Delft University of Technology

Economics for Technological Innovation

"When DICE meets the dice"

Integrated Economic and Climate Assessment under Uncertainty

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"Gouverner, c'est prévoir | To govern is to foresee "
- Émile de Girardin

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Preface

Before we begin, I'd like to take you on a paradigm shift. The same paradigm shift Claudia Sagastizabal took me on a year ago. Our shift takes us back to the year 1492, the year Christopher Columbus changed the world as we know it. In the years previous to his discovery, the trade with India flourished. Christopher, a man with a model, a very bad model, set voyage on a trip to India by sailing West instead of East. In his model, knowing the world was round, he set out to find a new route to India. Even though his model was a very bad one and he ended up in a different continent, he changed the world. Sometimes a model, even a very bad one, is the only thing we can hold on to in uncertain times.

Coming back from our paradigm shift, the model which will be discussed in this report is an oversimplified version of reality. Though its exact outcomes will not tell us anything, the changes we apply might tell us something about transient behaviour under certain circumstances. But remember: It is a model, and nothing more.

Symbol and abbreviation list

The following describes the major symbols and abbreviations used in the main text.

Symbol	Definition
Al_t	Total factory productivity
b	Constraint level
B_t	Cost of a renewable (backstop) technology
B_0	Initial cost of renewable (backstop) technology in 2010
c_t	Per capita consumption
C_t	Total consumption
db	Initial decline rate of backstop technology cost
d_s	Search direction
E_t	Total emission
$E_{eind,t}$	Industrial emissions
$E_{tree,t}$	Emissions from deforestation
$f_t^*(SoW_t)$	The optimal accumulated return function
$f_t(SoW_t)$	The accumulated return function
$f(x)$	Objective function
$F(x)$	Objective function
F_t	Increase in radiative forcing since 1900 at stage t
$F_{ex,t}$	Exogenous forcing from other greenhouse gasses at stage t
$g(x)$	Inequality constraints
$G_t(SoW_t, x_t)$	Transformation between states
G_{A0}	Initial growth rate of technological development
$h(x)$	Equality constraints
i	Scenario or series
I_t	Invested amount at stage t
j	Series
k	iteration counter
K_t	Capital stock
L_t	Population or labour inputs
$M_{atm,t}$	Carbon-concentration in the atmosphere
$M_{lo,t}$	Carbon-concentration in lower ocean
$M_{up,t}$	Carbon-concentration in upper ocean
n	Node in the decision tree or evaluated scenarios per stage
$\mathcal{N}$	Set of non-anticipativity constraints
N	Set of non-leaf nodes in the scenario tree
p_i	Probability of scenario i
P_{adj}	Projected growth rate of the population till 2050
P_{asym}	Asymmetric boundary of pollution
P_t	Fraction of emissions in control regime
Q	Quantity
$Q(SoW_{T+1})$	Last stage return

$\mathcal{Q}_t(x, \Xi)$	Value(function) of stage t
$Q_t(x, \xi)$	Approximated value(function) of stage t
r	Reduced gradient
r_s	Reduced gradient in search direction
R_t	Discount rate
$\mathcal{S}$	Number of evaluated scenarios
S_t	Gross savings rate as a fraction of gross world output
SoW_t	State of the world at period t
SoW_{t+1}	State of the world at period t+1
t	Time
T	Programming horizon
T_{2xCO_2}	Climate sensitivity parameter
$T_{atm,t}$	Atmospheric temperature increase since 1900 at stage t
$T_{ocean,t}$	Oceanic temperature increase since 1900 at stage t
T_{2xCO_2}	Transient sensitivity at equilibrium
u	multiplier
U_t	Utility
U_{sp}	Accumulated utility of EICE
U_{sa}	Scenario analysis averaged accumulated utility of DICE
V	Return or value function
W	Accumulated social welfare
$Y_{gross,t}$	Gross economic output
Y_t	Economic output
x	Decision variable
x_b	Basic decision variables
x_n	Non-basic decision variables
x_t	Decision variable at stage t
X	Feasible region
z	Objective
α	Elasticity of marginal utility
$\delta ga0$	Decline of the growth rate of technology development
δK	Depreciation of accumulated capital
η	Forcings of equilibrium doubling CO_2-concentrations
γ	Elasticity of output
ρ	Pure rate of social time preference
μ_t	Amount of abatement / Emission control rate at stage t
θ	Exponent of the cost control function
$\phi_{\cdot,\cdot}$	Carbon flow constants
$\phi_{\cdot}$	Separation sub-functions
Φ	Separation function
ξ	Realisation of uncertainty/ of the random variable
Ξ	Uncertainty (random variable) also used as the set of all realisation
ζ_1	Climate response of the atmosphere
ζ_2	Heat transfer between upper and lower stratum
ζ_3	Heat transfer between for the lower ocean
ζ_{10}	Initial value of ζ_1
ζ_β	Regression coefficient for ζ_1

Abbreviation	Definition
BATNEEC	Best available technology not entailing excessive costs
CO2	Carbon dioxide
COP21	Conference of Parties 21
D20	DICE model with time steps of twenty years
D20E(·)	D20 with a climate sensitivity parameter of (·)
D5	DICE model with time steps of five years
DICE	Dynamic Integrated model for Economics and Climate
DP	Dynamic programming
EICE	Extended Integrated model for Economics and Climate
GAMS	General Algebraic Modelling System
IPCC	Intergovernmental Panel on Climate Change
MC	Monte Carlo (approximation)
NLP	Nonlinear programming
$SP20_{(\cdot)}$	EICE with (·) evaluated stochastic stages
$SP20_{(\cdot)u}$	EICE with (·) evaluated stochastic stages and a uniform distribution
$SP20x(\cdot)$	EICE with (·) evaluated realisations
VSL	Value of statistical life
VSS	Value of stochastic solution

Summary

The decision is made to act upon climate change. The remaining question is: "How?". Based on economic theory, the transition to a carbon-neutral society is most efficient through market-based policies. These policies are partially based on Integrated Assessment Models (IAM), which combine the long term economics of climate change with a climate model. The model used in this study is the "Dynamic Integrated model of Climate and the Economy" (DICE) by W. Nordhaus. This influential model is heavily debated. One of the main objections is the exclusion of uncertainty. The influence of uncertainty is debated as some researchers even suggest that it marginalises the entire field of IAM. The focus of this study is on the inclusion of an uncertain climate response, represented by the climate sensitivity parameter. Showing the influence of this asymmetrically distributed parameter on the results of DICE is the subject of this thesis. The accompanying research question is:

"How does the advised mitigation policy by DICE respond to the influence of an uncertain climate sensitivity parameter?"

The uncertainty is modelled as exogenous, so it influences the decision at each stage. The uncertainty remains unknown to the decision-maker until a policy is decided upon. The resulting stochastic programming model continuously evaluates three scenarios at each of the eight stochastic stages. After these stages, the model evaluates seven more deterministic ones to show the effect of a far horizon (2300). The new version is named the "Extended Integrated model for Climate and Economy" (EICE) as it explicitly states all possible scenarios. EICE is subjected to four cases, representing the current understanding, a more extreme case and both cases at a higher level of uncertainty.

The conclusion from comparing these cases is that the model is sensitive to the distribution of the climate sensitivity parameter. In case of a more asymmetric (a.k.a. fatter) distribution, the model advices upon a stricter policy. Based on these cases, no instantaneous transition is required, as some literature suggests, as economic damages are within a few percent of the world gross output. In extend, it is found that EICE suggests a less strict mitigation policy than DICE. This underestimation is again a product of the inability to represent catastrophic damage.

From these findings can be concluded that the proposed mitigation policy by DICE does respond to the shape of the probability function. The fatter this distribution becomes, the stricter the advised policy. Decision makers following the current interpretation of climate science should follow the presented policy by DICE based on the findings of EICE. More risk averse policy makers can use the sensible, but more extreme case to justify their actions. Furthermore, as EICE is found to be less conservative than DICE, as a result of a damage function that is unable to represent extreme damages, it is advised to further explore policies which keep this notion in mind.

Chapter 1

Introduction to Climate Mitigation Policy

"We need to act!" - Barack Obama[52]. When it comes to climate change, the decision is made to act. The remaining question is: "How?", "How to act with respect to the complex mechanics binding an uncertain social-economic future and possibly an even more uncertain climate system?". In order to formulate an answer to this question, policy makers may call upon scientists to help weigh different policies. These climate mitigation policies are often (partially) based on integrated assessment models. The goal of such a model is to evaluate economic activity with respect to its effects on the environment. Their political relevance, their vast oversimplifications and inherent uncertainties make these models the subject of an ongoing debate[19][80].

This debate is the subject of this thesis. Before going into the discussion, this chapter starts with a rough time line of important climate science and policy moments. Sequentially, possible methods for formulating a policy are discussed. How these policies are implemented is (often) based on integrated assessment models. The model central to this thesis is DICE, which is introduced in section 1.3. This introduction is followed by an elaboration of its shortcomings in section 1.4. State-of-the-art methods for handling these shortcomings and their limitations are the subject of section 1.5. In line with these shortcomings of the model and the presented gaps in literature, a research question is formulated in section 1.6. The chapter ends with an outline of the rest of the chapters.

1.1 The rise of climate science

In 1824, the French scientist Jean Baptiste Fourier hypothesised that since the earth heats up under the influence of solar radiation and does not cool down to the absolute zero in its absence, it needed to store the energy, making it neither too hot nor too cold[28]. In 1896, the Swedish scientist Svante Arrhenius extended this idea by hypothesising that an increase of the carbon dioxide (CO_2) concentration in the atmosphere, due to carbon emissions, might influence its insulating properties and that the increased insulation may then result in an increase in the earth's overall temperature[4].

Since the time of Fourier, the atmospheric CO_2-concentration has increased from 280 ppm to over 400 ppm. This has resulted in a record breaking global average temperature increase of $0.99°C$ in 2016 since the reference temperature of 1880[46]. It is widely accepted within academic literature that this increase is (at least partially) a result of anthropogenic emissions. This conclusion is based on the changes since 1950 presented in figure 1.1. These changes have been labelled as unprecedented over decades to millennia, letting the Intergovernmental Panel on Climate Change [IPCC] to conclude that: "Human influence in the climate system is clear"[53].

The IPCC further concludes that continued emission of greenhouse gasses will cause further warming and long-lasting changes to the climate system, increasing the likelihood of extreme events (i.e. heat waves, droughts, floods, cyclones and wildfires[53]). To minimise the extent of possible climate damage in the future, emissions have to be reduced.

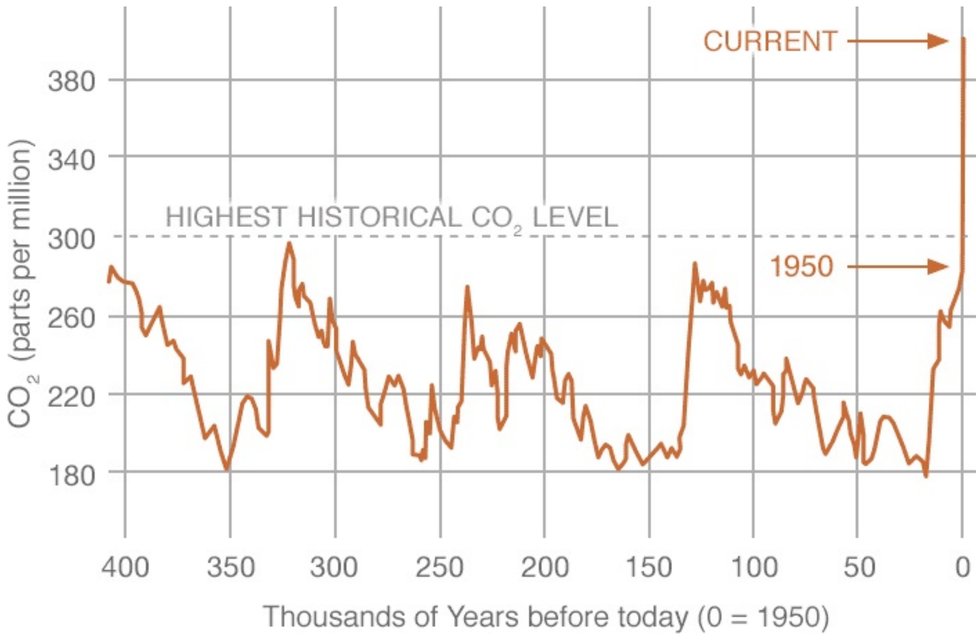

Figure 1.1: The increase of CO_2 in the atmosphere reconstructed paleo-archives[48]

The questions: "What percentage of future emissions should be reduced?", "What is the influence of these anthropogenic emissions?" and "What would be the result of these emissions?" are the subjects of an ongoing debate. The first major steps in solving these questions were made in the post-World War II era, when the advances in atmospheric science and the possibility of computer simulations led to the first World Climate Conference in 1979. During this conference, over 350 specialists gathered, resulting in data exchange programs, research programs and impact study alliances. Since then, a vast amount of conferences have been held. Important pinpoints on the timeline of climate change policy are listed as follows: 1. the foundation of the IPCC in 1987, 2. the Toronto Conference in 1988; which was the first to call upon actors to take specific actions to reduce the impending crisis caused by the pollution of the atmosphere, 3. the second World Climate conference in 1990, which set the basis for the earth Rio summit in 1992 and its "Rio Declaration on Environment Development", 4. the first conference of parties in 1995, 5. the conference regarding the Kyoto Protocol in 1997 and speeding up to 2015: 6. the Conference of Parties 21 (COP21) in Paris. COP21 resulted in a binding agreement of 195 nations to invest towards a carbon low future and steer to stay below a global average atmospheric surface temperature increase of two degrees Celsius with respect to pre-industrial levels. Following this, they promised to invest 100 billion U.S. dollars before 2020[74].[1]

1.2 Climate policy

In line with the previously stated questions, the question regarding this 100 billion U.S. dollars, is: "What is the most efficient policy for this transition?". Naturally, multiple approaches can be considered. Traditionally, standards (i.e. fuel quality, emission and environmental

[1] which is a factor of four lower than the yearly amount invested in fossil fuel production by the G20[75] and a factor of six lower than the yearly expenditure on military by the U.S. government[66]

quality standards) are used in environmental regulations to steer technology. These standards are commonly based on the current best available technology not entailing excessive costs (BATNEEC)[54]. A drawback to this approach is that it does not encourage continuous emissions reductions, something that can be accomplished by levying a tax. By setting a tax equal to the cost of CO_2 emissions to society, companies would have an incentive to invest in emission reductions and reduce the deadweight loss of overproduction as depicted in figure 1.2. In this figure, a supply-demand curve is presented. Here, the price of fossil fuel based products is depicted in red. Under these price conditions the amount of Q_1 will be produced. As this price only takes the private value into account, it does not represent the "real" cost to society. This lower price results in an overconsumption of $Q_1 - Q_2$. The additional consumption results in the deadweight loss arched in red. By levying a tax, the supply curve is lifted and a new equilibrium settles at point B, the socially optimal market equilibrium[44].

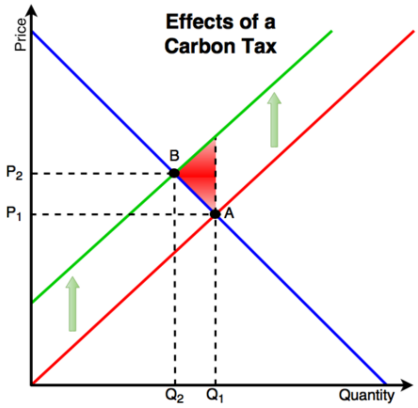

Figure 1.2: Social optimum as a result of levying a Pigouvian tax, thereby reducing the deadweight loss of overproduction[44]

Economic theory states that such an approach, one trough market-based policies, is the most efficient approach to combat climate change. Examples of relevant market-based policy tools are pollution taxes, transferable permits and subsidies.[2] Only (Pigouvian) pollution taxes have shown to reduce the link between the gross domestic product and emissions and are therefore the preferred solution[28][68]. The goal of such policies is to remove the disturbance from the market.

In the field of economics, climate change is seen as a market failure. It represents both a negative externality and an overuse of a common property resource.[3] [68] A negative externality arises when the production of a certain good has, in addition to its private cost, a negative effect on (public) social goods[28]. The goal of a pollution tax is to internalise these externalities.

An unregulated market for fossil fuels only includes private costs and neglects externalities. Such a market thus does not provide a social optimum as shown in figure 1.2.[4] A pollution tax, or carbon tax, internalises the damage done to the system based on a per unit tax. By levying a tax, firms are forced to operate at the point where their marginal abatement cost equals the set carbon tax[54]. As a result, firms emit less and a social optimum is achieved.

[2]From a political point of view introducing a tax or designing a complex system with transferable permits might be less preferable than providing subsidies to develop a backstop technology or to set standards that increase efficiency. The drawback to this is that the capital required for the subsidy cost at the cost of other commodities. Due to these shortcomings subsidies in further context will not be reviewed. Nonetheless a positive Pigouvain tax could be given to the CO_2-free technology

[3]The market failure of common property resource is not relevant to the further discussion and thus is left out of the further explanation, for more information see[28]

[4]In an optimal policy the carbon price equals the social cost of carbon[49].

By levying a tax to, for example, the fossil-fuel-based electricity, electricity produced by a backstop technology could tilt the market into renewable technologies. Additionally, the overall higher prices will give the consumers an incentive to reduce energy consumption. The raised funds, as a result of the tax, could be used to invest in the development of a backstop technology or to lower the financial burden of consumers[54].

The main question that climate-policy-makers face is the optimal level of the (global) carbon tax. To answer this question, economists generally make use of integrated assessment models that combine economic and climate cycles (e.g. Nordhaus[51], Stern[68]). These models are either used to evaluate a certain policy or to optimise it[49][19]. This thesis will focus on the optimisation of a climate mitigation policy and thus will use the latter.

Central to this thesis is the Dynamic Integrated model for Economics and Climate (DICE) by W. Nordhaus. This seminal integrated assessment model is an optimisation model and will be used for the optimisation of a certain carbon tax. Even though DICE is known as the standard Integrated Assessment Model, it is heavily debated[80][70][17]. The following section will further introduce this model after which the subsequent section elaborates on the discussion regarding its validity.

1.3 The Dynamic Integrated model for Economics and Climate

The idea behind DICE is to advise on a climate mitigation policy by combining the knowledge of (environmental) economics and climate science.[5] The goal of this model is to advise on an investment (or tax) policy, which optimises intergenerational welfare. It does this from the perspective of neoclassical growth theory[49].

In the standard neoclassical framework, the economy invests in technology and capital at the cost of current consumption. DICE adds to this by valuing the climate system. In the model, emissions from economic activity are included as negative capital. The amount of negative capital can be reduced, or completely avoided, by investing in fossil free backstop technologies. Thereby, reducing the current consumption, but increasing the long-term-welfare. The interlinking of these steps is visualised in figure 1.3[49].

In order to capture the effects of long term damage, the model is simulated over a period of 300 years. To keep the number of computations limited, the model assumes globally aggregated relations between the economic and climate variables[49].

By approximating (and oversimplifying) the major economic and climate relations, the model has the ability to estimate an optimal carbon tax, while describing climate-economic mechanics in a comprehensible manner. The ease with which the open access model can be (parametrically) altered, makes it an ideal model to bringing multiple fields relevant to the energy transition together, but is also cause for discussion[71].

[5] Remark: Since the early 90s, there have been multiple versions of DICE. This study will focus on the 2013R version of the model (revisions of earlier models can be found in the DICE103113r2 manual together with a more detailed elaboration of the current model [49]). This report will specifically focus the optimal form of the DICE vanilla GAMS version. This implies that the model will follow the optimal utility path which results from dynamically solving stage-wise relations. In addition to this, the focus will be on the theoretical relations provided in the manual instead of the practical relations used in the GAMS model. For a full description of the model and the original GAMS code, see appendix A.

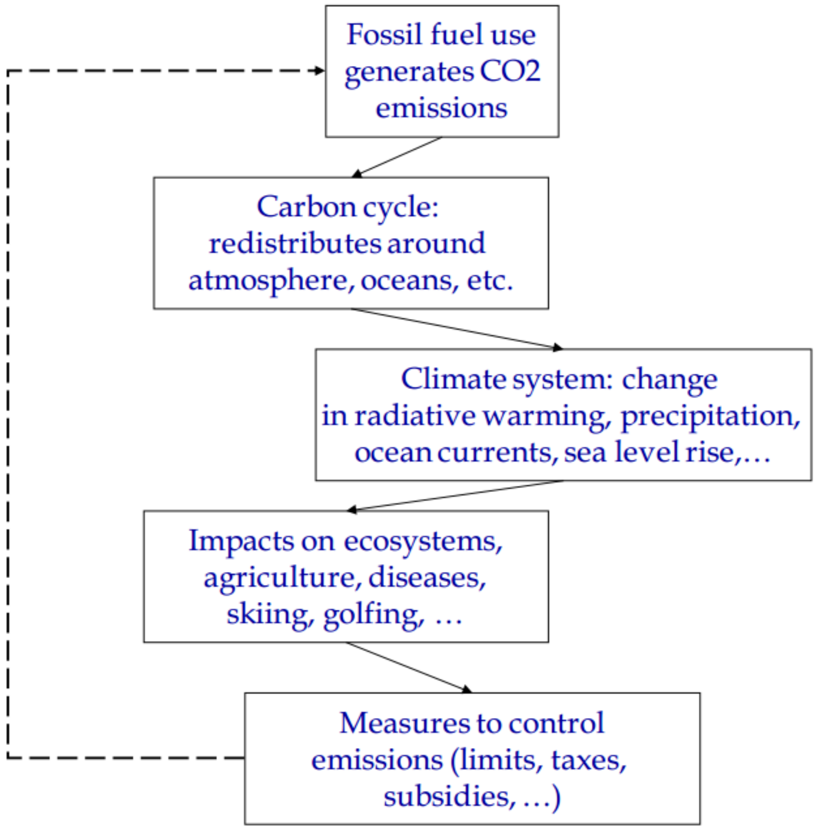

Figure 1.3: A visual representation of the interlinking modules within in DICE[49]

Furthermore, the model is of scientific interest for its political relevance. As, for example the United States Environmental Protection Agency, the environmental agency of the number two emitter, uses the model as an advisement tool, its scientific soundness is of the utmost importance.

1.4 Criticism on the DICE model

The use of DICE and other integrated assessment models is the matter of an ongoing debate. In addition to the standard ethical arguments against cost-benefit methods (i.e. the monetisation of human lives), these abridged models cover highly uncertain dynamics. In DICE, these uncertainties are not included and all results are based on mean-valued parameters. The result therefore consist of a single output and suggests a level of knowledge and precision that can be offsetting[56]. It is this uncertainty that will be the subject of this thesis.

As shown by various researchers (e.g. Stern[68], Traeger[70] and Pindyck[56]), the economics of climate change are very uncertain. This uncertainty has both normative and empirical roots, spanning from socio-economic factors to those of climate science. One of these socio-economic factors is a result of the assumption that climate damage mostly influences future generations. This makes the welfare of future generations of major importance to a proper policy[29]. The level of their welfare (partially) depends on the development of technology and the impact of current emissions, resulting in a fundamentally uncertain future. This fundamentally uncertain future makes it challenging to formulate a proper policy. To make matters worse, it is hard (or even impossible) to predict how society would respond to damages. For example, In the case of a rising sea level a response could be to either resort to geo-engineering or to migrate, of which both options could be executed in a well organised or more disruptive manner, making future damages unquantifiable and therefore the future even more uncertain[30].

As there is no empirical support for estimating the rise of the sea level, the development of technology nor the level of welfare overall, discounting these future generations becomes (even more than normal) a discussion on ethical grounds, as it can no longer be based on observations[30]. DICE enables this discussion by making the pure rate of time preference (i.e. the weight given to future generations) and the marginal utility of consumption (i.e. the aversion to inequality between generations), two of the many parameters that can be easily altered in the (open-source) model. Stern argues that based on ethical grounds this rate should be close to zero[68].[6][7] This assumption results in a ten times higher carbon price than suggested by Nordhaus, showing the sensitivity of the model to the modellers preference.

The evaluation of the discount rate or other exogenous parameters is very well suited for an approach called uncertainty propagation (a.k.a. scenario analysis, sensitivity analysis, Monte Carlo analysis or ex ante analysis). Here, the optimisation model is executed over a large number of possible parameter combinations. The results of these simulations may be combined in a weighted-average if a probability distribution is known[24]. This Monte Carlo type of approach is very popular in literature (i.g. Nordhaus[50], Dietz[21] and Ackerman[1]) and is appropriate for the discount factor when it is approached as a normative parameter.

For empirical, endogenous, uncertain parameters this method is less well suited. Whereas the discount rate can be seen as a product of the modellers preference, parameters defining climate damage, technological development and the overall response of the climate are defined by the real physical world[29]. These endogenous uncertainties materialise over time, but are unknown at the moment a policy is implemented. Therefore their probability distribution directly influences the decision making at each stage in time. This is different from a Monte Carlo type of approach where only a single realisation is evaluated at each stage[17]. The distinction between the two types of uncertainty is shown in figures 1.4 and 1.5. From figure 1.4 it is clear that endogenous uncertainty, visualised as multiple outcomes as a result of decision x, is experienced by the modeller at the time $(t, \dots, T)$ of execution. Here, the modeller is able to optimise the expected value of the problem instead of the average value of the problem, as in a Monte Carlo type of approach.

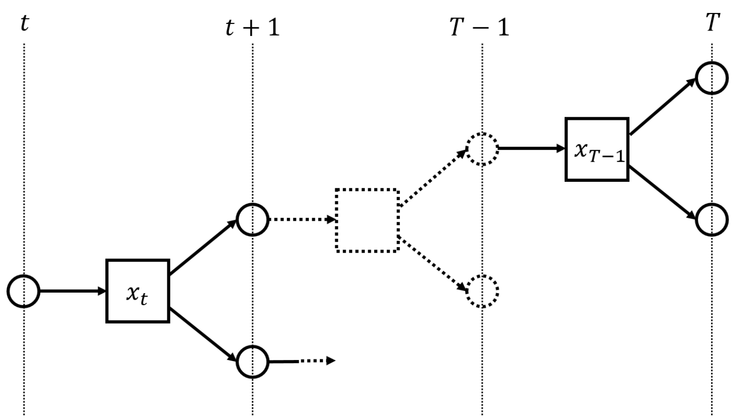

Figure 1.4: A stochastic tree describing the process of learning about uncertainty at times $t, \dots, T$ and making decisions $x_t, \dots, x_{T-1}$ in regard to the knowledge at that time.

Due to this influence the endogenous approach is more risk averse, as shown in figure 1.6. This figure shows that an ex ante (Monte Carlo) approach underestimates the abatement rate in

[6]these two components which together with the consumption growth rate make up the discount factor[49]
[7]No scaling is needed when altering the variables, unlike for example the size of the time steps.

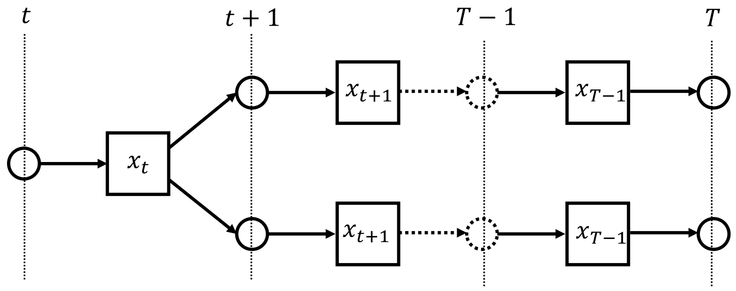

Figure 1.5: A stochastic tree describing scenario based decision making.

regard to the expected (expected draw) and endogenous approach (sim. 95% CI).

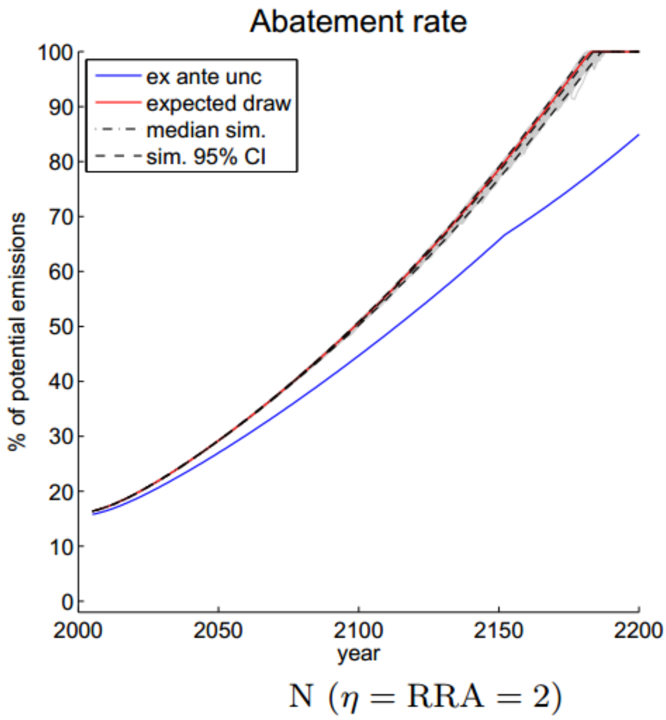

Figure 1.6: A comparison of the abatement rate in DICE between a deterministic expected approach (red), an ex ante approach (blue) and a stochastic approach with endogenous uncertainty (intermittent - black), showing the under estimation of the ex ante approach[16].

For this reason endogenous uncertainty is taken as the focus of this study. More specifically, the focus is on the endogenous uncertainty in the climate response since no quantitative estimates can be made regarding the resulting damage or the development of technology[30].[8]

As common in integrated assessment literature, DICE uses a climate sensitivity parameter to represent the response of the climate system to an increase in carbon concentration[80]. More specifically the climate sensitivity is defined to be: "the equilibrium temperature increase due to a doubling of CO_2 concentration in the atmosphere"[2]. In the DICE2013R model, the climate sensitivity parameter is set to be 2.9. Studies based on both palaeo-historical records, climate simulations and observed temperature deviations found a probability distribution that peaks

[8] An approach to study the influence of these unquantifiable uncertainties is by studying the possibility of climate tipping points (e.g. [14], [12], [65]). The same methodology could be used to simulate technological leaps.

around the same value, but skewed with a very long tail. Overall the distribution spans from extremely low probabilities that the climate sensitivity would be lower than $1°C$ and low probabilities that it would be higher than $6°C$, as presented in figure 1.7[62][53]. Within the scientific community there is a rough consensus that damage up to $3°C$ will be relatively moderate. Above that, only few quantitative estimates are available[53]. With a slight possibility that the global average temperature increases with $6°C$ or more, this asymmetric (fat tail) distribution imposes the problem of catastrophic events.

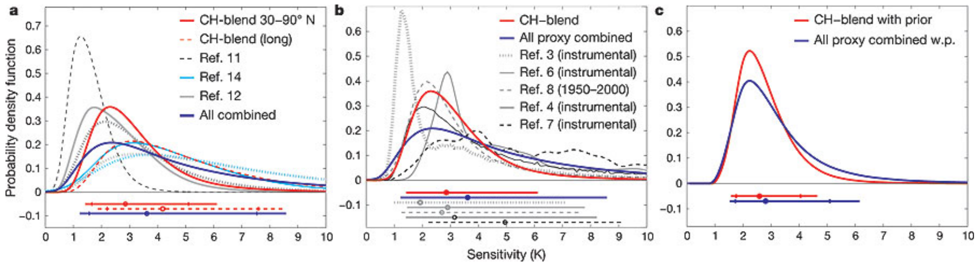

Figure 1.7: Estimations of the climate sensitivity parameter, with A: an estimation based on paleo-reconstructions using data from 1850 up to 2006, with B: a reconstruction based on instrumentally measured data from 1950 t0 2000 and with C: a combined estimation based on A and B[31].

Weitzman argues that the possibility of these extreme events possibly alters the provided answers to the questions stated in sections 1.1 and 1.2. His argument is based on the analogy with a standard cost-benefit analysis based policy, in which the value of a statistical life (VSL) highly influences the advised policy. In case of possible catastrophes, which are defined to be: " events with a very low probability of materialising, but when they do will produce a harm so great and sudden as to seem discontinuous with the flow of events that proceed it"[58], potentially unlimited downside exposure might occur. In the case of an integrated assessment model, such a VSL-like parameter thus represents: "something of the order of a catastrophic extinction of a civilisation or the value of the natural world as we know it"[80]. In case this parameter approaches an infinitely high value, no matter how small the risk, society would be infinitely willing to exchange today's consumption for the futures avoided cost[34]. This idea is summarised in the following theorem[80].

> **Dismal theorem**: *If the value of a statistical life (representing the rate of substitution of consumption and the mortality risk of a catastrophic extinction of civilisation) approaches infinity, then the amount of present consumption the current society would be willing to give up in the present time to obtain an additional sure unit of consumption in the future would also approach infinity.*

The asymmetrical distribution of the climate sensitivity parameter is a product of measurement errors and the accumulation of fundamental uncertainty regarding feedback loops. As these measurement errors are based on Bayesian learning, it is estimated the real value of the climate sensitivity parameter does not become certain this century[56]. Therefore, no learning regarding this parameter is assumed during this thesis.

The effects of feedback-loops will be examined according to their level of uncertainty. Feedback loops are processes like the thawing of the permafrost, the increase of water vapour in the atmosphere and the melting of the ice caps. These process may influence the current climate equilibrium[53]. Just as in a mechanical process, certain feedbacks can have a(n) (negative) influence on the stability of the process. And as in these mechanical processes, some climate feedbacks might set the process into overdrive. These feedbacks are fundamentally uncertain and again are likely to remain so for a number of centuries[51]. Based on the topology regarding uncertainty, this fundamental misunderstanding should be levelled at recognised ignorance[76]. Including recognised ignorance into the model would suggest that all possible realisations of the climate sensitivity parameter are equally weighted, placing more emphasis on the effects of extreme events.

1.5 Existing stochastic versions of DICE

The following state of the art focusses on publications regarding the inclusion of uncertainty in DICE. The idea of including uncertainty or catastrophic events within DICE is not novel and various researchers (i.g. Jerzy[23], Shayegh[65], Traeger[70], Golub[24], Cai[12], Chang[14], Kolstad[39], Webster[77], Ackerman[1]) have attempted to include this vital part into the decision process. This section focusses on the different approaches to including uncertainty and the regarded uncertainties in these evaluations.

Due to computational limitations, these models traditionally consist of two-stages and a few scenarios. As the impacts of climate change develops over time, a drawback of this approach is the coarse approximation of the transient behaviour. The goal of these evaluations is to optimise a single policy during the considered interval[77].

