

Random Taylor hypothesis and the behavior of local and convective accelerations in isotropic turbulence

Arkady Tsinober^{a)}

Faculty of Engineering, Tel Aviv University, Tel Aviv 69974, Israel

Prakash Vedula^{b)}

Laboratory of Aero and Hydrodynamics, Delft University of Technology, 2628 CA Delft, The Netherlands

P. K. Yeung^{c)}

School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332

(Received 3 November 2000; accepted 4 April 2001)

The properties of acceleration fluctuations in isotropic turbulence are studied in direct numerical simulations (DNS) by decomposing the acceleration as the sum of local and convective contributions ($\mathbf{a}_L = \partial \mathbf{u} / \partial t$ and $\mathbf{a}_C = \mathbf{u} \cdot \nabla \mathbf{u}$), or alternatively as the sum of irrotational and solenoidal contributions [$\mathbf{a}_I = -\nabla(p/\rho)$ and $\mathbf{a}_S = \nu \nabla^2 \mathbf{u}$]. The main emphasis is on the nature of the mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ which must occur in order for the acceleration ($\mathbf{a}$) to be small as predicted by the “random Taylor hypothesis” [Tennekes, J. Fluid Mech. **67**, 561 (1975)] of small eddies in turbulent flow being passively “swept” past a stationary Eulerian observer. Results at Taylor-scale Reynolds number up to 240 show that the random-Taylor scenario $\langle \mathbf{a}^2 \rangle \ll \langle \mathbf{a}_C^2 \rangle \approx \langle \mathbf{a}_L^2 \rangle$, accompanied by strong antialignment between the vectors $\mathbf{a}_L$ and $\mathbf{a}_C$, is indeed increasingly valid at higher Reynolds number. Mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ also leads to the solenoidal part of $\mathbf{a}$ being small compared to its irrotational part. Results for spectra in wave number space indicate that, at a given Reynolds number, the random Taylor hypothesis has greater validity at decreasing scale sizes. Finally, comparisons with DNS data in Gaussian random fields show that the mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ is essentially a kinematic effect, although the Reynolds number trends are made stronger by the dynamics implied in the Navier–Stokes equations.

© 2001 American Institute of Physics. [DOI: 10.1063/1.1375143]

I. INTRODUCTION

As the material derivative of the velocity vector, the fluid particle acceleration field in turbulent flow

$$\mathbf{a} \equiv \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \quad (1)$$

is a physical parameter of great interest for a variety of reasons, ranging from studies of fine-scale intermittency to applications in Lagrangian modeling of dispersion (see Refs. 1–21 and references therein). Clearly, the acceleration may be written as the sum of the local acceleration $\mathbf{a}_L \equiv \partial \mathbf{u} / \partial t$ expressing the unsteady rate of change at a fixed point, and the convective acceleration $\mathbf{a}_C \equiv \mathbf{u} \cdot \nabla \mathbf{u}$ which expresses the rate of change due to the spatial derivatives and also embodies nonlinearity effects. In addition, for flows governed by the Navier–Stokes equations without body forces the acceleration is also given by

$$\mathbf{a} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}, \quad (2)$$

i.e., as the sum of pressure gradient and viscous contributions which in incompressible flow are respectively irrota-

tional and solenoidal. In other words, in addition to $\mathbf{a} = \mathbf{a}_L + \mathbf{a}_C$, we can also write $\mathbf{a} = \mathbf{a}_I + \mathbf{a}_S$, where $\mathbf{a}_I \equiv -\nabla(p/\rho)$ and $\mathbf{a}_S \equiv \nu \nabla^2 \mathbf{u}$. The purpose of this article is to use data from direct numerical simulations (DNS) to explore several physical issues in turbulence based on these two alternative decompositions of the acceleration vector. As discussed later (Sec. III), we also refer to the irrotational and solenoidal parts of the convective acceleration (as $\mathbf{a}_C = \mathbf{a}_{C_I} + \mathbf{a}_{C_S}$), which are computed using the method described in Sec. II.

The first issue concerns a commonly used concept known as the *random Taylor hypothesis* or the *sweeping decorrelation hypothesis*. It was suggested by Tennekes¹¹ that in turbulence at high Reynolds number *the dissipative eddies flow past an Eulerian observer in a time frame much shorter than the time scale which characterizes their own dynamics*. In turn this implies that Taylor’s “frozen-turbulence” approximation would be valid for the analysis of the advection of the turbulence microstructure by the large-scale motions. To assess its validity it should be recognized that, in fact, Tennekes’ hypothesis consists of two ingredients. First, it is proposed that the Lagrangian acceleration ($\mathbf{a}$) of fluid particles is in some sense small, such that time scales measuring Eulerian and Lagrangian rates of change could be estimated by simply setting $\mathbf{a} = 0$ (which, of course, cannot be perfectly true). It is noteworthy that this assumption was formulated for the turbulent fluctuations

^{a)}Electronic mail: tsinober@eng.tau.ac.il

^{b)}Electronic mail: p.vedula@wbmt.tudelft.nl

^{c)}Electronic mail: yeung@peach.ae.gatech.edu

(which are local pointwise in space and time) instead of for statistical measures of these quantities. The second assumption made by Tennekes is of statistical nature, namely, that *the microstructure is statistically independent of the energy-containing eddies*.

A survey of data in the literature^{10,13,17,19,22,23} suggests that the random Taylor hypothesis is qualitatively correct but also subject to a number of quantitative deviations. However, because of the Reynolds number limitations on previous data (especially earlier work¹⁷ in DNS), it is important to understand in detail and quantify the Reynolds number dependence of the various statistical parameters involved. Here we draw upon a current DNS database for isotropic turbulence at ensemble-averaged Taylor-scale Reynolds numbers ranging from about 38 to 240 using up to 512³ grid points, where some limited inertial range properties can be observed.²⁴ While our primary interest in this article is on local and convective contributions to the acceleration, new information is also provided on pressure gradients and viscous acceleration which were studied recently using the same database by Vedula and Yeung.¹³

Several specific aspects of the acceleration field are studied in this work. The most important, and basic, issue is perhaps the nature of the mutual statistical cancellation between the local and convective accelerations underlying the argument of the total acceleration being small by comparison. Since these quantities are vectors the degree of this mutual cancellation can be studied in terms of the geometry of vector alignment. Furthermore, because the Reynolds number is a measure of the range of scales, the nature of Reynolds number effects can be expected to vary with scale size, which is conveniently studied in terms of spectra in wave number space. In homogeneous turbulent flows, including isotropic turbulence, it is well known that while one-point statistics are approximately Gaussian, two-point statistics are not. To highlight the effects of this non-Gaussianity we make comparisons between DNS data and those of Gaussian random fields which are constructed to have the same energy spectrum as that in DNS. Because these Gaussian fields are not evolved from the Navier–Stokes equations, these comparisons also allow us to distinguish between “kinematic” and “dynamic” effects in the present context.

The rest of the article is organized as follows. First, in Sec. II we give a brief description of the numerical procedures and the data used. In Sec. III we show DNS results for different contributions ($\mathbf{a}_L, \mathbf{a}_C, \mathbf{a}_I, \mathbf{a}_S$) to the total acceleration, in terms of single-point variances and correlation coefficients, geometric statistics of vector alignment, as well as scale-dependent quantities. Comparisons between velocity fields in DNS evolved from the Navier–Stokes equations and their Gaussian random field counterparts are given in Sec. IV. Finally, conclusions are summarized in Sec. V.

II. SIMULATION AND DATA ANALYSIS

We analyze velocity fields obtained from direct numerical simulations (DNS) carried out using the Fourier pseudo-spectral algorithm of Rogallo.²⁵ Differencing in time is explicit and second order. As in recent work,¹³ the velocity

TABLE I. Basic simulation parameters for the DNS database.

Grid	64 ³	128 ³	256 ³	512 ³
R_λ	38	90	141	243
L_1 / η	22	57	101	191
$k_{\max} \eta$	1.47	1.52	1.40	1.46
u_{rms}	1.60	1.26	1.44	1.41
λ	0.60	0.47	0.27	0.18
ν	0.025	0.006 546	0.0028	0.0011
$\langle \epsilon \rangle$	2.70	0.72	1.20	0.99

field considered is stationary isotropic turbulence with energy maintained by forcing at the large scales.²⁶ Data analysis is performed at each grid resolution in a pseudo-spectral manner for a number of velocity fields that were previously saved (as single-time snapshots) at time intervals approximately one eddy-turnover time apart. The statistical independence implied by this separation in time allows each data set to be taken as one of a number of different realizations for ensemble averaging. Aliasing errors associated with taking products in physical space are controlled by a combination of phase shifting and the truncation of aliased modes in wave number space. The actual grid resolutions and Reynolds numbers are listed in Table I. Other quantities also listed for reference include the ratio between longitudinal integral length scale L_1 and the Kolmogorov scale η , the nondimensional parameter $k_{\max} \eta$ ($k_{\max}$ being the highest wave number resolved by the grid) which measures the numerical resolution of the small scales, rms velocity fluctuation, Taylor microscale, kinematic viscosity and the averaged energy dissipation rate.

