MEASUREMENTS OF THE VELOCITY DISTRIBUTION IN THE BOUNDARY LAYER ALONG A PLANE SURFACE

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MEASUREMENTS OF THE VELOCITY DISTRIBUTION IN THE BOUNDARY LAYER ALONG A PLANE SURFACE

PROEFSCHRIFT TER VERKRIJGING VAN DEN GRAAD VAN DOCTOR IN DE TECHNISCHE WETENSCHAP AAN DE TECHNISCHE HOOGESCHOOL TE DELFT, OP GEZAG VAN DEN RECTOR-MAGNIFICUS C. FELDMANN, HOOGLEERAAR IN DE AFDEELING DER ELECTROTECHNIEK, VOOR EEN COMMISSIE UIT DEN SENAAT TE VERDEDIGEN OP DONDERDAG 9 OCTOBER 1924, DES NAMIDDAGS TE 3 UUR, DOOR

BERNARD GERARD VAN DER HEGGE ZIJNEN,

WERKTUIGKUNDIG INGENIEUR, GEBOREN TE NIJMEGEN

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DELFT — 1924.

Aan mijn Ouders.

Bij de voltooiing van dit proefschrift is het mij een behoefte om U, Hoogleeraren in de Afdeeling der Werktuigbouwkunde, dank te zeggen voor datgene wat ik in mijn studietijd van U mocht leeren; vooral U Hooggeleerde HOLST, bij wien ik het voorrecht had mijn studie te voltooien, breng ik gaarne een afzonderlijk woord van dank.

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CONTENTS.

LIST OF PRINCIPAL ABBREVATIONS USED IN THE TEXT.

ρ = density of the air (gr/cm^3).
μ = viscosity of the air (gr/cm sec).
ν = kinematical coefficient of viscosity $= \frac{\mu}{\rho}$ (cm^2/sec).
g = acceleration of gravity.
x = distance of a point from the leading edge of the glass plate (cm).
y = distance of a point from the surface of the glass plate (cm).
X = distance of the leading edge of the glass plate from the honey comb of the tunnel (cm).
l = length of the glass plate (167,5 cm, or 150 cm, as taken into account in the evaluation of the experiments).
b = width of glass plate = 40 cm.
V = velocity of the air outside of the boundary layer, parallel to the glass plate (cm/sec).
u = component of the velocity of the air in the boundary layer parallel to the glass plate (cm/sec).
δ = thickness of the boundary layer (cm).
a = velocity gradient at the surface $= \left(\frac{\partial u}{\partial y}\right)_{y=0}$ (cm/sec/cm).
τ_0 = shearing stress at the surface $= \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$ ($gr/cm\ sec^2$).
R = number of REYNOLDS $= \frac{Vx}{\nu}$ or $\frac{Vl}{\nu}$.
$R^* = \frac{V\delta}{\nu}$.

§ 1. INTRODUCTION.

a. Object of the experiments.

In all cases where the motion of a fluid may become turbulent, the motion is confined by the presence of solid boundaries or walls. In the cases most thoroughly studied the walls constitute a cylindrical tube or a channel, and in that part of the tube or channel where the motion has acquired a definite mean state, every part of the fluid experiences the influence of the whole wall around it or (as in the case of the channel) of the walls on both sides. There may be some interest in studying the effect of a single wall, which presents itself in the motion of a fluid in the boundary layer that develops along the surface of a rigid body immersed in a moving liquid. This case, moreover, has some practical interest as it in immediately related to the phenomena of „surface friction".

With this object in view the first problem that presents itself is to obtain data on the distribution of the velocity in such a boundary layer. As far as known to the author measurements of this kind have been executed only by CALVERT [1]) and by RIABOUCHINSKY [2]). These researches, however, do not supply sufficient data for a detailed study of the motion, and it seemed desirable to repeat them by a different method in a more complete form.

It was thought that this could be done by means of hot wire anemometers, as they easily allow to execute the measurements at very small distances from a solid wall. After a number of preliminary experiments, which were started in June 1922, on hot wire anemometers, on the velocity distribution in the boundary layer along a smaller glass plate, on the influence of the leading edge on the velocity distribution in the first part of the boundary layer along the smaller glass plate and along that one that is described in the present work, the definitive measurements were entered upon May 1923.

A provisional account of some of the measurements on this subject that were executed between July and October 1923 has already been given in a paper entitled:

„Preliminary Measurements of the Distribution of the Velocity of a Fluid in the immediate Neighbourhood of a plane smooth Surface" by J. M. BURGERS and B. G. VAN DER HEGGE ZIJNEN, published in the „Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam" (Mededeeling N^0. 5 uit het Laboratorium voor Aerodynamica en Hydrodynamica der Technische Hoogeschool te Delft). This paper will be called furtheron „Mededeeling 5".

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1) G. A. CALVERT, On the measurement of wake currents, Trans. Inst. Nav. Arch. 1893.

2) D. RIABOUCHINSKY, Étude expérimentale sur le frottement de l'air, Bull. Inst. Aérod. de Koutchino, fasc. V, p. 51, 1914.

b. Review of the principal theoretical data about the motion in the boundary layer.

Before entering upon a description of the measurements and their results, a short survey will be given of the theoretical aspects of the problem [3]).

The theory of the motion of a fluid along a single wall has been originated by PRANDTL, and has been developed by BLASIUS [4]), BOLTZE [5]), HIEMENZ [6]), VON KÁRMÁN [7]), POHLHAUSEN [8]). VON KÁRMÁN's researches are of great importance: he has given a very general equation by means of which the motion may be discussed, while at the same time special attention has been paid to the pecularities of the turbulent motion.

PRANDTL [9]) remarked that when a fluid moves tangentially along a wall the retarding influence exerted by this wall practically will extend only over a layer of finite thickness δ, the so called boundary layer. The thickness of this layer is least at the point where the current meets the surface; in the case of an infinitely thin, flat surface, mounted parallel to the original direction of the current, δ may be taken to be zero at the leading edge.

The motion of the fluid in this layer (which motion in most cases may be regarded as being limited to two dimensions, one tangential and one normal to the wall) may be either a laminar or a turbulent one, i. e. the velocity at a given point may be independent of time, and will be nearly parallel to the boundary; or the value and the direction of the velocity may change continually in some irregular manner. In the latter case a certain mean or principal motion exists, with an irregular relative motion superposed on it. The change from the laminar into the turbulent state occurs when the number of REYNOLDS $= R^* = \frac{V\delta}{\nu}$ surpasses a certain critical value. Direct researches on this change have not been published, but it may be inferred from the experiments on the motion of fluids in pipes, that the critical value of R^* will depend on the magnitude of the disturbances occurring in the laminar motion: the critical value decreases as the disturbances increase. However, a lower limit exists; hence with values of R^* smaller than this lower limit, every disturbance will be damped out. It may be expected that this lower limit will be of the same order of magnitude as that found in the case of the motion through a tube or between concentric cylinders, say in round numbers, 2000.

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3) Some parts of this paragraph have also been published in „Mededeeling 5", p. 4.
4) H. BLASIUS, Thesis Göttingen, 1907 (Zs. f. Math. u. Phys. **56**, p. 1, 1908).
5) E. BOLTZE, Thesis Göttingen, 1908.
6) K. HIEMENZ, Thesis Göttingen, 1911 (Dingler's polyt. Journ. **326**, p. 321, 1911).
7) TH. VON KÁRMÁN, Zs. f. angew. Math. u. Mech. **I** 1921.
8) K. POHLHAUSEN, Zs. f. angew. Math. u. Mech. **I** 1921.
9) L. PRANDTL, Verh. d. III[ten] Intern. Math. Kongresses, Heidelberg, 1904, p. 484.

PRANDTL [9]) has given a method to calculate the laminar motion in the boundary layer, if the thickness of this layer is small as compared to the dimensions and the radius of curvature of the surface. This method has been developed by BLASIUS and HIEMENZ for the case of the two dimensional motion, and by BOLTZE for the three dimensional motion. An approximate theory has been given by VON KÁRMÁN.

According to BLASIUS's formulae, the gradient of the velocity u in the immediate neighbourhood of a plane surface has the value:

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = a = 0{,}332\ V\sqrt{\frac{V}{\nu x}} \quad \ldots\ldots \quad (1)$$

From this formula the resistance experienced by one side of a wall of a length l and a breadth b (measured perpendicularly to the wall) is found to be:

$$W = b\int_0^l \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} dx = \frac{0{,}664}{\sqrt{R}}\,\rho\, b\, l\, V^2 \quad \ldots\ldots \quad (2)$$

Some values of the velocity u as calculated by BLASIUS are given here:

$y =$	$1{,}46\sqrt{\frac{\nu x}{V}}$	$2{,}18\sqrt{\frac{\nu x}{V}}$	$3{,}64\sqrt{\frac{\nu x}{V}}$
$u =$	$0{,}47\ V$	$0{,}68\ V$	$0{,}92\ V$

According to BLASIUS no definite value can be assigned to the thickness of the boundary layer, as u increases asymptotically to its limiting value V. An approximate value, however is:

$$\delta = \text{about } 5.5\sqrt{\frac{\nu x}{V}} \quad \ldots\ldots \quad (3)$$

which gives:

$$R^* = \text{about } 5{,}5\sqrt{R} \quad \ldots\ldots \quad (4)$$

If the motion is turbulent, it will be understood that u denotes the mean value of the component of the velocity parallel to the wall. The dependence of this mean value on y is derived by VON KÁRMÁN, by means of the theory of dimensions; VON KÁRMÁN discovered a formula which connects the velocity u, the distance in normal direction to the surface y and the resistance per unit of area of the wall (tangential stress). By making use of empirical data for the resistance, he deduced:

$$u = 8{,}7\left(\frac{\tau_0}{\rho}\right)^{4/7}\left(\frac{y}{\nu}\right)^{1/7} \quad \ldots\ldots \quad (5)$$

where τ_0 is the tangential stress per unit area of the wall.

The solution of this equation gives for the value of τ_0:

$$\tau_0 = 0{,}0225\ \rho\, u^{7/4}\left(\frac{\nu}{y}\right)^{1/4} \quad \ldots\ldots \quad (6)$$

In the immediate vicinity of the wall formula (5) cannot be applied, as it gives an infinite value to the gradient at the surface, $\left(\frac{\partial u}{\partial y}\right)_{y=0}$; it is supposed that in this region the turbulent state of motion disappears, and hence (5) has to be replaced by:

$$u = \frac{\tau_0}{\mu} y \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

By introducing the thickness δ of the boundary layer, in stead of (5) we may write:

$$u = V\left(\frac{y}{\delta}\right)^{1/7} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

The value of δ has been calculated by VON KÁRMÁN from the equation:

$$\frac{d}{dx}\int_0^\delta \rho u^2 dy - V\frac{d}{dx}\int_0^\delta \rho u\, dy = -\tau_0 \quad . \; . \; . \; . \quad (9)$$

which equation expresses the theorem of momentum as applied to an infinitely thin section of the boundary layer, perpendicular to the axis of x, in the absence of any pressure gradient, as is the case here.

If u and τ_0 are written as functions of δ (by the aid of (8) and (6)), equation (9) transforms into a differential equation of the first order for δ, the general integral of which is:

$$\delta = 0{,}371\,(x - x_0)^{4/5}\left(\frac{\nu}{V}\right)^{1/5} \quad . \; . \; . \; . \; . \; . \quad (10)$$

x_0 being the constant of integration.

Now the value of τ_0 becomes — from (6), (8) and (10) —:

$$\tau_0 = 0{,}029\,\rho V^{9/5}\left(\frac{\nu}{x - x_0}\right)^{1/5} \quad . \; . \; . \; . \; . \; . \quad (11)$$

If the motion in the boundary layer is turbulent from the beginning, it is natural to suppose that $x_0 = 0$ (as is done by VON KÁRMÁN).

Along a plane smooth wall, the leading edge of which has been sharpened in order to give rise to as less disturbances as possible, it is to be expected, however, that in the first part of the boundary layer the motion will be laminar, and a transition to the turbulent state cannot occur before δ has increased so much that $R^* = \frac{V\delta}{\nu}$ surpasses its critical value of about 2000. Once this limit having been surpassed, turbulence will set in sooner or later, according to the irregularities occurring in the current at the outside of the boundary layer being greater or less. In this case the value of x_0 will be different from zero, and the smaller the disturbances in the outer current, the greater x_0 will be.

The results of the first group of measurements executed with a velocity of the air passing over the surface of 800 cm/sec, which have been published, have confirmed this supposition: in the first part of the boundary layer the distribution of the velocity corresponds more or less to that calculated by BLASIUS, while in the second part it obeys the relation deduced by VON KÁRMÁN: $u \sim y^{1/7}$. Besides giving data on the influence of the disturbances in the outer current on the position of the region of transition, these measurements had shown that both in the laminar part and in the turbulent part in the immediate neighbourhood of the wall the velocity curve approximates to a straight line:

$$u = a\,y$$

$$\left(a = \lim_{y=0} \frac{\partial u}{\partial y}\right)$$

and it was possible to determine the value of the velocity gradient at the surface (a) with not too great an error. In this latter respect they confirm STANTON's [10] results for the case of the motion of a fluid (air) through a tube, obtained by a different method of measurement.

In the present work further researches in this direction will be described.

§ 2. METHOD OF MEASUREMENT. [11]

a. Experimental arrangement.

The measurements were executed in a current of air, by making use of the windchannel of the laboratory for Aerodynamics and Hydrodynamics of the Technical Highschool at Delft. The cross section of this channel measures 80 cm square; the portion available for the experiments has a length of 400 cm. A four bladed propeller or fan draws the air through the tunnel. The maximum air velocity that can be reached is about 3300 cm/sec and the lowest velocity that can be kept constant is about 120 cm/sec. Every value of V between these limits can be used. The velocity of the air in the working space is determined by a Pitot-tube connected to an alcohol pressure gauge (both constructed by FUESS). The propeller is driven by a direct current electromotor, the number of revolutions of which is regulated by the experimenter.

At the entrance of the tunnel a honeycomb is placed, consisting of brass blades, framing square cells of 8 cm diameter and having a length of 25 cm.

As smooth, plane, surface, a glass plate was chosen, 167,5 cm long, 40 cm wide and having a thickness of 1,2 cm. This plate was placed in the vertical plane of symmetry of the channel and parallel to the flow.

At the leading edge the plate was sharpened at both sides over a length of about 15 cm, with a radius of curvature of 75 cm (see fig. *a*, p. 6). This part is not absolutely smooth, as small irregularities caused

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[10]) T. E. STANTON, D. MARSHALL and C. N. BRYANT, Proc. Roy. Soc. London, A 97, p. 413, 1920.

[11]) Some parts of this chapter were also published in „Mededeeling 5", p. 10.

by the grinding could not be ameliorated; also the polishing could not be affected in such a way that this part became as smooth as the rest. The appearance of this part was somewhat like paraffine wax. [12])

The experimental arrangement is given by figure 1.

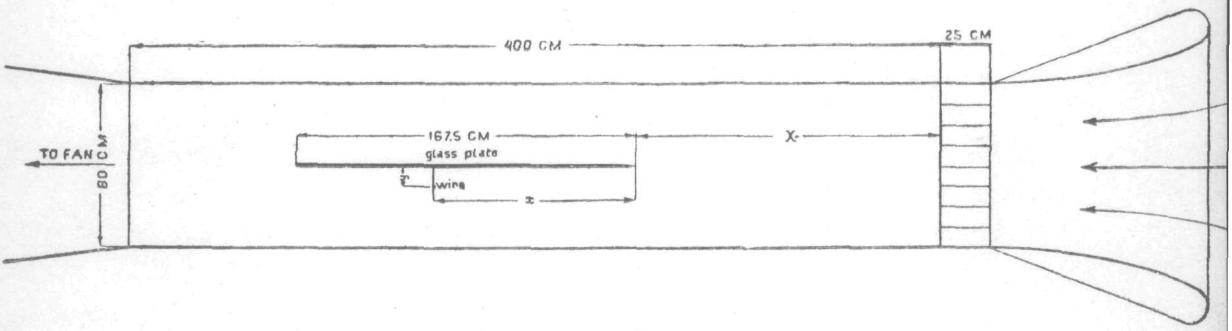

Fig. 1. Diagrammatic view of the position of the glass plate in the tunnel.

The velocity of the air in the boundary layer was determined by means of "hot wire anemometers", i. e. thin electrically heated wires of platinum or platinum-iridium. The principle of this method was brought to general knowledge by a detailed research by L. V. KING [13]); it is based on the cooling effect of the current of air: the electric current i is determined, which is necessary to keep the wire at a given and constant electrical resistance for any value of the velocity of the air. The relation between this electric current and the velocity is given by:

$$i^2 = c\sqrt{V} + b$$

where c and b are practically constants.

Before the measurements of the air velocity the anemometer was calibrated experimentally, which calibration was periodically repeated.

The new measurements to be described in the following paragraphs

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12) This grinding and polishing was done by the staff of the Aerodynamical Laboratory, as the firm that supplied the glass plate to us, could not do it so as was desired. The original and finished form of the glass plate are shown in figure *a*.

Fig. *a*.

Fig. *b*.

A new glass plate, supplied afterwards by the same firm, was ground and polished in their works and had the form of leading edge shown in figure *b*. Experiments carried out with this new plate, 210 cm long, 50 cm wide and 1,5 cm thick, showed that in this case the boundary layer, with $V = 800$ cm/sec, was almost turbulent from the beginning.

13) L. V. KING, On the convection of heat from small cylinders in a stream of fluid, Phil. Trans. **215**, p. 373, 1914.

have been executed in November and December 1923, using wire No. 25,9 the dimensions of which were:

Material of wire	Pt-Ir.
Diameter of wire	0,005 cm.
Length of wire	2,1 cm.
Temperature of wire in use	667° C.
Resistance at 17° C.	3,19 Ω
Resistance at T° C.	5,25 Ω

In the period mentioned the wire was calibrated four times, as it appeared that the values of the current corresponding to a given value of the velocity of the air changed with the time the wire had been in use.

The results of the calibrations are given in the table on p. 8 and 9.

If the values of i^2 are plotted against $\sqrt{V}$, nearly straight lines are obtained, in agreement with the results found by KING.

In working out the determinations of the velocity in the boundary layer along the glass plate, the preceding calibration was always used (the dates of the measurements have been mentioned in tables I, III—VII).

In order to be able to regulate the distance y of the wire from the surface of the plate, the anemometer was mounted on a screw micrometer. The zero of the scale reading had to be determined in an indirect way, as it was impossible to measure the distance from the wire to the surface of the glass plate by means of a measuring microscope. The more primitive method, which was used in the earlier experiments, was to regulate the distance of the wire so that the wire and its image in the glass appeared to be just one diameter apart, as estimated with the eye; the known value of the diameter of this wire gave then the zero reading of the micrometer. The distance was called zero when the wire touched the glass plate.

