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ESTIMATION OF THE NATURAL FREQUENCIES OF A SHIP'S DOUBLE BOTTOM BY MEANS OF A SANDWICH THEORY

(BENADERING VAN DE EIGEN-FREQUENTIES VAN DE DUBBELE BODEM VAN EEN SCHIP DOOR MIDDEL VAN EEN „SANDWICH" THEORIE)

by

Ir. S. HYLARIDES
(Netherlands Ship Model Basin)

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VOORWOORD

Bij de studie van scheepstrillingen, met als doel het trillingsgedrag van een schip te kunnen voorspellen, wordt nog algemeen de klassieke balktheorie toegepast. Bij deze theorie wordt het schip beschouwd als een vrij trillende zogenaamde Timoshenko balk.

Hoewel dit uitgangspunt, zeker bij de trillingen van lagere orde, betrekkelijk goede resultaten oplevert, is gebleken dat speciaal bij de trillingen van hogere orde aanzienlijke afwijkingen kunnen voorkomen. Dit zou gedeeltelijk verklaard kunnen worden door het feit, dat het elementaire balkmodel voor hogere orde trillingen minder betrouwbaar wordt. In dit verband wordt bijvoorbeeld verwezen naar rapport no. 75 S van het Nederlands Scheepsstudiecentrum TNO „Scheepstrillingen van het vracht- en passagiersschip m.s. „Oranje Nassau"" door ir. W. van Horssen.

Met het steeds toenemende belang van de trillingen van hogere orde stijgt ook de behoefte aan een betrouwbare methode voor het opstellen van een prognose hiervoor. Het verbeteren van de rekenmethoden kan in principe op twee verschillende manieren geschieden: ten eerste door het verfijnen en uitbreiden van de klassieke balktheorie; ten tweede door het loslaten van het balkmodel en het ontwikkelen van een nieuwe theorie. Het hier gepresenteerde rapport hoort in de eerste categorie thuis.

Een van de mogelijke oorzaken namelijk voor verschillen tussen het werkelijke trillingsgedrag van een schip en het langs theoretische weg, met de eenvoudige balktheorie bepaalde, kan de aanwezigheid zijn van grote massa's, die min of meer elastisch met de romp zijn verbonden. Een deel van een dubbele bodem van een schip tussen twee schotten bijvoorbeeld met de massa van de lading er boven op en de virtueel meetrillende watermassa, kan als een dergelijke verend bevestigde massa worden beschouwd.

In dit rapport nu wordt een rekenmethode ontwikkeld voor de bodem van een schip, uitgevoerd als dubbele bodem. Met deze rekenmethode kan de eigen-frequentie van een dubbele bodem „paneel" berekend worden. Als deze eigen-frequentie bekend is, kan de invloed van de verende massa worden verdisconteerd in de balkmethode. Voor de beschouwde rekenmethode is gebruik gemaakt van de eigenschappen van een „sandwich" constructie, dat wil zeggen twee dunne deklagen gescheiden door een dikkere kern.

Hoewel dit als een benadering beschouwd moet worden, zijn de resultaten die deze theorie geeft zeer redelijk.

Verificatie ervan door metingen aan modellen of constructies op ware grootte wordt aanbevolen.

HET NEDERLANDS SCHEEPSSTUDIECENTRUM TNO

PREFACE

For the study of ship vibrations, aiming of the prediction of the vibrational behaviour of a ship's hull, the classical beam theory is still generally applied. In this theory the hull is considered as a freely vibrating, so called Timoshenko beam.

Although this starting point, at least with the lower modes of vibration, yields relatively good results, it has been found that especially for the higher modes considerable deviations may occur. This could partly be explained by the fact that the elementary beam model becomes less reliable for these modes. In this respect may be referred, for instance, to report no. 75 S of the Netherlands Ship Research Centre TNO "Hull vibrations of the cargo-passenger motorship "Oranje Nassau"" by ir. W. van Horssen.

With the ever increasing importance of the higher modes of vibration the need for a reliable prediction method for these modes also increases. Improving the calculation methods may, in principle, proceed along two different lines: firstly by refining and extending the classical beam theory; secondly by abandoning the beam mcdel and developing a new theory. The report presented here belongs to the first category.

One of the reasons namely for the deviations between the real vibrational behaviour of a ship and that theoretically derived, with the simple beam model, may be the presence of large masses, more or less elastically attached to the ship's hull. A ship's bottom part between bulkheads, for instance, with a mass of cargo piled upon and with the virtually added mass of water, may be regarded as such a "sprung mass".

Now in this report a method of calculation is developed for ships' bottoms constructed as a double bottom. With this method the natural frequency of a double bottom panel can be computed. If this natural frequency is known, the effect of the sprung mass can be incorporated into the beam method. For the computation system under consideration use has been made of the properties of a "sandwich" structure, viz. two thin plates separated by a thicker core.

Although it must be regarded as an approximation the results of this theory are very reasonable.

Verification by model or full scale measurements is recommended.