The main numerical tasks involved in the calculation of acceleration quantities in this work are concerned with the convective acceleration and the pressure gradient. In the case of $\mathbf{a}_C$ we calculate terms of the type (in tensor notation) $u_j \partial u_i / \partial x_j$, where differentiation is performed in wave number space and products are taken in physical space. The pressure gradient and viscous acceleration are obtained in the same way as described in Ref. 13, with the former involving the recovery of pressure fluctuations by solving its Poisson equation in Fourier space. Finally, knowledge of $\mathbf{a}_C$, $\mathbf{a}_I$, and $\mathbf{a}_S$ together allows us to determine the local acceleration by

$$\mathbf{a}_L = \mathbf{a}_I + \mathbf{a}_S - \mathbf{a}_C. \quad (3)$$

Although this procedure for the calculation of $\mathbf{a}_L$ seems indirect, it is much more convenient than the task of taking a time derivative from two instantaneous velocity fields that would otherwise have to be saved at a very small time interval apart. For a test of accuracy we use the property that incompressibility requires the local acceleration must be solenoidal (since otherwise the velocity field computed from the Navier–Stokes equations would become nonsolenoidal as it evolves). It should be noted that for any vector $\mathbf{V}$ we can apply a Helmholtz decomposition in Fourier space, with the Fourier coefficients of irrotational and solenoidal parts being given respectively by

$$\hat{\mathbf{V}}^{(I)}(\mathbf{k}) = (\mathbf{k} \cdot \hat{\mathbf{V}}) \mathbf{k} / k^2; \quad \hat{\mathbf{V}}^{(S)} = \hat{\mathbf{V}} - \hat{\mathbf{V}}^{(I)}. \quad (4)$$

TABLE II. Acceleration variance and related quantities at different ensemble-averaged Reynolds numbers in DNS. All quantities are normalized by $\langle \epsilon \rangle^{3/2} \nu^{-1/2}$.

Grid	64 ³	128 ³	256 ³	512 ³
R_λ	38	90	141	243
$\text{Var}(\mathbf{a})$	1.26	2.33	2.75	3.54
$\text{Var}(\mathbf{a}_l)$	1.21	2.27	2.70	3.49
$\text{Var}(\mathbf{a}_s)$	0.05	0.05	0.05	0.05
$\text{Var}(\mathbf{a}_l)$	1.21	4.56	8.19	17.05
$\text{Var}(\mathbf{a}_c)$	2.47	6.89	10.96	20.61
$\text{Var}(\mathbf{a}_{c_l})$	1.21	2.27	2.69	3.49
$\text{Var}(\mathbf{a}_{c_s})$	1.26	4.61	8.24	17.10

The numerical procedures we use are capable of producing negligibly small values for the (nominally zero) divergence of the local acceleration vector.

Some remarks concerning the generation of Gaussian random fields noted in Sec. I are appropriate here. The goal is to construct velocity fields which are Gaussian in the sense of the velocity gradients being (artificially) Gaussian, but have the same form of the energy spectrum and Reynolds number corresponding to each grid resolution in Table I. This is accomplished by first taking an ensemble average of the energy spectrum function from “real” DNS data sets at each grid resolution, and then using a slight modification of the initialization procedure in Ref. 23 to create a randomized velocity field which has the desired statistical spectrum and is orthogonal to the wave number vector (hence divergence-free) for each Fourier mode. A Gaussian random number generator is used, and the method is tested by checking the skewness and flatness factors of velocity gradients versus the standard Gaussian values (0 and 3). Gaussianity is attained very closely except on a 512³ grid, where we found that (because of the large number of samples) the finite period of computerized pseudo-random number generators affects the quality of statistical sampling to a certain extent.

III. RESULTS FROM DNS

In this section we present DNS data bearing upon the validity of the random Taylor hypothesis involving mutual cancellation of local and convective accelerations. We cover several aspects, including conventional second-moment statistics, geometric vector alignment, and scale-size dependence as viewed in wave number space.

A. Variances and correlations

We begin with a basic characterization of various contributions ($\mathbf{a}_l, \mathbf{a}_c, \mathbf{a}_l, \mathbf{a}_s$) to the total acceleration [according to Eqs. (1) and (2)] in terms of simplest measures such as the variances of different contributions and the correlation coefficients as indicators of their statistical relationships to each other. Ensemble-averaged variances of these quantities at each grid resolution are shown in Table II. However, scatter plots showing one data point for each realization at its own Reynolds number (in a manner similar to Ref. 13, Fig. 1 therein) are perhaps more helpful in assessing the overall Reynolds number scaling behavior.

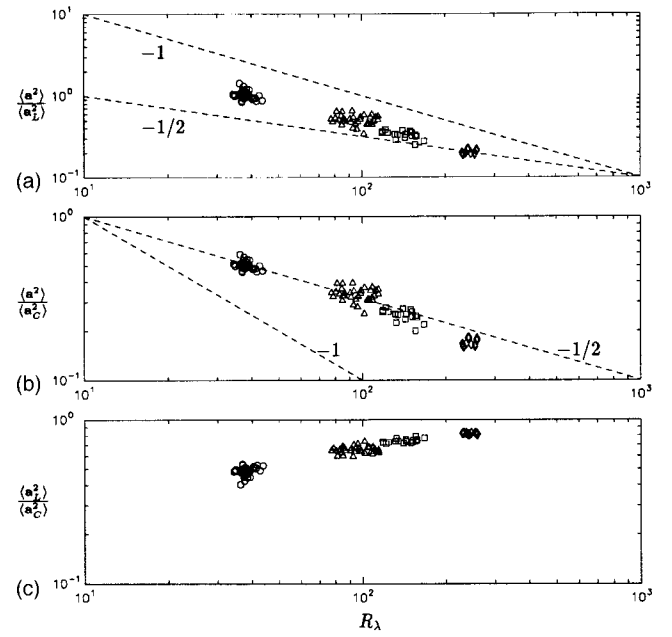


FIG. 1. Scatter plot of DNS data from multiple realizations on (a) $\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_l^2 \rangle$, (b) $\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_c^2 \rangle$, and (c) $\langle \mathbf{a}_l^2 \rangle / \langle \mathbf{a}_c^2 \rangle$, all vs Reynolds number (R_λ). Symbols for different grid resolutions are 64³ (○), 128³ (△), 256³ (□), and 512³ (◇). Dashed lines with slopes 1 and $-1/2$ are drawn to infer trends of scaling behavior.

As noted in Sec. I, a key assumption in Tennekes’ hypotheses is that the total acceleration should be small compared to its local and convective contributions, which in turn should have variances close to each other. At the second moment level this is illustrated directly by a comparison of variances. Figure 1 shows scatter plots for ratios between the variances of $\mathbf{a}$, $\mathbf{a}_l$, and $\mathbf{a}_c$, with different symbols for each grid resolution. The behavior predicted by Tennekes’ hypotheses is indeed observed in the figure, which also shows a definite Reynolds-number trend for each quantity—namely that (to a close approximation) both $\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_l^2 \rangle$ and $\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_c^2 \rangle$ vary as $R_\lambda^{-1/2}$, such that the ratio $\langle \mathbf{a}_l^2 \rangle / \langle \mathbf{a}_c^2 \rangle$ approaches a constant at high Reynolds numbers.

Since $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c$ the relative smallness of $\langle \mathbf{a}^2 \rangle$ noted above implies that $\mathbf{a}_l$ and $\mathbf{a}_c$ must be significantly negatively correlated. The correlation coefficients (ρ) among $\mathbf{a}$, $\mathbf{a}_l$, and $\mathbf{a}_c$ for the same data sets are shown in Fig. 2. In the bottom part of this figure we can see that the correlation coefficient $\rho(\mathbf{a}_l, \mathbf{a}_c)$ is about -0.7 at $R_\lambda \approx 40$, about -0.9 at $R_\lambda \approx 240$, and appears to approach the limiting value of -1.0 with increasing Reynolds number. However, as a consequence of the solenoidal nature of $\mathbf{a}_l$ it is important to note that $\mathbf{a}_l$ and $\mathbf{a}_c$ cannot cancel each other completely. To see this, we note that since $\mathbf{a}_l$ is solenoidal (with its irrotational part $\mathbf{a}_{l_i}$ being identically zero), adding up the irrotational and solenoidal parts of both $\mathbf{a}_l$ and $\mathbf{a}_c$ yields

$$\mathbf{a}_{c_i} = \mathbf{a}_l, \quad (5)$$

$$\mathbf{a}_l + \mathbf{a}_{c_s} = \mathbf{a}_s. \quad (6)$$

The smallness in variance of $\mathbf{a}_s$ versus those of $\mathbf{a}_l$ and $\mathbf{a}_{c_s}$ in Table II indicates an increasingly strong degree of cancella-

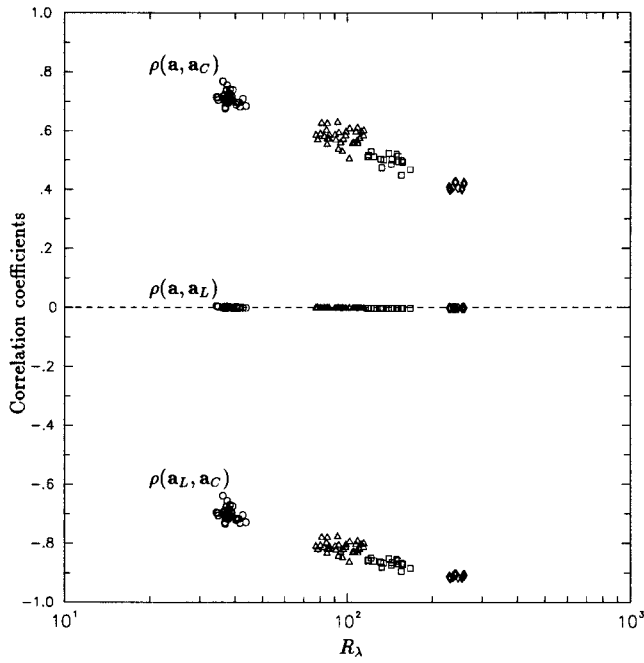


FIG. 2. Scatter plot of correlation coefficients $\rho(a, a_L)$, $\rho(a, a_C)$, and $\rho(a_L, a_C)$. Symbols are the same in Fig. 1.

tion between the solenoidal parts of $\mathbf{a}_L$ and $\mathbf{a}_C$ at high Reynolds number. However, cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ is not complete, since the irrotational part of their sum, i.e., $\mathbf{a}_I$ remains finite and is a dominant contributor to the variance of $\mathbf{a}$ at all Reynolds numbers.

