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S.I.: COMPUTATIONAL LOGISTICS IN FOOD AND DRINK INDUSTRY

Dynamic pricing and inventory control policies in a food supply chain of growing and deteriorating items

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Abstract

Revenue and inventory management play a crucial role in the operational efficiency of food supply chains. The current study investigates dynamic pricing and inventory control policies in a two-level Food Supply Chain (FSC) of growing and deteriorating inventory that involves a rearing farm as the supplier and a retailer where these slaughtered items are prone to deterioration. The rearing farm breeds newborn animals, then slaughters them and sends the items to the retailer. The negative impact of overbreeding is taken into account to preserve the items' quality and decrease food waste on the supply side. The model is analyzed under decentralized and centralized supply chain scenarios with a profit-sharing contract as the coordination tool in the centralized case. An analytic solution approach based on non-linear convex programming is developed to solve the problem. The developed structure is illustrated through experimental results with a real estimated growth function for broiler chickens. Sensitivity analysis is carried out to investigate the impact of different input parameters. It is shown that the centralized supply chain scenario not only enhances the profit of the supplier and the retailer but also is more desirable for the customers as the selling price of the items decreases in this setting. The results provide decision-makers of each echelon with insights into the features of the studied FSC, including their most influential input parameters, the areas that require further attention, and managerial suggestions under different scenarios.

Keywords Inventory control · Dynamic pricing · Food supply chain · Growth · Deterioration

1 Introduction

About two-thirds of food wastes occur during the processes of Food Supply Chains (FSCs), which has raised severe criticism against these chains' performance (Zhong et al., 2017). On the other hand, food quality is critical as it directly interfaces with health and safety

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issues. These illuminate the significance of operational efficiency in FSCs, which can lead to reduced food wastes, enhanced food quality, and considerable cost savings. Inventory management plays a crucial role in the operational efficiency of supply chains as its goal is to handle inventory in its most cost-efficient manner (Soleymanfar et al., 2015). In addition, there usually exists a strict connection between price and demand, such that lower prices can lead to higher demand. So, inventory management can achieve its ultimate purpose if optimal pricing policies are simultaneously taken into account. Then, joint inventory and revenue management policies are treated as influential tasks in enhancing the efficiency of FSCs.

Growth is a common concept in the food industry and is introduced as the natural development leading to physical changes such as size and weight increase. As the growing items enter the system, they start to flourish, resulting in their weight increase. This is usual in the farming, fisheries, poultry, and livestock industry. Each inventory cycle of a growing item involves two sub-cycles: breeding and consumption. During the breeding period, the inventory grows owing to feeding and nourishment. The inventory level keeps increasing until slaughtering time, which is the ending point of the breeding period. Then, the consumption period starts, and the slaughtered items are depleted to zero due to demand fulfillment and deterioration.

Growing items are the initial inputs of many FSCs, and thereby the managerial decisions on these items impact the entire chain (Westhoek et al., 2014). Taking the appropriate revenue and inventory management policies for these items is highly crucial as it is directly linked to food safety and quality. The growing inventory is prone to diseases and quality decrements. Accordingly, any inappropriate decision not only causes significant financial losses but also directly threatens consumers' health (Gebbers & Adamchuk, 2010). Besides, growing items tend to deteriorate after getting slaughtered, which brings considerable food wastes. So, in addition to food safety, optimizing their inventory and pricing decisions can effectively decrease food wastes and undesirable costs leading to the enhanced operational efficiency of FSCs.

Motivated by the significances above, this paper investigates joint dynamic pricing and inventory management policies in an FSC of growing-deteriorating items. We explore a two-level FSC involving a rearing farm as the supplier and a retailer. Newborn animals, such as broiler chickens enter the supplier's system and undergo growth during their breeding period. They are then slaughtered and sent to the retailer according to its ordering policy. To guarantee high food quality, the negative impact of overbreeding is taken into account by establishing a quality control process after slaughtering the items. The slaughtered inventory, such as chicken meat, starts to deteriorate during the retailer's consumption period. The problem is analyzed under decentralized and centralized supply chain scenarios. In the decentralized scenario, the chain members act independently and try to optimize their own profit. In contrast, in a centralized scenario, the chain is seen as a unified entity, and its total profit is optimized. We apply a profit-sharing contract as a coordination mechanism in the centralized supply chain scenario to provide the chain members with enough incentives to join this structure. An analytic solution approach based on non-linear convex programming is developed to solve the problem that uses first-order and second-order optimality conditions together with a numerical root-finding method for non-closed-form equations.

FSCs embrace some features that differentiate them from other SCs. Among those are food quality and safety concerns, inventory alterations and limited lifetime, coordination complexity, and the critical tradeoff between pricing and freshness (Zhong et al., 2017). These features are reflected in our paper by:

- Introducing growing items that are a present part of various FSCs.
- Implementing quality control and discarding low-quality slaughtered items.
- Taking the deterioration of slaughtered items into account.
- Applying a centralized supply chain scenario together with a profit-sharing contract to facilitate collaboration.
- Linking the customers' demands to the retailer's selling price and applying dynamic pricing to compensate for the decreased freshness of the items.

The study on pricing and inventory management of growing items is still in its infancy, and when it comes to the context of FSCs, the deficiency is even more highlighted. This research contributes to the existing literature by filling several existing gaps in the area, which are discussed comprehensively in Sect. 2. Explicitly, the contributions of this study are as follows:

- Modeling growth as a biological weight increase function and simultaneously considering the impact of the breeding period and the number of young-born animals entering the system.
- Taking the negative impact of overbreeding into account to preserve the quality of the growing items.
- Modeling price as a dynamic variable of initial selling price and discount rate to mitigate the impact of quality degradations.
- Investigating integrated inventory management and pricing decisions of growing-deteriorating items in the context of FSCs.
- Studying the impact of coordination mechanisms to enhance the performance of this FSC.

The rest of the paper is organized as follows. Section 2 provides a review of the related literature and existing research gaps. Section 3 presents the problem description, assumptions, and the mathematical model. The solution approach is outlined in Sect. 4. Section 5 provides numerical results, sensitivity analysis and managerial insights. Finally, the conclusion and future research directions are presented in Sect. 6.

2 Literature review

The food supply chain is a vast area of academic research with a wide variety of topics to be analyzed, including food safety, security and risks, inventory and revenue management, chain design, transportation, etc. Our research mainly falls into the area of inventory and revenue management in FSCs, the literature body of which is perused from two streams: deterioration and growth. Interested readers are referred to Blackmon et al. (2020) and Srivastava and Dashora (2021) for food safety and risks, Kwag et al. (2019) and Magale et al. (2020) for FSC design, Balster and Freidrich (2019), Sahinvazan et al. (2019) and Liu et al. (2020) for transportation problems.

2.1 Growth

Rezaei (2014) introduced growth in inventory management by developing an EOQ model for growing items. He modeled growth as weight increasing, which is a standard function in poultry and livestock literature. While the paper has incorporated several simplistic assumptions, it can be treated as an appropriate basis for later extensions. Zhang et al., (2016a, b) extended Rezaei (2014), considering environmental issues and carbon emission. Nobil et al.

(2019) generalized Rezaei (2014) to allow for shortages. To overcome the complexity raised by admissible shortages, their solution procedure treats the items' weight as a constant value independent of the period length, which is a significant shortcoming questioning the validity of their structure. In addition to Rezaei's (2014) findings, their results revealed that shortage cost has no impact on the breeding period.

Sebatjane and Adetunji (2019a) proposed an EOQ model for growing items with quality considerations. They treated the items' breeding period as a fixed value by defining a targeted final weight for each unit item. Investigating different growth functions is a noble feature of this study. Their results indicated that the logistic function could better project the growth procedure than split linear and linear functions. On the other hand, a split linear function can reduce the complexity of the equations and is still able to reflect growth better than a linear one. Sebatjane and Adetunji (2019b) and Sebatjane and Adetunji (2020) extended their previous paper by incorporating an incremental discount scheme and studying the problem in SC's context, respectively.

Khalilpourazari and Pasandideh (2019) provided a multi-item, multi-constrained EOQ model for growing items. They considered three operational constraints, including an on-hand budget, warehouse capacity, and total allowable holding cost. To solve the problem in small sizes, sequential quadratic programming is incorporated. Moreover, the paper uses two meta-heuristics in medium and large sizes. Malekitabar et al. (2019) proposed a novel model for Rainbow trout. They considered the items' initial inventory level to be known and the nature of the items to be unchanged through time. Expressly, the demand is for the growing items, not the slaughtered inventory. They optimized the system's periodic profit, which implicitly suggests that the problem is analyzed for a one-period case.

Gharaei and Almehdawe (2019) investigated optimal replenishment policies for growing inventory where a portion of the items die during growth. They considered the growth and mortality rates to be linear functions of time, which, as shown by Sebatjane and Adetunji (2019a), is unable to depict the growth of these items accurately. Pourmohammad-Zia and Karimi (2020) developed an economic quantity model for growing items in the presence of deterioration. They took the negative impact of overbreeding into account by incorporating a quality loss rate, which is an increasing function of the breeding period.

2.2 Deterioration

Management of deteriorating items through supply chains has not been investigated as profoundly as the single echelon case, and it still is regarded as a trending research area. This subsection contains studies that particularly consider deterioration in the context of SC. For further studies, the interested readers are referred to Feng et al. (2016), Zhang et al., (2016a, b), Janssen et al. (2016), Taleizadeh et al. (2019), Khan et al. (2020).

The two-echelon supply chain structure is a relatively active research area. Here we focus on the recent research works, which are to some extent related to our study. Jaggi et al. (2019) investigated replenishment and trade-credit policies in a two-level SC of deteriorating items. They studied both decentralized and centralized supply chain scenarios in their paper. Gupta et al. (2020) studied a similar problem where shortages are also allowed. Taleizadeh et al. (2020) studied mixed sales of deteriorating and serviceable products in a supplier-retailer scenario. They implemented a hybrid payment strategy involving advance payment and trade credit in the presence of shortages. Their results showed that their proposed payment strategy provides retailers with a powerful tool in real-world settings.

Supply chain contracts are potent implements of chain coordination, enhancing the benefits of distinct chain members, specifically, once the items are deteriorating. Zhang et al. (2015a) incorporated a revenue-sharing contract in a two-level SC. They also exploited cooperative investment in inventory holding technology to diminish the negative result of deterioration. Bai et al. (2017) investigated revenue and promotional cost-sharing contracts together with a two-part tariff contract in a supply chain of deteriorating items. They concluded that while both contracts can bring perfect coordination, the two-part tariff contract is more robust. He et al. (2018) also applied revenue-sharing and two-part tariff contracts in an SC with deteriorating items. Zhang et al. (2017) studied the impact of cooperative advertising contracts on the supply chain of deteriorating products. They showed that cooperative advertising improves the performance of their analyzed SC. Chernonog (2020) applied a wholesale price contract as a coordination mechanism.

Chao et al. (2019) studied integrated inventory, location, and routing problems in a food distribution network consisting of several manufacturers and distribution centers. They developed a heuristic to solve their proposed model. Gholami-Zanjani et al. (2021) developed a two-stage scenario-based mathematical model to optimally design a resilient FSC in the face of epidemic disruptions. In addition to perishability, they took discounted pricing into account and considered demand to be random. They developed a solution approach based on scenario reduction and Benders decomposition, which has shown good performance in their problem.

Dynamic pricing is a rigorous strategy to mitigate the negative impact of deterioration by stimulating demand. It has been extensively studied in company-level structures (see Liu et al. (2015), Zhang et al. (2015b), Rabbani et al. (2017), Feng et al. (2018), and Dye (2020)), while the research works in the context of SC are scarce. Dye et al. (2017) studied a two-level SC under a multi-period finite planning horizon. The selling price varies in different periods, which is referred to as dynamic pricing in their paper. Li and Wang (2017) investigated dynamic pricing and replenishment policies in an SC whose products' quality is dynamically traced. The price is exponentially discounted in time to boost demand. Chen and Chen (2020) studied a similar structure where the price is linearly discounted.

The three-echelon cases are more poorly investigated due to the complexity of their modeling and solution approach. Cai et al. (2013) explored a chain involving supplier, distributor, and retailer. They applied two coordination mechanisms and showed that there is no necessity for a price-discount contract to be accompanied by a buy-back contract. Daryanto et al. (2019) analyzed an integrated three-level SC under environmental considerations. They considered that transportation, warehousing, and deteriorated items disposal accompany carbon emission. Their results showed that integration could considerably reduce costs and carbon emissions.

