DUAL CASCADES IN TWO-DIMENSIONAL COMPRESSIBLE TURBULENCE

Alexei Kritsuk[1] & Gregory Falkovich [2]
[1]*Center for Astrophysics and Space Sciences, University of California, San Diego, La Jolla, USA*
[2]*Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot, Israel*

Abstract We report on results from a set of numerical simulations of isothermal compressible turbulence forced at an intermediate scale with grid resolutions up to 8192^2. At a moderate pumping rate, the total energy of a typical frictionless system, subject to periodic boundary conditions, exhibits linear growth characteristic of an inverse cascade followed—as in the incompressible case—by condensation. The energy capacity of the compressible condensate, however, is limited by shock dissipation. Effects of compressibility are discussed in both shaping the inverse energy cascade at moderate Mach numbers and mediating the energy condensate growth that involves the formation of strong shocks, connecting centers of large vortices and short-circuiting the energy cycle. In the saturated regime, the total energy exhibits strong irregular oscillations caused by metastability of the compressible condensate.

INTRODUCTION

Two-dimensional turbulence in incompressible fluids approximates, within certain limits, fluid dynamics in the atmosphere, oceans, laboratory, and computer simulations, implying its important practical applications. Two quantities conserved in 2D—kinetic energy and enstrophy—were conjectured by Kraichnan to cascade in a Kolmogorov fashion [1, 2]. The existence of such dual cascades with two different inertial ranges, where the energy is inversely transferred to large and enstrophy to small scales from the injection scale, has been largely supported both experimentally and numerically [3]. In the absence of large-scale friction, in a finite size system, energy condensation was predicted to occur [1]. In periodic domain simulations, the condensate is represented by a pair of large-scale vortices of opposite signs [3].

In astrophysical applications involving quasi-2D fluid dynamics, such as accretion disks or turbulent interstellar medium in disk-like galaxies, the likely presence of dual cascades, their observational signatures and potential role in the energy life-cycle have also been discussed [4, 5]. However in these environments, compressibility cannot be ignored, as the Mach numbers are approaching unity. Ref. [6] seems to be the only study of the inverse cascade in 2D compressible fluid turbulence (see also [7]), otherwise the subject remains a largely uncharted territory. Our contribution aims at exploring the feasibility of dual cascades and studying energy condensation phenomenon in compressible media using both theory and numerical simulation. Other potential applications include 2D turbulence in incompressible fluid layers (e.g. flowing soap films) with layer thickness playing the role of density.

SIMULATIONS AND RESULTS

A series of implicit large eddy simulations of 2D compressible turbulence in an isothermal fluid in a doubly-periodic domain is carried out using the Piecewise Parabolic method [8]. Energy is injected by a random solenoidal force, which is approximately white-noise in time, at an intermediate scale. The square domain $L \times L$, where $L = 1$, is covered by a Cartesian grid with a resolution of 512^2, 2048^2, or 8192^2 cells, depending on the model. The lowest resolution model is evolved to $t = 1500$ box sound-crossing times, while the other two for 500 and 30, respectively, starting with a uniform fluid at rest. (The highest resolution model is currently in production and will be evolved through $t = 200$ to capture the condensation event.) The only earlier study with a similar setup was limited to 128^2 grids and evolved through $t = 280$, reaching a Mach number of 0.22 [6].

Our numerical models cover different levels of compressibility and Mach number regimes up to $M = 0.7$. The simulations allow us to study the development of dual cascades in 2D, their scaling properties, as well as the physics of energy condensation in a compressible fluid. Some of the initial results are illustrated by Figures 1–3 below.

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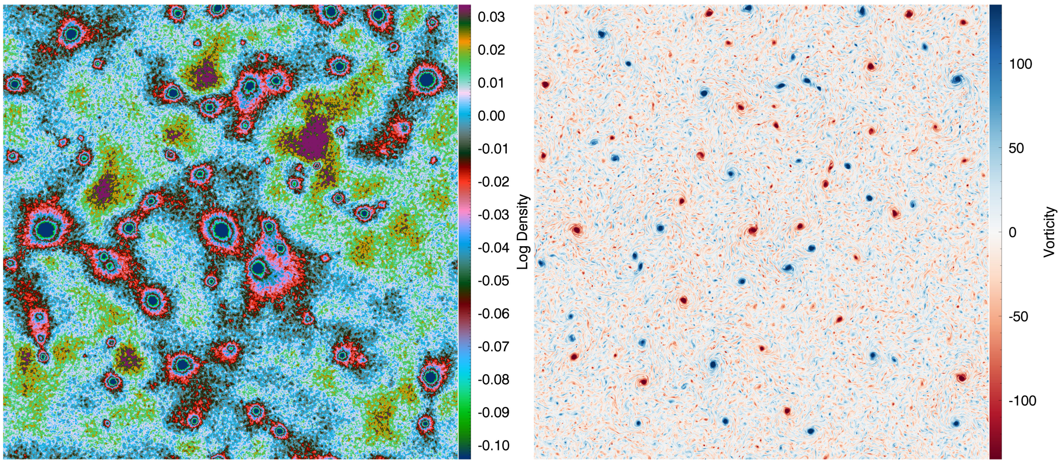

Figure 1. A snapshot of the density and vorticity fields from the 8192^2 simulation of isothermal compressible turbulence forced at $\sim 0.012L$ captured at $t = 28.3$. At this point, the rms Mach number $M = 0.22$ (as in Ref. [6]) and the total energy of the system continues to grow linearly with time, as the injected kinetic energy is transferred to large scales.

