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Highlights

- This paper presents a database of 287 reinforced concrete slabs failing in shear.
- Artificial neural networks are used to find a matrix-based expression.
- The developed expression can be used for the assessment of slab bridges.
- As mechanical models are lacking, the presented expression can be used to predict the shear capacity of slabs.
- The matrix-based expression gives insight in the sensitivity to certain parameters.

Neural network-based formula for shear capacity prediction of one-way slabs under concentrated loads

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Abstract

According to the current codes and guidelines, shear assessment of existing reinforced concrete slab bridges sometimes leads to the conclusion that the bridge under consideration has insufficient shear capacity. The calculated shear capacity, however, does not consider the transverse redistribution capacity of slabs, and thus leads to overly conservative values. While mechanics-based models have attempted to describe the problem of one-way shear in concrete slabs under concentrated loads, this problem is still not fully understood. Therefore, this paper proposes an artificial neural network (ANN)-based formula to come up with estimates of the shear capacity of one-way reinforced concrete slabs under a concentrated load that are as good as possible based on 287 test results obtained from the literature. The methods used for this purpose are: (i) the development of the database with experimental results from the literature, and (ii) the development of the ANN-based matrix formulation. For the latter purpose, many thousands of ANN models were generated, based on 475 distinct combinations of fifteen typical ANN features. The proposed "optimal" model yields maximum and mean relative errors of 0.0% for the 287 datapoints. Moreover, it was illustrated to clearly outperform (mean V_{test} / V_{ANN} =1.00) the Eurocode 2 provisions (mean $V_{E,EC} / V_{R,c}$ =1.59) for that dataset. A step-by-step assessment scheme for reinforced concrete slab bridges by means of the ANN-based model is also proposed in this work, which results in an improvement of the current assessment procedures.

Keywords: Artificial Neural Networks; Bridges; Design Formula; One-Way Slabs; Reinforced Concrete; Shear Capacity

1. Introduction

As the age of existing infrastructures is increasing, the question if existing structures are safe for further operation becomes important. To answer this question, an accurate assessment of the existing infrastructures is necessary. The assessment should not be overly conservative, so that unnecessary strengthening or replacement actions can be avoided. On the other hand, the assessment should be as accurate as possible, so that structural safety can be assured.

When reinforced concrete slab bridges are assessed, the estimated one-way shear capacity can be overly conservative, as transverse redistribution is not considered in the existing codes [1, 2]. In Europe, the live load model from NEN-EN 1991-2:2003 [3] uses a distributed lane load and design tandems. These tandems consist of large concentrated loads that are closely spaced, so that the load combination with the currently prescribed load model in Europe leads to large shear stresses at the support. As a result, a large number of reinforced concrete slab bridges are found to be insufficient for shear when assessed according to the currently governing codes [4]. While the shear provisions fulfil the purpose for design (i.e. ensuring that the designed bridge fails in flexure before shear), it does not fulfil the purpose for assessment (i.e. separating the bridges that pose a safety risk and require strengthening from those that are only shear-critical according to the conservative code formulas but, when analyzed further, fulfil the safety requirements when additional shear-carrying mechanisms are taken into account).

For more than a century [5-7], researchers have been debating the shear capacity of reinforced concrete members without shear reinforcement [8-10]. In slabs, the additional dimension of the width makes the problem three-dimensional [11, 12]. A plasticity-based model [13, 14] has been proposed to estimate the maximum load on a reinforced concrete slab bridge, but this method has the disadvantage that the calculation needs to be tailored to the geometry of the bridge under consideration. Nonlinear finite element models [15] combined with the appropriate safety formats [16-19] can be used for the assessment of existing reinforced concrete slab bridges, but this approach is quite time-consuming [20].

When a large number of bridges need to be assessed, computationally fast methods are necessary. To determine the sectional shear stresses and bending moments due to the applied load combination, automated procedures using linear finite element models can be. Determining the bending moment capacity can be based on the traditional flexural theory for reinforced concrete beams. For a more effective estimate of the shear capacity of one-way reinforced concrete slabs under a concentrated load, this paper proposes the use of artificial neural networks (ANN), a popular machine learning technique. This approach results in an improvement of the estimation of the shear capacity of reinforced concrete one-way slabs failing in shear. Moreover, the proposed ANN-based model can be used for the assessment of one-way reinforced concrete slab bridges. This paper contains a step-by-step approach for the assessment of such bridges.

Machine learning, one of the six disciplines of Artificial Intelligence (AI) without which the task of having machines acting humanly could not be accomplished, allows us to ‘teach’ computers how to perform tasks by providing examples of how they should be done [21]. When there is abundant data (also called examples or patterns) explaining a certain phenomenon, but its theory richness is poor, machine learning can be a perfect tool; as such its application to the problem of shear in one-way slabs is suitable and timely. The Artificial Neural Network (also referred in this manuscript as ANN or neural net) is the (i) oldest [22] and (ii) most powerful [23] technique of machine learning. ANNs also lead the number of practical applications, virtually covering any field of knowledge [24, 25]. In its most general form, an ANN is a mathematical model designed to perform a particular task, based in the way the human brain processes information, i.e. with the help of its processing units (the neurons). ANNs have been employed to perform several types of real-world basic tasks, and have been successfully applied to civil engineering problems [26-41]. Some efforts have also been geared towards using ANN-based prediction models for the problem related to shear in structural concrete, yet these models still have relatively large errors [42-49]. Concerning functional approximation, ANN-based solutions are frequently more accurate than those provided by traditional approaches, such as multi-variate nonlinear regression, besides not requiring a good knowledge of the function shape being modelled [50]. The proposed ANN was designed based on the 287 experimental results available to date in the literature.