Table 1 provides a general overview of the closely related models investigating growth and/or deterioration (in SC) in the literature.

To sum up, studies on growing items are very scarce. Even among the existing ones, the majority fails to consider the simultaneous effect of the breeding period and the initial number of newborn animals that enter the system. The negative impact of overbreeding that largely influences the quality of the items is almost overlooked. Despite their efficiency, integrated pricing and inventory control decisions are neglected in existing growth models. Although the growing items turn into deteriorating inventory after slaughtering in reality, this is only heeded in two research works. Moreover, there is only one paper in the context of supply chains that confronts several simplifying assumptions. Although deterioration is a rich area of academic research in the field of inventory management, studies are scarce in the context of supply chain management and when it comes to dynamic pricing.

Table 1 Brief overview of closely related models in the literature

Ref	Growth	Simultaneous impact of BP and IIL	Overbreeding effect	Deterioration	Dynamic Pricing	Structure		Coordination contracts
						CL	SC	
Rezaei (2014)	✓	✓				✓		
Zhang et al., (2016a, b)	✓					✓		
Nobil et al. (2019)	✓					✓		
Sebatjane and Adetunji (2019a)	✓					✓		
Sebatjane and Adetunji (2019b)	✓					✓		
Khalilpourazari and Pasandideh (2019)	✓	✓				✓		
Malekitabar et al. (2019)	✓			✓		✓		
Gharaei and Almehdawe (2019)	✓	✓				✓		
Pourmohammad-Zia and Karimi (2020)	✓	✓	✓	✓		✓		
Sebatjane and Adetunji (2020)	✓						✓	
Cai et al. (2013)				✓			✓	
Zhang et al. (2015a)				✓			✓	✓
Bai et al. (2017)				✓			✓	✓
Zhang et al. (2017)				✓			✓	✓
Dye et al. (2017)				✓	✓		✓	
Li and Wang (2017)				✓	✓		✓	

Table 1 (continued)

Ref	Growth	Simultaneous impact of BP and IIL	Overbreeding effect	Deterioration	Dynamic Pricing	Structure		Coordination contracts
						CL	SC	
He et al. (2018)				✓			✓	✓
Jaggi et al. (2019)				✓			✓	
Daryanto et al. (2019)				✓			✓	
Chen and Chen (2020)				✓	✓		✓	
Chao et al. (2019)				✓			✓	
Gupta et al. (2020)				✓			✓	
Taleizadeh et al. (2020)				✓			✓	
Chernonog (2020)				✓			✓	✓
Gholami-Zanjani et al. (2021)				✓	✓		✓	
This paper	✓	✓	✓	✓	✓		✓	✓

BP breeding period, *IIL* initial inventory level, *CL* company level, *SC* supply chain

3 Model development

3.1 Problem description

Consider a two-level FSC, including growing-deteriorating inventory, which is a prevalent scenario in the poultry and livestock industry. The rearing farm acts as the supplier of this chain. It buys newborn animals at the beginning of the inventory cycle, raises them during the breeding period, and then slaughters them (Rezaei, 2014). Finally, the slaughtered items are quality controlled and sent to the retailer, who sells them to the customers. As the inventory enters the retailer's system, it faces quality losses owing to deterioration. Accordingly, during the retailer's inventory cycle, which is called the consumption period, the inventory level depletes to zero due to both fulfilling the customers' demand and deterioration. Figure 1 provides a schematic view of the described FSC for broiler chickens.

4 Notations

The following notations are applied throughout the paper to formulate the problem:

Parameters	
P_S	Supplier's unit selling price
C_P	Supplier's unit purchasing cost
C_B	Supplier's breeding (feeding, nourishment, and holding) cost per unit item
C_{OS}	Supplier's fixed ordering cost per cycle
C_H	Retailer's unit holding cost per unit time
C_{OR}	Retailer's fixed ordering cost per cycle
θ	Constant deterioration rate at the retailer side
ρ	Profit-sharing ratio
Variables	
$P_R(t)$	Retailer's unit selling price at time t (decision variable)
T_S	Inventory cycle at the supplier (breeding period) (decision variable)
T_R	Inventory cycle at the retailer (consumption period) (decision variable)
$I_S(t)$	Supplier's inventory level at time t
$I_R(t)$	Retailer's inventory level at time t
Q_0	Supplier's order quantity (units)
Q_R	Retailer's order quantity (units)
y	Number of growing items purchased at the beginning of a cycle (unit items)
$D(P_R)$	Price-dependent demand at the retailer
$\lambda(.)$	Fraction of discarded items during quality control
TP_S	Supplier's total profit per unit time
TP_R	Retailer's total profit per unit time
OP_S	Supplier's obtained profit per unit time under the centralized supply chain scenario
OP_R	Retailer's obtained profit per unit time under the centralized supply chain scenario
TP	Total profit of the chain per unit time

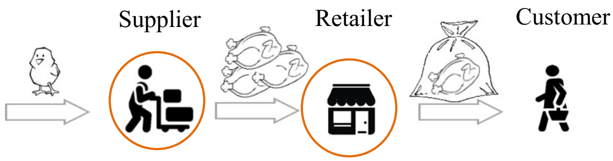

Fig. 1 The prescribed FSC

4.1 Assumptions

The assumptions that shape our model of the FSC are as follows:

1. A two-level FSC, including a rearing farm as the supplier of the growing items and a slaughtered items' retailer, is studied.
2. The planning horizon is infinite, and shortages are not admissible.
3. Replenishment at both sides is instantaneous with an infinite rate and negligible lead-time. However, non-zero lead-time (LT) does not impact the inventory level and total cost of the system. In this case, each order should be placed LT time units before the time that the inventory level reaches zero.
4. The growth starts to proceed as the items are effectively in stock and ends when the items are slaughtered at the supplier (Rezaei, 2014).
5. The deterioration occurs only at the retailer (during the consumption period). This is because it only affects slaughtered items.
6. The supplier applies an equal-size shipments policy, which is the most common ordering scheme in the literature, and delivers the slaughtered items to the retailer.
7. The growth is modeled as a biological weight increasing function (Richards, 1959), where the weight of a unit item at time t is formulated as $w_t = A(1 + be^{-lt})^{-1}$. A is the ultimate limiting value ($A > 0$), representing the maximum possible weight of the item. b is the integration constant, which is biologically unimportant since it reflects the choice of zero time ($b > 0$). l is a constant rate determining the growth curve's spread during the time axis ($0 < l < 1$). In this formulation, time is expressed in days. Since the time basis of our inventory model is in years, $k = 365.l$ is substituted to change the time basis. ($w_t = A(1 + be^{-kt})^{-1}$, t *in years*).
8. As the items grow, the ratio of useless weight (such as fat) to the whole weight increases (Jensen et al., 1974). The items might also lose quality standards due to illness and overbreeding. So, at the end of the breeding period, i.e., after slaughtering the items, quality control is performed, and a fraction of the inventory units are disposed. This process is assumed to be instantaneous.
9. While the items flourish in the supplier's system, their breeding costs increase. That is because the items' feeding costs rise as they grow in size and weight.
10. The demand rate is a function of the retailer's dynamic selling price as $D(P_R) = MB - \omega P_R(t)$. MB is the potential demand where the price is equal to zero, and $\omega > 0$ is the price sensitivity factor (Bernstein & Federgruen, 2003).

4.2 Mathematical formulation

The problem is investigated under decentralized and centralized supply chain scenarios. In the decentralized supply chain scenario, each entity optimizes its own inventory system.

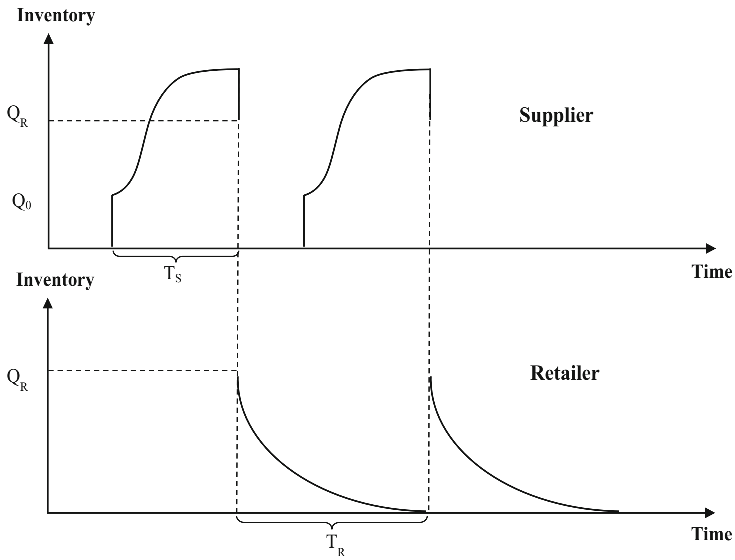

Fig. 2 The prescribed inventory system

Firstly, the retailer optimizes its profit by determining the appropriate retailing price and order quantity. Based on the ordered quantity, the supplier distinguishes its optimal breeding period and the initial ordering quantity of the newborn animals. In the centralized supply chain scenario, the whole supply chain is treated as a unified entity, and the optimal decisions of both sides are taken by considering the total profit of the chain. Figure 2 depicts the inventory system at the supplier and retailer sides. In subsequent subsections, we will first study the models of the retailer and supplier based on the provided description and then will proceed to explore the model under a centralized supply chain scenario.

4.2.1 The inventory model of the retailer

Dynamic pricing is a well-known strategy to mitigate the negative impact of deterioration. Three general approaches exist for dynamic pricing: Treating price as a control variable (Feng et al., 2015; Li et al., 2015), multi-period pricing (Chen et al., 2018; Dye et al., 2017), and dynamic discounting-markdown (Rabbani et al., 2014; Chen and Chen 2020). We incorporate the dynamic discounting scheme where the selling price is defined as a time-dependent function of the initial price and the discount rate. Accordingly, the dynamic price of the product at any time t is formulated as:

$$P_R(t) = P_0 e^{-\gamma t}, \quad 0 \le t \le T_R \tag{1}$$

where P_0 is the initial price of the product at the beginning of the inventory cycle, and $\gamma \in \Upsilon$, Υ={0.1,0.2,…,0.8,0.9} is the discounting rate.

During $[0, T_R]$ the inventory status is ruled by the following differential equation:

$$\frac{dI_R(t)}{dt} = -D(P_R) - \theta I_R(t), \quad 0 \le t \le T_R \tag{2}$$

With boundary condition $I_R(T_R) = 0$ solving Eq. (2) yields:

$$
\begin{aligned}
I_R(t) &= e^{-\theta t} \int_t^{T_R} D(P_R(u)) e^{\theta u} du \\
&= \frac{MB}{\theta}\left(e^{\theta(T_R - t)} - 1\right) - \frac{\omega P_0}{\theta - \gamma}\left(e^{(\theta-\gamma)T_R - \theta t} - e^{-\gamma t}\right), \quad 0 \le t \le T_R
\end{aligned} \tag{3}
$$

As such, the maximum inventory level is obtained as:

$$
I_R(0) = \frac{MB}{\theta}\left(e^{\theta T_R} - 1\right) - \frac{\omega P_0}{\theta - \gamma}\left(e^{(\theta-\gamma)T_R} - 1\right) \tag{4}
$$

Which is equal to the retailer's ordering quantity Q_R.

