K) \tag{21}$$

where

$$B(K) = \int_{-D}^{0} e^{K\zeta} \sqrt{D^2-\zeta^2}\, d\zeta$$

$$= \frac{\pi D}{2K} (I_1(KD)-L_1(KD)) \tag{22}$$

in which L_1 is the modified Struve function.

The amplitude of radiating wave A_1 at $|y| = \infty$ is obtained by

$$A_1 = \frac{2\omega|U|KB(K)}{g}$$

and if

$$U = -i\omega h_o e^{-i\omega t}$$

then

$$\bar{A} \equiv \frac{A_1}{h_o} = 2K^2 B(K) \tag{23}$$

Thus, the sway damping for demihull of a SWATH cross section can be obtained from Equations (8) and (23) as

$$\bar{b}_{22}^{(1)} = \frac{1}{\rho\omega S_A} \left(\frac{\rho\omega\bar{A}^2}{K^2}\right)$$

$$= \frac{4K^2B^2(K)}{S_A} \tag{24}$$

The interference effect from the other hull will be treated the same as in the previous case. That is, the far-field wave amplitude A_1 will be modified as $\beta_o A_1$ where β_o is given by Equation (13). Consequently, the sway-damping coefficient for the twin hulls is obtained by

$$\bar{b}_{22} = C_2 \beta_o^2 \bar{b}_{22}^{(1)} = \frac{4C_2(\beta_o KB(K))^2}{S_A} \tag{25}$$

where C_2 is a correction factor which should be determined as the need arises.

For the cross section without the struts, the sway damping is assumed to be similar to the heave damping given by Equation (19), i.e.,

$$\bar{b}_{22} = C_2 \frac{2\rho\omega K^2}{\rho\omega S_A} e^{-2Kd_c} \left(S_A + \frac{\mu_{22}}{\rho}\right)^2 (1+ \cos 2Kb)$$

$$= 2C_2 K^2 e^{-2Kd_c} \pi R^2 \left(2- \frac{R^2}{2b^2} + \frac{R^2}{2d_c^2}\right) (1+ \cos 2Kb) \tag{26}$$

where

$S_A = \pi R^2$

$\frac{\mu_{22}}{\rho}$ = one-half of the sway added mass of twin circles under a rigid surface

$= \pi R^2 \left(1- \frac{R^2}{2b^2} + \frac{R^2}{2d_c^2}\right)$

Roll Damping Approximation

The roll moment exerted on a SWATH cross section due to hydrodynamic pressures p can be expressed by

$$M_R = \oint p(yn_z - zn_y)d\ell \tag{27}$$

where n_y and n_z are, respectively, the y-component and the z-component of the unit normal vector pointing into the body on the submerged contour of the cross section, and $\oint d\ell$ is the integral over the submerged cross-section contour.

If ϕ_R denotes the velocity potential associated with a forced roll oscillation of the cross section, then, from the linearized Bernoulli equation,

$$p = i\rho\omega\phi_R$$

hence

$$M_R = i\rho\omega \oint \phi_R(yn_z - zn_y)d\ell \tag{28}$$

Assume that

$$\phi_R \doteq \phi_H y - \phi_S z \tag{29}$$

Then, substituting Equation (29) into Equation (28),

$$M_R \doteq i\rho\omega \left[\overline{y^2} \oint \phi_H n_z d\ell + \overline{z^2} \oint \phi_S n_y d\ell - \oint yz(\phi_S n_z + \phi_H n_y) d\ell \right] \tag{30}$$

where the mean-value theorem is used by defining

$$\overline{y^2} = \frac{\oint y^2 \phi_H n_z d\ell}{\oint \phi_H n_z d\ell}$$

$$\overline{z^2} = \frac{\oint z^2 \phi_S n_y d\ell}{\oint \phi_S n_y d\ell}$$

The last integral in Equation (30) can be approximated by

$$\oint yz\,(\phi_S n_z - \phi_H n_y)\,d\ell \doteq 2b \oint_{DH} z(\phi_S n_z - \phi_H n_y)\,d\ell$$

where $\oint_{DH}$ means the integral along the submerged contour of the demihull. From the far-field point of view, it can be approximated that ϕ_S is odd and ϕ_H is even with respect to the vertical centerline of the demihull. Then, due to the fact that z and n_z are even and n_y is odd, the integrand of the foregoing integral becomes odd, hence the integral vanishes.

Because the imaginary part of Equation (30) corresponds to the roll damping, the roll damping can be obtained by

$$b_{44} = \overline{y^2}\, b_{33} + \overline{z^2}\, b_{22}$$

or

$$\bar{b}_{44} \equiv \frac{b_{44}}{\rho\omega b^2 S_A}$$

$$= \frac{(\overline{y^2}\,\bar{b}_{33} + \overline{z^2}\,\bar{b}_{22})}{b^2}$$

$$\doteq \frac{\bar{b}_{33} + \bar{b}_{22}\,\overline{z^2}}{b^2} \qquad (31)$$

where it is assumed that $\overline{y^2} = b^2$. An approximate $\overline{z^2}$ is given by

$$\overline{z^2} \doteq \int_{-D}^{0} \frac{\zeta^2 e^{K\zeta} \sqrt{D^2-\zeta^2}\, d\zeta}{B(K)} \tag{32}$$

For the cross section without the struts, the roll damping is approximated by

$$\bar{b}_{44} = \bar{b}_{33} \left(1 + \frac{d_c^2}{b^2}\right) \tag{33}$$

where $\bar{b}_{33}$ corresponds to the one given by Equation (19).