The goal of this study is not to provide a full description of the mechanics underlying the behavior of one-way reinforced concrete slabs. While research efforts are being geared towards understanding the mechanics behind the shear capacity of reinforced concrete slabs, the proposed approach allows us to use the available experimental data in an optimal way, and to address the current need for the assessment of reinforced concrete slab bridges with computationally efficient tools.

2. Research significance

This work proposes a new way to determine the shear capacity of reinforced concrete one-way slabs based on artificial neural networks. For this study, a unique database (available in the public domain) of 287 experimental results is analyzed. The analysis is based on combining different possible features of artificial neural networks, and finding the best performing matrix-based expression. This new expression has a practical application as well: it can be used for the design and analysis of reinforced concrete slab bridges, as suitable physical models for this problem are currently not available.

3. Data Gathering

The dataset used for the development of the ANN simulations consists of 287 experimental results from (i) tests gathered from the literature reported in [51], consisting of references [52-85], (ii) the TU Delft slab shear tests [86], and (iii) recently reported experiments [87]. The collected experiments are on slabs and wide beams, under line loads and concentrated loads. The specimens are either cantilevers, simply supported slabs, continuously supported slabs, or experiments carried out on slab bridges. Analyzing the test results shows that the minimum value of $b/d_l = 0.57$, the maximum value is 46.30 and the average is 10.40, showing that the majority of the experiments would classify as slabs according to the ratio $b/d_l \geq 5$. Additionally, we considered that a certain amount of transverse load redistribution also occurs for wide beams subjected to a load that does not act over its full width. Eleven variables were adopted as input (independent) for the ANN-based shear capacity predictions, as described and illustrated in Table 1 and Fig. 1, respectively. Fig. 1 shows the resulting bending moment and shear diagrams when the slab is considered as a beam model, the supports are considered as point supports, and the load is considered as a point load.

Table 1 also gives the minimum and maximum value for each parameter in the database. As can be seen in Table 1, the range of parameters covers the geometry of laboratory-sized specimens to actual bridges tested to collapse in the field. The range of values for the concrete compressive strength covers from low strength concrete to high strength, which is often found in existing reinforced concrete slab bridges as the result of ongoing cement hydration. The reinforcement ratios in the database also encompass the values encountered in existing slab bridges. The large range of parameters for the loading conditions aims to reflect the assessment practice, where the main contribution to the sectional shear comes from the design tandem.