The total profit of the inventory system at the retailer comprises the following components:

$\mathbf{RE_R}$**:** The sales revenue

$$
\begin{aligned}
RE_R &= \int_0^{T_R} P_R(t) D(P_R) dt = \int_0^{T_R} P_0 e^{-\gamma t}\left(MB - \omega P_0 e^{-\gamma t}\right) dt \\
&= \frac{MBP_0}{\gamma}\left(1 - e^{-\gamma T_R}\right) + \frac{\omega P_0^2}{2\gamma}\left(e^{-2\gamma T_R} - 1\right)
\end{aligned} \tag{5}
$$

$\mathbf{PC_R}$**:** The purchasing cost

$$
PC_R = P_S Q_R = P_S \frac{MB}{\theta}\left(e^{\theta T_R} - 1\right) - P_S \frac{\omega P_0}{\theta - \gamma}\left(e^{(\theta-\gamma)T_R} - 1\right) \tag{6}
$$

$\mathbf{HC_R}$**:** The inventory holding cost

$$
\begin{aligned}
HC_R &= C_H \int_0^{T_R} I_R(t) dt = C_H \int_0^{T_R} \frac{MB}{\theta}\left(e^{\theta(T_R - t)} - 1\right) - \frac{\omega P_0}{\theta - \gamma}\left(e^{(\theta-\gamma)T_R - \theta t} - e^{-\gamma t}\right) dt \\
&= C_H \frac{MB}{\theta^2}\left(e^{\theta T_R} - \theta T_R - 1\right) + C_H \frac{\omega P_0}{\theta\gamma(\theta - \gamma)}\left((\gamma - \theta) e^{-\gamma T_R} - \gamma e^{(\theta-\gamma)T_R} + \theta\right)
\end{aligned} \tag{7}
$$

$\mathbf{OC_R}$**:** The ordering cost

$$
OC_R = C_{OR} \tag{8}
$$

Accordingly, the total profit per unit time of the retailer's inventory system is outlined as:

$$
TP_R = \frac{RE_R - PC_R - HC_R - OC_R}{T_R} \tag{9}
$$

4.2.2 The inventory model of the supplier

Suppose y unit items are ordered by the supplier at time zero. Since the weight of each unit item is $w_t = A(1 + be^{-kt})^{-1}$ at time t, the inventory level during $t \in [0, T_S)$ is illustrated by:

$$
I_S(t) = y w_t = yA(1 + be^{-kt})^{-1}, \quad 0 \le t < T_S \tag{10}
$$

Then, the initial inventory level (Supplier's ordering quantity) is:

$$
Q_0 = I_S(0) = yA(1 + b)^{-1} \tag{11}
$$

As y shows the number of newborn animals, it should take integer values, transforming the model into an integer non-linear programming. To overcome this intricacy, the equations could be reformulated to use Q_0 instead of y. Equation (11) gives $y = \frac{Q_0(1+b)}{A}$. So, Eq. (10) would be rewritten as:

$$I_S(t) = \frac{Q_0(1+b)}{\left(1+be^{-kt}\right)}, \quad 0 \le t < T_S \tag{12}$$

Recall that a fraction of inventory loses its quality during the breeding period, which is unfolded by quality control of items at the point T_S. Vividly, this fraction should be an increasing function of T_S. It should hold two more properties. First, in time zero, this fraction is negligible (i.e., $\lambda(0) = 0$). Second, as the breeding period takes very large values, this approaches one (i.e. $\lim\limits_{T_S \to \infty} \lambda(T_s) = 1$). The following function holds these features:

$$\lambda(T_S) = 1 - e^{-\alpha T_S}, \quad \alpha > 0 \tag{13}$$

The inventory level before inspection at the time T_S is depicted by $I'_S(T_S) = \frac{Q_0(1+b)}{\left(1+be^{-kT_S}\right)}$, then the disposal quantity after quality control would be expressed as:

$$(T_S)I'_S(T_S) = \left(1 - e^{-\alpha T_S}\right)\frac{Q_0(1+b)}{\left(1+be^{-kT_S}\right)} \tag{14}$$

Subsequently, the inventory level of inspected and useable items is outlined by:

$$I_S(T_S) = (1-\lambda(T_S))I_S^{'}(T_S) = (1-\lambda(T_S))\frac{Q_0(1+b)}{\left(1+be^{-kT_S}\right)} = Q_0\frac{e^{-\alpha T_S}(1+b)}{\left(1+be^{-kT_S}\right)} \tag{15}$$

This is the inventory quantity that enters the retailer's system. This inventory level $I_S(T_S)$ should match the retailer's ordering quantity Q_R i.e.

$$Q_0\frac{e^{-\alpha T_S}(1+b)}{\left(1+be^{-kT_S}\right)} = \frac{MB}{\theta}\left(e^{\theta T_R} - 1\right) - \frac{\omega P_0}{\theta-\gamma}\left(e^{(\theta-\gamma)T_R} - 1\right) \tag{16}$$

Using this equation, Q_0 can be expressed as a function of T_S and T_R:

$$Q_0 = \frac{MB}{\theta(1+b)}(1+be^{(-kT_S)})e^{\alpha T_S}(e^{\theta T_R} - 1) - \frac{\omega P_0}{(\theta-\gamma)(1+b)}(1+be^{(-kT_S)})e^{\alpha T_S}\left(e^{(\theta-\gamma)T_R} - 1\right) \tag{17}$$

The total profit of the inventory system at the supplier embodies the pursuant components:
$\mathbf{RE_S}$: The sales revenue

$$RE_S = P_S Q_R = P_S\frac{MB}{\theta}\left(e^{\theta T_R} - 1\right) - P_S\frac{\omega P_0}{\theta-\gamma}\left(e^{(\theta-\gamma)T_R} - 1\right) \tag{18}$$

$\mathbf{PC_S}$: The purchasing cost

$$\begin{aligned} PC_S = C_P Q_0 = C_P\frac{MB}{\theta(1+b)}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\theta T_R} - 1\right) \\ - C_P\frac{\omega P_0}{(\theta-\gamma)(1+b)}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{(\theta-\gamma)T_R} - 1\right) \end{aligned} \tag{19}$$

$\mathbf{BC_S}$: The breeding cost.

As mentioned, breeding costs rise as the items grow. The impact of this cost increase is regarded by a time-dependent function $B(t)$. There are several functions for $B(t)$ in existing literature, where polynomial and exponential functions are the most commonly applied ones (Goliomytis et al., 2003). In this paper, we use the exponential function ($B(t) = e^{\beta t}$, $\beta > 0$).

$$
\begin{aligned}
BC_S &= C_B y \int_0^{T_S} B(t)dt = C_B \frac{Q_0(1+b)}{A} \int_0^{T_S} e^{\beta t} dt \\
&= C_B \frac{MB}{\theta A \beta}\left(1 + be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\beta T_S} - 1\right)\left(e^{\theta T_R} - 1\right) \\
&\quad - C_B \frac{\omega P_0}{(\theta - \gamma)A\beta}\left(1 + be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\beta T_S} - 1\right)\left(e^{(\theta-\gamma)T_R} - 1\right)
\end{aligned} \tag{20}
$$

$\mathbf{OC_S}$: The ordering cost

$$OC_S = C_{OS} \tag{21}$$

As Fig. 2 depicts, the supplier's inventory cycle is repeated every T_R units of time. Accordingly, the total profit per unit time of the supplier's inventory system is obtained as:

$$TP_S = \frac{RE_S - PC_S - BC_S - OC_S}{T_R} \tag{22}$$

4.2.3 The centralized supply chain scenario

In the centralized supply chain scenario, the supply chain is considered one unit, and the decisions are made simultaneously for each of the chain echelons such that the total profit of the chain is optimized. Therefore:

$$TP = TP_R + TP_S \tag{23}$$

Although the cumulative profit through the chain increases in this case, one of the echelons might experience lower profit than the decentralized supply chain scenario. Then, this echelon will be reluctant to enter the centralized optimization structure. Profit-sharing is an effective coordination mechanism to convince it (Cachon, 2003). In this regard, the total profit of the chain is apportioned according to the profit-sharing ratio. The ratio can be specified based on the profit contribution of each level in the decentralized scenario. Therefore, the obtained profits per unit time by each of the SC echelons are equal to:

$$OP_S = \rho TP = \frac{TP_S}{TP_S + TP_R} TP \tag{24}$$

$$OP_R = (1-\rho)TP = \frac{TP_R}{TP_S + TP_R} TP \tag{25}$$

5 Solution approach

5.1 The inventory model of the retailer

For a fixed discount rate, the necessary conditions for TP_R to be optimal, are $\frac{\partial TP_R}{\partial T_R} = 0$ and $\frac{\partial TP_R}{\partial P_0} = 0$. Besides, it should be demonstrated that these equations give unique optimal solutions. Due to the complexity of the formulations, the concavity of the profit function cannot be demonstrated using the Hessian matrix, which is why we use an approach developed by Pentico and Drake (2009).

Lemma 1 *For fixed γ and known T_R, there exists a unique value $P_0{}^*$ which maximizes TP_R where* $\frac{\partial TP_R}{\partial P_0}\Big|_{P_0=P_0^*}=0$.

Proof The first-order optimality condition for the initial selling price gives:

$$\frac{\partial TP_R}{\partial P_0}=\frac{1}{T_R}\left[\begin{array}{l}\frac{MB}{\gamma}\left(1-e^{-\gamma T_R}\right)+\frac{\omega P_0}{\gamma}\left(e^{-2\gamma T_R}-1\right)+\frac{P_S\omega}{\theta-\gamma}\left(e^{(\theta-\gamma)T_R}-1\right)\\ -\frac{C_H\omega}{\theta\gamma(\theta-\gamma)}\left((\gamma-\theta)e^{-\gamma T_R}-\gamma e^{(\theta-\gamma)T_R}+\theta\right)\end{array}\right] \tag{26}$$

After some algebra, Eq. (26) can be rewritten as:

$$P_0=\frac{MB}{\omega}\frac{\left(1-e^{-\gamma T_R}\right)}{\left(1-e^{-2\gamma T_R}\right)}+\frac{P_S\gamma}{\theta-\gamma}\frac{\left(e^{(\theta-\gamma)T_R}-1\right)}{\left(1-e^{-2\gamma T_R}\right)}-\frac{C_H}{\theta(\theta-\gamma)}\frac{(\gamma-\theta)e^{-\gamma T_R}-\gamma e^{(\theta-\gamma)T_R}+\theta}{\left(1-e^{-2\gamma T_R}\right)} \tag{27}$$

To show that Eq. (27) gives the unique optimal value when γ and T_R are treated as fixed values, it is enough to demonstrate TP_R is concave with respect to P_0, which is:

$$\frac{\partial^2 TP_R}{\partial P_0^2}=\frac{\omega}{\gamma T_R}\left(e^{-2\gamma T_R}-1\right)\leq 0,\quad \forall 0\leq T_R<\infty,\gamma\in\Upsilon \tag{28}$$

Therefore, Lemma 1 is proven. □

Lemma 2 *For fixed γ and known P_0, there exists a unique value $T_R{}^*$ which maximizes TP_R* where $\frac{\partial TP_R}{\partial T_R}\Big|_{T_R=T_R{}^*}=0$.

Proof The first-order optimality condition for the consumption period gives:

$$\frac{\partial TP_R}{\partial T_R}=\frac{\left(RE_R'-PC_R'-HC_R'-OC_R'\right)T_R-\left(RE_R-PC_R-HC_R-OC_R\right)}{T_R^2}=0 \tag{29}$$

where:

$$RE_R'=MBP_0e^{-\gamma T_R}-\omega P_0^2e^{-2\gamma T_R} \tag{30}$$

$$PC_R'=P_SMBe^{\theta T_R}-P_S\omega P_0e^{(\theta-\gamma)T_R} \tag{31}$$

$$HC_R'=C_H\frac{MB}{\theta}\left(e^{\theta T_R}-1\right)+C_H\frac{\omega P_0}{\theta}\left(e^{-\gamma T_R}-e^{(\theta-\gamma)T_R}\right) \tag{32}$$

$$OC_R'=0 \tag{33}$$

Motivated by Eq. (29), the auxiliary function $F(T_R)$ is defined as:

$$F(T_R)=\left(RE_R'-PC_R'-HC_R'-OC_R'\right)T_R-\left(RE_R-PC_R-HC_R-OC_R\right) \tag{34}$$

Since $\frac{\partial TP_R}{\partial T_R}=0$ and $F(T_R)=0$ are equivalent, it is enough to demonstrate that $F(T_R)=0$ gives a unique solution. Then:

$$\frac{dF(T_R)}{dT_R}=\left(RE_R''-PC_R''-HC_R''\right)T_R \tag{35}$$

where

$$RE_R''=-MBP_0\gamma e^{-\gamma T_R}+2\omega P_0^2\gamma e^{-2\gamma T_R} \tag{36}$$

$$RE_R'' = -MBP_0\gamma e^{-\gamma T_R} + 2\omega P_0^2\gamma e^{-2\gamma T_R} \tag{37}$$

$$HC_R'' = C_H MBe^{\theta T_R} - C_H\frac{\omega P_0}{\theta}\left(\gamma e^{-\gamma T_R} + (\theta-\gamma)e^{(\theta-\gamma)T_R}\right) \tag{38}$$