Sway-Roll Coupling Damping Approximation

The same assumptions made in the approximation of the roll damping will yield

$$\bar{b}_{24}\ (= \bar{b}_{42}) = \frac{b_{22}}{\rho \omega b S_A} = \frac{b_{22}\bar{z}}{b} \tag{34}$$

where

$$\bar{z} \doteq \int_{-D}^{0} \frac{\zeta e^{K\zeta} \sqrt{D^2-\zeta^2}\, d\zeta}{B(K)} \tag{35}$$

For the cross section without the struts,

$$\bar{b}_{24} = \frac{\bar{b}_{22} d_c}{b} \tag{36}$$

where $\bar{b}_{22}$ is given by Equation (26).

Added Mass Approximation

Once the damping coefficients are determined, then the corresponding conjugate pairs, i.e., the added-mass coefficients can be obtained by the Kramers-Kronig relations. A convenient form for numerical manipulations is given in Reference 4 as

$$a_{jk}(\omega) - a_{jk}(\infty) = -\frac{2}{\pi}\int_0^\infty \cos \omega t \int_0^\infty \frac{b_{jk}(\omega')-b_{jk}(\infty)}{\omega'} \sin \omega' t \, d\omega' dt \qquad (37)$$

for j, k = 2, 3, and 4

where a_{jk} is the added-mass coefficient in the jth mode due to the motion in the kth mode, and $b_{jk}(\infty)$, in general, is zero because no surface wave can be generated at $\omega = \infty$. The nondimensional added-mass coefficients will be denoted by the bar sign, and are defined by

$$(\bar{a}_{22}, \bar{a}_{33}) \equiv \frac{(a_{22}, a_{33})}{\rho S_A} \qquad (38)$$

$$\bar{a}_{24} = \bar{a}_{42} \equiv \frac{a_{24}}{\rho b S_A} \qquad (39)$$

$$\bar{a}_{44} \equiv \frac{a_{44}}{\rho b^2 S_A} \qquad (40)$$

The added-mass coefficients at the infinite frequency are approximated by

$$\bar{a}_{22}(\infty) = \frac{2D^2}{\pi S_A} \qquad (41)$$

which is based on the flat plate results,

$$\bar{a}_{33}(\infty) = \frac{\pi}{S_A}\left(R^2 - \frac{b_o^{\,2}}{4}\right) \qquad (42)$$

$$\bar{a}_{44}(\infty) = \bar{a}_{33}(\infty) + \bar{a}_{22}(\infty)\,\frac{\pi^2 D^2}{32 b^2} \qquad (43)$$

where $\pi^2 D^2/32$ is obtained by $\bar{a}_{44}(\infty)/a_{22}(\infty)$ for a vertical plate having the draft D,[14]

$$\bar{a}_{42}(\infty) = \bar{a}_{22}(\infty)\,\frac{D}{2b} \qquad (44)$$

For the cross section without the struts, assume that

$$\bar{a}_{22}(\infty) = \left(1 - \frac{R^2}{2b^2} - \frac{R^2}{2d_c^{\,2}}\right) C_3 \qquad (45)$$

$$\bar{a}_{33}(\infty) = \left(1 + \frac{R^2}{2b^2} - \frac{R^2}{2d_c^{\,2}}\right) C_3 \qquad (46)$$

$$\bar{a}_{44}(\infty) = \bar{a}_{33}(\infty) + \frac{\pi^2}{32}\left(\frac{D}{b}\right)^2 \qquad (47)$$

$$\bar{a}_{42}(\infty) = \bar{a}_{22}\,\frac{d_c}{b} \qquad (48)$$

where C_3 is a correction factor which should be determined when the need arises. The expressions given by Equations (45) and (46) correspond to the cases that twin circles, separated by 2b between the centers, move with a constant velocity in the unbounded fluid in the direction opposite to each other, and in the same direction normal to the line connecting the centers of the circles, respectively.

DISCUSSION OF RESULTS

The numerical results obtained by the approximate methods described in the preceding sections are presented in Figures 2 through 9. The results of the approximate methods are compared with the results obtained by the accurate theory,[8] and with the available experimental results. Hereafter, the method described in Reference 8 shall be referred to as "accurate theory."

The damping coefficients of a heaving, two-dimensional SWATH cross section are presented in Figure 2. The heave damping b_{33} is made non-dimensional by the product of the displaced fluid mass (ρS_A) and the circular frequency of oscillation (ω). The relative dimensions of the section are as shown in the figure. The approximate heave damping coefficients are obtained by Equation (18) with $C_1 = 1$ and $\alpha_T = 0$. It means that the phase change of the transmitted wave beyond the vertical barrier is neglected. Inclusion of the phase change in Equation (18) resulted in poorer agreement with the accurate theoretical results. As can be seen, the three results agree reasonably well for the frequency number ($\omega^2 R/g$) less than 0.5.