THE NETHERLANDS SHIP RESEARCH CENTRE TNO

CONTENTS

LIST OF SYMBOLS

A_x	average surface of the material in the plane x = constant, carrying shear stresses perpendicular to the double bottom
a, b	length and breadth of the sandwich plate respectively
E	modulus of elasticity
f, f_n	frequency and natural frequency respectively
G, G^*	modulus of shear of cover plates and core respectively
h, H	thickness of cover plates and core respectively
M	total mass per unit of surface of the sandwich plate
m	mass of the loading per unit of surface of the sandwich plate
m_e	effective mass
$m_{\text{equiv.}}$	equivalent mass
u, v, w	displacements in the X-, Y- and Z-direction
x, y, z	Cartesian co-ordinates
ε	number, small compared with one
Δ	two-dimensional Laplace operator
θ, Ω	symbols representing $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ and $\frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$ respectively
ν	Poisson's ratio
ϱ, ϱ^*	mass density of cover plates and core respectively
σ, τ	normal and shear stress respectively
φ	phase lag
ω	circular frequency

ESTIMATION OF THE NATURAL FREQUENCIES OF A SHIP'S DOUBLE BOTTOM BY MEANS OF A SANDWICH THEORY *)

by

Ir. S. HYLARIDES

Summary

For the determination of its natural frequencies a double bottom is considered to be a sandwich plate. The equation of motion perpendicular to the plane of the sandwich plate is derived and solved by substitution of a double Fourier series for the vertical displacement. The natural frequencies corresponding with the appropriate boundary conditions are obtained from this solution, together with the normal mode patterns.

1 Introduction

For the calculation of the vertical and horizontal vibration a ship is, in general, considered as a beam with suitable distribution of mass, bending elasticity and shearing elasticity along the length. Measurements on ships reveal that the differences between the calculated and measured values of the natural frequencies increase with the number of nodes. So the representation of a ship by a beam appears to be acceptable for the calculation of the lowest natural frequencies only. For the calculation of the higher frequencies the method of calculation has to be changed. Therefore the method of representing the ship by a beam should either be abandoned or refined.

It is expected that the deviations between measured and calculated values of the natural frequencies are to a large extent caused by the fact that big mass lumps of the ship are elastically attached to the hull. This concerns especially the cargo in a hold stored on the double bottom, since these two represent a big mass connected to the hull by a certain spring, i.e. the double bottom. The effect of such a local structure on the vibrating ship can be represented by a sprung mass attached to the hull. Such a mass-spring-system has natural frequencies for the determination of which the rest of the ship is considered to be constrained. These frequencies are given by the effective stiffness of the double bottom and the effective mass consisting of that of the cargo, the double bottom and the added or hydrodynamic mass.

As indicated in [1] and [2] the effective mass m_e can be replaced by an equivalent mass, m_{equiv}, attached solidly to the hull. In the absence of damping the relation between these two masses is given by

$$m_{\text{equiv}} = \frac{m_e}{1-f^2/f_n^2}$$

where f is the frequency of the vibrating ship and f_n the natural frequency of the sprung mass. So the local vibrating structure can have a considerable influence, depending on its natural frequencies and effective mass.

Based on these features of a local structure, it is possible to incorporate its effect on the vibrating ship in the beam method. For that purpose the effective mass of a local structure should be regarded as being attached to the beam by a properly chosen spring. The point of attachment coincides with the centre of gravity of this sprung mass. In this way we restrict ourselves to the double bottom and the cargo stored, since this structure contributes considerably to the mass of the vibrating ship and it is expected that the fundamental frequency lies in the range of the natural frequencies of the ship.

Suppose the normal mode pattern and the natural frequency of a local structure to be known, then the effective mass, corresponding to the mode pattern considered, is determined as follows.

Consider the double bottom in free vibration and the rest of the ship to be constrained. For the bottom, with associated cargo and added mass, the kinetic energy is given by the known natural frequency f_n and the mode pattern $w(x,y)$; x and y represent the coordinates in the plane of the bottom. The amplitude at the centre of gravity is called $\bar{w} = w(x_{cg}, y_{cg})$. Then the kinetic energy T is given by

$$T = \iint m(x,y)\, w(x,y)^2\, \mathrm{d}x\mathrm{d}y$$

*) Publication no. 283, Netherlands Ship Model Basin.

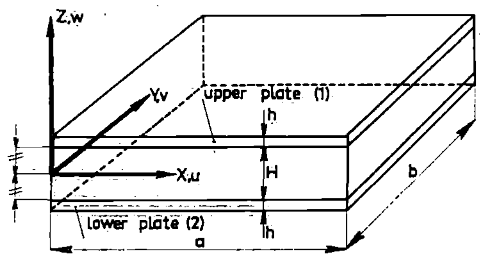

Fig. 1. The sign convention of the coordinate system XYZ and corresponding displacements u, v and w for the sandwich theory. The sandwich plate consists of a core of thickness H and of two cover plates of thickness h.

where $m(x,y)$ is the vibrating mass per unit of surface of the local structure [3].

This vibrating system is compared with a simple mass-spring-system which has the same natural frequency f_n and is in free vibration with amplitude $\bar{w}$. Its kinetic energy is equated with that of the vibrating double bottom. This leads to an equation for the mass of the mass-spring-system, which is apparently the effective mass m_e of the vibrating bottom

$$m_e = \frac{\iint m(x,y)\, w(x,y)^2 \, dxdy}{\bar{w}^2}$$

Once the effective mass has been evaluated, the effective spring constant k_e can be found from the relation

$$\omega_n = \sqrt{\frac{k_e}{m_e}}$$

where $\omega_n = 2\pi f_n$ is the natural circular frequency of the local structure.