In the worst condition if $P_0 \le \frac{MB}{2\omega}$ holds, we will have $\frac{dF(T_R)}{dT_R} < 0, \forall 0 < T_R < \infty$. *Consequently, under this condition, for any* $0 < T_R < \infty$, *$F(T_R)$is strictly decreasing function of T_R. Besides, $\lim_{T_R\to 0} F(T_R) = C_{OR} > 0$and $\lim_{T_R\to\infty} F(T_R) = -\infty < 0$. Thus, for a fixed discount ratio and known initial selling price, there exists a unique value of $T_R$for which* $F(T_R) = 0$. As $\frac{\partial TP_R}{\partial T_R} = \frac{F(T_R)}{T_R^2}$, *at the point $T_R = T_R{}^*$:*

$$\frac{\partial^2 TP_R}{\partial T_R^2}\Big|_{T_R=T_R^*} = \frac{F'T_R - 2F}{T_R^3}\Big|_{T_R=T_R^*} \frac{F'}{T_R^{*2}} < 0 \tag{39}$$

Hence Lemma 2 is proven. □

Equation (27) should be substituted into Eq. (29) to obtain the value of $T_R{}^*$. The equation does not represent a closed-form formula for $T_R{}^*$. Then, a numerical root-finding method, such as Newton–Raphson, should be applied. In sum, the solution procedure is depicted as follows:

Algorithm

Step 0- *i = 1,* $\gamma^i = 0.1$
Step 1- *If* $\gamma \le 0.9$, *go to step 2, otherwise go to step 6*
Step 2- *Substitute* Eq. (27) *into* Eq. (29) *and apply a numerical root-finding approach to solve* Eq. (29) *and obtain* T_R^{i*}.
Step 3- *Obtain the optimal initial selling price* (P_0^{i*}) *applying* Eq. (27) *where* $T_R = T_R^{i*}$.
Step 4- *Calculate* $TP_R^i\left(T_R^{i*}, P_0^{i*}\right)$ *by* Eq. (9).
Step 5- $i = i+1, \gamma^i = \gamma^{i-1} + 0.1$ *and go to step 1.*
Step 6- $i^* = \mathrm{argmax}_i(TP_R^i)$, $TP_R{}^* = TP_R{}^{i^*}$, $\left(T_R{}^*, P_0{}^*, \gamma^*\right) = \left(T_R^{i**}, P_0^{i**}, \gamma^{i**}\right)$.
Step 7- *End*.

5.2 The inventory model of the supplier

The necessary condition for TP_S to reach its optimal value is $\frac{dTP_S}{dT_S} = 0$. On the other hand, we need to demonstrate that this equation provides a unique optimal value for T_S. The first-order optimality condition gives:

$$\begin{aligned}\frac{dTP_S}{dT_S} &= \frac{C_P}{T_R}\left[\frac{MB}{\theta(1+b)}\left(e^{\theta T_R}-1\right) - \frac{\omega P_0}{(\theta-\gamma)(1+b)}\left(e^{(\theta-\gamma)T_R}-1\right)\right]e^{\alpha T_S}\left[\alpha\left(1+be^{-kT_S}\right) - bke^{-kT_s}\right] \\ &+ \frac{C_B}{T_R}\left[\begin{matrix}\frac{MB}{\theta A\beta}\left(e^{\theta T_R}-1\right) \\ -\frac{\omega P_0}{(\theta-\gamma)A\beta}\left(e^{(\theta-\gamma)T_R}-1\right)\end{matrix}\right]e^{\alpha T_S}\left[\begin{matrix}(\alpha+\beta)^2 e^{\beta T_s} + b(k-\alpha)(\alpha-k)e^{-kT_S} \\ +b(\alpha-k+\beta)^2 e^{-kT_S}e^{\beta T_s} - \alpha^2\end{matrix}\right]\end{aligned} \tag{40}$$

Then

$$C_P A\beta\left[\alpha + b(\alpha-k)e^{-kT_S}\right] + C_B(1+b)\left[\left(\alpha + b(\alpha-k)e^{-kT_S}\right)\left(e^{\beta T_S}-1\right)\right] = 0 \tag{41}$$

To illustrate that Eq. (41) provides a unique optimal solution, it is enough to show TP_S is concave:

$$\frac{d^2TP_S}{dT_S^2}=\frac{C_P}{T_R}\left[\frac{MB}{\theta(1+b)}\left(e^{\theta T_R}-1\right)-\frac{\omega P_0}{(\theta-\gamma)(1+b)}\left(e^{(\theta-\gamma)T_R}-1\right)\right]e^{\alpha T_S}\begin{bmatrix}\alpha^2+b\alpha(\alpha-k)e^{-kT_S}\\-bk(\alpha-k)e^{-kT_S}\end{bmatrix}$$
$$+\frac{C_B}{T_R}\begin{bmatrix}\frac{MB}{\theta A\beta}\left(e^{\theta T_R}-1\right)\\-\frac{\omega P_0}{(\theta-\gamma)A\beta}\left(e^{(\theta-\gamma)T_R}-1\right)\end{bmatrix}e^{\alpha T_S}\begin{bmatrix}(\alpha+\beta)^2e^{\beta T_S}+b(k-\alpha)(\alpha-k)e^{-kT_S}\\+b(\alpha-k+\beta)^2e^{-kT_S}e^{\beta T_S}-\alpha^2\end{bmatrix} \tag{42}$$

To guarantee the concavity of TP_S, Eq. (42) should be non-positive. It can be shown if $(2\alpha+\beta)(1+b)\geq 2bk$, this is met. Accordingly, Eq. (42) provides a unique optimal value for T_S. Again, Eq. (42) does not provide a closed-form formula for the optimal breeding period, and a numerical root-finding method should be used to solve this equation.

5.3 The centralized supply chain scenario

The necessary conditions for TP to be optimal are $\frac{\partial TP}{\partial T_S}=0$, $\frac{\partial TP}{\partial T_R}=0$ and $\frac{\partial TP}{\partial P_0}=0$. Again, the uniqueness of the optimal solutions is authenticated through the procedure which was provided by Pentico and Drake (2009). Optimal solution of T_S is obtained from Eq. (42), and the sufficient optimality condition of this decision variable is identical to the decentralized case. On the other hand, for P_0 and T_R we have the following Lemmas.

Lemma 3 *For fixed γ and known T_S and T_R, there exists a unique value ${P_0}^*$ which maximizes TP where $\frac{\partial TP}{\partial T_R}\Big|_{P_0={P_0}^*}=0$.*

Proof The first-order optimality condition for the selling price gives:

$$\frac{\partial TP}{\partial P_0}=\frac{1}{T_R}\begin{bmatrix}\frac{MB}{\gamma}\left(1-e^{-\gamma T_R}\right)+\frac{\omega P_0}{\gamma}\left(e^{-2\gamma T_R}-1\right)\\-\frac{C_H\omega}{\theta\gamma(\theta-\gamma)}\left((\gamma-\theta)e^{-\gamma T_R}-\gamma e^{(\theta-\gamma)T_R}+\theta\right)\\+C_P\frac{\omega}{(\theta-\gamma)(1+b)}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{(\theta-\gamma)T_R}-1\right)\\+C_B\frac{\omega}{(\theta-\gamma)A\beta}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\beta T_S}-1\right)\left(e^{(\theta-\gamma)T_R}-1\right)\end{bmatrix}=0 \tag{43}$$

After some algebra, Eq. (43) can be rewritten as:

$$P_0=\frac{MB}{\omega}\frac{1-e^{-\gamma T_R}}{1-e^{-2\gamma T_R}}-\frac{C_H}{\theta(\theta-\gamma)}\frac{(\gamma-\theta)e^{-\gamma T_R}-\gamma e^{(\theta-\gamma)T_R}+\theta}{1-e^{-2\gamma T_R}}$$
$$+\frac{\gamma}{(\theta-\gamma)}\frac{e^{(\theta-\gamma)T_R}-1}{1-e^{-2\gamma T_R}}\left[\frac{C_P}{(1+b)}+\frac{C_B}{A\beta}\left(e^{\beta T_S}-1\right)\right]\left(1+be^{-kT_S}\right)e^{\alpha T_S} \tag{44}$$

To show that Eq. (44) gives the unique optimal value when γ, and T_R are treated as fixed values, it is enough to demonstrate TP_R is concave with respect to P_0, which is:

$$\frac{\partial^2 TP}{\partial^2 P_0}=\frac{\omega}{\gamma T_R}(e^{-2\gamma T_R}-1)\leq 0,\quad \forall 0\leq T_R<\infty,\ \gamma\in\Upsilon \tag{45}$$

Hence Lemma 3 is proven. □

Lemma 4 *For fixed γ and known P_0 and T_S, there exists a unique value ${T_R}^*$ which maximizes TP where $\frac{\partial TP}{\partial T_R}\Big|_{T_R={T_R}^*}=0$.*

Proof The first-order optimality condition for the consumption period gives:

$$\frac{\partial TP}{\partial T_R}=\frac{\left(RE'_R-HC'_R-OC'_R-PC'_S-BC'_S-OC'_S\right)T_R-(RE_R-HC_R-OC_R-PC_S-BC_S-OC_S)}{T_R^2}=0 \tag{46}$$

where

$$RE'_R=MBP_0e^{-\gamma T_R}-\omega P_0^2e^{-2\gamma T_R} \tag{47}$$

$$HC'_R=C_H\frac{MB}{\theta}\left(e^{\theta T_R}-1\right)+C_H\frac{\omega P_0}{\theta}\left(-e^{-\gamma T_R}-e^{(\theta-\gamma)T_R}\right) \tag{48}$$

$$OC'_R=0 \tag{49}$$

$$PC'_S=C_P\frac{MB}{(1+b)}\left(1+be^{-kT_S}\right)e^{\alpha T_S}e^{\theta T_R}-C_P\frac{\omega P_0}{(1+b)}\left(1+be^{-kT_S}\right)e^{\alpha T_S}e^{(\theta-\gamma)T_R} \tag{50}$$

$$\begin{aligned}BC'_S=&\,C_B\frac{MB}{A\beta}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\beta T_S}-1\right)e^{\theta T_R}\\&-C_B\frac{\omega P_0}{A\beta}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\beta T_S}-1\right)e^{(\theta-\gamma)T_R}\end{aligned} \tag{51}$$

$$OC'_S=0 \tag{52}$$

Motivated by Eq. (46), the auxiliary function $F(T_R)$ is defined as:

$$\begin{aligned}F(T_R)=&\left(RE'_R-HC'_R-PC'_S-BC'_S\right)T_R\\&-(RE_R-HC_R-OC_R-PC_S-BC_S-OC_S)\end{aligned} \tag{53}$$

Since $\frac{\partial TP_R}{\partial T_R}=0$ and $F(T_R)=0$ are equivalent, it is enough to demonstrate that $F(T_R)=0$ gives a unique solution. Then:

$$\frac{dF(T_R)}{dT_R}=\left(RE''_R-HC''_R-PC''_S-BC''_S\right)T_R \tag{54}$$

where

$$RE''_R=-MBP_0\gamma e^{-\gamma T_R}+2\omega P_0^2\gamma e^{-2\gamma T_R} \tag{55}$$

$$HC''_R=C_HMBe^{\theta T_R}-C_H\frac{\omega P_0}{\theta}\left(\gamma e^{-\gamma T_R}+(\theta-\gamma)e^{(\theta-\gamma)T_R}\right) \tag{56}$$

$$PC''_S=C_P\frac{MB\theta}{(1+b)}\left(1+be^{-kT_S}\right)e^{\alpha T_S}e^{\theta T_R}-C_P\frac{\omega P_0(\theta-\gamma)}{(1+b)}\left(1+be^{-kT_S}\right)e^{\alpha T_S}e^{(\theta-\gamma)T_R} \tag{57}$$

$$\begin{aligned}BC''_S=&\,C_B\frac{MB\theta}{A\beta}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\beta T_S}-1\right)e^{\theta T_R}\\&-C_B\frac{\omega P_0(\theta-\gamma)}{A\beta}\left(1+be^{-kT_S}\right)e^{\alpha T_S}\left(e^{\beta T_S}-1\right)e^{(\theta-\gamma)T_R}\end{aligned} \tag{58}$$