Because the shortest wavelength of practical interest in SWATH motion at zero forward speed is about one-half of the ship length, the highest frequency number corresponding to this wavelength for a SWATH ship having the length-to-diameter ratio of 15 is about 0.8. Beyond this highest frequency limit, SWATH ships would barely respond to waves, which means that there is no need of computing the transfer function of the motion response beyond this frequency limit. Thus, the approximate results shown in Figure 2 would be quite reasonable for use in the computation of motion of SWATH ships in waves.

The heave added-mass coefficient for the SWATH section shown in Figure 2 is presented in Figure 3. The results from the approximate method are obtained from Equations (37) and (42). The integrals on the right-hand side of Equation (37), in fact, represent double Fourier transforms. A fast Fourier transform subroutine of the system library of the computer

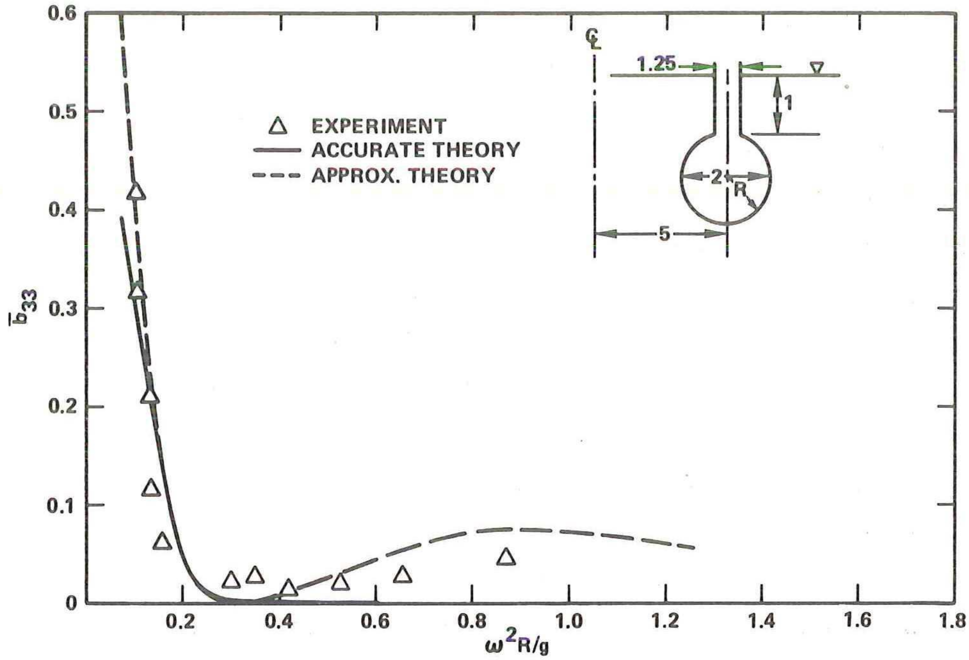

Figure 2 - Heave Damping Coefficient of a SWATH Section

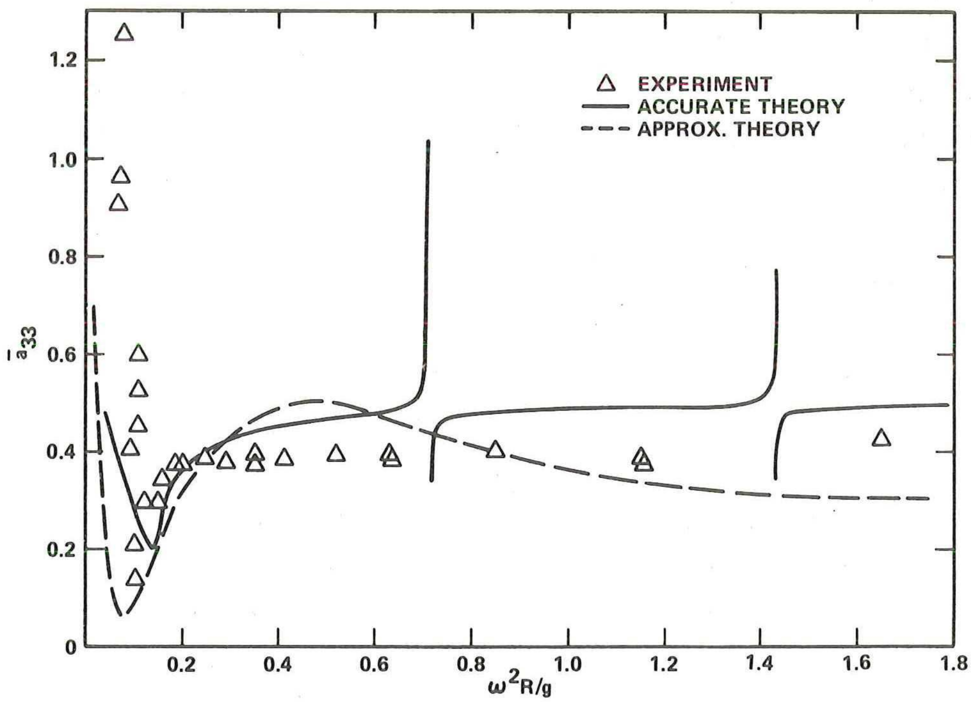

Figure 3 - Heave Added-Mass Coefficient of a SWATH Section

facility at the Center was used to evaluate the double Fourier transforms. The subroutine is called "FFT" which was originally developed at the Los Alamos Scientific Laboratory in California.