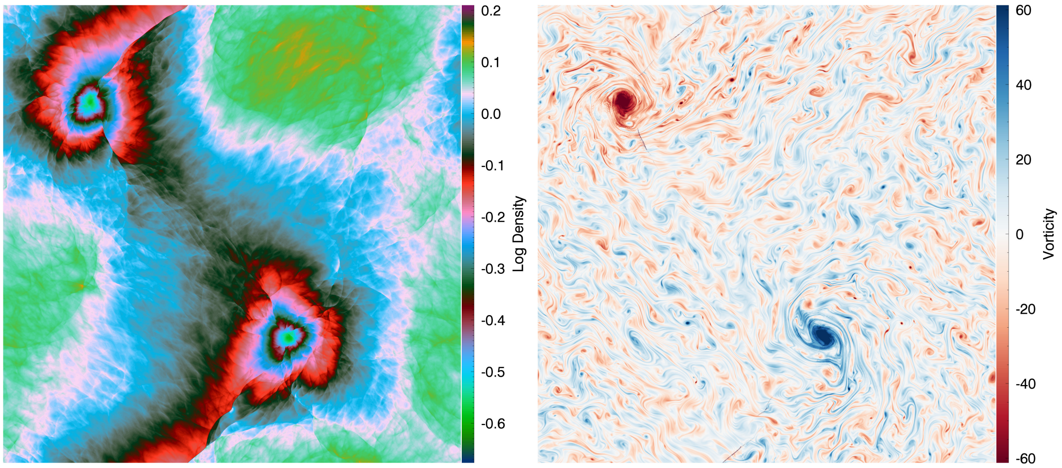

Figure 2. Same as Fig. 1, but for a smaller 2048^2 simulation forced at $\sim 0.047L$ after 460 sound crossing times of evolution. At this point, $M = 0.6$ and the total energy growth is saturated due to shock dissipation. The strongest shocks (visible as jumps in the density plot) are associated with the condensate represented by two largest counterrotating vortical structures.

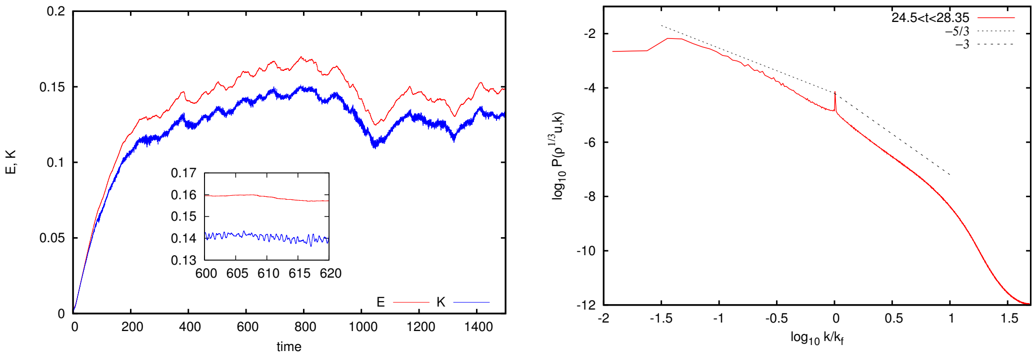

Figure 3. *Left:* Time evolution of the total energy, $E = \langle \rho u^2/2 + c^2 \rho \ln \rho/\rho_0 \rangle$, and kinetic energy, $K = \langle \rho u^2/2 \rangle$, in a 512^2 simulation forced at $\sim 0.047L$. As condensation occurs about $t = 90$, the condensate kinetic energy continues to grow linearly through $t \approx 200$, when shock dissipation saturates the growth in an oscillatory regime reminiscent of the DJIA behaviour. The inset illustrates coupling of kinetic and compressive components of the total energy through large-scale acoustic oscillations, representing purely compressible part of the condensate. *Right:* Time-average power spectrum of $\rho^{1/3}u$ [9] from the 8192^2 simulation (Fig. 1) follows the $k^{-5/3}$ scaling in the energy cascade range and has a slope of -3 in the enstrophy cascade at $M \approx 0.22$; the Kolmogorov constant $C \approx 6.46$.