In the worst condition if $P_0\le\frac{MB}{2\omega}$ holds, we will have $\frac{dF(T_R)}{dT_R}<0,\forall 0<T_R<\infty$. *Consequently, under this condition, for any* $0<T_R<\infty$, $F(T_R)$*is strictly decreasing function of* T_R. *Besides,* $\lim_{T_R\to 0}F(T_R)=C_{OR}+C_{OS}>0$*and* $\lim_{T_R\to\infty}F(T_R)=-\infty<0$. *Thus,* *for a fixed discount ratio and known selling price, there exists a unique value of $T_R$for which* $F(T_R) = 0$. As $\frac{\partial TP}{\partial T_R} = \frac{F(T_R)}{T_R^2}$*, at the point $T_R = T_R{}^*$:*

$$\frac{\partial^2 TP}{\partial T_R^2}\,|_{T_R=T_R^*} = \frac{F' T_R - 2F}{T_R^3}\,|_{T_R=T_R^*} \frac{F'}{T_R^{*2}} < 0 \tag{59}$$

Accordingly, Lemma 4 is proven. □

Equation (44) should be substituted into Eq. (46) to obtain the value of $T_R{}^*$. The equation does not represent a closed-form formula for $T_R{}^*$. Then, a numerical root-finding method should be applied. In sum, the solution procedure is depicted as follows:

Algorithm

Step 0- $i = 1$, $\gamma^i = 0.1$.
Step 1- *If* $\gamma \leq 0.9$, *go to step 2, otherwise go to step 7.*
Step 2- *Substitute Eq. (44) into Eq. (46)and apply a numerical root-finding approach to solve Eq. (46)and obtain* $T_R^{i\,*}$.
Step 3- *Obtain the optimal initial selling price* ($P_0^{i\,*}$) *applying Eq. (44)where* $T_R = T_R^{i\,*}$.
Step 4- *Obtain optimal breeding period* ($T_S^{i\,*}$) *applying Eq. (41)where* $P_0 = P_0^{i\,*}$ *and* $T_R = T_R^{i\,*}$.
Step 5- *Calculate* $TP^i\left(T_S^{i\,*}, T_R^{i\,*}, P_0^{i\,*}\right)$ *by* Eq. (23).
Step 6- $i = i + 1, \gamma^i = \gamma^{i-1} + 0.1$ *and go to step 1.*
Step 7- $i^* = \underset{i}{\operatorname{arg}max}(TP^i)$, $TP^* = TP^{i^*}$, $\left(T_S{}^*, T_R{}^*, P_0{}^*, \gamma^*\right) = \left(T_S^{i^{**}}, T_R^{i^{**}}, P_0^{i^{**}}, \gamma^{i^{**}}\right)$.
Step 8- *End.*

6 Results and analysis

The first step in applying the proposed model and the developed solution approach is to specify the items' weight function, which is dependent on the type of growing inventory. Richards (1959) showed that his suggested weight function could efficiently depict the growth of poultry. Accordingly, we carry out our analysis on a specific type of poultry known as "Broiler chickens" that are the main component of various FSCs. MATLAB R2019b is applied to solve the closed-form and non-closed-form equations of the problem, and the experiments are carried out on a computer with Intel® Core i7-8650U CPU 1.9 GHz, 2.11 GHz, and 7.88 GB memory available. Function approximation is applied to estimate the parameters of the weight function. Precisely, a data set including the weights of the broiler chickens during their lifetime in an industrial rearing farm located in the southeast of the Netherlands is used as the training input data in a neural network-based function approximation approach that provides the estimated weight function as follows:

$A = 3200$, $b = 69.4$ and $g = 0.12$, $k = 0.12 * 365 = 43.8$.

Then, the growth function is $w_t = 3200(1 + 69.4e^{-43.8t})^{-1}$. Moreover, the exponential breeding function $B(t)$ is ruled by: $B(t) = e^{86t}$.

Other identical parameters are taken from Rezaei (2014) and adapted to our model:

Supplier side: $C_P = 0.005$ €/gr, $C_B = 0.02$ €/unit item, $C_{OS} = 5000$ €/cycle, $P_S = 0.006$ €/gr and $\alpha = 1$

Retailer side: $C_H = 0.001$ €/gr/year, $C_{OR} = 400$ €/cycle, $\theta = 0.25$, $MB = 100 \times 10^6$ gr/year and $\omega = 300 \times 10^7$ €/gr.

As the solution approach is independent of the input setting, the average CPU time to solve the problem is 8–11 s in different problem instances and scenarios. Solving the outlined problem provides the following solutions:

6.1 Decentralized supply chain scenario

Supplier:	$T_S = 0.08176$ year, $Q_0 = 175703.82$ gr, $TP_S = 139543.69$€
Retailer:	$T_R = 0.12135$ year, $P_0 = 0.011479$ €/gr, $\gamma = 0.1$, $Q_R = 3886469.81$ g
	$TP_R = 160823.14$€

This indicates that the retailer orders 3886 kg of slaughtered inventory at the beginning of each cycle. On the other hand, based on the retailer's order quantity, the supplier buys 3912 newborn chickens at the beginning of the breeding period. These items are flourished during $T_S = 30$ days. So the final weight of each growing item reaches 1.091 kg. The items are slaughtered, and 7.85% of the inventory is disposed of after quality control as a useless portion. At the retailer, 2.5% of the inventory is deteriorated, which is equal to 95 kg of food waste. Therefore, the total food waste during the processes of our FSC is 399.930 kg.

6.2 Centralized supply chain scenario

Supplier:	$T_S = 0.08176$ year, $Q_0 = 1012086.59$ gr, $OP_S = 172069.27$€
Retailer:	$T_R = 0.43923$ year, $P_0 = 0.008824$ €/gr, $\gamma = 0.1$, $Q_R = 22386787.64$ g
	$OP_R = 201994.36$€

In comparison to the decentralized case, the total profit of the chain increases under the centralized scenario by 24.5%, which is highly desirable. The supplier and retailer experience 23.3% and 25.6% increases in their profit. This shows that our proposed profit-sharing mechanism efficiently takes individual rationality into account, which is the main barrier in supply chain coordination. Furthermore, the initial selling price of the retailer is considerably decreased. This suggests that the centralized supply chain scenario not only brings benefits to the systems of the supplier and retailer but also is desirable for the customers as they pay less for the same product.

6.3 Comparison with usual practice

In the EU (and some other regions), the average slaughter age of broiler chickens is 40–42 days (Mebratie et al., 2018), while our research suggests 30 days for the breeding period. Solving the problem of the supplier under known T_S=42 days yields the following results:

Supplier	$T_S = 0.1151$year	$Q_0 = 89729.6493$gr	$TP_S = 120317.2209$ €

The supplier buys 1994 newborn broiler chickens at the beginning of each cycle and raises them for 42 days when the final weight of each unit item reaches 2.21 kg. Figure 3 outlines a comparison of these two cases.

By breeding the items for 31 days, the initial order size increases compared to the usual practice (42 days), which is due to the decrease in the breeding period. Our proposed scheme brings improvements in terms of enhanced profit and lower food wastes. More precisely, in comparison to the usual practice, 15.97% increase in profit and 27.79% decrease in food wastes (discarded inventory after quality control) are obtained.

It should be noted that this 30-day breeding period is not a one-size-fits-all policy. The optimal breeding period largely depends on the broiler chickens' growth pattern (outlined as the weight function), which can vary for different rearing farms and growth conditions. In particular, this is optimal for our data-set and the estimated weight function. This highlights the significance of applying the exact optimization method instead of empirical practice. So the first step in using our model is to estimate the weight function of the growing items accurately.

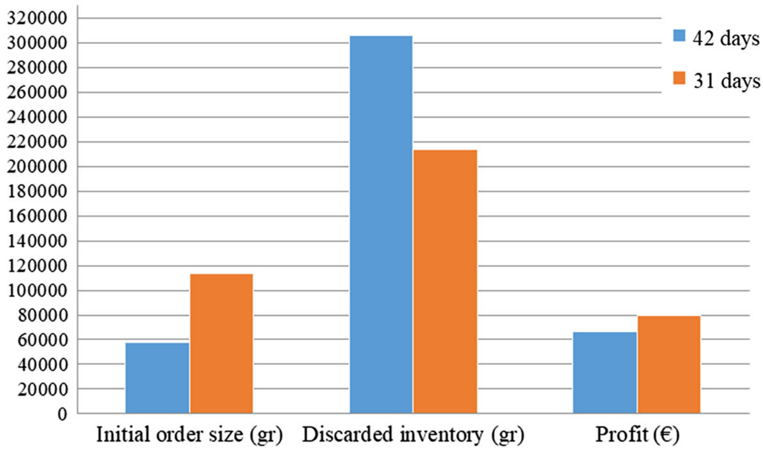

Fig. 3 Comparison of results between our approach and the usual practice

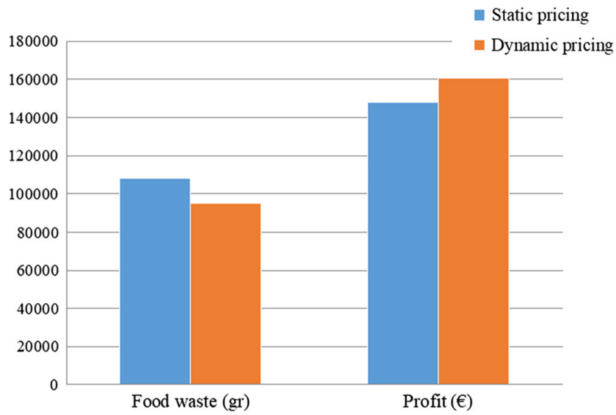

Fig. 4 Comparison of results between dynamic and static pricing

6.4 Comparison with static pricing

Dynamic pricing is a well-known revenue management policy in handling deteriorating items. To clarify its benefits in our problem, it is fruitful to compare our results with the case of static pricing:

Retailer: $T_R = 0.13494\ year$, $P_R = 0.011419$ €$/gr$, $Q_R = 4321452.93\ gr$, $TP_R = 147805.09$€.

The retailer orders 4321 kg of slaughtered meat at the beginning of each cycle to meet customer demand during its replenishment period. Figure 4 outlines a comparison of these two cases.

As Fig. 4 projects, dynamic pricing brings improvements in terms of enhanced profit and lower food wastes. More precisely, compared to the static pricing, 8.8% increase in profit and 12.03% decrease in food wastes (deteriorated inventory) are obtained.

6.5 The impact of profit-sharing ratio

A critical point in supply chain collaborations is to divide the benefits fairly among the involved partners. Individual rationality is a key principle, implying that the parties will be reluctant to collaborate if their benefits under collaboration are not as large as non-collaborative settings. Our proposed profit-sharing ratio, built based on the profits of the echelons under the decentralized scenario, takes this principle into account. In order to investigate this ratio in further detail, its value is changed by − 50%, − 25%, − 10%, + 10% + 25%, and + 50%, and the obtained profits under the decentralized and centralized scenarios are compared in Fig. 5.

Figure 5 shows that while individual rationality is not violated when the ratio remains in a close neighborhood of the proposed ratio, varying the profit-sharing ratio by larger magnitudes (e.g., − 50%, − 25%, + 25%, and + 50%) violates the individual rationality. Expressly, increasing the ratio by 25% and 50% results in a situation where the retailer's profit is lower than the decentralized scenario, and decreasing this value by the same amount brings profits lower than the decentralized scenario to the supplier. In either of these four settings, one of the two parties will be reluctant to enter the centralized scenario. This highlights

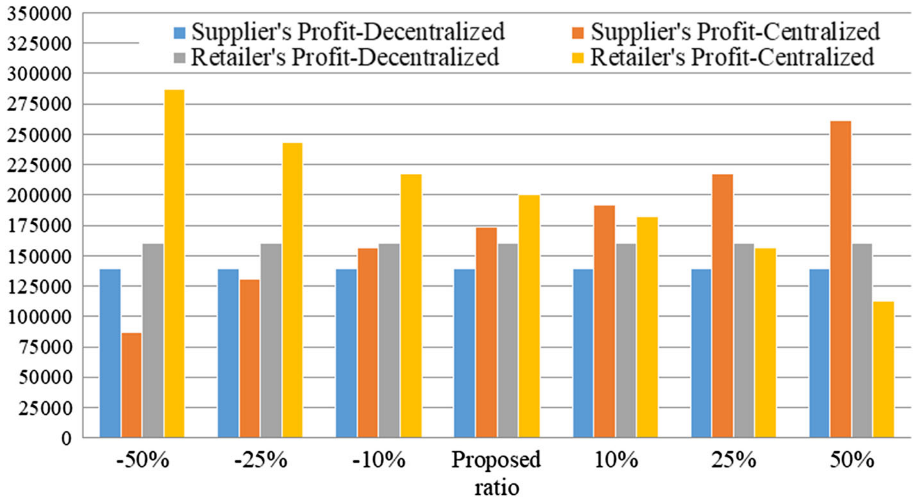

Fig. 5 The impact of different profit-sharing ratios

the importance of designing the appropriate profit-sharing contract in the success of the coordination mechanism.