As can be observed in Figure 3, an apparent discrepancy between the exact and approximate methods is at those frequencies at which the exact method shows abrupt discontinuities. So far as the problem is confined to a two-dimensional flow condition, the discontinuities exhibited by the exact method can be real physical phenomena*; although, the experimental results, unfortunately, were not taken at those frequencies. However, for three-dimensional twin bodies such as SWATH, there cannot be a complete trapping of waves between the two hulls; hence, no abrupt discontinuities of the hydrodynamic coefficients at certain frequencies should exist. In this respect, the approximate method, which does not create the discontinuities, may cause less errors at those frequencies at which the discontinuity in the hydrodynamic coefficient occurs.

The SWATH cross sections at far forward or aft portion of the body do not have vertical struts. For these sections, the cross-section view is completely submerged twin circles. The heave added mass and and damping coefficients of such a cross section are shown in Figure 4. For the damping, Equation (19) was used with $C_1 = e^{-2KR}$, and for the added mass, Equations (37) and (46) with $C_3 = 1.1$ were used. Agreement between the two methods appears good from a practical viewpoint for the prediction of SWATH motion. This is because a significantly larger magnitude of heave damping contributed by viscosity and stabilizing fins should be added to the wavemaking damping in the analytical prediction of SWATH motion; therefore, a 50 percent error, as can be observed in the heave damping in Figure 4, is not going to affect, significantly, the final results of the motion. There appears to be a maximum of about 5 percent error in the heave added mass calculated by the approximate method. Because the SWATH motion is far less sensitive to the added mass than to the damping, a 5 percent error in the added-mass coefficient is not going to affect, significantly, the predicted results of SWATH motion.

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*Such evidence is clearly demonstrated by the experiments for several two-dimensional twin bodies, as described in Reference 8.