An important problem is the evaluation of the normal modes of the double bottom with the total vibrating mass. The construction of a double bottom shows a reasonable resemblance to that of a sandwich plate. So for a first estimation of the lower natural frequencies and the corresponding modes of the double bottom we may try to take a sandwich plate as a mechanical model.

Two thin plates, separated by a core, together form a sandwich plate. In general a core consists of a honeycomblike structure of the same material as the two cover plates. Figure 1 shows the cartesian coordinate system, $OXYZ$, with the corresponding displacements u, v and w. The dimensions of the sandwich plate are given by

a = the length of the plate
b = the breadth of the plate
h = the thickness of the cover plates and
H = the thickness of the core

The quantities h and H are constant throughout the whole plate and they are small compared with a and b, whereas h is small compared with H. The displacements of and the stresses in the upper and lower plate have subscripts 1 and 2 respectively, while those of the core have no subscripts. In figure 2 the stresses in the core and the cover plates are defined.

Based on the characteristic geometry of a sandwich plate the following properties hold.

The core is especially effective in transmitting the shear stresses τ_{xz} and τ_{yz}, in comparison with these the other stresses in the core may be neglected.

Further the modulus of shear G^* of the core, corresponding with these stresses τ_{xz} and τ_{yz}, is small com-

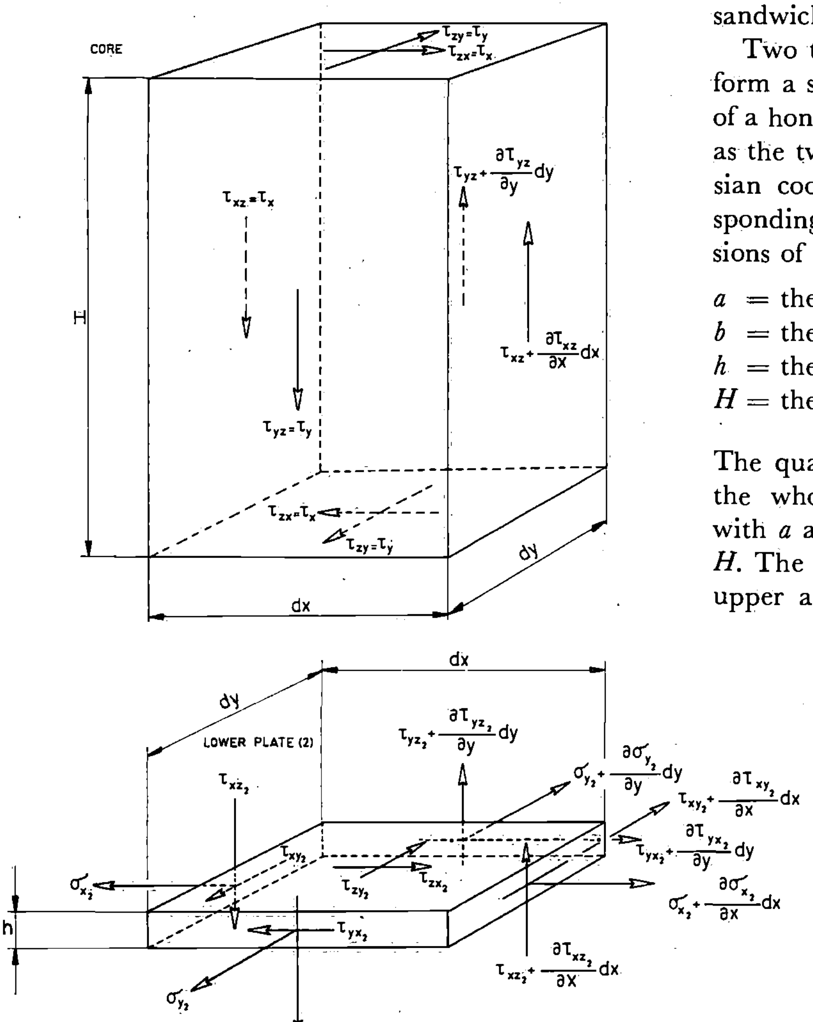

Fig. 2. The stresses on an element $dxdy$ of the core and on an element $dxdy$ of the lower plate.