This method can be extended for multiple stages, but quickly results in a computationally intractable problem. To (partially) avoid this problem multiple methods are at hand. An advanced method to include uncertainty is stochastic dynamic programming. In this case the program is dynamically and recursively solved, that is all stages are functions of both the first stage and the set policy, and the model is solved by recursively substituting sequential-stage solutions. In literature, two versions are often discussed; the stochastic tree and the approximation approach.

The stochastic tree approach is used in DICESP. This model reviews the economic activity of the DICE model in a period between 2015 and 2115, while evaluating the climate cycle until 2300. During this evaluation, a possible climate catastrophe is considered. The catastrophic damage is simulated by a tipping point with a certain probability. With an *act-then-learn* approach, acting before the uncertainty realises, the algorithm learns of the tipping point distribution at each stage. In case a catastrophic event occurs, the algorithm goes back up in the decision tree and hedges against the negative outcome. The resulting hedging strategy shows a steady increase in the expected abatement path, supporting the claims for a stricter mitigation policy[14].

Traeger uses Approximate Dynamic Programming for his model[70]. Such an approximation model uses basic functions to approximate future states on a rolling or finite horizon. This approach is extended to a two-step ahead model by Shalyegh, which focusses on the uncertainty of the climate sensitivity parameter in relation to the risk of hitting a climate tipping point[65].

These tipping points are the subject of a major field within economic climate studies. This field focusses on the influence of possible extreme events on policy making. In the case of Shalyegh, the climate tipping point is dependent on the climate sensitivity parameter, unlike in DSICE[12]. In the model by Shalyegh, a log-normal distribution of the climate sensitivity parameter is assumed with a mean of 1.1 and standard deviation of 0.5. As time progresses, the deviation is assumed to become narrower. Solving this model when only the uncertainty in the climate sensitivity parameter is included, results in a higher abatement policy in respect to the expected case. As Shalyegh focusses on the result of tipping points, he does not include the "pure" continuous response of the model to uncertainty in the climate sensitivity parameter.

An overall shortcoming of these studies is that they focus on damage shocks as a result of tipping points, instead of continuous damage due to temperature increase. In addition, they focus on the damage as a result of climate change. As the damage is deemed unquantifiable, it might be more suiting to evaluate the climate sensitivity parameter, as discussed in section 1.4, and work with temperature bounds.

Furthermore, the Dismal theorem is often cited in more abstract papers, but no numerical evaluations including endogenous uncertainty are at hand. These two shortcomings in scientific literature will be the focus of this thesis and lead to the research question of section 1.6. The contribution of this thesis is that it will give insight into the usability of DICE and its proposed climate mitigation policy in respect to uncertainty and extreme events.

1.6 Research question

Showing the influence of the asymmetric distribution and the level of uncertainty in respect to the suggested policy by Nordhaus will be the main goal of this thesis. This objective is reformulated in the following research question:

"How does the advised mitigation policy by DICE respond to the influence of an uncertain climate sensitivity parameter?"

In order to answer this question the following sub-questions are examined:

- Which design assumptions form the foundation of the DICE model and how do they influence its response?
- What is the influence of different possible integrations of uncertainty to the results of DICE?
- How can the climate sensitivity parameter be represented so it aligns with the current understanding of climate science?
- Do the found results support the claim of Weitzman's Dismal theorem?

The answers to these questions are not explicitly stated in the text, but form a basis for the overall underlying structure.

1.7 Outline of the report

This report continues by introducing the DICE model in chapter 2. Here, the focus is on the mathematical expressions that form the model and how to solve it. The chapter is roughly divided into three parts, one regarding the economic activity, a second regarding the climate part of the model and a third which covers the mathematical programming techniques needed to solve the model. Chapter 3 covers mathematical programming options for including uncertainty and provides the theory required for extending DICE into a stochastic dynamic nonlinear program. The following chapter implements this theory and states some critical design assumptions. One of these critical assumptions is the selection of uncertain cases. These cases are used to test the model and generate results, which are presented in chapter 5. The results are followed by the discussion in chapter 6. On the one hand this discussion focusses on the validation of the model and on the other on the results in respect to literature regarding the Dismal theorem. After the discussion, the report concludes and advises upon further research in chapters 7 and 8, respectively.

Chapter 2

Explanation of the DICE model

The DICE model forms the foundation of this report. Having insight into its core mechanics is therefore of vital importance for the chapters to come. As introduced in section 1.3, the objective of the DICE model is to maximise the intergenerational accumulated welfare. This welfare, expressed in utility, is a function of consumption, which in itself is a function of economic output. Hence, the goal of the model is to maximise intergenerational output. In the model, the output of the world is described by a single equation, the output function. As the labour force increases and its production becomes more efficient, the output increases[49]. This approach to economic growth is common within the neoclassical framework.

Nordhaus built on this model by adding a climate cycle. This cycle starts with emitting CO_2 is emitted into the atmosphere as a result of economic activity. These emissions accumulate and increase its concentration. As the atmosphere is connected to the upper oceans, which are connected to the lower oceans, carbon is passed trough, dampening the effect of carbon increase in the atmosphere. The remaining CO_2 influences the insulating properties of the atmosphere and results in an increase in atmospheric temperature. The climate damage resulting from this increased temperature reduces the economic output, closing the loop with the economic cycle.

As the temperature increase is a gradual process, this damage does not occur instantly. Here, the time factor connects to the intergenerational welfare, making an additional link with the economic cycle. In addition to climate damage, Nordhaus also added an abatement module. This module allows for current investment to avoid future damages. The resulting dynamic is the foundation of DICE. At every stage of this process the decision is made to invest into capital, to consume or to abate.

The following chapter will discuss these dynamics and how to solve them in greater detail. Additionally appendix A presents all these equations in the GAMS-format. The following section, section 2.1, elaborates on the economic model. This model mainly consists out of the social welfare function, the output function and the general accounting equations. Sequentially, the climate relations of the model are discussed in section 2.2. Solving the resulting nonlinear dynamic optimisation model is the subject of sections 2.3 and 2.4. Section 2.3 focusses on the intertemporal relations of the model in the form of dynamic programming. This framework is then extended to nonlinear models in section 2.4.

2.1 The economic model in DICE

As stated above, DICE is based on neoclassical economics. This predominant framework assumes that all actors have rational preferences, that they maximise utility and that all actions are based on full and transparent information[78]. This review starts with the used welfare function. This function is about maximising utility by varying consumption and investment.

Social welfare function

Today's policy has an effect on future generations. Hence, in order to value today's policy, the effects on future generations should be included. This gives rise to the problem of weighing intertemporal utility and consumption. From an utilitarian perspective, the aim might be to weigh the utility (U_t) of all generations equally. In the case where t represents generations until the horizon (T), the total welfare (W) would be equal to $\Sigma_{t=1}^{t=T} U_t$. As long as T has a finite value, the sum will converge[29]. Nordhaus argues that based on economic development, the intergenerational utility should be discounted, ranking alternative consumption sequences[50]. The amount of discounting is determined by the *pure rate of social time preference* (ρ). The value of this parameter is highly debated. When utility is discounted at the standard rate (R_t) of 3%, welfare of a hundred years from now only weighs in at 5%, thereby marginalising the welfare of future generations[68]. To counteract this marginalization Weitzman proposes to a use a declining discount rate[79]. The resulting discount rate would based on weighted information from an expert panel fitted to a Gamma-distribution. In this approach the discount rate declines over time, as it approaches the lowest proposed value. More variations, such as the inclusion of uncertainty in the growth rate of consumption and the representation of the social time preference as a choice problem are discussed in [30]. Since the focus of this thesis is on the uncertainty of the climate sensitivity parameter, the original social welfare function of DICE

$$W = \sum_{t=1}^{T} U_t R_t, \tag{2.1}$$

where

$$R_t = (1+\rho)^{-t} \tag{2.2}$$

is used. DICE combines this discounted approach with a Ramsey style growth model[49]. Here, utility is based on an iso-elastic expression of per capita consumption (c_t) expressed in trillions of 2005 dollars per person [$Tr\$_{2005}/pp$].[1] The goal of this expression is to steer away from overconsumption, avoiding its possible negative influence in the future. By setting the elasticity of marginal utility (α)[-] to be greater than one, the negative exponential shape results in risk averse behaviour. In addition, when defining U_t to be the resulting periodic utility function, where per capita consumption is aggregated into total consumption (C_t) by means of multiplication with the population (L_t) in millions of people [10^6 people] which grows according to the projected growth rate of the population until 2050 (P_{adj})[10^6 people] and the asymptotic population boundary (P_{asym})[10^6 people], the utility function becomes:

$$U_t(c_t, L_t) = L_t \frac{\frac{C_t}{L_t}^{1-\alpha}}{1-\alpha}, \tag{2.3}$$

with

$$L_{t+1} = L_t \left(\frac{P_{asym}}{L_t} \right)^{P_{adj}}, \tag{2.4}$$

where the growth of the population is exogenous.[2]

[1] In the following text [...] will be used express units. In case of a dimensionless number or a factor [-], will be used.

[2] L_t also represents the labour inputs

Output function

Within DICE, the economic output is calculated with an extended Cobb-Douglas production function[49]. The production function as proposed by Cobb and Douglas is a highly theoretical aggregated model[15]. In their model, economic output $(Y_t)[Tr\$_{2005}]$ is a function of the total factory productivity (AL_t)[-], the elasticity of output (γ)[-], the labour force and the capital stock $(K_t)[Tr\$_{2005}]$.[3] Here, AL_t represents the current level of technology, the development of this level is a function of its initial growth rate $(GA0)$[-] and the growth decline rate (δ_{ga0})[-][49]. The application of this standard neoclassical model is justified by the macro scale and the long-term optimisation horizon. As the Cobb-Douglas is based on the assumption of a continuous process and interchangeable variables, these two conditions are of major importance. The proposed relation by Cobb and Douglas for gross output $(Y_{gross,t})$ is:

$$Y_{gross,t} = AL_t L_t^{1-\gamma} K_t^{\gamma}, \tag{2.5}$$

where

$$AL_{t+1} = \frac{AL_t}{1 - GA0 \cdot e^{-\delta_{ga0} \cdot \Delta t \cdot t}}. \tag{2.6}$$

Nordhaus extends on equation (2.5) by correcting for emission abatement and the damage as a result of temperature increase. This is achieved by using a single unit to represent output, damage and abatement: trillions of 2005 dollars. The economic damage fraction is a function of the atmospheric temperature (T_{atm}) and is defined to be $1/(1 + a_1 T_{atm,t} + a_2 T_{atm,t}^{a_3})$. Here, a_1 to a_3 are fixed parameters based on estimates by [69]. The cost of mitigation is the product of the cost of a renewable (backstop) technology (B_t) in trillions of \$2005 per ton of $CO_2[\$_{2005}/tCO_2]$, the participation rate (P_t)[-] and the amount of abatement in that period (μ)[-], of which the last two are corrected with a control cost function (θ)[-]. At every stage, the cost of fossil fuel replacing (backstop) technologies is assumed to decline. This is in respect to its initial cost in 2010 (B_0), with an initial decline rate (db). These definitions combined result in the following relation for the nett output:

$$Y_t = \frac{AL_t L_t^{1-\gamma} K_t^{\gamma} (1 - P_t^{1-\theta} \mu_t^{\theta} B_t)}{1 + a_1 T_{atm,t} + a_2 T_{atm,t}^{a_3}}, \tag{2.7}$$

with

$$B_t = B_0 (1 - db)^{t-1}. \tag{2.8}$$

It is the nett output that is used to consume or invest in new stock. How the output is used is defined by the general accounting equations.

General accounting equations

These equations state that the sum of investment $(I_t)[Tr\$_{2005}]$ and consumption is equal to the nett economic output. Furthermore, they state that the non-consumed amount of output is therefore saved according to saving rate S_t[-]. The investment accumulates in capital, which is also subjected to depreciation (δ_K)[40]. These relations result the following equations:

$$I_t = Y_t - C_t, \tag{2.9}$$

$$I_t = Y_t S_t, \tag{2.10}$$

[3] In the original version by Cobb and Douglas there is also an elasticity of labour. In DICE this elasticity (α) is substituted by $1 - \gamma$. Which is justified by the standard values of 0.7 for γ and 0.3 for α

$$K_t \leq I_t + (1 - \delta_K)K_{t-1}. \tag{2.11}$$

2.2 The geographical model in DICE

The following section describes the geographical model in DICE. In this part of the model the CO_2-emissions from economic activity are converted into a climate response. Based on a stylised greenhouse model with a three-stage storage capability (representing the storage of carbon and energy in the atmosphere, upper and lower oceans), the economic activity is coupled to the carbon concentration in the reservoirs and the resulting increase in radiative forcing and temperature. Despite the fact that this model is highly simplified, it is in line with the fundamental processes described in climate science. The use of parsimonious representations is needed so the optimisation model is empirically and computationally tractable[49].

The described climate cycle is represented in figure 2.1. In this system the economy emits CO_2 by means of industry and deforestation. Other greenhouse gasses are excluded from the model, for they are more likely to be controlled in different ways[47]. As a result of these emissions, the atmospheric carbon concentration and the radiative force increases, resulting in a higher atmospheric temperature. This higher carbon concentration and temperature influence the concentration and temperature of the ocean. In DICE, the oceans are represented by a two stage model. These stages represent the upper and lower oceans. This classification is needed to represent the different speeds of mixing in the reservoirs. The upper oceans quickly mixes with the atmosphere, whereas the deeper oceans react extremely slow. This mixing results in mass transport in both directions in the so called "three reservoir model". Internally the reservoirs are assumed to be well mixed. This CO_2-cycle with temperature response is the focus of this section.

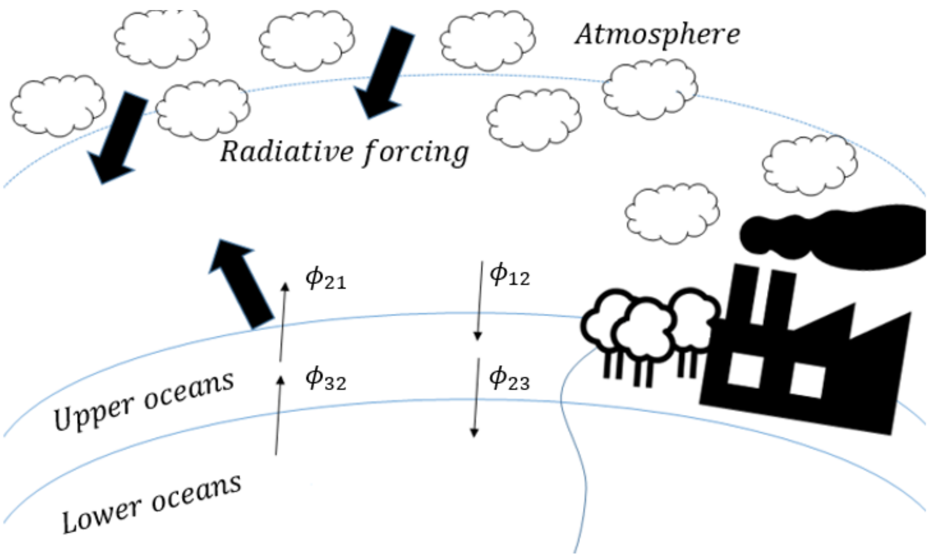

Figure 2.1: The greenhouse effect as described in DICE

The Carbon Cycle

The inputs of the carbon cycle are industrial emissions ($E_{eind,t}$) and emissions from deforestation ($E_{tree,t}$). These emissions, measured in gigatonnes of CO_2 per year [$GtCO_2/a$]. This amount is converted into tonnes of carbon by multiplication of the CO_2-equivalent-emission output ratio

(σ_t). Together they make up for the total carbon-emissions per year (E_t)][GtC/a]. This relation is expressed as:

$$E_t \geq \sigma_t Y_{gross,t}(1 - \mu_t) + E_{tree,t}. \tag{2.12}$$

As a result of these emissions the carbon-concentration in the atmosphere ($M_{atm,t}$)[GtC] increases. Due to the mixing of the atmosphere with the upper ocean ($M_{up,t}$)[GtC], a part of the carbon is transferred. The same holds for the flow of carbon between the upper and the lower oceans($M_{lo,t}$)[GtC]. The speed with which these reservoirs mix is determined by the carbon flow constants (ϕ), resulting in:

$$M_{atm,t} = E_t + \phi_{11} M_{atm,t-1} + \phi_{21} M_{up,t-1}, \tag{2.13a}$$
$$M_{up,t} = \phi_{12} M_{atm,t-1} + \phi_{22} M_{up,t-1} + \phi_{32} M_{lo,t-1}, \tag{2.13b}$$
$$M_{lo,t} = \phi_{23} M_{up,t-1} + \phi_{33} M_{lo,t-1}. \tag{2.13c}$$

Here, the carbon flow parameters are calibrated to more detailed global circulation models to mainly represent the expected behaviour until 2100. After 2100, the ocean reservoir absorbs more CO_2 in respect to the estimates by the global circulation models and thus overestimates possible consequences of atmospheric temperature increase.

Temperature response to carbon increase

As the emissions in the atmosphere accumulate, the radiative forcing increases due to the increased insulation of the atmospheric layers. This increase (F_t)[W/m^2 from 1900] is measured in respect to the level in 1750 and is defined to be the sum of the exogenous forcing ($F_{ex,t}$)[W/m^2 from 1900] and the logarithmic increase of concentration in carbon ($M_{atm,t}$)[GtC] with respect to its level in 1750, multiplied by the constant for equilibrium increase of forcing at a doubling of CO_2 (η)[$^\circ C/2xCO_2$], resulting in:

$$F_t = \eta(ln \frac{M_{atm,t}}{M_{atm,1750}}) + F_{ex,t}. \tag{2.14}$$

The higher radiative forcing increases the atmospheric temperature. The extent of this temperature increase is a function of the increased radiation, the forcing sensitivity, the climate sensitivity parameter (T_{2xCO_2}) expressed in $^\circ C$ per doubling of CO_2[$^\circ C/2xCO_2$] and the difference between the atmospheric ($T_{atm,t}$) and the oceanic ($T_{ocean,t}$) temperature increase[$^\circ C i.r.t.1900$]. In the summation of these variables, climate coefficients are used. Here, the (ζ_2) coefficient describes the heat transfer between the upper and lower stratum and (ζ_1) the climate response of the atmosphere itself. This last response is corrected from its initial value (ζ_{10}) with a regression coefficient (ζ_β) and the transient sensitivity at equilibrium ($T_{2xCO_{2,mean}}$)[$^\circ C/2xCO_2$] and are presented as:

$$T_{atm,t+1} = T_{atm,t} + \zeta_1 \left(F_t - \frac{\eta}{T_{2xCO_2}} T_{atm,t} - \zeta_2 (T_{atm,t} - T_{ocean,t}) \right), \tag{2.15}$$

with

$$\zeta_1 = \zeta_{10} + \zeta_{1\beta}(T_{2xCO_2} - T_{2xCO_{2,mean}}). \tag{2.16}$$

As a result of the mixing reservoirs, the temperature of the lower oceans also increases, this is proportional to ζ_3. As with the other transfer coefficients, these parameters are based on general circulation models and measurements[49]. Their relation is defined as:

$$T_{ocean,t+1} = T_{ocean,t} + \zeta_3 (T_{atm,t} - T_{ocean,t}). \tag{2.17}$$

The increase in atmospheric temperature results in a higher damage fraction in equation (2.7). It is this relation, that results in a trade-off between producing and abating, consuming today or in a later stage. This trade-off between climate action, investment and consumption together with the recursive formulation of the optimisation model that makes the problem dynamic programming problem[12], which is the subject of the next section.

2.3 Dynamic programming

The DICE model optimises the path to a carbon neutral economy in sixty stages, spanning a period of 300 years. At each stage the model calculates the best possible ratio between consumption, investment and abatement. In order to find the optimal path (or climate policy) a sequence of abatement decisions, which result in the highest discounted sum of periodic utility is made. Since the outcome at a certain stage is influenced by its predecessors, simply optimising every individual period would not result in an optimal policy. An optimal policy is provided by Richard Bellman's dynamic programming (DP) framework by taking recurrence relations into account[18]. The following section will go into this framework and show how DICE makes use of the provided structure to find an optimal abatement policy.

The classical approach to modelling multi-stage decision problems is to consider all possible solutions. This is done by collecting all feasible solutions and sequentially compute the return of each policy. In case of DICE, this would result in a very high computational burden due to the virtually unbounded range of variables and its sixty stages[5].

Bellman's approach to this problem is to reduce the size by stating that: it is sufficient to know what would determine the decision at a certain stage. This idea results in the basic idea of DP: an optimal policy is the one that determines all decisions in terms of the current state, which leads to Bellman's principle of optimality: "*An optimal policy has the property that whatever the initial stage and decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*"[5].

By making a decision (x_t) at each stage (t), until the discounted (R_t) horizon (T), regarding the state of the world at period t (SoW_t), the objective (z) and the return ($V(SoW_t, x_t)$, a transformation (G) projects the current state into the next (SoW_{t+1}). A problem of this form is generally expressed as[38][60]:

$$\begin{aligned} z \; &= \max_{x_t} \sum_{t=1}^{T} R_t V(SoW_t, x_t) \\ s.t. \; & x_t \in X_t, \qquad \forall t, \\ & SoW_{t+1} = G_t(SoW_t, x_t), \qquad \forall t. \end{aligned} \tag{2.18}$$

As the transition between stages is defined by $SoW_{t+1} = G_t(SoW_t, x_t)$, the objective function of (2.18) can be expressed with the help of a function F as:

$$z \; = \{\max_{x_t} \; [F(V_1(SoW_1, x_1), \ldots, V_T(SoW_T, x_T))] | x_t \in X_t, t = 1, \ldots, T\}, \tag{2.19}$$

can for some function Φ be rewritten into:

$$z \; = \{\max_{x_t} \; [\Phi(SoW_1, x_1, x_2, \ldots, x_T)] | x_t \in X_t, t = 1, \ldots, T\}. \tag{2.20}$$

Here, the objective is defined as a function of the initial state and the implemented policy. By applying the optimality principle to this equation, the first stage policy can be separated by introducing two functions: ϕ_1 and ϕ_2, which represents the response of the first and later stages. Including these functions converts equation (2.20) into:

$$z = \{ \max_{x_1 \in X_1} [\phi_1(V_1(SoW_1, x_1), \max_{x_2 \in X_2, \ldots, x_T \in X_T} \phi_2(V_2(SoW_2, x_2), \ldots, V_T(SoW_T, x_T)))]\}. \tag{2.21}$$

For equation (2.21) to be optimal, functions ϕ_1 and ϕ_2 have to exist and ϕ_1 should be monotonically non-decreasing in ϕ_2 for every possible V_1. As all stages are separable in corresponding way, this procedure can be expended upon. The result of this expansion is a model that can be recursively solved. With the introduction of $f_t(SoW_t)$ as the accumulated return function, $f_t^*(SoW_t)$ its optimal value and $Q(SoW_{T+1})$ as a last stage return, this model is generally presented as[38]:

$$\begin{aligned} f_t^*(SoW_t) &= \max_{x_t \in X_t} \phi_t(V_t(SoW_t, x_t), f_{t+1}(SoW_{t+1})) \\ s.t.\ SoW_{t+1} &= G_t(SoW_t, x_t), \\ f_{T+1}(SoW_{t+1}) &= Q(SoW_{T+1}). \end{aligned} \tag{2.22}$$

Applying the idea of dynamic programming to the DICE model is achieved by representing the state of the world by $(T_{atm,t}, T_{ocean,t}, M_{atm,t}, M_{up,t}, M_{lo,t}, K_t)$ and the decision to abate (μ_t) and to save (S_t) as decision variables. The other variables are a part of the transformation G, resulting in the following model:

$$W = \max_{(\mu_t, S_t)} \sum_{t=1}^{T} R_t U_t \tag{2.23a}$$

$$s.t. \quad U_t = L_t \frac{\left(\frac{C_t}{L_t}\right)^{1-\alpha}}{1-\alpha}, \tag{2.23b}$$

$$Y_t = \frac{A L_t L_t^{1-\gamma} K_t^{\gamma} (1 - P_t^{1-\theta} \mu_t^{\theta} B_t)}{1 + a_1 T_{atm,t} + a_2 T_{atm,t}^{a3}}, \tag{2.23c}$$

$$C_t = Y_t - I_t, \tag{2.23d}$$

$$I_t = S_t Y_t, \tag{2.23e}$$

$$K_t \leq I_t + (1 - \delta_K)^{\Delta t} K_{t-1}, \tag{2.23f}$$

$$E_t \geq \sigma(1 - \mu_t) A L_t L_t^{1-\gamma} K_t^{\gamma} + E_{tree,t}, \tag{2.23g}$$

$$M_{atm,t+1} = E_t + \phi_{11} M_{atm,t} + \phi_{21} M_{up,t}, \tag{2.23h}$$

$$M_{lo,t+1} = \phi_{33} M_{lo,t} + \phi_{23} M_{up,t}, \tag{2.23i}$$

$$M_{up,t+1} = \phi_{12} M_{atm,t} + \phi_{22} M_{up,t} + \phi_{32} M_{lo,t}, \tag{2.23j}$$

$$F_t = \eta(ln \frac{M_{atm,t}}{M_{atm,0}}) + F_{ex,t}, \tag{2.23k}$$

$$T_{atm,t+1} = T_{atm,t} + \zeta_1 \left(F_{t+1} - \frac{\eta}{t2xCO_2} T_{atm,t} - \zeta_2 (T_{atm,t} - T_{ocean,t}) \right), \tag{2.23l}$$

$$T_{ocean,t+1} = T_{ocean,t} + \zeta_3 (T_{atm,t} - T_{ocean,t}). \tag{2.23m}$$

$$\tag{2.23n}$$

In defining the feasible region of the decision variables and the state of the world, bounds are required. Table 2.1 states these bounds together with a lists of initial conditions describing the current state of the world according to Nordhaus[49]. By means of these bounds and conditions the model can be optimised. Maximising the problem at each individual stage is done by means of nonlinear programming (NLP). The next section elaborates on this method.

Table 2.1: Lower bounds, upper bounds and initial conditions

Variables[4]	Symbol	**Lower bounds**
Investment	I	0
Capital stock	K	1
Consumption	C	2
Gross savings rate	S	0
Per capita consumption	c	0.01
Gross world product of net A and D	Y	0
Gross world product of gross A and D	Y_{gross}	0
Carbon concentration atmosphere	M_{atm}	10
Carbon concentration shallow oceans	M_{lo}	100
Carbon concentration lower oceans	M_{up}	1000
Emission control rate GHGs	μ	0
Increase Temperature in atmosphere	T_{atm}	0
Increase temperature of lower oceans	T_{ocean}	-1

Variables	Symbol	**Upper bounds**
Cumulative carbon emissions	CCA	6000
Gross savings rate	S	1
Emission control rate GHGs	μ	1
Increase temperature lower oceans	T_{ocean}	20
Increase temperature of atmosphere	T_{atm}	9.1

Variables	Symbol	**initial conditions**
Cumulative carbon emissions	CCA	90
Capital stock	K	135
Carbon concentration atmosphere	M_{atm}	830.4
Carbon concentration shallow oceans	M_{up}	1527
Carbon concentration lower oceans	M_{lo}	10010
Increase temperature of atmosphere	T_{atm}	0.80
Increase temperature lower oceans	T_{ocean}	0.0068

2.4 Nonlinear programming

As the name suggest, NLP focusses on problems with nonlinear objectives and or constraints[32]. In the standard version of these problems, the objective function is represented by $f(x)$, which is subjected to equality ($h(x)$) and inequality ($g(x)$) constraints at level b. The domain bounded by these constraints is called the feasible region (X). The standard form of nonlinear models is presented as:

$$\begin{aligned} \underset{x}{\text{optimise}} \; & f(x) \\ s.t. \; & g(x) \leq b, \\ & h(x) = b, \\ & x \in X \subseteq \Re^n. \end{aligned} \tag{2.24}$$

The possibility of having multiple local optima within (i.e. polynomial) nonlinear functions, makes nonlinear programs difficult to solve. As a result of multiple local optima, the found solution might not be equivalent to the overall best (global) solution[10]. A global solution is guaranteed when the problem is convex. This is the case when a maximisation problem has a concave objective function and a convex feasible region. Based on this definition, the model from equation (2.23) is a non-convex optimisation problem. Fortunately, experimental studies concluded that global optimality is guaranteed and DICE is therefore defined as hidden-convex[49][67].

The solution of DICE is found by varying the decision variables: the savings rate (S_t) and emission control rate (μ_t). Since DICE is hidden convex, the decision variables can be optimised by means of a local optimisation algorithm. Nordhaus uses the CONOPT algorithm designed by A. Drud[49]. CONOPT uses a Newton based algorithm that finds the best fitting solution to problems by means of the generalised reduced gradient algorithm. This algorithm is presented in its generic form below.

The generalised reduced gradient algorithm of CONOPT

---

1. Convert the model input to:

$$\begin{aligned} \underset{x}{\text{optimise}} \; & f(x) \\ s.t. \; & g(x) \leq 0, \\ & h(x) = 0, \\ & lo < x < up. \end{aligned}$$

2. Find a feasible solution (x_0), evaluate $f(x_0)$ and set the iteration counter (k) to zero.

3. Evaluate the Jacobian $J^k = \dfrac{\partial f}{\partial x^k}$.

4. Use the pivots to create a set of n basic variables (x_b) such that the submatrix of the basic column of J (B) is non-singular. The remaining m·n variables (x_n) are named nonbasic.

5. Solve $B^T u = \dfrac{\partial f}{\partial x_b}$ to find the multipliers u.

6. Compute the reduced gradient: $r = \dfrac{\partial f}{\partial x} - J^T u$ with a value below zero for all the basic variables.

7. If the Karush- Kuhn- Tucker conditions (a-d) are satisfied (within a reasonable margin of error) then stop, the current point is close to the optimum, else continue.

   (a) $0 \in \partial f(x) + \sum_{i=1}^{m} u_i \partial g_i(x) + \sum_{j=1}^{r} v_j \partial h_j(x)$ (Stationary).

   (b) $u_i g_i = 0, \qquad \forall i$ (complementary slackness).

   (c) $g_i(x) \leq 0, \quad h_i(x) = 0 \qquad \forall i, j$ (primal feasibility).

   (d) $u_i \geq 0, \qquad \forall i$ (dual feasibility).

8. Define a set of superbasic variables (x_s) as a subset of x_n that can profitably be changed. Find a search direction (d_s) for x_s based on r_s and possibly some second order information.

9. Perform a one-dimensional line search in the direction of d_s via a Pseudo-Newton process.

10. Save the best solution and go to step 3.

In case of DICE, this results in a gradient of W in respect to both μ and S. This gradient steers the algorithm in the direction of the optimal solution. The algorithm iterates until this gradient becomes marginally small and the Karush-Kuhn-Tucker conditions are met.

Chapter 3

Programming under uncertainty

The focus of this chapter is on the theory required for the inclusion of uncertainty into the DICE model. In response to the critique regrading the use of an expected value (deterministic) approach, the first section will go into appropriate methods for including uncertainty into an optimisation model. This starts with a brief overview of possible methods, after which one method is selected. The selected method is elaborated in the second section, where both a two- and multi-stage portfolio investment problem are presented.

3.1 Options for modelling under uncertainty

As introduced in chapter 1, there are multiple ways to include uncertainty into the decision making process. Considering these options is the subject of this section. Following the project definition of section 1.4, the focus is on models with endogenous uncertainty.

The sequential process in a stochastic version of DICE, where the decision maker has only limited control over the future rewards, is known as a Markov Decision Process[22]. In such a process, a decision (x_t) is made in respect to the current state of the world (SoW_t). The information flow between these two is represented by the intermittent line. As the outcome of the decision is prone to uncertainty (Ξ), both the state of the world at the subsequent stage ($SoW_{t+1}(\xi)$) and the return ($V_t(\xi)$) of the current stage are uncertain, where ξ represents the realised uncertainty. A general example of such a process is presented in figure 3.1.

This uncertainty can be included into equation (2.24) by adding a random variable (Ξ) in both the objective function and the constraints. Here, the precautionary constraint is that uncertainty has to be on the statistical level. In such a case a probability distribution can be selected to represent the behavioural outcome. The uncertainty in the objective might represent a stochastic return, whereas the uncertain constrains represents either a uncertain response of the system or unknown boundary conditions[7].

$$
\begin{aligned}
\max_{x} \; & f(x, \Xi) \\
s.t. \; & g(x, \Xi) \leq b(\Xi), \\
& h(x, \Xi) = b(\Xi), \\
& x \in X \subseteq \Re^n.
\end{aligned}
\tag{3.1}
$$

A Markov Decision Process forms the foundation of stochastic programming. The essence of the resulting programs is that the information regarding random variable Ξ is incomplete at the time the decision is made[64]. Hence, x is taken before the realisation of Ξ. Important here is that Ξ itself is not a function of x. As a result of the inclusion of Ξ, equation

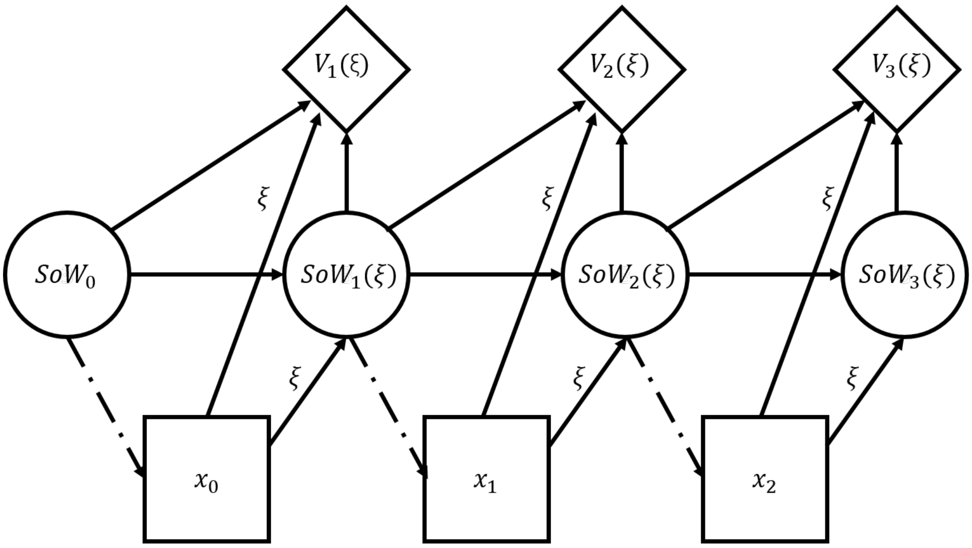

Figure 3.1: Schematic representation of a Markov Decision Process. Where a decision (x_t) is made in respect to state SoW_t and uncertainty Ξ to transition to state $SoW_{t+1}(\xi)$, where ξ is the realisation of Ξ, with the aim of optimising the (total) return V_t (based on the figure of [57]).