6.6 Sensitivity analysis on cost parameters

Investigating the result of changes in input parameters provides us with a better understanding of the problem, which can be fruitful for the decision-makers. In this regards, the changes in the values of C_P, C_B and C_{OS} as well as P_S, C_H and C_{OR} are analyzed. The sensitivity analysis is carried out by changing each parameter by − 50%, − 25%, + 25%, and + 50%, taking one parameter at a time and keeping the others constant. The related results for the decentralized and centralized supply chain scenarios are provided in the "Appendix". Figures 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16 depict the results graphically.

Analyzing the numerical results provides the following managerial insights:

- Apparently, decreasing the unit purchasing cost of the rearing farm (C_P), enhances its profit. In the decentralized scenario, varying C_P has no impact on the retailer's inventory system. As the supplier's purchasing cost rises, the initial order size gets smaller values. Then, the breeding period needs to get longer to meet the retailer's ordering quantity.
  In the centralized scenario, the retailer's ordering size and inventory cycle are also affected. The obtained profits of the retailer and the supplier are computed according to the total profit of the chain as well as the profit-sharing ratio (ρ). So the pattern of changes in obtained profits (OP_S and OP_R) cannot be studied with respect to changes in C_P, isolatedly. That is because, as C_P changes, TP_S and TP_R also, take different values altering the profit-sharing ratio.
- Similarly, changes in the unit item breeding cost of the rearing farm (C_B) have no impact on the retailer's inventory system. By decreasing C_B, an increase in the breeding period is observed. That is because by lowering C_B, the breeding costs during the growth period decrease. So the system has the chance to lengthen the growth period. As the retailer's order size is fixed, there exists a reverse link between the newborn chickens' order size

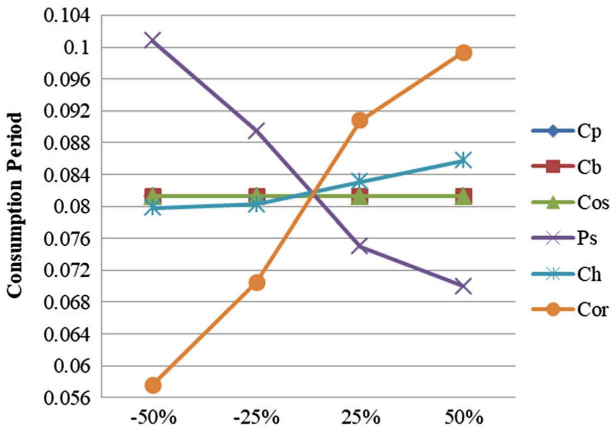

Fig. 6 Changes in the optimal T_S with variations in cost parameters-Decentralized scenario

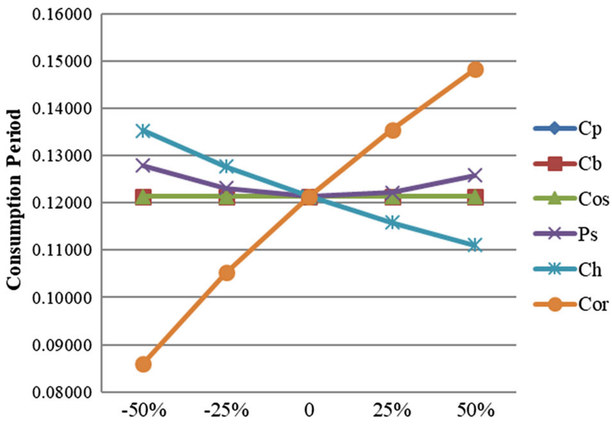

Fig. 7 Changes in the optimal T_R with variations in cost parameters-Decentralized scenario

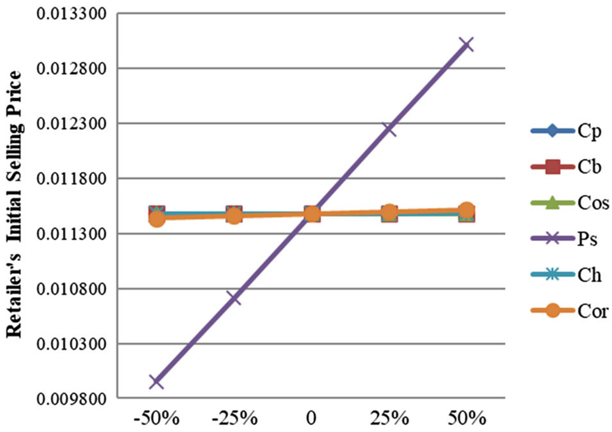

Fig. 8 Changes in the optimal P_0 with variations in cost parameters-Decentralized scenario

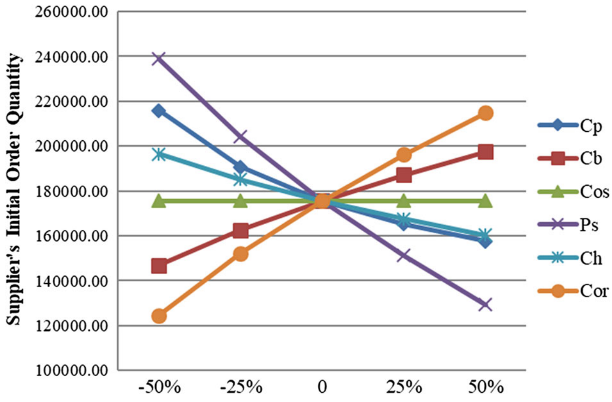

Fig. 9 Changes in the optimal Q_0 with variations in cost parameters-Decentralized scenario

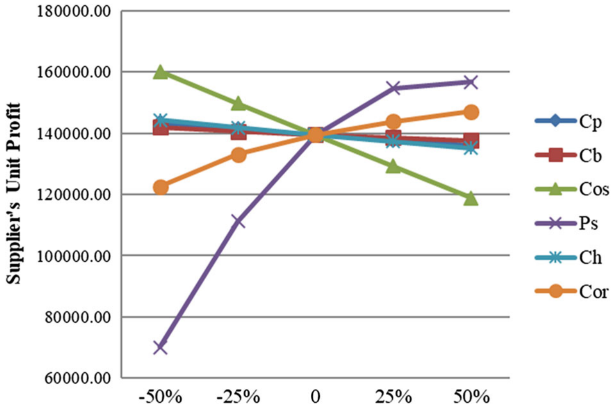

Fig. 10 Changes in the optimal TP_S with variations in cost parameters-Decentralized scenario

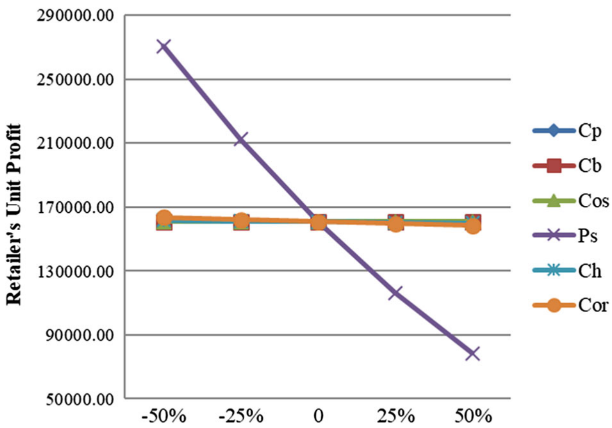

Fig. 11 Changes in the optimal TP_R with variations in cost parameters-Decentralized scenario

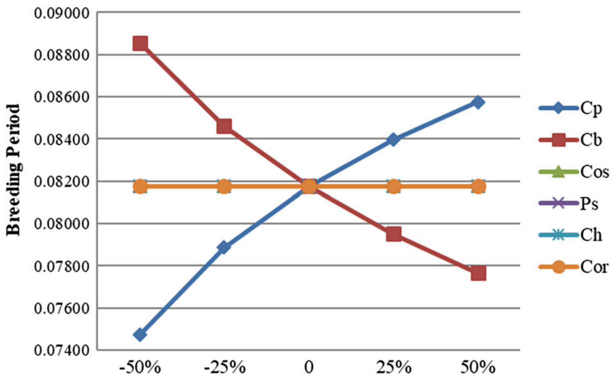

Fig. 12 Changes in the optimal T_S with variations in cost parameters-Centralized scenario

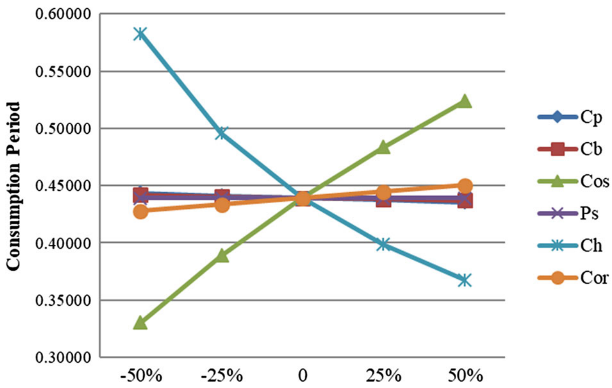

Fig. 13 Changes in the optimal T_R with variations in cost parameters-Centralized scenario

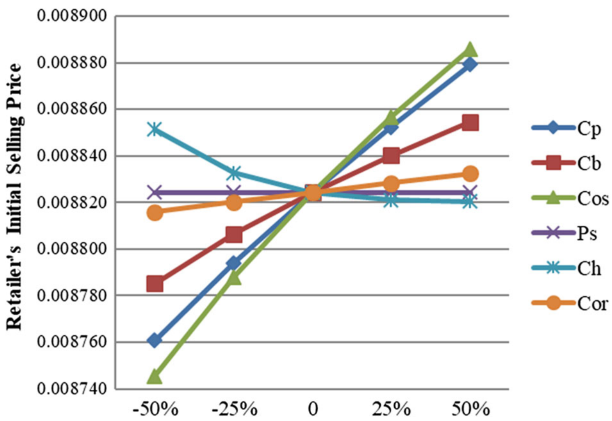

Fig. 14 Changes in the optimal P_0 with variations in cost parameters-Centralized scenario

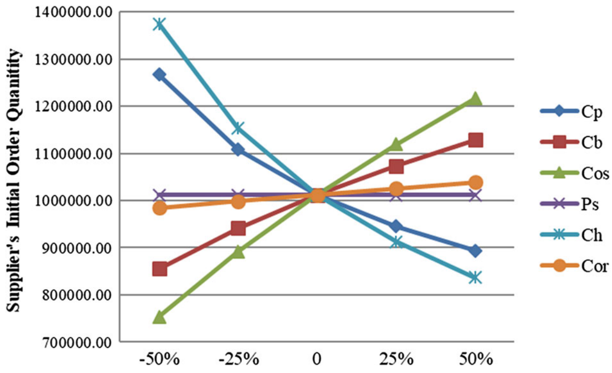

Fig. 15 Changes in the optimal Q_0 with variations in cost parameters-Centralized scenario

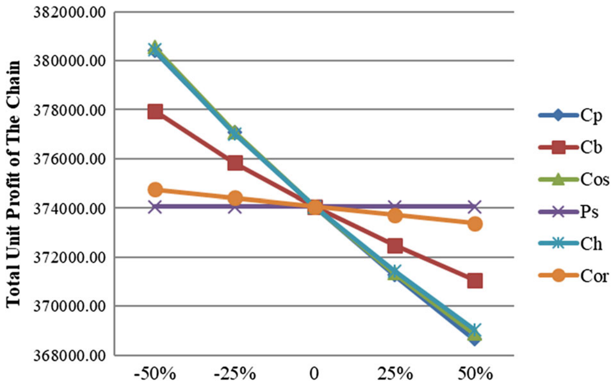

Fig. 16 Changes in the optimal TP with variations in cost parameters-Centralized scenario

and their breeding period. Thereby, the newborn chickens' order size gets smaller. This is in line with the pattern observed in Rezaei (2014).