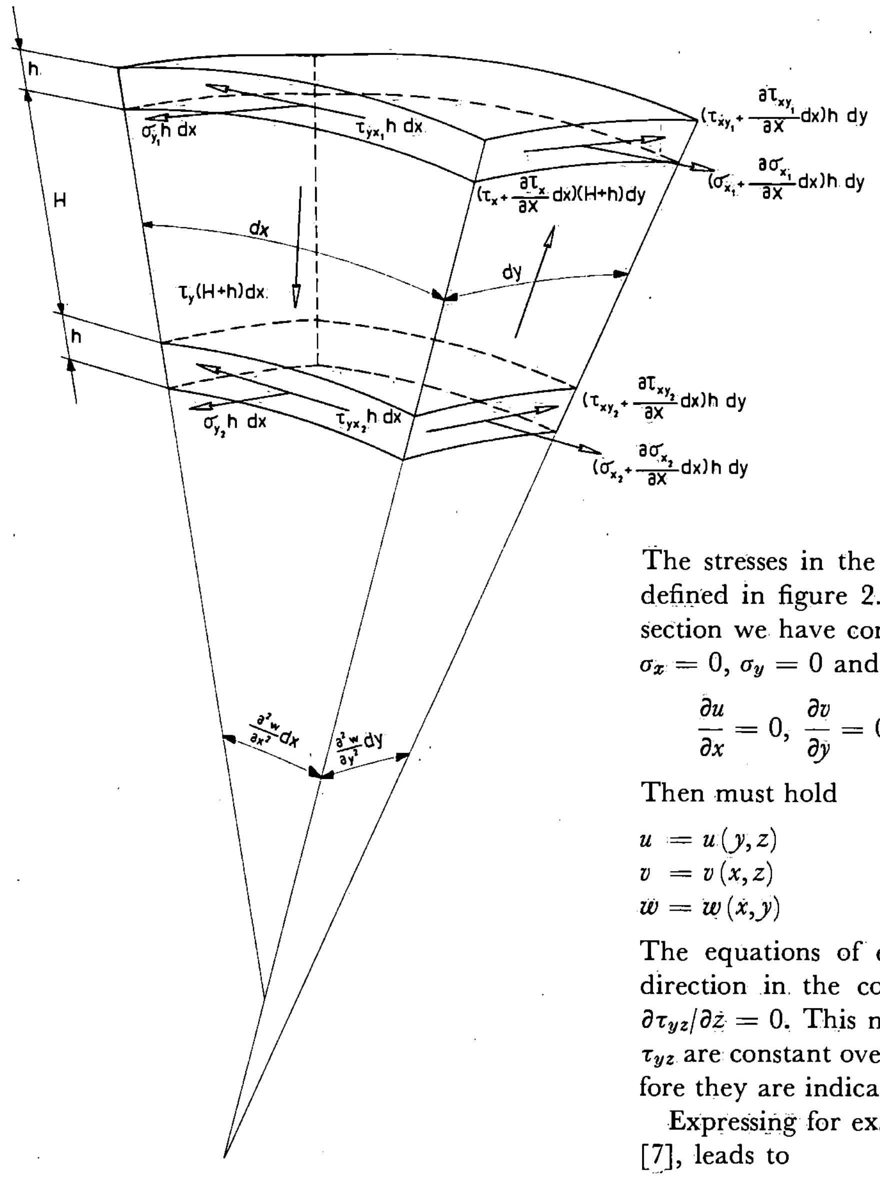

Fig. 3. The forces on an element $dxdy$ of the whole sandwich plate in deformed state. Only the stresses on the facing surfaces of the element are indicated.

pared with that of the material used. The rigidity against shear strain in the planes parallel to the plane of the sandwich plate G^{**}, is again smaller than G^*. So the shear stresses τ_{xy} are not considered at all.

With a view to the small thickness of the cover plates, the stresses and displacements are assumed to be constant over the height and taken equal to their average values.

2 The equation of motion

We are only interested in the motion of the double bottom in the vertical direction and, hence, restrict ourselves to these out-of-plane-motions for sandwich plates. Interaction between the in- and out-of-plane-motions will be negligible, as the stiffness in the x- and y- directions are considerably larger than in the z-direction.

The stresses in the core and the cover plates are defined in figure 2. At the end of the preceding section we have concluded that in the core holds $\sigma_x = 0$, $\sigma_y = 0$ and $\sigma_z = 0$, hence,

$$\frac{\partial u}{\partial x} = 0, \quad \frac{\partial v}{\partial y} = 0 \quad \text{and} \quad \frac{\partial w}{\partial z} = 0$$

Then must hold

$$u = u(y,z)$$
$$v = v(x,z)$$
$$w = w(x,y)$$

The equations of equilibrium in the x- and y-direction in the core result to $\partial\tau_{xz}/\partial z = 0$ and $\partial\tau_{yz}/\partial z = 0$. This means that the stresses τ_{xz} and τ_{yz} are constant over the height of the core, therefore they are indicated by $\tau_{xz} = \tau_x$ and $\tau_{yz} = \tau_y$.

Expressing for example τ_x in the displacements [7], leads to

$$\tau_x = G^* \left[\frac{\partial u(y,z)}{\partial z} + \frac{\partial w(x,y)}{\partial x}\right]$$

Hence, the first equation of equilibrium writes

$$\frac{\partial}{\partial z} G^* \left[\frac{\partial u(y,z)}{\partial z} + \frac{\partial w(x,y)}{\partial x}\right] = 0$$

Thus considering the modulus of shear of the core G^*, to be constant, we find

$$\frac{\partial^2 u(y,z)}{\partial z^2} = 0$$

This means that the displacement u in the core varies linearly with the height. The same holds for the displacement v.

Finally we see that the displacement w in the core is constant over the height of the core.

For the determination of the stresses τ_{xz} and τ_{yz} in the cover plates we consider an element $dxdy$ with thickness h of the lower plate (Fig. 2).