When the zero reading had been determined in this way, the current i_0 was measured which was necessary to keep the wire at the temperature T at different distances from the wall (with $V = 0$); these values were put down in a diagram as a function of y ("cooling curve"). It was found that i_0 increased very rapidly with decreasing values of y, and so it appeared possible to use this "cooling curve" to determine the zero reading, without making use of the image of the wire.

In the later experiments the zero reading was sometimes determined by diminishing the distance y until i_0 increased no longer, which occurs if the wire is in contact with the surface. This method gives somewhat greater risks as regards the wire, but it gave better results. Whenever necessary, the cooling curve was always used as a method of control.

The experiments were made in such a manner, that after the determination of the zero point, the hot wire anemometer was screwed as far as possible to the outside (about 5 cm from the plate). Then the electromotor was turned on and according to the anemometer indications, the velocity was brought up to the value of V as had been fixed for

Wire No. 25,9.

V cm/sec.	$\sqrt{V}$	i^2			
		6 Nov. '23.	20 Nov.'23.	6 Dec. '23.	18 Dec.'23.
0	0	0,203	0,203	0,203	0,202
128	11,3	0,429	0,433	0,423	0,424
155	12,5	0,450	0,461	0,449	0,452
179	13,4	0,477	0,475	0,469	0,475
200	14,1	0,491	0,487	0,479	0,490
219	14,8	—	0,503	0,493	0,504
253	15,9	0,530	0,521	0,520	0,520
283	16,8	0,549	0,546	0,545	0,546
310	17,6	0,564	0,561	0,560	0,561
346	18,6	—	—	0,578	0,579
400	20,0	0,616	0,605	0,607	0,608
448	21,2	—	0,627	0,629	0,637
490	22,1	0,656	0,648	0,654	0,653
530	23,0	—	0,654	0,664	0,669
566	23,8	0,691	0,672	0,674	0,684
639	25,3	0,723	0,702	0,706	0,707
693	26,3	0,741	0,726	0,726	0,736
748	27,3	0,760	0,743	0,753	0,753
800	28,3	0,776	0,773	0,759	0,773
890	29,9	0,812	0,792	0,792	0,792
980	31,3	0,843	0,814	—	0,828
1056	32,6	0,863	0,843	—	0,848
1136	33,7	0,884	0,861	—	0,869
1200	34,6	0,903	0,882	—	0,887
1260	35,5	0,912	0,901	—	0,903

V cm/sec.	$\sqrt{V}$	i^2			
		6 Nov. '23.	20 Nov. '23.	6 Dec. '23.	18 Dec. '23.
1335	36,5	0,925	—	—	0,922
1390	37,3	0,945	0,922	—	0,941
1440	38,0	0,958	0,939	—	0,951
1500	38,7	0,978	0,951	—	0,962
1550	39,4	0,982	0,962	—	0,978
1600	40,0	0,998	0,980	—	0,990
1700	41,2	1,020	1,000	—	—
1790	42,3	1,040	1,020	—	1,036
1875	43,3	—	1,040	—	1,055
1960	44,3	1,078	1,051	—	1,067
2040	45,2	1,094	1,065	—	1,075
2115	46,0	—	1,082	—	1,098
2190	46,8	1,122	1,100	—	1,113
2266	47,6	—	1,109	—	1,119
2390	48,3	—	1,119	—	1,132
2400	49,0	1,165	1,128	—	1,145
2530	50,3	—	1,162	—	1,166
2590	50,8	—	—	—	1,175
2650	51,5	—	—	—	1,188
2780	52,7	—	—	—	1,210
2895	53,8	—	—	—	1,225
2990	54,7	—	—	—	1,234
3100	55,7	—	—	—	1,254
3200	56,6	—	—	—	1,269
3295	57,4	—	—	—	1,296
	Bar.	752	746	758	761 mm.
	Temp.	13,2	18,7	16,3	19,1° C.

the respective series. Thus the value of V was the same for every value of x. The velocity was then kept constant by aid of a Pitot tube fitted up elsewhere in the tunnel. This method is followed contrarily to that described in „Mededeeling 5", in which the velocity outside the boundary layer sometimes deviated from 800 cm/sec, in which case the measured values of u were converted by means of a definite factor, in order to represent as precisely as possible the distribution of the velocity occurring with a constant value of V independent of x.

Once the velocity having been determined, the experiments were carried out in such a manner that the hot wire anemometer was transferred to the inside by small steps. Between two transfers i was noted. Thereupon the hot wire anemometer was moved away from the plate with the same intervals, until i certainly did no more increase with y. In proportion with the mutual differences of the results, the measurements were repeated more times, the minimum being three to four times.

The square of the average of the i values was then taken; $\sqrt{u}$ was calculated by means of the almost lineair experimental calibration curve and from this u was determined.

The distribution of the velocity was determined at a given value of x first with $V = 400$ cm/sec. Then V was brought up to 1200 cm/sec and the distribution of the velocity in the boundary layer determined again. Then experiments with $V = 1600$ and 2400 cm/sec were made and finally the hot wire anemometer was transferred to the next value of x.

b. Accuracy of the experiments.

1. The accuracy of the experiments made was influenced by the slightly irregular working of the propeller which supplied the current of air.

As stated, the velocity is not kept automatically constant, but is regulated by the experimenter; with some care the slow fluctuations in the velocity of the air could be kept below 1 % (as indicated by the alcohol micromanometer and the Pitot tube connected with the same).

2. The results of the previous experiments had shown that the region of unsteady motion behind the entrance of the tunnel exerted much influence on the place where the laminar boundary layer transferred into the turbulent one. This region of unsteady motion gets longer as the velocity V increases. For a comparison of the results, it is therefore necessary to know the position of the glass plate in the tunnel. The distance from the leading edge of the glass plate to the honeycomb of the windchannel, X cm, (see Fig. 1) has been determined in all series.

3. From the method of measuring with hot wire anemometers follows that the sensitiveness decreases as the value of V increases.

As the velocity is determined from the formula given by KING:

$$i^2 = c\sqrt{u} + b$$

(where c and b are nearly constant for each hot wire anemometer) the sensitiveness will be:

$$\frac{di}{dV} = \frac{c}{4\, i \sqrt{V}}.$$

(In the case of wire N^0. 25.9 c is about 0.019).

From this it appears that the accuracy obtainable is less at higher velocities. The fluctuations of the velocity of the air of high frequency will in general not influence the indications of the anemometer very much.

4. In the immediate vicinity of the wall, heat will be absorbed not only by the air, but also by the surface itself. So it may be expected that the hot wire anemometer will give too high a value of the velocity in the boundary layer. Taking this into account, a correction has to be applied to the read values of i. As it appears from the experiments, the absorbtion of heat by the wall has no more appreciable effect when $y \geqslant 0{,}2$ cm. Provisionally this correction for the cooling effect by the wall was applied in the following way: calling $i_{V(y)}$ the value of the electric current observed at the distance y, when the velocity of the air had the value V, $i_{0(y)}$ the current observed when the air was at rest, and $i_{0(\sim)}$ the same at a very great distance of the wall, the corrected value of i was calculated by means of the formula:

$$(i_{corr.})^2 = [i_{V(y)}]^2 - [i_{0(y)}]^2 + [i_{0(\sim)}]^2 \quad . \quad . \quad . \quad . \quad (13)$$

From $i_{corr.}$ the value of u was deduced by means of the experimental calibration curve.

However, it is possible that this correction depends on V; then it is to be expected that it decreases when V increases. Therefore the correction certainly will be zero for $y \geqslant 0{,}2$ cm, as it appeared that $i_{0(0,2)} = i_{0(\sim)} = b$. On the other hand for $y \to 0$ (where $u \to 0$) the cooling by the air current will be measured by $[i_{u(0)}]^2 - [i_{0(0)}]^2$, so that for small values of y we were led to put:

$$c\sqrt{u} = [i_{u(y)}]^2 - [i_{0(y)}]^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

which corresponds to formula (13) as:

$$[i_{corr.}]^2 = c\sqrt{u} + b = c\sqrt{u} + [i_{0(\sim)}]^2.$$

Hence it may be expected that the given formula will apply in the limiting regions; inaccuracies, however, may rise in the middle region.

The values of $i^2_{0(y)}$ have been given in the table on p. 12 and 13.

By means of this table and the calibration data of the wire, given on p. 8 and 9 it is possible to reconstruct the original values of i for all measurements.

This correction — which has been applied to all measurements in the same way — principally influences the value of the velocity gradient a at the surface. For this reason the results found have been discussed in § 5 by comparing the value of the friction deduced from them to the value found by various other methods. The general result is not disap-

y cm	V cm/sec.	x cm = 2,5	5	10	15	20	25
		i_0^2					
0,005	400	—	—	—	0,413	0,381	0,391
	800_2	0,338	0,350	0,445	0,362	0,368	0,386
	1200	0,401	0,371	0,468	0,375	0,392	0,396
	1600	—	—	0,458	0,389	0,388	0,396
	2400	—	—	0,458	0,389	0,388	0,396
0,010	400	—	—	—	0,346	0,331	0,332
	800_2	0,306	0,312	0,350	0,319	0,321	0,325
	1200	0,331	0,323	0,354	0,321	0,331	0,333
	1600	—	—	0,354	0,332	0,332	0,333
	2400	—	—	0,354	0,332	0,332	0,333
0,015	400	—	—	—	0,309	0,301	0,303
	800_2	0,284	0,292	0,314	0,295	0,297	0,297
	1200	0,298	0,295	0,314	0,297	0,303	0,304
	1600	—	—	0,310	0,304	0,301	0,304
	2400	—	—	0,310	0,304	0,302	0,304
0,020	400	—	—	—	0,288	0,284	0,281
	800_2	0,270	0,276	0,292	0,279	0,281	0,278
	1200	0,279	0,278	0,289	0,281	0,284	0,283
	1600	—	—	0,287	0,282	0,281	0,283
	2400	—	—	0,287	0,282	0,281	0,283

	x cm =				$V = 800_2$ cm/sec.**) All series.
	2,5*)—5	10-15-20 25-50-62,5	37,5-75 87,5	100-125 150*)	
	i_0^2				i_0^2
0,025	0,262 ± 2	0,272 ± 3	0,269 ± 2	0,281 ± 2	0,267 ± 3
0,030	0,255 ± 2	0,263 ± 2	0,258 ± 2	0,268 ± 1	0,257 ± 3
0,040	0,243 ± 2	0,249 ± 2	0,244 ± 1	0,251 ± 1	0,242 ± 2
0,050	0,233 ± 2	0,237 ± 2	0,234 ± 1	0,239 ± 2	0,233 ± 2

*) incl. $V = 800_2$ cm/sec.

**) excl. $V = 800_2$ cm/sec. at $x = 2,5$ and 150 cm.

	37,5	50	62,5	75	87,5	100	125	150
	0,375	0,366	0,378	0,360	0,348	0,423	0,423	0,490
	—	0,372	—	—	—	0,387	—	—
	0,387	0,366	0,378	0,360	0,355	0,423	0,490	0,490
	0,387	0,366	0,378	0,397	0,355	0,423	0,449	0,490
	0,387	0,408	0,378	0,397	0,355	0,423	0,423	0,490
	0,325	0,325	0,332	0,316	0,319	0,342	0,342	0,366
	—	0,325	—	—	—	0,335	—	0,359
	0,327	0,325	0,332	0,316	0,319	0,342	0,366	0,366
	0,327	0,325	0,332	0,348	0,319	0,342	0,366	0,366
	0,327	0,339	0,332	0,348	0,319	0,342	0,342	0,366
	0,297	0,303	0,301	0,294	0,297	0,318	0,318	0,325
	—	0,297	—	—	—	0,304	—	0,317
	0,304	0,303	0,301	0,294	0,297	0,318	0,325	0,325
	0,304	0,303	0,301	0,303	0,297	0,318	0,325	0,325
	0,304	0,306	0,301	0,303	0,297	0,318	0,318	0,325
	0,279	0,285	0,284	0,278	0,281	0,297	0,297	0,297
	—	0,280	—	—	—	0,283	—	0,296
	0,283	0,285	0,285	0,278	0,281	0,297	0,297	0,297
	0,283	0,285	0,285	0,284	0,281	0,297	0,297	0,297
	0,283	0,285	0,285	0,284	0,281	0,297	0,297	0,297

y cm	$V = 800_2$ cm/sec.; x cm =				
	30	40	60	70	80
	i_0^2				
,005	0,381	0,377	0,376	0,378	0,360
,010	0,326	0,328	0,323	0,325	0,318
,015	0,297	0,293	0,297	0,298	0,293
,020	0,280	0,279	0,277	0,277	0,276

All series.	
y cm	i_0^2
0,060	0,229 ± 2
0,075	0,221 ± 1
0,080	0,219 ± 2
0,100	0,213 ± 2
0,125	0,208 ± 2
0,150	0,205 ± 2
0,175	0,203 ± 1
0,200	0,202 ± 1

pointing. [14]) In this respect, however, it has to be remarked that at a low value of the velocity, say $V = 400$ cm/sec, the corrected $u(y)$ curve is clearly S shaped, as appears from Fig. 2 and Fig. 5. This must be due to the correction, and points evidently to a too high value of it. At a higher value of the outer current, say $V = 2400$ cm/sec, this phenomenon is not so clearly shown.

5. The distance y — this is the distance from the centre of the wire minus the radius of the section of the wire to the wall — has been established by the aid of the screw micrometer, wherein the hot wire anemometer was mounted. An investigation of this screw micrometer showed that the maximum error occurring in the screw thread surpassed not more than 0,001 cm per revolution, above or under the mean value. When the micrometer had been in use for some time, however, clearance in the moving parts made itself perceptible by relative differences of the measured values of the distance y when the anemometer was screwed in or out. The maximal error in the distance y caused by this clearance was 0,01 cm. By readjusting, this wear and tear was reduced as well as possible and by a repeated determination of the zero reading of the micrometer the effect of the clearance was eliminated. At the determination of a by drawing a straight line through the first points of the $u(y)$ curve, it appeared that this straight line in general did not pass through the zero of the diagram; this may be caused by a remaining constant error in the indication of the micrometer or by an erroneous determination of the zero of the micrometer readings.

This amount "f" has been mentioned at the foot of the tables where the results are collected.

In those cases where the first part of the $u(y)$ curve was distinctly S shaped (the series with $V = 400$ cm/sec), the position of this straight line is not certain and here the abnormal high value of the distance correction f has to be attributed to the uncertainty which follows from the correction applied to i.

6. It may be supposed that the glass plate would bend under influence of the forces exerted by the stream of air passing over it, and hence that the distance of the hot wire to the glass plate would be altered by the difference in pressure within and outside of the wind channel during the experiments. The possibility of bending has been taken into account and minimized by fixing the glass plate as tightly as possible: the glass plate was fixed by means of two wheels, at a relative distance of about 100 cm, to a rail screwed on the under surface of the tunnel, parallel to the axis of the wind channel. Two other wheels pressed on a rail at the top, and gave there also the required support. Moreover the glass plate was held in position by two or four horizontal bars ($^5/_8$″ diam.)

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14) It may be remembered that also STANTON had to apply an important correction to his measurements (mentioned above, p. 5) in order to deduce the value of a from them.

perpendicular to the axis of the channel and fixed to the same iron frame work that supported the micrometer screw and the anemometer. It was inevitable that these bars passed the stream of air on the same side as that on which the anemometer was mounted. The vertical distance of these bars was equal to the height of the glass plate, this is 40 cm; so they were mounted 20 cm above and under the anemometer; the distance in horizontal direction depended on the distance from the anemometer to the leading edge of the glass plate and on the distance from this leading edge to the honeycomb.

A separate experiment on the influence of these bars taught that the upstream pair of bars caused in the downstream region of the channel a rise in V of about 1,25 % (see p. 42); the second pair of bars was placed downstream relative to the anemometer and had by consequence no influence on the results of the experiments.

It has been mentioned already that the hot wire anemometer was not removed when the value of V was changed, and so it might be expected that an error in the zero reading of the micrometer, occurring once, would still remain the same at different values of V (say 400, 1200, 1600 and 2400 cm/sec) at a constant value of x (in general the removing of the hot wire anemometer along the glass plate will cause a change in the zero reading). Evidently this is not the case, and it will have to be attributed to the not entirely correct way wherein the i correction has been applied. However some doubts arise in the case of the series with $V = 3200$ cm/sec at $x = 150$ cm; here again the distance correction is abnormally great. It would be expected that f would not alter in the series with $V = 600$, 2000, 2800 and 3200 cm/sec, as they were not interrupted for a recalibration of the anemometer or to change x. These experiments were performed in two days. It should be noted that the series with $V = 800_2$ cm/sec, the series with artificially caused turbulence in the outer current, is executed apart and the values of f in this case may not be compared to those of the other series.

§ 3. RESULTS OF THE MEASUREMENTS.

a. Experiments performed.

In the paper „Mededeeling 5" only such experiments that were carried out with a velocity of 800 cm/sec outside of the boundary layer are described. From these measurements the influence of the position of the glass plate in the tunnel was deduced; the distribution of the velocity in the boundary layer at various values of x was compared to the theory of BLASIUS and to that of VON KáRMáN. From the measured velocity gradient at the surface the resistance experienced by the glass plate was calculated and compared to various formulae.

As a more complete review required more data on the flow in the boundary layer, the velocity distribution in the neighbourhood of the same surface was determined now at other values of V. Moreover the measurement, mentioned in „Mededeeling 5", with artificially strengthened turbulence of the stream of air (obtained by putting a screen with square meshes of 0,4 cm, diameter of the wire = 0,08 cm, immediately in front of the leading edge of the glass plate and covering the whole section of the wind channel) was completed by similar measurements at other values of x, so that at $V = 800$ cm/sec a complete series with purposely caused turbulence in the air current is obtained.

The series with artificially caused turbulence of the stream of air are denoted by $V = 800_2$ cm/sec, in contrary to the series where this was not the case: these are denoted by $V = 800_1$ cm/sec and have been published already.

An experiment was also carried out with purposely caused turbulence of the stream of air with $V = 1600$ cm/sec (at $x = 150$ cm); this is denoted by $V = 1600_2$ cm/sec.

In the following table the various values of x and of V, at which the experiments were carried out, are given, the series with $V = 800_1$ cm/sec being also mentioned.

V cm/sec	x cm														
400	—	—	—	—	15	20	25	37,5	50	62,5	75	87,5	100	125	150
600	—	—	—	—	—	—	—	—	—	—	—	—	—	—	150
800_1	2,5	5	7,5	10–12,5	15–17,5	20	25	30	40-50	62,5	75	80-85-90	100	125	150
800_2	2,5	5	—	10	15	20	25	30	40	50	60	70-80	100	—	150
1200	2,5	5	7,5	10	15	20	25	37,5	50	62,5	75	87,5	100	125	150
1600	—	—	—	10	15	20	25	37,5	50	62,5	75	87,5	100	125	150
1600_2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	150
2000	—	—	—	—	—	—	—	—	—	—	—	—	—	—	150
2400	—	—	—	10	15	20	25	37,5	50	62,5	75	87,5	100	125	150
2800	—	—	—	—	—	—	—	—	—	—	—	—	—	—	150
3200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	150

b. Summary of the results.