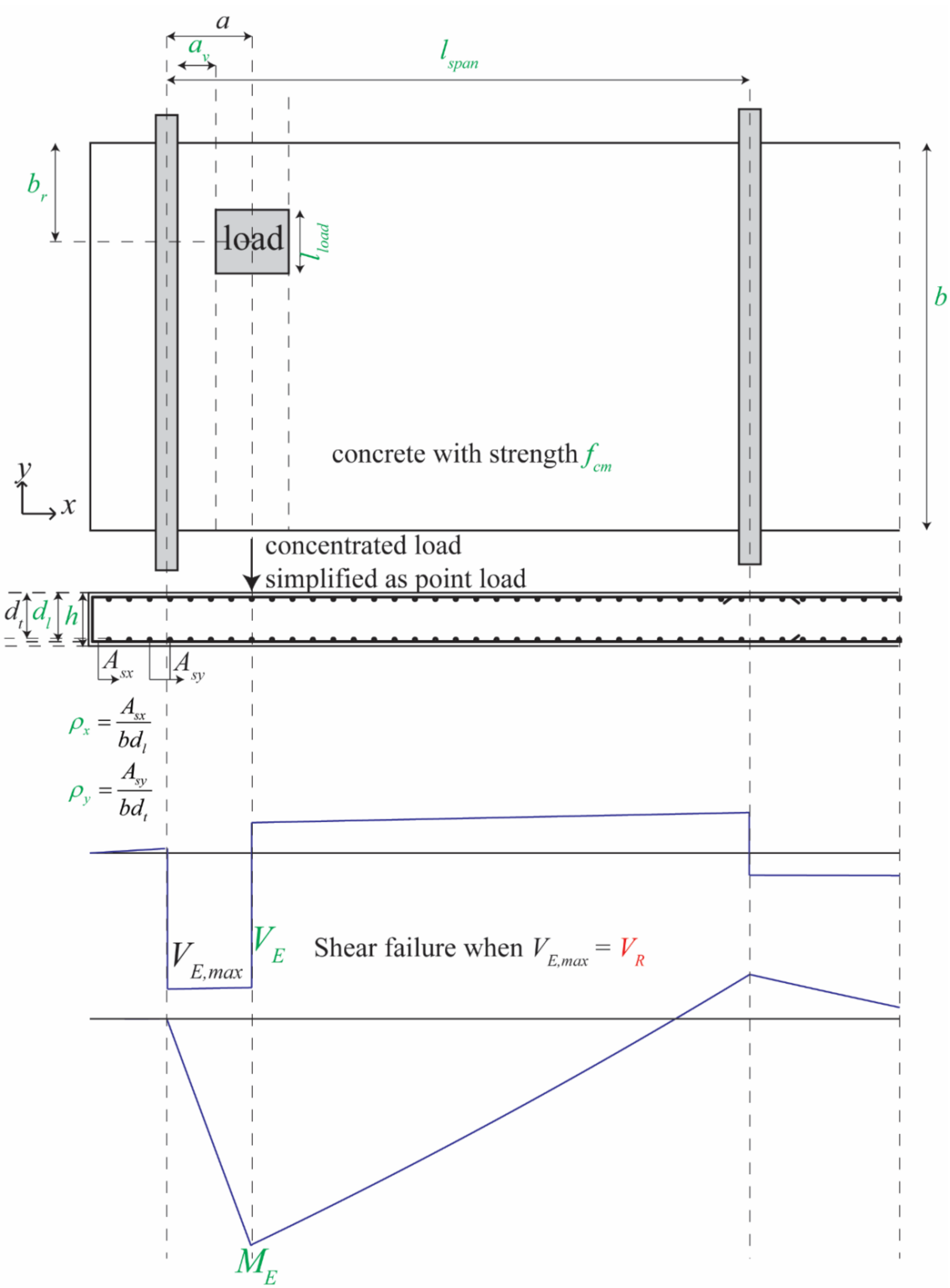

Fig. 1. Input (independent) and output (dependent) variables considered in ANN design.

Note that the proposed ANN features just 10 nodes in the first layer, which inputs have to be obtained as function of those eleven variables, as described in §3.7.1. For all experiments, the sectional shear and moment were calculated considering all loads, thus including the self-weight. For the case of a continuous slab shown in Fig. 1, the slight gradient in the shear diagram and the slight nonlinearity in the bending moment diagram are caused by the self-weight. All values of the concrete compressive strength are the cylinder compressive strength. This value was either reported in the original reference, or calculated as 82% of the cube compressive strength [88]. The corresponding 287-point dataset is publicly available [89], and was constructed by randomly ordering the collected experimental results.

Table 1. Variables adopted in the study, showing minimum and maximum values in the database.

Input Variables			min	max	Input
Slab geometry	$\boldsymbol{b}$ (m)	slab width	0.249	11.125	1
	$\boldsymbol{h}$ (m)	slab height	0.100	1.005	2
	$\boldsymbol{d_l}$ (m)	slab effective depth	0.080	0.916	6
	$\boldsymbol{l_{span}}$ (m)	span length	0.600	12.192	9
Material	$\boldsymbol{f_{cm}}$ (MPa)	average concrete cylinder compressive strength	12.4	77.7	3
Reinforcement	$\boldsymbol{\rho_x}$ (-)	longitudinal reinforcement ratio	0.003	0.028	4
	$\boldsymbol{\rho_y}$ (-)	transverse reinforcement ratio	0	0.015	5
Loading parameters	$\boldsymbol{b_r}$ (m)	distance from slab edge to the center of the load	0.125	5.563	7
	$\boldsymbol{l_{load}}$ (m)	dimension of the loading plate (wheel print)	0.070	2.519	8
	$\boldsymbol{M_E / V_E d_l}$ (-)	ratio of sectional moment to product of sectional shear and effective depth	0.14	10.75	10
	$\boldsymbol{a_v / d_l}$ **(-)**	ratio of clear shear span to effective depth	0.00	6.88	11

Output Variables				Output
V_R (kN)	shear capacity	35	2444	1

4. Artificial Neural Networks

4.1 General approach

The general ANN structure consists of several nodes in L vertical interconnected layers (input layer, hidden layers, and output layer). Between each node (or neuron) in layers 2 to L is a linear or nonlinear transfer function. All ANNs implemented in this work are feedforward. The neural network is developed through "learning": determining the synaptic weight of the connection between two nodes, and each neuron's bias. To find the optimal matrix-based expression for the problem under study, 15 ANN features were varied in this work. Tables 2-4 show the 15 ANN features that were varied in this study. The code was developed in MATLAB [90] with its neural network toolbox for using popular learning algorithms (1-3 from F13 in Table 4). Each parametric sub-analysis (SA) consists of running all feasible combinations of pre-selected methods for each ANN feature and finding the associated performance results for each designed net. This approach then allows selection of the "best" neural net for the problem under study. The best network is the one exhibiting the smallest average relative error for all learning data. The developed algorithm is validated [91]. The interested reader can find more background on the development of the algorithm to find the optimal neural network in [92].