On the surface x = constant the shear stress τ_{xz} varies from zero at the lower side to τ_x at the upper side of the element. The same holds for the stress τ_{yz} on the surfaces y = constant. As the thickness h of the cover plates is small, we may state the relations

$$\tau_{xz} = \tfrac{1}{2}\tau_x \text{ and } \tau_{yz} = \tfrac{1}{2}\tau_y$$

In figure 3 the forces on an element $dxdy$ of the sandwich plate are indicated. They are due to the vibrations only. By means of the linearity of the problem the statical loading and deformations may be left out of consideration. By considering small displacements only we may restrict ourselves to first-order terms. Then the equation of motion in vertical direction is

$$M\frac{\partial^2 w}{\partial t^2} = (H+h)\left(\frac{\partial \tau_x}{\partial x}+\frac{\partial \tau_y}{\partial y}\right) \quad . \; . \; . \; . \; . \quad (1)$$

where

$$M = 2\varrho h+\varrho^* H+m$$

and

ϱ = mass density of the cover plates
ϱ^* = average mass density of the core
m = vibrating mass per unit of surface of the sandwich plate, composed by the load and the added mass of water

For the core the relations between the shearing stresses and shearing strains are, [7]

$$\left.\begin{aligned}
\frac{\tau_x}{G^*} &= \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} = \frac{u_1-u_2}{H+h}+\frac{\partial w}{\partial x}\\
\frac{\tau_y}{G^*} &= \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y} = \frac{v_1-v_2}{H+h}+\frac{\partial w}{\partial y}\\
\frac{\tau_{xy}}{G^{**}} &= \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},
\end{aligned}\right\} \quad . \; . \; . \quad (2)$$

bearing in mind that u and v vary linearly with z and that these displacements are constant in the cover plates in z-direction, equalling their mean values in the plate and, hence, for the displacements $u_{H/2}$ and $v_{H/2}$ of the core in the boundary-plane with cover plate 1, for example, we write

$$u_{H/2} = \frac{H}{H+h}u_1 \quad \text{and}$$

$$v_{H/2} = \frac{H}{H+h}v_1$$

As we have already mentioned in section 1, the stresses τ_{xy} are small compared with τ_x and τ_y. In fact the rigidity of the sandwich plate against shear stresses τ_{xy} is provided by the cover plates. So we may omit the last equation of equations (2).

As the stresses σ_z in the core are equalling zero (section 1), we also may neglect the stresses σ_z in the cover plates in comparison with the other stresses, now, [7]

$$\sigma_z = \frac{E}{1+\nu}\left[\frac{\partial w}{\partial z}+\frac{\nu}{1-2\nu}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right] = 0,$$

from which follows

$$\frac{\partial w}{\partial z} = \frac{-\nu}{1-\nu}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

ν is called Poisson's ratio. Substituting this in the equations which relate the stresses with the deformations we find for the upper plate for example

$$\left.\begin{aligned}
\sigma_{x_1} &= \frac{E}{1-\nu^2}\left(\frac{\partial u_1}{\partial x}+\nu\frac{\partial v_1}{\partial y}\right)\\
\sigma_{y_1} &= \frac{E}{1-\nu^2}\left(\frac{\partial v_1}{\partial y}+\nu\frac{\partial u_1}{\partial x}\right)\\
\tau_{xy_1} &= G\left(\frac{\partial u_1}{\partial y}+\frac{\partial v_1}{\partial x}\right)
\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad (3)$$

where E is the modulus of elasticity and G is the modulus of shear of the cover plates. The expressions for τ_{xz_1} and τ_{yz_1} are not of importance since we have assumed, on account of the thickness of the plate, that $\tau_{xz_1} = \frac{1}{2}\tau_x$ and $\tau_{yz_1} = \frac{1}{2}\tau_y$. Analogous equations and considerations hold for the lower plate.

The equations of equilibrium of the cover plates in the x- and y-direction read, again for an element $dxdy$ of thickness h from the upper plate as an example:

$$\left.\begin{aligned}
&\frac{\partial \sigma_{x_1}}{\partial x}+\frac{\partial \tau_{xy_1}}{\partial y} = \frac{\tau_x}{h}\\
&\text{and}\\
&\frac{\partial \tau_{yx_1}}{\partial x}+\frac{\partial \sigma_{y_1}}{\partial y} = \frac{\tau_y}{h}
\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

as $\sigma_z = 0$ in the cover plates and τ_{xz_1} and τ_{yz_1} are constant. Due to the higher in-plane-stiffness of the sandwich plate, the in-plane-motions are small compared with the lateral motions, so in equations (4) the inertia forces have been neglected.

Substituting the value of τ_x from equation (2) and the values of σ_{x_1} and τ_{xy_1} from equations (3) in the first of equations (4) and using the relation between the moduli of elasticity and shear given by

$$G = \frac{E}{2(1+\nu)}$$

we find

$$\frac{E}{1-\nu^2}\left\{\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}\right)+\tfrac{1}{2}(1-\nu)\frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial y}-\frac{\partial v_1}{\partial x}\right)\right\}=$$
$$=\frac{G^*}{h}\left(\frac{2u_1}{H+h}+\frac{\partial w}{\partial x}\right)$$

where use is made of the fact that owing to the symmetry $u_2 = -u_1$.

Let

$$\left.\begin{aligned}&\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=\theta_1\\ &\text{and}\\ &\frac{\partial u_1}{\partial y}-\frac{\partial v_1}{\partial x}=2\Omega_1\end{aligned}\right\}\quad\ldots\ldots\ldots\ (5)$$

then the equation becomes

$$\frac{E}{1-\nu^2}\left\{\frac{\partial}{\partial x}\theta_1+(1-\nu)\frac{\partial}{\partial y}\Omega_1\right\}=\frac{G^*}{h}\left(\frac{2u_1}{H+h}+\frac{\partial w}{\partial x}\right)\ (6a)$$

In the same way the second of equations (4) may be written as

$$\frac{E}{1-\nu^2}\left\{\frac{\partial}{\partial y}\theta_1-(1-\nu)\frac{\partial}{\partial x}\Omega_1\right\}=\frac{G^*}{h}\left(\frac{2v_1}{H+h}+\frac{\partial w}{\partial y}\right)\ (6b)$$

as for reasons of symmetry $v_2 = -v_1$.