The results of these measurements are given in the tables I to VI; for the sake of completeness table II from „Mededeeling 5" (this is the series with $V = 800_1$ cm/sec where X had a not exactly known value, ranging from about 100 to 200 cm) also is given here as table II. [15])

As to table III (the results of the experiments with purposely caused turbulence of the stream of air), it has to be observed that in the first series, say to about $x = 10$ cm, the immediate vicinity of the wires composing the screen has influenced the results. As is shown in table III, at $x = 2,5$ cm the velocity of the air increases to 848 cm/sec and then

15) The measurements at $x = 1 — 1,5 — 60$ and 70 cm have been omited here.

decreases at increasing values of y again to 800 cm/sec. This has to be ascribed to the circumstance that the meshes of the screen give rise to regions of increased velocity, which regions extend for some distance (probable over about 7 or 8 cm) behind the screen.

In order to obtain a more complete oversight of the distribution of the velocity of the air in the turbulent boundary layer, further experiments were carried out at $x = 150$ cm with $V = 600$, 2000, 2800 and 3200 cm/sec; the results of which are, together with those of the series mentioned in the tables I to VI at $x = 150$ cm, given in table VII.

At the foot of the tables the dates of the experiments are given, and further the value of ν, calculated from the temperature of the air and from the barometric pressure occurring during the measurements by the aid of the diagram given by PRANDTL in „Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen" I 1921, p. 136.

§ 4. DISCUSSION OF THE RESULTS.

a. Determination of a.

In order to obtain a general oversight of the results of the measurements, the velocity gradient at the surface, a, was determined first. This was done in the following way: the values of u were put down in a diagram (u as a function of y) and then the lowest observed values of u were

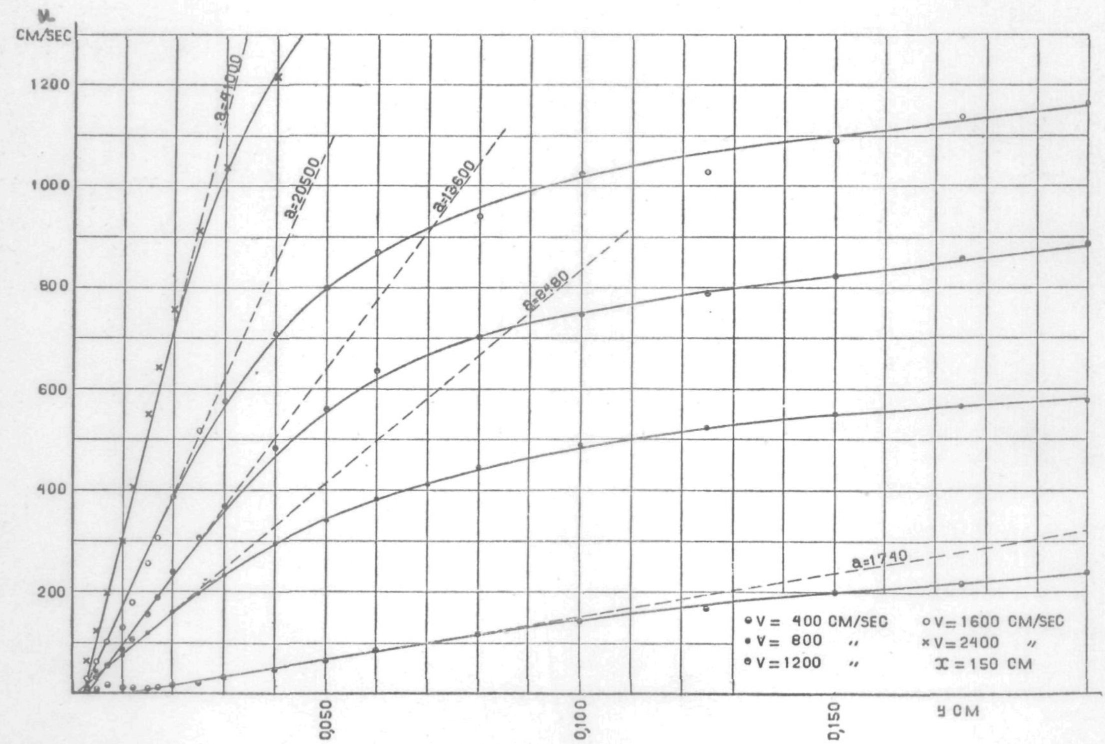

Fig. 2. Determination of a at $x = 150$ cm.

TAB

x cm	V cm/sec.					
	400		800_1		800_2	
	a sec^{-1}	δ cm	a sec^{-1}	δ cm	a sec^{-1}	δ cm
2,5	—	—	13520	0,15	14400	0,14
5	—	—	10400	0,18	12600	0,17
7,5	—	—	8000	0,20	—	—
10	—	—	7340	0,23	10400	0,30
15	2190	0,33	5780	0,28	8700	0,43
20	1950	0,40	5600	0,30	9300	0,50
25	1900	0,45	5560	0,35	10200	0,55
30	—	—	4740	0,40	10300	0,65
37,5	1720	0,58	—	—	—	—
40	—	—	4840	0,45	9700	0,90
50	1670	0,68	4960	0,60	9000	1,20
60	—	—	—	—	8600	1,40
62,5	1550	0,90	4920	0,75	—	—
70	—	—	—	—	8200	1,60
75	1480	1,05	6310	0,95	—	—
80	—	—	8890	1,10	7900	1,75
85	—	—	7060	1,20	—	—
87,5	1690	1,15	—	—	—	—
100	1480	1,25	8600	1,70	7600	2,00
125	1440	1,35	8350	1,95	—	—
150	1740	1,45	8480	2,30	7000	2,70

I.

1200		1600		2400	
a sec^{-1}	δ cm	a sec^{-1}	δ cm	a sec^{-1}	δ cm
26000	0,13	—	—	—	—
18800	0,15	—	—	—	—
15800	0,17	—	—	—	—
14400	0,19	24500	0,16	38200	0,17
11200	0,21	18000	0,19	33600	0,20
10300	0,23	17250	0,21	32500	0,21
10100	0,24	16750	0,26	31900	0,22
—	—	—	—	—	—
9600	0,36	16250	0,45	41000	0,45
—	—	—	—	—	—
11600	0,70	22500	0,85	56000	0,80
—	—	—	—	—	—
14400	0,95	26500	1,05	60000	0,90
—	—	—	—	—	—
16300	1,15	25800	1,25	59000	1,20
—	—	—	—	—	—
—	—	—	—	—	—
16000	1,40	24250	1,45	53000	1,35
14800	1,60	23000	1,55	48500	1,45
14100	1,95	21250	1,85	44000	1,65
13600	1,95	20500	1,90	41000	1,80

connected by a straight line, the inclination of which gave the required value of *a*. At the same time this line gives the distance correction *f*, mentioned in § 2 b 5.

As an example the lower parts of the curves for the series at $x = 150$ cm have been shown in figure 2.

In most of the measured $u(y)$ curves this line could be marked well, not, however, in the series with $V = 400$ cm/sec, which is undoubtedly caused by the *S* shaped $u(y)$ curve in the immediate vicinity of the wall. This gives rise to the supposition that the determined values of *a* of these series may possibly be too high, as follows from a more detailed research (see p. 43).

The experimental values of *a* are collected in table VIII.

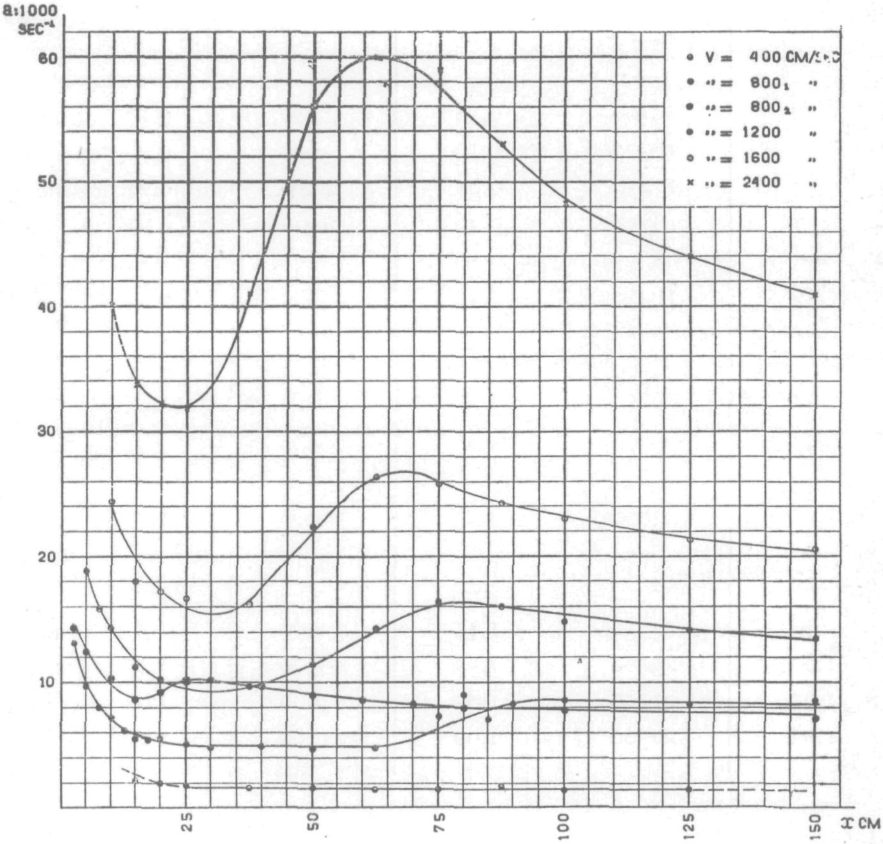

Fig. 3. Graphical representation of the values of the velocity gradient at the surface of the glass plate as a function of *x*.

In figure 3 the values of *a* are represented diagrammatically as a function of *x*.

In all measurements, except those with $V = 400$ cm/sec, it appears that *a* decreases first with increasing value of *x*, then increases over a longer or shorter period and then decreases again. This points to the fact that in the region where $\frac{\partial a}{\partial x}$ is positive, the stream of air changes character and passes from the laminar into the turbulent state of motion. This is confirmed by a consideration of the $u(y)$ curves: they cross one another in the region and transition, for the curves at a higher value of x are steeper at lower values of y and flatter at higher values of y. Outside the region of transition, however, the curves grow more flat as x increases.

In "Mededeeling 5" the researches have been discussed that were performed to prove that the laminar and the turbulent state of motion actually exist at the same time beside each other: the first at the lower values of x, the second at the higher. Furthermore, it is shown there that the presence of strong fluctuations in the flow of air in the tunnel gave rise to an early transition (that is to a transition at lower values of x). For this purpose experiments were carried out at various values of the distance X from the leading edge of the glass plate to the honeycomb of the tunnel; the smaller X was, the greater the fluctuations in the stream of air in the channel. [16])

This phenomenon has not been considered in the continued experiments; here the measurements were performed at a value of X that as much as possible was kept constant.

However, the mentioned research has been completed by the series with $V = 800_2$ cm/sec, with purposely caused turbulence. As it was expected, these series gave the most advanced region of transition.

In the series with $V = 400$ cm/sec only the curves at $x = 125$ cm and $x = 150$ cm show the crossing mentioned; so the transition from the laminar into the turbulent state of motion begins here at the end of the glass plate.

b. Determination of δ.

In order to determine the thickness of the boundary layer δ the values of u found at every value of x were put down in a diagram as a function of y. The series of points that was obtained in this way was connected by a smooth curve and the distance y where this curve passes into the line $u = V$ is called δ. It appears, however, that in the turbulent part of the boundary layer this curve intersects the line $u = V$ (see below, at e).

The diagrams show that in the neighbourhood of $y = \delta$ the values of u rather lie above this smooth curve, so that in the last part of the boundary layer the velocity has evidently been found too high. It is not impossible that this is due to experimental errors, for, on account of the obtainable accuracy of the method of measuring and the always more or less unsteady position of the measuring apparatus, it was difficult

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16) See Fig. 4, „Mededeeling 5", curves A, B and C.

to ascertain whether the value u had acquired its limiting value V or not.

The values of δ, graphically determined in this way, are also collected in table VIII, and have been represented in figure 4.

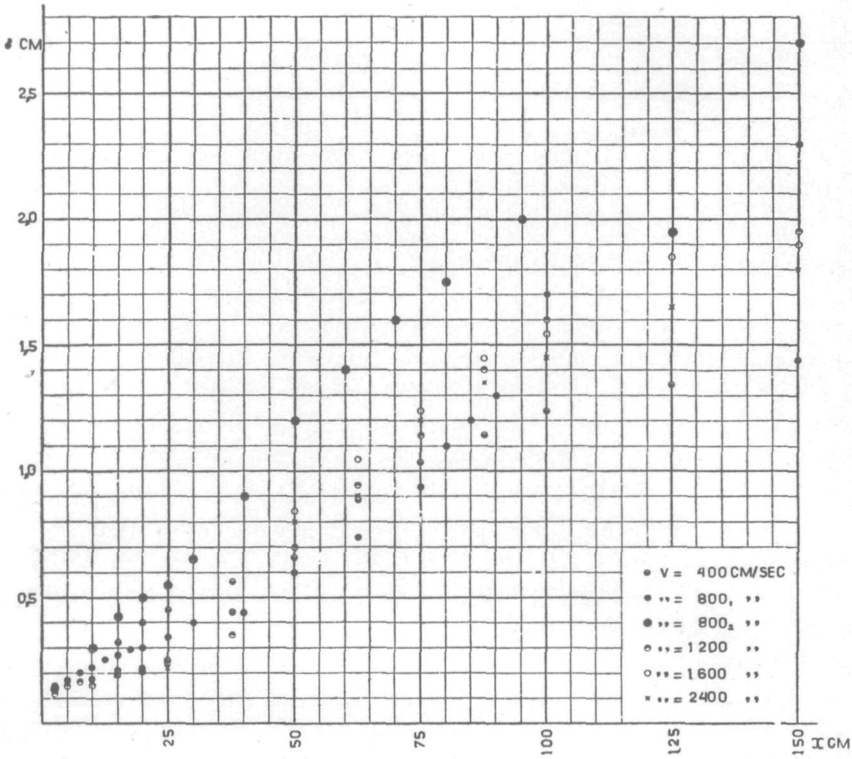

Fig. 4. Graphical representation of the values of δ as a function of x.

For a comparison of the results with VON KÁRMÁN's formula (8):

$$u = V\left(\frac{y}{\delta}\right)^{1/7}$$

and for calculations based upon this formula, it is desirable to make use of a value of δ determined from a logarithmic diagram (log. u as a function of log. y).

In those cases where it may be expected that the boundary layer is actually turbulent, such diagrams are given in figure 7 and 8. Connecting in these diagrams the observed values of u by a straight line with a gradient 1 : 7, the intersection of this line with the line $u = V$ will give the value of δ according to VON KÁRMÁN ($= \delta_k$). By means of Fig. 7 and 8 the following values of δ_k were found:

TABLE IX.

x cm.	δ_k cm. V cm/sec.								
	800_1	800_2	1200	1600	1600_2	2000	2400	2800	3200
75	0,88	1,72 mean	0,985	1,045	—	—	0,99	—	—
80	1,11	—	—	—	—	—	—	—	—
87,5	—	—	1,15	1,30	—	—	1,15	—	—
100	1,52	2,33	1,65	1,41	—	—	1,31	—	—
125	1,845	—	2,00	1,82	—	—	1,78	—	—
150	2,16	3,24	2,06	1,91	2,58	2,05	1,865	1,81	1,56

At the series with $V = 800_2$ cm/sec, no $u(y)$ curve is determined at $x = 75$ cm, but as this was done at $x = 70$ and $x = 80$ cm, the mean values of the velocity from both these series are put down in a diagram and by means of this diagram the value of δ_k at $x = 75$ cm was determined.

A comparison of the values of δ_k at $x = 150$ cm and those at $x = 125$ and at $x = 100$ cm gives the impression that $\delta_{k.150}$ at $V = 800_2$, 1200, 1600 and 2400 cm/sec is somewhat too small. Other results affirm this supposition, as the calculation of x_0 (see p. 29) and that of the loss of momentum (see p. 36). The values of $\delta_{k.150}$ might possibly be 10 to 16 % too small. The origin of this could, however, not be detected.

The high value of δ_k in the case of purposely caused turbulence of the stream of air ($V = 800_2$ and 1600_2 cm/sec) manifests itself strikingly and corresponds to all anticipations.

c. Laminar part of the boundary layer.

The series of the experiments with $V = 800_1$ cm/sec had shown that before the region of transition the measured values of the velocity u were fairly well in accordance with the theory of BLASIUS. As it was expected that this would be the case with $V = 400$ cm/sec as well, the measured values of u, taken from table I, are put down in a diagram as a function of $y/\sqrt{x}$ (represented in Fig. 5). When there is agreement with the theory of BLASIUS, all measured values of u should be on the full drawn curve.

Its appears that this is not the case, but one may say that the agreement is better when x is smaller [17]. In order to make the figure not too dense, some values of u below 25 cm/sec have been omitted. A similar figure (not given here) was constructed with $V = 1200$ cm/sec and with values of $x = 2{,}5$ 5, 10, 15, 20 and 25 cm. Although in this

[17] The same results are obtained by comparing the experimental values found for a with those calculated from form. (1).

diagram the values of u are lying above the line according to BLASIUS (except those with $x = 2{,}5$ cm and $y = 0{,}060$ to $y = 0{,}200$ cm) the correspondence has to be considered fair to the same extent as in the series with $V = 800_1$ cm/sec. (see "Mededeeling 5" figure 3).

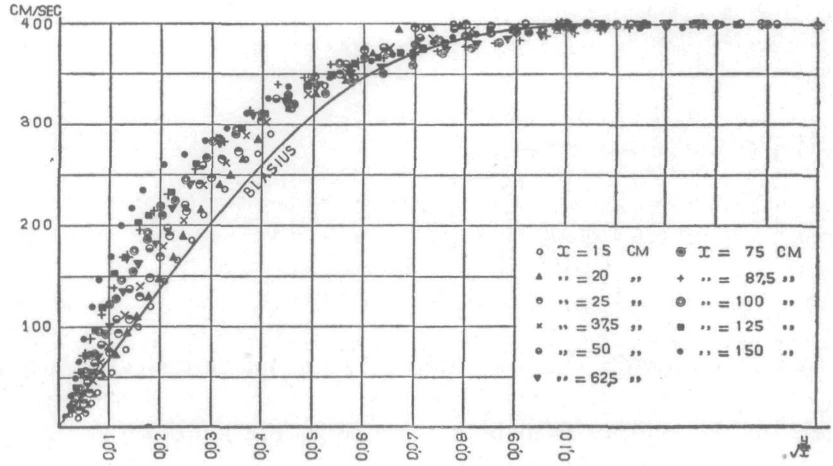

Fig. 5. Graphical representation of the velocity distribution in the boundary layer, with $V = 400$ cm/sec. The curve drawn is in accordance with the theory of BLASIUS.