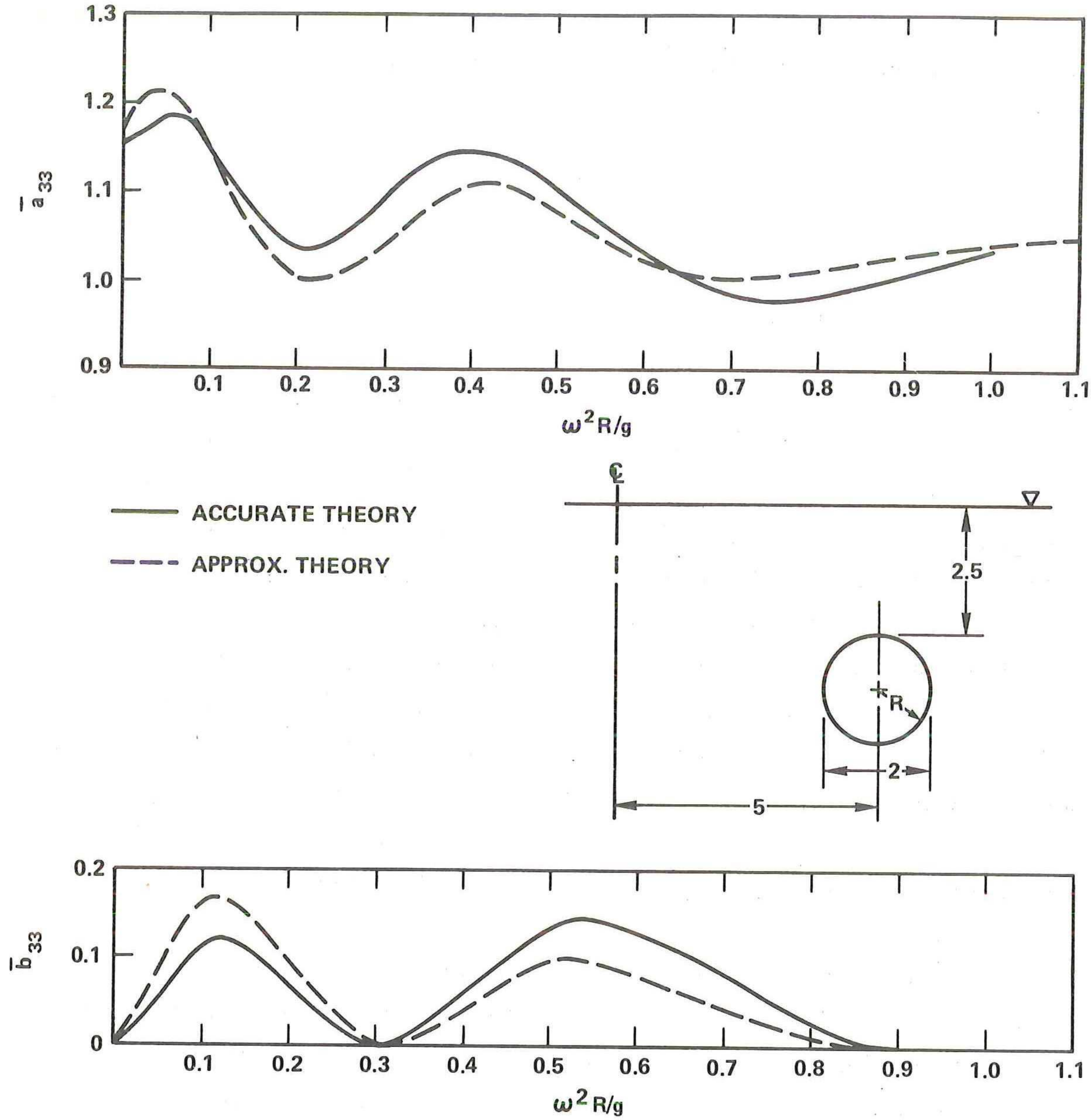

Figure 4 - Heave Added-Mass and Damping Coefficients of Submerged Twin Circles

The sway damping of a SWATH cross section is presented in Figure 5, together with the relative dimension of the cross section. The damping coefficients obtained by the approximate method are based on Equation (25) with the modification factor $C_2 = e^{(d_o/(2R)-KD)}$ where d_o, the depth of the centroid of the submerged cross sectional area, can be obtained by

$$d_o = \frac{1}{S_A} \left[\frac{1}{2} b_o d_S{}^2 + \pi R^2 (d_S + R)\right]$$

At present, there is no rational explanation for the derivation of the modification factor C_2 as it is except that such a factor seems to represent the behavior of the sway damping in the low and high frequency ranges reasonably well. Also, in the vicinity of the frequency where an abrupt discontinuity occurs, the modification factor appears to make the approximate method behave smoothly across the discontinuity predicted by the accurate theory. The chained curve shown in Figure 5 represents the results of the approximate method using $C_2 = 1$.

The sway added mass of the SWATH cross section shown in Figure 5 is presented in Figure 6. The approximate results were obtained by Equations (37) and (41). In the frequency range of $0.2 < \omega^2 R/g < 0.5$, the approximate results show a large discrepancy. It is not obvious at the present stage whether the sway added mass computed by the approximate method would necessarily result in poorer prediction of SWATH motion than that obtained by the accurate theory.

The sway added mass and damping coefficients for submerged twin circles are presented in Figure 7. Equation (26) was used to obtain the damping, and Equations (37) and (45) with $C_3 = 1.1$ were used to obtain the added mass. The trend of the results is very close to that of the heave added mass shown in Figure 4.

The roll damping of a SWATH cross section is presented in Figure 8. The dimensions of the cross section are also shown in the figure. Note that the center of roll is located above the calm water level by the