The corresponding equations of equilibrium of an element from the lower plate are

$$\frac{\partial\sigma_{x_2}}{\partial x}+\frac{\partial\tau_{xy_2}}{\partial y}=-\frac{\tau_x}{h}$$
$$\frac{\partial\tau_{yx_2}}{\partial x}+\frac{\partial\sigma_{y_2}}{\partial y}=-\frac{\tau_y}{h}$$

From the fact that $u_2 = -u_1$ and $v_2 = -v_1$ it follows that

$$\theta_2=\frac{\partial u_2}{\partial x}+\frac{\partial v_2}{\partial y}=-\theta_1$$
$$2\Omega_2=\frac{\partial u_2}{\partial y}-\frac{\partial v_2}{\partial x}=-2\Omega_1$$

Thus we find for the lower plate exactly the same equations as for the upper plate. Therefore it is allowable to omit the subscripts for θ and Ω. This follows already from reasons of symmetry.

Differentiating the equation (6a) with respect to y and (6b) with respect to x and subtracting these equations we find

$$\frac{E}{1+\nu}\Delta\Omega=\frac{4G^*}{H\cdot h}\Omega\quad\ldots\ldots\ldots\ (7a)$$

where

$$\Delta=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$$

Differentiating the equation (6a) with respect to x and (6b) with respect to y and adding these two equations leads to

$$\frac{Eh}{G^*(1-\nu^2)}\Delta\theta=\frac{2}{H+h}\theta+\Delta w\ .\ .\ .\ .\ .\ (7b)$$

Finally we substitute the equations (2) in (1) and obtain with the equations (5)

$$\frac{M}{G^*(H+h)}\frac{\partial^2 w}{\partial t^2}=\frac{2}{H+h}\theta+\Delta w\ \ .\ .\ .\ .\ (7c)$$

So we have obtained three equations (7a, b and c) describing the free vibrations of a sandwich plate. In the following section these equations are used in the determination of the fundamental frequency of a sandwich plate with special boundary conditions.

3 Determination of the natural frequencies

It follows from the equations (7a, b and c) that function Ω is not coupled to the other two mutually coupled functions w and θ, so we can concentrate ourselves, for the determination of the displacement w, to the solution of the equations (7b) and (7c). The problem consists in determining the functions w and θ which satisfy these equations and the appropriate boundary conditions. We restrict ourselves to a plate hinged at the boundaries (figure 4). Then the boundary conditions are for $x = 0$ or $x = a$

$$v_1 = 0,\ v_2 = 0$$
$$w = 0$$
$$\sigma_{x_1} = 0,\ \sigma_{x_2} = 0$$

and for the edges $y = 0$ or $y = b$

$$u_1 = 0,\ u_2 = 0$$
$$w = 0$$
$$\sigma_{y_1} = 0,\ \sigma_{y_2} = 0.$$

From the boundary condition $\sigma_{x_1} = 0$ we conclude that $(\partial u_1/\partial x)+\nu(\partial v_1/\partial y) = 0$ along the edges

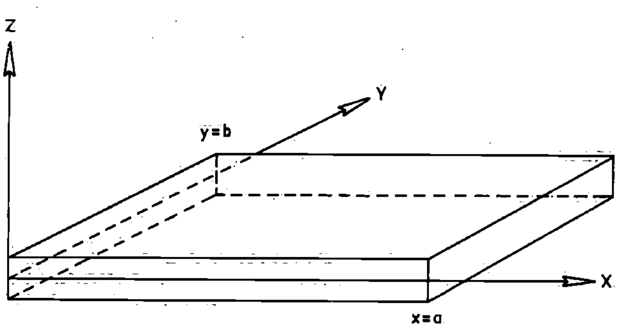

Fig. 4. Of a sandwich plate, hinged along its four sides and vibrating in a mode perpendicular to its plane, the boundary conditions are

$$\left.\begin{aligned}\text{for } x &= 0\\ x &= a\end{aligned}\right\}\ v_1 = v_2 = w = 0,\quad \sigma_{x_1} = \sigma_{x_2} = 0$$
$$\left.\begin{aligned}\text{for } y &= 0\\ y &= a\end{aligned}\right\}\ u_1 = u_2 = w = 0,\quad \sigma_{y_1} = \sigma_{y_2} = 0$$

$x = 0$ and $x = a$. But, from the fact that along these edges also $v = 0$, we conclude from the preceding equation that $\partial u_1/\partial x = 0$ and, hence, it must hold $\theta_1 = (\partial u_1/\partial x)+(\partial v_1/\partial y) = 0$. This holds also for the lower plate, so the subscripts may be omitted.

In a similar way we find along the boundaries y = constant also that $\theta = 0$. Thus along the whole boundary holds $w = 0$ and $\theta = 0$.

For obtaining an approximate solution, satisfying these boundary conditions we assume w and θ given by

$$\left.\begin{aligned} w &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} w_{ij} \sin i\frac{\pi x}{a} \sin j\frac{\pi y}{b} \sin \omega t \\ &\text{and} \\ \theta &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \theta_{ij} \sin i\frac{\pi x}{a} \sin j\frac{\pi y}{b} \sin (\omega t+\varphi) \end{aligned}\right\} \quad . \quad (8)$$

where the subscripts i and j refer to the deformations in the XOZ- and YOZ-plane respectively.