Other means for a comparison of the experimental results with the theory of BLASIUS are found by putting down in a diagram the measured values of δ as function of $\sqrt{\frac{\nu x}{V}}$; according to BLASIUS a straight line with a gradient 1 : 5,5 (see p. 3 form. (3)) has to be found.

This is represented in Fig. 6.

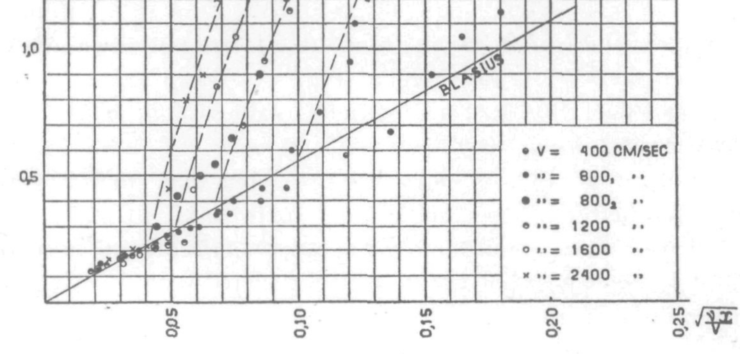

Fig. 6. Graphical representation of δ as a function of $\sqrt{\frac{\nu x}{V}}$, the full drawn curve is that given by the theory of BLASIUS.

For each point the appropriate value of ν (as given in tables I to VII) has been used.

It follows from Fig. 6 that with values of $\sqrt{\frac{\nu x}{V}}$ not exceeding 0,04, δ is fairly well in accordance with BLASIUS's theory and that with higher values of $\sqrt{\frac{\nu x}{V}}$, the $\delta\left(\sqrt{\frac{\nu x}{V}}\right)$ lines, mutually almost parallel, show a much larger inclination (they are slightly curved).

d. Region of transition.

It follows from the numerous researches on the appearance of the turbulent state of motion that the laminar state will pass into the turbulent one when R^* surpasses about 2000.

The critical value of R^* is calculated from the experimental results in question by taking the minimum value of a of Fig. 3 as a criterion for the transition and then determining R_c by the aid of the appropriate value of x.

Also R^*_c has been calculated from the results of Fig. 6, for the value of δ at which the transition seems to appear, follows rather clearly from this figure.

This gives the following results (table X):

TABLE X.

V cm/sec	400	800_1	800_2	1200	1600	2400
$x_{trans.}$ cm	125	60	15	33,75	30	22
$R_{trans.} = \frac{Vx_t}{\nu}$	332000	320000	83500	268000	318000	350000
$\delta_{trans.}$ cm	1,2	0,55	0,30	0,38	0,30	0,22
$R^*_t = \frac{V\delta_t}{\nu}$	3120	2950	1650	3000	3180	3480
$5{,}5\sqrt{R_t}$	3170	2980	1590	2840	3100	3255

Both values of R^*_c are fairly well in agreement, as might be expected from what is mentioned under **c**.

The mean value of R^* (about 3150) seems to be somewhat higher than one would expect from the experiments of COUETTE [18]) and SCHILLER [19]) which, though not for the same type of flow, gave values of R^*_c from 1900 to 2300 [20]).

The series with $V = 800_2$ cm/sec are exceptional; as was expected they gave a much lower value of R_c^*.

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18) COUETTE, Ann. de Chim. et de Phys. (6), **21**, p. 457, 1890.

19) L. SCHILLER, Zs. f. angew. Math. u. Mech. **I**, p. 436, 1921.

20) A. MALLOCK, Phil. Trans. A. 1896 p. 41, gives a higher value.

How the change from the laminar into the turbulent state takes place in the region of transition, can not yet be deduced with certainty. It is possible that within the boundary layer large vortices or waves are formed that originate the transition and that cleave the boundary layer: one part of the original laminar layer is forced out so that a greater part of the outer current is absorbed in the region of retarded motion, while another part is forced to the wall and will remain there as a laminar layer. This point of view is confirmed by the observation that the wire in the first part of the region of transition (with $V = 800_1$ cm/sec at x about 60 cm to 75 cm and $y = 0{,}040$ to about 0,100 cm) glitters visibly. Although after the region of transition having been passed the flow doubtless was turbulent, the visible glittering of the wire could no more be observed; evidently the fluctuations have so high a frequency there that their effect on the hot wire anemometer is no longer perceptible with the eye.

For a more detailed research of these phenomena it will be necessary to record the fluctuations of the velocity automatically; researches in this direction are in progress.

e. Turbulent part of the boundary layer.

The most important question is whether the distribution of the velocity in the turbulent boundary layer will satisfy the formula of VON KáRMáN:

$$u \sim y^{1/7}.$$

To decide this the values of log. u are plotted in the already mentioned figure 7 against the values of log y, at those values of x and V where it may be expected that the boundary layer is actually turbulent: this is the case with $V = 800_1$, 800_2, 1200, 1600, and 2400 cm/sec and x higher than 75 cm. As stated, in the series with $V = 800_2$ cm/sec in the diagram at $x = 75$ cm the mean value of u at $x = 70$ and 80 cm has been represented.

Moreover figure 8 gives a similar diagram at $x = 150$ cm, here all values of u from table VII are given.

It follows from these figures that indeed the relation $u \sim y^{1/7}$ is satisfied, provided that y is not too small (in § 6 an approximate calculation is given for the minimum value of y that satisfies VON KáRMáN's equation) with the series without artificial turbulence the errors are not greater than 2 %, hence it may be accepted that our result is a sufficient confirmation of the theory of VON KáRMáN.

It must be mentioned, however, that the relation $u \sim y^{1/7}$ is less distinct in such cases where the flow of air was purposely made turbulent. As a mean value, apparently u may be taken proportional to $y^{1/7}$, but deviations from this mean value are great at $V = 800_2$ cm/sec. In general the deviations do not surpass 6 %, except at $x = 75$ cm (mean) where

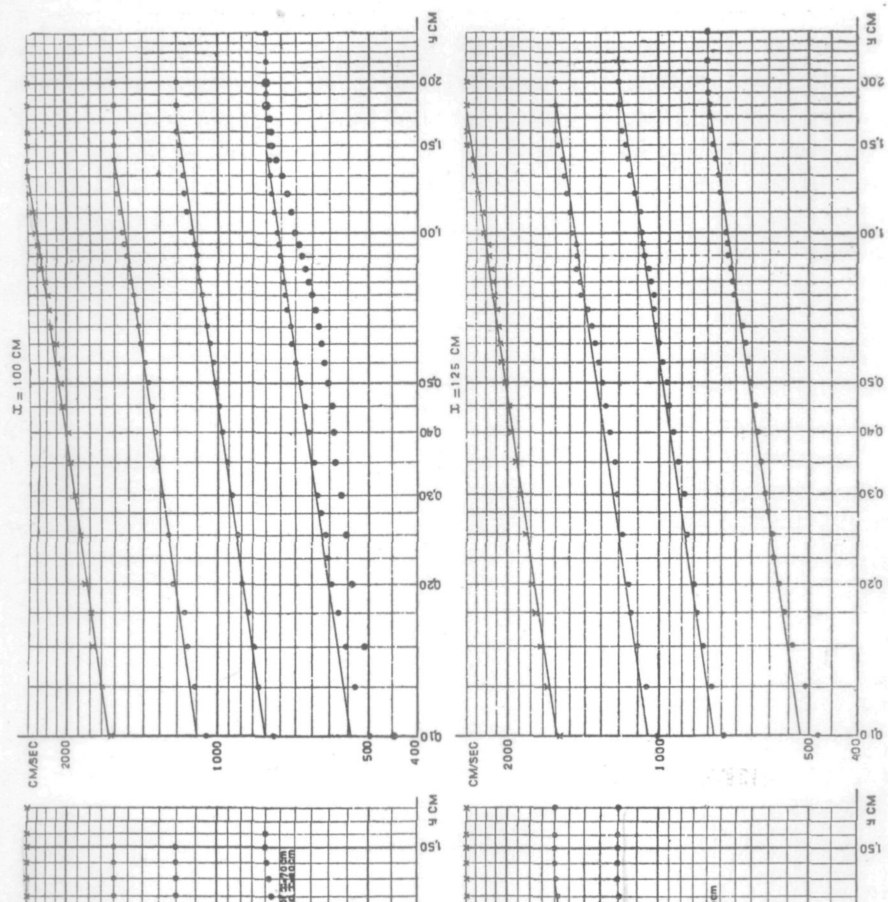

Fig. 7. Logarithmic diagrams of the velocity in the boundary layer as a function of y at the sections $x = 75$, $87{,}5$, 100 and 125 cm.

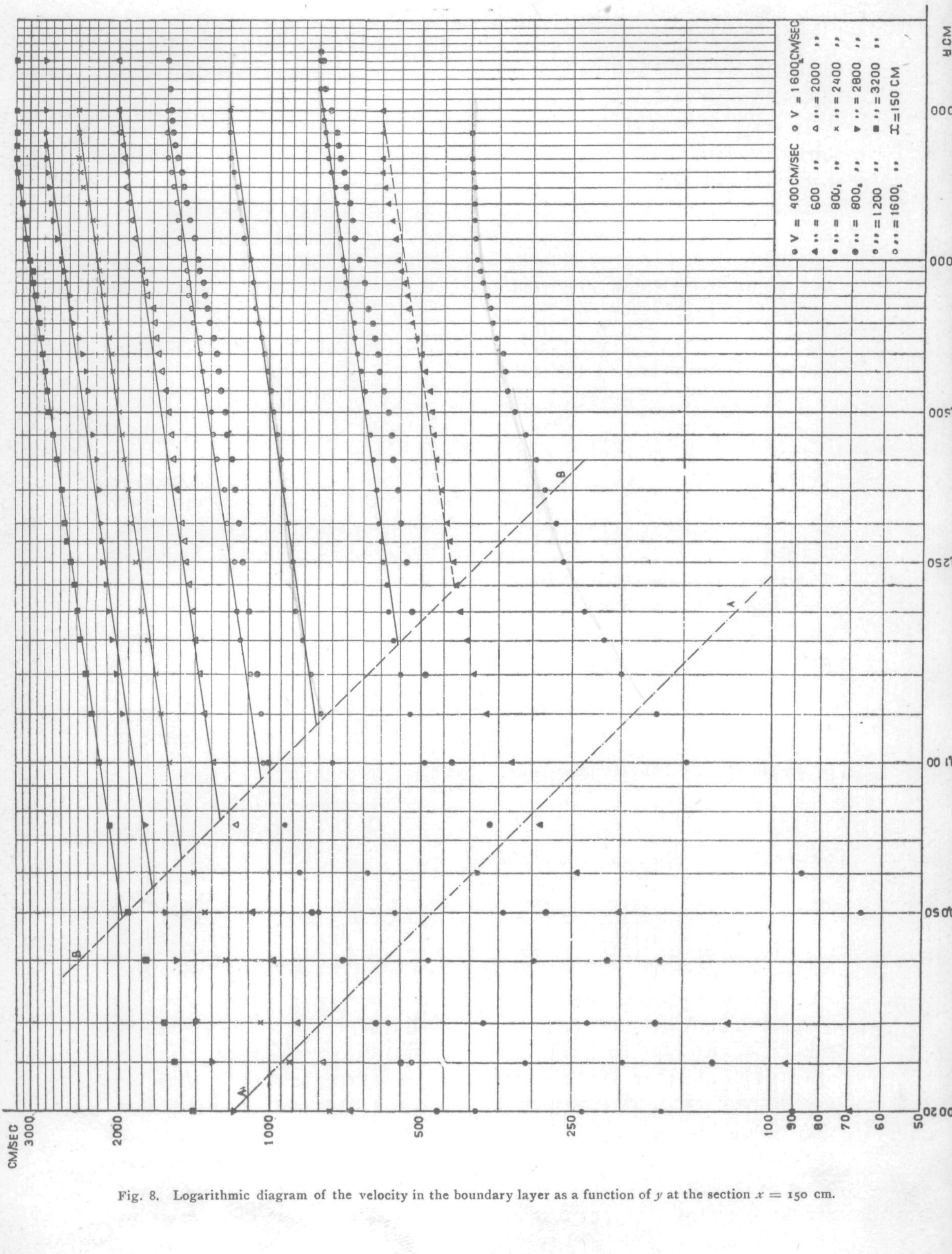

Fig. 8. Logarithmic diagram of the velocity in the boundary layer as a function of y at the section $x = 150$ cm.

differences to 16 % occur, against 5,5 % at $x = 100$ cm and 4,1 % at $x = 150$ cm.

The question might arise whether in these cases the relation $u \sim y^{1/6 \text{ to } 1/5}$ would not suit as well. In this respect the measurements described here do not supply sufficient data.

It may be deduced from figure 8 that the boundary layer is not yet turbulent with $V = 400$ cm/sec at $x = 150$ cm, at least there u proves not to satisfy the relation $u \sim y^{1/7}$.

Besides by the more wavy shape of the log. u (log. y) curves in the case of the series with artificial turbulence, the difference between these and other curves is marked by a parallel shift of the log. u (log. y) line in respect to the line that represents the conditions without purposely caused turbulence in the outer current. In the former case the transition will take place at a smaller value of x and consequently δ will be greater in the first case than in the second.

A remarkable peculiarity is that these diagrams do not give any certain indication of $\frac{\partial u}{\partial y}$ becoming zero at $y = \delta$; the curves seem to be rather discontinuous.

The small sensitiveness of the hot wire anemometer at higher velocities and the difficulties caused by the unsteadiness of the air current in the channel did not allow a more thorough research concerning this question.

An accurate determination of the form of the curve in the neighbourhood of $y = \delta$ would be of great theoretical importance as it would give data on the shape and the distribution of the vortices in the outer part of the boundary layer.

f. Determination of the constant x_0 in the formula for the thickness δ_k of the turbulent boundary layer.

Making use of the values of δ_k, mentioned at p. 23 (table IX), the parameter x_0 occurring in the formula

$$\delta = 0{,}371\ (x - x_0)^{4/5} \left(\frac{\nu}{V}\right)^{1/5}$$

(compare p. 4 form. (10)) has been calculated. For each calculation the appropriate value of ν was inserted. The results are given in table XI.

Notwithstanding rather large discrepancies shown by some numbers of this table, the following conclusions may be drawn from it:

1. The values of x_0 which ought to be independent of x, show an increase for values of x ranging from 75 cm to 125 cm; probably this may be ascribed to the circumstance that the turbulent state is not yet fully developed. This is followed by a larger and unexplained increase at $x = 150$ cm. The latter increase is connected with the circumstance (already stated at p. 23) that at $x = 150$ cm the value of δ seems to

TABLE XI.

V	800_1		800_2		1200		1600_1 cm/sec	
x	δ_k	x_0	δ_k	x_0	δ_k	x_0	δ_k	x_0 cm
75	0,88	50,1	1,72 mean	16,2	0,985	42,9	1,045	37,7
80	1,11	46,4	—	—	—	—	—	—
87,5	—	—	—	—	1,15	48,6	1,30	39,0
100	1,52	50,0	2,33	13,8	1,65	39,2	1,41	45,9
125	1,845	61,5	—	—	2,00	47,6	1,82	50,5
150	2,16	72,6	3,24	21,0	2,06	68,3	1,91	70,3

V	1600_2		2000		2400		2800		3200 cm/sec	
x	δ_k	x_0	δ_k	x_0	δ_k	x_0	δ_k	x_0	δ_k	x_0 cm
75	—	—	—	—	0,99	36,8	—	—	—	—
80	—	—	—	—	—	—	—	—	—	—
87,5	—	—	—	—	1,148	41,3	—	—	—	—
100	—	—	—	—	1,31	45,3	—	—	—	—
125	—	—	—	—	1,78	44,9	—	—	—	—
150	2,58	33,6	2,05	58,5	1,865	64,1	1,81	65,0	1,56	74,7

be too small. As will be seen from the dates mentioned in table VII, part of the measurements at $x = 150$ cm have been executed in November (8/9), another part in December (17/20). The differences between the values of x_0 given by both groups are not significant [21]).

2. There seems to be a very slight decrease of x_0 with increasing values of V, this effect being so small, however, that it can better be neglected, and that it may be stated that x_0 is independent of V (the value of X being the same for all values of V). The mean value of x_0 is for $x = 75$ to 125 cm about 45 cm and for $x = 150$ cm about 68 cm.

3. In accordance with what was to be expected x_0 decreases largely when the current is made turbulent on purpose: at $V = 800_2$ cm/sec it is about 17 cm, at $V = 1600_2$ cm/sec (calculated from δ_k at $x = 150$ cm) 34 cm.

The fact that even here x_0 is not zero, shows that in this case too the "surface friction" experienced by the plate will be smaller than what is calculated on the supposition of an everywhere turbulent boundary layer.

21) These values of x are situated in the region of transition; so the definite turbulent state has not yet been reached here.

g. Comparison of the value of a to that given by VON KÁRMÁN's formula for the resistance of a turbulent flow along a smooth wall.

The results of the determinations of a can be compared to those which are deduced from formula (6), p. 3:

$$a = \frac{\tau_0}{\mu} = 0{,}0225 \; \frac{V^2}{\nu} \left(\frac{V \delta_k}{\nu}\right)^{-1/4}$$

This formula forms the base of VON KÁRMÁN's calculations on the turbulent boundary layer; it has been deduced from the empirical data on the resistance of the motion of a fluid through a smooth walled tube, and is independent of x and x_0.

In calculating a_k, the values of δ_k mentioned in table IX have been used; the values of ν are those given at the foot of the tables I to VII. Variations of ν have a far greater influence than inaccuracies of δ_k.

In the following table XII the experimental values of a ($= a_e$) and those calculated according to the formula above ($= a_k$) are put together.

TABLE XII.

x cm	V cm/sec									
	800_1		800_2		1200		1600		2400	
	a_k sec^{-1}	a_e sec^{-1}	a_k sec^{-1}	a_e sec^{-1}	a_k sec^{-1}	a_e sec^{-1}	a_k sec^{-1}	a_e sec^{-1}	a_k sec^{-1}	a_e sec^{-1}
70	—	—	—	8200	—	—	—	—	—	—
75	11220	6310	10000	—	22850	16300	38050	25800	77100	59000
80	10950	8890	—	7900	—	—	—	—	—	—
87,5	—	—	—	—	22200	16000	35050	24250	74000	53000
100	10160	8600	9400	7600	19800	14800	35100	23000	72500	48500
125	9660	8350	—	—	18850	14100	33250	21250	67500	44000
150	9310	8480	8510	7000	19800	13600	33200	20500	69200	41000

The table shows that VON KÁRMÁN's formula gives a much higher value than is deduced from the experiments. The ratio a_k / a_e is approximately constant for the series:

at $V = 800_1$ cm/sec . . . mean value of $a_k / a_e = 1{,}15$
at $V = 800_2$ cm/sec . . . „ „ „ $a_k / a_e = 1{,}23$
at $V = 1200$ cm/sec . . . „ „ „ $a_k / a_e = 1{,}38$

For the series with $V = 1600$ cm/sec it shows a slight increase with x (from 1,47 at $x = 75$ cm to 1,62 at $x = 150$ cm), and for the series with $V = 2400$ cm/sec a very marked increase (from 1,3 at $x = 75$ cm to 1,7 at $x = 150$ cm). Comparison of the series with each other shows that the ratio increases with V.