Table 2. Implemented ANN features (F) 1-5.

FEATURE	F1	F2	F3	F4	F5
METHOD	Qualitative Var Represent	Dimensional Analysis	Input Dimensionality Reduction	% Train-Valid-Test	Input Normalization
1	Boolean Vectors	Yes	Linear Correlation	80-10-10	Linear Max Abs
2	Eq Spaced in]0,1]	No	Auto-Encoder	70-15-15	Linear [0, 1]
3	-	-	-	60-20-20	Linear [-1, 1]
4	-	-	Ortho Rand Proj	50-25-25	Nonlinear
5	-	-	Sparse Rand Proj	-	Lin Mean Std
6	-	-	No	-	No

Table 3. Implemented ANN features (F) 6-10.

FEATURE	F6	F7	F8	F9	F10
METHOD	Output Transfer	Output Normalization	Net Architecture	Hidden Layers	Connectivity
1	Logistic	Lin [a, b] = 0.7[φ_{min}, φ_{max}]	MLPN	1 HL	Adjacent Layers
2	-	Lin [a, b] = 0.6[φ_{min}, φ_{max}]	RBFN	2 HL	Adj Layers + In-Out
3	Hyperbolic Tang	Lin [a, b] = 0.5[φ_{min}, φ_{max}]	-	3 HL	Fully-Connected
4	-	Linear Mean Std	-	-	-
5	Bilinear	No	-	-	-
6	Compet	-	-	-	-
7	Identity	-	-	-	-

Abbreviations: MLPN = multi-layer perceptron net, RBFN = radial basis function net

Table 4. Implemented ANN features (F) 11-15.

FEATURE	F11	F12	F13	F14	F15
METHOD	Hidden Transfer	Parameter Initialization	Learning Algorithm	Performance Improvement	Training Mode

1	Logistic	Midpoint (W) + Rands (b)	BP	NNC	Batch
2	Identity-Logistic	Rands	BPA	-	Mini-Batch
3	Hyperbolic Tang	Randnc (W) + Rands (b)	LM	-	Online
4	Bipolar	Randnr (W) + Rands (b)	ELM	-	-
5	Bilinear	Randsmall	mb ELM	-	-
6	Positive Sat Linear	Rand [-*Δ*, *Δ*]	I-ELM	-	-
7	Sinusoid	SVD	CI-ELM	-	-
8	Thin-Plate Spline	MB SVD	-	-	-
9	Gaussian	-	-	-	-
10	Multiquadratic	-	-	-	-
11	Radbas	-	-	-	-

Abbreviations: SVD = singular value decomposition, MB SVD = mini batch SVD, BP = back propagation, BPA = back propagation with adaptive learning rate, LM = Levenberg-Marquardt, ELM = extreme learning machine, mb-ELM = mini batch ELM, I-ELM = incremental ELM, CI-ELM = convex incremental ELM, NNC = neural network composite.

4.2 Development of matrix-based expression for shear capacity of one-way slabs

To reduce the computational time by reducing the number of combos to be analyzed, the parametric simulation was divided into nine parametric SAs, where in each one, F7 takes a single value, see Table 5 (the numbers represent the method number as in Tables 2-4). Summing up the ANN feature combinations for all parametric SAs, a total of 475 combos were ran for this work. Table 6 shows the corresponding relevant results in terms of error, performance, and computational time. All results shown in Table 6 are based on target and output datasets computed in their original format, i.e. free of any transformations due to output normalization and/or dimensional analysis.

Table 5. ANN feature (F) methods used in the best combo from each parametric sub-analysis (SA).

SA	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
1	1	2	6	2	5	1	1	1	1	1	3	2	3	1	3
2	1	2	6	2	1	7	1	1	1	1	3	2	5	1	3
3	1	2	1	3	5	1	1	1	1	1	3	2	3	1	3
4	1	2	1	3	5	1	2	1	1	1	3	2	3	1	3
5	1	2	1	4	5	1	3	1	1	1	3	2	3	1	3
6	1	2	1	4	5	7	4	1	1	1	3	2	3	1	3
7	1	2	1	1	5	7	5	1	1	1	3	2	3	1	3
8	1	2	1	1	5	7	5	1	1	1	5	5	3	1	3
9	1	2	1	1	5	7	5	1	2	3	5	5	3	1	3

Table 6. Performance results for the best design from each parametric sub-analysis: (a) ANN, (b) NNC.