As the right sides of the equations (7b) and (7c) are the same, we may equate the left sides. Substituting the assumed solution for w and θ in this latter equation we find:

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \sin i\frac{\pi x}{a} \sin j\frac{\pi y}{b}\left[\frac{M(1-\nu^2)\omega^2}{(H+h)hE} w_{ij} \sin \omega t - \left(i^2\frac{\pi^2}{a^2}+j^2\frac{\pi^2}{b^2}\right)\theta_{ij} \sin (\omega t+\varphi)\right] = 0$$

This equation must hold for any value of x and y, so the term in square brackets should be zero for any value of t. Since we are not interested in the trivial solution $w_{ij} = \theta_{ij} = 0$ $(i = 1 \ldots . \infty$, $j = 1 \ldots . \infty)$, this is only possible if $\varphi = 0$.

Therefore

$$\frac{M(1-\nu^2)\omega^2}{(H+h)hE} w_{ij} - \left(i^2\frac{\pi^2}{a^2}+j^2\frac{\pi^2}{b^2}\right)\theta_{ij} = 0,$$

or

$$\omega^2 = \frac{(H+h)hE}{M(1-\nu^2)}\left(i^2\frac{\pi^2}{a^2}+j^2\frac{\pi^2}{b^2}\right)\frac{\theta_{ij}}{w_{ij}} \quad . \; . \; . \quad (9)$$

Substitution of the equations (8) in (7b) gives for the same reason as above

$$\frac{\theta_{ij}}{w_{ij}} = \frac{i^2\dfrac{\pi^2}{a^2}+j^2\dfrac{\pi^2}{b^2}}{\left(i^2\dfrac{\pi^2}{a^2}+j^2\dfrac{\pi^2}{b^2}\right)\dfrac{Eh}{G^*(1-\nu^2)}+\dfrac{2}{H+h}} \quad (10)$$

Substitution of this value for θ_{ij}/w_{ij} in equation (9) yields

$$\omega^2 = \frac{(H+h)\left(i^2\dfrac{\pi^2}{a^2}+j^2\dfrac{\pi^2}{b^2}\right)^2}{M\left\{\dfrac{1}{G^*}\left(i^2\dfrac{\pi^2}{a^2}+j^2\dfrac{\pi^2}{b^2}\right)+\dfrac{2(1-\nu^2)}{(H+h)hE}\right\}} \quad (11)$$

with $i = 1 \ldots . \infty$, $j = 1 \ldots . \infty$.

4 Comparison with the plate theory

The difference between the conventional plate theory and the sandwich theory is that in the latter we include shear deformations in planes perpendicular to the plate, whereas these deformations are zero in the conventional plate theory. Taking the rigidity of the core very large implies that we consider only bending stiffness and we obtain an equation comparable with that found by the conventional theory in which only bending stiffness is considered.

The equation of the out-of-plane-motion of the sandwich plate reads

$$\frac{M}{G^*(H+h)}\frac{\partial^2 w}{\partial t^2} = \frac{2}{H+h}\theta+\Delta w \quad . \; . \; . \; . \quad (7c)$$

while θ is coupled with w by

$$\frac{Eh}{G^*(1-\nu^2)}\Delta\theta = \frac{2}{H+h}\theta+\Delta w \quad . \; . \; . \; . \quad (7b)$$

Since the right sides are the same we can write

$$\frac{M}{G^*(H+h)}\frac{\partial^2 w}{\partial t^2} = \frac{Eh}{G^*(1-\nu^2)}\Delta\theta \quad . \; . \; . \; . \quad (12)$$

When we let G^* approach infinity equation (7b) leads to

$$0 = \frac{2}{H+h}\theta+\Delta w$$

so

$$\theta = -\frac{H+h}{2}\Delta w$$

Substituting this in equation (12) and dividing it by G^* we find

$$\frac{E(H+h)^2 h}{2(1-\nu^2)}\Delta\,\Delta w = -M\frac{\partial^2 w}{\partial t^2} \quad . \; . \; . \; . \; . \quad (13)$$

In the conventional theory the equation of a plate of thickness h^* loaded by a normal load p per unit of surface is

$$\frac{Eh^{*3}}{12(1-\nu^2)}\Delta\,\Delta w = p \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

The symbols E, ν, Δ and w are the same as in the sandwich theory.

Now we can consider the term $-M(\partial^2 w/\partial t^2)$ in equation (13) as the inertia forces forming a loading p^* perpendicular to the plate. Hence there is a close resemblance between the two equations (13) and (14). Equating the bending stiffness of both plates given in the equations (13) and (14) and putting for the sandwich plate $h = \varepsilon H$, with $\varepsilon << 1$, we find

$$\tfrac{1}{6}h^{*3} = H^3(1+\varepsilon)^2\varepsilon$$

Neglecting higher-order terms this equation becomes

$$h^* = H\sqrt[3]{6\varepsilon}$$

With $\varepsilon = 1/10$ the equivalent thickness h^* of the plate with only bending stiffness will be $h^* \approx 0.85H$ and with $\varepsilon = 1/100$ is $h^* \approx 0.39H$.