The deviation from unity is very great. If instead of a, V is calculated by the inverse formula:

$$V = 8{,}7 \; (\nu \, a)^{4/7} \left(\frac{\delta}{\nu}\right)^{1/7}$$

(p. 3, form. (5)), the deviation of course is less; it remains considerable, however.

It is not possible to ascribe this result at once to errors in the method of measurement with hot wire anemometers, or to the application of the correction for the cooling effect by the wall, as with all these series in the laminar part the experimental values of a are higher than those calculated from BLASIUS's formula (comp. f. i. the table given at p. 33 below). In the two parts of the boundary layer the deviations are in different directions, though the state of motion will be nearly the same in both cases, as in the immediate neighbourhood of the wall where $\frac{\partial u}{\partial y}$ has to be determined, the turbulence disappears.

Moreover the comparisons between the integral of the resistance, as calculated from the values of a, and the loss of momentum in the boundary layer, as calculated from the velocity curves, $u(y)$, gives a very satisfactory result (see § 5 below), and by no means does allow the supposition that the values of a_e are 30 % to 70 % in error. In order to try if a connection could be found between the values of a_e and δ_k, the values of $\frac{\nu a_e}{V^2}$ and $\frac{V \delta_k}{\nu}$ (both being abstract numbers) have been calculated and have been plotted on a logarithmic paper. The points obtained do not lie on one curve, and show a rather great dispersion. For the series $V = 2400$ cm/sec approximately the following formula was found:

$$\frac{a V^2}{\nu} = 0{,}00185 \left(\frac{V \delta}{10^4 \nu}\right)^{-1/2}$$

for the series with $V = 1600$ cm/sec:

$$\frac{a V^2}{\nu} = 0{,}00158 \left(\frac{V \delta}{10^4 \nu}\right)^{-2/5}$$

for the series with $V = 1200$ cm/sec:

$$\frac{a V^2}{\nu} = 0{,}00163 \left(\frac{V \delta}{10^4 \nu}\right)^{-1/4}$$

The last formula is proportional to VON KÁRMÁN's; the ratio of the coefficients being 0,0225/0,0163 = 1,38 as mentioned above.

§ 5. FURTHER PARTICULARS ON THE VALUE OF a AND CALCULATION OF THE FRICTIONAL RESISTANCE EXPERIENCED BY THE GLASS PLATE.

a. Frictional resistance calculated from a.

From the experimental values of the velocity gradient at the surface the resistance experienced by the glass plate can be found by integrating the values of a over the length of the plate.

TABLE I.

$V = 400$ cm/sec.

y cm	x cm										
	15	20	25	37,5	50	62,5	75	87,5	100	125	150
0,003	0	3	1	1	23	1	27	5	4	4	2
0,005	8	4	2	2	16	3	19	3	?	7	9
0,007	5	4	3	3	12	2	16	6	4	12	18
0,010	6	5	5	4	7	4	12	7	7	12	11
0,012	6	7	7	7	7	5	10	9	6	9	10
0,015	10	10	9	7	9	7	10	13	8	10	11
0,017	12	12	13	10	11	8	11	15	9	10	12
0,020	16	15	17	14	15	11	13	18	13	12	16
0,025	25	23	25	22	21	16	18	27	18	18	22
0,030	35	32	34	29	26	24	24	36	24	23	32
0,040	55	51	53	48	46	39	35	55	40	39	49
0,050	78	73	74	65	64	56	51	71	58	56	66
0,060	100	94	93	81	81	71	73	88	72	71	87
0,070	119	110	108	—	—	—	—	—	—	—	—
0,080	144	130	130	112	108	102	93	112	96	96	119
0,090	166	149	149	—	—	—	—	—	—	—	—
0,100	186	169	169	141	137	134	126	139	122	119	148
0,110	210	190	190	—	—	—	—	—	—	—	—
0,125	236	212	213	180	177	162	154	169	148	152	169
0,140	264	240	240	—	—	—	—	—	—	—	—
0,150	270	250	246	205	198	184	186	196	173	168	200
0,160	290	265	265	—	—	—	—	—	—	—	—
0,175	315	285	273	240	220	218	210	215	192	202	216
0,200	338	317	303	263	256	240	225	231	219	210	237
0,225	346	335	320	289	—	—	—	—	—	—	—
0,250	370	346	339	303	293	283	266	256	244	231	260
0,275	394	370	350	320	—	—	—	—	—	—	—
0,300	400	394	372	330	323	310	291	283	270	260	269
0,325	—	398	—	—	—	—	—	—	—	—	—
0,350	—	400	394	350	345	324	310	313	295	286	283
0,400	—	—	400	375	358	340	320	339	310	296	296
0,450	—	—	—	385	375	350	330	346	328	310	310
0,500	400	400	400	394	385	358	342	360	339	325	325
0,550	—	—	—	398	394	370	350	361	360	337	336
0,600	—	—	—	400	398	375	358	365	365	349	340
0,650	—	—	—	—	400	380	370	370	370	359	344
0,700	—	—	—	—	400	385	375	372	377	364	354
0,750	—	—	—	—	400	389	380	376	382	370	361
0,800	—	—	—	—	400	394	389	380	386	376	365
0,850	—	—	—	—	400	398	398	382	390	380	370
0,900	—	—	—	—	400	400	400	386	390	387	380
0,950	—	—	—	—	400	400	400	390	393	390	384
1,000	400	400	400	400	400	400	400	393	395	393	389
1,100	—	—	—	—	—	400	400	395	400	393	392
1,200	—	—	—	—	—	400	400	400	400	400	395
1,300	—	—	—	—	—	—	400	400	400	400	395
1,400	—	—	—	—	—	—	400	400	400	400	395
1,500	—	—	—	—	—	400	400	400	400	400	398
1,600	—	—	—	—	—	—	—	400	400	400	400
1,800	—	—	—	—	—	—	—	400	400	400	400
2,000	—	—	—	—	—	—	—	400	400	400	—
f cm	—0,014	—0,013	—0,011	—0,012	—0,012	—0,014	—0,013	—0,008	—0,012	—0,011	—0,011
X cm	225	225	225	225	175	175	175	175	225	225	225
ν cm²/sec.	0,143	0,147	0,148	0,151	0,147	0,148	0,146	0,148	0,151	0,151	0,144
	3 Dec.	30 Nov.	29 Nov.	28 Nov.	28 Nov.	22 Nov.	21 Nov.	17 Nov.	15 Nov.	13 Nov.	9 Nov. '23

TABLE II.

$V = 800_1$ cm/sec.

y cm	x cm: 2,5	5	7,5	10	12,5	15	17,5	20	25	30	40	50	62,5	75	80	85	90	100	125	150
0,005	70	53	48	41	34	33	33	33	33	32	30	29	26	37	43	49	49	48	45	48
0,010	133	104	85	74	58	56	58	55	60	54	49	49	45	63	79	76	91	85	81	80
0,015	199	151	119	108	85	84	85	81	83	82	71	75	71	90	117	116	132	128	119	124
0,020	263	203	158	148	120	114	117	108	109	101	96	99	95	127	159	151	180	172	160	164
0,025	324	247	200	182	148	143	146	142	137	124	121	123	119	160	194	191	201	206	197	196
0,030	391	297	240	222	176	171	172	169	164	148	144	142	143	182	225	222	244	246	233	232
0,040	508	391	322	286	238	226	224	220	217	195	194	182	193	233	278	288	296	303	295	297
0,050	601	483	409	355	305	287	283	270	267	242	230	227	235	278	329	338	350	349	337	342
0,060	664	552	489	425	379	350	356	342	322	298	289	267	280	328	374	383	395	407	377	385
0,070	705	626	548	490	435	410	408	388	360	338	330	318	314	339	421	408	—	413	408	415
0,080	741	662	610	550	495	452	455	440	406	387	365	347	345	385	455	450	—	449	435	447
0,100	774	730	670	625	578	520	526	514	484	462	438	409	402	461	488	490	500	498	481	490
0,125	789	768	740	701	660	617	605	599	577	547	510	483	468	490	—	502	—	531	508	525
0,150	796	788	770	735	733	696	645	680	643	607	562	532	519	540	582	571	—	555	541	550
0,175	800	797	796	775	763	745	731	732	694	660	605	581	570	562	—	590	—	572	561	568
0,200	800	800	800	790	780	772	750	752	737	700	660	632	600	629	617	620	598	594	576	577
0,225	800	800	800	795	790	782	788	775	768	732	688	654	636	615	—	—	—	604	584	585
0,250	800	800	800	800	798	797	800	795	778	743	720	680	655	640	658	658	—	610	594	595
0,275	—	—	—	800	800	800	800	797	790	757	740	701	683	662	—	—	—	621	604	600
0,300	—	—	—	800	800	800	800	800	795	770	750	729	691	680	662	667	645	630	611	606
0,350	—	—	—	800	800	800	800	800	800	790	772	743	714	701	—	—	—	644	622	613
0,400	—	—	—	800	800	800	800	800	800	795	792	754	739	715	685	690	670	657	632	622
0,450	—	—	—	—	—	—	—	800	800	800	800	764	755	739	—	—	—	669	640	630
0,500	—	—	—	—	—	—	—	800	800	800	800	786	765	743	713	716	694	681	653	641
0,550	—	—	—	—	—	—	—	—	—	800	800	795	775	748	—	—	—	699	662	647
0,600	—	—	—	—	—	—	—	—	—	800	800	800	781	757	741	732	720	713	670	657
0,650	—	—	—	—	—	—	—	—	—	—	—	800	783	759	—	—	—	715	680	670
0,700	—	—	—	—	—	—	—	—	—	—	—	800	788	778	755	760	740	727	694	677
0,750	—	—	—	—	—	—	—	—	—	—	—	800	793	—	—	—	—	732	705	680
0,800	—	—	—	—	—	—	—	—	—	—	—	800	798	797	762	772	759	740	713	692
0,850	—	—	—	—	—	—	—	—	—	—	—	—	800	—	—	—	—	747	719	700
0,900	—	—	—	—	—	—	—	—	—	—	—	—	800	800	—	—	—	753	726	708
0,950	—	—	—	—	—	—	—	—	—	—	—	—	800	—	—	—	—	758	730	713
1,000	—	—	—	—	—	—	—	—	—	—	—	—	800	800	787	790	780	763	736	717
1,100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	773	743	726
1,200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	800	800	800	783	752	732
1,300	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	786	760	741
1,400	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	788	769	748
1,500	—	—	—	—	—	—	—	—	—	—	—	—	—	—	800	800	800	790	778	760
1,600	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	792	786	768
1,700	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	794	791	771
1,800	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	798	796	778
1,900	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	800	799	784
2,000	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	800	800	789
2,200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	800	800	800
2,500	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	800	800	800
f cm	0	0	0	0	0	0	0	0	0	+ 0,001	0	+ 0,001	— 0,001	0	0	0	+ 0,002	+ 0,001	+ 0,001	+ 0,001
ν cm²/sec.	0,151	0,151	0,153	0,151	0,152	0,149	0,152	0,149	0,148	0,148	0,147	0,153	0,150	0,155	0,150	0,154	0,151	0,149	0,150	0,149

May to Sept. 1923.
$X = 100$ to 200 cm.

$V = 800_2$ cm/sec.
(With screen).

TABLE III.

y cm	x cm													
	2,5	5	10	15	20	25	30	40	50	60	70	80	100	150
0,005	162	68	19	40	34	42	39	33	33	29	28	29	20	—
0,010	243	128	51	68	78	81	64	58	61	55	52	58	40	25
0,012	286	156	65	84	98	82	85	79	79	70	66	74	57	39
0,015	320	194	97	108	126	127	114	106	108	96	92	100	76	58
0,017	346	218	121	132	142	131	132	126	124	114	110	116	96	73
0,020	395	256	155	155	173	178	166	155	151	140	140	140	119	90
0,025	455	320	205	198	220	227	209	196	191	172	174	170	156	130
0,030	515	367	255	238	255	273	247	239	230	214	212	207	188	169
0,040	597	470	331	312	331	342	320	306	293	272	272	270	251	222
0,050	712	561	406	369	390	406	375	355	349	326	323	317	310	280
0,060	—	620	475	430	450	456	427	415	385	359	355	349	—	—
0,075	747	—	555	483	514	518	441	464	436	416	400	396	390	361
0,100	788	764	668	581	578	581	565	530	489	480	460	458	445	435
0,125	812	780	718	641	—	625	—	—	—	—	—	—	—	—
0,150	840	795	742	690	678	660	642	594	559	540	518	518	510	490
0,175	840	809	761	726	—	689	—	—	—	—	—	—	—	—
0,200	848	823	780	745	712	705	680	620	606	565	555	535	540	517
0,250	848	—	800	780	740	741	700	650	630	588	571	555	555	535
0,300	848	842	800	790	777	769	735	680	650	606	588	569	568	550
0,350	848	—	800	800	790	787	752	691	670	621	606	580	580	555
0,400	848	839	800	800	800	795	768	709	680	640	621	600	586	569
0,450	842	—	800	—	800	800	785	730	699	652	632	610	591	571
0,500	839	820	800	800	800	800	790	740	711	670	645	621	601	580
0,550	832	—	—	—	—	800	800	752	730	680	652	632	611	595
0,600	832	809	—	—	—	800	800	768	740	699	670	645	621	605
0,650	—	—	—	—	—	—	—	780	752	710	680	652	630	610
0,700	809	802	—	—	—	—	—	785	768	721	691	662	640	617
0,750	—	—	—	—	—	—	—	790	777	730	701	679	650	625
0,800	800	—	—	—	—	—	—	800	781	740	715	691	659	636
0,850	—	—	—	—	—	—	—	800	785	752	724	701	668	—
0,900	—	—	—	—	—	—	—	800	790	760	735	715	680	650
0,950	—	—	—	—	—	—	—	—	—	777	745	730	691	—
1,000	800	800	—	800	800	800	800	800	800	785	758	740	701	666
1,100	—	—	—	—	—	—	—	—	—	790	777	751	715	679
1,200	—	—	—	—	—	—	—	—	—	800	785	777	730	688
1,300	—	—	—	—	—	—	—	—	—	800	790	785	748	699
1,400	—	—	—	—	—	—	—	—	—	800	800	790	768	710
1,500	—	—	—	—	—	—	—	—	—	800	800	800	780	715
1,600	—	—	—	—	—	—	—	—	—	—	—	800	785	727
1,700	—	—	—	—	—	—	—	—	—	—	—	—	790	735
1,800	—	—	—	—	—	—	—	—	—	—	—	—	800	740
2,000	—	—	—	—	—	—	—	—	—	—	800	800	800	760
2,100	—	—	—	—	—	—	—	—	—	—	—	—	—	773
2,200	—	—	—	—	—	—	—	—	—	—	—	—	—	779
2,300	—	—	—	—	—	—	—	—	—	—	—	—	—	780
2,400	—	—	—	—	—	—	—	—	—	—	—	—	—	785
2,500	—	—	—	—	—	—	—	—	—	—	—	—	—	790
2,600	—	—	—	—	—	—	—	—	—	—	—	—	—	800
2,800	—	—	—	—	—	—	—	—	—	—	—	—	—	800
f cm	+0,007	0	—0,005	—0,002	—0,002	—0,003	—0,004	—0,004	—0,003	—0,004	—0,003	—0,003	—0,005	—0,006
X cm	225	225	225	225	225	200	200	200	200	200	200	175	175	200
ν cm²/sec.	0,145	0,144	0,148	0,144	0,148	0,146	0,146	0,146	0,147	0,146	0,146	0,146	0,144	0,147
	10 Dec.	10 Dec.	10 Dec.	10 Dec. 11 Dec.	8 Dec.	12 Oct. 7 Dec.	7 Dec.	7 Dec.	6, 12, 13 Dec.	11 Dec.	11 Dec.	11 Dec.	12 Dec.	15, 17 Dec. '23.

TABLE IV.

$V = 1200$ cm/sec.

y cm	x cm 2,5	5	7,5	10	15	20	25	37,5	50	62,5	75	87,5	100	125	150
0,003	61	55	21	12	19	10	11	10	33	46	67	81	57	6	12
0,005	79	69	36	9	32	25	23	24	52	55	81	130	44	21	29
0,007	127	93	55	28	52	36	35	40	71	64	100	168	69	54	58
0,010	202	152	99	67	87	69	61	66	98	109	144	215	114	78	85
0,012	263	195	138	95	110	89	85	87	125	144	174	253	137	103	106
0,015	330	246	182	144	144	126	118	112	154	191	225	292	188	142	152
0,017	387	279	216	174	164	144	139	135	172	216	265	327	212	180	190
0,020	470	339	260	220	190	178	174	162	225	266	314	365	257	224	239
0,025	578	414	335	287	242	225	222	208	283	320	372	430	330	295	309
0,030	688	509	417	358	295	283	272	259	332	372	441	490	398	359	372
0,040	860	668	547	485	395	385	361	339	430	485	542	576	503	465	484
0,050	975	804	692	595	512	470	459	434	509	588	639	650	580	560	561
0,060	1040	928	809	738	611	576	555	519	590	658	678	680	635	611	636
0,070	1082	1015	900	830	720	669	631	—	—	—	—	—	—	—	—
0,080	1110	1078	1000	910	783	730	718	647	690	740	770	760	722	692	705
0,090	1158	1118	1060	960	870	810	735	—	—	—	—	—	—	—	—
0,100	1170	1160	1100	1040	939	870	796	770	798	815	810	800	770	740	750
0,110	1178	1178	1138	1098	988	950	882	—	—	—	—	—	—	—	—
0,125	1192	1192	1180	1122	1058	1010	980	890	880	882	872	838	825	785	790
0,150	1197	1197	1197	1190	1122	1122	1072	958	950	921	890	881	845	815	825
0,160	1197	1197	1197	1197	1142	1142	1102	—	—	—	—	—	—	—	—
0,175	1197	1197	1197	1197	1162	1152	1125	1020	1010	950	912	900	865	839	860
0,200	1200	1200	1200	1200	1200	1195	1168	1070	1042	970	945	940	895	850	888
0,225	—	—	—	—	—	1200	1195	1102	—	—	—	—	—	—	—
0,250	—	—	—	—	1200	—	1200	1142	1080	1025	980	961	915	880	900
0,300	—	—	—	—	—	—	—	1175	1110	1055	1015	1000	935	888	920
0,350	—	—	—	—	—	—	—	1195	1130	1080	1040	1020	955	910	939
0,400	—	—	—	—	—	—	—	1200	1142	1100	1060	1038	975	930	950
0,450	—	—	—	—	—	—	—	1200	1160	1115	1070	1045	995	950	961
0,500	—	—	—	—	—	—	—	1200	1170	1130	1082	1070	1010	961	980
0,550	—	—	—	—	—	—	—	1200	1180	1142	1100	1080	1020	980	992
0,600	—	—	—	—	—	—	—	1200	1185	1160	1110	1090	1035	1000	1005
0,650	—	—	—	—	—	—	—	—	1195	1170	1125	1105	1045	1010	1020
0,700	—	—	—	—	—	—	—	—	1200	1180	1140	1122	1060	1020	1035
0,750	—	—	—	—	—	—	—	—	1200	1185	1152	1130	1070	1020	1045
0,800	—	—	—	—	—	—	—	—	1200	1195	1162	1140	1085	1035	1065
0,850	—	—	—	—	—	—	—	—	1200	1200	1180	1145	1090	1045	—
0,900	—	—	—	—	—	—	—	—	1200	1200	1190	1150	1100	1070	1080
0,950	—	—	—	—	—	—	—	—	1200	1200	1200	1158	1110	1080	—
1,000	—	—	—	—	—	—	—	1200	1200	1200	1200	1170	1130	1080	1090
1,100	—	—	—	—	—	—	—	—	—	—	1200	1185	1150	1090	1125
1,200	—	—	—	—	—	—	—	—	—	—	1200	1195	1165	1120	1140
1,300	—	—	—	—	—	—	—	—	—	—	1200	1200	1170	1140	1145
1,400	—	—	—	—	—	—	—	—	—	—	1200	1200	1175	1150	1160
1,500	—	—	—	—	—	—	—	—	—	—	1200	1200	1185	1162	1180
1,600	—	—	—	—	—	—	—	—	—	—	—	1200	1200	1185	1195
1,800	—	—	—	—	—	—	—	—	—	—	—	1200	1200	1200	1200
2,000	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200
f cm	— 0,002	— 0,002	— 0,003	— 0,005	— 0,002	— 0,003	— 0,004	— 0,003	— 0,001	— 0,002	— 0,001	+ 0,003	— 0,002	— 0,004	— 0,003
X cm	225	225	225	225	225	225	225	225	175	175	175	175	225	225	225
ν cm²/sec.	0,148	0,150	0,152	0,149	0,148	0,144	0,148	0,151	0,147	0,146	0,151	0,149	0,153	0,154	0,142
	6 Dec.	5 Dec.	5 Dec.	4 Dec.	1 Dec.	30 Nov.	29 Nov.	28 Nov.	26 Nov.	23 Nov.	21 Nov.	16 Nov.	14 Nov.	9 Nov.	8 Nov. '23.