(3.1) not well defined as both the objective and the constraints can be interpreted in multiple ways.

The uncertain objective can be interpreted in two ways: the optimal policy can be taken in respect to the worst-case scenario or can be based on the expected value. These two options represent either the decision of the policy maker to maximise profit in the worst-case scenario or to come up with a policy that fits (almost) all realisations of Ξ and optimise their expected outcome. By taking ξ_w to be the least beneficial realisation of Ξ, the stochastic program representing the first case can be expressed as:

$$\begin{aligned} \max_x \; & f(x, \xi_w) \\ s.t. \; & g(x, \xi_w) \leq b(\xi_w), \\ & h(x, \xi_w) = b(\xi_w), \\ & x \in X \subseteq \Re^n. \end{aligned} \tag{3.2}$$

Taking $\mathbb{E}f(x, \xi)$ to be the probability weighted value of $f(x, \xi)$ and Ξ as the set of all possible realisations, the second case can be stated as:

$$\begin{aligned} \max_x \; & \mathbb{E}f(x, \xi) \\ s.t. \; & g(x, \xi) \leq b(\xi), \\ & h(x, \xi) = b(\xi), \\ & \xi \in \Xi, \\ & x \in X \subseteq \Re^n. \end{aligned} \tag{3.3}$$

These different approaches are respectively represented by the robust- or the stochastic programming approach. A drawback of robust optimisation is that it is deemed to be too conservative. That is, the focus on extreme tail events makes the proposed policy too financially unattractive. Stochastic programming is deemed more financially efficient in the case: the uncertainty is of a stochastic nature, there is an available probability distribution and a readiness to accept infeasibilities of tail events[6]. These conditions are assumed in this inquiry.

In extend, the decision maker can choose to always honour the constraints or to allow a certain margin of infeasibility. This last type is named the probabilistic approach and can be used to reduce the conservativeness of the model[72]. This approach requires a close approximation of the probability curve in order to relax constraints by a small margin, requiring a large number of samples. As the evaluated number of scenarios is equal to n^T, where n is the sample size and T the number of stage, this greatly increases the computational burden resulting in an intractable model. Furthermore, as the influence of the probability distribution and the possibility of extreme events on the advised policy is the main focus of this thesis, this approach is not a good fit.

Concluding, this chapter will further elaborate on the well defined stochastic programming approach without probabilistic constraints. Here the focus is on implementing this structure for multiple stages, where at each stage the decision x can be re-evaluated.[1]

3.2 Stochastic programming

The aim of this section is to explain the mechanics of stochastic programming. These dynamics are demonstrated by an investment problem, which just as DICE, is based upon acting before learning about the true mechanics of the system[14][7]. This section starts with a two-stage approach and later expands this into a multi-stage version.

For both versions the following example will be used: An investor starts with initial capital (SoW_t), which he can decide to invest (x_t) into multiple assets with a combined random return. In order to present uncluttered relations only a single decision option is presented in figure 3.2. The uncertainty regarding these assets is such that three types of realisations (ξ) are possible: high (ξ_1), medium (ξ_2) and low (ξ_3). As the uncertainty realises, so does the level of capital at that stage ($SoW_{t+1}(\xi_i)$) with a certain return ($V_{i,t}(\xi_i)$). For the multi-stage version this process of deciding whether to invest in which assets continues up to horizon T, as in figure 3.1 and more elaborate in 3.2.

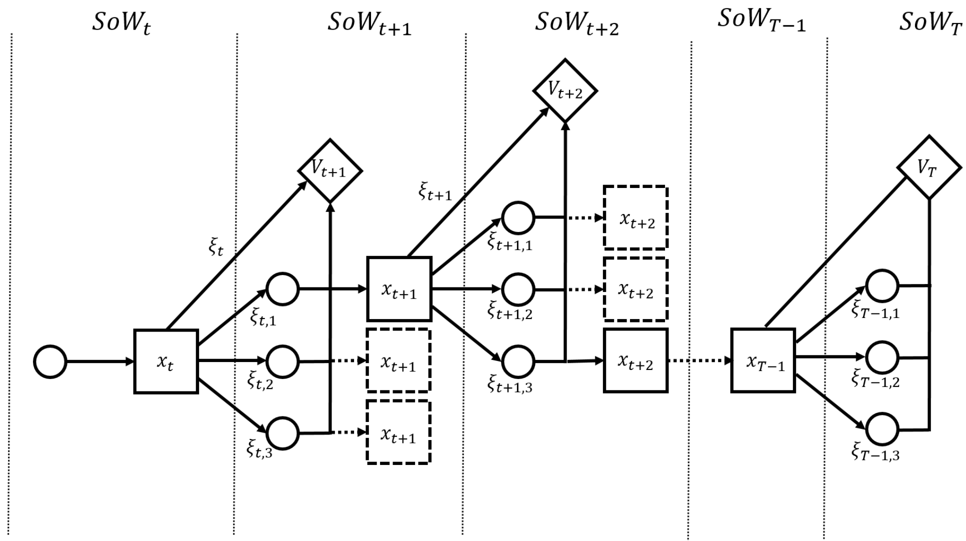

Figure 3.2: Schematic representation of the stochastic model with three realisations of uncertainty in every stage. At stage t until T-1, where T is the set horizon, the decision is made to act upon the uncertain future, which together with the realisation of Ξ results in the future state of the world and a certain return.

[1] In literature it is common to extend a stochastic model with a recourse action. In such a case the decision maker has the option to make sure the constraints are honoured by for example running a more expensive emergency production plan or to buy product in the market. This approach has the benefit of being more cost efficient, but since no such mechanism exists within DICE this extension is not possible.

3.2.1 Two-stage stochastic programming

The first two stages of figure 3.2 can mathematically be represented by separating equation (3.3) into two sub-problems. Looking at the first two stages of figure 3.2, it is clear that the decision to invest is taken before the uncertainty realises. The decision x_t is therefore known as a *first-stage* or a *here-and-now* decision variable. The first sub-problem covers the first-stage constraints, like limited initial wealth and has an objective function which estimates the value of the second stage ($\mathcal{Q}_2(x,\xi)$), or second stage value function, plus any first stage returns ($f_1(x)$). Note that ($f_1(x)$) and other returns are represented by V in figure 3.2. In the investment example no such returns exists, but in DICE it represents the current economic output. The value of the second stage can be estimated by a probability weighted (p_i) sum of all (three) possible realisations and the second stage function ($f_2(x,\xi)$). As there are a finite number of possible realisations, the problem can be stated in the so called *deterministic equivalent* form[7]:

$$\begin{aligned} \max_x \ & f_1(x) + \mathcal{Q}_2(x,\Xi) \\ s.t. \ & g_1(x) \le 0, \\ & h_1(x) = 0, \\ & x \in X \subseteq \Re^n, \end{aligned} \tag{3.4}$$

where

$$\mathcal{Q}_2(x,\Xi) = \mathbb{E}_\Xi Q_2(x,\Xi) \quad or \ \sum_i p_i[Q_2(x,\xi_i)], \tag{3.5}$$

and

$$\begin{aligned} Q_2(x,\xi_i) \ & = f_2(x,\xi_i), \\ s.t. \ & g_{2,i}(x,\xi_i) \le 0, \qquad \forall i, \\ & h_{2,i}(x,\xi_i) = 0, \qquad \forall i, \\ & \xi \in \Xi. \end{aligned} \tag{3.6}$$

Important here is that value of $\mathcal{Q}_2(x,\Xi)$ can only be calculated if $Q_2(x,\xi)$ is measurable. This is the case when: f_1 and g_1 are continuous and $f_2(\cdot,\xi)$ and $g_2(\cdot,\xi)$ are continuous for all realisations of Ξ. For this to hold, Ξ has to have a finite number of realisations[7]. As there are a finite number of realisations of Ξ the above problem can be rewritten into a single problem, the so called *extensive* form[7]:

$$\begin{aligned} \max_x \ & f_1(x) + \sum_i p_i[Q_2(x,\xi_i)] \\ s.t. \ & h_1(x) \le 0, \\ & g_1(x) = 0, \\ & \mathbb{E}_\Xi(x(\xi)) - x(\xi) = 0, \\ & h_{2,i}(x,\xi_i) \le 0, \qquad \forall i, \\ & g_{2,i}(x,\xi_i) = 0, \qquad \forall i, \\ & x \in X \subseteq \Re^n, \\ & \xi \in \Xi. \end{aligned} \tag{3.7}$$

Here the third constraint is known as a *non-anticipativity constraint* and implies that the decision x has to be made in the first stage. This is realised by stating that the expected decision, before the realisation of Ξ, is the same as the value of x after the realisation[7]. Hence, even though x is displayed as a function of ξ, it is not dependent on ξ.

The resulting program is a large (convex) model and can be solved with standard solvers[7]. A multi-stage model, of which the two-stage case is a special case, can also be solved by initially separating all stages in a deterministic equivalent form and sequentially define an extensive model. These conversions are the subject of the following subsection.

3.2.2 Multi-stage stochastic programming

As the investor is able to rebalance his portfolio at any stage, the resulting structure resembles that of a reoccurring two-stage model in which all stages are sequentially interlinked. As the probability distribution of Ξ is not influenced by previous stages or its present state it is said to be stochastically independent or Markovian. After separating all stages, the deterministic equivalent can be divided into three parts: one for the final stage, one for the intermediate stages and one for the first stage[64]. Thus the model can be expressed as:

$$\begin{aligned} Q_T(x_{T-1}, \xi_{T,i}) \; &= \max_{x_T} \; f_T(x_{T-1}, \xi_{T,i}) \\ &s.t. \; g_{T,i}(x_{T-1}, \xi_{T,i}) \leq 0, \qquad \forall i, \\ &\quad\; h_{T,i}(x_{T-1}, \xi_{T,i}) = 0, \qquad \forall i, \\ &\quad\; x_T \in X_T \subseteq \Re^{nT}, \\ &\quad\; \xi_T \in \Xi_T, \end{aligned} \tag{3.8}$$

and for stages $t = T-1, ..., 2$:

$$\begin{aligned} Q_t(x_{t-1}, \xi_{t,i}) = \; &\max_{x_t} \; f_t(x_t, \Xi_t) + \sum_i p_i[Q_{t+1}(x_t, \xi_{t+1,i})] \\ &s.t. \; g_{t,i}(x_{t-1}, \xi_t) \leq 0, \qquad \forall i, \\ &\quad\; h_{t,i}(x_{t-1}, \xi_t) = 0, \qquad \forall i, \\ &\quad\; x_t \in X_t \subseteq \Re^{nt}, \\ &\quad\; \xi_t \in \Xi_t, \end{aligned} \tag{3.9}$$

and for the first stage:

$$\begin{aligned} z = &\max_{x_1} \; f_1(x_1) + \sum_i p_i[Q_2(x_1, \xi_{2,i})] \\ &s.t. \; g_1(x) \leq 0, \\ &\quad\; h_1(x) = 0, \\ &\quad\; x \in X_1 \subseteq \Re^{n1}, \end{aligned} \tag{3.10}$$

which just as the dynamic programming problem of section 2.3 can be recursively solved[7][63][33].

Additionally, as with the two-stage model, the problem can also be solved by converting it into an extensive form. Different from the extensive form of the two-stage model is that the non-anticipativity constraints are not explicitly stated, but are defined in the set of resulting non-anticipativity solutions: $\mathcal{N} = \{(x_\xi)_{\xi \in \Xi} | x^t(\xi, n) - x_n = 0 \; \forall \xi \in B(n), \forall n \in N\}$, where n represents a node in the decision tree, $B(n)$ and the set of non-leaf nodes in the scenario tree is represented by N[33][63]. With this feasible region, the extensive form of the multi-stage model can be formulated as:

$$
\begin{aligned}
z = & \max_{x \in X \subset \mathcal{N}} f_1(x) + \sum_i p_i[f_2(x_1, \xi_2) + ... + \sum_i p_i[f_T(x_{T-1}, \xi_T)]] \\
s.t.\ & g_{t,i}(x_{t,i}(\xi), \xi_{t,i}) \leq 0, \qquad \forall t \ and \ i, \\
& h_{t,i}(x_{t,i}(\xi), \xi_{t,i}) = 0, \qquad \forall t \ and \ i, \\
& \xi \in \Xi.
\end{aligned}
\tag{3.11}
$$

It is this structure that is implemented in the GAMS extended mathematical programming architecture to solve a stochastic version of DICE[42]. The following chapter takes the presented theory and extends the existing DICE model. In addition it reformulates the model to make it computationally tractable and defines the set of scenarios that are used to generate results.

Chapter 4

Integrating Stochastic Programming in DICE: EICE

The aim of this chapter is to formulate a computationally tractable stochastic DICE-like model. As discussed in chapter 3, this stochastic model is solved by means of an extensive form, giving rise to the name: EICE (Extensive Integrated model of Climate and Economics). The tractability of this model depends on the required memory for storing data in (optimisation) vectors[36]. The size of these vectors depends on the number of evaluated scenarios ($\mathcal{S}$), which in itself is a function of the number of evaluated scenarios per stage (n) and the number of stages (T), as expressed in:

$$\mathcal{S} = n^T \tag{4.1}$$

In case of the current sixty stages, evaluating two scenarios per stage already results in over a quintillion (10^{18}) possible scenarios, making the model vastly intractable. It is therefore necessary to reduce the number of stages and keep the number of scenarios per stage to a minimum.

The transition from DICE to EICE is elaborated in the following section. Making this model computationally tractable is the subject of the rest of the chapter. Section 4.2 focusses on reducing the number of evaluated stages. The used approach in this section is to increase the time between stages and to define a stochastic tree with only a few uncertain stages. Representing the distribution of the climate sensitivity by a limited number of scenarios is the subject of section 4.3.

4.1 The Extended Integrated Model of Climate and Economy

Converting the model of section 2.3 into a stochastic model requires the inclusion of the theory of chapter 3. As explained in section 3.2.2, EICE would for any stage have to take all possible realisations of the climate sensitivity parameter and the resulting value function into account. Following this and defining X as the feasible region for both the saving rate and the emission control rate, the stochastic program of DICE can be formulated as:

$$\begin{aligned} W_t(SoW_t, \Xi_t) \; &= \max_{(\{\mu_t, S_t\} \in X_t)} \sum_{t=1}^{T} \{U_1(SoW_1, \mu_1, S_1) \\ &+ \sum_i p_i[R_2 U_2(SoW_2, \mu_2, S_2, \xi_i) + \ldots + \sum_i p_i[R_T U_T(SoW_T, \mu_T, S_T, \xi_i)]] \end{aligned} \tag{4.2}$$

which is subjected to $SoW_{t+1} = G(SoW_t, x_t, \Xi_t)$ for all values of t and is equivalent to:

$$\begin{aligned} W_t(SoW_t, \Xi_t) &= \max_{(\{\mu_t, S_t\} \in X_t)} \sum_{t=1}^{T} \{U_t(SoW_t, \mu_t, S_t) + R_{t+1} \mathbb{E}_{\Xi} U_{t+1}(SoW_{t+1} | SoW_t, x_t, \Xi_t)\}] \\ &s.t.\ SoW_{t+1} = G(SoW_t, x_t, \Xi_t), \qquad \forall t. \end{aligned} \tag{4.3}$$

Mathematically this relation describes how, at each stage, the decision maker decides to invest in new capital or into climate change mitigating measures under the (financial) limits of the current state of the world. Again, the function G describes the transition between different states of the world. This function is in EICE represented by equations (4.4b) till (4.4q), describing both the economic and climate cycle. In respect to the model of equation (2.23), the model below introduces six new relations (4.4c, 4.4d, 4.4i, 4.4j, 4.4n and 4.4o). These relations are used to calculate the global economic output and the temperature increase. These relations are added to give more insight into the response of the model. The only new variables in respect to the model of equation (2.23) are ζ_{10}, $\zeta_{1\beta}$ and Ξ, where Ξ represents the random climate sensitivity parameter and the ζ's are the reference heat transfer coefficients from equation (2.16).

$$W_t(SoW_t, \Xi_t) = \max_{(\{\mu_t, S_t\} \in X_t)} \sum_{t=1}^{T} \{U_t(SoW_t, \mu_t, S_t) + R_{t+1} \mathbb{E}_{\Xi} U_{t+1}(SoW_{t+1} | SoW_t, x_t, \Xi_t)\}] \tag{4.4a}$$

$$s.t.\ E_t \geq \sigma_t Y_{gross,t}(1 - \mu_t) + E_{tree,t}, \tag{4.4b}$$

$$Y_{gross,t} = A L_t L_t^{1-\gamma} k_t^{\gamma}, \tag{4.4c}$$

$$Y_{reduced,t} = Y_{gross,t}(1 - P_t^{1-\theta} \mu_t^{\theta} B_t), \tag{4.4d}$$

$$M_{atm,t+1} = E_t + \phi_{11} M_{atm,t} + \phi_{21} M_{up,t}, \tag{4.4e}$$

$$M_{up,t+1} = \phi_{12} M_{atm,t} + \phi_{22} M_{up,t} + \phi_{32} M_{lo,t}, \tag{4.4f}$$

$$M_{lo,t+1} = \phi_{23} M_{up,t} + \phi_{33} M_{lo,t}, \tag{4.4g}$$

$$F_t = \eta(ln \frac{M_{atm,t}}{M_{atm,1750}}) + F_{ex,t}, \tag{4.4h}$$

$$\zeta_1(\Xi_t) = \zeta_{10} + \zeta_{1\beta}(\Xi_t - \mathbb{E}(\Xi_t)), \tag{4.4i}$$

$$\tau(\Xi_t) = \frac{\eta}{\Xi_t}, \tag{4.4j}$$

$$T_{atm,t+1}(\Xi_t) = T_{atm,t} + \zeta_1(\Xi_t) \left(F_{t+1} - \tau(\Xi_t) T_{atm,t} - \zeta_2(T_{atm,t} - T_{ocean,t})\right), \tag{4.4k}$$

$$T_{ocean,t+1} = T_{ocean,t} + \zeta_3(T_{atm,t} - T_{ocean,t}), \tag{4.4l}$$

$$K_{t+1} \leq (1 - \delta k)^{\Delta t} K_t + Y_t S_t, \tag{4.4m}$$

$$D_t = 1 + a_1 T_{atm,t} + a_2 T_{atm,t}^{a_3}, \tag{4.4n}$$

$$Y_t = Y_{reduced,t} / D_t, \tag{4.4o}$$

$$C_t = Y_t(1 - S_t), \tag{4.4p}$$

$$U_t = L_t \frac{(\frac{C_t}{L_t} - 1)}{1 - \beta}, \tag{4.4q}$$

(4.4r)

And just as the model of equation (2.23), this model can be solved by honouring the bounds and initial conditions of table 2.1.

4.2 Limiting the number of uncertain stages

As introduced, this section reduces the number of stochastic stages by increasing the step size and defining a stochastic tree. This common approach (i.g. [23],[12]) has a counter part in the form of Approximate Dynamic Programming (ADP) (i.g. [65]). A drawback of ADP is the need of (complex) value function approximations, but in return it solves "the curses of dimensionality" by approaching the problem in a two-stage rolling horizon framework[59]. Nonetheless, the choice is made to change the number of stages for it helps to maintain the models accessibility. This section follows by first increasing the step size of the model, after which the stochastic tree is presented.

The time between stages should be a logical consequence of the time frame in which the fundamental process occurs. In the 2013 version of DICE, the five year time steps are based on the length of political cycles and do not representing any climate or investment cycles[49]. This misalignment can be used as an argument for the required extension of the time steps[12]. The extension is further justified as utility investment cycles cover decades and climate cycles cover even longer time spans[37][27].

Figure 4.1 shows that, according to Nordhaus, the transition to a renewable economy takes place during the next century. Based on this time window and the computational limitations, time steps of ten and twenty years are considered. Due to the burden of the remaining deterministic stages, the model with ten year time steps could only be solved for six stochastic stages. This model therefore did not cover the preferred period. The model with time steps of twenty years could be solved for eight stochastic stages, thereby covering the favoured one hundred years. An additional argument for using time steps of twenty years is that it fits the expected life time of common renewable technologies (i.e. wind turbines and photovoltaic cells) and medium-long term economic policy cycles[26][3][73]. Evidently, the model will be build to work with time steps of twenty years.

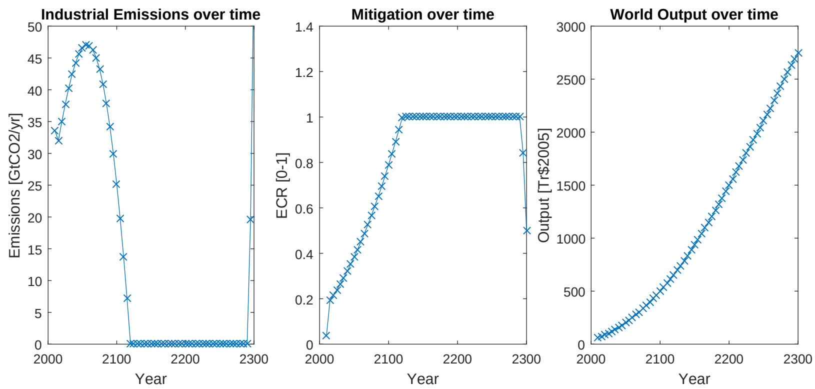

Figure 4.1: The industrial emissions, mitigation policy and the world output over time of the original DICE model[49]

Scaling the climate parameters regrading the mixing of the CO_2-reservoirs (ϕ), the temperature increase (ζ) and the decline rate of backstop technology cost (ΔBS), the original DICE model (D5) is converted into the model with time steps of twenty years (D20). The scaling of these parameters is presented in table 4.1. The results of two models are displayed in figure 4.2.

From this figure can be derived that the climate mitigation policy, aside from the deviation between 2010 and 2030, follows the same path for both models. This initial deviation is the result of the prolonged initial policy. A consequence of this is a somewhat higher emission rate in that same period and therefore, a rather higher atmospheric concentration. Additionally, due to an imperfect calibration of the climate parameters, the atmospheric temperature increases marginally faster during this period. In the periods after 2100, the atmospheric temperature falls slightly more, resulting in lower damages in this later period. The effect of this is a higher net world output, followed by an increase of investment in capital. Putting these deviations aside, it is clear that the D20 model has a similar system response as D5, making it fit for further experiments.

Table 4.1: Calibrated parameters for DICE20

	D5	D20
ϕ_{12}	0.088	0.0352
ϕ_{23}	0.0025	0.01
ζ_1	0.098	0.329
ζ_3	0.088	0.18
ζ_4	0.025	0.1
ΔBS	0.025	0.1

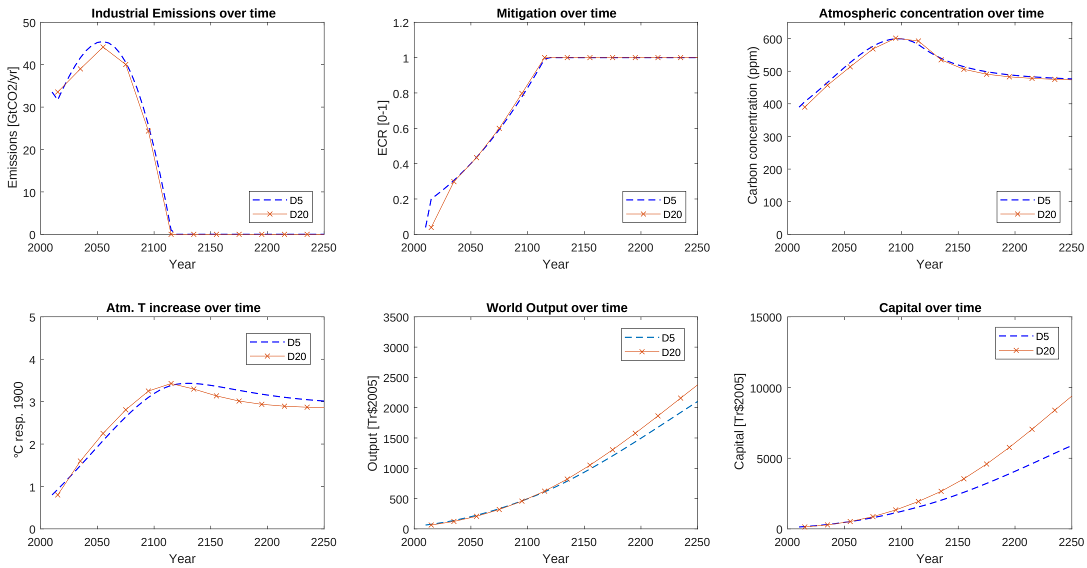

Figure 4.2: The industrial emissions, mitigation policy, atmospheric concentration, increase in atmospheric temperature, increase world output over time and the development of capital of the original DICE model (D5) versus that of the calibrated model with time steps of twenty years (D20).

In the case D20 is evaluated with only three realisations per stage, the solver would still have to evaluate over fourteen million scenarios, when covering all fifteen stages. It is therefore needed to limit the number of stochastic stages. Since the transition to a carbon neutral society is assumed to take place in the next hundred years, the focus is on the first stages of the model. This focus gives rise to the tree of figure 4.3. Section 5.2 will go into the sensitivity of the model to the number of uncertain stages. Again, here the focus will be regarding the uncertainty in the first stages.

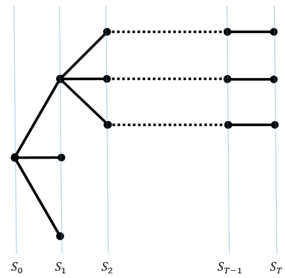

Figure 4.3: The stochastic tree structure used to reduce the computational burden of the EICE model

4.3 Limiting the number of scenarios per stage

As introduced in section 1.4, the main uncertainty in this study is the climate sensitivity parameter. The asymmetric distribution of this parameter gave rise to Weitzman's Dismal theorem[80][24]. This theorem proposes that for problems where the tail declines less quickly than in an exponential case, the expected resulting damage of the response becomes infinite and makes cost-benefit analysis an unfit approach[55]. Representing the probability distribution with only a few samples is the subject of this section.

Numerical experiments have shown that up to three scenarios can be evaluated per stage if stochastic stages are required to simulate more than a 100 years. A result of this limited number of samples is that only a crude and biased estimation can be made of the probability distribution and therefore its tail[20]. Perfectly representing this tail is even more challenging as the model is limited by a convexity constraint. This constraint states that the atmospheric temperature increase is limited to $9.1°C$, preventing the investigation of a nearly infinite climate sensitivity parameter[67]. Nonetheless, this section argues that within these limitations still sensible scenarios can be proposed to show the influence of such extreme tail events.

An argument in defence of a limited set of scenarios is the physical representation of the climate sensitivity parameter. As implied in section 1.4, it is believed that a temperature increase of only $6°C$ would already drastically change the world as we known it and therefore cause enormous economical damage[53]. From this perspective, it is therefore not necessary to evaluate extreme temperature rises, as only a small increase would cause immense damage. Further support for using a limited distribution is a lack of physical verification of such extreme claims, as they purely are a product of statistical error[8].

The scenarios which represent the climate sensitivity parameter are based on the current understanding of the matter and the sensitivity to its definition. The current consensus in literature estimates the climate sensitivity parameter to be likely (66-100% confident) between 1.5°C and 4.5°C, with an average around 3°C per doubling of pre-industrial CO_2 levels[53]. More specifically, it is assumed to be extremely unlikely (0-1%) to be less than 1°C. The reason for this is that it is assumed that internal feedbacks will have a positive influence on the 1.2°C increase of a system without feedback[62]. On the other side of the spectrum, it is also assumed to be very unlikely (0-10%) to be greater than 6°C[53]. Based on these characteristics and the limitation set by the curse of dimensionality, the model will be tested with the three realisations of the base case in table 4.2. The values in the base case of this table are based on the work by Golub[24].

The sampling bias, as a result of the crude estimation, is in this base case used to emphasise undesirable outcomes. An extreme case is constructed to further explore the influence of the shape of the tail. In this case the highest value of the discrete distribution is increased based on the work of Roe and Backer[62]. This case is also presented in table 4.2.

Table 4.2: Climate sensitivity scenarios

	Climate Sensitivity $[^{\circ}C/CO_{2,2x}]$	Probability [%]	
	Asymmetrical	Uniform	
Base case	2.2	25	33
	3.0	50	33
	4.3	25	33
Extreme case	2.2	25	33
	3.0	50	33
	8.0	25	33

In both the base and extreme case, it is assumed that the level of knowledge is developed enough to make assumptions about the probability distribution[62][56]. As feedback loops are not well understood and are deemed to be fundamentally uncertain, a case can be made to equally weigh all scenarios[76]. The uniform distribution in these fundamental uncertain cases puts (again) more emphasis on tail outcomes and thus approaches the ideas of Weitzman some more.

The following chapter will use these four cases, and show the model's response to uncertainty. This is done by generating results which can be used to debate Weitzman's claims regarding the use of cost-benefit analysis.

Chapter 5

Results

In order to answer the main question: "How does the advised mitigation policy by DICE respond to the influence of an uncertain climate sensitivity parameter?", the response of the EICE model is tested. This response will be examined by varying the duration and the intensity of uncertainty. The results of these tests are described in this chapter. While covering the test results, the focus is on the mitigation policy, the industrial emissions over time, the development of the atmospheric concentrations over time, the increase in atmospheric temperature in respect to 1900 over time and the development of net world output and capital. These variables are selected based on their representation of the state space and the decision variables. Herein, a visualisation of μ and K are given to represent climate policy and the saving rate, whereas E_{ind}, T_{atm} and Y describe the state of the system. Since the state variables: T_{ocean}, M_u and M_l are the result of fixed mechanics in the model and do not provide additional insight into the influence of uncertainty, they are left out in order to present uncluttered results. For all models, variations of the GAMS program in appendix B are used. Enlarged versions of the graphs, including an extension to the damage function, the savings rate and the carbon price, are presented in appendix C.

Before going into the response of the model, the model is verified in section 5.1. This verification is by means of evaluating the response in known and extreme outcomes. Sequentially, the sensitivity to the number of uncertain stages is tested in section 5.2. Due to computational limitations not all fifteen stages can be evaluated under multiple scenarios. The influence of this limitation is presented in this section.

The response of the model is dissected into two parts. The first part covers the overall response of the model to uncertainty, whereas the second part focusses on the influence of the shape of the probability curve. In section 5.3, the response of EICE under base case conditions is compared to a scenario analysis of D20. The aim of this test is to give insight into the differences between stochastic programming and scenario analysis and to compare the resulting hedging strategies. The following test, of section 5.4, compares the response of EICE with that of D20. To be able to compare results, the base case is evaluated in EICE and the expected value of this base case is implemented in D20. Here, the aim is to see whether the advised policy resulting form EICE differs from D20. The first test of the second part is presented in section 5.5. Here the base case is compared with the extreme case. After this comparison, both these cases are compared with their uniformly distributed equivalent in section 5.6. These two tests will give insight into the sensitivity to uncertainty and its level. The outcome of these tests are vital to the discussion regarding the Dismal debate, for they will give insight into the influence of possible catastrophic events.

The chapter concludes by evaluating the carbon price and the influence of uncertainty on utility in sections 5.7 and 5.8, respectively. As the carbon price is the main instrument of the climate policy, insight into the response to uncertainty of this variable will give insight into the response of advised policy to uncertainty. This evaluation will be about all previously presented cases. Since the climate mitigation policy is already evaluated under these cases, this test mainly has the function of verifying previous results. Evaluating the sensitivity of the utility function to uncertainty shows the response of the objective function and gives insight into possible benefits of the stochastic approach.

5.1 Validation of the new model

This section aims to verify the EICE model. This verification consists of proving that the outcome of this model corresponds to the outcome of the D20 model when only one realisation of the climate sensitivity parameter is evaluated. To show whether the response is the same, three tests are carried out:

1. a comparison of the D20 model and EICE with one possible realisation;
2. a comparison of the D20 model and EICE, where EICE has tree equally valued possible realisations;
3. a comparison of the D20 model and EICE, where extreme outcomes are evaluated.

During the tests, EICE with seven stochastic stages is used. For the tests, the climate sensitivity is set to 2.95, 1.00 and 6.00. The realisation of the climate sensitivity parameter at 2.95 is used for the first two tests, 1.00 and 6.00 are used for the third. These last two realisations are selected based on the probability curve proposed by the IPCC[53]. Here, it is assumed that the climate sensitivity parameter will be probably higher than $1[^{\circ}C]$ per doubling of CO_2-levels $[^{\circ}C/2xCO_2]$ and probably lower than $6[^{\circ}C/2xCO_2]$, as discussed in section 4.3.

Since the aim is to show that results do not differ, they are presented in numerals instead of graphs to show possible minor deviations. The results of these tests will be represented by the two decision variables, μ_t and S_t, and a representation of the state of the world by $T_{atm,t}$ and K_t.

Looking at tables 5.1 to 5.4, in which all three test are presented, it is clear that based on the three proposed tests the stochastic model is valid, as the results of the deterministic and the stochastic model are equal.

Table 5.1: Comparison of mitigation policies between the 15 stage deterministic (D) models and their stochastic (SP) counterparts with either 1 (SPx1) or 3 (SPx3) evaluated climate sensitivity (CS) realisations with a value of 2.95, 1.00 and 6.00.

		CS = 2.95			CS = 1.00		CS = 6.00	
stage	year	D	SPx1	SPx3	D	SPx3	D	SPx3
1	2015	0.039	0.039	0.039	0.039	0.039	0.039	0.039
2	2035	0.287	0.287	0.287	0.125	0.125	0.408	0.408
3	2055	0.417	0.417	0.417	0.174	0.174	0.604	0.604
4	2075	0.576	0.576	0.576	0.240	0.240	0.854	0.854
5	2095	0.765	0.765	0.765	0.307	0.307	1.000	1.000
6	2115	0.988	0.988	0.988	0.390	0.390	1.000	1.000
7	2135	1.000	1.000	1.000	0.480	0.480	1.000	1.000
8	2155	1.000	1.000	1.000	0.583	0.583	1.000	1.000
9	2175	1.000	1.000	1.000	0.694	0.694	1.000	1.000
10	2195	1.000	1.000	1.000	0.813	0.813	1.000	1.000
11	2215	1.000	1.000	1.000	0.930	0.930	1.000	1.000
12	2235	1.000	1.000	1.000	1.000	1.000	1.000	1.000
13	2255	1.000	1.000	1.000	1.000	1.000	1.000	1.000
14	2275	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	2295	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.2: Comparison of saving policies between the 15 stage deterministic (D) models and their stochastic (SP) counterparts with either 1 (SPx1) or 3 (SPx3) evaluated climate sensitivity (CS) realisations with a value of 2.95, 1.00 and 6.00.