SA	ANN				
	Max Error (%)	Performance All Data (%)	Errors > 3% (%)	Total Hidden Nodes	Running Time / Data Point (s)
1	0.0	0.0	0.0	44	2.13E-04
2	559.9	34.0	88.2	70	1.46E-04
3	0.0	0.0	0.0	37	2.43E-04
4	0.0	0.0	0.0	37	3.38E-04
5	0.0	0.0	0.0	37	1.77E-04
6	0.0	0.0	0.0	40	2.22E-04
7	171.0	5.8	30.3	29	1.48E-04
8	55.0	4.8	48.8	37	2.27E-04
9	66.7	6.5	62.0	30	1.59E-04

(a)

SA	NNC

	Max Error (%)	Performance All Data (%)	Errors > 3% (%)	Total Hidden Nodes	Running Time / Data Point (s)
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-
4	-	-	-	-	-
5	-	-	-	-	-
6	-	-	-	-	-
7	9.2	0.3	4.5	29	1.79E-04
8	54.3	4.8	48.4	37	2.40E-04
9	49.7	5.2	53.0	30	1.67E-04

(b)

Several SAs yielded approximately null errors, see Table 6. Therefore, the ANN having the least number of hidden nodes and the lowest running time per data point (SA 5) was selected as the optimal model. This model is developed with 50% of the data used for training, 25% for validation, and 25% for testing. To allow implementation of this model by any user, all variables/equations required for (i) data preprocessing, (ii) ANN simulation, and (iii) data post-processing, are available in the public domain [93]. The W and b arrays of the neural network are available as a spreadsheet in the public domain, allowing direct implementation of the proposed matrix-based formula [94]. The proposed model is a single MLPN with 3 layers and a distribution of nodes/layer of 10-37-1. The network is partially-connected, and the hidden and output transfer functions are all Hyperbolic Tangent and Logistic, respectively. The network was trained using the Levenberg-Marquardt algorithm (2565 epochs). The analysis showed that the column with d as input could be removed, resulting in 10 inputs. Fig. 2 depicts a simplified scheme of some of the network key features. The obtained ANN solution for every data point can be found in [89].

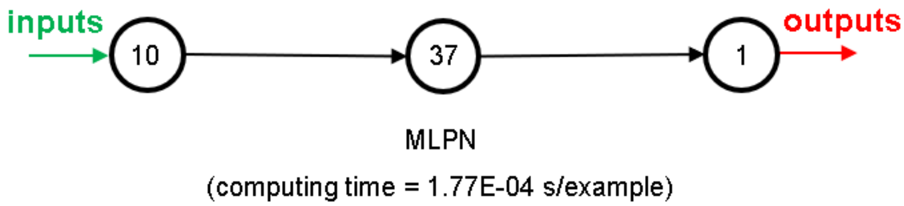

Fig. 2 Proposed 10-37-1 partially-connected MLPN – simplified scheme.

Finally, the results of the proposed ANN for the 287 datapoints, in terms of performance variables are presented as: (i) a regression plot (Fig. 3), where network target and output data are plotted, for each data point, as *x*- and *y*- coordinates respectively – a measure of linear correlation is given by the Pearson Correlation Coefficient (R); (ii) a performance plot, where performance (average error) values are displayed for several learning datasets, all of which give an error of 0%; and (iii) an error plot, where values concern all data (iii_1) maximum error and (iii_2) % of errors greater than 3% (both cases give 0%). All graphical results just mentioned are based on effective target and output values, i.e. computed in their original format (free of any transformations due to output normalization).

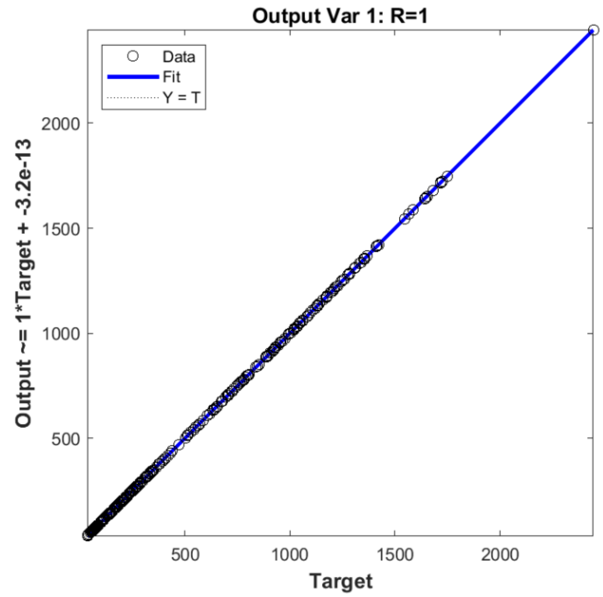

Fig. 3. Regression plot for the proposed ANN.

5. Sensitivity analysis

As suggested by [38], a sensitivity analysis can be carried out with the following expression, which determines the percentage effect of the i^{th} input variable on the output variable:

$$Q_i = \frac{\sum_{j=1}^{nhidden} \frac{w_{ji}}{\sum_{l=1}^{ninputs} \left| w_{jl} \right|} \times w_{oj}}{\sum_{i=1}^{ninputs} \left(\sum_{j=1}^{nhidden} \frac{w_{ji}}{\sum_{l=1}^{ninputs} \left| w_{jl} \right|} \times w_{oj} \right)} \quad (1)$$

The sum of the connection weights between the 10 input neurons and the 37 hidden neurons is:

$$\sum_{l=1}^{ninputs} \left| w_{jl} \right|$$

with w_{jl} the connection weight between the input neuron l and the hidden neuron j, and w_{oj} is the connection weight between the hidden neuron j and the output o.