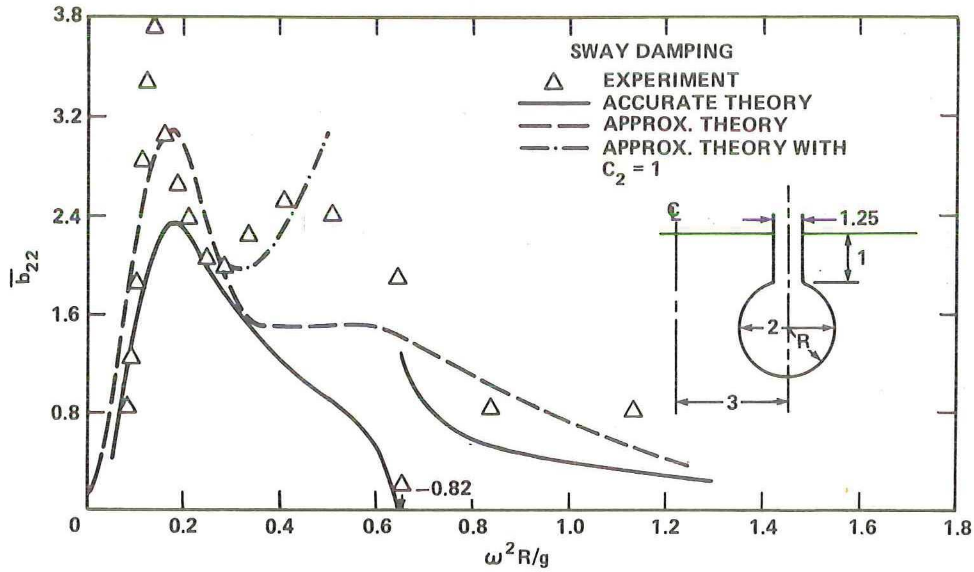

Figure 5 - Sway Damping Coefficient of a SWATH Section

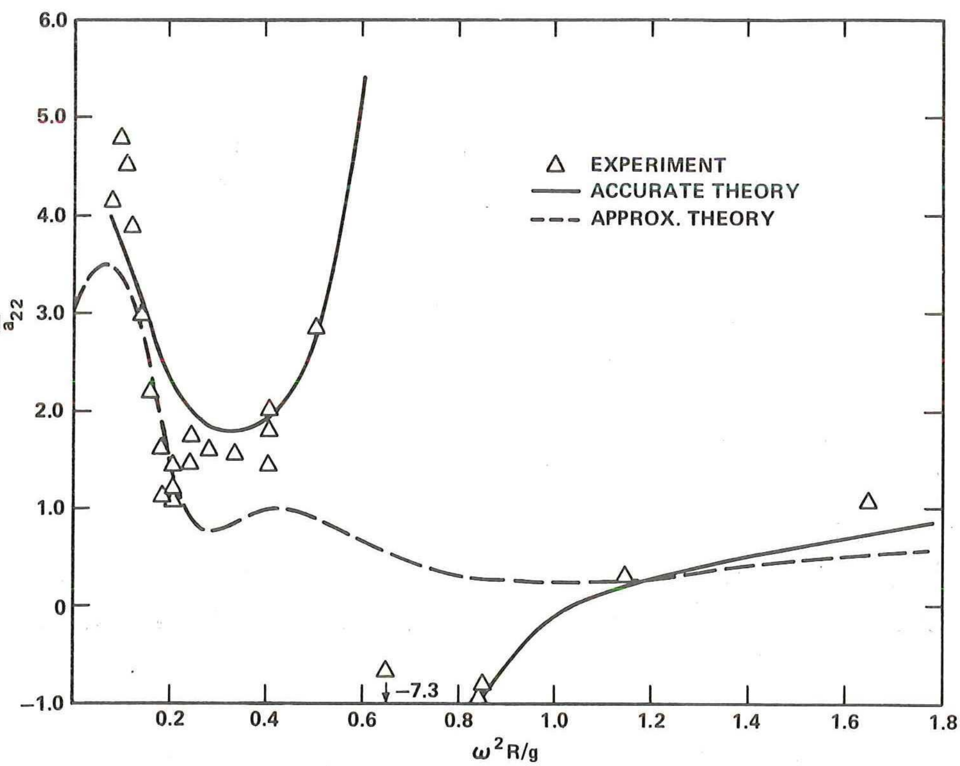

Figure 6 - Sway Added-Mass Coefficient of a SWATH Section

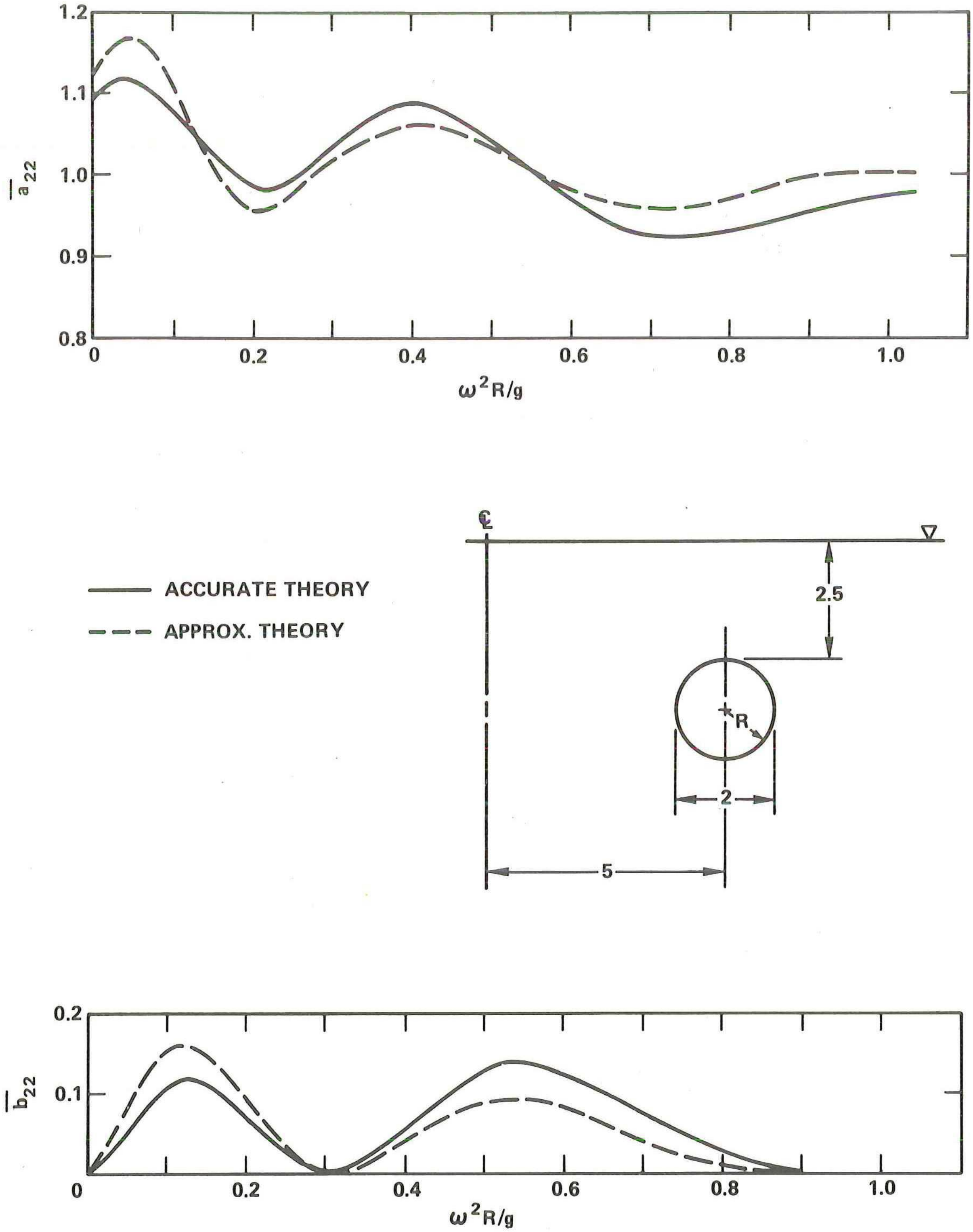

Figure 7 - Sway Added-Mass and Damping Coefficients of Submerged Twin Circles

distance of 3.1 times the radius of the cross section. The damping coefficients were obtained by Equation (31). Both the experimental and the accurate theoretical results show discontinuity at about $\omega^2 R/g = 0.65$. However, as in the case of heave and sway, the approximate results do not exhibit any discontinuity. The relative trends of the three different results are quite similar to the sway damping shown in Figure 5.