		CS = 2.95			CS = 1.00		CS = 6.00	
stage	year	D	SPx1	SPx3	D	SPx3	D	SPx3
1	2015	0.208	0.208	0.208	0.209	0.209	0.207	0.207
2	2035	0.197	0.197	0.197	0.198	0.198	0.198	0.198
3	2055	0.193	0.193	0.193	0.193	0.193	0.195	0.195
4	2075	0.193	0.193	0.193	0.192	0.192	0.194	0.194
5	2095	0.194	0.194	0.194	0.193	0.193	0.194	0.194
6	2115	0.194	0.194	0.194	0.194	0.194	0.195	0.195
7	2135	0.197	0.197	0.197	0.197	0.197	0.196	0.196
8	2155	0.198	0.198	0.198	0.198	0.198	0.198	0.198
9	2175	0.199	0.199	0.199	0.200	0.200	0.200	0.200
10	2195	0.201	0.201	0.201	0.201	0.201	0.201	0.201
11	2215	0.202	0.202	0.202	0.202	0.202	0.202	0.202
12	2235	0.202	0.202	0.202	0.202	0.202	0.202	0.202
13	2255	0.197	0.197	0.197	0.197	0.197	0.197	0.197
14	2275	0.258	0.258	0.258	0.258	0.258	0.258	0.258
15	2295	0.258	0.258	0.258	0.258	0.258	0.258	0.258

Table 5.3: Comparison of atmospheric temperature [°C i.r.t. 1900] increase between the 15 stage deterministic (D) models and their stochastic (SP) counterparts with either 1 (SPx1) or 3 (SPx3) evaluated climate sensitivity (CS) realisations with a value of 2.95, 1.00 and 6.00.

		CS = 2.95			CS = 1.00		CS = 6.00	
stage	year	D	SPx1	SPx3	D	SPx3	D	SPx3
1	2015	0.800	0.800	0.800	0.800	0.800	0.800	0.800
2	2035	1.560	1.560	1.560	0.773	0.773	1.766	1.766
3	2055	2.185	2.185	2.185	1.185	1.185	2.652	2.652
4	2075	2.728	2.728	2.728	1.360	1.360	3.417	3.417
5	2095	3.168	3.168	3.168	1.651	1.651	3.942	3.942
6	2115	3.365	3.365	3.365	1.766	1.766	4.139	4.139
7	2135	3.249	3.249	3.249	1.913	1.913	4.198	4.198
8	2155	3.085	3.085	3.085	1.974	1.974	4.207	4.207
9	2175	2.958	2.958	2.958	2.014	2.014	4.204	4.204
10	2195	2.877	2.877	2.877	1.999	1.999	4.202	4.202
11	2215	2.830	2.830	2.830	1.947	1.947	4.206	4.206
12	2235	2.804	2.804	2.804	1.858	1.858	4.215	4.215
13	2255	2.790	2.790	2.790	1.782	1.782	4.228	4.228
14	2275	2.782	2.782	2.782	1.757	1.757	4.243	4.243
15	2295	2.777	2.777	2.777	1.733	1.733	4.258	4.258

Table 5.4: Comparison of capital accumulation[Tr$\$_{2005}$] between the 15 stage deterministic (D) models and their stochastic (SP) counterparts with either 1 (SPx1) or 3(SPx3) evaluated climate sensitivity (CS) realisations with a value of 2.95, 1.00 and 6.00.

	CS = 2.95			CS = 1.00		CS = 6.00	
year	D	SPx1	SPx3	D	SPx3	D	SPx3
2015	135.000	135.000	135.000	135.000	135.000	135.000	135.000
2035	280.326	280.326	280.326	281.399	281.399	279.795	279.795
2055	517.568	517.568	517.568	523.369	523.369	516.189	516.189
2075	866.033	866.033	866.033	880.199	880.199	862.896	862.896
2095	1,339.678	1,339.678	1,339.678	1,369.372	1,369.372	1,318.544	1,318.544
2115	1,943.919	1,943.919	1,943.919	1,998.331	1,998.331	1,884.812	1,884.812
2135	2,663.343	2,663.343	2,663.343	2,775.201	2,775.201	2,599.702	2,599.702
2155	3,548.570	3,548.570	3,548.570	3,695.225	3,695.225	3,464.458	3,464.458
2175	4,591.965	4,591.965	4,591.965	4,753.261	4,753.261	4,470.273	4,470.273
2195	5,770.248	5,770.248	5,770.248	5,933.752	5,933.752	5,599.920	5,599.920
2215	7,053.653	7,053.653	7,053.653	7,216.611	7,216.611	6,828.774	6,828.774
2235	8,402.331	8,402.331	8,402.331	8,565.345	8,565.345	8,120.352	8,120.352
2255	9,738.882	9,738.882	9,738.882	9,909.991	9,909.991	9,400.389	9,400.389
2275	10,833.467	10,833.467	10,833.467	11,022.551	11,022.551	10,446.892	10,446.892
2295	15,394.257	15,394.257	15,394.257	15,664.829	15,664.829	14,830.585	14,830.585

5.2 Sensitivity to the number of uncertain stages

As introduced earlier, one of the major drawbacks to stochastic programming is the curse of dimensionality. Since the extent of uncertain stages is limited, it is important to know the influence of the duration of uncertainty. To show this influence, a test is set up where the number of uncertain stages are varied from four to eight. Models with more than eight stochastic stages result in infeasible solutions, as convergence is too slow. During the simulations, all models had the same base distribution from section 4.3. The expected climate sensitivity parameter after the uncertain stages is set to be 2.95. This value is based on the realised value of $\zeta(\xi_t)$. As a benchmark, these results are compared with the D20 model with the same expected climate sensitivity parameter.

As seen in figure 5.1 and its zoomed version in 5.2 or their enlarged versions in appendix C.1, the influence of ongoing uncertainty is a slightly more strict climate policy. In the case where only four uncertain stages are included ($SP20_4$), the emissions are fully controlled at the seventh stage in 2135, just as in the deterministic case. This transition is almost complete (0.997) a stage earlier, for the case with eight uncertain stages ($SP20_8$).

When looking at the mitigation over time graph, it can be stated that overall an increase in the duration of uncertainty results in an increasingly active policy. Note that this holds for all cases except the case with six uncertain stages ($SP20_6$). In that case the mitigation policy is less strict than all other evaluated cases. Important to note is that as the number of stages increase, the difference between stages becomes smaller.

When looking at the development of industrial emissions over time as a function of uncertain stages, it is clear that the more strict mitigation policy results in lower emissions. The main deviations between the simulations occurs just after the peak at 2055. Here, gradients vary from -0.07 for $SP20_4$ to -0.09 $GTCO_2/dec$ for $SP20_8$, increasing in steepness with the number of uncertain stages. The steeper mitigation policy also results in a lower carbon concentration peak. As a result of the increase in uncertain stages the peak of all cases is reduced with respect to the deterministic case. The deviation between stages with an ascending number of uncertain stages becomes less significant. The highest deviation in industrial emissions between the cases is at 2095, the fifth stage. After this stage the deviations become smaller tending to towards zero. This trend of diminishing variation is the result of full emission control in 2135.

As a result of almost full emission control of the $SP20_8$ case in 2115, the carbon concentration peaks in 2095, instead of in 2115. The major deviation between these cases is found at this stage. Here, gradients vastly differ and the cases with a higher number of uncertain stage cases do follow the deterministic case less closely. Up to the fourth stage in 2075, the concentration difference between the five cases is marginally small. They deviate until the 2135, after which they converge again. In the later stages of the model, all cases show equivalent behaviour. Overall the economic variables, net world output and capital, are marginally influenced by the number of uncertain stages as they are almost equal for all cases. There is marginal negative influence of uncertainty on the economic variables creating a deviation of two trillion USD in output and a deviation of seven trillion USD in capital in 2235.

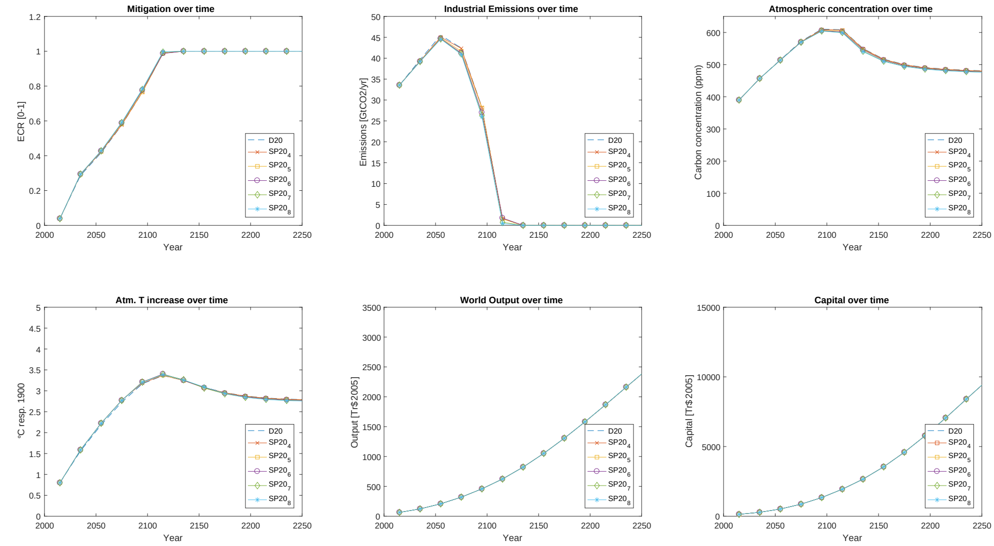

Figure 5.1: The influence of the number of stochastic stages to SP20 under base conditions

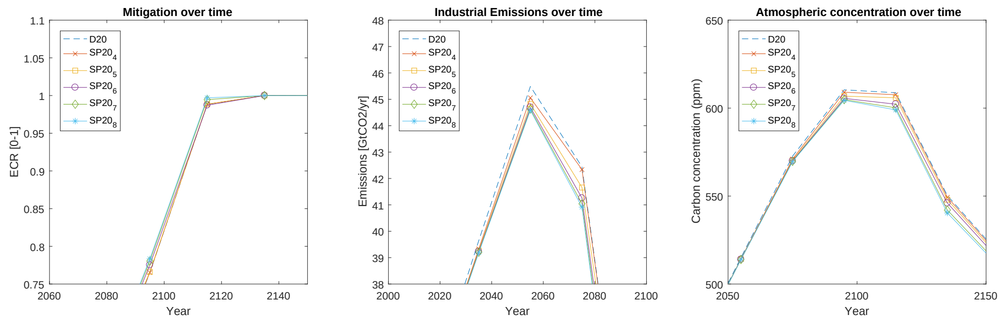

Figure 5.2: The influence of the number of stochastic stages to SP20 under base conditions

5.3 Scenario analysis versus stochastic programming

As the main goal of stochastic programming is to include uncertainty into the decision making process, it is interesting to see the response of the model in regard to the individual deterministic scenarios. In order to demonstrate these deviations, three deterministic cases are plotted against the EICE model under base conditions. The deterministic cases have a climate sensitivity of 2.2 (D20E220), 3.0 (D20E300) and 4.3 (D20E430). Thus, the same scenarios are evaluated as in the stochastic case. To represent the stochastic case, the model with eight uncertain stages is used, $SP20_8$. Again, here the expected climate sensitivity parameter of 2.95 is used.

As can be observed from figure 5.3 the transient behavior of all evaluated cases is equal. For the economic variables, net world output and capital development, deviations are marginal. Here, the stochastic case coincided with $D20E300$ and is exceeded by $D20E220$, but is higher than $D20E430$ for all stages. This framed relation holds for all other displayed variables. Slight deviations in the level of capital are a result of increased savings as presented in figure 5.4.

When looking at the mitigation graph, it is clear that a higher climate sensitivity results in a steeper emission control rate. While all cases demonstrate the same transient behavior, the climate sensitivity influences the time at which the emission control rate is almost under full control. A steeper mitigation policy results in a higher carbon price. This relative increase can be observed in figure 5.4. When comparing the stochastic case with $D20E300$, it can be seen that $SP20_8$ prefers a more strict climate policy early on, to be surpassed by $D20E300$ between 2095 and 2115. More on the difference between the stochastic and deterministic model is discussed in the successive section.

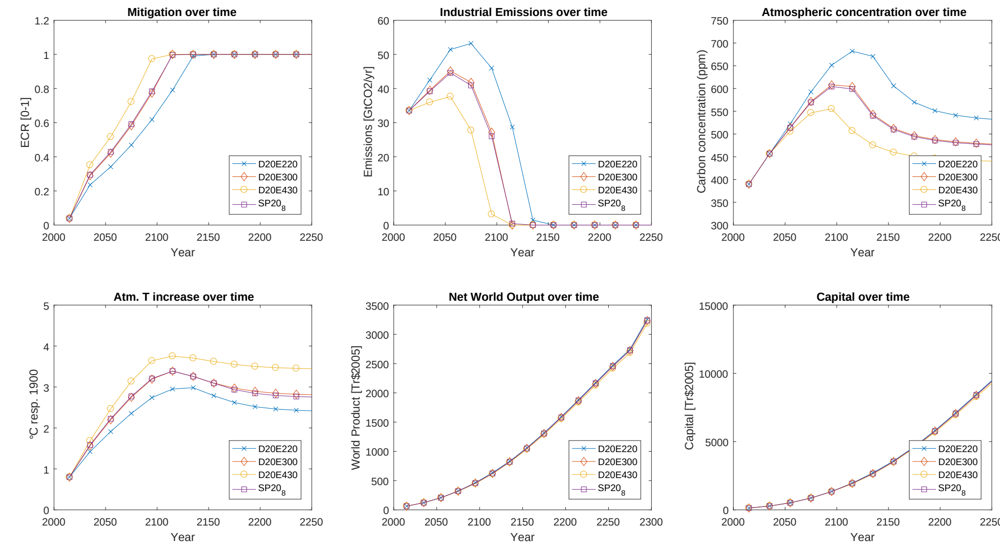

Figure 5.3: Comparison of the deterministic scenarios with CS:={2.2, 3.0, 4.3} and the stochastic program in the base case

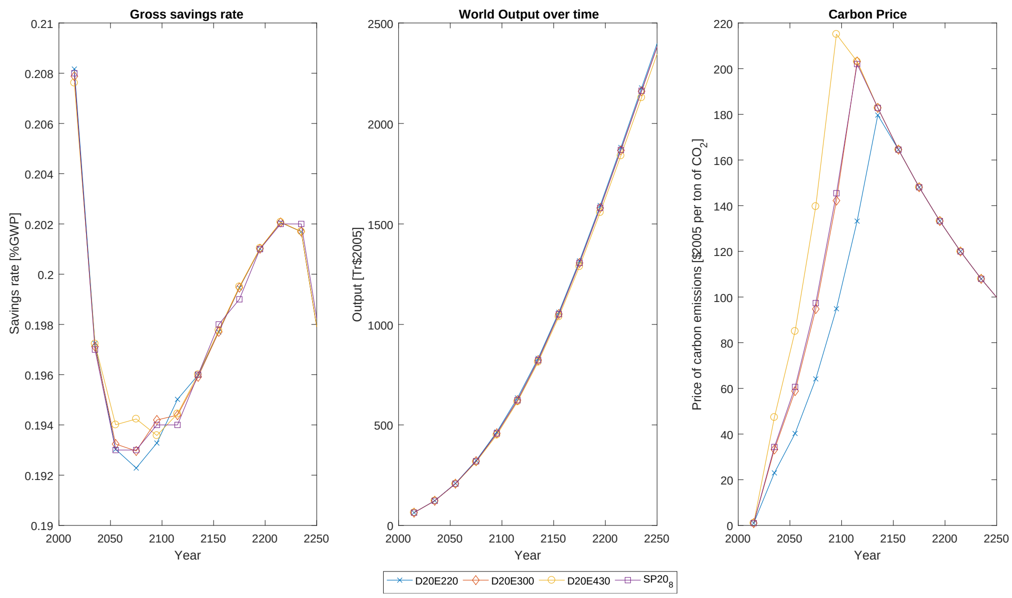

Figure 5.4: Comparison of the economic response from the deterministic scenarios with CS:={2.2, 3.0, 4.3} and the stochastic program in the base case

5.4 Comparison of the base and the expected case

The major argument for stochastic programming, is that a model which neglects uncertainty, results in a non-optimal policy. To test whether this statement is just, a comparison is made between the deterministic and the stochastic model. Based on the average value of the base case, the expected climate parameter is set to 3.125 in the deterministic model, $D20E3125$. The output of this model is compared with the stochastic $SP20_8$ model, in which the base case is used to represent uncertainty. The expected value of the climate sensitivity parameter after the eight uncertain stages, is again set at 2.95. In line with the results of Crost and Traeger it is expected that the deterministic case has a slightly more strict mitigation policy[16].

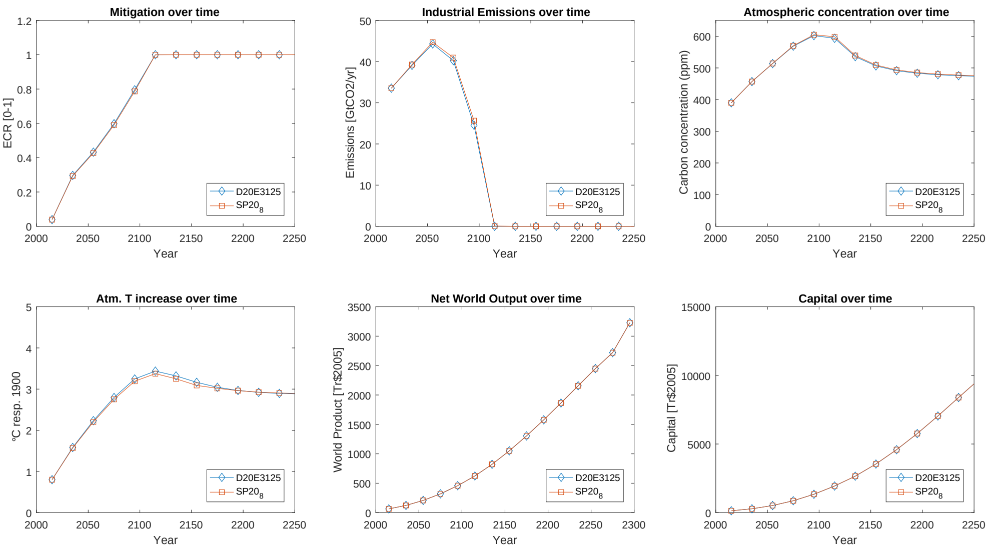

Figure 5.5: The deterministic D20 program with CS = 3.125 versus the stochastic $SP20_8$ model under the base case

Figure 5.5 shows a strong correlation ($\rho = 0.9998$) between the deterministic case and its stochastic extension. From the mitigation graph, it can be seen that the deterministic case has a slightly more stringent mitigation policy, which is in line with the results by Crost and Traeger. The stricter policy also becomes clear from the industrial emissions curve, where the results of $SP20_8$ are always above the curve of $D20E3125$. The same holds for the atmospheric concentration, unlike the temperature increase. Until 2075 temperature increase is relatively equal. After this stage, temperature rises more in the deterministic case. For both cases temperature increase peaks in 2115. After this stage the curves converge. As found in previous comparisons the economic variables, net world output and capital, are marginally negatively influenced by the uncertainty in climate response.

5.5 Comparison of the base and the extreme case

In order to add to the fat-tail debate, the base and the extreme case of section 4.3 are compared. Though the scenarios are based on the current understanding of the climate sensitivity parameter, this test is mainly useful to show the influence of the shape of the probability curve. For these tests it is expected that a "fatter" distribution supports claims for a more strict policy. The comparison is based on models with eight uncertain stages. From the ninth stage on, the climate sensitivity is set to 2.95 and 3.2, for the base and extreme case respectively. These values are again based on the realisation of the uncertainty in $\zeta(\xi_t)$.

As previously found, and again confirmed in the graphs of figure 5.6, the economic variables are marginally influenced by the uncertainty in the climate response. For both variables, the base case has a slightly higher value. A distinct difference in the other variables is the steeper increase in climate mitigation action. As a result the transition to an industrial emission free economy is achieved earlier with the extreme policy. In that case a complete transition is realised in 2115, with respect to 2135 in the base case. Nonetheless, the difference in industrial emissions is small at this stage. The deviation is much larger (around 40% of relative emissions) in 2075. As a result of the stricter climate policy, the carbon concentrations increase less in the extreme case. Even so, the atmospheric temperature increase is higher in the extreme case during the uncertain stages of the model. After industrial emission have stopped the atmospheric concentrations start to converge, in resemblance to the atmospheric temperatures.

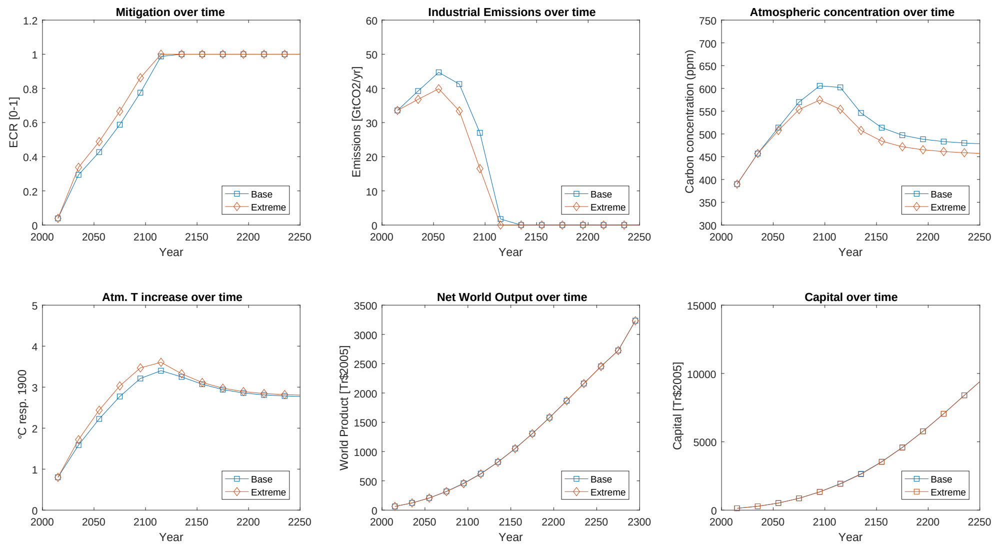

Figure 5.6: Comparison of the base and the extreme case

5.6 Comparison of the base and the extreme case and their uniform equivalent

As introduced in section 4.3, the debate regarding the fat-tail will be extended by including the level of uncertainty. In line with the previous hypothesis, it is assumed that the uniform distribution will support claims for a stricter policy. The proposed cases are simulated in models with seven uncertain stages. In figures 5.7 and 5.8, asymmetric cases are labelled $SP20_7$ and uniform cases as $SP20_{7u}$. In these models, after the seventh stage an estimated value is used. The estimated climate sensitivity for the base case is again set to 2.95 for both scenarios and 3.20 for both scenarios of the extreme case.

Looking at the influence of a higher level of uncertainty for the base case in figure 5.7, it becomes apparent that the level of uncertainty has nearly no influence on the advised policy in the base case. To indicate, how identical the mitigation response is, the industrial emissions only deviate by $0.16[GtC]$ at their production peak in 2055. The lower production of industrial emissions results in a lower atmospheric concentration, than the distributed case. Nonetheless, the atmospheric temperature of the uniformly distributed case is marginally higher ($0.015[°C]$ in 2115). As with all previous tests the influence on the economic variables is neglectable. When looking at the net world output, the uniform distribution is preferred. The opposite is true for the level of capital.

More significant deviations are presented in figure 5.8, where the extreme case is evaluated. The stricter climate policy for the uniformly distributed case results in minor deviations in industrial emissions. With a correlation factor of 0.9997, these deviations are not big enough to truly alter the advised policy. For example, no shift in transition time is observed. As a result of the lower emissions, the carbon concentration is less in respect to the asymmetrically distributed case. The increase in temperature for the uniformly distributed case is higher until 2135, after which the lower carbon concentration and the shared climate sensitivity parameter result in a relatively higher temperature increase for the asymmetrical distributed case. From an economic perspective, based on the net world output and the accumulation of capital, the asymmetrically distributed case is preferred.

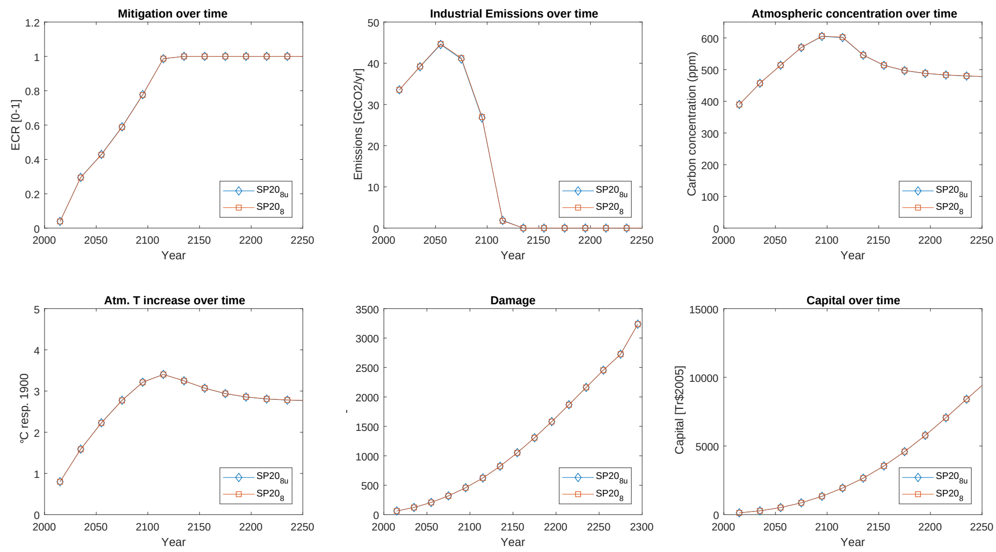

Figure 5.7: Comparison of the base case and its uniform extension

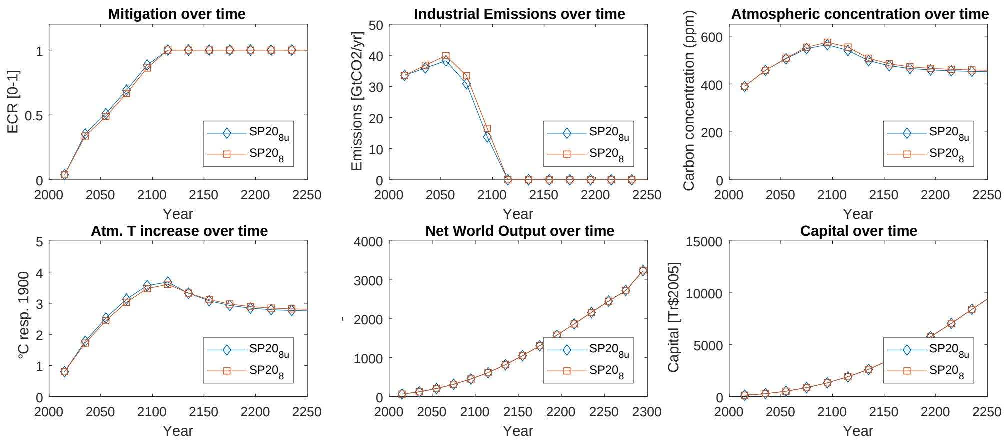

Figure 5.8: Comparison of the extreme case and its uniform extension

5.7 Fluctuations in the carbon price

Knowing how the carbon price will react to (the level of) uncertainty is of crucial importance for sound policy advice. Here the expectation is that a more uncertainty will lead to a higher carbon price. To show the influence of uncertainty, five cases are compared. These are the expected base case , the base case with an asymmetric and an uniform distribution and the extreme case with both distributions and an expected climate sensitivity parameter of 3.20. For all models seven stochastic stages are included. The results of the simulations are presented in figure 5.9.

This figure shows that in respect to the expected case, that for the base case the advice is for a marginally lower carbon tax and the extreme cases advice upon a notable higher carbon price. As the uncertainty increases, so does the carbon price. These deviations are relatively small for the base cases. The deviations are more significant for the extreme cases. Nonetheless, similar systemic behaviour is clear with an correlation factor of 0.9723 for the two most deviating cases, the expected and the uniformly distributed extreme case.

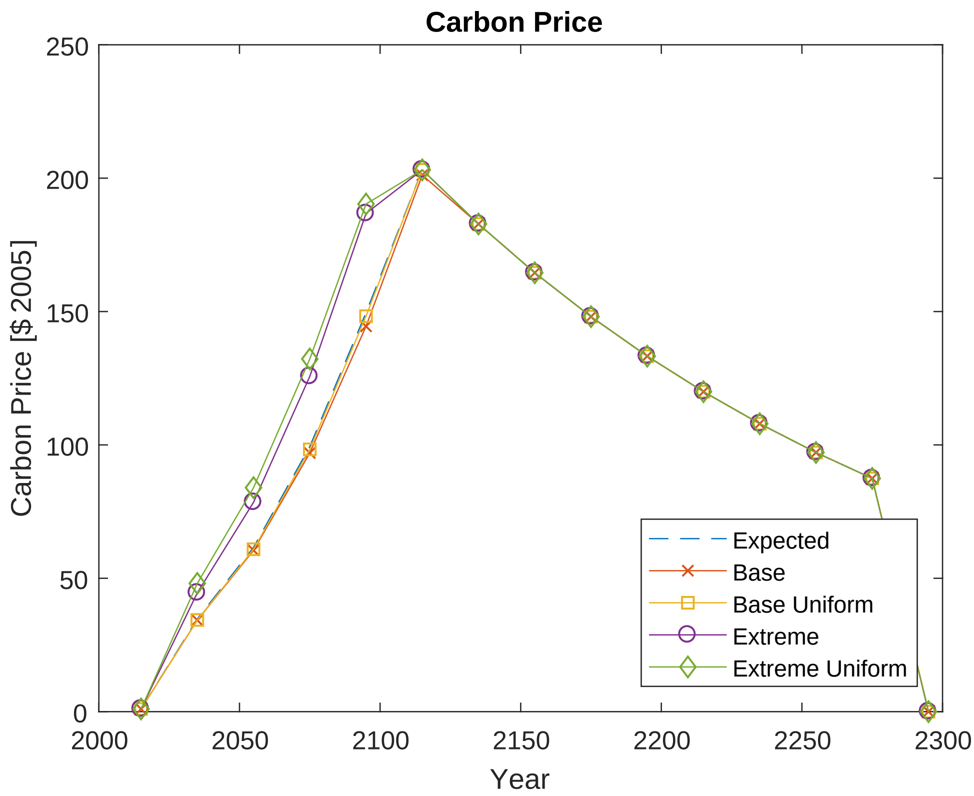

Figure 5.9: Comparison of the carbon price of the expected, base, base uniform, extreme and extreme uniform case

5.8 Sensitivity of utility to uncertainty

The objective of DICE or EICE is to maximize the utility function. The previously introduced cases are used to demonstrate the influence of uncertainty to the accumulation of utility.

Table 5.5 shows that fluctuations in the value of the climate sensitivity parameter have a minor influence on the outcome of the objective function. Additionally it shows that extending the number of uncertain stages results in a lower accumulated utility. The deviations between the cases decreases with the number of stages. In general, all values fall within the range of 3128 to 3191 with a modulus around 3175 utils. The lowest value is achieved in the worst case scenario, with a climate sensitivity parameter of 8.00 and its opposite in the 2.20 case.

Table 5.5: Accumulated utility of compared cases

	Utility	$\mathbb{E}CS$
$Expected$	3177.061	2.95
$Expected$	3190.168	2.20
$Expected$	3176.273	3.00
$Expected$	3159.761	4.30
$Expected$	3128.901	8.00
$Base_4$	3175.871	2.95
$Base_5$	3175.533	2.95
$Base_6$	3175.294	2.95
$Base_7$	3175.152	2.95
$Base_8$	3175.067	2.95
$Base_{6u}$	3174.775	2.95
$Extreme_6$	3164.144	3.20
$Extreme_{6u}$	3160.113	3.20

Chapter 6

Discussion

The aim of following discussion is to revisit the model, reflect on the results and debate possible implications of the found results. This discussion is divided into two sections covering these subjects. The resulting discussion will form the basis of the conclusions in chapter 7.

6.1 Discussing the EICE model

The following discussion will go into the design choices regarding EICE. Here, the debate focusses on the sensitivity of the result to the number of stochastic decisions, the sensitivity to the probability distribution and the influence of the inclusion of endogenous parameters. In addition, it states whether these decisions are justified.

Stochastic tree

The first design assumption that is discussed is the number of stochastic stages in the decision tree. Section 4.2 already discussed the influence of bigger time steps. Here the main findings were that a prolonged initial policy caused some deviations and that naturally the number of re-evaluations became less, resulting in a more abrupt function. Therefore, the selection of a longer time step resulted in a higher sensitivity to the initial policy. This section analyses the influence of the number of stochastic decisions in the model.

As seen in figure 5.1, the duration of uncertainty results in a slightly more stringent mitigation policy. From the same figure, the marginalisation of this influence also becomes apparent. The asymptotic behaviour justifies the design assumption that not all stages have to be evaluated under uncertainty, as proposed in section 4.2. The marginalisation of the influence of continuing uncertainty is a consequence of the end of industrial emissions in the seventh stage.

Successive stages with uncertainty can only influence the total energy transition time, instead of the current level of mitigation. The deterministic fat-tail scenario in the base case of $D20E430$ in figure 5.3 shows that this transition time will never be within five stages under assumed base conditions. The same figure also shows that changes in transition time are dominated by the value of the climate sensitivity parameter.

Concluding; results based on models with seven uncertain stages or higher are deemed to be "good enough". This conclusion is supported by the data of table 5.5. Here, an increase of the number of uncertain stages results in a reduction of the accumulation in utility. This reduction marginalises with an increase in the number of stochastic stages and supports the claim that models with more that six uncertain stages represent the systemic behaviour of a model with a fully stochastic tree. Therefore, the design assumption regarding the number of stochastic stages is justified.