In Fig. 2 it is also worth noting that $\mathbf{a}$ is positively correlated with $\mathbf{a}_C$ (although less so at higher Reynolds number) but practically uncorrelated with $\mathbf{a}_L$ regardless of the Reynolds number. This latter lack of correlation can also be explained by noting that, whereas $\mathbf{a}_L$ is solenoidal, $\mathbf{a}$ is—because of the dominance of $\mathbf{a}_I$ over $\mathbf{a}_S$ (as seen in Ref. 13)—nearly irrotational. Since irrotational and solenoidal vectors are uncorrelated in homogeneous turbulence, it follows that $\mathbf{a}$ and $\mathbf{a}_L$ are, as observed, nearly uncorrelated with each other.

It is clear from Eq. (6) that a small $\mathbf{a}_S$ can be interpreted as the result of strong mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_{C_S}$, i.e., these two terms must be nearly the same in magnitude but (as vectors) almost antiparallel to each other. These properties are evident in Fig. 3, which shows the ratio of variances between $\mathbf{a}_L$ and $\mathbf{a}_{C_S}$, their correlation coefficient, and the ratio between $\langle \mathbf{a}_S^2 \rangle$ and $\langle \mathbf{a}_I^2 \rangle$. It can be seen that as the Reynolds number increases, the ratio $\langle \mathbf{a}_I^2 \rangle / \langle \mathbf{a}_{C_S}^2 \rangle$ indeed approaches 1.0 whereas $\rho(a_L, a_{C_S})$ approaches -1.0 . On the other hand, even at the lowest Reynolds number in the figure, $\langle \mathbf{a}_S^2 \rangle$ is seen to be only about 5% of either $\langle \mathbf{a}_I^2 \rangle$ or $\langle \mathbf{a}_{C_S}^2 \rangle$, becoming smaller still at higher Reynolds numbers. In other words, the tendency of increasing mutual cancellation at higher Reynolds numbers between $\mathbf{a}_L$ and $\mathbf{a}_{C_S}$ tends to make the solenoidal part ($\mathbf{a}_S$) of the acceleration very small compared to its irrotational part ($\mathbf{a}_I$). This tendency may be called a *reduction of solenoidality* of the total acceleration, and is in contrast to an accompanying *enhancement* of sole-

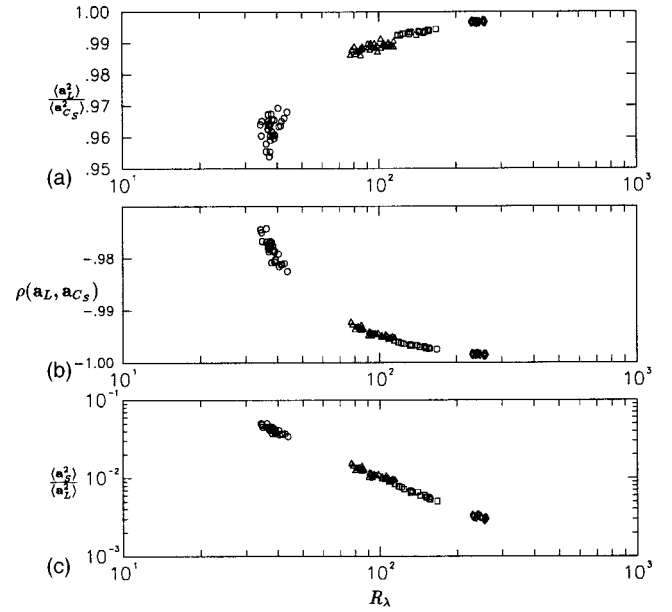


FIG. 3. Scatter plot of (a) $\langle \mathbf{a}_I^2 \rangle / \langle \mathbf{a}_{C_S}^2 \rangle$, (b) $\rho(a_L, a_{C_S})$, and (c) $\langle \mathbf{a}_S^2 \rangle / \langle \mathbf{a}_I^2 \rangle$. Symbols are the same as in Fig. 1.

noidality of the convective acceleration. The latter opposite trend can be seen in Table II by noting that the variance of the solenoidal part $\mathbf{a}_{C_S}$ becomes larger compared to that of the irrotational part ($\mathbf{a}_{C_I}$) of $\mathbf{a}_C$ as the Reynolds number increases.

Although we have focused on local and convective accelerations, it is useful to include a comparison with the alternative decomposition into irrotational and solenoidal accelerations. The dominance of $\mathbf{a}_I$ over $\mathbf{a}_S$ is well known (e.g., Ref. 27), has been verified recently in DNS,¹³ and can be

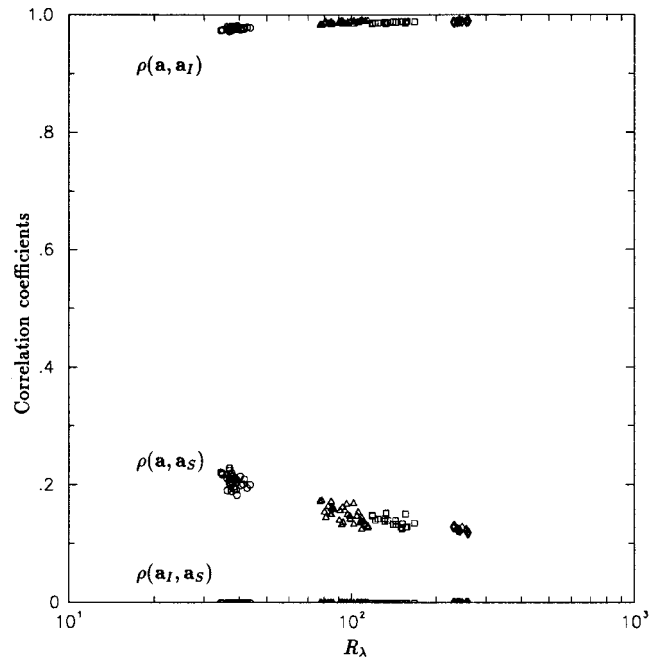


FIG. 4. Scatter plot of $\rho(a, a_I)$, $\rho(a, a_S)$, and $\rho(a_I, a_S)$. The last quantity is close to 0. Symbols are the same in Fig. 1.

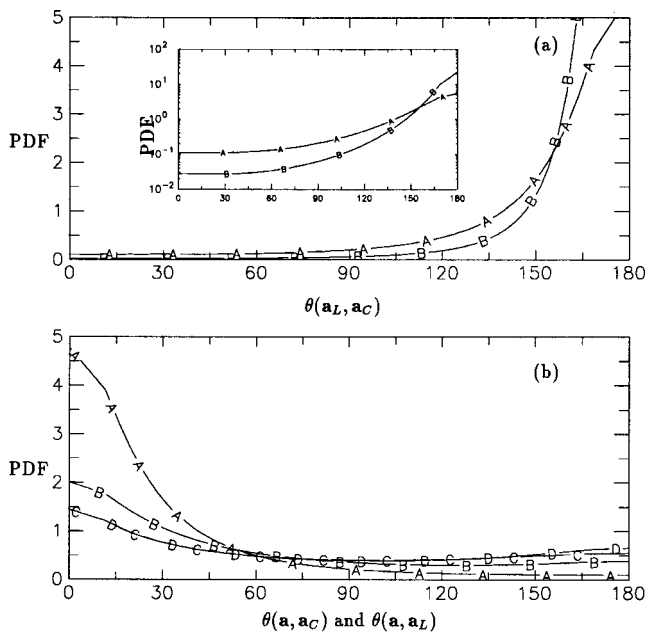


FIG. 5. (a) PDFs of $\theta(\mathbf{a}_L, \mathbf{a}_C)$ for R_λ 38 and 240 (lines A and B, respectively). The inset shows the same PDFs on a logarithmic scale. (b) PDFs of $\theta(\mathbf{a}, \mathbf{a}_C)$ (lines A, B) and $\theta(\mathbf{a}, \mathbf{a}_L)$ (lines C, D) at R_λ 38 and 240.

seen by taking the ratios of $\langle \mathbf{a}_I^2 \rangle$ to $\langle \mathbf{a}_S^2 \rangle$ in Table II. As noted in Ref. 13, the Reynolds number trend resides primarily in $\mathbf{a}_I$, whereas $\mathbf{a}_S$ is nearly universal when scaled by Kolmogorov variables. A scatter plot of correlation coefficients among $\mathbf{a}$, $\mathbf{a}_I$, and $\mathbf{a}_S$ is shown in Fig. 4, where it can be seen that $\mathbf{a}_I$ and $\mathbf{a}_S$ are (due to homogeneity) uncorrelated, whereas $\mathbf{a}$ is almost perfectly correlated with $\mathbf{a}_I$ but only weakly correlated with $\mathbf{a}_S$.