The roll added inertia coefficient of the cross section shown in Figure 8 is presented in Figure 9. The approximate results show a large discrepancy in $0.2 < \omega^2 R/g < 0.5$ as was already observed in Figure 6 for the sway added mass. The same remarks as given earlier on the sway added mass apply to the roll added inertia obtained by the approximate method.

The accurate theory, referred to frequently in this report, is based on the Green function method which leads to an integral equation involving the unknown strengths of the source distribution over the submerged contours of the cross section of the twin hulls. The numerical evaluation of the integral equation requires a segmentation of the integral along the cross-section contour into a finite number of integrals. Increasing the number of segments of the contour increases the accuracy of the results. For SWATH sections, it takes about fifteen to twenty segments on the contour of the demihull cross section to obtain accurate results. A typical computation by the CDC 6000-series computer at the Center for the sway and heave hydrodynamic coefficients for submerged twin circles for twenty frequencies took about 220 seconds of executing time.

On the other hand, the present approximate method took only about 16 seconds to obtain the sway, heave, roll, and sway-roll coupling coefficients for 256 frequencies.* Even if a discount is made for the large number of frequencies which are dictated by the usage of the fast Fourier transform technique but not by the practical necessity, the approximate theory seems to yield a time saving in the computation by an order of magnitude compared to the accurate theory.

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*Most of the fast Fourier transform subroutine requires that the function to be transformed should be given in numbers of 2^n where n is an integer. For the present work, it was found that $n = 8$ yields satisfactory results.