Climate sensitivity parameter

With respect to the number of uncertain stages, the model is more sensitive to the value of the climate sensitivity parameter as seen in figure 5.3 and table 5.5. As the influence of the sample size is already debated in section 4.3 and forms the subject of section 6.2, the following subsection will go into the sensitivity of the model to the distribution of the climate sensitivity parameter. Whether the selected cases are justified is treated in section 6.2.

From figure 5.1 can be deduced that the possibility of a high climate sensitivity suggests a notably more active mitigation policy. Contrariwise, figure 5.5 also shows that when evaluating the expected case with the stochastic version, the advise is on a slightly less strict policy. This result is the product of a slightly lower realisation of the climate sensitivity parameter, at 2.95 instead of the expected 3.125 for the base case. This realised version of the climate sensitivity is derived from ζ_1.[1] The stochastic tree, available in the IST.file of EICE, shows that all 729 (3^6) scenarios are evaluated. This deviation in expected climate sensitivity is therefore a result of internal mechanics of the model.

The current hypothesis is that; due to the limited climate damage as a result of temperature increase and the lack of strict bounds, the model finds that it is more efficient to "underestimate" the climate sensitivity parameter. The low sensitivity to uncertainty of the model can at least be partially explained by the little amount of negative capital resulting from emissions, equation (4.4n). When implementing this damage function, Nordhaus proposes the values of; $a_1 = 0$, $a_2 = 0.0026$ and $a_3 = 2$. With these settings and an extreme case, where the world is in an unrecognisable state at an atmospheric temperature increase of $8[^\circ C]$ in respect to 1900, the net world output (only) decreases by 15[%]. In the results of chapter 5, atmospheric temperature increase stays below $4[^\circ C]$ and therefore below a negative capital of 4[%] of the net world output. These relatively low percentages and the inability to inflict damage on existing levels of welfare dampen the effect of extreme outcomes. A revision of the damage function, justified by the statement that in excess of $3[^\circ C]$ no quantifiable estimates about the damage could be made, in section 1.4, could increase the sensitivity of the model to the shape of the probability curve. An other option, an possibly a more preferable one, is to set a temperature bound. Such a bound is preferable as future damages are defined to be unquantifiable in the same section.

A revision of the damage function could be justified by the statement that in excess of $3[^\circ C]$ no quantifiable estimates about the damage can be made, as stated in section 1.4. Such a revision could increase the sensitivity of the model to the shape of the probability curve. Nonetheless, as damages are deemed to be fundamentally uncertain, as stated in the same section, such a function can only be used to show the sensitivity of the model. In all likelihood such a revised damage function would spawn a new (ethical) debate. An other option, an possibly a more preferable one, is to set a temperature bound and avoid this ethical debate regarding catastrophic events.

Concluding, it is clear that the model is sensitive to the distribution of the climate sensitivity parameter and that an increase in uncertainty, a fatter distribution, advocates for a steeper mitigation curve. To achieve this, the model suggests a higher carbon price. The resulting cost has a negative effect on the accumulation of utility. Even so, no grave differences are found. Hence, the model's results are influenced by the expectation of extreme climate outcomes, but

[1] For the extreme case, the realisation of the climate sensitivity parameter is also underestimated, 3.2 versus 4.05. Hence can be concluded that, the model is influenced by variations in the distribution.

these might be smaller then expected. In extend, it is advised to view the results of the model in respect to the knowledge of the dampening damage function and advised to include temperature bounds while searching for an optimal policy.

Inclusion of uncertainty

The aim of the following discussion is the show the influence of the choice for endogenous uncertainty. Endogenous uncertainty directly led to stochastic programming and this has been defining for the project. The obvious alternative would have been to work with exogenous uncertainty in a Monte Carlo type of approach. Figure 5.5 shows that a scenario with the same expected value has a correlation of almost one. This high correlation is a result of excluding probabilistic bounds and a relative insensitivity to extreme values of the climate sensitivity parameter.

Regarding the use of probabilistic bounds, as only a coarse approximation of the probability curve is tractable, the implications of this technique are present as the end-of-the-tail realisations are not evaluated.

In addition to the information of figure 5.5, figure 5.3 shows a very simple scenario analysis with respect to EICE. From these graphs can be concluded that even though the models react relatively the same, the scenario analysis method only verifies possible policies instead of advising upon one. Nonetheless, when the objective values of the scenario analysis (U_{sa}) are compared with the stochastic solution (U_{sp}), where the utilities of all scenarios are summed with respect to their weights (p_i) and this summed utility is subtracted from the accumulated utility of the stochastic model, no additional value for the stochastic solution (VSS) is found[7]. In summary, the VSS is calculated according to:

$$VSS = U_{sp} - \Sigma_{i=1}^{n} p_i U_{sa,i}. \tag{6.1}$$

Since the VSS is marginally close to zero, the current solution does not offer a financial advantage over scenario analysis. Again, this is (partially) a result of the dampened effect of extreme outcomes by the damage function. Thus concluding, the current model does, in addition to providing actual policy advice, not give a hedging strategy that preforms better than the equivalent scenario analysis policy.

6.2 The results of EICE and its implications

The following section will reflect upon the results of chapter 5 in two ways. The first approach is to look at the results in respect to the abstract evaluations of DICE. Here, the focus is on the Dismal theorem and its counterarguments. This discussing is followed by a comparison of the results found in literature and the implications of the optimal control policy suggested by EICE.

Counterarguments to the Dismal theorem

The original DICE model argues for modest emission control in the nearby future. This control should be increased as time progresses, stretching the energy transition till 2150[49]. The resulting J-curve policy is often debated on three major points: 1) the discounting of future generations, 2) the possibility of abrupt climate change due to feedback loops and 3) the increasing concern of potential "tail" events and their catastrophic changes as a result of climate change[68][65][12]. This last field of interest is in line with the thesis of this report and the Dismal theorem.

When working with asymmetric distributions, the possibility of these extreme events cannot be ignored. Weitzman estimates, based on data from the IPCC's fifth assessment report, that the climate sensitivity parameter could be in the order of 10 to $20°C$[80]. Though these estimates are based on incorrect simplifications, they serve the purpose of creating awareness[51]. In extension to the Dismal theorem, Nordhaus proposed a topology defining certain levels of *tail dominance*, i.e. the level of influence of the shape of the tail on the advised policy[51]. These levels are defined as follows:

1. *Tail irrelevance*: when the distribution of the random variable has no or little influence on the advices policy or on the outcome of the model,

2. *Weak tail dominance*: where the outcomes or the policies of the model are effected by the tails of the distribution. Though the results are influenced, the outcome of the model does converge,

3. *Strong tail dominance*: the case when the outcome of the model does not converge when focussing on the tail, resulting in an infinite response.

The strong tail dominance is a result of the creeping convergence of the fat tail, slower then exponential[55]. When looking at a normal distributed random variable, an extreme event has a slightly more extreme counter part at just a slightly less probable state. In the case of a (truly) fat tailed distribution, the extreme event has a much more extreme event at a slightly less probable state. In other words, when looking at fat tailed distributions, the case with a relatively lower probability has a significantly (approaching infinity for very very fat tailed distributions) higher value[51]. Nordhaus finds that the distribution of the climate sensitivity parameter is not fat enough to fall latter in this category of non-converging cases. This conclusion supports the selected classes in section 4.3 and therefore the findings that "run-away" damage does not occur.

Reflections on literature and implications of EICE

Based on this conclusion and topology, the following subsection further dissects the implications of EICE. Connecting the topology of tail dominance to the results presented in chapter 5, it can be concluded that the base and extreme case respectively show the irrelevant tail and the weak tail level response. This result thereby verifies the propositions made by Nordhaus. As the base case is derived from the current understanding of the climate sensitivity parameter, it can be concluded that; in regard to assumed variations of the climate sensitivity parameter, EICE does not promote additional emission cuts. Hence, the EICE supports the claim of a moderate mitigation policy. Nonetheless, the extreme case does support a more strict policy, but without the non-converging consequences suggested by the Dismal theorem. This claim for a more strict climate mitigation policy is also recommended based on the inclusion of higher level uncertainty.

Extrapolating these findings to conclusions in literature can only be done indirectly. The reason for this is that evaluating the climate sensitivity parameter as the main uncertainty is uncommon in Integrated Assessment literature. The results are therefore compared with literature regarding uncertain damage. Damage is selected as the implications are expected to be the same. When uncertain damage in DICE is approached in the same way as in this thesis, by means of recursive dynamic programming, risk has a notable effect on the optimal policy. As in the current evaluation, uncertain damage results in the advice to abate slightly less under "base-like" conditions[16]. The possibility of a more extreme response is found to advocate increased optimal control rates[65]. Both these responses are present in EICE and therefore support its advised policy based on continuous evaluation of the prime uncertainty.

As DICE possibly underestimates damages, Weitzman proposed an alternative damage function[82].This alternative damage function is based on the findings of an expert panel and puts more emphasis of tail events. Implementing this function into DICE showed that the proposed optimal policy is very sensitive to its definition[9]. The need for such an alternative function and the found sensitivity to its definition aligns with the results of chapter 5. Therefore, when evaluating extreme events in the DICE model, results should be interpreted with this limitation in mind. When following the conditions stated by the IPCC, the results of the original DICE model can be assumed to be representative.

Concluding, synthesising a policy remains subjected to the modellers risk averseness, as both moderate and deeper cuts in emissions can be supported under reasonable assumptions. A side note here is that all modellers should take notice of the influence of the damage function. Modellers following the current interpretation of climate science, as defined by the IPCC, should follow the presented results in Nordhaus' DICE model.

Chapter 7

Conclusion

The goal of this thesis is to show the influence of climate uncertainty and the possible implications of the Dismal theorem on model based decision making. Here, the focus is on the climate sensitivity parameter, the response of the climate to CO_2-emissions, as it is adducted by literature to be the main source of uncertainty. The reason for its importance in literature is the fat-tail of its distribution. This asymmetric shape is a result of accumulated measurement errors and fundamental uncertainty regarding possible climate tipping points. For this reason the climate sensitivity parameter is taken to be the uncertainty of interest. Showing the influence of this asymmetrically distributed parameter on model based decision making is achieved by answering the main question:

"How does the advised mitigation policy by DICE respond to the influence of an uncertain climate sensitivity parameter?"

The response of the model to uncertainty depends on the way it is included. The first decision regarding this integration is: whether to view uncertainty as an endogenous or exogenous parameter. In case the climate sensitivity parameter is implemented as a exogenous parameter, its value is set before the model is run and therefore not experienced at the time the decision maker has to set a policy. With endogenous uncertainty this is the case. Here, the modeller is able to optimise the expected value instead of averaging the outcome after the simulations as with exogenous uncertainty. This approach therefore provides an actual policy advice, is more representative of the real situation and is found to be more risk averse.

As the focus is on endogenous uncertainty, Monte Carlo methods such as scenario analysis are incompatible. Two often used methods that do include endogenous uncertainty are Robust Optimisation and Stochastic Programming. In case of the foremost method, the focus is on the tail events of the probability curve. As the aim of the thesis is to look at the shape of the curve, this method is deemed ill-equipped. Stochastic Programming does look at the entire distribution of the probability curve and thus is the preferred method.

The main idea of stochastic programming is that decisions are made before (exact) information about the transition to a subsequent stage is known. Base on the probability curve, the stochastic programming approach recursively estimates the value of future stages are uses this information to advice on an optimal policy.

A drawback to this method is the exponential growth in size of the stochastic tree. In case the original model with sixty stages is evaluated with only three scenarios per stage, the model would have to evaluate a total of over a quintillion scenarios. This makes the model vastly intractable. In order to work with the limited size of the tree a model is formulated with time steps of 20 years, 7 stochastic stages, 8 deterministic stages and 3 scenarios per stage.

Based on the limited number of samples and the current understanding of the climate sensitivity parameter, the four scenarios of the table 7.1 are constructed. The data in this table is represents the current understanding of the climate sensitivity parameter (the asymmetrical base case), a more extreme/risk-averse case (the asymmetrical extreme case). In addition both cases are evaluated with a higher level of uncertainty as their uncertainty can be classified as fundamental. At this fundamental level no probability distribution is at hand and scenarios are weighted equally.

Table 7.1: Climate sensitivity scenarios

	Climate Sensitivity [°C/$CO_{2,2x}$]	Probability [%]	
	Asymmetrical	Uniform	
Base case	2.2	25	33
	3.0	50	33
	4.3	25	33
Extreme case	2.2	25	33
	3.0	50	33
	8.0	25	33

From comparing the results of these cases can be concluded that the model is sensitive to the distribution of the climate sensitivity parameter.

When the base case is used as input for the new model, EICE, the advised policy is slightly lower than that the expected policy by DICE. The less strict policy is probably a product of the damage function, which makes it profitable to allow damage in case of overconsumption. If the base case is reviewed from a higher level of uncertainty the optimal control rate and the carbon tax remain below the set value of the expected case. In both cases deviations are minute. Therefore, the base case is said to have "tail irrelevance", as the distribution of the random variable has little to no influence of the advised policy.

The extreme case does support notable alterations to the suggested optimal control rate. As can be seen in figure 7.1, both the asymmetrical and uniformly distributed extreme case support an increase in the carbon price, advocating a stricter mitigation policy. Though, they advocate more mitigation, they do not support the claim of an infeasible policy at tail-scenarios, as suggested in the Dismal theorem. The extreme case is therefore defined to have "weak tail dominance".

Overall can be concluded that both a "fatter" tail and a higher level of uncertainty support claims for a stricter mitigation policy. Nonetheless, due to a relatively low climate impact as a result of emissions, these policies might be expected to be even more strict.

Consequently, synthesising a policy remains subjected to the modellers risk averseness, as both moderate and deeper cuts in emissions can be supported by reasonable assumption regarding the climate sensitivity parameter. Policy makers following the current interpretation of climate science, as defined by the IPCC, should follow the presented results in Nordhaus' DICE model.

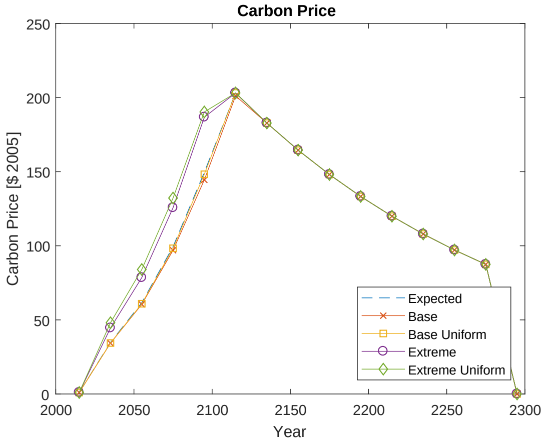

Figure 7.1: Comparison of the carbon price of the expected, base, base uniform, extreme and extreme uniform case

Chapter 8

Reflections and Recommendations

The following chapter reflects upon the design of the model and the process behind it. The chapter starts with a reflection of the project defining decisions. The sequential section reviews the path resulting in and from these decisions. A reflection of the results is already present in subsection 6.2. The chapter concludes with an advise for further research based on the encountered obstacles during the thesis.

8.1 Technical reflection

The design of the model is a product of a two major decisions. The following section will look at these decisions and reflect upon them. These decisions are to focus on uncertainty within the climate system and to look at endogenous uncertainty.

The decision to focus on the climate sensitivity parameter is one that can easily debated. The reason for this is that the economic uncertainty (specifically the economic damage) plays a bigger role in DICE. A logical alternative therefore would have been the damage function. This is the case as simulating climate damage makes use of the same strong points in DICE as the climate sensitivity parameter: the 300 year planning horizon. Thought the response of varying the climate sensitivity parameter is relatively weak in respect to the damage exponent, it is the more realistic thing to do. In addition, as already stated many times, such economic behaviour is unquantifiable and therefore it can only be use to show the sensitivity of the model. Adverse to this the (almost) fundamental uncertainty of the climate sensitivity parameter can be statistically approximated by means of measurements and thus can be used as a basis for policy advisement. Therefore, a model based on an uncertain climate sensitivity parameter can be used as a foundation for follow up experiments with bounded feasible regions that lay within the somewhat more "knowable" realm.

A consequence of working with an endogenous climate sensitivity parameter is stochastic programming itself. Therefore, the endogenous inclusion resulted in only being able to evaluate a single parameter, the need for a reduction in stages and a course approximation of the probability density function. These limitations themselves led to further limitations like being unable to use risk measures and probabilistic programming.

In order to enable these expansions, the aim has been to linearise the model. Unfortunately this has failed as the convex base function by Solak has been found to be non-convex. Alternatives to linearisation are presented later in section 8.3. With hindsight, converting DICE into one of these structures should have been a part of the process.

8.2 Process reflection

During this thesis I gained many new insights, too much to all individually mention. Therefore, in the following section, I will go into the main learning events on a process level. The following will be discussed: the structure of the research process, peer-support, time management and learning cycles.

The structure of the research process

The project started with the focus on modelling. This focus was the product of not knowing whether DICE could be simulated outside of Microsoft Excel. At this time no licence for GAMS was available and I found the program in Excel did not offer a good starting point. Therefore the aim in this initial period was to rebuild DICE in Matlab or Python. Furthermore, as the aim of the thesis was to include random variables in the model, and the most common way to achieve this is by Monte Carlo Analysis, I followed a course on the matter during the same period.

This initial focus on modelling steered away from the usual starting point of performing a literature study. When the study was executed, it showed that there are better ways of including uncertainty and led to contacts that could help with the project and a GAMS-licence.

In my opinion, this initial distortion is the greatest mistake of the project. In future projects I will be reminded of the wandering around during the first stages of the project and make sure to stay in line with the standard research process.

Peer-support

Not being linked to a research group is possibly the root of most of my challenges. Perhaps if I was part of a team, the initial phase would have been more structured and less time would have been lost. In this case, peers could have helped with formulating the problem as they would have been familiar with the jargon.

Peers would also have been very helpful with fixing minor errors. Being new to the GAMS-environment means simple problems could take days (or weeks) to solve. Being a part of a research group could have greatly reduced the lost time. In addition, not having the possibility to spar with colleagues working on the same matter, resulted in lasting misunderstandings and possibly missed opportunities.

Based on these experiences, a following project will start by defining or joining a research group. During the project I learned that being able to spar with peers, as in the later stages occasionally occurred with Germán, fits my way of working and greatly aids its quality.

Time management

In advance I knew the project could not be finished within the stated nine months, nonetheless it took longer then expected. One of these reasons can be classified as:"the other obligations". What I did not expect to take as much time, but did, was switching between work and the project. In the later stages of the project I was more strict with the time slots for both tasks, which greatly improved my efficiency. Therefore, in further projects I will aim to do the same.

In hindsight, little things like calibrating the model from D5 to D20, took way more time than estimated. The same holds for formulation text and small programming tasks. During the project I learned to recognize such tasks and was able to more closely estimate the required time. A personal pitfall in this regard is the need to show results. As all these little delays sneaked in, the meeting intervals increased, alienating myself further from my supervisors. In a following project I hope to be part of a research group and hope to have weekly (fixed) meetings. These meetings will force me to show the weekly progression and help to keep the pace.

In extend to the little things, there were of course also major setbacks. The biggest one, after meeting with Mathijs and Germán, was the inability to linearise the convex model by Solak. Even though, within a few weeks, this path seemed progression less, I clung to it. The more I put time in the subject, the more uncertain I became and the more I was unable to alter my path. The inevitable diversion came too late and delayed the process as a whole. Seeing whether a path is viable will always be a difficult part of the process. Again, discussing the matter with peers might help. For future projects, more "mile stones" and points of reflection are required in the planning to avoid such delays.

learning cycles

A personal pitfall regarding the mastery of a subject, is focussing to much on theoretical details or to stay to long in the realm of abstract structures. A good example of this is with solving simple multi-stage optimisation structures. Following the book "Introduction to Stochastic Programming" by J.R. Birge led me to nested decomposition and extensions thereof, which are all challenging to program. Working out a problem showed the potential for simpler solutions methods like the deterministic equivalent approach. From this experience, I learned to always work from an example an build upon that, providing both a better understanding of and feeling for the matter.

Overall, I would like to end on a positive note as being a part of this process has renewed my interest in engineering and science overall. After a disappointing time in the laboratory at the nuclear facility in Delft, I was not sure in which direction I wanted to go. Now, more than a year later, the experiences during this thesis have resulted in a wish to obtain a PhD and pursue a carrier in science.

8.3 Further developments of the EICE model

When further extending the EICE model, three tracks stand out. These tracks are the computability, the socio-logical response to climate change and the treatment of the probability tail. The following subsections will cover these subjects with the aim of sparking future research.

8.3.1 Computability

As stated in earlier chapters, stochastic models suffer from the curse of dimensionality. The limitations set by the exponential growth in size, with each additional step or added scenario, limits the possible number of stochastic stages and evaluated scenarios. Increasing the number of samples will give a more representative approximation of the probability curve and might give new insights into the mechanics of the system. In addition, more stochastic stages might be used to compensate the losses of section 4.2. The following subsection suggests the following research tracks: linearisation, decomposition and extensions thereof as these are common in literature.

Linearisation

As introduced in section 2.4, DICE is a non-convex optimisation problem. Nonetheless, empirical studies have shown that a local optimisation algorithm is able to find the same optimal solution as a global optimisation algorithm regardless the initial guess[49]. Due to this characteristic DICE is defined to be hidden-convex [67]. In case of a hidden-convex model an optima-preserving transformation is possible to a convex model. While this new model may have different a feasible region, it has the same optimal value[41].

The resulting convex model can form the basis of a linear model. The advantage of a linear model is the possibility of linear programming and its efficient solving techniques (i.e. simplex and the interior point algorithm). The difference between linear and non-linear programming is the need for derivative information in determining the search direction and the rate of convergence. These deriving steps are not needed in linear programming resulting in a lower computation time[43].

Based on this line of reasoning it is advised to search for an equivalent model to DICE. When building the extended model of chapter 4, the initial idea was to base it on a linearised version of the convex DICE model as proposed by Solak[67]. In this model the non-convex terms of DICE: the utility function, the net output function, the emissions function and the radiative forcing function are converted into convex functions. This conversion provides a model with only convex and affine relations and thus a convex model. Unfortunately the proposed model is found to be non-convex. As the emissions have a negative effect on the net output function, and thus on the objective, the feasible area has to be above the curve, figure 8.1 shows that this region is non-convex.

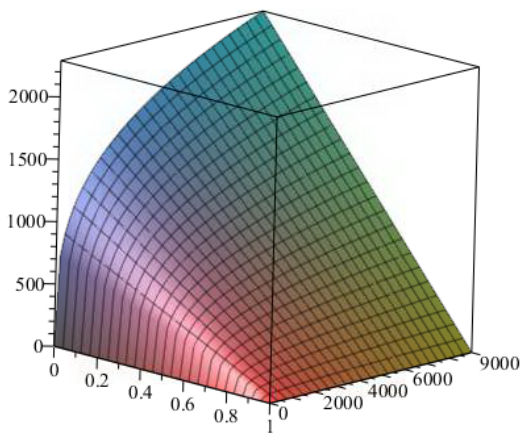

Figure 8.1: Feasible area of the emissions functions by Solak[67]. Here the vertical axis represents the emissions and the horizontal axis covers the mitigation policy (0-1) and the state of capital (0-9000)

As the proposed model by Solak cannot be linearised, there is the option of using specially ordered sets (SOS) and integer programming to obtain a computationally less demanding model[83]. Using type 2 SOS-variables, points on the emissions graph can be identified and linear combinations of two of these points can be used to approximate the curve. A drawback to this method is the relative increase in computational burden with respect to linear programming and the loss of information with respect to the (convex) non-linear model. Based on these arguments this solution method is not further explored.

Decomposition

Section 4.3 introduced the idea of a stochastic tree within stochastic programming. The idea behind decomposition is to make use of this repetitive structure to split the large deterministic equivalent problem into multiple (easier to solve) small problems[7]. Traditionally Benders decomposition is used to separate mixed integer programs into a master and a sub-problem. This idea can be extended to two-stage stochastic programming. By solving sub-problems in the second stage, feasibily and optimality cuts (bounds) can be generated for the first stage. Extending this idea to multi-stage models is the foundation of Nested Benders Decomposition[45]. By first solving the root-node problems feasibility cuts are generated. These cuts are sequentially passed back to the parent nodes, further constraining the model. As more cuts are passed up the tree, the nonlinear value function in the parent models are more closely approximated. Finally resulting in a master problem which is purely a function of the first stage decision variable.

GAMS' EMP framework provides the option of Nested Benders Decomposition. This option is available, when using the LINDO solver by LINDO Systems incorporated[11]. A comparison between the computational time of the standard deterministic and the Nested Benders approach showed no computational advantage for EICE. During this comparisons the standard cases of three samples and four till eight stages are evaluated. Additionally, the case with four uncertain stages and samples of six are tested without finding a reduction in computation time. A follow up study could look into how the advantages of decomposition could be used to reduce the computation burden of EICE.

In extend to the idea of decomposition, multiple daughter-techniques are developed. Often used techniques are: Approximate dynamic programming, Stochastic Dual Dynamic Programming and the Rolling Horizon approach, which all may provide computational benefits. These approaches either work by exploring the future states by means of approximations (i.e. linearisations of future stages), sampling future stages or by simulating with a nearby horizon and "roll" the horizon over for each stage[13][25]. Using decomposition and to a further extend apply one of the above mentioned techniques will possibly reduce the computational burden and allow for a finer simulation.

8.3.2 Socio-logical response to climate change

One of the main conclusions is that the DICE model is relatively insensitive to climate damage. This insensitivity could be partially due to the dampening effect of the damage function. Nordhaus knowledges this shortcoming and states that: "... the economic impact of climate change is the thorniest issue in climate-change economics"[50]. In order to overcome this effect the damage function could be replaced. Many researchers (i.g. Weitzman[82][81], Ackerman[1], Crost and Traeger[16], Hwang[35] and Pycroft[61]) already have followed this path. The challenge here is that quantifying damages becomes increasingly more difficult with rising temperatures. It is therefore a possibly more preferable option to work with temperature bounds. As stated by Heal, future damages are unquantifiable. Therefore, using such a bound will also avoid an unsolvable (ethical) debate and keep damages in the realm of the semi-quantifiable[30][49].

An interesting bound to review is the one at $2[^\circ C]$. This bounds is agreed upon by almost 200 nations at the COP21 in 2005. As a part of a preliminary investigation this option is explored with the $SP20_7$ model under base conditions. Adding the $2^\circ C$-constraint results in a locally infeasible solution. This infeasibility can be explained by looking at the suggested emission control rate of the deterministic case with a climate sensitivity parameter of $2.9[^\circ C/CO_{2,2x}]$ under a $2[^\circ C]$ restriction. Table 8.1 shows the suggested mitigation policy in this case. From this table can be derived that an near step response is needed in order to provide the optimal transition path. Setting the climate sensitivity at $4.3[°C/CO_{2,2x}]$ makes the problem insolvable.

Table 8.1: Climate mitigation policy under a $2[°C]$ restriction in the D20 model with a climate senstivity of $2.9[°C/CO_{2,2x}]$

Time	Level
2015	0.039
2035	0.589
2055	0.999
2075	0.972
2095	0.961
2115	0.972
2135	1.000
2155	1.000
2175	1.000

8.3.3 Treatment of the probability tail

The focus of this thesis has been on the sensitivity of the advised policy to the shape of the climate sensitivity's probability distribution. A logical follow up to this study would be to look at the sensitivity to risk measures. These measures can be used to reduce the variability of the return. Before these measures can be implemented a finer approximation of the probability curve is needed. Common approaches to achieve this are: variance reduction, value at risk (VaR) and conditional value at risk (CVaR)[72].

The first method aims to reduce the variance of the model. A disadvantage of this approach is that it penalises both the profits and the losses. VaR avoids this drawback by defining a "value at risk" at a certain probability and stating a constraint at that probability with a value no greater than the value at risk. A downside to this approach is that the measure does not have the property of subadditivity and makes the model non-convex. To avoid this problem, the conditional expectation over losses in access of the value at risk can be accounted for. This is the approach in CVaR[7]. Adding these risk measures to the model simulates the intolerance to annihilation and therefore allows the exploration of the effects of very extreme events.

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Appendix A

The DICE model in GAMS format

This appendix lists the code of the original 2013 Vanilla GAMS version of DICE in GAMS format. After the code is listed, a verbal explanation of the lines in the code is presented