Table 7 shows the results of this sensitivity analysis. The most important parameters are the total width b and the expression of the shear span to depth ratio based on the bending moment and shear diagram M_E/V_Ed_l. The third most important factor is the concrete compressive strength f_{cm}, which traditionally is considered as one of the most determining factors for the shear capacity. Of much less importance are the position of the load with respect to the width b_r and the total span length l_{span}.

Table 7. Sensitivity analysis with Q_i the sensitivity of the ith input value.

Input	Q_i (%)
b	47.05
h	7.01
f_{cm}	9.24
ρ_x	1.23
ρ_y	2.40
b_r	0.12
l_{load}	4.77
l_{span}	0.51
M_E/V_Ed_l	20.38
a_v/d_l	7.29

6. ANN-based vs. Existing Models

Since the focus of this study is the assessment of reinforced concrete slab bridges in Europe, this section demonstrates the improved prediction capability of the ANN-based analytical model proposed in section 4, as compared to the shear capacity of one-way slabs predicted by the provisions of Eurocode 2 (NEN-EN 1992-1-1:2005 [95]). The reduction of the contribution of loads close to the support ($a_v \leq 2d_l$, see Fig. 1) to the sectional shear force prescribed by the Eurocode is taken into account, resulting in $V_{E,EC}$. This reduction corresponds to an increase in the shear capacity for loads close to the support as a result of direct load transfer. Since this mechanism only occurs for loads applied on top of the cross-section and close to the support, the Eurocode 2 reduces the contribution of externally applied loads close to the support. As such, this provision allows for finding the sectional shear force for a combination of loads – a situation that occurs when assessing existing reinforced concrete slab bridges. The corresponding average shear capacity according to Eurocode 2 is determined as:

$$
\begin{gathered}
V_{R,c} = 0.15k\left(100\rho_x f_{cm}\right)^{1/3} b_{eff} d_l \geq 0.035 k^{3/2} \sqrt{f_{cm}} b_{eff} d_l \\
k = 1 + \sqrt{\frac{200mm}{d_l}} \leq 2
\end{gathered}
\quad , (2)
$$

with (i) k the size effect factor, (ii) ρ_x the longitudinal reinforcement ratio, (iii) f_{cm} the average concrete cylinder compressive strength in [MPa], (iv) b_{eff} the effective width for one-way shear, determined with a 45° horizontal load spreading from the loading plate back edge to the face of the support, and (v) d_l the effective depth to the longitudinal reinforcement. $C_{R,c}$ = 0.15 is used to find average values [96].

In addition to the Eurocode provisions, the shear provisions from ACI 318-14 [97] are also analyzed. The reader should note that these provisions are for building slabs, and thus not directly applicable to slab bridges. The design shear capacity according to ACI 318-14 equals:

$$
V_{ACI,d} = 0.167\sqrt{f_c'} b_w d \quad (3)
$$

with f_c' the specified concrete compressive strength in [MPa], b_w the web width, and d the effective depth. For application to slabs and for finding the average shear capacity $V_{ACI,m}$, f_c' is replaced with f_{cm}, b_w with b_{eff}, and d with d_l.

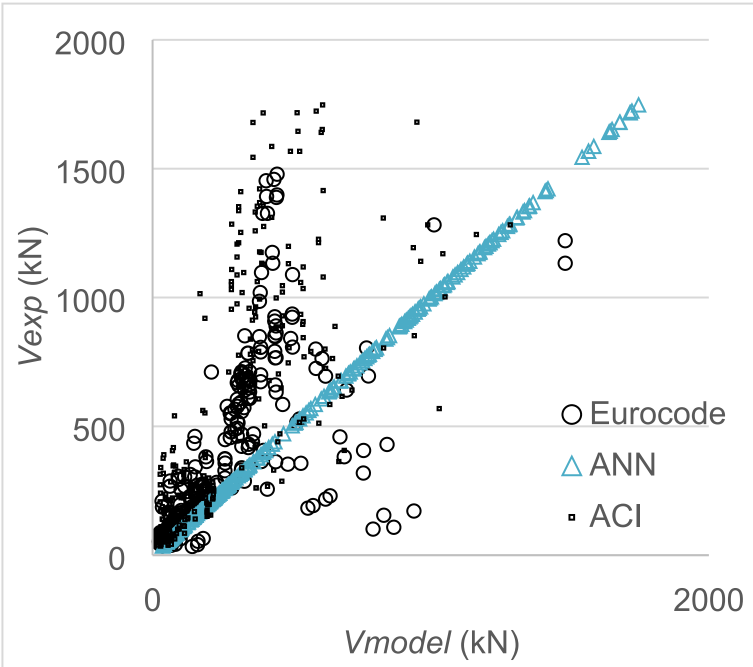

Fig. 4. Comparison between tested and predicted shear capacities: Eurocode 2 and ACI 318-14 vs. proposed ANN.