In the centralized scenario, the retailer's ordering size and inventory cycle are also affected.

- Variating the supplier's ordering cost parameter (C_{OS}), not only doesn't impact the retailer's system but also has no effect on the supplier's decision variables. This might seem odd, but it is entirely rational: The supplier's inventory system recurs based on the retailers' inventory cycle (T_R) and thereby its ordering cost is paid every T_R units of time. So, the changes in C_{OS} will not influence its decision variables.

  In the centralized scenario, the patterns differ. The breeding period is independent of changes in C_{OS}. On the other hand, as C_{OS} takes larger values, the consumption period gets longer, leading to an increase in the retailer's ordering quantity.
- The supplier's unit selling price (P_S) is the retailer's unit purchasing cost. Accordingly, its variations lead to changes in both systems. As P_S takes larger values, the unit profit of the supplier rises, while the retailer faces a drop in its profit. By increasing P_S, the retailer has to raise its initial selling price leading to a reduction in demand. Then, the retailer's order quantity and inventory cycle get smaller values, and the decline in the supplier's

order quantity is expected. As Figs. 10 and 11 project, P_S is the most influential parameter on changes in TP_S and TP_R in decentralized supply chain scenario.
In the centralized case, no changes are observed as P_S is not present in the total profit of the chain.

- As in classical inventory models, by decreasing the retailer's holding cost, the inventory cycle gets longer, which leads to a rise in the retailer's order sizes. The breeding period is insensitive to these changes. Therefore, we observe the same percentage of increase in the ordering size of the initial young-born chickens.
The pattern of changes is similar in the centralized supply chain scenario.
- By increasing C_{OR} in either of the chain structures (centralized and decentralized), the retailer's ordering quantity and inventory cycle increase. This is what we expect in classical inventory models as well.

6.7 Sensitivity analysis on food waste parameters

After slaughtering the broiler chickens, a fraction of inventory that is distinguished as non-consumable is discarded. Moreover, the slaughtered items at the retailer side undergo quality degradation. The deteriorated items are disposed during the retailer's inventory cycle. Therefore, we face food wastes both at the supplier and retailer. The impact of these food wastes is studied by analyzing the changes in food waste parameters, including α and θ in Table 2.

As the results depict, the breeding period is slightly affected by the changes in α. The changes in the initial newborn order size are more intense than the breeding period. By decreasing α, smaller amount of inventory is discarded as non-consumable. Therefore, the initial order size can decrease for the same required final inventory level. The system of the retailer is insensitive to changes in α under the decentralized scenario. This is while we observe a slight change in different variables of the retailer under the centralized scenario.

The changes in the deterioration rate affect the supplier's system as well. The breeding period is insensitive to changes in the deterioration rate. By decreasing θ, the consumption period gets longer that results in larger order sizes. Since the breeding period is constant, the retailer's larger order size leads to a larger initial order size of newborn chicks. The retail price is almost insensitive to the changes in θ.

It is noteworthy that the impact of changes in θ is stronger than α. This shows that food wastes at the retailer are higher than the supplier. The other interesting point is that the centralized (compared to decentralized) supply chain scenario is more robust to the changes in food waste parameters, which is highly desirable as it decreases the risk of large losses facing unexpected conditions.

6.8 Managerial implications

It is widely accepted that the centralized supply chain scenario can enhance the chain members' operational efficiency. Our results support this claim as we observe an increase in the profit of both echelons. Furthermore, it is shown that the retailer's selling price is reduced under the centralized scenario, which suggests that this scheme is even more favorable for the customers as they pay less for the same product. Then, retailers can gain a competitive advantage in absorbing customers' demands by entering this setting.

It should be noted that the key challenge in the success of the centralized scenario is the profit-sharing mechanism that divides the outcomes of collaboration among involved

Table 2 Sensitivity analysis on food waste parameters

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP_S	TP_R
Decentralized supply chain scenario									
α	− 50%	0.1	0.08204	0.12135	0.01148	167,328	3,886,470	144,125	160,823
			(+ 0.34%)	0%	0%	(− 4.77%)	0%	(+ 3.28%)	0%
	− 25%	0.1	0.0819	0.12135	0.01148	171,469	3,886,470	141,856	160,823
			(+ 0.17%)	0%	0%	(− 2.41%)	0%	(+ 1.65%)	0%
	+ 25%	0.1	0.08161	0.12135	0.01148	180,034	3,886,470	137,188	160,823
			(− 0.17%)	0%	0%	(+ 2.46%)	0%	(− 1.68%)	0%
	+ 50%	0.1	0.08147	0.12135	0.01148	184,460	3,886,470	134,788	160,823
			(− 0.35%)	0%	0%	(+ 4.98%)	0%	(− 3.41%)	0%
θ	− 50%	0.1	0.08176	0.14513	0.01148	209,364	4,631,023	145,627	170,475
			0%	(+ 19.60%)	0%	(+ 19.16%)	(+ 19.16%)	(+ 4.35%)	(+ 6.00%)
	− 25%	0.1	0.08176	0.13167	0.01148	190,335	4,210,093	142,471	166,916
			0%	(+ 8.51%)	0%	(+ 8.33%)	(+ 8.33%)	(+ 2.09%)	(+ 3.78%)
	+ 25%	0.1	0.08176	0.1131	0.01148	164,003	3,627,661	136,802	153,673
			0%	(− 6.79%)	0%	(− 6.66%)	(− 6.66%)	(− 1.96%)	(− 4.44%)
	52%	0.1	0.08176	0.10632	0.01148	154,370	3,414,574	134,214	150,185
			0%	(− 12.38%)	− 0.02%	(− 12.14%)	(− 12.14%)	(− 3.81%)	(− 6.61%)

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP	OP_S	OP_R
Centralized supply chain scenario										
α	− 50%	0.1	0.08204	0.43973	0.00882	965,890	2.2E + 07	175,407	200,367	174,426
			(+ 0.34%)	(+ 0.11%)	(− 0.08%)	(− 4.56%)	(+ 0.21%)	(+ 1.94%)		

Table 2 (continued)

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP	OP_S	OP_R
	− 25%	0.1	0.0819	0.43949	0.00882	988,753	2.2E + 07	174,083	200,325	174,107
			(+ 0.17%)	(+ 0.06%)	(− 0.04%)	(− 2.31%)	(+ 0.11%)	(+ 1.17%)		
	+ 25%	0.1	0.08161	0.43898	0.00883	1,035,897	2.2E + 07	169,987	200,238	173,450
			(− 0.17%)	(− 0.06%)	(+ 0.04%)	(+ 2.35%)	(− 0.11%)	(− 1.21%)		
	+ 50%	0.1	0.08147	0.43872	0.00883	1,060,193	2.2E + 07	168,680	200,194	173,113
			(− 0.35%)	(− 0.12%)	(+ 0.09%)	(+ 4.75%)	(− 0.22%)	(− 1.97%)		
θ	− 50%	0.1	0.08176	0.45602	0.00883	1,023,163	2.3E + 07	175,459	197,618	177,171
			0%	(+ 3.82%)	(+ 0.05%)	(+ 1.09%)	(+ 1.09%)	(+ 1.97%)		
	− 25%	0.1	0.08176	0.44744	0.00883	1,017,584	2.3E + 07	174,117	198,986	175,439
			0%	(+ 1.87%)	(+ 0.02%)	(+ 0.54%)	(+ 0.54%)	(+ 1.19%)		
	+ 25%	0.1	0.08176	0.43138	0.00882	1,006,670	2.2E + 07	169,970	201,520	172,185
			0%	(− 1.79%)	(− 0.02%)	(− 0.54%)	(− 0.54%)	(− 1.22%)		
	+ 50%	0.1	0.08176	0.42386	0.00882	1,001,336	2.2E + 07	168,611	202,710	170,640
			0%	(− 3.50%)	(− 0.04%)	(− 1.06%)	(− 1.06%)	(− 2.01%)		

partners. Our results indicate that if the profit-sharing ratio is not wisely selected, some chain members may lose the incentives to enter this setting. We showed that constructing this ratio based on the profits of the decentralized scenario guarantees the involved partners' individual rationality. However, this requires a smooth flow of information among the partners or between each involved partner and the centralized decision-maker. This indicates that the centralized scenario is an appropriate scheme in long-run business cooperation, where the trust is already built.

One significant advantage of the centralized scenario in our FSC that differentiates it from conventional SCs is its robustness in facing alterations in food waste parameters. This is highly important in the food industry as it decreases the risk of large financial losses when a specific health issue is spreading among the growing items or the holding facilities confront unexpected breakdowns. To put it in a nutshell, the centralized supply chain scheme accompanied by an appropriate profit-sharing contract is a rigorous coordination mechanism in FSCs with various benefits in terms of decreased food loss, increased profit, customer satisfaction, and lower financial risks.

The results provide the decision-makers with fruitful insights into FSCs under different scenarios. The most rigorous input parameters impacting the rearing farm's profit and the retailer are distinguished, helping each echelon's decision-makers prioritize their focus in improving the costs and revenues. Since the studies on growing items are still in their infancy, the findings can be particularly beneficial for the rearing farms as the suppliers of FSCs.

The results imply that if the rearing farm has a chance to choose among different hatcheries, selecting the one with the lowest purchasing cost not only reduces its costs but also shortens the breeding period. This, in turn, lowers the risk of poultry disease during growth, which is particularly advantageous under the conditions of a newly emerging disease among the broiler chickens. Moreover, if the rearing farm faces limitations in its periodic purchasing budget, the firm can manage its costs by applying better holding technologies and feeding processes, which decreases the unit-item breeding cost. The latter can be achieved by controlling feed ingredients, fermenting the feed, and incorporating grit and probiotics.

As the growth function imposes limitations on the speed of the weight increase and the ultimate weight, the breeding period is independent of the retailer's system. Furthermore, since quality standards and the negative impact of overbreeding are considered in the model, the changes in the supplier's purchasing cost and even breeding cost do not change the breeding period on a severe scale in comparison to other variables. This is while the breeding period is highly affected by the pattern of the growth, which is modeled as the growth function. This indicates that the first step in applying our proposed framework is estimating the growth parameters accurately. Any inaccurate estimation can lead to significant financial losses.

It is shown that most input parameters do not highly influence the retailing price. In most cases, regulations intervene in the pricing of these items (such as meat and chicken), and the companies face some restrictions in pricing. Moreover, as the items are almost identical, the market is very price-sensitive, and a slight increase in the retailing price can shift the customers from one seller to another. The results also shed light on the efficiency of dynamic pricing in FSCs. A decrease in food waste and profit increase is observed by shifting from static pricing to dynamic pricing. Accordingly, retailers can benefit more by switching from static pricing to dynamic pricing.

7 Conclusion

In this study, dynamic pricing and ordering policies were studied in a two-level FSC, including a rearing farm as the supplier and a retailer. The chain embraces a class of inventory known as growing items (such as poultry and livestock), which are central components in various FSCs. The problem is analyzed under decentralized and centralized supply chain scenarios. An analytic solution approach is developed and validated by experimental results. The results show that the centralized supply chain scheme accompanied by an appropriate profit-sharing contract is a rigorous coordination mechanism in FSCs with various benefits in terms of decreased food loss, increased profit, customer satisfaction, and lower financial risks. Our results indicate that the first important step in applying the proposed optimization scheme for FSCs with growing items is the accurate estimation of growth parameters that largely impact the optimal decisions. The results provide decision-makers of involved echelons with insights into the features of the studied FSC, including their most influential input parameters, the areas that require further attention, and managerial suggestions under different settings.

The current research faces some limitations that can be the focus of future research. In the real world, a portion of the broiler chickens dies during their lifetime. This influences the inventory model of the rearing farm. Then, taking the mortality rate of the items into account is a promising future direction. Furthermore, feeding conditions play a crucial role in the growth pattern of growing items. This can be heeded by taking the feeding level of the items as decision variables as a future research direction. As this research topic is still in its preliminary stage, further interdisciplinary studies are required to unravel all the characteristics of such FSCs. Other feeding functions and deterioration patterns might be applied in the future to investigate their impact on the system. Incorporating other marketing policies such as advertising and delay in payments might also help to illustrate the practical features of the problem. Finally, since part of the SC, after slaughtering the items, holds the features of a cold chain, investigating the related problems raised in cold supply chains such as decisions on holding and transportation facilities and temperature optimization can be regarded as a promising future direction.