A.1 Original DICE Vanilla 2013 GAMS code

```
 This is the DICE2013R model, version DICE2013R_100413_vanilla.gms, revised from
    April version.   The vanilla version includes only the optimal and baseline
    scenarios. These are determined by setting the "ifopt" control at 1 (optimal)
    or 0 (baseline). This version has write ("put") output but does not have
    subroutines ("include"). A full discussion is included in the "DICE 2013R
    Manual" on the web at dicemodel.net. As the GAMS Latex converter does not
    allow for the minus sign, negative values will be appointed with "n!".

$title DICE2013R October 2013

set t Time periods (5 years per period)                           /1*60/ ;

parameters
**Time Step
tstep Years per Period /5/

** If optimal control
ifopt If optimized 1 and if base is 0                             /1/

** Preferences
elasmu Elasticity of marginal utility of consumption              / 1.45 /
prstp Initial rate of social time preference per year             / .015 /

** Population and technology
gama Capital elasticity in production function                    /.300 /
pop0 Initial world population (millions)                          /6838 /
popadj Growth rate to calibrate to 2050 pop projection            /0.134 /
popasym Asymptotic population (millions)                          /10500 /
dk Depreciation rate on capital (per year)                        /.100 /
q0 Initial world gross output (trill 2005 USD)                    /63.69 /
k0 Initial capital value (trill 2005 USD)                         /135 /
a0 Initial level of total factor productivity                     /3.80 /
ga0 Initial growth rate for TFP per 5 years                       /0.079 /
dela Decline rate of TFP per 5 years                              /0.006 /

** Emissions parameters
gsigma1 Initial growth of sigma (continuous per year)             /n!0.01 /
dsig Decline rate of decarbonization per period                   /n!0.001 /
```

```
eland0 Carbon emissions from land 2010 (GtCO2 per year)          / 3.3 /
deland Decline rate of land emissions (per period)               / .2 /
e0 Industrial emissions 2010 (GtCO2 per year)                    /33.61 /
miu0 Initial emissions control rate for base case 2010           /.039 /

** Carbon cycle
* Initial Conditions
mat0 Initial Concentration in atmosphere 2010 (GtC)              /830.4 /
mu0 Initial Concentration in upper strata 2010 (GtC)             /1527. /
ml0 Initial Concentration in lower strata 2010 (GtC)             /10010. /
mateq Equilibrium concentration atmosphere (GtC)                 /588 /
mueq Equilibrium concentration in upper strata (GtC)             /1350 /
mleq Equilibrium concentration in lower strata (GtC)             /10000 /

* Flow paramaters
b12 Carbon cycle transition matrix                               /.088/
b23 Carbon cycle transition matrix                               /0.00250/

* These are for declaration and are defined later
b11 Carbon cycle transition matrix
b21 Carbon cycle transition matrix
b22 Carbon cycle transition matrix
b32 Carbon cycle transition matrix
b33 Carbon cycle transition matrix
sig0 Carbon intensity 2010 (kgCO2 per output 2005 USD 2010)

** Climate model parameters
t2xco2 Equilibrium temp impact (oC per doubling CO2)             / 2.9 /
fex0 2010 forcings of non CO2 GHG (Wm 2)                         / 0.25 /
fex1 2100 forcings of non CO2 GHG (Wm 2)                         / 0.70 /
tocean0 Initial lower stratum temp change (C from 1900)          /.0068 /
tatm0 Initial atmospheric temp change (C from 1900)              /0.80 /
c1 Climate equation coefficient for upper level                  /0.098 /
c3 Transfer coefficient upper to lower stratum                   /0.088 /
c4 Transfer coefficient for lower level                          /0.025 /
fco22x Forcings of equilibrium CO2 doubling (Wm 2)               /3.8 /

** Climate damage parameters
a1 Damage intercept                                              /0 /
a2 Damage quadratic term                                         /0.00267 /
a3 Damage exponent                                               /2.00 /

** Abatement cost
expcost2 Exponent of control cost function                       / 2.8 /
pback Cost of backstop 2005$ per tCO2 2010                       / 344 /
gback Initial cost decline backstop cost per period              / .025 /
limmiu Upper limit on control rate after 2150                    / 1.2 /
tnopol Period before which no emissions controls base            / 45 /
cprice0 Initial base carbon price (2005$ per tCO2)               / 1.0 /
gcprice Growth rate of base carbon price per year                /.02 /

** Participation parameters
periodfullpart Period at which have full participation           /21 /
partfract2010 Fraction of emissions under control in 2010        / 1 /
partfractfull F. of emissions under control at full time         / 1 /

** Availability of fossil fuels
fosslim Maximum cumulative extraction fossil fuels (GtC)         /6000/

** Scaling and inessential parameters
* Note that these are unnecessary for the calculations but are for convenience
scale1 Multiplicative scaling coefficient                  /0.016408662 /
scale2 Additive scaling coefficient                        /n!3855.106895/ ;
```

```
* Program control variables
sets tfirst(t), tlast(t), tearly(t), tlate(t);

PARAMETERS
L(t)              Level of population and labor
al(t)             Level of total factor productivity
sigma(t)          CO2 equivalent emissions output ratio
rr(t)             Average utility social discount rate
ga(t)             Growth rate of productivity from
forcoth(t)        Exogenous forcing for other greenhouse gases
gl(t)             Growth rate of labor
gcost1            Growth of cost factor
gsig(t)           Change in sigma (cumulative improvement of energy efficiency)
etree(t)          Emissions from deforestation
cost1(t)          Adjusted cost for backstop
partfract(t)      Fraction of emissions in control regime
gfacpop(t)        Growth factor population
pbacktime(t)      Backstop price

optlrsav          Optimal long run savings rate used for transversality
scc(t)            Social cost of carbon
cpricebase(t)     Carbon price in base case ;

* Program control definitions
tfirst(t) = yes$(t.val eq 1);
tlast(t) = yes$(t.val eq card(t));

* Parameters for long run consistency of carbon cycle
b11 + b12 = 1;
b21 = b12*MATEQ/MUEQ;
b22 + b21 + b23 = 1;
b32 = b23*mueq/mleq;
b33 + b32 = 1;

* Further definitions of parameters
sig0                    = e0/(q0*(1 n! miu0));
lam               = fco22x/ t2xco2;
L("1")            = pop0;
loop(t, L(t+1)=L(t););
loop(t, L(t+1)=L(t)*(popasym/L(t))**popadj ;);
ga(t)             = ga0*exp( dela*5*((t.val  n!1)));
al("1")           = a0; loop(t, al(t+1)=al(t)/((1  n! ga(t)));););
gsig("1")         = gsigma1; loop(t,gsig(t+1)=gsig(t)*((1+dsig)**tstep) ;);
sigma("1")        = sig0; loop(t,sigma(t+1)=(sigma(t)*exp(gsig(t)*tstep)););
pbacktime(t)      = pback*(1 gback)**(t.val  n!1);
cost1(t)          = pbacktime(t)*sigma(t)/expcost2/1000;
etree(t)          = eland0*(1 deland)**(t.val n!1);
rr(t)             = 1/((1+prstp)**(tstep*(t.val n!1)));
forcoth(t)        = fex0+ (1/18)*(fex1 fex0)*(t.val n!1)$(t.val lt 19)+ (fex1 fex0)$
    (t.val ge 19);
optlrsav          = (dk + .004)/(dk + .004*elasmu + prstp)*gama;
partfract(t)$(ord(T)>periodfullpart) = partfractfull;
partfract(t)$(ord(T)<periodfullpart+1) = partfract2010+(partfractfull partfract2010
    )*(ord(t) n!1)/periodfullpart;
partfract("1")    = partfract2010;
cpricebase(t)     = cprice0*(1+gcprice)**(5*(t.val n! 1));

VARIABLES
MIU(t)                  Emission control rate GHGs
FORC(t)                 Increase in radiative forcing (watts per m2 from 1900)
TATM(t)                 Increase temperature of atmosphere (degrees C from 1900)
TOCEAN(t)               Increase temperatureof lower oceans (degrees C from 1900)
```

```
MAT(t)                  Carbon concentration increase in atmosphere (GtC from 1750)
MU(t)                   Carbon concentration increase in shallow oceans (GtC from
    1750)
ML(t)                   Carbon concentration increase in lower oceans (GtC from
    1750)
E(t)                    Total CO2 emissions (GtCO2 per year)
EIND(t)                 Industrial emissions (GtCO2 per year)
C(t)                    Consumption (trillions 2005 US dollars per year)
K(t)                    Capital stock (trillions 2005 US dollars)
CPC(t)                  Per capita consumption (thousands 2005 USD per year)
I(t)                    Investment (trillions 2005 USD per year)
S(t)                    Gross savings rate as fraction of gross world product
RI(t)                   Real interest rate (per annum)
Y(t)                    Gross world product net of abatement and damages (trillions
    2005 USD per year)
YGROSS(t)               Gross world product GROSS of abatement and damages (
    trillions 2005 USD per year)
YNET(t)                 Output net of damages equation (trillions 2005 USD per year)
DAMAGES(t)              Damages (trillions 2005 USD per year)
DAMFRAC(t)              Damages as fraction of gross output
ABATECOST(t)            Cost of emissions reductions (trillions 2005 USD per year)
MCABATE(t)              Marginal cost of abatement (2005$ per ton CO2)
CCA(t)                  Cumulative industrial carbon emissions (GTC)
PERIODU(t)              One period utility function
CPRICE(t)               Carbon price (2005$ per ton of CO2)
CEMUTOTPER(t)           Period utility
UTILITY                 Welfare function;

NONNEGATIVE VARIABLES MIU, TATM, MAT, MU, ML, Y, YGROSS, C, K, I;

EQUATIONS
*Emissions and Damages
EEQ(t)                  Emissions equation
EINDEQ(t)               Industrial emissions
CCACCA(t)               Cumulative carbon emissions
FORCE(t)                Radiative forcing equation
DAMFRACEQ(t)            Equation for damage fraction
DAMEQ(t)                Damage equation
ABATEEQ(t)              Cost of emissions reductions equation
MCABATEEQ(t)            Equation for MC abatement
CARBPRICEEQ(t)          Carbon price equation from abatement

*Climate and carbon cycle
MMAT(t)                 Atmospheric concentration equation
MMU(t)                  Shallow ocean concentration
MML(t)                  Lower ocean concentration
TATMEQ(t)               Temperature climate equation for atmosphere
TOCEANEQ(t)             Temperature climate equation for lower oceans

*Economic variables
YGROSSEQ(t)             Output gross equation
YNETEQ(t)               Output net of damages equation
YY(t)                   Output net equation
CC(t)                   Consumption equation
CPCE(t)                 Per capita consumption definition
SEQ(t)                  Savings rate equation
KK(t)                   Capital balance equation
RIEQ(t)                 Interest rate equation

* Utility
CEMUTOTPEREQ(t)         Period utility
PERIODUEQ(t)            Instantaneous utility function equation
UTIL                    Objective function ;
```

```
** Equations of the model
*Emissions and Damages
eeq(t).. E(t)                     =E= EIND(t) + etree(t);
eindeq(t).. EIND(t)               =G= sigma(t) * YGROSS(t) * (1 (MIU(t)));
ccacca(t+1).. CCA(t+1)            =E= CCA(t)+ EIND(t)*5/3.666;
force(t).. FORC(t)                =E= fco22x * ((log((MAT(t)/588.000))/log(2))) +
    forcoth(t);
damfraceq(t) .. DAMFRAC(t)        =E= (a1*TATM(t))+(a2*TATM(t)**a3) ;
dameq(t).. DAMAGES(t)             =E= YGROSS(t) * DAMFRAC(t);
abateeq(t).. ABATECOST(t)         =E= YGROSS(t) * cost1(t) * (MIU(t)**expcost2) *
    (partfract(t)**(1 expcost2));
mcabateeq(t).. MCABATE(t)         =E= pbacktime(t) * MIU(t)**(expcost2 1);
carbpriceeq(t).. CPRICE(t)        =E= pbacktime(t) * (MIU(t)/partfract(t))**(
    expcost2 1);

*Climate and carbon cycle
mmat(t+1).. MAT(t+1)              =E= MAT(t)*b11 + MU(t)*b21 + (E(t)*(5/3.666));
mml(t+1).. ML(t+1)                =E= ML(t)*b33 + MU(t)*b23;
mmu(t+1).. MU(t+1)                =E= MAT(t)*b12 + MU(t)*b22 + ML(t)*b32;
tatmeq(t+1).. TATM(t+1)           =E= TATM(t) + c1 * ((FORC(t+1) n!(fco22x/t2xco2)
    *TATM(t)) n!(c3*(TATM(t) n!TOCEAN(t))));
toceaneq(t+1).. TOCEAN(t+1)       =E= TOCEAN(t) + c4*(TATM(t) n!TOCEAN(t));

*Economic variables
ygrosseq(t).. YGROSS(t)           =E= (al(t)*(L(t)/1000)**(1 GAMA))*(K(t)**GAMA);
yneteq(t).. YNET(t)               =E= YGROSS(t)*(1 n!damfrac(t));
yy(t).. Y(t)                      =E= YNET(t)  n! ABATECOST(t);
cc(t).. C(t)                      =E= Y(t)  n! I(t);
cpce(t).. CPC(t)                  =E= 1000 * C(t) / L(t);
seq(t).. I(t)                     =E= S(t) * Y(t);
kk(t+1).. K(t+1)                  =L= (1 n!dk)**tstep * K(t) + tstep * I(t);
rieq(t+1).. RI(t)                 =E= (1+prstp) * (CPC(t+1)/CPC(t))**(elasmu/tstep
    )  1;

*Utility
cemutotpereq(t).. CEMUTOTPER(t)   =E= PERIODU(t) * L(t) * rr(t);
periodueq(t).. PERIODU(t)         =E= ((C(T)*1000/L(T))**(1 n!elasmu) n!1)/(1 n!
    elasmu) n!1;
util.. UTILITY                    =E= tstep * scale1 * sum(t, CEMUTOTPER(t)) +
    scale2 ;

*Resource limit
CCA.up(t) = fosslim;

* Control rate limits
MIU.up(t) = limmiu*partfract(t);
MIU.up(t)$(t.val<30) = 1;

** Upper and lower bounds for stability
K.LO(t)                 = 1;
MAT.LO(t)               = 10;
MU.LO(t)                = 100;
ML.LO(t)                = 1000;
C.LO(t)                 = 2;
TOCEAN.UP(t)            = 20;
TOCEAN.LO(t)            =  n!1;
TATM.UP(t)              = 40;
CPC.LO(t)               = .01;

* Control variables
* Savings rate for asympotic equilibrium
S.FX(t)$(t.val>50) = optlrsav;
```

```
* Base carbon price if base, otherwise optimized
* Warning: If parameters are changed, the next equation might make base case
    infeasible.
* If so, reduce tnopol so that don't run out of resources.
cprice.up(t)$(ifopt=0) = cpricebase(t);
cprice.up(t)$(t.val>tnopol) = 1000;
cprice.up('1')=cpricebase('1');

* Initial conditions
CCA.FX(tfirst)    = 90;
K.FX(tfirst)      = k0;
MAT.FX(tfirst)    = mat0;
MU.FX(tfirst)     = mu0;
ML.FX(tfirst)     = ml0;
TATM.FX(tfirst)   = tatm0;
TOCEAN.FX(tfirst)       = tocean0;

** Solution options
option iterlim   = 99900;
option reslim    = 99999;
option solprint  = on;
option limrow    = 0;
option limcol    = 0;

model CO2 /all/;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;

** POST SOLVE
* Calculate social cost of carbon
scc(t) = 1000*eeq.m(t)/cc.m(t);

** Display at bottom of output for visual inspection
option decimals=2;
display tatm.l,scc,utility.l,cprice.l,y.l, cpc.l,cc.m;
option decimals=6;
display ri.l,utility.l,cc.m;

*Describes a file labeled 'results' with the filename "DiceResults.csv" in the
    current directory
file results /DiceResults.csv/; results.nd = 10 ; results.nw = 0 ; results.pw=1200;
     results.pc=5;
put results;

*Some sample results. For an include file which outputs ALL relevant information,
    see 'PutOutputAllT.gms' in the Include folder.
*You may likely use:
*$include Include\PutOutputAllT.gms
* ...if your directory contains this file.
put / "Period";
Loop (T, put T.val);
put / "Year" ;
Loop (T, put (2005+(TSTEP*T.val) ));
put / "* CLIMATE MODULE" ;
put / "Atmospheric Temperature (deg C above preindustrial)" ;
Loop (T, put TATM.l(T));
put / "Total Increase in Forcing (Watts per Meter2, preindustrial)" ;
Loop (T, put FORC.l(T));
put / "Lower Ocean Temperature (deg C above preindustrial)" ;
Loop (T, put TOCEAN.l(T));
putclose;
```

A.2 Verbatim elaboration of DICE 2013

The internal relations, also known as the *equations of the model* consists of 25 relations with 27 variables. Additionally previously defined parameters are included in the relations. The section below covers these relations in a pseudo mathematical style, in the program of table A.1 uses the mathematical representation as proposed by Nordhaus. The abbreviation in this tables are explained in the text below. The relations in the GAMS model are divided into four groups. The first group focuses on the emissions and corresponding damages, the second group deals with the physical aspects of the climate and the third group covers the economic variables. The objective function is defined in the fourth. For continuity purposes this order will be maintained.

The total CO_2 emissions per year(E(t))[GtCO_2/a] is equal to the sum of industrial emissions (EIND(t))[GtCO_2/a] and the emissions related to deforestation (etree(t))[GtCO_2/a]. Here E(t) and EIND(t) are presented in uppercase letters whereas etree(t) uses lowercase letters. This distinction is made to separate the variables (uppercase) from the parameters(lower case). The industrial emissions are formed by the product of the CO_2-equivalent-emissions output ratio (σ(t))[-], the gross world product of gross abatement and damages (YGROSS)[tr$.2005/a] and 1 - the emission control rate (MIU(t))[-]. The industrial emissions also play a role in the calculation of the cumulative industrial carbon emissions (CCA(t))[GtCO_2]. Here it is multiplied by a factor and added with the previously cumulated CO_2. The increase in radiative forcing as a result of the accumulated CO_2 since 1900 (FORC(t))[W/m^2] is defined to be equal to the forcings of equilibrium CO_2 doubling (foc22x) times the logarithmic function of the carbon concentration increase since 1970 in the atmosphere (MAT(t))[GtCO_2] and the exogenous forcing for other greenhouse gasses (forcoth(t))[-]. The damages inflicted due to emissions (DAMAGES(t))[tr$.2005/a] are a function of the gross world product of gross abatement and damages and the damage fraction (DAMFRACT(t))[-]. The damage fraction is set equal to the damage intercept (a1)[-] times the increase in temperature of the atmosphere since 1900 (TATM)[$°C$] plus the damage quadratic term (a2)[-] times the increase in temperature to the power of the damage exponent (a3)[-]. To battle this rise in temperature and the resulting damages emissions can be reduced. The price of this reduction is captured in the abatement cost (ABATECOST(t))[tr$.2005/a]. Abatement cost is a function of the gross world product of gross abatement and damages, the adjusted cost for a backstop technology (cost1(t)),the emission control rate to the power of the control cost function (expcost2)[-] and the fraction of emissions under control (partfract(t))[-] to the power 1 - the control cost function. The marginal cost of abatement (MCABATE(t)) is equal to the backstop price (pbacktime(t)) times the emission control rate to the power control cost function minus 1.

The climate and carbon cycle starts with a definition of the carbon concentration increase in the atmosphere since 1750 (MAT(t))[GtCO_2]. The increase is based on the previous level times a carbon cycle transition matrix(abriviation:cctm) (b11)[-] for long-run consistency of the carbon cycle. To this value the carbon concentration increase since 1750 of the shallow oceans is added (MU(t))[GtCO_2] which is multiplied by again a cctm (b21)[-]. The increase of the concentration within the shallow oceans in itself is a function of the concentration in the atmosphere times a cctm (b12)[-] plus the increase in carbon concentration in the lower oceans since 1750 (ML(t))[GtCO_2] times the cctm (b32)[-] and its own previous concentration times cctm (b22)[-]. The increase in concentration of carbon in the lower oceans is a function of its previous concentration time cctm (b33)[-] plus the increase in carbon concentration of the shallow oceans times cctm (b23)[-]. The resulting increase in temperature of the atmosphere since 1900 (TATM(t))[$°C$] is a function of its previous self plus the climate equation coefficient for the upper lever (c1)[-] times the subtraction of the increase of radiative forcing and radiative forcings of the equilibrium CO_2 doubling divided by the equilibrium temperature impact (t2xco2)[°C per doubling CO_2]. The product of this substraction is multiplied by the previous increase in temperature since 1900. Which as a whole is to be subtracted by product of the transfer coefficient of the upper to the lower stratum (c3)[-] and the previous temperature increase minus the increase in temperature of the lower oceans since 1900 (TOCEAN(t))[°C]. The temperature of the lower oceans is again a function of its previous self plus transfer coefficient for lower levels (c4)[-] times the difference between the previous increase in atmospheric temperature and the increase in the lower ocean temperature.

To conclude with the economics of the model, the gross world product of abatement and damages (YGROSS)[tr$.2005/a] is defined to be a function of the level of total productivity (al(t))[-] and the level of labour and populations (l(t))[-] of which the product to the power one - the capital elasticity in the production function (γ)[-] is multiplied the capital stock (K(t))[tr$.2005/a] to the power γ. The gross world product is used to calculated the net output of damages (YNET(t))[tr$.2005/a] by multiplying it by one - the damage fraction. The gross world product of net abatement and damages (Y(t))[tr$.2005/a] is formed by the subtraction of the net output of damages and the cost of emission reduction. The resulting consumption[1] (C(t))[tr$.2005/a] is equal to the gross world net product minus the investment (I(t))[tr$.2005/a]. Which in itself is a function of the Gross savings rate as a function the the gross world product (S(t))[-] and the gross world net product. Change in the capital stock (K(t))[tr$.2005/a] is calculated as the sum of investment and 1 minus the depreciation of capital (dk)[-] times the current capital stock. The economic variables conclude with the real interest rate (RI(t)) which is defined to be the sum of 1 and the initial rate of social time preference per year (prstp)[-] times the change in per capita consumption to the power of the division of the elasticity of marginal utility (elasmu)[-] and the number of years per period (tstep) minus one.

For optimization purposes the DICE defines three more functions under the name of utility (the objective function). The per period utility function (PERIODU(t)) is equal to the per capita consumption to the power of the elasticity of marginal utility. The result of this variable is multiplied by the level of population and labour and the average utility social discount rate (rr(t))[-] form the period utility (CEMUTOTPER(t)). Finally the welfare function (UTILITY(t))[-] is equal to the sum of this periods utility times the number of years per period and a multiplicative scaling coefficient (scale1)[-].

[1]Consumption per capita (CPC(t))[1000$.2005/a per person]

Table A.1: Equations of the model

Emissions and damages	
$E(t) =$	$EIND(t) + entree(t)$
$EIND(t) =$	$\sigma(t) \cdot YGROSS(t) \cdot (1 - MIU(t))$
$CCA(t) =$	$CCA(t-1) + EIND \cdot \frac{15}{11}$
$FORC(t) =$	$fco22x \cdot \frac{log \frac{MAT(t)}{588.000}}{log(2)} + forcoth(t)$
$DAMFRAC(t) =$	$a1 \cdot TATM(t) + (a2 \cdot TATm(t))^{a3}$
$DAMAGES(t) =$	$YGROSS(t) \cdot DAMFRAC(t)$
$ABATECOST(t) =$	$YGROSS(t) \cdot cost1(t) \cdot MIU(t)^{expcost2}$
	$\cdot partfract(t)^{1-expcost2}$
$MCABATE(t) =$	$pbacktime(t) \cdot MIU(t)^{expcost2-1}$
$CPRICE(t) =$	$pbacktime(t) \cdot \frac{MIU(t)}{partfract(t)}^{expcost2-1}$
Climate and carbon cycle	
$MAT(t) =$	$MAT(t-1) \cdot b11 + MU(t-1) \cdot b21 + E(t-1) \cdot \frac{15}{11}$
$ML(t) =$	$ML(t-1) \cdot b33 + MU(t-1) \cdot b23$
$MU(t) =$	$MAT(t-1) \cdot b12 + MU(t-1) \cdot b22 + ML(t-1) \cdot b32$
$TATM(t) =$	$TATM(t-1) + c1 \cdot (FORC(t) - \frac{fco22x}{t2xCO_2} \cdot TATM(t-1))$
	$-c3(TATM(t-1) - TOCEAN(t-1)$
$TOCEAN(t) =$	$TOCEAN(t-1) + c4 \cdot (TATM(t-1) - TOCEAN(t-1))$
Economic variables	
$YGROSS(t) =$	$(al(t)) \cdot \frac{l(t)}{1000})^{1-\gamma} \cdot K(t)^{\gamma}$
$YNET(t) =$	$YGROSS(t) \cdot (1 - damfrac(t))$
$Y(t) =$	$YNET(t) - ABATECOST(t)$
$C(t) =$	$Y(t) - I(t)$
$CPC(t) =$	$1000 \cdot \frac{C(t)}{l(t)}$
$I(t) =$	$S(t) \cdot Y(t)$
$K(t) <$	$(1 - dk)^{tstep} \cdot K(t-1) + tstep \cdot I(t-1)$
$RI(t) =$	$(1 + prstp) \cdot (\frac{CPC(t)}{CPC(t-1)})^{\frac{elasmu}{tstep}} - 1$
Utility	
$PERIODU(t) =$	$\frac{C(t) * 1000}{l(t)}^{\frac{(1-elasmu)-1}{1-elasmu}} - 1$
$CEMUTOTPER(t) =$	$PERIODU(t) \cdot l(t) \cdot rr(t)$
$UTILITY(t) =$	$tstep \cdot scale1 \cdot \int_0^t CEMUTOTPER(t)dt$