5 The fundamental frequency of a ship's bottom

Assuming that a ship's double bottom can be considered in its lowest mode as a sandwich plate hinged at the boundaries, equation (11) with $i = j = 1$ leads to the lowest natural frequency f_1

$$(2\pi f_1)^2 = \omega_1^2 = \frac{(H+h)\left(\dfrac{\pi^2}{a^2}+\dfrac{\pi^2}{b^2}\right)}{M\left\{\dfrac{1}{G^*}\left(\dfrac{\pi^2}{a^2}+\dfrac{\pi^2}{b^2}\right)+\dfrac{2(1-\nu^2)}{(H+h)hE}\right\}} \quad \ldots \quad (15)$$

For the calculation of the numerical value of f_1 we consider first the modulus of rigidity of the double bottom G^* and the total mass M per unit of surface of the bottom. The other quantities as H, h, a, b, E and ν are more or less evident, they are determined by the construction and the material used.

The core of the sandwich plate is formed by the longitudinal and transverse stiffeners of the double bottom. In general these stiffeners differ from each other, so the core will be orthotropic, whereas in the given derivation the core has been supposed to be isotropic. Yet the deduced expression for the natural frequencies is applicable as the orthotropy will have a small influence. This is ensured by the fact that the length of the double bottom in the hold of a ship between two bulkheads, in general will approximately equal the breadth. Furthermore the natural frequencies will be symmetrical functions of both the shear stiffnesses of the plate, as follows from the equations (4.13) and (4.14) from reference [6].

Therefore the orthotropic moduli of rigidity of the core can be replaced by their mean value, for the determination of which it is requisite to consider the longitudinal and transverse stiffeners separately.

For the calculation of the modulus of shear in the planes x = constant for example we determine first the total average area A_x of the material carrying shear stresses in the z-direction and divide this by the area of the core in this plane, i.e. Hb. The product of this quotient and the modulus of rigidity of the material used for the stiffness we define as the modulus of rigidity G_x^* of the core in the planes x = constant

$$G_x^* = \frac{A_x}{H \cdot b} \cdot G.$$

The same holds for the planes y = constant and, hence

$$G^* = \tfrac{1}{2}(G_x^* + G_y^*)$$

For the thickness of the core H and of the cover plates h we take mean values, whereas in h the effect of the stiffeners of bottom and tanktop plating is also included. Then the total mass per unit of surface of the double bottom M is composed of the mass density ϱ^* of the core multiplied by its height H; the mass density ϱ of the cover plates multiplied by their height h; the mass m_c of the cargo or machinery and the virtual mass of water m_a, vibrating with the plate, both per unit of surface and thus

$$M = \varrho^* H + 2\varrho h + m_c + m_a$$

where $m_a + m_c = m$, as previously used.

The first and the last of these quantities, i.e. ϱ^* and m_a, require our special attention.

We define the mass density ϱ^* of the core of the sandwich plate, as the total mass M^* of all the longitudinal and transverse stiffeners of the double bottom divided by the product of the surface ab of the double bottom and the mean height H of the stiffeners and, hence, $\varrho^* = M^*/(abH)$.

For the determination of the added mass of water we refer to the theory of Joosen and Sparenberg [4]. These authors consider a rectangular cylinder of infinite length, vibrating with a sinusoidal wave along the length of the cylinder.

For the m.v. "Koudekerk" of Messrs. N.V. Vereenigde Nederlandsche Scheepvaartmaatschappij at The Haque the fundamental frequency of the double bottom in the engine room has been calculated. As described above, in this case holds

$a = 20.8$ m
$b = 20$ m
$E = 20.6 \times 10^{10}$ N/m^2
$\varrho = 8 \times 10^3$ kgmass/m^3
$m_c = 2.15 \times 10^3$ kgmass/m^2
$H = 1.85$ m
$h = 1.76 \times 10^{-2}$m
$G^* = 38.2 \times 10^7$ N/m^2
$\varrho^* = 0.18 \times 10^3$ kgmass/m^3
$m_a = 5.03 \times 10^3$ kgmass/m^2

and, hence, $M = 7.818 \times 10^3$ kgmass/m^2

Substitution of these values in equation (15) gives

$f_1 = 5.9$ c/s

In general measurements on ships show that the fourth and fifth natural frequencies are about 6 to 8 c/s [2], [3] and [5]. So we may conclude that for frequencies above the third natural frequency the influence of the double bottom of the engine room of the ship considered must be taken into consideration in the calculation of the higher natural frequencies of the hull. As mentioned in section 1 this can be done by introducing the equivalent masses of the double bottom.

6 Discussion

It is open to question whether the ship's double bottom can be considered to be hinged on its boundaries. Certainly it may be expected that the double bottom is hinged along the sides of the ship, but on the bulkheads the support will be somewhere between hinged and clamped.

In this paper the boundary supports of the double bottom have been chosen such that the bottom is hinged everywhere, because this seams to be a reasonable assumption in the first consideration of the problem. Of course, the calculation might be carried out for a more general boundary condition where the bottom is neither completely hinged nor completely clamped. The present calculation provides, however, a lower estimate for the natural frequency. Measurements of the vibration pattern on a ship might indicate whether the condition chosen here is reasonable indeed.

7 Conclusions

It appears that the fundamental frequencies of the double bottom and the mass connected lie in the range of the principal higher-order natural frequencies of a ship. The method described gives an acceptable value for these fundamental frequencies and the corresponding modes. In the introduction it is outlined how to insert this fact in the calculation of ship vibration by means of the beam method.

It follows also from this paper that the evaluation of a more detailed method is justified.

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