$V = 1600$ cm/sec.

TABLE V.

y cm	x cm											
	10	15	20	25	37,5	50	62,5	75	87,5	100	125	150
0,003	23	25	25	20	32	100	79	65	121	94	30	29
0,005	34	55	49	44	54	126	124	97	192	78	66	63
0,007	64	87	83	76	89	166	173	130	252	116	95	103
0,010	114	144	132	126	140	247	266	196	324	196	142	129
0,012	176	180	166	169	166	280	324	266	369	245	178	178
0,015	250	231	219	214	205	340	394	354	445	324	250	256
0,017	283	265	250	246	250	400	460	398	488	375	305	306
0,020	346	322	300	292	310	468	520	480	543	435	380	389
0,025	466	390	385	361	364	561	610	572	625	494	445	520
0,030	558	485	449	450	430	650	710	660	701	612	571	578
0,040	750	625	606	582	592	770	810	798	810	753	702	709
0,050	911	790	736	730	730	850	895	870	881	838	785	800
0,060	1050	931	872	850	811	950	960	930	928	918	858	870
0,070	1200	1050	992	958	—	—	—	—	—	—	—	—
0,080	1300	1141	1078	1058	1015	1072	1072	1020	1020	981	950	942
0,090	1398	1250	1160	1140	—	—	—	—	—	—	—	—
0,100	1440	1320	1245	1218	1142	1142	1130	1083	1075	1050	1005	1025
0,110	1449	1400	1325	1270	—	—	—	—	—	—	—	—
0,125	1550	1460	1435	1365	1280	1225	1192	1135	1122	1110	1055	1032
0,140	1595	1505	1470	1445	—	—	—	—	—	—	—	—
0,150	1597	1541	1515	1485	1380	1282	1245	1170	1158	1140	1100	1090
0,160	1597	1592	1530	1500	—	—	—	—	—	—	—	—
0,175	1597	1597	1570	1525	1450	1320	1262	1200	1205	1160	1130	1138
0,190	—	1600	1595	—	—	—	—	—	—	—	—	—
0,200	1600	1600	1600	1570	1505	1375	1290	1232	1220	1220	1145	1160
0,225	—	—	—	1595	1535	—	—	—	—	—	—	—
0,250	—	1600	1600	1600	1570	1438	1322	1300	1255	1250	1179	1179
0,300	—	—	—	—	1590	1455	1375	1340	1290	1290	1205	1210
0,350	—	—	—	—	1595	1498	1400	1380	1315	1310	1220	1225
0,400	—	—	—	—	1600	1505	1450	1400	1350	1325	1250	1270
0,450	—	—	—	—	1600	1525	1480	1440	1380	1350	1270	1290
0,500	—	—	—	—	1600	1545	1505	1460	1410	1365	1295	1300
0,550	—	—	—	—	1600	1560	1525	1480	1425	1390	1310	1325
0,600	—	—	—	—	1600	1570	1545	1490	1445	1420	1335	1350
0,650	—	—	—	—	—	1580	1570	1500	1460	1430	1355	1365
0,700	—	—	—	—	—	1590	1590	1520	1475	1445	1380	1380
0,750	—	—	—	—	—	1595	1595	1525	1495	1460	1420	1410
0,800	—	—	—	—	—	1600	1600	1540	1505	1480	1430	1420
0,850	—	—	—	—	—	1600	1600	1555	1515	1492	1445	1440
0,900	—	—	—	—	—	1600	1600	1565	1525	1507	1452	1445
0,950	—	—	—	—	—	1600	1600	1575	1535	1525	1460	1460
1,000	—	—	—	—	—	1600	1600	1590	1545	1545	1485	1475
1,100	—	—	—	—	—	—	—	1600	1560	1560	1500	1505
1,200	—	—	—	—	—	—	—	1600	1580	1580	1515	1515
1,300	—	—	—	—	—	—	—	1600	1595	1590	1535	1525
1,400	—	—	—	—	—	—	—	1600	1600	1600	1550	1545
1,500	—	—	—	—	—	—	—	1600	1600	1600	1580	1570
1,600	—	—	—	—	—	—	—	—	1600	1600	1600	1595
1,800	—	—	—	—	—	—	—	—	1600	1600	1600	1600
2,000	—	—	—	—	—	—	—	—	1600	1600	1600	1600
f cm	— 0,005	— 0,002	— 0,002	— 0,002	— 0,002	0	0	— 0,002	+ 0,003	— 0,001	— 0,003	— 0,002
X cm	225	225	225	225	225	175	175	175	175	225	225	225
ν cm²/sec.	0,152	0,150	0,150	0,151	0,152	0,147	0,147	0,148	0,152	0,148	0,146	0,144
	4 Dec.	1 Dec.	30 Nov.	29 Nov.	28 Nov.	24 Nov.	23 Nov.	22 Nov.	16 Nov.	15 Nov.	12 Nov.	8 Nov. '23.

$V = 2400$ cm/sec.

TABLE VI.

y cm	x cm											
	10	15	20	25	37,5	50	62,5	75	87,5	100	125	150
0,003	53	96	96	81	172	158	196	137	400	169	139	64
0,005	100	166	158	134	260	256	321	225	539	184	174	123
0,007	168	239	225	198	363	385	436	334	650	282	234	200
0,010	270	350	328	317	490	586	608	490	760	466	364	300
0,012	361	414	393	380	592	690	720	612	821	558	413	408
0,015	493	501	495	480	698	825	841	729	909	682	580	552
0,017	560	571	542	545	786	909	912	808	970	730	670	645
0,020	680	668	651	611	869	1000	988	912	1058	830	762	758
0,025	860	802	780	785	970	1142	1102	1040	1165	960	910	912
0,030	1018	950	929	920	1138	1280	1235	1162	1278	1110	1040	1040
0,040	1310	1242	1170	1150	1305	1420	1390	1320	1362	1245	1220	1219
0,050	1640	1480	1385	1360	1470	1525	1480	1420	1450	1380	1310	1338
0,060	1840	1700	1625	1562	1600	1598	1570	1502	1502	1442	1398	1410
0,070	2015	1880	1795	1700	—	—	—	—	—	—	—	—
0,080	2135	2015	1922	1820	1780	1725	1675	1585	1600	1535	1470	1538
0,090	2250	2150	2050	1962	—	—	—	—	—	—	—	—
0,100	2295	2240	2145	2085	1960	1770	1740	1670	1675	1630	1570	1580
0,110	2345	2280	2195	2150	—	—	—	—	—	—	—	—
0,125	2375	2335	2290	2250	2100	1870	1810	1750	1745	1695	1670	1640
0,140	2390	2380	2345	2330	—	—	—	—	—	—	—	—
0,150	2395	2395	2365	2350	2165	1920	1875	1800	1795	1780	1715	1680
0,160	2395	2395	2385	2375	—	—	—	—	—	—	—	—
0,175	2395	2395	2395	2385	2245	1980	1920	1838	1840	1795	1755	1745
0,190	2400	2400	2400	—	—	—	—	—	—	—	—	—
0,200	2400	2400	2400	2400	2310	2025	2000	1885	1880	1840	1795	1800
0,250	—	2400	2400	2400	2345	2110	2055	1920	1905	1880	1840	1840
0,300	—	—	—	—	2375	2165	2110	2000	1965	1920	1885	1880
0,350	—	—	—	—	2390	2215	2150	2045	2005	1960	1925	1900
0,400	—	—	—	—	2400	2250	2200	2100	2055	1990	1965	1940
0,450	—	—	—	—	2400	2285	2242	2150	2100	2040	1985	1960
0,500	—	—	—	—	2400	2310	2260	2200	2140	2055	2010	1990
0,550	—	—	—	—	—	2345	2290	2250	2155	2090	2050	2005
0,600	—	—	—	—	—	2350	2325	2280	2170	2100	2060	2040
0,650	—	—	—	—	—	2365	2345	2300	2200	2150	2070	2055
0,700	—	—	—	—	—	2390	2365	2320	2240	2165	2085	2070
0,750	—	—	—	—	—	2400	2390	2340	2260	2190	2110	2100
0,800	—	—	—	—	—	2400	2400	2355	2290	2210	2130	2110
0,850	—	—	—	—	—	2400	2400	2370	2310	2260	2140	2140
0,900	—	—	—	—	—	2400	2400	2380	2340	2280	2170	2155
0,950	—	—	—	—	—	2400	—	2390	2355	2300	2180	2170
1,000	—	—	—	—	—	2400	2400	2400	2370	2320	2220	2190
1,100	—	—	—	—	—	—	—	2400	2390	2350	2240	2210
1,200	—	—	—	—	—	—	—	2400	2395	2390	2280	2250
1,300	—	—	—	—	—	—	—	2400	2400	2395	2310	2300
1,400	—	—	—	—	—	—	—	2400	2400	2400	2330	2355
1,500	—	—	—	—	—	—	—	2400	2400	2400	2400	2390
1,600	—	—	—	—	—	—	—	2400	2400	2400	2400	2400
1,800	—	—	—	—	—	—	—	2400	2400	2400	2400	2400
2,000	—	—	—	—	—	—	—	2400	2400	2400	2400	2400
f cm	— 0,003	0	0	— 0,001	+ 0,002	0	0	— 0,001	+ 0,005	0	— 0,002	— 0,002
X cm	225	225	225	225	225	175	175	175	175	225	225	225
ν cm²/sec.	0,152	0,150	0,149	0,151	0,152	0,150	0,151	0,150	0,151	0,149	0,148	0,143
	4 Dec.	1 Dec.	30 Nov.	29 Nov.	28 Nov.	24 Nov.	23 Nov.	22 Nov.	16 Nov.	14 Nov.	13 Nov.	8 Nov. '23.

TABLE VII.

$x = 150$ cm

X cm	225	150	200	225	225	225	200	150	225	150	150
V cm/sec.	400	600	800_1	800_2	1200	1600_1	1600_2	2000	2400	2800	3200
y cm											
0,005	9	7	48	—	29	63	110	375	123	561	769
0,010	11	23	80	25	85	129	250	400	300	841	1045
0,015	11	40	124	58	152	256	355	559	552	1045	1280
0,020	16	69	164	90	239	389	464	688	758	1180	1420
0,025	22	93	196	130	309	520	549	785	912	1300	1540
0,030	32	121	232	169	372	578	611	870	1040	1400	1611
0,040	49	166	297	222	484	709	712	980	1219	1530	1760
0,050	66	201	342	280	561	800	820	1075	1338	1611	1908
0,060	87	244	385	—	636	870	—	—	1410	—	—
0,075	—	289	—	361	—	—	930	1162	—	1761	2075
0,100	148	329	490	435	750	1025	1002	1282	1580	1879	2180
0,125	169	370	525	—	790	1032	—	1340	1640	1952	2250
0,150	200	393	550	490	825	1090	1060	1362	1680	2005	2315
0,175	216	405	568	—	860	1138	—	1398	1745	2050	2365
0,200	237	416	577	517	888	1160	1098	1410	1800	2085	2400
0,225	—	425	585	—	—	—	—	1432	—	2105	2440
0,250	260	430	595	535	900	1179	1125	1460	1840	2148	2490
0,275	—	437	600	—	—	—	—	1470	—	2151	2530
0,300	269	443	606	550	920	1210	1142	1485	1880	2155	2550
0,350	283	453	613	555	939	1225	1162	1520	1900	2165	2598
0,400	296	465	622	569	950	1270	1184	1542	1940	2200	2655
0,450	310	471	630	571	961	1290	1210	1560	1960	2245	2698
0,500	325	476	641	580	980	1300	1220	1580	1990	2275	2748
0,550	336	480	647	595	992	1325	1238	1600	2005	2300	2770
0,600	340	490	657	605	1005	1350	1258	1662	2040	2315	2798
0,650	344	500	670	610	1020	1365	1265	1642	2055	2348	2825
0,700	354	510	677	617	1035	1380	1288	1665	2070	2399	2851
0,750	361	520	680	625	1045	1410	1308	1682	2100	2440	2870
0,800	365	529	692	636	1065	1420	1322	1700	2110	2480	2900
0,850	370	534	700	—	—	1440	1340	1740	2140	2500	2920
0,900	380	541	708	650	1080	1445	1352	1762	2155	2545	2950
0,950	384	550	713	—	—	1460	1372	1790	2170	2570	2960
1,000	389	559	717	666	1090	1475	1382	1820	2190	2598	3000
1,100	392	564	726	679	1125	1505	1405	1840	2210	2655	3048
1,200	395	573	732	688	1140	1515	1440	1880	2250	2698	3055
1,300	395	581	741	699	1145	1525	1460	1900	2300	2725	3110
1,400	395	591	748	710	1160	1545	1480	1915	2355	2750	3152
1,500	398	600	760	715	1180	1570	1498	1920	2390	2784	3185
1,600	400	600	768	727	1195	1595	1520	1945	2400	2798	3200
1,700	—	—	771	735	—	—	1540	1960	—	2800	3200
1,800	400	600	778	740	1200	1600	1558	1995	2400	2800	3200
1,900	—	—	784	—	—	—	1565	—	—	—	—
2,000	—	600	789	760	1200	1600	1572	2000	2400	2800	3200
2,200	—	—	800	779	—	—	1585	—	—	—	—
2,500	—	—	800	790	—	—	1600	2000	—	2800	3200
2,600	—	—	—	800	—	—	—	—	—	—	—
f cm	— 0,011	— 0,008	+ 0,001	— 0,006	— 0,003	— 0,002	— 0,002	+ 0,002	— 0,002	+ 0,005	?
ν cm²/sec.	0,144	0,147	0,149	0,147	0,142	0,144	0,148	0,148	0,143	0,150	0,150
	9 Nov.	20 Dec.	10 July 10 Sept.	15 Dec. 17 Dec.	8 Nov.	8 Nov.	20 Dec.	19 Dec.	8 Nov.	19 Dec.	19 Dec. '23.

The resistance, with friction at one side is:

$$W_a = b\mu \int_0^l a\,dx = b\rho\, w_a \; . \; . \; . \; . \; . \; . \; . \; (14)$$

The resistance coefficient that follows from formula (14) is:

$$c_{fa} = \frac{W_a}{{}^1\!/_2\, \rho\, V^2 \times surface} = \frac{\nu \int_0^l a\,dx}{{}^1\!/_2\, l\, V^2} \; . \; . \; . \; . \; (15)$$

In the determination of $\int_0^l a\,dx$ the difficulty was encountered that in general no value of a was determined at smaller values of x than 5 to 15 cm.

Supposing the proportionality of a with $x^{-1/2}$ (according to BLASIUS) in the region $x = 0$ to $x = 5$ or 15 cm, the resistance in this part could be found from:

$$\int_0^x a\,dx = 2\,x\,a_{e(x)} \; . \; . \; . \; . \; . \; . \; . \; . \; (16)$$

Here $a_{e(x)}$ is the experimental value of a taken at that value of x that is taken as the upper limit of the integral. Now taking $a_{x(B)}$ to be the value of a according to the formula of BLASIUS at this value of x, the following numbers are found:

V	400	800_1	800_2	1200	1600	2400 cm/sec
x	15	5	5	5	10	10 cm
$a_{x(e)}$	2190	10400	12600	18800	24500	38200 cm/sec/cm
$a_{x(B)}$	1780	8660	8660	16000	17350	31900 cm/sec/cm

If the average of $a_{e(x)}$ and $a_{B(x)}$ is taken as basis for the determination of the resistance between $x = 0$ and $x = 5$ to 15 cm (the formula of BLASIUS gives apparently too low a value for the integral, whereas that calculated from $a_{e(x)}$ gives too high a value), the results given in table XIII are obtained, wherein the possible allowance made by making use of this average is given at the foot of the columns. From the values of $\nu \int_{x=0}^{x=150} a\,dx$ it appears that the uncertainty is maximally 2,5 %.

The value of the resistance for both sides of the glass plate together and for the resistance coefficient are (with $l = 150$ cm).