Appendix B

The EICE model in GAMS format

```
$title       Extended DICE2013R (EICE2013R) with rewritten equations and multi
    stage stochastic format. Before adjustment are made it is advised that the
    modeller consults the GAMS EMP stochatic programming manual. It is advised to
    first practice with the examples in this model before alterations to this model
     are made. As the GAMS Latex converter does not allow for the minus sign,
    negative values will be appointed with "n!".


set   t  Time periods (20 years per period)                        /1*15/
st(t)       recourse periods (following years)               /2*15/
cst(st) consumtion gradient time                             /3*15/
torg        Original number of periods (5years per period)       /1*60/;



parameters

** uncertainty parameters
t2xco2(st) Equilibrium temp impact (oC per doubling CO2)


**Time Step
tstep    Years per Period                                    /20/
tsteporg Years per Period                                    /5/
scale    new vs org

** If optimal control
ifopt    Indicator where optimized is 1 and base is 0        /1/

** Preferences
elasmu   Elasticity of marginal utility of consumption        /1.45/
prstp    Initial rate of social time preference per year      /.015/

** Population and technology
gama     Capital elasticity in production function            / 0.300/
pop0     Initial world population (millions)                  / 6838 /
popadj   Growth rate to calibrate to 2050 pop projection      / 0.134/
popasym  Asymptotic population (millions)                     / 10500/
dk       Depreciation rate on capital (per year)              / 0.100/
q0       Initial world gross output (trill 2005 USD)          / 63.69/
k0       Initial capital value (trill 2005 USD)               / 135  /
a0       Initial level of total factor productivity           / 3.80 /
ga0      Initial growth rate for TFP per 5 years              / 0.079/
dela     Decline rate of TFP per 5 years                      / 0.006/

** Emissions parameters
```

```
gsigma1  Initial growth of sigma (per year)                       / n!0.01 /
dsig     Decline rate of decarbonization (per period)             / n!0.001/
eland0   Carbon emissions from land 2010 (GtCO2 per year)         / 3.3  /
deland   Decline rate of land emissions (per period)              / 0.2  /
e0       Industrial emissions 2010 (GtCO2 per year)               / 33.61/
miu0     Initial emissions control rate for base case 2010        /.039  /

** Carbon cycle
* Initial Conditions
mat0   Initial Concentration in atmosphere 2010 (GtC)             / 830.4 /
mu0    Initial Concentration in upper strata 2010 (GtC)           / 1527. /
ml0    Initial Concentration in lower strata 2010 (GtC)           / 10010 /
mateq  Equilibrium concentration atmosphere  (GtC)                / 588   /
mueq   Equilibrium concentration in upper strata (GtC)            / 1350  /
mleq   Equilibrium concentration in lower strata (GtC)            / 10000 /

* Flow paramaters

b12      Carbon cycle transition matrix                           / 0.352 /
b23      Carbon cycle transition matrix                           / 0.01  /


* These are for declaration and are defined later
b11      Carbon cycle transition matrix
b21      Carbon cycle transition matrix
b22      Carbon cycle transition matrix
b32      Carbon cycle transition matrix
b33      Carbon cycle transition matrix
sig0     Carbon intensity 2010 (kgCO2 per output 2005 USD 2010)

** Climate model parameters
Ecs           Expected temperature increase                       / 4.05  /
fex0     2010 forcings of non CO2 GHG (Wm 2)                      / 0.25  /
fex1     2100 forcings of non CO2 GHG (Wm 2)                      / 0.70  /
tocean0  Initial lower stratum temp change (C from 1900)          / 0.0068/
tatm0    Initial atmospheric temp change (C from 1900)            / 0.80  /

c10      Initial climate equation coefficient for upper level     /0.392  /
c1beta   Regression slope coefficient(SoA~Equil TSC)              /0.04972/
c3       Transfer coefficient upper to lower stratum              / 0.18 /
c4       Transfer coefficient for lower level                     / 0.1  /
fco22x   Forcings of equilibrium CO2 doubling (Wm 2)              / 3.8  /

** Climate damage parameters
a10       Initial damage intercept                                /0        /
a20       Initial damage quadratic term                           /0.00267 /
a1        Damage intercept                                        /0        /
a2        Damage quadratic term                                   /0.00267 /
a3        Damage exponent                                         /2.00    /

** Abatement cost
expcost2  Exponent of control cost function                       / 2.8  /
pback     Cost of backstop 2005$ per tCO2 2010                    / 344  /
gback     Initial cost decline backstop cost per period           / .1 /
limmiu    Upper limit on control rate after 2150                  / 1.2  /
tnopol    Period before which no emissions controls base          / 45   /
cprice0   Initial base carbon price (2005$ per tCO2)              / 1.0  /
gcprice   Growth rate of base carbon price per year               /.02   /

** Participation parameters
periodfullpart Period at which have full participation            /21  /
partfract2010  Fraction of emissions under control in 2010        / 1  /
partfractfull  Fraction of emissions under control at full time   / 1  /
```

```
** Availability of fossil fuels
fosslim        Maximum cumulative extraction fossil fuels (GtC) /6000/

** Scaling and inessential parameters
* Note that these are unnecessary for the calculations but are for convenience
scale1      Multiplicative scaling coefficient              /0.016408662 /
scale2      Additive scaling coefficient                    / n!3855.106895/ ;

* Program control variables
sets     tfirst(t), tlast(t), tearly(t), tlate(t);

PARAMETERS
l(t)           Level of population and labor
lb(torg)       Level of population and labor
al(t)          Level of total factor productivity
alb(torg)      Level of total factor productivity
sigma(t)       CO2 equivalent emissions output ratio
rr(t)          Average utility social discount rate
ga(torg)       Growth rate of productivity from
forcoth(t)     Exogenous forcing for other greenhouse gases
gl(t)          Growth rate of labor
gcost1         Growth of cost factor
gsig(t)        Change in sigma (cumulative improvement of energy efficiency)
etree(t)       Emissions from deforestation
cost1(t)       Adjusted cost for backstop
partfract(t)   Fraction of emissions in control regime
lam            Climate model parameter
gfacpop(t)     Growth factor population
pbacktime(t)   Backstop price
optlrsav       Optimal long run savings rate used for transversality
scc(t)         Social cost of carbon
cpricebase(t) Carbon price in base case
photel(t)      Carbon Price under no damages (Hotelling rent condition);

* Timestep scaling parameter
scale = tstep/tsteporg;

* Program control definitions
tfirst(t) = yes$(t.val eq 1);
tlast(t)  = yes$(t.val eq card(t));

* Parameters for long run consistency of carbon cycle
b11 = 1   b12;
b21 = b12*MATEQ/MUEQ;
b22 = 1  n!b21   n!b23;
b32 = b23*mueq/mleq;
b33 = 1   n!b32 ;

* Further definitions of parameters
sig0 = e0/(q0*(1 n!miu0));

*Based on the original curve a new labour function is constructed
lb("1") = pop0;
loop(torg, lb(torg+1)=lb(torg););
loop(torg, lb(torg+1)=lb(torg)*(popasym/lb(torg))**popadj ;);
l("1")=lb("1"); l("2")=lb("5");  l("3")=lb("9"); l("4")=lb("13");
l("5")=lb("17");  l("6")=lb("21"); l("7")=lb("25"); l("8")=lb("29");
l("9")=lb("33"); l("10")=lb("37"); l("11")=lb("41"); l("12")=lb("45");
l("13")=lb("49"); l("14")=lb("53"); l("15")=lb("57");

 *Based on the original curve a new tecnology function is constructed
ga(torg)=ga0*exp( n!dela*5*(((torg.val n!1))));
```

```
alb("1") = a0; loop(torg, alb(torg+1)=alb(torg)/((1 n!ga(torg)));););
al("1") = alb("1"); al("2")=alb("5"); al("3")=alb("9"); al("4")=alb("13");
al("5")=alb("17"); al("6")=alb("21"); al("7")=alb("25"); al("8")=alb("29");
al("9")=alb("33"); al("10")=alb("37"); al("11")=alb("41"); al("12")=alb("45");
al("13")=alb("49"); al("14")=alb("53"); al("15")=alb("57");

gsig("1")=gsigma1; loop(t,gsig(t+1)=gsig(t)*((1+dsig)**tstep) ;);
sigma("1")=sig0;   loop(t,sigma(t+1)=(sigma(t)*exp(gsig(t)*tstep)););

pbacktime(t)=pback*(1 n!gback)**((t.val n!1));
cost1(t) = pbacktime(t)*sigma(t)/expcost2/1000;

etree(t) = eland0*(1 deland)**(scale*(t.val n!1));
rr(t) = 1/((1+prstp)**(tstep*((t.val 1))));
forcoth(t) = fex0+ (1/18)*(fex1 fex0)*scale*(t.val n!1)$(t.val lt 5)+ (fex1 fex0)$(
    t.val ge 5);
optlrsav = (dk + .004)/(dk + .004*elasmu + prstp)*gama;

partfract(t)$(ord(T)>periodfullpart) = partfractfull;
partfract(t)$(ord(T)<periodfullpart+1) = partfract2010+(partfractfull partfract2010
    )*(ord(t) 1)/periodfullpart;

partfract("1")= partfract2010;


*new
t2xco2('2')=ECS;
loop(st, t2xco2(st+1)=t2xco2(st););


*Base Case      Carbon Price
cpricebase(t)= cprice0*(1+gcprice)**(tstep*(t.val n!1));

VARIABLES
MIU(t)          Emission control rate GHGs
FORC(t)         Increase in radiative forcing (watts per m2 from 1900)
TATM(t)         Increase temperature of atmosphere (degrees C from 1900)
TOCEAN(t)       Increase temperatureof lower oceans (degrees C from 1900)
TCAL(t)         Adjustment for climate sensitivity
TINC(t)         Inclusion of uncertainty
MAT(t)          Carbon concentration increase in atmosphere (GtC from 1750)
MU(t)           Carbon concentration increase in shallow oceans (GtC from 1750)
ML(t)           Carbon concentration increase in lower oceans (GtC from 1750)
E(t)            Total CO2 emissions (GtCO2 per year)
EIND(t)         Industrial emissions (GtCO2 per year)
C(t)            Consumption (trillions 2005 US dollars per year)
K(t)            Capital stock (trillions 2005 US dollars)
CPC(t)          Per capita consumption (thousands 2005 USD per year)
I(t)            Investment (trillions 2005 USD per year)
S(t)            Gross savings rate as fraction of gross world product
RI(t)           Real interest rate (per annum)
Y(t)            Gross world product net of abatement and damages
                (trillions 2005 USD per year)
YGROSS(t)       Gross world product GROSS of abatement and damages
                (trillions 2005 USD per year)
YRED(t)         Reduced gross world product (Ygross Abatement)
DAMAGES(t)      Damages (trillions 2005 USD per year)
DAMFRAC(t)      Damages as fraction of gross output
ABATECOST(t)    Cost of emissions reductions  (trillions 2005 USD per year)
MCABATE(t)      Marginal cost of abatement (2005$ per ton CO2)
CCA(t)          Cumulative industrial carbon emissions (GTC)
PERIODU(t)      One period utility function
CPRICE(t)       Carbon price (2005$ per ton of CO2)
```

```
CEMUTOTPER(t)   Period utility
UTILITY         Welfare function
;

NONNEGATIVE VARIABLES  MIU, TATM, MAT, MU, ML, Y, YGROSS, C, K, I;

EQUATIONS
*Emissions and Damages
EEQ(t)           Emissions equation
EINDEQ(t)        Industrial emissions
CCACCA(t)        Cumulative carbon emissions

FORCE(t)         Radiative forcing equation
DAMFRACEQ(t)     Equation for damage fraction
DAMEQ(t)         Damage equation

ABATEEQ(t)       Cost of emissions reductions equation
MCABATEEQ(t)     Equation for MC abatement
CARBPRICEEQ(t)   Carbon price equation from abatement

*Climate and carbon cycle
MMAT(t)          Atmospheric concentration equation
MMU(t)           Shallow ocean concentration
MML(t)           Lower ocean concentration
TATMEQ(t)        Temperature climate equation for atmosphere
TCALEQ(st)       Climate sensitivity inclusion
TINCEQ(st)       Inclusion of uncertainty
TOCEANEQ(t)      Temperature climate equation for lower oceans

*Economic variables
YGROSSEQ(t)      Output gross equation
YREDEQ(t)        Output reduced equation
YY(t)            Output net equation
CC(t)            Consumption equation
CPCE(t)          Per capita consumption definition
SEQ(t)           Savings rate equation
KK(t)            Capital balance equation
RIEQ(t)          Interest rate equation

* Utility
CEMUTOTPEREQ(t)  Period utility
PERIODUEQ(t)     Instantaneous utility function equation
UTIL             Objective function      ;

** Equations of the model
*Emissions and Damages
eeq(t)..                E(t)              =E= EIND(t) + etree(t);
eindeq(t)..             EIND(t)           =G= sigma(t) * YGROSS(t) * (1 n!(MIU(t)));
ccacca(t)$st(t)..       CCA(t)            =E= CCA(t n!1)+ EIND(t n!1)*tstep/3.666;
force(t)..              FORC(t)           =E= fco22x * ((log((MAT(t)/588.000))/log
    (2))) + forcoth(t);
damfraceq(t)..          DAMFRAC(t)        =E= (a1*TATM(t))+(a2*TATM(t)**a3) ;
dameq(t)..              DAMAGES(t)        =E= DAMFRAC(t);
abateeq(t)..            ABATECOST(t)      =E= YGROSS(t) * cost1(t) * (MIU(t)**
    expcost2) * (partfract(t)**(1 n!expcost2));
mcabateeq(t)..          MCABATE(t)        =E= pbacktime(t) * MIU(t)**(expcost2 n!1);
carbpriceeq(t)..        CPRICE(t)         =E= pbacktime(t) * (MIU(t)/partfract(t))
    **(expcost2 n!1);

*Climate and carbon cycle
mmat(t)$st(t)..         MAT(t)            =E= MAT(t n!1)*b11 + MU(t n!1)*b21 + (E(t
    n!1)*(tstep/3.666));
mml(t)$st(t)..          ML(t)             =E= ML(t n!1)*b33  + MU(t n!1)*b23;
```

```
mmu(t)$st(t)..               MU(t)              =E= MAT(t n!1)*b12 + MU(t n!1)*b22 + ML(t n!1)*b32;
TCALEQ(st)..             TCAL(st)           =E= c10+ c1beta*(t2xco2(st) n!ECS);
TINCEQ(st)..             TINC(st)           =E= fco22x / t2xco2(st);
TATMEQ(t)$st(t)..        TATM(t)            =E= (TATM(t n!1)) +  Tcal(t)*(( FORC(t) n!TINC(t)*TATM(t n!1)) (c3*(TATM(t n!1) TOCEAN(t n!1))));
toceaneq(t)$st(t)..      TOCEAN(t)          =E= TOCEAN(t n!1) + c4*(TATM(t n!1) TOCEAN(t n!1));

*Economic variables
ygrosseq(t)..            YGROSS(t)          =E= (al(t)*(L(t)/1000)**(1 n!GAMA))*(K(t)**GAMA);
yredeq(t)..              YRED(t)            =E= YGROSS(t)   n!ABATECOST(t);
yy(t)..                  Y(t)               =E= YRED(t)/(1+DAMAGES(t));
cc(t)..                  C(t)               =E= Y(t)   n!I(t);
cpce(t)..                CPC(t)             =E= 1000 * C(t) / L(t);
seq(t)..                 I(t)               =E= S(t) * Y(t);
kk(t)$st(t)..            K(t)               =L= (1 n!dk)**tstep * K(t n!1) + tstep * I(t n!1);
rieq(t)$st(t)..          RI(t)              =E= (1+prstp) * (CPC(t)/CPC(t n!1))**(elasmu/tstep)   n!1;

*Utility
cemutotpereq(t)..        CEMUTOTPER(t)      =E= PERIODU(t) * L(t) * rr(t);
periodueq(t)..           PERIODU(t)         =E= ((C(t)*1000/L(t))**(1 n!elasmu) n!1)/(1 n!elasmu) n!1;
util..                   UTILITY            =E= tstep * scale1 * sum(t,  CEMUTOTPER(t)) + scale2 ;

*Resource limit
CCA.up(t)       = fosslim;

* Control rate limits
MIU.up(t)            = limmiu*partfract(t);
MIU.up(t)$(t.val<30) = 1;

**  Upper and lower bounds for stability
K.LO(t)         = 1;
MAT.LO(t)       = 10;
MU.LO(t)        = 100;
ML.LO(t)        = 1000;
C.LO(t)         = 2;
TOCEAN.UP(t)    = 20;
TOCEAN.LO(t)    =  n!1;
TATM.UP(t)      = 9.1;
TATM.lo(t)      = 0;
DAMAGES.lo(t)   = 0.001;
CPC.LO(t)       = .01;

* Control variables
* Set savings rate for steady state for last 10 periods
set lag10(t) ;
lag10(t) =  yes$(t.val gt card(t) 2);
S.FX(lag10(t)) = optlrsav;

* Initial conditions
CCA.FX('1')     = 90;
K.FX('1')       = k0;

MAT.FX('1')     = mat0;
MU.FX('1')      = mu0;
ML.FX('1')      = ml0;
TATM.FX('1')    = tatm0;
```

```
TOCEAN.FX('1') = tocean0;


** Solution options
option iterlim = 9990000;
option reslim = 9999999;
option solprint = on;
option limrow = 0;
option limcol = 0;

model  CO2 /all/;


miu.fx('1') = miu0;


*****************************
The following section covers the extension of the deterministic model. When
    simulation both a continuous and a discrete distribution can be used.
*****************************

*** for CONTINOUS disctributions
*** Use the LINDO solver
*$funclibin msllib lsadclib
*function    setSeed                / msllib.setSeed /
*            sampleUniform    / msllib.sampleLSUniform  /
*            getSampleValues / msllib.getSampleValues /;
*scalar scalK;
*scalK = sampleUniform(2.5,4,3);
*set g /1*3/; parameter sv1(g);
*loop(g,
*   sv1(g) = getSampleValues(scalK);
*);
*display sv1;

*file emp / '%emp.info%' /; put emp '* problem %gams.i%'/;
*put emp; emp.nd=4;
*put "randvar t2xco2('2') discrete "; loop(g, put (1/card(g)) ' ' sv1(g) ' ');
*put "randvar t2xco2('3') discrete "; loop(g, put (1/card(g)) ' ' sv1(g) ' ');
*put "randvar t2xco2('4') discrete "; loop(g, put (1/card(g)) ' ' sv1(g) ' ');
*put "randvar t2xco2('5') discrete "; loop(g, put (1/card(g)) ' ' sv1(g) ' ');
*put "randvar t2xco2('6') discrete "; loop(g, put (1/card(g)) ' ' sv1(g) ' ');
*put "randvar t2xco2('7') discrete "; loop(g, put (1/card(g)) ' ' sv1(g) ' ');

*** For DISCRETE distributions
*** Use the DE solver
file emp / '%emp.info%' /; put emp '* problem %gams.i%'/;
put emp; emp.nd=4;
put "randvar t2xco2('2') discrete ", 0.25 2.2 0.50 3 0.25 8 /;
put "randvar t2xco2('3') discrete ", 0.25 2.2 0.50 3 0.25 8 /;
put "randvar t2xco2('4') discrete ", 0.25 2.2 0.50 3 0.25 8 /;
put "randvar t2xco2('5') discrete ", 0.25 2.2 0.50 3 0.25 8 /;
put "randvar t2xco2('6') discrete ", 0.25 2.2 0.50 3 0.25 8 /;
put "randvar t2xco2('7') discrete ", 0.25 2.2 0.50 3 0.25 8 /;
** This line can be extended to incorporate more uncertain stages



$onput
*** stage > variable > equation
stage 1  E('1') EIND('1') MIU('1') k('1') Ygross('1') Yred('1') Y('1') FORC('1')
    ABATECOST('1') MCABATE('1')
```

```
CPRICE(’1’) C(’1’) CPC(’1’) I(’1’) S(’1’)  DAMAGES(’1’) DAMFRAC(’1’) PERIODU(’1’) CEMUTOTPER(’1’)
EEQ(’1’) EINDEQ(’1’)  FORCE(’1’) DAMFRACEQ(’1’) DAMEQ(’1’) ABATEEQ(’1’) MCABATEEQ(’1’) CARBPRICEEQ(’1’)
YGROSSEQ(’1’) YREDEQ(’1’) YY(’1’) CC(’1’) CPCE(’1’) SEQ(’1’) CEMUTOTPEREQ(’1’) PERIODUEQ(’1’)

stage 2 E(’2’) EIND(’2’) MIU(’2’) TATM(’2’) TOCEAN(’2’) k(’2’) Ygross(’2’) Yred(’2’) Y(’2’) MAT(’2’) ML(’2’) MU(’2’) FORC(’2’) ABATECOST(’2’) MCABATE(’2’) TCAL(’2’) TINC(’2’)
CCA(’2’) CPRICE(’2’) C(’2’) CPC(’2’) I(’2’) S(’2’)  DAMAGES(’2’) DAMFRAC(’2’) PERIODU(’2’) CEMUTOTPER(’2’) t2xco2(’2’)
EEQ(’2’) EINDEQ(’2’) CCACCA(’2’) FORCE(’2’) DAMFRACEQ(’2’) DAMEQ(’2’) ABATEEQ(’2’) MCABATEEQ(’2’) CARBPRICEEQ(’2’) TCALEQ(’2’) TINCEQ(’2’)
MMAT(’2’) MMU(’2’) MML(’2’) TATMEQ(’2’) TOCEANEQ(’2’) YGROSSEQ(’2’) YREDEQ(’2’) YY(’2’) CC(’2’) CPCE(’2’) RIEQ(’2’) SEQ(’2’) KK(’2’) CEMUTOTPEREQ(’2’) PERIODUEQ(’2’)

stage 3 E(’3’) EIND(’3’) MIU(’3’) TATM(’3’) TOCEAN(’3’) k(’3’) Ygross(’3’) Yred(’3’) Y(’3’) MAT(’3’) ML(’3’) MU(’3’) FORC(’3’) ABATECOST(’3’) MCABATE(’3’) TCAL(’3’) TINC(’3’)
CCA(’3’) CPRICE(’3’) C(’3’) CPC(’3’) I(’3’) S(’3’) RI(’3’) DAMAGES(’3’) DAMFRAC(’3’) PERIODU(’3’) CEMUTOTPER(’3’) t2xco2(’3’)
EEQ(’3’) EINDEQ(’3’) CCACCA(’3’) FORCE(’3’) DAMFRACEQ(’3’) DAMEQ(’3’) ABATEEQ(’3’) MCABATEEQ(’3’) CARBPRICEEQ(’3’) TCALEQ(’3’) TINCEQ(’3’)
MMAT(’3’) MMU(’3’) MML(’3’) TATMEQ(’3’) TOCEANEQ(’3’) YGROSSEQ(’3’) YREDEQ(’3’) YY(’3’) CC(’3’) CPCE(’3’) SEQ(’3’) KK(’3’) RIEQ(’3’) CEMUTOTPEREQ(’3’) PERIODUEQ(’3’)

stage 4 E(’4’) EIND(’4’) MIU(’4’) TATM(’4’) TOCEAN(’4’) k(’4’) Ygross(’4’) Yred(’4’) Y(’4’) MAT(’4’) ML(’4’) MU(’4’) FORC(’4’) ABATECOST(’4’) MCABATE(’4’) TCAL(’4’) TINC(’4’)
CCA(’4’) CPRICE(’4’) C(’4’) CPC(’4’) I(’4’) S(’4’) RI(’4’) DAMAGES(’4’) DAMFRAC(’4’) PERIODU(’4’) CEMUTOTPER(’4’) t2xco2(’4’)
EEQ(’4’) EINDEQ(’4’) CCACCA(’4’) FORCE(’4’) DAMFRACEQ(’4’) DAMEQ(’4’) ABATEEQ(’4’) MCABATEEQ(’4’) CARBPRICEEQ(’4’) TCALEQ(’4’) TINCEQ(’4’)
MMAT(’4’) MMU(’4’) MML(’4’) TATMEQ(’4’) TOCEANEQ(’4’) YGROSSEQ(’4’) YREDEQ(’4’) YY(’4’) CC(’4’) CPCE(’4’) SEQ(’4’) KK(’4’) RIEQ(’4’) CEMUTOTPEREQ(’4’) PERIODUEQ(’4’)

stage 5 E(’5’) EIND(’5’) MIU(’5’) TATM(’5’) TOCEAN(’5’) k(’5’) Ygross(’5’) Yred(’5’) Y(’5’) MAT(’5’) ML(’5’) MU(’5’) FORC(’5’) ABATECOST(’5’) MCABATE(’5’) TCAL(’5’) TINC(’5’)
CCA(’5’) CPRICE(’5’) C(’5’) CPC(’5’) I(’5’) S(’5’) RI(’5’) DAMAGES(’5’) DAMFRAC(’5’) PERIODU(’5’) CEMUTOTPER(’5’) t2xco2(’5’)
EEQ(’5’) EINDEQ(’5’) CCACCA(’5’) FORCE(’5’) DAMFRACEQ(’5’) DAMEQ(’5’) ABATEEQ(’5’) MCABATEEQ(’5’) CARBPRICEEQ(’5’) TCALEQ(’5’) TINCEQ(’5’)
MMAT(’5’) MMU(’5’) MML(’5’) TATMEQ(’5’) TOCEANEQ(’5’) YGROSSEQ(’5’) YREDEQ(’5’) YY(’5’) CC(’5’) CPCE(’5’) SEQ(’5’) KK(’5’) RIEQ(’5’) CEMUTOTPEREQ(’5’) PERIODUEQ(’5’)

stage 6 E(’6’) EIND(’6’) MIU(’6’) TATM(’6’) TOCEAN(’6’) k(’6’) Ygross(’6’) Yred(’6’) Y(’6’) MAT(’6’) ML(’6’) MU(’6’) FORC(’6’) ABATECOST(’6’) MCABATE(’6’) TCAL(’6’) TINC(’6’)
CCA(’6’) CPRICE(’6’) C(’6’) CPC(’6’) I(’6’) S(’6’) RI(’6’) DAMAGES(’6’) DAMFRAC(’6’) PERIODU(’6’) CEMUTOTPER(’6’) t2xco2(’6’)
EEQ(’6’) EINDEQ(’6’) CCACCA(’6’) FORCE(’6’) DAMFRACEQ(’6’) DAMEQ(’6’) ABATEEQ(’6’) MCABATEEQ(’6’) CARBPRICEEQ(’6’)   TCALEQ(’6’) TINCEQ(’6’)
MMAT(’6’) MMU(’6’) MML(’6’) TATMEQ(’6’) TOCEANEQ(’6’) YGROSSEQ(’6’) YREDEQ(’6’) YY(’6’) CC(’6’) CPCE(’6’) SEQ(’6’) KK(’6’) RIEQ(’6’) CEMUTOTPEREQ(’6’) PERIODUEQ(’6’)
```

```
stage 7 E('7') EIND('7') MIU('7') TATM('7') TOCEAN('7') k('7') Ygross('7') Yred('7') Y('7') MAT('7') ML('7') MU('7') FORC('7') ABATECOST('7') MCABATE('7') TCAL('7') TINC('7')
CCA('7') CPRICE('7') C('7') CPC('7') I('7') S('7') RI('7') DAMAGES('7') DAMFRAC('7') PERIODU('7') CEMUTOTPER('7')  t2xco2('7')
EEQ('7') EINDEQ('7') CCACCA('7') FORCE('7') DAMFRACEQ('7') DAMEQ('7') ABATEEQ('7') MCABATEEQ('7') CARBPRICEEQ('7')   TCALEQ('7') TINCEQ('7')
MMAT('7') MMU('7') MML('7') TATMEQ('7') TOCEANEQ('7') YGROSSEQ('7') YREDEQ('7') YY('7') CC('7') CPCE('7') SEQ('7') KK('7') RIEQ('7') CEMUTOTPEREQ('7') PERIODUEQ('7')

stage 8 E('8') EIND('8') MIU('8') TATM('8') TOCEAN('8') k('8') Ygross('8') Yred('8') Y('8') MAT('8') ML('8') MU('8') FORC('8') ABATECOST('8') MCABATE('8') TCAL('8') TINC('8')
CCA('8') CPRICE('8') C('8') CPC('8') I('8') S('8') RI('8') DAMAGES('8') DAMFRAC('8') PERIODU('8') CEMUTOTPER('8')
EEQ('8') EINDEQ('8') CCACCA('8') FORCE('8') DAMFRACEQ('8') DAMEQ('8') ABATEEQ('8') MCABATEEQ('8') CARBPRICEEQ('8')   TCALEQ('8') TINCEQ('8')
MMAT('8') MMU('8') MML('8') TATMEQ('8') TOCEANEQ('8') YGROSSEQ('8') YREDEQ('8') YY('8') CC('8') CPCE('8') SEQ('8') KK('8') RIEQ('8') CEMUTOTPEREQ('8') PERIODUEQ('8')

stage 9 E('9') EIND('9') MIU('9') TATM('9') TOCEAN('9') k('9') Ygross('9') Yred('9') Y('9') MAT('9') ML('9') MU('9') FORC('9') ABATECOST('9') MCABATE('9') TCAL('9') TINC('9')
CCA('9') CPRICE('9') C('9') CPC('9') I('9') S('9') RI('9') DAMAGES('9') DAMFRAC('9') PERIODU('9') CEMUTOTPER('9')
EEQ('9') EINDEQ('9') CCACCA('9') FORCE('9') DAMFRACEQ('9') DAMEQ('9') ABATEEQ('9') MCABATEEQ('9') CARBPRICEEQ('9')   TCALEQ('9') TINCEQ('9')
MMAT('9') MMU('9') MML('9') TATMEQ('9') TOCEANEQ('9') YGROSSEQ('9') YREDEQ('9') YY('9') CC('9') CPCE('9') SEQ('9') KK('9') RIEQ('9') CEMUTOTPEREQ('9') PERIODUEQ('9')

stage 10 E('10') EIND('10') MIU('10') TATM('10') TOCEAN('10') k('10') Ygross('10') Yred('10') Y('10') MAT('10') ML('10') MU('10') FORC('10') ABATECOST('10') MCABATE('10')  TCAL('10') TINC('10')
CCA('10') CPRICE('10') C('10') CPC('10') I('10') S('10') RI('10') DAMAGES('10') DAMFRAC('10') PERIODU('10') CEMUTOTPER('10')
EEQ('10') EINDEQ('10') CCACCA('10') FORCE('10') DAMFRACEQ('10') DAMEQ('10') ABATEEQ('10') MCABATEEQ('10') CARBPRICEEQ('10')   TCALEQ('10') TINCEQ('10')
MMAT('10') MMU('10') MML('10') TATMEQ('10') TOCEANEQ('10') YGROSSEQ('10') YREDEQ('10') YY('10') CC('10') CPCE('10') SEQ('10') KK('10') RIEQ('10') CEMUTOTPEREQ('10') PERIODUEQ('10')

stage 11 E('11') EIND('11') MIU('11') TATM('11') TOCEAN('11') k('11') Ygross('11') Yred('11') Y('11') MAT('11') ML('11') MU('11') FORC('11') ABATECOST('11') MCABATE('11')  TCAL('11') TINC('11')
CCA('11') CPRICE('11') C('11') CPC('11') I('11') S('11') RI('11') DAMAGES('11') DAMFRAC('11') PERIODU('11') CEMUTOTPER('11')
EEQ('11') EINDEQ('11') CCACCA('11') FORCE('11') DAMFRACEQ('11') DAMEQ('11') ABATEEQ('11') MCABATEEQ('11') CARBPRICEEQ('11')   TCALEQ('11') TINCEQ('11')
MMAT('11') MMU('11') MML('11') TATMEQ('11') TOCEANEQ('11') YGROSSEQ('11') YREDEQ('11') YY('11') CC('11') CPCE('11') SEQ('11') KK('11') RIEQ('11') CEMUTOTPEREQ('11') PERIODUEQ('11')

stage 12 E('12') EIND('12') MIU('12') TATM('12') TOCEAN('12') k('12') Ygross('12') Yred('12') Y('12') MAT('12') ML('12') MU('12') FORC('12') ABATECOST('12') MCABATE('12') TCAL('12') TINC('12')
CCA('12') CPRICE('12') C('12') CPC('12') I('12') S('12') RI('12') DAMAGES('12') DAMFRAC('12') PERIODU('12') CEMUTOTPER('12')
EEQ('12') EINDEQ('12') CCACCA('12') FORCE('12') DAMFRACEQ('12') DAMEQ('12') ABATEEQ('12') MCABATEEQ('12') CARBPRICEEQ('12') TCALEQ('12') TINCEQ('12')
```

```
MMAT('12') MMU('12') MML('12') TATMEQ('12') TOCEANEQ('12') YGROSSEQ('12') YREDEQ
    ('12') YY('12') CC('12') CPCE('12') SEQ('12') KK('12') RIEQ('12') CEMUTOTPEREQ
    ('12') PERIODUEQ('12')

stage 13 E('13') EIND('13') MIU('13') TATM('13') TOCEAN('13') k('13') Ygross('13')
    Yred('13') Y('13') MAT('13') ML('13') MU('13') FORC('13') ABATECOST('13')
    MCABATE('13') TCAL('13') TINC('13')
CCA('13') CPRICE('13') C('13') CPC('13') I('13') S('13') RI('13') DAMAGES('13')
    DAMFRAC('13') PERIODU('13') CEMUTOTPER('13')
EEQ('13') EINDEQ('13') CCACCA('13') FORCE('13') DAMFRACEQ('13') DAMEQ('13') ABATEEQ
    ('13') MCABATEEQ('13') CARBPRICEEQ('13') TCALEQ('13') TINCEQ('13')
MMAT('13') MMU('13') MML('13') TATMEQ('13') TOCEANEQ('13') YGROSSEQ('13') YREDEQ
    ('13') YY('13') CC('13') CPCE('13') SEQ('13') KK('13') RIEQ('13') CEMUTOTPEREQ
    ('13') PERIODUEQ('13')

stage 14 E('14') EIND('14') MIU('14') TATM('14') TOCEAN('14') k('14') Ygross('14')
    Yred('14') Y('14') MAT('14') ML('14') MU('14') FORC('14') ABATECOST('14')
    MCABATE('14') TCAL('14') TINC('14')
CCA('14') CPRICE('14') C('14') CPC('14') I('14') S('14') RI('14') DAMAGES('14')
    DAMFRAC('14') PERIODU('14') CEMUTOTPER('14')
EEQ('14') EINDEQ('14') CCACCA('14') FORCE('14') DAMFRACEQ('14') DAMEQ('14') ABATEEQ
    ('14') MCABATEEQ('14') CARBPRICEEQ('14') TCALEQ('14') TINCEQ('14')
MMAT('14') MMU('14') MML('14') TATMEQ('14') TOCEANEQ('14') YGROSSEQ('14') YREDEQ
    ('14') YY('14') CC('14') CPCE('14') SEQ('14') KK('14') RIEQ('14') CEMUTOTPEREQ
    ('14') PERIODUEQ('14')

stage 15 E('15') EIND('15') MIU('15') TATM('15') TOCEAN('15') k('15') Ygross('15')
    Yred('15') Y('15') MAT('15') ML('15') MU('15') FORC('15') ABATECOST('15')
    MCABATE('15') TCAL('15') TINC('15')
CCA('15') CPRICE('15') C('15') CPC('15') I('15') S('15') RI('15') DAMAGES('15')
    DAMFRAC('15') PERIODU('15') CEMUTOTPER('15')
EEQ('15') EINDEQ('15') CCACCA('15') FORCE('15') DAMFRACEQ('15') DAMEQ('15') ABATEEQ
    ('15') MCABATEEQ('15') CARBPRICEEQ('15') TCALEQ('15') TINCEQ('15')
MMAT('15') MMU('15') MML('15') TATMEQ('15') TOCEANEQ('15') YGROSSEQ('15') YREDEQ
    ('15') YY('15') CC('15') CPCE('15') SEQ('15') KK('15') RIEQ('15') CEMUTOTPEREQ
    ('15') PERIODUEQ('15')

$offput
putclose emp;

**number of scenarios
Set scen        Scenarios / s1*s1000000 /;



Parameter
s_cs(scen,st)
s_miu(scen,t)
s_E(scen,t)
s_Fo(scen,t)
s_TATM(scen,t)
s_TOCEAN(scen,t)
s_mat(scen,t)
s_mu(scen,t)
s_ml(scen,t)
s_eind(scen,t)
s_c(scen,t)
s_k(scen,t)
s_cpc(scen,t)
s_i(scen,t)
s_s(scen,t)
s_ri(scen,t)
s_y(scen,t)
```

```
s_ygross(scen,t)
s_yred(scen,t)
s_damages(scen,t)
s_damfrac(scen,t)
s_abatecost(scen,t)
s_mcabate(scen,t)
s_cca(scen,t)
s_periodu(scen,t)
s_cprice(scen,t)
s_cemutotper(scen,t)          ;

** Statement of which variable is uncertain
Set dict / scen .scenario.''
t2xco2   .randvar         .s_cs
MIU      .level           .s_miu
E        .level           .s_E
FORC     .level           .s_Fo
TINC     .level           .s_TINC
TCAL     .level           .s_TCAL
TATM     .level           .s_TATM
TOCEAN   .level           .s_TOCEAN
Mat      .level           .s_mat
MU       .level           .s_mu
ML       .level           .s_ml
EIND     .level           .S_eind
C        .level           .s_c
K        .level           .s_k
CPC      .level           .s_cpc
I        .level           .s_i
S        .level           .s_s
RI       .level           .s_ri
Y        .level           .s_y
Ygross   .level           .s_ygross
Yred     .level           .s_yred
Damages .level            .s_damages
Damfrac .level            .s_damfrac
Abatecost .level          .s_abatecost
Mcabate .level            .s_mcabate
CCA      .level           .s_cca
Periodu .level            .s_periodu
Cprice   .level           .s_cprice
Cemutotper      .level   .s_cemutotper
/;

** choose solver
option emp = de;


** limit number of evaluated stages
$onecho > de.opt
*maxnodes  10000000
$offecho
CO2.optfile=1;

Option DECIMALS=4;


** SOLVE
Solve CO2 max UTILITY using emp scenario dict ;

** Show computation time
scalar executiontime;
executiontime = timeElapsed;
```

```
display s_cs, s_miu, s_E, s_y, sigma, etree, executiontime, forcoth,rr, pbacktime, cost1, al, l, partfract ;



** POST SOLVE
* Calculate social cost of carbon
*scc(t) = 1000*eeq.m(t)/cc.m(t);


file results /DiceResultsDE_dt20Sam3_Extreme_st7.csv/;     results.nd = 10 ; results.nw = 0 ; results.pw=1200; results.pc=5;
put results;
put /"Results of DICE model run using model DICE2013RExtended imported sample DE solver";
put /"Number of samples: 4, distributed scenarios: 2, 3, 4,5, 6 ";
put /"Distribution: dicrete 0.33 2.2 0.33 3.0 0.33 8"
Loop (T, put T.val);
put / "Year" ;
Loop (T, put (2015+(TSTEP*(T.val 1) ) ));
put / "Industrial Emissions (GTCO2 per year)" ;
Loop (T, put EIND.l(T));
put / "Atmospheric concentration of carbon (ppm)" ;
Loop (T, put (MAT.l(T)/2.13));
put / "Atmospheric Temperature (deg C above preindustrial)" ;
Loop (T, put TATM.l(T));
put / "Output (Net of Damages and Abatement, trillion USD pa) " ;
Loop (T, put Y.l(T));
put / "Climate Damages (fraction of gross output)" ;
Loop (T, put DAMages.l(T));
put / "Consumption Per Capita (thousand USD per year)" ;
Loop (T, put CPC.l(T));
put / "Carbon Price (per t CO2)" ;
Loop (T, put cprice.l(T));
put / "Emissions Control Rate (total)" ;
Loop (T, put MIU.l(T));
put / "Social cost of carbon" ;
*Loop (T, put scc(T));
*put / "Interest Rate (Real Rate of Return)" ;
Loop (T, put RI.l(T));
put / "Capital" ;
Loop (T, put K.l(T));
put / "Gross Economic Output" ;
Loop (T, put YGROSS.l(T));
put / "Oceanic Temperature (deg C above perindustrial" ;
Loop (T, put TOCEAN.l(T));
put / "Sigma" ;
Loop (T, put Sigma(T));
put / "Consumption" ;
Loop (T, put C.l(T));
put / "MU" ;
Loop (T, put MU.l(T));
put / "ML" ;
Loop (T, put ML.l(T));
put / "AL" ;
Loop (T, put AL(T));
put / "L" ;
Loop (T, put L(T));
put / "Savings" ;
Loop (T, put S.l(T));

putclose;
```

Appendix C

Enlarged results

C.1 Figures regarding section 5.2

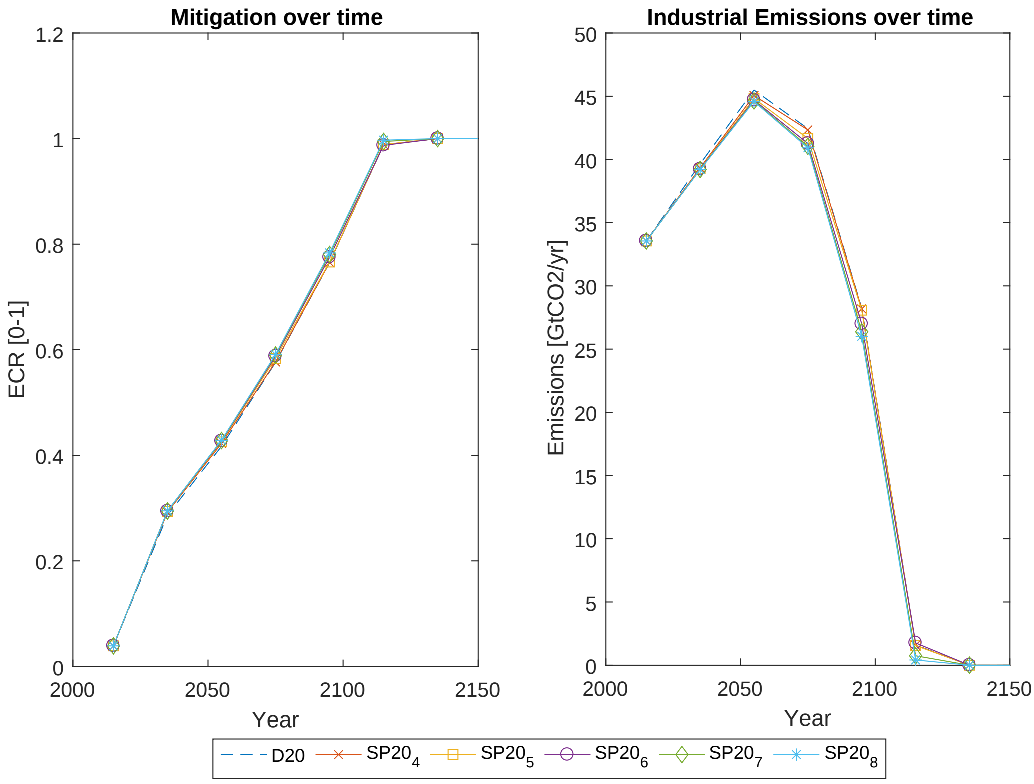

Figure C.1: The influence of the number of stochastic stages to SP20 under base conditions on the emissions

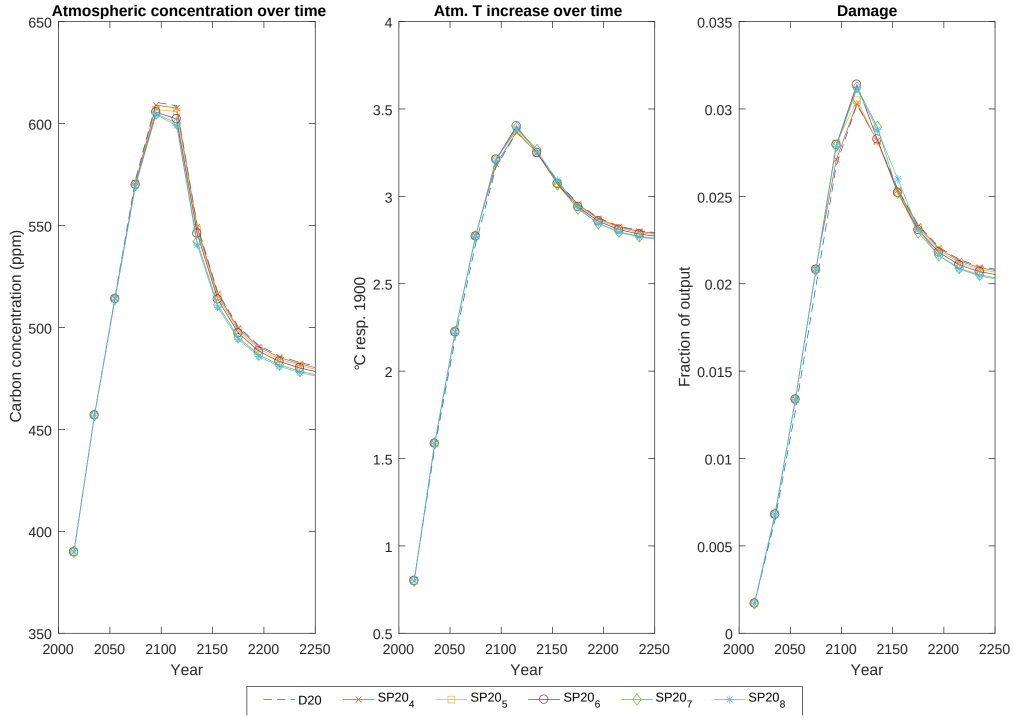

Figure C.2: The influence of the number of stochastic stages to SP20 under base conditions on the climate cycle

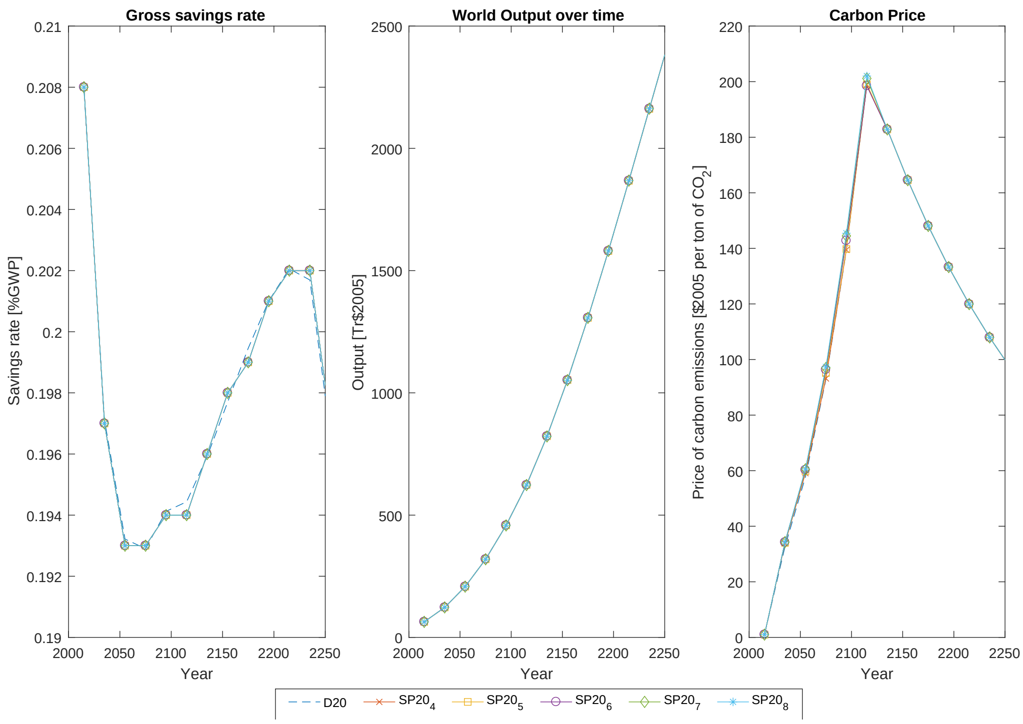

Figure C.3: The influence of the number of stochastic stages to SP20 under base conditions on the economic system