The average value of the ratio $V_{E,EC} / V_{R,c}$ (with $V_{E,EC}$ the sectional shear force taking into account the reduction of the contribution of loads close to the support, and $V_{R,c}$ according to Eq. 2) for the 287 experimental results is 1.59, with a standard deviation of 0.79 and a coefficient of variation of 49%. The reduction for loads close to the support applies to 151 datapoints of the database. The average value of the ratio $V_{test}/ V_{ACI,m}$ for the 287 experimental results is 2.36, with a standard deviation of 1.74 and a coefficient of variation of 74%. The poor performance of the ACI code is explained by the fact that direct load transfer is not taken into account. This observation was already made before [86]. For comparison, the average value of V_{test} / V_{ANN} (with V_{test} the sectional shear force at failure in each experiment, and V_{ANN} the ANN-based shear capacity) is 1.00, with a standard deviation of 5×10^{-14} and a coefficient of variation of 0.0%. The major improvement of the ANN as compared to the Eurocode and ACI code is also shown in Fig. 4, where the x-axis shows the predicted shear capacity V_{model} (V_{ANN} or $V_{R,c}$) and the y-axis shows the experimental result V_{exp}, which is $V_{E,EC}$ for comparison to the Eurocode shear capacity and V_{test} for comparison to the ANN-predicted shear capacity and ACI code. Fig. 4 shows the results for the 287 datapoints used in this study.

7. Discussion

The results in Fig. 4 show the major improvement, for the 287-point dataset used, of the proposed ANN-based model as compared to currently used Eurocode 2 and ACI 318 expressions for the shear capacity of reinforced concrete slabs in one-way shear. One critical observation should be made here: the ANN predictions are only valid within the input variable ranges of the employed 287-point dataset [89]. The number of experiments is rather limited, since slab shear tests are expensive to carry out. The user should keep this restriction in mind when predicting the shear capacity with the proposed ANN. The dataset covers a large number of variables that influence the shear assessment of reinforced concrete, but all tested slabs are rectangular. For skewed slabs, shear stress concentrations will result in the obtuse angle [98-100], making the skew angle an important factor for the shear assessment. Besides the Liverpool experiments on skewed slabs [101], which did not result in shear failures of the slabs, the authors are not aware of experiments on skewed slabs under concentrated loads failing in one-way shear. To extend this novel ANN-based design approach to new scenarios, experiments on skewed slabs failing in one-way shear should be carried out, and the skew angle should then be included as input variable for ANN design.

From Fig. 4, we can observe that there are 56 experiments for which Eurocode 2 EN 1992-1-1:2005 leads to unsafe predictions. There are three reasons for this observation. The first reason is related to the way in which concentrated loads on slabs are considered. For the calculations shown in this work, a 45-degree load spreading between the far side of the loading plate and the face of the support is used. We can observe that this approach seems not to perform equally well for loads close to the support as for loads further away from the support. The second reason is that the database contains slabs and wide beams in different loading schemes. Analyzing the results shows that the Eurocode-based approach tends to be unconservative for cantilever slabs, yet perform well for simply supported or continuously supported slabs. The third reason is that the sectional shear in this analysis is calculated based on the sectional shear caused by self-weight and the sectional shear caused by the load applied in the experiment, based on the principle of superposition. Analysis of the results show that large members, where the self-weight is considerable, lead to unsafe predictions. These observations should be considered in the next round of revisions of EN 1992-1-1:2005, and should lie at the basis of better methods for taking into account the contribution of concentrated loads in slabs.

To use the developed ANN formulation for the assessment of existing reinforced concrete one-way slab bridges, the following procedure is proposed:

1. Make a linear finite element model (LFEM) of the bridge under consideration.
2. Apply the superimposed dead load and live load model on the LFEM.
3. Make the factored load combination according to the governing code.
4. Find the governing sectional shear force v_u based on a distribution of the peak shear stress over $4d_l$ [102] and find the governing sectional moment m_E (including the effect of the twisting moments [103]) based on a distribution of the peak sectional moment over $2d_l$.

5. Determine the shear capacity with the proposed ANN (V_{ANN}), taking as input the characteristic material properties (where possible updated with measured values) and the value of $M_E / (V_E d_l)$ where this ratio is maximum. Divide V_{ANN} by $4d_l$ to find v_{ANN}.
6. Determine the bending moment capacity m_R based on the flexural theory of concrete elements.
7. Determine the Unity Check for shear: $UC_v = v_u/v_{ANN}$. If $UC_v \leq 1$, the requirements for shear are fulfilled.
8. Determine the Unity Check for bending moment: $UC_m = m_E/m_R$. If $UC_m \leq 1$, the requirements for bending moment are fulfilled.
9. If $UC_v > UC_m$ the bridge can be considered as shear-critical: shear failure is expected to occur before flexural failure.

When either UC_v or UC_m is found to be larger than 1, more refined methods, such as nonlinear finite element analysis or proof load testing, may be necessary for a sharper assessment of the bridge under consideration. The proposed method is fast, cheap, and computationally efficient, and as such it is especially suitable for cases where a large number of bridges need to be assessed.