The results of this subsection show that higher Reynolds numbers tend to produce reduced solenoidality of total acceleration but at the same time also enhanced solenoidality of the convective acceleration. The mutual cancellation between local and convective accelerations which is central to the random Taylor hypothesis has been observed to become stronger at the second-moment level as the Reynolds number is increased. However, it should be noted that the degree of statistical correlation between fluctuating vector quantities depends on both the properties of coordinate components and the geometric orientation (or alignment) of these vectors with each other. These alignment properties are studied in the next subsection.

B. Geometrical statistics

To provide information complementary to that in Sec. III A, we consider here the alignment properties of $\mathbf{a}$, $\mathbf{a}_L$, and $\mathbf{a}_C$ relative to one another, followed by the same for $\mathbf{a}$, $\mathbf{a}_I$, and $\mathbf{a}_S$. In each case we consider both the shape of the probability density function (PDF, in Figs. 5 and 6) of the angle, as well as the mean value of its cosine (Table III) which provides a useful quantitative measure. We use the notation $\theta(\mathbf{V}_1, \mathbf{V}_2)$ for the angle between any two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$.

If the magnitude of the total acceleration, $\mathbf{a} = \mathbf{a}_L + \mathbf{a}_C$, is to be small compared to those of $\mathbf{a}_L$ and $\mathbf{a}_C$, then the vectors

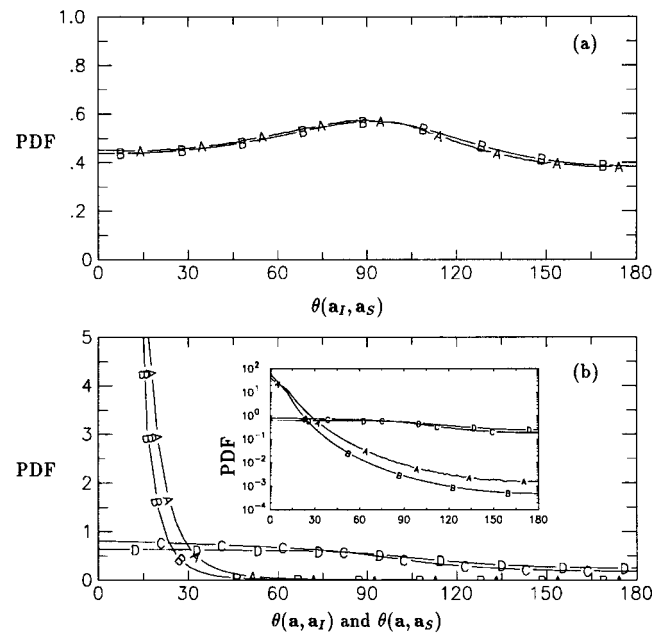


FIG. 6. (a) PDFs of $\theta(\mathbf{a}_I, \mathbf{a}_S)$ at R_λ 38 and 240 (lines A and B, respectively). (b) PDFs of $\theta(\mathbf{a}, \mathbf{a}_I)$ (lines A, B), and PDFs of $\theta(\mathbf{a}, \mathbf{a}_S)$ (lines C, D) at R_λ 38 and 240. The inset shows the same PDFs on a logarithmic scale.

$\mathbf{a}_L$ and $\mathbf{a}_C$ must be nearly antiparallel. This in turn implies that the angle between $\mathbf{a}_L$ and $\mathbf{a}_C$, denoted by $\theta(\mathbf{a}_L, \mathbf{a}_C)$, should have a high likelihood of being close to 180 degrees. In Fig. 5 we show ensemble-averaged PDFs at the lowest and highest Reynolds numbers in our calculations, for $\theta(\mathbf{a}_L, \mathbf{a}_C)$ (in the upper half of the figure) as well as $\theta(\mathbf{a}, \mathbf{a}_L)$ and $\theta(\mathbf{a}, \mathbf{a}_C)$ (in the lower half). A very strong peak in the PDF around 180° is indeed seen for $\theta(\mathbf{a}_L, \mathbf{a}_C)$, with a more complete view of data in the range close to 180° given in the inset using linear-log scales. It is clear that the degree of antialignment between $\mathbf{a}_L$ and $\mathbf{a}_C$ becomes stronger with increase in Reynolds number. This Reynolds number trend is also supported by the behavior of the mean of the cosine of the alignment angle, which in Table III is seen to become very close to -1.0 .

In the lower half of Fig. 5 we can observe that $\mathbf{a}$ is in general positively aligned with both of $\mathbf{a}_L$ and $\mathbf{a}_C$, with a modest peak of these PDFs being at zero degrees. Although the alignment with $\mathbf{a}_C$ appears to be stronger than that for $\mathbf{a}_L$, it also weakens significantly at higher Reynolds number. However, the alignment of $\mathbf{a}$ with $\mathbf{a}_I$ shows no appreciable Reynolds number dependence. Similarly, it can be seen in

TABLE III. Mean values of the cosines of the angles between $\mathbf{a}$, $\mathbf{a}_L$ and $\mathbf{a}_C$ and $\mathbf{a}$, $\mathbf{a}_I$, and $\mathbf{a}_S$ in DNS.

Grid	64 ³	128 ³	256 ³	512 ³
R_λ	38	90	141	243
$\cos(\mathbf{a}, \mathbf{a}_L)$	0.103	0.109	0.105	0.093
$\cos(\mathbf{a}, \mathbf{a}_C)$	0.568	0.427	0.353	0.267
$\cos(\mathbf{a}_L, \mathbf{a}_C)$	-0.581	-0.699	-0.762	-0.836
$\cos(\mathbf{a}, \mathbf{a}_I)$	0.951	0.971	0.973	0.977
$\cos(\mathbf{a}, \mathbf{a}_S)$	0.224	0.169	0.156	0.146
$\cos(\mathbf{a}_I, \mathbf{a}_S)$	0.022	0.018	0.014	0.013

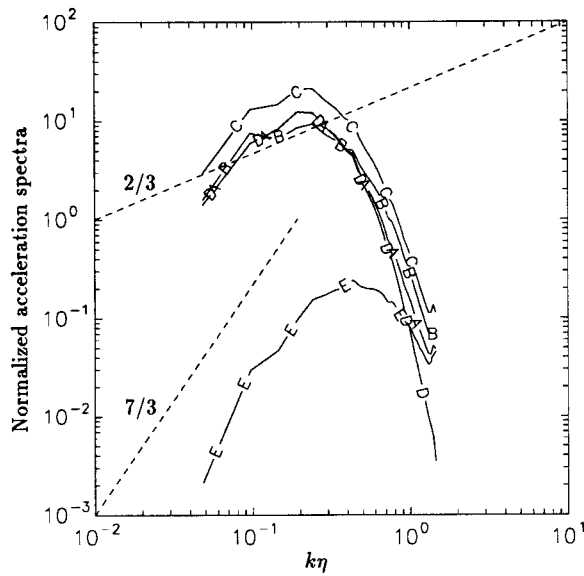


FIG. 7. Ensemble-averaged spectra of $\mathbf{a}$, $\mathbf{a}_L$, $\mathbf{a}_C$, $\mathbf{a}_I$, and $\mathbf{a}_S$ in wave number space normalized by $\nu^{1/4}\langle\epsilon\rangle^{5/4}$, denoted by lines A–E, respectively, for DNS data at R_λ 38. Dashed lines with slopes $\frac{2}{3}$ and $\frac{7}{3}$ are shown.

Table III that whereas $\langle\cos\theta(\mathbf{a},\mathbf{a}_C)\rangle$ decreases strongly with Reynolds number, $\langle\cos\theta(\mathbf{a},\mathbf{a}_L)\rangle$ remains at a nearly constant (but low) value of order 0.1. It is noteworthy that the strong antialignment between $\mathbf{a}_L$ and $\mathbf{a}_C$ was observed only recently in experiments in the laboratory²⁸ and in the atmospheric surface layer.²⁹

Figure 6 shows information analogous to that in Fig. 5, but for the vectors $\mathbf{a}$, $\mathbf{a}_I$, and $\mathbf{a}_S$. Because $\mathbf{a}_I$ and $\mathbf{a}_S$ are statistically orthogonal (in the sense that $\langle\mathbf{a}_I\cdot\mathbf{a}_S\rangle=0$) due to incompressibility and homogeneity, they are expected to have no net tendency for preferential alignment with each other. The PDF of $\theta(\mathbf{a}_I,\mathbf{a}_S)$ is seen to be relatively flat, with highest values at close to 90° , and no significant Reynolds number dependence. The closeness of this PDF to a uniform distribution is consistent with $\langle\cos\theta(\mathbf{a}_I,\mathbf{a}_S)\rangle$ being close to 0, as seen in Table III.

Because of the dominance of the irrotational part of the acceleration over its solenoidal part, it seems almost inevitable that $\mathbf{a}$ would be aligned much more closely with $\mathbf{a}_I$ than with $\mathbf{a}_S$. Indeed, together with the inset (on semi-log scales) in its lower half, Fig. 6 also shows that the PDF of $\theta(\mathbf{a},\mathbf{a}_I)$ has a high peak at zero, with the peak value increasing with Reynolds number. In contrast, the PDF of $\theta(\mathbf{a},\mathbf{a}_S)$ shows much weaker alignment, being nearly flat and increasingly so at higher Reynolds number.