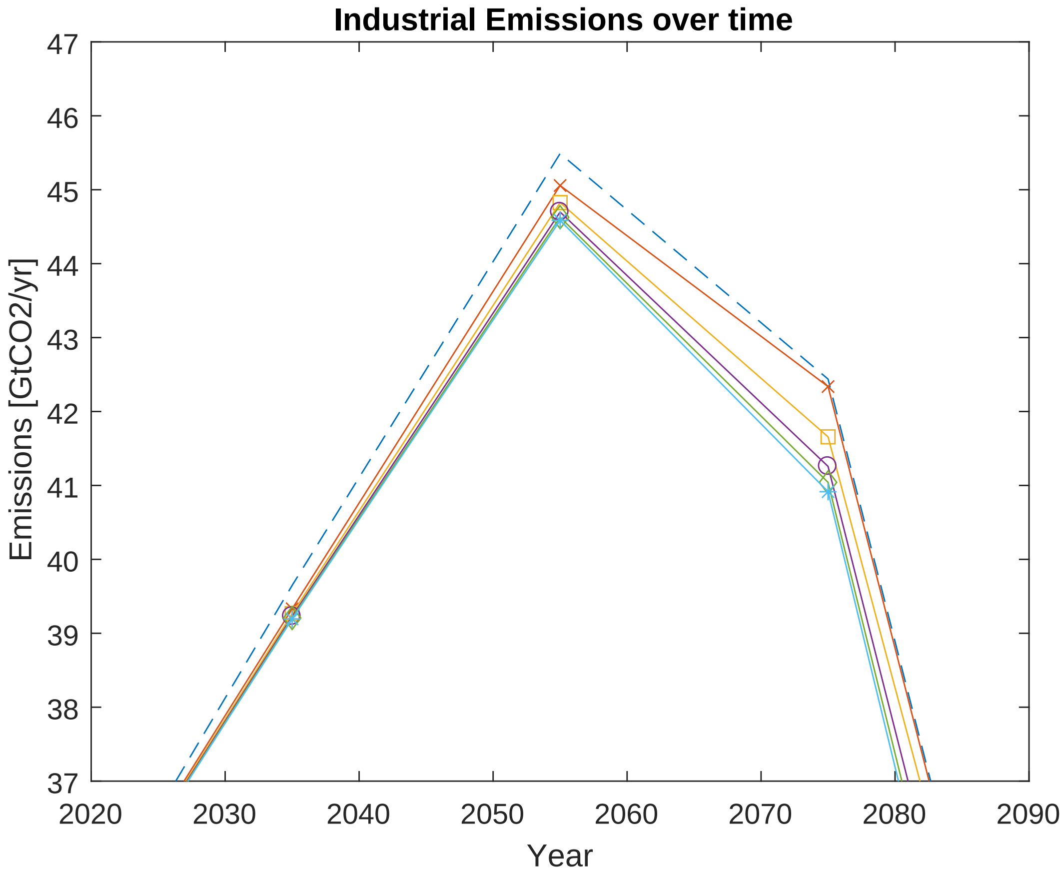

Figure C.4: Enlarged version of the emissions graph in figure C.1

C.2 Figures regarding section 5.3

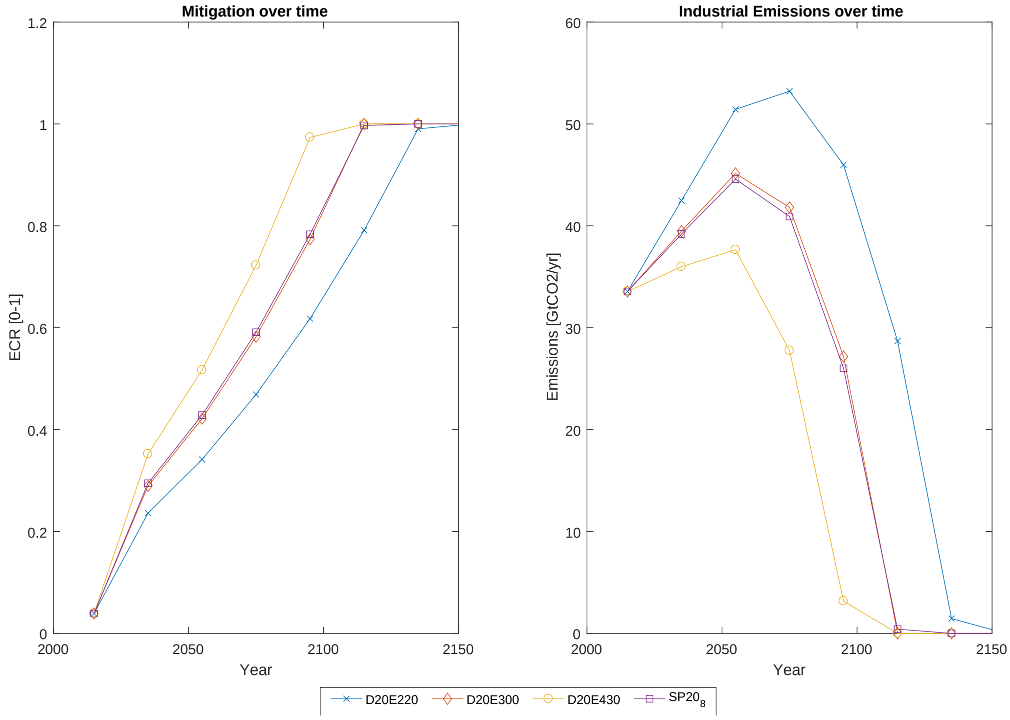

Figure C.5: Comparison of the emissions from the deterministic scenarios with CS:={2.2, 3.0, 4.3} and the stochastic program in the base case

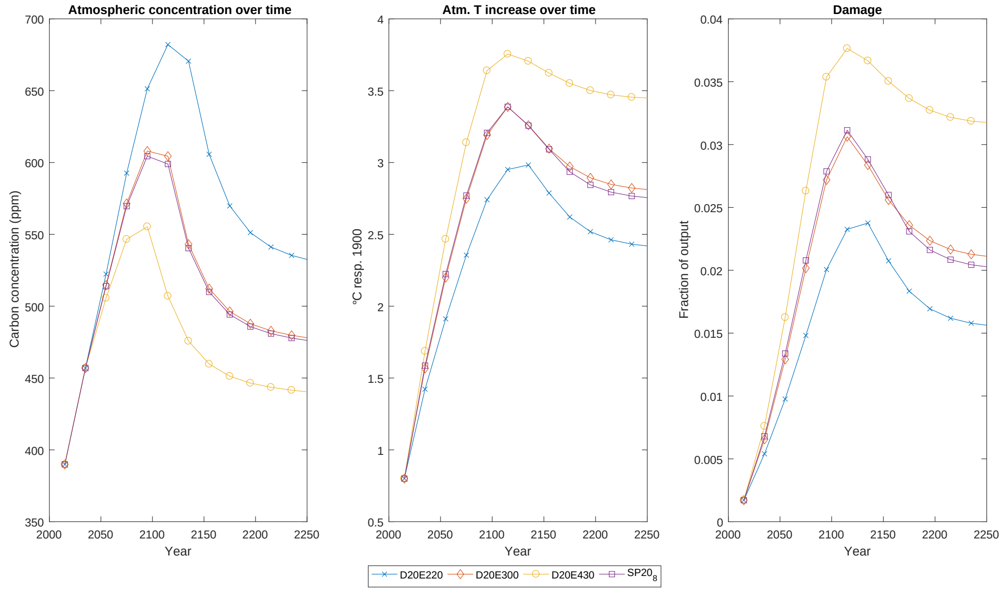

Figure C.6: Comparison of the climate dynamics from the deterministic scenarios with CS:={2.2, 3.0, 4.3} and the stochastic program in the base case

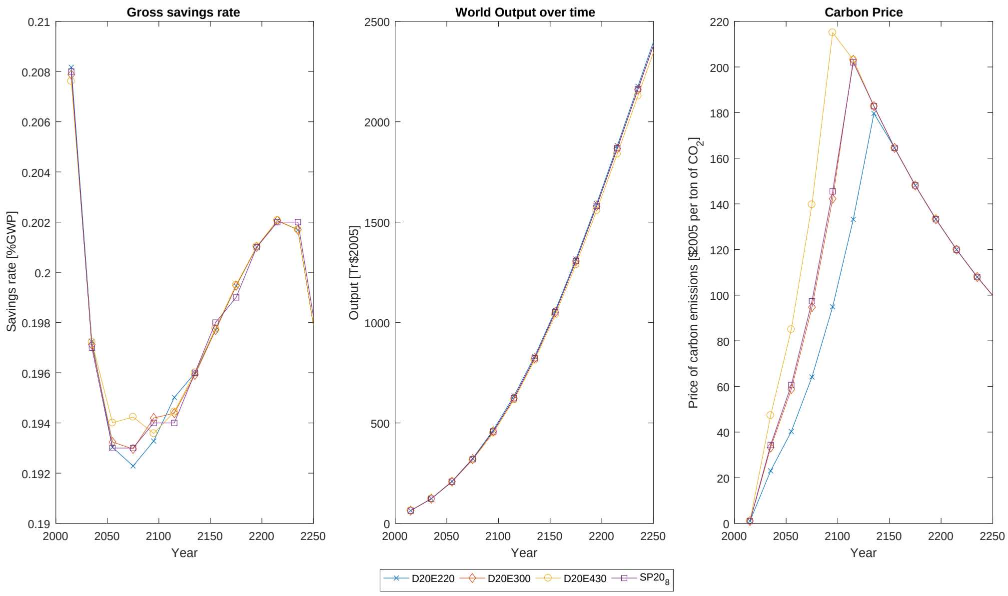

Figure C.7: Comparison of the economic response from the deterministic scenarios with CS:={2.2, 3.0, 4.3} and the stochastic program in the base case

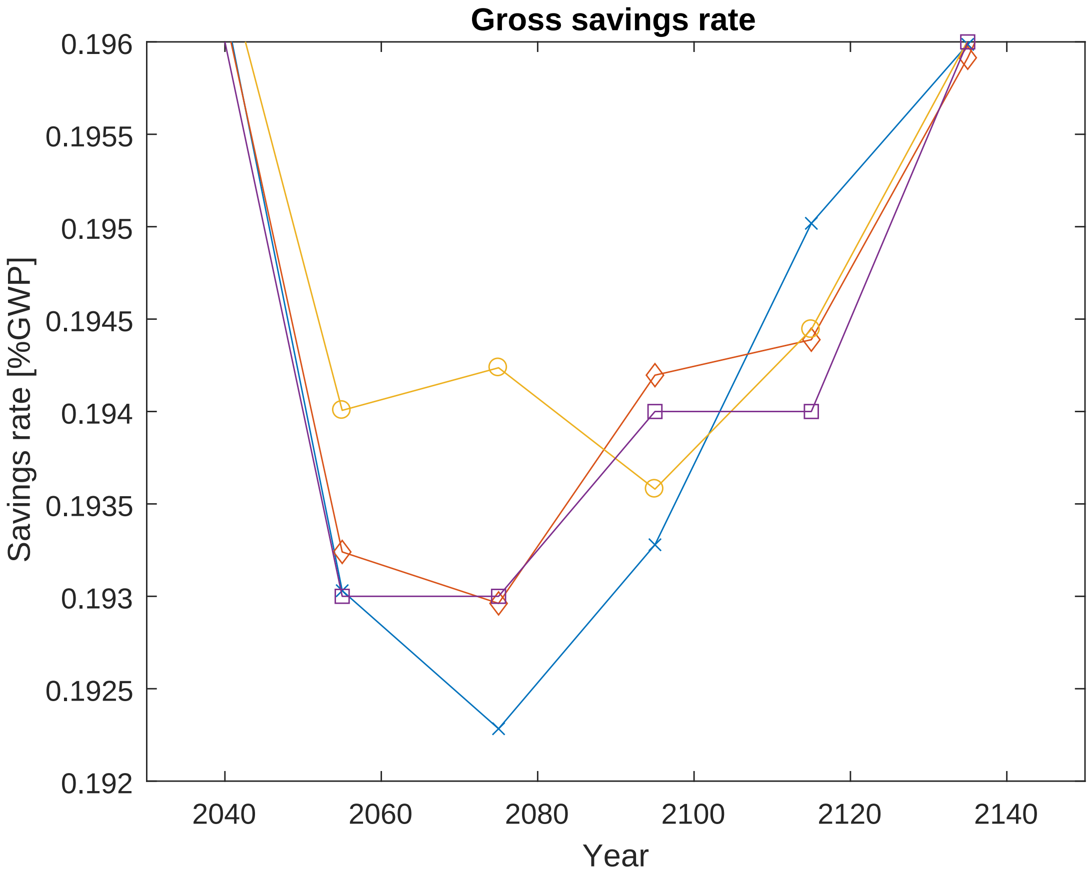

Figure C.8: Enlarged version of the savings graph in figure C.7

C.3 Figures regarding section 5.4

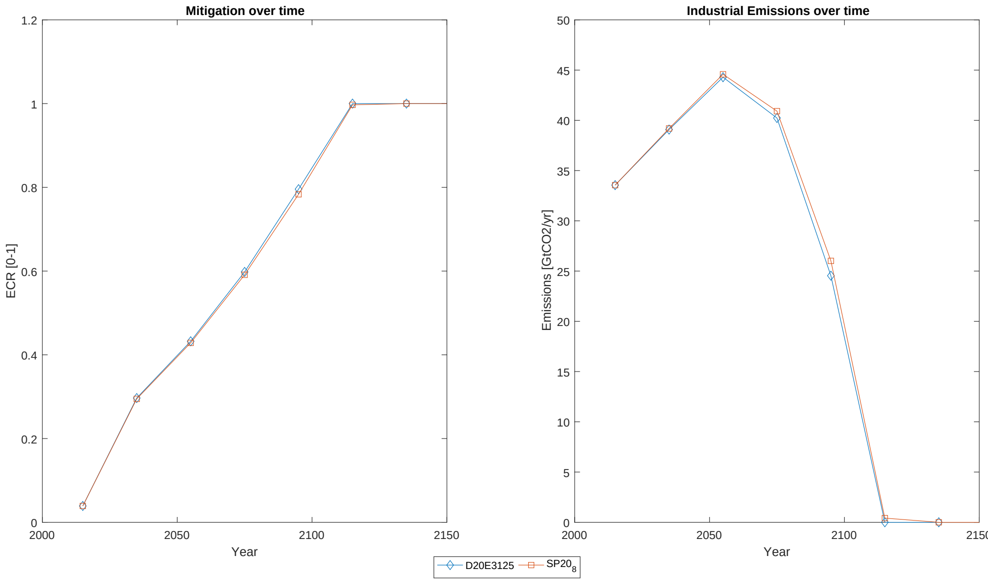

Figure C.9: The emissions and mitigation policy of the deterministic D20 program with CS = 3.125 versus the stochastic $SP20_8$ model under the base case

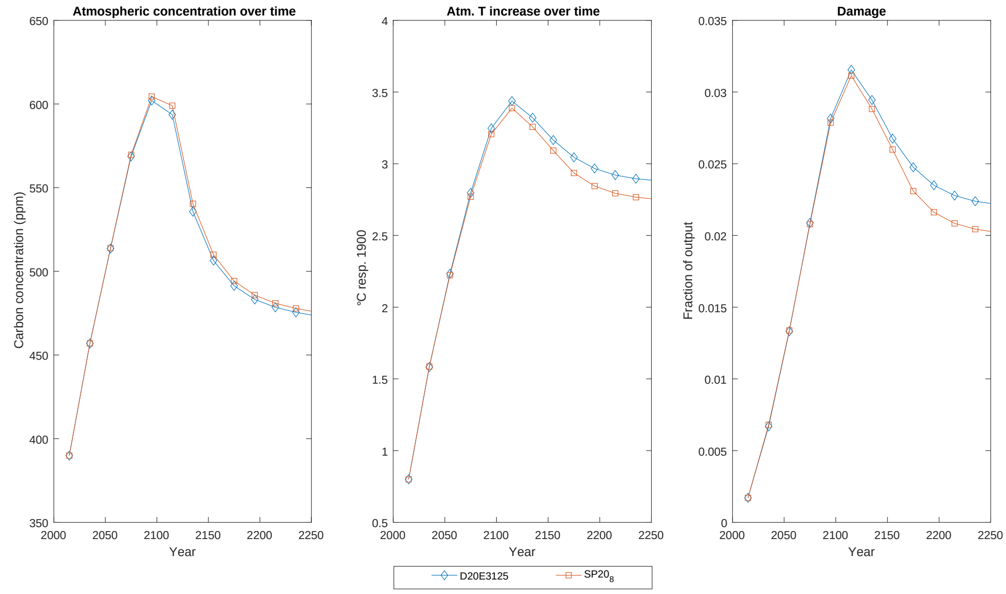

Figure C.10: The climate response and corresponding damage of the deterministic D20 program with CS = 3.125 versus the stochastic $SP20_8$ model under the base case

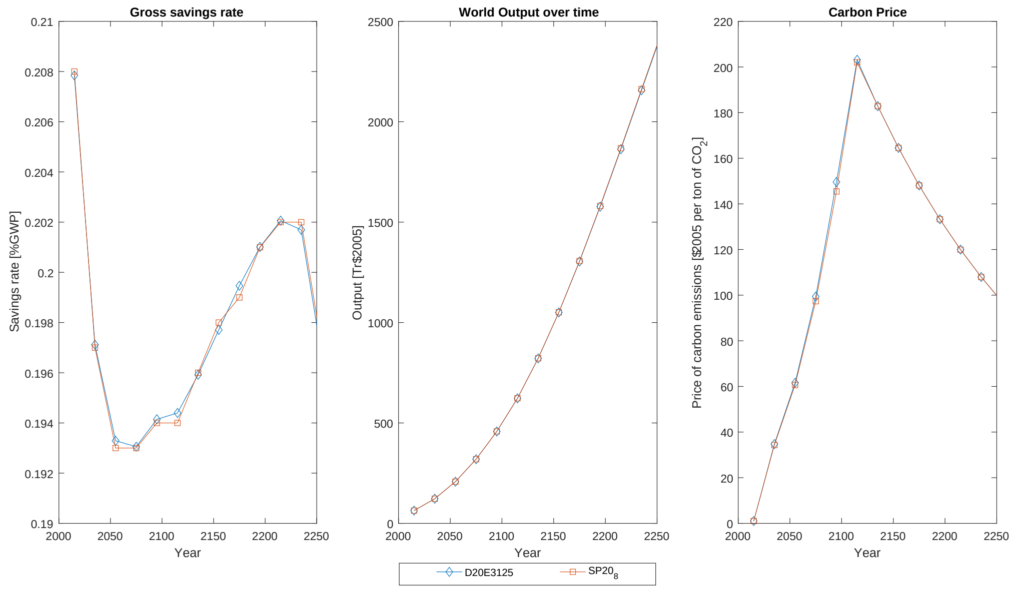

Figure C.11: The economic response of the deterministic D20 program with CS = 3.125 versus the stochastic $SP20_8$ model under the base case

C.4 Figures regarding section 5.5

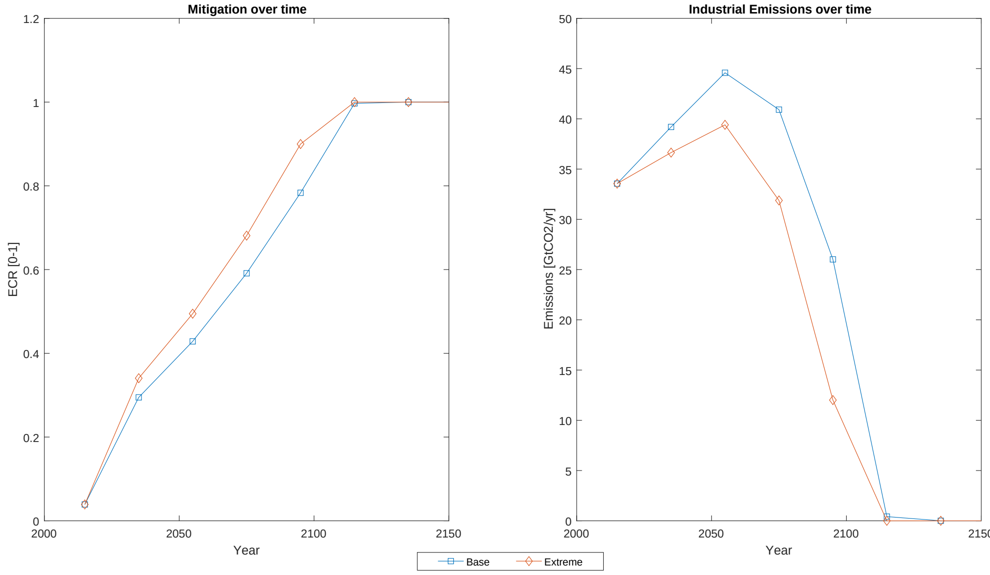

Figure C.12: Comparison of the emissions and mitigation policy of base and the extreme case

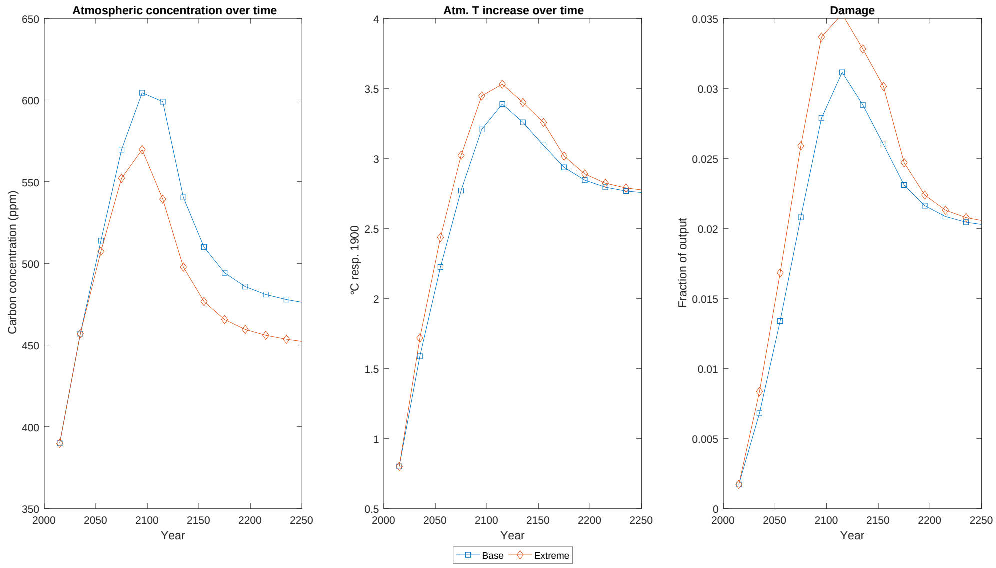

Figure C.13: Comparison of the emissions and mitigation policy of base and the extreme case

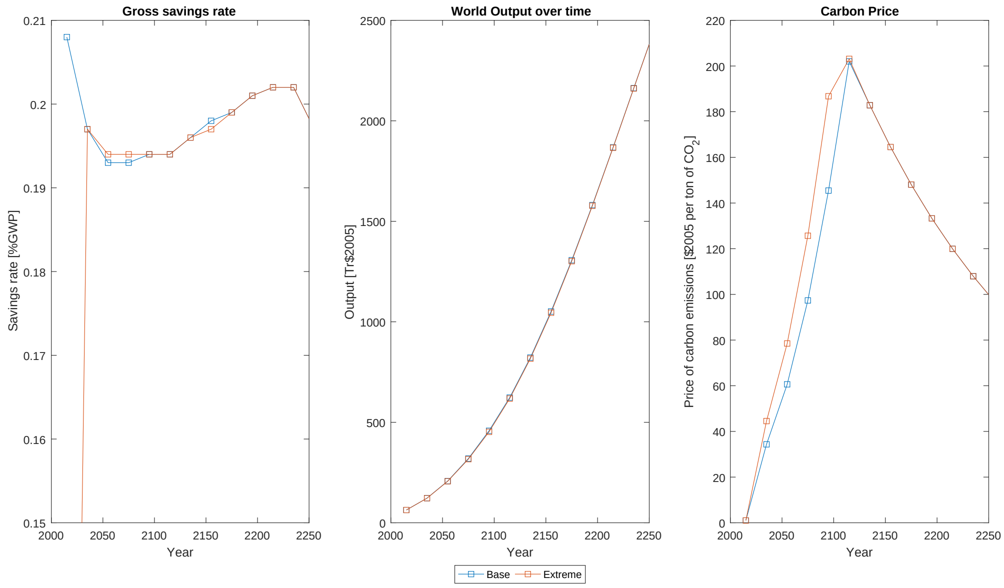

Figure C.14: Comparison of the emissions and mitigation policy of base and the extreme case

C.5 Figures regarding section 5.6

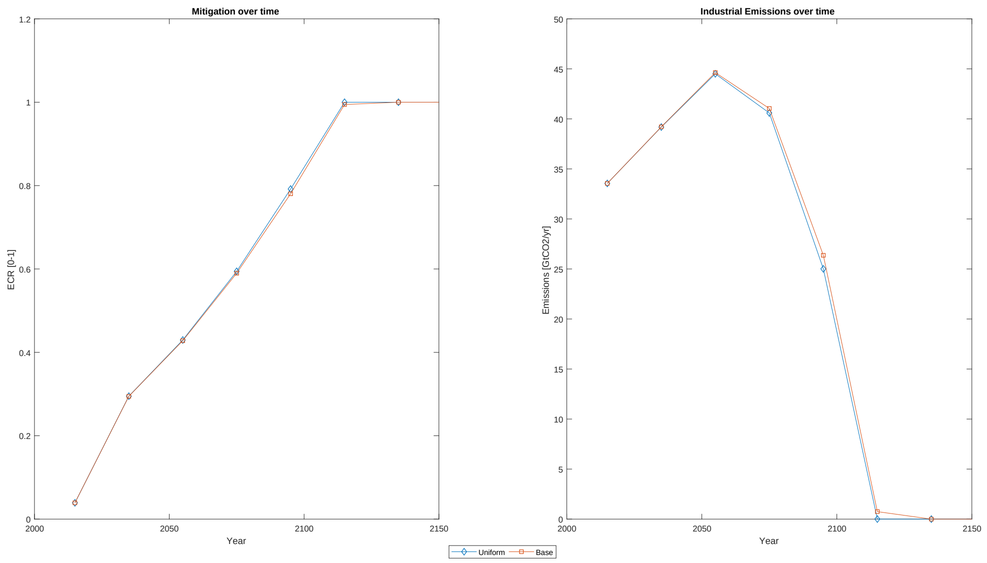

Figure C.15: Comparison of the emissions and mitigation policy of base case and its uniform extension

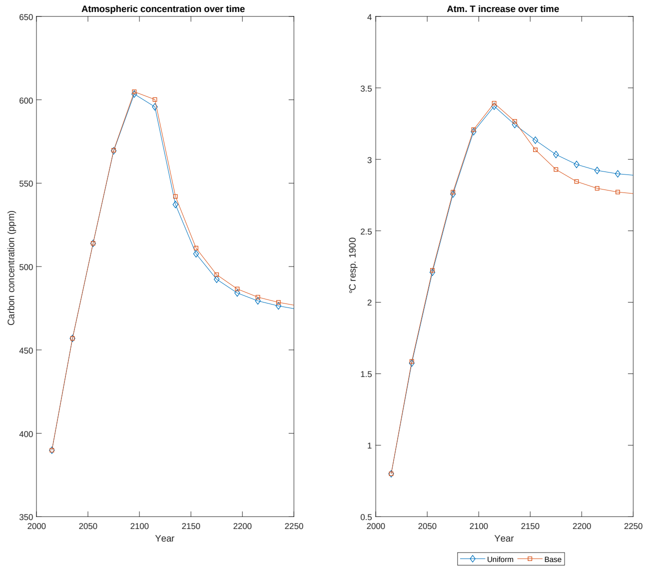

Figure C.16: Comparison of the climate response and the resulting damage of the base case and its uniform extension

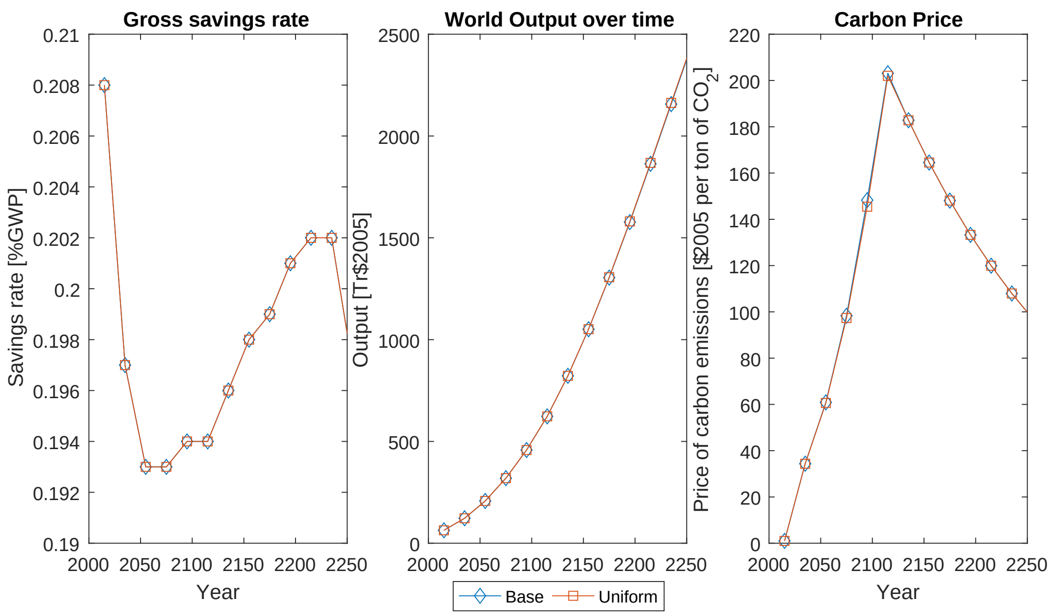

Figure C.17: Comparison of the economic response of base case and its uniform extension

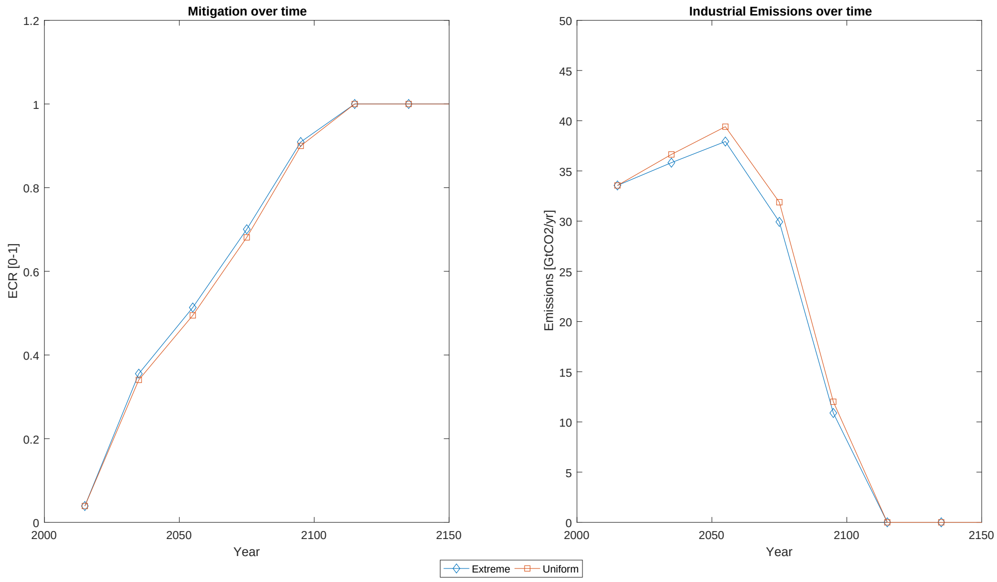

Figure C.18: Comparison of the emissions and mitigation policy of the extreme case and its uniform extension

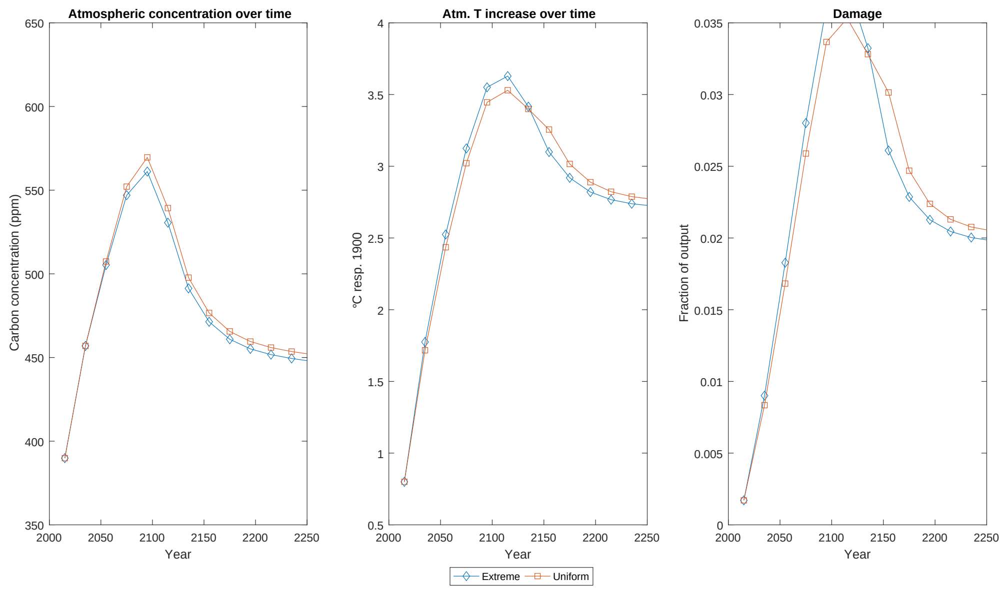

Figure C.19: Comparison of the climate response and the resulting damage of the extreme case and its uniform extension

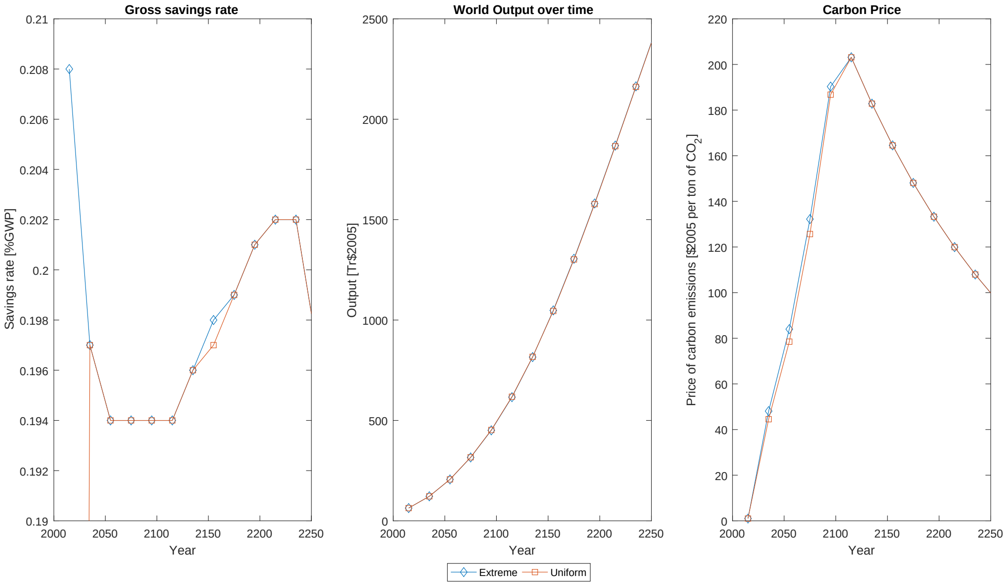

Figure C.20: Comparison of the economic response of extreme case and its uniform extension