V	400	800_1	800_2	1200	1600	2400 cm/sec
R	405000	800000	822000	1205000	1610000	2400000
W	4,1	16,4	19,9	31,6	51,3	108,9 gr
c_{fa}	0,0033	0,0034	0,0041	0,0029	0,0027	0,0025

TABL

x cm	V cm/sec.					
	400		800_1		800_2	
	$\nu \int_0^\infty a dx$	$w_{i(x)}$	$\nu \int_0^\infty a dx$	$w_{i(x)}$	$\nu \int_0^\infty a dx$	$w_{i(x)}$
5	—	—	14,3	13,0	15,5	—
7,5	—	—	17,8	14,5	—	—
10	—	—	20,6	15,2	28,3	16,6
15	8,8	6,6	25,6	20,5	34,4	22,7
20	10,4	7,4	29,8	21,6	41,6	26,8
25	11,8	7,9	34,0	24,8	48,1	27,8
30	—	—	37,8	28,6	55,6	35,2
37,5	15,2	11,1	—	—	—	—
40	—	—	45,0	33,1	70,1	53,4
50	18,3	12,2	52,4	35,6	83,9	65,3
60	—	—	59,6	47,5	96,6	86,4
62,5	21,3	15,6	—	—	—	—
70	—	—	—	—	109	101
75	24,1	18,1	71,9	55,9	—	—
87,5	27,0	18,9	—	—	—	—
100	29,9	19,1	101	91,2	143	139
125	35,3	20,8	133	115	—	—
150	41,1	23,5	164	135	197	190
	± 0,9		± 1,3		± 2,9	
	.1000	.1000	.1000	.1000	.1000	.1000

III.

1200		1600		2400	
$\nu \int_0^x a dx$	$w_{i(x)}$	$\nu \int_0^x a dx$	$w_{i(x)}$	$\nu \int_0^x a dx$	$w_{i(x)}$ cm^3/sec^2
26,0	22,8	—	—	—	—
32,0	25,5	—	—	—	—
38,1	29,5	62,7	47,5	105	91,4
47,5	36,8	78,6	58,8	132	105
55,6	38,9	91,8	66,0	157	119
63,1	44,2	105	71,1	174	125
—	—	—	—	—	—
81,4	57,0	135	88,8	250	183
—	—	—	—	—	—
101	81,5	171	158	340	366
—	—	—	—	—	—
126	108	216	198	449	423
—	—	—	—	—	—
154	138	265	253	560	531
184	163	312	336	665	636
214	221	356	338	760	745
267	273	440	433	934	943
318	250	516	446	1093	999
± 2,1		± 10,5		± 9,3	
.1000	.1000	.1000	.1000	.1000	.1000

The results could not be verified by direct measurements of the resistance, but the values of c_{fa} may be compared to the values given by various experimenters in the case of the flow over straight, smooth, surfaces. These values are represented in figure 9.

It is shown in this figure that the values of c_{fa} described here are rather low at high values of the velocity; at the values of the velocity of 400 and 800 cm/sec the values determined by us range between those given by the other experimenters.

For the sake of completeness the results obtained with $V = 800_1$ cm/sec with various degrees of turbulence in the outer current are mentioned in the following table („Mededeeling 5" p. 33):

$X = 250$ cm	$c_{fa} = 0{,}0030$
$X \cong 150$ cm	$c_{fa} = 0{,}00343$
$X = 25$ cm	$c_{fa} = 0{,}00384$
with screen	$c_{fa} = 0{,}00413$

b. Application of the theorem of momenta.

The resistance experienced by the glass plate may also be determined from the loss of momentum in the boundary layer at the end of the glass plate. The resistance deduced from this loss of momentum will be:

$$W_i = b\,\rho\,[\int_0^\delta u\,(V - u)\,dy\,]_{x=l} = b\,\rho\,w_i \quad . \quad . \quad . \quad . \quad (17)$$

If the turbulent state of motion in the boundary layer has been developed sufficiently, then, according to VON KÁRMÁN:

$$w_i = \frac{7}{72}\,V^2\,\delta_l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Taking into account the linear progress of the $u\,(y)$ curve in the laminar part of the boundary layer a somewhat greater value will be found; according to a calculation given in § 6 the loss of momentum will be then:

$$w_i = \frac{7}{72}\,V^2\,\delta_l + \frac{5}{72}\,\frac{V^3}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In the wind channel, however, V depends on x; the increase of the thickness of the boundary layers along the walls of the channel and along the glass plate as x increases will give rise to an increasing value of V.

The loss of momentum, determined from the velocity distribution at the end of the plate is influenced by this in two ways:

1^0. As V has a smaller value at the more advanced points, a will have a smaller value than is given in table VIII.

2^0. The increasing value of V along the axis of the tunnel causes a pressure drop that has the consequence that the pressure forces in the boundary layer have a resultant in the direction of motion. This pressure gradient reduces the resistance that is experienced by the flow in the boundary layer.

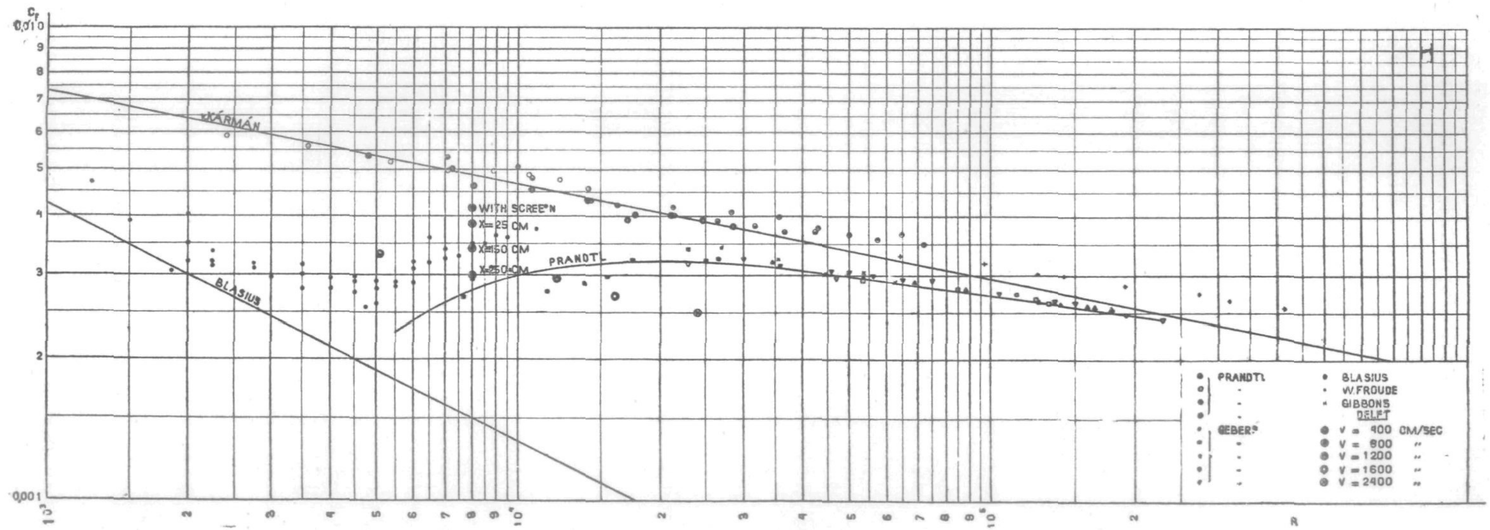

Fig. 9. Logarithmic diagram of the resistance coefficient as determined by various experimenters as a function of R.

Both influences together will have the consequence that w_i will be smaller then w_a. In table XIII the value of w_i is summarized next to the value of w_a : w_a has been determined from $\nu \int_0^x a\,dx$ in the supposition that V is not affected by the change in the value of x and w_i is calculated by integrating numerically the values of $u\,(V-u)$ between $y=0$

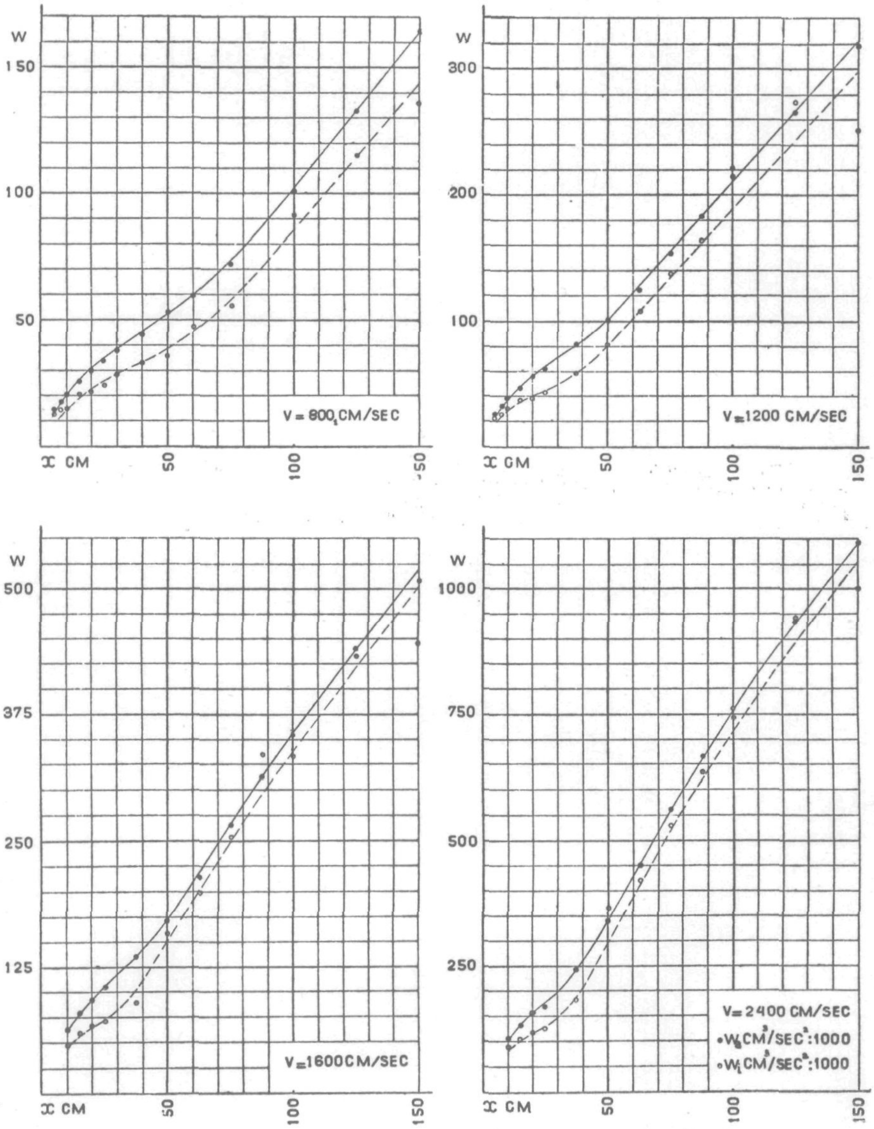

Fig. 10. Graphical representation of the values w_i and w_a as a function of x.

and $y=\delta$. The values of u used for this are taken from the tables I to VI.

Inaccuracies occurring in the values of u will cause irregularities in the behaviour of $w_i = \int_0^\delta u(V-u)\,dy$ when considering this value as a function of x. The values of w_a and w_i have been represented graphically in Fig. 10, for the series with $V = 800_1$, 1200, 1600 and 2400 cm/sec. The points for $V = 1200$ cm/sec show rather large discrepancies at $x = 100$, 125 and 150 cm; all four series show that the value of w_i at $x = 150$ cm is too low.

In every case, however, the mean curve drawn through the w_i points goes nearly parallel to the w_a curve; the former curve lying always below the latter one, in accordance with what has been said above.

In the following table the values of w_i at $x = 150$ cm are given as originally calculated by numerical integration from the $u(y)$ curve, as deduced from formula (25), and as derived from the smooth curves in figure 10:

V cm/sec	w_i	$\frac{7}{72} V^2 \delta + \frac{5}{72} \frac{V^3}{a}$	From Fig. 10
800_1	135.10^3	138.10^3	144.10^3
1200	250	298	296
1600	446	490	503
2400	999	1069	1055

As the third column comes nearer to the fourth than to the second, the smoothing of the $u(y)$ curve by assuming the $^1/_7$ power law seems to give the same results as the smoothing of the $w_i(x)$ curve in figure 10.

The differences between the smoothed values of w_i and the values of w_a at $x = 150$ cm amount to about 4 to 14 %.

In order to account for these differences according to the views exposed before, it is advantageous to bring the complete equation of momentum [22]) in the following form:

$$\left[\int_0^\delta u\,(V - u)\,dy\right]_{x=l} = \nu \int_0^l a_1\,dx + \alpha\,l \quad . \quad . \quad . \quad (19)$$

Here α is the mean of

$$\frac{dV}{dx} \int_0^\delta (V - u)\,dy\,.$$

This equation clearly shows the relation between the correction that has to be applied to the calculated values of the loss of momentum given in table XIII, and the velocity gradient along the axis of the tunnel. On the right hand side of formula (19) that value of a has to be

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22) Th. von Kármán, Zs. f. angew. Math. u. Mech. I 1921, p. 235.

introduced that belongs to the smaller value of the velocity $V_{(x)}$ existing at the point x, so

$$a_1 = a_{table} \cdot \frac{V_{(x)}}{V_{(l)}} .$$

The differences between w_a and w_i may be estimated as follows: In the laminar part of the boundary layer is:

$$\int_0^\delta (V - u)\, dy \cong {}^1/_3\, V \delta_x$$

in the turbulent part:

$$\int_0^\delta (V - u)\, dy = {}^1/_8\, V \delta_x .$$

Taking as a mean value ${}^1/_{10}\, V \delta_{(x = l)}$ and putting

$$l \frac{dV}{dx} = V_{x = 150} - V_{x = 0} = \beta V,$$

then the value of αl becomes:

$$\alpha l = {}^1/_{10}\, \beta V^2 \delta_l \cong \beta w_i .$$

On the other hand the correction to be applied to $\nu \int_0^\delta a\, dx$ is to be estimated at ${}^1/_2\, \beta w_a$.

From this follows that:

$$(1 - {}^1/_2\, \beta)\, w_a = (1 + \beta)\, w_i$$

or

$$\frac{w_a}{w_i} \cong 1 + 1{,}5\, \beta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

To explain the differences between the corrected values of w_i and w_a a value of β from 3 to 10 % had to be accepted.

In order to prove that this is really the case for the experimental arrangement used, a separate experiment was carried out in the following way:

The hot wire anemometer was withdrawn from the tunnel and a removable Pitot tube was installed instead; for the rest the experimental arrangement was not altered: the position of the bars, fixing the plate, was here almost the same as in the experiments with the hot wire anemometer at $x = 150$ cm.

A fixed Pitot tube was mounted in the front part of the tunnel (also existing in the experiments with the hot wire anemometer); call this one A ($X_A = 70$ cm, $y_A = -14$ cm, $z_A = +1$ cm), the removable Pitot tube B ($y_B = 12$ cm, $z_B = 0$). By the aid of two equal manometers the differences in the static and dynamic pressures of these Pitot tubes were determined simultaneously at various values of x (V being 2400 and 1200 cm/sec). Calling the static pressure of $A : p_A'$, the dynamic pressure p_A; the static pressure of $B : p_B'$ and the dynamic pressure p_B, then the differences of the values of ${}^1/_2\, \rho V^2$ at both places X_A and x_B can be found from the differences of the readings:

$$({}^1/_2\, \rho V^2)_B - ({}^1/_2\, \rho V^2)_A = (p_A - p_A') - (p_B - p_B') \quad . \; . \quad (21)$$

During the observings the differences of the dynamic pressure fluctuated considerably, the differences of the static pressure on the contrary were practically constant. On account of this the differences were determined during four minutes; the determination of the differences in the static and dynamic pressures at one value of x took place by reading every 10 sec. the manometers simultaneously; from these readings the mean value was taken and the velocity differences between X_A and x_B calculated. It should be noted that the number of revolutions of the air propeller was not regulated: the tunnel was left to itself.

Then the removable Pitot tube was adjusted at an other value of x and again a series of readings was taken.

As an example one series of readings follows here to show how the dynamic pressure varies:

Plate at a distance $X = 225$ cm from the honeycomb; velocity according to Pitot tube $A = 2400$ cm/sec; Pitot tube B at 372 cm from honeycomb:

time	Δp dyn.	Δp stat.	time	Δp dyn.	Δp stat.
0′ 0″	− 31	+ 113,5	2′ 10″	− 34	+ 111,5
10″	− 33	+ 113,5	20″	− 31	+ 113,5
20″	− 22	+ 112,5	30″	− 12	+ 109,5
30″	− 34	+ 111,5	40″	− 32	+ 111,5
40″	− 29	+ 113,5	50″	− 32	+ 112,5
50″	− 33	+ 113,5	3′ 0″	− 43	+ 113,5
1′ 0″	− 28	+ 113,5	10″	− 36	+ 111,5
10″	− 51	+ 112,5	20″	− 41	+ 112,5
20″	− 30	+ 111,5	30″	− 74	+ 111,5
30″	− 38	+ 111,5	40″	− 42	+ 111,5
40″	− 15	+ 113,5	50″	− 34	+ 112,5
50″	− 33	+ 113,5	4′ 0″	− 31	+ 110,5
2′ 0″	− 28	+ 111,5	mean value	− 34	+ 112 . 0,05 mm waterpressure.

The pressure difference $(p_B' - p_B) - (p_A' - p_A)$ is found to be $+ 146 \times 0{,}05$ mm water pressure. As $p_A' - p_A$ at $V = 2400$ cm/sec corresponds to $720 \times 0{,}05$ mm water pressure, the difference in the velocity between A and B is 9,5 %.

The experiments at $x = 100$ cm and $x = 147$ cm were performed with the bars that fixed the plate in transverse direction being present, these bars were placed at $x = 97$ cm; at the other experiments where the frame carrying the Pitot tube B had to be shifted in the upstream direction, the bars were withdrawn to coordinate these measurements with those that were carried out with the hot wire anemometer at $x = 150$ cm.

The presence of these bars proved to affect the experimental results in an appreciable way: when these bars (at $x_B = 147$ cm) were withdrawn the difference in the static pressure of A and B dropped with $18 \times 0,05$ mm water pressure, which is equivalent to a difference of 1,25 % in the measured velocity.

Ths experiments with $V = 2400$ and 1200 cm/sec gave the following result:

X glassplate = 225 cm V = 2400 cm/sec						
x cm	Δp dyn.	Δp stat.	$\Delta(\frac{1}{2}\rho V^2)$	ΔV in % of V_A	ΔV in % of $V = 2400$ cm/sec at $x = 150$ cm	
(X=70)	−39	−12	+27	1,9		
0	−34	+24	+58	4,0	−5,5	without bars
46	−35	+64	+99	6,6	−2,9	without bars
100	−33	+84	+117	7,1	−1,8	with bars
147	−34	+112	+146	9,7	0	with bars
	. 0,05 mm waterpressure					

X glassplate = 225 cm V = 1200 cm/sec						
x cm	Δp dyn.	Δp stat.	$\Delta(\frac{1}{2}\rho V^2)$	ΔV in % of V_A	ΔV in % of $V = 1200$ cm/sec at $x = 150$ cm	
(X=70)	−3,5	−2	+1,5	0,42		
0	−4,2	+6	+10,2	2,8	−5,8	without bars
47	−4,5	+17	+21,5	5,8	−3,1	without bars
100	−5	+21	+26	7,2	−1,6	with bars
147	−5	+27	+32	8,9	0	with bars
	. 0,05 mm waterpressure					

The mean value of β is now 5,5 % of $V_{x=150} = 2400$ cm/sec and 5,8 % of $V_{x=150} = 1200$ cm/sec.