C. Spectra in wave number space

To understand the contributions of different scale sizes to the behavior of the acceleration ($\mathbf{a}$) and its constituents ($\mathbf{a}_L,\mathbf{a}_C,\mathbf{a}_I,\mathbf{a}_S$) it is convenient to study the spectra of these quantities (denoted by S,S_L,S_C,S_I,S_S , respectively) in wave number space. The spectra of these quantities are shown in Figs. 7 and 8, for DNS data at the lowest and highest Reynolds number, respectively ($R_\lambda\sim 38$ and 240), and in a form normalized by Kolmogorov variables based on

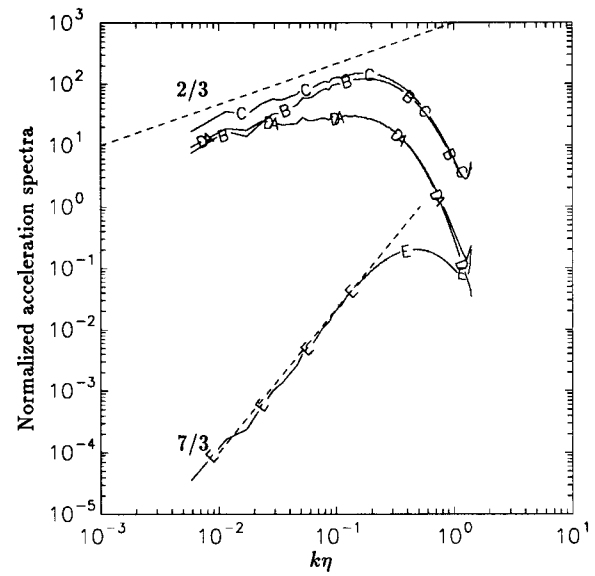


FIG. 8. Same as Fig. 7, but for R_λ 240.

viscosity and the energy dissipation rate. Generally, it can be seen that all except S_S have a peak at an intermediate wave number range, $k\eta\sim 0.1$ – 0.2 , and that S_S makes a significant contribution to S only at the small scales, i.e., for wave numbers at $k\eta\sim 1$ or higher.

The spectra of local and convective accelerations are of special interest here. It can be seen that the spectrum of $\mathbf{a}_C$ is monotonically above or equal to that of $\mathbf{a}_L$ at all wave numbers. However, especially at high Reynolds numbers, we find that the gap between these two spectra narrows substantially at higher wave numbers. At the same time, the spectrum of their sum (the acceleration itself) becomes much lower by comparison. In other words, for the small scales at high Reynolds number we find the comparison $S\ll S_L\approx S_C$, which corresponds to $\langle\mathbf{a}^2\rangle\ll\langle\mathbf{a}_L^2\rangle\approx\langle\mathbf{a}_C^2\rangle$, and hence to the mutual cancellation between the vectors $\mathbf{a}_L$ and $\mathbf{a}_C$ studied in the two previous subsections. The observations here also indicate that the mutual cancellation is more nearly complete at the small scales. In turn, this implies that the random Taylor hypothesis has greater validity at the small scales, and furthermore the range of scales over which the hypothesis is valid becomes wider (spreading from small towards intermediate length scales) with increasing Reynolds number. Extrapolation of our results towards Reynolds numbers higher than the DNS data range would suggest that S_L and S_C become coincident at nearly all wave numbers except perhaps the energy-containing range.

Another natural question concerning the data in Figs. 7 and 8 is whether there is evidence for inertial range behavior. It is clear that the spectrum of $\mathbf{a}_S$ (line E) has a $k^{7/3}$ scaling range, which is a direct result of $k^{-5/3}$ inertial range behavior in the energy spectrum.¹³ Other inferences are less definite. However, it does appear that, at high Reynolds number (Fig. 8), the convective acceleration has a $k^{2/3}$ scaling behavior over the range $k\eta\approx 0.02$ – 0.05 , which coincides approximately with the wave number range for inertial scaling in the energy spectrum.²⁴ While an explanation for this result is not

obvious, we also note that this scaling exponent is different from that of the acceleration spectrum which has a smaller slope. Since the spectra of $\mathbf{a}$, $\mathbf{a}_C$, and $\mathbf{a}_S$ all have the same physical dimensions, the differences observed between them also serve as a reminder that dimensional reasoning and traditional Kolmogorov similarity arguments are not sufficient for predicting the behavior of acceleration spectra.

The spectrum of $\mathbf{a}$ is (as seen in the results of Ref. 13) dominated by that of $\mathbf{a}_I$ except possibly at very high wave numbers, where the spectrum of $\mathbf{a}_S$ becomes comparable to that of $\mathbf{a}_I$. Plots comparing normalized acceleration spectra (Fig. 6 of Ref. 13) show that the strongest sensitivity to Reynolds numbers is in the spectrum of $\mathbf{a}_I$ at low wave numbers. Since the spectrum of $\mathbf{a}_I$ at low wave numbers contributes dominantly (via its integral in wave number space) to the variance of $\mathbf{a}_I$, this result is consistent with the trend seen in Table II that $\langle \mathbf{a}_I^2 \rangle$ and hence $\langle \mathbf{a}^2 \rangle$ increases with R_λ under Kolmogorov scaling.

IV. COMPARISONS WITH GAUSSIAN RANDOM FIELDS

The behavior of acceleration statistics studied in Sec. III can be regarded as due to both kinematic constraints as expressed by the continuity equation, and to the dynamics of momentum balance as expressed by the Navier–Stokes equations. In this section we attempt to separate the roles of kinematic versus dynamic effects, by comparing with Gaussian random fields (here denoted by GRFs for short) constructed to (see Sec. II) correspond to velocity fields in DNS having the same energy spectrum and Reynolds number. It should be noted that these Gaussian fields carry kinematic effects only, since they satisfy the continuity equation but have not evolved in time according to the dynamics of the Navier–Stokes equations. In some aspects like intermittency, kinematic and dynamic effects can differ considerably,^{23,30} and the contributions of kinematic effects can be very significant.^{31–33}

An effective way to illustrate possible differences in Reynolds number dependence for velocity fields from DNS versus GRF would be to examine results for the lowest and highest Reynolds numbers in the data available for both types of velocity fields. However, because of the limitations of our pseudo-random number generator (as stated in Sec. II) here we exclude the 512³ GRF data from our analysis and instead present comparisons of data at 64³ and 256³ resolutions, at R_λ respectively 38 and 141.

A comparison of variances and correlation coefficients involving the acceleration ($\mathbf{a}$) and its several distinct physical constituents ($\mathbf{a}_L$, $\mathbf{a}_C$, $\mathbf{a}_I$, and $\mathbf{a}_S$) is first given in Table IV, at the Reynolds numbers indicated above. It can be seen that the Reynolds number trends (e.g., the approach to unity of the ratio $\langle \mathbf{a}_L^2 \rangle / \langle \mathbf{a}_C^2 \rangle$ at higher Reynolds number) for the variances and correlation coefficients shown are qualitatively similar in both types of velocity fields. However, the magnitude of Reynolds number effects (again, say in the ratio $\langle \mathbf{a}_L^2 \rangle / \langle \mathbf{a}_C^2 \rangle$) is generally stronger in DNS.

At a given Reynolds number, the data in Table IV indicate, remarkably, that the random-Taylor scenario of $\langle \mathbf{a}^2 \rangle$

TABLE IV. Comparisons between acceleration variances and related quantities for DNS data and Gaussian random fields (GRFs).

Grid	64 ³ DNS	64 ³ GRF	256 ³ DNS	256 ³ GRF
R_λ	38	38	141	141
$\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_I^2 \rangle$	1.043	0.314	0.336	0.122
$\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_C^2 \rangle$	0.511	0.248	0.251	0.110
$\langle \mathbf{a}_I^2 \rangle / \langle \mathbf{a}_C^2 \rangle$	0.490	0.790	0.747	0.899
$\rho(\mathbf{a}, \mathbf{a}_I)$	0.003	0.039	0.0000	0.016
$\rho(\mathbf{a}, \mathbf{a}_C)$	0.713	0.463	0.502	0.317
$\rho(\mathbf{a}_L, \mathbf{a}_C)$	−0.699	−0.867	−0.867	−0.943
$\langle \mathbf{a}_C^2 \rangle / \langle \mathbf{a}_S^2 \rangle$	0.962	0.304	0.327	0.117
$\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_I^2 \rangle$	1.044	1.075	1.019	1.047
$\langle \mathbf{a}^2 \rangle / \langle \mathbf{a}_S^2 \rangle$	23.18	14.29	52.37	22.50
$\langle \mathbf{a}_I^2 \rangle / \langle \mathbf{a}_S^2 \rangle$	22.21	13.36	52.39	21.50
$\rho(\mathbf{a}, \mathbf{a}_I)$	0.951	0.964	0.989	0.977
$\rho(\mathbf{a}, \mathbf{a}_S)$	0.224	0.265	0.139	0.211

$\ll \langle \mathbf{a}_L^2 \rangle \approx \langle \mathbf{a}_C^2 \rangle$ is a better approximation for GRFs than in DNS. This suggests that the mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ is primarily a kinematic effect. This is in agreement with purely kinematic results³³ obtained via employing the Millionshtikov hypothesis. On the other hand, the dominance of $\langle \mathbf{a}_I^2 \rangle$ over $\langle \mathbf{a}_S^2 \rangle$ is stronger in DNS than for GRFs, suggesting that this is a true consequence of Navier–Stokes dynamics. It should be noted that both of these differences observed here appear to become weaker at the highest Reynolds number shown. However, it cannot yet be ascertained whether these differences would cease to exist at asymptotically high Reynolds numbers.