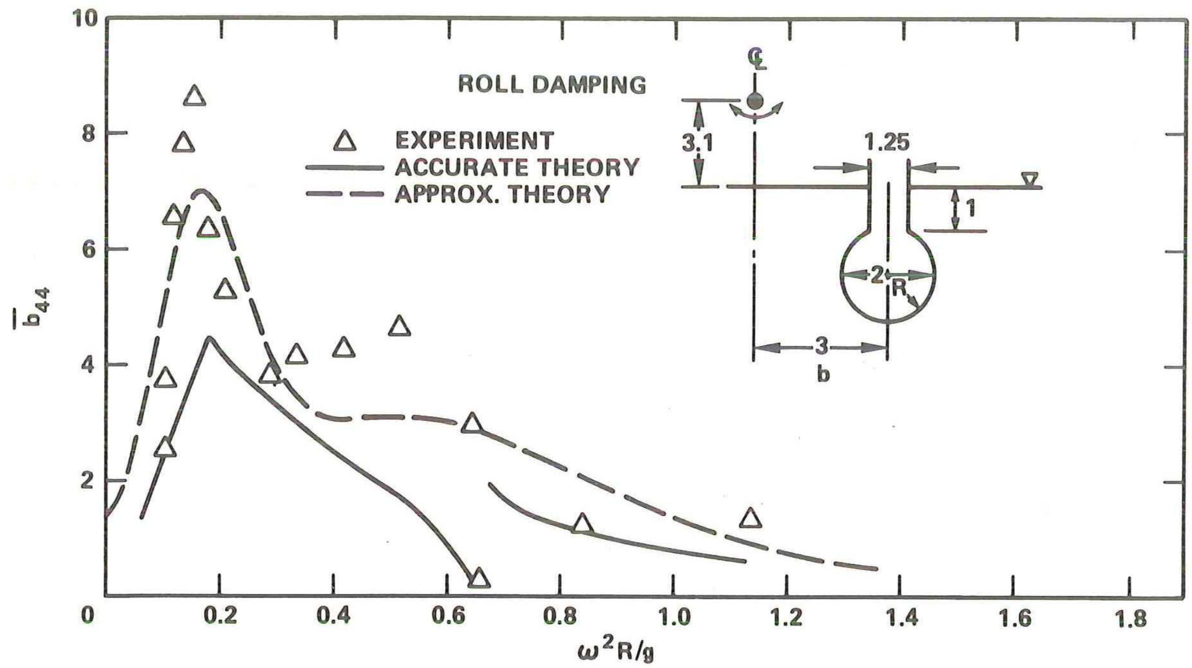

Figure 8 - Roll Damping Coefficient of a SWATH Section

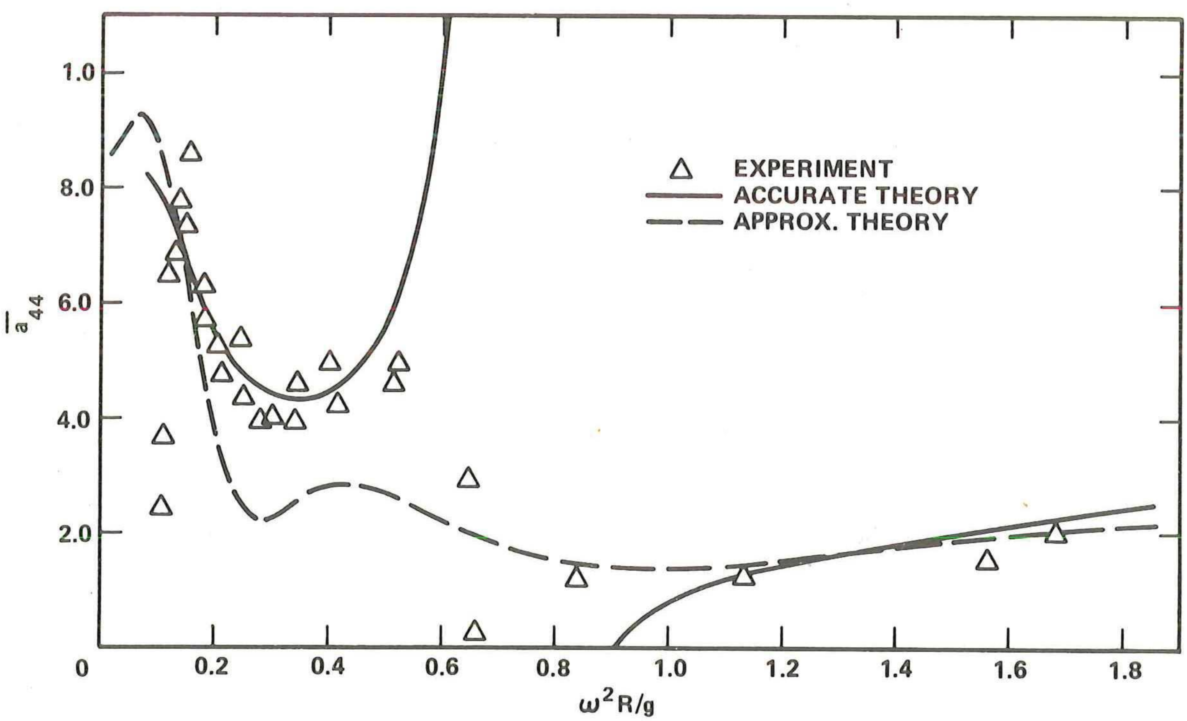

Figure 9 - Roll Added-Inertia Coefficient of a SWATH Section

SUMMARY AND CONCLUSIONS

To develop a simplified analytical prediction of SWATH motions in waves, approximate methods of computing the hydrodynamic coefficients of oscillating cross sections of a SWATH configuration are developed. The hydrodynamic coefficients studied are the added mass and the wavemaking damping coefficients in sway, heave, roll, and sway-roll coupling modes.

The approximate methods are based on the concept that the wavemaking damping coefficients can be obtained by the amplitudes of the outgoing waves at the far field. The outgoing waves at the far field can be represented by a pair of sources and of dipoles for the heave oscillation and by a dipole distribution on the vertical centerline of each hull for the sway and roll oscillations. The strength of the singularities are obtained under the assumption that the cross section is oscillating in an unbounded fluid.

The hydrodynamic interactions between the two hulls are treated as if one hull is replaced by a vertical barrier having the draft as the original form and, therefore, it blocks the passage of the waves generated by the other hull.

The added-mass coefficients are obtained by invoking the Kramers-Kronig relations with the known damping coefficients and the added-mass coefficients at the infinite frequency, as indicated by Equation (37). The evaluation of Equation (37) is carried out by a well-known numerical algorithm called fast Fourier transform technique. When needed, the approximate methods are augmented by the modification factors which are determined by a numerical trial-and-error approach.

The results from the approximate methods, the accurate theory based on the source distribution on the submerged contours,[8] and the experiments, if available, are compared in Figures 2 to 9. The agreements of the results in the frequency range of practical interest for SWATH motion are, in general, fair; however, the abrupt discontinuities of the hydrodynamic coefficients at certain frequencies exhibited by the accurate theory and the experiments are not reproduced by the approximate methods. The abrupt discontinuities are merely caused by the two-dimensional flow condition, and in reality, for three-dimensional bodies, they do not exist; hence, the continuity of the hydrodynamic coefficients at all frequencies given by the approximate methods could be more realistic.

The time saving in the computation of the hydrodynamic coefficients achieved by use of the approximate method is remarkable; the approximate method can provide the results in less than one-hundredth of the time which would be required by the accurate theory.

It is too early to judge whether the approximate methods developed here will provide reasonable prediction of SWATH motions in waves. The usefulness of the approximate methods will be realized only when they are shown to provide adequate predictions of the motions for use during the stage of conceptual hull design of SWATH configurations. However, it is believed that a final useful method of predicting SWATH motion can be developed on the basis of the approximate methods presented in this report by an intelligent usage of the modification factors associated with the approximate methods.

ACKNOWLEDGMENTS

The author would like to express his appreciation for the funding support given by the High Performance Vehicles Program Office of the Ship Performance Department at the Center. He also would like to extend his gratitude to Ms. Margaret D. Ochi and Mr. Grant R. Hagen for their encouragement during the course of this work.

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