By aid of the velocity gradient determined in this way, the correction which has to be applied to the values found from the theorem of momenta can be calculated accurately.

The loss of momentum is equal to the corrected value of $\nu \int_0^l a\,dx$ minus $\alpha\, l$.

At the series with $V = 2400$ cm/sec is found:

$\nu \int_0^l a\,dx$ (not corr.) = ... = 1093000 cm³/sec²

the correction is — 2,8 % . . . . = — 30600 cm³/sec²

the corrected value of $\nu \int_0^l a\,dx$ is now . . . . = 1062000 cm³/sec²

The value of αl is $0{,}055 \cdot 2400 \frac{1}{150} \sum_{x=0}^{x=150} \int_0^\delta (V - u)\, dy =$

$= 40800 \text{ cm}^3/\text{sec}^2$

so the value calculated for w_i is . . . . $= 1022000 \text{ cm}^3/\text{sec}^2$

By numerical integration of the $u(y)$ curve at $x = 150$ cm was found, however, $w_i = 999000 \text{ cm}^3/\text{sec}^2$; therefore the difference is 2 %. The agreement between the corrected values of w_i and w_a is to be considered fair. However, the graphically smoothed value of w_i is 4 % more than the calculated one.

With the series with $V = 1200$ cm/sec is found:

$\nu \int_0^l a\, dx$ (not corr.) = ... $= 318100 \text{ cm}^3/\text{sec}^2$

the correction is here — 2,9 % . . . . $= -9200 \text{ cm}^3/\text{sec}^2$

the corrected value of $\nu \int_0^l a\, dx$ is now . . . . . $= 308900 \text{ cm}^3/\text{sec}^2$

The value of αl is here: $0{,}058 \cdot 1200 \frac{1}{150} \sum_{x=0}^{x=150} \int_0^\delta (V - u)\, dy =$

$= 12100 \text{ cm}^3/\text{sec}^2$

so the value determined for w_i is . . . . . . $= 296800 \text{ cm}^3/\text{sec}^2$

By numerical integration of the $u(y)$ curves at $x = 150$ cm was found $w_i = 250000 \text{ cm}^3/\text{sec}^2$; this gives therefore an appreciable difference. The agreement between the values determined for w_i and w_a becomes much more favourable, however, when the graphically smoothed value of w_i is used instead of the calculated one.

These calculations lead to the supposition that the values of a derived from the $u(y)$ curves must be rather trustworthy and that errors of the magnitude mentioned in the considerations at p. 32 are improbable.

In these considerations the measurements with $V = 400$ cm/sec have been left aside. Here the differences between w_i and w_a are great, their ratio amounting to 1,75 at $x = 150$ cm.

The values of w_i nearly correspond to the values calculated from BLASIUS's resistance formula for laminar motion (form. (2) p. 3) as will be seen in the following table:

x cm	w_i cm^3/sec^2	$0{,}664\, V^{3/2} (l\nu)^{1/2}$ cm^3/sec^2
25	$7{,}9 \cdot 10^3$	$10{,}3 \cdot 10^3$
50	12,2	14,5
75	18,1	17,7
100	19,1	20,7
125	20,8	23,2
150	23,5	24,8

The difference at $x = 150$ cm is only 6 % which can be easily accounted for by the variations of V along the axis of the wind channel.

The conclusion seems to be allowed that the values of a are too high in this case. [23]

§ 6. APPROXIMATE FORMULA FOR THE DISTRIBUTION OF THE VELOCITY OVER THE TURBULENT BOUNDARY LAYER.

In the immediate vicinity of the wall the irregular velocity components superposed on the mean value of u will diminish to zero and so the turbulence will disappear; one can say therefore that a laminar layer must exist. Various authors have given an estimation of the thickness of this laminar layer which is situated between the wall and the turbulent region of motion in the boundary layer.

It will be easily seen that the thickness of this layer (δ_1) will be smaller than $\mu \frac{V}{\tau_0}$.

Starting from the formula given by VON KÁRMÁN a simple relation can be deduced by assuming that the thickness of this laminar layer δ_1 is determined by the point where the curves:

$$u = \frac{\tau_0}{\mu} y$$

and

$$u = V \left(\frac{y}{\delta}\right)^{1/7}$$

cross each other.

---

[23]) In the paper "Over het Omslaan van den Laminairen Stroomingstoestand in de Grenslaag in den Turbulenten Toestand" by J. M. BURGERS (Verslagen der Koninklijke Akademie van Wetenschappen te Amsterdam, 1923 deel XXXII N°. 8 p. 856), a calculation has been given of the loss of momentum in the neighbourhood of the point of transition for the measurements executed in Sept. 19/20 1923 with $V = 800_1$ cm/sec and x ranging from 50 to 100 cm. The numbers mentioned there have been recalculated since, as it has been found that some irregularities occur in the w_i values which make the error at the point in question rather large; the whole series is mentioned here:

x	a	$(w_a)_x - (w_a)_{50}$	$(w_i)_x - (w_i)_{50}$
50	4520	0	0
60	4640	$6{,}9 \cdot 10^3$	$3{,}3 \cdot 10^3$
70	5200	14,3	7,0
75	6040	18,5	11,5
80	7800	23,7	29,7
85	8080	29,7	34,3
90	8800	36,0	33,3
100	8800	49,2	46,4

If a smooth curve is drawn through the values of w_i, a good correspondence with the values of w_a is found, the difference being accounted for by the change of V along the plate.

The intersection of the line $u = \frac{\tau_0}{\mu} y$ and $u = V\left(\frac{y}{\delta}\right)^{1/7}$ gives the value of

$$\delta_1 = \left(\frac{\mu V}{\tau_0}\right)^{7/6} \delta_k^{-1/6}$$

and the velocity component u_1 in this point will be:

$$u_1 = \left(\frac{\mu}{\tau_0}\right)^{1/6} V^{7/6} \delta_k^{-1/6}$$

With the formula for τ_0 according to VON KáRMáN:

$$\tau_0 = 0{,}0225\, \rho V^2 \left(\frac{\nu}{V \delta_k}\right)^{1/4}$$

is found

$$\frac{u_1 \delta_1}{\nu} = R^{**} = \left(\frac{1}{0{,}0225}\right)^{4/3} = 157{,}3 \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This relation between u_1 and δ_1 is represented by the line $A - A$ in figure 8 (with ν is 0,15 cm²/sec).

The lower limit of y for which VON KáRMáN's formula applies is obtained approximately when a parabola is used in the lower part of the boundary layer in stead of the straight line.

This parabola has to satisfy the following conditions:

1. the velocity gradient at the surface must be $\frac{\tau_0}{\mu}$, according to VON KáRMáN;

2. the velocity curve must be continuous over the whole boundary layer, therefore the velocity gradient in the point where the parabola passes into the curve $u = V\left(\frac{y}{\delta_k}\right)^{1/7}$ (the point u_1, δ_1) must be the same for the parabola and the curve $u = V\left(\frac{y}{\delta_k}\right)^{1/7}$.

3. u_1 deduced from the parabola must be the same as that which follows from $u = V\left(\frac{y}{\delta_k}\right)^{1/7}$.

Be the equation of the parabola

$$u = P\left(\frac{2y}{p} - \frac{y^2}{p^2}\right)$$

then the evaluation of P and p leads to

$$\frac{u_1 \delta_1}{\nu_1} = \left(\frac{1{,}86}{0{,}0225}\right)^{4/3} = 360.$$

This relation between u_1 and δ_1 is represented in figure 8 by the line $B - B$ (with $\nu = 0{,}15$ cm²/sec).

A formula for the distribution of the velocity which can be used over the whole boundary layer (i.e. both for values of y comparable to δ and for very small values) is obtained by putting:

$$y = \frac{u}{a} + b u^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Here b is determined by the condition that for $u = V$, y becomes equal to δ, this gives:

$$y = \frac{u}{a} + \left(\delta - \frac{V}{a}\right)\left(\frac{u}{V}\right)^n \quad \ldots\ldots \quad (24)$$

In accordance with VON KÁRMÁN's theory n will be taken equal to 7. At $x = 150$ cm, we have the following values of δ_k and $\frac{V}{a}$.

$V =$	800_1	800_2	1200	1600	2400 cm sec
$\delta_k =$	2,16	3,24	2,06	1,91	1,865 cm
$\frac{V}{a} =$	0,095	0,11	0,09	0,08	0,06 cm

In order to show the approximation given by the formula, the following example has been calculated for $V = 800_1$, 1600 and 2400 cm sec and represented graphically in fig. 11. The equations of these $u(y)$ curves are at $x = 150$ cm:

$$V = 800_1 \text{ cm/sec} \quad \ldots\ldots \quad y = \frac{u}{8480} + 2{,}06\left(\frac{u}{V}\right)^7$$

$$V = 1600 \text{ cm/sec} \quad \ldots\ldots \quad y = \frac{u}{20500} + 1{,}83\left(\frac{u}{V}\right)^7$$

$$V = 2400 \text{ cm/sec} \quad \ldots\ldots \quad y = \frac{u}{40000} + 1{,}80\left(\frac{u}{V}\right)^7$$

The loss of momentum $w_i = \int_0^\delta u\,(V - u)\,dy$ becomes now with:

$$dy = \frac{1}{a}\,du + 7\left(\delta - \frac{V}{a}\right)\frac{u^6}{V^7}$$

$$w_i = \frac{7}{72}\,\delta\,V^2 + \frac{5}{72}\,\frac{V^3}{a} \quad \ldots\ldots \quad (25)$$

As $\frac{V}{a\delta}$ is a small quantity, the influence of the second term usually is only a few percents.

By introducing into VON KÁRMÁN's equation:

$$\frac{d\,w_i}{d\,x} = \nu\,a,$$

the formulae:

$$w_i = \frac{7}{72}\,\delta\,V^2 + \frac{5}{72}\,\frac{V^3}{a}$$

and $$a = c\,\frac{V^2}{\nu}\left(\frac{V\delta}{\nu}\right)^{-m}$$

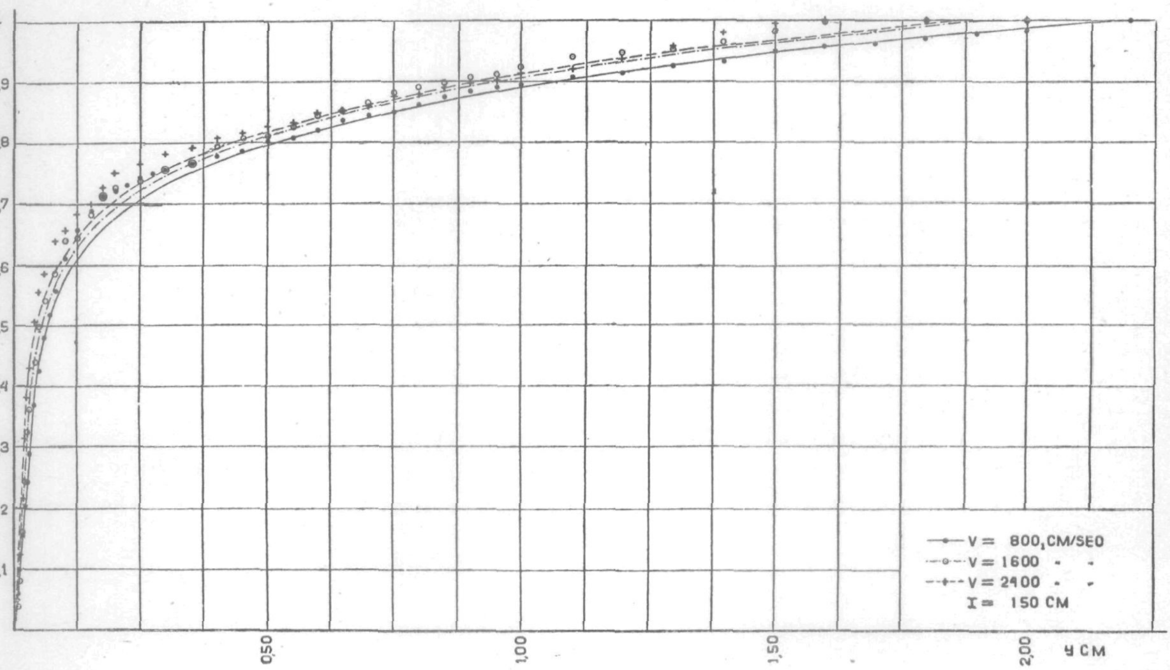

Fig. 11. Graphical representation of the $u(y)$ curves at the section $x = 150$ cm according to the approximate formula, compared to the experimental values of u.

(these relations being regarded as mere empirical interpolation formulae comp. § 4 g), we obtain:

$$\frac{d\delta}{dx}\left(\frac{7}{72}V^2 + \frac{5}{72}\frac{m}{c}\delta^{m-1}V^{m+1}\nu^{1-m}\right) = c\,\delta^{-m}V^{2-m}\nu^{m}$$

the integral of which is:

$$x - x_0 = \frac{7}{72\,c\,(m+1)}\delta^{m+1}V^{m}\nu^{-m} + \frac{5}{144\,c^2}\delta^{2m}V^{2m-1}\nu^{1-2m}.$$

In most of our series the second term is only 1 % of the first one. Neglecting it, the formula can be simplified into:

$$x - x_0 = \frac{7}{72\,(m+1)}\frac{V}{a}\left(\frac{V\delta}{\nu}\right) \quad . \; . \; . \; . \; . \quad (26)$$

In this form, which is very easy for calculations, the exponent m occurs only in the factor $(m+1)$, the coefficient c having disappeared altogether.

Applying formula (26) to the measurements executed with $V = 2400$ cm/sec and $m = 1/2$ according to p. 32 we get:

x cm	$\frac{V \delta_k}{\nu}$	a sec^{-1}	$x - x_0$ cm	x_0 cm
62,5	12200	60000	32	30
75	15800	59000	42	33
87,5	18200	53000	54	34
100	21100	48500	68	32
125	28900	44000	102	23
150	31300	41000	119	31

These values of x_0 are considerably less subjected to variations than that found at p. 30.

The only conclusion which can be drawn from this result is that the equation of momenta (loss of momentum = integral of friction) is fulfilled in a satisfactory manner, as has already been shown in the foregoing paragraph.

SUMMARY.

The principal results of the foregoing measurements may be summarized as follows:

1. It has been demonstrated that a permanent transition exists from the laminar to the turbulent motion; the position of the region of transition has been determined under various circumstances.
2. In the turbulent part of the boundary layer VON KÁRMÁN's formula $u \sim y^{1/7}$ has been proved to be correct; the deviations (most probably due to experimental errors) being at most 2 %.
3. In a less satisfactory way BLASIUS's formula for the laminar part has been verified.
4. The equation of momentum appears to be satisfied with sufficient accuracy.

The most prominent of the unexplained parts are:

1. the large differences between a_k and a_e (p. 31),
2. the rather low values of c_f at the high velocities.

STELLINGEN.

I.

Het gebruik der „Volt-thermometers", volgens THOMSON (beschreven door R. O. KING, „Engineering", 1924 No. 3034) verhoogt wel de bereikbare gevoeligheid der gloeidraadsnelheidsmeters, doch niet de absolute nauwkeurigheid dezer meetinstrumenten.

II.

De afwijkingen van de experimenteel bepaalde ijkkromme der gloeidraadsnelheidsmeters van het rechtlijnig verloop der i^2 $(\sqrt{V})$ lijnen zooals deze uit het experimenteele werk van L. V. KING (Phil. Trans. A 215, p. 373) blijken, zouden aannemelijk gemaakt kunnen worden uit de analogie met de verandering der weerstandscoefficient van cylinders boven een bepaalde waarde van het getal van REYNOLDS.

III.

Het is te verwachten dat metingen met behulp van gloeidraadsnelheidsmeters met draden van verschillende dikte en van verschillende temperatuur een oordeel kunnen geven omtrent de toelaatbaarheid van de op pag. 11 van dit proefschrift vermelde correctie op de directe afkoeling door den wand.

IV.

Bij de bepaling der krachten en momenten, die in luchttunnels op modellen worden uitgeoefend, verdient het aanbeveling een afzonderlijk onderzoek te wijden aan den invloed van den bouw van de tunnel op de strooming om deze modellen.

V.

De onderlinge afwijkingen van de door verschillende onderzoekers bepaalde waarden van de weerstandscoefficient voor gladde oppervlakken zouden waarschijnlijk verklaard kunnen worden uit de gebezigde wijze van onderzoek en uit den aard der oppervlakken.

VI.

De verschillende formules voor de snelheidsverdeeling in stroomend water (zie b.v. FORCHHEIMER „Hydraulik") hebben voor de practijk weinig waarde.

VII.

Het is niet waarschijnlijk dat de behandeling der problemen der niet-stationaire strooming om vliegtuig- of vogelvleugels met behulp der eigenschappen der wél-stationaire strooming tot een bevredigende oplossing leidt, zooals tot nu in de literatuur wordt aangenomen.

VIII.

Hoewel uit proeven gebleken is dat boven de kritische waarde van het getal van REYNOLDS de turbulente strooming voldoet aan de door VON KÁRMÁN afgeleide betrekking $u \sim y^{1/7}$ (pag. 26 van dit proefschrift), is er vooralsnog geen aanleiding om aan te nemen dat dit steeds het geval zal zijn.

IX.

De kritiek van GÜMBEL (Jahrbuch der Schiffbautechnischen Gesellschaft 1917 p. 262) op de theorie van SOMMERFELD (Zs. f. Math. und Phys. 1904) betreffende de smering is niet steekhoudend.

X.

Het is over het algemeen, in verband met de smering, wenschelijk om langzaamloopende en zwaar belaste assen met de daarbij behoorende metalen of bussen nauwkeuriger te bewerken dan snelloopende assen.

XI.

Het door Dr. Ing. G. KEMPF (Uittreksels der Voordrachten, Internationaal Congres voor Technische Mechanica, Delft, 1924, p. 125) genoemde bezwaar tegen formules gebaseerd op de uitkomsten der proeven met vlakke gesleepte platen, wordt slechts gedeeltelijk ondervangen door gebruik te maken van de uitkomsten met zeer lange gebogen oppervlakken verkregen.

XII.

Aan boord van motorzeeschepen is in het algemeen een electrische aandrijving der hulpwerktuigen te verkiezen boven een aandrijving door stoom.

XIII.

De vraag of hoogedruk- dan wel middeldrukmotoren voor de binnenvaart het meest geschikt zijn, kan slechts voor iedere onderneming afzonderlijk beantwoord worden.

XIV.

Voor aan een draaistroomnet aangesloten verbruikers kan de toepassing van een windmotor, aan een asynchrone motor-generator gekoppeld, een verlaging der stroomkosten teweeg brengen.

XV.

Voor de voortstuwing van schepen heeft het zuigerstoomwerktuig thans nog slechts reden van bestaan, toegepast op kleine vrachtschepen in de wilde vaart.