From a statistical point of view, a key difference between velocity fields in DNS and corresponding GRFs is that velocity gradients calculated in DNS are non-Gaussian and intermittent, in fact increasingly so at higher Reynolds number.³⁴ To characterize the effects of this non-Gaussianity on the probability distributions of the acceleration and its constituents, we make comparisons based on standardized PDFs in Figs. 9 and 10 and flatness factors in Table V. In view of isotropy the data have been averaged over three Cartesian coordinate components. Logarithms of the PDFs are also taken in order to give a clearer picture of low probability events.

It is important to note that, even for Gaussian random fields, both $\mathbf{a}_C$ and $\mathbf{a}_I$ are, because of nonlinearities in their definitions, inherently non-Gaussian. On the other hand, $\mathbf{a}_S$ is Gaussian because it is linear and can be considered in GRFs as approximated by a finite difference scheme involving linear combinations of independent Gaussian velocity fluctuations at neighboring grid points. Because of the dominance of $\mathbf{a}_I$ over $\mathbf{a}_S$, and because of imperfect cancellation between $\mathbf{a}_C$ and $\mathbf{a}_I$, both the total acceleration $\mathbf{a} = \mathbf{a}_I + \mathbf{a}_S$ and the local acceleration $\mathbf{a}_L = \mathbf{a} - \mathbf{a}_C$ are non-Gaussian. These properties are clearly demonstrated by the flatness factors in Table V. Furthermore, an exact result $\mathbf{a}_C$ can be deduced by noting that because uncorrelated Gaussian random variables are independent, each component of $\mathbf{a}_C$ (as $u_j \partial u_i / \partial x_j$) is proportional to the sum of three products of independent Gaussian variates. The flatness factor of $\mathbf{a}_C$ is thus the same as that of

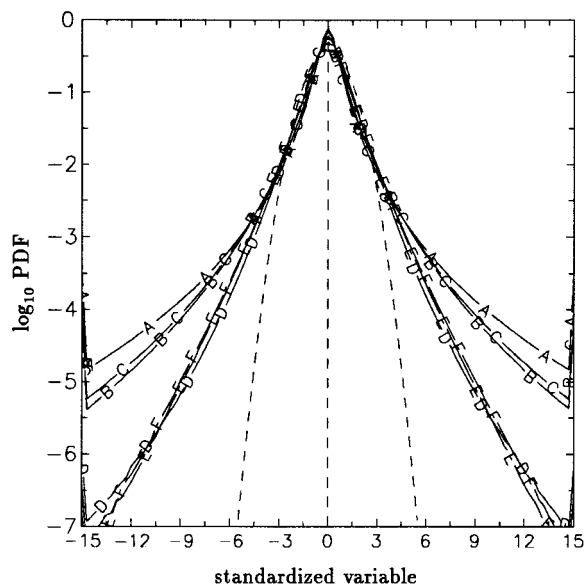


FIG. 9. Base-10 logarithms of the standardized PDFs of $\mathbf{a}$, $\mathbf{a}_L$, and $\mathbf{a}_C$ denoted by lines A, B, C for DNS and lines D, E, F for Gaussian random fields (GRF), all at R_λ 140. The dashed line represents a standardized Gaussian PDF for comparison.

a random variable $Z = Y_1 + Y_2 + Y_3$ where each of the latter three is the product of standardized Gaussian random variables and have the moments $\langle Y \rangle = 0$, $\langle Y^2 \rangle = 1$, and $\langle Y^4 \rangle = 9$. Expanding the moments of Z in terms of those of Y_1 , Y_2 , and Y_3 (which are also independent of each other) then leads to its flatness factor being $\langle Z^4 \rangle / \langle Z^2 \rangle^2 = 45 / (3)^2 = 5$. This value is very close to the GRF data in Table V.

It is clear from Fig. 9 that PDFs for the DNS data exhibit much wider tails, indicating greater intermittency compared to their Gaussian counterparts. It can also be seen that $\mathbf{a}_C$ is more intermittent than $\mathbf{a}_L$, with slightly wider tails in its PDF and a higher flatness factor. It is interesting to observe that whereas in DNS $\mathbf{a}$ is more intermittent than both of $\mathbf{a}_L$

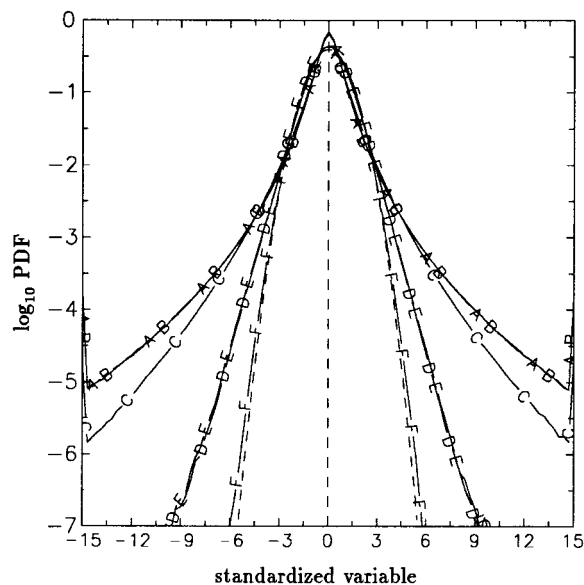


FIG. 10. Same as Fig. 9, but for $\mathbf{a}$, $\mathbf{a}_I$, and $\mathbf{a}_S$.

TABLE V. Comparison of flatness factors (μ_4) of acceleration and its constituents between DNS and GRFs.

R_λ	38 DNS	38 GRF	140 DNS	140 GRF
$\mathbf{a}$	7.9	3.9	24.4	4.0
$\mathbf{a}_L$	6.3	4.5	12.8	5.0
$\mathbf{a}_C$	7.6	5.1	15.3	5.3
$\mathbf{a}_I$	8.1	4.0	24.8	4.1
$\mathbf{a}_S$	5.9	3.0	11.4	3.1

and $\mathbf{a}_C$, the reverse is true for GRFs. In other words, the effects of mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ tend to promote intermittency of $\mathbf{a}$ in DNS but to reduce it for GRFs which contain no Navier–Stokes dynamics information. Another feature which is apparent from the flatness factors in Table V is a strong increase in intermittency with Reynolds number in DNS. This increase is in contrast to the GRF results, which are (within the limits of sampling uncertainties) insensitive to Reynolds number.

Similar characteristics of intermittency can be seen in Fig. 10, which shows PDFs for $\mathbf{a}$, $\mathbf{a}_I$, and $\mathbf{a}_S$. Both the PDFs shown and corresponding flatness factors in Table V indicate that $\mathbf{a}$ and $\mathbf{a}_I$ are (as already known) very close, and both are more intermittent than $\mathbf{a}_S$. In addition, we can observe that the PDFs in DNS have much wider tails than those of corresponding quantities in GRF, and are accompanied by larger flatness factors. In contrast, Gaussian random fields are by definition not intermittent; the non-Gaussianity of $\mathbf{a}_I$ (as for $\mathbf{a}_L$ and $\mathbf{a}_C$ noted above) is a result of its being nonlinear in the velocity fluctuations.

We continue comparisons between DNS and Gaussian fields by studying the alignment between $\mathbf{a}_L$ and $\mathbf{a}_C$ (Sec. III B) for these cases. Figure 11 shows, for both DNS and GRF data, PDFs of the angle $\theta(\mathbf{a}_L, \mathbf{a}_C)$ for simulations at