Appendix: Numerical results

See Tables 3 and 4.

Table 3 Sensitivity analysis on cost parameters-decentralized supply chain scenario

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP_S	TP_R
C_p	− 50%	0.1	0.07472	0.12135	0.01148	215,980	3,886,470	143,508	160,823
			(− 8.61%)	0%	0%	(+ 22.92%)	0%	(+ 2.84%)	0%
	− 25%	0.1	0.0789	0.12135	0.01148	190,773	3,886,470	141,426	160,823
			(− 3.53%)	0%	0%	(+ 8.58%)	0%	(+ 1.35%)	0%
	+ 25%	0.1	0.08397	0.12135	0.01148	165,376	3,886,470	137,790	160,823
			(+ 2.71%)	0%	0%	(− 5.88%)	0%	(− 1.26%)	0%
	+ 50%	0.1	0.08576	0.12135	0.01148	157,717	3,886,470	136,128	160,823
			(+ 4.89%)	0%	0%	(− 10.24%)	0%	(− 2.45%)	0%
C_B	− 50%	0.1	0.08855	0.12135	0.01148	146,895	3,886,470	141,979	160,823
			(+ 8.31%)	0%	0%	(− 16.40%)	0%	(+ 1.74%)	0%
	− 25%	0.1	0.0846	0.12135	0.01148	162,589	3,886,470	140,661	160,823
			(+ 3.48%)	0%	0%	(− 7.46%)	0%	(+ 0.8%)	0%
	+ 25%	0.1	0.07952	0.12135	0.01148	187,206	3,886,470	138,555	160,823
			(− 2.74%)	0%	0%	(+ 6.55%)	0%	(− 0.71%)	0%
	+ 50%	0.1	0.07767	0.12135	0.01148	197,581	3,886,470	137,657	160,823
			(− 5.00%)	0%	0%	(+ 12.45%)	0%	(− 1.35%)	0%
C_{OS}	− 50%	0.1	0.08176	0.12135	0.01148	175,704	3,886,470	160,146	160,823
			0%	0%	0%	0%	0%	(+ 14.76%)	0%
	− 25%	0.1	0.08176	0.12135	0.01148	175,704	3,886,470	149,845	160,823
			0%	0%	0%	0%	0%	(+ 7.38%)	0%
	+ 25%	0.1	0.08176	0.12135	0.01148	175,704	3,886,470	129,243	160,823
			0%	0%	0%	0%	0%	− 7.38%)	0%

Table 3 (continued)

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP_S	TP_R
	+ 50%	0.1	0.08176	0.12135	0.01148	175,704	3,886,470	118,942	160,823
			0%	0%	0%	0%	0%	– 14.76%)	0%
P_S	– 50%	0.2	0.08176	0.12786	0.00995	238,863	5,283,526	70,129.1	270,835
			0%	(+ 5.37%)	(– 13.29%)	(+ 35.95%)	(+ 35.95%)	(– 49.74%)	(+ 68.41%)
	– 25%	0.2	0.08176	0.1231	0.01072	204,100	4,514,570	111,325	212,344
			0%	(+ 1.46%)	(– 6.66%)	(+ 16.16%)	(+ 16.16%)	(– 20.22%)	(+ 32.04%)
	+ 25%	0.1	0.08176	0.1205	0.01225	151,295	3,346,570	154,727	116,261
			0%	(– 0.69%)	(+ 6.68%)	(– 13.89%)	(– 13.89%)	(+ 10.88%)	(– 27.71%)
	+ 50%	0.1	0.08176	0.1168	0.01302	129,364	2,861,457	156,824	78,662.7
			0%	(– 3.69%)	(+ 13.41%)	(– 26.37%)	(– 26.37%)	(+ 12.38%)	(– 51.09%)
C_H	– 50%	0.1	0.08176	0.13524	0.01148	196,473	4,345,866	144,380	161,848
			0%	(+ 11.45%)	0%	(+ 11.82%)	(+ 11.82%)	(+ 3.47%)	(+ 0.64%)
	– 25%	0.1	0.08176	0.12773	0.01148	185,229	4,097,153	141,885	161,321
			0%	(+ 5.26%)	0%	(+ 5.42%)	(+ 5.42%)	(+ 1.68%)	(+ 0.31%)
	+ 25%	0.1	0.08176	0.11585	0.01148	167,501	3,705,029	137,330	160,349
			0%	(– 4.53%)	(+ 0.01%)	(– 4.67%)	(– 4.67%)	(– 1.59%)	(– 0.29%)
	+ 50%	0.1	0.08176	0.11104	0.01148	160,341	3,546,644	135,227	159,897
			0%	(– 8.49%)	(+ 0.01%)	(– 8.74%)	(– 8.74%)	(– 3.09%)	(– 0.58%)
C_{OR}	– 50%	0.1	0.08176	0.08604	0.0114	124,547	2,754,905	122,591	163,716
			0%	(– 29.10%)	(– 0.37%)	(– 29.11%)	(– 29.12%)	(– 12.15%)	(+ 1.80%)
	– 25%	0.1	0.08176	0.10522	0.01146	152,335	3,369,565	133,209	162,147
			0%	(– 13.29%)	(– 0.17%)	(– 13.30%)	(– 13.30%)	(– 4.54%)	(+ 0.82%)

Table 3 (continued)

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP_S	TP_R
	+ 25%	0.1	0.08176	0.13552	0.0115	196,248	4,340,906	143,870	159,655
			0%	(+ 11.68%)	(+ 0.15%)	(+ 11.69%)	(+ 11.69%)	(+ 3.10%)	(− 0.73%)
	+ 50%	0.1	0.08176	0.1483	0.01151	214,787	4,750,969	147,066	158,598
			0%	(+ 22.22%)	(+ 0.28%)	(+ 22.24%)	(+ 22.24%)	(+ 5.39%)	(− 1.38%)

Table 4 Sensitivity analysis on cost parameters—centralized supply chain scenario

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP	OP_S	OP_R
C_P	− 50%	0.1	0.08397	0.44366	0.00876	1,267,357	2.3E + 07	380,399	179,378	201,021
			(+ 2.71%)	(+ 1.01%)	(− 0.72%)	(+ 25.22%)	(+ 1.87%)	(+ 1.69%)		
	− 25%	0.1	0.08576	0.4413	0.00879	1,108,549	2.3E + 07	377,065	176,433	200,632
			(+ 4.89%)	(+ 0.47%)	(− 0.34%)	(+ 9.53%)	(+ 0.88%)	(+ 0.80%)		
	+ 25%	0.1	0.07472	0.43735	0.00885	944,930	2.2E + 07	371,278	171,320	199,958
			(− 8.61%)	(− 0.43%)	(+ 0.32%)	(− 6.64%)	(− 0.80%)	(− 0.74%)		
	+ 50%	0.1	0.0789	0.4356	0.00888	894,351	2.2E + 07	368,647	168,995	199,653
			(− 3.53%)	(− 0.83%)	(+ 0.62%)	(− 11.63%)	(− 1.56%)	(− 1.45%)		
C_B	− 50%	0.1	0.08397	0.44192	0.00879	855,796	2.3E + 07	377,950	177,215	200,735
			(+ 2.71%)	(+ 0.61%)	(− 0.44%)	(− 15.44%)	(+ 1.14%)	(+ 1.04%)		
	− 25%	0.1	0.08576	0.44046	0.00881	941,410	2.3E + 07	375,843	175,354	200,489
			(+ 4.89%)	(+ 0.28%)	(− 0.20%)	(− 6.98%)	(+ 0.52%)	(+ 0.48%)		
	+ 25%	0.1	0.08855	0.43817	0.00884	1,073,434	2.2E + 07	372,492	172,393	200,099
			(+ 8.31%)	(− 0.24%)	(+ 0.18%)	(+ 6.06%)	(− 0.46%)	(− 0.42%)		
	+ 50%	0.1	0.0846	0.43721	0.00886	1,128,259	2.2E + 07	371,068	171,134	199,934
			(+ 3.48%)	(− 0.46%)	(+ 0.34%)	(+ 11.48%)	(− 0.87%)	(− 0.80%)		
C_{OS}	− 50%	0.1	0.07952	0.33028	0.00875	753,300	1.7E + 07	380,556	189,877	190,680
			(− 2.74%)	(− 24.80%)	(− 0.89%)	(− 25.57%)	(− 25.57%)	(+ 1.74%)		
	− 25%	0.1	0.07767	0.38902	0.00879	892,153	2E + 07	377,082	181,878	195,204
			(− 5.00%)	(− 11.43%)	(− 0.41%)	(− 11.85%)	(− 11.85%)	(+ 0.81%)		
	+ 25%	0.1	0.08176	0.48369	0.00886	1,119,211	2.5E + 07	371,355	165,462	205,893
			0%	(+ 10.12%)	(+ 0.37%)	(+ 10.58%)	(+ 10.58%)	(− 0.72%)		

Table 4 (continued)

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP	OP_S	OP_R
	+ 50%	0.1	0.08176	0.52393	0.00889	1,216,921	2.7E + 07	368,874	156,826	212,048
			0%	(+ 19.28%)	(+ 0.70%)	(+ 20.24%)	(+ 20.24%)	(– 1.39%)		
P_S	– 50%	0.1	0.08176	0.43923	0.00882	1,012,087	2.2E + 07	374,064	76,937	297,127
			0%	0%	0%	0%	0%	0%		
	– 25%	0.1	0.08176	0.4392	0.00882	1,012,087	2.2E + 07	374,064	128,658	245,406
			0%	0%	0%	0%	0%	0%		
	+ 25%	0.1	0.08176	0.4392	0.00882	1,012,087	2.2E + 07	374,064	213,581	160,483
			0%	0%	0%	0%	0%	0%		
	+ 50%	0.1	0.08176	0.4392	0.00882	1,012,087	2.2E + 07	374,064	249,110	124,953
			0%	0%	0%	0%	0%	0%		
C_H	– 50%	0.1	0.08176	0.58286	0.00885	1,374,280	3E + 07	380,431	179,365	201,066
			0%	(+ 32.70%)	(+ 0.31%)	(+ 35.79%)	(+ 35.79%)	(+ 1.70%)		
	– 25%	0.1	0.08176	0.49582	0.00883	1,153,158	2.6E + 07	377,010	176,421	200,589
			0%	(+ 12.88%)	(+ 0.10%)	(+ 13.94%)	(+ 13.94%)	(+ 0.79%)		
	+ 25%	0.1	0.08176	0.39867	0.00882	912,209	2E + 07	371,438	171,358	200,080
			0%	(– 9.24%)	(– 0.04%)	(– 9.87%)	– 9.87%)	(– 0.70%)		
	+ 50%	0.1	0.08176	0.36774	0.00882	836,767	1.9E + 07	369,047	169,099	199,948
			0%	(– 16.28%)	(– 0.04%)	(– 17.32%)	(– 17.32%)	(– 1.34%)		
C_{OR}	– 50%	0.1	0.08176	0.42778	0.0088	984,639	2.2E + 07	374,756	160,463	214,293
			0%	(– 2.61%)	(– 0.09%)	(– 2.71%)	(– 2.71%)	(+ 0.19%)		
	– 25%	0.1	0.08176	0.43355	0.00882	998,458	2.2E + 07	374,407	168,862	205,545
			0%	(– 1.29%)	(– 0.05%)	(– 1.35%)	(– 1.35%)	(+ 0.09%)		

Table 4 (continued)

Parameter	Changes (%)	γ	T_S	T_R	P_0	Q_0	Q_R	TP	OP_S	OP_R
	+ 25%	0.1	0.08176	0.44484	0.00883	1,025,534	2.3E + 07	373,724	177,144	196,580
			0%	(+ 1.28%)	(+ 0.05%)	(+ 1.33%)	(+ 1.33%)	(− 0.09%)		
	+ 50%	0.1	0.08176	0.4504	0.00883	1,038,807	2.3E + 07	373 ,389	179,651	193,739
			0%	(+ 2.53%)	(+ 0.09%)	(+ 2.64%)	(+ 2.64%)	(− 0.18%)		

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