The sensitivity analysis gives us a unique insight in the most important parameters for the shear capacity of reinforced concrete slabs failing in one-way shear. Based on these results, we find that the ratio M_E/V_Ed_l and the overall width b are the parameters that have the largest impact on the resulting shear capacity. In general, we can observe in Table 7 that the parameters related to the geometry of the load and the slab govern the shear behavior. This observation confirms the hypothesis from [86], which was one of the starting points for the lower-bound plasticity-based analysis method for one-way slabs failing in shear, the Extended Strip Model [13, 14].

The proposed approach is a tool to use the available experimental data to address a practical problem where mechanical models have not been able to yield good predictions yet. While it is of the utmost importance to develop better mechanical models so that we can understand the shear failure of reinforced concrete slabs, such a development may still require some time. Therefore, we propose our developed matrix-based formula to address the current need to estimate more accurately the shear capacity of reinforced concrete slab bridges.

The main novelty of this work is that the available experimental results are analyzed and that an accurate model is presented. Not only can this model be used to predict shear capacities in experiments; the range of parameters covers the range of practical values for short span reinforced concrete slab bridges. As such, the proposed model has a direct practical implication. To the authors' knowledge, no other ANN-based expression is available for estimating the shear capacity of reinforced concrete one-way slabs. From this point of view, the proposed method is the first in its kind. As compared to other ANN-based expressions for structural concrete applications, the large number of ANN features that we explored in this study are an improvement as compared to the applications we encountered in the literature, which are based on the features provided in the standard ANN toolbox of Matlab. The result of this approach is that our proposed ANN-based expression has lower errors than those reported for other ANN-based expressions for problems related to shear in structural concrete.

8. Summary and conclusions

This paper shows how artificial neural networks can be used to predict the shear capacity of one-way slabs under concentrated loads.

- For this purpose, a database with 287 experimental results was compiled. From this dataset, 10 governing parameters were identified as input variables and the sectional shear force at failure was considered the output variable.
- The proposed ANN-based analytical model (with 50% of the data used for training, 25% for validation and 25% for testing) yielded maximum and mean relative errors of 0.0% and 0.0% for those 287 points, respectively. Moreover, it was illustrated to clearly outperform (mean V_{test} / V_{ANN} =1.00) the Eurocode 2 provisions (mean $V_{E,EC} / V_{R,c}$ =1.59) and the ACI 318-14 provisions (mean V_{test}/ V_{ACI} = 2.36) for that dataset.
- A sensitivity analysis of the ANN-based model showed that the most important input parameters are the width of the slab, the effect of the shear span to depth ratio represented by the ratio of the sectional moment to the product of the sectional shear and effective depth, and the concrete compressive strength.
- Lastly, a step-by-step methodology for the assessment of existing reinforced concrete one-way slab bridges, based on the use of the developed ANN-based formula, was proposed.

The study carried out has not yet allowed a full description of the mechanics underlying the behavior of one-way reinforced concrete slabs, but parametric studies by means of ANN-based models make it possible to evaluate and improve existing mechanical models.

Notations

a center-to-center distance between load and support

a_v face-to-face distance between load and support

b width

b_{eff} effective width for concentrated loads on slabs

b_r distance from edge to load in the transverse direction

b_w web width

d effective depth

d_l effective depth to the longitudinal reinforcement

d_t effective depth to transverse reinforcement

f_c' specified concrete compressive strength

f_{cm} average concrete cylinder compressive strength

h height of cross-section

k size effect factor

l_{load} size of the loading plate, in the y-direction

l_{span} span length

m_E moment in slab

m_R moment resistance

v_{ANN} shear capacity (stress) derived from V_{ANN}

v_u shear in slab

w_{ji} connection weight between the neuron i and the neuron j

w_{jl} connection weight between the input neuron l and the hidden neuron j

w_{oj} connection weight between the hidden neuron j and the output o

A_{sx} area of steel in the longitudinal direction

A_{sy} area of steel in the transverse direction

M_E sectional moment caused by self-weight and loads applied during experiment

R pearson correlation coefficient

Q_i sensitivity of i-th parameter

UC_m unity check for bending moment

UC_v unity check for shear

$V_{ACI,d}$ design capacity according to ACI 318-14

$V_{ACI,m}$ average capacity based on design capacity from ACI 318-14

V_{ANN} shear capacity determined with ANN-based model

V_{exp} experimental shear capacity

V_E sectional shear caused by self-weight and loads applied during experiment

$V_{E,EC}$ governing sectional shear, keeping into consideration the reduction of loads close to the support prescribed in EN 1992-1-1:2005

$V_{E,max}$ sectional shear at governing cross-section, maximum absolute value of V_E

V_{model} shear capacity predicted with model

V_R shear resistance

$V_{R,c}$ mean shear resistance calculated with EN 1992-1-1:2005

V_{test} sectional shear force at failure in experiments

ρ_x amount of reinforcement in the longitudinal direction

ρ_y amount of reinforcement in the transverse direction

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Conflic of interest

The authors declare no conflict of interest.

Author statement

Section 1, 4, and 8: Abambres M.

Sections 1, 2, 3, 5, 6, 7, and 8: Lantsoght E.