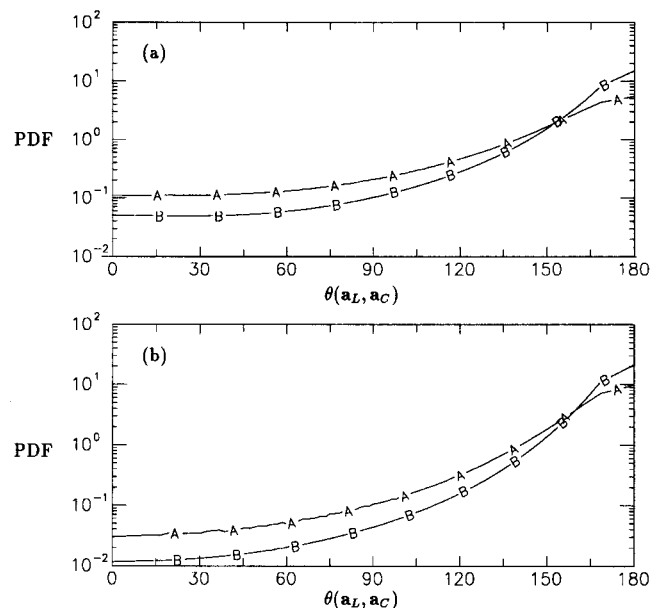


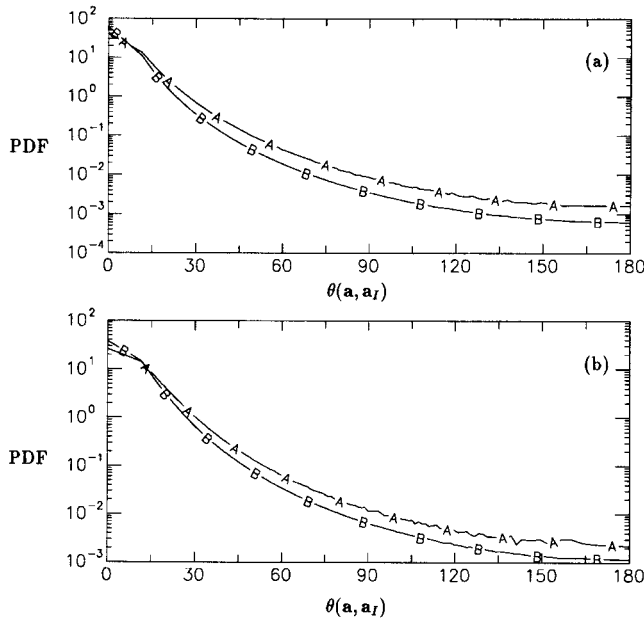
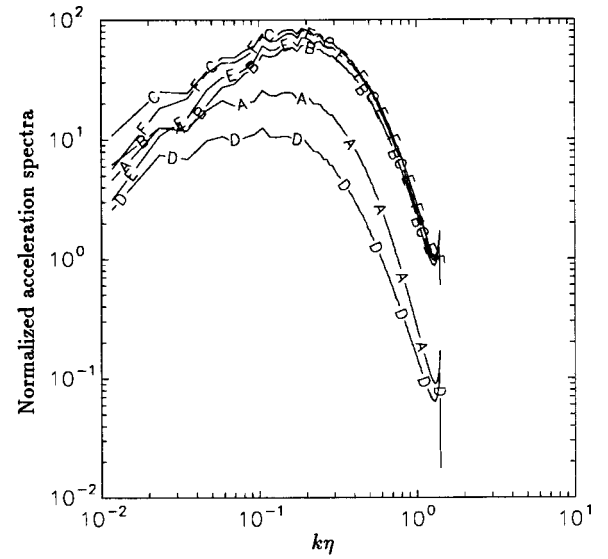
FIG. 11. PDFs of $\theta(\mathbf{a}_L, \mathbf{a}_C)$ at R_λ 38 and 140 (lines A and B). Part (a) of the figure is for DNS, part (b) for GRFs.

TABLE VI. Mean values of the cosines of the angles between $\mathbf{a}$, $\mathbf{a}_L$ and $\mathbf{a}_C$ and $\mathbf{a}$, $\mathbf{a}_I$ and $\mathbf{a}_S$ (similar to Table II) for Gaussian random fields.

Grid	64^3	128^3	256^3	512^3
R_λ	38	90	141	243
$\cos(\mathbf{a}, \mathbf{a}_L)$	0.133	0.109	0.098	0.108
$\cos(\mathbf{a}, \mathbf{a}_C)$	0.358	0.272	0.233	0.195
$\cos(\mathbf{a}_L, \mathbf{a}_C)$	-0.756	-0.836	-0.869	-0.883
$\cos(\mathbf{a}, \mathbf{a}_I)$	0.931	0.947	0.954	0.959
$\cos(\mathbf{a}, \mathbf{a}_S)$	0.249	0.216	0.198	0.178
$\cos(\mathbf{a}_I, \mathbf{a}_S)$	6.9×10^{-4}	1.3×10^{-4}	7.9×10^{-5}	-2.7×10^{-5}

$R_\lambda \approx 40$ and 140. Because these PDFs are highly peaked (near 180 degrees), semi-log scales are used on the PDF axis to display data over the entire range. Although there are quantitative differences (including the height of the peak), it is clear that the DNS and GRF data are very similar. These similarities suggest that, consistent with indications from variance ratios in Table IV, antiparallel alignment between $\mathbf{a}_L$ and $\mathbf{a}_C$ is primarily a kinematic effect. The Reynolds number trends for alignment between $\mathbf{a}_L$ and $\mathbf{a}_C$ are also similar, with the peak height of the PDF increasing with Reynolds number. Further information can be obtained by comparing the mean values of the cosines of the angles between $\mathbf{a}$, $\mathbf{a}_L$, and $\mathbf{a}_C$ in Table VI with those for DNS in Table III. From these tables one can infer that the vectorial anti-alignment between $\mathbf{a}_L$ and $\mathbf{a}_C$ is more sensitive to Reynolds number in DNS than for the Gaussian fields.

A similar comparison for vectorial alignment between $\mathbf{a}$ and $\mathbf{a}_I$ is given in Fig. 12, where again semi-log axes are used in view of a high peak (near 0 degrees). As for $\mathbf{a}_L$ and $\mathbf{a}_C$ in Fig. 11, it is clear that the alignment properties are qualitatively very similar for DNS and Gaussian fields. However, quantitative differences in the Reynolds number dependence also exist, which (because of semi-log scaling) are not obvious in the PDF plots but can be seen in the behavior

FIG. 12. Same as Fig. 11, but for $\theta(\mathbf{a}, \mathbf{a}_I)$.FIG. 13. Ensemble-averaged spectra of $\mathbf{a}$, $\mathbf{a}_L$, and $\mathbf{a}_C$ (normalized by $\nu^{1/4}(\epsilon)^{5/4}$) at R_λ 140 for DNS (lines A, B, C) and Gaussian random fields (D, E, F).

(Table VI) of mean values of the cosine of the angle between $\mathbf{a}$ and $\mathbf{a}_I$ at different Reynolds numbers. Indeed, comparison with DNS data in Table III shows that Reynolds number effects on the alignment between $\mathbf{a}$ and $\mathbf{a}_I$ are stronger for DNS data than for Gaussian fields. Taken together, the data in these figures and tables indicate that close alignment between $\mathbf{a}$ and $\mathbf{a}_I$ is also mainly a kinematic effect. On the other hand, the trend towards closer alignment between $\mathbf{a}$ and $\mathbf{a}_I$ due to the dominant contribution of $\mathbf{a}_I$ to $\mathbf{a}$ is essentially a consequence of the Navier–Stokes dynamics.

Finally, we compare the spectra of $\mathbf{a}$, $\mathbf{a}_L$, and $\mathbf{a}_C$ between DNS and Gaussian random fields in Fig. 13. It can be seen that the spectra have essentially the same shapes in each case. Except for some differences at the lowest wave numbers, the magnitudes of the spectra of $\mathbf{a}_L$ and $\mathbf{a}_C$ are comparable between DNS and GRF. However, the acceleration spectrum itself is significantly lower for GRFs, which is consistent with stronger mutual cancellation noted above based on analyses of other quantities.

V. SUMMARY AND DISCUSSION

In this article we have used data from direct numerical simulations of incompressible isotropic turbulence to investigate the validity of the random Taylor hypothesis,¹¹ in terms of statistical and geometrical properties of the acceleration vector and its local and convective constituents (as $\mathbf{a} = \mathbf{a}_L + \mathbf{a}_C$) compared with irrotational and solenoidal constituents (as $\mathbf{a} = \mathbf{a}_I + \mathbf{a}_S$). The data cover ensemble-averaged Taylor-scale Reynolds numbers in the range approximately 38 to 240 on grids from 64^3 to 512^3 , and are compared with results extracted from Gaussian random fields with the same energy spectrum and Reynolds number.

Our results show that the variance of the total acceleration $\mathbf{a}$ is indeed small compared to those of $\mathbf{a}_L = \partial \mathbf{u} / \partial t$ and $\mathbf{a}_C = (\mathbf{u} \cdot \nabla) \mathbf{u}$. This can be traced to $\mathbf{a}_L$ and $\mathbf{a}_C$ being strongly negatively correlated, with correlation coefficient close to

−0.9 at R_λ 240. At the same time, it is well known that $\mathbf{a}$ is close to $\mathbf{a}_I = -\nabla(p/\rho)$, which is much larger than $\mathbf{a}_S = \nu \nabla^2 \mathbf{u}$. By taking a Hemholtz decomposition of $\mathbf{a}_C$ we show that the fact of $\langle \mathbf{a}^2 \rangle$ being relatively small can be viewed as the result of strong mutual cancellation between $\mathbf{a}_L$ (which is solenoidal) and the solenoidal part of $\mathbf{a}_C$. This cancellation occurs in such a manner that $\mathbf{a}_I$ is still a dominant contribution to $\mathbf{a}$ despite being small in variance compared to $\mathbf{a}_L$ and $\mathbf{a}_C$.

It is important to note that the effects described in the preceding paragraph all become stronger at higher Reynolds number. Consequently at sufficiently high Reynolds numbers we may expect the relations

$$\langle \mathbf{a}_S^2 \rangle \ll \langle \mathbf{a}^2 \rangle \approx \langle \mathbf{a}_I^2 \rangle = \langle \mathbf{a}_{C_I}^2 \rangle \ll \langle \mathbf{a}_{C_S}^2 \rangle \approx \langle \mathbf{a}_C^2 \rangle \approx \langle \mathbf{a}_L^2 \rangle \quad (7)$$

to hold, where the “ $\ll$ ” sign is interpreted here as “at least an order of magnitude smaller than.” In particular, the relation $\langle \mathbf{a}^2 \rangle \ll \langle \mathbf{a}_L^2 \rangle \approx \langle \mathbf{a}_C^2 \rangle$ is in support of the random Taylor hypothesis. It may be appropriate, however, to give a more limited interpretation of this hypothesis in the sense that the microstructure is *statistically decorrelated* from the energy containing eddies. This is different from the original assumption by Tennekes¹¹ (and similar) that the *microstructure is statistically independent of the energy containing eddies*. There is a growing body of experimental evidence^{10,19,22–24,30} suggesting that large and small scales are not statistically independent despite being practically decorrelated. This issue, however, is beyond the scope of this article.

The DNS data show that local acceleration ($\mathbf{a}_I$) and the solenoidal part ($\mathbf{a}_{C_S}$) of the convective acceleration are mostly canceling each other, resulting in a very small solenoidal part ($\mathbf{a}_S$) of the total acceleration vector. However, we emphasize this does not mean that they are not separately important. For example, the main contribution to enstrophy production in the mean-squared vorticity budget is associated with $\mathbf{a}_{C_S}$, but not with $\mathbf{a}_L$. This issue is to be addressed separately elsewhere.

We also studied the statistics of acceleration from the perspectives of vectorial alignment in physical space and spectra in wave number space. Statistics of the angle between $\mathbf{a}_L$ and $\mathbf{a}_C$ show clear evidence of strong antialignment, with high probability in the range close to 180 degrees. On the other hand, strong positive alignment is observed between $\mathbf{a}$ and $\mathbf{a}_I$, and both of these effects become more pronounced at higher Reynolds number. In wave number space, we find that the spectrum of $\mathbf{a}$ is comparable to those of $\mathbf{a}_L$ and $\mathbf{a}_C$ at low wave number but much smaller in magnitude at high wave numbers. This dependence on wave number suggests that the random Taylor hypothesis has greater validity when applied to smaller scales being advected by the large-scale motions.

Finally, comparisons between velocity fields obtained in DNS and Gaussian random fields (GRF, which have not evolved according to the Navier–Stokes equations) at a given Reynolds number show that there is a large kinematic contribution to effects described above. In particular, we find considerable qualitative similarities between DNS and GRF results on the alignment between the vectors $\mathbf{a}$ and $\mathbf{a}_I$ and the

antiparallel alignment between $\mathbf{a}_L$ and $\mathbf{a}_C$, suggesting that these alignment properties are essentially kinematic in nature. These alignments and the mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ are, in fact, found to be stronger for GRFs. However, effects of Navier–Stokes dynamics are manifested in the form of Reynolds number dependence, which is much stronger for DNS data compared to Gaussian random fields. By considering PDFs of different quantities we also find that, within a general trend of increasing intermittency at higher Reynolds number, the effect of mutual cancellation between $\mathbf{a}_L$ and $\mathbf{a}_C$ is to make $\mathbf{a}$ more intermittent in DNS but less so for Gaussian random fields.

ACKNOWLEDGMENTS

P.V. and P.K.Y. gratefully acknowledge support from the National Science Foundation, via Grant No. CTS-9705678, and via NSF cooperative agreement ACI-9619020 through computing resources provided by the National Partnership for Advanced Computational Infrastructure at the San Diego Supercomputer Center.

- ¹F. H. Champagne, “The fine scale structure of the turbulent velocity field,” *J. Fluid Mech.* **86**, 67 (1978).
- ²W. J. A. Dahm and K. B. Southerland, “Experimental assessment of Taylor’s hypothesis and its applicability to dissipation estimates in turbulent flows,” *Phys. Fluids* **9**, 2101 (1997).
- ³E. Gledzer, “On the Taylor hypothesis corrections for measured energy spectra of turbulence,” *Physica D* **104**, 163 (1997).
- ⁴R. J. Hill and S. T. Thoroddsen, “Experimental evaluation of acceleration correlations for locally isotropic turbulence,” *Phys. Rev. E* **55**, 1600 (1997).
- ⁵Y. Kaneda, “Lagrangian and Eulerian time correlations in turbulence,” *Phys. Fluids A* **5**, 2835 (1993).
- ⁶V. S. L’vov, A. Pomyalov, and I. Procaccia, “Temporal surrogates of spatial turbulent statistics: The Taylor hypothesis revisited,” *Phys. Rev. E* **60**, 4175 (1999).
- ⁷A. M. Obukhov and A. M. Yaglom, “Microstructure of turbulent flow,” *Prikl. Mat. Mekh.* **15**, 3 (1951).
- ⁸J.-F. Pinton and R. Labbe, “Correction to the Taylor hypothesis in swirling flows,” *J. Phys. II* **4**, 1461 (1994).
- ⁹M. B. Pinsky and A. P. Khain, “Turbulence effects on cloud and precipitation formation—A review,” *J. Aerosol Sci.* **28**, 1177 (1997).
- ¹⁰A. A. Praskovsky, E. B. Gledzer, M. Yu. Karyakin, and Y. Zhou, “Fine-scale turbulence structure of intermittent shear flows,” *J. Fluid Mech.* **248**, 493 (1993).
- ¹¹H. Tennekes, “Eulerian and Lagrangian time microscales in isotropic turbulence,” *J. Fluid Mech.* **67**, 561 (1975).
- ¹²C. Tong, “Taylor’s hypothesis and two-point coherence measurements,” *Boundary-Layer Meteorol.* **81**, 399 (1966).
- ¹³P. Vedula and P. K. Yeung, “Similarity scaling of acceleration and pressure statistics in numerical simulations of isotropic turbulence,” *Phys. Fluids* **11**, 1208 (1999).
- ¹⁴G. Voth, K. Satyanarayan, and E. Bodenschatz, “Lagrangian acceleration measurements at large Reynold numbers,” *Phys. Fluids* **10**, 2268 (1998).
- ¹⁵P. K. Yeung, “Direct numerical simulation of two-particle relative diffusion in isotropic turbulence,” *Phys. Fluids* **6**, 3416 (1994).
- ¹⁶P. K. Yeung, “One- and two-particle Lagrangian acceleration correlations in numerically simulated homogeneous turbulence,” *Phys. Fluids* **9**, 2981 (1997).
- ¹⁷P. K. Yeung and S. B. Pope, “Lagrangian statistics from direct numerical simulations of isotropic turbulence,” *J. Fluid Mech.* **207**, 531 (1989).
- ¹⁸Y. Zhou and R. Rubinstein, “Sweeping and straining effects in sound generation by high Reynolds number isotropic turbulence,” *Phys. Fluids* **8**, 647 (1996).
- ¹⁹A. Tsinober, “How important are direct interactions between large and small scales in turbulent flows?” in *New Approaches and Concepts in Turbulence*, edited by T. Dracos and A. Tsinober (Birkhäuser, Basel, 1993), p. 141.

- ²⁰M. S. Borgas and B. L. Sawford, "The small scale structure of acceleration correlations and its role in the statistical theory of turbulent dispersion," *J. Fluid Mech.* **228**, 295 (1991).
- ²¹M. S. Borgas and P. K. Yeung, "Conditional fluid particle accelerations in turbulence," *Theor. Comput. Fluid Dyn.* **11**, 69 (1998).
- ²²K. R. Sreenivasan and B. Dhruva, "Is there scaling in high-Reynolds-number turbulence?" *Prog. Theor. Phys. Suppl.* **130**, 103 (1998).
- ²³M. Kholmyansky and A. Tsinober, "On the origins of intermittency in real turbulent flows," in *Intermittency in Turbulent Flows*, edited by J. C. Vassilicos (Cambridge University Press, Cambridge, UK, 2000), p. 183.
- ²⁴P. K. Yeung and Y. Zhou, "Universality of the Kolmogorov constant in numerical simulations of turbulence," *Phys. Rev. E* **56**, 1746 (1997).
- ²⁵R. S. Rogallo, "Numerical experiments in homogeneous turbulence," NASA Tech. Memo. 81315 (1981).
- ²⁶V. Eswaran and S. B. Pope, "An examination of forcing in direct numerical simulations of turbulence," *Comput. Fluids* **16**, 257 (1988).
- ²⁷M. Nelkin, "Universality and scaling in fully developed turbulence," *Adv. Phys.* **43**, 143 (1994).
- ²⁸B. Lüthi, U. Burr, W. Kinzelbach, and A. Tsinober, "Velocity derivatives in turbulent flow from 3D-PTV measurements," in *Proceedings of the 2nd International Symposium on Turbulence and Shear Flow Phenomena*, Stockholm, Sweden, 27–29 June 2001 (in press).
- ²⁹M. Kholmyansky, A. Tsinober, and S. Yorish, "Velocity derivatives in an atmospheric surface layer at $Re=10^4$. Further results," in *Proceedings of the 2nd International Symposium on Turbulence and Shear Flow Phenomena*, Stockholm, Sweden, 27–29 June 2001 (in press).
- ³⁰X. Shen and Z. Warhaft, "The anisotropy of the small scale structure in high Reynolds number ($Re_\lambda \sim 1000$) turbulent shear flow," *Phys. Fluids* **12**, 2976 (2000).
- ³¹A. Tsinober, "On one property of Lamb vector in isotropic turbulent flow," *Phys. Fluids A* **2**, 484 (1990).
- ³²L. Shtilman, M. Spector, and A. Tsinober, "On some kinematic versus dynamic properties of homogeneous turbulence," *J. Fluid Mech.* **247**, 65 (1993).
- ³³M. Pinsky, A. Khain, and A. Tsinober, "Accelerations in isotropic and homogeneous turbulence and Taylor's hypothesis," *Phys. Fluids* **12**, 3195 (2000).
- ³⁴K. R. Sreenivasan and R. A. Antonia, "The phenomenology of small-scale turbulence," *Annu. Rev. Fluid Mech.* **29**, 435 (1997).